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Research Article

Strong Attractor of Beam Equation with Structural Damping and Nonlinear Damping

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This paper is mainly concerned with the existence of a global strong attractor for the nonlinear extensible beam equation with structural damping and nonlinear external damping. This kind of problem arises from the model of an extensible vibration beam. By the asymptotic compactness of the related continuous semigroup, we prove the existence of a strong global attractor which is connected with phase space $D(\Delta^2) \times H_0^1(\Omega) \cap H^2(\Omega)$.

1. Introduction

Global attractor is a basic concept in the study of long-time behavior of nonlinear dissipative evolution equations with various dissipation. There have been many methods to prove the existence of the global attractor. It can be proved by the theory of α-contractions of the solution semigroup $S(t)$, such as [1–3] and the reference therein. It can also be proved by the decomposition of the solution semigroup $S(t)$ (see Hale [4], Temam [5], etc.).

In this paper, we use the method of the asymptotically compact property of the solution semigroup $S(t)$ which is different from the method of [1–5] to prove the existence of a strong global attractor for the Kirchhoff type equations with structural damping and nonlinear external damping which arises from the model of the nonlinear vibration beam

$$u_{tt} + \alpha\Delta^2 u + \gamma\Delta^2 u_t - \left(\beta + M\left(\int_\Omega |\nabla u|^2 dx\right) + N\left(\int_\Omega \nabla u \nabla u_t \, dx\right)\right)\Delta u + g(u) + f(u_t) = h(x), \quad \text{in } \Omega \times R^+, \tag{1}$$

$$u = \Delta u = 0 \quad \text{on } \partial\Omega \times R^+, \tag{2}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) \quad \text{in } \Omega, \tag{3}$$

where α, γ, and β are all positive constants, Ω is a bounded domain of R^N with smooth boundary $\Gamma = \partial\Omega$, $M(s)$, $N(s)$, $g(s)$, and $f(s)$ are nonlinear functions specified later, and $h \in L^2(\Omega)$ is an external force term. $u(t)$ represents the vertical deflection of the beam, and $u = u(x,t)$ is a real-valued function on $\Omega \times [0, +\infty)$.

In this context of problem (1), based on the vibrating beams equation

$$u_{tt} + u_{xxxx} - \left(\alpha + \beta \int_0^l |u_x(s,t)|^2 ds\right) u_{xx} = 0 \tag{4}$$

which is proposed by Woinowsky-krieger [6]; Ma and Narciso [7] considered problem (1) without structural damping and posed a weak global attractor in weak phase space $H_0^2(\Omega) \times L^2(\Omega)$. Eden and Milani [8] considered the existence of exponential attractor for problem (1) with $f(u) = 0$ and a linear weak damping $g(u_t) = u_t$, $M(\cdot)$ being a nonlinear function and without structural damping. Ball [9] presented the existence and uniqueness of global solutions for problem (1) with $f = g = h = 0$, $M(\cdot)$, $N(\cdot)$ are all linear functions.

On the other hand, the existence of the attractor for a related problem, with the boundary conditions $u = \Delta u = 0$

of (2) replaced with $u = \nabla u = 0$, was considered by Ma and Narciso [7], Eden and Milani [8] with a linear damping u_t or nonlinear damping $f(u_t)$ without structural damping, respectively. Chueshov and Lasiecka [10] considered a kind of boundary condition which is $u = \Delta u = 0$ but without structural damping.

Generally speaking, there have been many works on the long-time behavior for nonlinear beam equations [6–10]. But for the beam equation (1) with structural damping, in strong phase space $D(\Delta^2) \times H_0^1(\Omega) \cap H^2(\Omega)$, the global solutions and the strong global attractor have not still been proved until now.

The outline of this paper is arranged as follows: in Section 2 we give the existence and uniqueness of global solutions in space $C(R^+; D(\Delta^2) \times H_0^1(\Omega) \cap H^2(\Omega))$, in Section 3 we give the boundedness of solutions in phase space $D(\Delta^2) \times H_0^1(\Omega) \cap H^2(\Omega)$, and finally in Section 4, we give the proof of the existence of a strong global attractor in phase space $D(\Delta^2) \times H_0^1(\Omega) \cap H^2(\Omega)$.

2. Some Assumptions and Existence of Global Solution

In (1), we assume that damping term and the source term are in the form of

$$f(u_t) = |u_t|^r u_t, \qquad g(u) = |u|^\rho u \tag{5}$$

with

$$\begin{gathered} 0 < \rho, \qquad r \le \frac{2}{N-2} \quad \text{if } N \ge 3, \\ \rho, r > 0 \quad \text{if } N = 1, 2. \end{gathered} \tag{6}$$

We assume that the nonlinear functions $M, N : R^+ \to R^+$ are all class C^1, and satisfying $M(0) = 0$, $N(0) = 0$ and

$$\begin{gathered} M(s)s \ge \widehat{M}(s), \quad \text{where } \widehat{M}(s) = \int_0^s M(z)\,dz, \\ M(s) \ge 2s; \\ N(s) \ge s, \quad \forall s \in R. \end{gathered} \tag{7}$$

The functions $f, g : R \to R$ are also class C^1, with $f(0) = g(0) = 0$, $\alpha_1 \le f'(v) \le \alpha_2$, and $|g'(u)| \le k_0(1 + |u|^\rho)$ for all $u, v \in R$, where α_1, α_2, and k_0 are all constants. There also exists constants k_5, k_6 such that

$$\begin{gathered} |f(u) - f(v)| \le k_5 (1 + |u|^r + |v|^r) |u - v|, \quad \forall u, v \in R, \\ |g(u) - g(v)| \le k_6 (1 + |u|^\rho + |v|^\rho) |u - v|, \quad \forall u, v \in R. \end{gathered} \tag{8}$$

In addition, nonlinear function $g(\cdot)$ also satisfies

$$\begin{gathered} \varphi(u) + \frac{\alpha - \varepsilon\gamma}{8} \|u\|^2 \ge -k_1, \\ \int_\Omega g(u) u\,dx - C_1 \varphi(u) + \frac{\alpha}{4} \|u\|^2 \ge -k_2, \end{gathered} \tag{9}$$

where $\varphi(u) = \int_\Omega G(u)dx$, $G(u) = \int_\Omega g(u)du$, and k_1, k_2 are all constants, $C_1 \ge 1$.

Our analysis is based on the following Sobolev spaces: $H = L^2(\Omega)$, $V = H_0^1(\Omega) \cap H^2(\Omega)$, with the usual inner products and norms as follows, respectively:

$$\begin{gathered} (u, v) = \int_\Omega uv\,dx, \quad |u| = (u, u)^{1/2}, \quad \forall u, v \in L^2(\Omega), \\ (\Delta u, \Delta v) = \int_\Omega \Delta u \Delta v\,dx, \qquad \|u\| = (\Delta u, \Delta u)^{1/2}, \\ \forall u, v \in H_0^1(\Omega) \cap H^2(\Omega). \end{gathered} \tag{10}$$

Consider $D(\Delta^2) = \{u \mid u \in V, u \in H^4(\Omega), \Delta^2 u \in H, \Delta u|_{\partial\Omega} = 0\}$ with the inner products $(\Delta^2 u, \Delta^2 u)$ and the norms $|\Delta^2 u|^2 = (\Delta^2 u, \Delta^2 u)$.

Take $E_0 = H_0^1(\Omega) \cap H^2(\Omega) \times L^2(\Omega)$ and $E = D(\Delta^2) \times H^2(\Omega) \cap H_0^1(\Omega)$ with the inner products and norms as follows, respectively:

$$\begin{gathered} (y_1, y_2)_{E_0} = (\Delta u_1, \Delta u_2) + (v_1, v_2), \qquad |y|_{E_0} = (y, y)_{E_0}^{1/2}, \\ \forall y_i = (u_i, v_i)^T, \quad y = (u, v)^T \in E_0, \quad i = 1, 2, \\ (y_1, y_2)_E = (\Delta^2 u_1, \Delta^2 u_2) + (\Delta v_1, \Delta v_2), \qquad |y|_E = (y, y)_E^{1/2}, \\ \forall y_i = (u_i, v_i)^T, \quad y = (u, v)^T \in E, \quad i = 1, 2. \end{gathered} \tag{11}$$

Note that assumption (6) implies that $H_0^1(\Omega) \cap H^2(\Omega) \hookrightarrow H_0^1(\Omega) \hookrightarrow L^{2(p+1)}(\Omega)$, with $p = \rho$ or $p = r$.

Finally, we assume that λ, σ are the first eigenvalue of Δ^2 and Δ, respectively; then we have

$$\begin{gathered} \|u\|^2 \ge \sigma |u|^2, \quad \forall u \in V, \\ \left|\Delta^2 u\right|^2 \ge \lambda \|v\|^2, \quad \forall u \in D(\Delta^2). \end{gathered} \tag{12}$$

In the following, we state the result of the existence and uniqueness of the solutions for systems (1)–(3).

Theorem 1. *Assume that $(u_0, u_1) \in E$, $h \in L^2(\Omega)$, and the assumptions of these functions $M(\cdot)$, $N(\cdot)$, $f(\cdot)$, and $g(\cdot)$ hold; then problems (1)–(3) have unique solutions $(u, u_t) \in C([0,T]; D(\Delta^2)) \times C([0,T]; H_0^1(\Omega) \cap H^2(\Omega))$ depending continuously on initial data in E.*

By virtue of Galerkin method, we may prove Theorem 1 combined with the priori estimates of Section 3.

According by Theorem 1, for any $t > 0$, we may introduce the mapping

$$\{S(t), t \ge 0\} : \{u_0, u_1\} \longrightarrow \{u(t), u_t(t)\}. \tag{13}$$

It maps E into itself, and it enjoys the usual semigroup properties as follows:

$$\begin{gathered} S(0) = I, \\ S(t + \tau) = S(t) S(\tau), \quad \forall t \ge 0. \end{gathered} \tag{14}$$

And it is obvious that the map $\{S(t), t > 0\}$, for all $t \in R$, is continuous in space E. In the following, we will introduce the existence of bounded absorbing set and global attractor in space E for map $\{S(t), t \geq 0\}$.

3. The Existence of Bounded Absorbing Set in Space E

In this section, we will show boundedness of the solutions for systems (1)–(3).

Theorem 2. *Assume that these assumptions of Theorem 1 hold then for the dynamic system determined by problems* (1)–(3), *there exists the boundary absorbing set in space E.*

Proof. Taking the inner products of $v = u_t + \varepsilon u$ with both sides of (1) and then making summation, we have

$$\frac{1}{2}\frac{d}{dt}E(t) - \varepsilon|v|^2 + \varepsilon(\alpha - \varepsilon\gamma)\|u\|^2 + \varepsilon\beta|\nabla u|^2 + \gamma|v|^2 + \varepsilon^2(u, v) + \varepsilon\widehat{M}(z) + N(\dot{z})\dot{z} + (g(u), v) + (f(u_t), v) = (h, v), \quad (15)$$

where $\widehat{M}(z) = \int_0^z M(z)dz$, $z(t) = |\nabla u|^2$ and ε is fixed at arbitrary time, and here the energy function $E(t)$ is defined on E_0 by

$$E(t) = |v|^2 + (\alpha - \varepsilon\gamma)\|u\|^2 + \beta|\nabla u|^2 + \widehat{M}(z) + |\nabla u|^4. \quad (16)$$

Considering the assumption $\int_\Omega ug(u)dx - C_1\varphi(u) + (\alpha/4)\|u\|^2 \geq -k_2$, we have

$$\begin{aligned}(g(u), v) &= \frac{d}{dt}\varphi(u) + \varepsilon(g(u), u) \\ &\geq \frac{d}{dt}\varphi(u) + \varepsilon C_1\varphi(u) \\ &\quad - \frac{\varepsilon\alpha}{4}\|u\|^2 - \varepsilon k_2.\end{aligned} \quad (17)$$

With $|\varepsilon^2(u, v)| \leq (\varepsilon^2/\sigma^2)\|u\|^2 + (\varepsilon^2/4)|v|^2$, we have

$$\begin{aligned}&-\varepsilon|v|^2 + \varepsilon(\alpha - \varepsilon\gamma)\|u\|^2 + \varepsilon\beta|\nabla u|^2 + \gamma|v|^2 + \varepsilon^2(u, v) \\ &\geq \left(\varepsilon\alpha - \varepsilon^2\gamma - \frac{\varepsilon^2}{\sigma^2}\right)\|u\|^2 + \left(\gamma\lambda^2 - \varepsilon - \frac{\varepsilon^2}{4}\right)|v|^2 \\ &\quad + \varepsilon\beta|\nabla u|^2.\end{aligned} \quad (18)$$

With the assumptions $f(0) = 0$, $f \in C^1(R, R)$, and $\alpha_1 \leq f'(v) \leq \alpha_2$ and by using Mean Value Theorem and Mean Value inequality, we have

$$\begin{aligned}(f(u_t), v) &= \int f'(\xi)v^2dx - \varepsilon\int f'(\xi)uv\,dx \\ &\geq \left(\alpha_1 - \frac{\varepsilon\alpha_2}{2}\right)|v|^2 - \frac{\varepsilon\alpha_2}{2\sigma^2}\|u\|^2,\end{aligned} \quad (19)$$

where ξ among 0 and $v - \varepsilon u$. Set

$$\begin{aligned}\widetilde{E}(t) &= |v|^2 + (\alpha - \varepsilon\gamma)\|u\|^2 + \beta|\nabla u|^2 + \widehat{M}(z) \\ &\quad + |\nabla u|^4 + 2\varphi(u) + 2k_1.\end{aligned} \quad (20)$$

Consider

$$\begin{aligned}Y(t) &= \left(\varepsilon\alpha - \varepsilon^2\gamma - \frac{\varepsilon^2}{\sigma^2} - \frac{\varepsilon\alpha}{4} - \frac{\varepsilon\alpha_2}{2\sigma^2}\right)\|u\|^2 \\ &\quad + \left(\gamma\lambda^2 - \varepsilon - \frac{\varepsilon^2}{4} + \alpha_1 - \frac{\gamma\sigma^2}{4} - \frac{\varepsilon\alpha_2}{2}\right)|v|^2 \\ &\quad + \varepsilon\beta|\nabla u|^2 + \varepsilon\widehat{M}(z) + N(\dot{z})\dot{z} + \varepsilon C_1\varphi(u) + \varepsilon k_1.\end{aligned} \quad (21)$$

So (15) is transformed into

$$\frac{1}{2}\frac{d}{dt}\widetilde{E}(t) + Y(t) \leq \frac{1}{\gamma\sigma^2}|h|^2 + \varepsilon k_2 + \varepsilon k_1. \quad (22)$$

Considering the assumptions $M(s)s \geq \widehat{M}(s)$, $M(s) \geq 2s$, $N(s) > s$, $\|u\|^2 \geq \sigma^2|u|^2$, and $|\Delta^2 u|^2 \geq \lambda^2\|u\|^2$, $C_1 \geq 1$ and letting $0 < \varepsilon < \min\{(\alpha\sigma^2 + 2\alpha_2)/(2\gamma\sigma^2 + 4), -(3 + \alpha_2) + \sqrt{(3 + \alpha_2)^2 + (4\alpha_1 + 3\gamma\sigma^2)}\} = \varepsilon_0$, we have

$$\frac{2}{\varepsilon}Y(t) - \widetilde{E}(t) > 0. \quad (23)$$

Substituting (23) into (22), we have

$$\frac{1}{2}\frac{d}{dt}\widetilde{E}(t) + \frac{\varepsilon}{2}\widetilde{E}(t) \leq \frac{1}{\gamma\sigma^2}|h|^2 + \varepsilon k_2 + \varepsilon k_1. \quad (24)$$

On the one hand, applying the Gronwall inequality to (24), we get

$$\widetilde{E}(t) \leq \widetilde{E}(0)e^{-\varepsilon t} + \frac{2}{\varepsilon}\left(\frac{1}{\gamma\sigma^2}|h|^2 + \varepsilon k_2 + \varepsilon k_1\right), \quad t \geq 0. \quad (25)$$

Note that $\|u(0)\|$ and $|u_t(0)|$ are bounded; then there exists a positive constant $R > 0$ such that $\widetilde{E}(0) \leq R^2$ is bounded; so

$$\limsup_{t\to\infty}\widetilde{E}(t) \leq \rho_0^2 = \frac{2}{\varepsilon}\left(\frac{1}{\gamma\sigma^2}|h|^2 + \varepsilon k_2 + \varepsilon k_1\right). \quad (26)$$

On the other hand, considering that $\varphi(u) + ((\alpha - \varepsilon\gamma)/8)\|u\|^2 \geq -k_1$, fixing $\mu_0 > \rho_0$, and assuming that $\widetilde{E}(0) \leq R^2$, then as $t \geq t_0 = t_0(R, \rho_0) = (1/\varepsilon_0)\log(R/(\mu_0^2 - \rho_0^2))$, we have

$$\widetilde{E}(t) \leq \mu_0^2, \quad (27)$$

that is,

$$|v|^2 + \frac{\alpha - \varepsilon\gamma}{4}\|u\|^2 \leq \mu_0^2. \quad (28)$$

Take the inner products by $\Delta^2 v$ in both sides of (1); then make summation to get

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left(\|v\|^2+(\alpha-\varepsilon\gamma)\left|\Delta^2 u\right|^2\right)-\varepsilon\|v\|^2\\&\quad-(\beta+M(z(t))+N(\dot{z}(t)))\left(\Delta u,\Delta^2 v\right)\\&\quad+\gamma\left|\Delta^2 v\right|^2+\varepsilon^2\left(\Delta^2 u,v\right)+\varepsilon\widehat{M}(z)+N(\dot{z})\dot{z}\\&\quad+\left(g(u),\Delta^2 v\right)+\left(f(v-\varepsilon u),\Delta^2 v\right)=\left(h,\Delta^2 v\right).\end{aligned}\tag{29}$$

Considering the continuity of the functions $M'(\cdot)$ and $N'(\cdot)$, we have

$$\begin{aligned}&-(\beta+M(z(t))+N(\dot{z}(t)))\left(\Delta u,\Delta^2 v\right)\\&\quad\geq-\left(\beta+C_2\mu_0^2+C_3\mu_0^2\right)|\Delta u|\left|\Delta^2 v\right|\\&\quad\geq-\frac{\left(\beta+C_2\mu_0^2+C_3\mu_0^2\right)\mu_0^2}{\gamma}-\frac{\gamma}{4}\left|\Delta^2 v\right|^2,\end{aligned}\tag{30}$$

where C_2, C_3 are all positive constants. Also

$$\begin{aligned}&\varepsilon^2\left(\Delta^2 u,v\right)\geq-\frac{\varepsilon^2}{\sigma^2}\left|\Delta^2 u\right|^2-\frac{\varepsilon^2}{4}\|v\|^2,\\&\left(h,\Delta^2 v\right)=\frac{d}{dt}\left(h,\Delta^2 u\right)+\varepsilon\left(h,\Delta^2 u\right).\end{aligned}\tag{31}$$

In addition, with $|g'(u)|\leq k_0(1+|u|^{\rho})$, there exists a constant k_3 such that $|g(u)|_{L^\infty}\leq k_3$, $|g'(u)|_{L^\infty}\leq k_3$; so

$$\begin{aligned}&\left(g(u),\Delta^2 v\right)\\&\quad=\frac{d}{dt}\left(g(u),\Delta^2 v\right)-\left(g'(u)u_t,\Delta^2 u\right)+\varepsilon\left(g(u),\Delta^2 u\right)\\&\quad\geq\frac{d}{dt}\left(g(u),\Delta^2 u\right)+\varepsilon\left(g(u),\Delta^2 u\right)-\frac{\varepsilon^2}{8}\left|\Delta^2 u\right|^2-\frac{2k_3^2\mu_0^2}{\varepsilon^2}.\end{aligned}\tag{32}$$

Also by using Schwarz and Mean Value inequalities and Mean Value Theorem, we have

$$\begin{aligned}&\left(f(v-\varepsilon u),\Delta^2 v\right)\\&\quad\leq\frac{\gamma}{4}\left|\Delta^2 v\right|^2+\frac{1}{\gamma}\int\left(f'(\xi)\right)^2(v-\varepsilon u)\,dx\\&\quad\leq\frac{\gamma}{4}\left|\Delta^2 v\right|^2+\frac{\alpha_2^2}{\gamma}\left(1+3\varepsilon^2\right)\mu_0^2,\end{aligned}\tag{33}$$

where ξ among 0 and $v-\varepsilon u$. Set

$$\begin{aligned}Y_1(t)=&\left(\varepsilon\alpha-\varepsilon^2\gamma-\frac{\varepsilon^2}{\sigma^2}-\frac{\varepsilon^2}{8}\right)\left|\Delta^2 u\right|^2\\&+\left(\frac{\gamma\lambda^2}{2}-\varepsilon-\frac{\varepsilon^2}{4}\right)\|v\|^2+\varepsilon\left(g(u),\Delta^2 u\right)\\&+\varepsilon\left(h,\Delta^2 u\right),\end{aligned}\tag{34}$$

and write $M=(\alpha_2^2/\gamma)(1+3\varepsilon^2)\mu_0^2+((\beta+C_2\mu_0^2+C_3\mu_0^2)\mu_0^2)/\gamma+2k_3^2\mu_0^2/\varepsilon^2$; then (29) is transformed into

$$\frac{1}{2}\frac{d}{dt}E_1(t)+Y_1(t)\leq M.\tag{35}$$

Here the function

$$E_1(t)=\|v\|^2+(\alpha-\varepsilon\gamma)\left|\Delta^2 u\right|^2+2\left(g(u),\Delta^2 u\right)+2\left(h,\Delta^2 u\right)\tag{36}$$

is obtained by the energy function being changed slightly.

Let $0<\varepsilon\leq\min\{\varepsilon_0,2\alpha/(\gamma+(2/\sigma^2)+(1/4)),-3+\sqrt{9+2\gamma\lambda^2},\alpha/(\gamma+(1/8))\}$, we have $Y_1(t)\geq(\varepsilon/2)E_1(t)$, and so

$$\frac{1}{2}\frac{d}{dt}E_1(t)+\frac{\varepsilon}{2}E_1(t)\leq M,\quad\forall t\geq t_0(B).\tag{37}$$

Then an application of the Gronwall inequality leads to

$$E_1(t)\leq E_1(0)e^{[-\varepsilon(t-t_0)]}+\frac{2M}{\varepsilon},\quad\forall t\geq t_0(ß).\tag{38}$$

If $B\subset B_E(0,\rho)$, there exists a positive constant $R_1>0$ such that $E_1(t_0)\leq R_1^2$.

Putting t_1 satisfing $t_1-t_0>(1/\varepsilon)\log R_1^2$, then as $t\geq t_1$, we get

$$E_1(t)\leq R_1^2e^{-\varepsilon\times(1/\varepsilon)\log R_1^2}+\frac{2M}{\varepsilon}=1+\frac{2M}{\varepsilon}.\tag{39}$$

So

$$\begin{aligned}&\left(\alpha-\varepsilon\gamma-\frac{\varepsilon}{8}\right)\left|\Delta^2 u\right|^2+\|v\|^2\\&\quad\leq\frac{16}{\varepsilon}|h|^2+\frac{16}{\varepsilon}k_3^2|\Omega|+1+\frac{2M}{\varepsilon}.\end{aligned}\tag{40}$$

The global estimate (40) shows the existence of an absorbing set of $S(t)$. □

4. The Existence of Global Attractor in Space E

The general theory [11] indicates that the continuous semigroup $S(t)$ defined on a Banach space X has a global attractor which is connected when the following conditions are satisfied.

(i) There exists a bounded absorbing set $B\subset X$ such that for any bounded set $B_0\subset X$,

$$\operatorname{dist}(S(t)B_0,B)\longrightarrow 0,\quad\text{as } t\longrightarrow+\infty.\tag{41}$$

(ii) $S(t)$ is asymptotically compact; that is, for any bounded sequence $\{u_n\}$ in X and $\{t_n\}$ tending to ∞, there exists a subsequence $\{n'\}$ such that $\{S(t_{n'})u_{n'}\}$ is convergent as $n'\to\infty$.

Theorem 3. *Under the assumptions of Theorem 1, the continuous semigroup $S(t)$ has a global attractor which is connected to E.*

Proof. Let u, v be two solutions of Problems (1)–(3) in space $C(R^+; E)$ as shown above corresponding to the initial data (u_0, u_1) and (v_0, v_1) with $|(u_0, u_1)|_E^2 + |v_0, v_1|_E^2 \leq R^2$, respectively. Then $w = u - v$ satisfies

$$w_{tt} + \alpha\Delta^2 w + \gamma\Delta^2 w_t - \beta\Delta w$$
$$= \left(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v\right)$$
$$+ \left(N\left(\int \nabla u \nabla u_t\, dx\right)\Delta u - N\left(\int \nabla v \nabla v_t\, dx\right)\Delta v\right)$$
$$- \left(g(u) - g(v)\right) - \left(f(u_t) - f(v_t)\right), \tag{42}$$

$$\left|(w, w_t)\right|_{E_0}^2 \leq C\left(\mu_0^2\right). \tag{43}$$

Taking the inner products in both sides of (42) by w_t, Aw, and Aw_t, respectively, we have

$$\frac{1}{2}\frac{d}{dt}\left(|w_t|^2 + \alpha|\Delta w|^2 + \beta|\nabla w|^2\right) + \gamma|\Delta w_t|^2$$
$$= \left(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v, w_t\right)$$
$$+ \left(N\left(\int_\Omega \nabla u \nabla u_t\, dx\right)\Delta u - N\left(\int_\Omega \nabla v \nabla v_t\, dx\right)\Delta v, w_t\right)$$
$$+ \left(g(u) - g(v) + f(u_t) - f(v_t), w_t\right), \tag{44}$$

$$\frac{1}{2}\frac{d}{dt}\left(\gamma\left|\Delta^2 w\right|^2 + 2\left(\Delta w, \Delta w_t\right)\right) + \alpha\left|\Delta^2 w\right|^2$$
$$- \beta\left(\Delta w, \Delta^2 w\right) + |\Delta w_t|^2$$
$$= \left(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v, \Delta^2 w\right)$$
$$+ \left(N\left(\int_\Omega \nabla u \nabla u_t\, dx\right)\Delta u - N\left(\int_\Omega \nabla v \nabla v_t\, dx\right)\Delta v, \Delta^2 w\right)$$
$$+ \left(g(u) - g(v) + f(u_t) - f(v_t), \Delta^2 w\right), \tag{45}$$

$$\frac{1}{2}\frac{d}{dt}\left(|\Delta w_t|^2 + \alpha\left|\Delta^2 w\right|^2\right) + \gamma\left|\Delta^2 w_t\right|^2 - \beta\left(\Delta^2 w, \Delta w_t\right)$$
$$= \left(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v, \Delta^2 w_t\right)$$
$$+ \left(N\left(\int_\Omega \nabla u \nabla u_t\, dx\right)\Delta u - N\left(\int_\Omega \nabla v \nabla v_t\, dx\right)\Delta v, \Delta^2 w_t\right)$$
$$+ \left(g(u) - g(v) + f(u_t) - f(v_t), \Delta^2 w_t\right). \tag{46}$$

Equation (46)+ $\tilde{k} \times (45) + \tilde{\tilde{k}} \times (44)$ yields

$$\frac{1}{2}\frac{d}{dt}\Big(|\Delta w_t|^2 + \alpha\left|\Delta^2 w\right|^2$$
$$+ \tilde{k}\gamma\left|\Delta^2 w\right|^2 + 2\tilde{k}\left(\Delta^2 w, w_t\right)$$
$$+ \tilde{\tilde{k}}|w_t|^2 + \tilde{\tilde{k}}\alpha|\Delta w|^2 + \tilde{\tilde{k}}\beta|\nabla w|^2\Big)$$
$$+ \gamma\left|\Delta^2 w_t\right|^2 + \tilde{k}\alpha\left|\Delta^2 w\right|^2 + \tilde{k}|\Delta w_t|^2 + \tilde{\tilde{k}}\gamma|\Delta w_t|^2$$
$$= \left(g(u) - g(v) + f(u_t) - f(v_t), \Delta^2 w_t + \tilde{k}\Delta^2 w + \tilde{\tilde{k}} w_t\right)$$
$$+ \Big(M\left(|\nabla u|^2\right)\nabla u - M\left(|\nabla v|^2\right)\Delta v$$
$$+ N\left(\int_\Omega \nabla u \nabla u_t\, dx\right)\Delta u$$
$$- N\left(\int_\Omega \nabla v \nabla v_t\, dx\right)\Delta v, \Delta^2 w_t\Big)$$
$$+ \tilde{k}\Big(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v$$
$$+ N\left(\int_\Omega \nabla u \nabla u_t\, dx\right)\Delta u$$
$$- N\left(\int_\Omega \nabla v \nabla v_t\, dx\right)\Delta v, \Delta^2 w\Big)$$
$$+ \tilde{\tilde{k}}\Big(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v$$
$$+ N\left(\int_\Omega \nabla u \nabla u_t\, dx\right)\Delta u$$
$$- N\left(\int_\Omega \nabla v \nabla v_t\, dx\right)\Delta v, w_t\Big)$$
$$+ \left(\beta\Delta w, \Delta^2 w_t\right) + \tilde{k}\beta\left(\Delta w, \Delta^2 w\right). \tag{47}$$

Consider that

$$\left(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v, \Delta^2 w_t\right)$$
$$= M'(\eta_0)|\nabla u|^2 \int_\Omega \Delta w \Delta^2 w_t dx + M'(\eta_1)$$
$$\times \int_\Omega \nabla w(\Delta u + \nabla v)\, dx \int_\Omega \Delta v \Delta^2 w_t dx$$
$$\leq C_2\mu_0^2|\Delta w|\left|\Delta^2 w_t\right| + 2C_2\mu_0^2|\nabla w|\left|\Delta^2 w_t\right|$$
$$\leq 2\frac{\left(C_2\mu_0^2 + \left(2C_2\mu_0^2/\sigma\right)\right)^2}{\gamma}|\Delta w|^2 + \frac{\gamma}{8}\left|\Delta^2 w_t\right|^2; \tag{48}$$

$$\begin{aligned}
&\tilde{k}\left(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v, \Delta^2 w\right)\\
&\quad = \tilde{k}M'\left(\eta_0\right)|\nabla u|^2\int_\Omega \Delta w\Delta^2 w dx + \tilde{k}M'\left(\eta_1\right)\\
&\qquad \times \int_\Omega \nabla w\left(\Delta u + \nabla v\right)dx\int_\Omega \Delta v\Delta^2 w dx\\
&\quad \le \tilde{k}C_2\left(\mu_0^2|\Delta w|\left|\Delta^2 w\right| + \frac{2\mu_0^2}{\sigma}|\Delta w|\left|\Delta^2 w\right|\right)\\
&\quad \le \frac{\left[C_2\tilde{k}\mu_0^2\left(1+(2/\sigma)\right)\right]^2}{\tilde{k}\alpha}|\Delta w|^2 + \frac{\tilde{k}\alpha}{8}\left|\Delta^2 w\right|^2;
\end{aligned} \tag{49}$$

$$\begin{aligned}
&\tilde{\tilde{k}}\left(M\left(|\nabla u|^2\right)\Delta u - M\left(|\nabla v|^2\right)\Delta v, w_t\right)\\
&\quad = \tilde{\tilde{k}}M'\left(\eta_0\right)|\nabla u|^2\int_\Omega \Delta w w_t dx + \tilde{\tilde{k}}M'\left(\eta_1\right)\\
&\qquad \times \int_\Omega \nabla w\left(\Delta u + \nabla v\right)dx\int_\Omega \Delta v w_t dx\\
&\quad \le \tilde{\tilde{k}}C_2\left(\mu_0^2|\Delta w|\left|w_t\right| + \frac{2\mu_0^2}{\sigma}|\Delta w|\left|w_t\right|\right)\\
&\quad \le \frac{\left[C_2\tilde{\tilde{k}}\mu_0^2\left(1+(2/\sigma)\right)\right]^2}{2}|\Delta w|^2 + \left|w_t\right|^2,
\end{aligned} \tag{50}$$

where η_0 is among 0 and $|\nabla u|^2$, η_1 is among $|\nabla u|^2$ and $|\nabla v|^2$, and

$$\begin{aligned}
&\left(N\left(\int_\Omega \nabla u\nabla u_t\,dx\right)\Delta u - N\left(\int_\Omega \nabla v\nabla v_t\,dx\right)\Delta v, \Delta^2 w_t\right)\\
&\quad = N'\left(\xi_0\right)\int_\Omega \nabla u\nabla u_t\,dx\int_\Omega \Delta w\Delta^2 w_t\,dx + N'\left(\xi_1\right)\\
&\qquad \times\left(\int_\Omega \nabla v\nabla w_t\,dx + \int_\Omega \nabla w\nabla u_t\,dx\right)\\
&\qquad \times \int_\Omega \Delta v\Delta^2 w_t\,dx\\
&\quad \le C_3\left[\mu_0^2|\Delta w|\left|\Delta^2 w_t\right| + \left(\mu_0\left|w_t\right| + \mu_0|\Delta w|\right)\mu_0\left|\Delta^2 w_t\right|\right]\\
&\quad \le \frac{\left(4C_3\mu_0^2\right)^2}{\gamma}|\Delta w|^2 + \frac{\gamma}{8}\left|\Delta^2 w_t\right|^2 + \frac{\left(4C_3\mu_0^2\right)^2}{\gamma}\left|w_t\right|^2,
\end{aligned} \tag{51}$$

$$\begin{aligned}
&\tilde{k}\left(N\left(\int_\Omega \nabla u\nabla u_t\,dx\right)\Delta u - N\left(\int_\Omega \nabla v\nabla v_t\,dx\right)\Delta v, \Delta^2 w\right)\\
&\quad = \tilde{k}N'\left(\xi_0\right)\int_\Omega \nabla u\nabla u_t\,dx\int_\Omega \Delta w\Delta^2 w\,dx + \tilde{k}N'\left(\xi_1\right)\\
&\qquad \times\left(\int_\Omega \nabla v\nabla w_t\,dx + \int_\Omega \nabla w\nabla u_t\,dx\right)\\
&\qquad \times \int_\Omega \Delta v\Delta^2 w\,dx\\
&\quad \le \tilde{k}C_3\left[\mu_0^2|\Delta w|\left|\Delta^2 w\right| + \left(\mu_0\left|w_t\right| + \mu_0|\Delta w|\right)\mu_0\left|\Delta^2 w\right|\right]\\
&\quad \le \frac{\left(4C_3\mu_0^2\right)^2\tilde{k}}{\alpha}|\Delta w|^2 + \frac{\tilde{k}\alpha}{8}\left|\Delta^2 w\right|^2 + \frac{\left(4C_3\mu_0^2\right)^2\tilde{k}}{\alpha}\left|w_t\right|^2,
\end{aligned} \tag{52}$$

$$\begin{aligned}
&\tilde{\tilde{k}}\left(N\left(\int_\Omega \nabla u\nabla u_t\,dx\right)\Delta u - N\left(\int_\Omega \nabla v\nabla v_t\,dx\right)\Delta v, w_t\right)\\
&\quad = \tilde{\tilde{k}}N'\left(\xi_0\right)\int_\Omega \nabla u\nabla u_t\,dx\int_\Omega \Delta w w_t\,dx + \tilde{\tilde{k}}N'\left(\xi_1\right)\\
&\qquad \times\left(\int_\Omega \nabla v\nabla w_t\,dx + \int_\Omega \nabla w\nabla u_t\,dx\right)\\
&\qquad \times \int_\Omega \Delta v w_t\,dx\\
&\quad \le \tilde{\tilde{k}}C_3\left[\mu_0^2|\Delta w|\left|w_t\right| + \left(\mu_0\left|w_t\right| + \mu_0|\Delta w|\right)\mu_0\left|w_t\right|\right]\\
&\quad \le \tilde{\tilde{k}}C_3\mu_0^2|\Delta w|^2 + \tilde{\tilde{k}}C_3\mu_0^2\left|w_t\right|^2 + \tilde{\tilde{k}}C_3\mu_0^2\left|w_t\right|^2,
\end{aligned} \tag{53}$$

where ξ_0 is among 0 and $\int_\Omega \nabla u\nabla u_t dx$, ξ_1 is among $\int_\Omega \nabla u\nabla u_t dx$ and $\int_\Omega \nabla v\nabla v_t dx$.

Also considering $|g(u) - g(v)| \le k_6(1 + |u|^\rho + |v|^\rho)|u - v|$ for all $u, v \in R$, $|f(u) - f(v)| \le k_5(1 + |u|^r + |v|^r)|u - v|$, for all $u, v \in R$, and $\rho/(2(\rho+1)) + 1/(2(\rho+1)) + (1/2) = 1$, $r/(2(r+1)) + 1/(2(r+1)) + (1/2) = 1$, by Hölder inequality we have

$$\begin{aligned}
&-\left(g(u) - g(v) + f\left(u_t\right) - f\left(v_t\right), \Delta^2 w_t\right)\\
&\quad \le k_6\int_\Omega \left(1 + |u|^\rho + |v|^\rho\right)|w|\left|\Delta^2 w_t\right|dx\\
&\qquad + k_5\int_\Omega \left(1 + \left|u_t\right|^r + \left|v_t\right|^r\right)\left|w_t\right|\left|\Delta^2 w_t\right|dx\\
&\quad \le k_6\left[\int_\Omega \left(1 + |u|^\rho + |v|^\rho\right)^{2(\rho+1)/\rho}dx\right]^{\rho/(2(\rho+1))}\\
&\qquad \times |w|_{2(\rho+1)}\left|\Delta^2 w_t\right|\\
&\qquad + k_5\left[\int_\Omega \left(1 + \left|u_t\right|^r + \left|v_t\right|^r\right)^{2(r+1)/r}dx\right]^{r/(2(r+1))}\\
&\qquad \times \left|w_t\right|_{2(r+1)}\left|\Delta^2 w_t\right|\\
&\quad \le C\left(\mu_0\right)|\nabla w|\left|\Delta^2 w_t\right| + C\left(\mu_0\right)\left|\nabla w_t\right|\left|\Delta^2 w_t\right|;
\end{aligned} \tag{54}$$

$$\begin{aligned}
&-\left(g(u) - g(v) + f\left(u_t\right) - f\left(v_t\right), \tilde{k}\Delta^2 w\right)\\
&\quad \le \tilde{k}k_6\int_\Omega \left(1 + |u|^\rho + |v|^\rho\right)|w|\left|\Delta^2 w\right|dx\\
&\qquad + \tilde{k}k_5\int_\Omega \left(1 + \left|u_t\right|^r + \left|v_t\right|^r\right)\left|w_t\right|\left|\Delta^2 w\right|dx\\
&\quad \le \tilde{k}k_6\left[\int_\Omega \left(1 + |u|^\rho + |v|^\rho\right)^{2(\rho+1)/\rho}dx\right]^{\rho/(2(\rho+1))}\\
&\qquad \times |w|_{2(\rho+1)}\left|\Delta^2 w\right|\\
&\qquad + \tilde{k}k_5\left[\int_\Omega \left(1 + \left|u_t\right|^r + \left|v_t\right|^r\right)^{2(r+1)/r}dx\right]^{r/(2(r+1))}\\
&\qquad \times \left|w_t\right|_{2(r+1)}\left|\Delta^2 w\right|\\
&\quad \le \tilde{k}C\left(\mu_0\right)|w|_{2(\rho+1)}\left|\Delta^2 w\right| + \tilde{k}C\left(\mu_0\right)\left|w_t\right|_{2(r+1)}\left|\Delta^2 w\right|\\
&\quad \le \tilde{k}C\left(\mu_0\right)|\nabla w|\left|\Delta^2 w\right| + \tilde{k}C\left(\mu_0\right)\left|\nabla w_t\right|\left|\Delta^2 w\right|;
\end{aligned} \tag{55}$$

$$
\begin{aligned}
&-\left(g(u)-g(v)+f(u_t)-f(v_t),\tilde{\tilde{k}}w_t\right)\\
&\quad\leq \tilde{\tilde{k}}k_6\int_\Omega\left(1+|u|^\rho+|v|^\rho\right)|w|\,|w_t|\,dx\\
&\quad+\tilde{\tilde{k}}k_5\int_\Omega\left(1+|u_t|^r+|v_t|^r\right)|w_t|\,|w_t|\,dx\\
&\quad\leq \tilde{\tilde{k}}k_6\left[\int_\Omega\left(1+|u|^\rho+|v|^\rho\right)^{2(\rho+1)/\rho}dx\right]^{\rho/(2(\rho+1))}\\
&\qquad\times|w|_{2(\rho+1)}|w_t|\\
&\qquad+\tilde{\tilde{k}}k_5\left[\int_\Omega\left(1+|u_t|^r+|v_t|^r\right)^{2(r+1)/r}dx\right]^{r/(2(r+1))}\\
&\qquad\times|w_t|_{2(r+1)}|w_t|\\
&\quad\leq \tilde{\tilde{k}}C(\mu_0)|w|_{2(\rho+1)}|w_t|+\tilde{\tilde{k}}C(\mu_0)|w_t|_{2(r+1)}|w_t|\\
&\quad\leq \tilde{\tilde{k}}C(\mu_0)|\nabla w|\,|w_t|+\tilde{\tilde{k}}C(\mu_0)|\nabla w_t|\,|w_t|.
\end{aligned}
\tag{56}
$$

Setting

$$
\begin{aligned}
E_2(t)&=\left|\Delta w_t\right|^2+\alpha\left|\Delta^2 w\right|^2+\tilde{k}\gamma\left|\Delta^2 w\right|^2+2\tilde{k}\left(\Delta^2 w,w_t\right)\\
&\quad+\tilde{\tilde{k}}\left|w_t\right|^2+\tilde{\tilde{k}}\alpha|\Delta w|^2+\tilde{\tilde{k}}\beta|\nabla w|^2+\beta|\nabla\Delta w|^2,\\
Y_2(t)&=\frac{\gamma}{2}\left|\Delta^2 w_t\right|^2+\frac{\tilde{k}\alpha}{2}\left|\Delta^2 w\right|^2+\frac{\tilde{k}}{2}\left|\Delta w_t\right|^2\\
&\quad+\frac{\tilde{\tilde{k}}\gamma}{2}\left|\Delta w_t\right|^2+\tilde{k}\beta|\nabla\Delta w|^2,
\end{aligned}
\tag{57}
$$

then substituting (48)–(56) into (47), by Schwarz inequality and Young inequality, and taking $\tilde{k} > (4C^2(\mu_0))/\gamma\sigma^2$ and $\tilde{\tilde{k}} \geq (8\tilde{k}C^2(\mu_0))/\alpha\sigma^2\gamma$, we have

$$
\frac{1}{2}\frac{d}{dt}E_2(t)+Y_2(t)\leq C\left(|\Delta w|^2+\left|w_t\right|^2\right).
\tag{58}
$$

Again setting $\xi = \max\{4/\alpha + 4/\tilde{k} + 4\gamma/\alpha, 2/\tilde{k}, (4/\lambda^2 + 4/\alpha\lambda^4)(\tilde{\tilde{k}}/\tilde{k}), 2/\gamma\lambda^2 + (2/\gamma\lambda^2)(\tilde{k}/\tilde{\tilde{k}})\}$, and considering that $-2\tilde{k}(\Delta^2 w, w_t) \geq -\tilde{k}|\Delta^2 w|^2 - \tilde{k}|w_t|^2$, we have $\xi Y_2(t) - E_2(t) \geq 0$. On the one hand, from (58) we have

$$
\frac{1}{2}\frac{dE_2(t)}{dt}+\frac{1}{\xi}E_2(t)\leq C\left(|\Delta w|^2+\left|w_t\right|^2\right).
\tag{59}
$$

Applying the Gronwall inequality to (59), we get

$$
\begin{aligned}
E_2(t)&\leq E_2(0)e^{-(2/\xi)t}\\
&\quad+C\int_0^t e^{-(2/\xi)t}\left(\|w(\tau)\|^2+\left|w_t(\tau)\right|^2\right)d\tau.
\end{aligned}
\tag{60}
$$

On the other hand, with $(2\tilde{k}w_t, \Delta^2 w) \geq -(\tilde{k}\gamma/2)|\Delta^2 w|^2 - (4\tilde{k}/\gamma)|w_t|^2$ and setting $\tilde{\tilde{k}} > 4\tilde{k}/\gamma$, we get

$$
E_2(t)\geq\left|\Delta w_t\right|^2+\alpha\left|\Delta^2 w\right|^2.
\tag{61}
$$

Hence

$$
\begin{aligned}
\left|w,w_t\right|_E^2&\leq CE_2(0)e^{-2t/\xi}\\
&\quad+C\int_0^t e^{-(2/\xi)t}\left(|\Delta w(\tau)|^2+\left|w_t(\tau)\right|^2\right)d\tau.
\end{aligned}
\tag{62}
$$

Now, let $\{(u_{0m}, u_{1m})\}$ be a bound sequence in $B_0 \subset E$, and $\{u_m(t), u_{mt}(t)\}$ the corresponding solutions of problems (1)–(3) in $C(R^+, E)$. We assume $t_n > t_m$. Let $T > 0$ and t_n, $t_m > T$. Then, applying estimate (62) to $w^{m,n} = u_n(t + t_n - T) - u_m(t + t_m - T)$, $t \geq 0$, we have

$$
\begin{aligned}
&\left|(w^{m,n},w_t^{m,n})\right|_E^2\\
&\quad\leq CC(\mu_0)e^{-(2/\xi)t}+C\\
&\qquad\times\sup_{0\leq s\leq t}\left|\left(u_n(t_n-T+s)-u_m(t_m-T+s)\right),\right.\\
&\qquad\left.\left(u_{nt}(t_n-T+s)-u_{mt}(t_m-T+s)\right)\right|_{E_0}^2.
\end{aligned}
\tag{63}
$$

By taking $t = T$ in the above, we have

$$
\begin{aligned}
&\left|\left(u_n(t_n)-u_m(t_m),u_{nt}(t_n)-u_{mt}(t_m)\right)\right|_E^2\\
&\quad\leq CC(\mu_0)e^{-(2/\xi)T}+C(\mu_0)\\
&\qquad\times\sup_{0\leq s\leq T}\left|\left(u_n(t_n+s)-u_m(t_m+s)\right),\right.\\
&\qquad\left.\left(u_{nt}(t_n+s)-u_{mt}(t_m+s)\right)\right|_{E_0}^2.
\end{aligned}
\tag{64}
$$

By Sobolev embedding Theorem, for any $T > 0$, we can extract a subsequence $\{(u_{n'}, u_{n't}\}$which is convergent in $C([0,T]; E_0)$ for any $T > 0$. For any $\varepsilon > 0$, we first fix $T > 0$ such that

$$
CC(\mu_0)e^{-(2/\xi)T}<\frac{\varepsilon}{2},
\tag{65}
$$

And, next, taking large m', n', we have

$$
\begin{aligned}
&C(\mu_0)\sup_{0\leq s\leq T}\left|\left(u_n(t_n+s)-u_m(t_m+s)\right.\right.,\\
&\qquad\left.\left.u_{nt}(t_n+s)-u_{mt}(t_m+s)\right)\right|_{E_0}^2\leq\frac{\varepsilon}{2}.
\end{aligned}
\tag{66}
$$

Then by (62) we have that

$$
\left|\left(u_{n'}(t_{n'})-u_{m'}(t_{m'}),u_{n't}(t_{n'})-u_{m't}(t_{m'})\right)\right|_E^2\leq\varepsilon.
\tag{67}
$$

We conclude that $S(t)$ is asymptotically compact on E. The theorem is now proved. □

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