Hindawi Publishing Corporation
Indian Journal of Materials Science
Volume 2015, Article ID 532939, 8 pages
http://dx.doi.org/10.1155/2015/532939

Hindawi

Research Article

SH-Wave at a Plane Interface between Homogeneous and Inhomogeneous Fibre-Reinforced Elastic Half-Spaces

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Received 11 May 2015; Revised 23 June 2015; Accepted 24 June 2015

Academic Editor: Fernando Rubio-Marcos

The problem of reflection and refraction of *SH*-waves at a plane interface between the homogeneous and inhomogeneous fibre-reinforced elastic half-spaces has been investigated. Amplitude and energy ratios corresponding to the reflected and refracted *SH*-waves are derived using appropriate boundary conditions. These ratios are computed numerically for a particular model and the results are depicted graphically.

1. Introduction

Fibre-reinforced composite material with the reinforcement distributed continuously in concentric circles is a locally transversely isotropic material, with the circumferential direction as the preferred direction coinciding with fibres directions. The fibres may be continuous, in which each fibre extends through a body from one boundary to another, or discontinuous, but in the discontinuous case the length of the fibre must be large compared to its diameter. The elastic fibre-reinforced composite materials, for example, typical carbon fibre-epoxy resin composites, are not just anisotropic but also strongly anisotropic in the sense that the modulus for extension in the fibre direction greatly exceeds the moduli for extension in the transverse direction and for shear in the fibre or transverse direction. Fibres have an excellent potential to improve the mechanical properties of rapid-setting materials and could be used effectively to improve the performance of repairs. The behavior of fibre-reinforced rapid-setting materials is similar to that of Portland cement fibre-reinforced concrete. Spencer [1] gave the concept of deformation in fibre-reinforced elastic materials. Belfield et al. [2] discussed the problem of stress in elastic plates reinforced by fibers lying in concentric circles and explained the anisotropic characters of fibre-reinforced materials. Sengupta and Nath [3] studied the problem of surface waves in fibre-reinforced anisotropic elastic media and derived the frequency equation for surface wave. Chattopadhyay et al. [4] investigated the problem of reflection of quasi-*P* and quasi-*SV* waves at the plane-free and rigid boundaries of a fibre-reinforced elastic medium and obtained the phase velocity of quasi-*P* and quasi-*SV* waves. B. Singh and S. J. Singh [5] discussed the problem of reflection of plane waves at the free surface of a fibre-reinforced elastic half-space and obtained the closed form expression of the amplitude ratios for reflected *qSV* and *qP* waves at the free surface of a fibre-reinforced, anisotropic, homogeneous, elastic half-space. Singh [6] studied the problem of propagation of plane waves in thermally conducting linear fibre-reinforced composite materials and derived the frequency equation. Abbas [7] discussed a two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation and the results with energy dissipation and without energy dissipation are compared. Singh and Zorammuana [8] solved the problem of incident longitudinal wave at a fibre-reinforced thermoelastic half-space and obtained the amplitude and energy ratios of the reflected waves.

A number of researchers have attempted the problem of *SH*-waves in elastic media. Gutenberg [9, 10] explored the existence of low velocity layer in the earth mantle. Chattopadhyay and Keshri [11] discussed the propagation of shear waves along the plane surface between two different elastic media with initial stress. Singh and Tomar [12] investigated the problem of shear waves at a corrugated interface between two dissimilar fiber-reinforced elastic half-spaces using Rayleigh's method of approximation. Chattopadhyay and Singh [13] studied the problem of G-type seismic waves in fibre-reinforced media and obtained the dispersion equation for the propagation of G-type seismic wave in fibre-reinforced layer lying over an inhomogeneous fibre-reinforced elastic half-space. Literatures showed many problems of propagation of SH-waves and notable among them are Bath and Arroyo [14], Gupta [15], Tomar et al. [16], Kumar et al. [17], Chaudhary et al. [18, 19], Emets et al. [20], Chattopadhyay and Michel [21], Tomar and Singh [22], Tomar and Kaur [23, 24], Abd-Alla and Alsheikh [25], Chattopadhyay et al. [26], Singh [27], Wang and Zhao [28], and Sahu et al. [29].

Fibre-reinforced composite materials are very attractive in many engineering applications due to their high strength and low weight. In this paper, the problem of reflection and refraction of SH-waves in the homogeneous/inhomogeneous fibre-reinforced elastic half-spaces has been studied. We have obtained the amplitude and energy ratios corresponding to the reflected and refracted SH-waves using appropriate boundary conditions. These ratios are computed numerically for a particular model.

2. Basic Equations

The constitutive relation for a linearly fibre-reinforced elastic medium is given by Belfield et al. [2] as

$$\begin{aligned}\tau_{ij} &= \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j\right)\\ &\quad + 2\left(\mu_L - \mu_T\right)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right)\\ &\quad + \beta a_k a_m e_{km} a_i a_j, \quad (i, j, k, m = 1, 2, 3),\end{aligned} \tag{1}$$

where τ_{ij} is stress tensor, e_{ij} is strain tensor and is defined by

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{2}$$

λ, μ_T, α, β, and μ_L are elastic constants in which μ_T is identified as the shear modulus in transverse shear across the preferred direction and μ_L as the shear modulus in longitudinal shear in the preferred direction, α and β are specific stress components depending upon the concrete part of the composite materials, δ_{ij} is Kronecker delta, and a_i is the component of a unit vector $\mathbf{a} = (a_1, a_2, a_3)$ which gives the preferred direction of fibre-reinforcement and $a_1^2 + a_2^2 + a_3^2 = 1$.

The equation of motion for the fibre-reinforced elastic material without body forces may be written as

$$\tau_{ij,j} = \rho\frac{\partial^2 u_i}{\partial t^2}, \quad (i, j = 1, 2, 3), \tag{3}$$

where $u_i = (u_1, u_2, u_3)$ and $x_1 = x$, $x_2 = y$, $x_3 = z$.

Let us take two-dimensional problem of SH-wave propagation in xz-plane and the preferred direction of fibre-reinforcement is taken as $(a_1, 0, a_3)$. We may take $u_1 = u_3 = 0$, $u_2 = u_2(x, z, t)$. With the help of (1) and (3), we have

$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} = \rho\frac{\partial^2 u_2}{\partial t^2}, \tag{4}$$

where

$$\begin{aligned}\tau_{12} &= \mu_T\frac{\partial u_2}{\partial x} + \left(\mu_L - \mu_T\right)a_1\left(a_1\frac{\partial u_2}{\partial x} + a_3\frac{\partial u_2}{\partial z}\right),\\ \tau_{23} &= \mu_T\frac{\partial u_2}{\partial z} + \left(\mu_L - \mu_T\right)a_3\left(a_1\frac{\partial u_2}{\partial x} + a_3\frac{\partial u_2}{\partial z}\right).\end{aligned} \tag{5}$$

It may be noted that the second term in the expression of the stress tensors (τ_{12} and τ_{23}) contributes due to the effect of direction of reinforcement, that is, fibre orientation in the self-reinforced material.

Using the stress tensors into (4), we obtain the equation of motion for SH-wave propagation in the fibre-reinforced elastic medium as

$$P\frac{\partial^2 u_2}{\partial z^2} + Q\frac{\partial^2 u_2}{\partial x^2} + R\frac{\partial^2 u_2}{\partial x\partial z} = \rho\frac{\partial^2 u_2}{\partial t^2}, \tag{6}$$

where

$$\begin{aligned}P &= \mu_T + a_3^2\left(\mu_L - \mu_T\right),\\ Q &= \mu_T + a_1^2\left(\mu_L - \mu_T\right),\\ R &= 2a_1 a_3\left(\mu_L - \mu_T\right).\end{aligned} \tag{7}$$

3. Problem Formulation

Let x- and y-axes of the Cartesian coordinate system be on the horizontal plane and let z-axis be pointing vertically downward. Consider a homogeneous fibre-reinforced elastic half-space, $\{H; 0 \le z < \infty\}$ with ρ as density and μ_L, μ_T as elastic constants and a nonhomogeneous fibre-reinforced elastic half-space, $\{H'; -\infty < z \le 0\}$ with ρ' as density and μ_L', μ_T' as elastic constants. The elastic constants and density in the nonhomogeneous fibre-reinforced elastic half-space H' are defined as (see [13])

$$\begin{aligned}\mu_L' &= \mu_L^{(0)}\left(1 - \varepsilon\cos sz\right),\\ \mu_T' &= \mu_T^{(0)}\left(1 - \varepsilon\cos sz\right),\\ \rho' &= \rho_0\left(1 - \varepsilon\cos sz\right),\end{aligned} \tag{8}$$

where ε is small positive constant and s is real depth parameter.

The stress tensors in the nonhomogeneous fibre-reinforced elastic half-space (H') are

$$\begin{aligned}\tau_{12}' &= \mu_T'\frac{\partial u_2'}{\partial x} + \left(\mu_L' - \mu_T'\right)a_1\left(a_1\frac{\partial u_2'}{\partial x} + a_3\frac{\partial u_2'}{\partial z}\right),\\ \tau_{23}' &= \mu_T'\frac{\partial u_2'}{\partial z} + \left(\mu_L' - \mu_T'\right)a_3\left(a_1\frac{\partial u_2'}{\partial x} + a_3\frac{\partial u_2'}{\partial z}\right).\end{aligned} \tag{9}$$

The equation of motion for SH-wave in the half-space, H', may be written as

$$\begin{aligned} &P^{(0)}\left\{(1-\varepsilon\cos sz)\frac{\partial^2 u_2'}{\partial z^2}+\varepsilon s\sin sz\frac{\partial u_2'}{\partial z}\right\} \\ &\quad+Q^{(0)}(1-\varepsilon\cos sz)\frac{\partial^2 u_2'}{\partial x^2} \\ &\quad+R^{(0)}\left\{2(1-\varepsilon\cos sz)\frac{\partial^2 u_2'}{\partial x\partial z}+\varepsilon s\sin sz\frac{\partial u_2'}{\partial x}\right\} \\ &=\rho_0(1-\varepsilon\cos sz)\frac{\partial^2 u_2'}{\partial t^2}, \end{aligned} \tag{10}$$

where

$$\begin{aligned} P^{(0)}&=\mu_T^{(0)}+a_3^2\left(\mu_L^{(0)}-\mu_T^{(0)}\right), \\ Q^{(0)}&=\mu_T^{(0)}+a_1^2\left(\mu_L^{(0)}-\mu_T^{(0)}\right), \\ R^{(0)}&=a_1a_3\left(\mu_L^{(0)}-\mu_T^{(0)}\right). \end{aligned} \tag{11}$$

The displacement components in the half-spaces, H and H', may be represented by

$$\begin{aligned} &\left\langle u_2,u_2'\right\rangle(x,z,t) \\ &\quad=\langle A,B\rangle\exp\left[\imath\left\{\omega t-k\left(xp_1+zp_3\right)\right\}\right], \end{aligned} \tag{12}$$

where $\langle A,B\rangle$ are amplitude constants, $\omega(=kc)$ is angular frequency, and $\mathbf{p}=(p_1,0,p_3)$ is the unit propagation vector.

Using (12) into (6) and (10), the phase velocities of SH-waves in the half-spaces, H and H', are, respectively, given by

$$\rho c_2^2=Pp_3^2+Qp_1^2+Rp_1p_3, \tag{13}$$

$$\begin{aligned} &\rho_0c_2'^2 \\ &=P^{(0)}\left\{p_3^2+\imath\varepsilon sp_3\sin sz(1-\varepsilon\cos sz)^{-1}k^{-1}\right\} \\ &\quad+Q^{(0)}p_1^2 \\ &\quad+R^{(0)}\left\{2p_1p_3+\imath\varepsilon sp_1\sin sz(1-\varepsilon\cos sz)^{-1}k^{-1}\right\}. \end{aligned} \tag{14}$$

The expression of c_2' depending on ε may be written as

(i) zeroth order, $O(\varepsilon^0)$:

$$\rho_0c_2'^2=P^{(0)}p_3^2+Q^{(0)}p_1^2+R^{(0)}2p_1p_3, \tag{15}$$

(ii) first order, $O(\varepsilon^1)$:

$$\begin{aligned} \rho_0c_2'^2&=P^{(0)}\left[p_3^2\right. \\ &\quad\left.+\imath\varepsilon sp_3\left\{sz-\frac{(sz)^3}{3!}+\frac{(sz)^5}{5!}-\frac{(sz)^7}{7!}+\cdots\right\}k^{-1}\right] \\ &\quad+Q^{(0)}p_1^2+R^{(0)}\left[2p_1p_3\right. \\ &\quad\left.+\imath\varepsilon sp_1\left\{sz-\frac{(sz)^3}{3!}+\frac{(sz)^5}{5!}-\frac{(sz)^7}{7!}+\cdots\right\}k^{-1}\right], \end{aligned} \tag{16}$$

(iii) second order, $O(\varepsilon^2)$:

$$\begin{aligned} \rho_0c_2'^2&=P^{(0)}\left[p_3^2+\imath\varepsilon sp_3\left\{sz-\frac{(sz)^3}{3!}+\frac{(sz)^5}{5!}-\frac{(sz)^7}{7!}\right.\right. \\ &\quad\left.+\cdots\right\}k^{-1}+\imath\varepsilon^2sp_3\left[\left\{sz-\frac{(sz)^3}{2!}+\frac{(sz)^5}{4!}-\cdots\right\}\right. \\ &\quad+\left\{-\frac{(sz)^3}{3!}+\frac{(sz)^5}{3!2!}-\frac{(sz)^7}{3!4!}+\cdots\right\} \\ &\quad\left.\left.+\left\{\frac{(sz)^5}{5!}-\frac{(sz)^7}{5!2!}+\frac{(sz)^9}{5!4!}\cdots\right\}\right]k^{-1}\right]+Q^{(0)}p_1^2 \\ &\quad+R^{(0)}\left[2p_1p_3+\imath\varepsilon sp_1\left\{sz-\frac{(sz)^3}{3!}+\frac{(sz)^5}{5!}-\frac{(sz)^7}{7!}\right.\right. \\ &\quad\left.+\cdots\right\}k^{-1}+\imath\varepsilon^2sp_1\left[\left\{sz-\frac{(sz)^3}{2!}+\frac{(sz)^5}{4!}-\cdots\right\}\right. \\ &\quad+\left\{-\frac{(sz)^3}{3!}+\frac{(sz)^5}{3!2!}-\frac{(sz)^7}{3!4!}+\cdots\right\} \\ &\quad\left.\left.+\left\{\frac{(sz)^5}{5!}-\frac{(sz)^7}{5!2!}+\frac{(sz)^9}{5!4!}\cdots\right\}\right]k^{-1}\right], \end{aligned} \tag{17}$$

and so on. We will use expression (15) of c_2' for the numerical computation.

4. Wave Propagation

Suppose a plane SH-wave with amplitude, A_0, making an angle θ_0 with the normal be incident at the plane interface ($z=0$) between homogeneous and inhomogeneous fibre-reinforced elastic half-spaces. Such an incident SH-wave, due to the interface, give rises a reflected SH-wave in the half-space, H, and a refracted SH-wave in the half-space, H'. The geometry of the problem is given in Figure 1.

The total displacement due to incident and reflected SH-waves in the half-space, H, is given by

$$\begin{aligned} u_2&=A_0\exp\left[\imath\left\{\omega t-k_0\left(x\sin\theta_0+z\cos\theta_0\right)\right\}\right] \\ &\quad+A\exp\left[\imath\left\{\omega t-k_2\left(x\sin\theta_1-z\cos\theta_1\right)\right\}\right], \end{aligned} \tag{18}$$

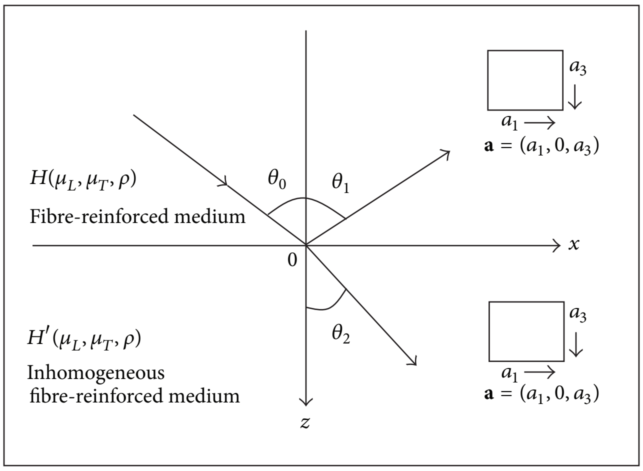


FIGURE 1: Geometry of the problem.

where A is the amplitude constant of the reflected SH-wave at an angle θ_1, $k_i = \omega/c_i$ $(i = 0, 2)$ is the wavenumber, ω is angular frequency, and c_i is the phase speed.

The displacement due to refracted SH-wave in the half-space, H', is given by

$$u_2' = B \exp\left[\imath\left\{\omega t - k_2'\left(x \sin\theta_2 + z\cos\theta_2\right)\right\}\right], \qquad (19)$$

where $k_2' = \omega/c_2'$ is the wavenumber with c_2' as the phase speed and B is the amplitude constant of the refracted SH-wave at an angle θ_2. Snell's law of this problem is given by

$$k_0 \sin\theta_0 = k_2 \sin\theta_1 = k_2' \sin\theta_2. \qquad (20)$$

We assume that the angle of incidence is equal to the angle of reflection.

5. Boundary Conditions

The boundary conditions are the continuities of the displacements and stress tensors at the interface. Mathematically, these conditions at $z = 0$ are

$$u_2 = u_2', \qquad (21)$$

$$\tau_{23} = \tau_{23}'. \qquad (22)$$

Equation (22) may be written in terms of displacement as

$$\mu_T \frac{\partial u_2}{\partial z} + \left(\mu_L - \mu_T\right) a_3 \left(a_1 \frac{\partial u_2}{\partial x} + a_3 \frac{\partial u_2}{\partial z}\right) = \mu_T' \frac{\partial u_2'}{\partial z} + \left(\mu_L' - \mu_T'\right) a_3 \left(a_1 \frac{\partial u_2'}{\partial x} + a_3 \frac{u_2'}{\partial z}\right). \qquad (23)$$

Using (18)–(20) into boundary conditions (21) and (23), we have

$$\frac{A}{A_0} - \frac{B}{A_0} = -1, \qquad r_1 \frac{A}{A_0} + r_2 \frac{B}{A_0} = r_0, \qquad (24)$$

where

$$\begin{aligned} r_0 &= \mu_T k_0 \cos\theta_0 + \left(\mu_L - \mu_T\right) a_3 \left(a_3 k_0 \cos\theta_0 + a_1 k_0 \sin\theta_0\right), \\ r_1 &= \mu_T k_2 \cos\theta_1 + \left(\mu_L - \mu_T\right) a_3 \left(a_3 k_2 \cos\theta_1 - a_1 k_0 \sin\theta_0\right), \\ r_2 &= \left\{\mu_T^{(0)} k_2' \cos\theta_2 + \left(\mu_L^{(0)} - \mu_T^{(0)}\right) a_3 \left(a_3 k_2' \cos\theta_2 + a_1 k_0 \sin\theta_0\right)\right\} (1 - \varepsilon). \end{aligned} \qquad (25)$$

These equations will give the amplitude ratios corresponding to the reflected and refracted SH-waves.

6. Amplitude and Energy Ratio

The amplitude ratios of the reflected and refracted SH-waves are defined as the ratio of the amplitudes corresponding to the reflected and refracted waves to that of the incident wave. Solving (24), we have

$$\frac{A}{A_0} = \frac{r_0 - r_2}{r_1 + r_2}, \qquad \frac{B}{A_0} = \frac{r_0 + r_1}{r_1 + r_2}, \qquad (26)$$

where A/A_0 is the amplitude ratios corresponding to the reflected SH-wave and B/A_0 is that of the refracted SH-wave. We have observed that these ratios depend on the elastic constants, fibre orientation, inhomogeneity parameter, and the angle of incidence.

Let us consider the energy partition of the reflected and refracted SH-waves due to the plane interface, $z = 0$. The energy transmission per unit area may be given as (see [30])

$$\wp^* = \tau_{23} \cdot \dot{u}_2 + \tau_{23}' \cdot \dot{u}_2'. \qquad (27)$$

The expression of the energy due to incident wave is given by

$$E_{\text{inc}} = J_0 \omega A_0^2 \exp\left[2\imath\left\{\omega t - k_0\left(x \sin\theta_0 + z\cos\theta_0\right)\right\}\right], \qquad (28)$$

where $J_0 = k_0\{\mu_T \cos\theta_0 + (\mu_L - \mu_T) a_3 (a_3 \cos\theta_0 + a_1 \sin\theta_0)\}$.

The energy ratio of the reflected and refracted SH-waves may be defined as the ratios of the energy corresponding to the reflected and refracted SH-waves to that of the incident wave. The modulus of energy ratios of the reflected and refracted SH-waves is given as

$$E_1 = \left|\frac{J_1}{J_0}\right| \left|\frac{A}{A_0}\right|^2, \qquad E_2 = \left|\frac{J_2}{J_0}\right| \left|\frac{B}{A_0}\right|^2, \qquad (29)$$

where E_1 is the energy ratio of the reflected SH-wave and E_2 is that of refracted SH-wave and the expressions of J_1 and J_2 are given by

$$\begin{aligned} J_1 &= -k_2 [\mu_T \cos\theta_1 \\ &\quad + (\mu_L - \mu_T) a_3 (a_3 \cos\theta_1 - a_1 \sin\theta_1)], \\ J_2 &= k_2' \left[\mu_T^{(0)} \cos\theta_2 \right. \\ &\quad \left. + \left(\mu_L^{(0)} - \mu_T^{(0)}\right) a_3 (a_3 \cos\theta_2 + a_1 \sin\theta_2)\right] (1 - \varepsilon). \end{aligned} \tag{30}$$

We come to know that the energy ratios are functions of the amplitude ratios, elastic constants, fibre orientation, inhomogeneity parameter, and the angle of incidence of the incident SH-wave.

7. Particular Cases

7.1. Case I. When the inhomogeneous fibre-reinforced elastic half-space, H', reduces to homogeneous fibre-reinforced elastic medium, the problem reduces to the reflection and refraction of SH-waves at the plane interface between the two dissimilar homogeneous fibre-reinforced elastic half-spaces. Under this condition, $\varepsilon = 0$ and (14) reduces to

$$\rho_0 c_2'^2 = P^{(0)} p_3^2 + Q^{(0)} p_1^2 + 2R^{(0)} p_1 p_3. \tag{31}$$

The amplitude and energy ratios corresponding to the reflected and refracted SH-waves are given by (26) and (29) with the following modified values:

$$\begin{aligned} r_0 &= \mu_T k_0 \cos\theta_0 + (\mu_L - \mu_T) a_3 (a_3 k_0 \cos\theta_0 \\ &\quad + a_1 k_0 \sin\theta_0), \\ r_1 &= \mu_T k_2 \cos\theta_1 + (\mu_L - \mu_T) a_3 (a_3 k_2 \cos\theta_1 \\ &\quad - a_1 k_2 \sin\theta_1), \\ r_2 &= \mu_T^{(0)} k_2' \cos\theta_2 + \left(\mu_L^{(0)} - \mu_T^{(0)}\right) a_3 \left(a_3 k_2' \cos\theta_2 \right. \\ &\quad \left. + a_1 k_2' \sin\theta_2\right), \\ J_1 &= -k_2 [\mu_T \cos\theta_1 \\ &\quad + (\mu_L - \mu_T) a_3 (a_3 \cos\theta_1 - a_1 \sin\theta_1)], \\ J_2 &= k_2' \left[\mu_T^{(0)} \cos\theta_2 \right. \\ &\quad \left. + \left(\mu_L^{(0)} - \mu_T^{(0)}\right) a_3 (a_3 \cos\theta_2 + a_1 \sin\theta_2)\right]. \end{aligned} \tag{32}$$

These results are similar to those of Singh and Tomar [12].

7.2. Case II. When the two half-spaces H and H' reduce to homogeneous isotropic elastic media, the problem reduces to the reflection and refraction of SH-waves between two dissimilar isotropic elastic half-spaces. Under this condition,

$$\begin{aligned} P^{(0)} &= Q^{(0)} = \mu_T^{(0)} = \mu_L^{(0)} = \mu_0, \\ P &= Q = \mu_T = \mu_L = \mu, \\ R^{(0)} &= R = 0, \\ c_2^2 &= \frac{\mu}{\rho}, \\ c_2'^2 &= \frac{\mu_0}{\rho_0}. \end{aligned} \tag{33}$$

The amplitude and energy ratios corresponding to the reflected and refracted SH-waves are given by (26) and (29) with the following modified values:

$$\begin{aligned} r_0 &= \mu k_0 \cos\theta_0, \\ r_1 &= \mu k_2 \cos\theta_1, \\ r_2 &= \mu_0 k_2' \cos\theta_2, \\ J_1 &= -k_2 \mu \cos\theta_1, \\ J_2 &= k_2' \mu_0 \cos\theta_2. \end{aligned} \tag{34}$$

These results exactly match with that of Achenbach [30].

8. Numerical Results and Discussion

For the numerical computations, we take the following relevant parameters for fibre-reinforced elastic half-space, H, and the nonhomogeneous fibre-reinforced elastic half-space, H' [13].

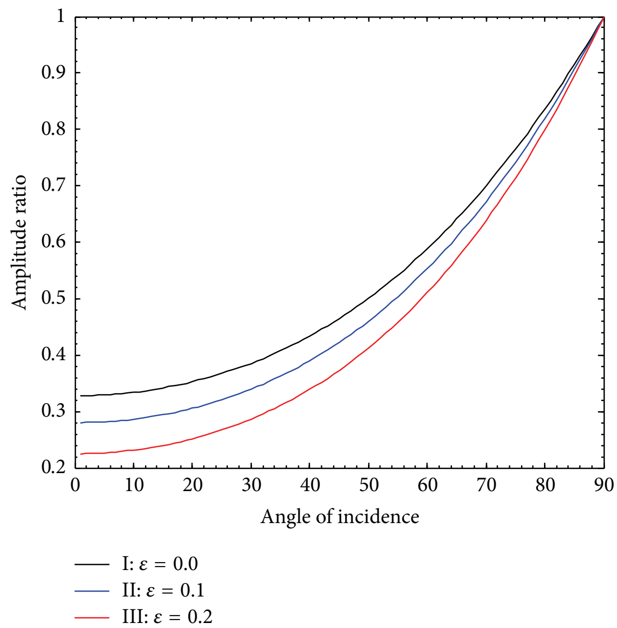
For the homogeneous half-space H,

$$\begin{aligned} \mu_L &= 7.07 \times 10^9\ \mathrm{N/m^2}, \\ \mu_T &= 3.5 \times 10^9\ \mathrm{N/m^2}, \\ \rho &= 1600\ \mathrm{Kg/m^3}. \end{aligned} \tag{35}$$

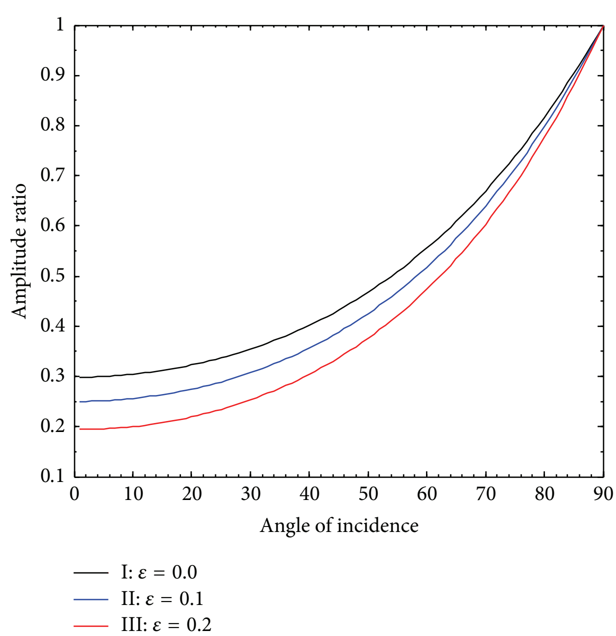
For the nonhomogeneous half-space H',

$$\begin{aligned} \mu_L^{(0)} &= 5.66 \times 10^9\ \mathrm{N/m^2}, \\ \mu_T^{(0)} &= 2.46 \times 10^9\ \mathrm{N/m^2}, \\ \rho_0 &= 7800\ \mathrm{Kg/m^3}, \\ a_1 &= 0.00316227, \\ \varepsilon &= 0.0, 0.1, 0.2. \end{aligned} \tag{36}$$

The variation of the modulus of amplitude ratios corresponding to the reflected and refracted SH-waves with the angle of incidence, θ_0, is depicted in Figures 2 and 3 with and without fibre-reinforcement at different values of inhomogeneity parameter, ε. In Figures 2(a) and 2(b), the amplitude

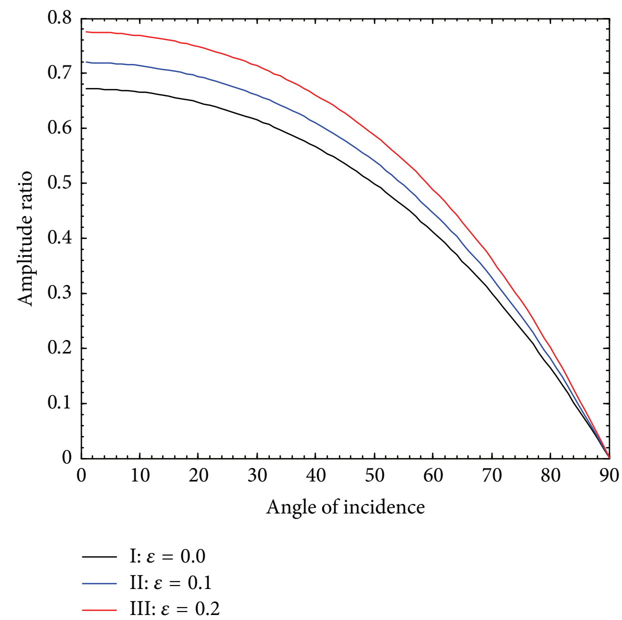


(a) In the presence of reinforcement

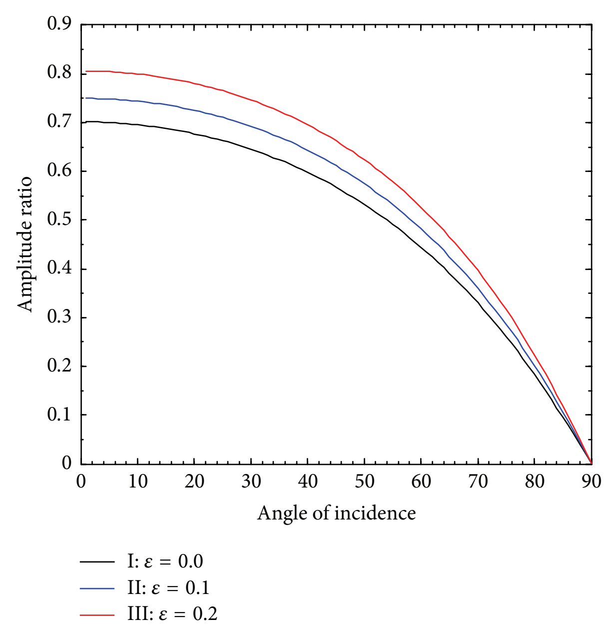


(b) In the absence of reinforcement

Figure 2: Variation of the modulus of amplitude ratio, A/A_0 with θ_0.



(a) In the presence of reinforcement



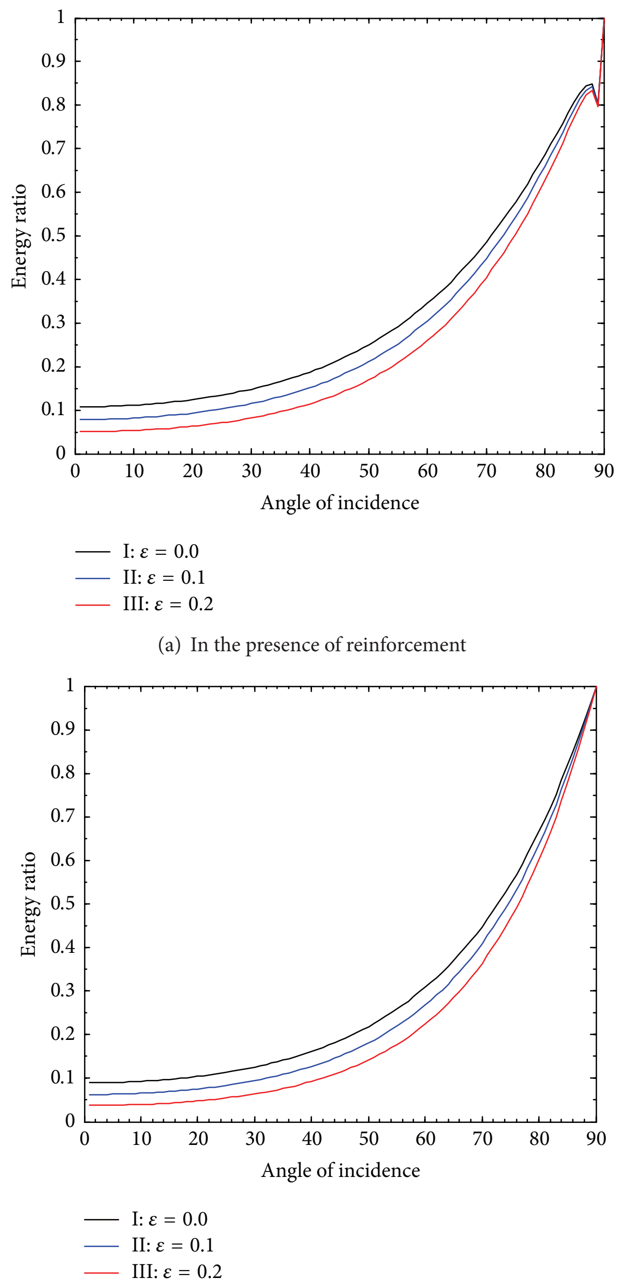
(b) In the absence of reinforcement

Figure 3: Variation of the modulus of amplitude ratio, B/A_0 with θ_0.

ratio, A/A_0, corresponding to the reflected *SH*-wave starts from certain values and increases with the increase of θ_0 which attains the maximum value at the grazing angle of incidence. The values of A/A_0 decrease with the increase of inhomogeneity parameter.

All Curves I, II, and III in Figures 3(a) and 3(b) show that B/A_0 corresponding to the refracted *SH*-wave decreases with the increase of θ_0 and attains the minimum value at the grazing angle of incidence. The values of B/A_0 increase with the increase of inhomogeneity parameter. Figures 4 and 5 show the variation of the modulus of energy ratios corresponding to the reflected and refracted *SH*-waves with the angle of incidence with and without fibre-reinforcement at different values of inhomogeneity parameter, ε. In Figures 4(a) and 4(b), E_1 corresponding to the reflected *SH*-wave starts from certain values and increases up to $\theta_0 = 88^\circ$ and decreases to $\theta_0 = 89^\circ$ which increases thereafter with the increase of θ_0.

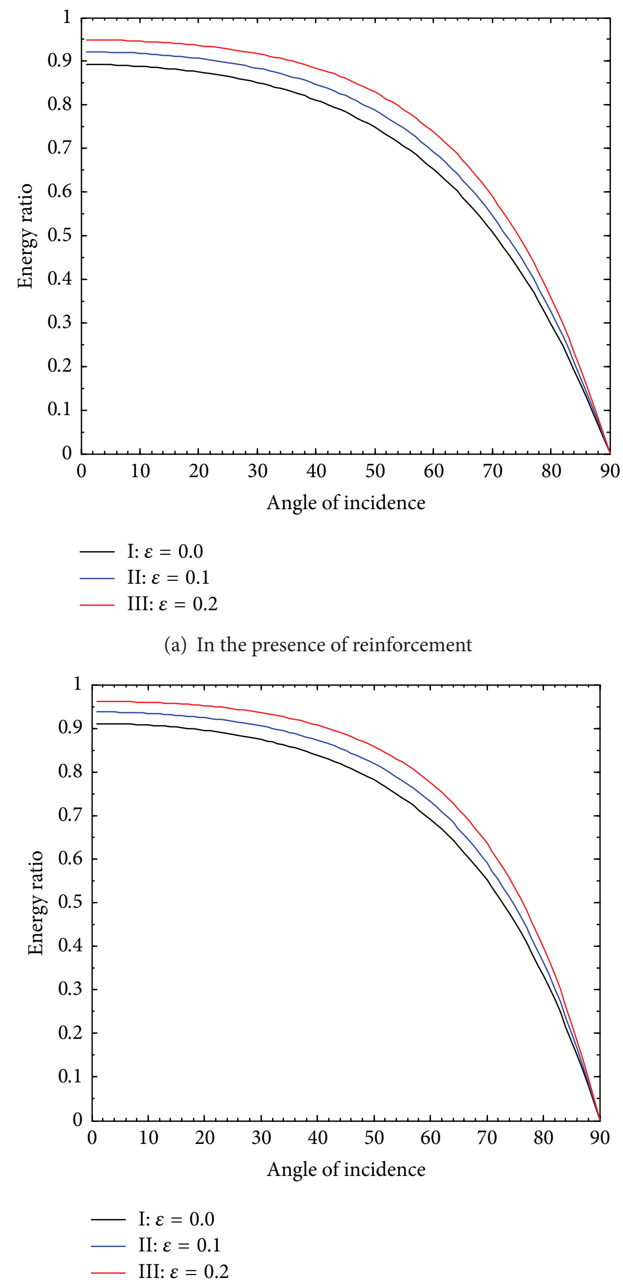
The modulus of energy ratio, E_2, corresponding to the refracted *SH*-wave in Figures 5(a) and 5(b) starts from certain



(a) In the presence of reinforcement

(b) In the absence of reinforcement

FIGURE 4: Variation of the modulus of energy ratio, E_1 with θ_0.



(a) In the presence of reinforcement

(b) In the absence of reinforcement

FIGURE 5: Variation of the modulus of energy ratio, E_2 with θ_0.

values and decreases with the increase of θ_0 and attains the minimum value at the grazing angle of incidence. We have observed that the sum of energy ratios is close to unity.

9. Conclusion

The problem of reflection and refraction of *SH*-waves at the plane interface between the homogeneous fibre-reinforced half-space and the nonhomogeneous fibre-reinforced half-space has been investigated. The amplitude and energy ratios corresponding to the reflected and refracted *SH*-waves have been obtained and computed numerically. We may conclude the following points:

(i) The amplitude and energy ratios corresponding to the reflected and refracted *SH*-waves are functions of elastic constants, fibre orientation, inhomogeneity parameter, and angle of incidence.

(ii) The amplitude ratio, A/A_0, and energy ratio, E_1, attain the maximum value at grazing angle of incidence.

(iii) The values of A/A_0 and E_1 decrease with the increase of ε.

(iv) The values of A/A_0 and E_1 with fibre-reinforcement are greater than those values without fibre-reinforcement.

(v) The values of B/A_0 and E_2 increase with the increase of ε.

(vi) The sum of energy ratios is close to unity.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

The author C. Zorammuana acknowledges UGC, New Delhi, for giving financial assistance through Rajiv Gandhi National Fellowship (RGNF) to complete this work.

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