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Research Article

Optimum Design of Gravity Retaining Walls Using Charged System Search Algorithm

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This study focuses on the optimum design retaining walls, as one of the familiar types of the retaining walls which may be constructed of stone masonry, unreinforced concrete, or reinforced concrete. The material cost is one of the major factors in the construction of gravity retaining walls therefore, minimizing the weight or volume of these systems can reduce the cost. To obtain an optimal seismic design of such structures, this paper proposes a method based on a novel meta-heuristic algorithm. The algorithm is inspired by the Coulomb's and Gauss's laws of electrostatics in physics, and it is called charged system search (CSS). In order to evaluate the efficiency of this algorithm, an example is utilized. Comparing the results of the retaining wall designs obtained by the other methods illustrates a good performance of the CSS. In this paper, we used the Mononobe-Okabe method which is one of the pseudostatic approaches to determine the dynamic earth pressure.

1. Introduction

Every time a product is created or designed to satisfy human needs, the creator tries to achieve the best solution for the task in hand and therefore performs optimization. This process is often manual, time consuming and involves a step by step approach to identify the right combination of the product and associated process parameters for the best solution. Often the manual approach does not allow a thorough exploration of the solution space to find the optimum design, resulting in suboptimal designs [1–3]. Therefore experienced engineers may be able to come up with solutions that fulfill some of the requirements on structural response, cost, aesthetics, and manufacturing but they will seldom be able to come up with the optimal structure.

One type of optimization methods is known as metaheuristic algorithms. These methods are suitable for global search due to their capability of exploring and finding promising regions in the search space at an affordable time. Meta-heuristic algorithms tend to perform well for most of the optimization problems [4–7]. As a new meta-heuristic approach, this paper utilizes charged system search algorithm (CSS) for the optimum design of gravity retaining walls subjected to seismic loading. Retaining walls is generally classified as gravity, semigravity (or conventional), nongravity cantilevered, and anchored. Gravity retaining walls are the walls which use their own weight to resist the lateral earth pressures. The main forces acting on gravity retaining walls are the vertical forces from the weight of the wall, the lateral earth pressure acting on the back face and the seismic loads. These forces are used herein to illustrate the design principles. If other forces are encountered, such as vehicular loads, they must also be included in the analysis. The lateral earth pressure is usually calculated by the Coulomb equation.

The paper is structured as follows. After this introduction, Section 2 recalls the optimization problem statement. Then review of CSS is presented in Section 3. Test case is presented in Section 4 while optimization and sensitivity analysis results are reported and discussed. Finally, Section 5 summarizes the main findings of this study, and conclusion is drawn based on the reported results.

2. The Optimization Problem

Gravity walls derive their capacity to resist lateral loads through dead weight of the wall. The earliest method for determining the combined static and dynamic earth pressure on a retaining wall was developed by Okabe [8] and Mononobe [9]. This method, generally referred to as the Mononobe-Okabe method, is based on plasticity theory and is essentially an extension of the Coulomb sliding wedge theory in which the transient earthquake forces are represented by an equivalent static force. Therefore the effect of the earthquake motion can be represented as inertial forces K_hW and K_vW acting at the centre of gravity of the mass [10]. The principle of this method is illustrated in Figure 1. The Mononobe-Okabe method was originally developed for a dry cohesion less material with the following two assumptions.

(1) The wall yields sufficiently such that a triangular soil wedge behind the wall is formed at the point of incipient failure, with the maximum shear strength mobilized along the sliding surface.

(2) The wall and the soil behave as a rigid body with the shear wave travelling at an infinite speed such that the acceleration effectively becomes uniform throughout the mass of the soil wedge.

The expression of the total dynamic force, P_{AE} (Figure 1) is given below:

$$P_{AE} = \frac{1}{2}\gamma H^2(1+K_v)C_{AE},$$
$$C_{AE} = \frac{\cos^2(\phi' - \theta - \beta)}{\cos\theta\cos^2\beta\cos(\delta+\beta+\theta)\left[1+\sqrt{\sin(\phi'+\delta)\sin(\phi'-\theta-i)/\cos(\delta+\beta+\theta)\cos(i-\beta)}\right]^2}. \tag{2.1}$$

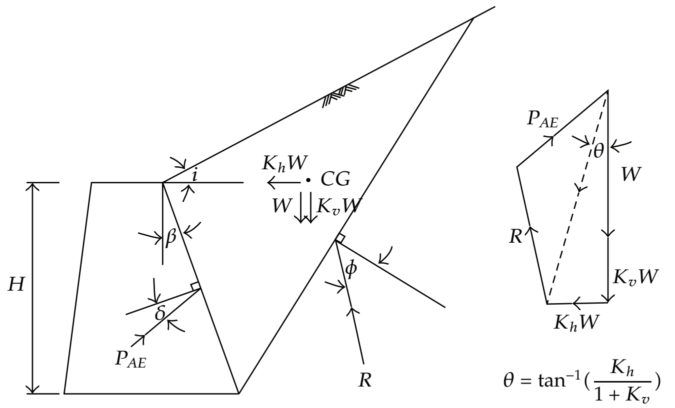

Figure 1: The Mononobe-Okabe method.

The pseudostatic approach can be visualized as effectively tilting the ground profile and wall geometry by an angle 0 (defined as above), with a new gravity, g', given by the following equation:

$$g' = \sqrt{(1+K_v)^2 + {K_h}^2}\, g. \tag{2.2}$$

It should be noted that the Mononobe-Okabe equation is applicable for retaining walls where the angle i is less than or equal to $(\phi' - \theta)$. This is because if the angle i is greater than $(\phi' - \theta)$, the sloping backfill behind the wall will be unstable unless the soil has sufficient cohesive strength. In the latter case, the more versatile analysis approaches should be adopted.

More advanced methods, such as dynamic response analysis and finite element method, are capable of allowing for the dynamic characteristics of the soil-structure system. However, these advanced methods are usually not justified for the analysis of conventional gravity retaining walls subjected to earthquake loading and the above simple methods are generally adequate as shown in [11]. Therefore, Mononobe-Okabe method is used herein to determine the dynamic earth pressure.

On the other hand, there are three different modes of instabilities, namely sliding, overturning, and bearing capacity, which should be checked [12]. The procedure for computing the dynamic factors of safety against sliding and overturning is same as that for static calculations, except that the inertia of the gravity wall itself must also be included when earthquake loading is considered [13]. Thus, the optimal seismic design problem of gravity retaining walls may be expressed as

Design variables

$$\mathbf{X} = [x_1, x_2, \ldots, x_6] \tag{2.3}$$

minimize

$$W(\mathbf{X}) = A_{\mathrm{cs}} \cdot \gamma \tag{2.4}$$

constraints

$$
\begin{aligned}
FS_o &\geq 2, \\
FS_s &\geq 1.2, \\
FS_b &\geq 3,
\end{aligned}
\tag{2.5}
$$

where $\mathbf{X}$ is the vector containing the design variables (see Figure 2); W is the weight of a unit length of wall; A_{cs} is the wall cross-section area; γ is the density of the material; FS_o, FS_s, and FS_b are the factors of safety against overturning, sliding, and bearing capacity, respectively.

3. Charged System Search Algorithm

The Charged System Search (CSS) algorithm is based on the Coulomb and Gauss laws from electrical physics and the governing laws of motion from the Newtonian mechanics. This algorithm can be considered as a multiagent approach, where each agent is a Charged Particle (CP). Each CP is considered as a charged sphere with radius a, having a uniform volume charge density and is equal to

$$
q_i = \frac{\text{fit}(i) - \text{fitworst}}{\text{fitbest} - \text{fitworst}} \quad i = 1, 2, \ldots, N. \tag{3.1}
$$

CPs can impose electric forces on the others, and its magnitude for the CP located in the inside of the sphere is proportional to the separation distance between the CPs, and for a CP located outside the sphere is inversely proportional to the square of the separation distance between the particles. The kind of the forces can be attractive or repelling, and it is determined by using ar_{ij}, the kind of force parameter, defined as

$$
ar_{ij} = \begin{cases} +1 & \text{w.p. } k_t, \\ -1 & \text{w.p. } 1 - k_t, \end{cases} \tag{3.2}
$$

where ar_{ij} determines the type of the force, +1 represents the attractive force, −1 denotes the repelling force, and k_t is a parameter to control the effect of the kind of force. In general, the attractive force collects the agents in a part of search space and the repelling force strives to disperse the agents. Therefore, the resultant force is redefined as

$$
\mathbf{F}_j = q_j \sum_{i, i \neq j} \left(\frac{q_i}{a^3} r_{ij} \cdot i_1 + \frac{q_i}{r_{ij}^2} \cdot i_2 \right) ar_{ij} p_{ij} \left(\mathbf{X}_i - \mathbf{X}_j \right) \quad \begin{cases} j = 1, 2, \ldots, N \\ i_1 = 1, \ i_2 = 0 \Longleftrightarrow r_{ij} < a \\ i_1 = 0, \ i_2 = 1 \Longleftrightarrow r_{ij} \geq a \end{cases} \tag{3.3}
$$

the separation distance between two charged particles defined as

$$
r_{ij} = \frac{\left\| \mathbf{X}_i - \mathbf{X}_j \right\|}{\left\| \left(\mathbf{X}_i + \mathbf{X}_j \right) / 2 - \mathbf{X}_{\text{best}} \right\| + \varepsilon}, \tag{3.4}
$$

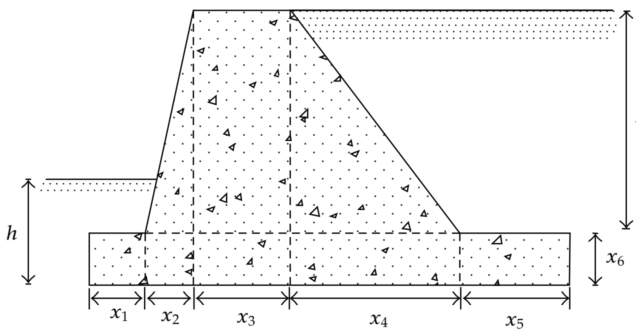

Figure 2: The design variables.

where ε is a small positive number to avoid singularity. The initial positions of CPs are determined randomly in the search space and the initial velocities of charged particles are assumed to be zero. P_{ij} determines the probability of moving each CP toward the others as

$$p_{ij} = \begin{cases} 1 & \dfrac{\text{fit}(i) - \text{fitbest}}{\text{fit}(j) - \text{fit}(i)} > \text{rand} \vee \text{fit}(j) < \text{fit}(i). \\ 0 & \text{otherwise}. \end{cases} \tag{3.5}$$

The resultant forces and the laws of the motion determine the new location of the CPs. At this stage, each CP moves towards its new position under the action of the resultant forces and its previous velocity as

$$\begin{gathered} \mathbf{X}_{j,\text{new}} = \text{rand}_{j1} \cdot k_a \cdot \frac{\mathbf{F}_j}{m_j} \cdot \Delta t^2 + \text{rand}_{j2} \cdot k_v \cdot \mathbf{V}_{j,\text{old}} \cdot \Delta t + \mathbf{X}_{j,\text{old}}, \\ \mathbf{V}_{j,\text{new}} = \frac{\mathbf{X}_{j,\text{new}} - \mathbf{X}_{j,\text{old}}}{\Delta t}, \end{gathered} \tag{3.6}$$

where k_a is the acceleration coefficient; k_v is the velocity coefficient to control the influence of the previous velocity; rand_{j1} and rand_{j2} are two random numbers uniformly distributed in the range $(0, 1)$. If each CP moves out of the search space, its position is corrected using the harmony search-based handling approach as described in [14]. In addition, to save the best design, a memory (charged memory) is utilized. The flowchart of the CSS algorithm is shown in Figure 3.

4. Numerical Example

In this section, an example is optimized with the proposed method. The final result is compared to the solution of the particle swarm optimization (PSO), big bang-big crunch algorithm (BB-BC), and heuristic big bang-big crunch (HBB-BC) [15] methods to demonstrate the efficiency of the present approach. For the example presented in this paper, the CSS algorithm parameters were set as follows: $k_a = 2.0$, $k_v = 1.5$, the number of agents is taken as 20, and the maximum number of searches is set to 500. The algorithms are coded in

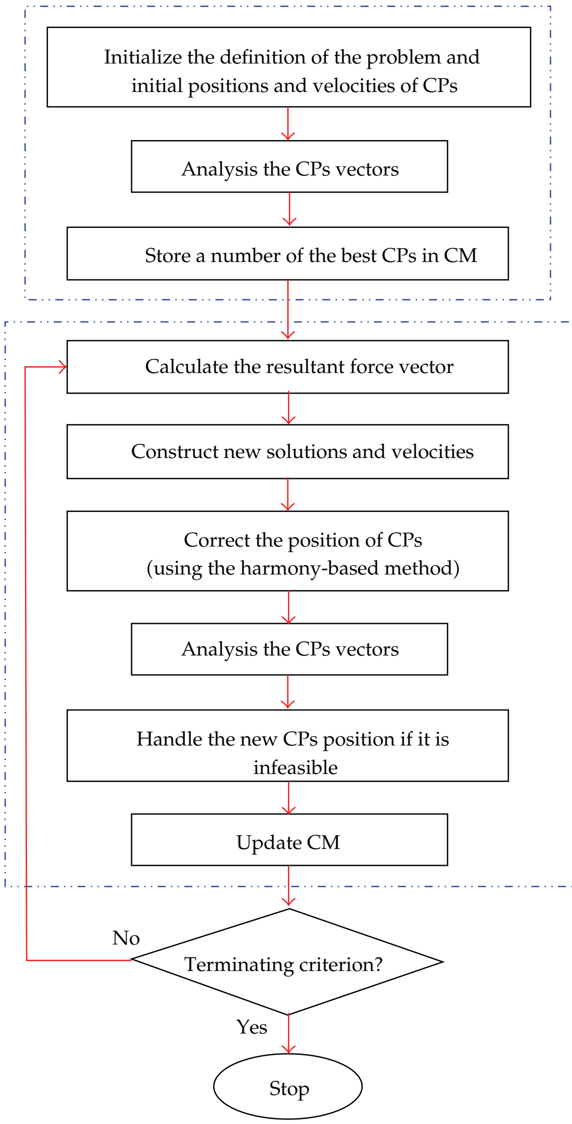

Figure 3: The flowchart for the CSS algorithm.

Matlab and in order to handle the constraints, a penalty approach is utilized. If the constraints are between the allowable limits, the penalty is zero; otherwise, the amount of penalty is obtained by dividing the violation of allowable limit to the limit itself.

The problem is the optimum seismic design of a wall with $H = 5.5\,\text{m}$ and $h = 1\,\text{m}$. The backfill has shear strength parameters of $c' = 0$, $\phi' = 30^\circ$, and $\gamma = 16\,\text{kN/m}^3$. The wall is founded on a soil with c' equals zero, $\phi' = 38^\circ$, and $\gamma = 18.5\,\text{kN/m}^3$. The horizontal and vertical ground acceleration coefficient (K_h and K_v) is 0.35 and 0.0. Also the material's density is 24 kN/m (concrete wall). In this example, the angle of wall friction is 15° and the inclination of ground surface behind wall to horizontal is zero.

The results of the seismic design optimization process for the CSS algorithm and the PSO, BB-BC, and HBB-BC are summarized in Table 1. As shown in this table, the result for

Table 1: The optimum seismic designs comparison for the gravity retaining wall.

Design variable	Optimal values (m)			
	CSS	HBB-BC [15]	BB-BC [15]	PSO [15]
x_1	1.137	1.734	0.903	0.566
x_2	0.440	0.553	0.650	2.000
x_3	0.353	0.467	0.689	0.200
x_4	3.200	3.014	2.691	2.000
x_5	2.261	0.719	0.515	0.645
x_6	0.200	0.200	0.200	0.200
Best weight (kN)	322.293	328.297	337.607	344.700
Average weight (kN)	329.893	337.860	345.652	351.189
Number of analysis	4,400	10,000	10,000	10,000
FS_o	2.152	2.504	2.003	2.097
FS_s	1.200	1.200	1.200	1.200
FS_b	7.154	7.459	5.921	7.182

the CSS algorithm is 322.293 kN, which is lighter than the result of the PSO, standard BB-BC, and HBB-BC algorithm. In addition, the average weight of 20 different runs for the CSS algorithm is 2.3%, 4.8%, and 6.1% lighter than the average results of the HBB-BC, BB-BC, and PSO algorithms, respectively. Comparing these results shows that the new algorithm not only improves the reliability property due to decrease in the mean of results but also enhances the quality of the results due to the decrease in the best of results. The convergence history for the CSS gravity retaining wall design is shown in Figure 4.

Among the design constraints, the safety factor of sliding is the active one and almost for all design of different studied algorithms, it is the most important while the factor of safety against bearing capacity is not active and it will not affect the optimum design.

Any optimum design problem involves a design vector and a set of problem parameters. In many cases, we would be interested in knowing the sensitivities or derivatives of the optimum design (design variables and objective function) with respect to the problem parameters because this is very useful to the designer, to know which data values are more influential on the design. Sensitivity of optimal responses to these parameters is one of the important issues in the optimum design of retaining walls.

Here, using sensitivity analysis, the effect of changes on the safety factor for sliding on optimum weight of a wall was studied. The factor of safety for sliding of the wall is defined as the resisting forces divided by the driving force, or

$$FS_s = \frac{\text{Sliding resistance force} + \text{Allowable passive resultant force}}{\text{Active earth pressure resultant force}}. \tag{4.1}$$

If the wall is found to be unsafe against sliding, shear key below the base is provided. Such a key develops passive pressure which resists completely the sliding tendency of the wall. The customary minimum safety factor against sliding is 1.2, with some agencies requiring more. In the determination of the, FS_s, the effect of passive lateral earth pressure resistance in front of a wall footing or a wall footing key will only be considered when competent soil or rock exists which will not be removed or eroded during the structure life.

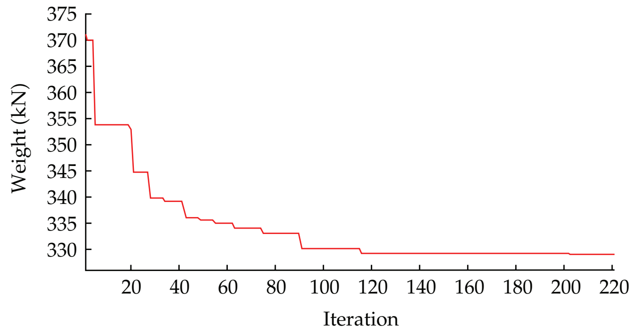

Figure 4: The convergence history of CSS algorithm (average of 20 different runs).

Not more than 50 percent of the available passive lateral earth pressure will be considered in determining the FS_s. In Figure 5, optimum weight variation against safety factor of sliding is depicted. It is interesting to emphasis that a small coefficient for $FS_s = 1.2$ causes an average decrease in cost of 43% as compared to a coefficient for $FS_s = 2$.

5. Concluding Remarks

Determining optimum weight and sensitivity analysis of gravity retaining walls subject to seismic loading is presented in detail, using the CSS algorithm. This algorithm contains three levels: initialization, search, and controlling the terminating criterion. In the initialization level, the parameters of the CSS algorithm, the primary location of the CPs, and their initial velocities are defined. Also in this level, a memory to store a number of the best CPs is introduced. The search level starts after the initialization level, where each CP moves toward the others considering the probability function, the magnitude of the attracting force vector, and the previous velocities. The moving process is defined in a way that it not only can perform more investigation in the search space, but also can improve the results. To fulfill this goal, some laws of physics containing the Coulomb and Gauss laws, and the governing laws of motion from Newtonian mechanics are utilized. The last level consists of controlling the termination.

Comparing the results of the retaining wall designs obtained by other meta-heuristic algorithms such as the PSO and the BB-BC shows a good balance between the exploration and exploitation abilities of the CSS; hence, its superior performance becomes evident. Both CSS and PSO are population-based algorithms in which the position of each agent is obtained by adding the agent's movement to its previous position; however, the movement strategies are different. The PSO algorithm utilizes a velocity term being a combination of the previous velocity movement in the direction of the local best, and movement in the direction of the global best, while the CSS approach uses the governing laws from electrical physics and the governing laws of motion from the Newtonian mechanics to determine the amount and the direction of a charged particle' movement. The potency of the PSO is summarized to find the direction of an agent's movement and therefore determining the acceleration constants becomes important. Similarly in the CSS method, updating is performed by considering the quality of the solutions and the separation distances between CPs. Therefore, not only the directions but also the amounts of movements are determined.

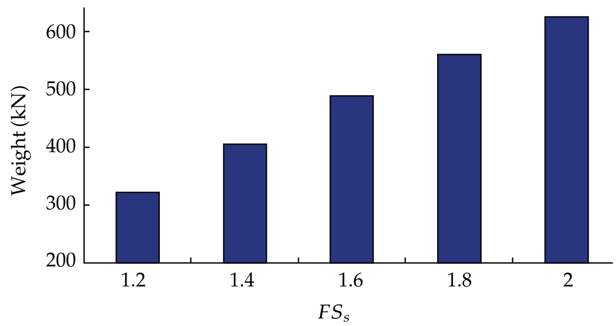

Figure 5: Weight variation against different FS_s.

Also a sensitivity analysis is performed for the optimum seismic design of gravity retaining wall parameters using the CSS algorithm in which the safety factor for sliding is concerned. The results related to the influence of the safety factors of sliding show that as expected, a large safety factor causes a costly wall compared to a small one.

Notation

W: Weight of the sliding wedge
K_h: Horizontal ground acceleration coefficient
K_v: Vertical ground acceleration coefficient
P_{AE}: Total dynamic force on the retaining wall
R: Reaction on soil wedge from the surrounding ground
H: Height of the wall
ϕ': Angle of shearing resistance of the soil
δ: The angle of wall friction
i: Inclination of ground surface behind wall to horizontal
β: Inclination of the back of wall to vertical
θ: Inclination of the resultant inertial force to the vertical $= \tan^{-1}(K_h/(1+K_v))$
C_{AE}: Horizontal seismic coefficient
fitbest: Best fitness of all the particles
fitworst: Worst fitness of all the particles
fit(i): Fitness of the agent i
N: Total number of CPs
$\mathbf{F}_j$: Resultant force acting on the jth CP
r_{ij}: Separation distance between two charged particles
$\mathbf{X}_i$: Positions of the ith CPs
$\mathbf{X}_{\text{best}}$: Position of the best current CP.

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