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Examining a renormalizable supersymmetric SO(10) model

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Abstract We examine a renormalizable SUSY SO(10) model without fine-tuning. We show how to construct MSSM doublets and to predict proton decay. We find that in the minimal set of Yukawa couplings the model is consistent with the experiments, while including 120_H to fit the data there are inconsistencies.

1 Introduction

Supersymmetric (SUSY) grand unification theories (GUTs) of SO(10) [1,2] are very important candidates for the new physics beyond the standard model (SM). As was firstly occurred in the SU(5) models, a very serious difficulty in all GUT models is the realization of doublet–triplet splitting (DTS) within the same Higgs multiplets. The two Higgs doublets of the minimal supersymmetric standard model (MSSM) have weak scale masses, while the color triplets and anti-triplets in the same representations need to have masses of the GUT scale. This is not only needed in the realization of gauge coupling unification [3–8], but also needed in the suppression of proton decay mediated by the colored Higgsinos [9–12].

In the models without natural DTS, the two Higgs doublets of the MSSM are generated through fine-tuning the doublet mass matrix [13–18]. The condition is highly nonlinear, so that it is difficult for these doublets to satisfy those constraints got by fitting the SM fermion masses and mixing. Consequently, the superpotential parameters are difficult to fix so that the models are difficult to make definite predictions on data like proton decay.

In the present work, we will discuss the MSSM doublets and proton decay in a renormalizable SUSY SO(10) model [19] in which the DTS is realized through the Dimopoulos–Wilczek (DW) mechanism [20–33] of miss-

ing vacuum expectation value (VEV). In this model, the MSSM doublets are linear combinations of the Higgs doublets from several different representations of SO(10). Consequently, the superpotential parameters are easily related to these doublets. Then the color-triplet Higgs mass matrix is determined, which makes the determination of proton decay feasible. Being a renormalizable model, the Z_2 subgroup of the SO(10) center Z_4 remains unbroken which acts as the matter parity, thus dangerous dimension-four operators of proton decay are eliminated, and the lightest SUSY particle (LSP) is stable which is a good candidate of the dark matter [34].

There are also other important advantages in the model [19]. Following [35–37], the type-I seesaw mechanism [38–44] for neutrino masses and mixing is incorporated without introducing a real scale. Instead, only a VEV smaller than the GUT scale is used, so that all heavy particles are given masses of the GUT scale. Consequently, the mass splitting among them is not large and thus the threshold effects of the GUT scale can be small, at least when we adjust the parameters which are not used in the present numerical study. In this sense the gauge coupling unification is maintained. Furthermore, the form of the color-triplet mass matrix exhibits proton decay suppression explicitly [36,37]. However, whether the model [19] is realistic or not depends on whether its prediction on proton decay is consistent with the data, and we will examine this in the following.

2 Superpotential

The model in [19] is a renormalizable SUSY SO(10) model in which the Yukawa couplings are given by the superpotential

$$W^{\text{Fermion}} = Y_{10}^{ij}\psi_i\psi_j H_1 + Y_{120}^{ij}\psi_i\psi_j D_1 + Y_{126}^{ij}\psi_i\psi_j \overline{\Delta}_1, \quad (1)$$

which is generally enough to fit all fermion masses and mixing. Here $\psi_i (i = 1, 2, 3)$ are the matter superfields, H_1, D_1

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and $\overline{\Delta}_1$ are the Higgs superfields in the 10, 120 and $\overline{126}$ representations of SO(10), respectively.

SO(10) symmetry is broken by $\Phi(210)$ and/or $A(45) + E(54)$ into $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$. To further break $U(1)_{B-L} \times U(1)_{I_{3R}}$ into $U(1)_Y$ of the MSSM symmetry, the SM singlets which carry nonzero $B - L$ quantum numbers need to have VEVs. In renormalizable models these SM singlets are contained in $\overline{\Delta}(\overline{126}) + \Delta(126)$. It has been studied in [15] that these VEVs $\overline{v} = v$, which is required by the D-flatness of SUSY, should be taken at the GUT scale 2×10^{16} GeV to avoid breaking gauge coupling unification. However, to generate low energy neutrino masses and mixing, the seesaw mechanism requires these VEVs to be of order $\sim 10^{14}$ GeV. To solve this discrepancy, we need to introduce two pairs of $\overline{\Delta} + \Delta$, one $\overline{\Delta}_1$ couples to the matter fields through (1) which has a smaller VEV $\overline{v_1} \sim 10^{14}$ GeV for the seesaw mechanism, the other VEVs are at the GUT scale to realize gauge coupling unification.

To be specific, we introduce the following superpotential:

$$(m_1 + \eta_1\Phi)\,\overline{\Delta}_1\Delta_2 + (m_2 + \eta_2\Phi)\,\overline{\Delta}_2\Delta_1 + \eta_3 Q\overline{\Delta}_2\Delta_2, \quad (2)$$

which contains all interactions relevant to the $U(1)_{B-L}$ breaking. The D-flatness condition of maintaining SUSY at the GUT scale is simply $|\overline{v_1}|^2 + |\overline{v_2}|^2 = |v_1|^2 + |v_2|^2$. The F-flatness conditions are

$$0 = \begin{pmatrix} \overline{v}_1 & \overline{v}_2 \end{pmatrix} \begin{pmatrix} 0 & m_1 + \eta_1\Phi_0 \\ m_2 + \eta_2\Phi_0 & \eta_3 Q \end{pmatrix},$$

$$0 = \begin{pmatrix} 0 & m_1 + \eta_1\Phi_0 \\ m_2 + \eta_2\Phi_0 & \eta_3 Q \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix},$$

where Φ_0 is a combination of the three VEVs in Φ, and the SO(10) singlet Q has a VEV $\sim 10^{-2}\Lambda_{\text{GUT}}$. This smaller VEV Q can be linked with the Planck scale by $Q \sim \Lambda_{\text{GUT}}^2/\Lambda_{\text{Planck}}$ through the Green–Schwarz mechanism [45–48], provided that the extra global symmetry is embedded into an anomalous U(1) symmetry [37]. One set of solutions of the above equations require $m_2 + \eta_2\Phi_0 = 0$, which gives

$$\overline{v}_1 = \overline{v}_2 \frac{\eta_3 Q}{\frac{\eta_1}{\eta_2}m_2 - m_1} \sim 10^{-2}\Lambda_{\text{GUT}} \quad (3)$$

and

$$v_2 = 0, \quad (4)$$

and $\overline{v}_2 \sim v_1 \sim \Lambda_{\text{GUT}}$ satisfying the D-flatness condition. Note that following (2), the color-triplet–anti-triplet mass matrix has the structure

$$\begin{pmatrix} 0 & \Lambda_{\text{GUT}} \\ \Lambda_{\text{GUT}} & Q \end{pmatrix}. \quad (5)$$

When only $\overline{\Delta}_1$ couples with the matter superfields, we can integrate out $\Delta_2 - \overline{\Delta}_2$ to generate the effective triplet mass matrix whose elements are $\sim \frac{\Lambda_{\text{GUT}}^2}{Q} \sim 10^2\Lambda_{\text{GUT}}$. Consequently, the dimension-five operators for proton decay mediated by the color triplet–anti-triplet of $\overline{\Delta}_1$ are suppressed accordingly. To suppress proton decay mediated by $D_1(120)$ of (1), we need also introduce a second $D_2(120)$ to get a mass matrix with the same structure as (5).

Now we include $H_1(10)$, which couples with the matter superfields. $H_1, D_1, \Delta_1, \overline{\Delta}_1$ couple with $D_2, \Delta_2, \overline{\Delta}_2$ through Φ. Note that in Φ there is also a pair of $SU(2)_L$ doublets. As will be seen in the next section, $v_2 = 0$ eliminates some possible mixing terms between the doublets of Φ and those of $H_1, D_1, \Delta_1, \overline{\Delta}_1$. This is crucial in generating a pair of massless doublets in the model. There are, however, also a pair of massless color triplet–anti-triplet. To give these triplet masses of the GUT scale, we need to use the DW mechanism using a second $H_2(10)$ and $A(45)$ with DW-type VEVs $A_1 = 0, A_2 \neq 0$. Here A_1 and A_2 are the VEVs of the SM singlets in the $(1,1,3)$ and $(15,1,1)$ directions, respectively, under the $SU(4)_c \times SU(2)_L \times SU(2)_R$ subgroup of SO(10). To suppress $H_1(10)$ mediated dimension-five operators for proton decay, we need another $A'(45)$ whose VEVs $A'_1 \neq 0, A'_2 = 0$, which are the complements to the DW (CDW) mechanism. In [19] it was found that the simplest method to realize both the DW and the CDW mechanisms is using the superpotential of the form

$$\xi_1 PAA' + (\xi_2 E + \xi_3 R)A'A'', \quad (6)$$

which contains all interactions for A' and the new $A''(45)$ contributing to their F-terms. Here P, R are SO(10) singlets playing the roles of masses and E is a 54 of SO(10). Solving the F-term conditions for A'' gives the CDW VEV $A'_2 = 0$, then solving the F-term conditions for A' gives the DW VEV $A_1 = 0$. There is another set of solutions with $A'_1 = 0$ and $A_2 = 0$, which are not used.

In the renormalizable models, the direct application of the DW mechanism does not work, since the couplings $H_1 A H_2$ and $D_1 A H_2$ exist simultaneously. The latter interaction invalidities the DW mechanism due to the coupling $D_1(15, 2, 2)A_2(15, 1, 1)H_2(1, 2, 2)$ using the notations under $SU(4)_c \times SU(2)_L \times SU(2)_R$. To avoid this interaction, the filter mechanism [33] can be used with the superpotential

$$PH_1\overline{h} + m_h h\overline{h} + AhH_2 + \frac{1}{2}m_{H_2}H_2^2 \quad (7)$$

where dimensionless couplings are suppressed. The singlet P is used as a filter to eliminate $D_1 PAH_2$ while keeping $H_1 PAH_2$. To apply the CDW mechanism to suppress proton

Table 1 Notations and $Z_{24} \times Z_4$ properties of all superfields. Here $\psi_i (i = 1, 2, 3)$ are the matter superfields

	A''	E	R	A'	P	A	E'	Φ	Q	ψ_i
SO(10)	45	54	1	45	1	45	54	210	1	16
Z_{24}	2	12	12	10	2	12	0	0	4	-1
Z_4	-1	0	0	1	1	2	0	0	0	0

	H_1	$\overline{h}$	h	H_2	H_3	D_1	$\overline{\Delta}_1$	Δ_1	D_2	$\overline{\Delta}_2$	Δ_2
SO(10)	10	10	10	10	10	120	$\overline{126}$	126	120	$\overline{126}$	126
Z_{24}	2	-4	4	8	6	2	2	2	-2	-2	-2
Z_4	0	-1	1	1	2	0	0	0	0	0	0

Table 2 Redefinitions of the couplings

Old	η'_1	η'_2	η'_3	η'_4	η'_5	η'_6	η'_7	η'_8	η'_9
New	$10\sqrt{6}\eta_1$	$10\sqrt{6}\eta_2$	η_3	$2\sqrt{30}\eta_4$	$2\sqrt{30}\eta_5$	$2\sqrt{30}\eta_6$	$2\sqrt{30}\eta_7$	η_8	$\frac{5\sqrt{2}}{2}\eta_9$
Old	η'_{10}	η'_{11}	η'_{12}	η'_{13}	η'_{14}	η'_{15}	λ'_1	λ'_2	
New	$\frac{5\sqrt{2}}{2}\eta_{10}$	$2\eta_{11}$	$\sqrt{5}\eta_{12}$	$\sqrt{5}\eta_{13}$	$\frac{3\sqrt{2}}{2}\eta_{14}$	$\frac{3\sqrt{6}}{2}\eta_{15}$	$\sqrt{6}\lambda_1$	$2\sqrt{2}\lambda_2$	

decay mediated by the color triplets of H_1, the last term in (7) is replaced by

$$A'H_2H_3 + \frac{1}{2}m_{H_3}H_3^2. \tag{8}$$

Consequently, H_1 mediated proton decay is fobidden, as can be seen in Sect. 3. In building realistic models, mass parameters can be replaced by VEVs of singlets and/or 54s.

When we use all the above superfields to build the model, the F-flatness conditions cannot be all consistent so that a new $E'(54)$ is introduced. All the superfields are summarized in Table 1. To avoid unwanted terms, we have enforced an extra symmetry $Z_{24} \times Z_4$ under which the transformation properties of all the particles are also listed in Table 1. Note that to generate the seesaw VEV $\sim Q$ through the Green–Schwarz mechanism, this discrete symmetry is the subgroup of the anomalous gauge U(1) groups [37], its symmetry breaking may not bring in the domain wall problem.

Then the full superpotential is

$$W^{\text{Higgs}} = W_{SB} + W_{D\Delta} + W_{\text{filter}} + W_{DW}, \tag{9}$$

where

$$W_{SB} = \frac{1}{2}m_\Phi\Phi^2 + \lambda'_1\Phi^3 + \lambda'_2E'\Phi^2 + \lambda'_3\Phi A^2$$
$$+ \frac{1}{2}m_{E'}E'^2 + \frac{1}{2}m_E E^2$$
$$+ \lambda'_4 E^2 E' + \lambda'_5 E'^3 + \frac{1}{2}m_A A^2 + \lambda'_6 E'A^2$$
$$+ \frac{1}{2}m_R R^2 + \lambda'_7 REE',$$
$$W_{D\Delta} = \left(\eta'_1\Phi + m_1\right)\overline{\Delta_1}\Delta_2 + \left(\eta'_2\Phi + m_2\right)\overline{\Delta_2}\Delta_1$$
$$+ \eta'_3 Q\Delta_2\overline{\Delta}_2 + \Phi D_1\left(\eta'_4\Delta_2 + \eta'_5\overline{\Delta}_2\right)$$

$$+ \Phi\left(\eta'_6\Delta_1 + \eta'_7\overline{\Delta_1}\right)D_2 + \eta'_8 QD_2^2$$
$$+ E'\left(\eta'_9\Delta_1\Delta_2 + \eta'_{10}\overline{\Delta_1\Delta_2}\right)$$
$$+ \Phi H_1\left(\eta'_{11}D_2 + \eta'_{12}\Delta_2 + \eta'_{13}\overline{\Delta_2}\right)$$
$$+ \left(m_D + \eta'_{14}E' + \eta'_{15}\Phi\right)D_1D_2,$$
$$W_{\text{filter}} = \kappa_1 PH_1\overline{h} + \left(\kappa_2 E' + m_h\right)h\overline{h} + \kappa_3 AhH_2$$
$$+ \kappa_4 A'H_2H_3$$
$$+ \frac{1}{2}(\kappa_5 R + \kappa_6 E)H_3^2.$$
$$W_{DW} = \xi_1 PAA' + (\xi_2 E + \xi_3 R)A'A''.$$

Here, the couplings with "'" follow the notations given in [16]. However, not all of them are normalized properly to be of order one. In Table 2, we redefine these couplings so that the unprimed couplings are of order one numerically.

Compared to [19], we have eliminated a reluctant 45 and its interactions. Although the superpotential (9) is complicated, it solves several major difficulties of SUSY SO(10) at the same time and thus can be taken as a prototype of realistic SUSY GUTs. The many representations used in building this model may bring in the question if they are allowed. To our knowledge, except the string model argument [49] based on perturbative study, there is no reason to exclude these representations in principle. Whether the model is minimal or not remains an open question.

3 The weak doublets and the color triplets

The mass matrix for the doublets can be read off from (9). To simplify the discussion, we neglect W_{filter} at first and consider its effects later. The mass matrix for the doublets is

$$M_D^{D\Delta} = \begin{pmatrix} 0_{6\times5} & A_{6\times5} \\ \hline B_{4\times5} & C_{4\times5} \end{pmatrix}, \tag{10}$$

where the columns are $(H_1^u, D_1^u, D_1'^u, \overline{\Delta}_1^u, \Delta_1^u; \Phi^u; \overline{\Delta}_2^u, \Delta_2^u, D_2^u, D_2'^u)$, and the rows are $(H_1^d, D_1^d, D_1'^d, \Delta_1^d, \overline{\Delta}_1^d; \Phi^d; \Delta_2^d, \overline{\Delta}_2^d, D_2^d, D_2'^d)$. The sixth row corresponds to Φ^d, and the first five entries in this row are proportional to v_2, which is zero. It is obvious that the upper-most six rows are not independent which combine into a massless eigenstate of H_d type, while the left-most five columns give a

$$a_{42} = m_2 + \frac{\eta_2\left(2\Phi_2 + \sqrt{2}\Phi_3\right)}{\sqrt{3}},$$

$$a_{61} = m_\Phi - \sqrt{\frac{3}{10}}E'\lambda_2 + \sqrt{3}\lambda_1\Phi_2 + \sqrt{\frac{3}{2}}\lambda_1\Phi_3,$$

and

$$B_{4\times5} = \begin{pmatrix} -\eta_{12}\left(\frac{\sqrt{2}}{2}\Phi_2 + \frac{1}{2}\Phi_3\right) & \frac{\eta_4\Phi_3}{2} & \eta_4\left(\frac{\Phi_1}{\sqrt{2}} + \frac{\Phi_3}{\sqrt{3}}\right) & b_{14} & \frac{1}{2}\sqrt{\frac{5}{6}}E'\eta_9 \\ \frac{1}{2}\eta_{13}\left(\sqrt{2}\Phi_2 - \Phi_3\right) & \frac{\eta_5\Phi_3}{2} & b_{23} & \frac{1}{2}\sqrt{\frac{5}{6}}E'\eta_{10} & b_{25} \\ -\eta_{11}\Phi_1 & b_{32} & \frac{\eta_{15}\Phi_3}{2\sqrt{2}} & \frac{\eta_7\Phi_3}{2} & \frac{\eta_6\Phi_3}{2} \\ -\frac{\eta_{11}\Phi_3}{\sqrt{2}} & \frac{\eta_{15}\Phi_3}{2\sqrt{2}} & b_{43} & \eta_7\left(\frac{\Phi_1}{\sqrt{2}} + \frac{\Phi_3}{\sqrt{3}}\right) & b_{45} \end{pmatrix},$$

H_u type massless eigenstate. Consequently, the massless doublets can easily be seen from (10),

$$H_u^0 = \alpha_u^1 H_1^u + \alpha_u^2 D_1^u + \alpha_u^3 D_1'^u + \alpha_u^4 \overline{\Delta}_1^u + \alpha_u^5 \Delta_1^u$$
$$H_d^0 = \alpha_d^1 H_1^d + \alpha_d^2 D_1^d + \alpha_d^3 D_1'^d + \alpha_d^4 \Delta_1^d$$
$$\qquad + \alpha_d^5 \overline{\Delta}_1^d + \alpha_d^6 \Phi_1^d \qquad (11)$$

satisfying the linear equations

$$(\alpha_d^1, \alpha_d^2, \alpha_d^3, \alpha_d^4, \alpha_d^5, \alpha_d^6)^* A_{6\times5} = 0,$$
$$B_{4\times5}(\alpha_u^1, \alpha_u^2, \alpha_u^3, \alpha_u^4, \alpha_u^5)^\dagger = 0 \qquad (12)$$

and the normalization conditions

$$1 = |\alpha_u^1|^2 + |\alpha_u^2|^2 + |\alpha_u^3|^2 + |\alpha_u^4|^2 + |\alpha_u^5|^2,$$
$$1 = |\alpha_d^1|^2 + |\alpha_d^2|^2 + |\alpha_d^3|^2 + |\alpha_d^4|^2 + |\alpha_d^5|^2 + |\alpha_d^6|^2. \qquad (13)$$

The explicit forms of $A_{6\times5}$ and $B_{4\times5}$ can be read off from the superpotential,

where

$$b_{14} = m_1 + \frac{\eta_1\left(2\Phi_2 + \sqrt{2}\Phi_3\right)}{\sqrt{3}},$$

$$b_{23} = \eta_5\left(\frac{\sqrt{2}}{2}\Phi_1 - \frac{\sqrt{3}}{3}\Phi_3\right),$$

$$b_{25} = m_2 + \frac{\eta_2\left(2\Phi_2 - \sqrt{2}\Phi_3\right)}{\sqrt{3}},$$

$$b_{32} = m_D + \frac{3}{2}\sqrt{\frac{3}{10}}E'\eta_{14},$$

$$b_{43} = m_D - \frac{E'\eta_{14}}{2\sqrt{30}} + \frac{\eta_{15}\Phi_2}{\sqrt{3}},$$

$$b_{45} = \eta_6\left(\frac{\sqrt{2}}{2}\Phi_1 - \frac{\sqrt{3}}{3}\Phi_3\right).$$

$$A_{6\times5} = \begin{pmatrix} -\eta_{13}\bar{v}_2 & a_{12} & \frac{1}{2}\eta_{12}\left(\sqrt{2}\Phi_2 - \Phi_3\right) & -\eta_{11}\Phi_1 & -\frac{\eta_{11}\Phi_3}{\sqrt{2}} \\ -\eta_5\bar{v}_2 & \frac{\eta_5\Phi_3}{2} & \frac{\eta_4\Phi_3}{2} & a_{24} & \frac{\eta_{15}\Phi_3}{2\sqrt{2}} \\ -\sqrt{3}\eta_5\bar{v}_2 & a_{32} & \frac{1}{6}\eta_4\left(3\sqrt{2}\Phi_1 - 2\sqrt{3}\Phi_3\right) & \frac{\eta_{15}\Phi_3}{2\sqrt{2}} & m_D - \frac{E'\eta_{14}}{2\sqrt{30}} + \frac{\eta_{15}\Phi_2}{\sqrt{3}} \\ \sqrt{6}\eta_2\bar{v}_2 & a_{42} & \frac{1}{2}\sqrt{\frac{5}{6}}E'\eta_9 & \frac{\eta_6\Phi_3}{2} & \eta_6\left(\frac{\Phi_1}{\sqrt{2}} + \frac{\Phi_3}{\sqrt{3}}\right) \\ 0 & \frac{1}{2}\sqrt{\frac{5}{6}}E'\eta_{10} & m_1 + \frac{\eta_1\left(2\Phi_2-\sqrt{2}\Phi_3\right)}{\sqrt{3}} & \frac{\eta_7\Phi_3}{2} & \frac{1}{6}\eta_7\left(3\sqrt{2}\Phi_1 - 2\sqrt{3}\Phi_3\right) \\ a_{61} & \sqrt{6}v_1\eta_2 & 0 & -v_1\eta_6 & -\sqrt{3}v_1\eta_6 \end{pmatrix},$$

where

$$a_{12} = -\frac{1}{2}\eta_{13}\left(\sqrt{2}\Phi_2 + \Phi_3\right),$$

$$a_{24} = m_D + \frac{3}{2}\sqrt{\frac{3}{10}}E'\eta_{14},$$

$$a_{32} = \eta_5\left(\frac{\Phi_1}{\sqrt{2}} + \frac{\Phi_3}{\sqrt{3}}\right),$$

Here we have used the fields to represent the VEVs of their SM singlets without introducing confusion, and $\Phi_{1,2,3}$ are the VEVs of Φ in the (1,1,1), (15,1,1) and (15,1,3) directions, respectively, under the $SU(4)_c \times SU(2)_L \times SU(2)_R$ subgroup.

Now we take into the effects of W_{filter}. Ordering both the columns and the rows as $(H_{u(d)}^0, \bar{h}, h, H_2, H_3)$ with $H_{u,d}^0$

given in (11), we have the doublet mass matrix

$$
M_D^{\text{filter}} = \begin{pmatrix} 0 & \alpha_d^1 \kappa_1 P & 0 & 0 & 0 \\ \alpha_u^1 \kappa_1 P & 0 & \kappa_2 E' + m_h & 0 & 0 \\ 0 & \kappa_2 E' + m_h & 0 & \kappa_3 A_1 = 0 & 0 \\ 0 & 0 & \kappa_3 A_1 = 0 & 0 & \kappa_4 A_1' \\ 0 & 0 & 0 & \kappa_4 A_1' & \kappa_5 R + \kappa_6 E \end{pmatrix},
$$

(14)

then we have a pair of massless eigenstates

$$
H_u = \frac{(\kappa_2 E' + m_h)H_u^0 - (\alpha_u^1 \kappa_1 P)h^u}{\sqrt{|\alpha_u^1 \kappa_1 P|^2 + |\kappa_2 E' + m_h|^2}},
$$

$$
H_d = \frac{(\kappa_2 E' + m_h)H_d^0 - (\alpha_d^1 \kappa_1 P)h^d}{\sqrt{|\alpha_d^1 \kappa_1 P|^2 + |\kappa_2 E' + m_h|^2}},
$$

(15)

which are the weak doublets in the MSSM. For P having a VEV of order Λ_{GUT} the components of $H_1^{u,d}$ in the MSSM doublet $H_{u,d}$ are not small, thus we can take (15) as pure normalization without significant numerical effects, and we will neglect these effects in the doublets to simplify our discussions.

In the absence of the effects from W_{filter}, there are two more pairs of color triplets from $\overline{\Delta}_{1,2} + \Delta_{1,2}$ comparing to the doublets, and the mass matrix for the triplets without the effects of W_{filter} is

$$
M_T^{D\Delta} = \begin{pmatrix} 0_{7\times 6} & A_{7\times 6} \\ B_{5\times 6} & C_{5\times 6} \end{pmatrix},
$$

(16)

where the columns are $(H_1^T, D_1^T, D_1'^T, \overline{\Delta}_1^T, \overline{\Delta}_1'^T, \Delta_1^T;$ $\Phi^T; \overline{\Delta}_2^T, \overline{\Delta}_2'^T, \Delta_2^T, D_2^T, D_2'^T)$, while the rows are similarly ordered. Again, there is a pair of massless triplets. We can re-write the mass matrix in (16) as

$$
M_T^{D\Delta} = \begin{pmatrix} 0_{6\times 6} & A'_{6\times 6} \\ B'_{6\times 6} & C'_{6\times 6} \end{pmatrix},
$$

(17)

so that only the upper-left sub-matrix may couple to the matter fields. The explicit forms of $A'_{6\times 6}$, $B'_{6\times 6}$ and $C'_{6\times 6}$ are

$$
A'_{6\times 6} = \begin{pmatrix} \eta_{13}\bar{v}_2 & a_{12} & -\sqrt{\frac{2}{3}}\eta_{13}\Phi_3 & a_{14} & -\frac{\eta_{11}\Phi_3}{\sqrt{3}} & -\sqrt{\frac{2}{3}}\eta_{11}\Phi_2 \\ -\sqrt{2}\eta_5\bar{v}_2 & \frac{\eta_5\Phi_3}{\sqrt{6}} & \sqrt{\frac{2}{3}}\eta_5\Phi_2 & \frac{\eta_4\Phi_3}{\sqrt{6}} & a_{25} & \frac{\eta_{15}\Phi_3}{\sqrt{6}} \\ \sqrt{2}\eta_5\bar{v}_2 & \frac{\eta_5\Phi_2}{\sqrt{3}} & \frac{\eta_5\Phi_3}{\sqrt{3}} & -\frac{\eta_4\Phi_2}{\sqrt{3}} & \frac{\eta_{15}\Phi_3}{\sqrt{6}} & a_{36} \\ -\sqrt{2}\eta_2\bar{v}_2 & m_2 & \frac{2\eta_2\Phi_3}{\sqrt{3}} & \sqrt{\frac{5}{6}}E'\eta_9 & \frac{\eta_6\Phi_3}{\sqrt{6}} & \frac{\eta_6\Phi_2}{\sqrt{3}} \\ -2\eta_2\bar{v}_2 & \frac{2\eta_2\Phi_3}{\sqrt{3}} & a_{53} & 0 & \sqrt{\frac{2}{3}}\eta_6\Phi_2 & \frac{\eta_6\Phi_3}{\sqrt{3}} \\ 0 & \sqrt{\frac{5}{6}}E'\eta_{10} & 0 & m_1 & \frac{\eta_7\Phi_3}{\sqrt{6}} & -\frac{\eta_7\Phi_2}{\sqrt{3}} \end{pmatrix},
$$

where

$$
a_{12} = -\frac{\eta_{13}\left(3\Phi_1 + \sqrt{3}\Phi_2\right)}{3\sqrt{2}},
$$

$$
a_{14} = \frac{\eta_{12}\left(-3\Phi_1 + \sqrt{3}\Phi_2\right)}{3\sqrt{2}},
$$

$$
a_{25} = m_D + \sqrt{\frac{2}{15}}E'\eta_{14} + \frac{\eta_{15}\Phi_1}{2},
$$

$$
a_{36} = m_D - \sqrt{\frac{3}{10}}E'\eta_{14} + \frac{\eta_{15}\Phi_2}{2\sqrt{3}},
$$

$$
a_{53} = m_2 + \eta_2\left(\Phi_1 + \frac{\Phi_2}{\sqrt{3}}\right),
$$

$$
B'_{6\times 6}
$$
$$
= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\eta_{12}\left(3\Phi_1 + \sqrt{3}\Phi_2\right)}{3\sqrt{2}} & \frac{\eta_4\Phi_3}{\sqrt{6}} & \frac{\eta_4\Phi_2}{\sqrt{3}} & m_1 & \frac{2\eta_1\Phi_3}{\sqrt{3}} & \sqrt{\frac{5}{6}}E'\eta_9 \\ -\sqrt{\frac{2}{3}}\eta_{12}\Phi_3 & \sqrt{\frac{2}{3}}\eta_4\Phi_2 & \frac{\eta_4\Phi_3}{\sqrt{3}} & \frac{2\eta_1\Phi_3}{\sqrt{3}} & b_{35} & 0 \\ \frac{\eta_{13}\left(-3\Phi_1 + \sqrt{3}\Phi_2\right)}{3\sqrt{2}} & \frac{\eta_5\Phi_3}{\sqrt{6}} & -\frac{\eta_5\Phi_2}{\sqrt{3}} & \sqrt{\frac{5}{6}}E'\eta_{10} & 0 & m_2 \\ -\frac{\eta_{11}\Phi_2}{\sqrt{3}} & b_{52} & \frac{\eta_{15}\Phi_3}{\sqrt{6}} & \frac{\eta_7\Phi_3}{\sqrt{6}} & \sqrt{\frac{2}{3}}\eta_7\Phi_2 & \frac{\eta_6\Phi_3}{\sqrt{6}} \\ -\sqrt{\frac{2}{3}}\eta_{11}\Phi_2 & \frac{\eta_{15}\Phi_3}{\sqrt{6}} & b_{63} & \frac{\eta_7\Phi_2}{\sqrt{3}} & \frac{\eta_7\Phi_3}{\sqrt{3}} & -\frac{\eta_6\Phi_2}{\sqrt{3}} \end{pmatrix},
$$

where

$$
b_{35} = m_1 + \eta_1\left(\Phi_1 + \frac{\Phi_2}{\sqrt{3}}\right),
$$

$$
b_{52} = m_D + \sqrt{\frac{2}{15}}E'\eta_{14} + \frac{\eta_{15}\Phi_1}{2},
$$

$$
b_{63} = m_D - \sqrt{\frac{3}{10}}E'\eta_{14} + \frac{\eta_{15}\Phi_2}{2\sqrt{3}},
$$

and

$$
C'_{6\times 6} = \begin{pmatrix} c_{11} & -\sqrt{2}v_1\eta_2 & -2v_1\eta_2 & 0 & -\sqrt{2}v_1\eta_6 & \sqrt{2}v_1\eta_6 \\ -\sqrt{2}\eta_1\bar{v}_1 & Q\eta_3 & 0 & 0 & 0 & 0 \\ -2\eta_1\bar{v}_1 & 0 & Q\eta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q\eta_3 & 0 & 0 \\ -\sqrt{2}\eta_7\bar{v}_1 & 0 & 0 & 0 & Q\eta_8 & 0 \\ \sqrt{2}\eta_7\bar{v}_1 & 0 & 0 & 0 & 0 & Q\eta_8 \end{pmatrix},
$$

where

$$
c_{11} = m_\Phi + \sqrt{\frac{2}{15}}E'\lambda_2 + \frac{1}{3}\lambda_1\left(3\Phi_1 + \sqrt{3}\Phi_2 + 2\sqrt{6}\Phi_3\right).
$$

In calculating proton decay rates mediated by the color-triplet Higgsinos, what is relevant is the effective mass matrix M^{Eff} which is got from the full mass matrix by integrating out those fields which do not couple with the matter fields. This effective mass matrix is $M^{\text{Eff}} = A'(C')^{-1}B'$. Proton decay amplitudes depend on the inverses of the eigenvalues of M^{Eff} so that small eigenvalues of C' are needed to suppress pro-

ton decay. $C'_{6 \times 6}$ contains five small eigenvalues which are not enough to generate six large eigenvalues for the effective mass matrix. This is cured by including the effects from W_{filter}, which gives

$$M_T^{\text{filter}} = \begin{pmatrix} 0 & \kappa_1 P & 0 & 0 & 0 \\ \kappa_1 P & 0 & \kappa_2 E' + m_h & 0 & 0 \\ 0 & \kappa_2 E' + m_h & 0 & \kappa_3 A_2 & 0 \\ 0 & 0 & \kappa_3 A_2 & 0 & \kappa_4 A'_2 = 0 \\ 0 & 0 & 0 & \kappa_4 A'_2 = 0 & \kappa_5 R + \kappa_6 E \end{pmatrix},$$

(18)

where the bases are $(H_1^T, \bar{h}^T, h^T, H_2^T, H_3^T)$ in the columns and similar for the anti-triplets in the rows. After integrating out the fields $\bar{h}^T, h^T, H_2^T, H_3^T$ and their conjugates, the effect is to give an infinite effective mass to $H_1^T H_1^{\overline{T}}$, or the (1,1) entry in $M_T^{D\Delta}$ of (17) is replaced by infinity. Now there are six large eigenvalues including an infinity in the effective triplet mass matrix which are supposed to be sufficient to suppress all proton decay amplitudes.

4 Determination of the parameters

In order to calculate the proton decay rates, we need to know the parameters in the color-triplet mass matrix. These parameters also appear in the weak doublet mass matrix which gives the two massless doublets of the MSSM and hence are linked to the matter masses and mixing. There are also constraints from the neutrino oscillations [17,50–62]. In the literature, however, since few people believe that the MSSM doublets are got through fine-tuning the doublet mass matrix, proton decays are calculated by simply adjusting parameters in the color-triplet mass matrix [17,63–67]. The fitting of the fermion masses and mixing can give constraints on the components of the weak doublets, but they are not linked to the color triplets in the absence of a realistic mechanism of naturally generating the weak doublets.

In the model of [19], the doublets are got without fine-tuning so that the parameters in the doublet and the triplet are closely related. Consequently, we need to consider the constraints from the doublets to determine the superpotential parameters. Instead of adjusting the superpotential parameters, then solving the weak doublets and requiring them to satisfy the low energy data, we find it is easy for the weak doublets to take their reasonable contents while the superpotential parameters are determined later. For those parameters unconstrained by the present data, we simply take them to be of order one as reasonable inputs.

Although there are several works on fitting the data, only in [17,61] the detailed results are presented. In [17] an unacceptably small component of 10_H in the MSSM doublet H_u is used so we will use the numerical results in [61]. The con-

straints on the contents of the MSSM doublets give the two ratios

$$r = \frac{\alpha_u^1}{\alpha_d^1}, \quad s = \frac{\alpha_u^4}{r \alpha_d^5}.$$

(19)

In [61] the results are given for $\tan \beta = 10, 38, 50$ for $10_H + \overline{126}_H$ coupling with the matter superfields, and $\tan \beta = 50$ only for $10_H + \overline{126}_H + 120_H$ coupling with the matter superfields. The former corresponds to taking $Y_{120} = 0$ in the latter case.

We take $\tan \beta = 10$ as an example which corresponds to $r = 13.1538$ and $s = 0.244325 + 0.0495071i$ in [61]. First, we input the reasonable contents of the MSSM doublets as

$$(\alpha_u^1, \alpha_u^4) = (0.8, 0.57)$$

(20)

which, together with Eq. (19), give

$$(\alpha_d^1, \alpha_d^5) = (0.0608189, 0.170365 - 0.0345208i).$$

(21)

Then we also use the reasonable inputs

$$(\alpha_u^2, \alpha_u^3) = (0.1, 0.1)$$

(22)

and

$$(\alpha_d^2, \alpha_d^3, \alpha_d^4) = (0.2, 0.5, 0.4).$$

(23)

From the normalization conditions (11), we have

$$\alpha_u^5 = 0.122882, \quad \alpha_d^6 = 0.718391.$$

(24)

Second, we require that all the GUT scale masses are of the order of 10^{16} GeV except Q, which is taken as $\sim 10^{14}$ GeV for the seesaw mechanism, and all the massless couplings are of order one. The massive parameters

$$m_D = 1.0, \quad m_1 = 0.6, \quad m_2 = 1.1, \quad m_\Phi = 1.2, \quad E' = 0.8,$$
$$\Phi_1 = -0.4, \quad \Phi_2 = -2, \quad \Phi_3 = -1.5, \quad v_1 = 0.5, \quad \bar{v}_2 = 0.5$$

(25)

are in units of 10^{16} GeV, where in the second line the VEVs are taken larger values due to normalizations. Note that in principal the VEVs in (25) should be determined through solving the F- and D-flatness conditions after all the superpotential parameters are fixed first. The vice versa case is also true, since there are extra free parameters in (9) not used in the numerical calculations.

Third, for the dimensionless couplings we take

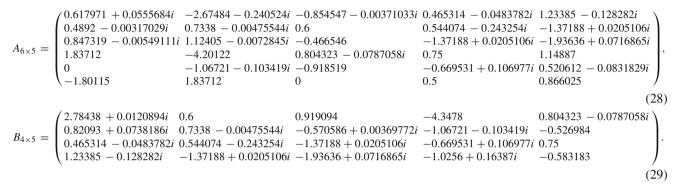$$\lambda_1 = 0.5, \quad \lambda_2 = 0.8, \quad \eta_3 = \eta_8 = 1.0$$

(26)

and

$$\eta_1 = 1.4, \ \eta_2 = 1.5, \ \eta_4 = -0.8, \ \eta_6 = -1.0. \qquad (27)$$

By solving the linear equations (12), we get

$$\eta_5 = -0.9784 + 0.00634058i, \quad \eta_7 = 0.892708 - 0.142636i, \quad \eta_9 = 2.20273 - 0.215545i, \quad \eta_{10} = -2.92267 - 0.283226i,$$
$$\eta_{11} = 1.16329 - 0.120945i, \quad \eta_{12} = 1.28655 + 0.00558605i, \quad \eta_{13} = -1.23594 - 0.111137i, \quad \eta_{14} = -0.693669 - 0.3701i,$$
$$\eta_{15} = 2.58683 - 0.0386752i.$$

Putting these parameters into the doublets, we get

$$A_{6\times5} = \begin{pmatrix} 0.617971 + 0.0555684i & -2.67484 - 0.240524i & -0.854547 - 0.00371033i & 0.465314 - 0.0483782i & 1.23385 - 0.128282i \\ 0.4892 - 0.00317029i & 0.7338 - 0.00475544i & 0.6 & 0.544074 - 0.243254i & -1.37188 + 0.0205106i \\ 0.847319 - 0.00549111i & 1.12405 - 0.0072845i & -0.466546 & -1.37188 + 0.0205106i & -1.93636 + 0.0716865i \\ 1.83712 & -4.20122 & 0.804323 - 0.0787058i & 0.75 & 1.14887 \\ 0 & -1.06721 - 0.103419i & -0.918519 & -0.669531 + 0.106977i & 0.520612 - 0.0831829i \\ -1.80115 & 1.83712 & 0 & 0.5 & 0.866025 \end{pmatrix}, \qquad (28)$$

$$B_{4\times5} = \begin{pmatrix} 2.78438 + 0.0120894i & 0.6 & 0.919094 & -4.3478 & 0.804323 - 0.0787058i \\ 0.82093 + 0.0738186i & 0.7338 - 0.00475544i & -0.570586 + 0.00369772i & -1.06721 - 0.103419i & -0.526984 \\ 0.465314 - 0.0483782i & 0.544074 - 0.243254i & -1.37188 + 0.0205106i & -0.669531 + 0.106977i & 0.75 \\ 1.23385 - 0.128282i & -1.37188 + 0.0205106i & -1.93636 + 0.0716865i & -1.0256 + 0.16387i & -0.583183 \end{pmatrix}. \qquad (29)$$

We also get $\bar{v}_1 = 0.0117188$ through (3) which gives masses to the right handed neutrinos.

Accordingly, for the color triplets, we have

$$A'_{6\times6} = \begin{pmatrix} -0.617971 - 0.0555684i & -1.35872 - 0.122177i & -1.51371 - 0.136114i & -0.686575 - 0.00298102i & 1.00744 - 0.104742i & 1.89964 - 0.197503i \\ 0.691833 - 0.00448347i & 0.599145 - 0.0038828i & 1.59772 - 0.0103541i & 0.489898 & 0.279999 - 0.100378i & -1.58411 + 0.0236836i \\ -0.691833 + 0.00448347i & 1.12976 - 0.00732148i & 0.847319 - 0.00549111i & -0.92376 & -1.58411 + 0.0236836i & -0.189559 + 0.184499i \\ -1.06066 & 1.1 & -2.59808 & 1.60865 - 0.157412i & 0.612372 & 1.1547 \\ -1.5 & -2.59808 & -1.23205 & 0 & 1.63299 & 0.866025 \\ 0 & -2.13442 - 0.206839i & 0 & 0.6 & -0.54667 + 0.0873464i & 1.03081 - 0.164702i \end{pmatrix}, \qquad (30)$$

$$B'_{6\times6} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1.41436 + 0.00614096i & 0.489898 & 0.92376 & 0.6 & -2.42487 & 1.60865 - 0.157412i \\ 1.5757 + 0.00684149i & 1.30639 & 0.69282 & -2.42487 & -1.57658 & 0 \\ 0.659566 + 0.0593086i & 0.599145 - 0.0038828i & -1.12976 + 0.00732148i & -2.13442 - 0.206839i & 0 & 1.1 \\ 1.34325 - 0.139656i & 0.279999 - 0.100378i & -1.58411 + 0.0236836i & -0.54667 + 0.0873464i & -1.45779 + 0.232924i & 0.612372 \\ 1.89964 - 0.197503i & -1.58411 + 0.0236836i & -0.189559 + 0.184499i & -1.03081 + 0.164702i & -0.773108 + 0.123526i & -1.1547 \end{pmatrix}, \qquad (31)$$

and

$$C'_{6\times6} = \begin{pmatrix} -0.5684 & -1.06066 & -1.5 & 0 & 0.707107 & -0.707107 \\ -0.0232019 & 0.01 & 0 & 0 & 0 & 0 \\ -0.0328125 & 0 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0 & 0 \\ -0.0147947 + 0.00236388i & 0 & 0 & 0 & 0.01 & 0 \\ 0.0147947 - 0.00236388i & 0 & 0 & 0 & 0 & 0.01 \end{pmatrix}. \qquad (32)$$

In choosing the numerical inputs above, we do not fine-tune any number besides solving the linear equations (12). Instead, we have imposed the constraints on the inputs that only few small or large matrix elements can exist and all the eigenvalues of the doublet and the triplet mass matrices are of order Λ_{GUT}. Then, no large splitting exists in the spectrum so that the GUT scale threshold effects are small. Consequently, the predictions on proton decay in the following are not tuned, which will be taken as these being natural estimations in the model [19].

5 The proton decay via dimension-five operator

In SUSY GUT models, proton decays are dominated by the baryon and lepton number violating operators of dimension-five,

$$W = C_L^{ijkl} Q^i Q^j Q^k L^l, \tag{33}$$

dressed mainly by the wino components of the charginos [12]. The coefficients are [14,17]

$$C_L^{ijkl} = \left(Y_{10}^{ij} \ Y_{126}^{ij} \ 0 \right) \left(M_C^{-1} \right) \begin{pmatrix} Y_{10}^{kl} \\ Y_{126}^{kl} \\ \sqrt{2} Y_{120}^{kl} \end{pmatrix}. \tag{34}$$

Here M_C is effective triplet mass matrix got by integrating out those fields which do not couple with Q or L in (17), (30)–(32). For $\tan \beta = 10$, we have

The dominant proton decay mode via the dimension-five operator with the Wino dressing diagram is $p \to K^+ \bar{\nu}$. The decay rate is approximately

$$\begin{aligned} \Gamma(p \to K^+ \bar{\nu}) &\simeq \Gamma(p \to K^+ \bar{\nu}_\tau) \\ &= \frac{m_p}{32\pi f_\pi^2} |\beta_H|^2 \times |A_L A_S|^2 \\ &\times \left(\frac{\alpha_2}{4\pi} \right)^2 \frac{1}{m_S^2} |C_L^{1123} - C_L^{1213} + \lambda (C_L^{1223} - C_L^{2213})|^2 \\ &\times 5.0 \times 10^{31} \ [\text{years}^{-1}/\text{GeV}], \end{aligned} \tag{37}$$

which gives the partial lifetime

$$\tau(p \to K^+ \bar{\nu}) = 3.88362 \times 10^{34} \ \text{years} \tag{38}$$

for $\tan \beta = 10$. In (37) we have used the MSSM and hadronic parameters taken from [17]. The above numerical predictions depend on the inputs (20)–(27). We have changed several VEVs by a factor of two around 2×10^{16} GeV, and we find that the proton partial lifetime varies in $10^{34} \sim 10^{36}$ years for $\tan \beta = 10$, consistent with the present data.

We also calculate the proton decay partial lifetimes with constraints given in [61],

$$\tau(p \to K^+ \bar{\nu}) = 5.52536 \times 10^{34} \ \text{years} \tag{39}$$

for $\tan \beta = 38$ and

$$\tau(p \to K^+ \bar{\nu}) = 6.85908 \times 10^{33} \ \text{years} \tag{40}$$

for $\tan \beta = 50$ with $10_H + \overline{126}_H$ giving fermion masses. As we can see, the proton decay can be suppressed even for large $\tan \beta$ and there is no obvious $\tan \beta$ dependence in the partial

$$(M_T^{\text{eff}})_{33} = \begin{pmatrix} \infty & -62.1295 - 90.5827i & -324.779 + 31.8573i \\ 164.562 - 5.04775i & -400.812 - 7.02252i & 133.955 - 12.2758i \\ -200.267 - 73.1462i & 411.089 + 161.251i & 525.038 + 62.3212i \end{pmatrix}. \tag{35}$$

It is obvious that proton decays are dominated by the contributions from $\overline{126}$ and/or 120. Then the decay rates are proportional to $|Y_{126}|^4$ or $|Y_{126} Y_{120}|^2$. These Yukawa couplings extracted from [61] are

lifetimes which are all consistent with the present lower limit 6.6×10^{33} years [68].

However, using the results by fitting the data with $10_H + \overline{126} + 120_H$ [61],

$$Y_{10} = \begin{pmatrix} 0.000180154 + 0.000194604i & 0.000928682 + 0.000572057i & 0.000667056 - 0.000681864i \\ 0.000928682 + 0.000572057i & 0.00559167 + 0.0000902319i & -0.00390991 - 0.0113164i \\ 0.000667056 - 0.000681864i & -0.00390991 - 0.0113164i & 1.02374 + 5.28625 \times 10^{-7}i \end{pmatrix}$$

and

$$Y_{126} = \begin{pmatrix} -0.000246286 - 0.000272057i & -0.00130341 - 0.000802886i & -0.000936218 + 0.000957003i \\ -0.00130341 - 0.000802886i & -0.00607135 - 0.00273387i & 0.00548759 + 0.0158826i \\ -0.000936218 + 0.000957003i & 0.00548759 + 0.0158826i & 0.00938846 - 0.0581996i \end{pmatrix}. \tag{36}$$

Table 3 Partial lifetimes of proton decay in years using different Higgs to fit fermion masses. Dimension-five operators with charged wino dressing are used only. Inputs (20)–(27) are used. The Yukawa couplings are taken from [61]

| Decay mode | Lower limit [68] | $10_H + \overline{126}_H$ | | | $10_H + \overline{126}_H + 120_H$ |
		$\tan \beta = 10$	$\tan \beta = 38$	$\tan \beta = 50$	$\tan \beta = 50$
$p \to K^+ \overline{\nu}$	6.6×10^{33}	3.88×10^{34}	5.53×10^{34}	6.86×10^{33}	3.60×10^{30}
$p \to K^0 e^+$	–	2.67×10^{39}	3.42×10^{38}	3.83×10^{37}	6.96×10^{35}
$p \to K^0 \mu^+$	6.6×10^{33}	1.01×10^{36}	1.35×10^{35}	1.60×10^{34}	2.69×10^{32}
$p \to \pi^+ \overline{\nu}$	3.9×10^{32}	2.55×10^{37}	2.85×10^{36}	2.67×10^{35}	2.71×10^{32}
$p \to \pi^0 e^+$	1.7×10^{34}	4.64×10^{40}	6.16×10^{39}	7.33×10^{38}	2.64×10^{36}
$p \to \pi^0 \mu^+$	7.8×10^{33}	1.75×10^{37}	2.43×10^{36}	3.06×10^{35}	1.02×10^{33}

$$\tau(p \to K^+ \overline{\nu}) = 3.59502 \times 10^{30} \text{ years,} \qquad (41)$$

which is much lower than the data [68]. This can be tracked back to the Yukwawa couplings given in [61]. Compared to the fitting without 120_H, all the entries in the Yukawa couplings Y_{126} and Y_{120} appearing in (34,37) are larger by one order of magnitude. Thus the results in this case is very difficult to understand, since without 120 the fitting is rather good except small values like m_e [53], thus 120 is probably playing a minor role in the fitting. Also, the top quark mass calculated using the results in [61] is generally larger than the input used by the same paper, which exhibits probably numerical inconsistency in [61].

There are also sub-dominant decay modes whose partial lifetimes are also calculated. The results are summarized in Table 3. Again, there are conflicts when 120_H is included to contribute to the fermion masses.

6 Summary

In this article we have examined the renormalizable SUSY SO(10) model [19]. Without any fine-tuning of the parameters, we have shown how to construct MSSM doublets, to determine the parameters of the model, and to predict on proton decay rates. We find that in the case using with $10_H + \overline{126}_H$ to fit fermion masses and mixing, proton decay lifetimes are consistent with the experiment. In the case using also 120_H to fit the data, proton decay too fast. However, we find the numerical results with 120_H may not be consistent, and independent check of the same study is highly called for.

As in all renormalizable SUSY GUT models, the large representations used in [19] contribute largely to the β-function of the GUT gauge coupling. Then the GUT gauge coupling blows up quickly above the unification scale and cause the non-perturbative problem. However, the universe was in the GUT symmetric phase at very high temperature in its very early stage. There occurred a phase transition and the GUT symmetry was broken when the universe was cooling down. However, this phase transition has only been well studied in very simple models in the perturbative region. Without definite conclusions on the phase transition, especially in the models in non-perturbative region, the running behavior of

the GUT gauge coupling before this phase transition may not be a real difficulty.

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