University of Kent

School of Economics Discussion Papers

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November 2011

KDPE 1107

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Abstract

In this paper we consider the option value of the environment employing a stochastic general equilibrium growth model. In our model, as in existing studies, because of irreversibility, the environment has significant real option value. However, unlike the existing literature, the value of the environment is endogenously determined in our general equilibrium setting. In our model, the elasticity of substitution between the environment and consumption not only has quantitative effects but also qualitative effects on the option value of the environment and the optimal allocation of land. We also show that the volatility of the exogenous shock process has quantitatively significant effects on the size of the option value which has important implications for the practical estimation of environmental option values.

KEYWORDS: Real option, environment, general equilibrium, elasticity of substitution, generalized isoelastic preference.

JEL CLASSIFICATION: Q38, G13, O13.

*[*] **Acknowledgements**: We thank Shiro Takeda for his comments at the semi-annual conference of the Japanese Economic Association in Kumamoto and we specially thank our colleagues Miguel Leon-Ledesma for his suggestion and Sylvain Barde for his technical advises.

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1 Introduction

Since one of the most important features of environmental economics is uncertainty and irreversibility (Pindyck, 2007) the application of real options is well established in the literature; to name a few, Arrow and Fisher (1974), Conrad (1997), Ulph and Ulph (1997), Bulte et al. (2002), Kassar and Lasserre (2004), Morgan et al. (2008), and Leroux et al. (2009). We add to this literature by investigating an optimal land conversion problem where reserved land (e.g., old growth forest) is converted to agricultural land. We explicitly take into account the irreversibility of land conversion in a simple stochastic growth model. The irreversibility of land conversion means that the government can convert the reserved land into agricultural land but it cannot reverse the process. Furthermore, due to stochastic technological growth, the future values of reserved land and agricultural land are uncertain. Hence, naturally and inevitably, the option value of unconverted reserved land plays the most important role in our analysis.

What is new in our study is that we investigate the real option in a general equilibrium framework. The most relevant papers to ours are Conrad (1997), Bulte et al. (2002) and Leroux et al. (2009) all of which analyze the option value of irreversible land conversion within a partial equilibrium setup. The benefit of constructing a general equilibrium model is that the (shadow) price of the reserved land is endogenized. In partial equilibrium models, to determine the value of the reserved land it is necessary to assume that the economic value of the service flow from the reserved land is given exogenously. In our model, by assuming that the exogenous shock emerges from the productivity of agricultural land, and that the quality of (the service flow from) the reserved land is unchanged, the shadow price of the reserved land is determined within the model. More specifically, we argue that an old forest that we see now is physically the same old forest a couple of centuries ago. What is really different now from the past is us. That is, as the productivity of farmland (or general alternative land use) improves, we become richer. And, as we become richer, we demand for more the environment (income effect). In other words, it is our way of perceiving the environment that generates the changes in the shadow price of the environment. This is very different from, say, computers; the value of computers has been changing rapidly, mainly because their quality has been indeed changing rapidly. The assumption taken in the existing partial equilibrium framework treats the value of the environment in a similar way to that of computers. Such a treatment can be preferable, for example, when we are motivated by the developments of the technology specialized to extract the economic values from the environment; on the contrary, what we try to capture is the change in the way of evaluating the environment as people become richer.

There are additional reasons for employing a general equilibrium model in this context. First, many decisions relating to environmental use are made at the national/regional level. Therefore, the government should not ignore general equilibrium feedbacks when it makes a decision of this kind. Second, we can exploit macroeconomic data and some technology concepts such as total factor productivity (TFP) in our estimation of the exogenous stochastic process. The partial equilibrium models typically estimate the value (or shadow price) of the environment as the exogenous stochastic process by using a proxy. For example, Conrad (1997) estimates the trend and volatility parameters of the value of a forest as a geometric Brownian motion (GBM) by using the numbers of visitors to the forest being examined. Bulte et al. (2002) also employ visitor numbers, in this case to Costa Rica. As noted by Forsyth (2000), this approach can be valid as long as benefits are broadly defined. In this paper, we use the TFP of the agricultural sector (agri-TFP) and TFP based on GDP (GDP-TFP); the former is motivated by the fact that farmland is the most important reason to convert reserved land, frequently forest, in many developing countries, while the latter is chosen because we can interpret agricultural land as general land use other than the reserved environment, which is perhaps more relevant to developed countries. Of course, although we refer to agricultural land in our theoretical model we do not need to interpret this construct literally as farmland in our empirical application.

In our model we show that the elasticity of substitution η between general consumption goods C and the service flow from the reserved land or environment R plays a key role not only quantitatively but also qualitatively. More specifically, if preferences are elastic ($\eta > 1$), then as society become richer, the optimal R decreases, while it increases if inelastic ($\eta < 1$). That is, the ultimate fate of the environment hinges on whether η is above or below 1. The key intuition behind this is that η represents the flexibility of the societal preferences in terms of the consumption-environment choice. In our model, the production of C requires farm land A as an input and its productivity W grows stochastically. On the one hand, the improvement of farmland productivity directly makes the conversion of R into A more attractive. On the other it also makes people richer, stimulating the demand for R. Intuitively, if people are flexible ($\eta > 1$), the reduction of the service flow from R is not very painful, since it can easily be compensated by additional C. Hence, the government optimally chooses more production of C by converting R into A. However, if preferences are not flexible ($\eta < 1$), the demand mix of C and R does not change very much. Since the demand for C does not increase very much as W improves, less A is necessary. In sum, while the shadow price of R always increases as W increases, the equilibrium level of R may increase or decrease, depending whether people feel strongly that the environment is irreplaceable ($\eta < 1$) or not ($\eta > 1$). This is essentially the same mechanism that generates Baumol's curse (Baumol, 1967), in which the key is the cost reduction effect of technological improvement. If the demand for a sector does not increase rapidly enough, as the productivity increases, the labour input for a sector decreases. Alternatively, we can frame this in terms of land use, the intensive margin (i.e., the productivity of W) increases stochastically, and if the demand for C does not increase rapidly enough the extensive margin (i.e., the land use for C) decreases.

The significance of η qualitatively is also in operation even without irreversibility. Although we examine η in depth because of its strong impact on the option value, its implication is more general. Indeed, the importance of η is not a new finding as it is well understood in the environmental literature (eg, Lopez, 1994, Rowthorn and Brown, 1999, and Heal, 2009) In addition, to ensure that there is no ambiguity in parameter interpretation that is caused by von Neumann and Morgenstern (vNM) type expected utility theory, we employ Epstein-Zin-Weil recursive preferences, (See Epstein and Zin 1989 and Weil 1990. See also Smith and Son, 2005 for an application in environmental economics) which are also known as generalized isoelastic preferences (GIE). Thus, in our model it is the elasticity of substitution between C and R, but not the elasticity of intertemporal substitution or the coefficient of relative risk aversion, that determines the fate of the environment.

Previewing our numerical results, we find that the effects of the option value quantitatively depends on the technology process. Specifically, the technology process has a small welfare effect when framed in relation to GDP-TFP, but its effects can be much larger with agri-TFP. This is simply because, as is often the case in many real option applications, the volatility of the stochastic process (relative to its trend growth rate) is quantitatively important in the model. In fact, compared to the trend growth rate, GDP-TFP exhibits relatively low levels of volatility while agri-TFP is much more volatile. Therefore, given relatively small share of the agricultural sector in developed countries, unless we assume some extreme parameter values, the option value of R is not very large. A similar result has previously been noted in the literature by Bulte et al. (2002) and others. Other parameters, such as the elasticity of intertemporal substitution θ and the coefficient of relative risk aversion γ also have significant effects on the model behavior, albeit to a lesser extent.

Another important result is that the extent of excessive land conversion can be fairly large even under GDP-TFP if the government ignores the option value of R. But, the combination of a large mistake in land allocation and a small welfare loss is due to the flat value function F with respect to R. That is, a flat value function implies that a large change in R (ΔR which is a mistake in this case) leads to only a small change in F (ΔF which is a welfare loss in this case), such that $\Delta F/\Delta R$ is small in absolute terms. Similarly, losing a small option value causes a small welfare loss (ΔF, since the value function is the sum of the non-option and option values), which is associated with a large mistake in land use (ΔR). This implies that it is important to choose which measure is used to evaluate the effects of a land conversion in practical policy terms: welfare loss or the physical extent of land conversion.

The plan of this paper is as follows. Section 2 introduces myopic and dynamic models and solves them. The former is effectively a static general equilibrium model without the irreversibility constraint; we use this as a benchmark to evaluate our full dynamic model. Also, the myopic model alone can illustrate how the value (shadow price) of the environment is endogenously determined and how η affects optimal land allocation. Section 3 presents and evaluates the numerical results of the dynamic model. Section 4 provides further discussions and Section 5 concludes.

2 Model

This section establishes and solves the dynamic and myopic models. For the dynamic model we explicitly analyze the irreversibility constraint of the reserved land R, while for the myopic model we eliminate it. For the former, Section 2.4 obtains the fundamental partial differential equation (PDE) by applying Ito's lemma (or the like) to the model, Section 2.5 finds the value function by integrating the fundamental PDE, and Section 2.7 calculates the barrier curve and the option value by using the boundary conditions. In the text presented, we mostly discuss the intuition based on the simple cases, because their analytical tractability delineates implications and model intuition more clearly. Extensive mathematical derivations have been placed in a technical appendix at the end of the paper.

2.1 Model Setup

We consider the optimal land conversion problem in a continuous time dynamic general equilibrium setting with GIE preferences. Within this setup we assume that the government maximizes the objective function (1) subject to constraints (2). That is,

$$\text{value function} \quad : \quad F_t\left(W_t, R_t\right) = \lim_{\Delta t \to 0} \max_{\nu_t} \left\{ U_t^{1-1/\theta} \Delta t + \frac{1}{1+\rho\Delta t} E_t \left[F_{t+\Delta t}^{1-\gamma} \right]^{\frac{1-1/\theta}{1-\gamma}} \right\}^{\frac{1}{1-1/\theta}} \tag{1a}$$

where

$$\text{flow utility} \quad : \quad U_t\left(C_t, R_t\right) = \left(C_t^{1-1/\eta} + \phi R_t^{1-1/\eta} \right)^{\frac{1}{1-1/\eta}} \tag{1b}$$

subject to

$$\text{total land available:} \quad 1 = A_t + R_t \tag{2a}$$

$$\text{agricultural production:} \quad Y_t = W_t A_t \tag{2b}$$

$$\text{resource constraint:} \quad Y_t = C_t + \kappa W_t v_t \tag{2c}$$

$$\text{technology growth:} \quad \Delta W_t = \tilde{\alpha} W_t \Delta t + \tilde{\sigma} W_t \Delta w_t \tag{2d}$$

$$\text{land conversion rate:} \quad \Delta A_t = -\Delta R_t = \nu_t \Delta t \geq 0 \tag{2e}$$

where Δt is an infinitesimally short time duration. The value function F_t is essentially the present value (PV) of the representative household's current and future flow utility U_t with discount rate ρ; here we formulate it in the recursive form. The government's choice is the optimal land conversion rate ν_t (1a). U_t is increasing in general consumption goods C_t and the service flow from the environment R_t (1b). Parameters $\eta > 0$, $\theta > 0$ and $\gamma > 0$ are the elasticity of intratemporal substitution between C_t and R_t, the elasticity of intertemporal substitution, and the coefficient of relative risk aversion, respectively. If $\theta = 1/\gamma$, this GIE formulation reduces to the expected utility model of vNM. By using GIE specification we can disentangle θ and γ, which are often regarded as economically different objects. Note that ϕ is the relative importance of the service flow from R_t, and it also absorbs the difference in measurement units.

Without loss of generality, the total land mass is normalized to be one (2a), which can be used as agricultural land A_t or as reserved environment R_t (e.g., old growth forest, conservation reserves, etc.). Output Y_t is produced by a linear technology (2b), in which the only production factor is A_t, and its productivity W_t follows a GBM (2d). The only source of uncertainty in our model is the shock $\mathrm{d}w_t$ to the technology, where $\tilde{\alpha}$ and $\tilde{\sigma}$ are the trend growth rate of W_t and its volatility, respectively. Output is consumed or used as land conversion cost $\kappa W_t v_t$ (2c), where we assume that the land conversion cost is proportional to conversion rate ν_t and the marginal cost of land conversion κW_t is linearly increasing in W_t, where $\kappa > 0$ is a parameter.[1]

[1]Often, the land conversion cost (per unit production capacity of the farmland) is supposed to be increasing over time because people start converting the most profitable land (i.e., cheap conversion cost relative to its production capacity) then gradually move to less profitable land. Though we do not model this sort of land heterogeneity explicitly, our assumption tries to capture this increasing marginal cost of land conversion. However, in our numerical experiments, we set $\kappa = 0$ since perhaps the conversion cost is anyway negligible (see Bulte et al. 2002 for this). This

We assume that land conversion is an irreversible decision (2e); $-\mathrm{d}R_t \geq 0$ implies that the government can reduce R_t but cannot increase it. We can restate this as $\nu_t \geq 0$. It is known that, since there is no upper bound for the conversion speed in our model, the optimal land conversion is either (a) convert no land ($\nu_t = 0$) if $R_t < R_*(W_t)$ or (b) jump[2] immediately to optimal $R_*(W_t)$ otherwise, where optimal land allocation $R_*(W_t)$ changes as technology changes, and hence it is a function of W_t. Since curve $R_*(W_t)$ demarcates the state space into conversion and non-conversion regions and the optimal locus "closely" follows $R_*(W_t)$, this type of optimal control problem is referred to as barrier control.[3] Preview Figure 3 to see how the barrier curve divide the state space.

2.2 Short Form

For simplicity, we substitute out several variables; the following short form notation is equivalent to the above model (1) and (2). All derivations in this section are based on this formulation.

$$F_t = \lim_{\Delta t \to 0} \max_{v_t} \left\{ \left(Z_t \left(1 - R_t - \kappa v_t\right)^{1-1/\eta} + \phi R_t^{1-1/\eta} \right)^{\frac{1-1/\theta}{1-1/\eta}} \Delta t + \frac{1}{1+\rho \Delta t} E_t \left[F_{t+\Delta t}^{1-\gamma} \right]^{\frac{1-1/\theta}{1-\gamma}} \right\}^{\frac{1}{1-1/\theta}} \quad (3a)$$

$$\Delta Z_t = \alpha Z_t \Delta t + \sigma Z_t \Delta w_t \quad (3b)$$

$$\Delta A_t = -\Delta R_t = v_t \Delta t \geq 0 \quad (3c)$$

where $\alpha = (1 - 1/\eta)\left(\tilde{\alpha} - \tilde{\sigma}^2/2\eta\right)$, $\sigma^2 = (1 - 1/\eta)^2 \tilde{\sigma}^2$ and $Z_t = W_t^{1-1/\eta}$. We always assume that $\alpha < \rho$, otherwise the value function $F_t(Z_t, R_t)$ explodes. The state variables in this formulation are transformed technology Z_t and reserved land R_t, and the only choice variable is ν_t. Since there is a one-to-one relationship between technology W_t and its transformation Z_t, as a function argument, we use them interchangeably in generic functions; e.g., we use both $F_t(W_t, R_t)$ and $F_t(Z_t, R_t)$.

In the special case where $\theta = 1/\gamma$, it is obvious that (3a) reduces to vNM additively time separable formulation. Recognizing that preference are invariant against any positive monotonic transform of F_t, we define $V_t = F_t^{1-1/\theta}/(1 - 1/\theta)$. Replacing Δt with $\mathrm{d}t$, (3a) can be rewritten as follows.

$$V_t(Z_t, R_t) = \max_{v_t} \frac{1}{1-1/\theta} \int_t^\infty e^{-\rho t} \left(Z_t \left(1 - R_t - \kappa v_t\right)^{1-1/\eta} + \phi R_t^{1-1/\eta} \right)^{\frac{1-1/\theta}{1-1/\eta}} \mathrm{d}t \quad (4a)$$

$$\mathrm{d}Z_t = \alpha Z_t \mathrm{d}t + \sigma Z_t \mathrm{d}w_t \quad (4b)$$

$$\mathrm{d}A_t = -\mathrm{d}R_t = v_t \mathrm{d}t \geq 0 \quad (4c)$$

assumption is also necessary to facilitate the comparability between dynamic and myopic models.

[2] To be precise, R_t does not jump, if we define jump as a discontinuous movement. The movement of R_t is continuous but non-differentiable. Its movement is non-differentiable because it is too zigzag. In a sense, it exhibits successive "twitches". This fact comes from our assumption that the stochastic term in GBM is a Wiener process. Knowing this, we still (ab)use the word "jump" to depict such a continuous but zigzag movement in R_t.

[3] The real option models can be classified into three types based on the way of exercising the option. For land conversion models, Conrad (1997) assumes that all land must be converted from R_t to A_t at once; in his model, since the possibility of partial land conversion is excluded, essentially it reduces to the optimal stopping model (i.e., only the timing of the conversion matters). The other two allow the government to convert a portion of land; Leroux et al. (2009) assumes that there is a maximum speed of land conversion, which is often called bang-bang model, while R_t can jump to the optimal level immediately in Bulte et al. (2002), which is a barrier control model like ours. For the latter, unlike the former, R_t jumps to the barrier curve, because there is no upper limit of the conversion speed.

2.3 Myopic Case

First we consider the model without irreversibility. Since this specification omits the irreversibility constraint, we call this the myopic model. However, the solution to the myopic model per se is rational within its framework. The myopic model provides a useful benchmark in evaluating the option value of R_t, as the dynamic version inherits many of the properties from it. Importantly, the myopic version reveals that (a) the shadow price of R_t is increasing in W_t, and (b) η determines the ultimate fate of R_t, indicating that these two results hold even without option values being considered. Assuming no conversion cost ($\kappa = 0$), the myopic model becomes static (or a sequence of static models) such that the optimization problem reduces to the maximization of the flow utility U_t

$$\max_R \left(C_t^{1-1/\eta} + \phi R_t^{1-1/\eta}\right)^{\frac{1}{1-1/\eta}} \tag{5}$$

$$\text{s.t. } C_t = W_t \left(1 - R_t\right) \tag{6}$$

The solution is given by

$$R_m \left(W_t\right) = \frac{\phi^\eta}{W_t^{\eta-1} + \phi^\eta} \tag{7}$$

$$P_t = -\frac{U_{R,t}}{U_{C,t}} = \phi \left(\frac{R_m \left(W_t\right)}{C_t}\right)^{\frac{-1}{\eta}} = \phi \left(\frac{R_m \left(W_t\right)}{W_t \left(1 - R_m \left(W_t\right)\right)}\right)^{\frac{-1}{\eta}} = W_t \tag{8}$$

where $R_m \left(W_t\right)$ is the optimal choice under the myopic assumptions. We define the shadow price P_t of the environment to be the slope of the indifference curve in equilibrium. Regardless of the parameter values, P_t is increasing in W_t; indeed, $P_t = W_t$ in this simple case. In contrast the optimal $R_m \left(W_t\right)$ can be both increasing and decreasing in W_t depending on η. That is, if $\eta < 1$ the optimal reserved land increases as technology improves, such that at the limit $t \to \infty$ all land is used as the reserved environment. However, if $\eta > 1$, all land is converted into A_t. The broken lines in Figure 3 shows the optimal land allocation for the myopic case.

2.4 Fundamental PDE

Here, we obtain the fundamental PDE from the model. In the vNM time separable case (4), we can simply apply Ito's lemma to obtain (9) i.e., if $\theta = 1/\gamma$, the solution techniques employed are in keeping with Dixit and Pindyck (1994). However, for GIE preferences (3), to obtain (9), we follow Svensson (1989), which is shown in the Appendix. The application of his method within environmental economics can be found in Epaulard and Pommeret (2003) and Smith and Son (2005) for example.

Redefining the value function as $V = F^{1-1/\theta} / \left(1 - 1/\theta\right)$, we find the following fundamental PDE:

$$\rho V = \frac{U^{1-1/\theta}}{1 - 1/\theta} + \alpha V_Z Z + \frac{\gamma \theta \sigma^2}{2} V_{ZZ} Z^2 - V_R v_* \tag{9}$$

where the lower subscripts show partial derivatives; $V_Z = \partial V/\partial Z$, $V_{ZZ} = \partial^2 V/\partial Z^2$ and so on.[4] To obtain the fundamental PDE for the vNM formulation, simply let $\theta = 1/\gamma$. Optimal conversion rate v_* can be zero or positive. Intuitively, ν_* is positive only if R is larger than the optimal level $R_*(Z)$. In this case, the government finds it optimal to change R to $R_*(Z)$ immediately. Similarly, it is zero if the current R is smaller than $R_*(Z)$. In this case, the government has no choice other than keeping the current level of R.

To further understanding we have five specific comments. First, since V is a monotonically increasing transformation of F, preferences are invariant against this transformation.[5] Second, the role of γ in our framework is to rescale the risk measure σ and nothing more than that. This immediately implies that γ cannot cause any *qualitative* change, in the sense that we can always reparameterize σ to eliminate γ, although it can have *quantitatively* important implications, which we will discuss later. Third, while σ is *objective* risk which measures the standard deviation of the technology shock as it is, we can interpret $\sqrt{\theta\gamma}\sigma$ as the *subjective* risk measure, which captures the idea that different people perceive one unit of risk differently, depending on their risk tolerance $1/\gamma$. Fourth, θ also affects the subjective risk measure. As Epaulard and Pommeret (2003) point out, θ also can be interpreted as a fluctuation aversion parameter. Subsequently we numerically show that the role of θ is both quantitatively and qualitatively quite similar to that of γ. Although θ also appears in the first term in (9), its effects are small. Fifth, it is obvious that, for the vNM formulation, there is no gap between objective and subjective risk measures.

2.5 Non-Conversion Region ($v_* = 0$)

In the region where $R < R_*(Z)$, the government does not convert the land; $v_* = 0$. In this region, the PDE (9) is not really a PDE but an ordinary differential equation (ODE). Using superscript 0 on V^0 to indicate the value function that is defined in the non-conversion region, we can now re-express equation (9) as:

$$\rho V^0(Z,R) = \alpha V_Z^0 Z + \frac{\gamma\theta\sigma^2}{2} V_{ZZ}^0 Z^2 + \frac{\left(Z(1-R)^{1-1/\eta} + \phi R^{1-1/\eta}\right)^{\frac{1-1/\theta}{1-1/\eta}}}{1-1/\theta} \tag{10}$$

Solving (i.e., integrating) ODE (10), we obtain (11) below. It is quite easy to confirm that (11a) is indeed the solution to (10) by checking that the derivatives of V satisfy (10). Unfortunately, the solution includes the nuisance integrals, though we can have a more simple solution form in the special case with $\theta = \eta$. Indeed, when $\theta = \eta$ (11b), the solution techniques employed below are

[4] From this point onward we drop time subscripts, such that all uppercase letters are time varying and lowercase Greek letters are time invariant parameters. The only exceptions are choice variable ν and Wiener process dw which do not appear below.

[5] Note that, for flow utility U, it is only increasing linear transformations that preserve the preference. Note that U is included in the expectation operator, implying it is stochastic, while V (or equivalently F) includes the expectation which per se is non-stochastic.

standard and in keeping with those explained in Dixit and Pindyck (1994, Ch11).

$$V^0(Z,R) = B(R)Z^\beta + G(Z,R) \tag{11a}$$

$$= B(R)Z^\beta + \frac{Z}{\rho-\alpha}\frac{(1-R)^{1-1/\eta}}{1-1/\eta} + \frac{\phi}{\rho}\frac{R^{1-1/\eta}}{1-1/\eta} \qquad \text{if } \theta=\eta \tag{11b}$$

where

$$G(Z,R) = G^0(Z,R) - G^1(Z,R)$$

$$G^0(Z,R) = \frac{Z^{\beta_0}}{\gamma\theta\sigma^2 a_2}\int^Z \frac{\left(\tilde{Z}(1-R)^{1-1/\eta} + \phi R^{1-1/\eta}\right)^{\frac{1-1/\theta}{1-1/\eta}}}{\tilde{Z}^{1+\beta_0}(1-1/\theta)}\mathrm{d}\tilde{Z}$$

$$G^1(Z,R) = \frac{Z^{\beta}}{\gamma\theta\sigma^2 a_2}\int^Z \frac{\left(\tilde{Z}(1-R)^{1-1/\eta} + \phi R^{1-1/\eta}\right)^{\frac{1-1/\theta}{1-1/\eta}}}{\tilde{Z}^{1+\beta}(1-1/\theta)}\mathrm{d}\tilde{Z}$$

$$\beta = a_1 + a_2 > 1 \quad \text{and} \quad \beta_0 = a_1 - a_2 < 0 \quad \text{with } a_1 = \frac{1}{2} - \frac{\alpha}{\gamma\theta\sigma^2} \text{ and } a_2 = \sqrt{a_1^2 + \frac{2\rho}{\gamma\theta\sigma^2}}$$

where $\tilde{Z}$ and Z indicate that we take integral only with respect to the former. We can provide some intuition regarding the above results.

First, in general, for this class of ODE (second order linear ODE), the characteristic polynomial of (10) has two roots β and β_0. One is greater than one and the other is negative. There are two elementary solutions corresponding to them (Z^β and Z^{β_0}), and the general solution is the linear combination of these two elementary solutions and particular solution G. Also, to eliminate term with Z^{β_0}, we have already used one boundary condition, which we discuss in Section 2.7.

Second, particular solution G can be interpreted as the value of land if the government does not change the land allocation from the current combination. This is particularly clear if $\theta = \eta$, for which the last two terms can be rewritten as

$$\int_0^\infty e^{-\rho t}\left\{\frac{Z(t)(1-R)^{1-1/\eta}}{1-1/\eta}\right\}\mathrm{d}t = \frac{1}{\rho-\alpha}\frac{(WA)^{1-1/\eta}}{1-1/\eta} = \frac{1}{\rho-\alpha}\frac{C^{1-1/\eta}}{1-1/\eta} \tag{12}$$

$$\int_0^\infty e^{-\rho t}\left\{\phi\frac{R^{1-1/\eta}}{1-1/\eta}\right\}\mathrm{d}t = \frac{\phi}{\rho}\frac{R^{1-1/\eta}}{1-1/\eta} \tag{13}$$

These shows the PVs of A and R, given fixed A and R.[6] Interestingly, without adding any special assumptions, our model naturally has two different effective discount rates: $\rho-\alpha$ for A and ρ for R. We will discuss this further, in relation to environmental discounting in Section 4.

[6] This is the so-called discounted dividend model (DDM) in accounting. The PV of an eternal constant dividend flow D is

$$\int_0^\infty e^{-\rho t}D\mathrm{d}t = \frac{D}{\rho}$$

If the dividend grows at the rate of α, $D(t) = e^{\alpha t}D$ where $D = D(0)$ is the dividend at $t=0$. Then, it follows that

$$\int_0^\infty e^{-\rho t}D(t)\,\mathrm{d}t = \int_0^\infty e^{-\rho t}e^{\alpha t}D\mathrm{d}t = \int_0^\infty e^{-(\rho-\alpha)t}D\mathrm{d}t = \frac{D}{\rho-\alpha}$$

Third, the remaining term $B(R)Z^{\beta} > 0$ represents the value of future possible land conversion, i.e., the option value. We will soon determine the functional form of $B(R)$ by using free boundary conditions, but for now it should be regarded as an anonymous function of R. Note, the option value of future land conversion is zero in the conversion region, in which the government converts R (i.e., exercises the real option) immediately and hence the remaining option value is zero.

Fourth, related to the third point, remember that often option values are explained as follows; *"If a decision is irreversible and the situation is uncertain, even though the net present value of an irreversible decision is positive, it may still be better to wait for the arrival of new information, rather than making such an irreversible decision right now."* This can be understood as follows in our case. Suppose for a while there are only two possible choices R_1 and R_2 where $R_1 < R_2$. Choosing R_2 is keeping an option in the sense that it is still possible to move to R_1 at some point in the future, while choosing R_1 is exercising an option in the sense that it is no longer possible to return back to R_2. In this case, even if the present value of having R_1 forever is higher than that of having R_2 forever (i.e., $dif > 0$ below), choosing R_1 may still not be optimal.

$$dif = \left\{ \frac{Z}{\rho - \alpha} \frac{(1-R_1)^{1-1/\eta}}{1-1/\eta} + \frac{\phi}{\rho} \frac{R_1^{1-1/\eta}}{1-1/\eta} \right\} - \left\{ \frac{Z}{\rho - \alpha} \frac{(1-R_2)^{1-1/\eta}}{1-1/\eta} + \frac{\phi}{\rho} \frac{R_2^{1-1/\eta}}{1-1/\eta} \right\}$$

This is precisely because of the difference in the option values; $B(R_1)Z^{\beta} < B(R_2)Z^{\beta}$. That is, since Z is stochastic, the government wants to wait for the arrival of new information about Z until the dif becomes large enough to make sure that the optimal choice is moving to R_1.

2.6 Conversion Region ($\nu_* > 0$)

In the conversion region ($R > R_*(Z)$), since R jumps to R_* immediately, the total value of land is the value of the optimal land allocation at the barrier curve $R_*(Z)$ minus the conversion cost (measured in utility terms). By letting F^1 be the value function for the conversion region, we have

$$\begin{aligned} V^1(Z,R) &= V^0(Z,R_*) - \text{Conversion Cost} \\ &= V^0(Z,R_*) - \kappa \int_R^{R_*(Z)} \Phi_\nu(Z,\mathrm{d}R) && \text{(14a)} \\ &= V^0(Z,R_*) + \kappa Z \left\{ \frac{(1-R)^{1-1/\eta}}{1-1/\eta} - \frac{(1-R_*(Z))^{1-1/\eta}}{1-1/\eta} \right\} \quad \text{if } \theta = \eta && \text{(14b)} \end{aligned}$$

where $\kappa\Phi_\nu(Z,R) < 0$ is the marginal value loss of land conversion, and $\Phi_\nu(Z,R)$ satisfies

$$\Phi_\nu(Z,R) = -\left(Z(1-R)^{1-1/\eta} + \phi R^{1-1/\eta}\right)^{\frac{1/\eta-1/\theta}{1-1/\eta}} Z(1-R)^{-1/\eta} < 0 \qquad (15)$$

$$\Phi(Z,R) = \frac{\left(Z(1-R)^{1-1/\eta} + \phi R^{1-1/\eta}\right)^{\frac{1-1/\theta}{1-1/\eta}}}{1-1/\theta} = \frac{\Psi(Z,R)^{\frac{1-1/\theta}{1-1/\eta}}}{1-1/\theta} \qquad (16)$$

Intuitively, $\Phi_\nu(Z,R)$ is the derivative of flow utility with respect to ν; i.e., Φ_ν is the marginal utility loss of land conversion cost. The total conversion cost is the integral of the marginal utility loss from current R to optimal $R_*(Z)$.

2.7 Boundary Conditions

We now consider our boundary conditions. In general, each real option problem shares the same or similar fundamental PDE as (9), but it is boundary conditions that characterize each of them. In this model, we have five boundary conditions; among them, the main boundary conditions are three free boundary conditions, which are imposed along the barrier curve $R_*(Z)$. The adjective "free" implies that the position of the boundary (the barrier curve in this case) is determined endogenously. We first explain the three free boundary conditions, and, after deriving key expressions from them, we then discuss the other two boundary conditions.

[Figure 1: Boundary Conditions around here]

2.7.1 Free Boundary Conditions

There are three conditions imposed on the free boundary $R_*(Z)$:

◊ *Boundary Condition 1:* The level matching condition (LM) implies that the values of F^0 and F^1 must be the same on the barrier curve. That is,

$$V^0(Z,R_*) = V^1(Z,R_*) \tag{LM}$$

We have already used this to derive V^1. Obviously (14a) implies that the (LM) holds when $R = R_*(Z)$.

◊ *Boundary Condition 2:* Following Dixit and Pindyck (1994), we use the value matching (VM) condition to signify the first order optimality condition (with respect to ν): $0 = V_R^0(Z,R_*) + \kappa\Phi_\nu(Z,R_*)$. Or equivalently,

$$\begin{aligned} B_R(R_*)Z^\beta + G_R(Z,R_*) &= -\kappa\Phi_\nu(Z,R_*) && \text{(VM)} \\ B_R(R_*)Z^\beta - \frac{Z}{\rho-\alpha}(1-R_*)^{-1/\eta} + \frac{\phi}{\rho}R_*^{-1/\eta} &= \kappa Z(1-R_*)^{-1/\eta} \text{ if } \theta=\eta && \text{(VM')} \end{aligned}$$

Heuristically, the left hand side of (VM) shows the marginal gain of converting one more unit of R to A and its right hand side shows the marginal cost of land conversion. If the government decides to convert the land, the marginal benefit must be equated to the marginal cost at the optimum (i.e., at $R = R_*(Z)$). If the left hand side is greater than the right hand side, then more land should be converted, and vice versa.

◊ *Boundary Condition 3:* The smooth pasting condition (SP) must also hold: $0 = F_{RZ}^0(Z,R_*) +$

$\Phi_{\nu Z}(Z, R_*)$, or

$$\beta B_R(R_*) Z^{\beta-1} + G_{RZ}(Z, R_*) = -\kappa \Phi_{\nu Z}(Z, R_*) \quad \text{(SP)}$$

$$\beta B_R(R_*) Z^{\beta-1} - \frac{1}{\rho-\alpha}(1-R_*)^{-1/\eta} = \kappa (1-R_*)^{-1/\eta} \text{ if } \theta = \eta \quad \text{(SP')}$$

This is the first derivative of (VM) with respect to Z. See Dixit and Pindyck (1994) for the intuition behind the smooth pasting condition.

2.7.2 Barrier Curve $R_*(Z)$

We now can obtain the barrier curve R_* as a function of Z (or equivalently W), by eliminating the unknown function $B_R(R_*)$ from (VM) and (SP). In general, we can only implicitly determine R_* as a solution to (17a), which can be solved only numerically.

$$0 = \beta G_R(Z, R_*) - Z G_{RZ}(Z, R_*) + \kappa \{\beta \Phi_\nu(Z, R_*) - Z \Phi_{\nu Z}(Z, R_*)\} \quad \text{(17a)}$$

$$0 = \beta \frac{\phi R_*^{-1/\eta}}{\rho} - (\beta-1)\frac{Z(1-R_*)^{-1/\eta}}{\rho-\alpha} - \kappa(\beta-1) Z (1-R_*)^{-1/\eta} \quad \text{if } \theta = \eta \quad \text{(17b)}$$

But, if $\theta = \eta$ (17b), we can express R_* explicitly.

$$R_* = R_*(Z) = \frac{\phi_2^\eta}{Z^\eta + \phi_2^\eta} = \frac{\phi_2^\eta}{W^{\eta-1} + \phi_2^\eta} \quad \text{if } \theta = \eta \quad (18)$$

$$\text{where } \phi_2 = \frac{\beta}{\beta-1}\frac{\phi/\rho}{\frac{1}{\rho-\alpha}+\kappa} \quad \left(= \frac{\beta}{\beta-1}\frac{\rho-\alpha}{\rho}\phi > \phi \text{ if } \kappa = 0\right)$$

To understand this result, consider the result when $\theta = \eta$ and assume $\kappa = 0$. First, compare (18) with the myopic solution (7). The functional forms are clearly identical except for the difference between ϕ_2 and ϕ. Hence, the qualitative nature of the barrier curve crucially hinges on η as the myopic choice does (see Figure 3). Second, it can be shown that $\phi_2 > \phi$, if κ is small enough.[7] Hence, if the government ignores the irreversibility constraint, it always converts too much reserve into agricultural land; i.e., $R_*(Z) > R_m(Z)$ for any Z. Third, excessive land conversion $R_* - R_m$ is larger when the ratio $\phi_2/\phi > 1$ is larger. It is easy to show that, if $\kappa = 0$, ϕ_2/ϕ is a function of only α/σ^2 and ρ/σ^2, and it is decreasing in both α/σ^2 and ρ/σ^2. That is, ϕ_2/ϕ is larger if the volatility σ is larger relative to α and ρ. This confirms our natural conjecture; i.e., gap $R_* - R_m$ is larger when the technology is more volatile. Hence, it is not surprising that the option value tends to be larger with more uncertainty. Although it is hard to show algebraically, our numerical examinations demonstrate that these observations hold for the general case (17a) for reasonable parameter ranges.

[7] To show this exactly holds for $\kappa = 0$, first apply l'Hospital's rule to show $\phi_2 > 1$ if $\rho = \alpha$ ($\phi_2 \to 1$ as $\alpha = \rho \to \infty$), and then show that ϕ_2 is increasing in ρ, given α. Note if $\rho > \alpha$ then a meaningful solution exists.

2.7.3 Option Value $B(R)Z^{\beta}$

To find the option value $B(R)Z^{\beta}$, we first eliminate $Z(R_*)$ from (VM) and (SP) to obtain $B_R(R_*)$ (20a), where $Z(R_*)$ is the inverse function of $R_*(Z)$. An inverse exists because $R_*(Z)$ is monotonic, indeed, if $\theta = \eta$, we can write it explicitly as $Z(R_*) = \phi_2 \left(\frac{1-R_*}{R_*}\right)^{1/\eta}$.

$$B_R(R_*) = \frac{-\kappa\Phi_{\nu}(Z(R_*), R_*) - G_R(Z(R_*), R_*)}{Z(R_*)^{\beta}} \tag{19a}$$

$$= \phi_3 \frac{R_*^{\frac{\beta-1}{\eta}}}{(1-R_*)^{\frac{\beta}{\eta}}} \text{ where } \phi_3 = \frac{\phi/\phi_2^{\beta}}{\rho(\beta-1)} \qquad \text{if } \theta = \eta \tag{19b}$$

Integrating this, we obtain $B(R)$.

$$B(R) = \int_0^R B_R\left(\tilde{R}\right) \mathrm{d}\tilde{R} \tag{20a}$$

$$= \phi_3 \int_0^R \frac{\tilde{R}^{\frac{\beta-1}{\eta}}}{(1-\tilde{R})^{\frac{\beta}{\eta}}} \mathrm{d}\tilde{R} \qquad \text{if } \theta = \eta \tag{20b}$$

Again the integral is with respect to $\tilde{R}$, and a tilde is added to $\tilde{R}$ to distinguish $\tilde{R}$ from its end value R.[8] With this, the option value is pinned down if we specify state variables Z and R by using $B(R)Z^{\beta}$. Note that, as mentioned above, this result holds only in the non-conversion region. In the conversion region ($R > R_*(Z)$), the real option is exercised, and the remaining option value is identically zero.

2.7.4 Additional Boundary Conditions

The two remaining boundary conditions are imposed along lines $Z = 0$ and $R = 0$.

◊ *Boundary Condition 4:* Due to the consideration at $Z = 0$, the integral constant on the negative root β_0 must be zero.

$$B^0(R) = 0 \text{ for any } R \tag{21}$$

A similar condition appears in most real option problems. First of all, remember that the general solution is the linear combination of two elementary solutions and one particular solution: $V^0(Z,R) = B(R)Z^{\beta} + B^0(R)Z^{\beta_0} + G(Z,R)$, where $B(R)$ and $B^0(R)$ are integral constants. Note that they are constants with respect to Z (because (10) has the derivatives of Z only), and in general they may not be constants with respect to R. Indeed, $B(R)$ has been already obtained in (20a). For $B^0(R)$, if it is non-zero, since $\beta_0 < 0$, term $B^0(R)Z^{\beta_0}$ dominates as $Z \to 0$. In this case, from (17), $R_* < R_m$, implying that the irreversibility leads to more land conversion, which is a contradiction.

◊ *Boundary Condition 5:* Along $R = 0$, we impose the condition that option value $B(R)Z^{\beta}$ is

[8] Note that, because $B(R)$ is independent from Z, we can integrate $B_R(\tilde{R})$ even if $\tilde{R}$ is not on the barrier curve. Obviously, $B_R(\tilde{R}(Z)) = B_R(R_*(Z'))$ as long as $\tilde{R}(Z) = R_*(Z')$ even if $Z \neq Z'$. In a sense, what we do care about here is only the shape of function $B_R(\cdot)$.

zero.

$$B(R) = 0 \text{ for } R = 0 \tag{22}$$

This is simply because, without land to be converted, there is no chance to exercise an option in the future. This condition is used in (20), in which integration is between 0 and R. The starting value 0 of the integration (i.e., lower subscript on $\int$) is determined by this zero option value condition.

3 Numerical Results

This section first shows the baseline case, where we extract some parameters from GDP-TFP. It also shows the results with agri-TFP and the sensitivity of the quantitative results to the changes in key parameters.

3.1 Parameter Selection

In our numerical exercise, we set the baseline parameter values as in Table 1. Our choice of value for the discount rate 4% is guided by the macroeconomic literature. It must be close to the long-run average of the risk-free short-term risk-free rate. Our choice is significantly less than the value, frequently 7%, employed in previous studies such as Bulte et al. (2002) and Leroux et al. (2009). We will discuss this difference further in Section 4.2, in relation to environmental discounting.

Trend technology growth rate $\tilde{\alpha}$ and its volatility $\tilde{\sigma}$ (standard deviation) are obtained from simple estimations of Japan, U.K. and U.S. production function. The average estimates are $\tilde{\alpha} = 0.0117$ (1.17% per annum) and $\tilde{\sigma} = 0.0112$. We label this as GDP-TFP. As a sensitivity analysis, we show the results with TFP growth rate of agricultural sector, since agricultural land use is perhaps the leading reason for land conversion for many developing countries. For this, we employ data from Huffman and Evenson's (1993) yielding $\tilde{\alpha} = 0.0211$ and $\tilde{\sigma} = 0.0604$. We also note from the agricultural productivity literature that many countries have annual TFP growth rates of approximately 2% (e.g., Heady et al, 2010), although volatility is not typically reported. We call this Agri-TFP case. We could also use the GDP growth rate and its volatility, instead of TFP, but the result does not differ very much from the GDP-TFP case (this is perhaps because σ^2/α does not change very much).

As discussed above, intratemporal substitution η plays a crucial role in our model. Model behaviour is qualitatively different between $\eta > 1$ and $\eta < 1$. Hence, our strategy is rather than employing one precise parameter value, we show results for two values: 5.0 and 0.7. As we will demonstrate, all parameters other than η only affect model behaviour quantitatively.[9]

Next we consider the value of the elasticity of intertemporal substitution θ. While there is a consensus about the importance of this parameter in the literature the range of values reported is wide. Indeed $\theta = 1$ is the key threshold for many economic properties (see Bansal and Yaron 2004, for example). Most empirical research investigates θ in relation to the sensitivity of consumption growth (C_{t+1}/C_t) to the the change in the real interest rate, and they often find $\theta < 1$ (see Yogo 2004). In

[9] At this point, it is also worth noting that, in the partial equilibrium models of Bulte et al. (2002) and Leroux et al. (2009), it is the agricultural returns to scale parameter that plays a similar role to our η.

more structured models, such as dynamic general stochastic equilibrium models, researchers tend to prefer (or estimate) θ that is equal to 1 or somewhat greater than it. In contrast, for the coefficient of relative risk aversion γ, there seems to be a consensus in the literature that $\gamma > 1$ is necessary to mimic the observed equity premium and other stylized economic facts. In financial economic papers, in which γ and θ are often disentangled under GIE preferences, it is common to have both γ and θ greater than 1 (e.g., Bansal and Yaron 2004). We choose $\theta = 2.0$ and $\gamma = 4.0$ as the baseline, following Barro (2009). However, since it is interesting to investigate the cases with $\theta < 1$ and $\gamma < 1$, we show the results with $\theta = 0.4$ and $\gamma = 0.8$ as part of our sensitivity analyses. We also show the results with $\theta = 6.0$ and $\gamma = 12.0$ in light of Mehra and Prescott (1985) who suggest in their seminal paper that a reasonable value for γ is 10 or less.

To allow for the relative importance of R in the flow utility ϕ, we simply assume that it is equal to 1.0; i.e., fifty-fifty weights on C and R. Note that this choice of ϕ is innocuous in the sense that its value depends on the choice of the measurement unit of Z. Hence, changing ϕ simply changes the scale unit of the W axis in the following figures (though in a non-linear way). In Section 4.3, we discuss how to estimate ϕ and η. Finally, we simply assume κ (coefficient on the marginal cost of land conversion) is zero. There are three reasons why $\kappa = 0$. First, it ensures the comparability with the myopic case. Second, it is in keeping with the existing literature such as Bulte et al. (2002). Third, we simply believe that conversion costs are negligible when modern technology is available.

3.2 Barrier Curve

Given our choice of parameter values, we are now able to plot the barrier curve. Due to the option value of unconverted land, the optimal level of the reserved environment R_* is always larger than myopic R_m, as shown Figure 3. As discussed above, the ultimate fate of the reserved land crucially depends on η; R_* approaches to zero if η is above 1, while it approaches the maximum possible value (1) if η is below 1. The *horizontal* gap between the solid and broken lines shows the mistake that the government or society would make if it ignores the option value of R. This gap is larger with agri-TFP than with GDP-TFP, simply because agri-TFP exhibits more volatile technology growth. Note that, for GDP-TFP with $\eta = 5.0$, although visually R_m and R_* are quite close, there is a sort of optical illusion. In fact its horizontal gap is larger than with $\eta = 0.7$ (see Figure 6.).

[Figure 3: Barrier Curves around here]

If we evaluate the size of mistakes by examining the *vertical* gap between the solid and broken lines, we obtain a further insight into the model. Consider the case with agri-TFP and $\eta = 0.7$, for example. Suppose that $R = 0.5$ and read the solid line, which shows that it is optimal when W is around 0.70; i.e., $0.5 = R_*(0.70)$. Similarly, we can read the broken line, which shows $R = 0.5$ is myopically optimal if $W = 1.00$; i.e., $0.5 = R_m(1.00)$. The vertical gap along line $R = 5.0$, i.e., the difference between 0.70 and 1.00, is very large, reflecting the steep slope of the barrier curve. On average, it takes 30.5 years for technology W to grow from 0.70 to 1.00.[10] Hence, if the government

[10] Ignoring the volatility term $\tilde{\sigma} dw$ in (2d), to evaluate a vertical gap in terms of time, solve $W_T = W_0 e^{\tilde{\alpha} T}$ for T; i.e., $T = \ln(W_T/W_0)/\tilde{\alpha}$, where W_0 and W_T are initial and end technology levels, respectively.

mistakenly chooses $R = 0.5$ when $W = 1$, then we can say that such a choice was optimal more than 30 years ago. Similarly, for agri-TFP with $\eta = 5.0$, we can numerically show that $0.5 = R_* (1.04)$ and $0.5 = R_m (1.00)$, and the technology gap between 1.00 and 1.04 corresponds to 3.4 years, which is not long because the barrier curve is flat in this case. Hence, if the government mistakenly chooses $R = 0.5$ when $W = 1.00$, then it will be optimal within 4 years. Though we do not discuss the vertical gap any further, this gives a hint to the question that "why does the government convert the land to $R_* (W)$ knowing that, if $\eta < 1$, such a R_* will result in too much land conversion in the future?". It is because, even if the government chooses $R > R_* (W)$, it will be optimal only in the very distant future.

3.3 Option Value

Figure 4 shows the option value of unconverted land R. Remember that the option value is zero in the conversion region. It is always true that the option value tends to be larger near the barrier curve, because future conversion is more likely. Also, the option value tends to be larger with $\eta = 5.0$ compared to the cases with $\eta = 0.7$.

Now suppose that an economy rests just to the left of the barrier curve. Since it is in the non-conversion region, it has a positive option value. Given an upward trend of technology ($\tilde{\alpha} > 0$), with a downward sloping barrier curve ($\eta > 1$) it is more likely to move from the non-conversion region to the conversion region than with an upward sloping barrier curve ($\eta < 1$). In a similar vein, the slope of the barrier curve is flatter with $\eta = 5.0$, which also increases the option value. Because of this, the option value tends to be larger with very small η and very large.[11] Thus, it takes its minimum value of zero at $\eta = 1$. If $\eta = 1$, the barrier curve is vertical at $R_* = \phi / (\phi + 1)$. In this case, there is no option value since there is no chance for W, which causes vertical movements only, to push an economy from the non-conversion region into the conversion region.

Not surprisingly, the option value is larger with agri-TFP because of its higher volatility. Together with the barrier curve (Figure 3), we find that (i) the volatility of W is important and (ii) given upward trend of W, the downward sloping barrier curve tends to generate a larger option value. Finally, regardless of the value of η, the option value is small compared to the magnitude of the value function. This can be understood by comparing the units of the z axis of Figures 4 and 5.

[Figure 4: Option Value around here]

3.4 Value Function

Figure 5 shows the shape of the value function over both conversion and non-conversion regions.[12] In all cases, given W (i.e., along the direction of the R axis), the value function is quite flat, and it

[11] It may appear strange that we cannot have a large option value even if we set η very close to zero. However, this slope effect is rather small and secondary. Even if we set $\eta = 0.25$, the magnitude of the option value is much smaller than with $\eta = 5.0$ (result not shown). Note that the condition that $\rho > \alpha$ is violated for η lower than 0.221...; if other parameters are set as in the baseline case, $\eta = 0.25$ is almost the lower bound for reliable computation.

[12] Under our standard formulation (1) or equivalently (3), V takes a negative values for $\theta < 1$ and vice versa, though this does not have any meaningful implications.

is flatter with $\eta > 1$ than $\eta < 1$. This property is inherited from the flow utility, which also shows a very flat shape with respect to R.

[Figure 5: Value Function around here]

Figure 6 shows the consumption equivalence of the welfare loss and the gap between R_m and R_*. For the latter, we can interpret the mistake that the government would make if it ignores the option value of R. Such a gap is around 5% of the optimal R_* even in the smallest case (i.e., GDP-TFP with $\eta = 0.7$), and it can be 80% of R_* (for agri-TFP with $\eta = 0.5$). However, the consumption equivalent welfare loss, which is defined as a permanent exogenous consumption compensation that is necessary to make up the welfare loss caused by choosing R_m, is negligible for GDP-TFP, though it could be almost 3% for agri-TFP. This finding that the welfare loss tends to be small is in keeping with those previously reported by Alders et al. (1996) and Bulte et al. (2002).

There are a couple of points worth discussing. First, it is not surprisingly, the consumption equivalent loss and the myopic mistake $R_* - R_m$ are both larger for agri-TFP than for GDP-TFP. This is because the option value is larger when the technology is more volatile. Second, for both technology types, for $\eta = 5.0$, we find the combination of small consumption equivalent loss and a large mistake in the land allocation. This is because of the flat shape of the value function along the R axis. That is, the flat value function implies that a large change in R is related to a small change in F (value loss or gain), and vice versa. Such a tendency is not peculiar for $\eta = 0.7$, with which the value function is less flat. Third, for both technology types, we find smaller welfare losses and larger gaps $R_* - R_m$ for $\eta = 5.0$ than for $\eta = 0.7$. This is intuitively because η shows the flexibility of preferences. If η is low, society is inflexible in consumption-environment choices ; i.e., for a certain change in economic conditions, preferable combination of C and R do not change very much. In the current context, even if society misperceives the irreversibility condition, their choice does not change very much. However, the inflexibility leads to a large welfare loss, and society requires a large compensation amount to make up the loss from the misallocation. We can say the opposite things for $\eta = 0.5$. Note, a flat value function (or utility function) is often associated with flexible preferences in general, and also it is partly because that the over-conversion of the land is not a simple waste of resources but it also contributes, though not optimally, to the welfare through generating C.

[Figure 6: Equivalent Consumption Loss around here]

3.5 Sensitivity Analyses

Before discussing the details, note that the myopic barrier curve is not affected by θ, γ or ρ, because the myopic choice is effectively static. Hence, the following changes in the gap between R_* and R_m and the welfare loss is caused by the change in R_* and not in R_m.

[Figure 7: Sensitivity Analysis for θ and γ around here]

Figure 7 shows the effect of changing θ and γ for the GDP-TFP case. Their effects are more or less similar. This is because they affect the model behaviour mainly through subjective risk measure

$\sqrt{\theta\gamma}\sigma$, where θ and γ enter in the same way. Note that, as Epaulard and Pommeret (2003) point out, θ also shows "the aversion to fluctuations". For θ, the effect transmitted through the power on the flow utility seem to be minor for this parameter assumption. The effects of γ materialize only through the subjective risk measure. The effects of changing θ is larger than those of γ for $\eta < 1$, and vice versa. For agri-TFP, we find similar results, but both consumption equivalence and the gap $R_* - R_m$ are much more sensitive to the changes in θ and γ (figure not shown).

[Figure 8: Sensitivity Analysis for ρ around here]

Figure 8 shows that, for lower ρ, horizontal gap $R_* - R_m$ and consumption equivalent loss are both larger, which implies that the option value is more important for lower ρ. The effects of changing the discount rate ρ appear mainly because it changes the value of R as an asset; i.e., the PV of the future service flow is higher with lower discount rate.[13] Both are more sensitive for $\eta = 0.7$ than for $\eta = 5.0$. This is perhaps because the expected life length of the unconverted land R is longer for $\eta = 0.7$ (see the discussion in Section 3.2). One exception is that the consumption equivalent loss for $\eta = 5.0$, for which higher ρ leads to more consumption equivalent loss (upper left panel). This is perhaps because lower ρ also affects the PV of the *permanent* consumption compensation. This effect is not strong, though it is always working. Indeed, in this case (i.e., upper left panel), the effect through R as an asset is overturned because the consumption equivalent loss takes extremely small values (its peak is around 0.005% or less of actual consumption).[14] These tendencies are unchanged even for agri-TFP. In summary, we can say that ρ affects model behaviour *mainly* by changing the asset value of R.

Finally, with vNM preferences, in which we set $\gamma = 1/\theta = 0.5$, the results are quite similar to the case with $\gamma = 0.8$.

3.6 Summary of Numerical Simulations

We summarize our findings as follows. First, intratemporal elasticity of substitution η between C and R plays the most crucial role in our model. It changes the shape of the barrier curve, but it also has quantitative effects as well. Since the barrier curve is downward sloping for $\eta < 1$, where, given an increasing trend of technology W, it is more likely that society will convert R in near future; i.e., graphically it is more likely to cross the barrier curve from below in Figure 3. Also, since it shows the flexibility in consumption-environment choice, with a lower η (i.e., less flexible), the gap between myopic R_m and the dynamically optimal R_* tends to be smaller, but the welfare loss by ignoring the option value of the reserved land tends to be larger.

Second, the effect of changing ρ is often supposed to appear through the change in the value of R as an asset. Basically, we find the same effects, that is, a lower discount factor ρ increases the

[13] See footnote 6.

[14] Because of this, in the upper left panel of Figure 8, the lines show unnatural bumps; actually, each of them should be a smooth bell-shaped curve. These bumps are simply due to the finite approximation error of the method involved here. In addition to this zoom-in effect, the level of consumption for low W is also low, and hence by normalizing the welfare loss such a small consumption the finite approximation error is amplified.

optimal reserved land R_* for all cases examined. In our analysis, however, its effects, especially on the welfare loss, are complicated and can be dependent on W. This complication for the consumption equivalence loss is perhaps an artifact of measuring it as a permanent consumption compensation, and its PV is also affected by ρ.

Third, the effects of increasing the elasticity of intertemporal substitution θ and increasing the coefficient of relative risk aversion γ are quite similar. As discussed above, θ is also regarded as a sort of "fluctuation aversion". The main channel of θ and γ affecting the results is through the subjective risk measure $\sqrt{\theta\gamma}\sigma$, for which $x\%$ change in θ has exactly the same effect as $x\%$ change in γ. Certainly, while γ affects the model behaviour solely through $\sqrt{\theta\gamma}\sigma$, θ also appears on the shoulder of the flow utility. However, the effects of θ through the latter seems to be very small. The welfare loss and the mistake in land allocation become more sensitive to the change in θ and γ under the assumption that the alternative land use is literally agricultural land, in which the volatility of technology growth is large.

Fourth, setting aside η, the most important factor that quantitatively affects the results is the volatility of technology growth $\tilde{\sigma}$, which is in line with most real option studies. In this analysis, agri-TFP is examined as an alternative assumption to GDP-TFP. In our numerical exercises, we use $\tilde{\sigma} = 0.0604$ for agri-TFP and $\tilde{\sigma} = 0.0112$ for GDP-TFP. This means that subjective risk $\sqrt{\theta\gamma}\sigma$ is almost 5.4 times larger under agri-TFP than that under GDP-TFP. To achieve the same effect by changing γ for example, we need to raise γ by more than 29 times larger than baseline $\gamma = 4$.[15] This means that we need $\gamma = 116$, which is perhaps implausible. To avoid possible misunderstanding that the square root on $\theta\gamma$ is the reason behind this observation, which we believe is a superficial issue, we rephrase our finding as follows; (i) we choose, as plausible upside alternatives, $\theta = 12$ following Mehra and Prescott (1985) and $\tilde{\sigma} = 0.0604$ using agri-TFP; and (ii) we find that the effects of changing $\tilde{\sigma}$ to 0.0604 has much larger effects than changing θ to 12.

Fifth, for GDP-TFP, even in the case with a large gap between R_* and R_m, the welfare loss of choosing R_m is quite small. Our value function is flat; i.e., it is insensitive to a change in R especially for $\eta = 5.0$. Hence, a large mistake in R is related to a small value loss, and vice versa. This finding is more or less consistent with the existing real option analyses of land conversion. However, for agri-TFP, the consumption equivalent loss can be as large as 1% for $\eta = 0.7$ and almost 3% for $\eta = 5.0$. Since we measure the consumption equivalent loss in terms of permanent consumption subsidy, these numbers are fairly significant. However, these results are sensitive to the volatility assumption which is also a typical finding in the real option literature. All in all, if we evaluate the policy effect by the consumption equivalent loss, the policy implication can be sensitive to the assumption of the volatility of W.

[15] It is calculated as $29.1... = (0.0604/0.0117)^2$.

4 Further Discussions

4.1 Initial State Dependence

It is interesting to investigate in more detail the dynamics of the barrier control model we have developed. Consider the left panel of Figure 9. Suppose that an economy starts from somewhere in the conversion region (north east of the barrier). Then, no sooner does the government find that it is located in the conversion region, it jumps to R^* horizontally. This means that if a positive shock hits this economy, then it again jumps to R^* horizontally. However, if a negative technology shock hits, it will be pushed into the non-conversion region, and so no government action will be taken. That is, a negative shock leads to no action, while a positive shock yields a horizontal jump. This results in a zigzag movement along the barrier curve, which is the reason why this type of dynamics is called barrier control. Note that, since, whatever the starting point is, all economies will be on the barrier curve in the end, there is little initial state dependence.

[Figure 9: Barrier Control Dynamics around here]

However, an interesting outcome occurs for $\eta < 1$. In this case, once an economy is pushed a long way into the non-conversion region, since a *technology is increasing on average*, it is likely to stay in that region forever. Of course, it is also possible that the economy is hit by a large negative shock and moves to left, but such a possibility is low. To be more precise, it is known that the drift term $\tilde{\alpha}dt$ dominates the stochastic term $\tilde{\sigma}dw$ in (2d) in the long run. Hence, once an economy is a long way inside the non-conversion region, it will not come back to the conversion region again. This dominance of the drift term over the stochastic term has previously been noted in the environmental economic option value literature (e.g., Bulte et al., 2002), albeit in relation to environmental values as opposed to technology shocks.

One important implication of this behaviour is that an upward sloping barrier curve exhibits strong initial state dependence, which is often regarded as something theoretically undesirable. For example, with initial state dependence for any starting point at the initial date, we can always ask ourselves what happened one day before the initial date. This type of question is not important if the dynamics are not dependent on the initial state. This is somewhat troublesome because, on the one hand, we find anecdotal support for a very low elasticity, but, on the other hand, having $\eta < 1$ causes this initial value dependence problem.

4.2 Environmental Discounting

If $\theta = \eta$ (11b), as already noted, (12) and (13) exhibit different (effective) discount rates for A and R as assets. That is, the effective discount rate for A (or the sequence of consumption flow C from it) is $\rho - \alpha$, while that for R is ρ in our model. Note that since α can be negative or positive depending on η, $\rho - \alpha$ can be larger or smaller than ρ. This result is strongly related to Gollier (2010). Although we cannot directly apply his propositions to our model mainly because we endogenize the shadow price of the environment, the several results in his paper reveal the importance of the curvature parameters such as η in determining the relative size of the effective discount rates for R and for C. Like Gollier (2010), the dual discount rates in our model do not require any particular assumptions and they appear even without irreversibility.

In this relation, remember that our model assumes that physically the quality of the environment does not change over time, while the production cost of C decreases as the productivity of A improves. Hence, only the effective discount rate for C (or A) is affected by α. If instead the economic value of the environment changes exogenously, because of the same reason, the effective discount rate for the environment can be higher or lower than deep parameter ρ. This means that, for example, if a model does not take into account the technology growth of alternative land use A, the discount rate for C should be chosen to make the numerical results empirically relevant. As such, our parameter assumption does not really contradict that of Leroux et al. (2009) and Bulte et al. (2002). While we assume there is no exogenous change in the value of R, they do not take into account the effect of the technology change in the alternative land use.

4.3 Empirical Implication

In the parameter selection, instead of pinning down the best guess of η, given its importance, we employed two possible values for η, greater and smaller than 1. Although most existing researches explicitly investigate the elasticity of substitution on the production side, it is a common observation that η is important in the fate of the environment even outside the real option literature. From the derivation process of our theoretical model, we find two (at least potentially) estimable equations for η. The first equation is given by (8).

$$\text{Myopic: } \ln P = \ln \phi - \frac{1}{\eta} (\ln R - \ln C) \tag{23}$$

This equation holds in the context that the irreversibility is, if not irrelevant, negligible. Regressing the log of the shadow price of R on $\ln R - \ln C$, we can obtain the estimates of $\ln \phi$ and $1/\eta$. In this equation, we define the shadow price P of the environment as the value of consumption compensation required to accept for the *temporary* reduction of (the service flow from) environment R. It is for the temporary reduction of R, because the irreversibility of R is negligible. We do not worry about the difference in measurement units, since ϕ absorbs it.

The second means by which we might estimate η is derived mainly from the value function (17b) and optimality condition (17b), where we assume $\theta = \eta$. See Appendix A.4 for the derivation and some additional remarks. It is useful, when the irreversibility constraint is under consideration.

$$\text{Dynamic: } \ln P = \left(\ln \phi + \ln \frac{1}{\rho} + \ln \frac{\beta}{\beta - 1} \right) - \frac{1}{\eta} (\ln R - \ln C) \tag{24}$$

In this case, the constant term is the mixture of many parameters, but, since we can calculate β from η, $\tilde{\alpha}$ and $\tilde{\sigma}$, all parameters are identifiable, provided that $\tilde{\alpha}$, $\tilde{\sigma}$ and ρ are already estimated or known. For (24), we define the shadow price as the value of a one-off consumption compensation to accept a one unit of *permanent* reduction in R. It is for the permanent reduction of R, because R is assumed be irreversible.

Interestingly, the coefficient on $\ln R - \ln C$ is the same for the both, and the difference emerges from the constant term. Intuitively, $\ln 1/\rho$ represents the difference between temporary and permanent reductions of R. Since we assume $\rho = 0.04$, the present value of a one unit permanent reduction of R is larger than that of a temporary reduction by a factor of $1/\rho = 25$.[16] In addition, the option value of R increases the shadow price of R by $\beta / (\beta - 1) > 1$. As is clear in Table 2, $1/\rho$ accounts the most of the difference between (23) and (24). For $\eta = 5.0$ with agri-TFP, however, the option value increases the shadow price P by almost 140%, which is significant, although its effect is negligible for $\eta = 0.7$ with GDP-TFP.

The obvious problem with this approach is that shadow price P is usually unobservable. However, we could employ non-market valuation methods to evaluate P. If such evaluation is available, say, by conducting a stated preference survey, then we can estimate either (23) or (24), depending on the context. For example, if individual level data in one region is available over a number of years, assuming that individual characteristics appear only in ϕ (i.e., assuming that all individuals share a common η), the following panel estimation may be implantable.

$$
\begin{aligned}
\ln P_{it} &= \ln \phi_{it} - \frac{1}{\eta} \left(\ln R_t - \ln C_{it} \right) \\
\phi_{it} &= \phi \left(X_{it} \right) \text{ where } X_{it} \text{ includes any individual characteristics, etc.}
\end{aligned}
$$

A problem with this proposal is that there are very few stated preferences studies that explicitly take account of temporal changes in value. In a survey of the literature Skourtos et al. (2010) identify one which examines the temporal reliability of stated preference estimates. However, in principle with the development of stated preference databases there is potential for the use of some form of meta analysis to reveal the shadow price P. Another caveat of this approach is that both (23) and (24) are derived at the societal (or government) level and not for individuals (see Appendix A.4 for details). However, we believe these equations are good proxies for individual environmental pricing. Finally, setting aside the option value of the environment,[17] as many researchers observe, the elasticity of substitution between reproductive goods and environmental goods plays a key role in many areas of environmental economics. Thus, the empirical approaches identified here, that emerge from our model, may well be worth pursuing as a means to estimate η in the future.

[16] See footnote 6.

[17] In our another on-going research, we find that, in a wide class of models, η is again the key to determine whether the environmental degradation progresses or not for an economy rich enough. At the same time, however, we also find that the threshold of η is not necessarily to be 1. For example, if we have capital accumulation in the model, the plausible threshold value for a flow pollutant is not far from 2 for a reasonable parameter set (especially for the reasonable assumption of the depreciation rate of capital). The threshold is 1 when (i) all output is consumed (there are no other demand components such as investment in the market clearing condition) and (ii) the production technology is Hicksian neutral, both of which hold in the model of this paper, but it seems to be a lower bound of reasonable range of the threshold for η.

5 Conclusion

In this paper, we consider the option value of the environment based on a simple general equilibrium growth model with stochastic technology shock. As in existing environmental option value studies, the environment can in principle have a significant real option value in our model because of its irreversibility. However, unlike the existing literature in which the uncertainty of the value of the environment is given exogenously, the value of the environment is endogenously determined in our general equilibrium setting. Since we have assumed that the environment is in limited supply, as society becomes richer, the relative (shadow) price of the environment increases, which is the mechanism of the change in the economic value of the environment in our model. This is consistent with the observation that people living in rich countries are more eager to conserve the environment than those in developing countries. In our model the quality (or productivity) of the environment per se is physically unchanged.

As found in many papers in environmental economics, the most crucial parameter in our model is the elasticity of substitution η between consumption and the service flow from the environment. The value of the environment is mainly dependent on how easily the environment can be substituted by consumption in society's consumption-environment choice. We have shown that by changing η we can significantly alter model behaviour not only quantitatively but also qualitatively. That is, when $\eta < 1$, the optimal amount of the environment is increasing over time as people becomes richer, and vice versa. This result is not dependent on the irreversibility of the environment and hence appears to hold in a wide class of economic models, and we offer some potentially estimable equations as by-products of our derivation process.

With the irreversibility constraint on the land conversion imposed, other parameters such as intertemporal substitution θ and coefficient of relative risk aversion γ can have quantitatively important effects. Under our model formulation, the effects of θ and γ are quite similar, because their effects appear mainly through the subjective risk measure $\sqrt{\theta\gamma}\sigma$. Employing GIE preferences, it is possible to have $\gamma \neq 1/\theta$, and hence the subjective risk measure can be different from the objective risk measure σ. We find that as θ and γ increase, the option value becomes larger. However, in our model, trend technology growth $\tilde{\alpha}$ and its volatility $\tilde{\sigma}$ seem to have more quantitatively significant impacts on the option value and welfare. More specifically, comparing GDP and agricultural sector TFPs, we find that the excessive land conversion and welfare loss are both larger, when the trend technology growth $\tilde{\alpha}$ and its volatility $\tilde{\sigma}$ are taken from the agricultural sector, since agricultural TFP is much more volatile in than that of the whole economy.

In terms of policy implications, we find that, depending on the parameter assumptions, a huge proportion of the environment can be mistakenly converted, if the option value of the reserved environment is ignored. For example, if we assume that the main alternative use of the environment is agricultural land, given large uncertainty in agricultural TFP, the area of mistakenly converted reserved land can be around 75% of the optimal reserved land if $\eta = 5.0$. However, if we measure the policy effect by consumption equivalent loss, its magnitude sharply depends on the level of uncertainty. Specifically, the welfare loss is almost negligible if we adopt GDP based TFP as the productivity shock to A, while it is much larger if we use agricultural TFP. The welfare loss is also dependent on the flexibility of the preference. If the preference is inflexible ($\eta < 1$), people feel strong pain when the government converts the environment excessively. Thus, when drawing policy implications, we need to be careful in choosing the preferred policy evaluation measure and these measures are sensitive to the parameter assumptions, especially those relating to $\tilde{a}$, $\tilde{\sigma}$ and η.

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A Appendix

A.1 Fundamental PDE for GIE

This subsection derives the fundamental PDE for GIE preferences (3a) by using Svensson's (1989) method. First, expand $F_{t+\Delta t}^{1-\gamma}$ in (3a) around $F_t^{1-\gamma}$. Note that this is analogous to a Taylor expansion and the higher order terms vanish in expectation (so we omit them).

$$\begin{aligned} F_{t+\Delta t}^{1-\gamma} &= F_t^{1-\gamma} + (1-\gamma) F_t^{-\gamma} F_{Z,t} Z_t (\alpha \Delta t + \sigma \Delta w_t) + (1-\gamma) F_t^{-\gamma} F_{R,t} \Delta R_t \\ &\quad -\gamma (1-\gamma) F_t^{-\gamma-1} \frac{F_{ZZ,t}}{2} Z_t^2 (\sigma \Delta w_t)^2 \end{aligned}$$

where $F_{Z,t} = \partial F_t / \partial Z_t$ and so on. Substituting this back into (3a) and rearranging it, we find

$$\begin{aligned} \lim_{\Delta t \to 0} (1+\rho \Delta t)^{\frac{1}{1-1/\theta}} &= \lim_{\Delta t \to 0} \max_{v_t} \left\{ \begin{array}{c} \left(\frac{U_t}{F_t}\right)^{1-1/\theta} (1+\rho\Delta t) \Delta t \\ + \left(1 + (1-\gamma) \left\{ \frac{F_{Z,t}}{F_t} Z_t \alpha - \gamma \frac{F_{ZZ,t}}{F_t^2} Z_t^2 \frac{\sigma^2}{2} - \frac{F_{R,t}}{F_t} v_t \right\} \Delta t \right)^{\frac{1-1/\theta}{1-\gamma}} \end{array} \right\}^{\frac{1}{1-1/\theta}} \\ &= \lim_{\Delta t \to 0} \max_{v_t} \left\{ \begin{array}{c} \left(\frac{U_t}{F_t}\right)^{1-1/\theta} (1+\rho\Delta t) \Delta t \\ + \left(1 + \left(1-\frac{1}{\theta}\right) \left\{ \frac{F_{Z,t}}{F_t} Z_t \alpha - \gamma \frac{F_{ZZ,t}}{F_t^2} Z_t^2 \frac{\sigma^2}{2} - \frac{F_{R,t}}{F_t} v_t \right\} \Delta t + o(\Delta t) \right) \end{array} \right\}^{\frac{1}{1-1/\theta}} \\ &= \lim_{\Delta t \to 0} \max_{v_t} \left\{ \begin{array}{c} \frac{1}{1-1/\theta} + \frac{1}{1-1/\theta} \left(\frac{U_t}{F_t}\right)^{1-1/\theta} (1+\rho\Delta t) \Delta t \\ + \left\{ \frac{F_{Z,t}}{F_t} Z_t \alpha - \gamma \frac{F_{ZZ,t}}{F_t^2} Z_t^2 \frac{\sigma^2}{2} - \frac{F_{R,t}}{F_t} v_t \right\} \Delta t + o(\Delta t) \end{array} \right\} \end{aligned}$$

where $\lim_{\Delta t \to 0} o(\Delta t) = 0$. Note that (i) $E_t [\Delta w_t^2] = \Delta t$ and (ii) $(1+\rho\Delta t)^{1/(1-1/\theta)} = \frac{1}{1-1/\theta} (1+\rho \Delta t) + o(\Delta t)$. Then, subtracting $1/(1-1/\theta)$ from the both sides, dividing the both sides by $\Delta t / (1-1/\theta)$, and taking the limit, we obtain

$$\rho = \max_{v_t} \left\{ \left(\frac{U_t}{F_t}\right)^{1-1/\theta} + \left(1-\frac{1}{\theta}\right) \left(\frac{F_{Z,t}}{F_t} Z_t \alpha - \gamma \frac{F_{ZZ,t}}{F_t^2} Z_t^2 \frac{\sigma^2}{2} - \frac{F_{R,t}}{F_t} v_t \right) \right\}$$

Let $V_t = F_t^{1-1/\theta} / (1-1/\theta)$. Then,

$$V_{Z,t} = \left(1-\frac{1}{\theta}\right) V_t \frac{F_{Z,t}}{F_t} \quad , \qquad V_{ZZ,t} = -\frac{1}{\theta} \left(1-\frac{1}{\theta}\right) V_t \frac{F_{ZZ,t}}{F_t^2} \quad , \qquad V_{R,t} = \left(1-\frac{1}{\theta}\right) V_t \frac{F_{R,t}}{F_t}$$

Hence, letting ν be its optimal value ν_*, we obtain (9) in the main text:

$$\rho V_t = \frac{U_t^{1-1/\theta}}{1-1/\theta} + \alpha V_{Z,t} Z_t + \frac{\gamma \theta \sigma^2}{2} V_{ZZ,t} Z_t^2 - V_R v_{*,t}$$

Note that, anticipating this transform from F_t to V_t, instead of (3a), often the value function is equivalently formulated as

$$
\begin{aligned}
V_t &= \lim_{\Delta t\to 0}\max_{v_t}\frac{\left(Z_t\left(1-R_t-\kappa v_t\right)^{1-1/\eta}+\phi R_t^{1-1/\eta}\right)^{\frac{1-1/\theta}{1-1/\eta}}}{1-1/\theta}\Delta t+\frac{1}{1+\rho\Delta t}E_t\left[V_{t+\Delta t}^{\frac{1-\gamma}{1-1/\theta}}\right]^{\frac{1-1/\theta}{1-\gamma}}\\
\Delta Z_t &= \alpha Z_t\Delta t+\sigma Z_t\Delta w_t\\
\Delta A_t &= -\Delta R_t = v_t\Delta t\geq 0
\end{aligned}
$$

A.2 Computation Tips

The computation is not trivial mainly because (a) some numerical integrals require a function solver *in each step* (see (20a)) and (b) they also include the power with β and β_0 which can be very large numbers. Here, we mainly discuss how to reduce the implementation of numerical integrals. Also, we show the special case, in which further analytical solution is available. For other techniques such as non-equidistant grids, readers can examine our Matlab codes which can be provided on request. Below, equation numbers with ".m" show the names of function m-files used in our Matlab codes.

A.2.1 Key Derivatives

This section summarizes the key notations and their derivatives. Note that the derivatives of G^1 can be obtained by replacing β_0 with β in those of G^1.

$$
\begin{aligned}
V^0(Z,R) &= B(R)Z^\beta+G(Z,R)\\
G(Z,R) &= G^0(Z,R)-G^1(Z,R) \qquad\qquad \text{(NoOpValG.m)}\\
&= \frac{Z^{\beta_0}}{\gamma\theta\sigma^2a_2}\int^Z\tilde{Z}^{-\beta_0-1}\Phi\left(\tilde{Z},R\right)\mathrm{d}\tilde{Z}-\frac{Z^{\beta}}{\gamma\theta\sigma^2a_2}\int^Z\tilde{Z}^{-\beta-1}\Phi\left(\tilde{Z},R\right)\mathrm{d}\tilde{Z}\\
\Phi(Z,R) &= \frac{\left(Z\left(1-R\right)^{1-1/\eta}+\phi R^{1-1/\eta}\right)^{\frac{1-1/\theta}{1-1/\eta}}}{1-1/\theta}=\frac{\Psi^{\frac{1-1/\theta}{1-1/\eta}}}{1-1/\theta}\\
\Psi(Z,R) &= Z\left(1-R\right)^{1-1/\eta}+\phi R^{1-1/\eta}
\end{aligned}
$$

$$
\begin{aligned}
G_Z^0(Z,R) &= \beta_0\frac{G^0(Z,R)}{Z}+\frac{\Phi(Z,R)}{\gamma\theta\sigma^2a_2Z}\\
G_{ZZ}^0(Z,R) &= \beta_0\left(\beta_0-1\right)\frac{G^0(Z,R)}{Z^2}+\beta_0\frac{\Phi(Z,R)}{\gamma\theta\sigma^2a_2Z^2}+\frac{\Phi_Z(Z,R)-\Phi(Z,R)/Z}{\gamma\theta\sigma^2a_2Z}\\
G_R^0(Z,R) &= \frac{Z^{\beta_0}}{\gamma\theta\sigma^2a_2}\int^Z\tilde{Z}^{-\beta_0-1}\Psi(Z,R)^{\frac{1/\eta-1/\theta}{1-1/\eta}}\left(\phi R^{-1/\eta}-\tilde{Z}\left(1-R\right)^{-1/\eta}\right)\mathrm{d}\tilde{Z}\\
G_{RZ}^0(Z,R) &= \beta_0\frac{G_R^0}{Z}+\frac{\Psi(Z,R)^{\frac{1/\eta-1/\theta}{1-1/\eta}}}{\gamma\theta\sigma^2a_2Z}\left(\phi R^{-1/\eta}-Z\left(1-R\right)^{-1/\eta}\right)\\
\Phi_\nu(Z,R) &= -\Psi(Z,R)^{\frac{1/\eta-1/\theta}{1-1/\eta}}Z\left(1-R\right)^{-1/\eta}\\
\Phi_{\nu Z}(Z,R) &= -\Psi(Z,R)^{\frac{1/\eta-1/\theta}{1-1/\eta}}\left(1-R\right)^{-1/\eta}-\frac{1/\eta-1/\theta}{1-1/\eta}\Psi(Z,R)^{\frac{1/\eta-1/\theta}{1-1/\eta}-1}Z\left(1-R\right)^{1-2/\eta}
\end{aligned}
$$

A.2.2 Solving NonOption Value G When $(1-1/\theta)/(1-1/\eta)=1,2,\cdots$

We can explicitly solve the integral in $G(Z,R)$ not only for $\theta=\eta$ but also for the case where $n=(1-1/\theta)/(1-1/\eta)$ is a natural number (i.e., $n=1,2,\cdots$), which is equivalently $\theta=1/(1-n(1-1/\eta))$. This result is useful for the debugging purpose. We show the result only, because it is easy to verify it.

$$\begin{aligned} G(Z,R) &= \frac{1}{1-1/\theta}\sum_{k=0}^{n}\frac{\binom{n}{k}\left(Z(1-R)^{1-1/\eta}\right)^{n-k}\left(\phi R^{1-1/\eta}\right)^{k}}{\rho-(n-k)\left(\alpha-\frac{1-n+k}{2}\gamma\theta\sigma^2\right)} \\ &= \frac{-2}{\gamma\theta\sigma^2}\frac{1}{1-1/\theta}\sum_{k=0}^{n}\frac{\binom{n}{k}\left(Z(1-R)^{1-1/\eta}\right)^{n-k}\left(\phi R^{1-1/\eta}\right)^{k}}{(n-k-\beta_0)(n-k-\beta)} \qquad \text{(NoOpValN.m)} \\ G^0(Z,R) &= \frac{1}{\gamma\theta\sigma^2 a_2(1-1/\theta)}\sum_{k=0}^{n}\frac{\binom{n}{k}\left(Z(1-R)^{1-1/\eta}\right)^{n-k}\left(\phi R^{1-1/\eta}\right)^{k}}{(n-k-\beta_0)} \end{aligned}$$

A.2.3 Solving (17a) Further for R_*

It is easy to obtain (17a) from $\beta\times$(VM)$-Z\times$(SP). Hence, for each Z,

$$\begin{aligned} 0 &= \beta G_R(Z,R_*)-ZG_{RZ}(Z,R_*)+\kappa\{\beta\Phi_\nu(Z,R_*)-Z\Phi_{\nu Z}(Z,R_*)\} \\ &= 2a_2G^0_R(Z,R_*)+\kappa\Phi_\nu(Z,R_*)\left\{\beta-\frac{nZ(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}{Z(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}\right\} \qquad \text{(OptRg.m)} \end{aligned}$$

where $n=(1-1/\theta)/(1-1/\eta)$, which is not necessarily a natural number in this case. For the second line, note that (i) G^0_{RZ} and G^1_{RZ} have the same second term, implying

$$\beta G_R-ZG_{RZ}=\beta G^0_R-\beta G^1_R-\beta_0G^0_R+\beta G^1_R=(\beta-\beta_0)G^0_R=2a_2G^0_R$$

and (ii)

$$\begin{aligned} \beta\Phi_\nu-Z\Phi_{\nu Z} &= \frac{1/\eta-1/\theta}{1-1/\eta}\Psi^{\frac{1/\eta-1/\theta}{1-1/\eta}-1}Z^2(1-R)^{1-2/\eta}-(\beta-1)\Phi_\nu(Z,R) \\ &= -\Psi^{\frac{1/\eta-1/\theta}{1-1/\eta}}Z(1-R)^{-1/\eta}\left((\beta-1)-\frac{1/\eta-1/\theta}{1-1/\eta}\Psi^{-1}Z(1-R)^{1-1/\eta}\right) \\ &= -\Psi^{\frac{1/\eta-1/\theta}{1-1/\eta}-1}Z(1-R)^{-1/\eta}\left(\left(\beta-\frac{1-1/\theta}{1-1/\eta}\right)Z(1-R)^{1-1/\eta}+(\beta-1)\phi R^{1-1/\eta}\right) \end{aligned}$$

If $n=(1-1/\theta)/(1-1/\eta)$ is a natural number, for each Z, R_* is such that

$$\begin{aligned} 0 &= \sum_{k=0}^{n}\frac{(\beta-n+k)(k/n-R_*)}{R_*(1-R_*)}\frac{\binom{n}{k}\left(Z(1-R_*)^{1-1/\eta}\right)^{n-k}\left(\phi R_*^{1-1/\eta}\right)^{k}}{\rho-(n-k)\left(\alpha-\frac{1-n+k}{2}\gamma\theta\sigma^2\right)} \\ &\quad +\kappa\{\beta\Phi_\nu(Z,R_*)-Z\Phi_{\nu Z}(Z,R_*)\} \qquad \text{(OptRn.m)} \end{aligned}$$

This can be obtained by either $2a_2 G_R^0$ or $\beta G_R - Z G_{RZ}$.

A.2.4 Solving (19a) Further for Option Value

Let $Z_* = Z(R_*)$. Using $-G_R^0 = \frac{\kappa}{2a_2}\Phi_\nu \left\{\beta - \frac{nZ(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}{Z(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}\right\}$ (see (OptRg.m)), (19a) can be rewritten as:

$$\begin{aligned} B_R(R_*) &= \frac{-\kappa\Phi_\nu(Z_*,R_*) - G_R(Z_*,R_*)}{Z_*^\beta} = \frac{-\kappa\Phi_\nu(Z_*,R_*) - G_R^0(Z_*,R_*) + G_R^1(Z_*,R_*)}{Z_*^\beta} \\ &= \frac{\kappa\Phi_\nu}{Z_*^\beta}\left\{\left(\frac{\beta}{2a_2}-1\right) - \frac{1}{2a_2}\frac{nZ_*(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}{Z_*(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}\right\} + \frac{G_R^1(Z_*,R_*)}{Z_*^\beta} \\ &= \frac{\kappa\Phi_\nu}{2a_2 Z_*^\beta}\left\{\beta_0 - \frac{nZ_*(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}{Z_*(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}\right\} + \frac{G_R^1(Z_*,R_*)}{Z_*^\beta} \qquad \text{(BprimG.m)} \end{aligned}$$

If $n = (1-1/\theta)/(1-1/\eta)$ is a natural number, we can avoid the integral in the second term of (19a).

$$\begin{aligned} B_R(R_*) &= \frac{\kappa\Phi_\nu}{2a_2 Z_*^\beta}\left\{\beta_0 - \frac{nZ_*(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}{Z_*(1-R_*)^{1-1/\eta}+\phi R_*^{1-1/\eta}}\right\} \\ &\quad + \frac{2/\gamma\theta\sigma^2}{2a_2 Z_*^\beta}\sum_{k=0}^{n}\frac{k/n - R_*}{R_*(1-R_*)}\frac{\binom{n}{k}\left(Z_*(1-R_*)^{1-1/\eta}\right)^{n-k}\left(\phi R_*^{1-1/\eta}\right)^k}{(n-k-\beta)} \qquad \text{(BprimN.m)} \end{aligned}$$

A.2.5 Consumption Equivalent Loss

In this version, to compute the consumption equivalent loss C_{eq}, we define it as *permanent* consumption that is given exogenously to make up the value loss caused if the government (or society) mistakenly chooses $R = R_m$ (not $R = R_*$). Since C_{eq} is measured in percentage term, the compensated consumption is $C = W(1-R)(1+C_{eq})$. But, instead of computing C_{eq} directly, it is efficient to redefine technology as $W^c = W(1+C_{eq})$ so that $C = W^c(1+R)$. From (1), it is obvious that this redefinition does not change the *functional form* of F, meaning that we do not need recompute everything. In sum, we define C_{eq} such that for each W

$$F(W^c, R_m(W)) = F(W, R_*(W)) \text{ where } W^c = W(1+C_{eq})$$

This implies that, in this version, we take into account the fact that C_{eq} changes the optimal response by the government, meaning that the option value $Z^\beta B(R)$ is also affected by C_{eq}. Also, this definition implies that the consumption compensation is given even after the myopic choice becomes optimal. While unusal if a rational government ignores the option value, it is logically straightforward in computation to assume that the mistake happens only once.

A.3 Estimation of TFP

While we employ the numbers from Huffman and Evenson (1993) for agricultural TFP, because we cannot find the growth rate of GDP-TFP and its volatility from the literature, we have estimated TFP for Japan, U.K. and U.S. employing the Solow residuals approach. Assuming a Cobb-Douglas aggregate production function, we can estimate TFP as something remaining after attributing the GDP growth rate to the contributions of capital and labour .

$$\frac{\mathrm{d}W}{W} = \frac{\mathrm{d}Y}{Y} - \phi_K \frac{\mathrm{d}K}{K} - \phi_L \frac{\mathrm{d}L}{L}$$

where W, K, L and Y are TFP, capital, labour and output, respectively. Parameters ϕ_K and ϕ_L show capital and labour shares. Table 3 shows the estimates of TFP based on (a) OLS estimates of ϕ_K and ϕ_L and (b) fixed ϕ_K and ϕ_L. In OLS estimates, constant returns to scale (CRS) is not satisfied for Japan and U.S., but imposing CRS as a restriction reduces estimation performance. We limit ourselves to this simple estimation equation, because using more elaborated methods is beyond our scope. Fixed factor shares are motivated by the idea of "calibration" that is popular in macroeconomics. The average of the estimated trend growth rate of TFP $\tilde{\alpha}$ and its volatility $\tilde{\sigma}$ are 0.0117 and 0.0112 for OLS estimates and 0.0131 and 0.0129 for fixed factor shares calculation. We adopt the average OLS estimates, but our model results change little if we employ the fixed factor shares assumption (mainly because $\tilde{\alpha}/\tilde{\sigma}^2$ are very close to each other in the both cases).

[Table 3: TFP around here]

A.4 Derivation of (24)

Here we assume that $\theta = \eta$. We define the price P of the environment as the amount of a one-time consumption increase (ΔC_0) to compensate the decrease in R forever (this decrease is permanent due to the irreversibility assumption). Rewrite (11b) by using (12) and 13.

$$\begin{aligned} V^0(R,Z) &= \frac{Z}{\rho-\alpha}\frac{(1-R)^{1-1/\eta}}{1-1/\eta} + \frac{\phi}{\rho}\frac{R^{1-1/\eta}}{1-1/\eta} + B(R)Z^\beta \\ &= \int_0^\infty e^{-\rho t}\frac{C_t^{1-1/\eta}}{1-1/\eta}\mathrm{d}t + \frac{\phi}{\rho}\frac{R^{1-1/\eta}}{1-1/\eta} + B(R)Z^\beta \\ \frac{\partial V^0}{\partial R} &= \frac{\phi}{\rho}R^{-1/\eta} + B'(R)Z^\beta \quad \text{given } C_t \text{ for } 0 \le t \\ \frac{\partial V^0}{\partial C_0} &= C_0^{-1/\eta} = (W(1-R))^{-1/\eta} = Z^{\frac{-1}{\eta-1}}(1-R)^{-1/\eta} \end{aligned}$$

We put time subscript on C_t to discriminate current consumption C_0 from future consumption in the integral. Note that all variables other than C_t show the current values; e.g., $R = R_0$. From (SP'), at optimum (i.e., $R = R^*$),

$$B_R(R)Z^\beta = \frac{W}{\beta(\rho-\alpha)}(W(1-R))^{-1/\eta} = \frac{WC_0^{-1/\eta}}{\beta(\rho-\alpha)}$$

By the implicit function theorem, for a fixed level of the value function $\bar{V}^0$

$$\begin{aligned} P &= -\frac{\partial \bar{V}^0/\partial R}{\partial \bar{V}^0/\partial C_0} = \frac{\phi}{\rho}\left(\frac{R}{C_0}\right)^{-1/\eta} + \frac{W}{\beta(\rho-\alpha)} \\ &= \frac{\phi}{\rho}\left(\frac{R}{W(1-R)}\right)^{-1/\eta} + \frac{1/\beta}{\rho-\alpha}W \\ &= \left(\frac{\phi/\phi_2}{\rho} + \frac{1/\beta}{\rho-\alpha}\right)W = \frac{W}{\rho-\alpha} \end{aligned}$$

where (17) is used to derive the second line. Hence, eliminating W from the first line by using the third line, we obtain:

$$P = \frac{\phi}{\rho}\left(\frac{R}{C_0}\right)^{-1/\eta} + \frac{P}{\beta} = \frac{\beta}{\beta-1}\frac{\phi}{\rho}\left(\frac{R}{C_0}\right)^{-1/\eta}$$

Omitting time subscript on C, and taking the logarithm, we obtain (18):

$$\ln P = \ln\phi + \ln\frac{\beta/\rho}{\beta-1} - \frac{1}{\eta}(\ln R - \ln C)$$

Our model suggests that the log of environmental price in this case must be higher than (23) by $\ln\frac{\beta/\rho}{\beta-1} = \ln(1/\rho) + \ln(\beta/(\beta-1)) > 0$.

The price of R in terms of C derived in this subsection is a concept very close to the equivalent consumption measure of the myopic loss in Figure 6. Unlike Figure 6, however, setting aside some rather technical differences, there are two major differences. First, here we do not assume that lost R is used as a production factor. This assumption is chosen to study the environmental pricing by individuals, because presumably they do not take into account the benefits from converted land. However, in Figure 6, even if society mistakenly converts too much R, such converted land is used for production, which mitigates the loss in the value function. As a result, the price increases due to the option value ($\beta/(\beta-1)$ in Table 2 are large especially for $\eta = 5$, while the equivalent consumption loss for ignoring the option value is negligible in Figure 6. Second, unlike the consumption equivalent loss, in which the consumption compensation is permanent; here, the consumption loss that an individual is willing to give up to save one marginal unit of the environment is one time. Finally, note that there is one caveat in using (24) and (23) for individual level data. That is, (24) is derived under the assumption that $R = R^*$ for society, which may not be true for all individuals in society. In other words, (24) and (23) are true only under the existence of aggregate utility, which, in general, conflicts with the variations observed in individual level data. In this sense, they are most suitable to time series data or cross country data, but obtaining such data is usually very costly.

B Tables and Figures

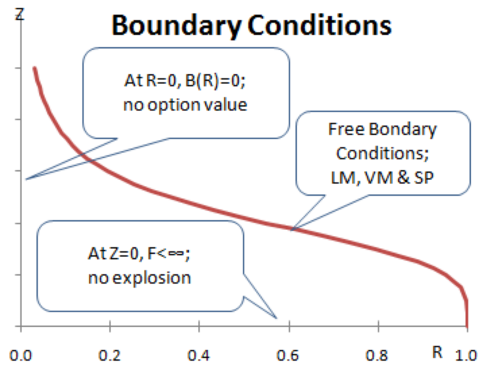

Figure 1: Boundary Conditions.

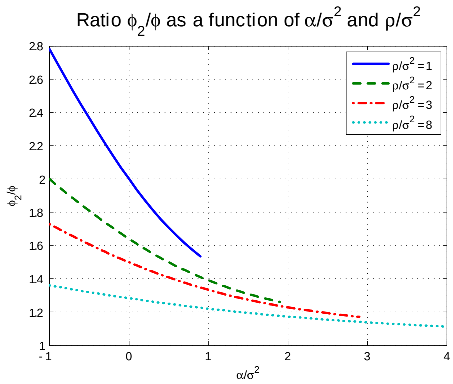

Figure 2: Ratio ϕ_2/ϕ as a function of ρ/σ^2 and α/σ^2.

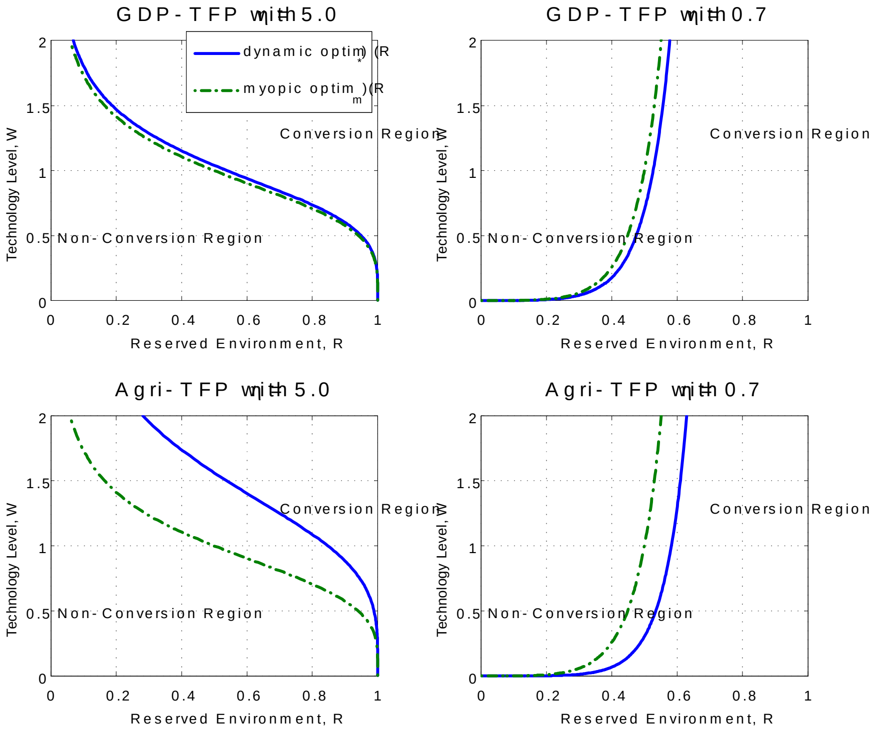

Figure 3: Myopic R_m and Optimal R_*.

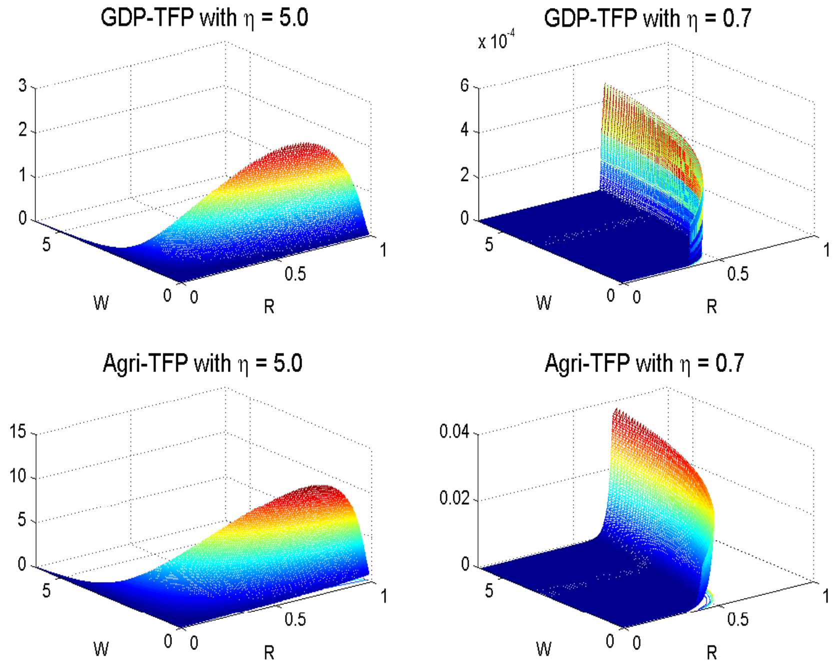

Figure 4: Option value $B(R)\,Z^{\beta}$.

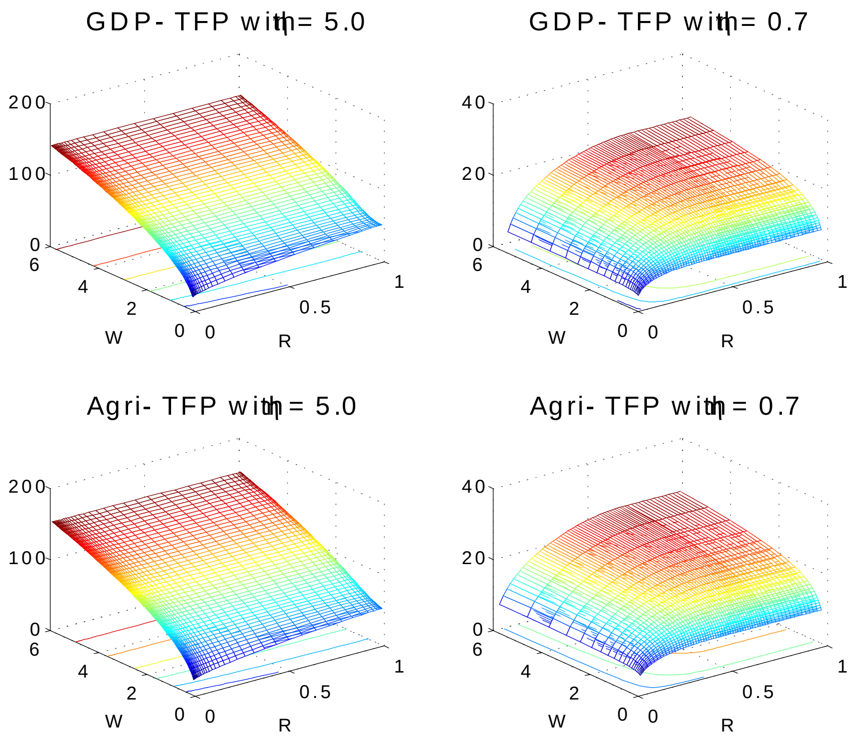

Figure 5: Value Function $V(W, R)$

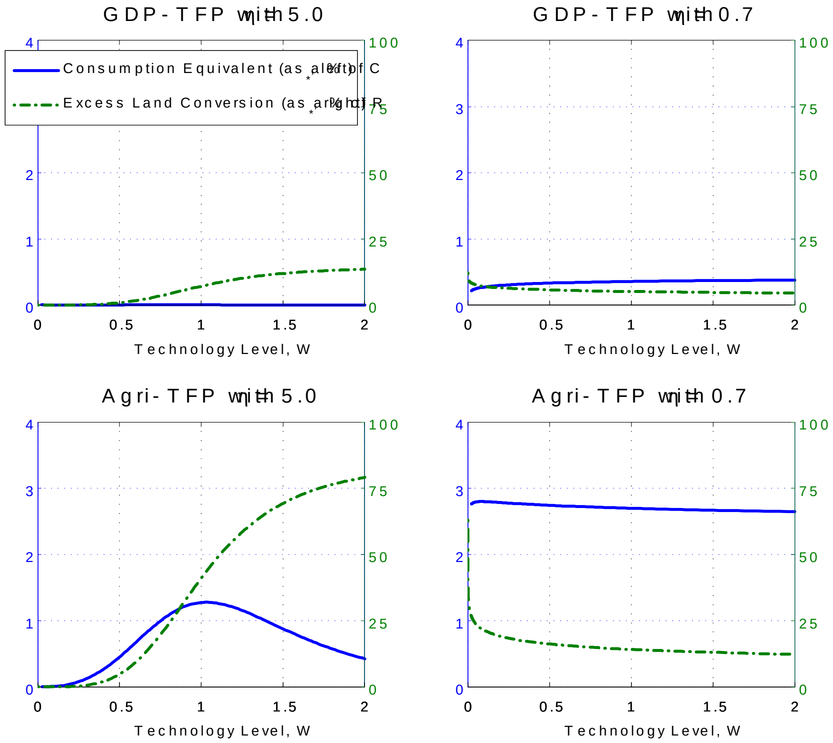

Figure 6: Equivalent consumption measure (as a % of C_*, left axis) and the gap between R_* and R_m (as a % of R_*, right axis).

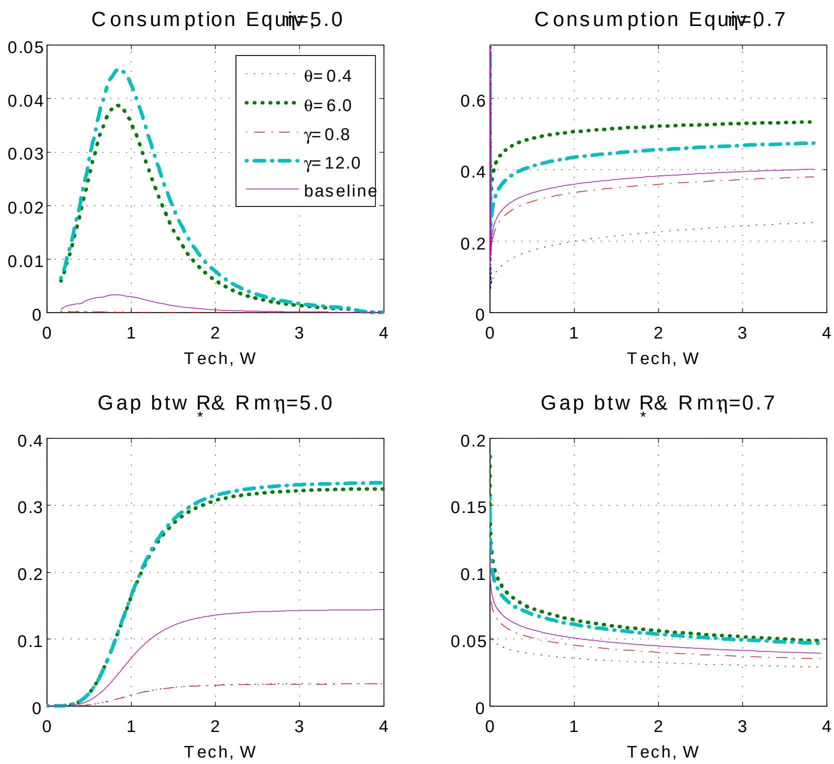

Figure 7: Sensitivity analysis for θ and γ with GDP-TFP.

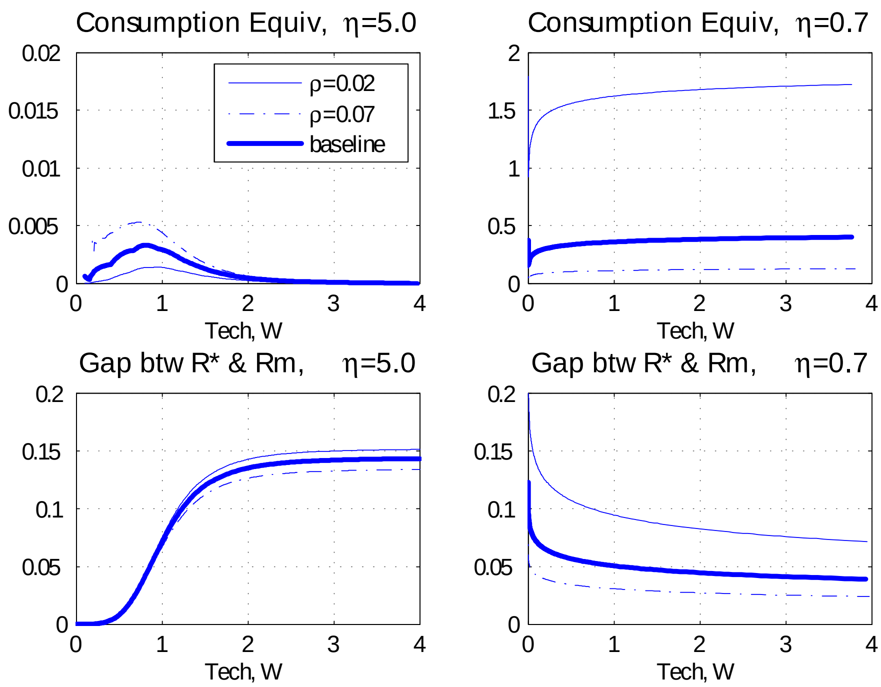

Figure 8: Sensitivity analysis for ρ with GDP-TFP.

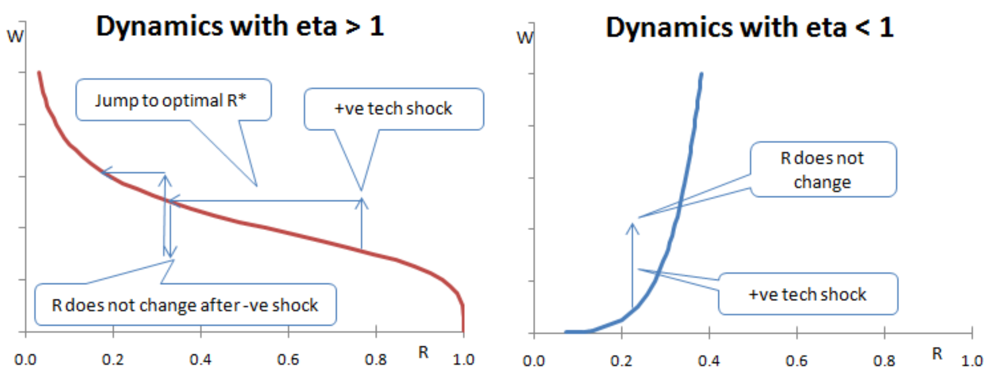

Figure 9: Conceptual diagram to show the dynamics of barrier control model.

Table 1: Deep Parameters

symbol	baseline	sensitivity analysis	meaning
ρ	0.04	0.02/0.07	discount rate
$\tilde{\alpha}$	0.0117	0.0211 (agri-TFP)	trend growth rate of W
$\tilde{\sigma}$	0.0112	0.0604 (agri-TFP)	volatility of W growth
η	5.0/0.7		elasticity of substitution between C and R
θ	2.0	6.0/0.4	elasticity of intertemporal substitution
γ	4.0	12.0/0.8	coefficient of relative risk aversion
ϕ	1.00		relative importance of service flow of R
κ	0.00		marginal cost of of land conversion

Table 2: Key Numbers in Base Line Simulation

		α	σ	β	ϕ_2/ϕ	$\ln \frac{\beta/\rho}{\beta-1}$	$1/\rho$	$\frac{\beta}{\beta-1}$
GDP TFP	$\eta = 5.0$	0.009	0.009	3.892	1.031	3.516	25.00	1.346
($\tilde{\alpha} = 0.0117$, $\tilde{\sigma} = 0.0112$)	$\eta = 0.7$	-0.005	0.005	61.99	1.143	3.235	25.00	1.016
Agri TFP	$\eta = 5.0$	0.017	0.048	1.718	1.401	4.092	25.00	2.394
($\tilde{\alpha} = 0.0211$, $\tilde{\sigma} = 0.0604$)	$\eta = 0.7$	-0.008	0.026	6.319	1.423	3.391	25.00	1.188

Table 3: Estimation of Solow Residuals

		OLS			Fixed Factor Shares		
		JPN	UK	US	JPN	UK	US
Constant	A	0.0073	0.0144	0.0131	0.0134	0.0141	0.0114
std er		0.004	0.011	0.008	-	-	-
Capital		0.4024	0.3284	0.1444	0.35	0.35	0.35
std er		0.123	0.492	0.333	-	-	-
Labour		1.1818	0.7160	0.9653	0.65	0.65	0.65
std er		0.184	0.153	0.094	-	-	-
SD(e)	B	0.0118	0.0130	0.0088	0.0153	0.0130	0.0104
R2 adj		0.792	0.516	0.822	-	-	-
Period		1980-09	1972-09	1970-09	1980-09	1972-09	1970-09
$\tilde{\alpha}$		0.0074	0.0145	0.0132	0.0135	0.0142	0.0115
$\tilde{\sigma}$		0.0118	0.0130	0.0088	0.0153	0.0130	0.0104

Notes: Data frequency is annual. "SD(e)" shows the standard deviation of residuals.
Our estimates are: $\tilde{\alpha} = A + B^2/2$ and $\tilde{\sigma} = B$. See Reed and Clarke (1990).