Dynamics and Performance of Decentralized Portfolios with Size-Induced Fund Flows

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Abstract

We examine the implications of fund sizes for portfolio dynamics and performance within a decentralized structure, where the chief investment officer optimally allocates capital to two fund managers who invest in multiple assets within their own asset classes. The managers experience fund inflows/outflows depending on not only their investment performances but also their fund sizes that are mainly driven by their optimal portfolios. We characterize these practical features through a two-layer dynamic optimization model where the managers maximize their size-dependent compensations. We solve the highly path-dependent optimization problem using a simulation-projection method with a multidimensional grid search and policy iteration. Our analysis provides new interpretations on the controversial

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scale effects on fund performance, along with insights into portfolio dynamics and management fees for bond funds and stock funds under different fund ages.
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1. Introduction

There is an enormous size of Total Net Assets (TNAs) managed by the investment management industry. For example, the TNA of U.S.-registered investment companies reached \$19.2 trillion in 2016 and this phenomenal size is mainly due to large capital inflows and asset appreciation.[1] Fund sizes matter in investment management since managers have incentives to attract cash inflows in order to increase their TNA and compensations. Motivated by the prevalent facts, we analyze the effects of fund sizes (*scale effects*) on managers' optimal portfolios and fund performance.

As fund sizes are large, the majority of funds delegate portfolios to multiple asset managers for their professional knowledge,[2] see, e.g., van Binsbergen et al. (2008), Blake et al. (2013). Being consistent with this industry practice, we use a model of decentralized structure to study the scale effects on the dynamics and performance of managers' optimal portfolios.[3]. In addition to practical relevance, the decentralized structure makes it tractable to solve the managers' path-dependent dynamic portfolio choice problems with multiple risky assets without incurring a serious 'curse of dimension'. To the best of our knowledge, this paper is the first one using a decentralized model to reveal the insights into the effects of fund sizes on funds' portfolios and

[1]See *2017 Investment Company Fact Book*, available at https://www.ici.org/.

[2]There is a growing body of literature on the asset pricing implications of the delegated intermediary investment structure, see He and Xiong (2013).

[3]Given the popularity of the decentralized structure, the implications of our analysis are essentially applicable to the investment management divisions of the institutions like banks, mutual funds, and pension funds with a decentralized structure locally or globally.

performances. It contributes to the literature in the following ways.

First, we provide a theoretical model that captures the relationship between fund sizes and fund flows in a decentralized structure, which fills the gap between the empirical studies and theoretical studies on funds' scale effects. On the one hand, the empirical literature examines the response of fund flows to the changes in fund sizes and finds that the fund size is one of the key determinants of fund flows (e.g. Pollet and Wilson, 2008). Although empirical methods are powerful in showing interesting findings, they have to omit managers' portfolio choice problems. On the other hand, prior theoretical studies assume that fund flows depend on funds' performances (e.g. Basak and Makarov, 2014; Koijen, 2014) and do not depend on fund sizes.

To capture the scale effects on funds, we model a *size-induced* specification where the fund inflow/outflow responses to both portfolio performance and fund size that are mainly driven by the manager's optimal dynamic portfolio weights. The mechanism is that the fund size affects the manager's portfolio weights through its dominant effect on the manager's fund flows and compensation. As a result, the fund manager makes portfolio decisions by simultaneously considering the short-term compensation and the long-term history of the fund size. This mechanism allows us to investigate the scale effects on the portfolio dynamics, fund performance, and management fees.

Second, we study the portfolio dynamics of bond funds and equity funds with the size-induced fund flows. For the portfolio dynamics of bond funds, little is known in the literature although bond funds constitute a significant part of the industry. We find that the bond manager diversifies portfolios well but maintains a substantial allocation to Baa-rated bonds. The underlying reasons are the close expected returns and substitutes in the bonds, a slightly high expected return on Baa-rated bonds, and the mismatched horizons between the long-maturity government bond and the short-term compensation. For the stock portfolio, the holdings of value stocks are sensitive to the changes in fund flows and the sensitivity enhances largely when the fund age increases. The driving force is that more experienced managers adjust their portfolio weights dramatically to hedge against the risk from the fluctuations of fund flows and to maintain their reputations.

Third, we reveal new insights into the controversial scale effects by running regressions on the data realized from the managers' optimal portfolio dynamics in the calibrated model.[4] For the bond funds, we find a *positive* relationship between bond fund performance and its lagged fund size, which implies that large sizes of bond funds bring cost advantages and is referred to as '**scale economies**'. The economic force is that the bonds are close substitutes and large bond funds are not hampered by large market positions and therefore they achieve scale economies. For the stock funds, the relationship becomes *negative*, which indicates the cost disadvantages of large stock funds. In addition, the management fee, which is the ratio of a manager's compensation to the TNA, increases with fund sizes for both bond and stock funds because fund managers have incentives to maximize their compensations. Moreover, the old stock funds achieve scale economies since

[4] Evidence on size-performance relationship is controversial. The classical paper (Carter, 1950) believe the advantages of large fund sizes such as more resources and lower commissions. By contrast, recent work seems to support the cost disadvantage of large stock funds (e.g. Chen et al., 2004; Pollet and Wilson, 2008).

more experienced managers attract more fund inflows by beating the benchmark, whereas the returns on young stock funds have a negative relationship with fund sizes.

We also contribute to the large body of literature on portfolio choices and hedging. Yang and Yang (2013) provide hedging strategies in an illiquid market. Ting and Ewald (2013) study the performance of risk-minimizing hedges in the Heston model. Palczewski et al. (2015) solve a problem of portfolio optimization with transaction costs and state-dependent drift. Wang et al. (2015) apply a dynamic portfolio strategy to hedge idiosyncratic risk. Ormos and Urbán (2013) analyze the performance of log-optimal portfolio strategies under transaction costs. González-Pedraz et al. (2015) introduce conditional copulas and skew preferences to commodity portfolio choices. Nystrup et al. (2017) solve a dynamic portfolio choice problem in hidden market regimes. Most papers in this field study one investor's portfolio strategy. We contribute to the literature by analyzing the dynamics of optimal portfolio weights of two delegated stock and bond managers considering predictable returns on multiple assets.

Finally, to obtain managers' life-cycle dynamic portfolio strategies, we solve the decentralized investment problem by applying the simulation-projection method advocated by Brandt et al. (2005) along with a multidimensional grid search and *policy* iteration. Extending this method to the decentralized structure is not straightforward because of a number of risky assets and the size-induced flow-performance specification. Given the highly path-dependent optimization problem, analytical solutions seem unfeasible and therefore we describe our numerical solution in Appendix A.[5]

The paper proceeds as follows. Section 2 presents the model. Section 3 analyzes portfolios. Section 4 discusses performances. Section 5 concludes.

2. Economic Setup and Investment Problems

2.1. Decentralized Structure and Financial Market

We first introduce a typical decentralized structure and a financial market following van Binsbergen et al. (2008) before we describe our specification for managers' compensations. The Chief Investment Officer (**CIO**) of the asset management company allocates funds between two market portfolios, and then delegates the portfolios to two specialized bond and stock **managers** who have professional knowledge and reallocate capital to multiple assets within their asset classes.

The asset menu includes a government bond index, Baa- and Aaa-rated corporate bond indices for the bond manager, and three stock portfolios ranked on their book-to-market ratios, namely growth, intermediate, and value stocks for the stock manager. The CIO's asset menu includes, e.g., the Standard & Poor's 500 Stock Index and the US Treasury Bond Index, which are also benchmarks for measuring the managers' relative investment performance. In addition, the CIO and managers have access to a riskless cash account.

[5]We provide a package of transparent Matlab programs producing numerical results and robustness checks upon request. In the paper, we present many results obtained by varying key variables, which show the strong robustness of our numerical solution.

The vector X_t of predictors of risk returns include the short rate, the 10-year nominal government bond yield, and the log dividend yield of the equity index. The dynamics of X_t is

$$dX_t = -\kappa_X X_t dt + \sigma_X^\top dz_t, \quad \kappa_X \in \mathbb{R}^{3\times 3}, \quad \sigma_X \in \mathbb{R}^{(2k+3)\times 3}, \tag{1}$$

where $X_t \in \mathbb{R}^{3\times 1}$ and dz denotes a $(2k+3)\times 1$ vector of independent Brownian motions. We consider $k = 3$ risky bonds and $k = 3$ risky stocks. The price of risk, Λ_t, is affine in the state variable X_t:

$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t, \quad \Lambda_0 \in \mathbb{R}^{(2k+3)\times 1}, \quad \Lambda_1 \in \mathbb{R}^{(2k+3)\times 3}. \tag{2}$$

There are $2k + 1$ assets with prices denoted by S_j, $j = 0, \ldots, 2k$. The first asset is a riskless cash account whose price at time t is given by

$$\frac{dS_{0t}}{S_{0t}} = r dt, \tag{3}$$

where r is the constant instantaneous interest rate. The remaining $2k$ assets in the menu are risky assets with the price dynamics

$$\frac{dS_{jt}}{S_{jt}} = (r + \sigma_j^\top \Lambda_t)dt + \sigma_j^\top dz_t, \quad \sigma_j \in \mathbb{R}^{(2k+3)\times 1}, \quad S_{j0} = 1. \tag{4}$$

Equation (4) shows that the expected excess returns $\sigma_j^\top \Lambda_t$ are an affine function of the predictor vector X_t, which captures time variation in risky asset returns. Define the full volatility matrix as $\Sigma \equiv (\Sigma_1, \Sigma_2, \sigma_X)$, where the loadings for the first k assets and the second k assets are $\Sigma_1 = (\sigma_1, \ldots, \sigma_k)$ and $\Sigma_2 = (\sigma_{k+1}, \ldots, \sigma_{2k})$ respectively.

The CIO's investment opportunities consist of cash and benchmark S_t^{Bi}:

$$\frac{dS_t^{Bi}}{S_t^{Bi}} = (r + \bar{\eta}_{Bi})dt + \bar{\sigma}_{Bi} dz_t^{Bi}, \quad S_0^{Bi} = 1, \tag{5}$$

where $i = 1, 2$ refers to bond and stock benchmarks. Following van Binsbergen et al. (2008), $\bar{\eta}_{Bi}$ and $\bar{\sigma}_{Bi}$ are the constant risk premium and standard deviation as the CIO does not have security picking skills. Then the state price density follows

$$\frac{d\varphi_t^B}{\varphi_t^B} = -rdt - \lambda_B dz_t^B, \quad \varphi_0^B = 1, \tag{6}$$

where $\lambda_B = \begin{bmatrix} \frac{\bar{\eta}_{B1}}{\bar{\sigma}_{B1}} & \frac{\bar{\eta}_{B2}}{\bar{\sigma}_{B2}} \end{bmatrix}$ and $z_t^B = \begin{bmatrix} z_t^{B1} & z_t^{B2} \end{bmatrix}^\top$.

2.2. The CIO's Objective

Following the industry practice, we assume that the CIO has a long investment horizon $\hat{T}$ years and acts in the best interests of the beneficiaries of the company. The investment objective is to maximize the expected constant relative risk aversion (CRRA) utility function over the terminal wealth $W_{\hat{T}}$ of the company:

$$\max_{\theta_{c,t} \in \mathcal{K}_{c,t}} \mathbb{E}[u(W_{\hat{T}})] = \max_{\theta_{c,t} \in \mathcal{K}_{c,t}} \mathbb{E}\left(\frac{W_{\hat{T}}^{1-\gamma_c}}{1-\gamma_c}\right), \tag{7}$$

where γ_c denotes the risk aversion level and the portfolio weights $\theta_{c,t} \in \mathbb{R}^{2\times 1}$ are the fractions of wealth invested in the bond and stock markets. The constraint $\mathcal{K}_{c,t}$ includes the standard borrowing and short-selling constraints: $\theta_{c,t} \geq 0$ and $1 - \mathbf{1}^\top \theta_{c,t} \geq 0$, where $\mathbf{1}$ is a 2-dimensional identity vector.

We denote the (gross) benchmark returns by $R_t^B \equiv (R_t^{B1}\ R_t^{B2})^\top$ and the return on cash by R_t^f. The dynamics of wealth with an initial $W_0 = 1$ is:

$$W_{t+1} = W_t[\theta_{c,t}^\top (R_{t+1}^B - \mathbf{1} R_{t+1}^f) + R_{t+1}^f]. \tag{8}$$

2.3. The Managers' Objectives

We introduce a *size-induced* fund-flow model to capture a significant empirical fact and to study controversial scale effects on funds. The prior models assume that fund flows depend only on fund performance (e.g. Basak and Makarov, 2014; Koijen, 2014). Nevertheless, empirical studies highlight the fact that the fund size is one of the key determinants of fund flows and they report controversial scale effects on fund performance (e.g. Jain and Wu, 2000; Pollet and Wilson, 2008). Motivated by these findings, we model the effects of both fund performance and fund sizes on fund flows, which reflects the practice that managers are concerned about not only their performances and compensations at the current year but also the historical realizations of their fund sizes driven by their portfolio weights. Therefore, our setup allows us to investigate the scale effects on the portfolio dynamics, fund performance, and management fees.

Denote the portfolio returns and benchmark returns for the manager i by R_t^{Ai} and R_t^{Bi}. If the relative performance R_t^{Ai}/R_t^{Bi} is higher than a threshold $\eta > 1$, the manager receives a *fund inflow* at a rate $f_t > 0$, which depends on both relative performance and fund size. The fund size is defined as the natural logarithm of one plus the TNA in the previous year, following the prevailing literature. By contrast, a manager who performances poorly incurs a *fund outflow* at a rate $f_t < 0$, which is sizable to the fund size.

$$f(R_t^{Ai}, R_t^{Bi}, TNA_{i,t-1}) = \begin{cases} \psi_0 + \psi_1 R_t^{Ai}/R_t^{Bi} + \psi_2 \log(TNA_{i,t-1}+1), & \text{if } R_t^{Ai}/R_t^{Bi} > \eta, \\ \psi_3 \log(TNA_{i,t-1}+1), & \text{if } R_t^{Ai}/R_t^{Bi} \leq \eta, \end{cases} \tag{9}$$

where ψ_0 to ψ_3 are parameters.

Denote the portfolios (risk exposures) by $\theta_{i,t} \in \mathbb{R}^{k\times 1}$ for the manager i, whose investment objective is to maximize the compensation as follows.

$$\max_{\theta_{i,t}\in\mathcal{K}_t} \mathbb{E}[u(Y_{iT})] = \max_{\theta_{i,t}\in\mathcal{K}_t} \mathbb{E}\left(\frac{Y_{iT}^{1-\gamma_i}}{1-\gamma_i}\right), \tag{10}$$

subject to the *path-dependent* dynamic compensation scheme Y_{iT}:

$$\begin{aligned} Y_{iT} &= \underbrace{\kappa_0 TNA_{i,T-1}}_{\text{base salary}} + \underbrace{\kappa_1(TNA_{i,T-1}R_T^{Ai} + TNA_{i,T-1}f(R_T^{Ai}, R_T^{Bi}, TNA_{i,T-1}))}_{\text{variable component (bonus)}} \\ &= TNA_{i,T-1}[\kappa_0 + \kappa_1(R_T^{Ai} + f(R_T^{Ai}, R_T^{Bi}, TNA_{i,T-1}))], \end{aligned} \tag{11}$$

where κ_0 to κ_1 are parameters. The TNA is largely determined by the previous TNA and the optimal portfolios. In turn, the fund size and fund flow affect the manager's portfolio through our dynamic compensation scheme. The constraint $\mathcal{K}_t$ includes the borrowing and short-selling constraints and a risk limit $\sigma_{i,t} = \sqrt{\theta_{i,t}^\top \Sigma_i \Sigma_i^\top \theta_{i,t}} \leq 1.25\bar{\sigma}_{Bi}$ that prevents the manager from deliberately taking high risk for the size-dependent compensation.

Note that our compensation scheme nonlinearly depends on the dynamic fund size in the prior period and hence is path dependent. This is in contrast to Koijen (2014), where the compensation is linearly proportional to the initial size $TNA_{i,0}$ that can be normalized to one. Since the compensation here is nonlinear and path dependent, an analytical solution to the optimization problem seems unfeasible.

Therefore, we implement a simulation-projection method with a multidimensional grid search and policy iteration to obtain the CIO and managers portfolio strategies, see Appendix A. Within the decentralized investment structure, we can solve their optimization problems separately. After solving the CIO's problem (7), we solve each manager's problem (10) with $k = 3$

risky assets instead of $2k$ assets. If a decentralized structure was not adopted, it would be infeasible to solve a highly path-dependent dynamic optimization problem with $2k = 6$ assets due to the serious 'curse of dimension'.

2.4. Model Calibration and Parameter Values

We take the values of standard parameters from recent studies on institutional investors' optimal portfolio choices and calibrate some parameters to the targets that are consistent with prior studies.

We use the estimations of van Binsbergen et al. (2008) for the *time-varying* financial market, see Table 1 here. The first six sources of risk are due to the risky asset prices S_t in (4) and the last three sources come from the state variables X_t in (1). The short rate is negatively correlated with the expected returns of all assets, except for government bonds. The 10-year bond yield negatively impacts on the expected stock returns, while it positively impacts on the expected bond returns. The dividend yield is positively correlated with the expected returns of all assets. The autoregressive parameters κ_X reflect the high persistence of the state variables.

Table 2 reports the instantaneous asset correlations and expected returns implied by the model. The expected return spread between the Baa's 0.096 and the Aaa's 0.087 is 0.9% in the fixed income class while it reaches up to 5% between the value stocks' 0.148 and the growth stocks' 0.098 in the equity class. The distinctive return spreads indicate homogeneous (heterogeneous) expected returns on the bonds (stocks) that largely affect managers' portfolio dynamics discussed later. In addition, the correlations within asset classes are high and range between 80% and 93%.

Table 1. Time-Varying Investment Opportunities

Sources of Risk	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
Λ_0	0.306	0.409	-0.020	0.089	0.498	0.310	0	0	0
			$\Sigma\Lambda_1$						
	Gov	Baa	Aaa	Growth	Interm	Value		κ_X	
Short rate	0.227	-0.964	-0.209	-0.270	-0.249	-0.012		0.36	
10Y yield	1.269	1.225	0.893	-0.778	-1.086	-1.010		0.12	
Div yield	0.020	0.071	0.038	0.132	0.121	0.130		0.052	
Σ	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
Gov. bonds	13.2%	0	0	0	0	0	0	0	0
Corp. bonds, Baa	7.7%	5.4%	0	0	0	0	0	0	0
Corp. bonds, Aaa	8.7%	2.6%	2.4%	0	0	0	0	0	0
Growth stocks	3.1%	5.8%	0.2%	16.5%	0	0	0	0	0
Int. stocks	2.9%	6.2%	0.1%	11.7%	7.2%	0	0	0	0
Value stocks	2.8%	7.1%	-0.2%	10.4%	6.7%	5.8%	0	0	0
Short rate	-1.1%	-0.1%	0.0%	0.3%	-0.1%	-0.1%	2.3%	0	0
10Y yield	0.0%	0.0%	0.0%	0.1%	0.1%	0.0%	0.0%	1.3%	0
Div yield	-3.0%	-6.7%	0.1%	-14.0%	-2.5%	-0.9%	0.0%	0.6%	4.7%

The parameter values are taken from van Binsbergen et al. (2008) in annual terms. The asset menu includes government bonds, Baa- and Aaa-rated corporate bonds, and three stocks ranked on their book-to-market ratios, namely growth, intermediate, and value stocks. The short rate, the 10-year government bond yield and the dividend yield are the three state variables. There are nine sources of risk in the model. The first six risks are due to the risky asset prices S_t in (4), and the last three risks are due to the state variables X_t in (1).

Table 2. Implied Parameters.

	Returns	Correlations					
Gov. bonds	0.090	1					
Corp. bonds, Baa	0.096	0.82	1				
Corp. bonds, Aaa	0.087	0.93	0.92	1			
Growth stocks	0.098	0.17	0.33	0.25	1		
intermediate stocks	0.131	0.19	0.39	0.29	0.87	1	
Value stocks	0.148	0.18	0.41	0.29	0.80	0.92	1

This table gives the expected asset returns and their correlations implied by the model using the parameter values from van Binsbergen et al. (2008). The assets include government bonds, Baa- and Aaa-rated corporate bonds, and three stocks ranked on their book-to-market ratios.

Table 3. Calibrated Parameters for Compensation and Benchmarks

	Symbol	Value		Symbol	Value
Coefficient of fund flows	ψ_0	-0.79	Interest rate	r	5%
Coefficient of fund flows	ψ_1	0.97	Pay-for-performance	κ_0	0.5%
Coefficient of fund flows	ψ_2	-0.04	Pay-for-performance	κ_1	1.4%
Coefficient of fund flows	ψ_3	-0.07	Performance threshold	η	1.015
Bond Benchmark			Stock Benchmark		
Risk premium	$\bar{\eta}_{B1}$	3.1%	Risk premium	$\bar{\eta}_{B2}$	5.4%
Volatility	$\bar{\sigma}_{B1}$	9.5%	Volatility	$\bar{\sigma}_{B2}$	13%

This table summarizes the calibrated parameter values for managers' compensation scheme and benchmarks in the decentralized investment structure. The text presents more details.

Table 3 summarizes the calibrated parameters for the compensation scheme and benchmarks. The two values of κ_0 and κ_1 are calibrated by two conditions. First, they target a reasonable pay-for-performance sensitivity, i.e. $\kappa_1/\kappa_0 = 2.28$, which is within the range of the 50^{th} quantile and the 75^{th} quantile of estimates in Koijen (2014). Second, they make the model produce an average expense ratio ranging between 1.8% – 1.85% of the TNA, which is in line with the leading practice (e.g. Huang et al., 2007).

About the calibration of fund flow coefficients, ψ_0 and ψ_1 are almost from Koijen (2014), where ψ_1 has to be large enough to ensure that there is inflow $f_T > 0$ when fund performance is higher than the relative performance threshold η. The empirical negative relationship between fund sizes and fund flows (e.g Pollet and Wilson, 2008) determines a negative ψ_2. The value of ψ_3 is calibrated by the relationship that a 1% of fund size in the case of underperformance results in a 0.07% of fund outflows. These settings produce average inflows of 9% – 13% for bond funds and 11% – 18% for equity funds that are consistent with the practice. The risk premiums ($\bar{\eta}_{B1}$ and $\bar{\eta}_{B2}$) and volatilities ($\bar{\sigma}_{B1}$ and $\bar{\sigma}_{B2}$) of bond and stock benchmarks are calibrated to obtain the corresponding Sharpe ratios 32.6% and 41.5%. The riskless cash pays an interest rate $r = 5\%$ per annum following the setup in van Binsbergen et al. (2008) who use this rate and data from January 1973 to November 2004 to determine the unconditional instantaneous price of risk, Λ_0, whose value is taken as an input parameter in our computation. Although this value of r seems high for the decade after the 2008 financial crisis, it is reasonable as it is near both mean and median of the US 3-month treasury bill during the history since 1973 until 2017.

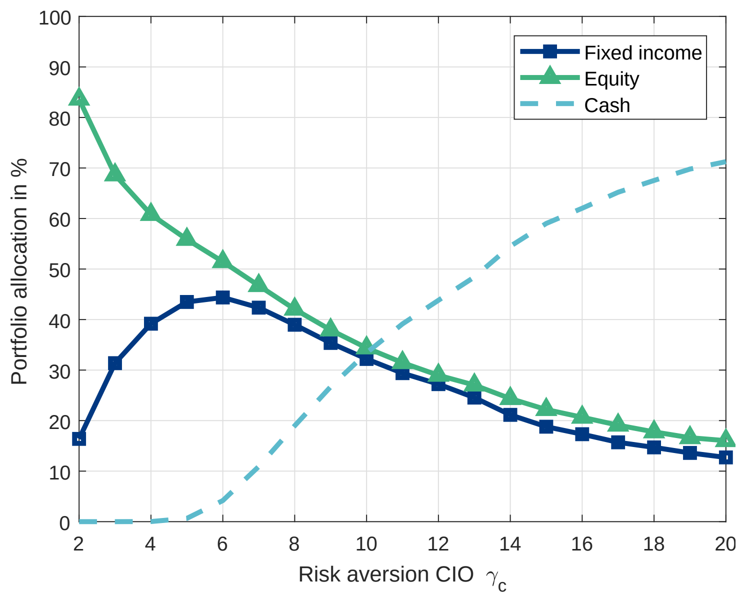


Figure 1. The CIO's optimal portfolio allocations.
The figure displays the CIO's optimal allocations to fixed-income and equity asset classes versus the risk aversion. The allocation to cash is one minus the sum of these quantities. The crosses of lines are expected as the risk aversion largely influences portfolio choices.

3. Dynamics of Decentralized Optimal Portfolios

3.1. The CIO's Optimal Portfolio

Figure 1 plots the optimal allocation of the CIO who has a $\hat{T} = 10$ years horizon and a risk aversion level γ_c varying from 2 to 20. The circle, triangle, and dashed lines represent the allocations to the fixed-income, equity, and cash account.

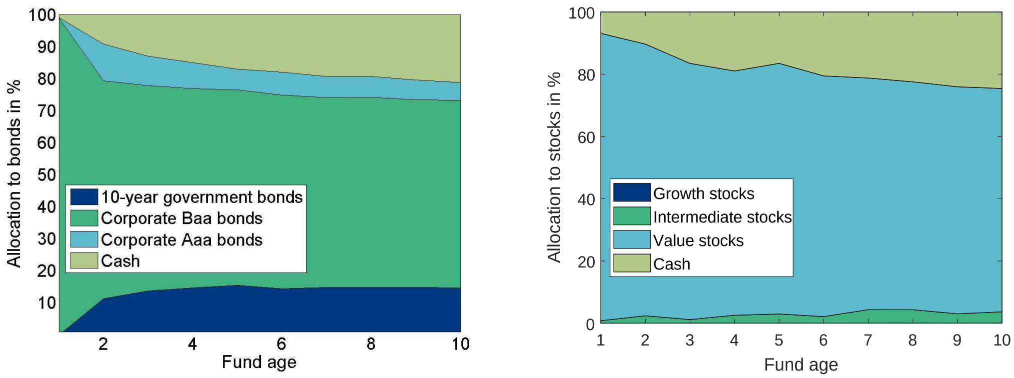
The stock allocation decreases with the risk aversion, but the bond allocation exhibits a hump-shaped pattern. When the CIO has a moderate degree of risk aversion around $\gamma_c = 5$, the bond allocation approaches the peak, though it is still 10% less than the stock allocation. Meanwhile, the CIO starts to raise cash holdings around the risk aversion $\gamma_c = 5$. Therefore, we choose a moderate $\gamma_c = 5$ throughout the paper to highlight both bond and stock managers' portfolio dynamics respectively.

Note that the CIO affects the managers' portfolios by allocating initial wealth to the managers once according to the CIO's optimal policy. Given the initial wealth, the managers trade forward in time and determine their optimal policies depending on the realized TNA and state variables. In short, through deciding the initial funds for each market, the CIO influences the managers' portfolio dynamics.

3.2. The Managers' Optimal Portfolios

The managers use the allocations from the CIO to make portfolios during the life-cycles of funds. Figure 2 illustrates the portfolio dynamics of the bond (stock) manager as the functions of the fund age in the left (right) panel.

The bond portfolio is well diversified over the life cycle. In year one, the bond manager exclusively allocates wealth to corporate Baa-rated bonds, which offer the highest expected returns (9.6%). The underlying economic mechanism is that the manager wants to beat the benchmark for attracting more fund inflows at early stages. This mechanism implies a positive relationship between fund flows and the allocation to Baa-rated bonds. When the fund getting older, the manager diversifies the portfolio by holding 15% 10-year government bonds, 5% corporate Aaa-rated bonds, 20% cash and meanwhile keeps a substantial 60% holdings in Baa-rated bonds. The underlying reasons are homogeneous expected returns on the bonds and the slightly high expected return on Baa-rated bonds, see Table 2.



(a) The Bond Portfolio (b) The Stock Portfolio

Figure 2. The asset managers' optimal portfolios as a function of the fund age.

The bond manager (a) and stock manager (b) implement their optimal portfolios depending on realized wealth levels and state variables after obtaining the CIO's initial fund allocation.

The stock manager takes a less diversified portfolio with predominant value stocks, though the allocation to value stocks decreases slightly over the life cycle. The manager allocates little capital to growth stocks because of their low returns yet high volatilities. In the end, the portfolio on average consists of about 71% value stocks, 4% intermediate stocks, 25% cash, and slight growth stocks. The underlying economic force is that the stock manager faces markedly heterogeneous expected returns ranging from 9.8% (growth stocks) to 14.8% (value stocks). Thus, the large allocation in value stocks can potentially bet the benchmark and can attract more cash inflows.

The equity portfolio comprises value, intermediate, and growth stocks, although the value stocks take a large proportion since the parameters for the time-varying market come from the estimations of van Binsbergen et al. (2008) based on the data from January 1973 to November 2004 when the expected return spread between the value stocks (14.8%) and the growth stocks (9.8%) is large at 5%. We fix the parameters at the estimations for the ten-year investment horizon following the practice of van Binsbergen et al. (2008). A ten-year horizon is shorter than the average fund age of 15.7 years reported by Chen et al. (2004). A shorter horizon does not change the large allocation to the value stocks given the parameters that we use, as shown in Figure 2 for the fund ages from 1 to 10 years. Admittedly, the growth stocks rather than the value stocks can be dominant during some short periods of time but integrating this case into the model will lead to a more complicated dynamics portfolio model with Bayesian updates of parameter values, which is beyond the scope of this paper. We study this interesting topic in another paper, considering the complexity of the current model with the decentralized structure with size-induced fund flows.

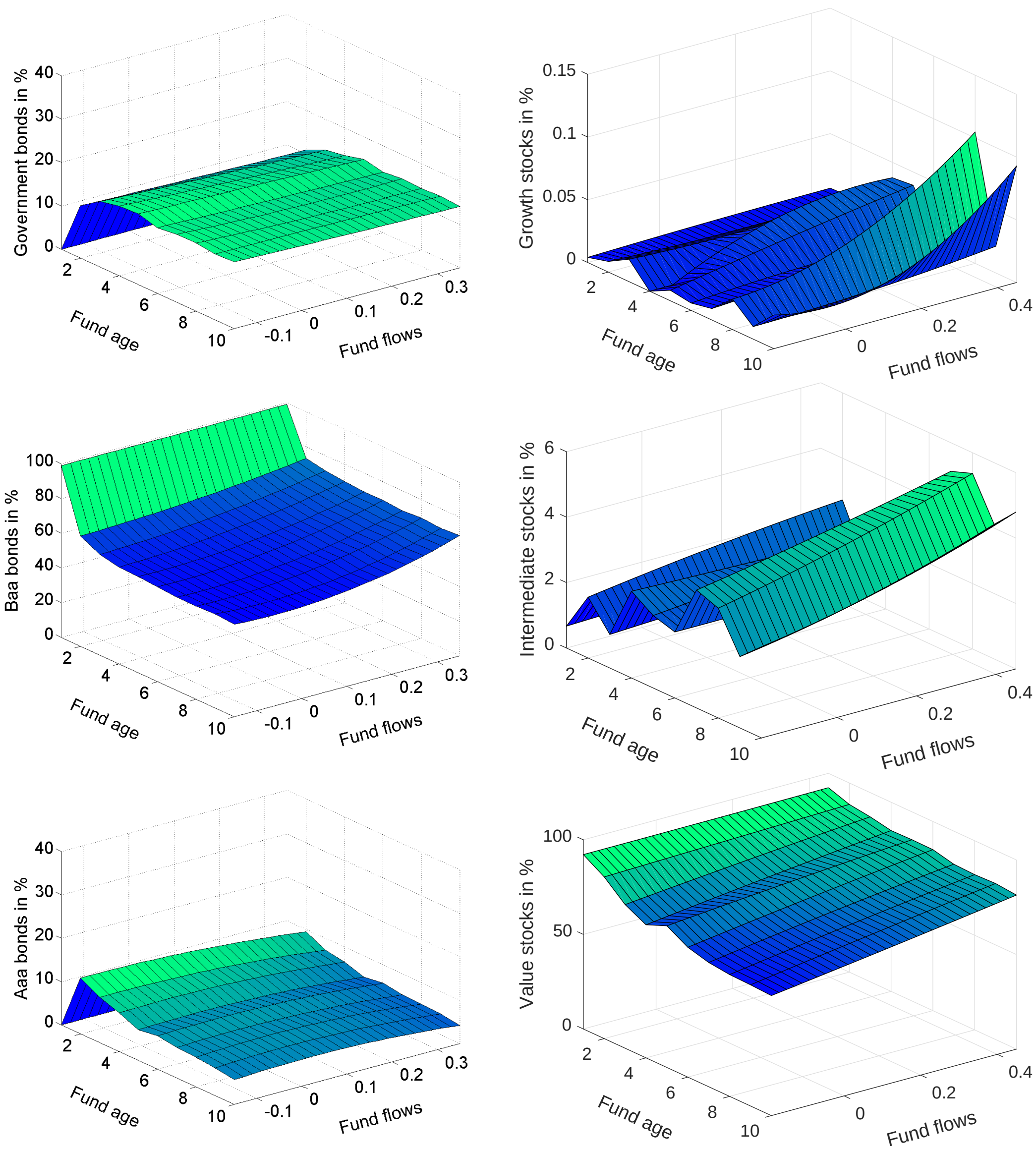
Meanwhile, the increasing cash holdings in the two sub-figures implies that both managers are reluctant to put their reputations at risk and becomes more conservative when the fund becomes old. The career concern is the underlying driving force. Previous studies on executive compensations point out that asset managers with greater career concerns become more conservative and demand higher incentives to take risky strategies (e.g. Nohel and Todd, 2005).

3.3. Scale Effects on Portfolio Dynamics

To examine the scale effects of fund flows on portfolio dynamics, Figure 3 displays the optimal allocations to bonds (stocks) in response to the changes in fund flows over the bond (stock) manager's life cycle in the left (right) panel. The figures are constructed by projecting the optimal allocations along all paths on a second-order polynomial expansion in the state space.

A key observation is that the allocation to Baa-rated bonds increases considerably in response to high fund inflows. For instance, at year six, the optimal allocation to Baa-rated bonds rises by about 10% when fund flows increase from -0.15 to 0.35. This feature confirms the aforementioned economic mechanism for the large allocation to Baa-rated bonds as well as the positive relationship between such allocation and fund flows. Another observation is that the allocation to 10-year government bonds is hardly affected by fund flows, because of the potential high mismatched risk from long-maturity bonds and short-term compensations.

About the stock portfolios, the three sub-figures in Panel (b) of Figure 3 show that a rise in fund flows induces an increase in the holdings of growth, intermediate, and value stocks. The relative changes of the growth stocks and the intermediate stocks are noticeable, although their increases are small compared with the increase in the value stocks. This is expected since the expected returns of value stocks are dominant in the history data that are used for the parameter estimations, as discussed in Section 3.2. Similarly, the slight fluctuations of portfolio weights in the growth stocks and the intermediate stocks along the dimension of the fund age are reasonable considering the small portfolio weights that are not closely related with the fund age.



(a) Bond allocations

(b) Stock allocations

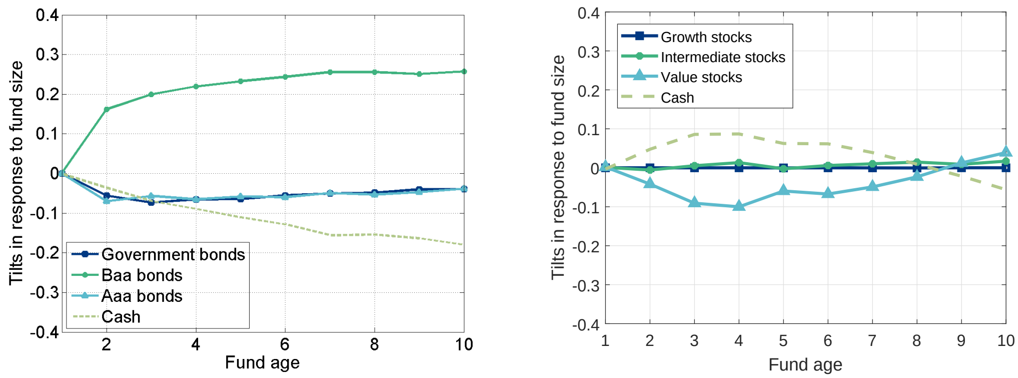
Figure 3. Portfolio dynamics versus fund flows over the life cycle. This figure depicts the portfolios dynamics of the bond manager (a) and the stock manager (b) in response to changes in fund flows over the life cycle of the fund. We project the optimal allocations along all paths on a second-order polynomial expansion in the state space.

Furthermore, the magnitudes of the increases in stock holdings become larger as the fund age increases. For instance, the flow-induced allocation spread for value stocks is 3% in year two, while the spread increases to 18% in year ten. This result reveals that a more experienced manager is more sensitive to changes in fund flows. The underlying economic force is that when the fund gets older, the manager has a greater incentive to hedge against risk resulting from fund flow variation. Thus, the risk-averse manager makes a larger adjustment in the holdings of value stocks that offer the highest expected returns.

To reveal the effects of the TNA sizes on portfolio dynamics, Figure 4 presents the bond (stock) allocation sensitivities in response to changes in TNA sizes in the left (right) panel. We measure the tilts in allocations as the difference in the optimal weights when the TNA size varies from its 25^{th} quantile to 75^{th} quantile at a time point. A positive (negative) tilt indicates an increase (decrease) in the asset weight if the fund size increases. The portfolio weight of Baa-rated bonds increases in fund size whereas the portfolio weight of value stocks reduces. These results imply that the fund size matters to bond and stock funds, which will be analyzed later.

4. Implications for Scale Effects

We evaluate the scale effects of fund size through two perspectives: directly measuring the relationship between the TNA and investment returns, and indirectly measuring the relationship between the TNA and management fees. To demonstrate that our model captures these interesting relations, we use a cross-sectional regression approach to the data produced by the model.



(a) Bond tilts (b) Stock tilts

Figure 4. Tilts in optimal portfolios induced by changes in TNA sizes. This figure depicts the tilts in the optimal portfolios in bonds (a) and stocks (b) in response to changes in TNA sizes. We measure the tilts in allocations as the difference in the optimal weights when the TNA size varies from its 25^{th} quantile to 75^{th} quantile. A positive (negative) tilt indicates an increase (decrease) in the asset weight if the TNA size increases.

The fund's gross investment returns (or management fees) are regressed on various one-year-lagged fund characteristics as follows:

$$\begin{aligned} FUNDRET_{i,t} &= \alpha_i + \beta_{i,1} LOGTNA_{i,t-1} \\ &+ \beta_{i,2}(LOGTNA_{i,t-1} - AVGLOGTNA_{i,t-1})^2 \\ &+ \beta_{i,3} FLOW_{i,t-1} + \beta_{i,4} MANAGFEE_{i,t-1} \\ &+ \beta_{i,5} LAGFUNDRET_{i,t-1} + \epsilon_{i,t}. \end{aligned} \tag{12}$$

$$\begin{aligned} MANAGFEE_{i,t} &= \alpha_i + \beta_{i,1} LOGTNA_{i,t-1} \\ &+ \beta_{i,2}(LOGTNA_{i,t-1} - AVGLOGTNA_{i,t-1})^2 \\ &+ \beta_{i,3} FLOW_{i,t-1} + \beta_{i,4} FUNDRET_{i,t-1} \\ &+ \beta_{i,5} LAGMANAGFEE_{i,t-1} + \epsilon_{i,t}. \end{aligned} \tag{13}$$

Following the literature, we define the variables as follows. $FUNDRET_{i,t}$ and $MANAGFEE_{i,t}$ are the gross returns and management fees of fund i in year t. The latter is a ratio of the compensation in (11) to the TNA. $LOGTNA_{i,t-1}$ measures the fund size, which is the natural logarithm of one plus the TNA. $AVGLOGTNA_{i,t-1}$ is the average size. $FLOW_{i,t-1}$ is the rate of fund flows in (9). $LAGFUNDRET_{i,t-1}$ and $LAGMANAGFEE_{i,t-1}$ are prior-period returns and management fees.

4.1. Scale Effects on Fund Performance

There is a debate about the controversial empirical findings on the relationship between fund sizes and fund performances (e.g. Carter, 1950; Chen et al., 2004; Pollet and Wilson, 2008). To analyze this relationship, we calculate 30 sets of 4,000 trajectories of the 10-year investment process. For each set, we estimate the cross-sectional regression (12) on the data produced by the model and then report the averages over the life cycle.

Table 4 reports the cross-sectional distribution of the t-statistics for the estimates of fund characteristics following the approach in the literature. We find that the fund size is *positively* associated with bond returns (Panel A), while it is highly *negatively* associated with stock returns (Panel B). The underlying reason for the distinct scale effects on bond and stock funds is the relatively homogeneous expected returns on bonds compared to stocks, see Table 2 where the expected return spread in bonds is just 0.9%. Since the three bonds are closer substitutes to each other, large bond funds are not be hampered by large market positions and therefore they realize scale economies (Philpot et al., 1998).

Table 4. Quantiles of the T-Statistics of Fund Performance

	Panel A: T-statistics for Bond Fund Performance						
	5%	10%	25%	50%	75%	90%	95%
$INTERCEPT$	9.965	10.597	11.326	12.167	13.029	13.925	14.414
$LOGTNA_{i,t-1}$	0.562	0.849	1.43	2.221	3.077	3.562	4.019
$(LOGTNA - AVGLOGTNA)^2$	-1.266	-0.999	-0.362	0.34	1.046	1.684	2.02
$FLOW_{i,t-1}$	-1.72	-1.427	-0.879	-0.113	0.694	1.199	1.434
$MANAGFEE_{i,t-1}$	-1.2	-0.777	-0.163	0.505	1.206	1.776	2.086
$LAGFUNDRET_{i,t-1}$	3.153	3.344	3.927	4.62	5.167	5.761	6.088
	Panel B: T-statistics for Stock Fund Performance						
	5%	10%	25%	50%	75%	90%	95%
$INTERCEPT$	12.02	12.432	13.172	14.222	15.042	15.9	16.364
$LOGTNA_{i,t-1}$	-5.449	-5.275	-4.708	-4.073	-3.405	-2.737	-2.346
$(LOGTNA - AVGLOGTNA)^2$	0.174	0.419	1.028	1.733	2.388	2.942	3.321
$FLOW_{i,t-1}$	0.742	1.063	1.595	2.23	2.884	3.385	3.815
$MANAGFEE_{i,t-1}$	-1.687	-1.446	-0.813	-0.14	0.56	1.164	1.486
$LAGFUNDRET_{i,t-1}$	-2.13	-1.755	-1.326	-0.73	-0.019	0.58	0.952

This table reports the quantiles of the t-statistics for the estimates of fund returns regressed on the lagged fund size and other characteristics. The dependent variable is the fund gross returns $FUNDRET_{i,t}$. $LOGTNA_{i,t-1}$ is the proxy of fund size defined as the natural logarithm of one plus the total net assets of fund i, where $i = 1, 2$ for bond and stock funds respectively. $(LOGTNA_{i,t-1} - AVGLOGTNA_{i,t-1})^2$ is the quadratic term. All independent variables are lagged one-year. $FLOW_{i,t-1}$ is the percentage of investment flows for fund i over the past one year as in (9). The management fee $MANAGFEE_{i,t-1}$ is a ratio of fund total expenses to the TNA. In our model, the expenses mainly come from the administrative expenses, i.e. the fund manager's annual compensations $Y_{i,t}$ as defined in (11). $LAGFUNDRET_{i,t-1}$ is the prior-period fund returns. The general form of the regression is:

$$\begin{aligned} FUNDRET_{i,t} &= \alpha_i + \beta_{i,1} LOGTNA_{i,t-1} + \beta_{i,2}(LOGTNA_{i,t-1} - AVGLOGTNA_{i,t-1})^2 \\ &\quad + \beta_{i,3} FLOW_{i,t-1} + \beta_{i,4} MANAGFEE_{i,t-1} + \beta_{i,5} LAGFUNDRET_{i,t-1} + \epsilon_{i,t}. \end{aligned}$$

We calculate 30 sets of 4,000 trajectories of the optimization process for 10 years. For each of the 30 sets of trajectories, we estimate the above regression on the simulated data, and then report the average over the 10-year life cycle of fund i. Panel A and B report the cross-sectional distribution of the t-statistics for bond and stock funds respectively.

In contrast, the scale effect erodes stock fund performance. In Panel B, the values of t-statistics of $LOGTNA$ are significantly negative and the values of the quadratic term $(LOGTNA - AVGLOGTNA)^2$ are positive. These results indicate that when the fund size increases, stock fund returns decline at an increasing rate, which is in line with the empirical results (e.g. Pollet and Wilson, 2008). The different scale effects also support that multiple assets funds can avoid the cost disadvantages of large stock funds.[6]

The stock manager attracts capital inflows by beating the benchmark. The regression provides a strong support for this conjecture: the prior-period fund flows are significantly and positively related to the subsequent stock fund returns. In particular, the t-statistic of the term $FLOW$ in the middle quantile is 2.23, which is significant at the 5% level. This result implies that a stock manager who performs better can distinguish herself from her counterparts by selecting superior stocks and attracts more cash inflows. The t-statistics of the term $FLOW$ for the bond fund, however, is not evident because of the close expected returns on bond investment opportunities.

4.2. Fund Performance and Fund Ages

The bond fund's returns are significantly positively related to the past returns but the stock fund does not show significant autocorrelation. We conjecture that the autocorrelation is canceled out by two opposite underlying forces across stock fund ages. To test this conjecture, we split the funds into young and old subgroups using year six as the threshold in Table 5.

[6] Cummings (2016) reports that a fund with allocations to multi-assets captures scale economies that are not captured by single-asset funds.

Table 5. Main Results for Fund Performance of Young and Old Funds

Panel A: Quantiles of the T-statistics for Bond Fund Performance

	YOUNG					OLD				
	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
$INTERCEPT$	9.999	11.227	11.943	12.932	14.159	11.852	13.263	14.139	14.936	16.379
$LOGTNA_{i,t-1}$	-0.401	0.837	1.33	2.145	3.263	2.712	3.71	4.31	5.078	5.739
$(LOGTNA - AVGLOGTNA)^2$	-1.114	-0.328	0.366	1.177	2.05	-1.456	-0.406	0.308	0.881	1.982
$FLOW_{i,t-1}$	-1.642	-0.777	-0.014	0.839	1.446	-1.817	-1.005	-0.236	0.514	1.419
$MANAGFEE_{i,t-1}$	-1.31	-0.261	0.407	1.027	1.876	-1.063	-0.041	0.628	1.429	2.348
$LAGFUNDRET_{i,t-1}$	2.189	2.962	3.602	4.136	4.909	4.357	5.134	5.892	6.456	7.56

Panel B: Quantiles of the T-statistics for Stock Fund Performance

	YOUNG					OLD				
	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
$INTERCEPT$	12.484	13.817	14.553	15.376	16.533	13.799	15.045	16.021	17.226	18.967
$LOGTNA_{i,t-1}$	-7.581	-6.694	-6.133	-5.51	-4.679	-5.462	-4.389	-3.739	-2.918	-2.048
$(LOGTNA - AVGLOGTNA)^2$	0.336	1.03	1.668	2.372	3.389	-0.028	1.026	1.815	2.407	3.237
$FLOW_{i,t-1}$	0.94	1.666	2.277	3.002	3.728	0.495	1.507	2.172	2.738	3.924
$MANAGFEE_{i,t-1}$	-1.945	-0.935	-0.252	0.387	1.157	-1.364	-0.661	-0.001	0.775	1.897
$LAGFUNDRET_{i,t-1}$	-3.516	-2.697	-2.166	-1.509	-0.611	-0.398	0.387	1.066	1.843	2.906

The table reports similar results as in Table 4, but separately for young and old funds. Young funds are the funds whose ages are equal to or below six years old, whereas old funds are more than six years old. We calculate 30 sets of 4,000 trajectories of the optimization process for 10 years. For each of the 30 sets of trajectories, we estimate the following regression on the simulated data and then report the averages for young and old funds. Panel A and B report the cross-sectional distribution of the t-statistics for bond and stock funds respectively. The general form of the regression is:

$$\begin{aligned} FUNDRET_{i,t} &= \alpha_i + \beta_{i,1} LOGTNA_{i,t-1} + \beta_{i,2}(LOGTNA_{i,t-1} - AVGLOGTNA_{i,t-1})^2 \\ &\quad + \beta_{i,3} FLOW_{i,t-1} + \beta_{i,4} MANAGFEE_{i,t-1} + \beta_{i,5} LAGFUNDRET_{i,t-1} + \epsilon_{i,t}. \end{aligned}$$

Young stock funds' returns are negatively correlated with their historical returns. The median of the t-statistics of young stock funds' lagged returns $LAGFUNDRET$ is -2.166. Suppose that young stock fund returns are hit by a positive shock. Then their next year returns are expected to decrease and they will attract less capital. The negative autocorrelation suggests that the managers of young funds lack experience and abilities in dealing with heterogeneous stock returns and volatile fund flows during start-up periods. This implication is also illustrated in Panel B of Figure 3 where the tilt of allocation to value stocks induced by fund flows in year two are small.

By contrast, the median of the t-statistics of $LAGFUNDRET$ for old stock funds is positive at 1.066. The opposite autocorrelations in stock fund returns suggest that it is important to control for fund ages when examining fund managers' investment abilities and the characteristics of funds.

4.3. Scale Effects on Management Fees

We investigate the impact of fund sizes on management fees calculated as a ratio of managers' compensations to funds' TNAs. The management fees are regressed against fund characteristics including the fund size in (13).

The results in Table 6 show that the t-statistics of the size term $LOGTNA$ is positive and statistically significant, implying that the management fees of both bond and stock funds increase with the fund size. This increasing pattern is reasonable since managers' compensations are proportional to the sizes of funds that they manage. As (10) indicates, managers have the incentives to maximize their compensations by making optimal portfolios. Such incentives lead to higher management fees as managers achieve more compensations in exchange for better professional management.

Table 6. Quantiles of the T-Statistics of Management Fees

	Panel A: T-statistics for Bond Management Fees						
	5%	10%	25%	50%	75%	90%	95%
$INTERCEPT$	17.088	18.166	19.416	20.917	22.498	23.8	24.629
$LOGTNA_{i,t-1}$	2.497	2.888	3.458	4.319	5.084	5.81	6.087
$(LOGTNA - AVGLOGTNA)^2$	0.824	1.152	1.722	2.492	3.378	4.017	4.449
$FLOW_{i,t-1}$	3.312	3.703	4.361	5.272	6.078	6.847	7.478
$FUNDRET_{i,t-1}$	-1.196	-0.977	-0.342	0.388	1.123	1.8	2.421
$LAGMANAGFEE_{i,t-1}$	5.819	6.257	6.916	7.851	8.883	9.907	10.536
	Panel B: T-statistics for Stock Management Fees						
	5%	10%	25%	50%	75%	90%	95%
$INTERCEPT$	20.655	21.392	22.749	24.014	25.573	27.024	27.658
$LOGTNA_{i,t-1}$	7.453	7.754	8.403	9.095	9.729	10.378	10.71
$(LOGTNA - AVGLOGTNA)^2$	-1.947	-1.608	-0.941	-0.192	0.593	1.322	1.542
$FLOW_{i,t-1}$	-0.792	-0.464	0.22	0.929	1.638	2.284	2.563
$FUNDRET_{i,t-1}$	0.966	1.348	1.918	2.651	3.342	3.821	4.109
$LAGMANAGFEE_{i,t-1}$	3.093	3.345	4.008	4.822	5.702	6.442	6.807

This table reports the quantiles of the t-statistics for the estimates of fund management fees regressed on the lagged fund size and other characteristics. The dependent variable is the total management fee $MANAGFEE_{i,t}$ calculated as investment expenses over the total net assets of fund i, where $i = 1, 2$ for bond and stock funds respectively. In our model, the expenses mainly come from the administrative fees, i.e. the fund manager's annual compensations $Y_{i,t}$ as defined in (11). $LOGTNA_{i,t-1}$ is the proxy of fund size defined as the natural logarithm of one plus the total net assets of fund i. $(LOGTNA_{i,t-1} - AVGLOGTNA_{i,t-1})^2$ is the quadratic term. All independent variables are lagged one-year. $FLOW_{i,t-1}$ is the percentage of investment flows for fund i over the past one year as in (9). $FUNDRET_{i,t-1}$ is the fund gross return. $LAGMANAGFEE_{i,t-1}$ is the prior-period management fee. The general form of the regression is:

$$\begin{aligned} MANAGFEE_{i,t} &= \alpha_i + \beta_{i,1} LOGTNA_{i,t-1} + \beta_{i,2}(LOGTNA_{i,t-1} - AVGLOGTNA_{i,t-1})^2 \\ &\quad + \beta_{i,3} FLOW_{i,t-1} + \beta_{i,4} FUNDRET_{i,t-1} + \beta_{i,5} LAGMANAGFEE_{i,t-1} + \epsilon_{i,t}. \end{aligned}$$

We calculate 30 sets of 4,000 trajectories of the optimization process for 10 years. For each of the 30 sets of trajectories, we estimate the above regression on the simulated data, and then report the average over the 10-year life cycle of fund i. Panel A and B report the cross-sectional distribution of the t-statistics for bond and stock funds' management fees respectively.

Table 7. Main Results for Management Fees of Young and Old Funds

	Panel A: Quantiles of the T-statistics for Bond Management Fees									
	YOUNG					OLD				
	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
$INTERCEPT$	18.774	20.726	22.216	23.51	25.589	18.285	20.991	22.714	24.315	26.799
$LOGTNA_{i,t-1}$	3.13	4.35	5.001	5.661	6.612	5.977	7.225	7.856	8.78	9.534
$(LOGTNA - AVGLOGTNA)^2$	-0.128	0.782	1.684	2.576	3.57	2.015	2.898	3.501	4.38	5.547
$FLOW_{i,t-1}$	1.666	2.622	3.519	4.254	5.722	5.368	6.536	7.464	8.359	9.672
$FUNDRET_{i,t-1}$	-0.9	-0.025	0.674	1.324	2.382	-1.567	-0.738	0.029	0.872	2.469
$LAGMANAGFEE_{i,t-1}$	4.136	5.092	5.979	6.819	8.054	7.923	9.195	10.19	11.464	13.639

	Panel B: Quantiles of the T-statistics for Stock Management Fees									
	YOUNG					OLD				
	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
$INTERCEPT$	21.568	23.531	24.535	26.211	28.421	22.407	25.277	26.998	28.973	31.011
$LOGTNA_{i,t-1}$	10.146	11.012	11.592	12.122	13.132	12.522	13.273	14.128	15.003	16.093
$(LOGTNA - AVGLOGTNA)^2$	-2.877	-1.8	-1.212	-0.525	0.509	-0.784	0.131	1.082	1.991	2.833
$FLOW_{i,t-1}$	-1.565	-0.747	0.004	0.595	1.521	0.173	1.431	2.086	2.942	3.867
$FUNDRET_{i,t-1}$	1.766	2.69	3.407	4.086	5.001	-0.034	0.952	1.705	2.411	2.994
$LAGMANAGFEE_{i,t-1}$	2.267	3.006	3.761	4.613	5.465	4.125	5.26	6.148	7.064	8.485

The table reports similar results as in Table 6, but separately for young and old funds. Young funds are the funds whose ages are equal to or below six years old, whereas old funds are more than six years old. We calculate 30 sets of 4,000 trajectories of the optimization process for 10 years. For each of the 30 sets of trajectories, we estimate the following regression on the simulated data and then report the averages for young and old funds. Panel A and B report the cross-sectional distribution of the t-statistics for bond and stock funds' management fees respectively. The general form of the regression is:

$$\begin{aligned} MANAGFEE_{i,t} &= \alpha_i + \beta_{i,1} LOGTNA_{i,t-1} + \beta_{i,2}(LOGTNA_{i,t-1} - AVGLOGTNA_{i,t-1})^2 \\ &\quad + \beta_{i,3} FLOW_{i,t-1} + \beta_{i,4} FUNDRET_{i,t-1} + \beta_{i,5} LAGMANAGFEE_{i,t-1} + \epsilon_{i,t}. \end{aligned}$$

In addition, the bond management fees are significantly and positively related to fund flows, yet the relationship to fund returns is statistically insignificant. These results imply that capital inflows mainly contribute to the management fees and bond returns have negligible effects on bond management fees. In contrast, returns on the stock funds are the main driving force for the increasing management fees of stock funds, as can be seen from the statistically significant and positive t-statistics across their quantiles.

4.4. Management Fees and Fund Ages

Turning to the effects of fund ages on management fees, we split the funds into young and old subgroups depending on whether the fund age is smaller or larger than six years old. Table 7 presents the results.

Fund sizes demonstrate different effects on the management fees of bond funds and stock funds. For the management fees of bond funds (Panel A), the aforementioned positive scale effects still hold after controlling for fund ages. The t-statistics values of the quadratic terms are significantly positive for both young and old bond funds.

Panel B shows that the stock management fees increase with respect to the average fund size at a *decreasing* (*increasing*) rate for the young (old) funds. The 50^{th} quantiles of the t-statistics of the quadratic terms are -1.212 and 1.082 respectively. Moreover, there is a significant positive relationship between fund flows and management fees in the old stock funds, while this relationship is insignificant for the young stock funds since the managers of young stock funds lack experience in taking the advantages of fund sizes. These features lead to large compensations and management fees to the managers of mature stock funds.

5. Conclusion

Despite the prevalence of fund size in empirical studies on fund flows and fund performance, little is known about the scale effects of size-induced fund flows on the dynamics of different managers' optimal portfolios and their fund performances in a theoretical model. To the best of our knowledge, this is the first paper to address the scale-related portfolio implications theoretically in a dynamic setting. The managers experience dynamic fund flows that depend on not only the relative performance, but also the total net assets under management. This innovative ingredient in our model reflects the empirical fact of size-dependent fund flows.

We contribute to the literature in three ways. First, we solve the highly path-dependent portfolio optimization problem and analyze the dynamics of optimal life-cycle investment strategies for bond and stock funds separately. Second, we provide new explanations for the debate over the controversial empirical findings on scale economies of funds through the lens of a theoretical optimization model within the decentralized management structure. Third, we reveal further implications of our optimal solution for the two types of fund management fees and fund ages.

Appendix A: Model Solution

We describe our method of solving the highly path-dependent multi-period portfolio optimization problem. A package of transparent Matlab programs producing all numerical results and more robustness checks is provided upon request. In fact, the results that we present in the paper, such as those obtained by varying risk aversions/fund ages/fund flows or controlling for young and old funds, and the detailed quantiles of regression results, demonstrate the strong robustness of our numerical solutions.

In the decentralized structure, the CIO first decides the optimal wealth between different markets and managers. Then the managers implement their optimal investment policies forward. For both agents, we use the simulation-projection method proposed by Brandt et al. (2005) along with a multidimensional grid search and *policy* iteration. The idea is to approximate conditional expectations by projecting the variables of interests on a second-order polynomial basis of state variables. The projection coefficients are estimated by cross-sectional regressions across simulated paths of asset returns and state variables.[7]

We simulate $N = 120,000$ paths of state variables and asset returns,[8] as defined in equation (1) and (4), over $\hat{T} = 10$ years. We discretize the space $[0,1]\times[0,1]$ for the CIO's $\theta_{c,t}(i)$-grid of portfolio weights, where $i = 1,2$ refers to the bond and stock benchmarks. For asset managers $i = 1,2$, we discretize the space $[0,1]\times[0,1]\times[0,1]$ for their $\theta_{i,t}(k)$-grid of portfolio weights, where $k = 1,2,3$ refers to the risky assets of each manager. The remainder is invested in the cash account.

For the CIO: the CIO's investment problem and budget constraint are formulated by (7) and (8). We normalize the initial wealth to be one, $W_0 = 1$, without loss of generality. For each point of $\theta_{c,t}(i)$-grid, we run an OLS

[7] The approach is inspired by the American options pricing of Longstaff and Schwartz (2001).

[8] Our robust check shows that raising the number of simulation paths does not improve the accuracy of numerical results at a reported level.

regression of N simulated utilities of $W_{\hat{T}}$ on a basis function of state variables, i.e.,

$$\mathbb{E}_{t=\hat{T}-1,\hat{T}-2,\dots,0}\left(\frac{W_{\hat{T}}^{1-\gamma_c}}{1-\gamma_c}\right) \simeq \alpha(\theta_{c,t}(i))^\top f(\varphi_t^B),$$

where $\alpha(\theta_{c,t}(i)) \in \mathbb{R}^{N\times 1}$ are regression coefficients and the basis function $f(\varphi_t^B)$ is:

$$f(\varphi_t^B) = \begin{bmatrix} 1 & \varphi_{t,1}^B & (\varphi_{t,1}^B)^2 \\ 1 & \varphi_{t,2}^B & (\varphi_{t,2}^B)^2 \\ \vdots & \vdots & \vdots \\ 1 & \varphi_{t,N}^B & (\varphi_{t,N}^B)^2 \end{bmatrix}_{t=\hat{T}-1,\hat{T}-2,\dots,0}.$$

The fitted value of regression provides the approximations of conditional expectations.

Since α relies on portfolio weights, we then repeat the procedure of regression for a grid search of portfolio weights $\theta_{c,t}(i)$. We select the optimal weights that maximize the expected utility in each period and along each path. The grid search method is robust and avoids several convergence issues that could occur when iterating to a solution based on the first-order conditions (van Binsbergen and Brandt, 2007). Follow the same procedure, we iterate backward recursively and store the optimal weights at $t = 0$. During the recursion, we obtain the realized values of utility function by iterating over the optimal policies in the following periods. Note that we use *policy* iteration rather than value function iteration to keep numerical stability.

For the managers: the managers maximize their compensations depending on relative investment performance and fund sizes, which is characterized by (9) to (11). Since their portfolio weights depend on the TNA, we construct a TNA(l)-grid, l = 1, . . ., L, with an exponential growth rate between the grid points. For each value of TNA(l) and each point of $\theta_{i,t}(k)$-grid, we run similar procedures of regression on another basis function given below. Since the time-varying investment opportunities are captured by the state variables X_t, the basis function here is the second-order polynomial expansion of X_t including cross terms:

$$f(X_t)_{t=\hat{T}-1,\hat{T}-2,\ldots,0} =$$

$$\begin{bmatrix} 1 & X_{1,1} & X_{2,1} & X_{3,1} & X_{1,1}^2 & X_{2,1}^2 & X_{3,1}^2 & X_{1,1}X_{2,1} & X_{1,1}X_{3,1} & X_{2,1}X_{3,1} & X_{1,1}X_{2,1}X_{3,1} \\ 1 & X_{1,2} & X_{2,2} & X_{3,2} & X_{1,2}^2 & X_{2,2}^2 & X_{3,2}^2 & X_{1,2}X_{2,2} & X_{1,2}X_{3,2} & X_{2,2}X_{3,2} & X_{1,2}X_{2,2}X_{3,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & X_{1,N} & X_{2,N} & X_{3,N} & X_{1,N}^2 & X_{2,N}^2 & X_{3,N}^2 & X_{1,N}X_{2,N} & X_{1,N}X_{3,N} & X_{2,N}X_{3,N} & X_{1,N}X_{2,N}X_{3,N} \end{bmatrix}.$$

Then we iterate backwards through time and follow the similar procedures of grid search and policy iteration within the managers' constraint set $\mathcal{K}_t$ described in Section 2.3.

The method produces the optimal portfolios at all $(N \times L)$ grid points at each t. The last step is to start from the initial wealth and to interpolate over the map from the TNA(l)-grid to the optimal $\theta^*_{i,t}(k)$ that are stored during the backward recursions. Note that the CIO affects the managers' portfolios through optimally allocating initial wealth. Depending on the initial wealth, the managers determine their optimal policies forward in time along with N paths. Finally, we report the average to illustrate the dynamics of portfolios.

Appendix B: Solution Verification and a General Utility

In this section, we examine the questions of solution verification and a general utility function.

Appendix B.1: Solution Verification

To verify the correctness of our solutions to the optimization problems of the CIO and managers, we can show that the optimization problems are convex problems and then the solutions are optimal. We discuss the CIO's problem only to save the space as the discussion on managers' problems is similar.

We re-formulate the CIO's maximization problem (7) as an equivalent standard form of a convex minimization problem below.

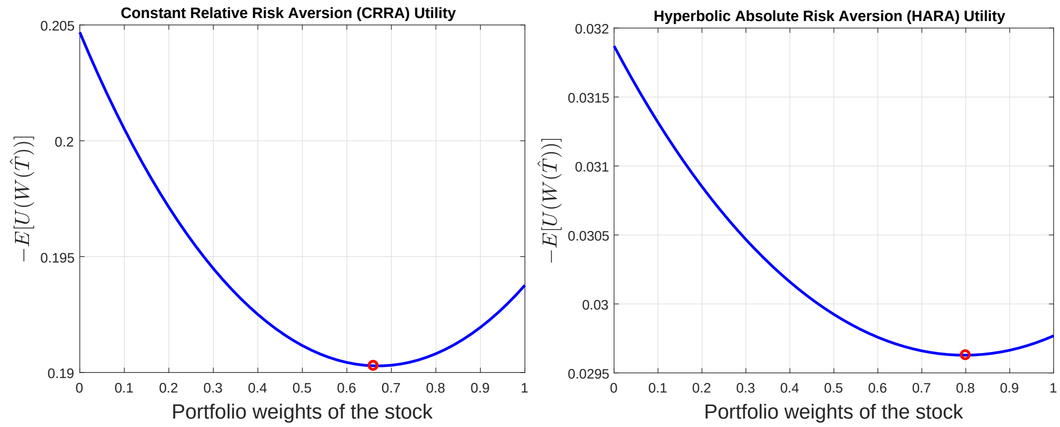
$$\min_{\theta_{c,t}\in\mathcal{K}_{c,t}} f(W_{\hat{T}}) = \min_{\theta_{c,t}\in\mathcal{K}_{c,t}} \mathbb{E}\left(\frac{-W_{\hat{T}}^{1-\gamma_c}}{1-\gamma_c}\right),$$

subject to the standard borrowing and short-selling constraints $\mathcal{K}_{c,t}$ that are re-formulated as:

$$g_1(W_{\hat{T}}) = -\theta_{c,t}W_{\hat{T}} \leq 0, \quad g_2(W_{\hat{T}}) = -(1-\mathbf{1}^\top\theta_{c,t})W_{\hat{T}} \leq 0.$$

The function f is a strictly convex function since the power utility function $W_{\hat{T}}^{1-\gamma_c}/(1-\gamma_c)$ is a well-defined strictly concave function of $W_{\hat{T}}$. The function g_i, $i = 1, 2$, in the inequality constraints are affine in $W_{\hat{T}}$ and hence they are convex as well.

For a convex minimization problem, there are the following well-known theories: (1) If a local minimum exists, then it is a global minimum. (2) For each strictly convex function, if the function has a minimum, then the minimum is unique. Therefore, the optimal solution obtained from our algorithm is the unique global optimal solution. In addition, Figure 5 portrays that the solutions we found are the correct solutions to the equivalent convex problems with the CRRA utility and the more general hyperbolic absolute risk aversion (HARA) utility given by (14).



(a) The CRRA Utility

(b) The HARA Utility

Figure 5. The convex problems and their minimization solutions of stock weights to $\min_{\theta_{c,t}\in\mathcal{K}_{c,t}} \mathbb{E}[-U(W_{\hat{T}})]$**.**
The red circles indicate the optimal solutions for the utility function U being (a) the constant relative risk aversion (CRRA) utility with $\gamma_c = 5$ and (b) the hyperbolic absolute risk aversion (HARA) utility (14) with $a = 10$, $b = 0.4$, $\gamma_h = 1 - \gamma_c = -4$.

Appendix B.2: A General Utility

Our model and solution method can be applied to mathematically more complex models by extending the utility to a broad class of hyperbolic absolute risk aversion (HARA) utility. We discuss the CIO's problem only to save the space and the way of extending managers' problems is similar.

We consider the HARA utility below, see, e.g., Ingersoll (1987).

$$U(W_{\hat{T}}) = \frac{1-\gamma_h}{\gamma_h}\left(\frac{a\,W_{\hat{T}}}{1-\gamma_h} + b\right)^{\gamma_h}, \tag{14}$$

where $a > 0$. The relative risk aversion increases with $W_{\hat{T}}$ if $b > 0$ for $\gamma_h \neq 1$. The general class of HARA utility includes many special utilities. It is linear and risk neutral if $\gamma_h = 1$, quadratic if $\gamma_h = 2$, exponential if $b = 1$ and

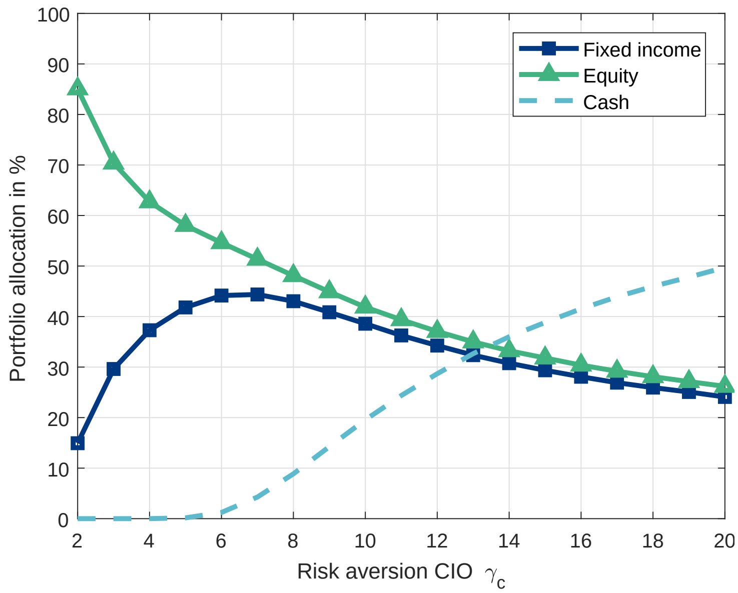


Figure 6. The CIO's optimal portfolio allocations with the HARA utility.

The figure displays the CIO's optimal allocations to fixed-income and equity asset classes by using the HARA utility (14) with $a = 10$, $b = 0.4\,\gamma_h = 1 - \gamma_c$ versus the γ_c in (7). The crosses of lines are expected as the risk aversion largely influences portfolio choices.

$\gamma_h \to -\infty$, power if $\gamma_h < 1$ and $a = 1 - \gamma_h$, and logarithmic if $a = 1$ and $\gamma_h \to 0$. For the last two cases, they turn to the CRRA utility if further $b = 0$.

We illustrate an example of the HARA utility in Figure 5 and Figure 6. Figure 5 displays that the CIO with the HARA utility takes more stock allocations than the CIO with the CRRA utility. Comparing with Figure 1 with the CRRA utility, Figure 6 shows that the CIO with the HARA utility makes more allocations to the risky fixed-income and equity asset classes.

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