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Real Estate valuation and forecasting in non-homogeneous markets: A case study in Greece during the financial crisis

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Abstract

In this paper we develop an automatic valuation model for property valuation using a large database of historical prices from Greece. The Greek property market is an inefficient, non-homogeneous market, still at its infancy and governed by lack of information. As a result modelling the Greek real estate market is a very interesting and challenging problem. The available data cover a wide range of properties across time and include the financial crisis period in Greece which led to tremendous changes in the dynamics of the real estate market. We formulate and compare linear and non-linear models based on regression, hedonic equations and artificial neural networks. The forecasting ability of each method is evaluated out-of-sample. Special care is given on measuring the success of the forecasts but also on identifying the property characteristics that lead to large forecasting errors. Finally, by examining the strengths and the performance of each method we apply a combined forecasting rule to improve forecasting accuracy. Our results indicate that the proposed methodology constitutes an accurate tool for property valuation in a non-homogeneous, newly developed market.

Keywords — Forecasting, property valuation, real estate, neural networks

1 Introduction

In recent years big financial institutions are interested in creating and maintaining property valuation models. The main objective is to use reliable historical data in order to be able to forecast the price of a new property in a comprehensive manner and provide some indication for the uncertainty around this forecast. In this paper we develop an automatic valuation model (AVM) for property valuation using a large database of historical prices from Greece. The Greek property market is an inefficient, non-homogeneous market, still at its infancy and governed by lack of information. As a result modelling the Greek real estate market is a very interesting and challenging problem.

The global crisis led to a significant decline in house prices. For years, financial institutions were eager to lend money to home buyers. As a result when the house market collapsed financial institutions were the ones most affected with major financial losses. The global financial crisis was followed by the Greek crisis and a long period of recession. At the moment the Greek market is experiencing an unprecedented situation regarding the current valuations and the future trends. The "domino effect" of the financial crisis became apparent to Greece at the end of 2008. The residential market in Greece has experienced significant contraction over the last 8 years. It is highlighted that since the start of the financial crisis the private construction activity in Greece, as this is depicted by the number of building permits, reduced by almost 80% and the house prices showed a cumulative decrease of 41% with the corresponding drop on prices, in metropolitan areas such as Athens and Thessaloniki, being 43.5% and 45.1% respectively. At the period 2008q1-2015q4, the ratio of non-performing loans to total bank loans increased by 30.9% (and by 38.4% if restructured loans are also taken into consideration), rising to 35.6% (and 43.5%, respectively) at the end of that period.

The need for unbiased, objective, systematic assessment of real property has always been important. This need is urgent now as banks need assurance that they have appraised a property on a fair value before issuing a loan and also as the government needs to know the fair market value of a property in order to determine accordingly the annual property tax. In 2016, Greeks were called to pay seven times more in property taxes compared to 2009, even though they had to

deal with a 25 percent drop in GDP and similar unemployment percentages. Greece is one of the countries with the highest taxation of real estate as a percentage of GDP. According to European Commission figures for 2015, the only countries with higher property taxes are France and Britain. In particular, in Greece, property owners are required to pay taxes that exceed 2.5% of GDP, when in Germany the figure is no more than 0.5% while citizens of neighbouring countries, such as Italy, Cyprus, Bulgaria and Turkey enjoy less property taxes. Furthermore, valuations determined for real property have further significant tax implications for current and new home owners and have to be verified in the courtroom in extreme cases.

Forecasters in the real-estate sector have to take into consideration the unique characteristics of a property market (Hoesli and MacGregor, 2000) such as heterogeneity, fixed locations, illiquidity, and the absence of a central marketplace. These characteristics make the real-estate market inefficient. For the above reasons, automatic mass appraisal approaches could assist in the science of valuation especially in a world where there is increased availability and use of data, and where failure to achieve an opinion of value which takes proper and balanced account of such information and analysis may result in greater exposure to expensive litigation. AVMs or mass appraisal systems can enhance experts' valuation with data-driven estimates. They can provide model-based valuations for properties using information about the property's location and characteristics, appropriate for risk management and big-data analytics. Last but not least, AVMs can be used for the redesign of the appraisal process. The automation features of the AVM can reduce the need for manual data collection and manipulation by the appraiser, while at the same time providing an independent estimated value. The role of the appraiser would be to evaluate the findings of the AVM in light of his own physical inspection of the property, verification of comparables and knowledge of local market conditions.

Traditional valuation methods include various expressions of linear regression (Caples et al., 1997; Coleman and Larsen, 1991; Lentz and Wang, 1998; McCluskey and Anand, 1999; Miller, 1982; Reichert, 1990; Schulz et al., 2014) including multiple, (Isakson, 2001; Mark and Goldberg, 1988; Pagourtzi et al., 2003) stepwise, (Pagourtzi et al., 2003) ridge, (Moore et al., 1984; Ferreira and Sirmans, 1988) rank, (Cronan et al., 1986) quantile, (Narula et al., 2012) robust (Janssen et al., 2001) and additive regression (Brunauer et al., 2013) approaches using hedonic models incorporating features of the property such as its age, square feet of living space, number of bedrooms, lot size, and others. The underlying hypothesis of these models is that the valuation of the residence can be related to a specific set of the property's characteristics (Kummerow, 2000).

Recently more advanced methodologies have been employed including neural networks (NNs), machine learning, fuzzy logic, regression trees, random forest, multi-criteria decision analysis, k-Nearest Neighbors and spatial analysis (Ahn et al., 2012; Antipov and Pokryshevskaya, 2012; Atkinson and Crocker, 1987; Aznar and Guijarro, 2007; Aragonés-Beltrán et al., 2008; Bagnoli and Smith, 1998; Ball and Srinivasan, 1994; Bitter et al., 2007; Brown and Uyar, 2004; Chica-Olmo, 2009; Curry et al., 2002; Fan et al., 2006; Filho and Bin, 2005; Helbich et al., 2014; Kaklauskas et al., 2007; Kauko et al., 2002; Kauko, 2003; Kilpatrick, 2011; Kontrimas and Verikas, 2011; Kusan et al., 2010; Landajo et al., 2012; Lins et al., 2005; McCluskey and Anand, 1999; McCluskey et al., 2013; Narula and Wellington, 2007; Narula et al., 2012; Nguyen and Cripps, 2001; Pace, 1995; Park and Bae, 2015; Peterson and Flanagan, 2009; Selim, 2009; Worzala et al., 1995; Zurada et al., 2011)

Initial results using non-linear techniques and especially NNs revealed unstable results, (Kontrimas and Verikas, 2011; Worzala et al., 1995). On the other hand NNs proved to be effective in the case of large heterogeneous datasets, (Nguyen and Cripps, 2001), to have better pricing precision out-of-sample and to extrapolate better from more volatile environments, (Peterson and Flanagan, 2009; McCluskey et al., 2013).

Although various studies have been published on mass appraisal systems, previous studies focus on large and already developed markets. Furthermore, the analysis is usually based on small samples (less than 500 properties) at regional level, (Landajo et al., 2012; Kilpatrick, 2011; Kusan et al., 2010; Selim, 2009; Kontrimas and Verikas, 2011; Narula et al., 2012; Brasington and Hite, 2008). Furthermore, Antipov and Pokryshevskaya (2012) argue that the existing literature do not take into account model diagnostics. Traditionaly, model quality is evaluated by the use of

aggregated diagnostic indicators. Antipov and Pokryshevskaya (2012) propose a segmentational approach for the diagnostics of mass appraisal models quality. Without such diagnostics, model quality is questionable, since it may give a much higher than average error when objects from particular segments are under consideration, (Antipov and Pokryshevskaya, 2012). In this study we address the aforementioned two issues.

More precisely, the main contributions of our approach are the following: we develop three mass appraisal systems and we compare their forecasting power in 4 non-overlapping out-of-sample sets. The systems are based on hedonic characteristics and professional property valuations. Very few papers have examined the accuracy of professional forecasts in real estate, (Papastamos et al., 2015). The first system is linear and based on multiple linear regressions. The second system is a novel valuation method that uses spatial information. It is based on similarity measures and geographical distances in order to derive the price of a property using local regression approch. Finally, we apply a non-linear automatic valuation method based on machine learning. More precisely, we apply an optimised NN in order to forecast real-estate prices based on hedonic characteristics. We give extra care in the construction of the NN. In contrast to previous studies, we apply statistical methods in order to select the appropriate number of hidden units as well as the statistical significant variables. Furthermore, we fine tune the NN using regularization methods in order to avoid overfitting. Secondly, in this study the proposed automatic valuation methods are tested in a new market still at its infancy with lots of unique characteristics. The Greek market is an inefficient, non-homogeneous market governed by lack of information and declining prices due to the recession, while the properties' characteristics are diverse both at regional and country level, as, for example, differences in urban and rural areas or touristic areas of high demand. Thirdly, we examine the forecasting performance of the proposed methods in a very large data set (over 35,000 properties) in country level. Given the large size of our data set, we derive significant conclusions about the strengths and weaknesses of each method as well as the dynamics that govern the Greek real estate property market. As this application illustrates automatic model valuations can be applied to both case-by-case valuations and batch processing of thousands of properties. Next, we highlight the importance of checking how well the model performs in different segments of data. The developed models are evaluated in different segments of data and time-periods including the financial crisis. Finally, we combine the forecasts from the three methods in order to obtain one "overall" forecast aiming at gaining predictive power from the different aspects the three methods can capture.

The proposed AVM tool can have significant impact on the decision making process as well as managerial implications to different stakeholders such as government, commercial banks and decision makers. We postpone this discussion until the last section.

The rest of the paper is organised as follows. In section 2 we present the three valuation methods. More precisely we present three methods based on 1) Multiple Linear Regression, 2) Similarity Valuation Method and 3) Neural Networks. The data are described in section 3. In section 4 we present our results. More precisely in section 4.1 the forecasting ability of each method is discussed while in section 4.2 an in-depth analysis of how the forecasting error changes when the characteristics of the properties change is presented. Section 5 concludes the paper.

2 Methodology

2.1 Multiple regression analysis

The first method used in this study is a typical hedonic regression model. Hedonic models assume that the price of a property reflects inherent characteristics valued by some implicit prices. In empirical studies, these implicit characteristic prices are coefficients that relate prices and the underlying attributes in a regression model.

The model takes the form

$$Y_i = \beta_0 + \sum_{j=1}^k \beta_j X_j^i + \epsilon_i$$

where X_{j}^{i} is the value of the *j*-th explanatory variable/characteristic for the *i*-th property, Y_{i} is the

logarithm of the value of the property translated to value of the present period and β_j , j = 0, ..., kare regression coefficients associated with the explanatory variables, β_0 being the intercept. The usual assumptions for the errors apply, namely zero mean and constant variance.

Estimation of the model was done using standard OLS approach. The variable selection approach however was not standard, and we will explain later. There are many reason for considering such a simple model. Regression model, while simple, can reveal useful information about the underlying structure. Being simple offers certain advantages as a) it is easy to use and interpret, b) provides easy and stable variable selection approaches, c) modification is rather simple, the same for its update and generalization, and finally d) inference is simple and hence insight can be generated rather simply. We emphasize that the interest lies on forecasting new properties and not identifying the important characteristics, so all our approaches are based on this.

We applied a variable selection approach in order to find the variables that are predictive for the value of the property. The aims were: first to check existing work and whether it needs simplification with simpler models to attain parsimony, second to formulate a meaningful model to use and third to be able to derive a comprehensive and simple model in order to see the variables that are deemed useful for the purpose of the prediction.

Since the aim of the approach was to predict new unseen properties, we modified the forward selection approach in order to use it for creating a predictive model. Standard forward selection selects the new variables to add to the model among the significant ones. The reason is that the model building aims at producing a descriptive (exploratory) model that can help to identify the variables with relationship to the response. In our case the interest lies on prediction and hence we want to find a model that predicts well while it is not necessarily the best for exploring the existing data.

Hence our approach is the following

- 1. Start from a model with only the constant.
- 2. Select as the variable to enter the model the one that minimizes the mean of the relative absolute prediction error for the model k, defined as

$$MAPE_k = \sum_{i=1}^{n_t} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

where y_i is the observed value of the property from the validation sample and \hat{y}_i is the predicted value of the model. In the above n_t is the cardinality of a validation.

- 3. With the selected variable in the model we go back to step 2 to find among the other candidates the one that minimizes the MAPE
- 4. Stop when no further decrease of MAPE is possible.

The final model is used to forecast the values of the properties out-of-sample. The approach mimics typical forward approaches but uses a criterion that selects a predictive model. An interesting note is that usually the MAPE after few steps almost stabilizes and further covariates create a small decrease. For predictive purposes this needs some care because it is known that for prediction the more covariates the more overfitting is achieved and hence the model may loose its value very quickly. We tried to keep the model parsimonious, i.e. without many covariates. Finally, note that other variable screening/selection approaches like LASSO could have been used but since mainly the covariates are categorical special amendments for the LASSO were needed.

2.2 Similarity measure valuation

In this section we propose and present the Similarity Measure Valuation (SMV) method. The SMV method is based on spatial information and a representative asset (RA). The RA is the "average" property derived from the database. This is a standard procedure in sales comparison methods since comparables have different characteristics, (Kauko and d'Amato, 2009). The value of each

property is converted to a Hedonic Value (HV) based on the characteristics of the property and the Index area. The role of the HV is to convert all properties into a representative property in terms of characteristics. So, each comparable in our database has each own HV based on their characteristics compared to the RA and is given by:

$$HV_i = X_S^{RA} \exp\left[\log\left(\frac{UV_i}{X_S^i}\right) + \sum_{j=1}^J \beta_{kj} \left(X_j^{RA} - X_j^i\right)\right]$$
(1)

where β_{kj} is the hedonic coefficient of variable j for the index area k where property i is located, X_j^i is the value of variable j for the property i, X_S^i is the size of the property in square meters and X_j^{RA} is the value of variable j for the RA. Finally, UV_i , is the updated value of property i. Our database consists of historical valuations, V_i , performed by experts. We are interested in updating each property's historical value to the current time where our method is used. Residential indices by region are used to update the values of comparables. The UV_i is given by:

$$UV_i = V_i \frac{ind_1}{ind_3} \left(\frac{ind_1}{ind_2}\right)^{\frac{m_1 - m_2}{3}}$$
(2)

where ind_1 is the residential index at the current quarter, ind_2 is the residential index of the previous quarter, ind_3 is the residential index of the initial quarter and m_1 and m_2 are the month of the quarter of valuation and the month of the quarter of the initial valuation respectively.

All available properties in the database are ranked based on their similarity with the property under consideration. A metric, W_{ij} , is defined to quantify the similarity:

$$W_{ij} = \exp\left[w_1 \log \frac{c_1}{d_{ij} + c_1} + w_2 I_{ij} \left(X_7\right) + w_3 I_{ij} \left(X_8\right)\right]$$
(3)

The above formula assesses the similarity of property i to another property j from the database by considering the geographical distance between properties i and j, the administrative sector and the type of the property where:

- d_{ij} is the geographical distance between properties i and j.
- X_7 , X_8 are the main characteristics of the properties as defined in section 3.
- $I_{ij}(x)$ is a 0-1 indicator which equals 1 if properties *i* and *j* are identical in terms of their characteristic *x* and 0 if they differ on that characteristic.
- w_i are weighting coefficients, which sum up to 1; they indicate the relative importance of the different characteristics of the properties in defining the above similarity metric.
- c_1 is a scaling parameter for the distance, it is used to map the difference between the properties being compared on a similarity scale common to all characteristics. They are scalars used to avoid numerical problems.

The weights and the scaling parameters are adjusted differently for each administrative index area and they have been defined on the basis of inputs obtained from experts. The higher the similarity metric W_{ij} is, the stronger the similarity between properties *i* and *j*.

After selecting the most suitable properties, a weighted RA value, WRAV, is obtained based on the following formula:

$$WRAV = \frac{\sum_{i=1}^{n} w_i H V_i}{\sum_{i=1}^{n} w_i} \tag{4}$$

Finally, we need to convert the WRAV into the weighted value based on the property's under valuation characteristics.

$$SMV = X_8^i e^{\ln\left(\frac{WRAV}{X_S^{RA}}\right) + \sum_{j=1}^J \beta_{kj} \left(X_j^{RA} - X_j^i\right)}$$
(5)

where, as before, β_{kj} is the hedonic coefficient of variable j for the index area k where property i is located and X_i^i is the value of variable j for the property i,

2.3 Neural networks

In this paper we treat NNs as the eminent expression of non-linear regression, which constitutes a very powerful approach, especially for financial applications. The main characteristic of NNs is their ability to approximate any non-linear function without making a priori assumptions about the nature of the process that created the available observations. A multilayer perceptron (MLP) is a feed-forward NN that utilizes a back-propagation learning algorithm in order to enhance the training of the network, (Rumelhart et al., 1986).

For this study we propose a three-layer NN. The lower layer is called the input layer and consists of the input variables. The middle layer is the hidden layer and consists of hidden units (HUs). Finally, the upper layer is called the output layer where the approximation of the target values is estimated. Often more hidden layers can be used. Each node in one layer connects to each node in the next layer with a weight w_{ij} , where ij is the connection between two nodes in adjacent layers within the network. The units of each layer receive their inputs from the units of the layers immediately below and send their outputs to the units of the layers lying directly above. The flow of information is done through the connections. A sigmoid activation function is used in the hidden layer while there is a linear connection between the neurons and the output nodes, (see Cybenko (1989)).

On each pass through, the NN calculates the loss between the predicted output \hat{y}_n at the output layer and the expected output y_n for the n^{th} iteration (epoch). The loss function used in this paper is the sum of squared errors, given by:

$$L_n = \frac{1}{2} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2 \tag{6}$$

where N represents the total number of training points. Once the loss has been calculated, the back-propagation step begins by tracking the output error back through the network. The errors from the loss function are then used to update the weights for each node in the network, such that the network converges. Therefore, minimising the loss function requires w_{ij} to be updated repeatedly using gradient descent, so we update the weights at step t + 1, $w_{ij,t+1}$, by:

$$w_{ij,t+1} = w_{ij,t} - \eta \frac{\partial L}{\partial w_{ij,t}} + \mu \Delta w_{ij,t}$$
⁽⁷⁾

2.3.1 Parameter tuning for neural network generalisation improvement

A small number of HUs will lead to underfitting of the NN to the data while a very large number of HUs will lead to overfitting. In this study the model selection and variable selection algorithms presented in Zapranis and Refenes (1999) and extended in Alexandridis and Zapranis (2013, 2014) were adapted. Analytical presentation of the model identification framework is beyond the scope of this paper. We refer the interested reader to Alexandridis and Zapranis (2013, 2014). One method for improving network generalization is to use a network that is just large enough to provide an adequate fit. Unfortunately, it is difficult to know beforehand how large a network should be for a specific application. In this study two methods for improving generalization are implemented: regularization and early stopping.

The default method for improving the generalization ability of a NN is called early stopping. In early stopping a relative large number of HUs is used in the construction of the network. The number of weights roughly defines the degrees of freedom of the network. If the training phase continues more than the appropriate iterations and the weights grow very large on the training phase then the network will start to learn the noise part of the data and will become over-fitted. As a result the generalization ability of the network will be lost. Hence, it is not appropriate to use such a NN in predicting new unseen data. On the other hand, if the training is stopped at an appropriate point, it is possible to avoid over-fitting.

A common practice to overcome the above problems is the use of a validation sample. The in-sample data consists of property valuations in the period January 2012 - December 2015. In

order to train a neural network the in-sample data were split into two samples. The first one is called the training sample which is used for computing the gradient and updating the network weights and biases as described in the previous section. The second subset is the validation set and is used to measure the generalisation ability of the network. The data were split randomly. The train sample consists of 85% of the in-sample data while the validation set of the 15% of the in-sample data. This ratio allows for a large enough sample for training for all Index areas and a large enough set for validation.

At each iteration, the NN is trained using the training sample. Then the cost function between the training data and the network output is estimated and it is used for the adjustment of the weights. The generalization ability of the network is measured using the validation sample. More precisely, the network is used to forecast the target values of the validation sample using the unseen input data of the validation sample. The error between the network output and the target data of the validation sample is calculated. At the beginning of the training phase the errors of both the training and the validation sample will start to decrease as the network weights are adjusted to the training data. After a particular iteration the network will start to learn the noise part of the data. As a result the error of the validation sample will start to increase. This is an indication that the network is starting to lose its generalization ability and the training phase must be stopped, (Anders and Korn, 1999; Dimopoulos et al., 1995). The network weights and biases are saved at the minimum of the validation set error. The network is trained using the Levenberg-Marquardt (LM) algorithm, (Samarasinghe, 2006). The LM alogorithm is very fast but is less efficient for large networks with thousands of weights. In this study the proposed network is very small and only few parameters are used for the minimisation of the fitness function.

Another approach to avoid over-fitting is regularization. In regularization methods the weights of the network are trained in order to minimize the loss function plus a penalty term. Regularization is attempting to keep the overall growth of weights to a minimum by allowing only the important weights to grow. The rest of the weights are pulled towards zero. The regularization method tries to minimizes the sum:

$$W = L_n + \delta \sum_{j=1}^J w_j^2 \tag{8}$$

where the second term is the penalty term, w_j is a weight, J is the total number of weights in the network architecture and δ is a regularization parameter.

It is desirable to determine the optimal regularization parameters in an automated fashion. We apply the Bayesian framework of MacKay (1992). A discussion of Bayesian regularization is beyond the scope of this study. For detailed expositions of the Bayesian regularizations and the LM training algorithm we refer to, for example, Dan Foresee and Hagan (1997). The Bayesian regularization provides a measure of the number of the weights that are effectively being used by the network. The effective number of weights should remain approximately the same, no matter how large the number of parameters in the network becomes. This assumes that the network has been trained for a sufficient number of iterations to ensure convergence.

3 Data description

This study focuses in the Greek property market. The Greek property market is of special interest as it is an inefficient, non- homogeneous market, still at its infancy and governed by lack of information. As a result modelling the Greek real estate market is a very interesting and challenging problem. The available data cover a big range of properties across time and include the financial crisis period in Greece which led to tremendous changes in the dynamics of the real estate market.

The data were provided by the Eurobank Property Services S.A. which is the real estate arm of Eurobank group and is one of the largest real estate service providers in Greece as well as in South East Europe. The database represents the hedonic characteristics of real estate properties. Only the hedonic characteristics of each property were obtained and not any personal data of the clients. We have enriched our dataset with new variables by transformations and interactions between the initial variables. Due to confidentiality reasons we cannot report the new variables that have been resulted due to the transformations or the interaction between the initial variables. The original variables are reported in Table 1.

The sample consists of 36,527 properties that have been professionally evaluated in the period 2012 – 2016. In the majority of the studies regarding real estate value forecasting, very small datasets are used for testing the various methodologies (Landajo et al., 2012; Kilpatrick, 2011; Kusan et al., 2010; Selim, 2009; Kontrimas and Verikas, 2011; Narula et al., 2012; Brasington and Hite, 2008). An exception are the studies from Zurada et al. (2011); Peterson and Flanagan (2009). Given the large size of our datasets, we expect to derive significant conclusions about the strengths and weaknesses of each method as well as the dynamics that govern the Greek real estate property market.

The properties belong to 240 different administrative sectors covering all areas in Greece. In Figure 1, a map with all the properties used in this study is presented. Most of the properties refer to urban areas. It can be also seen that in some areas there are few properties which makes the forecasting a challenging option. An interesting novelty of the current paper is that we attempt to model such a inhomogeneous portfolio of properties.

The various characteristics of each property are presented in Tabel 1. We are interested in forecasting the valuation price of each property (V06). In Table 2 the descriptive statistics of each variable are presented. The values of the properties range from $\notin 15,000$ to $\notin 1,000,000$. Similarly, there is a lot of variation in the year of construction (from 1800 to 2016) and the size of the properties (from $12m^2$ to $400m^2$).

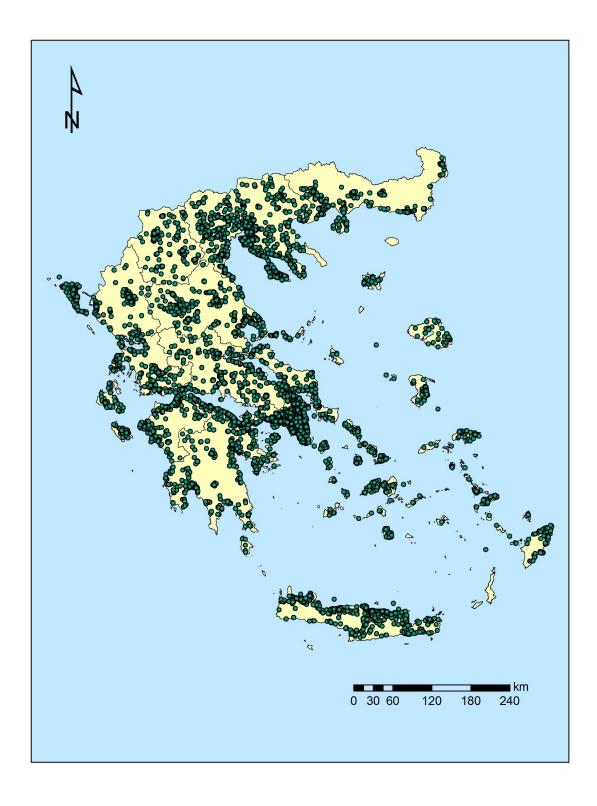
Valuations in the period of 2012 to December 2015 will constitute the in-sample period that will be used for the estimation and fitting of all models. Then, property prices in the first quarter of 2016 are forecasted. Finally, we follow a recursive procedure to forecast the property prices in the remaining quarters of 2016. Hence, the in-sample consists of 32,477 properties while the out-of-sample contains 4,050 properties.

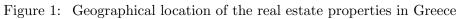
Some sectors contain only a few observations, hence, we group the data into 32 aggregated administrative areas (Index areas). The majority of the properties are located in the capital or in large cities. This is expected as around 50% of the population in Greece lives in two cities, the capital – Athens – and Thessaloniki. Similarly, around 84.5% of the properties are flats while 6.5% are houses, 5.4% maisonettes and only the 3.6% of properties are of type douplex. In the supplementary material the number of properties per Index area, year of valuation, urban classification and type of property are presented.

Code	Characteristic	Code	Characteristic
V01	ID	V12	Floor
V02	Year of valuation	V13	Total number of floors
V03	Month of valuation	V14	Existence of parking space (Y/N)
V04	Administrative sector	V15	Type of parking (Indoor/Outdoor)
V05	Urban classification	V16	Type of heating (categorical)
V06	Survey value (in euros)	V17	Quality of construction (categorical)
V07	Type of residence (categorical)	V18	Number of bedrooms
V08	Usable residence area (in sq. m.)	V19	Touristic hotspot (Y/N)
V09	Land area (in sq.m)	V20	Elevator (Y/N)
V10	Year of construction	V21	View (Y/N)
V11	Distance from centre (in kms)	V22	Number of bathrooms

Table 1: Explanation of the initial set of variables

A final comment about the data applies. The properties are those existing in the database of the bank and refer to those that asked for evaluation from the bank. So it is neither a random sample of any hypothetical population nor a product of some controlled experiment.





4 Empirical results

4.1 Out-of-sample real estate valuation

In this section an out-of-sample validation of the proposed methodologies is provided. The three models are evaluated to 4 non-overlapping samples corresponding to the four quarters of 2016. We

Numerical	Mean	St.Dev	Max	Min
V06	$123,\!592$	112,403	1,000,000	15,000
V08	97.64	52.42	400	12
V09	79.87	882.82	86000	0
V10	1988	16.88	2016	1800
V11	28.33	45.38	321.20	0
V12	1.81	1.59	14	-1
V13	3.44	1.75	25	0
V18	2.10	0.87	6	0
V22	1.46	0.68	6	0
Categorical	Mode	Value		
V07	Flat	2		
V16	Low efficiency	1		
V17	Good	2		
Binary	Proportion 0	Proportion 1		
V14	87%	13%		
V15	97%	3%		
V19	87%	13%		
V20	69%	31%		
V21	92%	8%		

Table 2: Descriptive statistics

apply a recursive window forecasting scheme. Initially, the in-sample data consists of the property's valuations between the 1^{st} quarter of 2012 and the 4^{th} quarter of 2015 (2012q1 - 2015q4). The out-of-sample data consists of the property's valuations that took place during the 1^{st} quarter of 2016 (2016q1). In the next step, 2016q1 is included in the in-sample data set and we forecast the 2^{nd} quarter of 2016 (2016q2). Similarly for the remaining quarters. The total out-of-sample set consists of 4,050 observations.

Three criteria are used for the evaluation of the forecasting ability of each method. The first one is the Mean Absolute Percentage Error (MAPE) and it is given by

$$MAPE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

The second error criterion, denoted by P20, measures the percentage of the cases where the MAPE is less than 20%. This is a standard metric used in real-estate valuations, (see, e.g. Rossini and Kershaw (2008)) and it is given by

$$P20 = \frac{100}{N} \sum_{i=1}^{N} 1_{|PE_i| \le 0.2}$$

where PE is the percentage error and it is given by

$$PE = \frac{y_i - \hat{y}_i}{y_i}$$

and $1_{|PE_i| \le 0.2}$ is an indicator function where

$$1_{|PE_i| \le 0.2} = \begin{cases} 1 & if \quad |PE_i| \le 0.2\\ 0 & if \quad |PE_i| > 0.2 \end{cases}$$

Finally, we calculate the squared correlation coefficient, R^2 , between the predicted and the real prices. In this definition of R^2 , the value is not directly a measure of how good the modeled values are, but rather a measure of how good a predictor might be constructed from the modeled values

In Table 3 a summary of the results are presented. More precisely, the Average PE, the standard deviation of the PE, the average MAPE, the average P20 and the R^2 are presented for each of the four quarters of 2016.

A closer inspection of Table 3 reveals that NN constantly outperforms the alternative methodologies. Interestingly, there is an indication that the results from NNs are more stable. They produce similar forecasting errors for all quarters. The MAPE ranges from 15.05% in the first quarter to 17.67% in the last. The MRA produce similar but slightly worse results for the 1^{st} quarter but the MAPE increases significantly for the remaining quarters. More precisely the MAPE increases from 15.34% to 20.72%. Finally, SMV seems to produces the largest out-of-sample forecasting errors ranging from 18.15% in the third quarter to 22.64% in the fourth.

Similarly, the P20 is always higher when NN are used, followed by MRA while SMV ranks last. With an exception of the last quarter, the P20 is always above 70% for the NN while for the MRA is above 70% for the first two quarters. Finally, it is always below 70% for the SMV. The R^2 is always higher when the NN is used. The MRA has a higher R^2 for the first and fourth quarter but it is lower for the second. Finally, MRA and SMV have the same R^2 in the third quarter.

In general, the MAPE increases in the third and fourth quarter indicating a change in the dynamics of the Greek housing market. However, one must be careful in the interpretation of the results since in each quarter a different set of properties is included. For example, in the 4th quarter the number of properties with land area is doubled and they have significantly larger land area on average. As an example we report a property with $86,000m^2$ land area while in the historical data set the average land area was around $1,200m^2$. Properties with zero land area where excluded from the calculation of the mean. As it is shown in the next section, for all methods the MAPE is higher when properties with land area are considered. This is due to the fact that only 6.5% of the properties have land area.

Next we focus on combining the results. Two simple averaging methods are used. The first method is to take the average of the two best methods, the NN and MRA since SMV produces significantly higher errors. The second method is to compute the average of all three methods.

In general both averaging schemes improves the results. Surprisingly, including the SMV method, further reduces the MAPE. More precisely, the NN+MRA produce the best results, outperforming the NN, in the first quarter with a MAPE of only 14.54% and a P20 of 77.00%. For quarters 2 and 3 the best results are given by the SMV+NN+MRA approach with a MAPE of 15.70% and 15.89% respectively while the P20 is 71.76% and 71.33%. Finally, for the last quarter neither averaging technique can outperform the NN with respect to the MAPE although the P20 when all three methods are used was increased to 69.03% from 68.28% in the case of NN. Analytically results of the MAPE and the P20 for each index for all 5 approaches are presented in Supplementary Material that accompany the online version of this paper.

Table 4 shows the number of best predictive performance of the three main methods with (bot) and without (top) averaging methods, i.e. in how many index areas each method outperform all the other in each quarter. In summary, the NN outperformed the alternative methods in 63 cases out of the 126. The MRA method produced the most accurate forecasts 39 times while the SMV only 24. A closer inspection of Table 4 revelas that the NN outperform the other methods in all quarters while MRA and SMV always rank second and third respectively. Taking into consideration the two averaging techniques, SMV+NN+MRA produces the lower MAPE in 40 cases while NN in 38. The MRA outperform all the other methods in 25 methods while the SMV in 16. Finally, the NN+MRA give the best forecasts only in 9 index areas. Breaking down our results by quarter we observe that in the first quarter the NN method ranks first and the NN+MRA+SMV ranks second while it is the opposite in the fourth quarter. In the second and third quarter both methods rank first.

Recently, artificial intelligence based methods have been proposed as an alternative for mass assessment. However, there are mixed results. Guan et al. (2009) find no improvement when advance machine learning techniques are used while Worzala et al. (1995) find that NNs based methods are inferior to traditional regression methods. More precisely Worzala et al. (1995) find that NN-based methods do not produce results that are notably better than those of MRA except when more homogeneous data are used. In contrast, in this study, where NNs are fine tuned and extra care is taken to avoid overfitting our results indicate that NNs can significantly outperform traditional valuation methods. Our results indicate the importance of constructing the architecture of the NNs based on statistical methods. Finally, our results are in line with Nguyen and Cripps (2001), Peterson and Flanagan (2009) and McCluskey et al. (2013) where NNs proved to be effective in the case of large heterogeneous datasets.

	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
2016q1					
Av. PE	6.93%	2.05%	1.25%	1.65%	3.41%
Std. PE	0.1400	0.0423	0.0452	0.0392	0.0510
MAPE	19.73%	15.05%	15.34%	14.54%	14.86%
P20	66.63%	75.54%	75.42%	77.00%	75.99%
R^2	81.13%	$\mathbf{86.98\%}$	86.85%	88.31%	88.11%
2016q2					
Av. PE	0.67%	1.86%	1.01%	1.43%	1.18%
Std. PE	0.0722	0.0532	0.0581	0.0500	0.0474
MAPE	18.30%	16.22%	17.46%	16.19%	15.70%
P20	67.27%	72.06%	68.06%	71.06%	71.76%
R^2	81.71%	85.71%	78.18%	84.14%	85.62%
2016q3					
Av. PE	3.20%	1.61%	0.10%	0.85%	1.63%
Std. PE	0.0661	0.0511	0.0604	0.0502	0.0487
MAPE	18.15%	16.67%	18.13%	16.48%	15.89%
P20	66.19%	70.97%	65.95%	69.65%	$\mathbf{71.33\%}$
R^2	84.44%	$\mathbf{85.64\%}$	84.44%	87.03%	$\boldsymbol{87.70\%}$
2016q4					
Av. PE	10.18%	3.91%	2.45%	3.18%	5.51%
Std. PE	0.7240	0.2448	0.2807	0.2390	0.3227
MAPE	22.64%	17.67%	20.72%	17.80%	18.10%
P20	65.33%	68.28%	60.80%	67.15%	$\mathbf{69.03\%}$
R^2	78.65%	$\boldsymbol{88.25\%}$	80.08%	87.75%	88.29%

Table 3: Out-of-sample performance for the four quarters of 2016

Table 4: Number of best predictive performance of the three main methods with (right) and without (left) averaging methods

	Main Methods			All Methods				
Quarter	SMV	NN	MRA	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
Q1	2	18	11	2	11	8	1	9
Q2	8	15	9	3	9	8	3	9
$\mathbf{Q3}$	8	15	8	6	10	3	2	10
Q4	6	15	11	2	8	7	3	12

4.2 Analysis of the forecasting errors

In this section we analyse the forecasting errors of each methodology. More precisely, we examine how the forecasting error changes when the characteristics of the properties change. Due to space limitation we cannot present the results for all the characteristics, however the results are available from the authors upon request. Hence, we focus on the characteristics where the analysis is more interesting. Our analysis is based on the complete out-of-sample period (2016q1-2016q4) and consists of 4,050 properties. In Figure 2 the MAPE for each Index area is presented. In addition the number of observations for each Index area is depicted in Figure 2. A closer inspection of Figure 2 reveals that the variation of the MAPE is greater in the case of SMV compared to the other methods. For all methods the MAPE is greater when only few observations are present while the lower MAPE for all indices is obtained when the average of the three methods is considered. Finally, the MAPE is similar across all indices for the NN, the MRA and the averaging method while is quite different for the SMV method. Note also that all methods perform badly for some indices just because they refer to few properties and rather inhomogeneous areas. For example index 30 refers to islands in the Aegean sea which contains both some very touristic islands but also some other islands with no tourism.

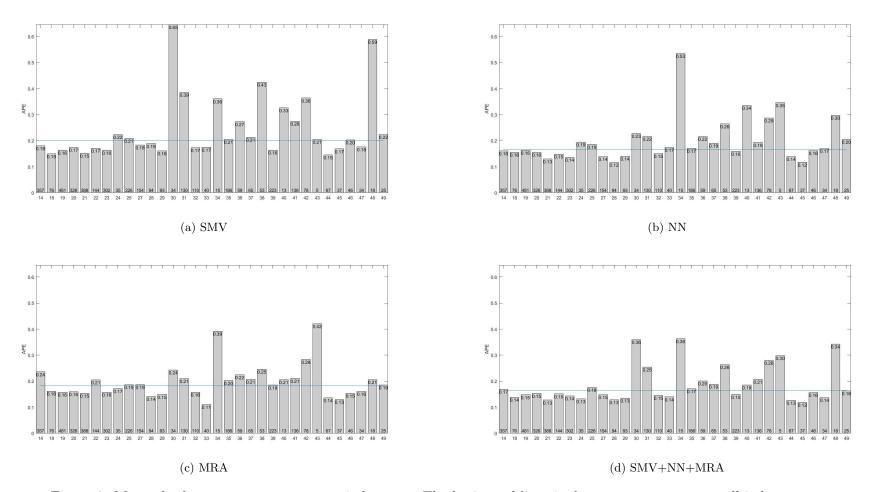


Figure 2: Mean absolute percentage error per index area. The horizontal lines is the average error across all index areas.

Next, we examine the contribution to the error of the following characteristics: urban classification, property type, usable residence area, land area, age and number of bedrooms. Due to space limitation these figures are presented in the supplementary material. The MAPE per urban classification is similar for all methods. The MAPE is higher for rural areas and small towns while it is significantly lower for small, medium and large cities and the capital. While the number of observations per urban classification is the same in the out-of-sample set (except the capital) this is not the case in the in-sample. The majority of the properties are located in the capital (18,123) while there are 3,483 and 2,571 for large and medium size cities. On the other hand there are only 1,615 properties in rural areas in the in-sample where the out-of-sample MAPE is greater. We obtain similar results when we examine the MAPE per type of property. More precisely the MAPE is lower for flats while it is large for houses. The majority of the properties are flats, 3,181, while only 356, 256 and 304 properties are of type house, duplex and maisonette respectively.

Regarding the usable residence area, the error is minimised for properties between $50m^2$ and $80m^2$ when the SVM method is used while it is significantly larger for any other category. For the NN and the MRA the results are similar. The MAPE is lower for properties up to $120m^2$ and then it increases as the area increases. Finally, the MAPE for the NN is smaller for every category. When the land area is considered all methods produce significantly higher errors when land area is included in the property. However, the SMV produce significantly higher errors. More precisely, when land area is included the MAPE for the SMV is 0.40 while it is only 0.29 for the remaining methods. When the properties do not have any land the MAPE falls to 0.18 and 0.17 for the SMV and the MRA respectively while it is only 0.15 for the NN and the averaging method. Again the lower errors for each category are obtained by the NN and the averaging method.

Next, we examine the effect of the age of the property to the forecasting ability of the models. The MAPE is higher for properties constructed before 1970. Also the variation for the SMV is higher compared to the other methods while it is relative stable for the remaining methods. Again, the lower MAPE per category is obtained by the NNs. Finally, the MAPE per number of bedrooms is examined. The MAPE is high for the SMV, MRA and the averaging method when properties with 0 or 5 bedrooms are considered. On the other hand, for the NNs the MAPE increases for properties with 5 or 6 bedrooms. A closer inspection reveals that the majority of the properties have 1–3 bedrooms. Again, the best results for all categories are obtained for the NN and the averaging methods.

A final interesting remark arises from Figure 3 where the smoothed (using LOESS smoother) average predicted value over all the properties predicted at 2016 is presented for the property size (a) and the age at time of valuation (b). Each line corresponds to one of the methods, but we also depict their combination. It is interesting to see that the different methods model a different effect, while their combination, since it compromises the different effects, improves prediction. From Figure 3b one can see a very steep decline in the price after the age of 30 years. As far as the size of the property one an see that there is an increasing trend up to 300 square meters, the SVM method based on a linear (in the logarithmic scale) trend leads to much larger predicted values. The plots reveal the different dynamics of the different approaches.

5 Conclusions

In this study we developed three mass appraisal systems for the automatic valuation of real estate properties in Greece. The Greek property market is a new market still at its infancy with lots of unique characteristics. It is an inefficient, non-homogeneous market governed by lack of information. Also, there are declining prices due to the recession while the properties characteristics are diverse both at regional and country level. We formulate and compare linear and non-linear models based on regression, hedonic equations, spatial analysis and artificial NNs.

We perform an extensive out-of-sample analysis in four non-overlapping data sets. In contrast to previous studies, our results indicate that NNs constantly outperform traditional valuation methods. In this study the proposed NN was fine tuned and extra care was taken to avoid overfitting. The MRA method ranks second while the SMV method ranks third. The forecasting accuracy can

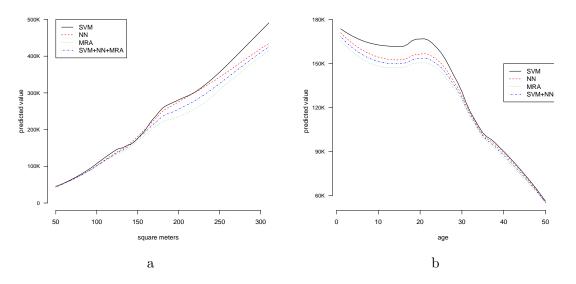


Figure 3: Smoothed average predicted value for the entire 2016, based on all methods

be further improved by employing averaging techniques. A simple average of the three methods performs as well as, and in some cases outperforms, the NN.

Finally, we try to identify the property characteristics that lead to large forecasting errors. Our results indicate that the forecasting error increases when the residence area is above $120m^2$ or the property is a house or large land area is included. Similarly, very old properties (built before 1950) lead to larger forecasting errors. However, it is worth to mention that our analysis revealed that NNs are less sensitive to the changes of these characteristics compared to the SMV or the MRA.

Our results indicate that the proposed methodology constitutes an accurate tool for property valuation in non-homogeneous, newly developed markets. The results of this study can potentially have significant policy and fiscal implications. It can help both the government and the public sectors like commercial banks.

To start with it can help the central and local governments in planning and implementing their fiscal policies, both at micro and macro level and can promote economic and development sustainability. For example, governments can set accurately the fair market value of a property and determine accordingly fair property taxes (see, e.g. McCluskey et al. (1996)). In Greece the properties taxation system is based on objective values set by experts but they may not reflect the true market value, especially after few years. The proposed AVM can be the basis for deriving proper objective real estate values that reflect the current market values and thus be more fair while at the same time the government collects fair taxes. In addition, such a system can assist governments in implementing properties indices and performance measurements and consequently the operation of the markets can be more transparent and efficient for both investors and stakeholders. This can also promote cooperation towards public-private partnerships and projects of common interest.

Second, it can have significant impact in operational efficiency of commercial banks. The proposed AVM can be adapted in applications such as mortgage quality control or appraisal review, loss mitigation analysis, portfolio valuation and appraisal process redesign. For example, the mortgage quality control entails validation or verification of appraisals conducted to determine the market value of collateral properties securing purchase money or refinancing loans. Conventional quality control methods typically entail manual review of a random sample of completed appraisals. Application of an AVM to this process offers the advantages of increased speed, reduced subjectivity, limiting the need for manual review to cases identified by the AVM as exceptional. In loss mitigation analysis, AVMs can be applied to estimate the current loan to value (LTV) ratio on non-performing loans to assist the lender or guarantor in determining optimal foreclosure strategies. Portfolio valuation is a natural application of AVM providing an efficient means of marking-to-market a large number of property values and is most closely aligned with the underlying statistical methods. Unlike real valuations based on expert opinions, automatic mass valuation can reduce operational costs since it is inexpensive and can be performed in a regular basis. In addition performing reevaluation of properties in a regular basis can potentially shed additional light to inefficiencies of real estate markets.

Finally, it can be used as an administrative tool in monitoring the trends of the property market and especially the level of mortgage risk that the commercial banks are likely to be exposed in terms of their LTV ratio. This administrative tool can also be used to take proactive measures, where the economic environment is turbulent and the economies are experiencing abnormal conditions.

We also point out some limitations in the proposed approach. First of all, we use a large data base with properties spanning over a certain time period. So, implicitly we assume that there is some homogeneity across time and that neither large structural changes have been made nor the data contains systematic inconsistencies (e.g. potential change in the definition of variables or in the way of their measurement). In particular, dealing with a rather unstable financial period for Greece, led us to assume that the level of such market inefficiencies is controllable and that we can handle it within the model approach. Second, as the data refer to estimations made by a large number of different professional valuators and not to transactional values, we assume that the criteria provided by the bank were followed in a correct way and no systematic errors and biases have entered the database. Furthermore, our database is not homogeneous. There are geographical sectors with very few properties. In order to overcome this we aggregated the neighbouring sectors. This implies an exchangeability assumption, meaning that information from neighbouring areas is useful and can be used. However, as we pointed out in our analysis, in areas with very few properties and great inhomogeneity large errors may incur. The present study is one of the few dealing with so massive database of properties providing valuable insights.

Finally, we have identified certain areas for future research. To start with, spatial modelling could help improving the models. In this paper we used the spatial information in an implicit way either by using a local regression approach or by considering some distance measures between properties. More refined spatial models, as for example geographically weighted hedonic models (Geniaux and Napoléone, 2008) could help to improve this aspect in an explicit way. Also, since we used data from the entire country some more refined approach that segments the properties not only with respect to their proximity could be used. To another direction, in this paper we considered an ensemble of methods to end up with a combination of them. Proper weighting for such a forecasting is ongoing research. Finally, relatively to managerial implications, we are currently working on measuring risk based on our models as well as the impact and the feasibility of deriving objective values for properties for taxation properties based on such an approach.

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Supplementary material

1 Supplementary Material: Tables

Index	SMV	NN	MRA	NN + MRA	SMV+NN+MR
14	17.24%	13.91%	17.82%	14.40%	14.17%
18	10.42%	15.00%	12.85%	13.93%	12.39%
19	17.86%	16.38%	13.90%	14.34%	14.89%
20	16.91%	12.91%	11.72%	12.14%	13.51%
21	13.09%	12.14%	13.58%	12.25%	11.25%
22	24.97%	14.57%	19.62%	16.27%	15.23%
23	11.67%	10.29%	10.42%	9.69%	9.55%
24	28.35%	15.51%	10.03%	10.90%	14.53%
25	18.75%	14.66%	15.78%	14.65%	14.68%
27	16.56%	14.50%	15.20%	14.49%	12.90%
28	12.58%	6.38%	9.49%	7.49%	7.49%
29	17.28%	17.71%	13.12%	14.76%	14.78%
30	106.28%	26.12%	14.53%	19.17%	46.40%
31	22.35%	22.26%	22.13%	22.03%	21.66%
32	17.30%	13.84%	11.33%	11.51%	12.14%
33	26.83%	11.66%	14.00%	12.14%	16.49%
34	6.05%	28.38%	21.55%	17.18%	9.43%
35	21.10%	18.32%	19.37%	17.98%	16.94%
36	27.27%	18.92%	20.42%	19.35%	19.06%
37	20.67%	18.52%	22.60%	20.56%	18.62%
38	23.79%	15.12%	17.43%	13.94%	12.91%
39	18.33%	11.74%	12.39%	12.04%	11.79%
40	42.75%	20.97%	24.79%	22.88%	21.88%
41	23.45%	23.29%	26.00%	24.19%	22.39%
42	45.19%	27.72%	22.13%	24.88%	29.69%
44	10.87%	12.47%	10.06%	10.37%	9.38%
45	11.62%	11.41%	5.85%	6.93%	7.38%
46	23.11%	10.25%	14.53%	10.88%	14.71%
47	23.37%	10.43%	15.07%	11.74%	14.66%
48	10.45%	12.37%	9.71%	8.81%	8.77%
49	13.00%	10.11%	12.54%	10.18%	11.12%
Average	19.73%	15.05%	15.34%	14.54%	14.86%

Table 1: 2016q1 MAPE per Index

Index	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
14	18.38%	16.99%	24.32%	19.87%	18.31%
18	10.85%	14.88%	14.05%	13.92%	10.59%
19	15.70%	16.09%	15.71%	15.55%	14.58%
20	16.97%	17.34%	17.77%	17.13%	16.39%
21	16.90%	11.40%	14.93%	11.99%	12.31%
22	12.93%	14.07%	17.75%	15.64%	13.36%
23	19.34%	16.41%	12.89%	14.26%	15.28%
24	15.74%	19.90%	15.86%	14.78%	12.49%
25	22.62%	23.51%	22.54%	22.19%	21.77%
27	14.77%	13.33%	16.31%	13.94%	12.79%
28	19.95%	12.59%	10.78%	11.58%	12.25%
29	13.07%	11.32%	16.27%	13.44%	13.03%
30	26.60%	25.28%	28.88%	26.67%	25.99%
31	16.76%	17.24%	18.53%	17.83%	15.02%
32	16.88%	15.40%	16.13%	14.99%	14.85%
33	10.96%	15.25%	5.90%	10.15%	10.38%
34	32.75%	30.30%	48.18%	39.24%	37.07%
35	20.97%	15.77%	16.23%	15.24%	15.28%
36	14.69%	14.17%	20.70%	16.86%	13.92%
37	21.77%	19.34%	22.65%	20.68%	19.98%
38	12.53%	20.63%	24.81%	22.06%	16.71%
39	15.00%	18.70%	21.36%	19.73%	17.90%
40	41.98%	31.46%	22.49%	26.79%	27.62%
41	29.58%	13.58%	13.86%	12.53%	17.09%
42	26.62%	22.76%	26.92%	22.84%	23.38%
43	42.83%	6.36%	11.72%	9.04%	20.30%
44	16.99%	11.38%	10.83%	10.85%	11.73%
45	22.93%	8.90%	12.35%	8.70%	11.60%
46	11.31%	19.99%	9.55%	13.75%	9.86%
47	22.11%	22.15%	12.92%	17.06%	16.24%
48	30.25%	10.48%	16.47%	13.47%	15.24%
49	20.64%	25.86%	16.83%	20.35%	18.59%
Average	18.30%	16.22%	17.46%	16.19%	15.70%

Table 2: 2016q2 MAPE per Index

Index	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
14	14.39%	14.40%	17.23%	14.16%	12.87%
18	14.96%	11.85%	14.75%	12.75%	12.68%
19	15.50%	14.98%	14.27%	13.81%	13.63%
20	19.39%	19.10%	18.35%	18.61%	18.47%
21	14.70%	12.91%	14.65%	13.18%	12.25%
22	21.02%	17.64%	21.74%	19.46%	18.79%
23	18.35%	15.84%	16.93%	15.88%	16.21%
24	22.90%	18.71%	25.73%	15.99%	15.56%
25	24.29%	20.86%	20.35%	19.69%	20.25%
27	23.69%	16.21%	21.49%	18.32%	19.23%
28	18.23%	10.82%	14.99%	12.13%	13.53%
29	13.54%	15.83%	16.89%	15.24%	13.92%
30	25.80%	13.43%	14.61%	14.02%	17.95%
31	17.27%	19.65%	23.96%	21.18%	18.22%
32	14.65%	10.23%	10.75%	10.32%	10.72%
33	15.41%	26.22%	23.69%	24.86%	20.98%
34	46.84%	51.64%	49.15%	50.40%	49.21%
35	13.44%	17.07%	20.40%	18.06%	15.67%
36	9.46%	14.11%	12.81%	11.86%	7.45%
37	14.21%	14.18%	14.24%	13.76%	12.75%
38	26.75%	19.92%	25.58%	19.35%	16.93%
39	19.27%	17.93%	22.74%	18.66%	16.15%
40	25.81%	20.75%	20.12%	17.60%	13.54%
41	25.58%	23.69%	20.69%	19.65%	21.01%
42	36.75%	34.07%	31.33%	29.55%	29.20%
44	15.65%	16.82%	20.48%	18.52%	17.41%
45	17.45%	10.86%	10.43%	10.46%	11.47%
46	25.84%	16.64%	17.49%	16.99%	19.68%
47	14.95%	9.84%	22.02%	14.98%	13.83%
48	49.74%	62.98%	45.57%	54.27%	52.76%
49	20.79%	16.93%	29.46%	21.30%	19.48%
Average	18.15%	16.67%	18.13%	16.48%	15.89%

Table 3: 2016q3 MAPE per Index

Index	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
14	19.38%	17.11%	28.91%	21.26%	18.92%
18	23.13%	20.93%	22.57%	21.12%	20.42%
19	16.26%	16.95%	17.26%	16.17%	15.40%
20	17.39%	15.28%	17.25%	15.53%	14.98%
$\frac{1}{21}$	15.32%	14.35%	16.95%	14.91%	14.05%
22	14.33%	13.57%	23.25%	16.84%	14.93%
$23^{}$	15.42%	11.95%	19.61%	15.18%	14.80%
24	25.78%	20.86%	16.06%	13.89%	13.19%
25	20.33%	17.61%	17.87%	16.60%	16.11%
27	19.29%	13.03%	21.52%	16.71%	15.51%
28	22.51%	15.42%	21.11%	17.38%	17.66%
29	21.12%	14.18%	14.14%	12.96%	13.85%
30	79.61%	20.15%	31.52%	23.88%	42.04%
31	93.61%	27.30%	21.04%	21.96%	45.65%
32	19.18%	18.94%	22.78%	19.29%	18.68%
33	10.77%	16.92%	6.19%	9.35%	9.04%
34	45.20%	75.29%	38.58%	45.86%	41.02%
35	27.75%	16.93%	25.08%	19.41%	20.83%
36	45.64%	33.10%	31.42%	26.22%	32.24%
37	31.93%	26.82%	24.29%	25.45%	26.26%
38	73.64%	38.25%	27.14%	28.07%	42.41%
39	16.15%	16.87%	19.24%	16.96%	15.64%
40	29.62%	47.43%	17.85%	27.69%	14.79%
41	28.59%	18.31%	34.35%	24.86%	24.73%
42	27.58%	27.66%	33.34%	28.92%	24.71%
43	15.29%	42.27%	50.21%	41.37%	32.68%
44	17.91%	14.16%	14.91%	12.80%	12.83%
45	13.47%	13.37%	19.63%	14.46%	12.77%
46	14.50%	15.54%	15.88%	13.65%	12.73%
47	8.57%	21.57%	18.80%	16.79%	10.66%
48	80.91%	34.30%	16.90%	22.79%	42.16%
49	43.99%	31.32%	3.65%	13.95%	8.60%
Average	22.64%	17.67%	20.72%	17.80%	18.10%

Table 4: 2016q4 MAPE per Index

Index	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
14	67.61%	71.83%	59.15%	70.42%	77.46%
18	86.67%	93.33%	86.67%	86.67%	93.33%
19	68.49%	69.86%	75.34%	69.86%	72.60%
20	70.59%	81.18%	83.53%	85.88%	82.35%
21	81.16%	81.16%	78.26%	81.16%	84.06%
22	59.38%	75.00%	75.00%	71.88%	81.25%
23	84.00%	84.00%	84.00%	82.00%	86.00%
24	50.00%	75.00%	75.00%	75.00%	75.00%
25	67.27%	74.55%	80.00%	78.18%	76.36%
27	68.57%	80.00%	77.14%	82.86%	80.00%
28	87.50%	100.00%	87.50%	95.83%	95.83%
29	73.68%	57.89%	68.42%	73.68%	68.42%
30	44.44%	55.56%	77.78%	55.56%	55.56%
31	51.28%	56.41%	61.54%	61.54%	56.41%
32	65.22%	82.61%	82.61%	82.61%	78.26%
33	53.85%	84.62%	76.92%	76.92%	76.92%
34	100.00%	50.00%	50.00%	50.00%	100.00%
35	56.00%	66.00%	70.00%	74.00%	70.00%
36	43.48%	65.22%	73.91%	69.57%	60.87%
37	68.75%	81.25%	75.00%	81.25%	81.25%
38	50.00%	75.00%	75.00%	75.00%	75.00%
39	66.10%	84.75%	88.14%	88.14%	76.27%
40	0.00%	50.00%	50.00%	50.00%	50.00%
41	60.87%	65.22%	52.17%	56.52%	56.52%
42	34.48%	41.38%	48.28%	48.28%	31.03%
44	80.95%	90.48%	95.24%	95.24%	95.24%
45	85.71%	85.71%	100.00%	85.71%	100.00%
46	44.44%	100.00%	77.78%	77.78%	77.78%
47	44.44%	77.78%	77.78%	88.89%	77.78%
48	100.00%	100.00%	66.67%	100.00%	100.00%
49	83.33%	100.00%	66.67%	100.00%	100.00%
Average	66.63%	75.54%	75.42%	77.00%	76.0%

Table 5: 2016q1 P20 per Index

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18 86.96% 78.26% 69.57% 73.91% 82.61% 19 71.03% 70.09% 71.96% 70.09% 73.83% 20 67.61% 64.79% 64.79% 63.38% 21 78.82% 84.71% 75.29% 83.53% 80.00% 22 77.27% 75.00% 68.18% 77.27% 77.27% 23 71.62% 78.38% 72.97% 75.68% 74.32% 24 60.00% 70.00% 70.00% 70.00% 80.00% 25 63.64% 65.91% 65.91% 68.18% 88.78% 27 64.86% 75.68% 72.97% 81.08% 83.78% 28 73.33% 83.33% 83.33% 80.00% 29 76.00% 84.00% 68.00% 76.00% 80.00% 30 45.45% 45.45% 36.36% 45.45% 27.27% 31 69.44% 66.67% 55.56% 61.11% 75.00% 32 75.00% 70.45% 75.00% 77.27% 79.55% 33 80.00% 100.00% 80.00% 80.00% 34 25.00% 25.00% 0.00% 25.00% 55.56% 35 51.16% 69.77% 65.12% 65.12% 62.79% 36 66.67% 77.78% 66.67% 55.56% 58.89% 37 61.90% 55.26% 55.56% 55.56% 39 77.27% 55.56% 55.56% 55.56% 39 77	Index	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
19 71.03% 70.09% 71.96% 70.09% 73.83% 20 67.61% 64.79% 64.79% 64.79% 63.38% 21 78.82% 84.71% 75.29% 83.53% 80.00% 22 77.27% 75.00% 68.18% 77.27% 77.27% 23 71.62% 78.38% 72.97% 75.68% 74.32% 24 60.00% 70.00% 70.00% 70.00% 80.00% 25 63.64% 65.91% 65.91% 68.18% 68.18% 27 64.86% 75.68% 72.97% 81.08% 83.78% 28 73.33% 73.33% 83.33% 83.33% 80.00% 29 76.00% 84.00% 68.00% 76.00% 80.00% 30 45.45% 45.45% 36.36% 45.45% 27.27% 31 69.44% 66.67% 55.56% 61.11% 75.00% 32 75.00% 70.045% 75.00% 77.27% 79.55% 33 80.00% 80.00% 80.00% 80.00% 34 25.00% 25.00% 0.00% 25.00% 25.00% 35 51.16% 69.77% 65.12% 65.12% 62.79% 36 66.67% 75.78% 55.56% 55.56% 39 77.27% 65.91% 52.27% 61.36% 72.73% 40 33.33% 33.64% 81.82% 83.64% 72.73% 41 32.73% 83.64% 81.82% <td>14</td> <td>63.54%</td> <td>70.83%</td> <td>53.13%</td> <td>58.33%</td> <td>60.42%</td>	14	63.54%	70.83%	53.13%	58.33%	60.42%
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22 $77.27%$ $75.00%$ $68.18%$ $77.27%$ $77.27%$ 23 $71.62%$ $78.38%$ $72.97%$ $75.68%$ $74.32%$ 24 $60.00%$ $70.00%$ $70.00%$ $70.00%$ $80.00%$ 25 $63.64%$ $65.91%$ $65.91%$ $68.18%$ $68.18%$ 27 $64.86%$ $75.68%$ $72.97%$ $81.08%$ $83.78%$ 28 $73.33%$ $73.33%$ $83.33%$ $83.33%$ $80.00%$ 29 $76.00%$ $84.00%$ $68.00%$ $76.00%$ $80.00%$ 30 $45.45%$ $45.45%$ $36.36%$ $45.45%$ $27.27%$ 31 $69.44%$ $66.67%$ $55.56%$ $61.11%$ $75.00%$ 32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $56.25%$ $50.00%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$	20	67.61%	64.79%	64.79%	64.79%	63.38%
23 71.62% 78.38% 72.97% 75.68% 74.32% 24 60.00% 70.00% 70.00% 70.00% 80.00% 25 63.64% 65.91% 65.91% 68.18% 68.18% 27 64.86% 75.68% 72.97% 81.08% 83.78% 28 73.33% 73.33% 83.33% 83.33% 80.00% 29 76.00% 84.00% 68.00% 76.00% 80.00% 30 45.45% 45.45% 36.36% 45.45% 27.27% 31 69.44% 66.67% 55.56% 61.11% 75.00% 32 75.00% 70.45% 75.00% 77.27% 79.55% 33 80.00% 80.00% 100.00% 80.00% 80.00% 34 25.00% 25.00% 0.00% 25.00% 25.00% 35 51.16% 69.77% 65.12% 62.79% 36 66.67% 77.78% 66.67% 51.90% 37 61.90% 61.90% 66.67% 61.90% 39 77.27% 65.91% 52.27% 61.36% 72.73% 40 33.33% 33.33% 66.67% 66.67% 66.67% 41 32.73% 83.64% 81.82% 83.64% 72.73% 42 56.25% 56.25% 56.25% 52.00% 50.00% 43 0.00% 100.00% 100.00% 80.00% 44 71.43% 85.71% 71.43% 85.71% 45 4	21	78.82%	84.71%	75.29%	83.53%	80.00%
24 $60.00%$ $70.00%$ $70.00%$ $70.00%$ $80.00%$ 25 $63.64%$ $65.91%$ $65.91%$ $68.18%$ $68.18%$ 27 $64.86%$ $75.68%$ $72.97%$ $81.08%$ $83.78%$ 28 $73.33%$ $73.33%$ $83.33%$ $83.33%$ $80.00%$ 29 $76.00%$ $84.00%$ $68.00%$ $76.00%$ $80.00%$ 30 $45.45%$ $45.45%$ $36.36%$ $45.45%$ $27.27%$ 31 $69.44%$ $66.67%$ $55.56%$ $61.11%$ $75.00%$ 32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $57.14%$ 38 $88.89%$ $55.56%$ $55.56%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.34%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $56.25%$ $52.50%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $71.43%$ $85.71%$ 45 $42.86%$ $85.71%$	22	77.27%	75.00%	68.18%	77.27%	77.27%
25 $63.64%$ $65.91%$ $65.91%$ $68.18%$ $68.18%$ 27 $64.86%$ $75.68%$ $72.97%$ $81.08%$ $83.78%$ 28 $73.33%$ $73.33%$ $83.33%$ $80.00%$ 29 $76.00%$ $84.00%$ $68.00%$ $76.00%$ $80.00%$ 30 $45.45%$ $45.45%$ $27.27%$ 31 $69.44%$ $66.67%$ $55.56%$ $61.11%$ $75.00%$ 32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $88.89%$ 37 $61.90%$ $61.90%$ $66.67%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $56.25%$ $50.00%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $71.43%$ $85.71%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ 46 $90.00%$ $80.00%$ $80.00%$ $80.00%$ 47 <td>23</td> <td>71.62%</td> <td>78.38%</td> <td>72.97%</td> <td>75.68%</td> <td>74.32%</td>	23	71.62%	78.38%	72.97%	75.68%	74.32%
27 $64.86%$ $75.68%$ $72.97%$ $81.08%$ $83.78%$ 28 $73.33%$ $73.33%$ $83.33%$ $83.33%$ $80.00%$ 29 $76.00%$ $84.00%$ $68.00%$ $76.00%$ $80.00%$ 30 $45.45%$ $45.45%$ $36.36%$ $45.45%$ $27.27%$ 31 $69.44%$ $66.67%$ $55.56%$ $61.11%$ $75.00%$ 32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $88.89%$ 37 $61.90%$ $61.90%$ $66.67%$ $51.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $62.50%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $85.71%$ $85.71%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $85.71%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $85.00%$ 45 $42.86%$ $85.71%$	24	60.00%	70.00%	70.00%	70.00%	80.00%
28 $73.33%$ $73.33%$ $83.33%$ $83.33%$ $80.00%$ 29 $76.00%$ $84.00%$ $68.00%$ $76.00%$ $80.00%$ 30 $45.45%$ $45.45%$ $36.36%$ $45.45%$ $27.27%$ 31 $69.44%$ $66.67%$ $55.56%$ $61.11%$ $75.00%$ 32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $88.89%$ 37 $61.90%$ $61.90%$ $66.67%$ $51.2%$ $62.79%$ 38 $88.89%$ $55.56%$ $55.56%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $62.50%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $85.71%$ $85.71%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $80.00%$ 45 $42.86%$ $80.00%$ $90.00%$ $80.00%$ $80.00%$ 46 $90.00%$ $80.00%$	25	63.64%	65.91%	65.91%	68.18%	68.18%
29 $76.00%$ $84.00%$ $68.00%$ $76.00%$ $80.00%$ 30 $45.45%$ $45.45%$ $36.36%$ $45.45%$ $27.27%$ 31 $69.44%$ $66.67%$ $55.56%$ $61.11%$ $75.00%$ 32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $51.56%$ $88.89%$ 37 $61.90%$ $61.90%$ $66.67%$ $61.90%$ $57.14%$ 38 $88.89%$ $55.56%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $62.50%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $71.43%$ $85.71%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $80.00%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $80.00%$ 45 $42.86%$ $80.00%$ $80.00%$ $80.00%$ $80.00%$ 45 $42.86%$ $80.00%$ $80.00%$	27	64.86%	75.68%	72.97%	81.08%	83.78%
30 $45.45%$ $45.45%$ $36.36%$ $45.45%$ $27.27%$ 31 $69.44%$ $66.67%$ $55.56%$ $61.11%$ $75.00%$ 32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $88.89%$ 37 $61.90%$ $61.90%$ $66.67%$ $61.90%$ $57.14%$ 38 $88.89%$ $55.56%$ $55.56%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $62.50%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $71.43%$ $85.71%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $80.00%$ 45 $42.86%$ $80.00%$ $90.00%$ $80.00%$ $80.00%$ 47 $50.00%$ $80.00%$ $80.00%$ $80.00%$ 48 $60.00%$ $80.00%$ $80.00%$ $80.00%$ 49 $77.78%$ $55.56%$ $77.78%$ $55.56%$	28	73.33%	73.33%	83.33%	83.33%	80.00%
31 $69.44%$ $66.67%$ $55.56%$ $61.11%$ $75.00%$ 32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $88.89%$ 37 $61.90%$ $61.90%$ $66.67%$ $51.2%$ $57.14%$ 38 $88.89%$ $55.56%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $56.25%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $75.71%$ $85.71%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $80.00%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $80.00%$ 47 $50.00%$ $80.00%$ $80.00%$ $80.00%$ $80.00%$ 48 $60.00%$ $80.00%$ $80.00%$ $80.00%$ 49 $77.78%$ $55.56%$ $77.78%$ $55.56%$ $77.78%$	29	76.00%	84.00%	68.00%	76.00%	80.00%
32 $75.00%$ $70.45%$ $75.00%$ $77.27%$ $79.55%$ 33 $80.00%$ $80.00%$ $100.00%$ $80.00%$ $80.00%$ 34 $25.00%$ $25.00%$ $0.00%$ $25.00%$ $25.00%$ 35 $51.16%$ $69.77%$ $65.12%$ $62.79%$ 36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $88.89%$ 37 $61.90%$ $61.90%$ $66.67%$ $61.90%$ $57.14%$ 38 $88.89%$ $55.56%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $56.25%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $78.57%$ $85.71%$ $92.86%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $80.00%$ 46 $90.00%$ $80.00%$ $90.00%$ $80.00%$ $80.00%$ 47 $50.00%$ $40.00%$ $90.00%$ $80.00%$ $80.00%$ 48 $60.00%$ $80.00%$ $60.00%$ $80.00%$ 49 $77.78%$ $55.56%$ $77.78%$ $55.56%$ $77.78%$	30	45.45%	45.45%	36.36%	45.45%	27.27%
33 80.00% 80.00% 100.00% 80.00% 80.00% 34 25.00% 25.00% 0.00% 25.00% 25.00% 35 51.16% 69.77% 65.12% 65.12% 62.79% 36 66.67% 77.78% 66.67% 55.56% 88.89% 37 61.90% 61.90% 66.67% 61.90% 57.14% 38 88.89% 55.56% 55.56% 55.56% 55.56% 39 77.27% 65.91% 52.27% 61.36% 72.73% 40 33.33% 33.33% 66.67% 66.67% 66.67% 41 32.73% 83.64% 81.82% 83.64% 72.73% 42 56.25% 56.25% 56.25% 62.50% 50.00% 43 0.00% 100.00% 100.00% 0.00% 44 71.43% 85.71% 71.43% 85.71% 45 42.86% 85.71% 71.43% 85.71% 46 90.00% 90.00% 80.00% 80.00% 47 50.00% 40.00% 90.00% 80.00% 80.00% 48 60.00% 80.00% 60.00% 80.00% 49 77.78% 55.56% 77.78% 55.56% 77.78%	31	69.44%	66.67%	55.56%	61.11%	75.00%
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	32	75.00%	70.45%	75.00%	77.27%	79.55%
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	33	80.00%	80.00%	100.00%	80.00%	80.00%
36 $66.67%$ $77.78%$ $66.67%$ $55.56%$ $88.89%$ 37 $61.90%$ $61.90%$ $66.67%$ $61.90%$ $57.14%$ 38 $88.89%$ $55.56%$ $55.56%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $56.25%$ $62.50%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $78.57%$ $85.71%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $85.71%$ 46 $90.00%$ $80.00%$ $90.00%$ $80.00%$ $60.00%$ 47 $50.00%$ $40.00%$ $90.00%$ $80.00%$ $80.00%$ 48 $60.00%$ $80.00%$ $60.00%$ $80.00%$ 49 49 $77.78%$ $55.56%$ $77.78%$ $55.56%$ $77.78%$	34	25.00%	25.00%	0.00%	25.00%	25.00%
37 $61.90%$ $61.90%$ $66.67%$ $61.90%$ $57.14%$ 38 $88.89%$ $55.56%$ $55.56%$ $55.56%$ $55.56%$ 39 $77.27%$ $65.91%$ $52.27%$ $61.36%$ $72.73%$ 40 $33.33%$ $33.33%$ $66.67%$ $66.67%$ $66.67%$ 41 $32.73%$ $83.64%$ $81.82%$ $83.64%$ $72.73%$ 42 $56.25%$ $56.25%$ $56.25%$ $62.50%$ $50.00%$ 43 $0.00%$ $100.00%$ $100.00%$ $100.00%$ $0.00%$ 44 $71.43%$ $85.71%$ $78.57%$ $85.71%$ $92.86%$ 45 $42.86%$ $85.71%$ $71.43%$ $85.71%$ $85.71%$ 46 $90.00%$ $80.00%$ $90.00%$ $80.00%$ $80.00%$ 47 $50.00%$ $40.00%$ $90.00%$ $80.00%$ $80.00%$ 48 $60.00%$ $80.00%$ $60.00%$ $80.00%$ 49 49 $77.78%$ $55.56%$ $77.78%$ $55.56%$ $77.78%$	35	51.16%	69.77%	65.12%	65.12%	62.79%
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	36	66.67%	77.78%	66.67%	55.56%	88.89%
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	37	61.90%	61.90%	66.67%	61.90%	57.14%
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	38	88.89%	55.56%	55.56%	55.56%	55.56%
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	39	77.27%	65.91%	52.27%	61.36%	72.73%
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	40	33.33%	33.33%	66.67%	66.67%	66.67%
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	41	32.73%	83.64%	81.82%	83.64%	72.73%
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	42	56.25%	56.25%	56.25%	62.50%	50.00%
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	43	0.00%	100.00%	100.00%	100.00%	0.00%
4690.00%80.00%90.00%80.00%80.00%4750.00%40.00%90.00%80.00%60.00%4860.00%80.00%60.00%80.00%80.00%4977.78%55.56%77.78%55.56%77.78%	44	71.43%	85.71%	78.57%	85.71%	92.86%
4750.00%40.00%90.00%80.00%60.00%4860.00%80.00%60.00%80.00%80.00%4977.78%55.56%77.78%55.56%77.78%	45	42.86%	85.71%	71.43%	85.71%	85.71%
4860.00%80.00%60.00%80.00%80.00%4977.78%55.56%77.78%55.56%77.78%	46	90.00%	80.00%	90.00%	80.00%	80.00%
49 77.78% 55.56% 77.78% 55.56% 77.78%	47	50.00%	40.00%	90.00%	80.00%	60.00%
	48	60.00%	80.00%	60.00%	80.00%	80.00%
Average 67.27% 72.06% 68.06% 71.06% 71.76%	49	77.78%	55.56%	77.78%	55.56%	77.78%
	Average	67.27%	72.06%	68.06%	71.06%	71.76%

Table 6: 2016q2 P20 per Index

Index	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
14	76.67%	66.67%	70.00%	75.00%	81.67%
18	65.00%	85.00%	65.00%	75.00%	80.00%
19	69.81%	71.70%	75.47%	72.64%	73.58%
20	61.29%	64.52%	66.13%	67.74%	64.52%
21	73.91%	81.52%	75.00%	80.43%	83.70%
22	63.16%	68.42%	57.89%	63.16%	68.42%
23	73.08%	78.21%	65.38%	70.51%	75.64%
24	50.00%	66.67%	66.67%	83.33%	83.33%
25	56.52%	63.04%	65.22%	65.22%	60.87%
27	51.85%	62.96%	59.26%	59.26%	55.56%
28	81.82%	90.91%	72.73%	81.82%	81.82%
29	75.00%	75.00%	68.75%	68.75%	68.75%
30	50.00%	100.00%	100.00%	100.00%	100.00%
31	66.67%	61.90%	57.14%	52.38%	71.43%
32	66.67%	100.00%	88.89%	94.44%	88.89%
33	57.14%	28.57%	42.86%	42.86%	42.86%
34	50.00%	50.00%	0.00%	0.00%	0.00%
35	82.00%	70.00%	62.00%	62.00%	68.00%
36	90.91%	90.91%	72.73%	81.82%	100.00%
37	70.59%	76.47%	76.47%	76.47%	76.47%
38	40.00%	60.00%	50.00%	50.00%	50.00%
39	67.39%	65.22%	54.35%	71.74%	76.09%
40	33.33%	66.67%	33.33%	33.33%	66.67%
41	42.86%	54.29%	65.71%	71.43%	54.29%
42	30.77%	46.15%	38.46%	30.77%	46.15%
44	66.67%	60.00%	40.00%	60.00%	46.67%
45	61.54%	84.62%	84.62%	76.92%	84.62%
46	35.29%	70.59%	52.94%	58.82%	47.06%
47	66.67%	100.00%	50.00%	83.33%	100.00%
48	0.00%	0.00%	0.00%	0.00%	0.00%
49	57.14%	71.43%	42.86%	57.14%	57.14%
Average	66.19%	70.97%	65.95%	69.65%	71.33%

Table 7: 2016q3 P20 per Index

Index	SMV	NN	MRA	NN + MRA	SMV+NN+MRA
14	70.00%	65.38%	45.38%	55.38%	60.77%
18	61.11%	66.67%	50.00%	55.56%	66.67%
19	70.77%	69.23%	67.18%	71.79%	73.33%
20	64.81%	75.00%	77.78%	77.78%	80.56%
21	71.13%	78.17%	67.61%	74.65%	80.99%
22	79.59%	79.59%	44.90%	63.27%	71.43%
23	77.00%	80.00%	54.00%	73.00%	73.00%
24	73.33%	66.67%	80.00%	73.33%	86.67%
25	58.02%	71.60%	72.84%	74.07%	71.60%
27	58.18%	76.36%	65.45%	69.09%	67.27%
28	58.62%	72.41%	58.62%	68.97%	75.86%
29	63.64%	69.70%	78.79%	81.82%	75.76%
30	8.33%	50.00%	16.67%	50.00%	16.67%
31	50.00%	50.00%	50.00%	44.12%	41.18%
32	76.00%	56.00%	56.00%	72.00%	68.00%
33	86.67%	60.00%	100.00%	86.67%	93.33%
34	28.57%	0.00%	14.29%	28.57%	42.86%
35	53.49%	72.09%	51.16%	65.12%	60.47%
36	37.50%	43.75%	43.75%	37.50%	43.75%
37	36.36%	36.36%	63.64%	45.45%	45.45%
38	27.27%	27.27%	36.36%	36.36%	36.36%
39	72.97%	70.27%	60.81%	70.27%	74.32%
40	40.00%	20.00%	40.00%	40.00%	80.00%
41	52.17%	60.87%	52.17%	65.22%	52.17%
42	50.00%	35.00%	35.00%	25.00%	35.00%
43	75.00%	25.00%	0.00%	25.00%	25.00%
44	76.47%	76.47%	70.59%	70.59%	70.59%
45	80.00%	80.00%	80.00%	90.00%	90.00%
46	70.00%	70.00%	60.00%	80.00%	70.00%
47	100.00%	66.67%	66.67%	66.67%	88.89%
48	0.00%	22.22%	66.67%	44.44%	11.11%
49	33.33%	66.67%	100.00%	66.67%	100.00%
Average	65.33%	68.28%	60.80%	67.15%	69.03%

Table 8: 2016q4 P20 per Index

Index	All	In	Out	Year	All	In	Out	Urban Classifications	All	In	Out	Type of Property	All	In	Out
14	3977	3620	357	2012	3951	3951	-	Rural	1976	1615	361	House	2395	2037	358
18	925	849	76	2013	13316	13316	-	Small town	2959	2580	379	Flat	30838	27657	3181
19	3442	2961	481	2014	10017	10017	-	Small city	2894	2571	323	Duplex	1315	1108	207
20	3157	2831	326	2015	5193	5193	-	Medium size city	3857	3483	374	Maisonette	1979	1675	304
21	3354	2966	388	2016	4050	-	4050	Larger city	4510	4105	405				
22	1063	919	144					Capital	20331	18123	2208				
23	2657	2355	302												
24	425	390	35												
25	1652	1426	226												
27	1999	1845	154												
28	1016	922	94												
29	641	548	93												
30	309	275	34												
31	1441	1311	130												
32	1106	996	110												
33	517	477	40												
34	216	201	15												
35	1363	1177	186												
36	595	536	59												
37	820	755	65												
38	413	360	53												
39	1250	1027	223												
40	153	140	13												
41	838	702	136												
42	387	309	78												
43	73	68 765	5												
44	832	765	67 27												
45	384 507	347 461	37 46												
46	507 422	461	46												
47	$432 \\ 253$	$\frac{398}{235}$	$\frac{34}{18}$												
48	$\frac{253}{330}$	$\frac{235}{305}$	$\frac{18}{25}$												
49	330	305	$_{20}$												

Table 9: Number of observations per Index area, year, urban classification and type of property

2 Supplementary Material: Figures

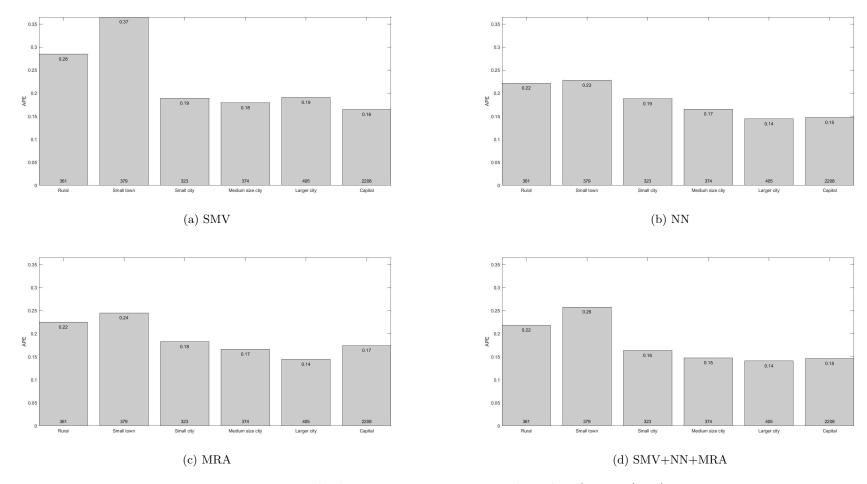


Figure 1: Mean absolute percentage error per urban classification (V05).

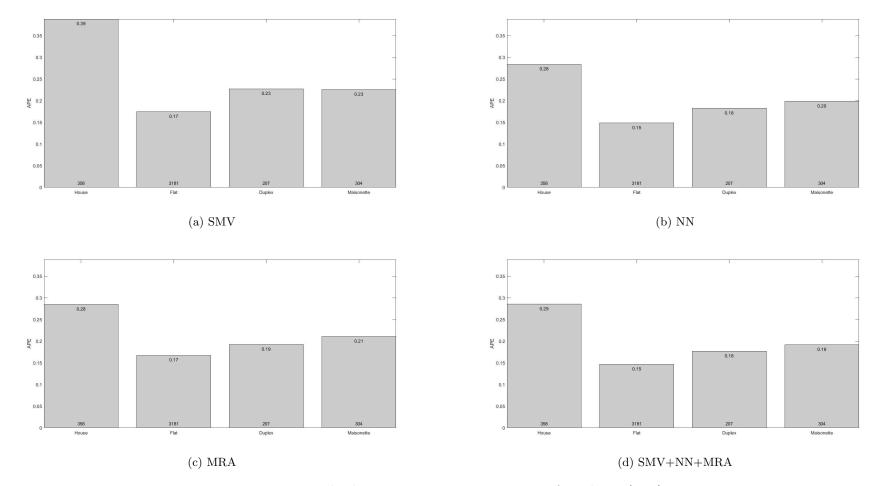


Figure 2: Mean absolute percentage error per type of residence (V07).

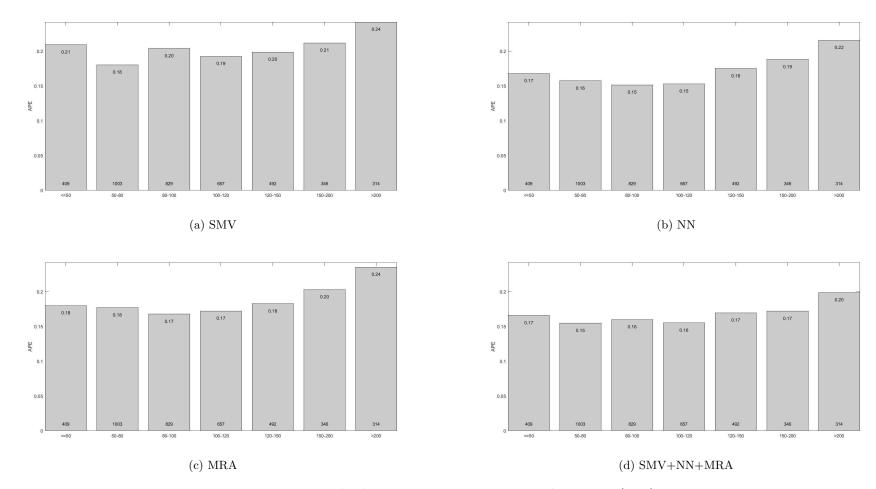


Figure 3: Mean absolute percentage error per residence area (V08).

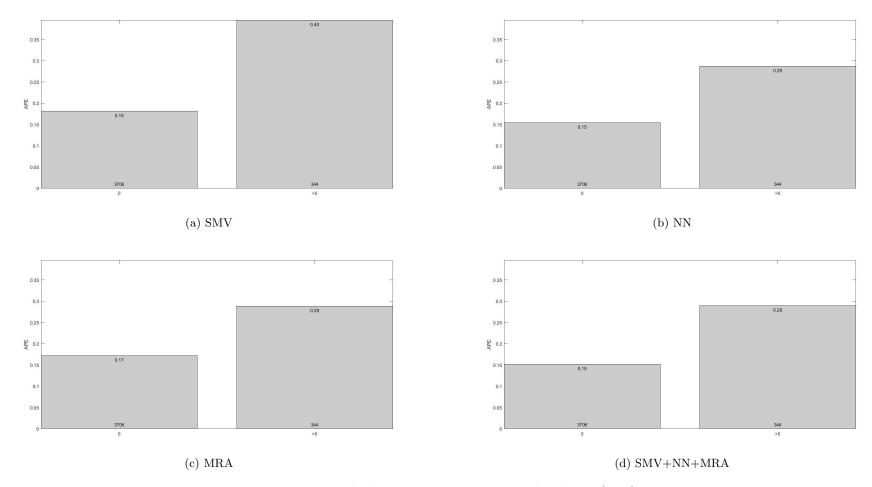


Figure 4: Mean absolute percentage error per land area (V09).

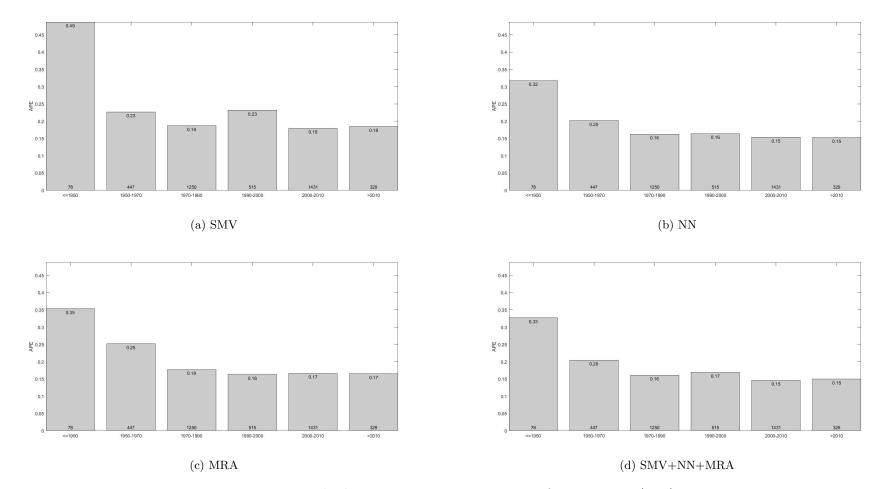


Figure 5: Mean absolute percentage error per year of construction (V10).

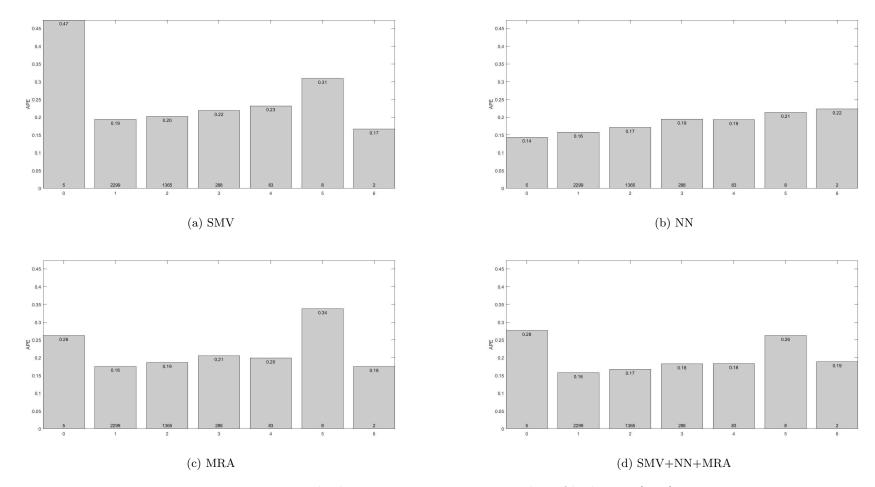


Figure 6: Mean absolute percentage error per number of bedrooms (V22).