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Skyrmions in the Moore-Read state at $\nu = 5/2$

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We study spinful excitations in the Moore-Read state. Energetics of the skyrmion based on a spin-wave picture support the existence of skyrmion excitations in the plateau below $\nu = 5/2$. This prediction is then tested numerically. We construct trial skyrmion wavefunctions for general FQHE states, and obtain significant overlaps for the predicted skyrmions of $\nu = 5/2$. The case of $\nu = 5/2$ is particularly interesting as skyrmions have twice the charge of quasiparticles (qp's). As the spin polarization of the system is tuned from full to none, we observe a transition between qp- and skyrmion-like behaviour of the excitation spectrum that can be interpreted as binding of qp's. Our ED results confirm that skyrmion states are energetically competitive with quasiparticles at low Zeeman coupling. Disorder and large density of quasiparticles are discussed as further mechanisms for depolarization.

The $\nu = 5/2$ state has been intensely studied in recent years due to accumulating evidence that it realizes a non-abelian phase of matter [1], that could serve as a basis for topological quantum computation [2]. A crucial step towards establishing the experimental $\nu = 5/2$ state as in the weakly paired Moore-Read phase [1, 3, 4], is to show that the groundstate is spin-polarized. This has been achieved theoretically by numerical simulations of model systems [5–7]. However, recent experiments [8, 9] cast doubt on the realization of a polarized quantum liquid at $\nu = 5/2$. Experiments by Piczuk *et al.*[8] are consistent even with a completely unpolarized Hall state at this filling. It appears crucial to resolve the discrepancy between the current theoretical understanding of the $\nu = 5/2$ as a spin-polarized Hall state and these puzzling experimental findings.

In this Letter, we analyze the possibility whether skyrmion excitations exist, and contribute to depleting the spin-polarization at $\nu = 5/2$. An important factor influencing the spin spectrum, that was ignored in previous studies [5], is the role of finite layer width w. Indeed, the leading even pseudopotentials V_0, V_2 of the Coulomb interaction in the second Landau level soften considerably upon increasing w. In line with this observation, it was found for integer quantum Hall states in higher LL that skyrmions are stabilized only in finite w [10]. We shall therefore include variations of w in all our considerations, below [*** define somewhere the width-model used, or add reference ***]. At finite $w \gtrsim \lambda$ (where $\lambda = [\hbar c/eB]^{1/2}$ is the magnetic length), we find hole-like skyrmion excitations to exist, and to be stable. These skyrmions are found to be very different from the skyrmions of integer QH states or at $\nu = 1/3$. As their charge is twice that of the fundamental charged excitations, skyrmions may break up into pairs of quasiparticles. This has important consequences for high quasiparticle densities or in the presence of disorder, where qp's can spontaneously bind and acquire a spin texture to minimize their energy.

Our study proceeds as follows. We first estimate the energy of skyrmions based on an estimate of the spin-stiffness at $\nu = 5/2$. This leads us to predict that skyrmions are favoured over pairs of quasiholes. We then establish this result in finite systems on the sphere based on exact diagonalization techniques. In particular, we formulate trial wavefunctions for general skyrmions and obtain satisfactory overlaps with the exact states. An alternative construction of these trial states using model Hamiltonians is also given. In light of these results, we then consider the competition of skyrmions and quasiparticles, which is found to confirm the conclusions obtained from spin-wave theory. Finally, we discuss how the interplay of $e/2$ skyrmions and $e/4$ quasiparticles intriguingly influences transport properties.

Skyrmions in quantum Hall liquids are topological excitations created by adding a single flux quantum to a fully spin polarized state while creating a spin texture that absorbs the additional degree of freedom. Given the spin-stiffness ρ_s of a polarized electron gas, the energy of an infinitely extended [lowest energy] skyrmion is $E_{\text{sk}} = 4\pi\rho_s$. However, ρ_s can also be found from the single-spinwave spectrum of the groundstate. We considered such spectra in exact numerical diagonalizations (ED) of finite systems on the sphere for the Moore-Read state at flux $N_\phi = 2N - 3$ and spin $S = N/2 - 1$. At long wavelength, spinwaves have a quadratic dispersion which reads, expressed for the 2nd LL and on the sphere, $E_L = \frac{8\pi}{\nu N_\phi}\rho_s L(L+1)$ (*** N_ϕ or $N_\phi - 2$ in denominator? ***). In the spectra for systems of size $N = 10, \ldots, 16$ (check!) that could be calculated in ED, only the $L = 1$ state is well separated from the further excitation spectrum. Therefore, we estimate ρ_s by regression of $E_{L=1} = [E^{S=N/2}_{L=1} - E^{\text{pol}}]/N$ as a function of the system size, as shown in Fig. 1a). The resulting skyrmion energy has to be compared against the energy-cost to creating a pair of quasiholes (electrons) upon adding (removing) one flux quantum to the polarized $\nu = 5/2$ liquid. While the most common quantity to be evaluated is the charge-gap $\Delta = \epsilon_{qe} + \epsilon_{qh}$, care has to be taken to determine how this energy-cost is distributed between these two particles. When comparing different types of quasiparticles, the relevant quantity is the neutral quasipar-

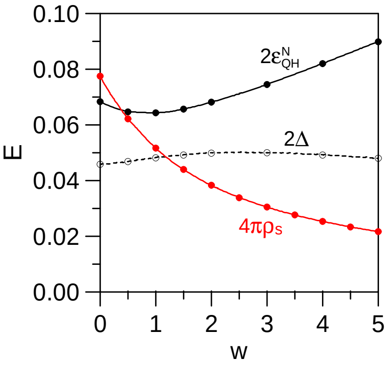

FIG. 1: AREK: could you add the extrapolation of E_1 to extract ρ_s as panel a)? a) Extraction of the spin-stiffness ρ_s from the long-wavelength part of the spectra with a single spinflip. b) Skyrmion energy from spin-stiffness, in comparison to the neutral quasihole energy ϵ^N_{qh}, establishing the skyrmion as the lowest energy hole-like excitation of the $\nu = 5/2$ state at finite width.

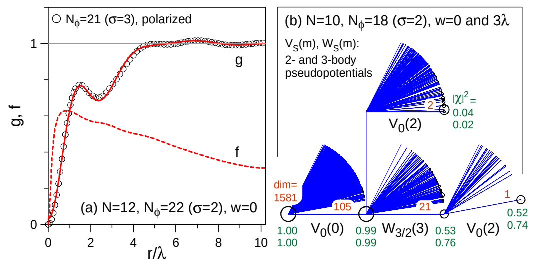

FIG. 2: a) Spin correlations of the skyrmion state at $N_\phi = 2N - 2$ in terms of guiding center coordinates (left) and electron coordinates (right). [for clarity, show a single correlation function for 2N-2, maybe $w = 0$, and $w = 3$, but maybe one is sufficient] b) Splitting of the $L = S = 0$ Hilbert space for $N = 10$ and $2l = 18$ under the consecutive action of pair and triplet pseudopotentials, $V_S(m)$ and $W_S(m)$. At each step, green labels give squared projections of the Coulomb ground state onto the lowest subspace (upper and lower value: $w = 0$ and $w = 3\lambda$) with dimension indicated in red. The nondegenerate ground state obtained by the successive application of $V_0(0)$, $W_{3/2}(3)$, and $V_0(2)$ is an approximation to the skyrmion formed in the pfaffian state.

ticle energy [11, 12]

$$\epsilon^N_{qh(e)} = \tilde{\epsilon}_{qh(e)} \mp \frac{\nu}{2k}\xi_\nu, \tag{1}$$

which takes into account the gross quasiparticle energy $\tilde{\epsilon}_{qh(e)}$ that expresses a change in correlation energy, as well as the change of potential energy associated with modifying the number of particles in the liquid. The latter is related to the energy per particle in the infinite system ξ_ν, and we have added the possibility of creating k quasiparticles per flux quantum. Using known quasiparticle- and groundstate-energies for the Moore-Read state [13, 14], we obtain a qh energy ϵ^N_{qh} of the order of the gap, while ϵ^N_{qe} is slightly negative [20]. Our results, as a function of layer width w, are summarized in Fig. 1b), and show a cross-over from the quasihole to the skyrmion as the lowest energy excitation at small but finite $w \approx \frac{\lambda}{2}$.

The non-abelian statistics of the Moore-Read state allows for two distinct fusion channels $(1, \psi)$ for a pair of quasiholes. For completeness, let us also consider states in the ψ-channel, which occurs at odd N. As the groundstate energies for both odd and even N extrapolate to the same thermodynamic limit, we consider the energy cost of adding a neutral fermion

$$\Delta_{nf}[N] = (E_{N+1}/(N+1) - E_N/N)N \tag{2}$$

for states with two qh at $N_\phi = 2N-2$. The scaling of $\Delta_{nf}[N]$ reveals a positive and roughly constant (in w) gap towards adding a neutral fermion. We will therefore focus on even particle numbers N, below.

Let us analyze the exact numerical spectra of small model systems to verify if the predictions of spin-wave theory for the stability of skyrmions are born out. While we considered the infinite skyrmion above, the largest skyrmion that can be created on a finite sphere is the one which covers its entire surface, and has quantum numbers $S = L = 0$. In order to obtain the $L = S = 0$ many body states for finite N in exact diagonalization, we use a projected Lanczos procedure [15] that constrains S^2 to remain minimized between iterations (while simulating in a subspace with L_z, S_z fixed). We are thus able to generate spin-projected groundstates efficiently, with Hilbert-space dimensions reaching 1.4×10^9 in the case of $N = 12$ at $N_\phi = 26$. Skyrmions potentially exist at a field of one flux quantum above/below an incompressible state. For $\nu = 5/2$ this mother state could be either the pfaffian Pf or antipfaffian $\overline{\mathrm{Pf}}$ [16, 17], with shifts on the sphere of $\sigma = 3$ or $\sigma = -1$ respectively [with the shift σ defined by $N_\phi = \nu^{-1}N - \sigma$]. There are four possible skyrmion states with $\sigma = 4, 2, 0$ and -2, of which the state at $\sigma = 2$ appears most energetically favourable in extrapolations of the respective GS energies from the ED spectra of systems with up to $N = 12$ particles. Let us tentatively name these states Ψ_σ by their shift σ, and distinguish skyrmions $\mathcal{S}^+$ (increasing flux) or anti-skyrmions $\mathcal{S}^-$ (decreasing N_ϕ), such that $\Psi_4 = \mathcal{S}^+ Pf$, $\Psi_2 = \mathcal{S}^- Pf$, $\Psi_0 = \mathcal{S}^+ Pf^*$, and $\Psi_{-2} = \mathcal{S}^- Pf^*$.

Features of the correlation functions of $\Psi_{4,2,0,-2}$ are consistent with the interpretation of these spin-singlet groundstate as skyrmions. Let us discuss the case of Ψ_2, shown in Fig. 2. Remarkably, its charge correlations $g(r) = g_{\uparrow\uparrow} + g_{\uparrow\downarrow}$ are virtually identical to the correlation function of the spin-polarized Moore-Read state at shift $\sigma = 3$. In addition however, there is a distinct spin texture revealed by decreasing spin correlations $f = g_{\uparrow\uparrow} - g_{\uparrow\downarrow}$, i.e. antiparallel spins are favoured on long distances, while parallel spins dominate at small r. Similar features are consistently found for the other three possible skyrmions with $\sigma = 4, 0$, and -2. To further consolidate our interpretation of this state as a skyrmion of $\nu = 5/2$, we formulate a trial wavefunction for an idealised skyrmion state,

N	$d(\mathcal{H}_{L=0})$	$\mathcal{O}_{MR}$	$\mathcal{O}_{\text{CF-BCS}}$	$\mathcal{O}_{MR}$	$\mathcal{O}_{\text{CF-BCS}}$
		$w=0$		$w=3\ell_0$	
8	285	0.788(9)	0.802(9)	0.81(2)	0.84(3)
10	6996	0.51(3)	0.54(3)	0.71(1)	0.72(1)

TABLE I: If there is space, this table will be kept, otherwise a few values of the overlaps may just be added in the main text. If kept, should add overlaps obtained with polarized Coulomb GS, also.

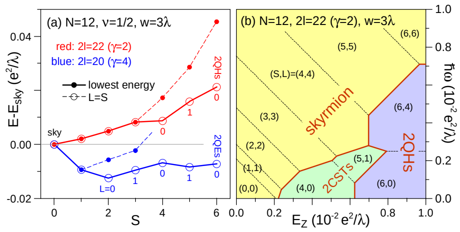

FIG. 3: (a) Dependence of energy E on the total spin S for $N=12$ unpolarized electrons at $2l=20$ and 22 (i.e., at $\nu=1/2$ and $\gamma=4$ or 2). Solid and dashed lines connect the ground states and the lowest $L=S$ states at each S. The energies include electrostatic corrections defined in the text. (b) Phase diagram in the plane of Zeeman energy E_Z and lateral harmonic confinement $\hbar\omega$, showing transitions between various states of a pair of $e/4$-charged quasiholes (QHs) or spin textures (CSTs), and of the positive skyrmions ($L=S$).

and consider its overlaps with the states from ED.

In the case of the integer quantum Hall effect at $\nu=1$, it is known that the wavefunction factors into a part describing the charge correlations of a filled LL multiplying a factor Ψ_B describing the spin-texture of the skyrmion [18]. Ψ_B can be obtained formally as a many-body states of spinful *bosons* experiencing the added flux $\delta N_\phi = N_\phi - N^{\text{pol}}_{\phi,\nu=1}$. At small δN_ϕ, such a state is uniquely defined by its angular-momentum and spin quantum numbers L, L_z, S and S_z. We construct general skyrmion states by analogy, multiplying a general polarized state $\Psi^{\nu_0}_{\text{pol}}$ by a spin texture to yield

$$\Psi^{\nu_0}_{\text{sk}}(\{z_i,\chi_i\}) = \Psi^{\text{pol}}_{\nu_p}(\{z_i\}) \times \Psi^{S,L}_B(\{z_i,\chi_i\}) \quad (3)$$

$$\overset{S=0}{=} \mathcal{P}_{S=0}\left[\Psi^{\text{pol}}_{\nu_p}(\{z_i\}) \times \begin{pmatrix} u_i \\ v_i \end{pmatrix}\right], \quad (4)$$

where particle coordinates are denoted by positions $z_i = x_i + y_i \leftrightarrow (u_i\, v_i)$ and spinors χ_i. Up to a projection $\mathcal{P}_{S=0}$ into a global spin singlet, the spin of electrons in an infinite skyrmion (4) simply points radially outwards at all points on the sphere. We calculate overlaps of these trial states with the exact eigenstates in a standard way [19], where the polarized state can be taken either as the Moore-Read state, an optimized weakly paired state [3] or as the exact Coulomb eigenstate of the polarized system. The resulting overlaps, shown in Table I, are found be moderately large. Further information as to where inaccuracies in our modelling arise can be extracted from a construction of skyrmion states using simple model Hamiltonians build from short distance two- and three-body repulsions as represented in Fig. 2. Beginning with the full $L=S=0$ Hilbert space $\mathcal{H}$ (dimension $d=1581$ at $N=10$), we first construct a zero-energy subspace $\mathcal{H}'$ ($d=105$) of the pair interaction with a single pseudopotential coefficient, $V_S(m)=\delta_{S,0}\delta_{m,0}$. Next, within $\mathcal{H}'$ we apply a triplet interaction $W_S(m)=W_{3/2}(3)$, to obtain a zero-energy subspace $\mathcal{H}''$ ($d=21$) retaining unpolarized states with pfaffian-like correlations at short range. Finally, inside $\mathcal{H}''$ we use minimization of $V_0(2)$ to select the state with the longest spin wave length, i.e., the infinite skyrmion formed in an exact pfaffian parent. Indeed, the single state obtained in this procedure is very similar to the trial state (3) based on the Moore-Read pfaffian, with a relative overlap of $0.96(1)$. Monitoring the total overlap of the exact Coulomb groundstate with vectors in $\mathcal{H}^{(n)}$, it becomes evident that overlap is lost at the stage of enforcing the pair-correlations of the pfaffian. This is hardly surprising given that even in polarized systems, the pfaffian is not a very accurate description of the exact Coulomb groundstate [3, 6]. In line with this analogy, the overlap increases in finite well width [i.e., relatively higher $V_0(0)$].

Having identified the spin-singlet states $\Psi_{2(4)}$ as skyrmions of the MR pfaffian, let us now analyze the entire spin-spectrum including partial spin-polarizations. For general skyrmions of size K on the sphere [18], the angular momentum satisfies $L=S=\frac{N}{2}-K>0$, with the 'infinite' skyrmion studied above being $K=\frac{N}{2}$. Considering the lowest energy Coulomb states with these quantum numbers at angular momentum projection $L_z=S_z=L=S$ we confirm that these correspond to states with an accumulation of charge $(e/2)$ and spin up within an area $\propto K$ (???) near the north pole.

Are these skyrmions stable, however? Carrying twice the charge ($q_{\text{sk}}=e/2$) of the elementary spinless quasiparticles of the Moore–Read state ($q_{qp}=e/4$), they must combine a pair of like-charged quasiholes (QHs) or quasielectrons (QEs). It would therefore be somewhat of a surprise should such skyrmions be stable at small sizes K. Indeed, as shown in Fig. 3(a) for $N=12$ particles in a system of finite width $w=3\lambda$, the energy of the Coulomb-groundstate in the Hilbert-subspace with fixed S is *lower* than the energy of the lowest state with the quantum numbers of the skyrmion for $K\lesssim\frac{N}{4}$. This break-up of a unique $e/2$ skyrmion into two separate objects is clearly evidenced by a low total angular momentum (shown in Fig. 3, wherever $L\neq S$) of the Coulomb state. Only a pair of two separate objects whose individual angular momenta are counter-aligned can explain such small L. We also refer to these intermediate states composed of two separate charged entities as charge-spin textures (CST). A further subtlety comes into play when comparing skyrmions of different sizes: finite-size effects caused by the concentration of charge in form of two QEs/QHs/CSTs have to be compensated for. This is achieved by applying the standard electrostatic correction δE [13] for quasiparticle states at $S = \frac{N}{2}$, and by using cubic interpolation of $(2S/N)^3\delta E$ at the intermediate polarizations. A marked asymmetry between $\mathcal{S}^-$ and $\mathcal{S}^+$ is evident. While quasielectrons are energetically more favourable than antiskyrmions, skyrmions are clearly preferred over quasiholes. These finite size results therefore confirm our above analysis based on extrapolation of the skyrmion energy from the spin-stiffness.

In realistic systems, skyrmion stability will be reduced by the Zeeman splitting E_Z. However, quite unlike for cases with $q_{\rm sk} = q_{qp}$, the stability of skyrmions may be *enhanced* by the presence of disorder, here. In the usual case, disorder acts to reduce the size of pinned charge carriers, i.e., it disfavours large skyrmions. For nu=5/2 on the other hand, disorder can act to bring two $e/4$ quasiholes together to form a single skyrmion. It can thus have the opposite effect of making skyrmions *more* favourable, by overcoming the Coulomb repulsion of the quasiparticles. We model this effect by assuming a simple lateral harmonic confinement of frequency ω that modulates the energy of available states as a function of their localization of charge. We thus convert the map of Coulomb energy $E(L,S)$ into the phase diagram for the stability of $\mathcal{S}^+$ syrmions shown in Fig. 3(b). While its details are certainly affected by the finite size $N = 12$ and choice of disorder, the emergence and relative position of the polarized 2QH phase and the unpolarized 2CST and skyrmion phases are expected to remain valid in realistic systems. [Need to say what the values of E_Z and $\hbar\omega$ correspond to in terms of more obvious system parameters]

Another concern that might be raised about the stability of skyrmions in the $\nu = 5/2$ states concerns the presence of the second fusion channel ψ. If at odd particle number, a different low-energy state involving a neutral fermion would have lower energy than the skyrmion obtained as a spin-texture of the vacuum channel, our conclusions would fail. In analogy of our above comparison of the two fusion channels for the polarized system, we have scaled the energy per particle for the systems with the smallest possible polarization, $S = \frac{1}{2}$ for N odd versus $S = 0$ for N even according to Eq. (2). While our results are limited to two data points for $N = 9$ and $N = 11$ due to the large Hilbert spaces at low spin and we cannot confidently extrapolate to large N, the data obtained are consistent with the ψ channel being higher in energy. [**overlaps for odd N**]

If realized in experimental samples, the presence of skyrmions due to pinning by disorder might have an unusual effect on the transport properties of the system. Typically, increasing the Zeeman energy also increases the transport gap, as it increases the creation energy of the charge carrier. However, here, if the skyrmions are present only due to pinning in the disorder, and the elementary charge carriers are $e/4$ QPs, then one can expect the opposite effect. Namely, if pinned quasiparticles are bound into skyrmions, increasing the Zeeman energy decreases their binding energy and would therefore *decrease* the activation energy associated with the unbinding of these QPs.

(** Discuss effect of skyrmions on NA interferometer **)

A large density of quasiparticles far from the centre of the quantum Hall plateau can have a similar effect, as it forces qp's to come close.

Conclusions

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[19] G. Möller, S. H. Simon, and E. H. Rezayi, Phys. Rev. B **79**, 125106 (2009), see App. C-2.
[20] In line with the results found for $\nu = \frac{1}{3}, \frac{1}{5}$ in Ref. [12].