

SUPPLY CHAIN MANAGEMENT FOR THE PROCESS INDUSTRY



Songsong Liu

Department of Chemical Engineering

University College London

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DECLARATION

I, Songsong Liu, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Signature:

Date:

ABSTRACT

This thesis investigates some important problems in the supply chain management (SCM) for the process industry to fill the gap in the literature work, covering production planning and scheduling, production, distribution planning under uncertainty, multiobjective supply chain optimisation and water resources management in the water supply chain planning. To solve these problems, models and solution approaches are developed using mathematical programming, especially mixed-integer linear programming (MILP), techniques.

First, the medium-term planning of continuous multiproduct plants with sequence-dependent changeovers is addressed. An MILP model is developed using Travelling Salesman Problem (TSP) classic formulation. A rolling horizon approach is also proposed for large instances. Compared with several literature models, the proposed models and approaches show significant computational advantage.

Then, the short-term scheduling of batch multiproduct plants is considered. TSP-based formulation is adapted to model the sequence-dependent changeovers between product groups. An edible-oil deodoriser case study is investigated.

Later, the proposed TSP-based formulation is incorporated into the supply chain planning with sequence-dependent changeovers and demand elasticity of price. Model predictive control (MPC) is applied to the production, distribution and inventory planning of supply chains under demand uncertainty.

A multiobjective optimisation problem for the production, distribution and capacity planning of a global supply chain of agrochemicals is also addressed, considering cost, responsiveness and customer service level as objectives simultaneously. Both ϵ -constraint method and lexicographic minimax method are used to find the Pareto-optimal solutions

Finally, the integrated water resources management in the water supply chain management is addressed, considering desalinated water, wastewater and reclaimed water, simultaneously. The optimal production, distribution and storage systems are determined by the proposed MILP model. Real cases of two Greek islands are studied.

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CONTENTS

Declaration	2
Abstract	3
Acknowledgements	5
Contents	6
List of Figures	12
List of Tables	14
Chapter 1 General Introduction	16
1.1 Introduction to SCM	16
1.1.1 What is SCM.....	17
1.1.2 Key Elements in SCM.....	18
1.1.3 Hierarchical Levels in SCM.....	19
1.2 SCM in the Process Industry.....	20
1.2.1 What is Process Industry.....	20
1.2.2 Process Industry SCM.....	20
1.3 Mathematical Programming Techniques	21
1.4 Scope of This Thesis	23
1.4.1 Production Planning and Scheduling	23
1.4.2 Production and Distribution Planning under Uncertainty.....	24
1.4.3 Multiobjective Supply Chain Optimisation	24
1.4.4 Water Supply Chain Design and Planning.....	25
1.5 Thesis Overview.....	25
Chapter 2 Medium-Term Planning of Single-Stage Multiproduct Continuous Plants	27
2.1 Introduction and Literature Review	27
2.2 Problem Description.....	32
2.3 Mathematical Formulation	33
2.3.1 Nomenclature	34
2.3.2 Assignment Constraints	36
2.3.3 Changeover Constraints	37

2.3.4 Subtour Elimination Constraints	38
2.3.5 Timing Constraints	41
2.3.6 Production Constraints	41
2.3.7 Backlog Constraints	41
2.3.8 Inventory Constraints	42
2.3.9 Objective Function	42
2.3.10 Summary	42
2.4 Illustrative Examples.....	42
2.4.1 Illustrative Example 1	43
2.4.1.1 Data	43
2.4.1.2 Results and Discussion.....	44
2.4.1.3 Changeover Assumptions.....	48
2.4.2 Illustrative Example 2	49
2.4.2.1 Data	49
2.4.2.2 Results and Discussion.....	50
2.5 Rolling Horizon Algorithm	53
2.5.1 Algorithm Description	53
2.5.2 Illustrative Example 2 Revisited.....	55
2.6 Comparison with Literature Models	57
2.6.1 Literature Model Modifications	57
2.6.2 Model Size Comparison.....	59
2.6.3 Example A.....	60
2.6.4 Example B.....	61
2.6.5 Example C.....	62
2.6.6 Example D.....	63
2.7 Concluding Remarks.....	65
Chapter 3 Short-Term Scheduling of Single-Stage Multiproduct Batch Plants..66	
3.1 Introduction and Literature Review	66
3.2 Problem Statement	69
3.3 Mathematical Formulation	71
3.3.1 Nomenclature	71
3.3.2 Model for Scenario 1: DEO-S1.....	73
3.3.2.1 Assignment and Sequencing Constraints.....	73
3.3.2.2 Subtour Elimination Constraints	74
3.3.2.3 Processing Timing Constraints	74
3.3.2.4 Processing Amount Constraints	75
3.3.2.5 Inventory Constraints	75
3.3.2.6 Demand Constraints	75
3.3.2.7 Objective Function	75
3.3.2.8 Summary	76
3.3.3 Model for Scenario 2: DEO-S2.....	76
3.3.3.1 Demand and Backlog Constraints.....	76

3.3.3.2 Objective Function	76
3.3.3.3 Summary	77
3.4 Case Study.....	77
3.4.1 Data	77
3.4.2 Computational Results of Model DEO-S1.....	80
3.4.3 Computational Results of Model DEO-S2.....	85
3.5 Comparison with Other Approaches.....	90
3.5.1 Comparison with a Heuristic Approach.....	91
3.5.2 Comparison with a Literature Model	93
3.6 Concluding Remarks.....	94
Chapter 4 An MPC Approach for Supply Chain Planning under Uncertainty...96	
4.1 Introduction and Literature Review	96
4.2 Problem Statement	99
4.3 Mathematical Formulation	101
4.3.1 Nomenclature	101
4.3.2 Production Sequence Constraints.....	104
4.3.3 Production Time and Amount Constraints.....	106
4.3.4 Inventory Constraints	106
4.3.5 Price Elasticity of Demand Constraints	107
4.3.6 Lost Sales Constraints.....	108
4.3.7 Pricing Constraints	108
4.3.8 Inventory Deviation Constraints	108
4.3.9 Price Change Constraints	109
4.3.9.1 Price Change from the Previous Week Prices	109
4.3.9.2 Price Change from the Initial Prices	109
4.3.10 Profit.....	110
4.3.11 Weighed Total Inventory Deviation.....	111
4.3.12 Weighed Total Price Change	111
4.3.13 Objective Function	111
4.3.14 Summary	112
4.4 MPC Approach	112
4.5 An Numerical Example.....	114
4.6 Results and Discussion.....	122
4.6.1 Length of the Control Horizon.....	122
4.6.2 Inventory and Inventory Deviation	123
4.6.3 Effect of Weights	125
4.6.4 Pricing Strategies	126
4.6.5 Changeovers.....	128
4.7 Concluding Remarks.....	131

Chapter 5 Multiobjective Optimisation of Supply Chain Planning with Capacity Expansion.....	132
5.1 Introduction and Literature Review	132
5.2 Problem Statement	136
5.3 Mathematical Formulation	139
5.3.1 Nomenclature	139
5.3.2 Production and Flow Constraints	141
5.3.3 Inventory Constraints	142
5.3.4 Inventory Shortage Constraints	142
5.3.5 Lost Sales Constraints	142
5.3.6 PCE Constraints	143
5.3.7 CCE Constraints	143
5.3.8 Capacity Utilisation Constraints	143
5.3.9 Logical Constraints	143
5.3.10 Objective Functions	144
5.3.10.1 Total Cost	144
5.3.10.2 Total Flow Time	145
5.3.10.3 Total Lost Sales	145
5.3.11 Summary	146
5.4 Solution Approaches	146
5.4.1 The ε -Constraint Method	147
5.4.1.1 Method Overview	147
5.4.1.2 Method Implementation	147
5.4.2 The Lexicographic Minimax Method	149
5.4.2.1 Method Overview	149
5.4.2.2 Method Implementation	152
5.5 A Numerical Example	153
5.6 Results and Discussion	159
5.6.1 The ε -Constraint Method	159
5.6.2 The Lexicographic Minimax Method	161
5.6.3 PCE vs CCE	162
5.6.4 Cost Minimisation vs Flow Time Minimisation	163
5.7 Concluding Remarks	169
Chapter 6 Optimisation of Integrated Water Resources Management in Water Supply Chains.....	170
6.1 Introduction and Literature Review	170
6.2 Problem Statement	174
6.3 Mathematical Formulation	176
6.3.1 Nomenclature	177
6.3.2 Velocity Calculation	180
6.3.3 Mass Balance Constraints	180

6.3.4 Flow Constraints	182
6.3.5 Pipeline Network Constraints	183
6.3.6 Pumping Station Constraints.....	183
6.3.7 Storage Tank Constraints	184
6.3.8 Plant Capacity Constraints	184
6.3.9 Plant Production Constraints.....	184
6.3.10 Pumping Energy Constraints	185
6.3.11 Logical Constraints	185
6.3.12 Objective Function	186
6.3.13 Summary	187
6.4 Case Studies	188
6.4.1 Case Study I – Syros Island	188
6.4.1.1 Background and Data.....	188
6.4.1.2 Results and Discussion.....	192
6.4.1.3 Alternative Scenarios	196
6.4.2 Case Study II – Paros Island	198
6.4.2.1 Background and Data.....	198
6.4.2.2 Results and Discussion.....	201
6.4.2.3 Alternative Scenarios	205
6.5 Concluding Remarks.....	207
Chapter 7 Conclusions and Future Work.....	208
7.1 Concluding Remarks.....	208
7.2 Directions for the Future Work	210
Appendix A Model E-D&G1	212
A.1 Nomenclature	212
A.2 Mathematical Formulation	213
A.2.1 Objective Function	213
A.2.2 Assignment and Processing Times Constraints	213
A.2.3 Transitions Constraints.....	214
A.2.4 Timing Relations Constraints.....	214
A.2.5 Inventory Constraints	214
A.2.6 Demand Constraints	214
A.2.7 Degeneracy Prevention Constraints	214
Appendix B Model CPP.....	216
B.1 Nomenclature	216
B.2 Mathematical Formulation	217
B.2.1 Objective Function	217
B.2.2 Assignment Constraints.....	217
B.2.3 Timing Constraints	218

B.2.4 Transition Constraints	218
B.2.5 Process and Storage Capacity Constraints	218
B.2.6 Inventory and Demand Constraints	218
B.2.7 Degeneracy Prevention Constraints	218
Appendix C Model E-D&G2	219
C.1 Nomenclature	219
C.2 Upper Level Problem	221
C.2.1 Objective Function	221
C.2.2 Assignment and Production Constraints	222
C.2.3 Inventory Balance and Costs Constraints	222
C.2.4 Demand Constraints	222
C.2.5 Sequencing Constraints	222
C.2.6 Time Balance Constraints	223
C.2.7 Integer Cuts Constraints	223
C.3 Lower Level Problem	223
C.3.1 Objective Function	223
C.3.2 Assignment and Processing Times Constraints	223
C.3.3 Transitions Constraints	224
C.3.4 Timing Relations Constraints	224
C.3.5 Inventory Balance and Costs Constraints	224
C.3.6 Demand Constraints	224
C.3.7 Degeneracy Prevention Constraints	224
C.3.8 Subset of Products by the Upper Level Problem	225
Appendix D Model K&Z	226
D.1 Original Model K&Z	226
D.1.1 Nomenclature	226
D.1.2 Mathematical Formulation	227
D.2 Modified Model	227
D.2.1 Nomenclature	227
D.2.2 New Mathematical Formulation	228
D.2.2.1 Objective Function	228
D.2.2.2 Constraints	228
Appendix E Publications	230
References	232

LIST OF FIGURES

Figure 1.1 Structure of a supply chain.	17
Figure 2.1 A multiproduct continuous plant with parallel units.	32
Figure 2.2 Hybrid discrete/continuous time representation.	33
Figure 2.3 Assignments and changeovers within 1 week.	37
Figure 2.4 Assignments and changeovers between 2 weeks.	38
Figure 2.5 Schedule example. (a) feasible schedule, (b) infeasible schedule with a subtour.	39
Figure 2.6 Order indices within 1 week.	39
Figure 2.7 Gantt chart of the optimal schedule for the 4-week case of Example 1. .	45
Figure 2.8 Gantt chart of the optimal schedule for the 6-week case of Example 1. .	46
Figure 2.9 Gantt chart of the optimal schedule for the 8-week case of Example 1. .	46
Figure 2.10 Gantt chart of the optimal schedule for the 6-week case of Example 2. (a) M1, (b) M2, (c) M3, (d) M4.	52
Figure 2.11 RH approach.	53
Figure 2.12 RH approach for the 24-week case.	55
Figure 3.1 Orders, products and product groups.	70
Figure 3.2 Aggregated time periods.	77
Figure 3.3 Time window and demand (in ton) of each order.	78
Figure 3.4 Gantt chart of the optimal schedule by model DEO-S1.	81
Figure 3.5 Demand and production levels of products by model DEO-S1.	82
Figure 3.6 Demand and production levels of orders by model DEO-S1.	83
Figure 3.7 Gantt chart of the optimal schedule by model DEO-S2.	86
Figure 3.8 Demand and production levels of products by model DEO-S2.	87
Figure 3.9 Demand and production levels of orders by model DEO-S2.	88
Figure 4.1 The structure of the supply chain network.	99
Figure 4.2 The MPC strategy. $u(t)$: inputs, $w(t)$: reference trajectory, $y(t)$: outputs.	112
Figure 4.3 The structure of MPC.	113
Figure 4.4 The structure of the supply chain in the example.	115
Figure 4.5 The average inventory deviation in each echelon.	123
Figure 4.6 The average inventory levels for each product. (a) plants, (b) DCs, (c) markets.	124
Figure 4.7 Effect of weights on profit and inventory deviation.	125
Figure 4.8 The average price comparison.	127
Figure 4.9 The average price for each product at all markets. (a) PS1, (b) PS3, (c) PS4.	128

Figure 5.1 Supply chain network of an agrochemical company.....	137
Figure 5.2 Capacity expansion strategies comparison: PCE vs CCE.	138
Figure 5.3 Formulation plants in the supply chain example.	154
Figure 5.4 Region markets in the supply chain example.	155
Figure 5.5 Total annual demand of each product in each market.	155
Figure 5.6 The Pareto-optimal solutions from the ε -constraint method.....	159
Figure 5.7 The lexicographic minimax solutions on the Pareto-optimal curves....	162
Figure 5.8 Capacity comparisons with the minimum cost ($\mu=1\%$).....	163
Figure 5.9 Capacity comparisons with the minimum flow time ($\mu=1\%$).....	163
Figure 5.10 Capacity comparisons after 100% PCE ($\mu=1\%$).....	164
Figure 5.11 Capacity comparisons after 100% CCE ($\mu=1\%$).....	164
Figure 5.12 Annual flows with the minimum cost after PCE ($\mu=1\%$).....	165
Figure 5.13 Annual flows with the minimum flow time after PCE ($\mu=1\%$).....	166
Figure 5.14 Annual flows with the minimum cost after CCE ($\mu=1\%$).	167
Figure 5.15 Annual flows with the minimum flow time after CCE ($\mu=1\%$).	168
Figure 6.1 Schematic graph for the definition of the terms: “distance”, “pumping distance” and “pumping elevation”.....	175
Figure 6.2 Schematic graph of potable and non-potable water systems.	181
Figure 6.3 Locations of the islands of Syros and Paros.	188
Figure 6.4 Subdivision of Syros island into 6 regions.	189
Figure 6.5 Breakdown of the optimal annualised total cost for case study I.	192
Figure 6.6 Optimal plant locations and pipeline networks for case study I.....	193
Figure 6.7 Desalination plant production for case study I.	194
Figure 6.8 Water reclamation and treated wastewater disposal daily volumes after treatment for case study I.	194
Figure 6.9 Non-potable daily water demand for case study I.	195
Figure 6.10 Potable daily water demand for case study I.	196
Figure 6.11 Optimal plant locations and pipeline networks in all scenarios for case study I. (a) Scenario 1, (b) Scenario 2, (c) Scenario 3, (d) Scenario 4.....	197
Figure 6.12 Subdivision of Paros and Antiparos islands into 8 regions.	199
Figure 6.13 Breakdown of the optimal annualised total cost for case study II.	201
Figure 6.14 Optimal plant locations and pipeline networks for case study II.....	202
Figure 6.15 Desalination plant production for case study II.....	203
Figure 6.16 Wastewater reclamation and disposal daily volumes after treatment for case study II.....	203
Figure 6.17 Non-potable daily water demand for case study II.....	204
Figure 6.18 Potable daily water demand for case study II.....	204
Figure 6.19 Optimal plant locations and pipeline networks in all scenarios for case study II. (a) Scenario 1, (b) Scenario 2, (c) Scenario 3, (d) Scenario 4.	206

LIST OF TABLES

Table 2.1 Weekly customer demands of Example 1 (ton).	43
Table 2.2 Changeover times of Example 1 (min).....	44
Table 2.3 Product selling prices of Example 1(\$/ton).....	44
Table 2.4 Solution results of 4, 6 and 8-week cases of Example 1.	45
Table 2.5 Optimal aggregate sales and backlogs of 8-week case of Example 1(ton).	47
Table 2.6 Objectives of the proposed approach and the hierarchical scheme of Example 1.....	48
Table 2.7 The optimal profits of Example 1 under two changeover assumptions. ...	48
Table 2.8 Products assignment on each unit of Example 2.....	49
Table 2.9 Weekly demands by the customers of Example 2 (ton).....	50
Table 2.10 Solution results of 6, 12, 18 and 24-week cases of Example 2.	51
Table 2.11 Computational results of single-level MILP and RH for Example 2.	56
Table 2.12 Model and solution statistics of four models for Example A.....	61
Table 2.13 Model and solution statistics of four models for Example B.	62
Table 2.14 Model and solution statistics of E-D&G2 and proposed MILP for Example C.....	63
Table 2.15 Model and solution statistics of E-D&G2, proposed MILP, and RH for Example D.....	64
Table 3.1 Details of each product group, product and order.	79
Table 3.2 Changeover matrix.	80
Table 3.3 Breakdown of the optimal profit of model DEO-S1 (k\$).	80
Table 3.4 Demands, sales and service levels by model DEO-S1.....	84
Table 3.5 Breakdown of the optimal profit by model DEO-S2 (k\$).....	85
Table 3.6 Demands, sales and service levels by model DEO-S2.....	89
Table 3.7 Backlog levels by model DEO-S2 (ton).....	90
Table 3.8 Comparison between MILP model and heuristic approach.	93
Table 3.9 model sizes of the proposed model and literature model.	94
Table 3.10 Comparison between the proposed model and literature model.	94
Table 4.1 Suitability of plants, DCs and markets.....	116
Table 4.2 Sequence-dependent changeover times (hours).	117
Table 4.3 Production rates (ton/hour).....	117
Table 4.4 Unit production costs (k\$/ton).....	117
Table 4.5 Transportation times from plant to DC (weeks).....	117
Table 4.6 Transportation times from DC to market (weeks).....	118
Table 4.7 Inventory trajectories at all sites (ton).....	119

Table 4.8	Price elasticity coefficients.....	120
Table 4.9	Available price levels (k\$/ton).	121
Table 4.10	Comparisons of the three cases with different control horizon lengths.	123
Table 4.11	Comparison of different pricing strategies.....	126
Table 4.12	Comparison between the MILP model and hierarchical approach.	130
Table 5.1	Formulation capability and capacity of each formulation plant.....	154
Table 5.2	Products in each group.	154
Table 5.3	Unit raw material cost (cu/mu).....	156
Table 5.4	Unit variable formulation cost (cu/mu).	157
Table 5.5	Transportation times from formulation plants to markets (week).....	158
Table 5.6	Duty rates from plants to markets (%).	158
Table 5.7	Unit inventory cost in region markets (cu/mu).	158
Table 5.8	Maximum and minimum values of the Pareto-optimal solution curves.	160
Table 5.9	Lexicographic minimax solutions.	161
Table 6.1	Distances, pumping distances and elevations between two regions of case study I.....	189
Table 6.2	Estimated water demands and wastewater productions of case study I..	190
Table 6.3	Plant capital costs (k\$).	190
Table 6.4	Unit energy consumption of water production and treatment (kWh/m ³).	190
Table 6.5	Optional pipes and corresponding flow rates.....	191
Table 6.6	Flow rates, costs, maximum pumping height and efficiencies of optional pumps.	191
Table 6.7	Sizes and costs of optional concrete storage tanks.....	191
Table 6.8	Solution details for each established link for case study I.	193
Table 6.9	Solution details of each scenario for case study I.	197
Table 6.10	Distances, pumping distances, elevations between two population centres of case study II.	199
Table 6.11	Distances, pumping distances and elevations from potential desalination plant locations to population centres of case study II.	199
Table 6.12	Estimated water demands, wastewater productions and available groundwater supplies for case study II.....	200
Table 6.13	Solution details for each established link for case study II.	202
Table 6.14	Solution details of each scenario for case study II.	205

GENERAL INTRODUCTION

A supply chain is defined as “a network of organisations that are involved, through upstream and downstream linkages in the different processes and activities that produce value in the form of products and services in the hand of the ultimate consumer.” (Christopher, 1998) Successful supply chains can significantly benefit the competitiveness of the firms. Thus, the supply chain management (SCM) is a crucial problem in the process industry. This thesis aims to address some key problems in the process industry SCM by developing optimisation-based models, approaches and solution procedures using mathematical programming techniques.

1.1 Introduction to SCM

A supply chain may contain all activities that transform raw materials to final products and deliver them to the customers. A number of stages are involved in a supply chain, typically including suppliers, manufacturers, warehouses, distribution centres, retailers, and customers.

From Fig. 1.1, the material flows go through the supply chain from suppliers to customers, while the information flows of orders and demands are in an opposite direction. In today’s highly competitive and complex marketplace, a company with a more effective and efficient supply chain can have more advantage than its competitors. Thus, supply chain management, as a source of competitive advantage (Mentzer, 2004), has become a big challenge for the companies in different industries.

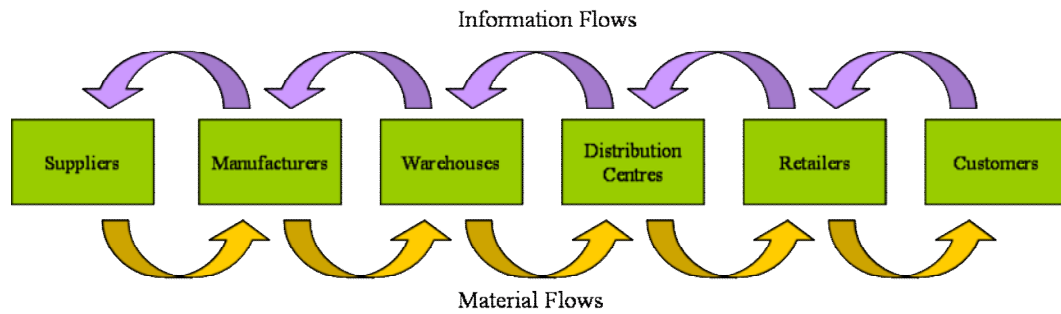


Figure 1.1 Structure of a supply chain.

1.1.1 What is SCM

The fundamental concepts of the SCM can be tracked back to channels research (Bucklin, 1966) and systems integration research (Optner, 1960; Forrester, 1969) in 1960s. The term “supply chain management”, extending beyond the concept of “logistics” (Cooper et al., 1997), first appeared in the literature in 1980s (Keith and Webber, 1982; Houlihan, 1985; Jones and Riley, 1985), and has become a widespread use and attracted enthusiasm from both industry and academia since 1990s. Now, there is still no consistent definition of the SCM. The official definition given by the Council of Supply Chain Management Professionals (Vitasek, 2010) is as follows:

“Supply chain management encompasses the planning and management of all activities involved in sourcing, procurement, conversion, and all logistics management activities. Importantly, it also includes the coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies. Supply chain management is an integrating function with primary responsibility for linking major business functions and business processes within and across companies into a cohesive and high-performing business model. It includes all the logistics management noted above, as well as manufacturing operations, and it drives coordination of processes and activities with and across marketing, sales, product design, finance and information technology.”

In the APICS Dictionary (Cox and Blackstone, 2005), SCM is described as

“the design, planning, execution, control, and monitoring of supply chain activities with the objective of creating net value, building a competitive infrastructure, leveraging worldwide logistics, synchronizing supply with demand, and measuring performance globally.”

There are other similar definitions which are commonly accepted. For example, Simchi-Levi et al. (2003) considered SCM as

“a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements.”

Christopher (2005) defined SCM as

“the management of upstream and downstream relationships with suppliers and customers to deliver superior customer value at less cost to the supply chain as a whole.”

In the definition of Stadtler (2008), SCM is

“the task of integrating organizational units along a supply chain and coordinating material, information and financial flows in order to fulfill (ultimate) customer demands with the aim of improving the competitiveness of a supply chain as a whole.”

1.1.2 Key Elements in SCM

From the above definitions, SCM comprises of a lot of issues related to different stages in the supply chain. The six key elements in the SCM (Cappello et al., 2006) and the coordination and integration between them have been given extensive research attention:

- Service level management, including customer segmentation (Chen, 2001), service level management (Boyaci, 1998; Yoo et al., 2009), etc.;
- Order and demand management, including sales demand planning and forecasting (Aviv, 2001; Liang and Huang, 2006), inventory management (Lee and Billington, 1992; Cachon and Fisher, 2000; Minner, 2003), order entry and fulfillment (Akhil and Sharman, 1992; Lin and Shaw, 1998; Chan et al., 2006), etc.;
- Production management, including network configuration/rationalisation (Pyke and Cohen, 1994; Tuma, 1998), production planning and scheduling (Shapiro, 1993; Shah, 1998; Kallrath, 2002b; Kreipl and Pinedo, 2004; Maravelias and Sung, 2009), production execution (Dickersbach, 2009), etc.;
- Supply management, including procurement planning (Kingsman, 1986; Bonser and Wu, 2001), supplier performance management (Verma and Pullman, 1998; Prahinski and Benton, 2004), etc.;
- Distribution management, including network configuration/rationalisation (Chopra, 2003; Jayaraman and Rose, 2003; Amiri, 2006), warehousing (Landers

et al., 2000; Frazelle, 2003), transportation (Morash and Clinton, 1997; Wilson, 2007), etc.;

- Integrated SCM planning and execution (Chandra and Fisher, 1994; Thomas and Griffin, 1996; Erenguc et al., 1999; Frohlich and Westbrook, 2001; Gunasekaran and Ngai, 2004; Power, 2005; Arshinder et al., 2008), which is enabled by the SCM processes, IT systems, organisation and performance measurement.

1.1.3 Hierarchical Levels in SCM

The activities in the SCM can be classified into three hierarchical levels (Simchi-Levi et al., 2003): strategic level, tactical level, and operational level, with the time horizons ranging from several years to a few hours.

The strategic level management involves long-term decision making for the supply chain, which determines the objective of the supply chain and prepares the resources to achieve this objective (Shapiro, 2004), such as the supply chain network design (Tsiakis et al., 2001; Santoso et al., 2005; Georgiadis et al., 2011), facilities locations (Owen and Daskin, 1998; Snyder, 2006; Tsiakis and Papageorgiou, 2008), etc. Decisions at this level have a significant impact on the supply chain lasting for a relatively long time, usually several years, or even tens of years.

The tactical level management deals with medium-term decisions about how to do in the supply chain to ensure the effective and efficient utilisation of the resources from the strategic level decisions. The typical tactical level decisions, which are updated from once a few weeks to once a few year, include production and distribution planning (Timpe and Kallrath, 2000; Lee and Kim, 2002; Park, 2005; Mula et al., 2006; Selim et al., 2008), inventory policies (Gupta et al., 2000; Disney and Towill, 2003), etc.

At the operational level, short-term decisions with high details are made to implement the operations and tasks in order to fulfill the objective at the tactical level. The operational level decisions, such as production and transportation scheduling (Cetinkaya and Lee, 2000; Hall and Potts, 2003; Higgins et al., 2006), are usually updated on a daily or weekly basis.

The SCM problems addressed in this thesis will cover the decision makings in all the above three levels.

1.2 SCM in the Process Industry

1.2.1 What is Process Industry

In the process industry, raw materials are transformed into finished products on a commercial scale using a sequence of physical and chemical conversions and changes (Brennan, 1998). The process industry includes the “manufacturers that produce products by mixing, separating, forming, and/or performing chemical reactions” (Cox and Blackstone, 2005), such as the chemical, pharmaceutical, petrochemical, food and beverages, pulp and paper, textiles, rubber and plastics, glass, metal, cement, electricity, coal, tobacco, wood, water treatment, and associated industries. All these industries provide primary products and commodities that are fundamental and essential to our everyday life.

Different from discrete industry (e.g., automotive, construction, engineering, and high-tech industry.) and service industry (e.g., media, communication, financial, and education industry), the process industry is characterised by the production in process that can be convergent and divergent as well. The products of the process industry can be the intermediate and final products at the same time, which can be sold to ultimate customers or used to produce other products (Kannegiesser, 2008).

The process industry is also a key portion in the world economy. According to the statistics from the European Chemical Industry Council, the world chemical sales (excluding pharmaceuticals) were valued 1871 billion Euro in 2009, increased from the value of 1166 billion Euro in 1999 (Hadhri, 2010).

1.2.2 Process Industry SCM

SCM is one of the major issues in the process industry, which deals with large and complex supply chain networks (Grossmann, 2004; Kallrath, 2005). In the literature, there is a lot of research work on the SCM in almost all branches of the process industry, such as chemical (Kallrath 2002a; Berning et al., 2004; Laínez et al., 2007), pharmaceutical (Papageorgiou et al., 2001; Shah, 2004; Meijboom and Obel, 2007;

Amaro and Barbosa-Povoa, 2008; Sousa et al., 2011), agrochemical (Sousa et al., 2008), petrochemical (Neiro and Pinto, 2004; Lababidi et al., 2004; Kuo and Chang, 2008; Rocha et al., 2009), food (van der Vorst et al., 2000; Wein and Liu, 2005, Bongers and Bakker, 2006; Ahumada and Villalobos, 2009), pulp and paper (Philpott and Everett, 2001; Carlsson et al., 2009), textiles (Perry et al., 1999; Bruce et al., 2004), glass (He et al., 1996; Richard and Proust, 2000; Almada-Lobo et al., 2008), wood (Vila et al., 2006), and rubber industry (de Haan et al., 2003), etc.

Shah (2005) classified the supply chain problems in the process industry into three categories: supply chain network design, supply chain simulation and policy analysis and supply chain planning, and reviewed the state of the art of research in these areas. Grossmann (2005) gave an overview and highlighted some major challenges in a new emerging area of enterprise-wide optimisation, which is considered to significantly overlap with the SCM in the process industry. Papageorgiou (2009) presented a review of the mathematical programming models for the supply chain optimisation problems for the process industry, and divided the key issues in the SCM into three categories, including supply chain design, supply chain planning and scheduling and supply control. This review proposed that the future challenges in the area include the optimisation under uncertainties, multiscale optimisation, development of efficient solution procedures, multiobjective optimisation with environmental impacts, and new types of supply chains associated with sustainability and healthcare.

1.3 Mathematical Programming Techniques

Currently, the optimisation-based mathematical programming approaches are main methodologies used in the process industry SCM (Papageorgiou, 2009). A brief introduction of mathematical programming is presented here.

Mathematical programming, also referred as mathematical optimisation (Kallrath and Wilson, 1997), is a technique for determining the values of a set of decision variables to optimise an objective function subjective to a number of mathematical constraints (Lev and Weiss, 1982). It is used to obtain the optimal allocations of limited resources among competing activities, under a number of constraints imposed by the

nature of the problem being studied (Bradley et al., 1977). A typical mathematical programming problem (or optimisation) problem is as follows:

$$\begin{aligned}
 \min \quad & f(\mathbf{x}) \\
 \text{s.t.} \quad & g(\mathbf{x}) \leq 0 \\
 & h(\mathbf{x}) = 0 \\
 & \mathbf{x} \in X
 \end{aligned} \tag{1.1}$$

where $\mathbf{x} \in X \subseteq \mathbb{R}^q$ is the decision variable; $f(\mathbf{x})$ is the objective function, i.e. the function to be optimised; $g(\mathbf{x}) \in \mathbb{R}^r$ and $h(\mathbf{x}) \in \mathbb{R}^s$ are r inequality constraints and s equality constraints, respectively. These constraints and the subset X determine the feasible region within which the optimal decision variable is searched for.

Based on the nature of equations for the objective function and the constraints, the mathematical programming problems can be classified into two categories:

- Linear programming (LP), in which the objective function and all the constraints are linear functions of the variables;
- Nonlinear programming (NLP), in which there exists at least one function among the objective function and the constraints that is nonlinear function of the variables.

If some of the variables are restricted to integer or discrete values in a mathematical programming problem, the problem is called mixed-integer programming (MIP) problem, which can be classified into mixed-integer linear programming (MILP) and mixed-integer nonlinear programming (MINLP). In certain MIP problems, each integer variable can only take value of 0 or 1, i.e. binary variable. The work in this thesis will use MILP-based models and approaches to model and solve the considered SCM problems

Mathematical programming was developed based on the introduction of linear programming (Kantorovich, 1939). As one of the most important branches in the area of operational research (or management science), it has been widely studied in the research literature and commonly applied in the real world, e.g. engineering, business, management, and social sciences.

Large mathematical programming models are difficult to solve, e.g., LP can only be solved in weakly polynomial time, and NLP and MIP problems are generally NP-complete. Thus, a lot of efforts have been made to find effective and efficient

solution methods for the optimisation problems. Since the invention of the simplex algorithm for LP problems by Dantzig in 1947 (Dantzig and Thapa, 2003), a number of solution methods have been proposed for different mathematical programming models, e.g. branch & bound method (Land and Doig, 1960), cutting plane method (Gomory, 1958), interior point method (Karmarkar, 1984), quasi-Newton method (Davidon, 1959). Also, a number of metaheuristics, including genetic algorithm (Holland, 1975), local search (Kuhlen and Hamburger, 1963), simulated annealing (Kirkpatrick et al., 1983; Černý, 1985), tabu search (Glover., 1986), etc., have also been developed.

With the recent rapid computational development, a number of commercial softwares have been available for implementing the mathematical programming problems, including CPLEX Optimiser (ILOG, 2007), GAMS (Brooke et al., 2008), Gurobi Optimiser (Gurobi, 2011), LINGO (Schrage, 2006), MOSEK (MOSEK, 2011), Xpress Optimiser (FICO, 2009), etc. It is worth noting that unless stated specially, all the implementations in this thesis are done in GAMS 22.8 (Brooke et al., 2008) using MILP solver CPLEX 11.1 (ILOG, 2007) in a Windows XP environment on a Pentium 4 3.40 GHz, 1.00 GB RAM machine.

1.4 Scope of This Thesis

Despite of rapid advances in the past decades, there is still a large unexplored research area in the process industry SCM, which cannot be all covered by this thesis. The purpose of this thesis is to fill the gap in the current literature work on some key issues in all three decision levels and investigate several real-world case studies in the SCM for the process industry using mathematical programming techniques, especially by developing MILP-based models, approaches and solution procedures. The issues covered in this thesis and the contributions of this work are presented below.

1.4.1 Production Planning and Scheduling

The modelling of the production planning and scheduling is one of the major challenges in the process industry supply chain problems (Grossmann, 2005). More studies are required for the development of novel optimisation models for the planning and scheduling of both batch and continuous processes with sequence-

dependent changeovers to overcome the computational complexity (Allahverdi et al., 1999). In the real-work industrial practice, some large-size problems need to be further investigated as well.

The work in this thesis will address both medium-term planning and short-term scheduling of multiproduct plants with sequence-dependent changeovers, using novel and efficient MILP-based models and approaches, and investigate a real case study of a batch edible-oil deodoriser scheduling.

1.4.2 Production and Distribution Planning under Uncertainty

The coordination between production planning and distribution planning can benefit the performance of the multi-site supply chain with faster response to customer needs. Efficiently modelling the complex production and distribution network is crucial in the SCM (Chandra and Fisher, 1994; Erengüç et al., 1999; Chen, 2010). With the demand uncertainty, model predictive control (MPC) is a commonly used tool to maintain a desired stock level is crucial to the supply chains facing fluctuations of uncertain demands (Babbar and Prasad, 1998; Toomey, 2000; Syntetos et al., 2009; Fiestras-Janeiro et al., 2011). Meanwhile, the control the price fluctuation is missing in the literature in the present of demand elasticity of demand.

In this thesis, we will address the production and distribution planning problem with the price elasticity of demand under demand uncertainty. An MILP optimisation-based MPC approach is developed to maintain both the inventory and price levels by minimising the inventory deviation and price change in the objective function.

1.4.3 Multiobjective Supply Chain Optimisation

Apart from the performance measure of supply chains based on financial aspects (cost, profit, etc.), other measures such as the responsiveness and customer service level, are also critical in the supply chain optimisation, but have received much less attention (Chan, 2003).

In order to solve a real case study of an agrochemical global supply chain planning, a multiobjective optimisation framework for supply chain production, distribution and

capacity planning will be developed with cost, flow time and lost sales as optimisation objectives, and two solution methods are adapted to solve the problem.

1.4.4 Water Supply Chain Design and Planning

The design of the “supply chains of the future” is one of future challenges in the SCM (Shah, 2005). The design and planning of water supply chain for the integrated non-conventional water resources management in insular areas with water deficiency has not been covered in the literature.

The work in this thesis will develop an optimisation framework for the integrated management of desalinated seawater, wastewater and reclaimed water to investigate real case studies of two Greek islands, with the consideration of production, conveyance and storage infrastructures, as well as the water production and distribution planning.

By addressing the above problems, this thesis will improve current literature models for the existing problems in process industry (production planning and scheduling), address new problems on process industry SCM (production and distribution planning considering inventory deviation and price change, multiobjective optimisation with aforementioned three objectives, integrated water resources management), and study real industrial cases (deodoriser scheduling, multiobjective agrochemical supply chain planning, and water resources management in Greek islands). Meanwhile, all three levels decision makings are covered by this thesis, including the strategic level (water supply chain network design, capacity planning), tactical level (production and distribution planning), and operational level (production scheduling).

1.5 Thesis Overview

The rest of this thesis is organised as follows:

The medium-term planning problem of single-stage multiproduct continuous plants with sequence dependent changeovers is addressed in Chapter 2. An MILP model is proposed, as well as a solution approach for large-scale problems. Comparative study with other literature approaches is investigated.

In Chapter 3, the case study of the short-term scheduling problem of a single-stage multiproduct batch edible-oil deodoriser is studied. Two MILP models are proposed for two scenarios considering with and without backlog, which are compared with a heuristics approach and a literature model.

Chapter 4 proposes an MPC approach for a multi-site multiproduct supply chain planning problem under demand uncertainty. The proposed MPC approach is to maximise the profit with the maintenance of the desired inventory levels and stable prices. The discussion about several aspects of the solution results is also made.

In Chapter 5, the multiobjective optimisation of a global supply chain production, distribution and capacity planning problem is addressed. Considering two different capacity expansion strategies, a multiobjective MILP model is proposed, and is solved by two methods, the ε -constraint method and the lexicographic minimax method.

The integrated water resources management of desalinated water, reclaimed water and wastewater in the water supply chains is addressed in Chapter 6. An MILP optimisation model is proposed for the maximisation of the annualised total cost, and is applied to two Greek islands for real case studies.

Finally, Chapter 7 provides conclusions and recommendations for the future work directions.

MEDIUM-TERM PLANNING OF SINGLE-STAGE MULTIPRODUCT CONTINUOUS PLANTS

Production planning and scheduling involve the procedures and processes of allocating available resources and equipment over a period of time to perform a series of tasks required to manufacture one or more products.

Production planning and scheduling improve the performance of multiproduct facilities by tackling rapid-changing demands and various production constraints, and benefit the overall supply chain. In the presence of significant sequence-dependent changeovers, the utilisation of the processing units is significantly influenced by the production sequence. Although a large number of literature models and approaches have been proposed on production planning and scheduling, efficient models and solution techniques for large instances still need further investigation.

In this chapter, we aim to develop efficient MILP-based approaches for medium-term planning of multiproduct continuous plants with sequence-dependent changeovers.

2.1 Introduction and Literature Review

In the literature, most of the research in planning and scheduling has focused on the area of batch/discrete processes (e.g. Pinto and Grossmann, 1995; Bassett et al., 1996; Papageorgiou and Pantelides, 1996; Cerdá et al., 1997; Karimi and McDonald, 1997; Zhu and Majozi, 2001; Chen et al., 2002; Castro and Grossmann, 2006;

Erdirik-Dogan and Grossmann, 2007, 2008b; Castro et al., 2008; He and Hui, 2008; Marchetti and Cerdá, 2009a, b). On the other hand, continuous processes are not discussed as much as batch processes, although continuous processes play an important role in the chemical process industry.

Sahinidis and Grossmann (1991) developed a large-scale MINLP model for the problem of cyclic multiproduct production scheduling on continuous parallel lines. A solution method based on generalised Benders decomposition was developed. Kondili et al. (1993b) addressed the problem of short-term scheduling of multiproduct energy-intensive continuous plants to minimise the total cost of energy and changeovers, while satisfying customer orders within given deadlines. An MILP model was proposed considering changeover costs and delays when switching a mill from one type of cement to another. Pinto and Grossmann (1994) extended the work of Sahinidis and Grossmann (1991), addressing the problem of optimising cyclic schedules of multiproduct continuous plants with several stages interconnected by intermediate inventory tanks. The proposed large-scale MINLP model was able to handle intermediate storage as well as sequence-dependent changeovers.

Karimi and McDonald (1997) presented two MILP formulations for the detailed short-term scheduling of a single-stage multiproduct facility with multiple parallel semicontinuous processors, based on a continuous time representation to minimise inventory, transition, and shortage costs. Ierapetritou and Floudas (1998) presented a continuous-time MILP formulation based on the state-task network (STN) representation for short-term scheduling for multistage continuous processes, as well as mixed production facilities involving batch and continuous processes. The formulation was proven capable of handling limited storage and cleanup requirements. Mockus and Reklaitis (1999) considered a general MINLP formulation for planning the operation of multiproduct/multipurpose batch and continuous plants with a goal of maximisation of profit, using the STN representation. Lee et al. (2002) addressed scheduling problems in single-stage and continuous multiproduct processes on parallel lines with intermediate due dates and especially restrictions on minimum run lengths. The proposed MILP formulation significantly reduced the model size and computation time compared with previous approaches (Karimi and McDonald, 1997; Ierapetritou and Floudas, 1998). Giannelos and Georgiadis (2002)

introduced a novel event-based MILP formulation to the scheduling problem of multipurpose continuous processes of arbitrary STN structure, sequence-dependent changeovers, and flexible finite storage requirements.

Alle and Pinto (2002) proposed an MINLP model for the simultaneous scheduling and optimisation of the operating conditions of continuous multistage multiproduct plants with intermediate storage, which was based on the Travelling Salesman Problem (TSP) formulation. The proposed formulation showed to be faster and able to solve larger problems than the model proposed by Pinto and Grossmann (1994). Also, a linearisation approach was presented to discretise nonlinear variables and compared to the direct solution of the original MINLP model, with the results showing that nonlinear restrictions were more effective than linear discrete ones. Alle et al. (2004) extended the models in the work of Pinto and Grossmann (1994) and Alle and Pinto (2002), and proposed an MINLP model for cyclic scheduling of cleaning and production operations in multiproduct multistage plants with performance decay, based on a continuous time representation.

Méndez and Cerdá (2002b) developed an MILP continuous-time short-term scheduling formulation considering sequence-dependent changeover times and specific due dates for export orders in a make-and-pack continuous production plant to meet all end-product demands with minimum make-span. In their other work (Méndez and Cerdá, 2002a), an MILP mathematical formulation for the short-term scheduling of resource-constrained multiproduct plants with continuous processes is presented, based on a continuous time representation that accounts for sequence-dependent changeover times and storage limitations. The objective is to maximise the revenue from production sales while satisfying specified minimum product requirements. Munawar et al. (2003) considered the cyclic scheduling of continuous multistage multiproduct plants operating in a hybrid flowshop, in which the operation in the plant is a combination of sequential and parallel modes. A generalised simultaneous scheduling and operational optimisation MINLP model for such plants was developed, accounting for sequence- and equipment-dependent transition times.

Erdirik-Dogan and Grossmann (2006) proposed a bi-level decomposition procedure that allows the optimisation and integration of the planning and scheduling of single-

stage single-unit multiproduct continuous plants producing several products that were subject to sequence-dependent changeovers. Shaik and Floudas (2007) presented an MILP model for short-term scheduling of continuous processes using unit-specific event-based continuous-time representation based on the STN representation. The model accounted for various storage requirements such as dedicated, finite, unlimited, and no intermediate storage policies, and allows for unit-dependent variable processing rates, sequence-dependent changeovers, and the option of bypassing storage. Shaik et al. (2009) extended the work of Shaik and Floudas (2007) to develop a systematic framework for short-term and medium-term scheduling of a large-scale industrial continuous plant to adapt to the specific requirements of the plant. A variant of a literature rolling-horizon based decomposition scheme was also introduced to solve the overall medium-term scheduling problem effectively.

Castro and Novais (2007) used a new multiple-time-grid MINLP formulation based on the resource-task network (RTN) process representation for the periodic scheduling of multistage, multiproduct continuous plants with parallel equipment units that were subject to sequence-dependent changeovers. Chen et al. (2008) proposed a slot-based MILP model for medium-term planning of single-stage single-unit continuous multiproduct plants based on a hybrid discrete/continuous time representation. Erdirik-Dogan and Grossmann (2008a) extended their own work (Erdirik-Dogan and Grossmann, 2006) from single-unit to parallel units. A detailed slot-based MILP was proposed that accounts for sequence-dependent transition times and costs. An upper-level MILP model was based on a relaxation of the original model to generate a bi-level decomposition scheme to overcome the computational expense for large problems with long time horizons.

Castro et al. (2009a) proposed a RTN-based continuous-time formulation for the optimal periodic scheduling of a continuous tissue paper mill, to find the optimal plant profit, the corresponding schedule and also the optimal cycle time for a given recycling policy. Bose and Bhattacharya (2009) developed an MILP model for the optimal scheduling operations in cascaded continuous processing units with finite intermediate storage, multiple upliftment dates and simultaneous arrival of input based on STN representation. Castro et al. (2009b) developed a RTN-based

continuous-time model for the scheduling of continuous plants under variable utility availability costs/profiles and multiple intermediate due dates, to minimise the total energy cost subject to constraints on resource availability. Lima et al. (2011) addressed the long-term scheduling of a real-world multiproduct single-stage single-unit continuous process for manufacturing glass. To overcome the computational complexity from the proposed large-scale MILP model, three different rolling horizon approaches were also developed. Kopanos et al. (2011) integrated three different modelling approaches, including discrete-time, continuous-time and lot-sizing approaches in the developed MILP formulation for the production planning and scheduling of single-stage parallel continuous processes with sequence-dependent changeovers for product families.

Many planning and scheduling problems discussed above are based on continuous time representations. Recently published papers adopted a discrete/continuous time representation. Westerlund et al. (2007) presented a mixed-time formulation for large-scale industrial scheduling problems. Chen et al. (2008) proposed an MILP model for medium-term planning of single-stage single-unit continuous multiproduct plants using a hybrid discrete/continuous time representation based on the work of Casas-Liza and Pinto (2005). In particular, the weeks of the planning horizon are modelled with a discrete time representation while within each week a continuous time representation is employed. This work also adopts a similar hybrid time approach for the planning horizon but a different formulation is proposed.

Usually in the literature, time slots are postulated in each time period (Erdirik-Dogan and Grossmann, 2006, 2008a; Chen et al., 2008). However, the introduction of binary variables to assign a number of products to time slots during each week increases significantly the size of the resulting optimisation models, and then affects their computational performance. These slot-based models always become intractable when a long planning horizon is considered. Thus, some recent papers (Alle and Pinto, 2002; Alle et al., 2004) proposed TSP-based formulations, where binary variables to represent changeovers are used in a way similar to the classic formulation used to model TSP.

The objective of the work in this chapter is to develop a compact and efficient MILP formulation for the medium-term planning of single-stage multiproduct continuous plants that are subject to sequence-dependent changeovers based on a classic TSP formulation, using a hybrid discrete/continuous time representation.

2.2 Problem Description

This work considers the optimal medium-term planning of a single-stage plant. The plant manufactures several types of products on one processing unit or multiple parallel processing units (see the example in Fig. 2.1). The total planning horizon lasts from several weeks to several months.

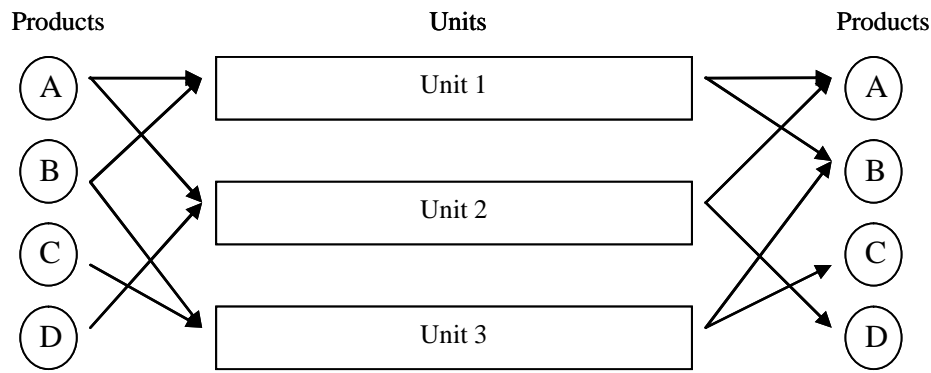


Figure 2.1 A multiproduct continuous plant with parallel units.

The customers place orders for one or more products. These demands are allowed to be delivered only at the end of each week, which is a key difference from the economic lot scheduling problem (ELSP), in which continuous demand rates are considered. If there are deliveries within each week, the whole planning horizon can be divided into multiple discrete time periods with varying lengths based on the delivery times. Thus, the assumption that the demand is delivered at the end of each time period is still valid. The weekly demands allow the use of hybrid discrete/continuous time representation (Fig. 2.2). If the demand is not fulfilled at the desired time, late delivery is allowed. At the same time, backlog penalties are imposed on the plant operation. The plant can also manufacture a larger amount of products than the demand in a time period. The limited inventory is allowed for product storage before sales. Sequence-dependent changeover times and costs occur when switching production between different products.

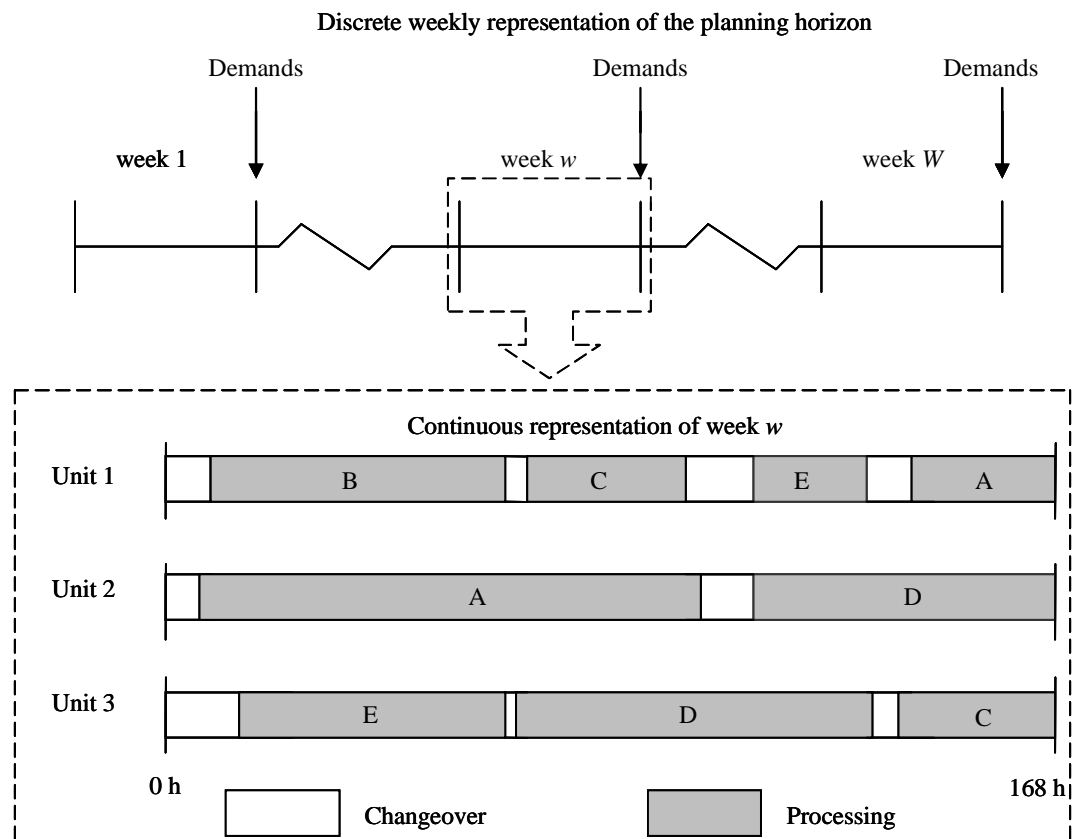


Figure 2.2 Hybrid discrete/continuous time representation.

The problem can be stated as follows: Given are the demands, prices, processing rates, changeover unit costs and times, unit penalty costs, and inventory costs for each product. Here, the main optimisation variables include decisions on the products to be produced during each week, processing schedule, production times, production amounts, and inventory and backlog levels over the planning horizon. The objective is to maximise the total profit, involving sales revenue, product changeover cost, backlog penalty cost and inventory cost.

2.3 Mathematical Formulation

A TSP-based MILP model for the medium-term planning of single-stage multiproduct continuous plants is described in this section. Due to the nature of the problem, a hybrid discrete/continuous time representation (Fig. 2.2), based on the models of Casas-Liza and Pinto (2005) and Chen et al. (2008), is applied over a planning horizon, in which the weeks of the planning horizon are modelled with a discrete time formulation and each week is represented by a continuous time formulation.

One key characteristic of the problem is that the sequence-dependent changeovers occur when switching from one product to another. Because of the sequence-dependent changeover times and costs, different sequences of the processing products generate different total profits, even if the processing times are fixed. Here, the planning of multiproduct plants can be taken as a TSP problem. In the classic TSP problem, a salesman is required to visit a number of cities in a sequence that minimises the overall cost or time, and in the classic TSP formulation binary variables are used to represent the transition from one city to another (Kallrath and Wilson, 1997). Similarly, on a processing unit, a number of products must be produced in a sequence that maximises profits. So, similar to the binary variables in classic TSP formulation, binary variables Z_{ijmw} and ZF_{ijmw} are introduced to model the changeovers from the production of product i to that of product j on unit m in week w and between two consecutive weeks $w-1$ and w , respectively.

Also, in order to avoid the occurrence of subtours in the sequence of the products, product ordering variables O_{imw} are introduced together with additional mathematical constraints to eliminate product subtours generation at the optimal solution. These constraints consider the order of each product in the production sequence. Subtour elimination constraints have been used in the classic TSP formulation, but are uncommon to scheduling models in process system engineering.

2.3.1 Nomenclature

Indices

c	customer
i, j	product
i^*	pseudo product
m	unit
w	week

Sets

C	set of customers
I	set of products
I_m	set of products that can be processed on unit m , including pseudo product

\bar{I}_m	set of real products that can be processed on unit m , excluding pseudo product
M	set of units
M_i	set of units that can process product i
W	set of weeks

Parameters

CB_{ic}	unit backlog penalty cost of product i to customer c
CC_{ijm}	changeover cost from product i to product j on unit m
CI_i	unit inventory cost of product i
D_{ciw}	demand of product i from customer c in week w
INV_i^{\max}	maximum inventory capacity of product i
INV_i^{\min}	minimum inventory capacity of product i
N	a large number
Pr_{ic}	unit selling price of product i to customer c
r_{im}	processing rate of product i on unit m
θ^L	lower bound for processing time in a week
θ^U	upper bound for processing time in a week
τ_{ijm}	changeover time from product i to product j on unit m

Binary Variables

E_{imw}	1 if product i is processed on unit m in week w , 0 otherwise
F_{imw}	1 if product i is the first one on unit m in week w , 0 otherwise
L_{imw}	1 if product i is the last one on unit m in week w , 0 otherwise
Z_{ijmw}	1 if product i immediately precedes product j on unit m in week w , 0 otherwise
ZF_{ijmw}	1 if product i on week $w-1$ immediately precedes product j in week w on unit m , 0 otherwise

Continuous Variables

$CT1_{mw}$	time elapsed within week w in a changeover starting in the previous week on unit m
$CT2_{mw}$	time elapsed within week w in a changeover completing in the next week on unit m
INV_{iw}	inventory volume of product i at the end of week w
OI_{imw}	order index of product i on unit m in week w
P_{imw}	amount of product i produced on unit m in week w
PT_{imw}	processing time of product i on unit m in week w
Sa_{ciw}	sales volume of product i to customer c in week w
Δ_{ciw}	backlog of product i to customer c at the end of week w
Π	total profit, the objective

2.3.2 Assignment Constraints

Assuming that each week comprises the processing of at least one product on each unit, the first and last products to be processed during each week are assigned:

$$\sum_{i \in I_m} F_{imw} = 1, \quad \forall m \in M, w \in W \quad (2.1)$$

$$\sum_{i \in I_m} L_{imw} = 1, \quad \forall m \in M, w \in W \quad (2.2)$$

The above one product per unit assumption can always be valid by introducing a pseudo product i^* , whose changeover times and costs are 0, and fixing Z_{i^*jmw} and Z_{ji^*mw} to 0, for every j, m and w . If a unit is occupied by a pseudo product in a week in the optimal solution, it implies that the unit is idle in the week.

A product cannot be assigned as the first or last one on a unit in a week, if the product is not processed in the same week, i.e., if $E_{imw} = 0$, then F_{imw} and L_{imw} should be forced to be 0:

$$F_{imw} \leq E_{imw}, \quad \forall m \in M, i \in I_m, w \in W \quad (2.3)$$

$$L_{imw} \leq E_{imw}, \quad \forall m \in M, i \in I_m, w \in W \quad (2.4)$$

2.3.3 Changeover Constraints

Changeovers refer to production switches between two different types of products. In the planning horizon, changeovers may occur within a week or between two consecutive weeks.

For changeovers within a week, if a product is the first one processed on one unit and in a week, then no product is processed precedent to this product on the unit and in the week. Also, if a product is to be processed, but is not the first one, then there is exactly one product precedent to this product on the unit and in the week:

$$\sum_{i \in I_m \setminus \{j\}} Z_{ijmw} = E_{jmw} - F_{jmw}, \quad \forall m \in M, j \in I_m, w \in W \quad (2.5)$$

If a product is the last one processed on one unit and in a week, then no product is processed following this product on the unit and in the same week. Also, if a product is to be processed, but is not the last one, then there is exactly one product following this product on the unit and in the week:

$$\sum_{j \in I_m \setminus \{i\}} Z_{ijmw} = E_{imw} - L_{imw}, \quad \forall m \in M, i \in I_m, w \in W \quad (2.6)$$

Note that from Eqs. (2.5) and (2.6), there is no changeover from or to a product that is not processed. Fig. 2.3 shows an example of changeover with two products A and B within week w on unit m .

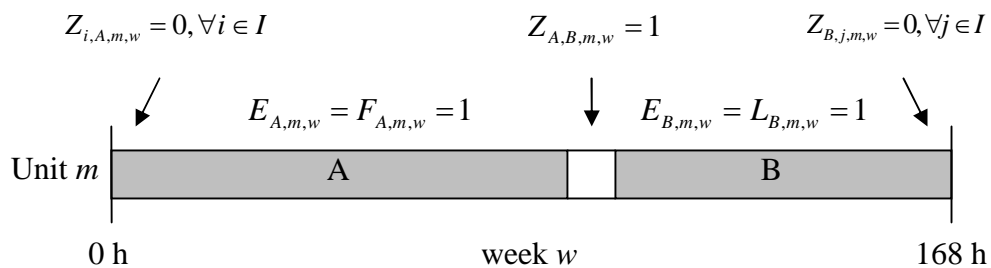


Figure 2.3 Assignments and changeovers within 1 week.

For changeovers between two consecutive weeks, if product j is the first one to be processed on one unit and in week w , there is exactly one changeover from a product at week $w-1$ to product j on the unit. Also, if product i is the last one to be processed in week $w-1$ on one unit, there is exactly one changeover to a product at the beginning of week w in the unit. If a product is not the first or the last one processed

on one unit, then there is no changeover involving the products between two weeks on one unit.

$$\sum_{i \in I_m} ZF_{ijmw} = F_{jmw}, \quad \forall m \in M, j \in I_m, w \in W \setminus \{1\} \quad (2.7)$$

$$\sum_{j \in I_m} ZF_{ijmw} = L_{im,w-1}, \quad \forall m \in M, i \in I_m, w \in W \setminus \{1\} \quad (2.8)$$

Here, it is assumed that the changeover between week $w-1$ and w on each unit occurs at the beginning of week w . Fig. 2.4 is an example of changeover from product A to B between weeks $w-1$ and w on unit m .

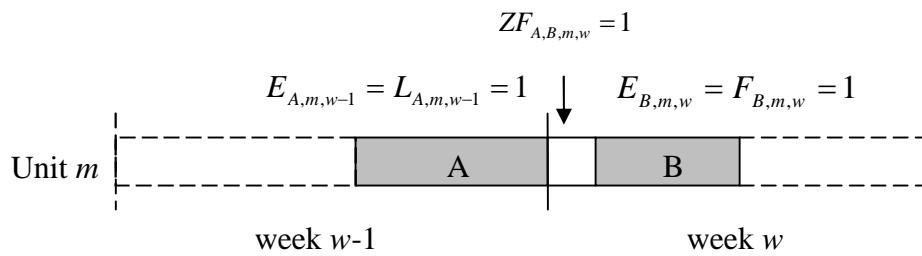


Figure 2.4 Assignments and changeovers between 2 weeks.

It should be noted that the last product processed in week $w-1$ may be the same product as the one processed first in week w in unit m . In such cases, the production process of the product continuously proceeds from week $w-1$ to week w , so no changeover time and cost occurs. Here, variables ZF_{ijmw} are treated as continuous, $0 \leq ZF_{ijmw} \leq 1$, as the relevant changeover terms are minimised in the objective function.

2.3.4 Subtour Elimination Constraints

The above mentioned constraints have the potential drawback of generating solutions with subtours. When a subtour is present, the solution of the model is an infeasible schedule (Fig. 2.5b). So, subtour elimination constraints are needed to generate feasible schedules (Fig. 2.5a).

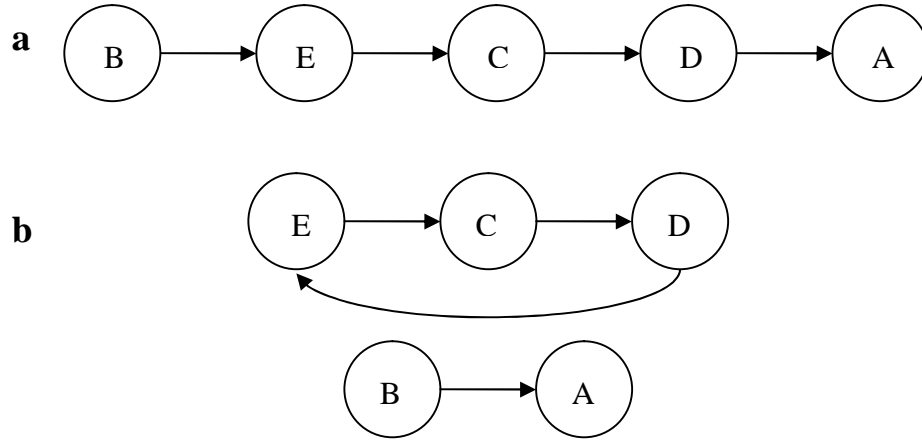


Figure 2.5 Schedule example. (a) feasible schedule, (b) infeasible schedule with a subtour.

In order to avoid subtours, positive integer variables OI_{imw} are introduced to define the order in which each product is processed in a week on the same unit. The later a product is processed, the greater its order index is, as shown in Fig. 2.6.

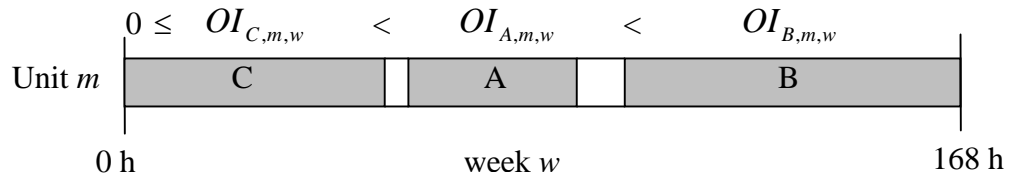


Figure 2.6 Order indices within 1 week

Here, it is assumed that if product i is processed precedent to product j on unit m in week w , the order index of product j is at least one higher than that of product i :

$$OI_{jmw} - (OI_{imw} + 1) \geq -N \cdot (1 - Z_{ijmw}), \quad \forall m \in M, i \in \bar{I}_m, j \in \bar{I}_m, j \neq i, w \in W \quad (2.9)$$

Also, when a product is not processed on unit m , its order index is equal to zero:

$$OI_{imw} \leq N \cdot E_{imw}, \quad \forall m \in M, i \in \bar{I}_m, w \in W \quad (2.10)$$

In Eqs. (2.9) and (2.10), N is a large number and is an upper bound of OI_{imw} . We can also use $\max_{m \in M} |I_m|$, the maximum cardinality of set I_m , as the upper bound of OI_{imw} .

Eq. (2.9) guarantees that no subtour exists in any feasible optimal solution. Thus, we have the following theorem on the effect of Eq. (2.9):

Theorem 2.1: Eq. (2.9) eliminates subtours in the feasible solutions.

Proof: Assume in a feasible solution, there is a cyclic sequence, consisting of k products i_1, i_2, \dots, i_k , on unit m in week w , where $k \geq 2$.

So, we have

$$Z_{i_1, i_2, m, w} = Z_{i_2, i_3, m, w} = \dots = Z_{i_{k-1}, i_k, m, w} = Z_{i_k, i_1, m, w} = 1.$$

From Eq. (2.10), we obtain

$$OI_{i_2, mw} - OI_{i_1, mw} \geq 1,$$

$$OI_{i_3, mw} - OI_{i_2, mw} \geq 1,$$

\vdots

$$OI_{i_k, mw} - OI_{i_{k-1}, mw} \geq 1,$$

$$OI_{i_1, mw} - OI_{i_k, mw} \geq 1.$$

By adding the above k constraints together, we get

$$OI_{i_1, mw} - OI_{i_1, mw} = 0 \geq k,$$

which is a contradiction. So, there is no subtour in the feasible solutions. \square

Note that it is the first time that the above subtour elimination constraints used in the classic TSP formulations (Kallrath and Wilson, 1997; Öncan et al., 2009) are applied to the production planning and scheduling in the process industry. It is worth mentioning that the order indices obtained from Eqs. (2.9) and (2.10) do not guarantee values of successive integers. If the latter is required, the following constraints should be included:

$$F_{imw} \leq OI_{imw} \leq \sum_{j \in \bar{I}_m} E_{jmw}, \quad \forall m \in M, i \in \bar{I}_m, w \in W. \quad (2.11)$$

Note the Eqs. (2.9) and (2.11) force the product order indices to take successive values starting from 1 for selected products.

Alternatively to Eqs. (2.10) and (2.11), the following term can be subtracted by the objective function:

$$\varepsilon \cdot \sum_{m \in M} \sum_{i \in \bar{I}_m} \sum_{w \in W} OI_{imw}$$

where ε is a small number.

2.3.5 Timing Constraints

For each product processed in a week, its processing time must be restricted between the lower and upper availability bounds (θ^L and θ^U , respectively). Meanwhile, if a product is not assigned to a unit, i.e. $E_{imw} = 0$, the processing time should be zero.

$$\theta^L \cdot E_{imw} \leq PT_{imw} \leq \theta^U \cdot E_{imw}, \quad \forall m \in M, i \in \bar{I}_m, w \in W \quad (2.12)$$

Also, the total processing and changeover time on a unit in a week should not exceed the total available time in each week:

$$\sum_{i \in \bar{I}_m} PT_{imw} + \sum_{i \in \bar{I}_m} \sum_{j \in \bar{I}_m} (Z_{ijmw} + ZF_{ijmw}) \cdot \tau_{ijm} \leq \theta^U, \quad \forall m \in M, w \in W \setminus \{1\} \quad (2.13)$$

$$\sum_{i \in \bar{I}_m} PT_{imw} + \sum_{i \in \bar{I}_m} \sum_{j \in \bar{I}_m} Z_{ijmw} \cdot \tau_{ijm} \leq \theta^U, \quad \forall m \in M, w \in \{1\} \quad (2.14)$$

Alternatively, if we assume that a changeover between two consecutive weeks can start and complete in different weeks, i.e. partial changeovers in each week are allowed, Eqs. (2.13) and (2.14) can be replaced by the following two equations proposed by Kapanos et al. (2011):

$$CT1_{mw} + CT2_{m,w-1} = \sum_{i \in \bar{I}_m} \sum_{j \in \bar{I}_m} ZF_{ijmw} \cdot \tau_{ijm}, \quad \forall m \in M, w \in W \setminus \{1\} \quad (2.15)$$

$$\sum_{i \in \bar{I}_m} PT_{imw} + \sum_{i \in \bar{I}_m} \sum_{j \in \bar{I}_m} Z_{ijmw} \cdot \tau_{ijm} + CT1_{mw}|_{w>1} + CT2_{mw}|_{w<|W|} \leq \theta^U, \quad \forall m \in M, w \in W \quad (2.16)$$

The difference between the two assumptions will be discussed later in this chapter.

2.3.6 Production Constraints

The product amount produced on one unit per week is simply given by:

$$P_{imw} = r_{im} \cdot PT_{imw}, \quad \forall m \in M, i \in \bar{I}_m, w \in W \quad (2.17)$$

2.3.7 Backlog Constraints

The backlog of a product to a customer in a week is defined as the backlog at the previous week plus the demand in this week, minus the sales volume to the customer:

$$\Delta_{ciw} = \Delta_{c,i,w-1}|_{w>1} + D_{ciw} - Sa_{ciw}, \quad \forall c \in C, i \in I, w \in W \quad (2.18)$$

2.3.8 Inventory Constraints

The inventory of a product in a week is defined as the inventory at the previous week plus the total amount produced on all units, minus the total sales volume of the product to all customers:

$$INV_{iw} = INV_{i,w-1} \Big|_{w>1} + \sum_{m \in M_i} P_{imw} - \sum_{c \in C} Sa_{ciw}, \quad \forall i \in I, w \in W \quad (2.19)$$

The amounts of products to be stored are limited by minimum and maximum capacities:

$$INV_i^{\min} \leq INV_{iw} \leq INV_i^{\max}, \quad \forall i \in I, w \in W \quad (2.20)$$

2.3.9 Objective Function

The profit of the plant is equal to the sales revenue minus operating costs involving changeover, backlog, and inventory costs. The backlog cost includes all costs generated by the backlog, including the increased shipment cost due to the backlog. The inventory cost in each week is calculated from the inventory level at the end of each week, multiplied by the unit inventory cost for each product. It is an underestimate of the actual inventory cost, which will not affect the decisions on the production schedules and amounts and sales.

$$\begin{aligned} \Pi = & \sum_{c \in C} \sum_{i \in I} \sum_{w \in W} Pr_{ic} \cdot Sa_{ciw} - \sum_{m \in M} \sum_{i \in I_m} \sum_{j \in I_m} \sum_{w \in W} CC_{ijm} \cdot Z_{ijmw} - \\ & \sum_{m \in M} \sum_{i \in I_m} \sum_{j \in I_m} \sum_{w \in W \setminus \{1\}} CC_{ijm} \cdot ZF_{ijmw} - \sum_{c \in C} \sum_{i \in I} \sum_{w \in W} CB_{ic} \cdot \Delta_{ciw} - \sum_{i \in I} \sum_{w \in W} CI_i \cdot INV_{iw} \end{aligned} \quad (2.21)$$

2.3.10 Summary

The planning of single-stage multiproduct plants is formulated as an MILP model that is described by Eqs. (2.1)–(2.10), (2.12)–(2.14), (2.17)–(2.20) with Eq. (2.21) as the objective function. The proposed model can be applied to the cases with parallel units, as well as the ones with single unit.

2.4 Illustrative Examples

To illustrate the applicability of the proposed model, the model is applied to two illustrative examples in this section. Example 1 considers a real-world polymer processing plant with one processing unit, which is an extension of the example

discussed in Chen et al. (2008). Example 2 considers a polymer processing plant consisting of 4 parallel processing units.

It should be added that all the implementations in this chapter are done in GAMS 22.6 (Brooke et al., 2008) using solver CPLEX 11.0 (ILOG, 2007). The optimality gap is set to 0%, and the computational time is limited to 3,600 s.

2.4.1 Illustrative Example 1

2.4.1.1 Data

In Example 1, 10 types of products (A–J) are manufactured by a single-unit plant. Weekly demands for each product (Table 2.1) are ordered from 10 customers (C1–C10) for a period of 8 weeks.

Table 2.1 Weekly customer demands of Example 1 (ton).

Customers	Products	Weekly demands							
		1	2	3	4	5	6	7	8
C1, C5	A	5				5			
	C	2	2	2	2	3	3	3	3
	D	3		3		3		3	
C2, C6	E	5		5		5		5	
	H		12				12		12
	B	4				4			
C3, C7, C9	G			5					
	J		6		6		6		6
	A	7				7			
C4, C8, C10	B		5		5			5	
	C	5			5			5	
	D	10					10		
	E	11		11		11		11	
	F	8			8			8	
	G	4		4		4		4	
	H	1	1	1	3	3	3	1	1
	I	5	5	5	5	5	5	5	5
	J		3		3	3		3	

The processing rate is 110 ton/week for each product. The total available processing time in each week is 168 h. The minimum processing time for a product in each week is 5 h. The changeover times (in minutes) are shown in Table 2.2. The changeover costs are proportional to the changeover times (in hours) by a factor of 10. For example, the changeover cost from product A to B is $45 \times 10 \div 60 = \$7.5$.

Table 2.2 Changeover times of Example 1 (min).

From/To	A	B	C	D	E	F	G	H	I	J
A		45	45	45	60	80	30	25	70	55
B	55		55	40	60	80	80	30	30	55
C	60	100		100	75	60	80	80	75	75
D	60	100	30		45	45	45	60	80	100
E	60	60	55	30		35	30	35	60	90
F	75	75	60	100	75		100	75	100	60
G	80	100	30	60	100	85		60	100	65
H	60	60	60	60	60	60	60		60	60
I	80	80	30	30	60	70	55	85		100
J	100	100	60	80	80	30	45	100	100	

Table 2.3 shows the product prices for all customers, except for customer C10 who is 50% higher. The unit inventory and backlog costs are 10% and 20% of product prices, respectively.

Table 2.3 Product selling prices of Example 1(\$/ton).

	A	B	C	D	E	F	G	H	I	J
Prices	10	12	13	12	15	10	8	14	7	15

2.4.1.2 Results and Discussion

Here, we consider three cases of the example, with planning horizons of four, six and eight weeks, respectively. In the implementations, only one unit is considered in the proposed model, i.e., $|M|=1$. Pseudo product is not considered in this example.

The solution results are shown in Table 2.4, and the detailed schedules corresponding to the optimal solutions of three cases are shown in Figs. 2.7–2.9, from which we can see that the proposed model is able to generate optimal schedules within three minutes, even for the case with a planning horizon of 8 weeks.

Table 2.4 Solution results of 4, 6 and 8-week cases of Example 1.

Time horizon (week)	4	Sales revenue (\$)	6,050.2
No. of equations	1,193	Changeover cost (\$)	114.2
No. of continuous variables	1,261	Backlog cost (\$)	493.7
No. of binary variables	480	Inventory cost (\$)	3.5
CPU (s)	3.5	Total profit (\$)	5,438.8
Time horizon (week)	6	Sales revenue (\$)	9,111.6
No. of equations	1,799	Changeover cost (\$)	185.8
No. of continuous variables	1,941	Backlog cost (\$)	781.3
No. of binary variables	720	Inventory cost (\$)	9.7
CPU (s)	28	Total profit (\$)	8,134.8
Time horizon (week)	8	Sales revenue (\$)	12,035.3
No. of equations	2,405	Changeover cost (\$)	254.2
No. of continuous variables	2,621	Backlog cost (\$)	1,125.7
No. of binary variables	960	Inventory cost (\$)	0.6
CPU (s)	160	Total profit (\$)	10,654.9

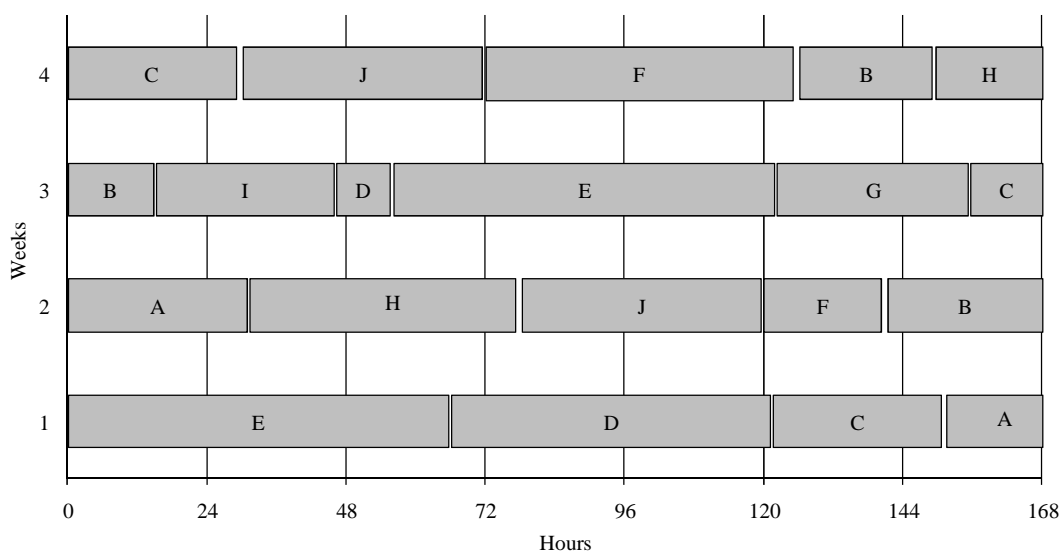


Figure 2.7 Gantt chart of the optimal schedule for the 4-week case of Example 1.

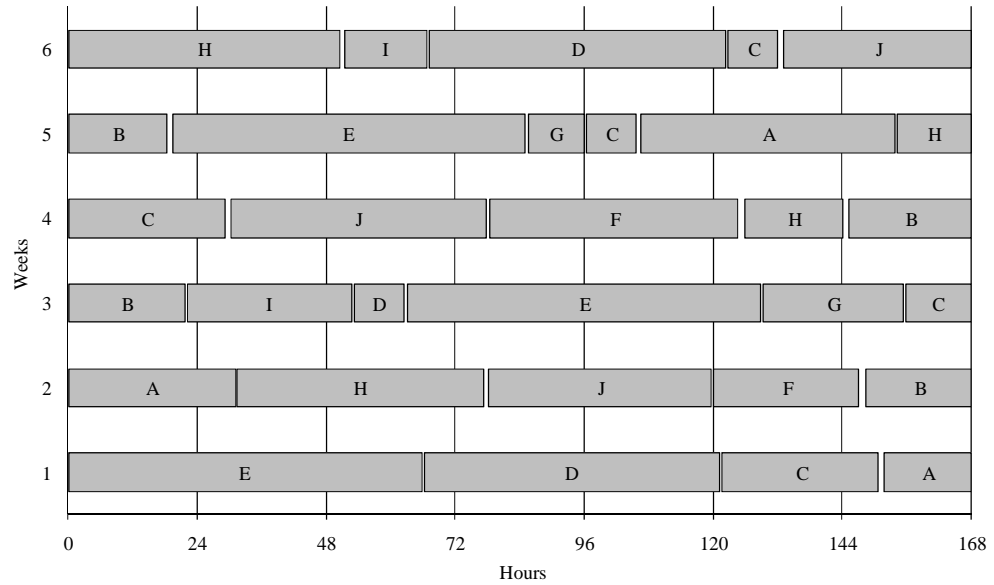


Figure 2.8 Gantt chart of the optimal schedule for the 6-week case of Example 1.

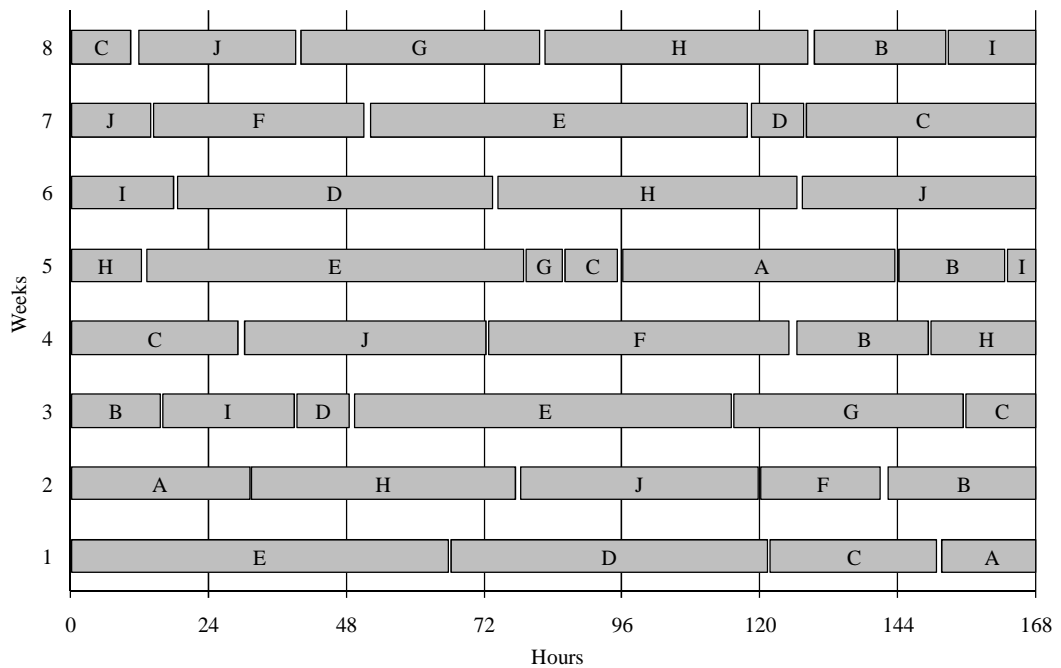


Figure 2.9 Gantt chart of the optimal schedule for the 8-week case of Example 1.

In the optimal solution of the 8-week case, only product J has an inventory of 0.42 ton at the end of week 4. In Table 2.5, the optimal weekly aggregate sales and backlogs of the 8-week case are shown.

Table 2.5 Optimal aggregate sales and backlogs of 8-week case of Example 1(ton).

		Weeks							
		1	2	3	4	5	6	7	8
Optimal weekly aggregate sales	A	10.7	20.3			31.0			
	B		16.9	10.1	15.0	12.0			15.0
	C	19.0		8.0	19.0	6.0		26.2	6.8
	D	36.0		6.0			36.0	6.0	
	E	43.0		43.0		43.0		43.0	
	F		13.7		34.3			24.0	
	G			26.1		4.0			27.3
	H		30.0		12.0	7.9	34.1		30.0
	I			15.0		3.3	11.7		10.0
	J		27.0		27.0	0.4	26.6	9.0	18.0
Optimal weekly aggregate backlogs	A	20.3							
	B	12.0	10.1					15.0	
	C		4.0				6.0	0.8	
	D					6.0			
	E								
	F	24.0	10.3	10.3					
	G	12.0	12.0	12.9	12.9	20.9	20.9	32.9	5.6
	H	3.0		3.0		1.1		3.0	
	I	15.0	30.0	30.0	45.0	56.7	60.0	75.0	80.0
	J					8.6			

Now, we focus on the optimal schedules over the first 4 weeks of all 3 cases. From Figs. 2.7–2.9, the sequence of the products over the first 4 weeks of the 6-week case is different from those of the 4-week and 8-week cases, and the differences result from the last two products processed in week 4. In the 4-week and 8-week cases, product H is the last one produced in week 4, while product B is the last one produced in week 4 in the 6-week case.

Also, except for the products B and H in week 4, the optimal sequences over the first 4 weeks are the same in all three cases, while the processing times are different for the same product in different cases, such as products F and B in week 2, products B, I and G in week 3, and products J and F in week 4. The reason for such differences in sequences and processing times is that the length of the overall planning horizon and associated product demands affect the scheduling decisions.

Based on the above observations, the advantages of the proposed single-level MILP approach are emphasised by applying the following hierarchical scheme:

STEP 1. Solve the 4-week case;

STEP 2. Fix the schedule (sequence and timings) obtained for 4 weeks;

STEP 3. Solve 6-week and 8-week cases in reduced spaces.

The comparative results between the proposed approach and the hierarchical scheme are shown in Table 2.6. It can clearly be seen that the profit decreases in both cases. Thus, the proposed model performs better than the hierarchical scheme.

Table 2.6 Objectives of the proposed approach and the hierarchical scheme of Example 1.

	Proposed approach	Hierarchical scheme
6-week case	8,134.8	8,131.5
8-week case	10,654.9	10,647.3

2.4.1.3 Changeover Assumptions

In the problem discussed in this chapter, it is assumed that the changeover between two consecutive weeks occurs at the latter week, i.e. partial changeovers in both two weeks are not allowed. However, if we assume the partial changeovers are allowed, Eqs. (2.15) and (2.16) are implemented instead of Eqs. (2.13) and (2.14). Here, we compare the above two assumptions: (1) partial changeovers are not allowed; (2) partial changeovers are allowed. Table 2.7 shows the differences between the optimal profits under the two assumptions are very small (lower than 1%) in all three cases of Example 1. The comparison results prove that the changeovers between two consecutive weeks do not affect the optimal solution significantly, and our assumption on the changeovers does not impair the performance of the model.

Table 2.7 The optimal profits of Example 1 under two changeover assumptions.

	Partial changeovers not allowed	Partial changeovers allowed
4-week case	5438.8	5461.5
6-week case	8134.8	8183.5
8-week case	10,654.9	10,732.2

2.4.2 Illustrative Example 2

2.4.2.1 Data

In Example 2, the single-stage plant manufactures 10 types of products (A–J) on 4 parallel units (M1–M4). Each unit can process 5 out of the 10 products (Table 2.8).

Table 2.8 Products assignment on each unit of Example 2.

Unit	Products									
	A	B	C	D	E	F	G	H	I	J
M1	√ ^a	√	√	√	√					
M2			√	√	√	√	√			
M3					√	√	√	√	√	
M4						√	√	√	√	√

^a The product can be assigned for production on the unit.

The total available processing time in each week is 168 h. The changeover times on different units are the same, which are as the same as those in illustrative Example 1 (Table 2.2). The changeover costs are proportional to the changeover times (in hours) by a factor of 10.

Weekly demands for each product (Table 2.9) are ordered from ten customers (C1–C10) for a period of 24 weeks. The processing rate is 110 ton/week for each product on all units.

The product prices are the same as those in Example 1, which are given in Table 2.3. Also, the unit inventory and backlog costs are 10% and 20% of product prices, respectively.

Table 2.9 Weekly demands by the customers of Example 2 (ton).

Customers	Products	Weekly demands											
		1	2	3	4	5	6	7	8	9	10	11	12
C1, C5	A	20				20				20			
	C	8	8	8	8	12	12	12	12	12	8	8	8
C2, C6	D	12		12		12		12		12		12	
	E	20		20		20		20		20		20	
	H		48				48		48				48
C3, C7, C9	B	16				16				16			
	G			20									
	J		24		24		24		24		24		24
C4, C8, C10	A	28				28				28			
	B		20		20			20		20			20
	C	20			20			20			20		
	D	40					40			40			
	E	44		44		44		44		44		44	
	F	32			32			32			32		
	G	16		16		16		16		16		16	
	H	4	4	4	12	12	12	4	4	4	12	12	12
	I	20	20	20	20	20	20	20	20	20	20	20	20
	J		12		12	12		12		12	12		12
Customers	Products	Weekly demands											
		13	14	15	16	17	18	19	20	21	22	23	24
C1, C5	A	20				20				20			
	C	8	12	12	12	12	12	8	8	8	8	12	12
C2, C6	D	12		12		12		12		12		12	
	E	20		20		20		20		20		20	
	H		48				48		48				48
C3, C7, C9	B	16				16				16			
	G	20										20	
	J		24		24		24		24		24		24
C4, C8, C10	A	28				28				28			
	B		20		20			20		20			20
	C	20			20			20			20		
	D	40				40				40			
	E	44		44		44		44		44		44	
	F	32			32			32			32		
	G	16		16		16		16		16		16	
	H	4	4	4	12	12	12	4	4	4	12	12	12
	I	20	20	20	20	20	20	20	20	20	20	20	20
	J		12	12		12		12	12		12		12

2.4.2.2 Results and Discussion

Here, we consider 4 cases of Example 2, with planning horizons of 6, 12, 18 and 24 weeks, respectively. Here, we also consider one pseudo product for each case. The

solution results are shown in Table 2.10. The proposed MILP model can find the global optimal solution within the specified time limit only for the 6-week case. For the other 3 cases with planning horizons of 12, 18 and 24 weeks, although the solutions obtained in the specified time limit are not global optimal, the model also provides very good feasible solutions. The gap between the profit given by the proposed MILP model and the global optimal one is within 1% for each case.

Table 2.10 Solution results of 6, 12, 18 and 24-week cases of Example 2.

Time horizon (week)	6	Sales revenue (\$)	36,691
No. of equations	2,509	Changeover cost (\$)	277
No. of continuous variables	2,581	Backlog cost (\$)	2,856
No. of binary variables	912	Inventory cost (\$)	8
CPU (s)	154	Total profit (\$)	33,550 (0.00% ^a)
Time horizon (week)	12	Sales revenue (\$)	72,735
No. of equations	5,065	Changeover cost (\$)	547
No. of continuous variables	5,305	Backlog cost (\$)	7,276
No. of binary variables	1,824	Inventory cost (\$)	72
CPU (s)	3,600	Total profit (\$)	64,841 (0.27% ^a)
Time horizon (week)	18	Sales revenue (\$)	109,597
No. of equations	7,621	Changeover cost (\$)	823
No. of continuous variables	8,029	Backlog cost (\$)	13,680
No. of binary variables	2,736	Inventory cost (\$)	212
CPU (s)	3,600	Total profit (\$)	94,882 (0.44% ^a)
Time horizon (week)	24	Sales revenue (\$)	145,629
No. of equations	10,177	Changeover cost (\$)	1,089
No. of continuous variables	10,753	Backlog cost (\$)	21,594
No. of binary variables	3,648	Inventory cost (\$)	220
CPU (s)	3,600	Total profit (\$)	122,725 (0.85% ^a)

^a Gap between current solution and best possible solution.

The detailed optimal schedule corresponding to the global optimal solution for the 6-week case is shown in Fig. 2.10.

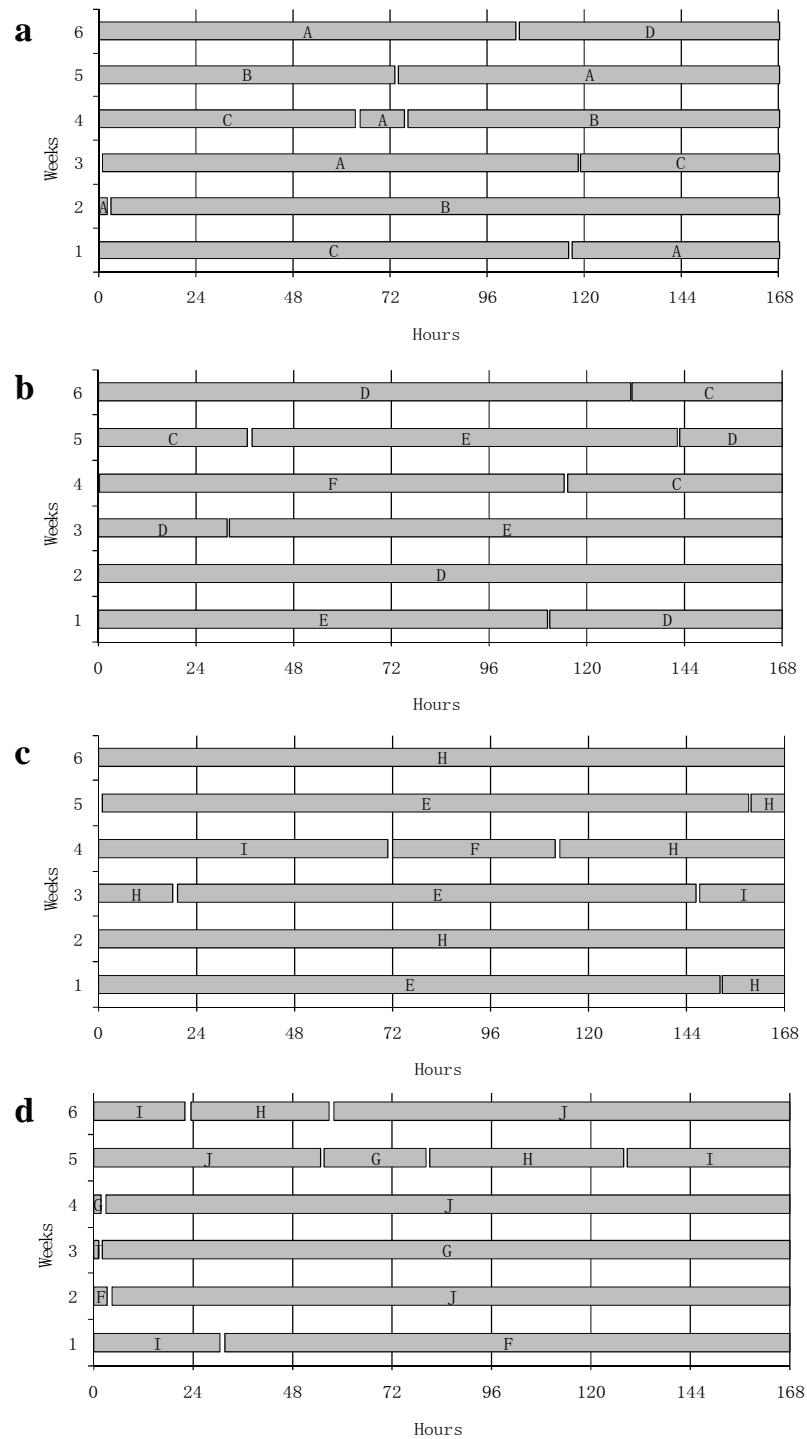


Figure 2.10 Gantt chart of the optimal schedule for the 6-week case of Example 2. (a) M1, (b) M2, (c) M3, (d) M4.

In the optimal schedule for the 6-week case, the pseudo product is not processed, which means that no unit is idle in any week. M4 is the only unit that processes all its assigned 5 products. Only 4 products are processed on M1, M2 and M3. Although product E is assigned to M1, M2 and M3, it is only processed on M2 and M3 in the optimal schedule. Also, only M4 processes product G, which is assigned to M2 and

M3 as well. Furthermore, product F is processed on all 3 units it can be assigned to, M2, M3 and M4.

2.5 Rolling Horizon Algorithm

The proposed single-level MILP model was solved directly and obtained global optimal solutions for horizons of up to 6 weeks in Example 2. However, because of the exponential growth in the computational effort when planning horizons and model sizes increase, we consider a rolling horizon (RH) algorithm, which can be used to reduce the computational effort.

2.5.1 Algorithm Description

In the RH algorithm, the problem considered is divided into a set of subproblems which are solved iteratively. The planning horizon of each subproblem (W_S) grows successively by a pre-specified number of weeks, while the length of periods (W_F) with fixed binary variables, including E_{imw} , F_{imw} , L_{imw} , Z_{ijmw} and ZF_{ijmw} , increases by the same time increment. The continuous variables in the fixed time periods (W_F) and all variables in the time periods without fixed variables (W_{NF}) are to be optimised in each subproblem. This iterative scheme stops when the entire planning horizon (W_T) is covered. The solution of the last subproblem is considered as an approximate optimal solution of the full problem. (Fig. 2.11)

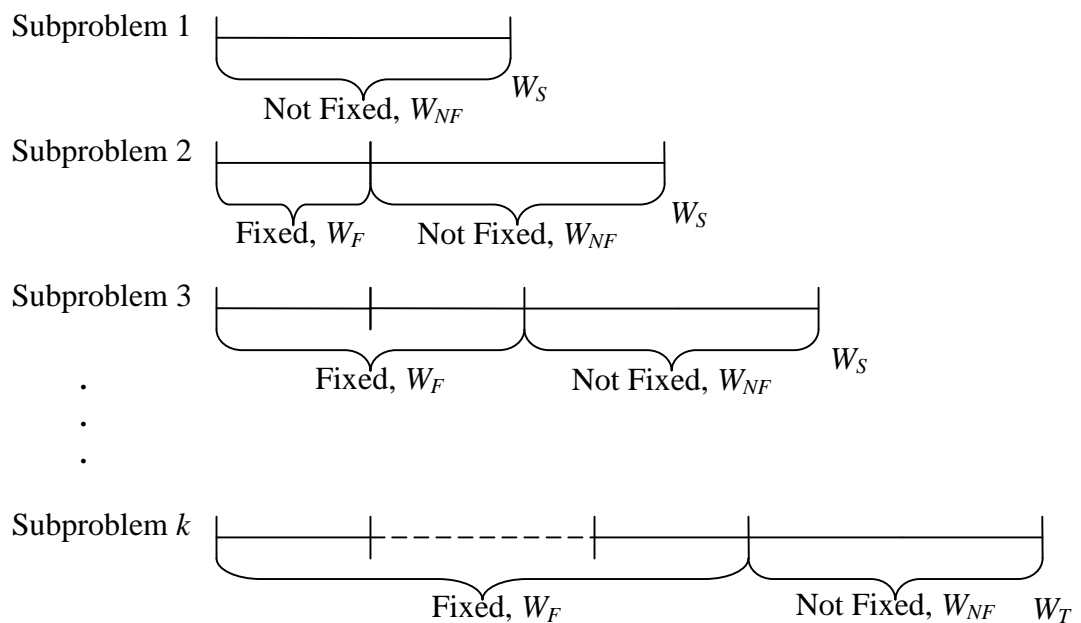


Figure 2.11 RH approach.

The proposed RH algorithm procedure can be outlined as follows:

- STEP 4. Initialise the length of time horizon without fixed binary variables in each subproblem W_{NF} , the length of the time horizon fixed in subproblem 1, $W_F = 0$, the length of planning horizon for subproblem 1, $W_S = W_F + W_{NF}$, and the increment of planning horizon between two subproblems, W_I , such that $W_I < W_S \leq W_T$, where W_T is the length of total planning horizon, Initialise $k=1$;
- STEP 5. Fix the binary variables within the planning horizon of W_F weeks to the values obtained in subproblem $k-1$;
- STEP 6. Solve subproblem k with a planning horizon of W_S weeks;
- STEP 7. If $W_S = W_T$, Stop, Otherwise, go to STEP 5;
- STEP 8. Let $k=k+1$, $W_F = W_F + W_I$, $W_S = W_S + W_I$, if $W_S > W_T$, let $W_S = W_T$, then go to STEP 2.

From the above procedure, in each subproblem, the values of binary variables newly fixed in the next subproblem are determined by tanking the demands in the next a few weeks into account. So although each subproblem is solved with a shorter horizon, the proposed RH approach is able to foresee some demand information in the next periods.

When implementing the above RH algorithm, in each iteration, we fix the values of all binary variables within W_F weeks, including E_{imw} , F_{imw} , L_{imw} , Z_{ijmw} and ZF_{ijmw} , as the same as the optimal ones obtained in the previous subproblem. For each subproblem, the continuous variables, especially T_{imw} , V_{iw} and S_{ciw} , within the whole horizon of that subproblem, are to be determined by the model. Thus, the RH approach has more flexibility when encountering unexpected high demands.

The performance of the proposed RH algorithm can be significantly affected by the values of W_{NF} and W_I . Usually, there is a tradeoff between the accuracy of the solution and the computational effort of the algorithm. The decision for each problem depends on the computational time limit and the tolerance required.

2.5.2 Illustrative Example 2 Revisited

To illustrate the applicability and computational efficiency of the proposed RH approach, we apply the RH approach to the 4 cases of the illustrative Example 2 discussed in Section 2.4.2. In the RH approach, we initialise $W_{NF} = 4$ and $W_I = 1$. Thus, the 6-week case is divided into 3 subproblems; the 12-week case is divided into 9 subproblems; the 18-week case is divided into 15 subproblems; and the 24-week problem is divided into 21 subproblems. See Fig. 2.12 for the subproblems in the 24-week problem.

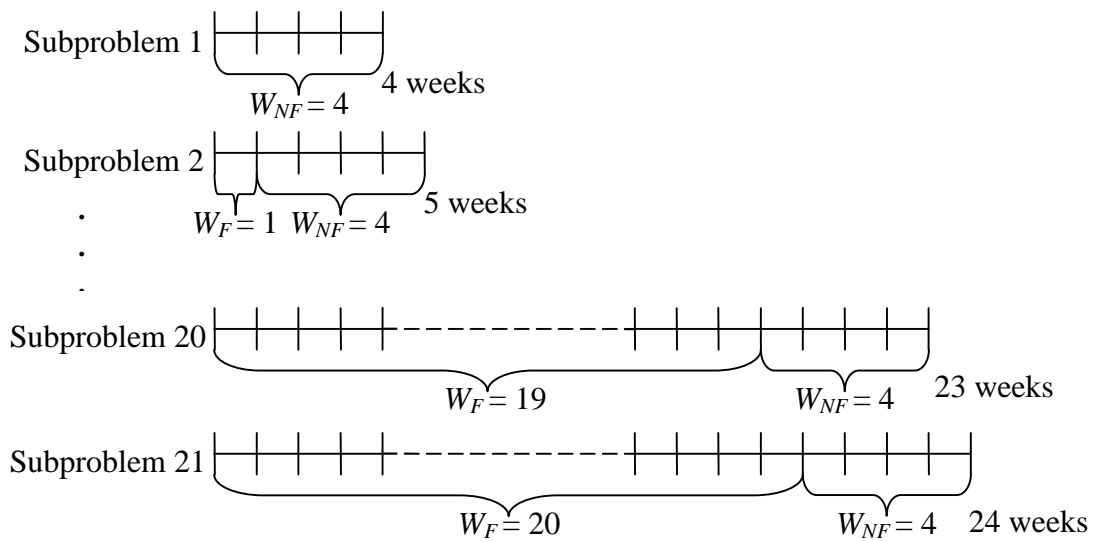


Figure 2.12 RH approach for the 24-week case.

Table 2.11 gives the computational results of the proposed single-level MILP model and RH algorithm. For the 6-week case, the proposed RH approach takes only 77 s to find a feasible solution which is almost the same as the global optimal one given by the single-level MILP model, with only few differences in the schedule on M1 in the first 3 weeks. Although the solution from the proposed RH approach is not as good as the feasible solution from single-level MILP model for the 12-week case, the gap between the two solutions is very small, around 0.01%. Moreover, the RH approach takes 401 s, while the single-level MILP model takes 3,600 s. The superior performance of the proposed RH approach becomes more apparent when the planning horizon of the example increases. For the 18-week case, the profit of the schedule given by RH approach is \$94,903, greater than \$94,882 obtained from the single-level MILP model. For the 24-week case, the RH approach takes 21 iterations

and a total of 892 s to generate a solution with an objective of 123,027, which is better than that of the MILP model, 122,725.

Table 2.11 Computational results of single-level MILP and RH for Example 2.

Model	Proposed MILP	Proposed RH
Time horizon (week)	6	6
No. of equations	2,509	2,253 ^a
No. of continuous variables	2,581	2,197 ^a
No. of binary variables	912	608 ^a
Total profit (\$)	33,550	33,550
Optimality gap (%)	0.00	0.00 ^b
CPU (s)	154	77
Time horizon (week)	12	12
No. of equations	5,065	3,897 ^a
No. of continuous variables	5,305	3,817 ^a
No. of binary variables	1,824	608 ^a
Total profit (\$)	64,841	64,830
Optimality gap (%)	0.27	0.00 ^b
CPU (s)	3,600	401
Time horizon (week)	18	18
No. of equations	7,621	5,541 ^a
No. of continuous variables	8,029	5,437 ^a
No. of binary variables	2,736	608 ^a
Total profit (\$)	94,882	94,903
Optimality gap (%)	0.44	0.00 ^b
CPU (s)	3,600	673
Time horizon (week)	24	24
No. of equations	10,177	7,185 ^a
No. of continuous variables	10,753	7,057 ^a
No. of binary variables	3,648	608 ^a
Total profit (\$)	122,725	123,027
Optimality gap (%)	0.85	0.00 ^b
CPU (s)	3,600	892

^a For the last subproblem in RH approach .

^b For each subproblem in RH approach.

2.6 Comparison with Literature Models

In this section, the computational efficiencies of the proposed MILP model and the RH approach are demonstrated by comparing them with those introduced by Erdirik-Dogan and Grossmann (2006, 2008a) and Chen et al. (2008). Erdirik-Dogan and Grossmann (2006) and Chen et al. (2008) proposed MILP models for the scheduling of single-stage single-unit continuous multiproduct plant, while the Erdirik-Dogan and Grossmann (2008a) proposed a bi-level decomposition approach for the scheduling and planning of continuous multiproduct plant with parallel units. Here, we compare the proposed model with the first iteration of the decomposition approach. The details of the models proposed by Erdirik-Dogan and Grossmann (2006) (E-D&G1 for short) and Chen et al. (2008) (CPP for short) are described in Appendices A and B, respectively. The details of the upper and lower level problems and integer cuts proposed by Erdirik-Dogan and Grossmann (2008a) (E-D&G2 for short) are described in Appendix C. It should be added that the lower level problem in Appendix C is an extension of the model in Appendix A.

Here, we make the comparison using four examples (A–D). The first two examples both consider single processing unit. We compare the single-unit case of the proposed model to the three literature models, including the single-unit case of the E-D&G2 model. Example A was introduced by Erdirik-Dogan and Grossmann (2006). Example B is the illustrative Example 1 in Section 2.4.1. The other two examples consider parallel units. The proposed MILP model is compared with model E-D&G2 using Example C, which was introduced by Erdirik-Dogan and Grossmann (2008a). Example D is the illustrative Example 2 in Section 2.4.2, which will be used to compare the proposed MILP model and the RH approach with model E-D&G2.

2.6.1 Literature Model Modifications

For the same representation and a fair comparison of their solution performance among the four MILP models, few modifications are made to the three literature models.

First, because of the similar nature of models E-D&G1 and E-D&G2, we compare the proposed model with model E-D&G1 and the upper level problem of model E-D&G2 simultaneously. There are six differences between the proposed model and

the other two models. Three involve the revenue and cost terms in the objective function. The others involve the sales, inventory and time constraints. These differences include:

- Both models E-D&G1 and E-D&G2 contain processing cost, which is not involved in the proposed model;
- The proposed model considers backlog cost term in the objective function and backlog constraints (Eq. (2.18)), while all demands in the models E-D&G1 and E-D&G2 must be satisfied (Eqs.(A.17) and (C.8));
- Models E-D&G1 and E-D&G2 do not consider multiple customers, while the proposed model considers the revenue and backlog cost from multiple customers;
- The proposed model represents the inventory constraints on a weekly basis (Eq. (2.19)), while the models E-D&G1 and E-D&G2 both utilise a linear overestimate of the inventory curve (Eqs. (A.13)–(A.16) and (C.4)–(C.7));
- The proposed model forces the processing time for a product in a week to exceed the minimum processing time (Eq. (2.12)), while there is no such constraint in models E-D&G1;
- Model E-D&G1 does not allow the production idle time except changeover (Eq. (A.10)), while the proposed model has no restriction on it.

To make a precise comparison, five modifications are made to both model E-D&G1 and the upper level problem of model E-D&G2, which include:

- The operating cost terms are removed from the each objective function;
- A backlog cost term is added to the each objective function, and Eqs. (A.17) and (C.8) are replaced by Eq. (2.18);
- Multiple customers are considered in the revenue term of each objective function;
- The inventory constraints Eqs. (A.13)–(A.16) and (C.4)–(C.7) are both replaced by Eq. (2.19), and the inventory cost term in the objective function is modified;
- The following constraints are added to model E-D&G1:

$$\theta_{it} \geq \theta^L \cdot YOP_{it}, \quad \forall i \in N, t \in HTot \quad (2.22)$$

Thus, after the above modifications, the objective function Eq. (A.1) of model E-D&G1 becomes:

$$z^p = \sum_c \sum_i \sum_t p_{cit} \cdot S_{cit} - \sum_i \sum_t c_i^{inv} \cdot V_{it} - \sum_c \sum_i \sum_t c_{ic}^{bac} \cdot \Delta_{cit} - \sum_i \sum_k \sum_l \sum_t c_{ik}^{trans} \cdot Z_{iklt} - \sum_i \sum_k \sum_t c_{ik}^{trans} \cdot TRT_{ikt} \quad (2.23)$$

The objective function of the upper level problem of model E-D&G2, Eq. (C.1), becomes:

$$\begin{aligned} \text{Profit} = & \sum_c \sum_i \sum_t CP_{cit} \cdot S_{cit} - \sum_i \sum_t CIN_{it} \cdot V_{it} - \sum_c \sum_i \sum_t CB_{ic} \cdot \Delta_{cit} - \\ & \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} CTRANS_{ikm} \cdot ZP_{ikmt} - \\ & \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} CTRANS_{ikm} \cdot (ZZZ_{ikmt} - ZZZP_{ikmt}) \end{aligned} \quad (2.24)$$

The first four modifications made to model E-D&G1 are also the modifications to the lower level problem of model E-D&G2. The objective function Eq. (C.25) becomes:

$$\begin{aligned} \text{Profit} = & \sum_c \sum_i \sum_t CP_{cit} \cdot S_{cit} - \sum_i \sum_t CIN_{it} \cdot V_{it} - \sum_c \sum_i \sum_t CB_{ic} \cdot \Delta_{cit} - \\ & \sum_m \sum_{i \in I_m} \sum_{k \in I_m} \sum_t \sum_l (CTTRANS_{ikm} \cdot Z_{ikmll} + CTRTRANS_{ikm} \cdot TRT_{ikmll}) \end{aligned} \quad (2.25)$$

In order to allow idle time in the schedule, another modification only added to model E-D&G1 is that Eq. (A.10) is modified as

$$Te_{ll} + \sum_i \sum_k \tau_{ik} \cdot TRT_{ikt} \leq Ts_{ll,t+1}, \quad \forall t \in HTot, l = N, ll = 1 \quad (2.26)$$

There is no difference between the presentations of the proposed model and model CPP, so no modification is made to model CPP. It should be added that pseudo product is not considered in all the following implementations in this section for a fair comparison as pseudo product is not considered in the literature models.

2.6.2 Model Size Comparison

Here, we compare the model sizes of the proposed model and three literature models after modifications. There are $|I_m|^2 \cdot |M| \cdot |W| + |C| \cdot |I_m| \cdot |W| + O(|I_m| \cdot |M| \cdot |W|)$ constraints in the proposed model, while model D-E&G1 for single-unit case has $O(|I|^2 \cdot |L| \cdot |T| + |C| \cdot |I| \cdot |T|)$ constraints, where $|W| = |T|$ and $|I| = |L|$. Thus, when

$|M|=1$, the proposed model size has fewer orders of magnitude. In model CPP for single-unit case, it has $4 \cdot |I| \cdot |K_w| \cdot |W| + |C| \cdot |I| \cdot |W| + o(|I| \cdot |K_w| \cdot |W|)$ constraints, where $|I| = |K_w|$. In model E-D&G2, the constraint number in its upper level problem is $2 \cdot |I_m|^2 \cdot |M| \cdot |T| + |C| \cdot |I_m| \cdot |T| + O(|I_m| \cdot |M| \cdot |T|)$, while constraint number in its lower level problem is $6 \cdot |I_m| \cdot |M| \cdot |L_m| \cdot |T| + |C| \cdot |I_m| \cdot |T| + o(|I_m| \cdot |M| \cdot |L_m| \cdot |T|)$, where $|W| = |T|$ and $|I_m| = |L_m|$. Thus, the proposed model has the same orders of magnitude of model size as model CPP when $|M|=1$, and as model E-D&G2. However, $|I_m|^2 \cdot |M| \cdot |W|$ comprises a large portion of constraint number, even the reduction in its coefficient produce a large decrease in model size. From the above comparison, we can see that the proposed model has advantage in model size than three literature model.

2.6.3 Example A

Example A, which was discussed in the work of Erdirik-Dogan and Grossmann (2006), consists of 5 types of products (A–E). The problem has a set of high demands and a set of low demands for a period of 8 weeks. Only the set of high demands is used in the comparison. The original example does not include backlog penalty cost, which is assumed to be 20% of product prices in the comparison. Two cases, with a planning horizon of four and eight weeks, respectively, are considered. Table 2.12 shows the solution results of the four models. It is observed that for the 4-week case, all models are able to achieve global optimality. The same optimal objective value obtained by the four models. At the same time, model E-D&G1 uses over 1,000 s to find the optimal solution, model CPP takes over 40 s to reach optimality, and the bi-level approach E-D&G2 requires around two seconds, while the proposed model requires only less than 1 second to find the globally optimal schedule. Both models E-D&G1 and CPP cannot find the global optimal solution of 8-week case in the specified time limit, although model CPP generates a very good approximation of the optimal schedule. However, model E-D&G2 and the proposed model reach global optimality, in which the former takes over 130 s and the latter uses about 80 s. The results show that the proposed model has superior computational performance.

Table 2.12 Model and solution statistics of four models for Example A.

Model	E-D&G1	CPP	E-D&G2 (upper / lower level)	Proposed MILP
Time horizon (week)	4	4	4	4
No. of equations	1,139	479	456 / 779	323
No. of continuous variables	936	696	205 / 961	196
No. of binary variables	120	120	260 / 120	140
Total profit (\$)	235,550	235,550	235,550 (235,550 / 235,550)	235,550
Optimality gap (%)	0.0	0.0	0.0 / 0.0	0.0
CPU (s)	1321	43	1.9 (1.5 / 0.4)	0.7
Time horizon (week)	8	8	8	8
No. of equations	2,303	967	916 / 1563	655
No. of continuous variables	2,136	1,396	409 / 1921	416
No. of binary variables	240	240	520 / 240	280
Total profit (\$)	470,520	471,330	471,350 (471,350 / 471,350)	471,350
Optimality gap (%)	1.1	0.4	0.0 / 0.0	0.0
CPU (s)	3,600	3,600	135.7 (135 / 0.7)	83

2.6.4 Example B

In Example B (see its details in Section 2.4.1), we also consider three cases with a planning horizon of 4, 6, and 8 weeks. The solution results of the four models are shown in Table 2.13. From the comparison, we can see that the proposed model is capable of finding the global optimal solution to all three cases within 200 seconds, even for the 8-week case. However, models E-D&G1 and CPP cannot reach global optimality within the specified time limit, even for the smallest-size case with a 4-week planning horizon. Comparing the aforementioned two models, model CPP has shown a better computational performance than model E-D&G1 for all cases. Model E-D&G2 only generates the global optimal schedule for the 4-week case, while for the other two cases, although the upper level problems can be solved in less than 120 seconds, the lower level problems cannot automatically terminate within the specified time limit. The E-D&G2 model can find better solution than models E-D&G1 and CPP.

Table 2.13 Model and solution statistics of four models for Example B.

Model	E-D&G1	CPP	E-D&G2 (upper / lower level)	Proposed MILP
Time horizon (week)	4	4	4	4
No. of equations	6,384	1,909	1,651 / 2,904	1,193
No. of continuous variables	6,141	5,311	1,325 / 6,141	1,261
No. of binary variables	440	440	920 / 440	480
Total profit (\$)	5,354	5,422	5,439 (5,448 / 5,439)	5,439
Optimality gap (%)	6.0	1.4	0.0 / 0.0	0.0
CPU (s)	3,600	3,600	390 (10 / 380)	3.5
Time horizon (week)	6	6	6	6
No. of equations	9,626	2,873	2,481 / 4,366	1,799
No. of continuous variables	9,261	7,971	1,987 / 9,261	1,941
No. of binary variables	660	660	1,380 / 660	720
Total profit (\$)	7,889	8,045	8,102 (8,148 / 8,102)	8,135
Optimality gap (%)	8.3	3.2	0.0 / 1.6	0.0
CPU (s)	3,600	3,600	3,639 (39 / 3,600)	28
Time horizon (week)	8	8	8	8
No. of equations	12,868	3,837	3,311 / 5,828	2,405
No. of continuous variables	12,381	10,631	2,649 / 12,381	2,621
No. of binary variables	880	880	1,840 / 880	960
Total profit (\$)	10,110	10,531	10,642 (10,667 / 10,642)	10,655
Optimality gap (%)	11.0	4.1	0.0 / 1.7	0.0
CPU (s)	3,600	3,600	3,713 (113 / 3,600)	160

Here, when implementing the models in GAMS, variables ZF_{ijw} in the proposed model, variables Z_{iklt} and TRT_{ikt} in the E-D&G1 model, variables Z_{iklt} in model CPP, variables ZZZ_{ikmt} in model E-D&G2 are treated as continuous variables between 0 and 1. Model statistics in Tables 2.12 and 2.13 show that the proposed model has much fewer equations and continuous variables than the other three models, especially model E-D&G1. These models have similar number of binary variables, except for the upper level problem of model E-D&G2.

2.6.5 Example C

Example C, which was discussed in the work of Erdirik-Dogan and Grossmann (2008a), consists of 8 types of products (A–H) and 3 units (M1–M3). The original problem considers a total planning horizon of 24 weeks. However, because of the

limited information provided in the paper, only two cases, with a planning horizon of four and eight weeks, respectively, are considered.

Table 2.14 shows the solution results of the two models.

Table 2.14 Model and solution statistics of E-D&G2 and proposed MILP for Example C.

Model	E-D&G2 (upper / lower level)	Proposed MILP
Time horizon (week)	4	4
No. of equations	965 / 1,490	701
No. of continuous variables	385 / 1,489	385
No. of binary variables	528 / 240	288
Total profit (\$)	633,851 (633,851 / 633,851)	633,851
CPU (s)	1.5 (1.3 / 0.2)	0.4
Time horizon (week)	8	8
No. of equations	1,953 / 3,006	1,425
No. of continuous variables	781 / 3025	817
No. of binary variables	1,056 / 480	576
Total profit (\$)	112,7163 (1,127,163 / 1,127,163)	1,127,163
CPU (s)	116.2 (116 / 0.2)	89

In both 4-week and 8-week cases, we can see that both approaches can find global solutions. However, the E-D&G2 model takes more CPU time to reach the global optimum than the proposed model. Especially in the 8-week case, the E-D&G2 model takes 116 s while the proposed model takes 1/4 less time, which is 89 s.

2.6.6 Example D

In Example D (see its details in Section 2.4.2), we consider 4 cases with a planning horizon of 6, 12, 18 and 24 weeks. We initialise $W_{NF} = 4$ and $W_I = 1$, and apply the proposed RH approach to the 4 cases.

From Table 2.15, except for the 6-week case, both model E-D&G2 and the proposed single-level MILP model cannot terminate within the specified time limit. However, the proposed MILP model yields better feasible solutions than those obtained by model E-D&G2, and takes only half of CPU time than model E-D&G2.

Table 2.15 Model and solution statistics of E-D&G2, proposed MILP, and RH for Example D.

Model	E-D&G2 (upper / lower level)	Proposed MILP	Proposed RH
Time horizon (week)	6	6	6
No. of equations	3,141 / 4,841	2,373	2,157 ^a
No. of continuous variables	2,105 / 6,201	2,121	2,021 ^a
No. of binary variables	1,560 / 720	840	560 ^a
Total profit (\$)	33,526 (33,555 / 33,526)	33,550	33,550
Optimality gap (%)	0.00 / 0.00	0.00	0.00 ^b
CPU (s)	2,186 (512 / 1,674)	260	76
Time horizon (week)	12	12	12
No. of equations	6,321 / 9,725	4,785	3,801 ^a
No. of continuous variables	4,229 / 12,501	4,341	3,641 ^a
No. of binary variables	3,120 / 1,440	1,680	560 ^a
Total profit (\$)	64,813 (64,850 / 64,813)	64,833	64,830
Optimality gap (%)	0.28 / 0.22	0.25	0.00 ^b
CPU (s)	7,200 (3,600 / 3,600)	3,600	430
Time horizon (week)	18	18	18
No. of equations	9,501 / 14,609	7,197	5,445 ^a
No. of continuous variables	6,353 / 18,801	6,561	5,261 ^a
No. of binary variables	4,680 / 2,160	2,520	560 ^a
Total profit (\$)	94,768 (94,875 / 94,768)	94,807	94,903
Optimality gap (%)	0.48 / 0.56	0.53	0.00 ^b
CPU (s)	7,200 (3,600 / 3,600)	3,600	621
Time horizon (week)	24	24	24
No. of equations	12,681 / 19,493	9,609	7,089 ^a
No. of continuous variables	8,477 / 25,101	8,781	6,881 ^a
No. of binary variables	6,240 / 2,880	3,360	560 ^a
Total profit (\$)	122,600 (122,764 / 122,600)	122,745	123,027
Optimality gap (%)	0.82 / 0.52	0.83	0.00 ^b
CPU (s)	7,200 (3,600 / 3,600)	3,600	808

^aFor the last subproblem in RH approach.^bFor each subproblem in RH approach.

For the 6-week case, the proposed MILP model takes only 260 s to get the global optimal solution, while model E-D&G2 totally takes over 8 times CPU time than the proposed MILP model. Moreover, for the 6-week case, the solution of model E-D&G2 is worse than the other two approaches. Because subtours occur in the solution of its upper level problem, the objective given by its upper level problem is

an upper bound of the global optimal one, and the solution given by its lower level problem is not a global optimum.

Moreover, for all cases, the RH approach also takes much less CPU time and finds better feasible solutions than those of model E-D&G2. It should be noticed that in the 12-, 18- and 24-week cases, the upper level problem of model E-D&G2 terminates when the computation time reaches the time limit, 3,600s. From Table 2.15, we can see that there is a gap between the obtained solution and the optimal solution of the upper level problem of model E-D&G2, which is also the upper bound of the problem. Moreover, the solutions of RH approach are better than those of the upper problem for the 18- and 24-week cases. It is worth noting that subtours still occur in the obtained solutions of the upper level problem of model E-D&G2, which yield infeasible production sequences of products.

2.7 Concluding Remarks

A novel MILP model for medium-term planning of single-stage continuous multiproduct plants has been presented in this chapter. The model is based on a hybrid discrete/continuous time representation. Because of the similar nature of the problem with the TSP, a formulation similar to the one used to model changeovers in the classic TSP has been proposed. Also, in order to eliminate subtours in the schedule, integer variables representing the sequence of the products and the subtour elimination constraints have been introduced. Illustrative examples of polymer processing plants have been used to illustrate the applicability of the proposed model.

In order to overcome the computational expense of solving large problems, we have proposed a rolling horizon approach, which significantly reduces the computational time with a good feasible solution. Finally, the proposed MILP model and RH algorithm have been compared favourably with models from recent literature (Erdirik-Dogan and Grossmann, 2006, 2008a; Chen et al., 2008), exhibiting a much improved computational performance for the examples investigated.

The TSP-based formulation proposed in this chapter will also be adapted in the next chapter to tackle the short-term scheduling of single-stage multiproduct batch plants.

SHORT-TERM SCHEDULING OF SINGLE-STAGE MULTIPRODUCT BATCH PLANTS

In the previous chapter, we have investigated the medium-term production planning problem, while in this chapter the short-term production scheduling problem of a single-stage batch plant is considered. The work in this chapter is inspired by the real-world industrial case study of edible-oil batch deodoriser discussed in Kelly and Zyngier (2007), in which the processing changeovers only occur while switching from one product group to another. In this case, the production schedule of product groups, rather than products, is of higher concern. The discrete-time model by Kelly and Zyngier (2007) is still very computational expensive for large instances with a higher number of orders, products and product groups and a longer planning horizon.

In this chapter we aim to adapt the TSP-based formulations in the previous chapter to develop efficient MILP models for the short-term scheduling of single-stage batch edible-oil batch deodoriser with sequence-dependent changeovers between product groups, and apply the proposed models for the real-world case study with a planning horizon of several days.

3.1 Introduction and Literature Review

As referred to in Chapter 2, a large number of optimisation models and approaches have been proposed for the planning and scheduling of multiproduct batch plants in the past two decades (see detailed reviews from Pinto and Grossmann, 1998; Kallrath, 2002b; Floudas and Lin, 2004; Burkard and Hatzl, 2005; Méndez et al., 2006; Pan et al., 2009).

At first, discrete-time formulation models using STN (Kondili et al., 1993a) or RTN representations (Pantelides, 1994) were used for batch scheduling problems. Because discrete-time formulations become extremely large for a large-size problem and a finer discretisation, several techniques have been proposed to reduce the computational effort of the large discrete-time MILP models (Shah et al., 1993; Bassett et al., 1996; Elkamel et al., 1997).

Increasing attention has been paid to the continuous-time formulations to overcome the difficulties from the discrete-time formulations. Pinto and Grossmann (1995) proposed a continuous-time MILP model for the short-term scheduling of batch plants with multiple stages. This work was improved with the assumption of pre-ordering of orders in Pinto and Grossmann (1996). Zhang and Sargent (1996) used the RTN representation to develop a MINLP formulation for the scheduling of general plant topologies and then solved the problem with iterative MILP models. Cerdá et al. (1997) developed an MILP model for the short-term scheduling of a single-stage batch multiproduct plant with nonidentical parallel units/lines based on continuous-time domain representation. Karimi and McDonald (1997) developed slot-based MILP formulations for the short-term scheduling of single-stage multiproduct plants with parallel semicontinuous units. Ierapetritou and Floudas (1998) presented an MILP formulation for the short-term scheduling of multiproduct/multipurpose batch processes based on STN representation.

Méndez et al. (2000) presented a two-stage approach for the batching and scheduling problem of single-stage multiproduct batch plants. Hui and Gupta (2001) proposed a general formulation for short-term scheduling of single-stage multiproduct batch plants with nonidentical parallel units with order sequence-dependent constraints. Neumann et al. (2002) put the detailed production scheduling phase into the framework of an Advanced Planning System, and decomposed a detailed production scheduling problem into a batching problem and a batch scheduling problem. Chen et al. (2002) introduced an MILP formulation for the short-term scheduling of multiproduct batch plants with parallel units, as well as two heuristic rules to reduce model size. Lim and Karimi (2003) considered both batching and scheduling in a slot-based MILP formulation for the short-term scheduling of single-stage batch plants with parallel units and multiple orders per product. Castro and Grossmann

(2006) proposed a multiple-time-grid, continuous-time MILP model for the short-term scheduling of single stage multiproduct plants. He and Hui (2006) proposed an evolutionary approach for the single-stage multiproduct scheduling with parallel units. The authors extended their own work by constructing a new set of heuristic rules (He and Hui, 2007) and proposing a heuristic rule-based genetic algorithm (He and Hui, 2008).

Erdirik-Dogan and Grossmann (2007) proposed two production planning models and a rolling horizon algorithm for the production planning of parallel multiproduct batch reactors with sequence-dependent changeovers. Liu and Karimi (2007a, b, 2008) proposed a series of slot-based and sequence-based MILP models for the scheduling of multistage multiproduct batch plants with parallel units, as well as unlimited and no intermediate storage. Prasad and Maravelias (2008), and Sundaramoorthy and Maravelias (2008) both considered the simultaneous batching and scheduling of multistage multiproduct processes in MILP formulations. Erdirik-Dogan and Grossmann (2008b) proposed a slot-based continuous time MILP formulation and a bi-level decomposition scheme for the short-term scheduling of multistage multiproduct batch plants.

Shaik and Floudas (2008) improved the model of Ierapetritou and Floudas (1998) and proposed a RTN-based unit-specific event-based model for short-term scheduling of batch plants. Castro et al. (2008) aggregated all batches of a product into a single task instead of considering one processing task per batch for the short-term batching and scheduling of single-stage multiproduct plants. Marchetti and Cerdá (2009a) presented an MILP formulation for the short-term scheduling of single-stage multiproduct batch plants with parallel units using a unit-dependent precedence-based representation. The same authors (Marchetti and Cerdá, 2009b) also proposed an MILP sequential approach for the short-term scheduling of multistage batch plants with sequence-dependent changeover times and intermediate due dates. Kopanos et al. (2009) proposed a new continuous-time MILP scheduling framework for dealing with sequence-dependent changeover time and/or cost issues in batch plants, based on the unit-specific general precedence concept. The proposed model solved medium-sized scheduling problems with relatively lower computational effort than literature precedence-based models. Castro and Novais

(2009) proposed a new RTN-based multiple-time-grid MILP formulation for the short-term scheduling of multistage batch plants with multiple product batches and sequence-dependent changeovers.

Kopanos et al. (2010b) addressed the production scheduling and lot-sizing in a multiproduct yogurt production line of a dairy plant and proposed a mixed discrete/continuous-time MILP model based on product families. Verderame and Floudas (2010) extended their previous work (Verderame and Floudas, 2008) to integrate of operational planning and medium-term scheduling of large-scale industrial batch plants under demand due date and amount uncertainty by means of a rolling horizon framework. Marchetti et al. (2010) presented two sequence-based continuous-time MILP models for the simultaneous lot-sizing and scheduling of single-stage multiproduct batch facilities. The computational study shows that cluster-based approach is more efficient to solve large-scale problems. Kopanos et al. (2010a) developed a two-step MILP-based solution approach for large-scale scheduling problems in multiproduct multistage batch plants and examined its performance by studying a real-world multiproduct multistage pharmaceutical batch plant. Subbiah et al. (2011) developed an approach based on the framework of timed automata to model the multistage, multiproduct batch scheduling problems with sequence-dependent changeovers, where the resources, recipes, and additional timing constraints are formulated independently as sets of (priced) timed automata.

The objective of the work in this chapter is to develop efficient MILP optimisation approaches for the short-term scheduling of single-unit batch plants, especially of an edible-oil batch deodoriser case study. The processing of products is incorporated into that of product groups, and the schedule of products groups is firstly considered. The processing of a product group involves the processing of multiple products in the group, and the processed products are used to satisfy the demands of the orders.

3.2 Problem Statement

In this problem, we consider a single-stage multiproduct batch deodoriser that processes multiple products. There are multiple customer orders for each product that belongs to certain product group (Fig. 3.1). Each order has its release time and due date. The total planning horizon is of several days. The single deodorisation tray in

the deodoriser cannot contain different products at the same time, which means that the deodoriser can only process one product in one batch. Sequence-dependent down-time restrictions occur when switching from one product group to another. The following assumptions have been made in the problem:

- Each product belongs to only one group;
- Each order is specific to only one product;
- Each order is released/due at the beginning/end of a time period;
- No order can be processed before its release time;
- Different orders of the same product can be processed together;
- Single batch time is fixed;
- Multiple deliveries are allowed for each order after its release time.

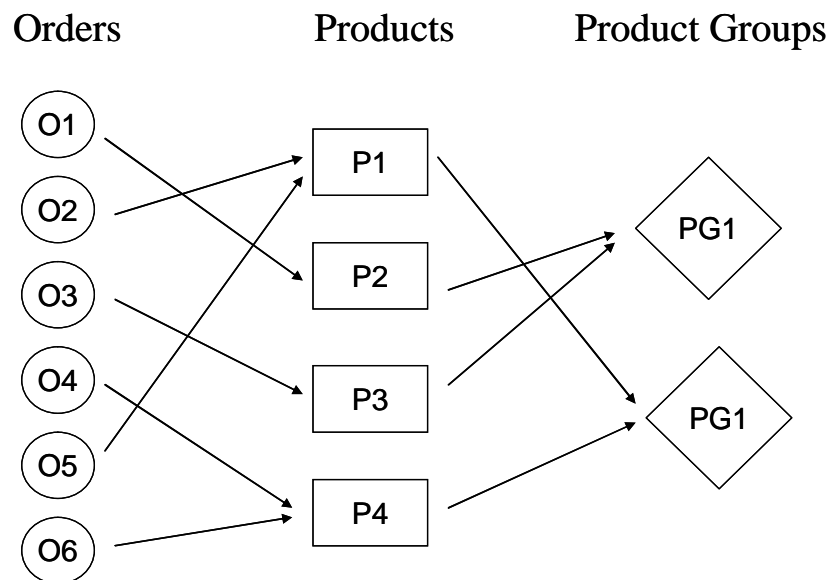


Figure 3.1 Orders, products and product groups.

In this scheduling problem, given are the product groups, products, orders, release time, due date and demand of each order, changeover times, batch time, minimum and maximum batch sizes, to determine the processing sequence and times of product groups, processing amount and batch number of each product, inventory levels, and deliveries/sales for each order, so as to maximise the total profit, involving sales revenue, processing cost, changeover cost, inventory cost and backlog cost, if backlog is allowed.

3.3 Mathematical Formulation

The proposed models for the batch edible-oil deodoriser scheduling problem are MILP formulations. Similar to the work in Chapter 2, we introduce the ordering index variable and use the classic TSP formulation, based on a discrete/continuous time representation.

For the batch scheduling problem, we consider two scenarios. In scenario 1, no backlog is allowed, and all orders should be processed and delivered within their time windows. In scenario 2, backlog is allowed, and the orders can be processed and delivered after the due dates. Note that the models proposed are for single-unit cases, which can be extended to tackle the multiple-unit cases.

3.3.1 Nomenclature

Indices

g, g'	product group
i	product
o	order
t, t'	time period

Sets

G	product groups
G_t	product groups whose windows contain time period t : $G_t = \{g \mid \min_{i \in I_g, o \in O_i} RT_o \leq t \leq \max_{i \in I_g, o \in O_i} DT_o\}$
\tilde{G}_t	product groups whose windows start before time period t : $\tilde{G}_t = \{g \mid t \geq \min_{o \in I_g} RT_o\}$
I	products
I_g	products in group g
I_t	products whose windows contain time period t : $I_t = \{i \mid \min_{o \in O_i} RT_o \leq t \leq \max_{o \in O_i} DT_o\}$
\tilde{I}_t	products whose window start before time period t : $\tilde{I}_t = \{i \mid t \geq \min_{o \in O_i} RT_o\}$
O	orders

O_i	orders for product i
O_t	orders whose windows contain time period t : $O_t = \{o \mid RT_o \leq t \leq DT_o\}$
\tilde{O}_t	orders whose windows start before time period t : $\tilde{O}_t = \{o \mid t \geq RT_o\}$
T	time periods

Parameters

BC_o	backlog cost of order o
BN_t^{\max}	maximum batch number during time period t
BS^{\max}	maximum batch size
BS^{\min}	minimum batch size
BT	batch time
D_o	demand of order o
DT_o	time period of due date of order o
IC_i	inventory cost of product i
INV_i^{\max}	maximum inventory capacity of product i
N	a large number
PC_i	processing cost of product i
Pr_i	price of product i
RT_o	time period of release time of order o
θ_t^U	upper bound of processing time in time period t
$\tau_{gg'}$	changeover time from group g to group g'

Binary Variables

E_{gt}	1 if group g is processed during time period t , 0 otherwise
F_{gt}	1 if group g is the first one in time period t , 0 otherwise
L_{gt}	1 if group g is the last one in time period t , 0 otherwise
$Z_{gg't}$	1 if group g immediately precedes group g' in time period t , 0 otherwise
$ZF_{gg't}$	1 if group g in period $t-1$ immediately precedes group g' in time period t , 0 otherwise

Integer Variables

N_{it} number of batches of product i during time period t

Continuous Variables

INV_{it} inventory amount for product i at the end of time period t

OI_{gt} ordering index of group g during time period t

P_{it} amount of product i processed during time period t

PT_{gt} processing time for group g during time period t

Q_{ot} product amount processed for order o during time period t

Sa_{ot} sales amount for order o in time period t

Δ_{ot} backlog amount for order o in time period t

Π total profit, the objective

3.3.2 Model for Scenario 1: DEO-S1

In scenario 1, as backlogs and processing/deliveries after the due dates of the orders are not allowed, only product group $g \in G_t$, product $i \in I_t$ and order $o \in O_t$ can be assigned to time period t for processing.

3.3.2.1 Assignment and Sequencing Constraints

Assuming that each time period comprises the processing of at least one product group, only one product group can be the first or the last one in each time period:

$$\sum_{g \in G_t} F_{gt} = 1, \quad \forall t \in T \quad (3.1)$$

$$\sum_{g \in G_t} L_{gt} = 1, \quad \forall t \in T \quad (3.2)$$

If a product group is not processed in a time period, then it can not be either the first or the last one in the time period:

$$F_{gt} \leq E_{gt}, \quad \forall t \in T, g \in G_t \quad (3.3)$$

$$L_{gt} \leq E_{gt}, \quad \forall t \in T, g \in G_t \quad (3.4)$$

During each time period, each product group is processed following another product group, except the first one, while each product group is processed preceding another product group, except the last one.

$$\sum_{g \in G_t, g \neq g'} Z_{gg't} = E_{g't} - F_{g't}, \quad \forall t \in T, g' \in G_t \quad (3.5)$$

$$\sum_{g' \in G_t, g' \neq g} Z_{gg't} = E_{gt} - L_{gt}, \quad \forall t \in T, g \in G_t \quad (3.6)$$

Considering two consecutive time periods, there is a changeover from the last processed product in the previous time period to the first processed product in the next time period.

$$\sum_{g \in G_{t-1}} ZF_{gg't} = F_{g't}, \quad \forall t \in T \setminus \{1\}, g' \in G_t \quad (3.7)$$

$$\sum_{g' \in G_{t-1}} ZF_{gg't} = L_{g,t-1}, \quad \forall t \in T \setminus \{1\}, g \in G_t \quad (3.8)$$

3.3.2.2 Subtour Elimination Constraints

The ordering index of a later processed product group is larger than an earlier one.

$$OI_{g't} - (OI_{gt} + 1) \geq -M \cdot (1 - Z_{gg't}), \quad \forall t \in T, g, g' \in G_t, g \neq g' \quad (3.9)$$

If a product group is not processed in a time period, then its order index is 0.

$$OI_{gt} \leq N \cdot E_{gt}, \quad \forall t \in T, g \in G_t \quad (3.10)$$

where the maximum of cardinality of set G_t , $\max_t |G_t|$, can be used as N . From

Theorem 2.1, the above constraints avoid the subtours in the feasible schedules.

3.3.2.3 Processing Timing Constraints

There should be at least one batch to be processed if a product group is assigned to a period. Otherwise, no batch of all its products is processed.

$$E_{gt} \leq \sum_{i \in I_g \cap I_t} N_{it} \leq BN_t^{\max} \cdot E_{gt}, \quad \forall t \in T, g \in G_t \quad (3.11)$$

The processing time of a product group is the total batches multiplied by the batch processing time.

$$PT_{gt} = BT \cdot \sum_{i \in I_g \cap I_t} N_{it}, \quad \forall t \in T, g \in G_t \quad (3.12)$$

The total processing and changeover time is limited by the total available time in each time period.

$$\sum_{g \in G_t} PT_{gt} + \sum_{g \in G_t} \sum_{g' \in G_t} Z_{gg't} \cdot \tau_{gg'} + \sum_{g \in G_{t-1}} \sum_{g' \in G_t} ZF_{gg't} \cdot \tau_{gg'} \leq \theta_t^U, \quad \forall t \in T \setminus \{1\} \quad (3.13)$$

$$\sum_{g \in G_t} PT_{gt} + \sum_{g \in G_t} \sum_{g' \in G_t} Z_{gg't} \cdot \tau_{gg'} \leq \theta_t^U, \quad \forall t \in \{1\} \quad (3.14)$$

3.3.2.4 Processing Amount Constraints

For each product, its processing amount in a time period is limited by the number of batches multiplied by the minimum and maximum batch sizes.

$$BS^{\min} \cdot N_{it} \leq P_{it} \leq BS^{\max} \cdot N_{it}, \quad \forall t \in T, i \in I_t \quad (3.15)$$

The process amount for each product in a time period is the summation of the process amounts for the related orders which can be processed in the time period.

$$P_{it} = \sum_{o \in O_i \cap O_t} Q_{ot}, \quad \forall t \in T, i \in I_t \quad (3.16)$$

3.3.2.5 Inventory Constraints

The inventory level for an order in a time period is the inventory in the previous time period, plus the production amount, minus the sales, which only occur within the time window.

$$INV_{ot} = INV_{o,t-1} \Big|_{t > RT_o} + (Q_{ot} - Sa_{ot}) \Big|_{o \in O_t}, \quad \forall t \in T, o \in \tilde{O}_t \quad (3.17)$$

The inventory level of each product is limited by its maximum capacity.

$$\sum_{o \in O_i \cap \tilde{O}_t} INV_{ot} \leq INV_i^{\max}, \quad \forall t \in T, i \in \tilde{I}_t \quad (3.18)$$

3.3.2.6 Demand Constraints

The sales for each order should only take place within the time window, and the total sales should be no more than its demand.

$$\sum_{t=RT_o}^{DT_o} Sa_{ot} \leq D_o, \quad \forall o \in O \quad (3.19)$$

3.3.2.7 Objective Function

The objective is to maximise the total profit, involving the sales revenue, processing cost, inventory cost and changeover cost.

$$\begin{aligned} \Pi = & \sum_{i \in I} \sum_{t \in T} \sum_{o \in O_t \cap \tilde{O}_t} Pr_i \cdot Sa_{ot} - \sum_{t \in T} \sum_{i \in I_t} PC_i \cdot P_{it} - \sum_{i \in I} \sum_{t \in T} \sum_{o \in O_t \cap \tilde{O}_t} IC_i \cdot INV_{ot} - \\ & \sum_{t \in T} \sum_{g \in \tilde{G}_t, g' \in \tilde{G}_t} CC_{gg'} \cdot Z_{gg't} - \sum_{t \in T \setminus \{1\}} \sum_{g \in \tilde{G}_{t-1}, g' \in \tilde{G}_t} CC_{gg'} \cdot ZF_{gg't} \end{aligned} \quad (3.20)$$

3.3.2.8 Summary

In summary, model DEO-S1 for scenario 1 of the problem is described by Eqs. (3.1)–(3.19) with Eq. (3.20) as the objective function.

3.3.3 Model for Scenario 2: DEO-S2

In scenario 2, backlogs and processing/deliveries after the due dates of the orders are allowed, so product group $g \in \tilde{G}_t$, product $i \in \tilde{I}_t$ and order $o \in \tilde{O}_t$ can be assigned to each time period t to process. By replacing the sets G_t , I_t and O_t in Eqs. (3.1)–(3.17) by the sets \tilde{G}_t , \tilde{I}_t and \tilde{O}_t , respectively, we can obtain the constraints for the model of scenario 2. Eq. (3.18) can be used in the model for scenario 2 without any change.

3.3.3.1 Demand and Backlog Constraints

The backlog of an order is only activated in the time periods after its due date. At a time period t , the backlog of each order is equal to its demand minus the total sales until time period t .

$$\Delta_{ot} = D_o - \sum_{t'=RT_o}^t Sa_{ot'}, \quad \forall o \in O, t \geq DT_o \quad (3.21)$$

The sale of each order can be in any time period after its release time.

$$\sum_{t \geq RT_o} Sa_{ot} \leq D_o, \quad \forall o \in O \quad (3.22)$$

3.3.3.2 Objective Function

The backlog cost is also included in the objective, besides the cost terms included in Eq. (3.20).

$$\begin{aligned} \Pi = & \sum_{i \in I} \sum_{t \in T} \sum_{o \in O_t \cap \tilde{O}_t} Pr_i \cdot Sa_{ot} - \sum_{t \in T} \sum_{i \in I_t} PC_i \cdot P_{it} - \sum_{i \in I} \sum_{t \in T} \sum_{o \in O_t \cap \tilde{O}_t} IC_i \cdot INV_{ot} - \\ & \sum_{t \in T} \sum_{o \in \tilde{O}_t} BC_o \cdot \Delta_{ot} - \sum_{t \in T} \sum_{g \in \tilde{G}_t, g' \in \tilde{G}_t} CC_{gg'} \cdot Z_{gg't} - \sum_{t \in T \setminus \{1\}} \sum_{g \in \tilde{G}_{t-1}, g' \in \tilde{G}_t} CC_{gg'} \cdot ZF_{gg't} \end{aligned} \quad (3.23)$$

3.3.3.3 Summary

In summary, model DEO-S2 for scenario 2 of the problem is described by Eqs. (3.1)–(3.17) after modification and Eqs. (3.18), (3.21) and (3.22) with Eq. (3.23) as the objective function.

3.4 Case Study

3.4.1 Data

In this section, we apply the proposed models to the real-world industrial edible-oil deodoriser scheduling problem. A planning horizon of 128 hours is considered. There are 70 orders (O1–O70) for 30 products (P1–P30) that belong to 7 different groups (PG1–PG7). The total demand is 4156 ton, and the demand for each order is given in Table 3.1. The release time and due date of each order (Table 3.1) are only at 8 am and 6 pm during each day. The total planning horizon is divided into 11 time periods illustrated in Fig. 3.2. For each order, its time window is shown in Fig. 3.3 and Table 3.1. The numbers in Fig. 3.3 indicate the order demands.

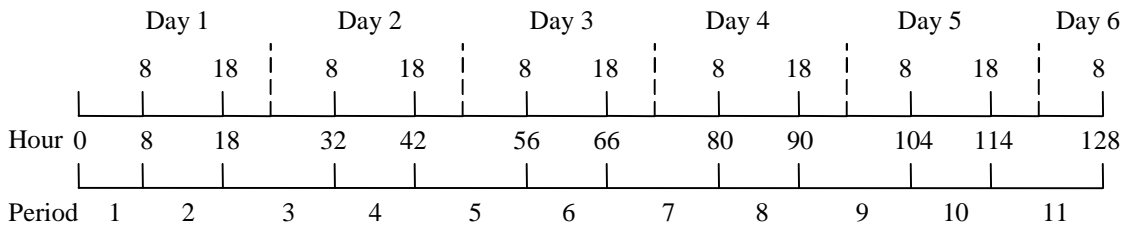


Figure 3.2 Aggregated time periods.

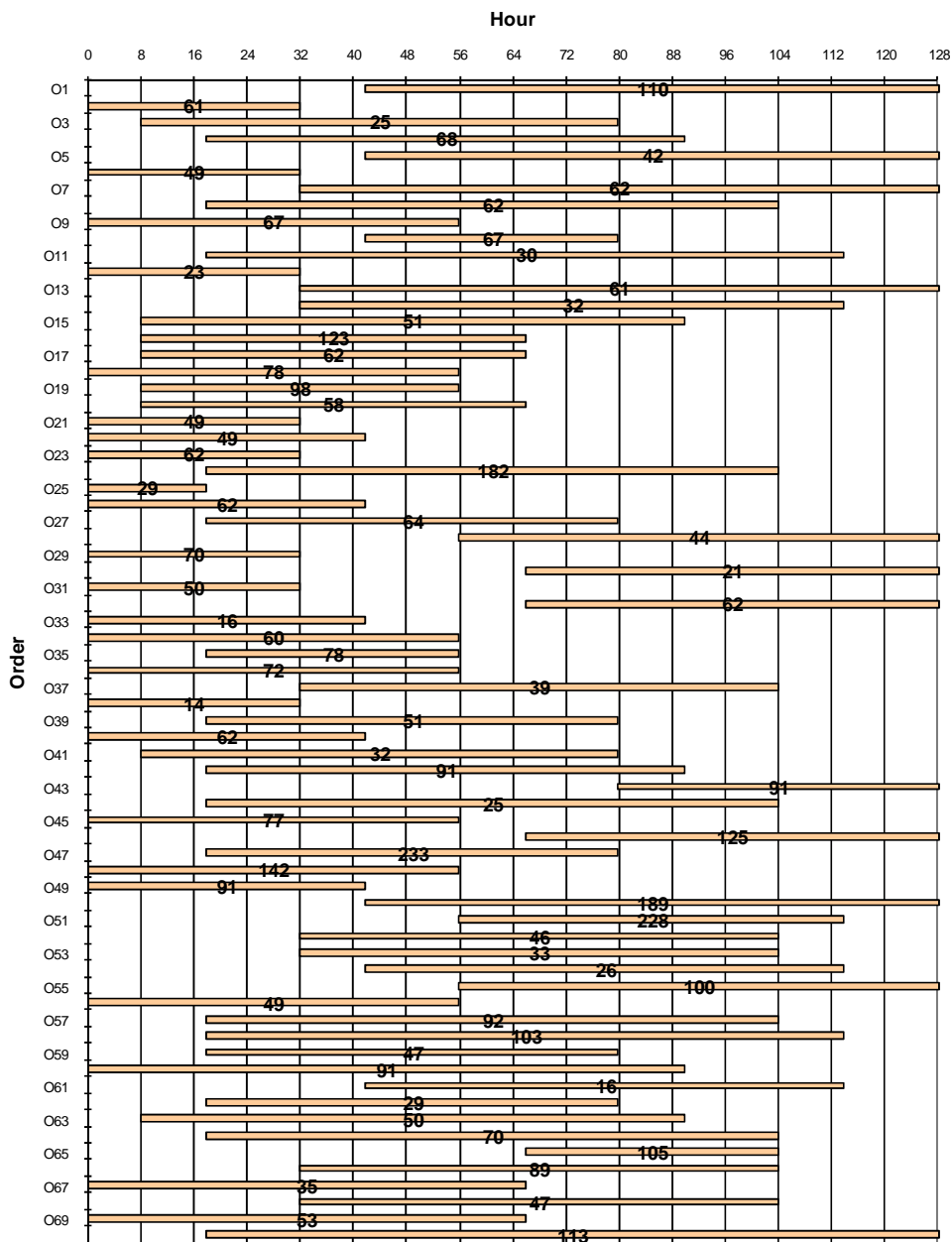


Figure 3.3 Time window and demand (in ton) of each order.

Table 3.1 Details of each product group, product and order.

Group	Product	Order	Release time (hour)	Due date (hour)	Demand (ton)	Group	Product	Order	Release time (hour)	Due date (hour)	Demand (ton)
PG1	P1	O5	42	128	36	PG3	P6	O13	32	128	52
	P3	O14	32	114	27		P12	O6	0	32	42
		O31	0	32	43		PG4	P17	O20	8	66
	P4	O22	0	42	42	P18		O10	42	80	57
		O33	0	42	14	O68		32	104	40	
	P14	O3	8	80	21	P19	O9	0	56	57	
		O54	42	114	22	O64	18	104	60		
		O60	0	90	77	PG5	P22	O25	0	18	25
		O70	18	128	96		P29	O41	8	80	27
	P16	O11	18	114	26	P10	O17	8	66	53	
		O59	18	80	40		O28	56	128	37	
		O69	0	66	45		O29	0	32	60	
	P20	O4	18	90	58		O30	66	128	18	
		O65	66	104	89		O37	32	104	33	
		O67	0	66	30		O46	66	128	106	
	P21	O44	18	104	21		O47	18	80	198	
		O45	0	56	65		O48	0	56	121	
	P27	O21	0	32	42		O49	0	42	77	
		O52	32	104	39		O50	42	128	161	
	P30	O23	0	32	53	PG6	O51	56	114	194	
		O36	0	56	61		O61	42	114	14	
O38		0	32	12	O26	0	42	53			
P5	O7	32	128	53	O27	18	80	54			
P7	O42	18	90	77	P11	O34	0	56	51		
	O57	18	104	78		O35	18	56	66		
P8	O19	8	56	83		O66	32	104	76		
	O53	32	104	28	P25	O24	18	104	155		
P9	O15	8	90	43		O63	8	90	43		
	O55	56	128	85	P26	O39	18	80	43		
P13	O1	42	128	94		O40	0	42	53		
P15	O2	0	32	52	PG7	P2	O12	0	32	20	
	O58	18	114	88		P23	O18	0	56	66	
P28	O43	80	128	77		O32	66	128	53		
	O56	0	56	42		P24	O16	8	66	105	
PG3	P6	O8	18	104			53	O62	18	80	25

The deodoriser can process a maximum batch size of 7.5 ton of products, with a fixed processing time of 15 min (0.25 hr). The processing time for each product should be fixed to values that are multiples of 0.25 hr, i.e., 0.25 multiplied by the number of batches (hrs). For each batch, the minimum batch size is 3.75 ton, half of the maximum batch size (7.5 ton). The down-time is 15 min (0.25 hr) for emptying or washing trays when switching from one product group to another (Table 3.2),

while the changeover cost is 10 k\$ for each changeover. The price of each product is 1 k\$/ton. The unit processing cost is 0.2 k\$/ton and the unit inventory and backlog costs are 0.1 k\$/ton.

Table 3.2 Changeover matrix.

Groups	PG1	PG2	PG3	PG4	PG5	PG6	PG7
PG1		√ ^a	√	√	√	√	√
PG2			√	√		√	√
PG3					√	√	√
PG4							√
PG5							√
PG6							√
PG7		√	√	√	√	√	

^a Changeovers are allowed to occur between the pair of the product groups.

All the runs in this section are done in Windows XP environment on an Intel Core Duo 3.40 GHz, 3.44 GB RAM machine. The optimality gap is set to 2.0%.

3.4.2 Computational Results of Model DEO-S1

Model DEO-S1, with 2,829 equations, 2,255 continuous variables and 766 binary/integer variables, is solved in 20 s. The obtained objective function value is 3,016.0, whose optimality gap is 1.8%. The breakdown of the optimal profit is given in Table 3.3.

Table 3.3 Breakdown of the optimal profit of model DEO-S1 (k\$).

Profit	3,016.0
Sales revenue	3,807.5
Processing cost	761.5
Inventory cost	0.0
Changeover cost	30.0

The Gantt chart of the optimal schedule obtained from model DEO-S1 is given in Fig. 3.4, which shows that there are total 3 changeovers in the planning horizon. Colors indicate the different product groups and each bar contains one or more products. Note that each batch production may satisfy multiple orders. The production levels of products and orders are given in Figs. 3.5 and 3.6, respectively. For each product/order, the cumulative production is given, as well as the demand.

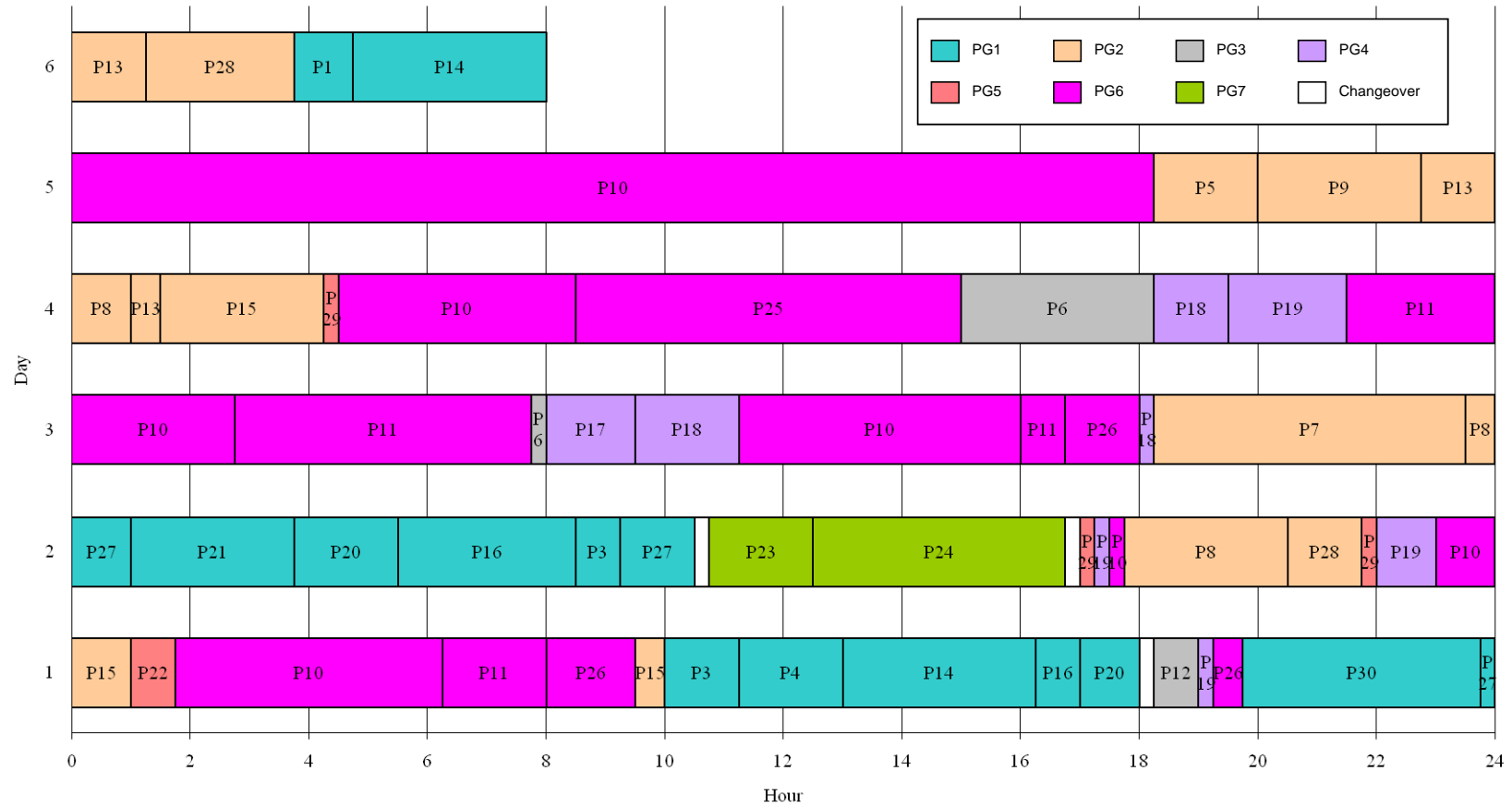


Figure 3.4 Gantt chart of the optimal schedule by model DEO-S1.

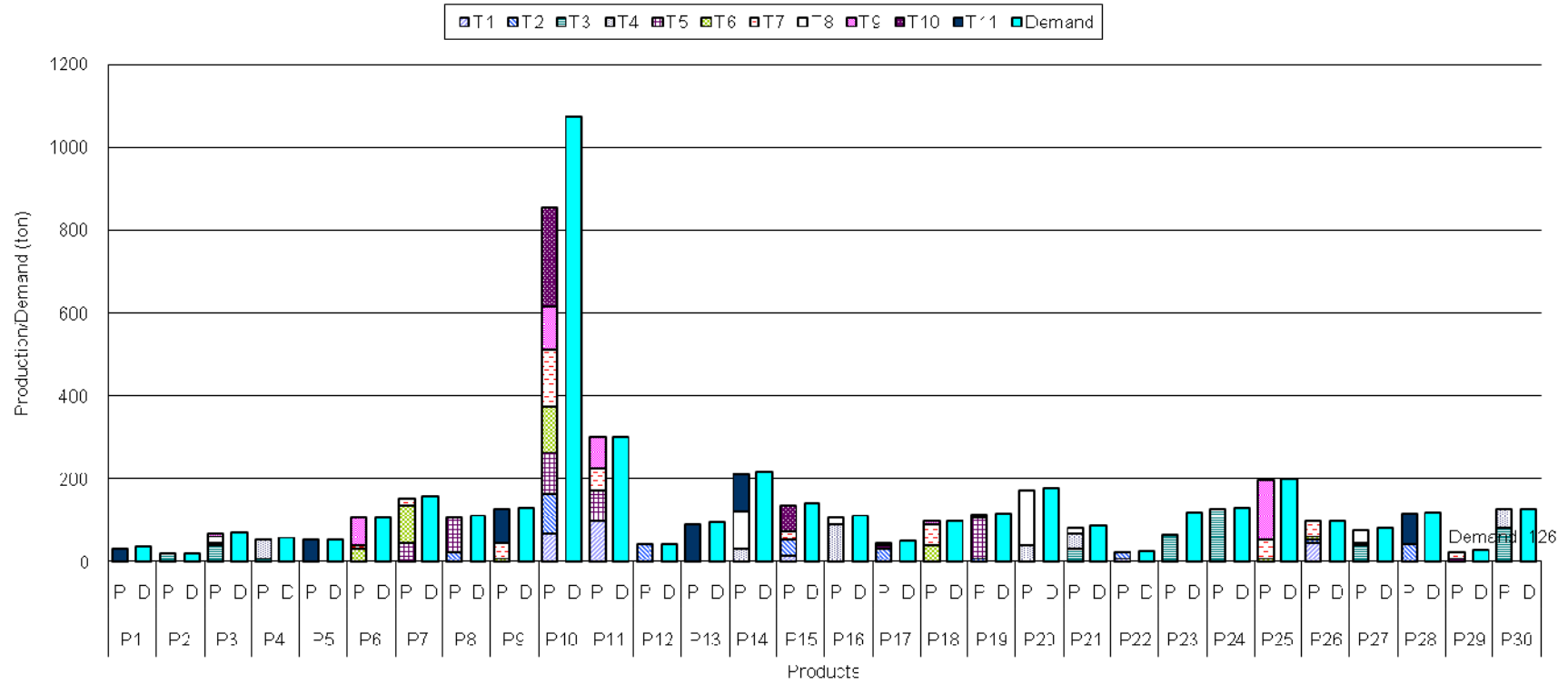


Figure 3.5 Demand and production levels of products by model DEO-S1.

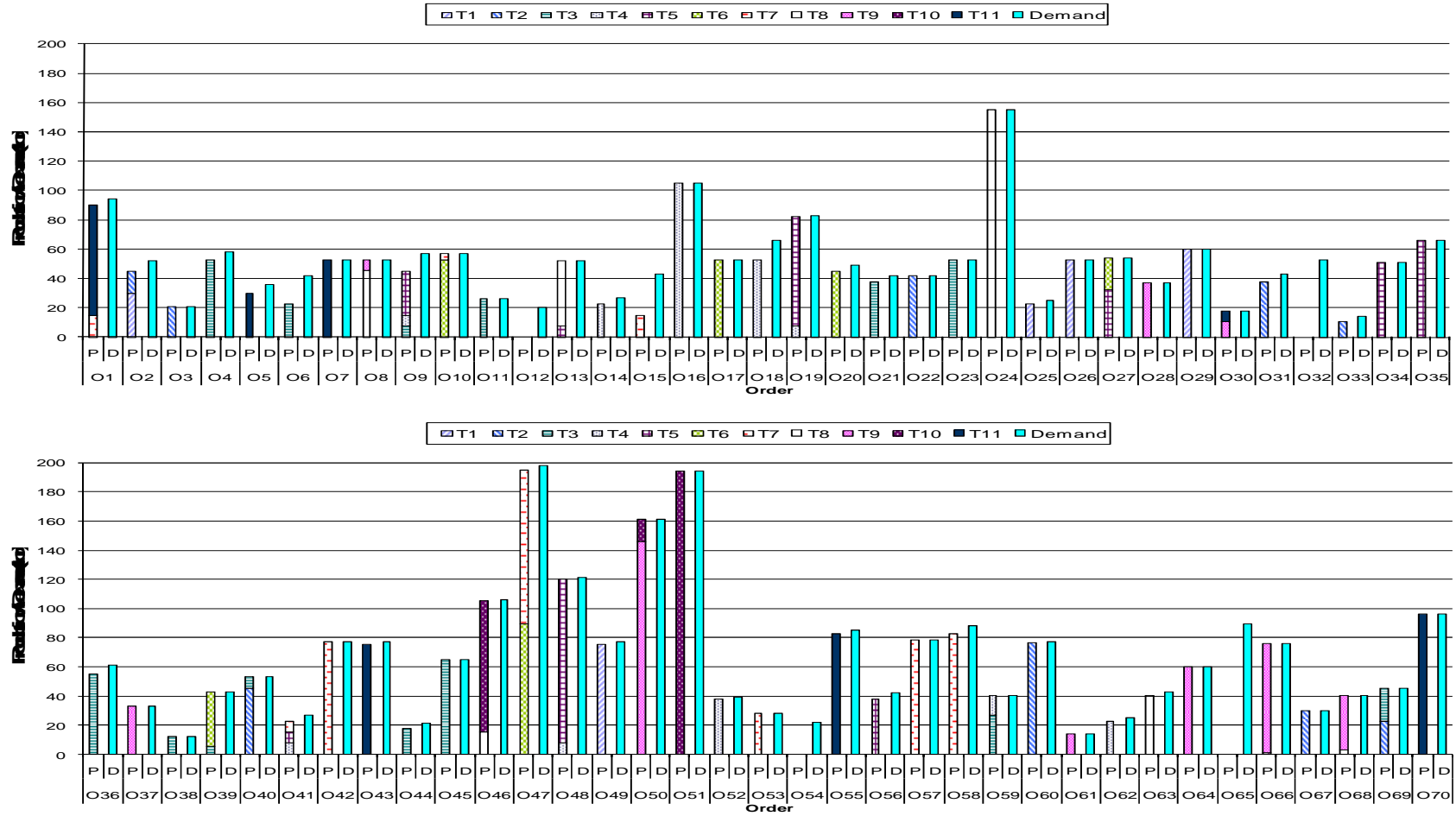


Figure 3.6 Demand and production levels of orders by model DEO-S1.

The above figures show not only how the demands are satisfied, but also the production time periods and amounts for each product/order. As there is no inventory in the optimal solution, which means that products are processed and delivered in the same week of the processing, the sale of each order at each time period can also be seen in Fig. 3.6.

In the optimal solution, out of 70 orders, 66 orders (94.3%) are either fully or partially satisfied, in which a total of 34 orders are fully satisfied by their due dates (numbers in bold in Table 3.4). Most of the partially satisfied orders (59.4%) have service levels above 90%. There are only 4 orders (5.7%) that are not satisfied at all. There is a total sale of 3,807.5 ton, and the aggregated service level is 91.6%.

Table 3.4 Demands, sales and service levels by model DEO-S1.

Order	Sales (ton)	Demand (ton)	Service level (%)	Order	Sales (ton)	Demand (ton)	Service level (%)
O1	90	94	95.7	O36	55	61	90.2
O2	45	52	86.5	O37	33	33	100
O3	21	21	100	O38	12	12	100
O4	52.5	58	90.5	O39	43	43	100
O5	30	36	83.3	O40	53	53	100
O6	22.5	42	53.6	O41	22.5	27	83.3
O7	52.5	53	99.1	O42	77	77	100
O8	53	53	100	O43	75	77	97.4
O9	45	57	78.9	O44	17.5	21	83.3
O10	57	57	100	O45	65	65	100
O11	26	26	100	O46	105.5	106	99.5
O12	0	20	0	O47	194.5	198	98.2
O13	52	52	100	O48	120	121	99.2
O14	22.5	27	83.3	O49	75	77	97.4
O15	15	43	34.9	O50	161	161	100
O16	105	105	100	O51	194	194	100
O17	53	53	100	O52	37.5	39	96.2
O18	52.5	66	79.5	O53	28	28	100
O19	82.5	83	99.4	O54	0	22	0
O20	45	49	91.8	O55	82.5	85	97.1
O21	37.5	42	89.3	O56	37.5	42	89.3
O22	42	42	100	O57	78	78	100
O23	53	53	100	O58	82.5	88	93.8
O24	155	155	100	O59	40	40	100
O25	22.5	25	90.0	O60	76.5	77	99.4
O26	52.5	53	99.1	O61	14	14	100
O27	54	54	100	O62	22.5	25	90.0
O28	37	37	100	O63	40	43	93.0
O29	60	60	100	O64	60	60	100
O30	18	18	100	O65	0	89	0
O31	37.5	43	87.2	O66	76	76	100
O32	0	53	0	O67	30	30	100
O33	10.5	14	75.0	O68	40	40	100
O34	51	51	100	O69	45	45	100
O35	66	66	100	O70	96	96	100

3.4.3 Computational Results of Model DEO-S2

Model DEO-S2, with 3,515 equations, 3,268 continuous variables and 922 binary/integer variables, is solved in 1,075 s. The obtained optimal objective is 2,959.1, whose optimality gap is 2.0%. The breakdown of the optimal profit is given in Table 3.5:

Table 3.5 Breakdown of the optimal profit by model DEO-S2 (k\$).

Profit	2,959.1
Sales revenue	3,803.5
Processing cost	760.7
Inventory cost	0.0
Backlog cost	53.7
Changeover cost	30.0

Although there is no inventory cost in the optimal solution of this case, inventory cost may occur for the cases with higher minimum batch sizes. The Gantt chart of the optimal schedule obtained from model DEO-S2 is given in Fig. 3.7. As the same as the optimal solution of scenario 1, 3 changeovers occur in the scenario 2 as well. The productions of each product/order in each time period are shown in Figs. 3.8 and 3.9. Similar to DEO-S1, Fig. 3.9 also provides the information about the sales at each time period.

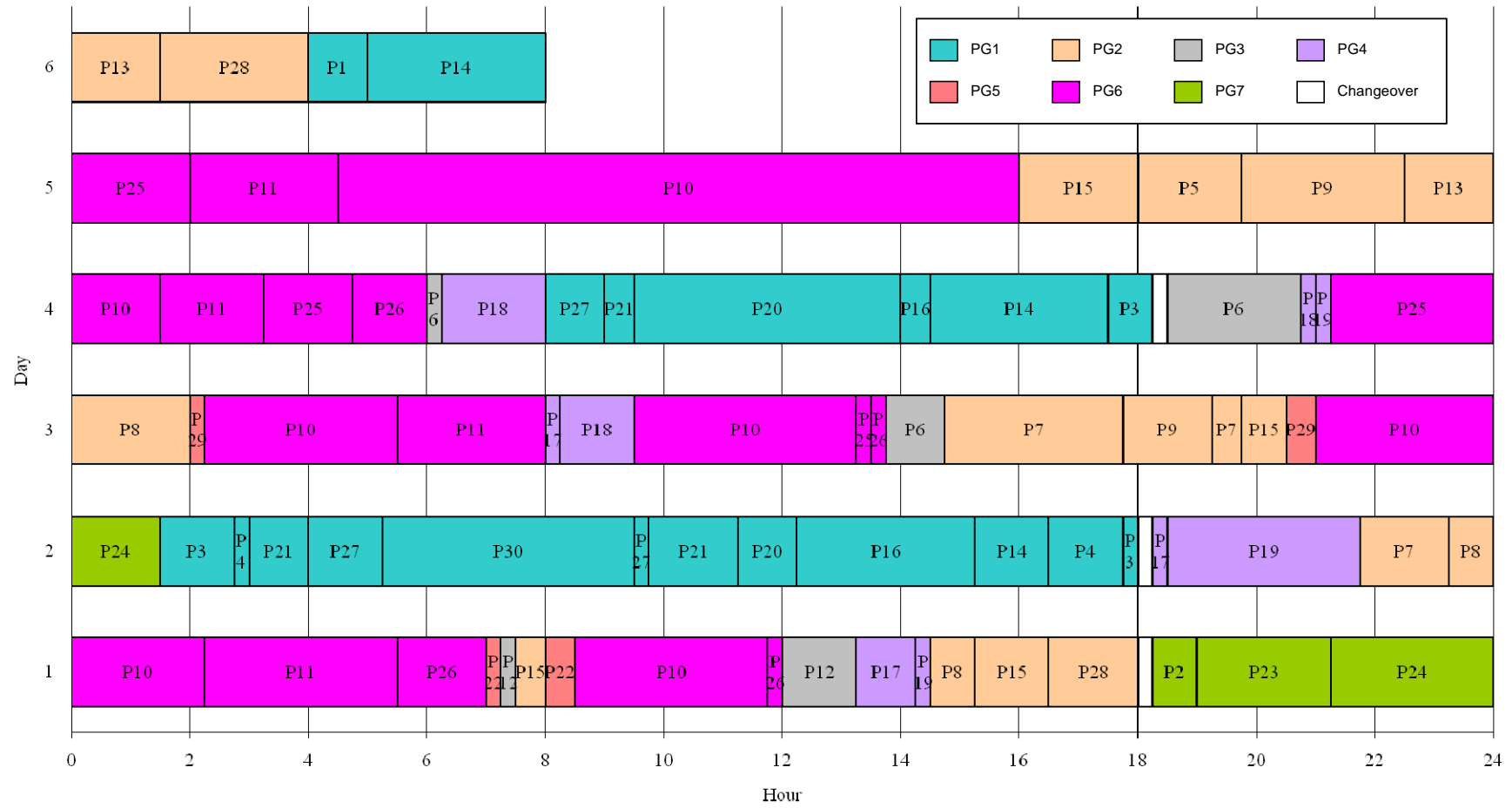


Figure 3.7 Gantt chart of the optimal schedule by model DEO-S2.

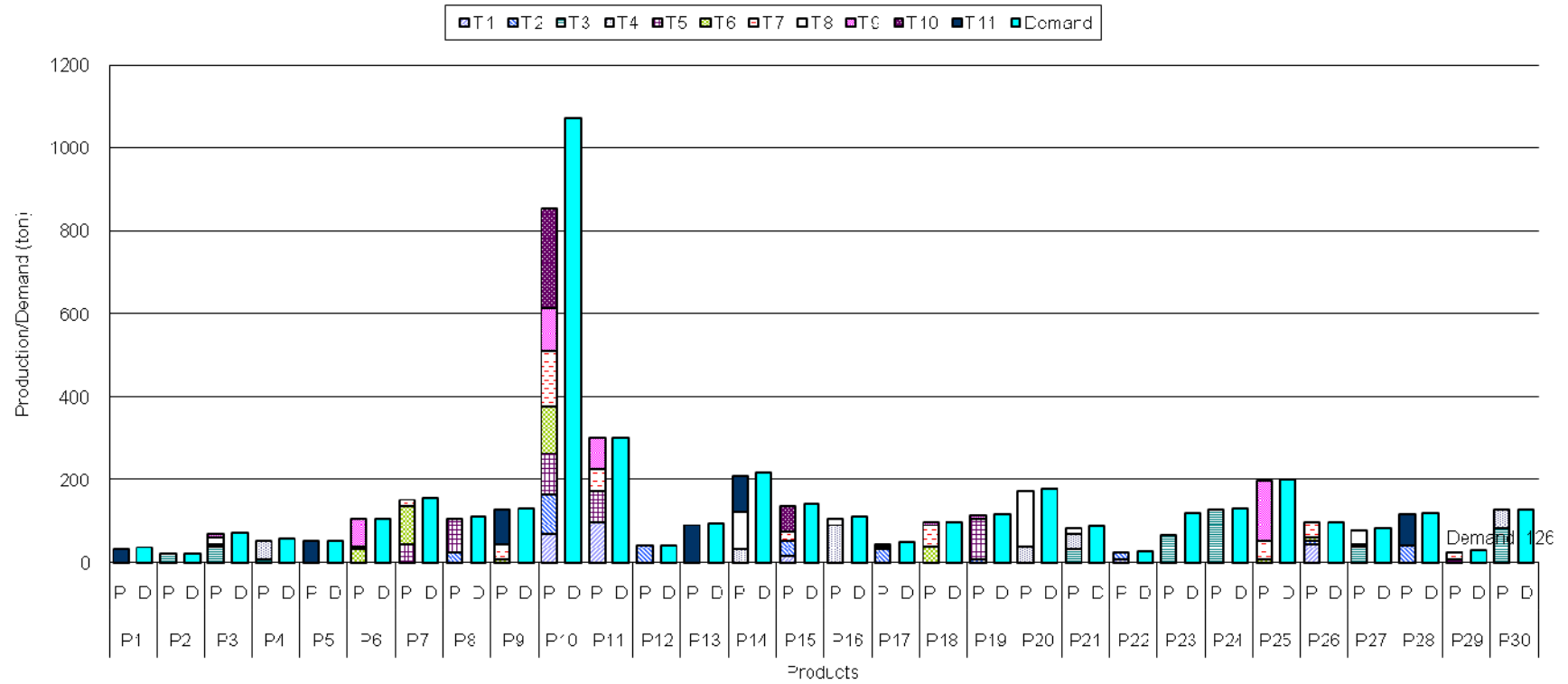


Figure 3.8 Demand and production levels of products by model DEO-S2.

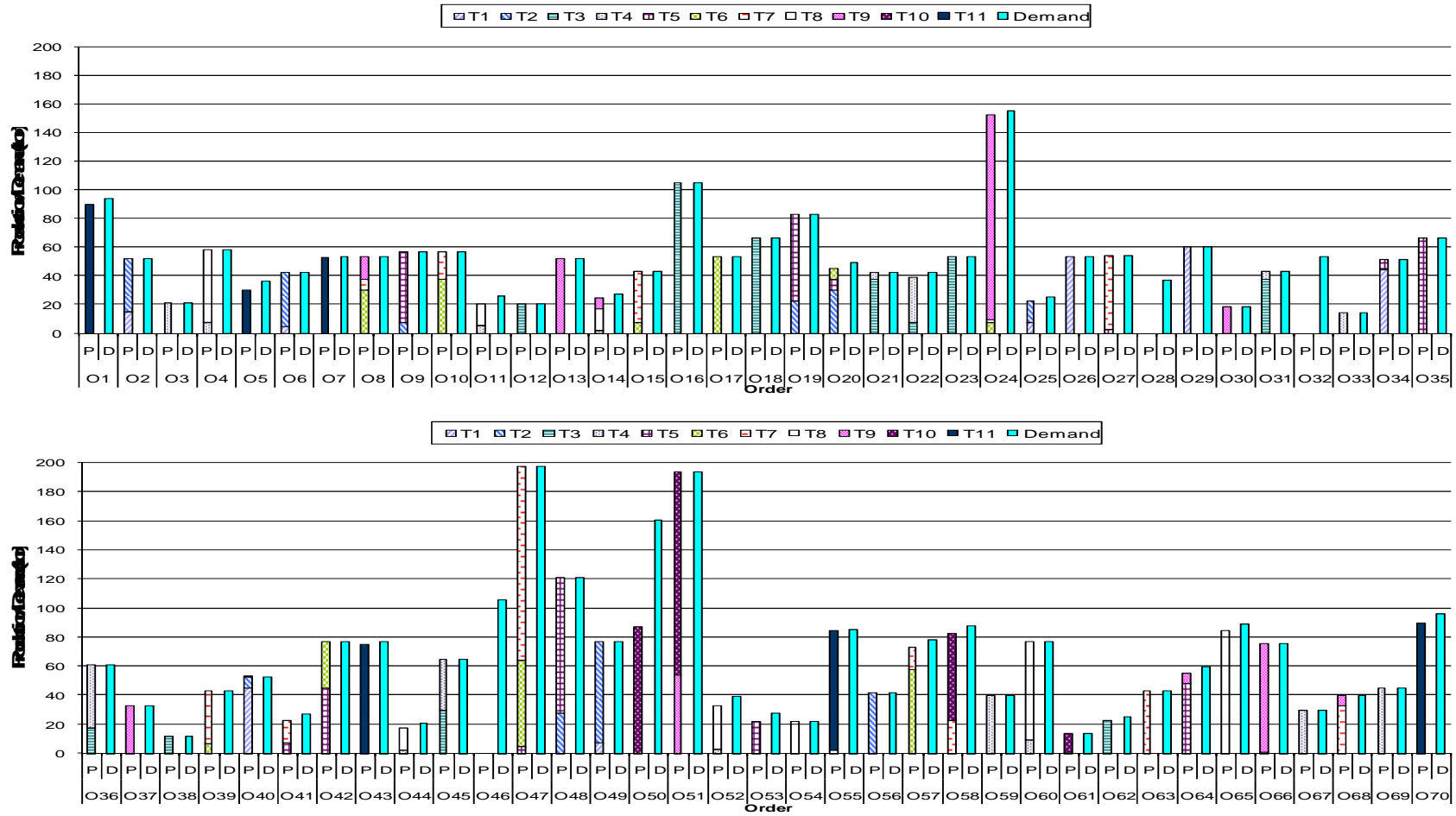


Figure 3.9 Demand and production levels of orders by model DEO-S2.

Out of the total 70 orders, 67 orders (95.7%) are fully or partially satisfied, in which 45 orders are fully satisfied. It should be mentioned that in the 45 fully satisfied orders, 42 orders are fully satisfied at their due dates and 3 orders are satisfied at later dates. The total sale is 3,803.5 ton, and the aggregated service level is 91.5%. The service level of each order is given in Table 3.6, in which even the partially satisfied orders have high service levels.

Table 3.6 Demands, sales and service levels by model DEO-S2.

Order	Sale (ton)	Demand (ton)	Service level (%)	Order	Sale (ton)	Demand (ton)	Service level (%)
O1	90	94	95.7	O36	61	61	100
O2	52	52	100 ^a	O37	33	33	100
O3	21	21	100	O38	12	12	100
O4	58	58	100	O39	43	43	100
O5	30	36	83.3	O40	53	53	100
O6	42	42	100	O41	22.5	27	83.3
O7	52.5	53	99.1	O42	77	77	100
O8	53	53	100	O43	75	77	97.4
O9	57	57	100	O44	17.5	21	83.3
O10	57	57	100	O45	65	65	100
O11	20	26	76.9	O46	0	106	0
O12	20	20	100	O47	198	198	100
O13	52	52	100	O48	121	121	100
O14	24.5	27	90.7	O49	77	77	100
O15	43	43	100	O50	87	161	54.0
O16	105	105	100	O51	194	194	100
O17	53	53	100	O52	33	39	84.6
O18	66	66	100	O53	22	28	78.6
O19	83	83	100	O54	22	22	100
O20	45	49	91.8	O55	84.5	85	99.4
O21	42	42	100	O56	42	42	100
O22	38.5	42	91.7	O57	73	78	93.6
O23	53	53	100	O58	82.5	88	93.8
O24	152	155	98.1	O59	40	40	100
O25	22.5	25	90.0	O60	77	77	100
O26	53	53	100	O61	14	14	100
O27	54	54	100	O62	22.5	25	90.0
O28	0	37	0	O63	43	43	100
O29	60	60	100	O64	55.5	60	92.5
O30	18	18	100	O65	84.5	89	94.9
O31	43	43	100	O66	76	76	100
O32	0	53	0	O67	30	30	100
O33	14	14	100	O68	40	40	100
O34	51	51	100	O69	45	45	100
O35	66	66	100	O70	90	96	93.8

^aThe bold numbers indicate that the corresponding orders are fulfilled by their due dates.

The backlog of each order at the end of each time period is given in Table 3.7. In each line, the first column with reported backlog level is the due date of the corresponding order. The decrease of the backlog level means that the corresponding

order is being partially or fully satisfied. From Table 3.7, there are 3 orders (O21, O31 and O40) that are not satisfied by their due dates, but later by the end of the planning horizon. There are 25 orders with backlogs at the end of planning horizon, and the total backlog amount is 352.5 ton.

Table 3.7 Backlog levels by model DEO-S2 (ton).

	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11
O1											4
O5											6
O7											0.5
O11										6	6
O14										2.5	2.5
O20						4	4	4	4	4	4
O21			4.5	0	0	0	0	0	0	0	0
O22				3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5
O24									3	3	3
O25		2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
O28											37
O31			5.5	0	0	0	0	0	0	0	0
O32											53
O40				0.5	0.5	0	0	0	0	0	0
O41							4.5	4.5	4.5	4.5	4.5
O43											2
O44									3.5	3.5	3.5
O46											106
O50											74
O52									6	6	6
O53									6	6	6
O55											0.5
O57									5	5	5
O58										5.5	5.5
O62							2.5	2.5	2.5	2.5	2.5
O64									4.5	4.5	4.5
O65									4.5	4.5	4.5
O70											6
Total	0	2.5	12.5	6.5	6.5	10	17	17	49.5	63.5	352.5

3.5 Comparison with Other Approaches

In this section, the efficiency and effectiveness of the proposed models are examined by comparing with a heuristic approach and a literature model. We use these approaches to solve the scenario 1 of the case study given in Section 3.4 for comparison.

3.5.1 Comparison with a Heuristic Approach

As no backlog is allowed in scenario 1, in the heuristics the demands of the orders are assumed to be equally divided into time periods within their time windows. For each time period, the groups with demands are selected and their sequence is solved by a TSP model, which is described by Eqs. (3.1)–(3.10), with the following Eq. (3.24) as the objective to minimise the total changeover cost:

$$\sum_{t \in T} \sum_{g \in \tilde{G}_t} \sum_{g' \in \tilde{G}_t} CC_{gg'} \cdot Z_{gg't} + \sum_{t \in T-1} \sum_{g \in \tilde{G}_{t-1}} \sum_{g' \in \tilde{G}_t} CC_{gg'} \cdot ZF_{gg't} \quad (3.24)$$

Then the production capacity of each time period can be obtained by subtracting the changeover times. In each time period, the product with the highest demand is selected for each participating group. Then to satisfy the production capacity, the productions of the selected products are reduced, or another product is selected, in order of highest to lowest demand. Overall, there is at least one product for each participating group in each time period and the production is restricted by the capacity. The details of the algorithm description are as follows:

STEP 0. The total demand of order o is distributed equally into the time periods in

its time window, i.e., $\bar{D}_{ot} = \frac{D_o}{DT_o - RT_o + 1}$, the demand of product i in each

time period is $\bar{D}_{it} = \sum_{o \in O_i} \bar{D}_{ot}$, the demand of group g in each time period is

$$\bar{D}_{gt} = \sum_{i \in I_g} \bar{D}_{it}. \text{ The required batch number of } \bar{D}_{gt} \text{ is } \bar{N}_{gt} = \left\lceil \frac{\bar{D}_{gt}}{BS^{\max}} \right\rceil;$$

STEP 1. Initialise the set of selected groups $SG_t = \{g: \max_{i \in I_g} \bar{D}_{it} \geq BS^{\min}\}$;

STEP 2. Solve the TSP model with fixed groups SG_t in each time period to minimise the total changeovers; then fix the values of $Z_{gg't}$ and $ZF_{gg't}$ given in the solution, and the maximum available batch number for time period t , $AN_t = [\theta_t^U - (\sum_{g \in SG_t} \sum_{g' \in SG_t} Z_{gg't} + \sum_{g \in SG_{t-1}} \sum_{g' \in SG_t} ZF_{gg't}) \cdot \tau_{gg'}] / BT$;

STEP 3. Initialise the set of selected products $SI_t = \{i: i = \arg \max_{i \in I_g, g \in SG_t} \bar{D}_{it}\}$, the set of candidate/non-selected products $CI_t = \{i: \bar{D}_{it} \geq BS^{\min}\} - SI_t$. Initialise the

production amount and batch number of product i , $P_{it} = \bar{D}_{it}$, $N_{it} = \bar{N}_{it}$ for $i \in SI_t$, and $P_{it} = N_{it} = 0$ for $i \in CI_t$. Initialise $t = 0$;

STEP 4. If $t = T$, STOP. If $t < T$, $t = t + 1$, go to STEP 5;

STEP 5. The required total batch number of the selected products $RN_t = \sum_{i \in SI_t} N_{it}$. If $RN_t > AN_t$, go to STEP 6; if $RN_t < AN_t$, go to STEP 7; otherwise, go to STEP 4;

STEP 6. Let $\bar{i} = \arg \max_{i \in SI_t} \bar{D}_{it}$, $N_{\bar{i}t} = N_{\bar{i}t} - 1$, $P_{\bar{i}t} = N_{\bar{i}t} \cdot BS$, then go to STEP 5;

STEP 7. Let $\bar{i} = \arg \max_{i \in CI_t} \bar{D}_{it}$, update sets $SI_t = SI_t \cup \{\bar{i}\}$, $CI_t = CI_t - \{\bar{i}\}$. $N_{\bar{i}t} = \min(\bar{N}_{\bar{i}t}, AN_t - RN_t)$, $P_{\bar{i}t} = N_{\bar{i}t} \cdot BS$. If $CI_t = \emptyset$, go to STEP 4; otherwise, go to STEP 5.

It should be noted that in the case studied in Section 3.4, the minimum batch size BS^{\min} is half of the maximum batch size BS^{\max} . A full batch and a small batch whose size is less than BS^{\min} can be reallocated to two batches that both are greater than BS^{\min} . So in the initial set of candidate products CI_t , the products whose weekly demands are less than BS^{\min} are excluded, and all products in the set can fulfill the requirement of the minimum batch size.

In Table 3.8, the profit, revenue and costs of the optimal solution from MILP model DEO-S1 are compared to the corresponding values obtained from the heuristic approach. The objective of model DEO-S1 is 30% higher than that of the heuristic approach, resulting from 25.5% higher production and service level of model DEO-S1. Meanwhile, 12 changeovers from the heuristic approach incurs more changeover cost more than that of the 3 changeovers from model DEO-S1,. Although the computational time of the heuristic approach is slightly less, the proposed model DEO-S1 can obtain a much better solution, and has a better performance than the heuristic approach.

Table 3.8 Comparison between MILP model and heuristic approach.

	MILP model DEO-S1	Heuristic approach
Profit (k\$)	3,016.0	2,306.8
Sales revenue (k\$)	3,807.5	3,033.5
Processing cost (k\$)	761.5	606.7
Inventory cost (k\$)	0.0	0.0
Changeover cost (k\$)	30.0	120.0
Service level (%)	91.6	73.0
CPU (s)	20.0	17.5

3.5.2 Comparison with a Literature Model

Kelly and Zyngier (2007) proposed an MILP model (K&Z for short) to represent the sequence-dependent changeovers for uniform discrete-time scheduling problems, and applied it to both batch- and continuous-process units. In the third illustrative example presented in their paper, a case study of an edible-oil deodoriser is considered. Their case study only considered a planning horizon of 3 days and total 45 orders.

As there are only sequencing constraints presented in their paper, we add our proposed objective function and constraints for production, inventory and sales to the literature model for comparison. The details of model K&Z and added constraints are presented in Appendix D.

As the batch time and changeover time in the case study are 15 min, the length of each discrete slot used for the case study is 15 min, and there are a total of 512 slots used in the model for this case study. The modified literature model is implemented under the same computational environment and same termination criteria as given in Section 3.4.

The model sizes of models DEO-S1 and K&Z are shown in Table 3.9, from which we can see that the proposed model has a much smaller model size than the model K&Z.

Table 3.9 model sizes of the proposed model and literature model.

	Proposed model DEO-S1	K&Z
No. of equations	2,829	775,749
No. of continuous variables	2,255	540,369
No. of binary variables	766	15,360

In Table 3.10, the profit, revenue and costs of the optimal solution from model DEO-S1 are compared to the respective values obtained from model K&Z. The literature model is terminated by the CPU limit, and takes 3,604 s to find a solution with an objective value of 2,321.6. On the other hand, the proposed model finds a solution of 3,016 in only 20 s. The service level obtained from model K&Z is only 69.8%, compared with 91.6% from the proposed model. From the comparison results, it is obvious that the proposed model has a significantly better computational performance.

Table 3.10 Comparison between the proposed model and literature model.

	Proposed model DEO-S1	K&Z
Profit (k\$)	3,016.0	2,321.6
Sales revenue (k\$)	3,807.5	2,902.0
Processing cost (k\$)	761.5	580.4
Inventory cost (k\$)	0.0	0.0
Changeover cost (k\$)	30.0	0.0
Service level (%)	91.6	69.8
Optimality gap (%)	1.8	32.3
CPU (s)	20.0	3,603.9

3.6 Concluding Remarks

In this chapter, the short-term scheduling problem of a single-stage batch edible-oil deodoriser has been investigated. TSP-based MILP models have been developed for two scenarios: without and with backlog. The novelty of the proposed models extended from the work in Chapter 2 is that the processing sequence of the product groups is considered instead of that of the products. Meanwhile, the orders with release times and due dates are tackled in this problem. The proposed models have been successfully applied to the deodoriser scheduling problem with 70 orders. At

last, the effectiveness of the models is shown by comparing with a heuristic approach and a discrete time literature model (Kelly and Zyngier, 2007). The proposed optimisation framework for the scheduling problem exhibits efficient computational performance.

The TSP-based formulations used in Chapters 1 and 2 for the production planning and scheduling will be adapted in the next chapter to deal with the supply chain production planning problem in multiple production sites.

AN MPC APPROACH FOR SUPPLY CHAIN PLANNING UNDER UNCERTAINTY

Due to the dynamic characteristics of the supply chains, control theory has been widely used in the SCM to facilitate the design, optimisation and simulation of the supply chain networks (Morari and Lee, 1999; Ortega and Lin, 2004; Choi et al., 2006). MPC, also referred to as model based predictive control, receding horizon control or moving horizon optimal control (Bemporad and Morari, 1999), is the most commonly used advanced control technique in the process industry for over 30 years. (Muske and Rawlings, 1993; Henson, 1998; Pannocchia and Rawlings, 2003; Nagy and Braatz, 2003; Liu et al., 2009).

In this chapter, we aim to develop an MILP-based MPC approach for a supply chain planning problem considering both inventory deviations and pricing fluctuations, adapting the TSP-based formulations introduced in the previous chapters.

4.1 Introduction and Literature Review

MPC has been largely investigated in the literature and successfully applied to supply chains during the past decade. See the detailed review in Sarimveis et al. (2008).

Bose and Pekny (2000) presented a model predictive approach to capture the supply chain dynamics under uncertainty. A forecasting-optimisation-simulation framework is proposed to integrate forecasting, optimisation and simulation modules. Perea-López et al. (2003) proposed a dynamic MILP model for a multiproduct, multiechelon global supply chain for profit maximisation which was implemented with an MPC strategy. The centralised and decentralised management approaches

were compared and the advantage of the former was shown. This work is acknowledged as the only work on the supply chain planning which has considered sequence-dependent changeovers in MPC approach. However, the changeover times are neglected, while only changeover costs are considered. Here, the formulations in Chapters 2 and 3 are adapted to model the sequence-dependent changeovers in the production sites. Moreover, in their MPC approach, only the economic performance of the supply chain is optimised in MPC, while in this work, the inventory and price are considered in the optimisation problem of MPC as well.

Seferlis and Giannelos (2004) developed a two-layer optimisation-based control approach for multiproduct, multi-echelon supply chains. The optimisation-based controller is proposed for customer satisfaction maximisation with the least operating costs under both deterministic and stochastic demand variations. Mestan et al. (2006) modelled the multiproduct supply chains using the mixed logical dynamical (MLD) system. The overall profit was optimised within three MPC configurations: centralised, fully decentralised, and semi-decentralised. Lei et al. (2006) described a MPC-based simulation method for the optimal profit in multiproduct, multi-echelon dairy supply chains. Comparisons were made between the MPC strategy and static optimisation, and between the centralised and decentralised management approaches. Wang et al. (2007) addressed the application of MPC to three benchmark SCM problems in semiconductor manufacturing, including the basic problem with backlog, the problem with stochastic manufacturing splits, and the multiproduct problem with shared capacity. The effects of tuning, model parameters, and capacity were investigated as well.

Doganis et al. (2008) incorporated a neural network time series forecasting model into the MPC strategy and proposed a complete SCM framework for production-inventory systems. Among all the investigated linear and nonlinear forecasting methodologies, the forecasting model used was the most accurate. The corresponding MPC configuration was proven to be the most efficient for the inventory control problem. Aggelogiannaki et al. (2008) proposed an adaptive MPC configuration for production - inventory systems to determine the optimal order volume at each discrete time, in which the inventory levels were predicted by the adapted model

along with a smoothed estimation of the future customer demand. Compared with non-adapted approaches, their proposed approach was proven to be superior.

Puigjaner and Laínez (2008) proposed an MILP model to incorporate financial considerations into the supply chain design and planning. The MPC strategy and a scenario based multi-stage stochastic MILP model were integrated with the expected corporate value as the objective. Later, this work was extended by Puigjaner et al. (2009) to integrate a design-planning model and a scheduling formulation. A Lagrangean decomposition was used to reduce the computational complexity. A robust MPC approach was presented by Li and Martin (2009) for the optimal closed-loop economic performance of supply chains. In the approach, a closed-loop model was used for prediction and a controller model was used by a constrained bi-level stochastic optimisation problem. An interior point method was used to solve a number of deterministic conic optimisation problems, which were transformed from the stochastic optimisation problem. Yüzgeç et al. (2010) proposed an MPC strategy to determine the optimal control decisions for the short-term refinery scheduling problem to minimise the total operating cost. Three case studies were investigated and the proposed strategy exhibited a good performance for all examples.

The pricing strategy is a very important issue to the supply chain, especially when the price elasticity of demand is high, i.e., the price has a significant effect on the product demands. Thus, how to make the correct pricing decisions is crucial in SCM. Some literature work has been done to investigate the supply chains with the price elasticity of demand (Viswanathan and Wang, 2003; Seferlis and Pechlivanos, 2004, 2006; Wang et al., 2004; Ray et al., 2005; Levis and Papageorgiou, 2007; Hsieh et al., 2010; Kaplan et al., 2010). As one of the main reasons for the bullwhip effect in the supply chains, (Lee et al., 1997; Simchi-Levi et al., 2003; Özelkan and Çakanyıldırım, 2009) price fluctuations also need to be considered when making the pricing strategy, but was ignored in the literature work (Seferlis and Pechlivanos, 2004, 2006).

The purpose of the work in this chapter is to incorporate the pricing strategies for products with price elasticity of demand into the MPC approach for the production, distribution planning and inventory control of a multi-echelon multiproduct supply chain with sequence-dependent changeovers under demand uncertainty.

4.2 Problem Statement

In this problem, we consider a supply chain network with three echelons, including plants, distribution centres (DCs) and markets (Fig. 4.1). The whole planning horizon of the problem is divided into multiple time periods. In the plants, multiple products are produced with the occurrences of sequence-dependent changeovers. The processed final products are shipped to several DCs. Then the final products are transported from DCs to the markets for sales. It is assumed that all the deliveries are done at the end of each time period. When the sale amount of a product is less than its actual demand, the unmet demand is lost. The costs of production, transportation, changeovers and lost sales occur during the above processes.

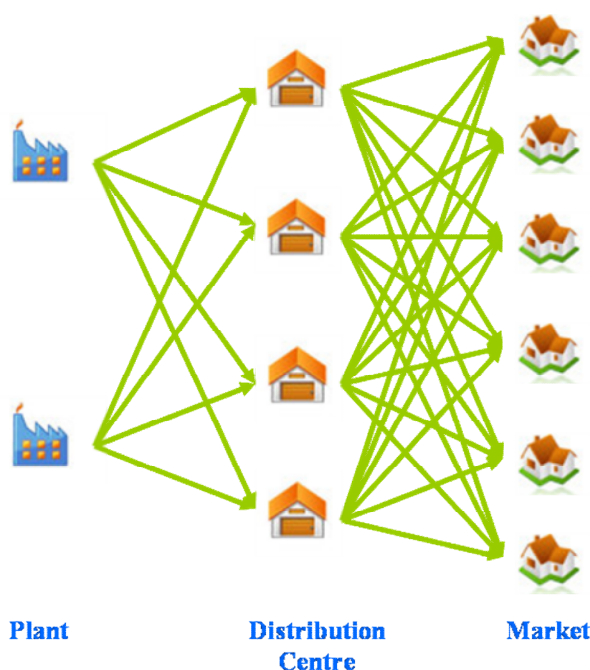


Figure 4.1 The structure of the supply chain network.

The demands of each product in each market are affected by the product's prices in the market by the price elasticity of demand. For each product, there is an initial demand in a time period corresponding to the product's initial price at each market. In each time period, there are several price levels to be selected for each product at one market. If the selected price is higher than the initial price, the actual final demand will become lower than the initial demand; while if a lower price level is selected, the actual final demand will be higher than the initial demand. The demand change rate is determined by the price elasticity coefficient. In this problem, the uncertainty comes from the initial demands, which are assumed to follow a uniform

distribution to allow higher probability of occurrence of demand in extreme condition. Before the initial demands are realised, the forecasts of initial demands can be predicted. The initial demands are realised at the beginning of each time period. When the pricing decisions are made, actual final demands can be known accordingly. In order to maintain a stable price level to avoid large price fluctuations at the markets, the price changes are considered as well.

Each final product is stored at all suitable sites including plants, DCs and markets. There is a reference inventory trajectory for each product at each site. The inventory trajectory is determined to avoid the risk of the occurrence of lost sales facing the uncertain demands. The aim of the inventory control is to control the inventory to be as close to the inventory trajectory as possible, i.e. to keep the inventory deviation from the inventory trajectory as small as possible. In this case, the inventory cost is not included in the total cost. Otherwise, the profit maximisation, which results in inventory cost minimisation, will conflict with the inventory control.

In the supply chain production, distribution and inventory planning problem, the following are given:

- plants, DCs and markets and their suitabilities and connections between them;
- unit production costs and changeover costs and times;
- unit transportation costs;
- unit inventory costs and inventory trajectories;
- unit lost sales costs;
- initial forecast demands;
- available product price levels and price elasticity coefficients;
- minimum and maximum inventories;

to determine

- production times, amounts and sequences;
- transportation flows;
- inventory levels and inventory deviations;
- sales and lost sales amounts;
- product prices and price changes;

so as to maximise the total profit with the maintenance of the inventory levels and price levels.

4.3 Mathematical Formulation

The supply chain planning problem is formulated as an MILP problem, the details of which are described below in this section.

4.3.1 Nomenclature

Indices

c	distribution centre (DC)
i, j	product
k	price level
m	market
s	plant
t	time period
t^*	the current time period in the control horizon

Sets

C	set of DCs
C_i	set of DCs that can store product i
C_m	set of DCs connected to market m
C_s	set of DCs connected to plant s
I	set of products
I_c	set of products that can be stored in DC c
I_m	set of products are demanded in market m ,
I_s	set of products that can be processed in plant s
K	set of available price levels
M	set of markets
M_c	set of markets connected to DC c
M_i	set of markets that demand product i
S	set of plants
S_c	set of plants connected to DC c
S_i	set of plants that can process product i
T	set of time periods
T^C	set of time periods in the control horizon

Parameters

CC_{ijs}	unit changeover cost from product i to j at plant s
CLS_{im}	unit lost sales cost of product i at market m
CP_{is}	unit production cost of product i at plant s
CT_{icm}^{CM}	unit transportation cost of product i from DC c to market m
CT_{isc}^{SC}	unit transportation cost of product i from plant s to DC c
$IniD_{imt}$	initial demand of product i at market m in time period t
$IniD_{imt}^F$	initial forecast demand of product i at market m in time period t
$IniINV_{ic}^C$	initial inventory of product i at DC c
$IniINV_{im}^M$	initial inventory of product i at market m
$IniINV_{is}^S$	initial inventory of product i at plant s
$IniPr_{im}$	initial price of product i at market m
$INVT_{ict}^C$	inventory trajectory of product i at DC c in time period t
$INVT_{imt}^M$	inventory trajectory of product i at market m in time period t
$INVT_{ist}^S$	inventory trajectory of product i at plant s in time period t
L^{CH}	length of the control horizon
$MaxINV_{ic}^C$	maximum inventory capacity of product i at DC c
$MaxINV_{im}^M$	maximum inventory capacity of product i at market m
$MaxINV_{is}^S$	maximum inventory capacity of product i at plant s
$MinINV_{ic}^C$	minimum inventory capacity of product i at DC c
$MinINV_{im}^M$	minimum inventory capacity of product i at market m
$MinINV_{is}^S$	minimum inventory capacity of product i at plant s
N	a large number
\overline{Pr}_{imk}	price at level k of product i at market m
PE_{im}	price elasticity coefficient of product i at market m
r_{is}	processing rate of product i in plant s
w^C	control weight for inventory deviation at DCs

w^M	control weight for inventory deviation at markets
w^P	control weight for price change
w^S	control weight for inventory deviation at plants
α_{imt}	forecast error of initial demand of product i at market m in time period t
θ^L	lower bound for processing time in a time period
θ^U	upper bound for processing time in a time period
τ_{ijs}	changeover time from product i to product j in plant s
τ_{icm}^{CM}	transportation time of product i from DC c to market m
τ_{isc}^{SC}	transportation time of product i from plant s to DC c

Binary Variables

E_{ist}	1 if product i is processed in plant s in time period t , 0 otherwise
F_{ist}	1 if product i is the first one in plant s in time period t , 0 otherwise
L_{ist}	1 if product i is the last one in plant s in time period t , 0 otherwise
Y_{imtk}	1 if price level k is selected for the product i in market m in time period t , 0 otherwise
Z_{ijst}	1 if product i immediately precedes product j in plant s in time period t , 0 otherwise
ZF_{ijst}	1 if product i in time period $t-1$ immediately precedes product j in time period t in plant s , 0 otherwise

Continuous Variables

$CT1_{st}$	time elapsed within time period t in a changeover starting in the previous time period at plant s
$CT2_{st}$	time elapsed within time period t in a changeover completing in the next time period at plant s
D_{imt}	actual demand of product i at market m in time period t
F_{icmt}^{CM}	flow of product i from DC c to market m at plant s in time period t
F_{isct}^{SC}	flow of product i from plant s to DC c in time period t
INV_{ict}^C	inventory of product i at DC c of time period t

INV_{imt}^M	inventory of product i at market m of time period t
INV_{ist}^S	inventory of product i at plant s of time period t
$INVD_{ict}^C$	inventory deviation to trajectory of product i at DC c of time period t
$INVD_{imt}^M$	inventory deviation to trajectory of product i at market m of time period t
$INVD_{ist}^S$	inventory deviation to trajectory of product i at plant s of time period t
LS_{imt}	lost sales amount of product i at market m in time period t
OI_{ist}	order index of product i in plant s in time period t
Pr_{imt}	price of product i at market m in time period t
PC_{imt}	price change from the previous time period of product i at market m in time period t
P_{ist}	production amount of product i at plant s in time period t
PT_{ist}	processing time of product i at plant s in time period t
Sa_{imt}	sales volume of product i to market m in time period t
SY_{imtk}	auxiliary variable for the linearisation of $Sa_{imt} \cdot Y_{imtk}$
$TotCC$	total changeover cost
$TotPC$	total production cost
$TotRev$	total revenue
$TotTC$	total transportation cost
$TotLSC$	total lost sales cost
Φ_1	total profit
Φ_2	total weighted inventory deviation
Φ_3	total weighted price change
Π	objective function

4.3.2 Production Sequence Constraints

The following constraints for the production sequences in multiple plants are adapted from the MILP model for the medium-term planning of multiproduct continuous plants with parallel units in Chapter 2.

At each plant, there is one product assigned to the first or last one to process in each time period, based on the assumption that at least one product is processed at each plant in each time period:

$$\sum_{i \in I_s} F_{ist} = 1, \quad \forall s \in S, t \in T \quad (4.1)$$

$$\sum_{i \in I_s} L_{ist} = 1, \quad \forall s \in S, t \in T \quad (4.2)$$

For any product assigned to be processed at one plant in one time period, there is only one product assigned immediate before (or after it), except for the first one (or the last one):

$$\sum_{i \in I_s, i \neq j} Z_{ijst} = E_{jst} - F_{jst}, \quad \forall s \in S, j \in I_s, t \in T \quad (4.3)$$

$$\sum_{j \in I_s, j \neq i} Z_{ijst} = E_{ist} - L_{ist}, \quad \forall s \in S, i \in I_s, t \in T \quad (4.4)$$

If a product is the first one (or the last one) to be processed at one plant in a time period, there is exactly one changeover from the last product in the previous time period (or to the first product in the next time period).

$$\sum_{i \in I_s} ZF_{ijst} = F_{jst}, \quad \forall s \in S, j \in I_s, t \in T \setminus \{1\} \quad (4.5)$$

$$\sum_{j \in I_s} ZF_{ijst} = L_{ist}, \quad \forall s \in S, i \in I_s, t \in T \setminus \{1\} \quad (4.6)$$

If product i is processed precedent to product j at one plant in one time period, the order index of product j is higher than that of product i ; otherwise if the product is not processed at one plant in one time period, the corresponding order index is zero:

$$OI_{jst} - (OI_{ist} + 1) \geq -N \cdot (1 - Z_{ijst}), \quad \forall s \in S, i \in I_s, j \in I_s, j \neq i, t \in T \quad (4.7)$$

$$OI_{ist} \leq N \cdot E_{ist}, \quad \forall s \in S, i \in I_s, t \in T \quad (4.8)$$

where N is the maximum number of products that one plant can process, i.e., $\max_{s \in S} |I_s|$. According to the Theorem 2.1, Eq. (4.7) can avoid the occurrences of subtours in the optimal production sequences.

4.3.3 Production Time and Amount Constraints

The production time of one product at one plant in each time period is limited between the upper and lower bounds.

$$\theta^L \cdot E_{ist} \leq PT_{ist} \leq \theta^U \cdot E_{ist}, \quad \forall s \in S, i \in I_s, t \in T \quad (4.9)$$

The changeover time between two consecutive time periods can be split into two parts in different time periods.

$$CT1_{st} + CT2_{s,t-1} = \sum_{i \in I_s} \sum_{j \in I_s} ZF_{ijst} \cdot \tau_{ijs}, \quad \forall s \in S, t \in T \setminus \{1\} \quad (4.10)$$

At each plant, the total production time plus the total changeover time should not exceed the total available time in each time period:

$$\sum_{i \in I_s} PT_{ist} + \sum_{i \in I_s} \sum_{j \in I_s} Z_{ijst} \cdot \tau_{ijs} + CT1_{st} \Big|_{t>1} + CT2_{st} \Big|_{t<|T|} \leq \theta^U, \quad \forall s \in S, t \in T \quad (4.11)$$

It needs to be mentioned that variables $CT1_{st}$ and $CT2_{st}$, and Eqs. (4.10) and (4.11) are adapted from the model by Kopanos et al. (2011).

The production amount of one product at one plant in each time period is equal to its production time multiplied by the corresponding processing rate:

$$P_{ist} = r_{is} \cdot PT_{ist}, \quad \forall s \in S, i \in I_s, t \in T \quad (4.12)$$

4.3.4 Inventory Constraints

At each plant, the inventory level of one product in one time period is equal to its inventory in the previous time period, plus the production amount, minus the total flows to all the connected distribution centres:

$$INV_{ist}^S = INV_{is,t-1}^S \Big|_{t>1} + IniINV_{is}^S \Big|_{t=1} + P_{ist} - \sum_{c \in C_s \cap C_i} F_{isc}^{SC}, \quad \forall s \in S, i \in I_s, t \in T \quad (4.13)$$

At each distribution centre, the inventory level of one product in one time period is equal to its inventory in the previous time period, plus the total incoming flows from connected plants, minus the total outgoing flows to all the connected markets:

$$INV_{ict}^C = INV_{is,t-1}^C \Big|_{t>1} + IniINV_{is}^C \Big|_{t=1} + \sum_{s \in S_c \cap S_i} F_{isc,t-\tau_{isc}}^{SC} - \sum_{m \in M_c \cap M_i} F_{icmt}^{CM}, \quad \forall c \in C, i \in I_c, t \in T \quad (4.14)$$

At each market, the inventory level of one product in one time period is equal to its inventory in the previous time period, plus the total incoming flows from the connected plants, minus the total sales volume :

$$INV_{imt}^M = INV_{im,t-1}^M \Big|_{t>1} + IniINV_{im}^M \Big|_{t=1} + \sum_{c \in C_m \cap C_i} F_{icm,t-t_{icm}^{CM}}^{CM} - Sa_{imt}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.15)$$

The inventory level of each product at one site in one time period is limited between the corresponding upper and lower bounds:

$$MinINV_{is}^S \leq INV_{ist}^S \leq MaxINV_{is}^S, \quad \forall s \in S, i \in I_s, t \in T \quad (4.16)$$

$$MinINV_{ic}^C \leq INV_{ict}^C \leq MaxINV_{ic}^C, \quad \forall c \in C, i \in I_c, t \in T \quad (4.17)$$

$$MinINV_{im}^M \leq INV_{imt}^M \leq MaxINV_{im}^M, \quad \forall m \in M, i \in I_m, t \in T \quad (4.18)$$

4.3.5 Price Elasticity of Demand Constraints

Price elasticity is the concept that determines the relationship between product price and its demand, which is used to measure the degree of responsiveness of demand to change in price (Lysons and Farrington, 2006). The price elasticities are almost always negative by the law of demand (Webster, 2003), which means that a decrease in product price leads to increase in product demand, and vice versa, although the price elasticities may be positive in some special cases (Gillespie, 2007). The price elasticity coefficient of product i at market m is defined as the division of percentage change in quantity of the product demanded by the percentage change in the price (Gwartney et al., 2008):

$$PE_{im} = \frac{\text{percentage change in demand}}{\text{percentage change in price}}$$

Based on the above equality, the relationship between the product price and its final demand is formulated as follows:

$$(D_{imt} - IniD_{imt}) / IniD_{imt} = PE_{im} \cdot (Pr_{imt} - IniPr_{im}) / IniPr_{im}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.19)$$

where the initial demand, $IniD_{imt}$, is uncertain disturbance. It follows a uniform distribution between $(1 - \alpha_{imt}) \cdot IniD_{imt}^F$ and $(1 + \alpha_{imt}) \cdot IniD_{imt}^F$, where the $IniD_{imt}^F$ is the expected value of $IniD_{imt}$, as well as the forecast initial demand, and $\alpha_{imt} \in (0,1)$ is the forecast error of $IniD_{imt}$. When the initial demand, $IniD_{imt}$, initial price,

$IniPr_{im}$, and price elasticity, PE_{im} , are known, the final demand, D_{imt} , is determined after the pricing decision, Pr_{imt} , is made.

4.3.6 Lost Sales Constraints

The lost sales amount is equal to the demand minus the sales of each product at each market in each time period:

$$LS_{imt} = D_{imt} - Sa_{imt}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.20)$$

4.3.7 Pricing Constraints

Among all available price levels, only one price level should be selected for each product at each market in each time period:

$$\sum_{k \in K} Y_{imtk} = 1, \quad \forall m \in M, i \in I_m, t \in T \quad (4.21)$$

$$Pr_{imt} = \sum_{k \in K} \overline{Pr}_{imk} \cdot Y_{imtk}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.22)$$

4.3.8 Inventory Deviation Constraints

The inventory deviation from the corresponding inventory trajectory is the absolute value of the difference between the inventory and inventory trajectory. Eq. (4.23) is for the inventory deviations at plants; Eq. (4.24) is for the inventory deviations at DCs; Eq. (4.25) is for the inventory deviations at markets.

$$INVD_{ist}^S = \left| INVT_{ist}^S - INV_{ist}^S \right|, \quad \forall s \in S, i \in I_s, t \in T \quad (4.23)$$

$$INVD_{ict}^C = \left| INVT_{ict}^C - INV_{ict}^C \right|, \quad \forall c \in C, i \in I_c, t \in T \quad (4.24)$$

$$INVD_{imt}^M = \left| INVT_{imt}^M - INV_{imt}^M \right|, \quad \forall m \in M, i \in I_m, t \in T \quad (4.25)$$

Here, we use the L_1 rather than L_2 norm to maintain model linearity and to avoid any overemphasis on outlier values of inventory which are not patently damaging to the system (contrast to process control applications).

As the absolute value functions in the above three constraints are nonlinear, we rewrite each of them using two linear inequalities. As the inventory deviation is minimised in the objective function, which will be introduced later, Eq. (4.23) can be

rewritten as Eqs. (4.26) and (4.27); Eq. (4.24) can be rewritten as Eqs. (4.28) and (4.29); Eq. (4.25) can be rewritten as Eqs. (4.30) and (4.31):

$$INVD_{ist}^S \geq INVT_{ist}^S - INV_{ist}^S, \quad \forall s \in S, i \in I_s, t \in T \quad (4.26)$$

$$INVD_{ist}^S \geq INV_{ist}^S - INVT_{ist}^S, \quad \forall s \in S, i \in I_s, t \in T \quad (4.27)$$

$$INVD_{ict}^C \geq INVT_{ict}^C - INV_{ict}^C, \quad \forall c \in C, i \in I_c, t \in T \quad (4.28)$$

$$INVD_{ict}^C \geq INV_{ict}^C - INVT_{ict}^C, \quad \forall c \in C, i \in I_c, t \in T \quad (4.29)$$

$$INVD_{imt}^M \geq INVT_{imt}^M - INV_{imt}^M, \quad \forall m \in M, i \in I_m, t \in T \quad (4.30)$$

$$INVD_{imt}^M \geq INV_{imt}^M - INVT_{imt}^M, \quad \forall m \in M, i \in I_m, t \in T \quad (4.31)$$

4.3.9 Price Change Constraints

In order to keep the price fluctuation at a low level, here we consider two types of price change.

4.3.9.1 Price Change from the Previous Week Prices

At first, the price change of each product at each market can be defined as the absolute difference between the prices in two consecutive time periods, which is given as:

$$PC_{imt} = |Pr_{imt} - Pr_{im,t-1}|, \quad \forall m \in M, i \in I_m, t \in T \quad (4.32)$$

where $Pr_{im,0} = IniPr_{im}$, i.e. in the first time period, the price change is the difference from the initial price, $IniPr_{im}$.

Similar to the inventory deviation, Eq. (4.32) can be rewritten as the following two inequalities, Eqs. (4.33) and (4.34):

$$PC_{imt} \geq Pr_{imt} - Pr_{im,t-1}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.33)$$

$$PC_{imt} \geq Pr_{im,t-1} - Pr_{imt}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.34)$$

4.3.9.2 Price Change from the Initial Prices

An alternative pricing strategy considers the price change from the initial prices. In this case, instead of the definition in Eq. (4.32), the price change of each product can be defined as the absolute difference between the current price and the initial price, alternatively, as given in Eq. (4.35):

$$PC_{imt} = |Pr_{imt} - IniPr_{im}|, \quad \forall m \in M, i \in I_m, t \in T \quad (4.35)$$

which can be rewritten as the following two inequalities, Eqs. (4.36) and (4.37):

$$PC_{imt} \geq Pr_{imt} - IniPr_{im}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.36)$$

$$PC_{imt} \geq IniPr_{im} - Pr_{imt}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.37)$$

4.3.10 Profit

The total profit is calculated by the sales revenue, production cost, changeover cost, transportation cost, and lost sales cost.

$$\Phi_1 = TotRev - TotPC - TotCC - TotTC - TotLSC \quad (4.38)$$

It is worth noting that the total inventory cost is not considered in the profit to avoid the conflict between the inventory control and profit maximisation.

The total revenue is the summation of sales multiplied by the price:

$$TotRev = \sum_{t \in T} \sum_{m \in M} \sum_{i \in I_m} Sa_{imt} \cdot Pr_{imt} \quad (4.39)$$

Incorporating with Eq. (4.22), Eq. (4.39) can be rewritten as:

$$TotRev = \sum_{t \in T} \sum_{m \in M} \sum_{i \in I_m} \sum_{k \in K} Sa_{imt} \cdot \bar{P}_{imk} \cdot Y_{imtk} \quad (4.40)$$

In Eq. (4.40), the nonlinear term $Sa_{imt} \cdot Y_{imtk}$ can be substituted by the introduced auxiliary positive variable SY_{imtk} with the following two constraints to enforce

$$SY_{imtk} \equiv Sa_{imt} \cdot Y_{imtk} :$$

$$SY_{imtk} \leq N \cdot Y_{imtk}, \quad \forall m \in M, i \in I_m, t \in T, k \in K \quad (4.41)$$

$$Sa_{imt} = \sum_{k \in K} SY_{imtk}, \quad \forall m \in M, i \in I_m, t \in T \quad (4.42)$$

where N is a large number, can be the upper bound of the sales at time period t . So the following constraint is equivalent to Eq. (4.40):

$$TotRev = \sum_{t \in T} \sum_{m \in M} \sum_{i \in I_m} \sum_{k \in K} SY_{imtk} \cdot \bar{P}_{imk} \quad (4.43)$$

The total production cost is calculated by the production amount multiplied by the corresponding production cost.

$$TotPC = \sum_{t \in T} \sum_{s \in S} \sum_{i \in I_s} CP_{is} \cdot P_{ist} \quad (4.44)$$

The total changeover cost is the summation of the costs of all occurred changeovers.

$$TotCC = \sum_{t \in T} \sum_{s \in S} \sum_{i \in I_s} \sum_{j \in I_s} CC_{ijs} \cdot Z_{ijst} + \sum_{t \in T \setminus \{1\}} \sum_{s \in S} \sum_{i \in I_s} \sum_{j \in I_s} CC_{ijs} \cdot ZF_{ijst} \quad (4.45)$$

The total transportation cost is the summation of the transportation costs from plants to distribution centres and from distribution centres to markets, which is equal to the summation of unit transportation cost multiplied by the product flows:

$$TotTC = \sum_{t \in T} \sum_{s \in S} \sum_{c \in C_s} \sum_{i \in I_s \cap I_c} CT_{isc}^{SC} \cdot F_{icst}^{SC} + \sum_{t \in T} \sum_{c \in C} \sum_{m \in M_c} \sum_{i \in I_c \cap I_m} CT_{icm}^{CM} \cdot F_{icmt}^{CM} \quad (4.46)$$

The total lost sales cost is determined by the unit lost sales cost and the lost sales amounts:

$$TotLSC = \sum_{t \in T} \sum_{m \in M} \sum_{i \in I_m} CLS_{im} \cdot LS_{imt} \quad (4.47)$$

4.3.11 Weighed Total Inventory Deviation

The weighed total inventory deviation is the summation of the total inventory deviation in each echelon multiplied by the corresponding weight, which could be the unit inventory cost in the practice:

$$\Phi_2 = w^S \cdot \sum_{t \in T} \sum_{s \in S} \sum_{i \in I_s} INVD_{ist}^S + w^C \cdot \sum_{t \in T} \sum_{c \in C} \sum_{i \in I_c} INVD_{ict}^C + w^M \cdot \sum_{t \in T} \sum_{m \in M} \sum_{i \in I_m} INVD_{imt}^M \quad (4.48)$$

4.3.12 Weighed Total Price Change

The weighed total price change is the summation of the total price change multiplied by the corresponding weight:

$$\Phi_3 = w^P \cdot \sum_{t \in T} \sum_{m \in M} \sum_{i \in I_m} PC_{imt} \quad (4.49)$$

4.3.13 Objective Function

The objective of the model is to maximise the profit with the maintenance of the inventory levels and price levels. So, the profit is penalised by the weighed inventory deviation and price change are in the objective function:

$$\Pi = \Phi_1 - \Phi_2 - \Phi_3 \quad (4.50)$$

4.3.14 Summary

Overall, the production, distribution and inventory planning problem has been formulated as an MILP model with Eqs. (4.1)–(4.22), (4.26)–(4.31), (4.33), (4.34), (4.38) and (4.41)–(4.49) as the constraints and Eq. (4.50) as the objective function, when the pricing strategy considering the price change from the previous week prices. If the alternative pricing strategy considering the price change from initial prices is applied, Eqs. (4.33) and (4.34) are replaced by Eqs. (4.36) and (4.37).

4.4 MPC Approach

To treat the uncertainty within the deterministic supply chain optimisation model, an MPC approach is suggested, in which the supply chain performance is optimised in a finite horizon using the current initial demands and future initial demand forecasts. The basic principle of MPC is to transform the control problem into an optimisation one (Scattolini, 2009). The main idea of MPC is to choose the control action by repeatedly solving online an optimal control problem, aiming to optimise a performance criterion, which consists of the deviation of the future controlled process from a reference trajectory over a future horizon. See Fig. 4.2 for the MPC strategy.

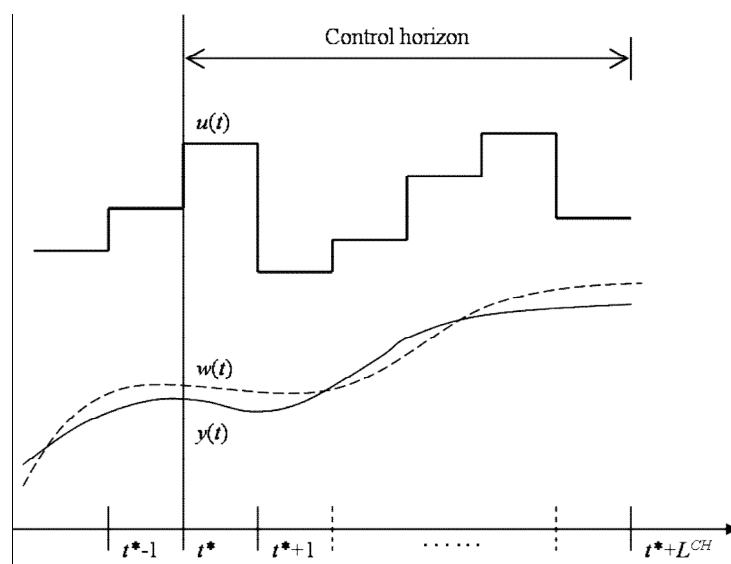


Figure 4.2 The MPC strategy. $u(t)$: inputs, $w(t)$: reference trajectory, $y(t)$: outputs.

The two fundamental parts of the MPC controller are the process model and the optimiser. At time t^* , the process model predicts the future outputs based on the past and current values and the proposed control actions. Then the optimiser calculates

the optimal future control actions over the control horizon $T^C = [t^*, t^* + L^{CH} - 1]$ by optimising the cost function to keep the process as close to the reference trajectory as possible, subject to the constraints on the manipulated inputs and outputs. Only the first step of the future control actions is implemented. At time t^*+1 , the calculations are repeated, yielding new control actions and new predicted outputs. Camacho and Bordons (2004) described the approach in more details. The structure of MPC is shown in Fig 4.3.

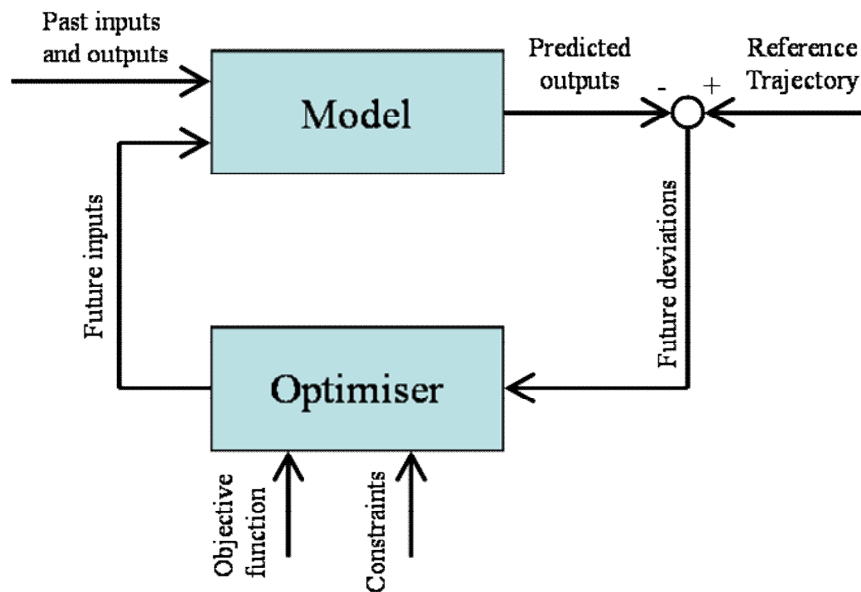


Figure 4.3 The structure of MPC.

Here, due to the weekly demand uncertainty nature of the supply chain planning problem discussed in this chapter, the iterative MPC approach is applied. In the MPC approach, the disturbance is the initial demand, $IniD_{int}$. The inputs of the process model include the production sequences, times and amounts, flow amounts, inventory at plants and DCs, and product prices, while the outputs are the inventory at markets, sales and lost sales. The process model comprises Eqs. (4.1)–(4.22) which are used to predict the future outputs.

The optimisation problem in the MPC approach optimises the inputs within the control horizon, T^C , correspond to a number of MILP problems. In the optimisation problem at the time period t^* , the initial demands at the current time period, t^* , are realised, while all the future demands in the control horizon are unknown. So,

forecast initial demands, $IniD_{imt}^F$, in the future time periods, $t^* < t < t^* + L^{CH}$, are used in the optimisation MILP model, while the actual initial demands, $IniD_{imt^*}$, are generated for the current time period, t^* . In this case, in the optimisation problem of MPC approach, Eq. (4.19) is rewritten as Eqs. (4.51) and (4.52):

$$(D_{imt} - IniD_{imt}) / IniD_{imt} = PE_{im} \cdot (Pr_{imt} - IniPr_{im}) / IniPr_{im}, \quad \forall m \in M, i \in I_m, t = t^* \quad (4.51)$$

$$(D_{imt} - IniD_{imt}^F) / IniD_{imt}^F = PE_{im} \cdot (Pr_{imt} - IniPr_{im}) / IniPr_{im}, \quad \forall m \in M, i \in I_m, t \in T^C \setminus \{t^*\} \quad (4.52)$$

The MILP model for the control horizon, T^C , is described as follows:

$$\max \quad \Pi = \Phi_1 - \Phi_2 - \Phi_3 \quad (4.53)$$

s.t. Eqs. (4.1)–(4.18), (4.20)–(4.22), (4.26)–(4.31), (4.33), (4.34),
(4.38), (4.41)–(4.49), (4.51) and (4.52) specified for $T = T^C$

Note that unless stated specifically, the price change definition by Eq. (4.32) is considered in the MPC.

The MPC approach implemented for the supply chain planning problem is described as follows:

STEP 1. Initialise the current time period $t^* = 1$;

STEP 2. Update the control horizon $T^C = [t^*, t^* + L^{CH} - 1]$;

STEP 3. Generate the initial demand for the current time period, t^* ,

$$IniD_{imt^*} = \text{Uniform}[(1 - \alpha_{imt^*}) \cdot IniD_{imt^*}^F, (1 - \alpha_{imt^*}) \cdot IniD_{imt^*}^F];$$

STEP 4. Solve the MILP model (4.53) for the control horizon;

STEP 5. Fix the values of the all variables at current time period t^* ;

STEP 6. If $t^* = |T|$, STOP; Otherwise, let $t^* = t^* + 1$, go to STEP 2.

4.5 An Numerical Example

The supply chain example considered here has 3 echelons with 3 plants (S1–S3), 8 distribution centres (C1–C8), 16 markets (M1–M16). See Fig. 4.4 for the structure of the supply chain.

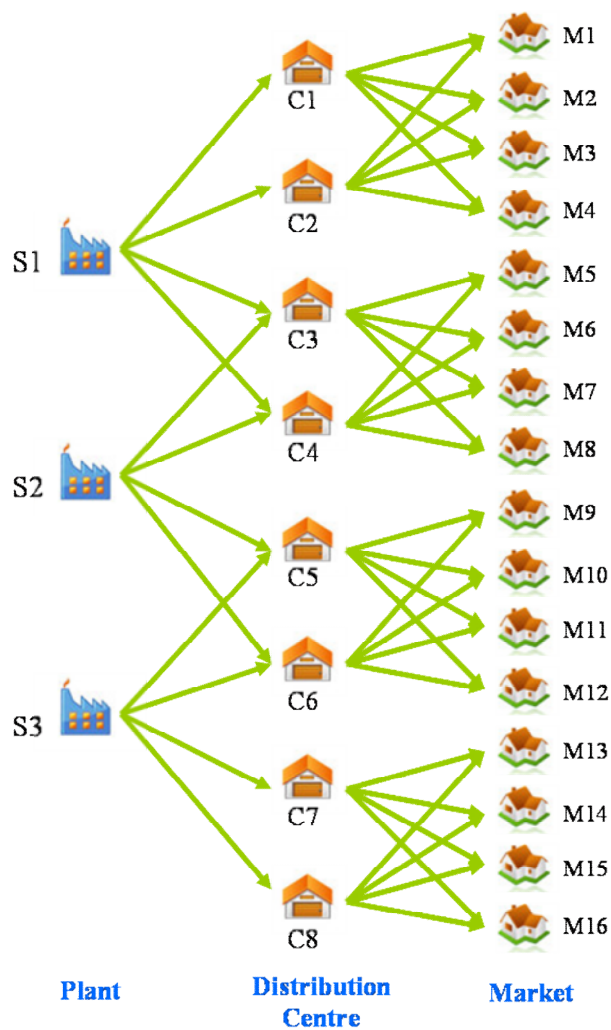


Figure 4.4 The structure of the supply chain in the example.

There are 10 products (I1–I10) in the supply chain. Table 4.1 shows the suitability of plants, DCs, and markets. We consider a planning horizon of one year, which is divided into 52 weeks. The minimum production time in each week is 5 hours.

Table 4.1 Suitability of plants, DCs and markets.

		Products									
		I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
Plants	S1	√ ^a	√	√	√	√	√				
	S2			√	√	√	√	√	√		
	S3					√	√	√	√	√	√
DCs	C1	√	√	√	√						
	C2	√	√	√	√						
	C3			√	√	√	√				
	C4			√	√	√	√				
	C5					√	√	√	√		
	C6					√	√	√	√		
	C7							√	√	√	√
	C8							√	√	√	√
Markets	M1	√	√	√	√						
	M2	√	√	√	√						
	M3	√	√	√	√						
	M4	√	√	√	√						
	M5			√	√	√	√				
	M6			√	√	√	√				
	M7			√	√	√	√				
	M8			√	√	√	√				
	M9					√	√	√	√		
	M10					√	√	√	√		
	M11					√	√	√	√		
	M12					√	√	√	√		
	M13							√	√	√	√
	M14							√	√	√	√
	M15							√	√	√	√
	M16							√	√	√	√

^a The product can be assigned for the plant, DC, or market.

We assume that the sequence-dependent changeover times and costs between two products occurring at different plants are the same. The changeover times (in hours) are presented in Table 4.2. The unit changeover cost is 60 k\$/hour. Thus, the value of each changeover cost in the unit of k\$ is equal to the value of the corresponding changeover time in the unit of hours multiplied by 60, e.g. the changeover cost from I1 to I2 is $60 \times 2.25 = 135$ k\$. The production rates and unit production costs at suitable plants are given in Tables 4.3 and 4.4, respectively.

Table 4.2 Sequence-dependent changeover times (hours).

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
I1	0	2.25	3	4	2.5	3	- ^a	-	-	-
I2	3.5	0	1.5	2.75	1.5	2	-	-	-	-
I3	3	2.25	0	3.5	3.25	2.5	3.5	4	-	-
I4	3.5	3	3.25	0	5	1.5	3	2.5	-	-
I5	2.5	3.5	3	4.5	0	4	2.75	3.25	4	4
I6	4.5	4	3.5	3.25	2.75	0	4	1.5	4.5	5
I7	-	-	3	4	2	2.25	0	2.5	2.5	3
I8	-	-	1.5	3	2.75	5	3.5	0	3.5	4
I9	-	-	-	-	4	3.5	3	3.25	0	4
I10	-	-	-	-	4.5	5	1.5	2	5	0

^aNot applicable.**Table 4.3** Production rates (ton/hour).

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
S1	3.5	3	3.5	4	3.5	4	- ^a	-	-	-
S2	-	-	4	4.5	3	3.5	4.5	5	-	-
S3	- ^a	-	-	-	3.5	4	3.5	5.5	3.5	4

^aNot applicable.**Table 4.4** Unit production costs (k\$/ton).

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
S1	2	1	3	2	2.5	5	- ^a	-	-	-
S2	-	-	1	2.5	3	4	2	3	-	-
S3	-	-	-	-	2	4.5	2	3.5	1	2

^aNot applicable.

The transportation times from plants to DCs and from DCs to markets are shown in Tables 4.5 and 4.6. The unit transportation cost for one week is 1 k\$/ton. Thus, the values of transportation costs in the unit of k\$/ton are equal to the values of the corresponding transportation times in the unit of week.

Table 4.5 Transportation times from plant to DC (weeks).

	C1	C2	C3	C4	C5	C6	C7	C8
S1	1	1	1	2	- ^a	-	-	-
S2	-	-	1	2	1	2	-	-
S3	-	-	-	-	1	1	2	1

^aNot applicable.

Table 4.6 Transportation times from DC to market (weeks).

	M1	M2	M3	M4	M5	M6	M7	M8
C1	1	1	0	1	- ^a	-	-	-
C2	0	1	1	1	-	-	-	-
C3	-	-	-	-	0	1	1	1
C4	-	-	-	-	1	1	1	0
	M9	M10	M11	M12	M13	M14	M15	M16
C5	1	1	0	1	-	-	-	-
C6	1	1	1	0	-	-	-	-
C7	-	-	-	-	1	1	1	0
C8	-	-	-	-	0	1	1	1

^aNot applicable.

For each product, the inventory trajectories at markets in each week are set to 2 times of its maximum forecast initial demand at the market; the inventory trajectories at DCs are set to 4 times of maximum forecast initial demand of the product at one market; the inventory trajectories at markets are set to 8 times of maximum forecast initial demand of the product at one market. The inventory trajectories at the suitable sites are given in Table 4.7. It is assumed that the initial inventories at the beginning of the planning horizon are the same as the corresponding inventory trajectories, i.e. $IniINV_{is}^S = INVT_{is}^S$, $IniINV_{ic}^C = INVT_{ic}^C$ and $IniINV_{im}^M = INVT_{im}^M$ to avoid any inventory deviation at the beginning of the planning horizon.

The product initial demand in each week at each market is uncertain and follows a uniform distribution between the known specific upper and lower bounds. Before the initial demand realisation, their forecasts, the expected values of actual demands, are known and used in the optimisation problem of MPC to predict future outputs. The total forecast initial demand is 69,460 ton. In each market, the maximum forecast initial demand for one product in one time period is 40 ton, while the minimum forecast initial demand is 5 ton. The forecast error, α_{imt} , varies among different products and markets, and its maximum value is 20%.

Table 4.7 Inventory trajectories at all sites (ton).

		I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
Plant	S1	160	160	240	160	320	320	- ^a	-	-	-
	S2	-	-	240	160	320	320	320	160	-	-
	S3	-	-	-	-	320	320	320	160	240	240
DC	C1	80	80	120	80	-	-	-	-	-	-
	C2	80	80	120	80	-	-	-	-	-	-
	C3	-	-	120	80	160	160	-	-	-	-
	C4	-	-	120	80	160	160	-	-	-	-
	C5	-	-	-	-	160	160	160	80	-	-
	C6	-	-	-	-	160	160	160	80	-	-
	C7	-	-	-	-	-	-	160	80	120	120
	C8	-	-	-	-	-	-	160	80	120	120
Market	M1	40	40	60	40	-	-	-	-	-	-
	M2	40	40	60	40	-	-	-	-	-	-
	M3	40	40	60	40	-	-	-	-	-	-
	M4	40	40	58	40	-	-	-	-	-	-
	M5	-	-	60	40	80	80	-	-	-	-
	M6	-	-	58	40	80	80	-	-	-	-
	M7	-	-	58	38	78	80	-	-	-	-
	M8	-	-	60	40	80	80	-	-	-	-
	M9	-	-	-	-	80	80	80	38	-	-
	M10	-	-	-	-	80	80	78	40	-	-
	M11	-	-	-	-	80	80	78	40	-	-
	M12	-	-	-	-	80	78	78	40	-	-
	M13	-	-	-	-	-	-	78	40	60	60
	M14	-	-	-	-	-	-	78	40	60	58
	M15	-	-	-	-	-	-	80	38	60	60
	M16	-	-	-	-	-	-	80	40	58	58

^a Not applicable.

The actual demands are determined by the price elasticity, initial demand and selected price obtained from the optimisation problem in MPC. The price elasticity coefficient for each product in each market is given in Table 4.8.

Table 4.8 Price elasticity coefficients.

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
M1	-1.68	-1.04	-1.44	-1.36	- ^a	-	-	-	-	-
M2	-1.76	-1.12	-1.44	-1.44	-	-	-	-	-	-
M3	-1.60	-1.28	-1.36	-1.28	-	-	-	-	-	-
M4	-1.84	-1.20	-1.36	-1.36	-	-	-	-	-	-
M5	-	-	-1.44	-1.52	-1.04	-1.36	-	-	-	-
M6	-	-	-1.28	-1.36	-1.12	-1.44	-	-	-	-
M7	-	-	-1.44	-1.44	-1.12	-1.44	-	-	-	-
M8	-	-	-1.28	-1.36	-1.04	-1.36	-	-	-	-
M9	-	-	-	-	-0.96	-1.44	-1.76	-1.20	-	-
M10	-	-	-	-	-0.96	-1.44	-2.00	-1.28	-	-
M11	-	-	-	-	-1.04	-1.52	-1.68	-1.28	-	-
M12	-	-	-	-	-0.88	-1.44	-1.60	-1.20	-	-
M13	-	-	-	-	-	-	-1.92	-1.36	-2.08	-1.44
M14	-	-	-	-	-	-	-1.76	-1.36	-2.32	-1.52
M15	-	-	-	-	-	-	-1.84	-1.28	-2.08	-1.44
M16	-	-	-	-	-	-	-1.68	-1.36	-2.00	-1.36

^a Not applicable.

Table 4.9 shows the available price levels (K1–K5) for selection, in which the prices at level K3 (in bold) are the initial prices. The unit lost sales cost of each product is assumed to be half of its initial price at the market.

Table 4.9 Available price levels (k\$/ton).

		Price levels					Price levels						
		K1	K2	K3^a	K4	K5	K1	K2	K3^a	K4	K5		
I1	M1	8	9.5	11	12.5	14	I6	M5	7	8.5	10	11.5	13
	M2	6	7.5	9	10.5	12		M6	8	9.5	11	12.5	14
	M3	7	8.5	10	11.5	13		M7	9	10.5	12	13.5	15
	M4	6	7.5	9	10.5	12		M8	8	9.5	11	12.5	14
I2	M1	4	5	6	7	8	I6	M9	7	8.5	10	11.5	13
	M2	3	4	5	6	7		M10	8	9.5	11	12.5	14
	M3	4	5	6	7	8		M11	7	8.5	10	11.5	13
	M4	3	4	5	6	7		M12	7	8.5	10	11.5	13
I3	M1	8	9.5	11	12.5	14	I7	M9	6	7.5	9	10.5	12
	M2	7	8.5	10	11.5	13		M10	5	6.5	8	9.5	11
	M3	9	10.5	12	13.5	15		M11	6	7.5	9	10.5	12
	M4	8	9.5	11	12.5	14		M12	5	6.5	8	9.5	11
	M5	7	8.5	10	11.5	13		M13	5	6.5	8	9.5	11
	M6	8	9.5	11	12.5	14		M14	5	6.5	8	9.5	11
	M7	9	10.5	12	13.5	15		M15	6	7.5	9	10.5	12
	M8	7	8.5	10	11.5	13		M16	5	6.5	8	9.5	11
I4	M1	10	12	14	16	18	I8	M9	8	9.5	11	12.5	14
	M2	9	11	13	15	17		M10	7	8.5	10	11.5	13
	M3	8	10	12	14	16		M11	8	9.5	11	12.5	14
	M4	9	11	13	15	17		M12	8	9.5	11	12.5	14
	M5	8	10	12	14	16		M13	8	9.5	11	12.5	14
	M6	9	11	13	15	17		M14	9	10.5	12	13.5	15
	M7	8	10	12	14	16		M15	8	9.5	11	12.5	14
	M8	8	10	12	14	16		M16	8	9.5	11	12.5	14
I5	M5	6	7.5	9	10.5	12	I9	M13	4	5	6	7	8
	M6	5	6.5	8	9.5	11		M14	4	5	6	7	8
	M7	4	5.5	7	8.5	10		M15	3	4	5	6	7
	M8	6	7.5	9	10.5	12		M16	3	4	5	6	7
	M9	5	6.5	8	9.5	11	I10	M13	13	15	17	19	21
	M10	6	7.5	9	10.5	12		M14	12	14	16	18	20
	M11	6	7.5	9	10.5	12		M15	13	15	17	19	21
	M12	4	5.5	7	8.5	10		M16	14	16	18	20	22

^a Price level K3 is the initial price.

4.6 Results and Discussion

For the supply chain example given above, there are 52 MILP models in total to implement in the MPC. The optimality gap for each MILP model is 5%. The CPU time limit of each MILP model is 3,600 seconds.

Unless stated specifically, in the objective function, the weights for the inventory deviations are set to 2.5, i.e., $w^S = w^C = w^M = 2.5$, and the weight for the price change is set to 10, i.e., $w^P = 10$.

In this section, we will investigate the computational results of the example by MPC and discuss the effects of the length of the control horizon, inventory, effect of weights, pricing strategy and changeovers on the solutions.

4.6.1 Length of the Control Horizon

Here, we consider three approaches with different lengths of control horizon, which are 4, 5 and 6 weeks.

The breakdowns of the objective values for all three approaches are presented in Table 4.10. The approach with $L^{CH} = 4$ has the worst performance among all the three approaches, as its objective value is only 70% of those of the other two approaches which results from the much higher inventory deviation. The approaches with $L^{CH} = 5$ and $L^{CH} = 6$ have similar objective values, profit, inventory deviation and price change. However, as the approach with longer control horizon takes much more CPU time, the approach with $L^{CH} = 5$, which takes only about 1/4 of CPU time taken by the approach $L^{CH} = 6$, is considered as the best option. We use the approach with $L^{CH} = 5$ for the further discussion. All the results discussed later in this chapter are obtained from the case with $L^{CH} = 5$.

Moreover, in all three cases, the total actual final demand is less than the total initial demand, which is 69,260 ton, which implies the average selected prices are higher than the initial prices. It can also be seen that when a longer control horizon is used, a higher actual final demand is realised after pricing decisions are made. So, lower prices are selected for the products at the markets.

Table 4.10 Comparisons of the three cases with different control horizon lengths.

		$L^{CH} = 4$	$L^{CH} = 5$	$L^{CH} = 6$
Objective		210,220	295,915	293,379
CPU (s)		723	2,045	7,953
Profit (k\$)		348,511	340,379	337,864
Revenue (k\$)		623,759	640,785	645,883
Production cost (k\$)		134,476	145,227	149,225
Changeover cost (k\$)		45,045	49,230	43,725
Transportation cost (k\$)		95,585	105,778	114,836
Lost sales cost (k\$)		142	171	233
Inventory deviation (ton)	Plant	273	1,319	2,641
	DC	22,715	7,206	5,130
	Market	30,972	7,668	8,245
Price change ^a (k\$/ton)		339	398	445
Actual final demand (ton)		56,394	58,911	59,872

^a Total absolute price change between two consecutive time periods.

4.6.2 Inventory and Inventory Deviation

Fig. 4.5 shows the average inventory deviations in percentage at all three echelons. We can see that the average inventory deviations at all the echelons are very small. The inventory deviation at the plants is the closest to zero, within 4% in all the weeks. At the markets, the average inventory deviations are within 4%, apart from the first three weeks. The average inventory deviations at the DC are the highest, but still within 10% except the first two weeks.

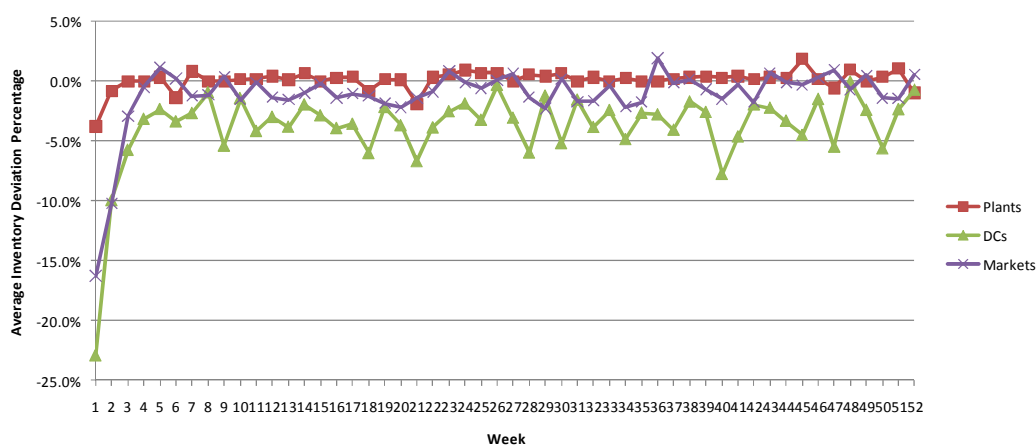


Figure 4.5 The average inventory deviation in each echelon.

Considering the inventory fluctuation of each product, Fig. 4.6 shows the average inventory levels of each product at all three echelons. Products I5, I6 and I10 have the largest fluctuations in inventory. Overall, the inventories at all echelons are maintained at stable levels, and the inventory fluctuation is not significant.

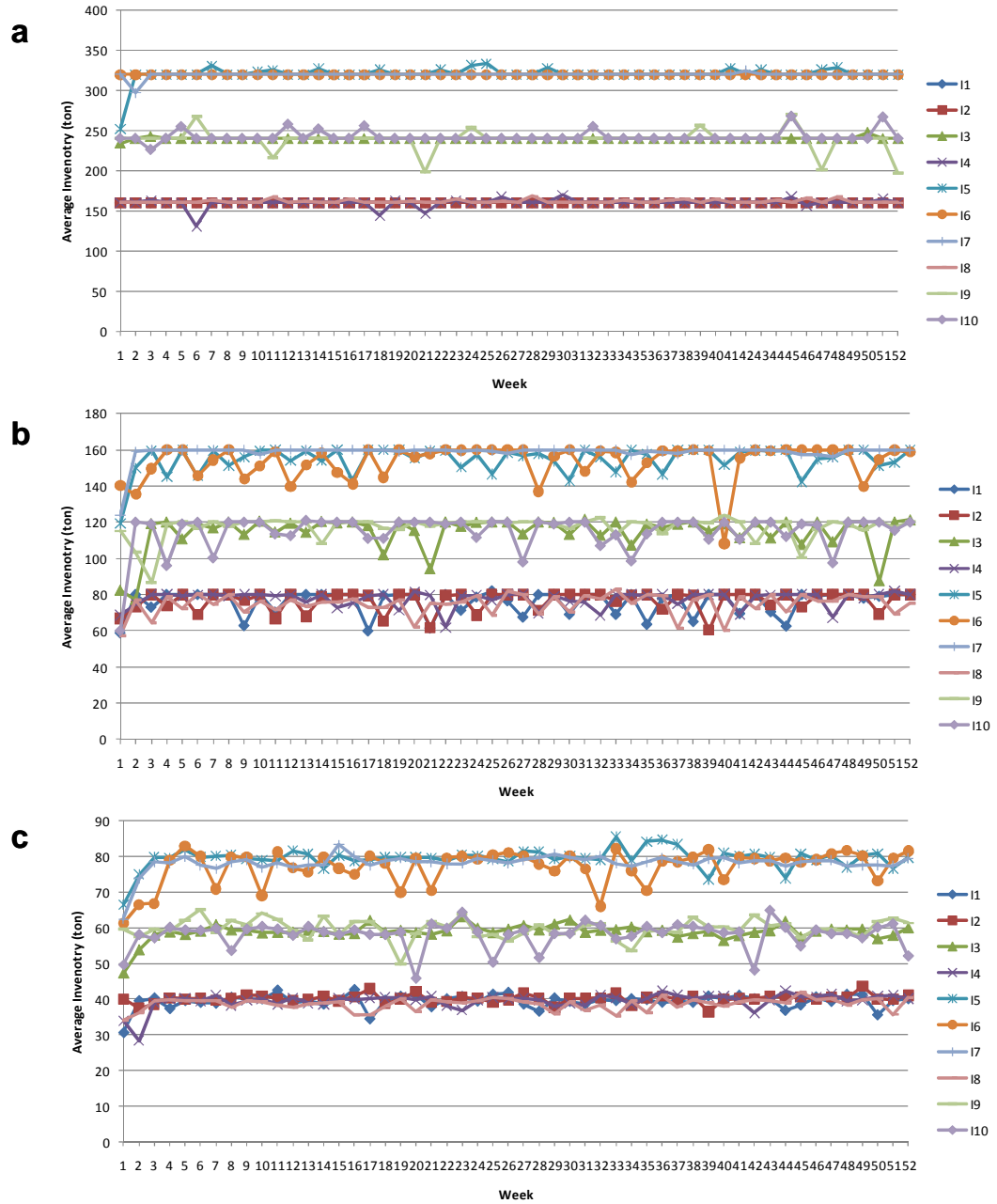


Figure 4.6 The average inventory levels for each product. (a) plants, (b) DCs, (c) markets.

4.6.3 Effect of Weights

Now, we examine the effect of values of weights for inventory deviations and price change on the profit and inventory deviation. The profit is expressed by Φ_1 in Eq. (4.38), while the total inventory deviation is expressed by Φ'_2 as follows:

$$\Phi'_2 = \sum_{t \in T} \sum_{s \in S} \sum_{i \in I_s} INVD_{ist}^S + \sum_{t \in T} \sum_{c \in C} \sum_{i \in I_c} INVD_{ict}^C + \sum_{t \in T} \sum_{m \in C} \sum_{i \in I_m} INVD_{imt}^M \quad (4.54)$$

It is assumed that the inventory deviation weights for different echelons are the same, i.e. $w^S = w^C = w^M$, whose value is denoted by w^{INV} . Here, we consider that the value of w^{INV} varies from 1 to 3 by a step length of 0.5, and the value of w^P is equal to 10 and 50. The fixed pricing strategy, where the prices are fixed to their initial values, are also investigated, which can be considered as a special case with a very large value of w^P .

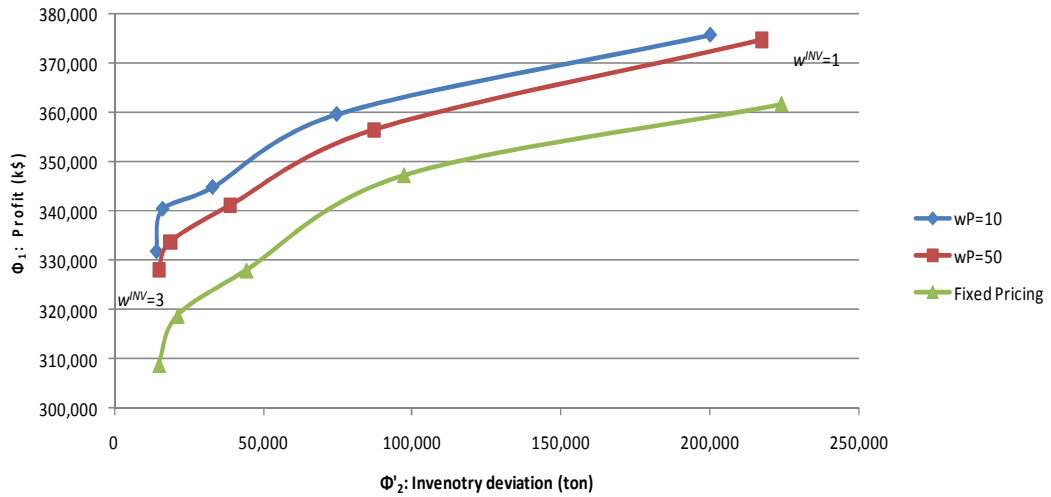


Figure 4.7 Effect of weights on profit and inventory deviation.

In Fig. 4.7, different values of w^P generate different curves. On each curve, the left end node represents the case with the largest value of w^{INV} , i.e., $w^{INV} = 3$, while the right end node represents the case with the smallest value of w^{INV} , i.e., $w^{INV} = 1$. The other points on the curve in Fig. 4.7 represent the solutions using different values of w^{INV} , which decrease from left to right. For a fixed value of w^P , with an increased penalty on inventory deviation, the inventory deviation decrease. In order to maintain a stable inventory level, the supply chain earns less profit. So a higher value of w^{INV} has a negative effect on both profit and inventory deviation, as shown in Fig. 4.7.

When the value of w^{INV} is fixed, a higher value of w^P can lead to a lower profit and a larger inventory deviation, as the less flexibility on pricing impacts the supply chain performance.

4.6.4 Pricing Strategies

To examine the effect of the price elasticity of demand on the solutions, we investigate the four pricing strategies (PS1–PS4):

- PS1: Free pricing, where no penalty on the pricing decisions, i.e., $w^P = 0$;
- PS2: Fixed pricing, i.e. no price elasticity of demand, where the prices are fixed to their initial values, i.e., $P_{imt} = IniP_{im}, \forall i, m, t$;
- PS3: Pricing considering price change from previous time period, i.e., Eqs. (4.33) and (4.34) are included in the optimisation model;
- PS4: Pricing considering price change from the initial price, i.e., Eqs. (4.36) and (4.37) are included in the optimisation model.

Comparing the four pricing strategies obtained by MPC, their solutions determined by MPC are given in Table 4.11.

Table 4.11 Comparison of different pricing strategies.

		PS1	PS2	PS3	PS4
Objective		300,237	265,889	295,915	280,174
Profit (k\$)		341,836	318,744	340,379	328,922
Revenue (k\$)		615,151	675,522	640,784	662,852
Production cost (k\$)		135,468	172,310	145,227	161,791
Changeover cost (k\$)		39,705	59,160	49,230	55,095
Transportation cost (k\$)		97,608	122,501	105,778	116,830
Lost sales cost (k\$)		534	2,807	171	214
Inventory deviation (ton)	Plant	986	1,247	1,319	1,006
	DC	8,160	10,281	7,206	7,164
	Market	7,494	9,614	7,668	6,940
Price change (k\$/ton)		- ^a	0	398 ^b	1,098 ^c

^aNo price change considered in the objective function.

^bTotal absolute price change between two consecutive time periods.

^cTotal absolute price change from the initial prices.

Among all the pricing strategies, PS1 generates the highest objective value, as there is no penalty on the pricing. However, PS1 generates the largest price fluctuation

(Fig. 4.8), which is not recommended. Although a stable price level under PS2 is maintained, PS2 generates the lowest objective value, lowest profit and highest inventory deviation, due to the lack of flexibility for pricing. PS3 and PS4 have similar performances. Although PS3 has a higher objective value and profit, the inventory deviations are higher than PS4. From Fig. 4.8, both PS3 and PS4 obtain stable price levels, and the price fluctuation under PS3 is smaller, but the selected prices of PS4 are lower and closer to the initial prices. As PS4 selects lower prices than PS3, the total final demand under PS4 is higher than PS3 (64,509 ton vs. 58,911 ton), and is slightly lower than the total initial demand (69,260 ton).

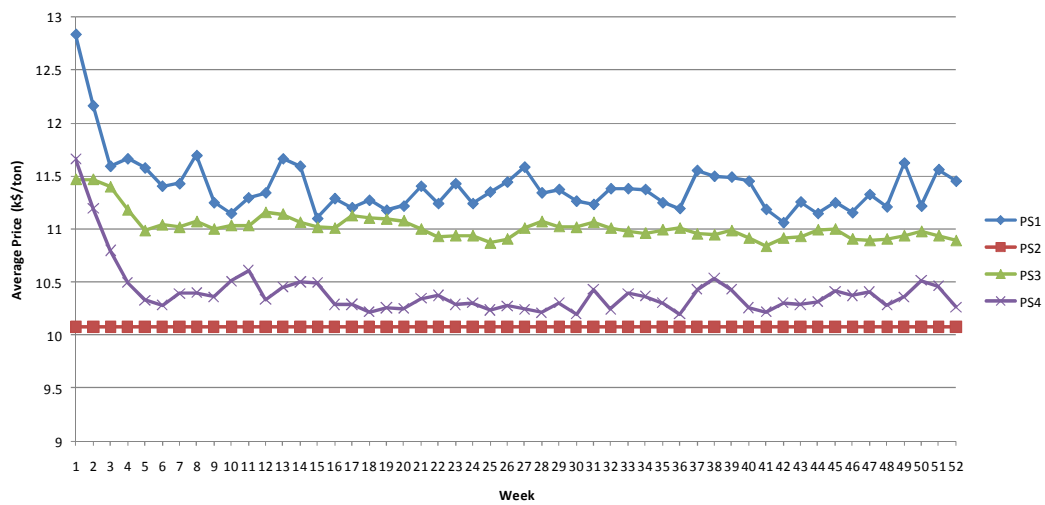


Figure 4.8 The average price comparison.

Fig. 4.9 shows the average price of each product under pricing strategies PS1, PS3 and PS4. The prices under PS2 are ignored here as there is no price fluctuation. The price fluctuation under PS1 (Fig. 4.9a) is much greater, while the other two have smaller fluctuations (Fig. 4.9b, c). It can be concluded that there are lower price changes and fluctuations when w^P is positive in the objective function. Both the proposed two pricing strategies with price change control have a good perform to reduce the risk of the supply chain brought by the great price fluctuations.

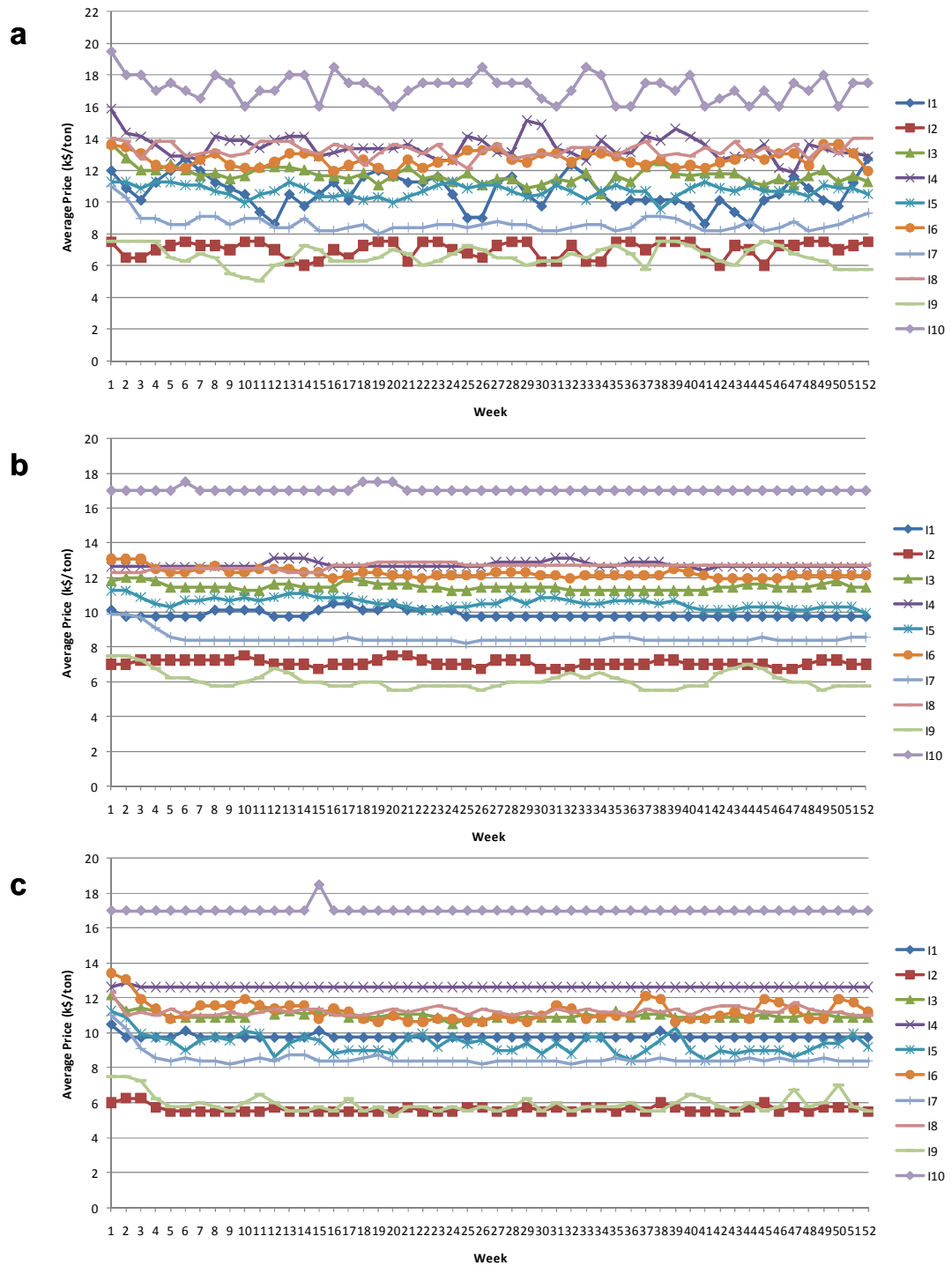


Figure 4.9 The average price for each product at all markets. (a) PS1, (b) PS3, (c) PS4.

4.6.5 Changeovers

Although the sequence-dependent changeovers are considered in the proposed MILP model. The constraints for the sequence-dependent changeovers in the MILP model

are heavy and increase the computational complexity of the proposed model. The necessity for considering changeovers in the MILP model will be verified below.

In order to examine whether the sequence-dependent changeover is crucial to be considered simultaneously with other constraints in the proposed MILP at the cost of the computational time, we proposed a hierarchical approach as another way to tackle the sequence-dependent changeovers. In the hierarchical approach, we firstly solve a simpler MILP model which only considers the production allocations, but not the production sequences and the changeovers. Its differences from the proposed original MILP model are as follows:

- The terms for changeover costs are not included in the objective function;
- Eqs. (4.1)–(4.8) and (4.10) are omitted as the constraints;
- The Eq. (4.11) is replaced by the following constraint:

$$\sum_{i \in I_s} PT_{ist} \leq \theta^U, \quad \forall s \in S, t \in T \quad (4.55)$$

In the MPC, the first MILP model in the hierarchical approach is given by:

$$\begin{aligned} \max \Pi = & \sum_{t \in T^C} \sum_{m \in M} \sum_{i \in I_m} \sum_{k \in K} SY_{imtk} \cdot \bar{P}_{imk} - \sum_{t \in T^C} \sum_{s \in S} \sum_{i \in I_s} CP_{is} \cdot PR_{ist} - \\ & \sum_{t \in T^C} \sum_{s \in S} \sum_{c \in C_s} \sum_{i \in I_s \cap I_c} CT_{isc}^{SC} \cdot F_{ist}^{SC} - \sum_{t \in T^C} \sum_{c \in C} \sum_{m \in M_c} \sum_{i \in I_c \cap I_m} CT_{icm}^{CM} \cdot F_{icmt}^{CM} - \\ & \sum_{t \in T^C} \sum_{m \in M} \sum_{i \in I_m} CU_{im} \cdot U_{imt} - \Phi_2 - \Phi_3 \end{aligned} \quad (4.56)$$

s.t. Eqs. (4.9), (4.12)–(4.18), (4.20)–(4.22), (4.26)–(4.31), (4.33), (4.34), (4.41),
(4.42), (4.48), (4.49), (4.51), (4.52) and (4.55) specified for $T = T^C$

Then, the optimal production sequences can be determined by minimising the total changeover time with the following MILP model:

$$\min \sum_{t \in T^C} \sum_{s \in S} \sum_{i \in I_s} \sum_{j \in I_s} \tau_{ijs} \cdot Z_{ijst} + \sum_{t \in T^C \setminus \{1\}} \sum_{s \in S} \sum_{i \in I_s} \sum_{j \in I_s} \tau_{ijs} \cdot ZF_{ijst} \quad (4.57)$$

s.t. Eqs. (4.1)–(4.8), (4.10) specified for $T = T^C$

Finally, the obtained production allocations and sequences are fixed before solving the reduced original MILP model (4.53) to obtain the final solution.

Overall, we use the following steps instead of STEP 4 in the MPC approach to implement the hierarchical approach:

- STEP 4.1. Solve the MILP model (4.56) without production sequences for the control horizon;
- STEP 4.2. Fix the binary variable E_{ist} in the control horizon;
- STEP 4.3. Solve the MILP model (4.57) to minimise changeover times;
- STEP 4.4. Fix the binary variables, F_{ist} , L_{ist} , Z_{ijst} and ZF_{ijst} , in the control horizon;
- STEP 4.5. Solve the reduced MILP model (4.53) for the control horizon;
- STEP 4.6. Free all binary variables E_{ist} , F_{ist} , L_{ist} , Z_{ijst} and ZF_{ijst} in the future time periods in the control horizon, $t^* < t < t^* + L^{CH}$.

Table 4.12 Comparison between the MILP model and hierarchical approach.

		MILP	Hierarchical
Objective		295,915	241,117
CPU (s)		2,045	161
Profit (k\$)		340,379	283,029
Revenue (k\$)		640,785	629,021
Production cost (k\$)		145,227	146,202
Changeover cost (k\$)		49,230	92,220
Transportation cost (k\$)		105,778	106,988
Lost sales cost (k\$)		171	581
Inventory deviation (ton)	Plant	1,319	111
	DC	7,206	2,333
	Market	7,668	5,646
Price change ^a (k\$/ton)		398	2,169

^a Total absolute price change between two consecutive time periods.

From the comparison in Table 4.12, the CPU time of the hierarchical approach is much faster than the original MILP model as expected, and the inventory deviations are lower in all three echelons. However, the optimal objective value and profit obtained from the hierarchical approach are both around 20% lower, and the price change is about 5 times higher, compared with the single-level MILP model. Also, there is much more lost sales from the hierarchical approach. It is also worth noting that the total changeover cost in the optimal solution of the hierarchical approach is almost doubled. It is obvious that the hierarchical approach generates much more changeovers, which result in much less profit and objective value. Thus, it is proved

that the consideration of the sequence-dependent changeovers simultaneously with other constraints in the MILP model is necessary, in spite of at the cost of computational complexity.

4.7 Concluding Remarks

In this chapter, an MPC approach for a multi-echelon, multiproduct supply chain has been presented to maintain the inventory and price levels at the maximum profit under demand uncertainty with price elasticity of demand and sequence-dependent changeovers. In the MPC approach, an MILP model has been proposed with an objective including the profit, inventory deviations from the trajectories, and price changes, in which the production, changeover, transportation and lost sales costs are considered.

The proposed MPC approach has been applied to a supply chain example. The length of control horizon with the best performance was selected. From the results, the inventory deviations at all three echelon of the supply chain are small. The effect of weights on both the profit and inventory deviation was investigated. The increased weights on inventory deviation and on price change both have a negative effect on the profit, while they have opposite effects on the inventory deviation. Comparing four pricing strategies, the proposed pricing strategies with price change control, which avoid the great fluctuation of the prices, were recommended, instead of the free pricing and fixed pricing strategies. Moreover, comparing with a hierarchical approach, the importance of changeover constraints in the proposed MILP model is verified. Overall, the proposed MPC approach successfully maximise the supply chain profit with the maintenance of stable inventory levels and price levels.

MULTIOBJECTIVE OPTIMISATION OF SUPPLY CHAIN PLANNING WITH CAPACITY EXPANSION

The criterion of a successful supply chain is more than one. Usually a supply chain considers multiple performance measures to direct their decision makings, in which cost, responsiveness, and customer service level are the crucial ones. The work in this chapter is inspired by a real-world case study of a global supply chain of an agrochemical company that considers cost, responsiveness and customer service level as separate criteria for the optimal production, distribution and capacity planning.

In this chapter, we aim to develop a multiobjective optimisation framework for a production, distribution and capacity planning of a global supply chain for agrochemicals, with cost, responsiveness and customer service level as the objective functions.

5.1 Introduction and Literature Review

Most literature models only consider single criterion for the supply chain planning and optimisation, such as cost (Tsiakis et al., 2001; Yılmaz and Çatay, 2006; Georgiadis et al., 2011), profit (Verderame and Floudas 2009) and net present value (NPV) (Papageorgiou et al., 2001; Laínez et al., 2009; You et al., 2010).

In the literature, cost is the most commonly used criterion for supply chain performance. The profit of a firm is directly affected by the cost of its operations.

Thus, its importance and influence to the whole performance is quite obvious and is the most significant direct kind of measurement (Chan, 2003).

Responsiveness is regarded as another important performance measure of a supply chain in a rapid changing market environment. A firm with a responsive supply chain can meet the market demand in shorter lead times and react quickly to the customer needs. How to develop a responsive supply chain has been widely studied (Gunasekaran et al., 2008). It is commonly regarded that the responsiveness and cost-efficiency conflict with each other. A responsive supply chain usually has a higher cost, while a cost-efficient supply chain often operates at the expense of market responsiveness (Randall et al., 2003).

Another fundamental characteristic determining the performance of a supply chain is customer service level (Wang, 2001), which measures the percentage of customer demand satisfied on time. A low customer service level may cause the loss of sales or customers, which results in profit loss for the whole supply chain.

One of the earliest papers using multiobjective method for supply chain is from Web and Current (1993), who proposed a multiobjective approach for vendor selection, considering three objectives including the purchases cost, number of late deliveries, and rejected units.

In the past decades, a large number of multiobjective optimisation problems and solution methods have been presented in the literature work on supply chain management, including classic supply chains and sustainable supply chains (Barbosa-Póvoa, 2009). Jayaraman (1999) developed a weighted multi-objective model for a service facility location problem to evaluate tradeoff between demand coverage and the number of facilities. Gjerdrum et al. (2001) aimed to reduce operating cost, while maintaining customer order fulfilment at a high level for a supply chain network. Mathematical programming model is developed to determine the production schedules in the supply chain, while multi-agent techniques are used to determine tactical decisions to simulate and control the supply chain network.

Chen et al. (2003) formulated a multiobjective MINLP production and distribution planning model for a fair profit distribution in a supply chain network. In this work,

the profit of each participant enterprise, customer service level and safe inventory level were treated as objectives, and a two-phase fuzzy decision-making method was proposed as the solution procedure. The authors later extended their own work by taking into account uncertain product prices and demands (Chen and Lee, 2004). A fourth objective of robustness of selected objectives to demand uncertainties were also considered. Hugo et al. (2005) proposed an MILP-based multiobjective model for the strategic investment planning and design of hydrogen supply chains, considering both investment and environmental criteria. Hugo and Pistikopoulos (2005) considered the life circle assessment together with the strategic investment decisions for the design and planning of supply chain networks. The proposed multiobjective MILP model was reformulated as a multi-parametric problem and solved by parametric optimisation algorithms (Dua and Pistikopoulos, 2000).

Amodeo et al. (2007) developed a simulation-based multi-objective optimisation method for the optimisation problem of the inventory policies of supply chains with two objectives for total inventory cost and service level. Roghanian et al. (2007) considered a probabilistic bi-level linear multiobjective programming problem for a supply chain planning and applied fuzzy programming technique adapted from Osman et al. (2004) to solve the this problem. Chern and Hsieh (2007) proposed a heuristic algorithm to solve master planning (MP) problems for a supply chain network, with three objectives including delay penalties, the use of outsourcing capacity, and the total cost. Lakhdar et al. (2007) developed a multi-objective long-term planning MILP model for biopharmaceutical manufacture in multiple facilities via goal programming, with cost, service level and capacity utilisation as objectives.

Pokharel (2008) optimised the operation cost and delivery reliability in a two-objective decision-making model for a supply chain network design problem using STEP algorithm proposed by Benayoun et al., (1971). Raj and Lakshminarayanan (2008) improved the performance of supply chain networks by optimising three multifaceted performance characteristics, such as customer satisfaction, back order and excess inventory, using hyper-space diagonal counting method introduced by Agrawal et al. (2004). Selim et al. (2008) developed a multi-objective MILP model for collaborative production-distribution planning problem in decentralised supply chains using fuzzy goal programming approach. The objectives used included profits

of the manufacturer and distribution centres, costs and backlogs of retailers. Liang (2008) developed a fuzzy multi-objective LP model with piecewise linear membership function to simultaneously minimise total cost and total delivery time of a multiproduct and multi-time period supply chain, adopting the fuzzy goal programming method from Hannan (1981).

Extending their previous work (Torabi and Hassini, 2008), Torabi and Hassini (2009) considered four objectives, including the total cost of logistics, the total value of purchasing, the defective items and the late deliveries, in a multi-echelon supply chain planning problem. A fuzzy goal programming-based approach was proposed, based on the work of Bellman and Zadeh (1970) and Zimmermann (1978). Sabio et al. (2010) addressed the strategic planning of hydrogen supply chains for vehicle use under uncertainty in the operating costs. A multiobjective multi-scenario stochastic MILP formulation was proposed to consider the minimisation of the expected total discounted cost and the worst case value. A two-step sequential approach was presented in which the problem was decomposed into two hierarchical levels. Pinto-Varela et al. (2011) used an optimisation approach adapted from symmetric fuzzy linear programming (SFLP) (Zimmermann, 1978) to solve a bi-objective MILP model for the planning and design of supply chains considering both economic and environmental aspects.

Apart from the solution methods mentioned above, the ε -constraint method has widely been used in the literature to generate Pareto-optimal solutions for multiobjective supply chain planning problems. Sobri and Beamon (2000) developed an integrated multiobjective model for simultaneous strategic and operational planning of a four-echelon supply chain. A deterministic strategic sub-model is developed to optimise the SC configuration and material flow and a stochastic operational level sub-model is integrated to accommodate uncertainty with cost, customer service level, and delivery flexibility as objectives. The ε -constraint method was used to solve the multiobjective problem. Guillén et al. (2005) used NPV, demand satisfaction and financial risk as objectives in the proposed two-stage MILP stochastic model for a supply chain design problem under demand uncertainty, which was solved by ε -constraint method. You and Grossmann (2008) proposed a multi-period MINLP model for supply chain design and planning under both responsive

and economic criteria with demand uncertainties. The ε -constraint method was also used to generate the Pareto-optimal curve with respect to the net present value and expected lead time of the whole supply chain network. The same authors extended their own work (You and Grossmann, 2011) to model the multi-echelon stochastic inventory system of a supply chain with the incorporation of the concept of guaranteed service approach. Guillén-Gosálbez and Grossmann (2009) addressed the optimal design and planning of chemical supply chains under uncertainty in the life cycle inventory. The proposed bi-objective deterministic MINLP model was formulated as a parametric model using the ε -constraint method and then solved by a decomposition technique.

The optimal design and planning problem of hydrogen supply chain production-distribution network for vehicle use was addressed by Guillén-Gosálbez et al. (2010), using the ε -constraint method to solve a proposed MILP model to minimise cost and environmental impact. Franca et al. (2010) used the ε -constraint method to solve a multi-objective stochastic model maximising both profit and Sigma quality function (by minimising the total number of defects in raw material obtained from the suppliers) of the supply chain. Duque et al. (2010) incorporated the eco-indicator 99 methodology into a model for the design and planning of industrial networks. The proposed MILP model was solved by the ε -constraint method to assert the economic and environmental optimal trade-off solution.

Form the above literature review, little work has done to consider three important performance measures, cost, responsiveness and customer service level, simultaneously, which are all crucial to the supply chain design and planning. The objective of the work in this chapter is to develop a multiobjective MILP-based optimisation model and solution procedures for a global supply chain planning problem considering the above three measure criteria.

5.2 Problem Statement

The global supply chain network of an agrochemical company consists of one active ingredient (AI) production plant, several formulation plants in different regions and a number of market regions. The products are divided into several product groups. Each plant can produce products in suitable product groups.

The production and transportation costs of AI are included in the raw material cost, which also includes the cost of other ingredients of final products. In the plants, the final products are formulated. Transportation costs and times occur when the products are shipped from plants to market regions for sale. When the products are imported into the market, duties are also charged. It is assumed that all inventories are held at the markets. The supply chain network is illustrated in Fig 5.1.

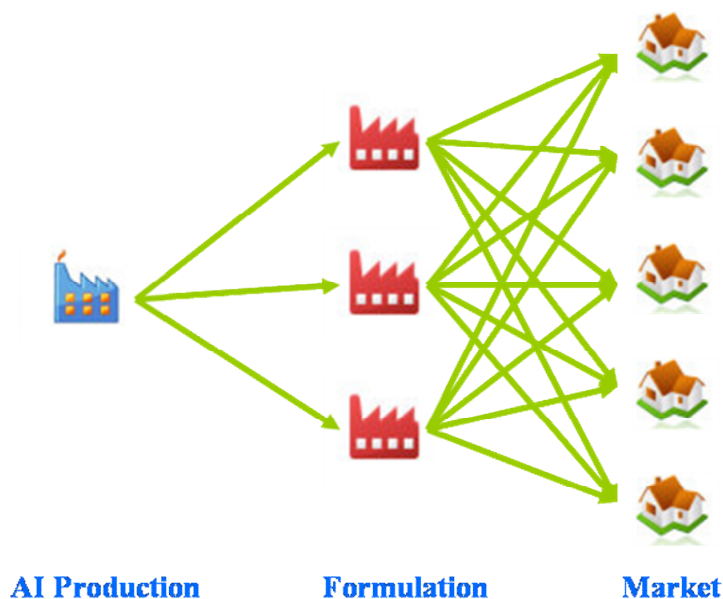


Figure 5.1 Supply chain network of an agrochemical company.

In this problem, we consider the production and distribution planning of an agrochemical supply chain. It is assumed that the original capacities of formulation plants can not satisfy the requirement of rapidly increased demand. So, the capacity planning is also considered here. There are two optional capacity expansion strategies: proportional and cumulative capacity expansion. In the proportional capacity expansion (PCE), the maximum capacity increment of each formulation plant is proportional to its capacity before expansion, which means that the formulation plants with larger capacities before expansion have more ability for expansion. In the cumulative capacity expansion (CCE), the cumulative capacity increment of all formulation plants is limited with respect to the cumulative capacity before expansion. The capacity of each formulation plant after expansion is independent of its capacity before expansion. The new cumulative capacity is reallocated to all formulation plants. A $x\%$ proportional (or cumulative) capacity

expansion will make the capacity increment of single plant (or multiple plants) be up to $x\%$ of the original capacity of single plant (or multiple plants).

To clarify the difference between the two expansion strategies, we take the example below. Formulation plants F1 and F2 have original capacities of 500 and 1000 mu (mass units). A 50% PCE, will allow F1's capacity to be up to 750 mu, and F2's capacity to be up to 1500 mu. While a 50% CCE will allow the total capacity of two plants to be up to 2250 mu. So after CCE, F1's capacity can be up to 1250 mu (capacity 1 in Fig 5.2), and F2's capacity can be up to 1750 mu (capacity 2 in Fig. 5.2). Note that under both two expansion strategies, the capacity of each plant cannot be reduced.

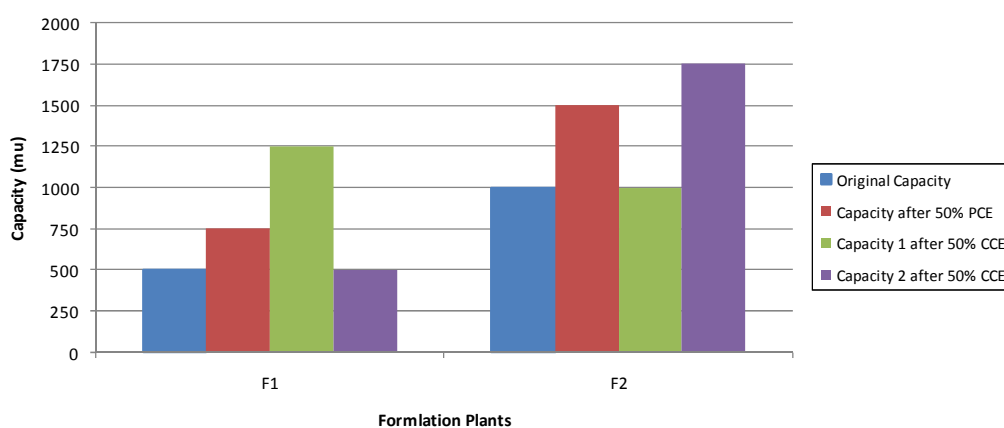


Figure 5.2 Capacity expansion strategies comparison: PCE vs CCE.

The objective of this problem is to find the optimal production, distribution and capacity planning of the supply chain network considering the cost, responsiveness and customer service level simultaneously. For the cost, we consider the total cost of the supply chain, including the raw material cost, formulation cost, transportation cost, inventory cost, and duties cost. It should be mentioned that due to that the capital cost of capacity expansion is not much dependent on the formulation plant locations, the long-term capacity expansion decisions is not affected by the capital cost. Thus, this work provides a strategic insight for the long-term capacity expansion planning decisions without considering the capacity expansion capital cost. To find a responsive supply chain, the total flow time is optimised in the model, which is equal to the product flow multiplied by the corresponding transportation

time from formulation plants to markets. Also, the total lost sales is minimised to obtain a better customer service level.

In this problem, given are the products, groups, formulation plants, markets, weekly demands, capacities and capabilities of formulation plants, unit raw material costs, and formulation costs of products, unit transportation costs/times and duties from plants to markets, initial inventory and inventory limits, and safety stocks, to determine the optimal productions, flows, inventory levels, and sales, so as to minimise:

- the total cost, including raw material cost, formulation cost, transportation cost, inventory cost and duties;
- the total flow time;
- and the total lost sales.

5.3 Mathematical Formulation

The supply chain planning problem is formulated as a multiobjective MILP problem, with the notations, constraints and objective functions as follows.

5.3.1 Nomenclature

Indices

g	product group
i	product
s	formulation plant
m	market
t	time period

Sets

G_s	set of product groups which can be processed at formulation plant s
I_g	set of products in product group g
M_i	set of markets for product i
M_s	set of markets which are served by formulation plant s
S_g	set of formulation plants which can process product group g
S_m	set of formulation plants which serve market m

Parameters

Cap_s	capacity at plant s before expansion
D_{ist}	demand of product i in market m in time period t
DC_{ism}	unit duties cost product i from plant s to market m
FFC_{is}	fixed formulation cost of product i at plant s
FTC_{ism}	fixed transportation cost of product i from plant s to market m
INV_{in}^0	initial inventory of product i at market m
INV_{in}^{\max}	maximum inventory capacity of product i at market m
IC_{im}	inventory cost of product i at market m
MC_{is}	unit material cost of product i at plant s
NN	safety stock coverage (in time periods)
r_{ism}	duty rate of product i from plant s to market m
SS_{imt}	safety stock requirement of product i at market m in time period t
T	total time periods
TDC	total duties cost
TFC	total formulation cost
TIC	total inventory cost
TMC	total raw material cost
TTC	total transportation cost
VFC_{is}	unit variable formulation cost of product i at formulation plant s
VTC_{ism}	unit variable transportation cost of product i from plant s to market m
α_i	coefficient for material cost for product i in the duty function
β_i	coefficient for variable formulation cost for product i in the duty function
γ_i	coefficient for variable transportation cost for product i in the duty function
σ^{\max}	maximum CCE rate
σ^{\min}	minimum CCE rate
θ_s^{\max}	maximum PCE rate for each plant s
θ_s^{\min}	minimum PCE rate for each plant s
ϕ_s	minimum capacity utilisation factor for each plant s
τ_{sm}	transportation time from plant s to market m

Binary Variables

E_{is}	1 if product i is produced in formulation plant s , 0 otherwise
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W_{ist}	1 if product i is assigned to formulation plant s for formulation in time period t , 0 otherwise
X_{ism}	1 if product i is assigned to the shipment from formulation plant s to market m , 0 otherwise
Y_{ismt}	1 if product i is shipped from formulation plant s to market m in time period t , 0 otherwise

Continuous Variables

$ECap_s$	capacity of formulation plant s after expansion
F_{ismt}	flow of product i from formulation plant s to market m in time period t
IS_{imt}	inventory shortage of product i at market m in time period t
INV_{imt}	inventory of product i at market m in time period t
LS_{imt}	lost sale of product i at market m in time period t
P_{ist}	amount of product i manufactured at formulation plant s in time period t
Sa_{imt}	sales of product i at market m in time period t
z_1	objective, total cost
z_2	objective, total flow time
z_3	objective, total lost sales

5.3.2 Production and Flow Constraints

If product i is allocated to formulation plant s for production during time period t , the formulated amount should be limited by the minimum and maximum production limits:

$$P_{is}^{\min} \cdot W_{ist} \leq P_{ist} \leq P_{is}^{\max} \cdot W_{ist}, \quad \forall s, g \in G_s, i \in I_g, t \quad (5.1)$$

As there is no inventory available at the plant, the amount of product i manufactured at formulation plant s during time period t equals to the total flows shipped from this formulation plant to all markets with demands.

$$P_{ist} = \sum_{M \in M_i \cap M_s} F_{ismt}, \quad \forall s, g \in G_s, i \in I_g, t \quad (5.2)$$

If product i is shipped from formulation plant s to market m during time period t , the shipment volume should be limited the minimum and maximum flow limits; otherwise, i.e. $Y_{ismt} = 0$, it is forced to zero:

$$F_{ism}^{\min} \cdot Y_{ismt} \leq F_{ismt} \leq F_{ism}^{\max} \cdot Y_{ismt}, \quad \forall s, g \in G_s, i \in I_g, M \in M_i, t \quad (5.3)$$

5.3.3 Inventory Constraints

The inventory of product i at market m at the end of time period t is equal to the inventory at the previous time period plus any incoming flows, and minus sales.

$$INV_{imt} = INV_{im}^0 \Big|_{t=1} + INV_{im,t-1} \Big|_{t>1} + \sum_{s \in S_m \cap S_g} F_{ism,t-\tau_{sm}} - S_{imt}, \quad \forall g, i \in I_g, m \in M_i, t \quad (5.4)$$

The inventory of a product at a market should not exceed the maximum capacity.

$$INV_{imt} \leq INV_{im}^{\max}, \quad \forall i, m \in M_i, t \quad (5.5)$$

5.3.4 Inventory Shortage Constraints

It is required that the safety stock of product i at market m at the end of time period t should cover its demands in the following NN time periods, where NN is pre-determined. So, the safety stock is defined as:

$$SS_{imt} = \sum_{t'=t+1}^{t+NN} D_{imt'}, \quad \forall i, m \in M_i, t \quad (5.6)$$

The inventory shortage of each product at each market at the end of each time period is the shortage of inventory level from its safety stock. In this problem, in order to guarantee that the inventory at the planning horizon is sufficient to cover the forthcoming demands, the inventory shortage is only allowed for the first $T-NN$ time periods, but not for the last NN time periods of the planning horizon. The inventory shortages at the end of the first $T-NN$ time periods are calculated by the safety stocks and the inventory levels (Eq. 5.7), and the inventory from the time period $T-NN+1$ should be no less than the safety stock (Eq. 5.8):

$$IS_{imt} \geq SS_{imt} - INV_{imt}, \quad \forall i, m \in M_i, t \leq T - NN \quad (5.7)$$

$$INV_{imt} \geq SS_{imt}, \quad \forall i, m \in M_i, t > T - NN \quad (5.8)$$

5.3.5 Lost Sales Constraints

The sales of each product at each market during each time period should not exceed its corresponding demand:

$$Sa_{imt} \leq D_{imt}, \quad \forall i, m \in M_i, t \quad (5.9)$$

If the sales of product i at market m during time period t is less than its corresponding demand, the unsatisfied amount is lost:

$$LS_{imt} \geq D_{imt} - Sa_{imt}, \quad \forall i, m \in M_i, t \quad (5.10)$$

5.3.6 PCE Constraints

If a formulation plant is proportionally expanded, its capacity increment should be no less than a lower bound, and no greater than an upper bound, which are determined by the capacity before expansion together with the minimum and maximum expansion rates, respectively.

$$\theta_s^{\min} \cdot Cap_s \leq \Delta Cap_s \leq \theta_s^{\max} \cdot Cap_s, \quad \forall s \quad (5.11)$$

5.3.7 CCE Constraints

Under the CCE strategy, the total capacity increment is no more than the total capacity before expansion multiplied by the maximum expansion rate, and is no less than the total current capacity before expansion multiplied by the minimum expansion rate:

$$\sigma^{\min} \cdot \sum_s Cap_s \leq \sum_s \Delta Cap_s \leq \sigma^{\max} \cdot \sum_s Cap_s, \quad (5.12)$$

5.3.8 Capacity Utilisation Constraints

The capacity after expansion is the capacity before expansion plus the capacity increment.

$$ECap_s = Cap_s + \Delta Cap_s, \quad \forall s \quad (5.13)$$

The total production of all products at each formulation plant is not only limited by its capacity after expansion, but also not less than the minimum capacity utilisation.

$$\phi_s \cdot ECap_s \leq \sum_{g \in G_s} \sum_{i \in I_g} P_{ist} \leq ECap_s, \quad \forall s, t \quad (5.14)$$

where ϕ_s is the minimum capacity utilisation factor between the interval $[0, 1]$.

5.3.9 Logical Constraints

If product i is not assigned to formulation plant s , then it should not be produced in formulation plant s during any time period.

$$\sum_t W_{ist} \leq T \cdot E_{is}, \quad \forall s, g \in G_s, i \in I_g \quad (5.15)$$

If the transportation link for product i from formulation plant s to market m is not set up, product i cannot be shipped in any time period.

$$\sum_t Y_{ismt} \leq T \cdot X_{ism}, \quad \forall s, g \in G_s, i \in I_g, m \in M_i \quad (5.16)$$

If product i is not assigned to formulation plant s , its transportation links from formulation plant s to any market m should not be set up.

$$\sum_{M \in M_i \cap M_s} X_{ism} \leq |K_i| \cdot E_{is}, \quad \forall s, g \in G_s, i \in I_g \quad (5.17)$$

5.3.10 Objective Functions

In this problem, three objectives are taken into account:

- z_1 : total cost, including the raw material cost, formulation cost (fixed and variable), transportation cost (fixed and variable), inventory cost and duties;
- z_2 : total flow time, which is equal to the summation of flows from plants to market regions multiplied by their corresponding transportation times;
- z_3 : total lost sales, which is the total unsatisfied demand.

5.3.10.1 Total Cost

Total raw material cost is the summation of unit raw material cost of a product at a formulation site multiplied by its total production volume.

$$TMC = \sum_t \sum_s \sum_{g \in G_s} \sum_{i \in I_g} MC_{is} \cdot P_{ist} \quad (5.18)$$

The formulation cost includes the fixed and variable formulation costs. The fixed formulation cost occurs if a product is allocated to a plant for formulation. The variable formulation cost is determined by the unit variable cost and formulation volume. Thus, the total formulation cost is given by:

$$TFC = \sum_s \sum_{g \in G_s} \sum_{i \in I_g} FFC_{is} \cdot E_{is} + \sum_t \sum_s \sum_{g \in G_s} \sum_{i \in I_g} VFC_{is} \cdot P_{ist} \quad (5.19)$$

The transportation cost consists of the fixed and variable parts as well, which are the fixed transportation cost and link assignments, and the unit variable transportation cost and the flows on the corresponding link, respectively.

$$TTC = \sum_s \sum_{g \in G_s} \sum_{i \in I_g} \sum_{m \in M_i \cap M_s} FTC_{ism} \cdot X_{ism} + \sum_t \sum_s \sum_{g \in G_s} \sum_{i \in I_g} \sum_{m \in M_i \cap M_s} VTC_{ism} \cdot F_{ismt} \quad (5.20)$$

The duties are charged when products are imported into the market counties. The total duty equals the summation of unit duty cost of each product on each link multiplied by its imported amount, i.e. the flows.

$$TDC = \sum_t \sum_s \sum_{g \in G_s} \sum_{i \in I_g} \sum_{m \in M_i \cap M_s} DC_{ism} \cdot F_{ismt} \quad (5.21)$$

The unit duties cost, DC_{ism} , is a function of the raw material cost, variable production cost and variable transportation cost and duty rate:

$$DC_{ism} = r_{ism} \cdot (\alpha_i \cdot MC_{is} + \beta_i \cdot VFC_{is} + \gamma_i \cdot VTC_{ism}), \forall s, g \in G_s, i \in I_g, m \in M_i \cap M_s \quad (5.22)$$

where α_i , β_i and γ_i are coefficients for raw material cost, variable production cost and variable transportation cost, respectively.

The inventory cost is the summation the unit inventory cost of each product at each market multiplied by the inventory level at the end of each time period.

$$TIC = \sum_t \sum_i \sum_{m \in M_i} IC_{im} \cdot INV_{imt} \quad (5.23)$$

The objective function of the total cost is the summation of each cost term given by Eqs. (5.18)–(5.21) and (5.23).

$$z_1 = TMC + TFC + TTC + TDC + TIC \quad (5.24)$$

5.3.10.2 Total Flow Time

The flow time is defined as the flow multiplied by its transportation time. The objective function of total flow time of all products on all links in all time periods is given as follows:

$$z_2 = \sum_t \sum_s \sum_{g \in G_s} \sum_{i \in I_g} \sum_{m \in M_i \cap M_s} \tau_{sm} \cdot F_{ismt} \quad (5.25)$$

5.3.10.3 Total Lost Sales

The objective function of total lost sales is the summation of the lost sales of each product at each market in each time period:

$$z_3 = \sum_t \sum_i \sum_{m \in M_i} LS_{imt} \quad (5.26)$$

5.3.11 Summary

The multiobjective optimisation problem can be expressed as:

$$\min_{\mathbf{x} \in Q} \{z_1(\mathbf{x}), z_2(\mathbf{x}), z_3(\mathbf{x})\} \quad (5.27)$$

where \mathbf{x} is the vector of decision variables and Q is the space of feasible solutions defined by Eqs. (5.1)–(5.5), (5.7)–(5.21) and (5.23)–(5.26). Eq. (5.11) is used only for the PCE strategy, while Eq. (5.12) is used only for the CCE strategy.

5.4 Solution Approaches

A number of solution methods have been developed for multiobjective optimisation problems. These methods can be classified into five categories, including scalar methods, interactive methods, fuzzy methods, metaheuristic methods, and decision aided methods (Collette and Siarry, 2003). The classical methods include ε -constraint, weighted sum, weighted metric, goal programming, lexicographic, etc. (Debb, 2001). Here, we apply two of them; the ε -constraint method and the lexicographic minimax method.

We first review the Pareto optimality in the multiobjective optimisation. Considering a multiobjective optimisation problem with K objective functions as below:

$$\min_{\mathbf{x} \in \Omega} \{\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_K(\mathbf{x}))\} \quad (5.28)$$

where $\mathbf{x} \in \mathbb{R}^q$ is the q -dimensional vector of variables, $\mathbf{f}(\mathbf{x})$ is the vector of K objective functions, and $\Omega \subset \mathbb{R}^q$ is the space of feasible solutions. In most cases, the objective functions conflict with each other, and no solution exists which can optimise all objective functions simultaneously. Thus, the solutions of a multiobjective problem are called as the Pareto-optimal solutions (Pareto, 1906), whose definition is as below:

Definition 5.1 $\mathbf{x}^* \in \Omega$ is called a Pareto-optimal (efficient, non-inferior, or non-dominated) solution of multiobjective problem (5.28), if there does not exist another feasible solution \mathbf{x} such that $f_k(\mathbf{x}) \leq f_k(\mathbf{x}^*)$, $\forall k \in \{1, 2, \dots, K\}$, and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one $j \in \{1, 2, \dots, K\}$.

5.4.1 The ε -Constraint Method

5.4.1.1 Method Overview

In the ε -constraint method, introduced by Haimes et al. (1971) and extensively discussed by Chankong and Haimes (1983), all but one objective are converted into constraints by setting an upper or lower bound to each of them, and only one objective is to be optimised. The multiobjective optimisation problem (5.28) is transformed as follows:

$$\begin{aligned} \min_{\mathbf{x} \in \Omega} \quad & f_k(\mathbf{x}) \\ \text{s.t.} \quad & f_j(\mathbf{x}) \leq \varepsilon_j, \quad \forall j \in \{1, 2, \dots, K\} \setminus \{k\} \end{aligned} \quad (5.29)$$

where only the objective function $f_k(\mathbf{x})$ is minimised, while all the other objective functions are constrained by the corresponding upper bounds.

The Pareto optimality of the solutions of the problem (5.29) follows from the following theorems (Miettinen, 1999):

Theorem 5.1 $\mathbf{x}^* \in \Omega$ is Pareto-optimal if and only if it is the optimal solution of the optimisation problem (5.29) for every $k \in \{1, 2, \dots, K\}$ with $\varepsilon_j = f_j(\mathbf{x}^*)$, $\forall j \in \{1, 2, \dots, K\} \setminus \{k\}$.

Theorem 5.2 $\mathbf{x}^* \in \Omega$ is Pareto-optimal if it is an unique optimal solution of the optimisation problem (5.29) for some k with $\varepsilon_j = f_j(\mathbf{x}^*)$, $\forall j \in \{1, 2, \dots, K\} \setminus \{k\}$.

Theorem 5.3 $\mathbf{x}^* \in \Omega$ is Pareto-optimal if it is an unique optimal solution of the optimisation problem (5.29) for any given upper bound vector $\varepsilon = (\varepsilon_1, \dots, \varepsilon_{k-1}, \varepsilon_{k+1}, \dots, \varepsilon_K)$.

5.4.1.2 Method Implementation

Implementing the ε -constraint method to the proposed multiobjective problem (5.27), we only use z_1 as the objective function, while z_2 and z_3 are transformed into constraints with ε_2 and ε_3 , respectively. Thus, the multiobjective problem (5.27) is transformed into the following single-objective problem:

$$\begin{aligned}
 & \min_{\mathbf{x} \in Q} z_1(\mathbf{x}) \\
 & s.t. \quad z_2(\mathbf{x}) \leq \varepsilon_2 \\
 & \quad \quad z_3(\mathbf{x}) \leq \varepsilon_3
 \end{aligned} \tag{5.30}$$

The value of ε_3 is defined as follows:

$$\varepsilon_3 = \mu \cdot \sum_t \sum_i \sum_{m \in M_i} D_{imt} \tag{5.31}$$

where $\mu \in [0, 1]$ indicates the maximum allowed percentage of total lost sales to total demand.

In order to guarantee that problem (5.30) is feasible, the value of ε_2 is determined based on the value of μ and its corresponding value of ε_3 . The following two subproblems are solved to obtain the maximum and minimum values of ε_2 :

$$\begin{aligned}
 & \min_{\mathbf{x} \in Q} z_1(\mathbf{x}) \\
 & s.t. \quad z_3(\mathbf{x}) \leq \varepsilon_3
 \end{aligned} \tag{5.32}$$

$$\begin{aligned}
 & \min_{\mathbf{x} \in Q} z_2(\mathbf{x}) \\
 & s.t. \quad z_3(\mathbf{x}) \leq \varepsilon_3
 \end{aligned} \tag{5.33}$$

In both problems (5.32) and (5.33), z_3 is limited by ε_3 in the constraints. In problem (5.32), z_1 is the objective, while z_2 is not considered. In problem (5.33), z_1 is not taken into account and z_2 is the only objective. Thus, by solving problem (5.32), the maximum value of z_2 is obtained; while the minimum value of z_2 is determined by problem (5.33) for a given value of ε_3 .

The following approach is implemented to generate several discrete values of ε_2 and the value of ε_3 :

STEP 1. Determine the value of ε_3 ;

STEP 1.1. Initialise L and μ ;

STEP 1.2. Obtain the value of ε_3 by Eq. (5.31);

STEP 2. Determine $L+1$ values of ε_2 ;

STEP 2.1. Initialise $l = 0, w = 0$;

STEP 2.2. Solve problem (5.32) and obtain its optimal solution $\bar{\mathbf{x}}$; let

$$z_2^{\max} = z_2(\bar{\mathbf{x}});$$

STEP 2.3. Solve problem (5.33) and obtain its optimal solution $\hat{\mathbf{x}}$; let

$$z_2^{\min} = z_2(\hat{\mathbf{x}});$$

STEP 2.4. Let $\varepsilon_2^l = w \cdot z_2^{\max} + (1 - w) \cdot z_2^{\min}$;

STEP 2.5. If $w = 1$, stop; else, $l = l + 1$, $w = w + 1/L$, go to STEP 2.4.

Thus, for each value of μ , we can have one value of ε_3 from Eq. (5.31) and $L+1$ values of ε_2 , ε_2^l , $l = 1, \dots, L+1$, from the above approach. By solving the single-objective problem (5.30) with generated pairs of ε_2 and ε_3 , $L+1$ solutions of multiobjective optimisation problem (5.27) are obtained for each scenario. According to Theorem 5.1, each above solution \mathbf{x}^* is Pareto-optimal if it is the solution of the both following two problems:

$$\begin{aligned} \min_{\mathbf{x} \in Q} \quad & z_1(\mathbf{x}) \\ \text{s.t.} \quad & z_2(\mathbf{x}) \leq \varepsilon_2 = z_2(\mathbf{x}^*) \\ & z_3(\mathbf{x}) \leq \varepsilon_3 = z_3(\mathbf{x}^*) \end{aligned} \quad (5.34)$$

$$\begin{aligned} \min_{\mathbf{x} \in Q} \quad & z_2(\mathbf{x}) \\ \text{s.t.} \quad & z_1(\mathbf{x}) \leq \varepsilon_1 = z_1(\mathbf{x}^*) \\ & z_3(\mathbf{x}) \leq \varepsilon_3 = z_3(\mathbf{x}^*) \end{aligned} \quad (5.35)$$

5.4.2 The Lexicographic Minimax Method

5.4.2.1 Method Overview

For some multiobjective optimisation problems, the decision makers do not have preference to any objective, i.e., all the objectives are equally important. In this case, decision makers would like to implement an equitable solution, in which all scaled objective values are equal to each other. As the ε -constraint method discussed above cannot precisely generate such kind of equitable solutions, here we use the lexicographic minimax method, a special case of the ordered weighted averaging (OWA) aggregation (Yager, 1988; Kostreva et al., 2004), to find equitable solutions.

Considering the K objectives are in the same scale, a feasible solution of the multiobjective problem (5.28) is called its minimax solution, if it is an optimal solution to the minimax problem,

$$\min_{\mathbf{x} \in Q} \{ \max_{k=1, \dots, K} f_k(\mathbf{x}) \} \quad (5.36)$$

However, the disadvantage of the minimax problem is that the optimal solution is not unique, and some of them may not be Pareto-optimal. To guarantee that we only select the Pareto-optimal solution from the optimal minimax solutions set, we can solve the following lexicographic minimax problem,

$$\text{lex min}_{x \in \Omega} \{\Theta(\mathbf{f}(x))\} \quad (5.37)$$

where $\Theta: \mathbb{R}^K \rightarrow \mathbb{R}^K$ is a mapping function that nonincreasingly orders the components of vectors. Given a vector $\mathbf{e} = (e_1, \dots, e_K)$, $\Theta(\mathbf{e}) = (\theta_1(\mathbf{e}), \dots, \theta_K(\mathbf{e}))$, where $\theta_k(\mathbf{e}) \in \{e_1, \dots, e_K\}$ is the k th component in vector $\Theta(\mathbf{e})$ and $\theta_1(\mathbf{e}) \geq \dots \geq \theta_K(\mathbf{e})$. For example, if $\mathbf{e} = (5, 3, 8)$, $\Theta(\mathbf{e}) = (8, 5, 3)$. In the lexicographic minimax problem, we minimise the worst objective value firstly, then sequentially minimise the second worst objective value, the third worst objective value, and so on. To connect the problems (5.36) and (5.37), we have:

Theorem 5.4 Each optimal solution of the problem (5.37) is also the optimal solution of the problem (5.36).

The lexicographic minimax solutions satisfy the principles of Pareto-optimality (efficiency) and perfect equity (Ogryczak, 1997). So we have the following theorem:

Theorem 5.5 $\mathbf{x}^* \in \Omega$ is Pareto-optimal with perfect equity $f_1(\mathbf{x}^*) = \dots = f_K(\mathbf{x}^*)$, if it is an optimal solution of the optimisation problem (5.37).

The lexicographic minimax method has been popularly used for a number of allocation problems (Luss, 1999), including resources allocation problem (Klein et al., 1992), bandwidth allocation (Ogryczak et al., 2008; Luss, 2010), waster resources allocation (Wang et al., 2008) and waste management (Erkut et al., 2008).

Ogryczak et al. (2005) transferred lexicographic maximin problem to a lexicographic maximisation problem. Similarly, Erkut et al. (2008) proposed a formulation that transfers a lexicographic minimax problem to a lexicographic minimisation problem. Here, we develop an approach to transfer the lexicographic minimax problem (37) to a minimisation optimisation problem.

First, we define an aggregated criterion $\Phi_n(\mathbf{e}) = \sum_{k=1}^n \theta_k(\mathbf{e})$, $n=1, \dots, K$, which expresses the summation of the first (largest) n components of the vector $\Theta(\mathbf{e})$. Here we let $\Psi(\mathbf{e})$ be the summation of $\Phi_n(\mathbf{e})$. Then, we have

$$\Psi(\mathbf{e}) = \sum_{n=1}^K \Phi_n(\mathbf{e}) = \sum_{n=1}^K \sum_{k=1}^n \theta_k(\mathbf{e}) = \sum_{k=1}^K (K - k + 1) \cdot \theta_k(\mathbf{e}) \quad (5.38)$$

Adapting the formulation by Erkut et al. (2008) which expresses $\Phi_n(\mathbf{e})$ as the objective function of an optimisation problem separately, here for a given vector \mathbf{e} , we formulate $\Psi(\mathbf{e})$ as the optimal objective value of the following optimisation problem:

$$\begin{aligned} \max \quad & \sum_{k=1}^K \sum_{n=1}^K e_m \cdot w_{kn} \\ \text{s.t.} \quad & \sum_{k=1}^K w_{kn} = n, \quad \forall n = 1, \dots, K \\ & w_{kn} \in \{0, 1\}, \quad \forall k, n = 1, \dots, K \end{aligned} \quad (5.39)$$

where w_{kn} is a binary variable and can be relaxed to a continuous variable, i.e. $0 \leq w_{kn} \leq 1$. In order to convert the above maximisation problem to a minimisation problem, we use its dual formulation as follows:

$$\begin{aligned} \min \quad & \sum_{n=1}^K n \cdot \lambda_n + \sum_{k=1}^K \sum_{n=1}^K d_{kn} \\ \text{s.t.} \quad & \lambda_n + d_{kn} \geq e_k, \quad \forall k, n = 1, \dots, K \\ & d_{kn} \geq 0, \quad \forall k, n = 1, \dots, K \end{aligned} \quad (5.40)$$

It should be mentioned that when \mathbf{e} is a variable, the above dual formulation can also overcome the nonlinearity in optimisation problem (5.39).

Thus, $\Psi(\mathbf{f}(\mathbf{x}))$ can be expressed as follows:

$$\Psi(\mathbf{f}(\mathbf{x})) = \min \left\{ \sum_{n=1}^K n \cdot \lambda_n + \sum_{k=1}^K \sum_{n=1}^K d_{kn} : \lambda_n + d_{kn} \geq f_k(\mathbf{x}), d_{kn} \geq 0, \forall k, n = 1, \dots, K \right\} \quad (5.41)$$

From the definition given in Eq. (5.38), for any two vectors \mathbf{e}_1 and $\mathbf{e}_2 \in \mathfrak{R}^K$, $\Psi(\mathbf{e}_1) \leq \Psi(\mathbf{e}_2)$, if and only if there exists $k \in \{1, \dots, K\}$, such that $\theta_k(\mathbf{e}_1) \leq \theta_k(\mathbf{e}_2)$ and $\theta_j(\mathbf{e}_1) = \theta_j(\mathbf{e}_2)$ for all $j < k$. Thus, we have the following theorem:

Theorem 5.6 $\mathbf{x}^* \in \mathcal{Q}$ is an optimal solution of the lexicographic minimax problem (5.37) if and only if it is the optimal solution of the optimisation problem

$$\begin{aligned} \min_{\mathbf{x} \in \mathcal{Q}} \quad & \sum_{n=1}^K n \cdot \lambda_n + \sum_{k=1}^K \sum_{n=1}^K d_{kn} \\ \text{s.t.} \quad & \lambda_n + d_{kn} \geq f_k(\mathbf{x}), \quad \forall k, n = 1, \dots, K \\ & d_{kn} \geq 0, \quad \forall k, n = 1, \dots, K \end{aligned} \quad (5.42)$$

Thus, the lexicographic minimax problem is converted into an optimisation problem, instead of a lexicographic minimisation problem as in Erkut et al. (2008), which needs to solve K optimisation problems iteratively. The proposed approach exhibits computational advantage, especially when the number of objective functions, K , is large.

5.4.2.2 Method Implementation

In the proposed multiobjective problem, a high customer service level is crucial to the company's reputation and long-term benefit. Thus, the customer service level, z_3 , is more important than the other two objective functions, while the other two objective functions share the same importance. In this case, we need an equitable solution between the cost and flow time based on a pre-determined custom service level. So, we firstly transfer the problem (5.27) into a bi-objective problem (5.38) as follows:

$$\begin{aligned} \min_{\mathbf{x} \in \mathcal{Q}} \quad & \hat{\mathbf{z}}(\mathbf{x}) = (\hat{z}_1(\mathbf{x}), \hat{z}_2(\mathbf{x})) \\ \text{s.t.} \quad & z_3(\mathbf{x}) \leq \varepsilon_3 \end{aligned} \quad (5.43)$$

where $\hat{\mathbf{z}}(\mathbf{x})$ is the vector of $\hat{z}_1(\mathbf{x})$ and $\hat{z}_2(\mathbf{x})$, the normalisation of $z_1(\mathbf{x})$ and $z_2(\mathbf{x})$, respectively. The value of ε_3 is determined by Eq. (5.31).

Here, we apply the lexicographic minimax method to have an equitable solution between cost and flow time. A fair Pareto-optimal solution of the above bi-objective problem (5.43) is the solution of lexicographic minimax problem (5.44):

$$\begin{aligned} \text{lex min}_{\mathbf{x} \in \mathcal{Q}} \quad & \Theta(\hat{\mathbf{z}}(\mathbf{x})) \\ \text{s.t.} \quad & z_3(\mathbf{x}) \leq \varepsilon_3 \end{aligned} \quad (5.44)$$

where $\Theta: \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$, $\Theta(\hat{\mathbf{z}}(\mathbf{x})) = (\theta_1(\hat{\mathbf{z}}(\mathbf{x})), \theta_2(\hat{\mathbf{z}}(\mathbf{x})))$ with $\theta_1(\hat{\mathbf{z}}(\mathbf{x})) \geq \theta_2(\hat{\mathbf{z}}(\mathbf{x}))$.

Following the approach discussed above, the lexicographic minimax problem (5.44) can be transformed into the following minimisation problem:

$$\begin{aligned}
\min_{\mathbf{x} \in Q} \quad & \sum_{n=1}^2 n \cdot \lambda_n + \sum_{k=1}^2 \sum_{n=1}^2 d_{kn} \\
\text{s.t.} \quad & \lambda_n + d_{kn} \geq \hat{z}_k(\mathbf{x}), \quad \forall k, n = 1, 2 \\
& d_{kn} \geq 0, \quad \forall k, n = 1, 2 \\
& z_3(\mathbf{x}) \leq \varepsilon_3
\end{aligned} \tag{5.45}$$

Given the value of ε_3 , the following approach is implemented to solve the lexicographic minimax problem:

STEP 1. Normalise $z_1(\mathbf{x})$ and $z_2(\mathbf{x})$;

STEP 1.1. Solve problem (5.32) and obtain its optimal solution $\bar{\mathbf{x}}$; let

$$z_1^{\min} = z_1(\bar{\mathbf{x}}), \quad z_2^{\max} = z_2(\bar{\mathbf{x}});$$

STEP 1.2. Solve problem (5.33) and obtain its optimal solution $\hat{\mathbf{x}}$; let

$$z_1^{\max} = z_1(\hat{\mathbf{x}}), \quad z_2^{\min} = z_2(\hat{\mathbf{x}});$$

STEP 1.3. Define $\hat{z}_1(\mathbf{x}) = \frac{z_1(\mathbf{x}) - z_1^{\min}}{z_1^{\max} - z_1^{\min}}$ and $\hat{z}_2(\mathbf{x}) = \frac{z_2(\mathbf{x}) - z_2^{\min}}{z_2^{\max} - z_2^{\min}}$;

STEP 2. Solve the minimisation problem (5.45).

Following the above steps, we can obtain one equitable Pareto-optimal solution with the two objective values, z_1 and z_2 , which are equal to each other after scaling for a given custom service level.

5.5 A Numerical Example

In this section, we consider an example from a real agrochemical supply chain to illustrate the application of the proposed model and solution approach. In this supply chain example, there are 8 formulation plants worldwide (F1–F8) (Fig. 5.3) for 10 product groups (G1–G10). The formulation capability and capacity before expansion of each plant are presented in Table 5.1. There are 32 products (P1–P32) in the 10 groups (Table 5.2) with demands in 10 region markets (R1–R10) (Fig. 5.4).

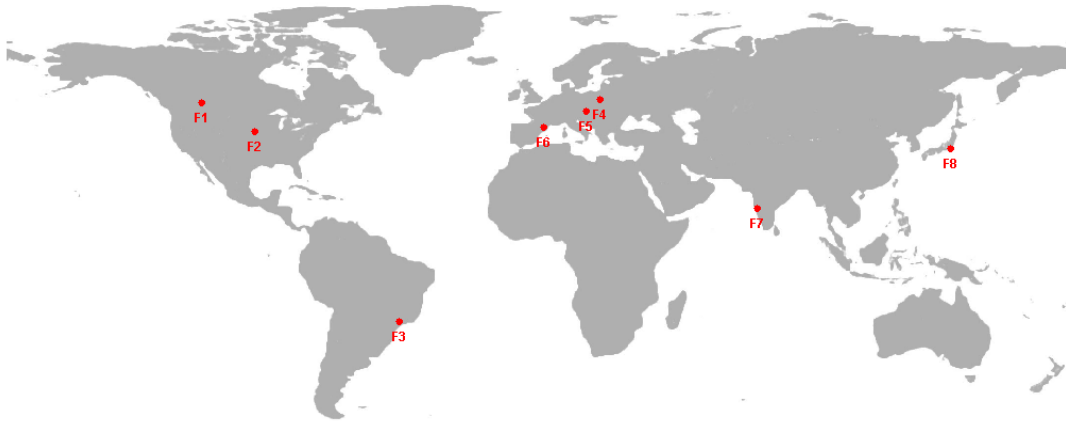


Figure 5.3 Formulation plants in the supply chain example.

Table 5.1 Formulation capability and capacity of each formulation plant.

	F1	F2	F3	F4	F5	F6	F7	F8
G1				√ ^a		√		√
G2	√	√					√	
G3		√	√	√			√	√
G4	√				√	√		
G5		√				√		
G6		√			√			
G7	√		√	√				√
G8						√	√	
G9			√		√			
G10	√					√		√
Capacity (mu/week)	48.1	173.1	38.5	115.4	144.2	115.4	38.5	48.1

^a The product group can be assigned to the formulation plant for production.

Table 5.2 Products in each group.

Group	Product
G1	P1–P4
G2	P5–P6
G3	P7–P10
G4	P11–P14
G5	P15–P20
G6	P21–P22
G7	P23–P25
G8	P26–P27
G9	P28–P30
G10	P31–P32

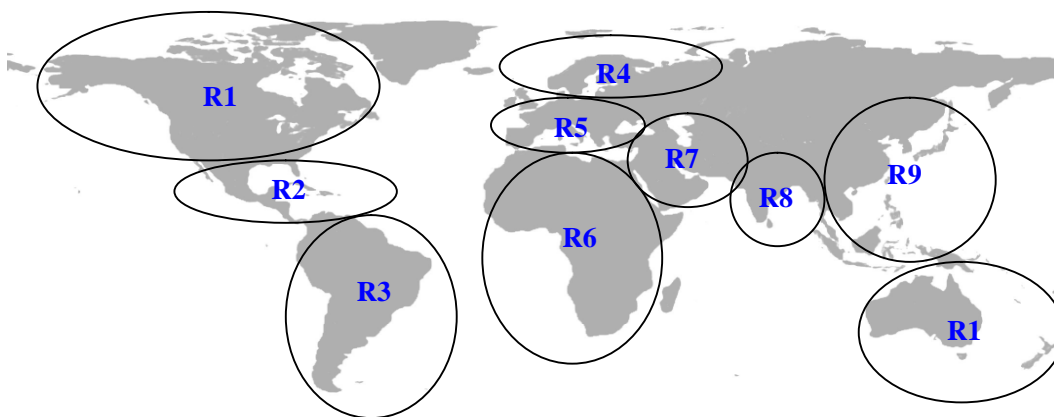


Figure 5.4 Region markets in the supply chain example.

Here, we have weekly demands in a planning horizon of one year, which consists of 52 weeks (time periods). The annual total demand in each market is given in Fig. 5.5. The annual total demand of all products is 59,683.8 mu, while the annual total capacity of all formulation plants (calculated from 5.1) 37,507.6 mu. In order to accommodate all the demand, we assume the capacity increment can be up to the current capacity before expansion (both PCE and CCE), i.e., the maximum expansion rate is equal to 100% for each strategy.

The unit raw material cost of each product at each formulation plant is given in Table 5.3. Table 5.4 presents the unit variable formulation cost of each product at each formulation plant. The fixed formulation cost $FFC = VFC \times 10$ cu.

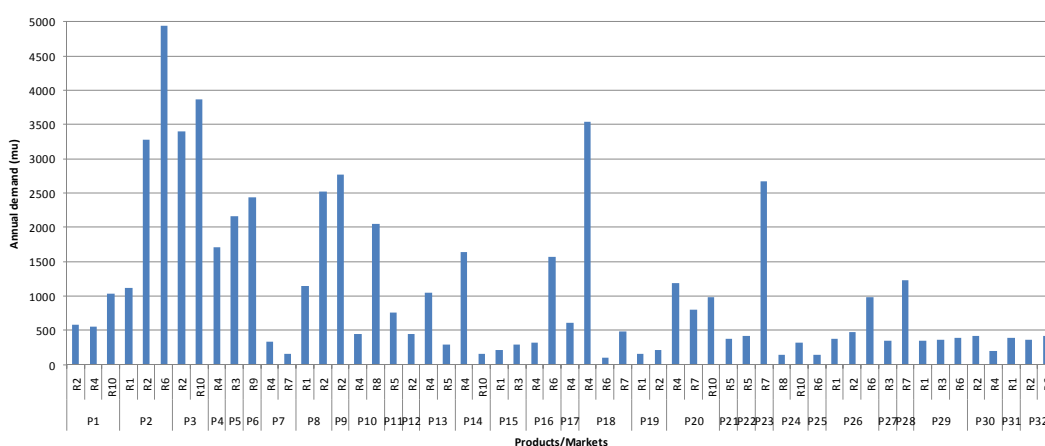


Figure 5.5 Total annual demand of each product in each market.

Table 5.3 Unit raw material cost (cu/mu).

	F1	F2	F3	F4	F5	F6	F7	F8
P1	- ^a	-	61	37	-	86.1	-	63
P2	-	-	62	34	-	84.4	-	63.7
P3	-	-	61	35	-	83.5	-	67.3
P4	-	-	60	30	-	81.5	-	61.4
P5	51.2	56.5	-	-	-	-	20.2	-
P6	40.4	56.6	-	-	-	-	22.5	-
P7	-	52.8	18.7	20	-	-	17.9	47
P8	-	59.1	25.9	24	-	-	22.5	50
P9	-	45.3	30.8	25	-	-	29.8	49
P10	-	53.5	30.8	23	-	-	36	52
P11	25.9	-	-	-	55.2	40.5	-	-
P12	22.5	-	-	-	68.4	30.4	-	-
P13	26.6	-	-	-	69.8	20.4	-	-
P14	20.5	-	-	-	56.4	35.4	-	-
P15	-	30.8	-	-	-	69.1	-	-
P16	-	46.9	-	-	-	56.2	-	-
P17	-	41.6	-	-	-	69.6	-	-
P18	-	35.2	-	-	-	62	-	-
P19	-	33.4	-	-	-	67	-	-
P20	-	35.1	-	-	-	55	-	-
P21	-	37.9	-	-	76.8	-	-	-
P22	-	28.5	-	-	87.1	-	-	-
P23	40.3	-	36.4	86.5	-	-	-	98.5
P24	47.2	-	32.8	84.6	-	-	-	82.6
P25	43.4	-	33.5	80.1	-	-	-	80.3
P26	-	-	-	-	-	29.9	40.6	-
P27	-	-	-	-	-	20	53.1	-
P28	-	-	39.6	-	56.7	-	-	-
P29	-	-	31.5	-	47.5	-	-	-
P30	-	-	39	-	62.9	-	-	-
P31	41.3	-	-	-	-	42.4	-	39.7
P32	37.5	-	-	-	-	37.1	-	33.6

^aNot applicable.

Table 5.4 Unit variable formulation cost (cu/mu).

	F1	F2	F3	F4	F5	F6	F7	F8
P1	- ^a	-	14.4	9	-	12.6	-	5
P2	-	-	16.4	10.6	-	16.6	-	5.4
P3	-	-	13.4	10.2	-	13.2	-	4.6
P4	-	-	16	10.4	-	13	-	5.2
P5	15.2	15.2	-	-	-	-	10.6	-
P6	13.2	16.6	-	-	-	-	12.8	-
P7	-	6.8	9	18.2	-	-	7.2	20.2
P8	-	9.8	8.8	18.4	-	-	8.4	20.6
P9	-	12.8	13.2	18.6	-	-	10.6	21
P10	-	18.6	16.6	19	-	-	22.4	19.6
P11	14.4	-	-	-	13.2	16.6	-	-
P12	15.8	-	-	-	15	18.2	-	-
P13	20.8	-	-	-	21	17	-	-
P14	15.2	-	-	-	12	15	-	-
P15	-	12.8	-	-	-	15	-	-
P16	-	19	-	-	-	15.2	-	-
P17	-	16.4	-	-	-	14.8	-	-
P18	-	15	-	-	-	14.6	-	-
P19	-	15	-	-	-	13.8	-	-
P20	-	12.8	-	-	-	12	-	-
P21	-	21.6	-	-	23.6	-	-	-
P22	-	14.8	-	-	15.4	-	-	-
P23	24.8	-	19.6	18.4	-	-	-	20.2
P24	17	-	19.8	19.4	-	-	-	17.2
P25	11.6	-	14.2	13.8	-	-	-	14.6
P26	-	-	-	-	-	14	12	-
P27	-	-	-	-	-	10.8	10.6	-
P28	-	-	13.4	-	13.2	-	-	-
P29	-	-	13.8	-	15.4	-	-	-
P30	-	-	13.4	-	13.6	-	-	-
P31	31.6	-	-	-	-	35.8	-	33.2
P32	31.2	-	-	-	-	39.4	-	30.2

^aNot applicable.

The transportation time from each formulation plant to each region market is presented in Table 5.5. The value of transportation cost in the unit of cu (currency units) is equal to that of the corresponding transportation time in the unit of week. The fixed transportation cost $FTC = VTC \times 10$ cu.

Table 5.5 Transportation times from formulation plants to markets (week).

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
F1	1	2	3	4	4	5	5	6	6	7
F2	1	2	3	4	4	5	5	6	6	7
F3	3	2	1	2	2	4	4	5	6	6
F4	4	4	2	1	1	3	3	4	5	5
F5	4	4	2	1	1	3	3	4	5	5
F6	4	4	2	1	1	3	3	3	5	5
F7	6	5	4	3	3	3	2	1	2	3
F8	6	6	5	5	5	4	3	2	1	2

The coefficients in the duty function (Eq. 5.22) are all equal to one. Here, it is assumed that all products share the same duty rate if their formulation plants and markets are the same. See Table 5.6 for the duty rates. The unit inventory costs of different products in the same market are assumed to be the same, which are given in Table 5.7. The safety stock at each week should cover the demands for 4 weeks, i.e. $NN = 4$.

Table 5.6 Duty rates from plants to markets (%).

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
F1	0	0	8	6	6	0	0	20	5	0
F2	0	0	8	6	6	0	0	20	5	0
F3	6.5	3	0	2	2	4	4	0	0	0
F4	6.5	3	8	0	0	12	7	20	8	0
F5	6.5	3	8	0	0	12	7	20	8	0
F6	6.5	3	8	0	0	12	7	20	8	0
F7	6.5	3	6.5	6	6	0	0	0	30	0
F8	6.5	3	8	6	6	0	0	20	0	0

Table 5.7 Unit inventory cost in region markets (cu/mu).

R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
1	0.9	0.8	1	0.9	0.5	0.6	0.7	0.8	0.9

5.6 Results and Discussion

We use the two solution methods, the ε -constraint and lexicographic methods, described in the previous section to solve the multiobjective supply chain planning problem. We have used three lost sales levels with $\mu = 1\%$, 3% and 5% , for both 100% PCE and 100% CCE strategies. So, totally six scenarios are investigated. The optimality gap is set to 0.1%.

5.6.1 The ε -Constraint Method

In the ε -constraint method, we let $L = 10$ and obtained 11 solutions by solving Eq. (5.30) with determined ε_2 and ε_3 for each scenario, which are proved to be Pareto-optimal by solving problems (5.34) and (5.35). Fig. 5.6 shows the Pareto-optimal solutions under both the PCE and CCE strategies. It should be noted that the total lost sales, z_3 , of different solutions in the same curve are the same. From the figure, if higher lost sales are allowed, both the total cost and flow time are reduced under both two expansion strategies, which are due to that there are less production and flows, causing lower cost and flow time and higher lost sales.

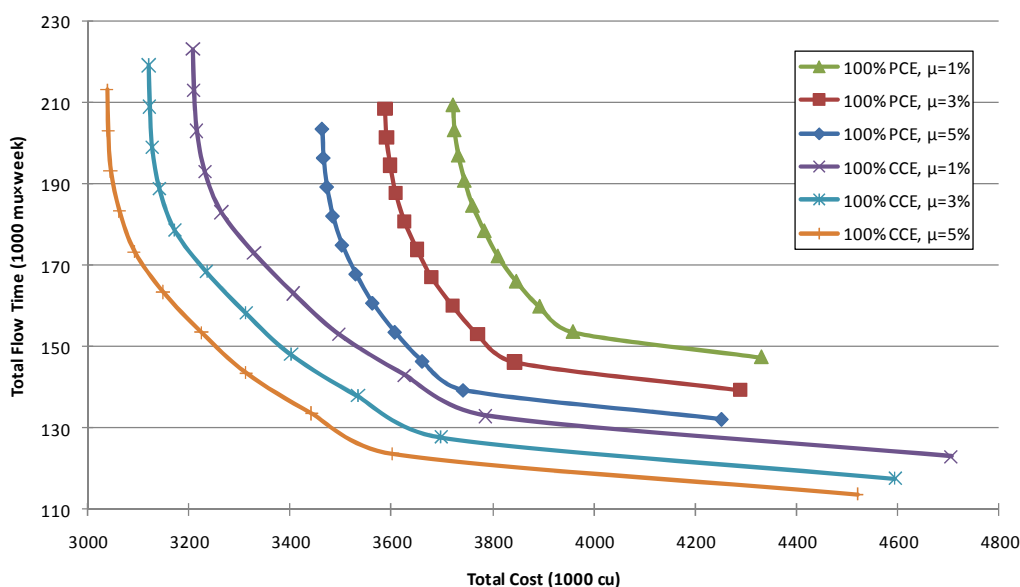


Figure 5.6 The Pareto-optimal solutions from the ε -constraint method.

Here, we examine the two end points on each curve of the Pareto-optimal solutions. The left end of each curve is the solution of single-objective problem (5.32) with the minimum total cost but the maximum total flow time, while the right end of the

curve is the solution of single-objective problem (5.33) with the minimum total flow time but the maximum total cost. In Table 5.8, under the CCE strategy, the difference between the two ends of the curves of Pareto-optimal solutions is much higher than the corresponding difference under the PCE strategy. Thus, the CCE strategy is more sensitive to the trade off between the objectives. Also, in each scenario, comparing the differences of z_1 and z_2 , we can see that the total flow time, z_2 , has a larger difference between the two ends. So, z_2 is more sensitive to the choice of w in the ε -constraint method approach.

Table 5.8 Maximum and minimum values of the Pareto-optimal solution curves.

Capacity expansion strategy	Scenario	Objective	Maximum value	Minimum value	Difference
100% PCE	$\mu=1\%$	z_1 (cu)	4,330,262	3,722,773	14.03%
		z_2 (mu×week)	209,413	147,463	29.58%
	$\mu=3\%$	z_1 (cu)	4,289,884	3,587,889	16.36%
		z_2 (mu×week)	208,555	139,391	33.16%
	$\mu=5\%$	z_1 (cu)	4,251,432	3,462,760	18.55%
		z_2 (mu×week)	203,635	132,256	35.05%
100% CCE	$\mu=1\%$	z_1 (cu)	4,705,432	3,207,789	31.83%
		z_2 (mu×week)	223,198	123,004	44.89%
	$\mu=3\%$	z_1 (cu)	4,595,372	3,120,692	32.09%
		z_2 (mu×week)	219,282	117,654	46.35%
	$\mu=5\%$	z_1 (cu)	4,520,213	3,038,066	32.79%
		z_2 (mu×week)	213,194	113,828	46.61%

In Fig. 5.6, the three Pareto-optimal solution curves under the CCE strategy in all three scenarios lie below the corresponding curves under the PCE strategy. Thus, with the same lost sales level, the CCE strategy can generate solutions with lower cost and lower flow time than the PCE strategy. As the CCE strategy allows the reallocation of the capacity increments with more flexibility, better solutions can be obtained under this strategy.

5.6.2 The Lexicographic Minimax Method

In order to get an equitable trade-off between cost and responsiveness, now we use the lexicographic minimax approach to determine which solution on the Pareto-optimal curve in Fig. 5.6 to be implemented by solving model (5.45). The objective values of the lexicographic minimax solutions are given in Table 5.9.

Table 5.9 Lexicographic minimax solutions.

Capacity expansion strategy	Scenario	Objective values		Scaled objective values	
		z_1 (cu)	z_2 (mu×week)	\hat{z}_1	\hat{z}_2
100% PCE	$\mu=1\%$	3,869,755	162,458	0.2420	0.2420
	$\mu=3\%$	3,751,081	155,470	0.2325	0.2325
	$\mu=5\%$	3,642,510	148,524	0.2279	0.2279
100% CCE	$\mu=1\%$	3,568,900	147,163	0.2411	0.2411
	$\mu=3\%$	3,476,511	142,176	0.2413	0.2413
	$\mu=5\%$	3,390,102	137,429	0.2375	0.2375

Comparing \hat{z}_1 and \hat{z}_2 , the two scaled objectives are equal to each other, which means the two objectives z_1 and z_2 are close to their minimum values equally in term of normalisation. The perfect equality of the scaled objective values is consistent with the conclusion of Theorem 5.5. Fig. 5.7 shows that in all scenarios, the lexicographic minimax solutions are on the Pareto-optimal solution curves, which is justified by Theorem 5.5 and other theoretical work (Marchi and Ovideo, 1992).

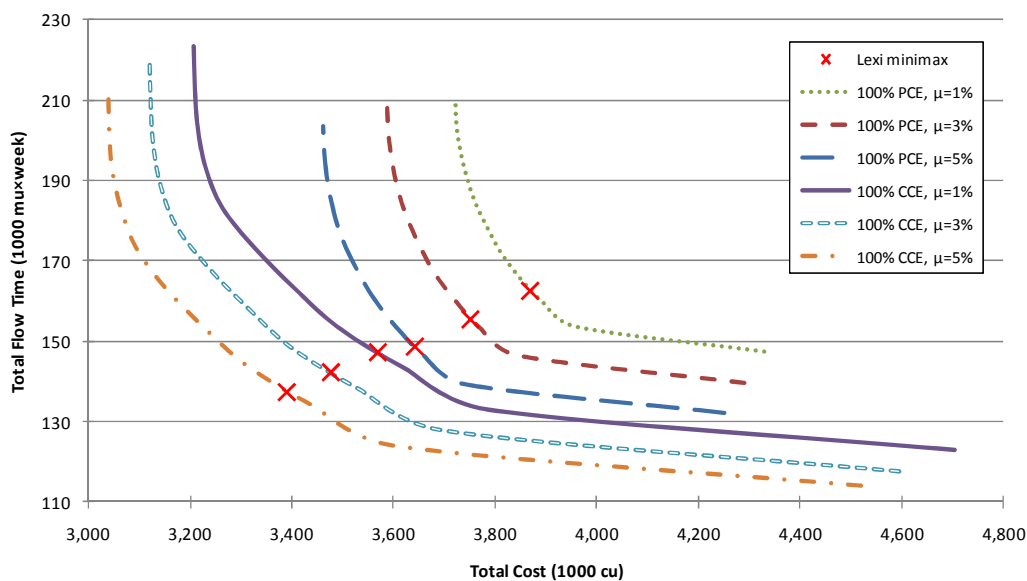


Figure 5.7 The lexicographic minimax solutions on the Pareto-optimal curves.

5.6.3 PCE vs CCE

Next, we examine the capacities of formulation plants under different expansion strategies. We only consider the scenario with $\mu=1\%$. By solving the problem (5.32), the capacity of each formulation plant under two expansion strategies with minimum total cost is given in Fig. 5.8. When minimising the total cost, the capacity of each plant after PCE is doubled, except F5 whose capacity keeps the same. Under the CCE strategy, although the cumulative capacity is doubled, some plants do not have any capacity increment, such as F1, F5, F6 and F8, while some plants, F3, F4, and F7, increase two or three times of their capacities before expansion. Under the PCE strategy, F2, F4 and F6 are the most capacitated formulation plants, while under CCE, F2, F4 and F7 have more capacities than any other plant.

Fig. 5.9 shows the capacities of the plants in the solutions of problem (5.33), i.e., with the minimum flow time. In this case, under the PCE strategy, all the plants expand their capacities. F1, F2 and F5 have the lowest expansion rates, while the other plants have the full 100% expansion. The capacities of F2, F4 and F6 are over 200 mu/week. Under the CCE strategy, the capacities of F1, F2 and F5 do not have any increment. The other plants have more increments compared with those under the PCE strategy. F6 becomes the only plant whose capacity is more than 200 mu/week. Also, both F1 and F5 are not preferred under CCE in both criteria.

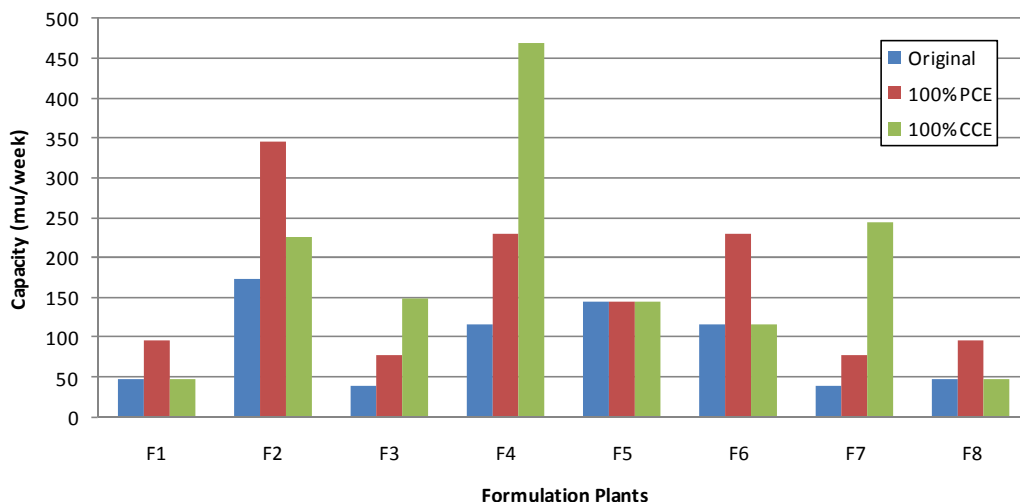


Figure 5.8 Capacity comparisons with the minimum cost ($\mu=1\%$).

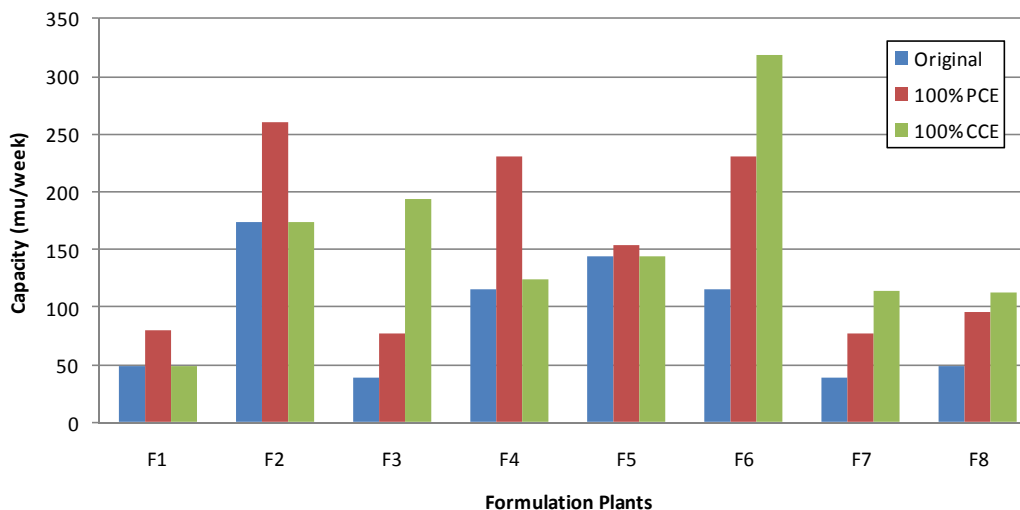


Figure 5.9 Capacity comparisons with the minimum flow time ($\mu=1\%$).

5.6.4 Cost Minimisation vs Flow Time Minimisation

The difference in each formulation plant’s capacity between the solutions with the minimum total cost and total flow time under each capacity expansion strategy is shown in Figs. 5.10 and 5.11. Under the PCE strategy, two criteria generate similar capacities, as there are more limitations on PCE. The significant difference comes from formulation plant F2, which has a higher capacity in the case with the minimum total cost. Under the CCE strategy, different minimisation criteria can generate significant different capacities for formulation plants. We can see that under CCE, F4

has a larger advantage in cost, while F6 contribute most to the flow time minimisation.

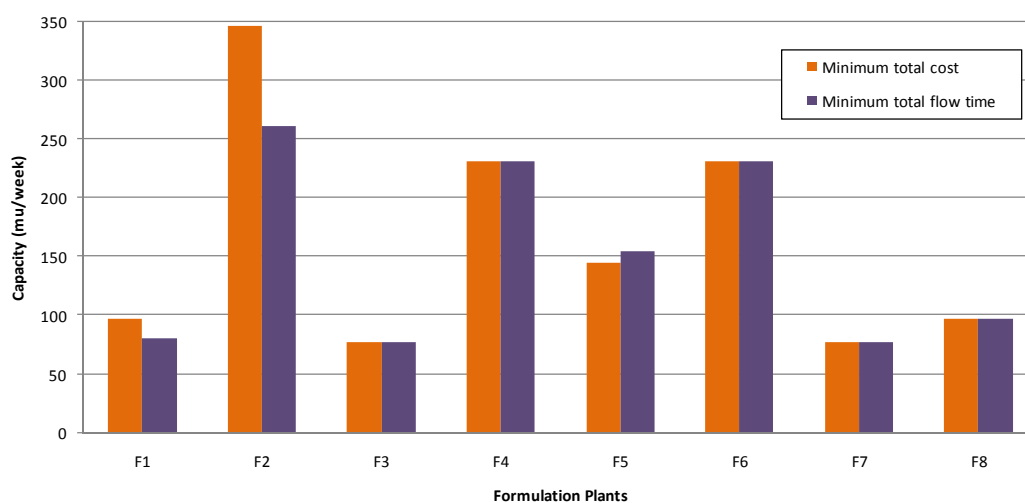


Figure 5.10 Capacity comparisons after 100% PCE ($\mu=1\%$).

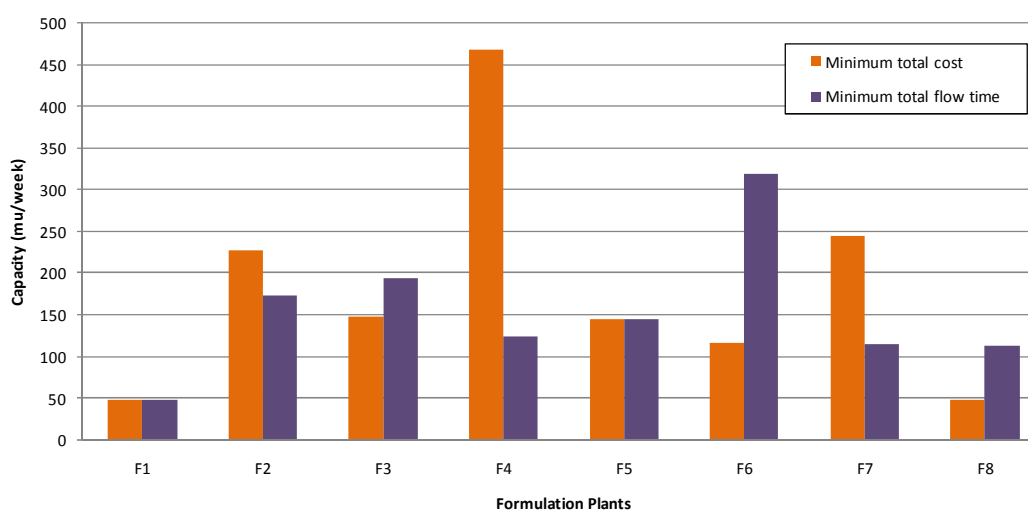


Figure 5.11 Capacity comparisons after 100% CCE ($\mu=1\%$).

Total cost minimisation and total flow time minimisation can also give different optimal flows in the solutions. In Figs. 5.12–5.15, the optimal annual flows with the minimum cost and flow time under both capacity expansion strategies in the scenario $\mu=1\%$ are presented. The solutions with the minimum flow time have fewer long distance flows than those with the minimum cost under both capacity expansions. Meanwhile, the flows with the minimum flow time under the cumulative capacity expansion strategy (Fig. 5.15) have the shortest transportation distance among all the cases, which is another example to show the advantage of CCE.

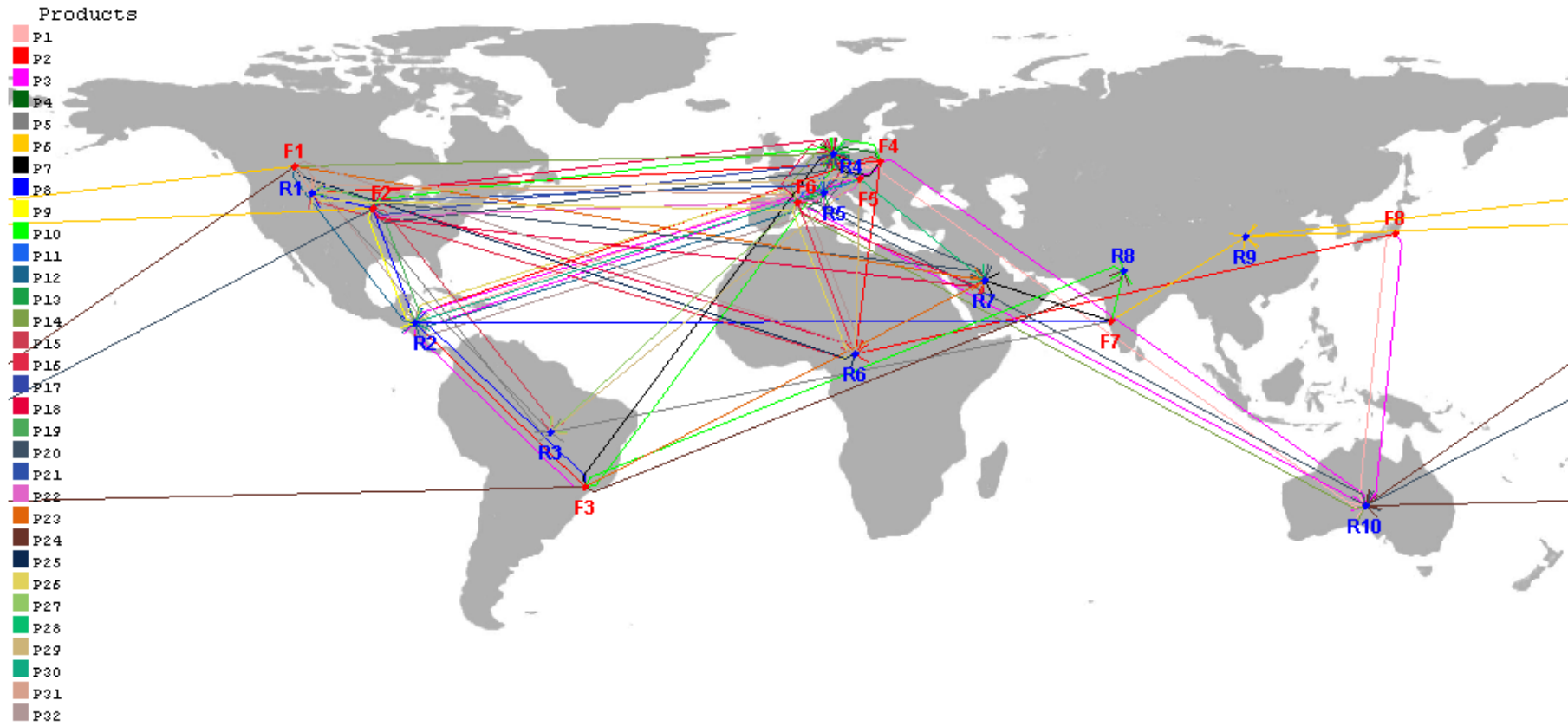


Figure 5.12 Annual flows with the minimum cost after PCE ($\mu=1\%$).

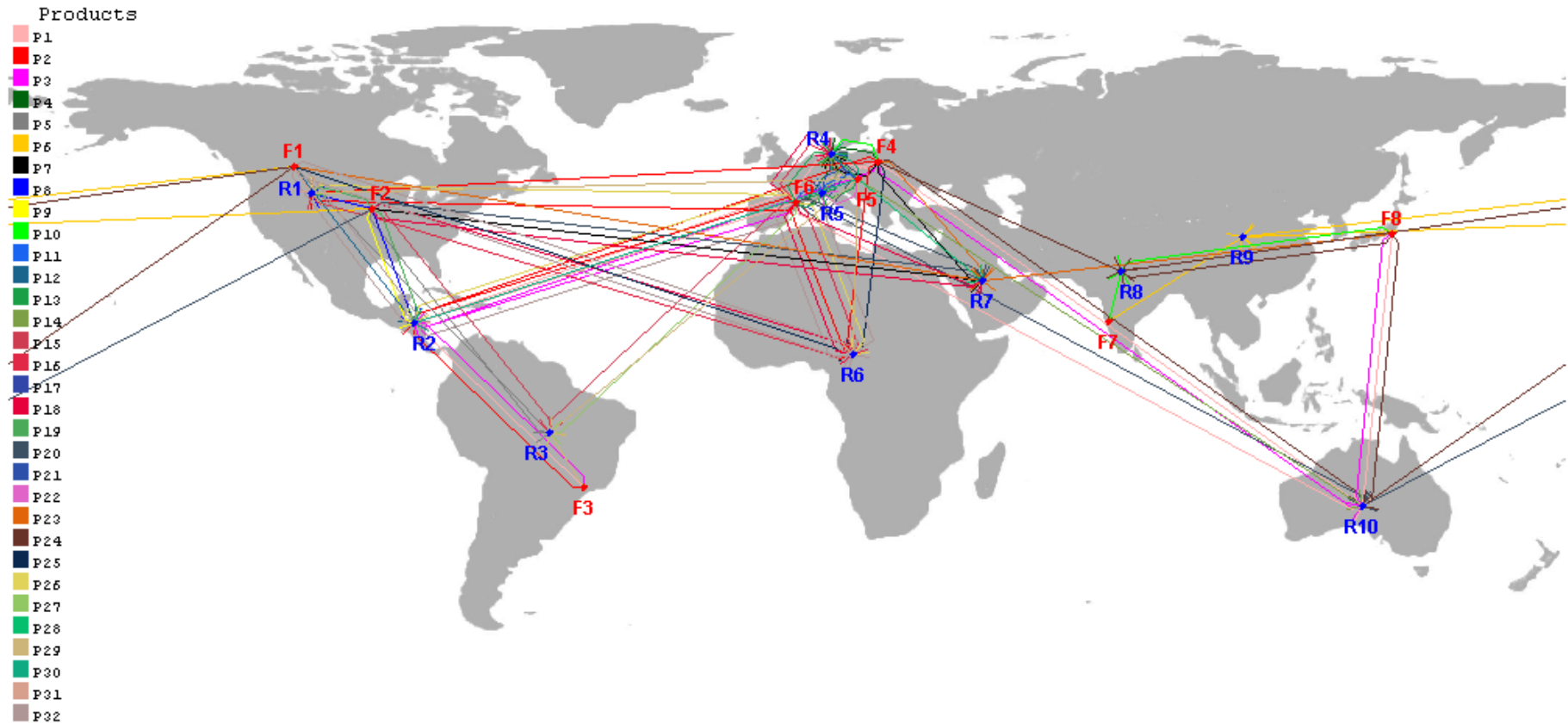


Figure 5.13 Annual flows with the minimum flow time after PCE ($\mu=1\%$).

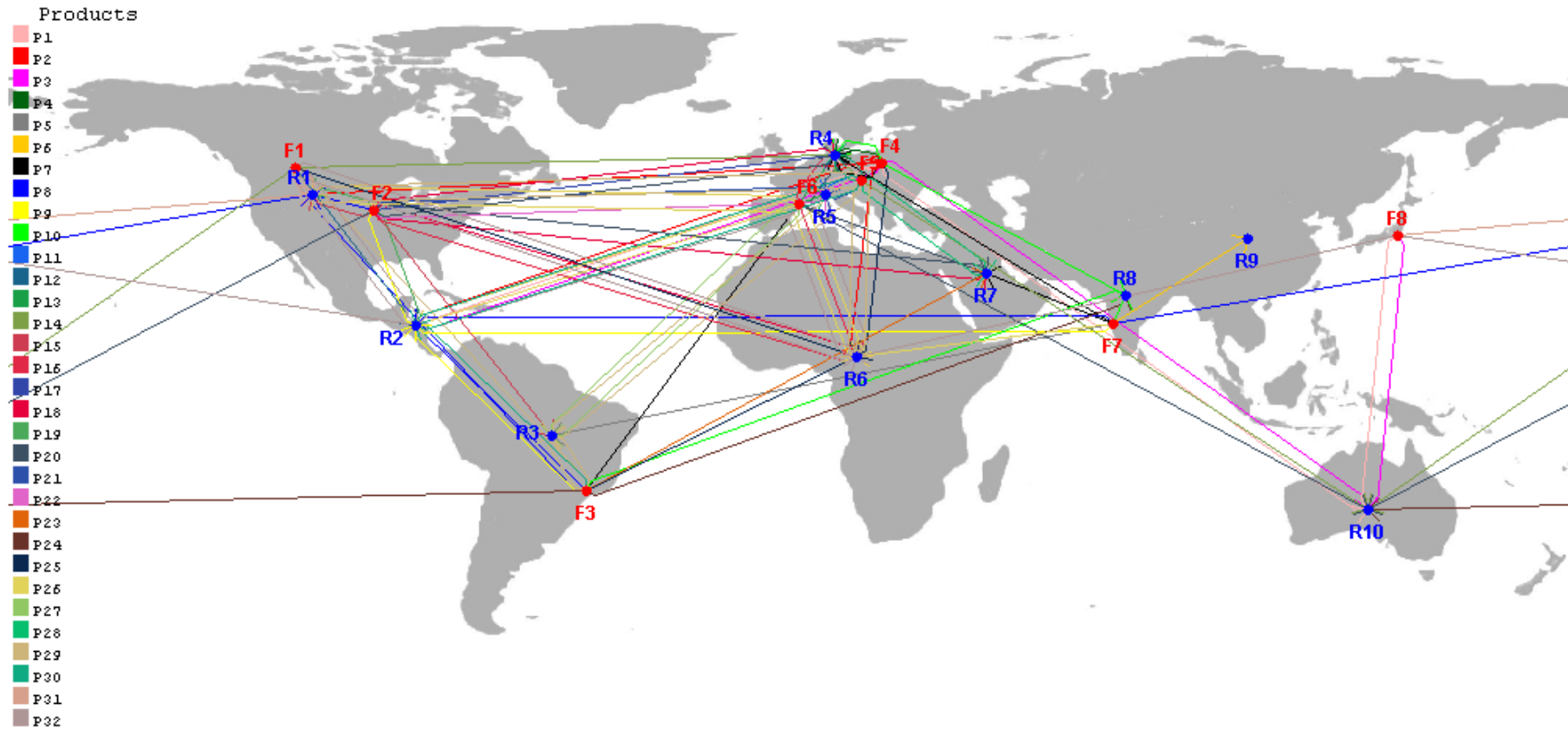


Figure 5.14 Annual flows with the minimum cost after CCE ($\mu=1\%$).

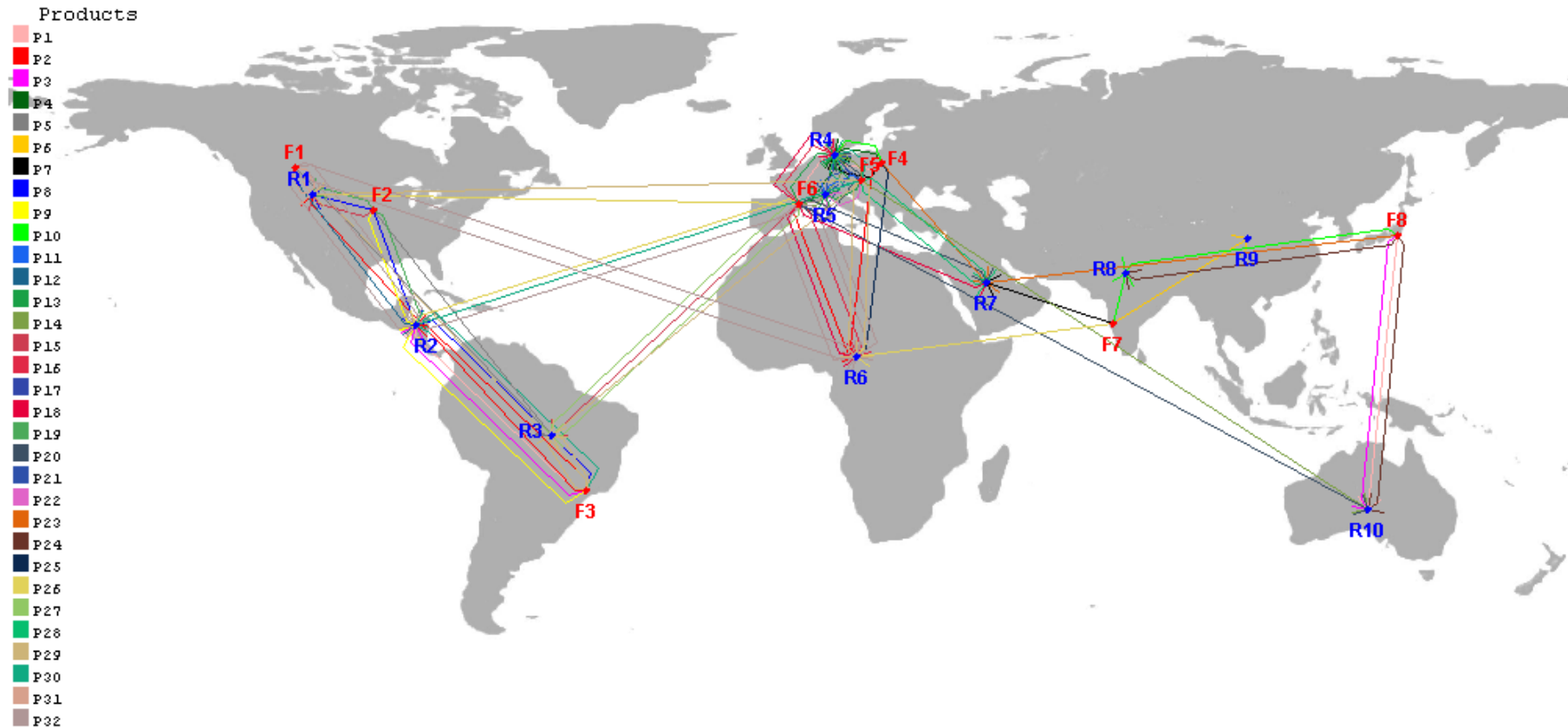


Figure 5.15 Annual flows with the minimum flow time after CCE ($\mu=1\%$).

5.7 Concluding Remarks

In this chapter, a multiobjective MILP model for a global agrochemical supply chain optimisation problem has been proposed. The production, distribution, and capacity expansion decisions have been optimised, considering total cost, total flow time and total lost sales as objectives. Two capacity expansion strategies (proportional and cumulative capacity expansions) have been taken into account.

The ε -constraint method has been adopted to solve the multiobjective optimisation problem, in which total cost is the only single objective to be optimised and total flow time and total lost sales were transformed into constraints. With different levels of total lost sales, the Pareto-optimal solutions between total cost and total flow time were obtained. To obtain an equitable solution, the lexicographic minimax method was also implemented. Adapting literature approaches, a new approach has been developed to transfer lexicographic minimax problem to a minimisation problem. Through a numerical example, we have examined the two capacity expansion strategies. The computational results showed that cumulative capacity expansion generates a better solution. Also, the solutions with the minimum total cost and the minimum flow time have been compared, whose differences showed the advantage of each plant in either cost minimisation or flow time minimisation.

OPTIMISATION OF INTEGRATED WATER RESOURCES MANAGEMENT IN WATER SUPPLY CHAINS

Water is an essential natural resource to the lives on the planet. With the rapid population increase and economic development, more water is needed to meet the increasing demands for irrigation, industry and food, and to satisfy the higher living standards of people (Bouwer, 2000). Lately, water shortage has become a major issue for achieving high living standards and for development, and is regarded as one of the two most worrying problems for this millennium (Kirby, 2000). Management and optimisation of water supply chains is regarded as one of the most difficult and urgent problems, due to the significantly varying water demand and availability (Kondili et al., 2010).

In this chapter, we aim to propose an optimisation-based approach for the integrated water resources management in water supply chains. The proposed approach will be used to apply to real-world case studies of two Greek islands, whose local governments concern the management of the non-conventional water resources at the minimum cost.

6.1 Introduction and Literature Review

To overcome the worldwide water shortage problems, an integrated approach for the sustainable exploitation of all potential water sources is needed. The integrated approach for water resources management is more pronounced in arid or semi-arid water deficit areas, especially in insular areas, where there are few alternatives for

water management (Lazarova et al., 2001). Groundwater is often limited and of poor quality, if it exists, thus it is usually not sufficient to cover increasing water demands (White et al., 2007). Fresh water importation from the mainland using tank boats is a particularly expensive and non-sustainable option (Gikas and Tchobanoglous, 2009b). Non-conventional water resources are expected to play an important role in water management (Gikas and Angelakis, 2009), as water conservation (Bakir, 2001) is usually unable to solve entirely the problem, while massive runoff collection is often expensive, time-consuming, and may also need valuable land if artificial lagoons are to be constructed (Hellenic Ministry for Agriculture, 2002). Thus, desalinated seawater (Khawaji et al., 2008) or brackish water (Jaber and Ahmed, 2004) and reclaimed water from wastewater (Kalavrouziotis and Apostolopoulos, 2007) are the alternative options which may be considered, in conjunction with groundwater.

The existing water treatment technologies are capable of producing even potable water from wastewater (Law, 2003), but it may be expensive and often not acceptable by the public for potable use (Manners and Dowson, 2010). Desalinated and reclaimed water could rather be used in a synergic way. Desalination yields water of potable quality, at a relatively higher cost, both in environmental and in money terms (Karagiannis and Soldatos, 2008), while reclaimed water can be used in non-potable urban, industrial and agricultural applications in relation to its qualitative characteristics (World Health Organisation, 2006), at production cost significantly lower than that of desalinated water (Gikas and Tchobanoglous, 2009b), and is considered as a sustainable, long-term solution to the challenges presented by the growing demand for water (Miller, 2006).

Gikas and Tchobanoglous (2009b) estimated the cost of desalinated and reclaimed water for the islands of the Aegean Sea in Greece, as a function of plant capacity and reclaimed water quality. Reclaimed water storage facilities and distribution network may have a significant contribution to the cost of reclaimed water. Literature work has indicated that decentralised and satellite strategies in water resources management can be particularly beneficial to achieving the optimal management (Gikas and Tchobanoglous, 2009a). However, if reclaimed water is to be used, a dual distribution system should be established (Okun, 1997). Reclaimed water quality is of critical importance for configuring the characteristics of water reclamation plant.

Usually, the design of reclaimed water systems is based on experience and existing data. However, if such data is not readily available, pilot studies may be required (Aggeli et al., 2009).

In the past decade, optimisation techniques have become a valuable tool in the water resources management. Reça et al. (2001) proposed an optimisation model for water optimal allocation planning in complex deficit agricultural water resources systems to maximise overall economic benefits obtained. Georgopoulou et al. (2001) considered brackish water desalination and wastewater treatment, together with aquifer recharge by treated wastewater as an alternative water supply strategy, and developed a decision aid tool for the investigation of the feasibility and applicability of the alternative strategy to be used for economic evaluation of the overall scheme. Wang and Jamieson (2002) presented an objective approach to regional wastewater treatment planning based on the combined use of genetic algorithm (GA) and artificial neural networks (ANN) to minimise the total cost of wastewater treatment with a fixed-emission standard or in-stream water quality requirements.

Voivontas et al. (2003) proposed a mathematical model to identify the economically optimal water supply enhancement to the existing infrastructure of Paros island in Greece. Draper et al. (2003) presented an economic-engineering optimisation model of California's major water supply system. The model was used to suggest water facility operations and allocations so as to maximise the economic value of agricultural and urban water use in California's main intertied water supply system. Later, Medellín-Azuara et al. (2007) applied the same economic model to explore and integrate water management alternatives, such as water markets, reuse and seawater desalination, in Ensenada, Mexico. Leitão et al. (2005) developed a decision support model to trace and locate regional wastewater systems, in terms of number, capacities and locations of wastewater treatment plants and the length of main sewers, based on geographic information systems (GIS) and location models.

Zechman and Ranjithan (2007) applied an extended evolutionary algorithm to generate alternatives (EAGA) to a regional wastewater treatment network design problem. Joksimovic et al. (2008) developed a decision support software (DSS) for water treatment for reuse with network distribution, in which a GA approach is used for the best selection of customers. Han et al. (2008) presented a multiobjective LP

model to allocate various water resources, including groundwater, surface water, reclaimed water, rainwater, seawater, etc., among multiusers and applied it for the water supply and demand in Dalian, China. This work was later extended by incorporating uncertain factors in the model (Han et al., 2011). Cunha et al. (2009) presented an MINLP model for regional wastewater systems planning, as well as the simulated annealing (SA) algorithm developed for solving the model to optimise the layout of sewer networks, the locations of treatment plants, etc., for the wastewater system in a region. Li et al. (2009) developed an inexact multistage joint-probabilistic programming (IMJP) method for the water resources management with uncertainties within a multi-stream, multi-reservoir and multi-period context with facility of MILP techniques.

Liu et al. (2010) presented an optimisation model for the water resources allocation in saltwater intrusion areas, considering three objectives: economic interest, social satisfaction and polluted water amounts. The GA approach was used to solve the model, which was applied to the Pearl River Delta in China. Ray et al. (2010) proposed a static and deterministic LP model to optimise the minimum cost configuration of future water supply, wastewater disposal, and reuse options for a semiarid coastal city, where reclaimed water was included as one viable option for water supply. The integrated optimisation model was applied to Beirut, Lebanon, and the optimal water and wastewater systems were obtained for different scenarios. Kondili et al. (2010) proposed a systemic approach for the optimal planning of water systems with multiple supply sources and multiple users. The benefit from water users and the cost from water sources are considered in the objective function, but the cost for water distribution was not included.

To the best of our knowledge, no literature work so far has considered the management of the production, distribution and storage of desalinated and reclaimed water, as well as the collection and treatment of wastewater, simultaneously, with the integration between potable and non-potable water systems. In this chapter, we consider the management of several water resources, including desalinated seawater, wastewater and reclaimed water. The locations and capacities of the desalination, wastewater treatment and water reclamation plants, the pipeline main networks, and

number and types the pumps and storage tanks for all desalinated seawater, wastewater and reclaimed water are to be optimised.

6.2 Problem Statement

In this problem, we consider an insular and geographically isolated area which is water deficient. The demands can only be satisfied by desalinated seawater, reclaimed water from wastewater and limited groundwater. All other options including freshwater importation and runoff collection are not taken into account.

Based on the population distribution and land terrain, the whole area is divided into several sub-regions. We assume that all the population in each region is located at the relative population centre, with given seasonal needs for potable and non-potable water. In addition, we consider several potential water/wastewater plant locations. The population centres and potential plant locations are called as “nodes” in this chapter. The optimal locations and capacities of desalination, wastewater treatment and water reclamation plants need to be determined in the problem.

The whole water system in the area is divided into non-potable water and potable water systems. In the non-potable water system, wastewater is collected from all possible regions. The collected wastewater undergoes primary and secondary treatment in wastewater treatment plants according to specific quality requirements. Then, part of treated wastewater may need further treatment, at an extra cost, for reclamation, while the rest is disposed into the sea. The reclaimed water could be distributed to other regions to satisfy only non-potable water demands for irrigation, industry, agriculture, etc. In the potable water system, the desalinated water from desalination plants can be distributed to satisfy both potable and non-potable water demands. Groundwater may be used to satisfy both potable and non-potable demands, if available. We assume that there is no water loss during all the processes.

The water demands (potable and non-potable) and wastewater productions vary throughout a year. Based on the demand volumes, the whole year can be divided into a number of time periods. In our case studies, two such time periods have been used: high-demand and low-demand seasons. The daily water demands and wastewater productions are assumed to be the same within each time period.

It is assumed that both qualities of water, and wastewater, are allowed to be distributed to most regions, in order to satisfy all the water demands at the minimum cost. Thus, the infrastructure needs for water distribution and storage, including the pipeline main networks between nodes, pumping stations, and storage tanks, are also optimised in the problem. The pipeline for groundwater conveyance is assumed as existing. However, the fraction of the groundwater pipelines, which could be utilised for desalinated water conveyance, is not considered, as flow directions to population centres are usually opposite (from the sea to population centres for desalinated water, from the hills to the population centres for groundwater). It should be noted that the local water distribution and storage infrastructure within each region is not considered.

Between any two nodes allowed to be connected, “distances”, “pumping distances” and “pumping elevations” are given. In Fig. 6.1, we consider the flow direction from node A to B. The length of the pipeline between A and B is called “distance” (= $a+b+c+d+e+f+g+h+i$ in Fig. 6.1), which is used to calculate the pipe lengths and pipeline cost. The length of the pressurised pipeline is called “pumping distance” (= $a+b+c+d+e$ in Fig. 6.1), and the maximum height that the liquid has to be pumped is called “pumping elevation” (= P_h in Fig. 6.1). The pairwise pumping distances and elevations are required for the calculation of the pumping cost and pumping station cost. Fig. 6.1 also illustrates that the pumping distances and elevations can be positive in both directions of a link.

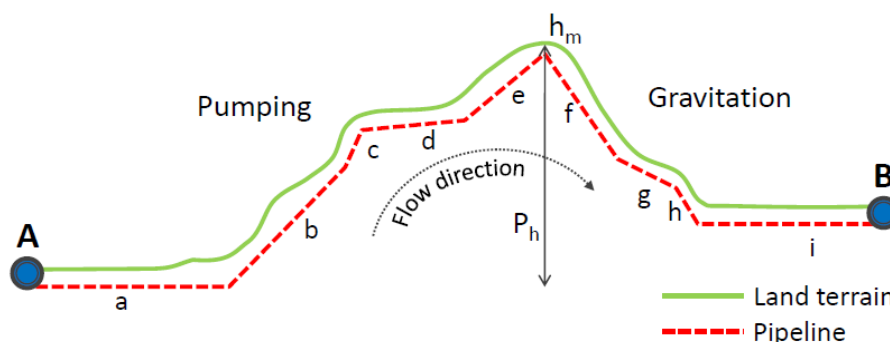


Figure 6.1 Schematic graph for the definition of the terms: “distance”, “pumping distance” and “pumping elevation”.

In the optimisation problem of integrated water resources management, the following are given:

- regions, nodes (population centres and potential plant locations), pairwise distances, pumping distances and elevations between the nodes;
- potable and non-potable water demands, wastewater productions, and available groundwater during each time period;
- capital investment capital costs of desalination, wastewater treatment and reclamation plants at multiple plant capacity levels;
- unit energy consumptions of desalinated water, wastewater treatment and reclaimed water production (additional treatment after wastewater treatment), at multiple production volume levels;
- unit costs of pipelines, dependent on pipe diameter;
- capital costs of storage tank, dependent on tank size;
- types, costs and efficiencies of pumps;
- unit cost of electricity;

to determine:

- locations and capacities of desalination, wastewater treatment, and water reclamation plants;
- pipeline main networks for desalinated water, wastewater and reclaimed water, including piping diameters;
- production volumes of desalinated water, treated wastewater and reclaimed water at plants during each time period;
- main flows of desalinated water, wastewater and reclaimed water during each time period;
- number, types and operating fractions of pumps for each established link;
- number, locations and sizes of storage tanks for potable and non-potable water;

so as to minimise the annualised total cost, including capital and operating costs. The capital cost includes the investment cost for plants, pipelines, pumps, and storage tanks, while the operating cost comprises of plant production operating cost and pumping cost.

6.3 Mathematical Formulation

The integrated water resources management problem is formulated as an MILP optimisation problem. In the proposed MILP model, to avoid the repetition of similar constraints for different types of plants or water/wastewater, superscript w is used to

indicate different types of plants or water/wastewater. Here, w can be dw (for desalinated water or desalination plant), ww (for wastewater or wastewater treatment plant), or rw (for reclaimed water or water reclamation plant).

6.3.1 Nomenclature

Indices

i, j	node
k	breakpoint of piecewise linear function
m	storage tank type
p	pipe type
s	pump type
t	time period
t_i^*	time period with highest water demand in node i
w	water/wastewater (plant) type, = dw , ww or rw

Sets

I	set of nodes
I^{gw}	set of nodes with available groundwater
I^p	set of nodes with potable water demands
I^{np}	set of nodes with non-potable water demands
I^w	set of nodes which are the potential locations of plants w
I^{wp}	set of nodes with wastewater productions
L^w	set of allowed links $\{i, j\}$ for water/wastewater w
K	set of breakpoints
M	set of storage tank types
P	set of pipe types
PL^w	set of allowed links for water/wastewater w where pumps are needed
S	set of pump types
T	set of time periods
W	set of water/wastewater (plant) types, = $\{dw, ww, rw\}$

Parameters

a	conversion factor for flow rate
A_{it}^{gw}	daily available groundwater at node i during time period t (m^3/day)

\tilde{A}_k^w	capacity of plant w at the breakpoint k (m^3/day)
b	conversion constant in the Hazen-Williams equation
B_{it}^{gnp}	1 if groundwater can satisfy non-potable water demand at node i in time period t
B_{it}^{gwp}	1 if groundwater can satisfy potable water demand at node i in time period t
C	roughness coefficient for plastic pipe
CC_k^w	capital cost of plant w at breakpoint k (\$)
D_{it}^{npw}	daily demand of non-potable water at node i during time period t (m^3/day)
D_{it}^{pw}	daily demand of potable water at node i during time period t (m^3/day)
d_p	diameter of pipe in type p (inch)
EC	unit electricity cost (\$/kWh)
g	standard gravity (m/s^2)
H_{ij}	pumping elevation from node i and j (m)
L_{ij}	distance from node i to j (m)
N	a large number
n	duration of project (year)
ND_t	duration of time period t (day/year)
\tilde{P}_k^w	daily production volume of plant w at breakpoint k (m^3/day)
PEC_k^w	energy consumption of plant w at breakpoint k (kWh/m^3)
PLC_p	unit pipeline cost for pipe type p (\$/m)
\tilde{Q}_p^w	flow rate of water/wastewater w in pipe of type p (m^3/day)
r	interest rate
S_{it}^{ww}	daily wastewater supply at node i during time period t (m^3/day)
TC_m	capital cost of one storage tank of type m (\$)
TS_m	size of storage tank of type m (m^3)
v^w	velocity of water/wastewater w (m/s)
α_{ij}	pumping distance from node i to j (m)
β^w	efficiency of pumps for water/wastewater w
θ_s^w	maximum pumping height for pump of type s for water/wastewater w (m)

λ	shell of one pumping station (\$)
μ_s^w	maximum flow rate of pump of type s for water/wastewater w (m^3/day)
ρ	density of water (kg/m^3)
σ_s^w	cost for one pump of type s for water/wastewater w (\$)
τ	water storage coverage time (day)
φ^U	upper bound of groundwater usage fraction

Binary Variables

E_i^w	1 if plant w is allocated at node i , 0 otherwise
X_{it}^w	1 if there is production of plant w at node i during time period t , 0 otherwise
Y_{ijp}^w	1 if pipe of type p is selected for water/wastewater w from node i to j , 0 otherwise
Z_{ijs}^w	1 if pump of type s is selected for water/wastewater w from node i to j , 0 otherwise

Integer Variables

N_{ijs}^w	operating pump number of type s for water/wastewater w from node i to j
TN_{im}^{npw}	storage tank number of type m for non-potable water at node i
TN_{im}^{pw}	storage tank number of type m for potable water at node i

Continuous Variables

A_i^w	capacity of plant w at node i (m^3/day)
$APrOC^w$	annual production operating cost of plant w (\$/year)
$APuOC$	annual pumping operating cost (\$/year)
ATC	annualised total cost, the objective (\$/year)
DS_{it}^{ww}	daily volume of wastewater disposed to the sea at node i during time period t (m^3/day)
O_{it}	daily flow of potable water to non-potable water system from node i to j during time period t (m^3/day)
P_{it}^w	daily production volume of plant w in node i during time period t (m^3/day)
PCC^w	capital cost of plant w (\$)

$PLCC$	pipeline capital cost (\$)
$PSCC$	pumping station capital cost (\$)
$STCC$	storage tank capital cost (\$)
PE_t^w	daily pumping energy for water/wastewater w during time period t (kWh/day)
Q_{ijt}^w	daily flow of water/wastewater w from node i to j during time period t (m ³ /day)
S_{it}^{gnpw}	daily groundwater supply for non-potable water at node i during period t (m ³ /day)
S_{it}^{gpw}	daily groundwater supply for potable water at node i during period t (m ³ /day)
YG_{ijpt}^w	auxiliary variable for the linearization of $Y_{ijp}^w \cdot \gamma_{ijt}^w$
γ_{ijt}^w	operating fraction of pumps for water/wastewater w from node i to j during time period t
ΔH_{ij}^w	head loss of water/wastewater w from node i to j (m)
λ_{ik}^w	SOS2 variable at breakpoint k for capital cost function of plant w at node i
ξ_{itk}^w	SOS2 variable at breakpoint k for production cost function of plant w at node i during time period t
ϕ_{it}	groundwater usage fraction at node i during time period t

6.3.2 Velocity Calculation

At first, the parameter of flow rate of water/wastewater in a pipe, which is related to the velocity of water/wastewater, pipe diameter, is calculated by the following equation:

$$\tilde{Q}_p^w = a \cdot v^w \cdot \pi \cdot \frac{d_p^2}{4}, \quad \forall w \in W, p \in P \quad (6.1)$$

6.3.3 Mass Balance Constraints

Fig. 6.2 illustrates the flow mass balance in both potable and non-potable water systems.

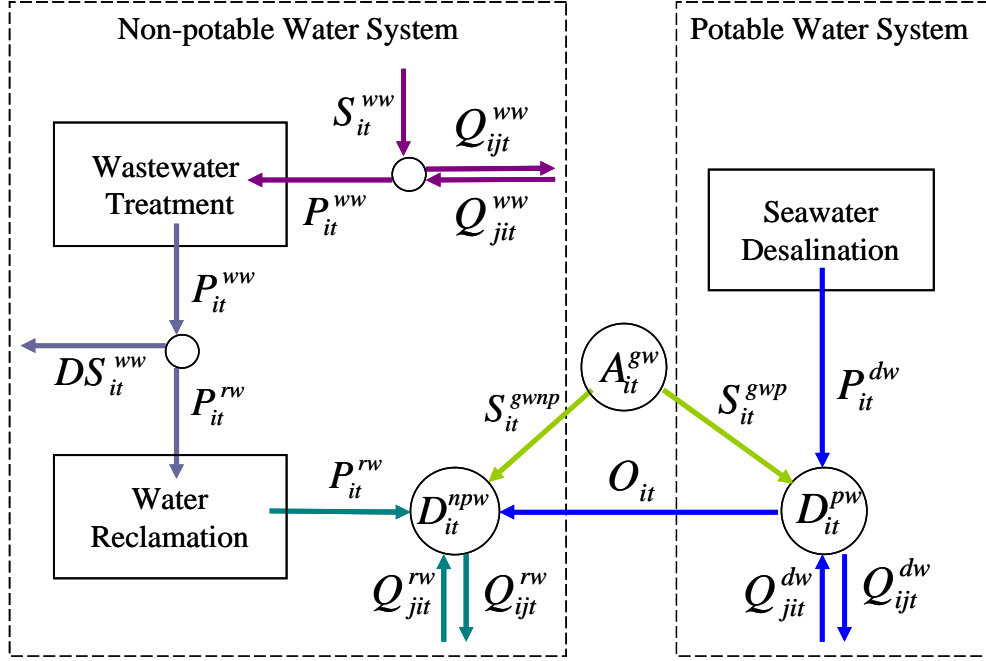


Figure 6.2 Schematic graph of potable and non-potable water systems.

At any node, the desalinated water production and the groundwater supply, plus all incoming/outgoing desalinated water flows, minus the flows to non-potable water system, is equal to the local potable water demand:

$$S_{it}^{gwnp} \Big|_{i \in I^{gwnp}} + P_{it}^{dw} \Big|_{i \in I^{dw}} + \sum_{j: \{j,i\} \in L^{dw}} Q_{jit}^{dw} - \sum_{j: \{i,j\} \in L^{dw}} Q_{ijt}^{dw} - O_{it} = D_{it}^{pw}, \quad \forall i \in I, t \in T \quad (6.2)$$

At any node, the summation of the daily wastewater supply and all incoming/outgoing wastewater flows should be equal to the amount of wastewater treated by the primary and secondary treatment systems:

$$S_{it}^{ww} + \sum_{j: \{j,i\} \in L^{ww}} Q_{jit}^{ww} - \sum_{j: \{i,j\} \in L^{ww}} Q_{ijt}^{ww} = P_{it}^{ww} \Big|_{i \in I^{ww}}, \quad \forall i \in I, t \in T \quad (6.3)$$

At any potential wastewater treatment plant location, the treated wastewater flow is equal to the volume of disposed treated wastewater plus the local reclaimed water production volume:

$$P_{it}^{ww} = DS_{it}^{ww} + P_{it}^{rw} \Big|_{i \in I^{rw}}, \quad \forall i \in I^{ww}, t \in T \quad (6.4)$$

At any node, the reclaimed water production plus the incoming/outgoing reclaimed water and the flows from potable water system is equal to the local non-potable demand:

$$S_{it}^{gwnp} \Big|_{i \in I^{sw}} + P_{it}^{rw} \Big|_{i \in I^{rw}} + O_{it} + \sum_{j:\{j,i\} \in L^{rw}} Q_{jit}^{rw} - \sum_{j:\{i,j\} \in L^{rw}} Q_{ijt}^{rw} = D_{it}^{npw}, \quad \forall i \in I, t \in T \quad (6.5)$$

At any node with groundwater supply, the total exploited groundwater is equal to the maximum available groundwater multiplied by the local groundwater usage rate.

$$S_{it}^{gwp} + S_{it}^{gwnp} = \varphi_{it} \cdot A_{it}^{gw}, \quad \forall i \in I^{sw}, t \in T \quad (6.6)$$

To avoid overexploitation of the aquifer, the groundwater usage fraction, φ_{it} , is limited by an upper bound, $\varphi^U \leq 1$. Also, it is assumed that the local groundwater can only be used for local demand. So, D_{it}^{pw} and D_{it}^{npw} are the upper bounds of S_{it}^{gwp} and S_{it}^{gwnp} , respectively.

6.3.4 Flow Constraints

Here, we introduce γ_{ijt}^w to indicate the pump operating fraction, i.e., the proportion of operating time of a pump during a day. The daily water/wastewater flow at each pumping link, where pumps are needed, is equal to the corresponding flow rate in m^3/day , multiplied by pump time operating fraction.

$$Q_{ijt}^w = \gamma_{ijt}^w \cdot \sum_{p \in P} \tilde{Q}_p^w \cdot Y_{ijp}^w = \sum_{p \in P} \tilde{Q}_p^w \cdot \gamma_{ijt}^w \cdot Y_{ijp}^w, \quad \forall w \in W, \{i, j\} \in PL^w, t \in T \quad (6.7)$$

For the other links where no pump is needed, we use simpler constraints to guarantee that the actual flow does not exceed the allowed flow rate in the selected pipe.

$$Q_{ijt}^w \leq \sum_{p \in P} \tilde{Q}_p^w \cdot Y_{ijp}^w, \quad \forall w \in W, \{i, j\} \in L^w \setminus PL^w, t \in T \quad (6.8)$$

The above nonlinear term $\gamma_{ijt}^w \cdot Y_{ijp}^w$ in Eq. (6.7) can be linearised. Auxiliary continuous variables $YG_{ijpt}^w \equiv Y_{ijp}^w \cdot \gamma_{ijt}^w$ are used to replace the nonlinear term. So Eq. (6.7) is equivalent to the following reformulated constraints, Eqs. (6.9)–(6.11):

$$Q_{ijt}^w = \sum_{p \in P} \tilde{Q}_p^w \cdot YG_{ijpt}^w, \quad \forall w \in W, \{i, j\} \in PL^w, t \in T \quad (6.9)$$

$$YG_{ijpt}^w \leq Y_{ijp}^w, \quad \forall w \in W, \{i, j\} \in PL^w, p \in P, t \in T \quad (6.10)$$

$$\gamma_{ijt}^w = \sum_{p \in P} YG_{ijpt}^w, \quad \forall w \in W, \{i, j\} \in PL^w, t \in T \quad (6.11)$$

6.3.5 Pipeline Network Constraints

There are three individual pipeline main networks to be determined for desalinated water, wastewater, and reclaimed water, respectively. In all pipeline networks, at most one pipe type p can be selected for each link:

$$\sum_{p \in P} Y_{ijp}^w \leq 1, \quad \forall w \in W, \{i, j\} \in L^w : \{j, i\} \in L^w, i < j, \text{ or } \{j, i\} \notin L^w \quad (6.12)$$

The pipeline from node i to j is also the one from node j to i , so Y_{ijp}^w and Y_{jip}^w should always have the same value:

$$Y_{ijp}^w = Y_{jip}^w, \quad \forall w \in W, p \in P, \{i, j\} \in L^w : \{j, i\} \in L^w, i < j, \quad (6.13)$$

6.3.6 Pumping Station Constraints

If there is no pipeline from node i to j , no pumping station should be installed at this link. Also, on each pumping link, at most one type of pump should be used.

$$\sum_{s \in S} Z_{ijs}^w \leq \sum_{p \in P} Y_{ijp}^w, \quad \forall w \in W, \{i, j\} \in PL^w \quad (6.14)$$

If there is no pump installed on the pumping link from node i to j , the corresponding pump operating fraction is zero.

$$\sum_{s \in S} Z_{ijs}^w \geq \gamma_{ijt}^w, \quad \forall w \in W, \{i, j\} \in PL^w, t \in T \quad (6.15)$$

The maximum flow rate of the selected pump should be no less than the flow rate on the corresponding link.

$$\sum_{s \in S} \mu_s^w \cdot Z_{ijs}^w \geq \sum_{p \in P} \tilde{Q}_p^w \cdot Y_{ijp}^w - N \cdot (1 - \sum_{s \in S} Z_{ijs}^w), \quad \forall w \in W, \{i, j\} \in PL^w \quad (6.16)$$

where N is the upper bound of the flow rate, which is equal to $\max_{p, w} \tilde{Q}_p^w$.

Also, the summation of the maximum pumping heights of all pumps in one direction should be no less than the corresponding pumping elevation plus the head loss.

$$\theta_s^w \cdot N_{ijs}^w \geq H_{ij} + \Delta H_{ij}^w - N \cdot (1 - Z_{ijs}^w), \quad \forall w \in W, \{i, j\} \in PL^w, s \in S \quad (6.17)$$

The head loss, ΔH_{ij}^w , is calculated by the Hazen-Williams equation:

$$\Delta H_{ij}^w = b \cdot \alpha_{ij} \cdot \sum_{p \in P} \left(\frac{\tilde{Q}_p^w}{C} \right)^{1.852} \cdot \frac{Y_{ijp}^w}{d_p^{4.87}}, \quad \forall w \in W, \{i, j\} \in PL^w \quad (6.18)$$

where b is a numerical conversion constant, which depends on the units used, and C is the roughness constant, whose value depends on the pipe material. N in Eq. (6.17) is determined by the upper bound of the pumping elevations and head losses. From Eq. (6.18), the head loss from node i to j depends on the pipe diameter selected for the link, so binary variable, Y_{ijp}^w , is included in the equation. Pumping for groundwater is not considered, as it often flows by gravity, after extraction.

6.3.7 Storage Tank Constraints

Storage is considered only for desalinated and reclaimed water. The total selected storage tank sizes should be able to cover demands for the given storage coverage time, τ .

$$\sum_{m \in M} TN_{im}^{pw} \cdot TS_m \geq \tau \cdot D_{it}^{pw} \quad \forall i \in I^{pw}, t = t_i^* \quad (6.19)$$

$$\sum_{m \in M} TN_{im}^{npw} \cdot TS_m \geq \tau \cdot D_{it}^{npw} \quad \forall i \in I^{npw}, t = t_i^* \quad (6.20)$$

6.3.8 Plant Capacity Constraints

The capacity of a plant can be expressed as a linear combination of the capacities at breakpoints:

$$A_i^w = \sum_{k \in K} \tilde{A}_k^w \cdot \lambda_{ik}^w, \quad \forall w \in W, i \in I^w, t \in T \quad (6.21)$$

where λ_{ik}^w is a SOS2 variable and is only activated when the plant is placed at node i :

$$\sum_{k \in K} \lambda_{ik}^w = E_i^w, \quad \forall w \in W, i \in I^w \quad (6.22)$$

6.3.9 Plant Production Constraints

The plant production volume should be limited by its capacity.

$$P_{it}^w \leq A_i^w, \quad \forall w \in W, i \in I^w, t \in T \quad (6.23)$$

Similarly to the plant capacity, the production volume can be expressed as follows:

$$P_{it}^w = \sum_{k \in K} \tilde{P}_k^w \cdot \xi_{ikt}^w, \quad \forall w \in W, i \in I^w, t \in T \quad (6.24)$$

where ξ_{ikt}^w is a SOS2 variable, which is restricted by the following constraint:

$$\sum_{k \in K} \xi_{ik}^w = X_{it}^w, \quad \forall w \in W, i \in I^w, t \in T \quad (6.25)$$

6.3.10 Pumping Energy Constraints

The daily required pumping energy is equal to the energy required to pump the water/wastewater to the pumping elevation plus the head loss, divided by the pump efficiency.

$$PE_t^w = \frac{1}{\beta^w} \cdot \rho \cdot g \cdot \sum_{\{i,j\} \in PL^w} (H_{ij} + \Delta H_{ij}^w) \cdot Q_{ijt}^w, \quad \forall w \in W, t \in T \quad (6.26)$$

In order to linearise the nonlinear term $\Delta H_{ij}^w \cdot Q_{ijt}^w$ in Eq. (6.26), we replace the term ΔH_{ij}^w by rhs of Eq. (6.18), and the term Q_{ijt}^w by rhs of Eq. (6.9). The following constraint, Eq. (6.27), is equivalent to Eq. (6.26):

$$PE_t^w = \frac{1}{\beta^w} \cdot \rho \cdot g \cdot \sum_{\{i,j\} \in PL^w} \sum_{p \in P} [H_{ij} + b \cdot \alpha_{ij} \cdot \sum_{p \in P} \left(\frac{\tilde{Q}_p^w}{C}\right)^{1.852} \cdot \frac{Y_{ijp}^w}{d_p^{4.87}}] \cdot \tilde{Q}_p^w \cdot YG_{ijpt}^w, \quad \forall w \in W, t \in T \quad (6.27)$$

From the definition of YG_{ijpt}^w and the nature of binary variables, it is obvious to obtain that $Y_{ijp}^w \cdot YG_{ijpt}^w = YG_{ijpt}^w$. Thus, we have:

$$PE_t^w = \frac{1}{\beta^w} \cdot \rho \cdot g \cdot \sum_{\{i,j\} \in PL^w} \sum_{p \in P} [H_{ij} + \frac{b \cdot \alpha_{ij}}{d_p^{4.87}} \cdot \left(\frac{\tilde{Q}_p^w}{C}\right)^{1.852}] \cdot \tilde{Q}_p^w \cdot YG_{ijpt}^w, \quad \forall w \in W, t \in T \quad (6.28)$$

6.3.11 Logical Constraints

If plant w is not installed at node i , i.e. $E_i^w = 0$, there is no production in any period:

$$\sum_{t \in T} X_{it}^w \leq N \cdot E_i^w, \quad \forall w \in W, i \in I^w \quad (6.29)$$

where the value of N can be the cardinality of set T , $|T|$, i.e. the total number of time periods.

At a node with potable water demand, if the groundwater is not enough to cover the demand, there should be a desalination plant or desalinated water pipelines connected to other nodes.

$$E_i^{dw} \Big|_{i \in I^{dw}} + \sum_{j: \{j,i\} \in L^{dw}} \sum_{p \in P} Y_{jip}^{dw} + B^{gwp} \geq 1, \quad \forall i \in I^{pw} \quad (6.30)$$

At a node with non-potable water demand, if the groundwater is not enough to cover the non-potable water demand, there should be desalination/reclamation plants, or desalinated/reclaimed water pipelines connected to other nodes.

$$E_i^{dw} \Big|_{i \in I^{dw}} + \sum_{j:\{j,i\} \in L^{dw}} \sum_{p \in P} Y_{jip}^{dw} + E_i^{rw} \Big|_{i \in I^{rw}} + \sum_{j:\{j,i\} \in L^{rw}} \sum_{p \in P} Y_{jip}^{rw} + B^{gnpw} \geq 1, \quad \forall i \in I^{npw} \quad (6.31)$$

At a node with wastewater production, there should be a wastewater treatment plant, or wastewater pipelines connected to other nodes.

$$E_i^{ww} \Big|_{i \in I^{ww}} + \sum_{j:\{i,j\} \in L^{ww}} \sum_{p \in P} Y_{ijp}^{ww} \geq 1, \quad \forall i \in I^{wp} \quad (6.32)$$

If a desalination plant is allocated to a node without potable water demand, the desalinated water pipelines must be built to distribute the desalinated water to other nodes (Eq. (6.33)). Similar constraints are also developed for reclamation plant sites without non-potable water demand (Eq. (6.34)), and wastewater treatment plant sites without wastewater production (Eq. (6.35)).

$$E_i^{dw} \leq \sum_{j:\{i,j\} \in L^{dw}} \sum_{p \in P} Y_{ijp}^{dw}, \quad \forall i \in I^{dw} \setminus I^{pw} \quad (6.33)$$

$$E_i^{rw} \leq \sum_{j:\{i,j\} \in L^{rw}} \sum_{p \in P} Y_{ijp}^{rw}, \quad \forall i \in I^{rw} \setminus I^{npw} \quad (6.34)$$

$$E_i^{ww} \leq \sum_{j:\{i,j\} \in L^{ww}} \sum_{p \in P} Y_{ijp}^{ww}, \quad \forall i \in I^{ww} \setminus I^{wp} \quad (6.35)$$

6.3.12 Objective Function

The annualised total cost in the objective includes the capital and operating costs. In the capital cost, there are following terms:

- Pipeline capital cost, determined by the pipe length and unit cost of each installed pipe, at its selected diameter:

$$PLCC = \sum_{w \in W} \sum_{p \in P} PLC_p \cdot \left(\sum_{\{i,j\} \in L^w : \{j,i\} \in L^w, i < j} L_{ij} \cdot Y_{ijp}^w + \sum_{\{i,j\} \in L^w : \{j,i\} \notin L^w} L_{ij} \cdot Y_{ijp}^w \right) \quad (6.36)$$

- Pumping station capital cost, determined by the number and cost of each pumping station, which includes the cost for two pumps (one for operating and the other for standby) and the shell of the pumping station:

$$PSCC = \sum_{w \in W} \sum_{\{i,j\} \in PL^w} \sum_{s \in S} (2 \cdot \sigma_s^w + \lambda) \cdot N_{ijs}^w \quad (6.37)$$

- Storage tank capital cost, determined by number and cost of each storage tank for both potable and non-potable water:

$$STCC = \sum_{i \in I^{pw}} \sum_{m \in M} TN_{im}^{pw} \cdot TC_m + \sum_{i \in I^{npw}} \sum_{m \in M} TN_{im}^{npw} \cdot TC_m \quad (6.38)$$

- Plant capital cost, as a piecewise linear function of the plant capacity, which is expressed in Eqs (6.21) and (6.22), given capital cost at breakpoints:

$$PCC^w = \sum_{i \in I^w} \sum_{k \in K} CC_k^w \cdot \lambda_{ik}^w, \quad \forall w \in W \quad (6.39)$$

To calculate the annualised capital cost, the capital cost is multiplied by the Capital Recovery Factor (CRF), $r \cdot (1+r)^n / ((1+r)^n - 1)$, where r is the interest rate and n is the project duration.

In the operating cost, there are the following terms:

- Annual pumping operating cost is the summation of daily pumping cost throughout the whole year, which equals to the daily pumping energy multiplied by the electricity cost:

$$APuOC = \sum_{w \in W} \sum_{t \in T} ND_t \cdot EC \cdot PE_t^w \quad (6.40)$$

- Annual production operating cost, as a piecewise linear function of production volume (Eqs. (6.24) and (6.25)), is the summation of daily pumping production cost throughout the whole year, which is the corresponding energy consumption and the electricity cost:

$$APrOC^w = \sum_{i \in I^w} \sum_{k \in K} \sum_{t \in T} ND_t \cdot EC \cdot PEC_k^w \cdot \tilde{P}_k^w \cdot \xi_{itk}^w, \quad \forall w \in W \quad (6.41)$$

The annualised total cost is given as below:

$$ATC = \sum_{w \in W} APrOC^w + APuOC + \left(\sum_{w \in W} PCC^w + PLCC + PSCC + STCC \right) \cdot \frac{r \cdot (1+r)^n}{(1+r)^n - 1} \quad (6.42)$$

6.3.13 Summary

Overall, the discussed integrated water resources management problem is formulated as an MILP model, described by Eqs (6.2)–(6.6), (6.8)–(6.25) and (6.28)–(6.41) as constraints and Eq. (6.42) as the objective function.

6.4 Case Studies

Two Greek islands of Aegean Sea, Syros and Paros (with the neighbouring island of Antiparos) (Fig. 6.3) are investigated as case studies to demonstrate the applicability of the proposed optimisation approach. For each case study, its background and data given at first. Then the optimal solution is presented and discussed. Finally, several alternative scenarios are further investigated. Note that the optimality gap is set to be 2% during all implementations in this chapter.



Figure 6.3 Locations of the islands of Syros and Paros.

6.4.1 Case Study I – Syros Island

6.4.1.1 Background and Data

On Syros island, potable water comes almost exclusively from seawater desalination plants currently. While in areas connected to sewerage system, the wastewater is disposed to the sea after appropriate treatment. Water reclamation does not currently practice on the island. However, the existing infrastructure is not taken into consideration, as the problem is solved on "ground basis". Imported freshwater and groundwater (which in any case is minimal and of non-potable quality) are also not taken into account in this case study. Water demands and wastewater productions

vary with season (with high values occurring during summer and lower during winter).



Figure 6.4 Subdivision of Syros island into 6 regions.

In order to estimate the optimal scenario for Syros island, it is subdivided to 6 regions (Fig. 6.4). All the plants and storage tanks are assumed to be installed in the population centres of the regions. The population centre for each region is at sea level, apart from R1 which is at an elevation of 250 m. The distances, pumping distances and elevations between the population centres of each couple of regions are given in Table 6.1.

Table 6.1 Distances, pumping distances and elevations between two regions of case study I.

	Distance / pumping distance / pumping elevation (km)					
	R1	R2	R3	R4	R5	R6
R1		8/0/0	- ^a	-	-	3.3/0/0
R2	8/8/0.25		5.2/2.3/0.12	9/3.3/0.15	7.3/4/0.18	5.3/3.7/0.26
R3	-	5.2/2.9/0.12		5.3/2/0.02	-	-
R4	-	9/5.7/0.15	5.3/3.3/0.02		5.7/3.7/0.05	-
R5	-	7.3/3.3/0.18	-	5.7/2/0.05		4.2/1.7/0.12
R6	3.3/3.3/0.25	5.3/1.3/0.26	-	-	4.2/2.5/0.12	

^aThe link between these regions is *a priori* not allowed.

Estimated values of seasonal water demand and wastewater production (Vakondios, 2009) are shown in Table 6.2. Here two distinct values are considered: high daily volumes which last for four months, from June to September (summer, 122 days) and low daily volumes, which last for the rest eight months (winter, 243 days).

Table 6.2 Estimated water demands and wastewater productions of case study I.

	Volume per day (summer/winter) (m ³ /day)					
	R1	R2	R3	R4	R5	R6
Potable water demand	150/ 50	4,000/ 2,800	500/ 250	650/ 350	500/ 200	500/ 300
Non-potable water demand	250/ 0	900/ 100	600/ 50	880/ 30	580/ 30	380/ 30
Wastewater production	150/ 50	3,700/ 2,600	200/ 100	300/ 150	300/ 150	450/ 250

The capital costs of plants and unit production energy consumptions at different breakpoints in the piecewise linear functions are given in Tables 6.3 and 6.4, respectively.

Table 6.3 Plant capital costs (k\$).

Volumetric capacity (m ³ /day)	Desalination plant	Wastewater treatment plant	Reclamation plant
100	100	190	80
1000	650	1,300	320
2500	1,500	2,400	800
5000	2,300	5,100	1,200
10000	3,200	10,000	1,600

Table 6.4 Unit energy consumption of water production and treatment (kWh/m³).

Volumetric production (m ³ /day)	Desalination	Wastewater treatment	Reclamation ^a
50	10.0	0.30	0.15
1000	5.0	0.25	0.12
2500	4.0	0.20	0.08
5000	3.5	0.15	0.05
10000	3.0	0.10	0.03

^a Additional cost following standard wastewater treatment.

For the pipeline main network, four potential types of plastic pipes with different diameters and unit installed costs (Gikas and Tchobanoglous, 2009b) have been considered for selection. The flow rates in different pipes (Table 6.5) are calculated

by Eq. (6.1) based on pipe diameters, water/wastewater velocities (0.8m/s and 1.0m/s, respectively) and conversion factor a ($24 \times 3,600 \times 0.0254^2 \approx 55.74$).

Table 6.5 Optional pipes and corresponding flow rates.

Pipe diameter (in)		2.5	4	6	10
Pipe Cost (installed) (\$/m)		55	60	65	70
Corresponding flow rate (m ³ /day)	Desalinated water	218.9	560.4	1,260.9	3,502.4
	Wastewater	273.6	700.5	1,576.1	4,378.0
	Reclaimed water	218.9	560.4	1,260.9	3,502.4

In addition, we have considered four potential sizes of pumps for water (desalinated water and reclaimed water) and wastewater, respectively. Their flow rates, costs, maximum pumping heights and efficiencies are shown in Table 6.6. It is assumed that the shell of each pumping station costs \$11,000.

Table 6.6 Flow rates, costs, maximum pumping height and efficiencies of optional pumps.

Pump flow rate (m ³ /day)		240	720	1,200	2,400
Pump cost (\$)		5,000	10,000	14,000	19,000
Water pump	Maximum pumping height (m)	400	400	400	400
	Efficiency (%)	70	70	70	70
Pump cost (\$)		6,000	19,000	28,000	56,000
Wastewater pump	Maximum pumping height (m)	50	50	50	50
	Efficiency (%)	55	55	55	55

There are also four types of concrete storage tanks to cover the 2-day water demands. The storage tank and costs are given in Table 6.7.

Table 6.7 Sizes and costs of optional concrete storage tanks.

Size (m ³)	50	100	200	500
Cost (\$)	9,500	16,000	41,000	7,6000

The unit electricity cost is \$0.15/kWh. In the Hazen-Williams equation, the roughness constant C for the plastic pipe is equal to 150, and the conversion factor b is equal to $167.5/24^2 \approx 0.452$ (Fujiwara and Khang, 1990). We consider the project duration over a 20-year period with an interest rate of 5%.

6.4.1.2 Results and Discussion

The MILP model for the Syros island case study has 1,624 constraints and 1,891 variables (including 810 binary variables). After a CPU time of 2,120 s, the obtained optimal solution gives an annualised total cost of 2,298,907 \$/year. The breakdown of the optimal annualised total cost is given in Fig. 6.5.

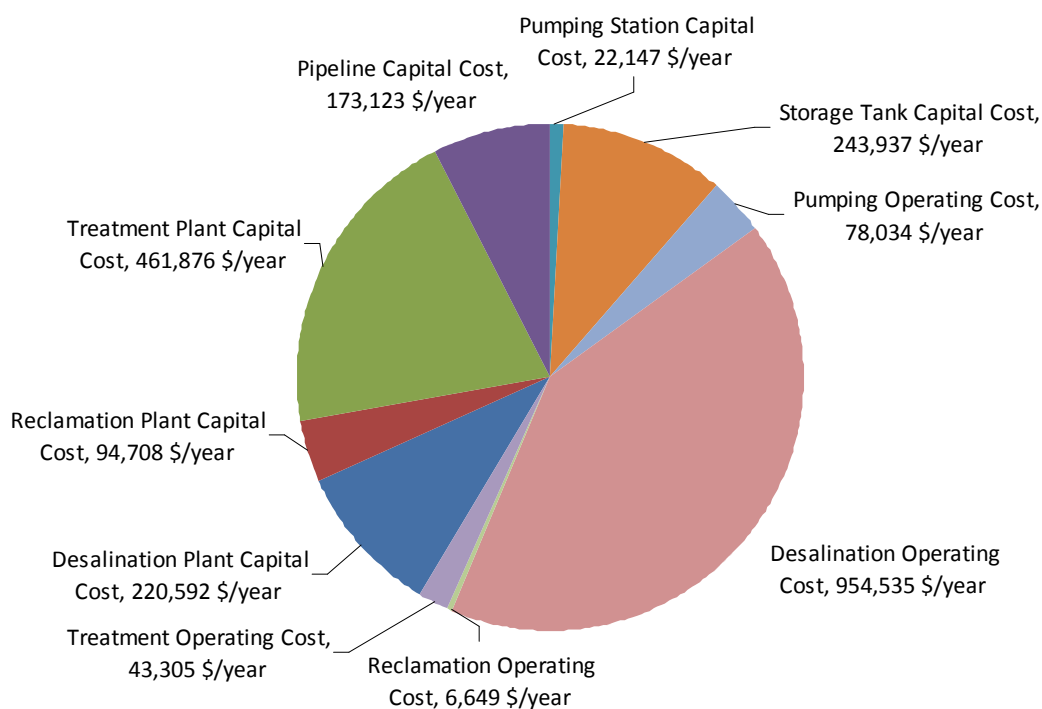


Figure 6.5 Breakdown of the optimal annualised total cost for case study I.

In the optimal solution (Fig. 6.6), the desalination plants are allocated in R1 and R2, and the wastewater treatment plants and reclamation plants are required for all regions, except R3 where no reclamation plant is allocated, which is in agreement with the study of Gikas and Tchobanoglous (2009a). There is no wastewater pipeline network in the optimal solution. The details of the optimal solution are shown in Table 6.8, including information for each established link (water type, pipe type, flow direction, type, number and operating fraction of pumps, and flow volume).

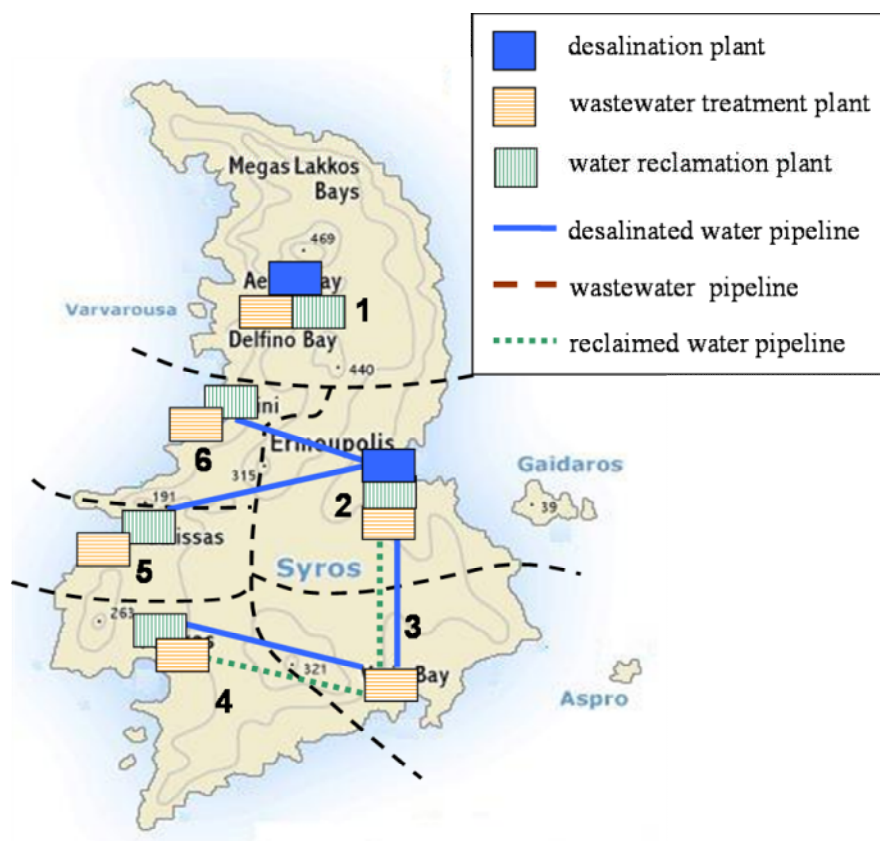


Figure 6.6 Optimal plant locations and pipeline networks for case study I.

Table 6.8 Solution details for each established link for case study I.

Link	Water type	Pipe diameter (in)	Flow direction	Pump max flow rate (m ³ /day)	No. of operating pumps	Pump operating fraction		Flow volume (m ³ /day)	
						Summer	Winter	Summer	Winter
R2--R3	dw ^a	6	R2→R3	2,400	1	0.91	0.48	1,150.0	600.0
R2--R5	dw	6	R2→R5	2,400	1	0.62	0.18	780.0	230.0
R2--R6	dw	4	R2→R6	720	1	0.89	0.59	500.0	330.0
R3--R4	dw	6	R3→R4	2,400	1	0.52	0.28	650.0	330.0
R2--R3	rw ^b	6	R2→R3	2,400	1	0.94	0.06	1,180.0	80.0
R3--R4	rw	6	R3→R4	2,400	1	0.46	0.02	580.0	30.0

^a dw: desalinated water.

^b rw: reclaimed water.

The daily production of desalinated water is shown in Fig. 6.7, in which most desalinated water is generated in R2. The reclaimed water and disposed treated wastewater daily volumes are shown in Fig. 6.8. The wastewater treatment plant in R2 has the highest treatment capacity, 3700 m³/day in summer and 2600 m³/day in winter. In R3, all treated wastewater is disposed.

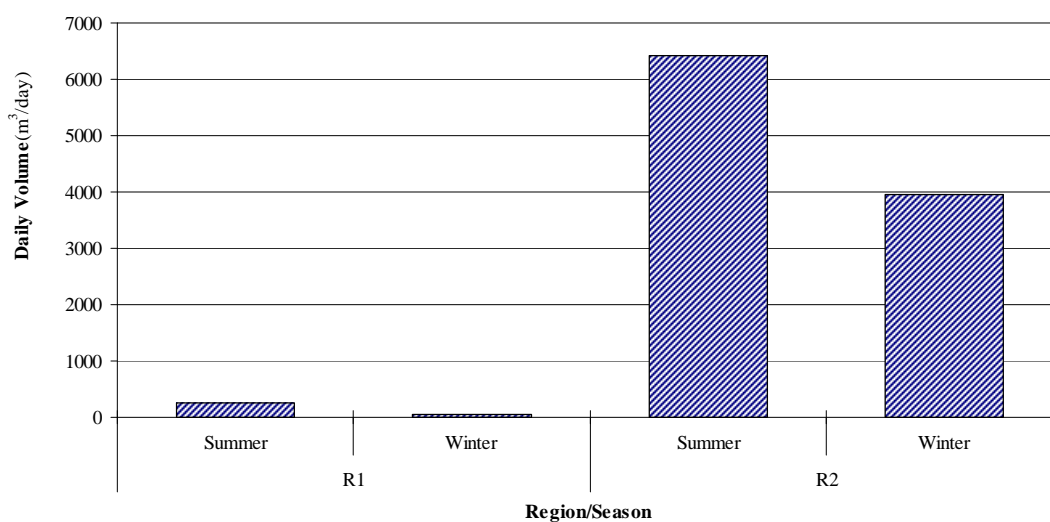


Figure 6.7 Desalination plant production for case study I.

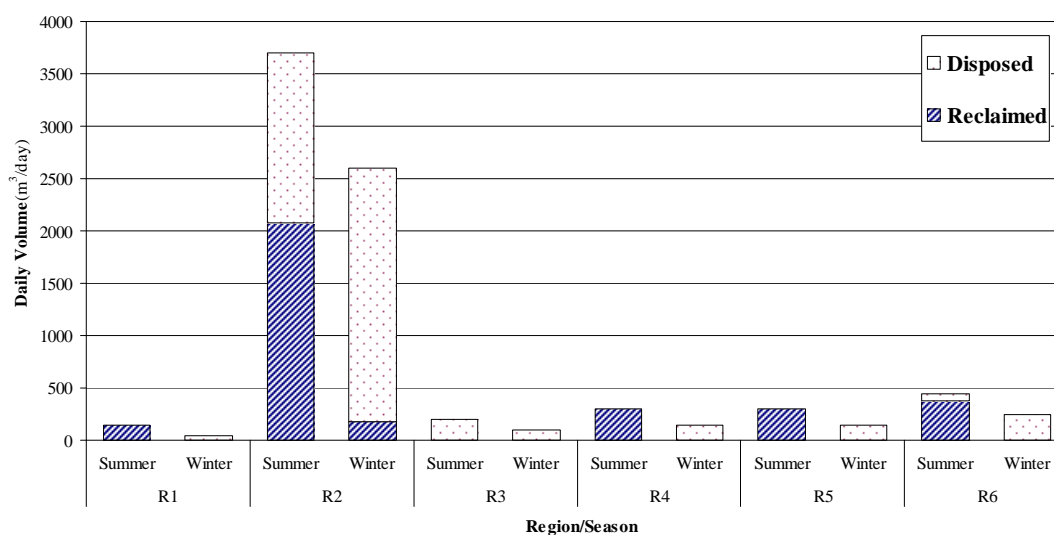


Figure 6.8 Water reclamation and treated wastewater disposal daily volumes after treatment for case study I.

From the above results, there are fewer desalination plants than the wastewater treatment and reclamation plants installed in the optimal solution, which is due to the higher cost of the desalination plant capital cost and unit production cost. The production of desalinated water is centralised in only two plants, in which the plant in R2 does almost all productions, because R2 is the capital and most populous region of the island. As a result of their lower costs, the wastewater treatment and reclamation plants are distributed in all regions, in order to avoid the cost on the

distribution system (pipelines and pumps). So all wastewater is treated locally, and no wastewater pipeline is established.

The non-potable water demand at each region can be satisfied by local reclaimed water (rw local), local desalinated water (dw local), imported reclaimed water (rw imported) and imported desalinated water (dw imported) from other regions (Fig. 6.9). In the optimal solution, there are flows of desalinated water to non-potable water system in R1, R5 and R6. Obviously, in the latter two regions, it is financially more beneficial to use desalinated water for non-potable applications, than to convey reclaimed water from other regions. Among all the four possible sources of non-potable demand, most demand is satisfied by the local reclaimed water production or imported reclaimed water. Due to its higher cost, most of desalinated water is chosen to satisfy the demand of potable water instead of non-potable water.

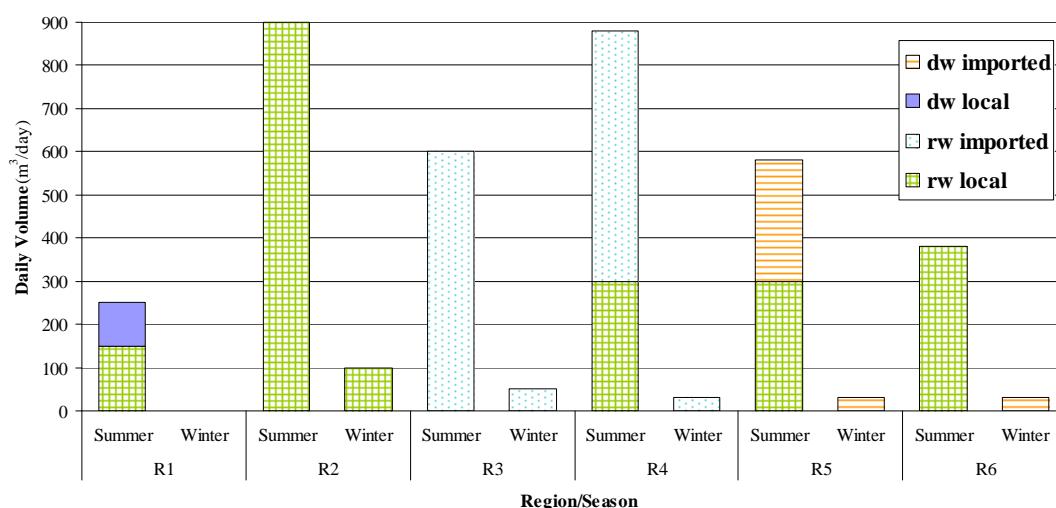


Figure 6.9 Non-potable daily water demand for case study I.

The potable water demand can be satisfied by either local desalinated water production (dw local) or imported desalinated water flows (dw imported). The desalination plant in R2 provides potable water for all other regions, apart from R1, where the potable water is satisfied locally (Fig. 6.10).

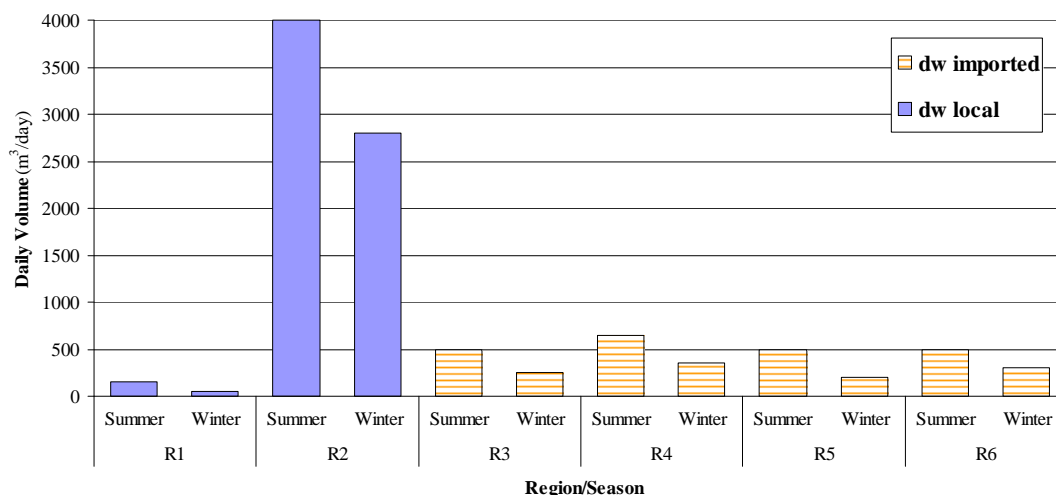


Figure 6.10 Potable daily water demand for case study I.

6.4.1.3 Alternative Scenarios

Here, four alternative scenarios of the problem are considered:

1. “Current locations”: Currently, every region on the island, except R1, has desalination plant; the sole wastewater treatment on the island is located in R2; No water reclamation facility is on the island.
2. “No reclamation”: Water reclamation does not practice on the island, i.e., no reclamation plants is installed. Thus, all water demands (potable and non-potable quality) are satisfied by desalinated seawater, while all wastewater after secondary treatment is disposed into the sea.
3. “Centralised”: Plants are only installed in R2, the capital and the most populous region of the island.
4. “No pipeline”: No water or wastewater main pipeline between the population centres is allowed. Thus, each region has to satisfy its water needs and wastewater treatment obligations.

The optimal objective value, locations and capacities of the plants in each scenario are given in Table 6.9. The corresponding pipeline networks are presented in Fig. 6.11.

Table 6.9 Solution details of each scenario for case study I.

	Optimal	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Annualised total cost (\$/year)	2,298,907	2,798,477	2,423,099	2,545,327	2,441,568
Objective difference	0%	21.7%	5.4%	10.7%	6.2%
Desalination plant	R1 (250)	R2 (6,240)	R2 (9,621)	R1 (6,590)	R1 (250)
	R2 (6,430)	R3 (540)	R4 (269)		R2 (4,000)
		R4 (1,530)			R3 (900)
		R5 (1,080)			R4 (1,230)
		R6 (500)			R5 (780)
					R6 (500)
Location and capacity (m ³ /day)	R1 (150)	R2 (5,100)	R1 (150)	R2 (5,100)	R1 (150)
	R2 (3,700)		R2 (3,700)		R2 (3,700)
	R3 (200)		R3 (200)		R3 (200)
	R4 (300)		R4 (300)		R4 (300)
	R5 (300)		R5 (300)		R5 (300)
	R6 (450)		R6 (450)		R6 (450)
Water reclamation plant	R1 (150)	None	None	R2 (3,300)	R1 (150)
	R2 (2,080)				R2 (900)
	R4 (300)				R3 (200)
	R5 (300)				R4 (300)
	R6 (380)				R5 (300)
					R6 (380)

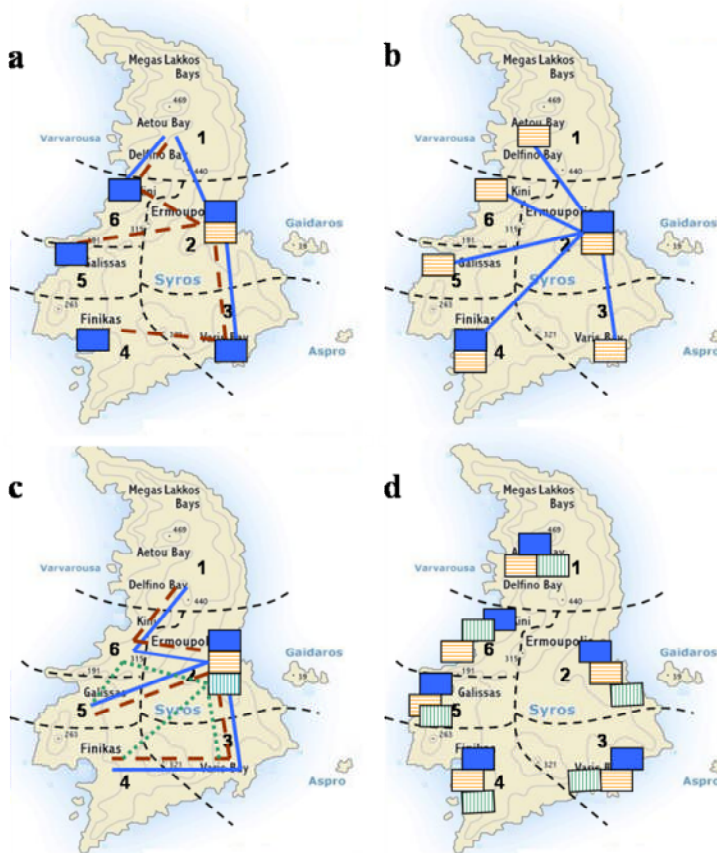
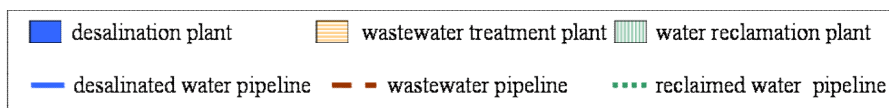


Figure 6.11 Optimal plant locations and pipeline networks in all scenarios for case study I. (a) Scenario 1, (b) Scenario 2, (c) Scenario 3, (d) Scenario 4.

The optimal solution is more than 5% better than all the examined scenarios. Scenario 1 has the worst performance, which means the current practice on the island can be improved very significantly. Both scenarios 3 and 4 give solutions more than 6% of the optimal solution, so the locations of the plants decided easily do not perform as well as the solution from the MILP model. The best scenario is scenario 2 among all those investigated, in which no reclamation is allowed. However, the increased production of desalinated water generates higher cost than the optimal solution and proves the benefit and necessity of the practice of reclamation.

6.4.2 Case Study II – Paros Island

6.4.2.1 Background and Data

Here, we consider Paros island, along with the neighboring Antiparos island. Currently, groundwater and desalinated seawater are used for potable and non-potable water applications on both islands. Similarly to Syros island, no reclamation facility has been installed on both islands. However, only the existing infrastructure for groundwater conveyance on the islands is considered (the existing seawater desalination and wastewater treatment plants are not considered). A previous study on water resources management for Paros island has concluded that the optimal water management for the island is a combination of groundwater and desalinated water (Voivontas et al., 2003). However, the use of reclaimed water was not examined by the aforementioned study.

It is assumed that the water systems on the two islands are not connected to each other. Thus, the two islands are considered as two independent systems. The whole area is divided into eight regions (R1–R8), in which R8 refers to the whole Antiparos island. Each region represents a sub-municipality administration district (Fig. 6.12). There are seven potential desalination plant locations at sea side (D1–D3, D4–5, D6–D8), and the wastewater treatment plants, reclamation plants and storage tanks are assumed to be at the population centre of each region (P1–P8). Thus, in this case study, we consider 15 nodes in total. The distances, pumping distances and elevations (see Fig. 6.1 for definitions) between population centres (Table 6.10) and from potential desalination plant locations to population centres (Table 6.11) are given.

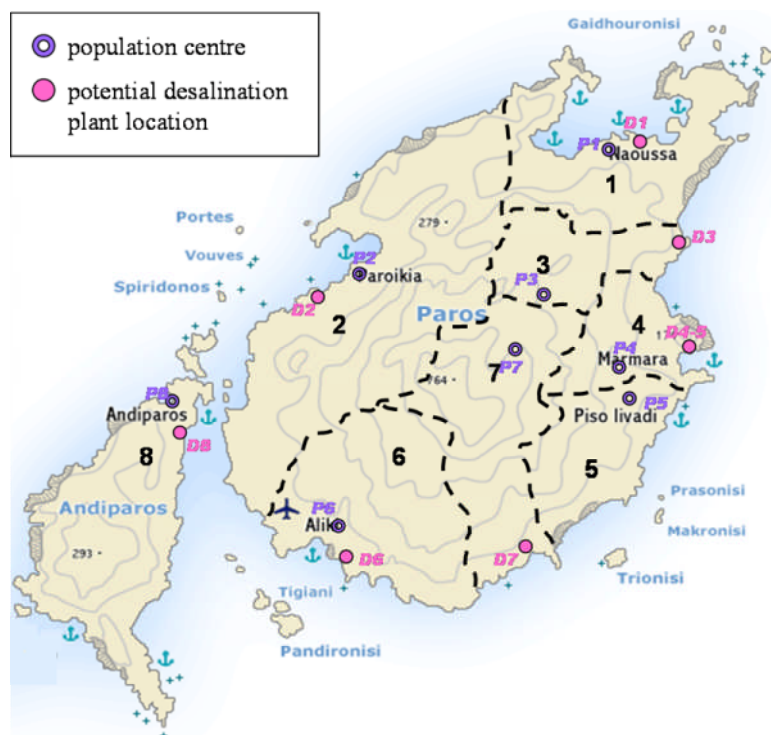


Figure 6.12 Subdivision of Paros and Antiparos islands into 8 regions.

Table 6.10 Distances, pumping distances, elevations between two population centres of case study II.

	Distance / pumping distance / pumping elevation (km)							
	P1	P2	P3	P4	P5	P6	P7	P8
P1		9/4/0.14	5.4/3.6/0.21	7.8/0.5/0.03	- ^a	-	-	-
P2	9/5/0.14		5.8/3.5/0.24	-	-	9/4.4/0.29	5.8/1.5/0.35	-
P3	5.4/0.7/0.05	5.8/0.7/0.10		3.8/0/0	-	-	2.6/2.6/0.08	-
P4	7.8/4.2/0.03	-	3.8/3.8/0.16		1/1/0.02	-	-	-
P5	-	-	-	1/0/0		10.7/4/0.4	4/4/0.22	-
P6	-	9/4.5/0.29	-	-	10.7/4.7/0.42		-	-
P7	-	5.8/2.5/0.11	2.6/0/0	-	4/0/0	-		-
P8	-	-	-	-	-	-	-	

^a The link between these population centres is *a priori* not allowed.

Table 6.11 Distances, pumping distances and elevations from potential desalination plant locations to population centres of case study II.

	Distance / pumping distance / pumping elevation (km)							
	P1	P2	P3	P4	P5	P6	P7	P8
D1	0.9/0/0	- ^a	-	-	-	-	-	-
D2	-	3/0/0	-	-	-	-	-	-
D3	-	-	4.4/4.4/0.15	-	-	-	-	-
D4-5	-	-	-	2.5/2.5/0.01	3/3/0.04	-	5.7/5.7/0.25	-
D6	-	-	-	-	-	0.6/0/0	-	-
D7	-	-	-	-	-	6/3.6/0.21	-	-
D8	-	-	-	-	-	-	-	0.7/0/0

^a The link between the desalination plant location and population centre is *a priori* not allowed.

On Paros island, we take into account groundwater, which can be used for both potable and non-potable water needs. The estimated values of seasonal water demand, wastewater production and available groundwater are shown in Table 6.12. Seasonal water demands have been based on population distribution (ESYE-Hellenic Statistical Authority, 2001) and localised tourist visit data (Hellenic Chamber of Hotels, 2010; Greek Tourist Organizer, 2010), assuming 300/200 L per capita per day (Malamos and Nalbandis, 2005) for summer/winter use, while groundwater availability is based on current groundwater abstraction (Mavri, 2010). The estimated theoretical monthly water consumption is also enlarged by 25% due to the losses of the supply network. The potable water demand is assumed as 60% of the total water demand, and the non-potable water demand is assumed to account for the remaining 40%. It is assumed that all the wastewater from potable water system is collectable for wastewater treatment. 75% of the non-potable water use is for irrigation (and thus lost to the environment), while the rest 25% is collected for treatment. Thus, total wastewater collected for treatment accounts for 70% of total water demand. It is also assumed that the exploited groundwater in each population centre is no more than 80% of the groundwater that is currently exploited in an attempt to avoid aquifer overexploitation.

Table 6.12 Estimated water demands, wastewater productions and available groundwater supplies for case study II.

	Volume per day (summer/winter) (m ³ /day)							
	P1	P2	P3	P4	P5	P6	P7	P8
Total water demand	2,842/ 821	4,238/ 1,511	183/ 91	398/ 222	1,398/ 292	831/ 258	385/ 182	834/ 271
Potable water demand	1,705.2/ 492.6	2,542.8/ 906.6	109.8/ 54.6	238.8/ 133.2	838.8/ 175.2	498.6/ 154.8	231/ 109.2	500.4/ 162.6
Non-potable water demand	1,136.8/ 328.4	1,695.2/ 604.4	73.2/ 36.4	159.2/ 88.8	559.2/ 116.8	332.4/ 103.2	154/ 72.8	333.6/ 108.4
Wastewater production	1,989.4/ 574.7	2,966.6/ 1,057.7	128.1/ 63.7	278.6/ 155.4	978.6/ 294.4	581.7/ 180.6	269.5/ 127.4	583.8/ 189.7
Available groundwater	1,568/ 755	2,043/ 1,090	306/ 123	298/ 95	511/ 296	566/ 295	246/ 134	0/ 0

Other assumptions and problem data about plants, pipes, pumps and storage tanks are the same as those in Syros case study.

6.4.2.2 Results and Discussion

The MILP model for the Paros case study, comprising 2,096 constraints and 2,489 variables (including 1,049 binary variables), takes 2,640 CPUs to find the optimal solution with an annualised total cost of 1,686,618 \$/year. The breakdown of the optimal annualised total cost is given in Fig. 6.13.

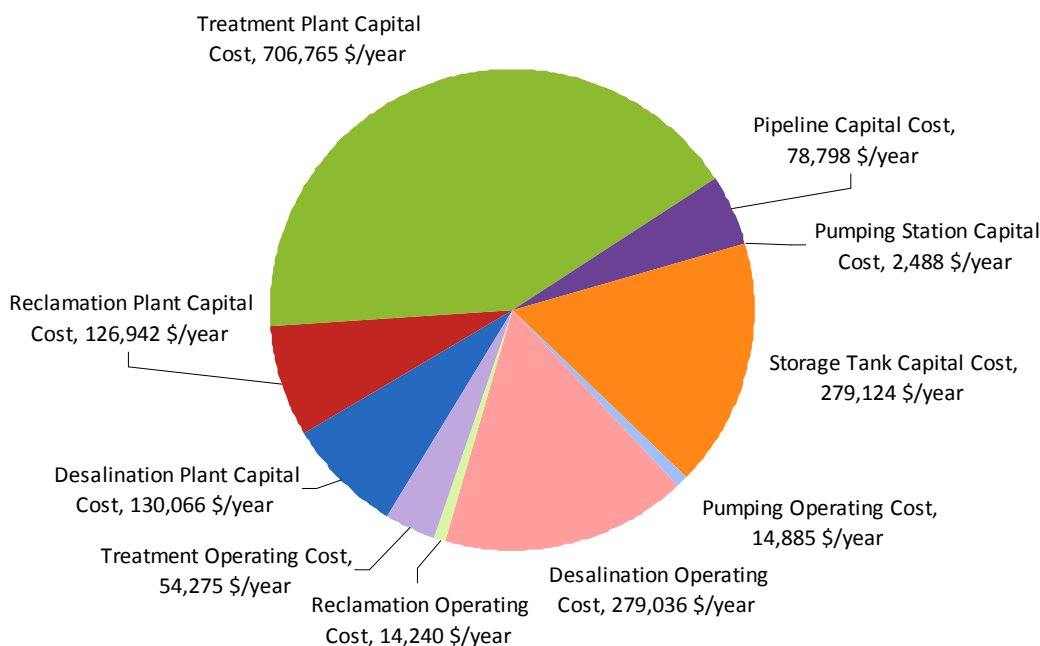


Figure 6.13 Breakdown of the optimal annualised total cost for case study II.

In the optimal solution (Fig. 6.14), four locations are selected as the desalination plant sites: D1, D2, D6 and D8. Wastewater treatment plants are allocated at all population centres, while water reclamation plants are installed at all population centres apart from P3. Concerning the pipeline networks, it should be mentioned that the pipelines are only for desalinated water. Table 6.13 provides water flow details of the optimal solution, in which only one operating pump is required in the solution, as all other flows are facilitated by gravity.

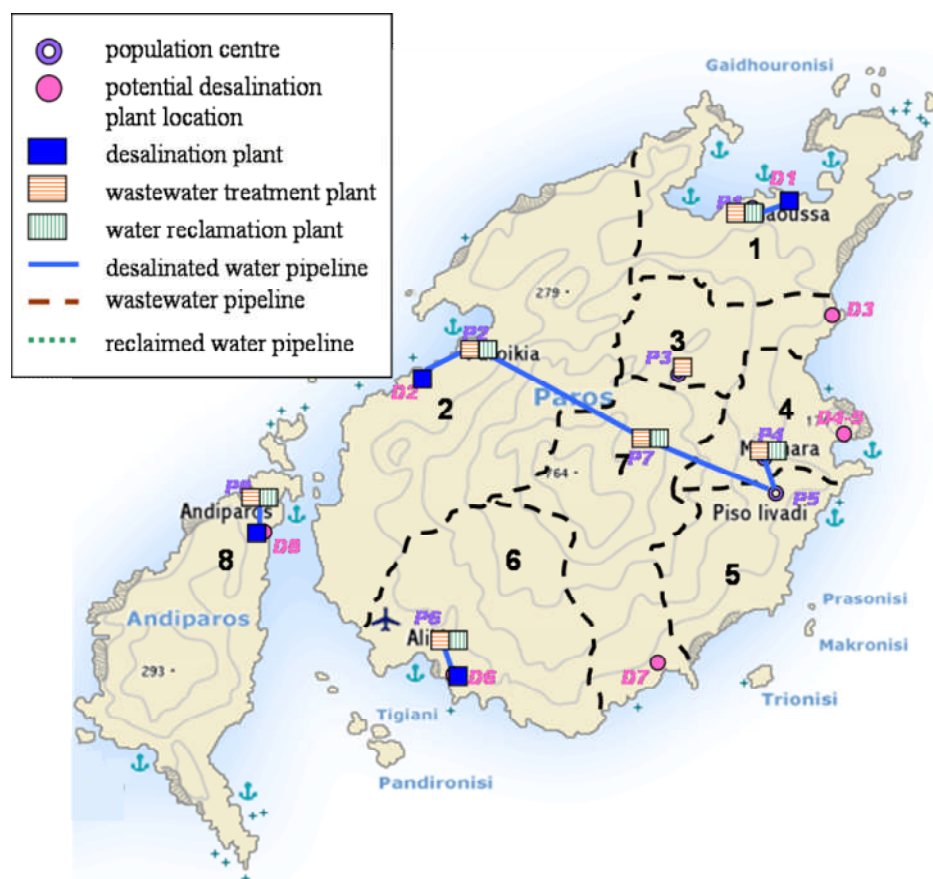


Figure 6.14 Optimal plant locations and pipeline networks for case study II.

Table 6.13 Solution details for each established link for case study II.

Link	Water type	Pipe diameter (in)	Flow direction	Pump max flow rate (m ³ /day)	No. of operating pumps	Pump operating fraction		Flow volume (m ³ /day)	
						Summer	Winter	Summer	Winter
D1--P1	dw ^a	4	D1→P1	- ^b	-	-	-	451.2	0.0
D2--P2	dw	10	D2→P2	-	-	-	-	1,373.4	94.0
D6--P6	dw	2.5	D6→P6	-	-	-	-	50.0	0.0
D8--P8	dw	4	D8→P8	-	-	-	-	500.4	162.6
P2--P7	dw	4	P2→P7	720	1	0.83	0.11	464.6	59.4
P4--P5	dw	2.5	P5→P4	-	-	-	-	0.8	57.2
P5--P7	dw	4	P7→P5	-	-	-	-	430.6	57.2

^a dw: desalinated water.

^b No pump installed.

The daily volumes of desalinated water production are shown in Fig. 6.15. Desalination plants at D1 and D6 only operate in summer, while plants at D2 and D8 operate year around. The plant at D2 has the most production. The details of

wastewater reclamation and disposed daily volumes are given in Fig. 6.16, from which we can see that all treated wastewater from P3 is disposed.

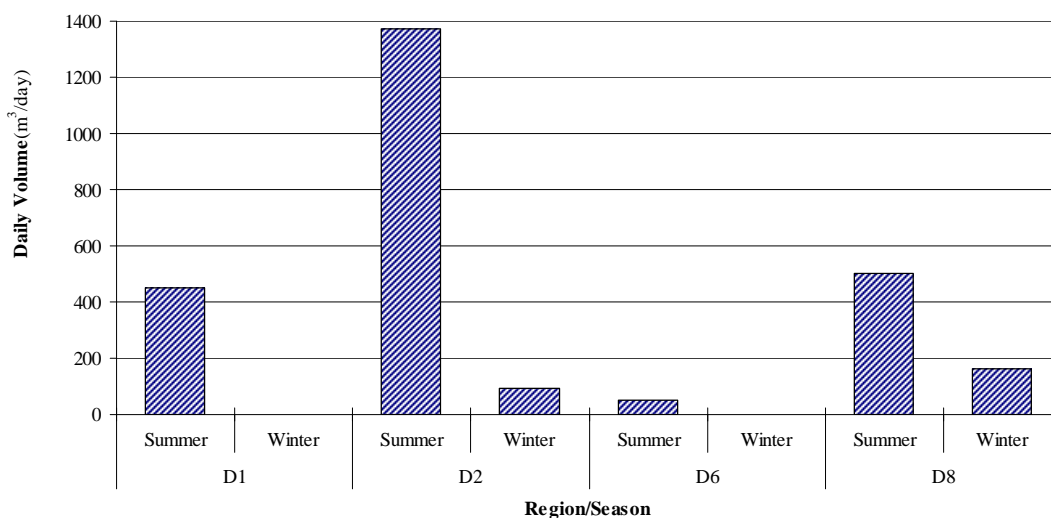


Figure 6.15 Desalination plant production for case study II.

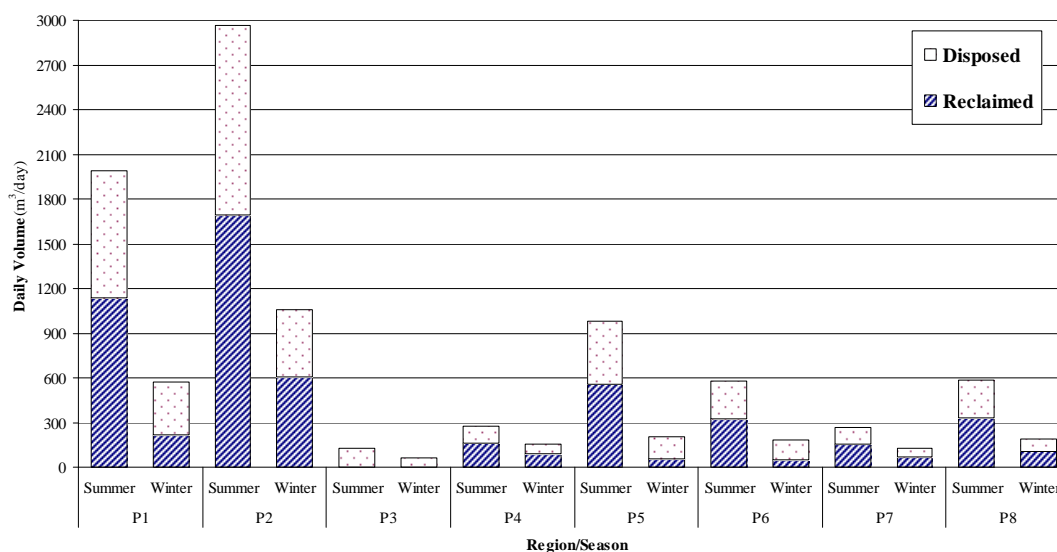


Figure 6.16 Wastewater reclamation and disposal daily volumes after treatment for case study II.

Similar to the Syros case, the production of desalinated water is centralised in a few plants, while the production of treated wastewater and reclaimed water is distributed in almost all the regions. The plants in R2 have the most productions, as P2 is the capital of the island with the most water demand.

Local groundwater supply (gw local) exists in both the non-potable (Fig. 6.17) and the potable (Fig. 6.18) water systems. In the optimal solution, the non-potable water

sources are local reclaimed water and local groundwater supply only (not desalinated seawater). In the potable water system, there are imported desalinated water flows, which exist at P4, P5 and P7. It can be seen that most of groundwater supply is used as potable water, as a substitution of the more expensive option, desalinated water. In all regions apart from R8, more local groundwater is used in the potable water system than the desalinated water, while the local reclaimed water production has the largest proportion in the non-potable water system, and the groundwater is used as non-potable water only at P1, P3, P5 and P6.

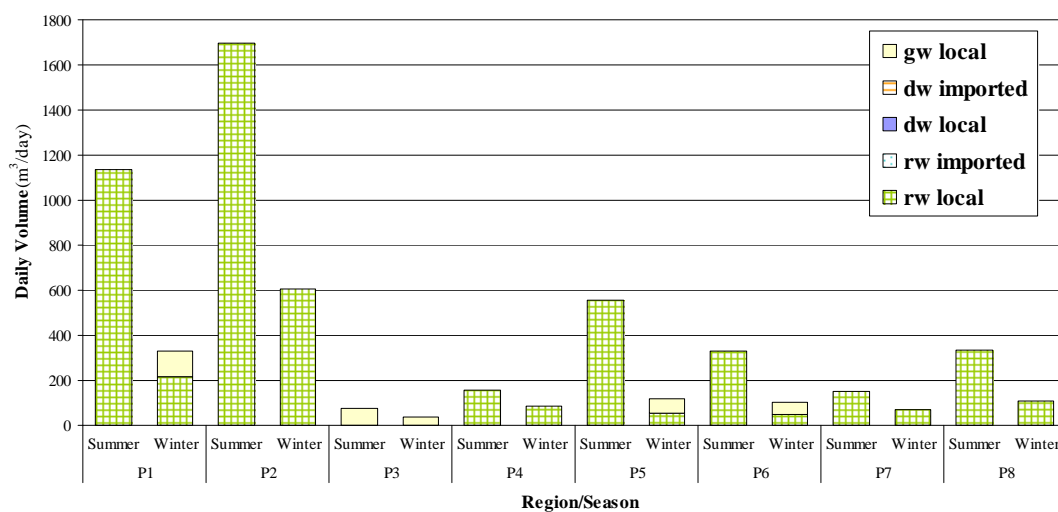


Figure 6.17 Non-potable daily water demand for case study II.

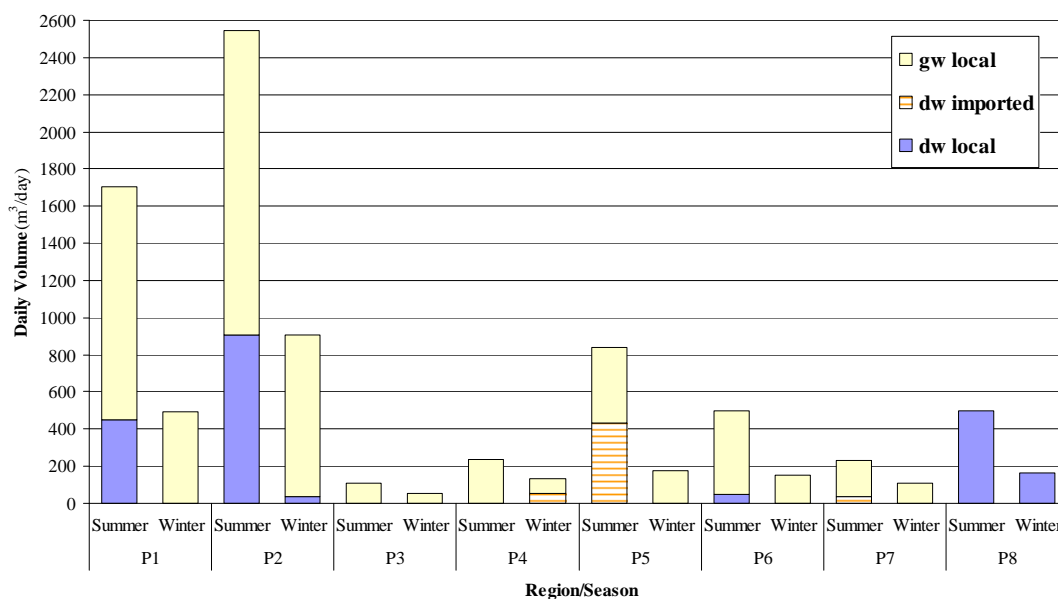


Figure 6.18 Potable daily water demand for case study II.

6.4.2.3 Alternative Scenarios

As the desalination plant locations are not at the population centres, the scenario “no pipe” discussed earlier, in case study I, is not applicable to the Paros case study. Thus, another scenario “no groundwater” is investigated here. Overall, four scenarios are considered:

1. “Current locations”: Currently on Paros island, desalination plants exist at D1 and D4-5; the wastewater treatment plants are located at P1, P2, and P5; and no water reclamation has practiced;
2. “No reclamation”: No water reclamation plant is installed on both islands;
3. “Centralised”: On Paros island, plants are only installed in R2, i.e. at D2 or P2, as P2 is the capital of the island;
4. “No groundwater”: No groundwater supply is available on both islands. Thus, desalinated and reclaimed water are the only sources for all demands.

Table 6.14 Solution details of each scenario for case study II.

	Optimal	Scenario 1	Scenario 2	Scenario 3	Scenario 4	
Annualised total cost (\$/year)	1,686,618	2,502,749	2,274,755	2,357,685	2,559,991	
Objective difference	0%	48.4%	34.9%	39.8%	51.8%	
Location and capacity (m ³ /day)	Desalination plant	D1 (451)	D1(3,502)	D1 (2,737)	D2 (2,203)	D1 (2,783)
		D2 (1,373)	D4-5 (2,405)	D2 (2,792)	D8 (500)	D2 (2,884)
		D6 (100)	D8 (500)	D6 (378)		D6 (499)
		D8 (500)		D8 (834)		D8 (500)
	Wastewater treatment plant	P1 (1,989)	P1 (1,989)	P1 (1,989)	P2 (7,193)	P1 (1,989)
		P2 (2,967)	P2 (3,548)	P2 (2,967)	P8 (584)	P2 (2,967)
		P3 (128)	P5 (1,655)	P3 (128)		P3 (128)
		P4 (279)	P8 (584)	P4 (1,257)		P4 (1,257)
		P5 (979)		P6 (582)		P6 (582)
		P6 (582)		P7 (270)		P7 (270)
		P7 (270)		P8 (584)		P8 (584)
		P8 (584)				
	Water reclamation plant	P1 (1,137)	P8 (334)	None	P2 (3,704)	P1 (1,137)
		P2 (1,695)			P8 (334)	P2 (1,695)
		P4 (159)				P3 (100)
		P5 (559)				P4 (718)
		P6 (328)				P6 (332)
		P7 (154)				P7 (154)
P8 (334)				P8 (334)		

The optimal objective value, plant locations and capacities for each scenario are provided in Table 6.14. The optimal plant locations and pipeline networks are shown in Fig. 6.19. It can be clearly seen that the advantage of the optimal solution is very significant. All the examined scenarios provide objective values over 30% higher

than the optimal one. In scenario 1, there are only two desalination plants and three wastewater treatment plants on Paros island. The higher cost results from pipeline, pumps and pumping. Scenario 2 (without reclamation) results in much higher production of desalinated water, the most expensive option. In scenario 3, all the plants on Paros island are located in R2, which generate smaller cost on plants and production, but much higher cost to distribute the collected wastewater and the produced desalinated and reclaimed water. The worst alternative is scenario 4, as more desalinated water is required to cover the lack of groundwater supply. But it is worth noting that the groundwater is a limited resource, and the exploitation of groundwater should be controlled to make it sustainable. It should be mentioned that there are desalinated water flows to the non-potable water system, i.e. positive values of O_{it} , in all these four scenarios.

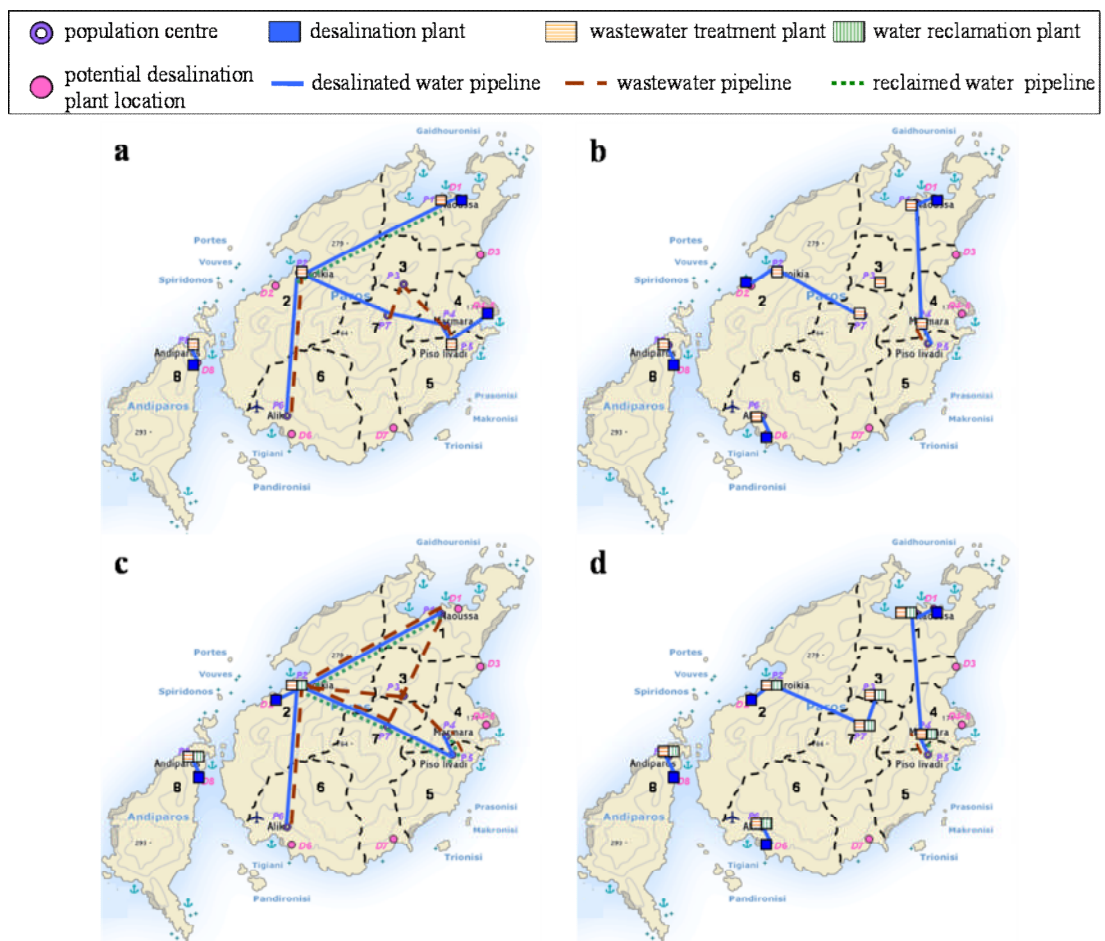


Figure 6.19 Optimal plant locations and pipeline networks in all scenarios for case study II. (a) Scenario 1, (b) Scenario 2, (c) Scenario 3, (d) Scenario 4.

6.5 Concluding Remarks

This chapter addresses the integrated management of desalinated water, wastewater and reclaimed water in water deficient areas. Based on the water demands and wastewater production on the subdivided regions, the geographic characteristics of each region, and the related unit cost parameters, an MILP model has been proposed to determine the optimal locations, capacities and production amounts of desalination, wastewater treatment and reclamation plants, and the optimal water conveyance infrastructure, such as pipeline main networks, pumps, storage tanks, etc., with an objective of minimum annualised total cost including capital costs of plants, pumps, pipelines and storage tanks, and operating costs of production and pumping.

The optimisation approach has been applied to the cases of Syros and Paros-Antiparos, and several scenarios have been examined. The results prove the applicability of the proposed model and show that the optimal solution obtained by the proposed model provides significant benefit when compared with the solutions from all other scenarios.

CONCLUSIONS AND FUTURE WORK

This thesis has addressed several SCM problems in the process industry, including production planning and scheduling, production and distribution planning under uncertainty, multiobjective supply chain optimisation and water supply chain design and planning, to fill the gap in the literature work.

In this chapter, we aim to conclude the work presented in this thesis and provide the potential research directions for the future work.

7.1 Concluding Remarks

In this thesis, MILP-based models and solution approaches have been proposed for several SCM problems in the process industry.

In Chapter 1, a general introduction has been given for the general SCM, process industry SCM and mathematical programming. Moreover, the scope and overview of this thesis have been presented.

In Chapter 2 an MILP model has been proposed for the medium-term planning problem of single-stage multiproduct continuous plants with sequence-dependent changeovers under a hybrid discrete/continuous time representation. In order to avoid the subtours in the optimal solution, a TSP classic formulation has been adopted. A rolling horizon approach has also been developed to deal with large-scale problems. After investigating four literature examples, the proposed approaches have been proven to be much more computational efficient than three literature approaches (Edirik-Dogan and Grossmann, 2006, 2008b; Chen et al., 2008). In addition, the rolling horizon approach contributes a lot in the reduction of the computational complexity.

The production sequencing constraints proposed in Chapter 2 have been adapted in Chapter 3 for the short-term scheduling of a single-stage multiproduct batch edible-oil deodoriser. The two proposed MILP models have considered two cases without and with backlog, respectively. The case study of the deodoriser considers a scheduling problem with 70 orders of 30 products within 7 product groups in a planning horizon of 128 hours. The efficiency of the proposed models is demonstrated by comparing it with a heuristics approach and a literature model (Kelly and Zyngier, 2007).

In Chapter 4, an MPC approach has been developed for the production and distribution planning of a multi-site multiproduct supply chain. Adapting the constraints in Chapters 1 and 2 for the parallel multisite production, an optimisation model has been proposed for the MPC approach to maintain of the desired inventory levels and stable prices. In the result discussion, the optimal control horizon length has been determined. Also, four pricing strategies have been investigated for the products with price elasticity of demand. Comparative study with a hierarchical approach shows the benefit of the inclusion of the sequence-dependent production changeovers in the single-level MILP optimisation model.

In Chapter 5, a multiobjective MILP model has been presented for a global supply chain production, distribution and capacity planning problem. Three criteria for the supply chain have been considered in the problem, including total cost, total flow time and total lost sales. Two different capacity expansion strategies, i.e. proportional and cumulative expansion strategies, have been considered. Two solution approaches have been applied to the proposed multiobjective problem, i.e., the ε -constraint method for the Pareto curve, and the lexicographic minimax method for an equitable solution. A new approach has been developed to transform a lexicographic minimax problem to a minimisation problem, adapting from literature models. From the computational results, the cumulative expansion obtains lower cost and flow time than the proportional expansion, given a predetermined customer service level.

In Chapter 6, an MILP model has been proposed for the integrated water resources management in the water supply chain planning. To maximise the annualised total cost, an MILP model has been developed to determine the allocations and capacities

of desalination plants, wastewater treatment plants and water reclamation plants, the distribution systems (pipelines and pumps) of the desalinated water, treated wastewater and reclaimed water, the storage tanks for potable and non-potable water, and the flows of water/wastewater between regions, based on the GIS-based information and water demand estimation. The proposed model has been successfully applied to Syros and Paros islands in Aegean Sea, and used to investigate several scenarios. The comparative study shows that the optimal scenario saves much in the annualised total cost than other scenarios.

From the work presented in this thesis, the mathematical programming techniques, especially MILP optimisation techniques, can be widely applied to the SCM problems. The proposed MILP approaches have successfully dealt with the supply chain problems discussed in this thesis. The work in this thesis, which not only has developed some novel approaches to literature problems, but also some problems not investigated before, is a complement to the literature research work on the process industry supply chains. A number of publications have arisen from the work presented in this thesis. See the list of the publications in Appendix E.

7.2 Directions for the Future Work

The work in this thesis has covered a number of problems in the SCM, and there are still several research directions for the future work as the extension of the current study.

A future development of the work in this thesis could be the incorporation of uncertainty issues. Although a large number of models have been developed, more investigations are still needed to overcome the limitations of current models. (Kallrath, 2005; Mula et al., 2006; Peidro et al., 2009; Verderame et al., 2010). In this thesis, only Chapter 4 has considered the demand uncertainty, which was tackled by an MPC approach. The uncertainty issues can also be considered in the production planning scheduling, global supply chain planning and water supply chain planning. The possible uncertain factors could be product demands and prices, raw materials availability and prices, production rates and times, changeover times and cost, transportation time and cost, etc. The incorporation of one or several factors

discussed above into the proposed models will be a good research direction following the work in this thesis.

The investigation of efficient solution procedures for tackling large-scale optimisation models constitutes another valuable research direction. Although a rolling horizon approach has been introduced to facilitate the computation of large-scale models in Chapter 2, other solution procedures are still worth being studied to tackle the larger-scale production and scheduling problems, such as decomposition approach (such as bi-level, Lagrangian, etc.), construction-based approach, and other heuristics. The development of an efficient solution procedure will also benefit the work in Chapter 6. The proposed MILP model can successfully tackle the case studies with 6 to 8 regions. However, the single-level MILP model may have more difficulties in solving the integrated water resources management problem for an area with a dozen of sub-regions or more. Thus, methods to overcome the computational complexity of larger instances are worth being investigated.

Another direction for the future work is the multiscale modelling. The integration of the medium-term planning and short-term scheduling for multiproduct continuous/batch plants can be studied by extending the work in Chapters 2 and 3. The integration of global supply chain planning and production scheduling will incorporate the scheduling problems into the work in Chapter 5. The supply chain design problem can be considered simultaneously with the production and distribution planning problem as well. The decisions at different levels considered simultaneously will definitely benefit the overall performance of the supply chains considered.

Several other minor extensions of the present work could be the extension of single-stage planning and scheduling in Chapters 2 and 3 to multistage planning and scheduling, the study on the demand forecasting and the incorporation of backlog level in the MPC approach in Chapter 4, the investigation of other efficient solution approaches for the multiobjective optimisation problems in Chapter 5, and the consideration of more than one offshore pipelines the examples with more than more islands, e.g. Paros, in Chapter 6.

MODEL E-D&G1

The model proposed by Erdirik-Dogan and Grossmann (2006), for the simultaneous planning and scheduling of single-stage single-unit continuous multiproduct plants is a multiperiod MILP model based on a continuous time representation.

A.1 Nomenclature

Indices

- i, k product indices, $i, k = 1, \dots, N$
 l, ll time slot indices, $l, ll = 1, \dots, N$
 t time period indices, $t = 1, \dots, HTot$

Parameters

- c_{inv} inventory cost
 c_{it}^{oper} operating cost for product i in period t
 c_{ik}^{trans} transition cost from product i to k
 d_{it} demand of product i in period t
 H_t duration of the t th time period
 $HTot$ time at the end of the planning horizon
 $INVO_{i0}$ initial inventory level of product i
 p_{it} selling price of product i in period t
 r_i production rates of product i
 τ_{ik} transition time from product i to product k

Binary Variables

- TRT_{ikt} 1 if product i is followed by product k at the end of period t , 0 otherwise

W_{ilt}	1 if product i is assigned to slot l of period t , 0 otherwise
YOP_{it}	1 if product i is assigned to period t , 0 otherwise
Z_{iklt}	1 if product i is followed by product k in slot l of period t , 0 otherwise

Continuous Variables

$Area_{it}$	area below the inventory time graph for product i at period t
INV_{it}	inventory level of product i at the end of time period t
$INVO_{it}$	final inventory of product i at time t after the demands are satisfied
NY_{it}	number of slots that product i is assigned in period t
S_{it}	sales of product i in period t
Te_{lt}	end time of slot l in period t
Ts_{lt}	start time of slot l in period t
X_{it}	amount produced of product i in period t
\tilde{X}_{ilt}	amount produced of product i in slot l of period t
z^p	total profit over a given time horizon
θ_{it}	production time of product i in period t
$\tilde{\theta}_{ilt}$	production time of product i in slot l of period t

A.2 Mathematical Formulation

A.2.1 Objective Function

$$\begin{aligned}
 z^p = & \sum_i \sum_t p_{it} S_{it} - c_{inv} \sum_i \sum_t Area_{it} - \sum_i \sum_t c_{it}^{oper} X_{it} - \\
 & \sum_i \sum_k \sum_l \sum_t c_{ik}^{trans} Z_{iklt} - \sum_t \sum_i \sum_k c_{ik}^{trans} TRT_{ikt}
 \end{aligned} \tag{A.1}$$

A.2.2 Assignment and Processing Times Constraints

$$\sum_i W_{ilt} = 1 \quad l \in N, t \in HTot \tag{A.2}$$

$$0 \leq \tilde{\theta}_{ilt} \leq H_t W_{ilt} \quad i \in N, l \in N, t \in HTot \tag{A.3}$$

$$\theta_{it} = \sum_l \tilde{\theta}_{ilt} \quad i \in N, t \in HTot \tag{A.4}$$

$$\tilde{X}_{ilt} = r_i \tilde{\theta}_{ilt} \quad i \in N, l \in N, t \in HTot \quad (A.5)$$

$$X_{it} = \sum_l \tilde{X}_{ilt} \quad i \in N, t \in HTot \quad (A.6)$$

A.2.3 Transitions Constraints

$$Z_{iklt} \geq W_{ilt} + W_{k,l+1,t} - 1 \quad i \in N, k \in N, l \in N, t \in HTot \quad (A.7)$$

A.2.4 Timing Relations Constraints

$$Te_{lt} = Ts_{lt} + \sum_i \tilde{\theta}_{ilt} + \sum_i \sum_k \tau_{ik} Z_{iklt} \quad l \in N, t \in HTot \quad (A.8)$$

$$TRT_{ikt} \geq W_{ilt} + W_{k,l,t+1} - 1 \quad i \in N, k \in N, l = N, ll = 1 \quad (A.9)$$

$$Te_{lt} + \sum_i \sum_k \tau_{ik} TRT_{ikt} = Ts_{ll,t+1} \quad t \in HTot, l = N, ll = 1 \quad (A.10)$$

$$Te_{lt} = Ts_{l+1,t} \quad l \neq N, t \in HTot \quad (A.11)$$

$$Te_{Nt} \leq HT_t \quad t \in HTot \quad (A.12)$$

A.2.5 Inventory Constraints

$$INV_{it} = INVI_{i0} + \sum_l r_i \tilde{\theta}_{ilt} \quad i \in N, t = 1 \quad (A.13)$$

$$INV_{it} = INVO_{i,t-1} + \sum_l r_i \tilde{\theta}_{ilt} \quad i \in N, t \neq 1 \quad (A.14)$$

$$INVO_{it} = INV_{it} - S_{it} \quad i \in N, t \in HTot \quad (A.15)$$

$$Area_{it} \geq INVO_{i,t-1} H_t + r_i \theta_{it} H_t \quad i \in N, t \in HTot \quad (A.16)$$

A.2.6 Demand Constraints

$$S_{it} \geq d_{it} \quad i \in N, t \in HTot \quad (A.17)$$

A.2.7 Degeneracy Prevention Constraints

$$NY_{it} = \sum_l W_{ilt} \quad i \in N, t \in HTot \quad (A.18)$$

$$YOP_{it} \geq W_{ilt} \quad i \in N, l \in N, t \in HTot \quad (A.19)$$

$$YOP_{it} \leq NY_{it} \leq NYOP_{it} \quad i \in N, t \in HTot \quad (A.20)$$

$$NY_{it} \geq N - \left[\left(\sum_i YOP_{it} \right) - 1 \right] - M(1 - W_{i,l,t}) \quad i \in N, t \in HTot \quad (\text{A.21})$$

$$NY_{it} \leq N - \left[\left(\sum_i YOP_{it} \right) - 1 \right] + M(1 - W_{i,l,t}) \quad i \in N, t \in HTot \quad (\text{A.22})$$

MODEL CPP

The model proposed by Chen et al. (2008) for the medium-term planning of single-stage single-unit continuous multiproduct plants is an MILP model based on a hybrid discrete/continuous time representation.

B.1 Nomenclature

Indices

c	customer
i, j	product
k	time slot
w	week

Sets

C	customers
I, J	products
K_w	time slots in week w
W	weeks

Parameters

$CB_{c,i}$	backlog cost of product i to customer c
$CI_{i,w}$	inventory cost of product i in week w
$CT_{i,j}$	transition cost from product i to product j
$D_{c,i,w}$	demand of product i from customer c in week w
$PS_{c,i}$	price of product i to customer c
r_i	processing rate of product i

V_i^{\max}	maximum storage of product i
V_i^{\min}	minimum storage of product i
θ^L	lower bound for the processing time
θ^U	upper bound for the processing time
$\tau_{i,j}$	changeover time from product i to product j

Binary Variables

$E_{i,w}$	1 if product i is produced in week w , 0 otherwise
$y_{i,k,w}$	1 if product i is processed in time slot k during week w , 0 otherwise
$Z_{i,j,k,w}$	1 if product i (slot $k-1$) precedes product j (slot k) in week w , 0 otherwise

Continuous Variables

$P_{i,w}$	production of product i in week w
Pro	operating profit
$S_{c,i,w}$	sales of product i to customer c in week w
$T_{k,w}$	end time of slot k in week w
$V_{i,w}$	volume of product i in week w
$\Delta_{c,i,w}$	backlog of product i for customer c in week w
$\theta_{i,k,w}$	processing time of product i in slot k during week w

B.2 Mathematical Formulation

B.2.1 Objective Function

$$Pro = \sum_i \sum_w \left[\sum_c (PS_{i,c} S_{c,i,w} - CB_{i,c} \Delta_{c,i,w}) - \left(\sum_j \sum_{K \in K_w} CT_{i,j} Z_{i,j,k,w} + CI_{i,w} V_{i,w} \right) \right] \quad (B.1)$$

B.2.2 Assignment Constraints

$$\sum_i y_{i,k,w} = 1 \quad k \in K_w, w \in W \quad (B.2)$$

B.2.3 Timing Constraints

$$T_{0,w} = 0, \quad T_{|K_w|,w} = 168 \quad w \in W \quad (\text{B.3})$$

$$0 \leq \theta_{i,k,w} \leq \theta^U \cdot y_{i,k,w} \quad i \in I, k \in K_w, w \in W \quad (\text{B.4})$$

$$\sum_{k \in K_w} \theta_{i,k,w} \geq \theta^L \cdot E_{i,w} \quad i \in I, w \in W \quad (\text{B.5})$$

$$T_{k,w} - T_{k-1,w} = \sum_i (\theta_{i,k,w} + \sum_j \tau_{j,i} \cdot Z_{j,i,k,w}) \quad \forall k \in K_w, w \in W \quad (\text{B.6})$$

B.2.4 Transition Constraints

$$\sum_j Z_{i,j,k,w} = y_{i,k-1,w} \quad i \in I, k \in K_w - \{1\}, w \in W \quad (\text{B.7})$$

$$\sum_i Z_{i,j,k,w} = y_{j,k,w} \quad j \in J, k \in K_w - \{1\}, w \in W \quad (\text{B.8})$$

$$\sum_j Z_{i,j,1,w+1} = y_{i,K_w,w} \quad i \in I, w \in W \quad (\text{B.9})$$

$$\sum_i Z_{i,j,1,w+1} = y_{j,1,w+1} \quad j \in J, w \in W \quad (\text{B.10})$$

B.2.5 Process and Storage Capacity Constraints

$$P_{i,w} = r_i \cdot \sum_{k \in K_w} \theta_{i,k,w} \quad i \in I, w \in W \quad (\text{B.11})$$

$$V_i^{\min} \leq V_{i,w} \leq V_i^{\max} \quad i \in I, w \in W \quad (\text{B.12})$$

B.2.6 Inventory and Demand Constraints

$$V_{i,w} = V_{i,w-1} + P_{i,w} - \sum_c S_{c,i,w} \quad i \in I, w \in W \quad (\text{B.13})$$

$$\Delta_{c,i,w} = \Delta_{c,i,w-1} + D_{c,i,w} - S_{c,i,w} \quad c \in C, i \in I, w \in W \quad (\text{B.14})$$

B.2.7 Degeneracy Prevention Constraints

$$\sum_k y_{i,k,w} \leq E_{i,w} + (K_w - 1) \cdot y_{i,K_w,w} \quad i \in I, w \in W \quad (\text{B.15})$$

$$E_{i,w} \geq y_{i,K_w,w} \quad i \in I, w \in W \quad (\text{B.16})$$

$$\sum_{j \neq i} \sum_k (Z_{i,j,k,w} + Z_{j,i,k,w}) \leq 2 - y_{i,K_w,w} \quad i \in I, w \in W \quad (\text{B.17})$$

MODEL E-D&G2

In the bi-level decomposition algorithm proposed by Erdirik-Dogan and Grossmann (2008a), the original MILP model of simultaneous planning and scheduling of single-stage multiproduct continuous plants with parallel units is decomposed into an upper level planning and a lower level scheduling problem, in which the latter is an extension of the single unit model proposed by Erdirik-Dogan and Grossmann (2006) in Appendix A. The two sub-problems are solved iteratively. Integer cuts are used to exclude the current assignment and generate new solutions. Finally, the solution of lower level problem becomes the final solution after convergence is achieved.

It should be noticed that for the single-unit case in Chapter 2, the number of units considered is 1, i.e. $|m|=1$, and all products can be processed on the unit, i.e. $|I_m|=I$.

C.1 Nomenclature

Indices

i, k	product
l	slot
\bar{l}_m	last slot of unit m
m	unit
t	time period
\bar{t}	last time period

Sets

I_m	set of products that can be processed on unit m
-------	---

L_m set of slots that belong to unit m
 M_i set of units that can process product i

Parameters

$CINV_{it}$ inventory cost of product i in period t
 COP_{it} operating cost of product i in period t
 CP_{it} selling price of product i in period t
 $CTRANS_{ikm}$ transition cost of changing the production from product i to k in unit m
 d_{it} demand of product i at the end of period t
 H_t duration of the t th time period
 HT_t time at the end of the t th time period
 $INVI_i$ initial inventory of product i
 MRT_{im} minimum run lengths
 N_m number of slots postulated for unit m
 r_{im} production rate of product i in unit m
 τ_{ikm} transition time from product i to product k in unit m

Binary Variables

TRT_{ikmt} 1 if product i is followed by product k at the end of time period t , 0 otherwise
 W_{imlt} 1 the assignment of product i to slot l of unit m during time period t , 0 otherwise
 XF_{imt} 1 if product i the first product in unit m during time period t , 0 otherwise
 XL_{imt} 1 if product i the last product in unit m during time period t , 0 otherwise
 YOP_{imt} 1 if product i is assigned to unit m during time period t , 0 otherwise
 YP_{imt} 1 the assignment of product i to unit m during time period t , 0 otherwise
 Z_{ikmlt} 1 if product i is followed by product k in slot l of unit m during time period t , 0 otherwise
 ZP_{ikmt} to denote if product i precedes product k in unit m during time period t
 ZZP_{ikmt} to denote if the link between products i and k is broken

ZZZ_{ikmt} transition variable denoting the changeovers across adjacent periods

Continuous Variables

$Area_{it}$ overestimate of the area below the inventory time graph for product i at the end of time period t

INV_{it} inventory level of product i at the end of time period t

$INVO_{it}$ inventory level of product i at the end of time period t after demands are satisfied

NY_{imt} total number of slots that are allocated for product i in unit m during time period t

S_{it} sales of product i at the end of period t

Te_{mlt} end time of slot l of unit m during time period t

$TRNP_{mt}$ total transition time for unit m within each time period

Ts_{mlt} start time of slot l of unit m during time period t

X_{imlt} amount of product i produced in slot l of unit m during time period t

\tilde{X}_{imt} amount of product i produced in unit m during time period t

Θ_{imlt} production time of product i in slot l of unit m during time period t

$\tilde{\theta}_{imt}$ production time of product i in unit m during time period t

C.2 Upper Level Problem

In the decomposition approach, the upper level problem yields a valid upper bound on the profit.

C.2.1 Objective Function

$$\begin{aligned}
 \text{Profit} = & \sum_i \sum_t CP_{it} \cdot S_{it} - \sum_i \sum_t CINV_{it} \cdot Area_{it} - \sum_t \sum_m \sum_{i \in I_m} COP_{it} \cdot \tilde{X}_{imt} - \\
 & \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} CTRANS_{ikm} \cdot (ZP_{ikmt} - ZZP_{ikmt}) - \\
 & \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} CTRANS_{ikm} \cdot ZZZ_{ikmt}
 \end{aligned} \tag{C.1}$$

C.2.2 Assignment and Production Constraints

$$\tilde{\theta}_{imt} \leq H_i \cdot YP_{imt} \quad \forall i \in I_m, m, t \quad (C.2)$$

$$\tilde{X}_{imt} = r_{im} \cdot \tilde{\theta}_{imt} \quad \forall i \in I_m, m, t \quad (C.3)$$

C.2.3 Inventory Balance and Costs Constraints

$$INV_{it} = INVI_i + \sum_{m \in M_i} r_{im} \cdot \tilde{\theta}_{imt} \quad \forall i, t = 1 \quad (C.4)$$

$$INV_{it} = INVO_{i,t-1} + \sum_{m \in M_i} r_{im} \cdot \tilde{\theta}_{imt} \quad \forall i, t \neq 1 \quad (C.5)$$

$$INVO_{it} = INV_{it} - S_{it} \quad \forall i, t \quad (C.6)$$

$$Area_{it} \geq INVO_{i,t-1} \cdot H_t + \left(\sum_{m \in M_i} r_{im} \cdot \tilde{\theta}_{imt} \right) \cdot H_t \quad \forall i, t \quad (C.7)$$

C.2.4 Demand Constraints

$$S_{it} \geq d_{it} \quad \forall i, t \quad (C.8)$$

C.2.5 Sequencing Constraints

$$YP_{imt} = \sum_{k \in I_m} ZP_{ikmt} \quad \forall i \in I_m, m, t \quad (C.9)$$

$$YP_{kmt} = \sum_{i \in I_m} ZP_{ikmt} \quad \forall k \in I_m, m, t \quad (C.10)$$

$$\sum_{i \in I_m} \sum_{k \in I_m} ZP_{ikmt} = 1 \quad \forall m, t \quad (C.11)$$

$$ZP_{ikmt} \leq ZP_{ikmt} \quad \forall i \in I_m, k \in I_m, m, t \quad (C.12)$$

$$YP_{imt} \geq ZP_{iimt} \quad \forall i \in I_m, m, t \quad (C.13)$$

$$ZP_{iimt} + YP_{kmt} \leq 1 \quad \forall i \in I_m, k \in I_m, i \neq k, m, t \quad (C.14)$$

$$ZP_{iimt} \geq YP_{imt} - \sum_{k \neq i, k \in I_m} YP_{kw} \quad \forall i \in I_m, m, t \quad (C.15)$$

$$TRNP_{mt} = \sum_{i \in I_m} \sum_{k \in I_m} \tau_{ikm} \cdot ZP_{ikmt} - \sum_{i \in I_m} \sum_{k \in I_m} \tau_{ikm} \cdot ZP_{ikmt} \quad \forall m, t \quad (C.16)$$

$$XF_{kmt} \geq \sum_{i \in I_m} ZP_{ikmt} \quad \forall k \in I_m, m, t \quad (C.17)$$

$$XL_{imt} \geq \sum_{k \in I_m} ZP_{ikmt} \quad \forall i \in I_m, m, t \quad (C.18)$$

$$\sum_{i \in I_m} XF_{imt} = 1 \quad \forall m, t \quad (C.19)$$

$$\sum_{i \in I_m} XL_{imt} = 1 \quad \forall m, t \quad (C.20)$$

$$\sum_{k \in I_m} ZZZ_{ikmt} = XL_{imt} \quad \forall i \in I_m, m, t \quad (C.21)$$

$$\sum_{i \in I_m} ZZZ_{ikmt} = XF_{k,m,t+1} \quad \forall k \in I_m, m, t \in T - \{\bar{t}\} \quad (C.22)$$

C.2.6 Time Balance Constraints

$$\sum_{i \in I_m} \tilde{\theta}_{imt} + TRNP_{mt} - \sum_{i \in I_m} \sum_{k \in I_m} (\tau_{ikm} \cdot ZZZ_{ikmt}) \leq H_t \quad \forall m, t \quad (C.23)$$

C.2.7 Integer Cuts Constraints

$$\sum_{(i,t) \in Z'_1} YP_{imt} - \sum_{(i,t) \in Z'_0} YP_{imt} \leq |Z'_0| - 1 \quad (C.24)$$

where $Z'_0 = \{i, t \mid YP_{imt}^r = 0\}$ and $Z'_1 = \{i, t \mid YP_{imt}^r = 1\}$.

C.3 Lower Level Problem

The lower level problem is solved to yield a lower bound on the profit, by excluding the products that were not selected by the upper level problem for each unit at each period.

C.3.1 Objective Function

$$\begin{aligned} \text{Profit} = & \sum_i \sum_t CP_{it} \cdot S_{it} - \sum_i \sum_t CINV_{it} \cdot Area_{it} - \sum_t \sum_m \sum_{i \in I_m} \sum_l COP_{it} \cdot X_{imlt} - \\ & \sum_m \sum_{i \in I_m} \sum_{k \in I_m} \sum_t \sum_l (CTTRANS_{ikm} \cdot Z_{ikm} + CTTRANS_{ikm} \cdot TRT_{ikmt}) \end{aligned} \quad (C.25)$$

C.3.2 Assignment and Processing Times Constraints

$$\sum_{i \in I_m} W_{imlt} = 1 \quad \forall m, l \in L_m, t \quad (C.26)$$

$$\Theta_{imlt} \leq H_t \cdot W_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \quad (C.27)$$

$$\Theta_{imlt} \geq MRT_{im} \cdot W_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \quad (C.28)$$

$$X_{imlt} = r_{im} \cdot \Theta_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \quad (C.29)$$

C.3.3 Transitions Constraints

$$\sum_{k \in I_m} Z_{ikmlt} = W_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \quad (\text{C.30})$$

$$\sum_{i \in I_m} Z_{ikmlt} = W_{k,m,l+1,t} \quad \forall k \in I_m, m, l \in L_m - \{\bar{l}_m\}, t \quad (\text{C.31})$$

$$\sum_{k \in I_m} TRT_{ikmt} = W_{imlt} \quad \forall i \in I_m, m, l = \bar{l}_m, t \quad (\text{C.32})$$

$$\sum_{i \in I_m} TRT_{ikmt} = W_{kmlt} \quad \forall k \in I_m, m, l = 1, t = T - \{\bar{t}\} \quad (\text{C.33})$$

C.3.4 Timing Relations Constraints

$$Te_{mlt} = Ts_{mlt} + \sum_{i \in I_m} \Theta_{imlt} + \sum_{i \in I_m} \sum_{k \in I_m} \tau_{ikm} \cdot Z_{ikmlt} \quad \forall m, l = L_m, t \quad (\text{C.34})$$

$$Ts_{mlt+1} \geq Te_{ml't} + \sum_{i \in I_m} \sum_{k \in I_m} \tau_{ikm} \cdot TRT_{ikmt} \quad \forall m, l = 1, l' = \bar{l}_m, t = T - \{\bar{t}\} \quad (\text{C.35})$$

$$Te_{mlt} = Ts_{m,l+1,t} \quad \forall m, l = L - \{\bar{l}_m\}, t \quad (\text{C.36})$$

$$Te_{mlt} \leq HT_t \quad \forall m, l = \bar{l}_m, t \quad (\text{C.37})$$

C.3.5 Inventory Balance and Costs Constraints

$$INV_{it} = INVI_t + \sum_{m \in M_i} r_{im} \cdot \sum_{l \in L_m} \Theta_{imlt} \quad \forall i, t = 1 \quad (\text{C.38})$$

$$INV_{it} = INVO_{i,t-1} + \sum_{m \in M_i} r_{im} \cdot \sum_{l \in L_m} \Theta_{imlt} \quad \forall i, t \neq 1 \quad (\text{C.39})$$

$$INVO_{it} = INV_{it} - S_{it} \quad \forall i, t \quad (\text{C.40})$$

$$Area_{it} \geq INVO_{i,t-1} \cdot H_t + \left(\sum_{m \in M_i} r_{im} \cdot \sum_{l \in L_m} \Theta_{imlt} \right) \cdot H_t \quad \forall i, t \quad (\text{C.41})$$

C.3.6 Demand Constraints

$$S_{it} \geq d_{it} \quad \forall i, t \quad (\text{C.42})$$

C.3.7 Degeneracy Prevention Constraints

$$YOP_{imt} \geq W_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \quad (\text{C.43})$$

$$YOP_{imt} \leq NY_{imt} \leq N_m \cdot YOP_{imt} \quad \forall i \in I_m, m, t \quad (\text{C.44})$$

$$NY_{imt} \geq N_m - \left[\left(\sum_{i \in I_m} YOP_{imt} \right) - 1 \right] - M \cdot (1 - W_{imlt}) \quad \forall i \in I_m, m, t \quad (\text{C.45})$$

$$NY_{imt} \leq N_m - \left[\left(\sum_{i \in I_m} YOP_{imt} \right) - 1 \right] + M \cdot (1 + W_{imt}) \quad \forall i \in I_m, m, t \quad (\text{C.46})$$

C.3.8 Subset of Products by the Upper Level Problem

$$YOP_{imt} \leq YP_{imt} \quad \forall i \in I_m, m, t \quad (\text{C.47})$$

MODEL K&Z

Kelly and Zyngier (2007) presented an MILP formulation for modelling sequence-dependent changeovers for discrete-time scheduling problems. The formulation can be applied to both batch and continuous process units. For fair comparison, some new constraints for backlog, inventory and sales and objective function are added to the original formulation.

D.1 Original Model K&Z

The original model K&Z used four dependent binary logic variables, startup, shutdown, switchover-to-itself and memory operation logic variables, for each independent mode operation changeover logic variable on a continuous-process unit and on fixed batch-size, variable batch-time batch-process units.

D.1.1 Nomenclature

Indices

i, j operation
 t, tt time period

Parameters

τ_i batch time for operation i
 τ_{ij} switchover time from operation i to j

Binary Variables

sd_{it} 1 for the shutdown of mode operation i at time period t , 0 otherwise
 su_{it} 1 for the startup of mode operation i at time period t , 0 otherwise

sw_{ijt} 1 for the switchover from mode operation i to mode operation j at period t , 0 otherwise

y_{it} 1 for the changeover of mode operation i at time period t , 0 otherwise

yy_{it} 1 for the memory variable of mode operation i at time period t , 0 otherwise

D.1.2 Mathematical Formulation

$$\sum_i y_{it} \leq 1, \quad \forall t \quad (\text{D.1})$$

$$y_{it} = \sum_{tt=t-\tau_i+1}^t su_{i,tt}, \quad \forall t \quad (\text{D.2})$$

$$sd_{it} = su_{i,t-\tau_i+1}, \quad \forall t \quad (\text{D.3})$$

$$\sum_i yy_{it} = 1, \quad \forall t \quad (\text{D.4})$$

$$y_{it} - yy_{it} \leq 0, \quad \forall i, t \quad (\text{D.5})$$

$$yy_{it} - yy_{it-1} - su_{it} \leq 0, \quad \forall i, t \quad (\text{D.6})$$

$$\sum_j sw_{ijt} = yy_{it-1}, \quad \forall i, t \quad (\text{D.7})$$

$$\sum_i sw_{ijt} = yy_{jt}, \quad \forall j, t \quad (\text{D.8})$$

$$su_{jt} + sd_{i,tt} \leq 1, \quad \forall i \neq j, t - tt = 0, \dots, \tau_{ij} \quad (\text{D.9})$$

It should be mentioned that in the above model, except for variable su_{it} , all variables can be relaxed as continuous variables in interval $[0, 1]$.

D.2 Modified Model

To compare with the above literature model, operation i in the above equations is regarded as the processing operation for product i . Moreover, the following indices, sets, parameters, variables and constraints are added to the original model.

D.2.1 Nomenclature

Indices

d due date

Sets

O_i set of orders for product i

Parameters

B_i^L lower bound of batch size for operation i

B_i^U upper bound of batch size time for operation i

CC_{ij} changeover cost from product i to j

D_o demand of order o

DT_o due date of order o

H_d time of due date d

IC_i inventory cost of product i

K_d number of slots by due date d

PC_i processing cost of product i

Pr_i price of product i

RT_o release time of order o

V_i^U upper bound of inventory of product i

Continuous Variables

B_{it} batch size for operation i at time period t

P_{ot} processed amount for order o at time period t

S_{od} sales of order o at due date d

V_{od} inventory amount for order o at due date d

D.2.2 New Mathematical Formulation

D.2.2.1 Objective Function

In the modified model, we take the profit as the objective:

$$\begin{aligned} \text{Profit} = & \sum_t \sum_{o \in O_i} \sum_{d: RT_o \leq H_d \leq DT_o} Pr_i \cdot S_{od} - \sum_t \sum_i PC_i \cdot B_{it} - \sum_i \sum_{o \in O_i} \sum_{d: H_d \geq RT_o} IC_i \cdot V_{od} - \\ & \sum_t \sum_i \sum_{j \neq i} CC_{ij} \cdot sw_{ijt} \end{aligned} \quad (\text{D.10})$$

D.2.2.2 Constraints

The following constraints are considered for backlog, inventory and sales:

$$B_{it} \leq B_i^U \cdot y_{it}, \quad \forall i, t \quad (\text{D.11})$$

$$B_{it} \geq B_i^L \cdot y_{it}, \quad \forall i, t \quad (\text{D.12})$$

$$B_{it} = \sum_{o \in O_i \cap \{H_i \leq DT_o\}} P_{ot}, \quad \forall i, t \quad (\text{D.13})$$

$$V_{od} = V_{o,d-1} + \left(\sum_{i \in I_o} \sum_{j \in J_i, t=K_{d-1}+1}^{K_d} P_{ot} - S_{od} \right) \Big|_{H_d \leq DT_o}, \quad \forall o, d : H_d \geq RT_o \quad (\text{D.14})$$

$$\sum_{o \in O_i \cap \{o: H_d \geq RT_o\}} V_{od} \leq V_i^U, \quad \forall i, d \quad (\text{D.15})$$

$$\sum_{d: RT_o \leq H_d \leq DT_o} S_{od} \leq D_o, \quad \forall o \in O \quad (\text{D.16})$$

PUBLICATIONS

The following is the list of the publications arising from the work in this thesis:

Articles in Refereed Journals

- [1] Liu, S., Pinto, J.M., Papageorgiou, L.G. (2008) A TSP-based MILP model for medium-term planning of single-stage continuous multiproduct plants. *Industrial & Engineering Chemistry Research*, 47, 7733–7743.
- [2] Liu, S., Pinto, J.M., Papageorgiou, L.G. (2010) MILP-based approaches for medium-term planning of single-stage continuous multiproduct plants with parallel units. *Computational Management Science*, 7, 407–435.
- [3] Liu, S., Pinto, J.M., Papageorgiou, L.G. (2010) Single-stage scheduling of multiproduct batch plants: an edible-oil deodorizer case study. *Industrial & Engineering Chemistry Research*, 49, 8657–8669.
- [4] Konstantopoulou, F., Liu, S., Papageorgiou, L.G., Gikas, P. (2011) Water resources management for Paros island, Greece. *International Journal of Sustainable Water and Environmental Systems*, 2, 1–6.
- [5] Liu, S., Konstantopoulou, F., Gikas, P., Papageorgiou, L.G. (2011) A mixed integer optimisation approach for integrated water resources management. *Computers & Chemical Engineering*, 35, 858–875.
- [6] Liu, S., Papageorgiou, L.G., Gikas, P. (2011) Management of desalinated seawater, wastewater and reclaimed water in insular and geographically isolated areas using optimisation techniques. *Desalination and Water Treatment*, in press.
- [7] Liu, S., Papageorgiou, L.G., Gikas, P. (2011) Integrated management of non-conventional water resources in anhydrous islands. *Water Resources Management*, in press.

- [8] Liu, S., Shah, N., Papageorgiou, L.G. (2011) Multi-echelon supply chain planning with sequence-dependent changeovers and price elasticity of demand under uncertainty. *AIChE Journal*, submitted.
- [9] Liu, S., Papageorgiou, L.G. (2011) Multiobjective optimisation of production, distribution and capacity planning of global supply chains in the process industry. *Omega*, submitted.

Articles in Refereed Conference Proceedings

- [10] Liu, S., Pinto, J.M., Papageorgiou, L.G. (2009) Medium-term planning of multistage multiproduct continuous plants using mixed integer optimisation. In: Jeowski, J., Thullie, J. (eds.) *19th European Symposium on Computer Aided Process Engineering*, Computer Aided Chemical Engineering, vol. 26. Amsterdam: Elsevier. pp. 393–398.
- [11] Gikas, P., Liu, S., Papageorgiou, L.G. (2009) Optimisation of water resources management for insular and geographically isolated areas. In: Kungolos, A., Aravossis, K., Karagiannidis, A., Samaras, P. (eds.) *Proceedings of the 2nd International Conference on Environmental Management, Engineering, Planning and Economics (CEMEPE) & SECOTOX Conference, Mykonos, Greece, 21–26 June 2009*. Thessaloniki: Grafima. pp. 1625–1630.
- [12] Liu, S., Gikas, P., Papageorgiou, L.G. (2010) An optimisation-based approach for integrated water resources management. In: Pierucci, S., Buzzi Ferraris, G. (eds.) *20th European Symposium on Computer Aided Process Engineering*, Computer Aided Chemical Engineering, vol. 28. Amsterdam: Elsevier. pp. 1075–1080.

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