Department of Economics

A multi-agent methodology to assess the effectiveness of alternative systemic risk adjusted capital requirements

Andrea Gurgone
Macro-Financial Division,
Central Bank of Ireland, Dublin

Giulia Iori[1]
Department of Economics,
City, University of London

**Department of Economics
Discussion Paper Series
No. 19/05**

[1] Corresponding author: Giulia Iori, Department of Economics, City, University of London, Northampton Square, London EC1V 0HB, UK.
Email: G.Iori@city.ac.uk

A multi-agent methodology to assess the effectiveness of alternative systemic risk adjusted capital requirements

Andrea Gurgone[a] and Giulia Iori[b]

[a] *Macro-Financial-Division, Central Bank of Ireland, Dublin*
[b] *Department of Economics, City, University of London*

Abstract

We propose a multi-agent approach to compare the effectiveness of macro-prudential capital requirements, where banks are embedded in an artificial macroeconomy. Capital requirements are derived from systemic-risk metrics that reflect both the vulnerability or impact of financial institutions. Our objective is to explore how systemic-risk measures could be translated in capital requirements and test them in a comprehensive framework. Based on our counterfactual scenarios, we find that macro-prudential capital requirements can mitigate systemic risk, but there is a trade-off between market- and balance-sheet-based policies in terms of banks' losses and credit supply.

This is a preliminary draft. Please do not cite or distribute without permission of the authors.

1 Introduction

The concept of *systemic risk* (SR) is relatively recent in economic and financial literature. The first appearance in scientific articles dates back to the early '90s, even if citations reveal that most of these contributions have been revived after 2008, when the term regained strength with the crisis. *Kaufman and Scott (2003, p. 371)* provide a general definition: "*Systemic risk refers to the risk or probability of breakdowns in an entire system, as opposed to breakdowns in individual parts or components, and is evidenced by comovements (correlation) among most or all the parts.*"

Macroprudential policies are the instrument through which regulators could mitigate systemic risk. They should be viewed as complementary to microprudential policies and be designed to improve the resistance of the whole financial system to unforeseen events. However, such policies are hard to implement, *inter alia* because they should be built on a reliable measure of systemic risk: it is unclear which metric performs better and under what circumstances. The task is more intricate given that systemic events are observed infrequently, as a banking crisis is observed on average every 35 years for OECD countries *(Danielsson et al., 2016)*.

Regulators require globally systemically important banks (G-SIBs) to set aside enough capital to cover unexpected losses and keep themselves solvent in a crisis. G-SIBs' surcharges have been in the spotlight in the financial literature with some studies suggesting they are inadequate to prevent the spread of systemic crisis. In this article we explore the effectiveness of capital surcharges derived from different systemic risk measures. By assuming that banks adopt different capital rules within a multi-agent macro-economic model, we quantify the impact of such policies in a stress-test scenario-based analysis.

Many methods to measure systemic risk have been proposed so far, but there is no consensus among scholars on which is most appropriate. We consider two alternative classes, namely *market-based* and *balance-sheet* approaches. A further distinction between them is what they measure: vulnerability or impact. Vulnerability focuses on the effect of a systemic event on the capital of banks, while impact captures the losses produced by the distress of banks on the rest of the financial system.

The first measure of vulnerability is based on LRMES (Long Run Marginal Expected Shortfall) *(Acharya et al., 2012)*, which contributes to compute SRISK. The second is derived from a stress test based on DebtRank, in such a way to obtain an expected shortfall on individual vulnerabilities. The indicators for impact are $\Delta CoVaR$ *(Adrian and Brunnermeier, 2016)* and DebtRank in strict sense.[1]

Counterfactual policy experiments are conducted on an agent-based model of the economy based on *Gurgone et al. (2018)*. The original model is expanded to allow banks to employ systemic-risk measures to determine their capital re-

---

[1]The DebtRank framework provides both measures of vulnerability and impacts. However the term "*DebtRank*" specifically refers to the systemic impact of banks in its original formulation. For further details see *Battiston et al. (2016)*

quirements. In the first set of experiments we assume that capital requirements are set on the basis of vulnerability metrics, so that more fragile banks are required to hold a proper equity capital. However, this might not be satisfactory, as it does not operate on systemic impact of banks. Hence in the second set of experiments capital requirements depend on the impact of banks on the system, or the extent of externalities they produce in case of default.

We find that putting this kind of macroprudential policy in place is preferable to than not to have it, as systemic-capital requirements are able to stabilize the economy. Moreover, market and balance-sheet policies differ in some aspects: the former are more robust as their ranks are less volatile over time, but balance-sheet measures are better to capture the build-up of systemic risk. We also find a trade-off between the two sets of measures in terms of banks' losses and lending capacity.

This paper is the first attempt to (i) suggest how to incorporate heterogeneous systemic-risk metrics into banks' capital requirements; (ii) compare market-based measures with balance-sheet measures of systemic risk, both from the perspective of vulnerability of single institutions and the individual impacts on the financial system; (iii) another divergence from the existing literature is the method of assessment. Our analysis is performed by means of simulated data, generated by a multi-agents model, rather than empirically observed data that, given the rare occurrence of systemic crisis, are scant. Our simulated economy produces data on returns on equities and at the same time includes a network structure of interlocked balance sheets, thus it allows for a double comparison.

The paper is organized as follows: Section 2 presents the related literature. Section 3 describes the modelling framework, distress dynamics, systemic-risk measures and macro-prudential policies. Section 4 goes through the results of the simulations and the policy experiments. Conclusions are in Section 5.

2 Related literature

Several studies have compared systemic risk measures so far. Part of the literature aims to compare different measures of systemic risk by means of econometric methods. *Benoit et al. (2013)* provide a theoretical and an empirical comparison of three market-based measures of systemic risk, namely MES, SRISK and $\Delta CoVaR$. They find that there is no measure able to fully account for multiple aspects of SR, but SRISK is fairly good for describing both the too-big-to-fail and too-interconnected-to-fail dimensions. *Kleinow et al. (2017)* empirically compare four widespread measures of systemic risk, namely MES, Co-Risk, $\Delta CoVaR$ and LTD using data on US financial institutions. Their estimates point out that the four metrics are not consistent with each other over time, hence it is not possible to fully rely on a single measure. *Rodríguez-Moreno and Peña (2013)* consider six measures of systemic risk using data from stock, credit and derivative markets. They quantitatively evaluate such metrics through a "horse race", exploiting a sample composed of the biggest European and US

banks. Their results favour SR measures based on simple indicators obtained from credit derivatives and interbank rates, rather than more complex metrics whose performance is not as satisfactory. Similarly *Pankoke (2014)* opposes sophisticated to simple measures of systemic risk and concludes that simple measures have more explanatory power.

Other studies assume that the regulator is disposed to tolerate a systemic-wide risk level and aims to reach the most parsimonious feasible capitalization at the aggregate level. Such objective is formally translated into a constrained optimization problem, whose solution includes both the unique level of capital in the banking system and its distribution across banks. *Tarashev et al. (2010)* find that if capital surcharges are set in order to equalize individual contributions to systemic risk, then a lower level of aggregate capital is needed to reach the system-wide risk objective. *Webber and Willison (2011)* find that optimal systemic capital requirements increase in balance sheet size and in the value of interbank obligations. However, they are also found to be strongly pro-cyclical.

Another set of contributions presents balance-sheet and network approaches to quantify systemic risk. *Battiston et al. (2016)* propose a network-based stress test building on the DebtRank algorithm. The framework is flexible enough to account for impact and vulnerability of banks, as well as to decompose the transmission of financial distress in various rounds of contagion and to estimate the distribution of losses. They perform a stress-test on a panel of European banks. The outcome indicates the importance of including contagion effects (or indirect effects) in future stress-tests of the financial system, so as not to underestimate systemic risk. *Alter et al. (2014)* study a reallocation mechanism of capital in a model of interbank contagion. They compare systemic risk mitigation approaches based on risk portfolio models with reallocation rules based on network centrality metrics and show that allocation rules based on centrality measures outperform credit risk measures. *Gauthier et al. (2010)* compare capital allocation rules derived from five different measures of systemic risk by means of a network-based model of interbank relations applied to a dataset including the six greatest banks of Canada. They also employ an iterative optimization process to solve the optimal allocation of capital surcharges that minimizes total risk, while keeping constant the total amount of capital to be kept aside. The adopted framework leads to a reduction of the probability of systemic crises of about 25%; however, results are sensitive to including derivatives and cross shareholdings in the data.

Poledna et al. (2017) propose to introduce a tax on individual transactions that may lead to an increase in systemic risk. The amount of the tax is determined by the marginal contribution of each transaction to systemic risk, as quantified by the DebtRank methodology. This approach reduces the probability of a large-scale cascading event by re-shaping the topology of the interbank networks. While the tax deters banks from borrowing from systemically important institutions, it does not alter the efficiency of the financial network, measured by the overall volume of interbank loans. The scheme is implemented in a macro-financial agent-based model, and the authors show that capital surcharges for G-SIBs could reduce systemic risk, but they would have to be sub-

stantially larger than those specified in the current Basel III proposal in order to have a measurable impact. Our framework is similar to *Poledna et al. (2017)*, but we consider a wider set of systemic risk measures and compare their effectiveness against each other.

3   The model

3.1   Macroeconomic Model

The macroeconomy is based on an amended version of the ABM in *Gurgone et al. (2018)*. The economy is composed of several types of agents: households, firms, banks, a government, a central bank and a special agency. The (discrete) numbers of households, firms and banks are N^H, N^F, N^B respectively.

Interactions take place in different markets: firms and households meet on markets for goods and for labour, while firms borrow from banks on the credit market and banks exchange liquidity on the interbank market. The CB buys government-issued bills on the bond market. The role of the government is to make transfer payments to the household sector. The governmental budget is balanced, namely the transfers are funded by taxes while the level of the public debt is maintained at a steady level. The CB generates liquidity by buying government bills and providing advances to those banks that require them; it furthermore holds banks' reserve deposits in its reserve account. Households work and and buy consumption goods by spending their disposable income. It is made up of wage and asset incomes after taxes and transfers. In the labour market, households are represented by unions in their wage negotiations with firms, while on the capital market, they own firms and banks, receiving a share of profits as part of their asset income. Firms borrow from banks in order to pay their wage bills in advance, hire workers, produce and sell their output on the goods market. The banking sector provides credit to firms, subject to regulatory constraints. In each period every bank tries to anticipate its liquidity needs and accesses the interbank market as a lender or a borrower. If a bank is short of liquidity, it seeks an advance from the CB.

The special agency was not present in *Gurgone et al. (2018)*. It has been introduced as a convenient way to model the secondary market for loans. It acts as a liquidator when banks default or when banks exceed the regulatory constraint and thus must de-leverage. The assets in its portfolio are then put on the market and can be purchased by those banks that have a positive credit supply. Further details about the working of the special agency are described below.

3.2   Distress dynamics

Banks and firms default if their equity turns negative. Distress propagates through defaults in the credit and interbank markets and banks' deposits. The transmission begins when firms cannot re-pay loans due to a negative outcome in

the goods market. Shocks propagate from firms to banks, within the interbank market and from banks to firms.[2] The process is illustrated in Fig. 1 and terminates only when there are no new losses.
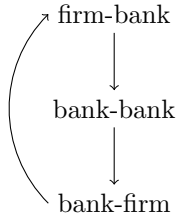
firm-bank

bank-bank

bank-firm

Figure 1: Diagram of the distress transmission. The distress is transmitted through the credit market (firm-bank), the interbank market (bank-bank) and banks' deposits (bank-firms).

Liquidation of assets The contagion dynamic is enhanced by the forced liquidation of assets sold by defaulted banks in order to repay creditors. The role of liquidator is operated by a special agency that buys the assets of bank i at price p:

$$p_\tau = p_{\tau-1} \left(1 - \frac{\Delta q_{i,\tau}}{q_t} \frac{1}{\epsilon} \right) \tag{1}$$

where $\Delta q_{i,\tau}$ is the quantity of loans that bank i needs to liquidate,[3] ϵ is the asset price elasticity, q_t is the total quantity of loans in period t. Banks that need liquidity enter the market in a random order represented by the subscript τ; we assume that at the end of each period of the simulation, the initial asset price is set again at $p_0 = 1$. The assets purchased by the agency are then put on sale before the credit market opens (lending to firms). Banks with positive net worth and complying with regulatory leverage rate can buy them at their net present value.

Recovery rates The effective loss on a generic asset A_{ij} owed by j to i is $A_{ij}(0)(1-\varphi_{ij}(t))$, where φ is the recovery rate. Each of j's creditors can recover $\varphi_{ij} = \frac{\mathcal{A}_j}{\mathcal{L}_j}$, *i.e.* the ratio of borrower's assets ($\mathcal{A}$) to liabilities ($\mathcal{L}$). However, the nominal value of illiquid assets is not immediately convertible in cash and must

---

[2]If the net worth of a bank is negative, it defaults on its liabilities including the deposits of firms and households.

[3]Banks first determine their liquidity need, then compute the fair value of their portfolio loan by loan. Next they determine Δq taking into account eq. (1). Lastly, they choose which loans should be liquidated to reach their objective.

The loans for sale are evaluated at their fair market value by discounting cash flows:

$$L_{ij}^{fv} = \frac{L_{ij}(1 + Sr^f)(1 - \rho_j^f)}{\underline{r}^S}$$

where $L_{i,j}$ is the book value of the loan of bank i to firm j, S is the residual maturity, r^f is the interest rate on the loan, ρ^f is the default probability of firm j, and $\underline{r}$ is the risk-free rate.

be first liquidated to compensate creditors. We denote the liquidation value of the assets of bank j with $\mathcal{A}_{j,t}^{liq}$, with $\mathcal{A}_{j,t}^{liq} \leq \mathcal{A}_{j,t}$. The actual recovery rate can be written as:

$$\varphi_{ij} \equiv \frac{\mathcal{A}_j^{liq}}{\mathcal{L}_j}$$

Furthermore, we assume that there is a pecking order of creditors, so that they are not equal from the viewpoint of bankruptcy law: the most guaranteed is the central bank, then depositors and finally banks with interbank loans. For instance, those creditors who claim interbank loans towards the defaulted bank j recover the part of j's assets left after the other creditors have been compensated. The recovery rate on an interbank loan, can be expressed as:

$$\varphi_{ij} = \max\left(0, \ \frac{\mathcal{A}_j^{liq} - A_j^{CB} - D_j}{\mathcal{L}_j - A_j^{CB} - D_j}\right) \tag{2}$$

where A^{CB} are central bank's loans to j and D are j's deposits.

It is worth noticing that *loss given default* is $LGD \equiv 1 - \varphi$, so that the net worth of creditor i updates as $nw_{i,t}^B = nw_{i,t-1}^B - LGD_{ij,t} I_{i,t}^l$.

3.3 Measuring systemic risk

Before defining systemic risk adjusted capital requirements (SCR) we need to introduce how we measure SR. We do it along two dimensions, that is the vulnerability of banks to a systemic crisis and their systemic impact on the financial sector.

Balance-sheet measures: DebtRank

The DebtRank algorithm can provide both measures of vulnerability and impact of banks (see Sect. 6.2 for details), that are respectively denoted by DR^{vul} and DR^{imp}.

Each measure is computed by repeating DebtRank 500 times. In each run we randomly draw from the distribution of recovery rates, which are extracted from a vector of data generated by the benchmark model. At the end, we compute the average out of the 500 realizations after removing the $1st$ and the $99th$ percentiles. Further details about the calibration procedure are detailed in Sect. 6.1.

Individual vulnerabilities produced by the stress test are expressed in terms of the relative equity loss of each bank at the last step of the algorithm ($\tau = T$) after we impose a shock on assets.

$$h_{i,T} \equiv \frac{nw_{i,T}^B - nw_{i,0}^B}{nw_{i,0}^B} \tag{3}$$

7

The impact of each bank on the rest of the system is the overall loss in capital produced by the default of bank i. The values for each institution are obtained by imposing its default at the beginning of the algorithm.

$$g_i = \sum_{j=1}^{N^b} h_{j,T} \ nw_{i,0}^B \qquad (4)$$

Market-based measures: SRISK and ΔCoVaR

SRISK *(Brownlees and Engle, 2012)* is a widespread measure of systemic risk based on the idea that capital shortages in case of a systemic crisis can be inferred from the tail of the distribution of negative equity returns during normal days, given that a crisis is a rare event whose data are barely available. Systemic risk arises when the financial system as a whole is under-capitalized, leading to externalities for the real sector.

We add SRISK to our model by following the approach in *Brownlees and Engle (2012)*. The SRISK of a financial firm i is defined as the quantity of capital needed to re-capitalize it conditional to a systemic crisis:

$$SRISK_{i,t} = \max \left[0, \ \frac{1}{\lambda}\mathcal{L}_i - \left(1 - \frac{1}{\lambda} \right) nw_{i,t}^B MES_{i,t+h|t}^{sys} \right] \qquad (5)$$

where $\mathcal{L}_i$ are i's liabilities and $MES_{i,t+h|t}^{Sys} = E\left(r_{i,t+h|t}|r < \Omega \right)$ is the tail expectation of i's equity returns conditional on a systemic event that happens when r is less than a threshold value Ω from $t-h$ to t.

Starting from 50 periods prior the external shock each bank computes its own SRISK on a time window made of the last 200 periods. The required information are the individual and market returns. The first are computed as returns on equity (ROE) of bank i, that correspond to the relative change in i's net worth during each step of the simulation.[4] The same logic is applied to obtain market returns, but they are weighted by net worth of each bank.

Another well-known measure of systemic risk is $\Delta CoVaR$, which quantifies the systemic distress conditional to the distress of a specific financial firm, namely it accounts for the impact of a bank on the financial system.

$CoVaR$ is implicitly defined as the VaR of the financial system (sys) conditional on an event $C(r_{i,t})$ of institution i:

$$Pr \left[r_{sys,t} \leq CoVaR^{sys|C(r_i)} \mid C(r_{i,t}) \right] = \alpha \qquad (6)$$

where r represents ROE and the conditioning event $C(r_i)$ corresponds to a loss of i equal or above to its Var_α^i level.

---

[4]We account the final value of the new worth before a bank is recapitalized, otherwise returns would be upwards biased by shareholders' capital.

$\Delta CoVaR$ is a statistical measure of tail-dependency between market returns and individual returns, which is able to capture co-movements of variables in the tails and account for both spillovers and common exposures. $\Delta CoVaR$ is the part of systemic risk that can be attributed to i: it measures the change in value at risk of the financial system at α level when the institution i shifts from its normal state (measured with losses equal to its median Var) to a distressed state (losses greater or equal to its Var).

$$\Delta CoVaR_{\alpha}^{sys|i} = CoVaR_{\alpha}^{sys|r_i = VaR_{i,\alpha}} - CoVaR_{\alpha}^{sys|r_i = VaR_{i,0.5}} \qquad (7)$$

A flaw of $\Delta CoVaR$ is its (at best) contemporaneity with systemic risk: it fails to capture the build-up of risk over time and suffers of procyclicality. Furthermore, contemporaneous measures lead to the "*volatility paradox*" *(Brunnermeier and Sannikov, 2014)*, inducing banks to increase the leverage target when contemporaneous measured volatility is low. A workaround would be to substitute contemporaneous with a forward-looking version of $\Delta CoVaR$ *(Adrian and Brunnermeier, 2016, p.1725)*. The latter is obtained by projecting on the regressors of $\Delta CoVaR$ their estimated coefficients, where the independent variables include individual banks' characteristics and macro-state variables. Nevertheless our model lacks of the wide range of variables that can be employed in empirical works, as a results our measure of forward $\Delta CoVaR$ turns out to be strongly proportional to the VaR of banks, thus failing to capture the build up of systemic risk.

3.4 Adjusted Capital Requirements

The comparability of policy experiments under different SR indicators requires a comprehensive definition of systemic risk adjusted capital requirements (SCR). In all cases capital requirements are expressed in an intuitive way, as SR indexes are normalized in the interval $[0, 1]$. Banks must hold a minimum net worth equal to a fraction of their risk weighted assets (RWA),[5] $nw_{i,t}^B \geq \psi_{i,t} RWA_{i,t}$, where ψ is a parameter determined by SR metrics. If a systemic risk measure equals 0, then $\psi = \frac{1}{\lambda}$ and a bank must have a capital greater or equal than a standard regulatory threshold, namely $nw^B \geq \frac{1}{\lambda} RWA$. When it equals 1, then $\psi = 1$ and capital requirements are as strict as possible, so that equity should equal assets, $nw^B = RWA$.

Banks comply with capital requirements in two ways: (i) if their actual leverage ratio is below the maximum admitted, they set the loan supply L^s and interbank supply I^s accordingly; (ii) if the actual leverage rate exceeds the maximum value, they de-leverage by selling the assets in excess to the special agency.

---

[5]For the sake of simplicity we assign a weight equal to 1 to loans to firms and to interbank lending, while liquidity is assumed to be riskless, hence its weight is 0. Risk weighted assets of bank i can be expressed as $RWA_{it} = 1 \times (L_{it}^F + I_{it}^l) + 0 \times R_{it} = L_{it}^F + I_{it}^l$. Total assets can be written as $\mathcal{A}_{i,t} = RWA_{i,t} + R_{i,t}$.

Banks supply loans up to a multiple of their net worth (nw^B), net of outstanding loans (L) at the beginning of t, assuming that all the interbank loans have been settled. The loan supply of bank i is:

$$L_{i,t}^s \leq \frac{1}{\psi} nw_{i,t}^B - L_{i,t} \tag{8}$$

Our unifying procedure to compare systemic capital requirements builds on the approach of *Acharya et al. (2012)*. Expected capital shortfall (CS) is the difference between minimum regulatory capital expressed as a fraction $\frac{1}{\lambda}$ of risk weighted assets (RWA) and the book value of equity in case of a crisis. It is the capital needed to restore capital adequacy ratio to the value set by the regulator:

$$
\begin{aligned}
CS_{i,t+\tau|t} &= \max\left\{0,\ E_t\left[\frac{1}{\lambda}RWA_{i,t+\tau} - nw_{i,t+\tau}^B|\ crisis_{t+tau}\right]\right\} \\
&= \max\left\{0,\ E_t\left[\frac{1}{\lambda}\mathcal{L}_{i,t+\tau} - R_{i,t+\tau}|\ crisis_{t+\tau}\right] + \right. \\
&\qquad \left. -E_t\left[\left(1 - \frac{1}{\lambda}\right)nw_{i,t+\tau}^B|\ crisis_{t+\tau}\right]\right\}
\end{aligned}
\tag{9}
$$

By assumption debt and liquidity are unchanged in case of crisis,[6] hence $E_t\left[\mathcal{L}_{i,t+\tau} - R_{i,t}|\ crisis_{t+\tau}\right] = \mathcal{L}_{i,t} - R_{i,t+\tau}$.

It turns out that

$$
\begin{aligned}
CS_{i,t+\tau|t} &= \max\left\{0,\ \frac{1}{\lambda}\left(\mathcal{L}_{i,t} - R_{i,t}\right) - E_t\left[\left(1 - \frac{1}{\lambda}\right)nw_{i,t+\tau}^B|\ crisis_{t+\tau}\right]\right\} \\
&= \max\left\{0,\ \frac{1}{\lambda}\left(RWA_{i,t} - nw_{i,t}^B\right) - E_t\left[\left(1 - \frac{1}{\lambda}\right)nw_{i,t+\tau}^B|\ crisis_{t+\tau}\right]\right\}
\end{aligned}
$$

$$\tag{10}$$

Vulnerability adjusted capital requirements

Adjusted capital requirement based on vulnerability are obtained under the assumption that the conditional expectation is:

$$E_t\left[nw_{i,t+\tau}^B|\ crisis_{t+\tau}\right] = (1 - vul_{i,t})nw_{i,t}^B \tag{11}$$

where $vul = \{LRMES,\ DR^{vul}\}$.

---

[6]In other words we assume that in case of crisis the major change in net worth is derived from losses on loans while cash flows and liabilities are constant. Even if there were a change in liabilities, for instance in deposits, the difference $\mathcal{L} - R$ remains constant because deposits and R would be reduced by the same amount.

$LRMES$ is the Long Run Marginal Expected Shortfall that is used to compute SRISK in *Acharya et al. (2012)* (see Sect. 6.3). DR^{vul} is the vulnerability index of financial institutions described in 3.3.

Capital requirements for bank i are then obtained by imposing $SRISK = 0$, so that it should always maintain a capital buffer great enough to avoid recapitalization during periods of distress:

$$nw_{i,t}^{B} \geq \frac{\frac{1}{\lambda}}{1 - (1 - \frac{1}{\lambda})LRMES_{i,t}} RWA_{i,t} \qquad (12)$$

Impact adjusted capital requirements

We adopt a top-down approach to ensure consistency with the previous rule and that based on impact. Adjusted capital requirements are defined starting from the aggregate and then deriving the individual requirements that each bank must satisfy to reach the aggregate objective. The idea is that banks contribute to the aggregate expected capital shortage in proportion to their systemic impact. To this end, we rewrite the expected capital shortage in aggregate terms.

$$\sum_{i=1}^{N^b} CS_{i,t+\tau|t} = \frac{1}{\lambda} \left(\sum_{i=1}^{N^b} RWA_{i,t} - \sum_{i=1}^{N^b} nw_{i,t}^{B} \right) + \\ -E_t \left[\left(1 - \frac{1}{\lambda}\right) \sum_{i=1}^{N^b} nw_{i,t+\tau}^{B} \mid crisis_{t+\tau} \right] \qquad (13)$$

We assume that each bank contributes to the expected aggregate capital shortage proportionally to its impact. To keep internal consistency and to avoid aggregation issues we also assume that the aggregate capital shortage is the sum of individual capital shortages values, computed with the same procedure of Sect. 3.4 (respectively by $LRMES$ and DR^{vul}).

Each bank should contribute to expected capital shortage in proportion to its systemic importance. That means that the additional capital required for each bank is

$$nw_{i,t}^{+} = \frac{imp_{i,t}}{\sum_{i=1}^{N^b} imp_{i,t}} \sum_{i=1}^{N^b} CS_{i,t+\tau|t} \qquad (14)$$

where $imp = \{\Delta CoVaR_t^{sys|i}, \ DR^{imp}\}$.

Hence the target level of capital for bank i is given by the minimum regulatory level of capital plus the additional capital:

$$nw_{i,t}^{tag} = \frac{1}{\lambda} RWA_{i,t} + nw_{i,t}^{+} \qquad (15)$$

A rule consistent with those descending from vulnerability is:

$$nw_{i,t}^{tag} \geq \frac{\frac{1}{\lambda}}{1 - (1 - \frac{1}{\lambda})\zeta_i} RWA_{i,t} \tag{16}$$

with $\zeta_i = \frac{nw_{i,t}^+}{(1-\frac{1}{\lambda})(\frac{1}{\lambda}RWA_{i,t}+nw_{i,t}^+)}$ [7]

4   Results

This section presents the results of our simulations and the policy experiments, whose aim is to show what would happen if measures of systemic risk were employed to determine SCR of banks. We compare the benchmark scenario, where all banks are subject to the same fixed regulatory ratio $\lambda^{max} = 0.08$, to those where SCR are derived from macroprudential measures of vulnerability or impact of financial institutions, as described in Section 3.4. We run a set of 100 Monte Carlo simulations for each scenario under different seeds of the pseudo-random numbers generator.
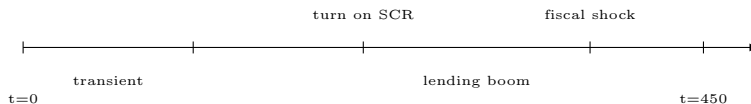

Figure 2: Timeline of the simulations.

The simulations are based on a variant of the macroeconomic model in *Gurgone et al. (2018)* in which the wage-price dynamics is dampened by setting the wage rate constant, so that business-cycle fluctuations are eliminated and the model converges to a quasi-steady-state after a transient period. Moreover, we supply to the lack of fluctuations of credit by simulating a lending boom, that is increasing the credit demand of firms in the 50 periods before an external shock. It increases the exposures of banks and contributes to the build-up of the risk. At the beginning of the lending boom stage we turn on systemic-capital requirements, so that the macroprudential regulation is binding. We finally impose a fiscal-shock for 10 periods that consists in a progressive reduction of transfers to the household sector. The purpose of the shock is to reduce the disposable income of households, that in turn affects consumption and firms' profits. Firms with negative equity then cannot repay their debts to the banking sector, thus the initial shock triggers a series of losses through the interlocked balance sheets of agents. At $S = shock$, transfers are reduced by 20% and then by an additional 5% per period with respect to the period before the shock $(S - 1)$. Fig. 2 summarizes what happens during each simulation.

A snapshot of the behaviour of SR measures over time is shown in Section 4.1. Autocorrelation is analysed in Section 4.2, finally empirical distributions arising from the simulations are presented in Section 4.3.

---

[7]If $nw_{i,t}^+ = 0 \Rightarrow \zeta_{i,t} = 0$ and $nw_{i,t}^{tag} = \frac{1}{\lambda}RWA_{i,t}$. Moreover we assume that $\zeta \in [0, 1]$, so that banks must hold at most an amount of capital equal to their assets.

4.1 SR measures over time

The next lines provide a qualitative analysis of market and balance-sheet based metrics to observe their behaviour on the periods over the shock.

Fig.s 3 and 4 depict the measured value of systemic risk at different times. It is worth to note that SCR are not active in this context, but we show the values of systemic risk as it is measured by the set of indicators described in 3.3.

Market based measures are in Fig. 3, where vulnerability (LRMES) is on the horizontal axis and the vertical axis reports impact (ΔCoVaR). In Fig. 4 vulnerability and impact are measured respectively by DR-vul and DR-imp. The size of the circles represent the leverage rate of banks (loans to equity), and colour reflects equity.

In Fig. 3 banks tend to move towards the right part of the graph on the horizontal axis as time approaches to $t = 400$, but observed impacts are low. ΔCoVaR remains near 0 until the shock turns towards the end. During the next periods SR metrics are more heterogeneous, but leverage and net worth are far lower in the aftermath of the crisis. A different picture of systemic risk is provided by Fig. 4, where the impact of bank measured by DR-imp stays high and rather constant until $t = 405$. Vulnerability moves slowly rightwards until $t = 400$, when almost all banks have non-zero values. A large number of banks is systemic from the viewpoint of impact, but their potential could be realized only if they become vulnerable and suffer losses. After $t = 10$, balance-sheet based measures shift towards the left-lower corner, meaning that banks are neither vulnerable or dangerous because there is no more capital to be lost.

There are two remarkable differences between market and balance-sheet based metrics. First let's look at the time evolution: market-based indexes move slowly and the larger movements are observed only after the shock at $t = 400$. They are quite stable over time and show high persistence, although ΔCoVaR is pro-cyclical and turns out to be ineffective as an early warning signal to predict the crisis. Their value does not decline after the end of the shock. Balance-sheet indexes offer a better dynamical representation of systemic risk, that is high impacts and vulnerabilities before the crisis but declining after. Second, the size and the leverages of banks are differently associated to systemic risk in each set of measures. In particular, all leveraged institutions are represented in the right part of the graphs in the periods before the crisis by DebtRank. This is not the case for market-based measures and identifies leveraged banks as the most vulnerable and in some cases also high-impact.

We can conclude that market based metrics offer a nearly-static but robust representation of the evolution of systemic risk as banks' ranking are more autocorrelated (see Sect 4.2) than in DebtRank. At the opposite, balance-sheet based measures seem to better reflect the evolution of risk, but display more volatility as they are sensible to small changes in the network of exposures.
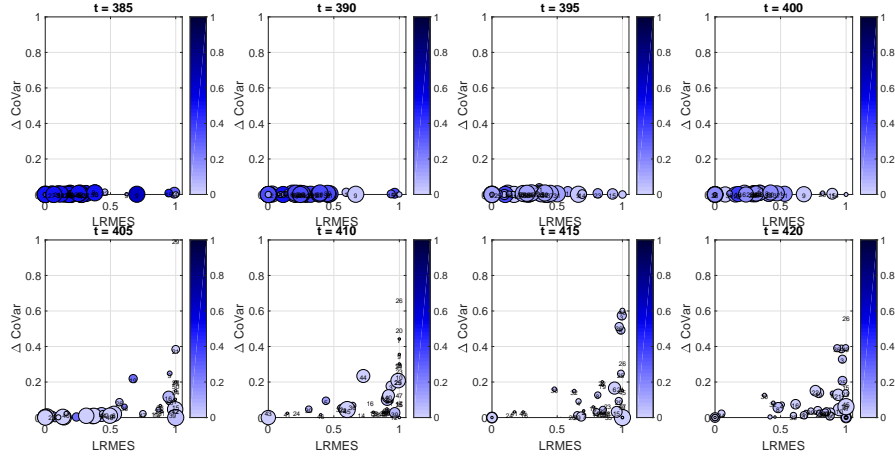
Figure 3: Market based SR measures over time. Circles' size represents assets to equity ratio, colour is the normalized equity.
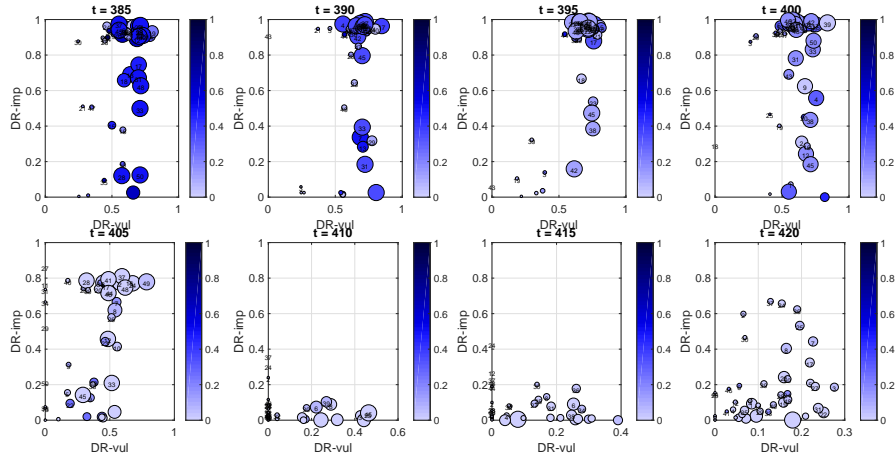

Figure 4: Balance-sheet based SR measures over time. Circles' size represents assets to equity ratio, colour is the normalized equity.

4.2 Rank correlation

A desirable property of SR measures would be to be stable over time, so that the ranking of systemically important financial institutions has no high variability and identifies the same set of subjects in a given time span. We study the auto-correlation of SR metrics to understand how stable they are.

We consider a measure of rank correlation, Kendall's tau (τ^k), which is a non-parametric measure of correlation between pairs of ranked variables with values between -1 and 1. If two variables are perfectly correlated $\tau^k = 1$, otherwise if there is no correlation at all $\tau^k = 0$.

$$\tau^k = \frac{C - D}{n(n-1)/2}$$

where C and D are the total number of concordant and discordant pairs and n is the sample size. Moreover when two variables are statistical independent, a z statistics built on τ^k tends to distribute as a standard normal, therefore it can be tested the null of no correlation versus the alternative of non-zero correlation.

We compute τ^k between the rank of SR measures of each bank and its lagged values. Results are reported in Tab. 1. When market-based measures are considered, the ranking has a high and persistent autocorrelation. On the other hand balance-sheet based measures are autocorrelated to a lower extent. The difference could be explained in terms of construction, as market-based measures are obtained from conditional variances (or conditional VaR), which in turn are estimated through a TGARCH model, where conditional variances are assumed to follow an autoregressive process (see Section 6.3). Conversely, balance-sheet based measures do not assume any dependence on past values, rather they depend on the network structure and credit-debt relationships, so that the outcome of the DebtRank algorithm might change as a result of small variations in configuration of the network.

Kendall's tau

	Lags			
SR metric	+1	+5	+10	+15
DR-vul	0.150	0.058	0.027	0.016
	(0.620)	(0.800)	(0.940)	(0.900)
LRMES	0.646	0.286	0.086	-0.022
	(0.020)	(0.280)	(0.640)	(0.720)
DR-imp	0.056	0.028	-0.004	0.004
	(0.800)	(0.900)	(0.940)	(0.980)
$\Delta CoVaR$	0.543	0.151	0.073	0.002
	(0.000)	(0.560)	(0.820)	(0.800)

Table 1: Kendall's correlation coefficients. Reported statistics refers to the average of τ^k computed for each bank. The share of p-values exceeding 0.05 are reported in parenthesis.

.

4.3 Policy Experiments

We present here the results of the policy experiments from 100 Monte-Carlo simulations for each scenario, in which SCR are activated. The artificial data considered in the graphs refer to periods 350-410, namely those following the credit boom and the shock. We cleaned the data to remove the outliers by trimming the observations above (below) the third (first) quartile plus (minus) 1.5 times the interquartile range.

Fig.s 5-9 are boxplots comparing the effects of SCR on selected variables.[8]

---

[8] The red line represents the sample median, the blue lines below and above the median

Systemic capital requirements succeed in reducing the leverage rate in the economy (Fig. 5), however, this comes at the cost of reduced lending capacity of banks. They also contain the losses of banks (except ΔCoVaR), as shown in Figs. 6 and 7. Balance-sheet based measures do a better job in limiting the losses of banks from the firm sector, while market-based measures are more effective on interbank losses and further rounds of contagion. The difference could be explained in terms of reduced lending on the interbank market: as balance-sheet measures impose stricter capital requirements before the shock (compare Fig.s 3 and 4), banks reduce lending to firms and interbank funding. On one hand, this results in a reduction of credit risk and losses inflicted by firms. On the other, borrower on the interbank market should turn to the lending facility of the central bank. By borrowing at an high interest rate they reduce their profitability and equities and thus increase the probability to be insolvent on interbank loans (see defaults in Fig. 8). This result suggests to assign to short-term interbank funding a lower weight compared to loans to firms in risk weighted assets. The worsening of rationing on the interbank market could lead to undesirable outcomes, that is increased cost of funds or regulatory arbitrage. Finally, Fig. 9 illustrates the losses in the firm sector. Similarly to above, balance-sheet-measures limit the exposures of firms to banks but worsen the defaults of banks, which spill-over into the firm sector.

By comparing vulnerability and impact based policies it does not result a notable difference, even if general measures based on vulnerability are slightly better to reduce the right skewness of the empirical distributions. This descends from the construction of impact-based capital requirements, that assign the expected capital loss induced by each bank top-down. In other words, the capital required by each institution is a function of the aggregate expected capital shortfall computed by metrics based on vulnerability. In this sense, the mapping of systemic-risk measures to capital requirements produce better results for vulnerability.

---

are the first and third quartiles of the sample. The black lines above and below the box are the whiskers, which extend from the nearest quartile to 1.5 times the interquantile range. Observations above (below) the whiskers are outliers represented by dots. Notches display a confidence interval above and below the median

$$median \; \pm 1.57 \times \frac{Inter \; Quantile \; Range}{\sqrt{n}}$$

If the notches of a pair of boxplots do not overlap, we can reject the null that the medians come from the same population with 95% confidence, namely their difference is statistically significant *(McGill et al., 1978)*.
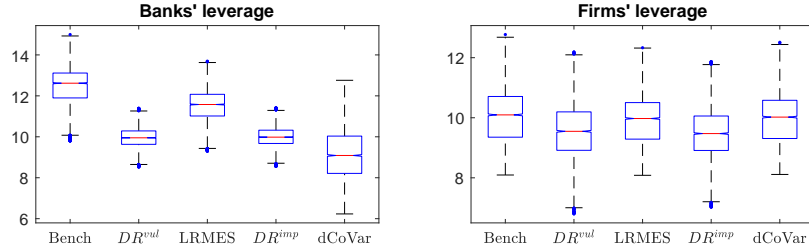
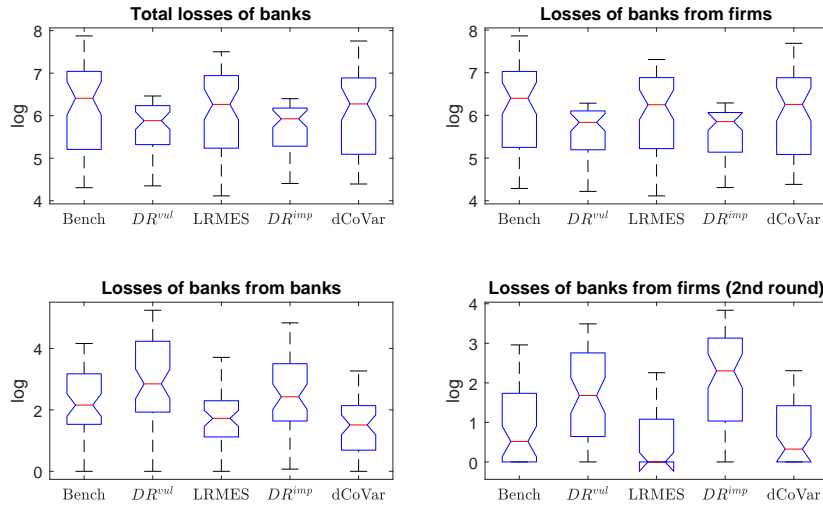Figure 5: Average leverage ratio per period of banks (exposures/equity) and firms (debt/equity).

Figure 6: Average losses of banks per period. Total losses (top-left), losses produced by insolvent firms (top-right), losses produced by insolvent banks on interbank lending (bottom-left), losses produced by second-round (and further) contagion.
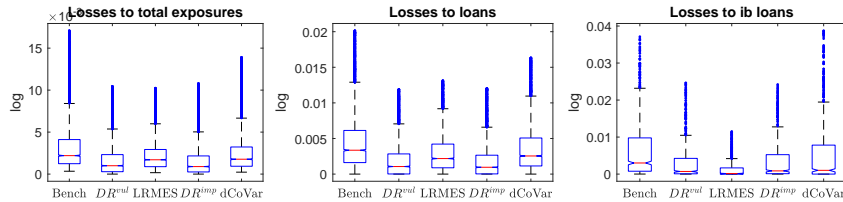

Figure 7: Average losses to exposures of banks per period. Losses to total exposure (left), losses to firms' loans (middle), losses to interbank loans (right).
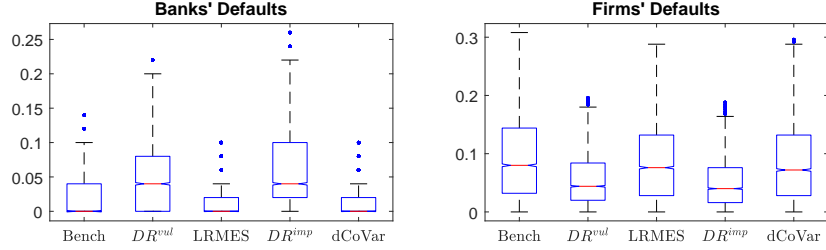
17

Figure 8: Average default rate per period (default/n. agents) of banks and firms.
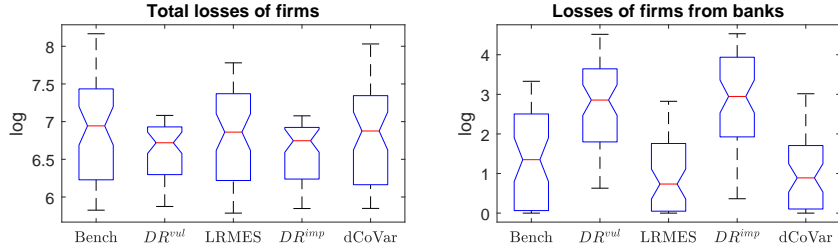

Figure 9: Average losses of firms per period. Total losses produced on the goods market and on deposits (left), and losses produced on deposits by banks insolvencies (right).

5   Concluding remarks

We presented a policy experiment built on a macro-agent-based model as a methodology to compare a set of lender-targeted macro-prudential rules in which banks are subject to capital requirements built on systemic risk measures. Four metrics are considered: the first set is composed by two market-based measures ($LRMES$ and $\Delta CoVaR$), while the second one includes balance-sheet-based measures (DR-vul and DR-imp). Each set contains a metric for vulnerability, which states how much a financial institution is systemically vulnerable to an adverse shock, and one measure for impact, which accounts for the effects of distress of single banks on the financial system. Capital requirements are obtained so that required capital is proportional to each bank's expected (or induced) capital shortage, which in turn depend on the SR measures.

In Section 4 we qualitatively and quantitatively analysed and compared macroprudential rules. We find that balance-sheet-based measures are more sensible to the build-up of systemic risk, conversely they are much more volatile than their market-based counterparts. Moreover $\Delta CoVaR$ turns out to be pro-

cyclical, that is it provides a measure of the systemic importance of banks only after a crisis, hence it is unable to anticipate the extent to which a bank can be systemic. Finally, we compared the empirical distributions of selected variables generated by the ABM. We find a trade-off between the two set of measures: balance-sheet metrics are better at reducing the risk arising from the exposures to firms, but are worse with respect to interbank contagion. This is because capital requirements based on DebtRank turn out to be tighter than their alternatives: they reduce the exposures and the losses of banks at the cost of limiting banks' and interbank lending.

The mapping from SR metrics to capital requirements is central to understand our results: alternative measures of systemic risk might be different for the same financial institution, therefore they could determine disparate behaviours about de-leveraging and credit supply. Even if the values of alternative measures have the same distribution, they could be associated heterogeneously to banks. So, a vulnerability index of 1 could translate in antipodean behaviours for a small unleveraged bank and a large leveraged one. This explains why we observe a higher credit rationing under balance-sheet based policies: DebtRank accounts for banks' interconnections in the credit and interbank market. In other words, it constraints more those banks with the greatest degrees in the network, but by doing so it limits their lending capacity (the credit market and the interbank market are modelled by static networks). Ultimately, this results in the association of high values of vulnerability to banks that are leveraged because they have multiple connection with the firms and/or banks, but doing so implies to affect the overall credit supply in the economy. The same is not happening with market-based-measures because they do not account for the structure of interlocked balance sheets but are derived from market indexes based on observed returns.

The last remark is about calibration. The policy experiments build on the idea to compare the sets of measures on a common ground, which translates in a calibration of DebtRank based on the assumption underlying LRMES, where a systemic crisis correspond to a decline of market returns of 40% in six months. Still, this might not work well for DebtRank. An alternative comparison could be conducted based on a calibration of systemic-risk metrics which is optimal for each of them. For instance, the shock to firms' assets in DebtRank (or threshold value of market decline in LRMES) could be determined based on the minimization of a loss function based on the realization of the economic variables in the model.

The issue of calibration could be studied in future research, together with other shortcomings. For instance the derivation of SCR from indicators of impact could be improved or combined with vulnerability. Another extension regards the analysis of systemic risk in a model capable to generate endogenous business cycles. This would permit to examine the time-dimension and the procyclicality of macroprudential policy. Furthermore, our concept of systemic risk is mainly related to banks. Although firms are also included in the financial network, measures of vulnerability and impact are only referred to banks. Firms might be systemic as well, at least from the point of view of credit-debt relation-

ships with banks. For this reasons macro-prudential polices should keep into account the financial network as a whole without excluding any of the relevant agents.

References

Acharya, V., Engle, R., and Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulating systemic risks. *The American Economic Review*, 102(3):59–64.

Adrian, T. and Brunnermeier, M. K. (2016). Covar. *The American Economic Review*, 106(7):1705–1741.

Alter, A., Craig, B. R., and Raupach, P. (2014). Centrality-based capital allocations.

Bardoscia, M., Battiston, S., Caccioli, F., and Caldarelli, G. (2015). Debtrank: A microscopic foundation for shock propagation. *PloS one*, 10(6):e0130406.

Bardoscia, M., Caccioli, F., Perotti, J. I., Vivaldo, G., and Caldarelli, G. (2016). Distress propagation in complex networks: the case of non-linear debtrank. *PloS one*, 11(10):e0163825.

Battiston, S., Caldarelli, G., D'Errico, M., and Gurciullo, S. (2016). Leveraging the network: a stress-test framework based on debtrank. *Statistics & Risk Modeling*, 33(3-4):117–138.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012). Debtrank: Too central to fail? financial networks, the fed and systemic risk. *Scientific reports*, 2:541.

Benoit, S., Colletaz, G., Hurlin, C., and Pérignon, C. (2013). A theoretical and empirical comparison of systemic risk measures.

Brownlees, C. T. and Engle, R. (2012). Volatility, correlation and tails for systemic risk measurement. *Available at SSRN*, 1611229.

Brunnermeier, M. K. and Sannikov, Y. (2014). A macroeconomic model with a financial sector. *The American Economic Review*, 104(2):379–421.

Danielsson, J., Valenzuela, M., and Zer, I. (2016). Learning from history: volatility and financial crises.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3):339–350.

Gauthier, C., Lehar, A., and Souissi, M. (2010). Macroprudential regulation and systemic capital requirements. Technical report, Bank of Canada Working Paper.

Gurgone, A., Iori, G., and Jafarey, S. (2018). The effects of interbank networks on efficiency and stability in a macroeconomic agent-based model. *Journal of Economic Dynamics and Control*.

Kaufman, G. G. and Scott, K. E. (2003). What is systemic risk, and do bank regulators retard or contribute to it? *The Independent Review*, 7(3):371–391.

Kleinow, J., Moreira, F., Strobl, S., and Vähämaa, S. (2017). Measuring systemic risk: A comparison of alternative market-based approaches. *Finance Research Letters*, 21:40–46.

Koenker, R. and Bassett Jr, G. (1978). Regression quantiles. *Econometrica: journal of the Econometric Society*, pages 33–50.

McGill, R., Tukey, J. W., and Larsen, W. A. (1978). Variations of box plots. *The American Statistician*, 32(1):12–16.

Pankoke, D. (2014). Sophisticated vs. Simple Systemic Risk Measures. Working Papers on Finance 1422, University of St. Gallen, School of Finance.

Poledna, S., Bochmann, O., and Thurner, S. (2017). Basel iii capital surcharges for g-sibs are far less effective in managing systemic risk in comparison to network-based, systemic risk-dependent financial transaction taxes. *Journal of Economic Dynamics and Control*, 77:230–246.

Rabemananjara, R. and Zakoian, J.-M. (1993). Threshold arch models and asymmetries in volatility. *Journal of Applied Econometrics*, 8(1):31–49.

Rodríguez-Moreno, M. and Peña, J. I. (2013). Systemic risk measures: The simpler the better? *Journal of Banking & Finance*, 37(6):1817–1831.

Tarashev, N. A., Borio, C. E., and Tsatsaronis, K. (2010). Attributing systemic risk to individual institutions.

Webber, L. and Willison, M. (2011). Systemic capital requirements.

6 Appendix

6.1 Calibration of DebtRank

In general, our approach is similar to that adopted in *Battiston et al. (2016)*, but we have adapted the algorithm to account for the structure of the underlying macro-model, as described in greater detail in Sect. 6.2. Given that the macro-environment includes firms, we first impose the shock on firms' assets to compute the systemic vulnerability index DR^{vul}. Next the induced distress transmits linearly to the assets of creditors (i.e. banks). This allows to capture the specific dynamics of the distress process.

Our calibration strategy aims to compare market and balance-sheet based measures on a common ground. To do so, we apply to DebtRank the definition of systemic crisis employed in the SRISK framework. SRISK is computed by LRMES, which represents the expected equity loss of a bank in case of a systemic event. This is represented by a decline of market returns of 40% over the next six months. We run 100 Monte-Carlo simulations of the macro-model, record the market ROE and the firms' losses to equity ratio. Then we compute the change in market ROE over the past 180 periods (approximately six months). Finally we construct a vector of the losses of firms to their equities in those periods where the ROE declined at least by -40%.

To compute vulnerabilities by DebtRank we randomly sample from the vector of the empirical distribution of losses/equity at each repetition of the algorithm. Finally we obtain DR^{vul} for each bank as an average of the realized values, after removing the 1*st* and the 99*th* percentiles.
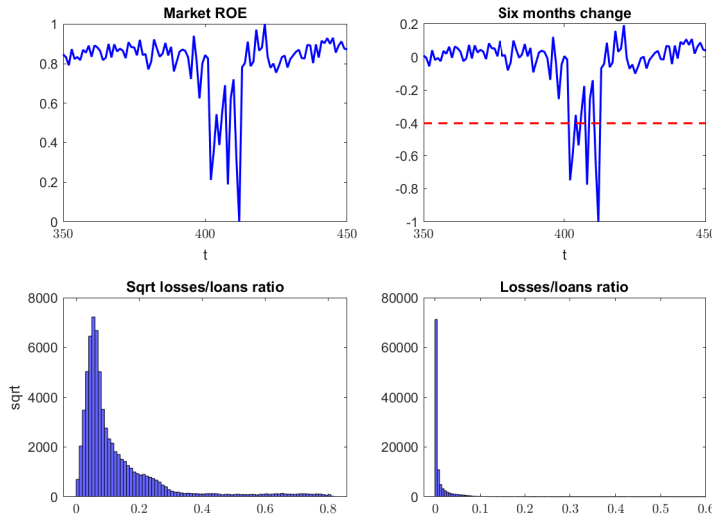

Figure 10: *(Top-left)* rescaled market ROE from a random Monte-Carlo run. *(Top-right)* Six month chance of market ROE. The red dashed line represents the threshold of -40%. *(Bottom-left)* Histogram of the square root of the losses/loans ratio of firms, where values equal to zero are ignored. *(Bottom-right)* Histogram of the losses/loans ratio of firms.

6.2 DebtRank

We employ a differential version of the DebtRank algorithm in order to provide a network measure of systemic risk. Differential DebtRank *(Bardoscia et al., 2015)* is a generalization of

the original DebtRank *(Battiston et al., 2012)* which improves the latter by allowing agents to transmit distress more than once. Moreover our formulation has similarities with *Battiston et al. (2016)*, where it is assumed a sequential process of distress propagation. In our case we first impose an external shock on firms' assets, then we sequentially account for the propagation to the banking sector through insolvencies on loans, to the interbank network and to firms' deposits.

The relative equity loss for banks (h) and firms (f) is defined as the change in their net worth (respectively nw^B, and nw^F) from $\tau = 0$ to τ with respect to their initial net worth. In particular the initial relative equity loss of firms happens at $\tau = 1$ due to an external shock on deposits:

$$h_i(\tau) = \min \left[\frac{nw_i^B(0) - nw_i^B(\tau)}{nw_i^B(0)} \right]$$

$$f_j(\tau) = \min \left[\frac{nw_j^F(0) - nw_j^F(\tau)}{nw_j^F(0)} \right]$$

The dynamics of the relative equity loss in firms and banks sectors is described by the sequence:

- Shock on deposits in the firms sector:

$$f_j(1) = \min \left[1, \ \frac{D_j^F(0) - D_j^F(1)}{nw_j^F(0)} \right] = \min \left[1, \ \frac{loss_j(1)}{nw_j^F(0)} \right]$$

- Banks' losses on firms' loans:

$$h_i(\tau + 1) = \min \left[1, \ h_i(\tau) + \sum_{j \in J} \Lambda_{ij}^{fb} (1 - \varphi_j^{loan})(p_j(\tau) - p_j(\tau - 1)) \right]$$

- Banks' losses on interbank loans:

$$h_i(\tau + 1) = \min \left[1, \ h_i(\tau) + \sum_{k \in K} \Lambda_{ik}^{bb} (1 - \varphi_k^{ib})(p_k(\tau) - p_k(\tau - 1)) \right]$$

- Firms' losses on deposits:

$$f_j(\tau + 1) = \min \left[1, f_j(\tau) + \Lambda_{jk}^{fb} (1 - \varphi_k^{dep})(p_k(\tau) - p_k(\tau - 1)) \right]$$

Where p_j is the default probability of debtor j and φ^i, $i = \{loan, ib, dep\}$ is the recovery rate on loans, interbank loans and deposits. Recovery rates on each kind of assets are randomly extracted from a vector of observations generated by the benchmark model.

For the sake of simplicity we can define it as linear in f_j (h_k for banks), so that $p_j(\tau) = h(\tau)$ [9]. Λ is the exposure matrix that represents credit/debt relationships in the firms-banks

---

[9]In a more realistic setting the default probability could be written as

$$p_j(\tau) = f_j(\tau) \exp(\alpha(h_j(\tau)) - 1)$$

where if $\alpha = 0$ it corresponds to the linear DebtRank, while if $\alpha \to \infty$ it is the Furfine algorithm *(Bardoscia et al., 2016)*. Moreover we can assume that deposits are not marked-to-market, but they respond to the Furfine algorithm, in other words the distress propagates only in case of default of the debtor. For deposits it might be reasonable to assume

$$p_j^D(\tau - 1) = \begin{cases} 1 & if \ h_k(\tau - 1) = 1 \\ 0 & otherwise \end{cases}$$

network. It is written as a block matrix, where Λ^{bb} refers to the interbank market, Λ^{bf} refers to deposits, Λ^{fb} refers to firm loans and Λ^{ff} is a matrix of zeros.

$$\Lambda = \begin{bmatrix} \Lambda^{bb} & \Lambda^{bf} \\ \Lambda^{fb} & \Lambda^{ff} \end{bmatrix}$$

The exposure matrix Λ represents potential losses over equity related to each asset at the beginning of the cycle, where each element has the value of assets at the numerator and the denominator is the net worth of the related creditor. in our specification firms have no intra-sector links, hence $\Lambda^{ff} = 0$. In case there are $N^b = 2$ banks and $N^f = 3$ firms, the matrix Λ looks like:

$$\Lambda = \begin{bmatrix} 0 & \frac{Ib_{12}}{nw_2^B} & \frac{D_{13}}{nw_1^F} & \frac{D_{12}}{nw_2^F} & \frac{D_{15}}{nw_3^F} \\ \frac{Ib_{21}}{nw_1^B} & 0 & \frac{D_{23}}{nw_1^F} & \frac{D_{24}}{nw_2^F} & \frac{D_{25}}{nw_3^F} \\ \frac{L_{31}^f}{nw_1^B} & \frac{L_{32}^f}{nw_2^B} & 0 & 0 & 0 \\ \frac{L_{41}^f}{nw_1^B} & \frac{L_{42}^f}{nw_2^B} & 0 & 0 & 0 \\ \frac{L_{51}^f}{nw_1^B} & \frac{L_{52}^f}{nw_2^B} & 0 & 0 & 0 \end{bmatrix}$$

6.3 SRISK

SRISK *(Brownlees and Engle, 2012)* is a widespread measure of systemic risk based on the idea that the latter arises when the financial system as a whole is under-capitalized, leading to externalities for the real sector. To apply the measure to our model we follow the approach of *Brownlees and Engle (2012)*. The SRISK of a financial firm i is defined as the quantity of capital needed to re-capitalize a bank conditional to a systemic crisis

$$SRISK_{i,t} = \min \left[0, \ \frac{1}{\lambda} \mathcal{L}_i - \left(1 - \frac{1}{\lambda} \right) nw_{i,t}^B (1 - MES_{i,t+h|t}^{Sys}) \right]$$

where $MES_{i,t+h|t}^{Sys} = E\left(r_{i,t+h|t} | r < \Omega \right)$ is the tail expectation of the firm equity returns conditional on a systemic event, that happens when i's equity returns r from $t - h$ to t are less than a threshold value Ω.

Acharya et al. (2012) propose to approximate MES^{Sys} with its *Long Run Marginal Expected Shortfall* (LRMES), defined as a

$$LRMES_{i,t} = 1 - \exp\{-18MES_{i,t}^{2\%}\}$$

LRMES represents the expected loss on equity value in case the market return drops by 40% over the next six months. Such approximation is obtained through extreme value theory, by means of the value of MES that would be if the daily market return drops by -2%.

The bivariate process driving firms' (r_i) and market (r_m) returns is

$$r_{m,t} = \sigma_{m,t} \epsilon_{m,t}$$
$$r_{m,t} = \sigma_{i,t} \rho_{i,t} \epsilon_{m,t} + \sigma_{i,t} \sqrt{1 - \rho_{i,t}^2} \xi i, t$$
$$(\xi_{i,t}, \epsilon_{m,t}) \sim F$$

where $\sigma_{m,t}$ is the conditional standard deviation of market returns, $\sigma_{i,t}$ is the conditional standard deviation of firms' returns, $\rho_{i,t}$ is the conditional market/firm correlation and ϵ and ξ are i.i.d. shocks with unit variance and zero covariance ϵ and ξ are i.i.d. shocks with unit variance and zero covariance.

$MES^{2\%}$ is expressed setting $\Omega = -2\%$:

$$MES_{i,t-1}^{\Omega} = \sigma_{i,t}\rho_{i,t}E_{t-1}\left(\epsilon_{m,t}|\epsilon_{m,t} < \frac{\Omega}{\sigma_{m,t}}\right) + \sigma_{it}\sqrt{1-\rho_{i,t}^2}E_{t-1}\left(\xi_{i,t}|\epsilon_{m,t} < \frac{\Omega}{\sigma_{m,t}}\right)$$

Conditional variances $\sigma_{m,t}^2$, $\sigma_{i,t}^2$ are modelled with a TGARCH model from the GARCH family *(Rabemananjara and Zakoian, 1993)*. Such specification captures the tendency of volatility to increase more when there are bad news:

$$\sigma_{m,t}^2 = \omega_m + \alpha_m r_{m,t-1}^2 + \gamma_m r_{m,t-1}^2 I_{m,t-1}^- + \beta_m \sigma_{m,t-1}^2$$
$$\sigma_{i,t}^2 = \omega_i + \alpha_i r_{i,t-1}^2 + \gamma_i r_{i,t-1}^2 I_{i,t-1}^- + \beta_i \sigma_{i,t-1}^2$$

$I_{m,t}^- = 1$ if $r_{m,t} < 0$ and $I_{i,t}^- = 1$ when $r_{i,t} < 0$, 0 otherwise.

Conditional correlation ρ is estimated by means of a symmetric DCC model *(Engle, 2002)*. Moreover to obtain the MES it is necessary to estimate tail expectations. This is performed with a non-parametric kernel estimation method *(see Brownlees and Engle, 2012)*.

Open-source Matlab code is available thanks to *Sylvain Benoit*, and *Gilbert Colletaz*, *Christophe Hurlin*, who developed it in *Benoit et al. (2013)*.

6.4 ΔCoVaR

Following *Adrian and Brunnermeier (2016)* $\Delta CoVaR$ is estimated through a quantile regression (Koenker and Bassett Jr (1978)) on the αth quantile, where r_{sys} and r_i are respectively market-wide returns on equity and bank i's returns. Quantile regression estimates the αth percentile of the distribution of the independent variable given the regressors, rather than the mean of the distribution of the dependent variable as in standard OLS regressions. This allows to compare how different quantiles of the dependent variables might affect the regressand, hence it is suitable to analyse tail events. While *Adrian and Brunnermeier (2016)* employ an estimator based on an augmented regression, we further simplify the estimation of $\Delta CoVaR$ following the approach in *Benoit et al. (2013)*, which is consistent with the original formulation.

First we regress individual returns on market returns:

$$r_{sys,t} = \gamma_1 + \gamma_2 r_{i,t} + \varepsilon_{\alpha,t}^{sys|i}$$

The estimated coefficients (denoted by $\widehat{}$) are employed to build CoVaR. The conditional VaR of bank i ($Var_{\alpha,t}^i$) is obtained from the quasi maximum likelihood estimates of conditional variance generated by the same TGARCH model described above *(see Benoit et al., 2013, p.38)*.

$$CoVar_{\alpha,t}^{sys|i} = \widehat{\gamma}_1 + \widehat{\gamma}_2 Var_{\alpha,t}^i$$

Finally $\Delta CoVar$ is obtained from the difference between the αth and the median quantile of $CoVar$.

$$\Delta CoVar_{\alpha,t}^{sys|i} = CoVar_{\alpha,t}^{sys|i} - CoVar_{0.5,t}^{sys|i}$$
$$\Delta CoVar_{\alpha,t}^{sys|i} = \hat{\gamma}_2\left(VaR_{\alpha,t}^i - VaR_{0.5,t}^i\right)$$