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The effects of precision teaching and self-regulated learning on early multiplication fluency.

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Abstract

Fluent recall of basic facts is essential to the development of more complex math skills. Therefore, failure to develop fluency with basic facts may impede the development of these skills. The present study used a between groups experimental design to investigate whether a basic facts fluency program, implemented within a self-regulated learner (SRL) framework, could lead to increased fluency with multiplication facts for Year 5 and Year 6 New Zealand students (9–10 years old). This study also investigated the extent to which the SRL program altered students' basic facts practice behavior outside of school hours. The study found that the SRL program resulted in rapid fluency development that was maintained over time. Nomothetic and idiographic analysis confirmed that the program was suitable for use within Tier 1 of the response to intervention framework. In addition, the study also found that students who received the program altered their practice behavior outside school hours. The results from this study show how elements of self-regulated learning and precision teaching can be successfully combined to enhance students' mathematics achievement.

Key words: Experimental design, multiplication facts, precision teaching, primary/elementary students, self-regulated learning.

The effects of precision teaching and self-regulated learning on early multiplication fluency

Fluent recall of basic facts is essential for the development of more advanced mathematics skills (Burns, Zaslofsky, Maki, & Kwong, 2016; Johnson & Street, 2013). By reducing the demand on working memory, fluent recall of basic facts enables students to devote more attention to the overall purpose of a mathematics problem (Burns, Kanive, & DeGrande, 2012; Burns et al., 2016; Neill, 2008; Sweller, Ayres, & Kalyuga, 2011), which facilitates better understanding (Gross, Duhon, Shutte, & Rowland, 2016). Compared to their less fluent peers, students who are fluent with the recall of basic facts also enjoy more opportunities to respond to complex mathematics tasks (McCallum, Skinner, Turner, & Saecker, 2006), show an increased willingness to exert effort, and increased student motivation (Codding, Chan-lannetta, Palmer, & Lukito, 2009). Despite the importance of fluent basic facts recall, many students find learning multiplication facts difficult (Steel & Funnell, 2001; Tait-McCutcheon, Drake, & Sherley, 2011), and as the mathematics curriculum increases in complexity, those less fluent are more likely to fall further behind their peers; a phenomenon commonly referred to as the Matthew effect (Merton, 1968). This emphasizes the need for effective Tier 1 programs that can increase students' fluency with basic facts. Tier 1 programs should be effective for 80–90% of students (Johnson & Street, 2013) and, according to the response to intervention (RTI) model, include evidence-based instructional practices in the core curriculum and continuous monitoring (Tunmer & Greaney, 2010).

Mathematics studies have found that students benefit most from relatively short but frequent basic facts practice sessions (Burns, 2005; Burns et al., 2016). Despite the efficacy of this approach, other research has found that effort and motivation are barriers to frequent practice (Ericsson, Krampe, & Tesch-Römer, 1993). In fact, even students who frequently engage in purposeful practice consider it to be less enjoyable and more effortful than other teaching/learning strategies (Duckworth, Kirby, Tsukayama, Berstein, & Ericsson, 2011). Self-regulated learner (SRL) programs may provide a means to overcome these barriers.

Instructional Approaches to Enhance Multiplication Fluency

Self-Regulated Learning. Research with school-aged students has shown that SRL programs can lead to improved goal orientation (Stoeger & Ziegler, 2008; Tzohar-Rozen & Kramarski, 2014), increased on-task behavior (Ness & Middleton, 2012), and increased willingness to exert effort (Stoeger & Ziegler, 2008). In addition, SRL programs in mathematics have also facilitated a reduction in negative emotions, including anxiety (Kramarski, Weisse, & Kololshi-Minsker, 2010) and helplessness (Stoeger & Ziegler, 2008), whilst increasing a number of positive emotions such as a sense of personal control and improved self-confidence (Lee, Yeo, & Hong, 2014). SRL programs have also been found to have a more positive effect on students' achievement in mathematics than other subject areas (Dignath, Buettner, & Langfeldt, 2008).

SRL interventions are typically composed of three stages (Zimmerman, 2000). This study adopted the terms used by Schmitz and Wiese (2006) of pre-action, action, and post-action, for each stage respectively. Each stage is comprised of a number of cognitive, metacognitive, and motivational processes, including goal setting, self-monitoring, self-evaluation, adaption of self-regulatory processes, and strategy use (Schunk & Ertmer, 2000). Many of the key strategies and processes that were incorporated in the SRL program used in this study were taken from *Detect, Practice, and Repair* (DPR) and *Precision Teaching* (PT) literature.

Detect, Practice, Repair. DPR is an evidence-based Tier 1 strategy that has been used to help students increase their basic facts fluency (Musti-Rao & Plati, 2015; Poncy, Fontenelle IV, & Skinner, 2013; Poncy, Skinner, & Axtell, 2010). One of the main benefits of DPR is that it allows for individualized basic facts instruction within a class-wide setting

(Musti-Rao & Plati, 2015; Poncy et al., 2013). The detect phase uses a timed pre-test to identify the facts that each student needs to practice (Poncy, Duhon, Lee, & Key, 2010; Poncy et al., 2013). The ability to administer the detect phase within a whole class setting is a key component of the DPR program. DPR is suitable for use within the action stage of an SRL program, as this stage focuses on effective strategies to organize, learn, and retain information (Schunk & Ertmer, 2000).

Within the practice phase, students apply the *Cover, Copy, and Compare* procedures with only the first five problems they did not complete, or completed incorrectly, during the detect phase. During the cover procedure, students look at the basic fact and its answer, then they cover the fact so that it cannot be seen. Next, for the copy procedure, students write the fact from memory, and finally, they compare the fact they have written to the original fact. If the fact was written incorrectly an error drill is often implemented. This may involve looking at the target fact and writing the fact correctly a small number of times (Becker, McLaughlin, Weber, & Gower, 2009; Codding et al., 2009).

The third phase of the DPR program is the repair phase. During this phase, students complete timed-assessments of the basic facts they have been practicing, with an alternate form of the detect assessment. The use of explicit timing procedures is a key component of many evidence-based programs (Miller, Skinner, Gibby, Galyon, & Meadows-Allen, 2011; Poncy et al., 2013; Schutte et al., 2015). In fact, it has been suggested that explicit timing procedures should be applied to all fluency practices (Gross et al., 2014). Research has established that collecting rate per minute data is a reliable and sensitive form of progress monitoring (Musti-Rao & Plati, 2015). This rate is typically expressed as correct answers per minute (Becker et al., 2009; Fuchs et al., 2010; Smith, Marchand-Martella, & Martella, 2011; Wong & Evans, 2007) or correct digits per minute. The repair phase finishes with students graphing their own performance. Self-graphing is a key component of the post-action stage of

SRL programs, as this stage emphasizes the importance of self-monitoring and self-reflection. The use of self-graphing procedures is an important ingredient in many basic facts fluency programs (Bryant et al., 2015; Gross et al., 2014; Schutte et al., 2015). Studies have found that self-graphing leads to enhanced basic facts performance (Figarola et al., 2008; Gross et al., 2014). It is also described favorably by both teachers (Gross et al., 2016) and students (Bryant et al., 2015).

Precision Teaching. DPR's emphasis on fluency and self-graphing procedures is particularly congruous with the aims of precision teaching (PT). PT is a measurement tool for learning (Gallagher, 2006) that encourages large amounts of practice within short time periods (Johnson & Street, 2013). Binder (1996) defines fluency as "the fluid combination of accuracy plus speed that characterizes competent performance" (p. 164). A number of positive outcomes are associated with fluent performance, including maintenance, endurance, stability, application, and generativity. The acronym MESsAGe is commonly used to refer to these outcomes (Johnson & Street, 2013). Fluency advocates believe that these outcomes are only observed when students are taught under fluency, rather than mastery, conditions. Typically, mastery instruction emphasizes teaching until set accuracy criteria are met, whereas, fluency instruction is concerned with accuracy and speed. Fluency programs involve instruction until accuracy criteria are met within a specified timeframe (Binder & Watkins, 2013; Singer-Dudek & Greer, 2005). Singer-Dudek and Greer (2005) compared students taught under these conditions and found that the gains in basic facts proficiency made by students under fluency instruction were better maintained over time than by students taught under mastery conditions. Fluency programs have also led to large improvements in academic performance (e.g., two or more grade levels within one year), and are helpful for students of all levels of ability (Binder, 1996; Binder & Watkins, 2013; Gallagher, 2006).

A key feature of PT is the use of Standard Celeration Charts (SCC) to record the frequency of behavior over time (Johnson & Street, 2013), formatted with a linear x-axis for calendar time and logarithmic scale on the y-axis for frequency of responses (Binder, 1996). These charts allow the user to measure the acceleration or deceleration of a behavior over time. Charts that use absolute frequency underestimate performance at low frequencies and overestimate performance at high frequencies. Unlike absolute frequency charts, the SCC measures student growth that is proportional to their previous growth. This more accurately reflects how people learn (Johnson & Street, 2013). These charts have been successfully used to monitor student progress with basic facts in studies with participants similar in age to those in this study (Chiesa & Robertson, 2000; Gallagher, 2006; Strømgren, Berg-Mortensen, & Tangen, 2014). PT's emphasis on progress monitoring makes it particularly suitable for use within the post-action stage of SRL programs.

Fluency aims and celeration rates are used in conjunction with the SCC. Both Binder and Watkins (2013) and Johnson and Street (2013) suggest aiming for celeration rates of ×2.0 per week. This represents a doubling in the frequency of the observed behavior. For example, if a student could correctly answer 15 basic facts per minute at the start of the week and 30 basic facts per minute at the end of the week, their celeration rate would be ×2.0 per week. Kubina and Yurich (2012) classify a celeration rate of ×2.0 per week as "exceptional", noting that a celeration rate of ×1.4 per week (40% growth in rate) still represents robust growth. The use of celeration rates enables teachers to monitor progress. When progress rates are below an acceptable celeration rate this alerts the teacher that changes need to be made to the teaching method or materials (Chiesa & Robertson, 2000). A student's failure to match an expected celeration rate is considered to represent a deficiency in instruction, rather than a deficit within the student.

Purpose of the Current Study

The purpose of the current study was to address two gaps in the literature. First, it sought to answer whether a basic facts fluency program based on an SRL framework could effectively incorporate DPR and PT methodologies. Specifically, it aimed to identify whether this program would lead to increased basic facts fluency with Year 5 and 6 students (ages 9 to 10 years) when compared with students receiving regular classroom instruction. Second, it sought to investigate the extent to which the SRL program might alter students' basic facts practice behavior outside of school hours. Whilst there is general agreement around the need for students to engage in high quality practice to develop basic facts fluency, there is a paucity of research on how a classroom-based intervention can influence students' practice behavior outside of school hours. These research aims were examined in a repeated measures between group (traditional instruction control group versus SRL instruction group) experiment within a single primary school in the South Island of Aotearoa New Zealand. Both students and teachers were randomly assigned to the experimental or control group condition.

Method

Participants and Setting

Participants in this study came from one full primary school (Years 1–8; aged 5–12 years) located in an above average socioeconomic community in Christchurch, New Zealand. Once permission to conduct the study from the author's Institutional Review Board was obtained, all the Year 5/6 students (children aged 9–10 years) at this school were invited to take part in the study. Of the 50 students and families eligible, 47 (94%) agreed to take part in the study. The majority of the participating students (64%) identified as New Zealand European/Pākeha. A smaller percentage (19%) identified as both New Zealand European/Pākeha and Māori and the remainder identified with a number of different cultures. There was a relatively even mix of male (47%) and female students (53%). Analyses revealed that there were no significant between group differences across any of the sample sociodemographic characteristics. Although no students withdrew during the course of the intervention, two students were absent from school for the follow up assessment that took place five weeks after the program. These students were absent for medical reasons. The study took place in two classrooms at the participating school. These classrooms were made available for the study during the final 15 minutes of the participating students' normal mathematics time. At this time, participating students moved to one of the two classrooms allocated to their group.

The two participating teachers were female, of comparable age, and had in excess of 20 years teaching experience. In both cases, they had spent the majority of their teaching career educating students similar in age to the participants in this study. One teacher identified as New Zealand European and the other teacher identified as American of British descent. Both teachers met the experienced registered teaching criteria and had been teaching at this school for over five-and-a-half years.

Materials

Both the SRL and control conditions received program packs that were similar in appearance. The SRL pack contained flash cards, a timer, a diary, and a pen. Students used five-minute sand timers (accuracy verified) to ensure they allocated a full five minutes to the action stage of the program when practicing outside of school hours. These items were stored inside plastic bags that were clearly named. The control pack contained a diary and a pen. Both teachers were provided with spare equipment which was added to packs when required.

Flash cards. Students in the SRL group were provided with a pack of flash cards based on analysis of their one-minute multiplication probe, which was administered prior to the program commencing. Initially, these packs contained either the 1, 2, and 10 times tables (referred to as Set 1), the 1, 2, 3, 4, 5, and 10 times tables (referred to as Set 2), or the 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 times tables (referred to as Set 3). As no commutative facts were included (e.g., if 2 × 1 was included then 1 × 2 was not included), Set 3 contained a total of 52 facts. These were the same 52 facts that were included in the Test A one-minute multiplication probe. Students had cards either removed or added to these packs according to the data obtained from their celeration charts.

Diaries. The SRL and control groups were both provided with 10cm by 16.5cm spiral bound notebooks. The diary pages varied for each condition. After every practice session, participants were asked to complete a page in their diaries. The control condition recorded how long they practiced in minutes and selected, from a list, what type of practice they engaged in. The control participants could choose from the following options: *playing math games, testing by another person, using flash cards, completing activity sheets*, or *other*. They were asked to describe the type of practice they engaged in if they chose the option *other*. The participants in the SRL group were asked to record how confident they were that they could instantly answer the facts they had been learning during that practice session. They responded by selecting either *not confident at all*, *a little bit confident, fairly confident, very confident*, or *extremely confident*. The SRL participants were also asked to record whether they used setting selection, setting modification, and attention deployment. They did this by selecting from one of four options: *no*; *yes, but it was not helpful*; *yes, it was somewhat helpful*; or *yes, it was very helpful*. The notebooks fitted inside plastic bags that were provided as part of the program.

Experimental Design

This study employed a repeated measures experimental design with a treatment-as-usual control group. This type of design controls for many threats to validity, including history, maturation, and regression effects (Tuckman & Harper, 2012). All students were randomly assigned to either the SRL or control condition. The participating teachers were also randomly assigned to one of the two conditions. The students worked only with the teacher that was assigned to their group. The random assignment of students and teachers occurred prior to any data collection. Data were collected at three time points, including; prior to the program, at the end of the program (after four weeks), and at a five-week follow-up after the end of the program. From this point forward, these time periods are referred to as T1, T2, and T3, respectively. T1 equates to the beginning of the second school term, which starts in May. The school year is composed of four school terms. These terms run for approximately 10 school weeks.

Dependent Variables

One-minute multiplication probes. The one-minute multiplication probes were paper and pencil tests, composed of 90 questions, which came from a pool of 52 multiplication facts. All questions were presented horizontally with a line next to every problem on which students recorded their answers. This pool of facts included all multiplication facts from 2 × 1 to 9 × 10. As no commutative facts were included, this created a pool of 52 problems. The probes were developed in Microsoft Excel®. This enabled the presentation order for the 90 problems, from the 52-item pool, to be randomized in all administrations of the one-minute multiplication probes. The probes were designed so that every question was randomly selected from the 52-item pool (facilitated through the use of a formula in Microsoft Excel®). As every question in each probe was randomly selected from the 52-item pool, some questions occurred more than once. Prior testing revealed that no student could complete all 90 problems within one minute. Two versions of the one-minute multiplication probes were created. These were labelled Test A and Test B. The main dependent variable for the SRL and control groups was participants' correct answers per minute (CAPM) on the one-minute multiplication probes. These scores were collected from the tests administered at T1, T2, and T3.

Test A consisted of all 52 multiplication fact problems. Variations of Test A were administered at T1, T2, and T3. The only difference between the Test A one-minute multiplication probes, administered throughout the study, was the order in which the facts were presented. Test B contained commutative facts for all the facts presented in Test A. For example, Test A contained 2 × 7, and Test B contained 7 × 2. Variations of Test B were also administered at all three time periods, shortly after Test A. As with Test A, the order that the multiplication facts were presented in differed for all three administrations of Test B. Test B was developed to test whether fluent performance of the 52 multiplication facts learned during the program, generalized to their associated commutative facts. Analysis of Test A and Test B data at T1 showed good reliability for these parallel forms (r =.85). A warm-up multiplication probe consisting of 90 problems was also administered; however, this was not used as a dependent variable. For this reason, these probes were not marked or analyzed.

Questionnaire. A brief researcher-developed questionnaire was administered at T1 and T2 (see Appendix). The questionnaire investigated the students' basic facts practice behavior outside of school hours. The students were asked to reflect on only their basic facts practice behavior over the prior seven days. The first question asked students to record the number of days on which they practiced their basic facts. Eight options were presented, ranging from *0 days* to *7 days*. The next question asked the students to record how long they practiced their basic facts for in a typical practice session. Students were given the following options: *0 minutes, 5 minutes, 10 minutes, 15 minutes, 20 minutes,* or *more than 20 minutes.* If students selected *more than 20 minutes*, they were asked to record the number of minutes they practiced for. The final question asked students to record what type of practice they engaged in. They were provided with the following options: *math games, testing by another person, using flash cards, completing an activity sheet,* and *other*. Students were asked to describe the type of practice they engaged in if they selected the *other* option. The questionnaire administered at T2 in the final week of the intervention included two additional questions. The first of these questions asked students “How enjoyable did you find this basic facts program?” Response options included: *not enjoyable, a little bit enjoyable, fairly enjoyable, very enjoyable, extremely enjoyable.* The final question asked the participants “How much do you think this program helped improve your basic facts recall?” Response options included: *not helpful, a little bit helpful, fairly helpful, very helpful, extremely helpful.*

Self-control and basic facts confidence. The diaries from SRL participants were analyzed to determine whether these students continued to use the self-control strategies that they had been taught, outside of school hours. The diaries were also used to determine how helpful the SRL participants found these self-control strategies. This analysis was only conducted for the SRL participants, as they were the only group taught this specific self-control strategy. A detailed description of how the SRL participants recorded their responses to these questions is provided in the materials section of this paper.

Independent Variables

Control condition: Traditional instruction group. Students assigned to the control group received regular classroom instruction. Specifically, this instruction focused on basic multiplication facts. The New Zealand Ministry of Education provides a range of Numeracy Project books that supplement the National Curriculum. Book 4 is titled *Teaching Number Knowledge* (Ministry of Education, 2008). This book provides a number of activities designed to improve students’ understanding and recall of basic multiplication facts. Every lesson includes a learning intention, a detailed description of the activity, and a link to the materials required for the lesson. Prior to the study, the first author met with the teacher who had been assigned to the control condition. At this meeting, the teacher selected four lessons from Book 4 to teach during the study. These were lessons that the teacher would normally teach to students of this age to help them learn their multiplication basic facts. The lessons encouraged students to explore the relationships between multiplication facts, in small mixed ability groups, through game-like activities. This reflected typical multiplication fact practice at this school.

Experimental condition: SRL group. Many SRL studies designed to improve students' mathematical achievement structure their programs according to Zimmerman's (2000) self-regulatory cycle (Digiacomo & Chen, 2016; Kramarski, Itzhak, & Sarit, 2013; Labuhn, Zimmerman, & Hasselhorn, 2010). The self-regulatory cycle is composed of three stages termed forethought, performance, and self-reflection. However, this study employed the commonly used terms pre-action, action, and post-action to refer to these stages (Schmitz & Wiese, 2006), as these terms could be more readily understood by participants of this age.

Pre-action stage. Self-regulated learning is comprised of many processes that are designed to enhance learning and motivation (Schunk & Ertmer, 2000). The pre-action stage focused on two of these processes. Specifically, it focused on goal directed learning with performance monitoring and the use of strategies to maintain attention during the learning task. During the pre-action stage, students were taught the first three strategies from the process model of self-control (Duckworth, Gendler, & Gross, 2014), which was specifically designed for use with school-aged children.

The first three strategies are termed *situation selection*, *situation modification*, and *attention deployment*. This study used the term *setting* instead of situation, as this was more easily understood by the participants in this study. The setting selection strategy focused on choosing to work in places or with people that facilitate self-control. The SRL students were taught to avoid spaces in the room where they might find it difficult to maintain attention. For example, some students chose to avoid sitting near windows where they might be distracted by activities occurring outside the classroom. Although students completed the practice phase independently, they were still taught to consider who they would sit near. Because students could work at desks or on the floor, students were able to find a practice space away from other students. Setting modification involved altering the physical or social situation to enhance self-control. These students were primarily taught how to alter the space where they had chosen to work to avoid distractions. For example, some students chose to place the rubber band and flash card packaging behind them, as they thought that they may be tempted to play with these items during the practice phase. *Attention deployment* required students to selectively focus on those things within their environment that enhanced self-control, rather than those that undermine it. To help students with this strategy, students were taught to create "if/then" plans. Students were first asked to imagine one thing that could occur during the practice phase that might distract them from their practice. They then decided on how they would respond if that distraction occurred. The plans used the following format, "If … happens I will …".

The students' medium-term goal was to reach a set target fluency rate (TFR), expressed as CAPM, for the multiplication facts they were practicing on their daily sprint sheets. The sprint sheet results provided feedback to students on the progress they were making and were not used as a dependent variable in this study. The TFR used in this program was based on existing literature and data obtained from older fluent performers within the school, as well as three fluent adults. In order to find fluent performers within the school, students were selected from classrooms with children two or three years older than the participants. The TFR was set at 45. This was also consistent with the TFR used in other studies (Chiesa & Robertson, 2000; Gallagher, 2006; Strømgren et al., 2014). During the study, it became apparent that some students lacked the fine motor skills required to achieve this TFR. These students completed a digit writing task to establish the number of digits they could write within a minute. This was then compared to a small group of fluent performers within the SRL group. On the basis of these data, a lower limit TFR, of 40 CAPM, was accepted for 15 students who lacked the fine motor skills to achieve a TFR of 45. A TFR of 40 has also been accepted as the lower limit in other PT studies (Chiesa & Robertson, 2000; Gallagher, 2006; Strømgren et al., 2014). Students were taught to identify their own daily goals. These daily goals were based on the celeration line that was drawn on their celeration chart. Throughout the study, students aimed to meet or exceed the CAPM depicted by this line, on every daily sprint. All celeration lines started from each student's median data point from the first three days of data collection. From this point, the line increased at a rate of ×1.6 CAPM per week. Prior to the program, a celeration rate of ×1.6 was set as the benchmark for acceptable growth. According to Kubina and Yurich (2012), celeration rates between ×1.4 and ×1.8 represent robust growth. Celeration lines were also recalculated after any program change. The students identified their goal by locating the practice day on the x-axis then moving vertically up the chart until they reached the celeration line. They then moved to the y-axis to identify their CAPM goal for the day's session. Some students, whose progress exceeded the celeration line, were taught to set slightly more ambitious CAPM goals by the teacher who had been assigned to the SRL condition. Students were provided with additional facts once they had met or exceeded the TFR for two out of three days.

The primary considerations when teaching simple facts were identifying how many new facts should be taught and how frequently they should be rehearsed (Kameenui & Simmons, 1990). On this basis, program adaptations involved reducing the number of new facts a child was exposed to if their progress fell below the celeration rate of ×1.6. Program changes were also made when a student met one of the other six change criteria. These included: (a) the student met the TFR for two out of three days, (b) acceleration data were decelerating, (c) deceleration data were accelerating, (d) there were four days of flat data, (e) the celeration rate fell below the projected aim for three or more data points, (f) additional information pertinent to improving the learner's performance was obtained.

Action stage. The use of effective strategies to organize, learn, and retain information is a key component of self-regulated learning (Schunk & Ertmer, 2000). In addition to progress monitoring, this was the main focus of the action stage. This action stage was composed of three phases similar to that used in detect, practice, repair procedures by Poncy et al. (2013). Students started the action stage by going through all the cards in their pack. This was referred to as the detect phase of the action stage. They read the card out loud, instantly said the answer, and then turned the card over to check if their answer was correct. If the student answered the question correctly they put the card in the “got it” pile. If the student did not answer the question correctly, or could not answer the question instantly, they put the card in the “not yet” pile. Students were required to answer the facts instantly to discourage counting procedures.

The second phase of the action stage was called practice. In this phase, students selected the top four cards from their not yet pile. Students read the question out loud, checked the answer, said the answer out loud, and wrote the fact three times on the reverse page of their notebook diary. They then read the question from the card again, but this time they said the answer before they turned the card over. If they correctly answered the question, they moved onto the next card and followed the same procedure. If they answered the card incorrectly, they repeated the procedure with this fact. Students went through all four cards three times, or until they could instantly answer each of the four questions.

The final phase of the action stage was called repair. This phase involved integrating the newly learned facts into a slightly larger pool of problems. Students randomly selected six facts from their got it pile and mixed them with the four not yet facts they had been practicing. They then went through these 10 facts following the same procedure they used for all the cards during the detect phase. If, at the end of this phase, students had placed cards in a not yet pile, they repeated the practice phase procedure with these facts. If all 10 cards were placed in the got it pile, they selected four new cards from the original not yet pile. In the event that there were no cards left in this pile, students practiced answering all the cards in their pack as quickly as they could for the remainder of the five minutes.

Post-action stage. The post-action stage focused on self-monitoring and self-reflection. Given the cyclical nature of the SRL model, this stage also informed the following day's action stage. The post-action stage incorporated sprints, self-graphing, and reflection. At the end of the 5-minute action stage, students took part in a sprint. Sprint sheets contained only the multiplication fact questions students had been practicing in their own flash card packs. Every student collected their own sprint sheet and placed this sheet upside down where they were working. The teacher instructed them to turn the sheets face up, put their finger on the first question and their pencil in the air. The teacher then indicated the start of the one-minute sprint using the phrase "On your marks, get set, go". Students aimed to answer as many questions as they could within one minute. Once the time period was up, they handed their sheet to their assigned partner who marked their sheet using the answer sheets provided. Students then graphed their number of correct answers and errors on their own celeration chart. At the end of every lesson, the celeration charts and sprint sheets were collected. Sprint sheets were reviewed to ensure students had marked and graphed the data accurately. If a result was not recorded accurately it was corrected by the researcher. A removable label was then applied to every celeration chart. This label provided specific praise relating to the student's progress. It also directed the student's attention to a fact, or series of facts, that required extra attention during the following day's program. For example, "Well done! You beat your goal for today. Focus on the following facts when you practice tomorrow: (facts inserted here)". Students were encouraged to reflect on this feedback during the subsequent day's pre-action stage. If the student had met the TFR for the previous three days, additional facts were added to their packs. The sprint sheet that they were provided the following day also reflected this change.

At the end of every session the teacher encouraged students to reflect on their learning and self-evaluate their performance. The reflection was led by the teacher and occurred after students had graphed their results on their celeration charts. Students were then advised to continue using the SRL program outside of school hours. To facilitate this, students were asked to identify a time that night when they would practice their basic facts.

Procedures. A randomized control trial was used to evaluate the efficacy of the intervention. The program took place during the final 15-minutes of the students' regular mathematics time. The mathematics program at this school ran every day of the week (five days). A typical lesson for these students lasted 60 minutes and included a 15-minute focus on basic facts and number knowledge. The study replaced this part of the students' normal mathematics lesson. At this time, the participating students moved from their regular instruction room to the room they had been randomly assigned to. In one of these rooms the students received the SRL program and in the other room the students received the control program.

To avoid contamination, no student in the SRL group was able to take celeration charts or sprint sheets away from the class. This ensured that no student in the control group could access the materials used by the students in the SRL group. The study was designed to mitigate the risk of demoralization. This was achieved by ensuring that the only variable that differed between the two conditions was the program that they were exposed to. For example, both groups received a program pack, were taught by a teacher in a classroom, engaged in their assigned program for the same duration over the same number of days, and saw the first author conducting fidelity checks in their classroom.

Training. Prior to the commencement of the program, a 30 minute training session was provided for the teacher in charge of the SRL group. In this session, the teacher was introduced to the program script and key elements of the SRL program. Student training occurred during the program. The student training was based on the four-phase model for the development of self-regulation competence (Schunk & Zimmerman, 2007). In particular, the training emphasized the role of modelling and emulation along the pathway to self-control and self-regulation. The teacher had an opportunity to run through the program and was encouraged to ask questions throughout the training.

Pre-program (T1). Prior to the study, the school provided student demographic information for all students who consented to participate in the study. The data were retrieved from the school's student management system and provided to the first author. In addition, all participants completed the warm-up multiplication probe and Test A and Test B of the one-minute multiplication probes at T1. The probes were administered by the researcher who followed a scripted administration procedure. The researcher then took away all probes for marking and analysis. Students in both groups also completed the questionnaire. The researcher emphasized to students that they were only to reflect on the basic facts practice they had engaged in over the previous seven days. To support students with the questionnaire, the researcher read all instructions and possible answers aloud. The researcher also supported any student who was unable to complete the written response to Question 3, which required students to describe the type of practice they engaged in if they selected the *other* option.

Post-program (T2). On the last day of the program and after completing the warm-up multiplication probe, all students completed a second version of Test A and Test B of the one-minute multiplication probes and T2 questionnaire. The first author conducted these assessments following the same administration and marking procedures employed at T1.

Maintenance (T3). Five weeks after the intervention, students completed a third version of the warm-up multiplication probe and Test A and Test B of the one-minute multiplication probes. The first author conducted these assessments following the same administration and marking procedures employed at T1.

Interobserver Agreement

Interobserver agreement (IOA) was conducted on 21.3% of all the T1 and T2 one-minute multiplication probes and 22.2% of the T3 one-minute multiplication probes. Agreement was defined as scoring the same response for each question. IOA was calculated by dividing the number of agreements on responses by the number of agreements and disagreements on responses and multiplying by 100. Agreement between the first author and second marker was 100% for the tests conducted at T1, T2, and T3. The interobserver checks confirmed that there was also 100% agreement on the total test score for every test checked at T1, T2, and T3. Total scores for each student were calculated by adding all the correct responses on a student's test. Cohen's kappa was not calculated due to the 100% agreement.

Treatment Fidelity

The first author conducted fidelity checks for both the SRL and control conditions based on similar checks used by Odom et al. (2010) in their evaluation of curriculum implementation in preschool classes. Both the procedural integrity and the level of student engagement in key components of the respective programs were measured. Scores for the engagement items ranged from *1*, which meant that 10 or more students were not engaged, to *5* which meant that up to two students were not engaged. Content fidelity was also measured on a scale from *1* to *5*. A score of *1* meant that the feature was not presented. A score of *5* meant that the program component was clearly presented in an engaging manner. The scores for each fidelity check were then summed and expressed as a percentage. Fidelity checks were conducted on 35% of all SRL group lessons. The median fidelity score across these observations was 86%, with a range of 80–88%. Fidelity checks were also conducted on 25% of all control group lessons. The median fidelity score across the observations was 90%, with a range of 60–95%. The fidelity checks were designed to measure the procedural integrity with which the programs were implemented. Because the programs varied, the fidelity scores should not be interpreted to mean that one program was implemented with greater fidelity than the other. Instead, the checks confirm that both the SRL and control programs were implemented with acceptable fidelity.

Analyses

The first set of analyses examined the possibility of significant differences between the two groups in terms of their basic facts and basic facts practice characteristics (frequency and duration) at baseline with independent samples *t*-tests. In addition, an exact significance test for Pearson's chi-square was conducted to investigate whether the type of practice students reported engaging in differed by condition at baseline. One-way between-groups analysis of covariance (ANCOVA) were used to investigate the differences between the two groups on the one-minute multiplication probes administered at T2 and T3. A repeated measures analysis of variance (RMANOVA) was also used to investigate group changes over time. In addition to these nomothetic analyses, idiographic analyses were also undertaken using the reliable change index (RCI). Unlike nomothetic analysis, idiographic analysis does not infer a within-individual effect from between person data. Thus, the advantage of the idiographic analysis is that it allows investigation into whether differences between the groups is due to improvement in multiplication fluency for children in the experimental condition across the range of individual differences. Without this analysis it is not possible to rule out that between-group differences could be due to improvements made by a small group of children who were struggling at the baseline assessment, who then showed substantial improvement. The RCI was calculated from the one-minute multiplication probe scores collected at T1, T2, and T3, for all students in the SRL and control groups, using the method described in Jacobson and Truax (1991). Calculation of the RCI allowed the identification of one-minute multiplication probe scores which showed a reliable improvement or decrement from T1 to T2 and T3. We also investigated whether students' basic facts practice behavior changed as a result of the SRL program. ANCOVAs were used to investigate whether the two groups differed significantly on the duration and frequency of practice they reported engaging in at T2. A multi-dimensional chi-square test was used to investigate whether the SRL and control group differed in the type of practice they engaged in at T2. The final set of analyses investigated social validity. Mann–Whitney U tests were used to investigate whether there was a significant difference in how enjoyable and helpful the two groups had found their respective programs. The Mann–Whitney U test was selected because the data was not normally distributed.

Results

Pre-Program Group Differences

All students who were randomly assigned to experimental conditions (SRL or control) completed the T1 assessments. Analysis confirmed that that there was no statistically significant difference between students in the SRL (M = 19.8, SD = 8.2) and control groups (M = 21.0, SD = 8.4) on Test A at T1 (t = -.499, df = 45, p = .620, two-tailed). There was also no statistically significant difference between the two groups on Test B (SRL M = 20.8, SD = 9.7; control M = 21.2, SD = 9.4; t = -.121, df = 45, p = .904). In addition, analyses also confirmed that the SRL and control groups did not differ significantly in either the reported number of practice days (t = .282, df = 45, p = .779), practice duration (t = .513, df = 45, p = .611), or the type of basic facts practice students engaged in outside of school hours (χ^2 (4, N = 45) = .671, exact p = .961). In summary, these analyses confirmed that the SRL group and control group did not differ significantly on any measure prior to the program commencing.

Multiplication Fact Fluency

Nomothetic analysis for basic facts (Test A). An ANCOVA was conducted to examine if there were any group differences (SRL and control) on Test A performance at T2. Test A performance at T1 was entered as the covariate. As expected, Test A performance at T1 was significantly related to Test A performance at T2: $F(1,44) = 80.52$, $p < .05$, partial $\eta^2 = .65$. After adjusting for this covariate, there remained a statistically significant difference between the two groups: $F\ (1,44) = 49.58$, $p < .05$, partial $\eta^2 = .53$; see Table 1). This difference exceeded Cohen's (1988) convention for a large effect size.

Nomothetic analysis for basic facts (Test B). In similar fashion, an ANCOVA was conducted to examine group differences for Test B (composed of the commutative facts) performance at T2 (see Table 1). Test B at T1 was entered as the covariate and the results again showed that Test B performance at T1 was significantly related to Test B performance at T2 ($F(1,44) = 103.56$, $p < .05$, partial $\eta^2 = .70$). After adjusting for this covariate, there remained a statistically significant difference between the two groups ($F(1,44) = 9.04$, $p < .05$, partial $\eta^2 = .17$). This difference also exceeded Cohen's (1988) convention for a large effect size.

Idiographic analysis for basic facts (Test A & B) at T2. Idiographic analysis was also undertaken for Test A and B (see Figure 1) using the method described by Jacobson and Truax (1991). Calculation of the RCI indicated that 21 (87.5%) students in the SRL group showed a reliable improvement from T1 to T2 on Test A. Of the 23 students in the control group, two students (8.7%) showed a reliable improvement and one student (4%) showed a reliable decrement in performance during this time period. The same analysis on Test B data showed that 12 (50%) students in the SRL group and six students in the control group (26.1%) showed a reliable improvement from T1 to T2. These analyses confirm that a higher number of students in the SRL group made substantially more progress with their multiplication facts than their peers in the control group.

(INSERT FIGURE 1 ABOUT HERE)

Maintenance of Multiplication Fact Fluency

Nomothetic analysis for basic facts. At the 5-week follow-up, all children completed a one-minute multiplication probe. Similar ANCOVA analyses showed that the improved performance experienced by the SRL group was maintained, and that the control group failed to substantially catch-up (see Table 1). These results were evident across both Test A (F (1,42) = 29.53, $p < .05$, partial $\eta^2 = .41$) and Test B (F (1,42) = 14.96, $p < .05$, partial $\eta^2 = .26$). Both of these results exceeded Cohen's (1988) convention for a large effect size.

(INSERT TABLE 1 ABOUT HERE)

A second analytic strategy was also employed to examine within group changes over time. Repeated measures analysis of variance (RMANOVA) were employed separately for Test A and Test B, with all three multiplication tests (T1, T2, and T3) as the within-group variable, and group assignment as the between-subjects variable. We anticipated a significant interaction across the factors with the improvement in multiplication fluency over time being more pronounced for the SRL group compared to the control group. The results for both Test A and Test B are shown in Figures 2 and 3. As can be seen, both groups showed an increase in multiplication fluency that was largely maintained at T3 (but did not increase). However, this change in fluency was much greater for the SRL group (significant linear Group X Time interaction: Test A, F (1,44) = 28.75, $p < .001$, partial $\eta^2 = .40$; Test B, F (1,44) = 14.70, $p < .001$, partial $\eta^2 = .26$.

(INSERT FIGURE 2 AND 3 ABOUT HERE)

Idiographic analysis for basic facts (Test A & B) at T3. Idiographic analysis was again undertaken for Test A and B at T3. Calculation of the RCI indicated that the progress made by students in the SRL group was largely maintained over time. Seventeen (71%) students in the SRL group showed a reliable improvement from T1 to T3 on Test A and 13 (54%) students showed a reliable improvement on Test B (see Figure 4). In contrast, only three students in the control group (14%), showed a reliable improvement on Test A and three (14%) students showed a reliable improvement of Test B (see Figure 4).

(INSERT FIGURE 4 ABOUT HERE)

Effects on Basic Facts Practice

Practice frequency and duration. Two separate ANCOVAs were run to investigate whether the SRL and control groups differed in their reported frequency and duration of practice at T2. The first analysis found that there was no significant difference between the frequency with which the SRL and control groups practiced at T2 after adjusting for frequency of practice at T1 ($F(1,43) = .467$, $p = .498$). The second analysis found that there was also no significant difference in the duration with which the groups practiced at T2 after adjusting for duration of practice at T1 ($F(1,44) = .875$, $p = .355$).

Type of practice. Prior to the study, both the SRL and control groups reported that they used math games to practice their basic facts more frequently than any other strategy. A multi-dimensional chi-square test was conducted to investigate whether the SRL group and control groups differed significantly in the type of practice they reported engaging in at T2. This analysis confirmed that there was a significant relationship between the conditions and the type of basic facts practice students engaged in outside of school hours: χ^2 $(4, N = 47) = 28.560$, exact $p < .05$. The main change observed at T2 was the number of SRL students who reported using flash cards as their most common form of basic facts practice, which is not surprising as flash cards were included in the students' study materials. At T2, 79.2% of SRL students reported using flash cards. This was substantially larger than the number of control students who reported using flash cards as their primary practice method at T2 (4.3%). It was also substantially higher than the number of SRL (13.6%) and control students (8.7%) who reported using flash cards at T1.

Social Validity

Use of self-control strategies. At the end of every practice session students in the SRL group recorded how confident they felt answering the basic fact questions they had worked on. These frequencies were calculated by summing all student responses across all the SRL participants. For example, in total, 260 responses were received from students in the SRL group. On 99 (38%) of these occasions, students were extremely confident they could instantly answer the facts they had learned in that session. These analyses show that after 72% of all the SRL group's home-practice sessions, students felt either very or extremely confident answering the basic facts they had been practicing. These findings suggest that students believed the practice sessions they completed helped them to become more fluent with their multiplication fact recall.

Students in the SRL group also recorded how helpful they found the self-control strategies. These frequencies were calculated by summing all student responses across all the SRL participants. The analyses confirmed that students chose to use the self-control strategies at home (see Table 2). Environmental strategies (setting selection and setting modification) were the most popular self-control strategies that students chose to employ. Students also reported finding the environmental self-control strategies more helpful than the cognitive strategy (attention deployment).

(INSERT TABLE 2 ABOUT HERE)

Enjoyment. The T2 questionnaire asked students in the SRL and control groups to rate how enjoyable they found their respective basic facts programs. Analysis of this question revealed that 83.3% of SRL students rated the SRL program as being either very enjoyable or extremely enjoyable. This differed from the control group where only 56.5% of students rated their program as very enjoyable or extremely enjoyable. In fact, 21.7% of students in the control group selected one of the lowest two categories to depict their level of enjoyment, compared to only 8.3% of students in the SRL group. A Mann–Whitney U test confirmed that students' enjoyment ratings differed significantly by condition (U = 168.500, N_1 = 24, N_2 = 23, p < .05, two-tailed).

Helpfulness. The T2 questionnaire also asked students in the SRL and control groups to rate how helpful they found their basic facts program. A substantial difference in perceived helpfulness by condition was observed. All of the SRL students rated their program as either very helpful or extremely helpful. In contrast, only 26.1% of students in the control group perceived their program to be either very helpful or extremely helpful. In fact, 19.2% of students in the control group selected one of the two lowest categories: *a little bit helpful* or *not helpful*. A Mann-Whitney U test confirmed that students' enjoyment ratings differed significantly by condition (U = 66.000, N_1 = 24, N_2 = 23, p < .05, two-tailed).

Teacher reflection. The teacher in the SRL class completed the social validity questionnaire at T2. Through her responses, the teacher indicated that she thought the program was extremely helpful for improving students' fluency with basic facts. She had a positive perception of the program and indicated that she is likely to use the program again and elements of the program in other curriculum areas.

Discussion

The current study used a repeated measures experimental design to evaluate the effectiveness of a basic facts fluency program based on DPR, self-regulated learning, and PT delivered in a classroom setting. The experimental group made significantly greater gains than the control group on the one-minute multiplication facts probes that were administered immediately after the intervention and at a five-week follow-up. Idiographic analysis also confirmed that the between-group differences were not due to a small group of children who showed substantial improvement. Instead, the RCI confirmed that progress was made by students of all ability levels. This was a key finding, as the program was designed for use within Tier 1 of the RTI framework. The study also confirmed that the SRL students continued to use the strategy that was taught in class, outside of school hours.

Previous studies have shown that PT (Chiesa & Robertson, 2000; Gallagher, 2006; Strømgren et al., 2014) and DPR (Musti-Rao & Plati, 2015; Poncy, Duhon, et al., 2010; Poncy et al., 2013) programs can increase students' fluency with basic facts. Studies have also shown that SRL programs can lead to enhanced student achievement in other math skills, with students of a similar age to those in study (Camahalan, 2006; Özcan & Erktin, 2015; Ramdass & Zimmerman, 2008). The results from this program support the findings from these studies and provide an important addition to this literature. Specifically, the results from this study indicate that key elements of DPR, PT, and self-regulated learning can be successfully combined to enhance students' fluency with basic multiplication facts. In fact, the results from this study compare favorably to other Tier 1 studies using alternative instructional approaches (Knowles, 2010; Wong & Evans, 2007; Woodward, 2006).

This study also provides a valuable addition to the literature on effective Tier 1 interventions for basic facts. Tier 1 programs should be effective for approximately 80–90% of the population (Johnson & Street, 2013). In order to ascertain this, results must be reported at the idiographic level. To date, few Tier 1 studies have reported their results at this level. In addition, few studies have used randomized control trials in typical school settings with students of all ability levels (Author, 2018). This design promotes confidence that this program reflects an empirically sound program that could be implemented within a typical school context.

The nomothetic analyses showed that the SRL program had a significant and large effect on the SRL students' basic facts recall. It could be argued that it is not surprising that a basic facts fluency program led to gains in multiplication facts fluency. What is of particular interest is the amount of progress made, especially when this is compared to the progress made by students in the control group who participated in typical basic facts instruction over the same period. The control condition continued to engage in typical basic facts practice every day, for the same duration as students in the SRL intervention. The lessons they engaged in were taken from Book 4 of the Numeracy Project (Ministry of Education, 2008) and represented typical practice in New Zealand schools. The findings indicate that the SRL program led to substantially greater gains for students in the SRL condition than those in the control condition. The idiographic analysis confirms that the progress made was not limited to a small group of students who made substantial progress over the course of the program. Instead, the analysis confirms that the program was effective for students of all ability levels. This indicates that the SRL program may be a far more efficient form of basic facts practice than that typically used in New Zealand schools. This finding is consistent with other SRL studies. Specifically, these studies found that the implementation of self-regulated learner programs in mathematics with students similar in age to those in this study, resulted in more efficient learning (Camahalan, 2006; Lee et al., 2014; Núñez, Rosário, Vallejo, & González-Pienda, 2013; Özcan & Erktin, 2015). The current study's findings should be of particular interest to educators as they indicate that the SRL program may result in greater and more rapid progress than those made using typical instruction practices. In summary, the results confirm our first hypothesis, that students in the SRL condition would perform significantly better on the one-minute multiplication probes administered after the intervention than the students in the control condition.

This study also investigated whether fluency with some or all of the 52 multiplication facts generalized to their respective commutative facts. Analyses confirmed that whilst students made slightly less progress with the commutative facts presented in Test B, they still exhibited substantial growth over the course of the program. It is possible that students' progress with the commutative facts was mediated by their understanding of the commutative property of multiplication. Unfortunately, this hypothesis could not be investigated due to inconsistencies in the screening data that were obtained from teachers.

The second hypothesis predicted that students in the SRL group would practice their basic facts more frequently and for longer durations, than they did prior to the program and in comparison, to their peers in the control group. The second part of this hypothesis predicted that, unlike students in the control group, SRL students would choose to engage in practice with flash cards over other types of practice. The data obtained during this study confirmed that students in the SRL condition chose to engage in practice with flash cards over other types of practice. This finding was of particular importance as previous research has found that the quality of practice that students engage in is more important than the quantity of practice they complete (Plant, Ericsson, Hill, & Asberg, 2005; Ramdass & Zimmerman, 2011; Rosario, Nunez, Valle, Gonzalez-Pienda, & Lourenco, 2013). In contrast to this finding, the students in the SRL group did not choose to practice more frequently or for longer durations than their peers in the control condition. Given how enjoyable and helpful students in the SRL group rated the program, it seems somewhat counterintuitive that these students did not then choose to engage in any additional practice outside of school hours. One possible explanation for this is that students did not find the program as enjoyable as the myriad of other activities they could engage in, outside of school hours. This hypothesis is consistent with research by Duckworth et al. (2011) who found that even high achieving students, who engaged in considerable practice, did not consider practice to be any more enjoyable than lower performing students who completed less practice. Alternatively, it is possible that students did not choose to engage in additional practice outside of school hours as almost all students were able to maintain robust progress throughout the program during their school practice sessions and could see this effect in their celeration chart. Thus, they may have felt there was no need for further practice.

Limitations and Future Research

The results from this study confirm that the SRL program resulted in a substantial improvement in multiplication fact fluency. Future research should look to replicate the findings from this study in different contexts and different geographical locations. These future studies could also conduct fidelity checks on a greater proportion of the SRL lessons than those that were reviewed in this study. In addition, future research should investigate whether this model can also be applied to other subject areas and more complex types of learning, such as composite skills. These studies could also increase the number of weeks in which the program is administered. This would enable investigation into whether the robust rates of progress observed in this study, could be maintained by students until the TFR is met and if the improved fluency performance allows students to more quickly learn more complex mathematics, such as long multiplication and division. Assessing these types of variables would enable future researchers to establish whether the basic facts fluency gains resulted in more generalized math gains. In addition, future studies could also include more distal measures of student performance. The current study established that gains were maintained over a period of five weeks after completion of the program. Future research could investigate maintenance over longer time frames.

This study used self-report data to investigate student practice behavior outside of school hours. In order to overcome the limitations associated with self-report data, future studies could supplement these data with direct observations or parent reports. These studies could also investigate enjoyment and helpfulness data at a lower level of abstraction by asking students to evaluate key components of the program rather than the program as a whole. Additionally, future studies should assess students understanding of the commutative property of multiplication, separately from teacher records. As previously noted, the impact of this variable could not be investigated due to the poor quality of the data received.

Conclusion

The findings from this study suggest that elements of self-regulated learning and PT can be successfully combined to enhance student achievement. As a result of this program, students also adopted more beneficial basic facts practice behaviors outside school hours. A number of features should appeal to educators. First, this program was able to facilitate robust and enduring basic facts progress. This means that educators should be able to allocate additional time to teaching more complex mathematics behavior. Second, this program was effective for students of all abilities. These findings, in addition to the program's high social validity, emphasize the utility of this program for use at the Tier 1 level. While this program showed good evidence of validity in a single school, future research will need to see if this can be repeated and extended to a wider demographic of students.

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References

Author. (2018). [Title omitted for blind review]. (Unpublished master's thesis). University of Canterbury, New Zealand.

Becker, A., McLaughlin, T., Weber, K. P., & Gower, J. (2009). The effects of cover, copy, and compare with and without additional error drill on multiplication fact fluency and accuracy. *Electronic Journal of Research in Educational Psychology, 7*, 747–760.

Binder, C. (1996). Behavioral fluency: Evolution of a new paradigm. *The Behavior Analyst, 19*, 163–197.

Binder, C., & Watkins, C. (2013). Precision teaching and direct instruction: Measurably superior instructional technology in schools. *Performance Improvement Quarterly, 26*(2), 73–115. https://doi.org/10.1002/piq.21145

Bryant, B. R., Ok, M., Kang, E. Y., Kim, M. K., Lang, R., Bryant, D. P., & Pfannestiel, K. (2015). Performance of fourth-grade students with learning disabilities on multiplication facts comparing teacher-mediated and technology-mediated interventions: A preliminary investigation. *Journal of Behavioral Education, 24*, 255–272.

Burns, M. K. (2005). Using incremental rehearsal to increase fluency of single-digit multiplication facts with children identified as learning disabled in mathematics computation. *Education and Treatment of Children, 28*, 237–249.

Burns, M. K., Kanive, R., & DeGrande, M. (2012). Effect of a computer-delivered math fact intervention as a supplemental intervention for math in third and fourth grades. *Remedial and Special Education, 33*, 184–191. https://doi.org/10.1177/0741932510381652

Burns, M. K., Zaslofsky, A. F., Maki, K. E., & Kwong, E. (2016). Effect of modifying intervention set size with acquisition rate data while practicing single-digit multiplication facts. *Assessment for Effective Intervention, 41*, 131–140. https://doi.org/10.1177/1534508415593529

Camahalan, F. M. G. (2006). Effects of self-regulated learning on mathematics achievement of selected Southeast Asian children. *Journal of Instructional Psychology, 33*, 194–205.

Chiesa, M., & Robertson, A. (2000). Precision teaching and fluency training: Making maths easier for pupils and teachers. *Educational Psychology in Practice, 16*, 297–310. https://doi.org/10.1080/713666088

Codding, R. S., Chan-Iannetta, L., Palmer, M., & Lukito, G. (2009). Examining a classwide application of cover-copy-compare with and without goal setting to enhance mathematics fluency. *School Psychology Quarterly, 24*, 173–185. https://doi.org/10.1037/a0017192

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: L. Erlbaum Associates.

DiGiacomo, G., & Chen, P. (2016). Enhancing self-regulatory skills through an intervention embedded in a middle school mathematics curriculum. *Psychology in the Schools, 53*, 601–616. https://doi.org/10.1002/pits.21929

Dignath, C., Buettner, G., & Langfeldt, H.-P. (2008). Review: How can primary school students learn self-regulated learning strategies most effectively? A meta-analysis on self-regulation training programmes. *Educational Research Review, 3*, 101–129.

Duckworth, A., Gendler, T., & Gross, J. (2014). Self-control in school-age children. *Educational Psychologist, 49*, 199–217. https://doi.org/10.1080/00461520.2014.926225

Duckworth, A., Kirby, T., Tsukayama, E., Berstein, H., & Ericsson, K. (2011). Deliberate practice spells success: Why grittier competitors triumph at the National Spelling Bee.

Social Psychological and Personality Science, 2, 174–181. https://doi.org/10.1177/1948550610385872

Ericsson, K., Krampe, R., & Tesch-Römer, C. (1993). The role of deliberate practice in the acquisition of expert performance. *Psychological Review, 100*, 363–406. https://doi.org/10.1037/0033-295X.100.3.363

Fuchs, L. S., Powell, S. R., Seethaler, P. M., Cirino, P. T., Fletcher, J. M., Fuchs, D., & Hamlett, C. L. (2010). The effects of strategic counting instruction, with and without deliberate practice, on number combination skill among students with mathematics difficulties. *Learning and Individual Differences, 20*, 89–100. https://doi.org/10.1016/j.lindif.2009.09.003

Gallagher, E. (2006). Improving a mathematical key skill using precision teaching. *Irish Educational Studies*, *25*, 303–319. https://doi.org/10.1080/03323310600913757

Gross, T. J., Duhon, G. J., Hansen, B., Rowland, J. E., Schutte, G., & Williams, J. (2014). The effect of goal-line presentation and goal selection on first-grader subtraction fluency. *The Journal of Experimental Education, 82*, 555–571. https://doi.org/10.1080/00220973.2013.813369

Gross, T. J., Duhon, G. J., Shutte, G., & Rowland, J. E. (2016). A comparison of group-oriented contingencies for addition fluency. *Contemporary School Psychology, 20*, 130–141. https://doi.org/10.1007/s40688-015-0054-x

Jacobson, N. S., & Truax, P. (1991). Clinical significance: A statistical approach to defining meaningful change in psychotherapy research. *Journal of Consulting and Clinical Psychology, 59*, 12–19. https://doi.org/10.1037/0022-006X.59.1.12

Johnson, K., & Street, E. (2013). *Response to intervention and precision teaching: Creating synergy in the classroom*. New York, NY: Guilford.

Kameenui, E., & Simmons, D. (1990). *Designing instructional strategies: The prevention of academic learning problems.* Columbus, OH: Merrill Publishing.

Knowles, N. (2010). *The relationship between timed drill practice and the increase of automaticity of basic multiplication facts for regular education sixth graders.* (Doctoral dissertation, Walden University). Retrieved from http://scholarworks.waldenu.edu/dissertations/825/

Kramarski, B., Itzhak, W., & Sarit, S. (2013). Generic versus context-specific prompts for supporting self-regulation in mathematical problem solving among students with low or high prior knowledge. *Journal of Cognitive Education and Psychology, 12*, 197–214.

Kramarski, B., Weisse, I., & Kololshi-Minsker, I. (2010). How can self-regulated learning support the problem solving of third-grade students with mathematics anxiety? *ZDM, 42*, 179–193. https://doi.org/10.1007/s11858-009-0202-8

Kubina, R., & Yurich, K. (2012). *The precision teaching book*. Lemont, PA: Greatness Achieved Publishing.

Labuhn, A. S., Zimmerman, B. J., & Hasselhorn, M. (2010). Enhancing students' self-regulation and mathematics performance: The influence of feedback and self-evaluative standards. *Metacognition and Learning, 5*, 173–194. https://doi.org/10.1007/s11409-010-9056-2

Lee, N. H., Yeo, D. J. S., & Hong, S. E. (2014). A metacognitive-based instruction for primary four students to approach non-routine mathematical word problems. *ZDM, 46*, 465–480. https://doi.org/10.1007/s11858-014-0599-6

McCallum, E., Skinner, C. H., Turner, H., & Saecker, L. (2006). The taped-problems intervention: Increasing multiplication fact fluency using a low-tech, classwide, time-delay intervention. *School Psychology Review, 35*, 419–434.

Merton, R. (1968). The Matthew effect in science. *Science*, *159*, 56–63. https://doi.org/10.1126/science.159.3810.56

Miller, K. C., Skinner, C. H., Gibby, L., Galyon, C. E., & Meadows-Allen, S. (2011). Evaluating generalization of addition-fact fluency using the taped-problems procedure in a second-grade classroom. *Journal of Behavioral Education, 20*, 203–220. https://doi.org/10.1007/s10864-011-9126-9

Ministry of Education. (2008). *Book 4: Teaching number knowledge*. Wellington, New Zealand: Author.

Musti-Rao, S., & Plati, E. (2015). Comparing two classwide interventions: Implications of using technology for increasing multiplication fact fluency. *Journal of Behavioral Education*, *24*, 418–437. https://doi.org/10.1007/s10864-015-9228-x

Neill, A. (2008). Basic facts: Start with strategies, move on to memorisation. *Set: Research Information for Teachers*, 19–24.

Ness, B. M., & Middleton, M. J. (2012). A framework for implementing individualized self-regulated learning strategies in the classroom. *Intervention in School and Clinic*, *47*, 267–275. https://doi.org/10.1177/1053451211430120

Núñez, J. C., Rosário, P., Vallejo, G., & González-Pienda, J. A. (2013). A longitudinal assessment of the effectiveness of a school-based mentoring program in middle school. *Contemporary Educational Psychology*, *38*, 11–21. https://doi.org/10.1016/j.cedpsych.2012.10.002

Odom, S. L., Fleming, K., Diamond, K., Lieber, J., Hanson, M., Butera, G., . . . Marquis, J. (2010). Examining different forms of implementation and in early childhood curriculum research. *Early Childhood Research Quarterly*, *25*, 314–328. https://doi.org/10.1016/j.ecresq.2010.03.001

Özcan, Z. Ç., & Erktin, E. (2015). Enhancing mathematics achievement of elementary school students through homework assignments enriched with metacognitive questions. *EURASIA Journal of Mathematics, Science & Technology Education*, *11*, 1415–1427. https://doi.org/10.12973/eurasia.2015.1402a

Plant, E., Ericsson, K., Hill, L., & Asberg, K. (2005). Why study time does not predict grade point average across college students: Implications of deliberate practice for academic performance. *Contemporary Educational Psychology*, *30*, 96–116. https://doi.org/10.1016/j.cedpsych.2004.06.001

Poncy, B. C., Duhon, G. J., Lee, S. B., & Key, A. (2010). Evaluation of techniques to promote generalization with basic math fact skills. *Journal of Behavioral Education*, *19*, 76–92. https://doi.org/10.1007/s10864-010-9101-x

Poncy, B. C., Fontenelle IV, S. F., & Skinner, C. H. (2013). Using detect, practice, and repair (DPR) to differentiate and individualize math fact instruction in a class-wide setting. *Journal of Behavioral Education*, *22*, 211–228. https://doi.org/10.1007/s10864-013-9171-7

Poncy, B. C., Skinner, C. H., & Axtell, P. K. (2010). An investigation of detect, practice, and repair to remedy math-fact deficits in a group of third-grade students. *Psychology in the Schools*, *47*, 342–353. https://doi.org/10.1002/pits.20474

Ramdass, D., & Zimmerman, B. J. (2008). Effects of self-correction strategy training on middle school students' self-efficacy, self-evaluation, and mathematics division learning. *Journal of Advanced Academics*, *20*, 18–41. https://doi.org/10.4219/jaa-2008-869

Ramdass, D., & Zimmerman, B. J. (2011). Developing self-regulation skills: The important role of homework. *Journal of Advanced Academics*, *22*, 194–218. https://doi.org/10.1177%2F1932202X1102200202

Rosario, P., Nunez, J. C., Valle, A., Gonzalez-Pienda, J., & Lourenco, A. (2013). Grade level, study time, and grade retention and their effects on motivation, self-regulated learning strategies, and mathematics achievement: A structural equation model. *European Journal of Psychology of Education 28*, 1311–1331. https://doi.org/10.1007/s10212-012-0167-9

Schmitz, B., & Wiese, B. S. (2006). New perspectives for the evaluation of training sessions in self-regulated learning: Time-series analyses of diary data. *Contemporary Educational Psychology*, *31*, 64–96. https://doi.org/10.1016/j.cedpsych.2005.02.002

Schunk, D., & Ertmer, P. (2000). Self-regulation and academic learning self-efficacy enhancing interventions. In M. Boekaerts, P. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation* (pp. 631–646). San Diego, CA: Elsevier Academic Press.

Schunk, D., & Zimmerman, B. (2007). Influencing children's self-efficacy and self-regulation of reading and writing through modeling. *Reading & Writing Quarterly: Overcoming Learning Difficulties*, *23*, 7–25. https://doi.org/10.1080/10573560600837578

Schutte, G. M., Duhon, G. J., Solomon, B. G., Poncy, B. C., Moore, K., & Story, B. (2015). A comparative analysis of massed vs. distributed practice on basic math fact fluency growth rates. *Journal of School Psychology*, *53*, 149–159. https://doi.org/10.1016/j.jsp.2014.12.003

Singer-Dudek, J., & Greer, R. D. (2005). A long-term analysis of the relationship between fluency and the training and maintenance of complex math skills. *The Psychological Record*, *55*, 361–376.

Smith, C. R., Marchand-Martella, N. E., & Martella, R. C. (2011). Assessing the effects of the rocket math program with a primary elementary school student at risk for school failure: A case study. *Education and Treatment of Children, 34*, 247–258. https://doi.org/10.1353/etc.2011.0011

Steel, S., & Funnell, E. (2001). Learning multiplication facts: A study of children taught by discovery methods in England. *Journal of Experimental Child Psychology, 79*, 37–55. https://doi.org/10.1006/jecp.2000.2579

Stoeger, H., & Ziegler, A. (2008). Evaluation of a classroom based training to improve self-regulation in time management tasks during homework activities with fourth graders. *Metacognition & Learning, 3*, 207–230. https://doi.org/10.1007/s11409-008-9027-z

Strømgren, B., Berg-Mortensen, C., & Tangen, L. (2014). The use of precision teaching to teach basic math facts. *European Journal of Behavior Analysis*, *15*, 225–240. https://doi.org/10.1080/15021149.2014.11434723

Sweller, J., Ayres, P. L., & Kalyuga, S. (2011). *Cognitive load theory*. New York, NY: Springer.

Tait-McCutcheon, S., Drake, M., & Sherley, B. (2011). From direct instruction to active construction: Teaching and learning basic facts. *Mathematics Education Research Journal*, *23*, 321–345. https://doi.org/10.1007/s13394-011-0018-z

Tuckman, B. W., & Harper, B. E. (2012). *Conducting educational research* (6th ed.). Lanham, MD: Rowman & Littlefield.

Tunmer, W., & Greaney, K. (2010). Defining dyslexia. *Journal of Learning Disabilities, 43*, 229–243. https://doi.org/10.1177/0022219409345009

Tzohar-Rozen, M., & Kramarski, B. (2014). Metacognition, motivation and emotions: Contribution of self-regulated learning to solving mathematical problems. *Global Education Review, 1*(4), 76–95.

Wong, M., & Evans, D. (2007). Improving basic multiplication fact recall for primary school students. *Mathematics Education Research Journal*, *19*, 89–106. https://doi.org/10.1007/BF03217451

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly, 29*, 269–289. https://doi.org/10.2307/30035554

Zimmerman, B. J. (2000). Attaining self-regulation. A social cognitive perspective. In M. Boekaerts, P. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation* (pp. 13–35). San Diego, CA: Elsevier Academic Press.

Appendix

Basic Facts Questionnaire

Basic fact questionnaire

Please complete this questionnaire as honestly as possible. Only the researcher will see the answers you put on this questionnaire.

Name (first and last): ……………………………………………………………………………………………………

1) On how many days did you practise your basic facts at home over the last week?

Please circle the most appropriate answer below.

0 days	1 day	2 days	3 days	4 days	5 days	6 days	7 days

2) On average, how long did you practise them for?

Please circle the most appropriate answer below.

0 minutes	5 minutes	10 minutes	15 minutes	20 minutes	More than 20 minutes. Please write the number of minutes here: ……………………

3) What was the **most common** strategy you used for learning your basic facts?

Please circle the most appropriate answer below.

Maths games	Tested by another person	Using flash cards	Completing activity sheets	Other (please state below)

If you selected other, please describe how you practised your basic facts: ……………………………………….

……

4) How enjoyable did you find this basic fact program?

Not enjoyable	A little bit enjoyable	Fairly enjoyable	Very enjoyable	Extremely enjoyable

5) How much do you think this programme helped improve your basic fact recall?

Not helpful	A little bit helpful	Fairly helpful	Very helpful	Extremely helpful

Figure 1

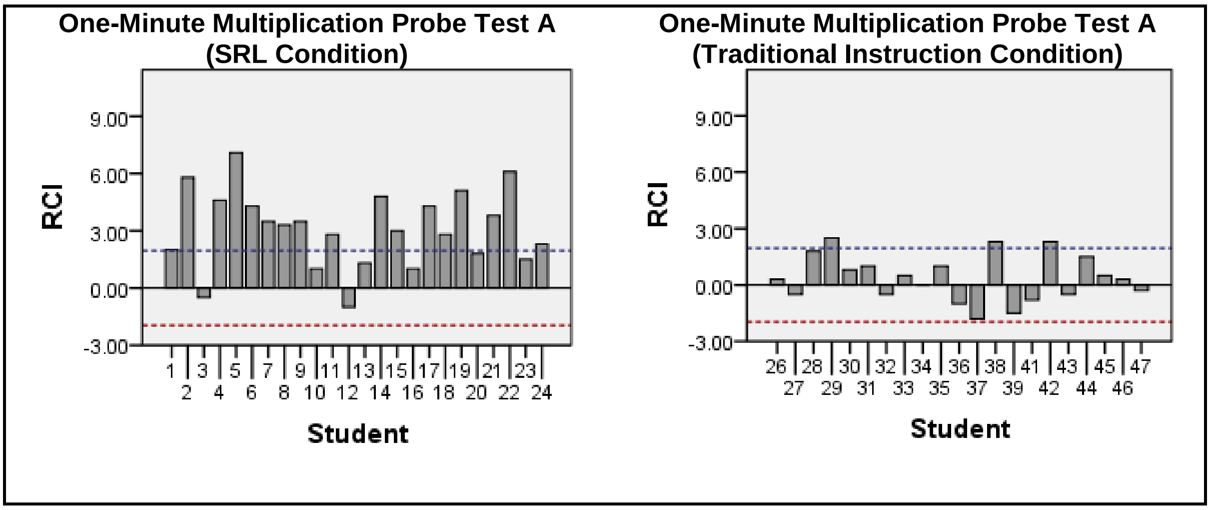

Figure 1. Reliable change scores from T1 to T3 for the SRL and control groups on Test A.

Note. The blue dotted line represents the threshold for reliable increase and the red dotted line represents the threshold for reliable decrease.

Figure 2

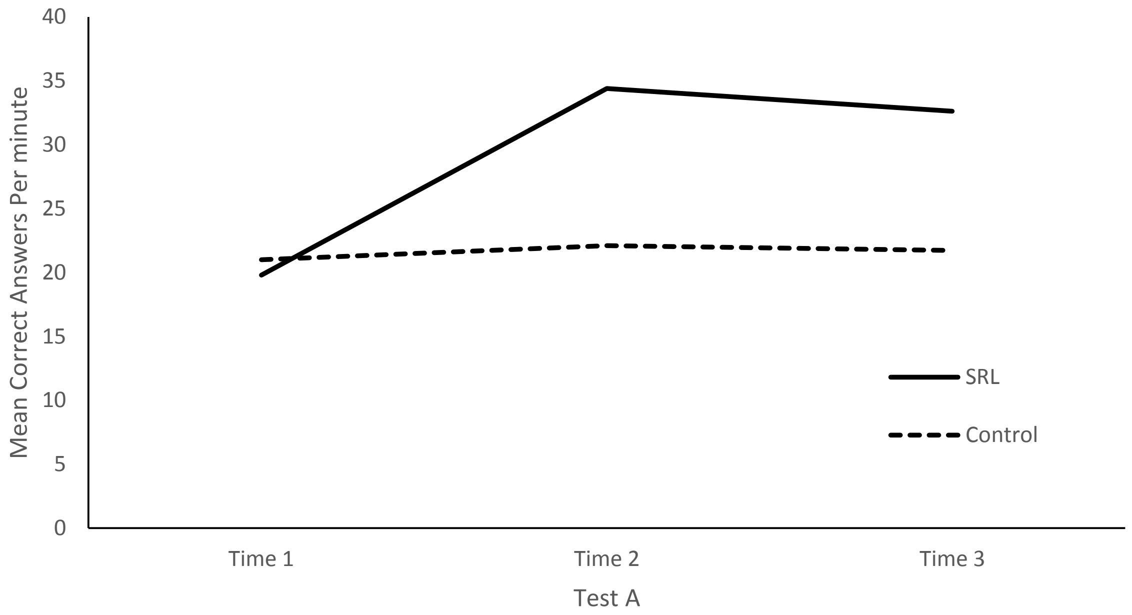

Figure 2. Mean correct answers per minute over time by condition for One-Minute Multiplication Probe Test A.

Figure 3

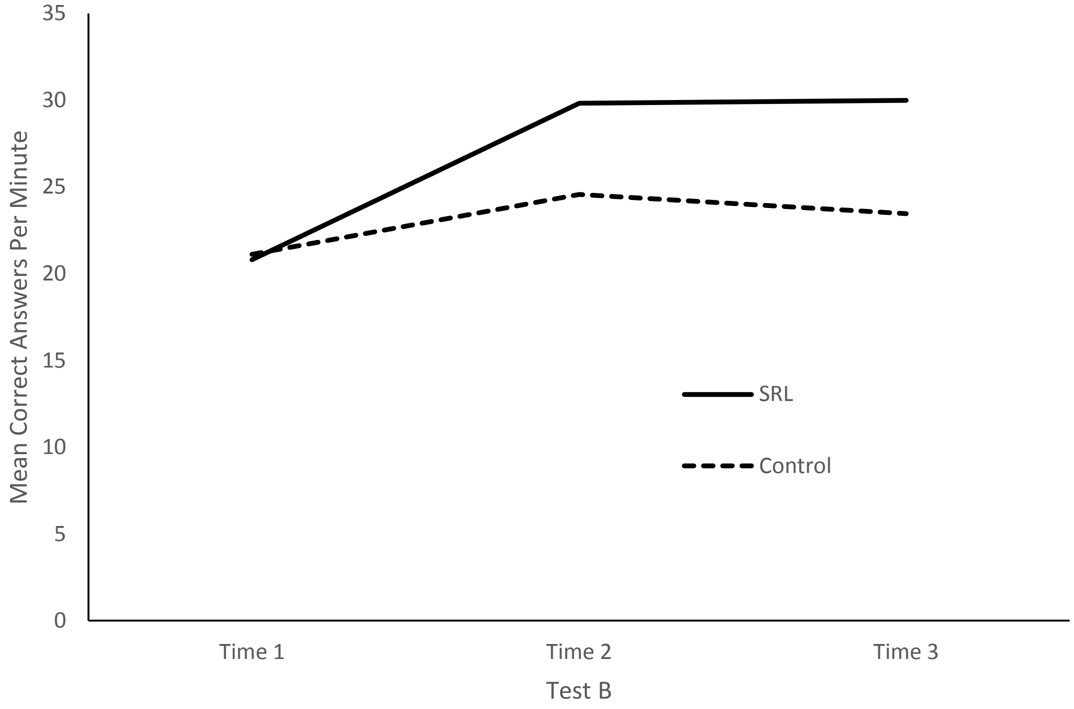

Figure 3. Mean correct answers per minute over time by condition for One-Minute Multiplication Probe Test B.

Figure 4

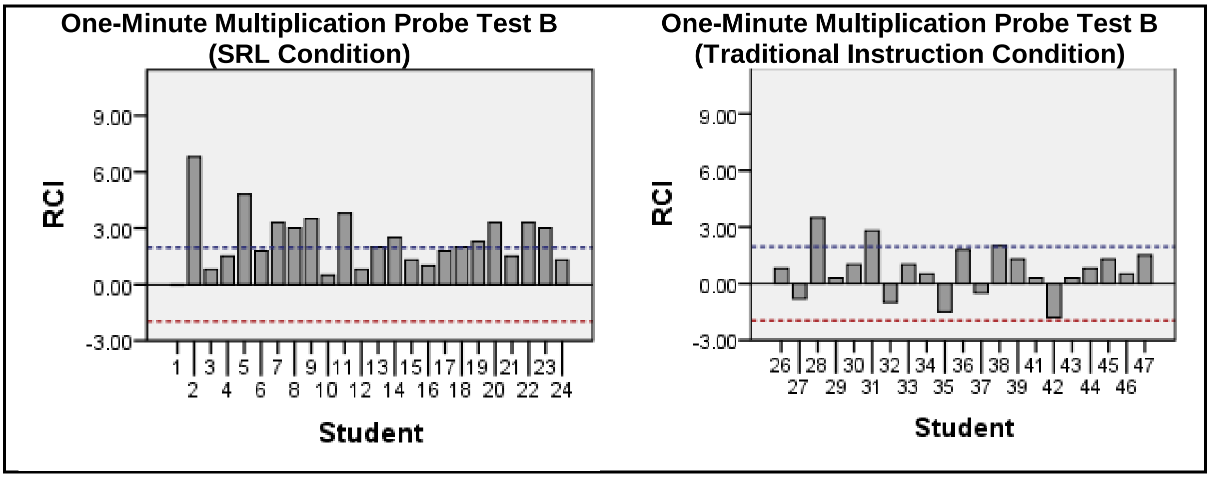

Figure 4. Reliable change scores from T1 to T3 for the SRL and control groups on Test B.

Note. The blue dotted line represents the threshold for reliable increase and the red dotted line represents the threshold for reliable decrease.

Table 1

SRL and Control Group Performance on the One-Minute Multiplication Probes at T2 and T3

Time period and test	Group[a]	*M* CAPM	*SE*	95% CI LL	95% CI UL
T2 Test A	SRL	34.40	1.22	31.94	36.86
	Control	22.10	1.25	19.59	24.62
T2 Test B	SRL	29.82	1.22	27.36	32.28
	Control	24.58	1.25	22.06	27.09
T3 Test A	SRL	32.61	1.37	29.85	35.37
	Control	21.73	1.46	18.79	24.68
T3 Test B	SRL	29.98	1.15	27.65	32.31
	Control	23.45	1.23	20.96	25.94

Note. a. T2 SRL *n* = 24; T2 control *n* = 23; T3 SRL *n* = 24; T3 control *n* = 21.

Table 2

SRL Group's Reported Use of Self-Control Strategies During Sessions Outside of School Hours

Strategy type		Did not use strategy	Used strategy but not helpful	Strategy was somewhat helpful	Strategy was very helpful	Strategy was extremely helpful	Total
Setting	*n*	20	2	99	138	1	260
selection	%	8	1	38	53	0	100
Setting	*n*	32	7	91	129	1	260
modification	%	12	3	35	50	0	100
Attention	*n*	67	4	91	96	0	258
deployment	%	26	2	35	37	0	100