COMPRESSION AND SHEAR BUCKLING PERFORMANCE OF INFINITELY LONG AND FINITE LENGTH PLATES WITH BENDING-TWISTING COUPLING.

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Presentation Contents

- Background, motivation and context.

- Design Heuristics:
    - ply angle,
    - ply percentage and
    - ply contiguity.

- Overview of databases for symmetric and non-symmetric *warp-free* laminates ($\boldsymbol{B} = \boldsymbol{0}$):
    - *Bending-Twisting* coupled laminates (balanced and symmetric?),
    - *Extension-Shearing*, *Bending-Twisting* coupled laminates (unbalanced and symmetric?).

- Lamination parameter point clouds are used to interrogate the effect of design heuristics on the design space. Each point represents a unique and practical design.

- Compression and shear buckling strength contours are mapped onto the design space, from which optimum designs can be readily identified.

- Conclusions.

Background

Compression buckling strength may be **overestimated** (unsafe) **if** the effects of ***Bending-Twisting*** **coupling** are **ignored**.

Shear buckling strength may be **underestimated** (over-designed) or overestimated **if** the effects of ***Bending-Twisting*** **coupling** are **ignored**.

In this study, the effect of *Bending-Twisting* coupling on the buckling strength of **infinitely long laminated plates with simply supported edges** is investigated….

> ….it complements an **extensive literature** on the subject, where the focus is primarily **on finite length plates**. Infinitely long plates represent useful lower-bound solutions for preliminary design.

With very few exceptions, the study of *Bending-Twisting* coupling effects has focussed entirely on symmetric designs….

> ….the relative buckling performance of non-symmetric designs are now revealed.

Laminate databases

Databases have been developed[1,2], which contain listings for UD material with standard (or non-standard) fibre angle orientations, i.e.:

0, 90 and/or ±45° (= ±θ°).

The derivations involved the restrictions that *each layer in the laminate*:

- *has identical material properties;*
- *has identical thickness and;*
- *differs only by its orientation.*

Databases contain both symmetric and non-symmetric *warp-free* laminates ($\boldsymbol{B} = \boldsymbol{0}$) for :

- ***Bending-Twisting*** coupled laminates[1],
  and
- *Extension-Shearing*, ***Bending-Twisting*** coupled laminates[2].

Databases also contain lamination parameter information for standard ply angle designs.

[1] C. B. York and S. F. M. Almeida, On extension-shearing bending-twisting coupled laminates, *Composite Structures*, 164, 2017, pp. 10-22 (doi: 10.1016/j.compstruct.2016.12.041).
[2] C. B. York, On bending-twisting coupled laminates, *Composite Structures*, 160, 2017, pp. 887-900 (doi: 10.1016/j.compstruct.2016.10.063).

Lamination Parameter Design Space for Interrogation of *Bending-Twisting* coupled designs.

Lamination parameters ($\xi_\Delta^A, \xi_R^A, \xi_{\Delta c}^A$) = ($\xi_1$, ξ_2, ξ_3) are related to the associated **extensional** stiffnesses through the following expression:

$$[\mathbf{A}] = H\begin{bmatrix} U_E + \xi_\Delta^A U_\Delta + \xi_R^A U_R & U_E - 2U_G - \xi_R^A U_R & \xi_{\Delta c}^A U_\Delta / 2 + \xi_{Rc}^A U_R \\ U_E - 2U_G - \xi_R^A U_R & U_E - \xi_\Delta^A U_\Delta + \xi_R^A U_R & \xi_{\Delta c}^A U_\Delta / 2 - \xi_{Rc}^A U_R \\ \xi_{\Delta c}^A U_\Delta / 2 + \xi_{Rc}^A U_R & \xi_{\Delta c}^A U_\Delta / 2 - \xi_{Rc}^A U_R & U_G - \xi_R^A U_R \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{xy} & A_{yy} & A_{ys} \\ A_{xy} & A_{ys} & A_{ss} \end{bmatrix} \quad (1)$$

Note that $\xi_{Rc}^A = 0$ for standard ply orientations, hence the [**A**] matrix is described by a **three dimensional lamination parameter coordinate** for all designs considered here.

The laminate invariant properties are defined in terms of the reduced stiffnesses, Q_{ij}:

$$\begin{aligned} U_E &= (3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})/8 \\ U_G &= (Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})/8 \\ U_\Delta &= (Q_{11} - Q_{22})/2 \\ U_R &= (Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})/8 \end{aligned} \qquad (U_E \equiv U_1,\ U_G \equiv U_5,\ U_\Delta \equiv U_2,\ U_R \equiv U_3) \quad (2)$$

Lamination Parameter Design Space for Interrogation of *Bending-Twisting* coupled designs.

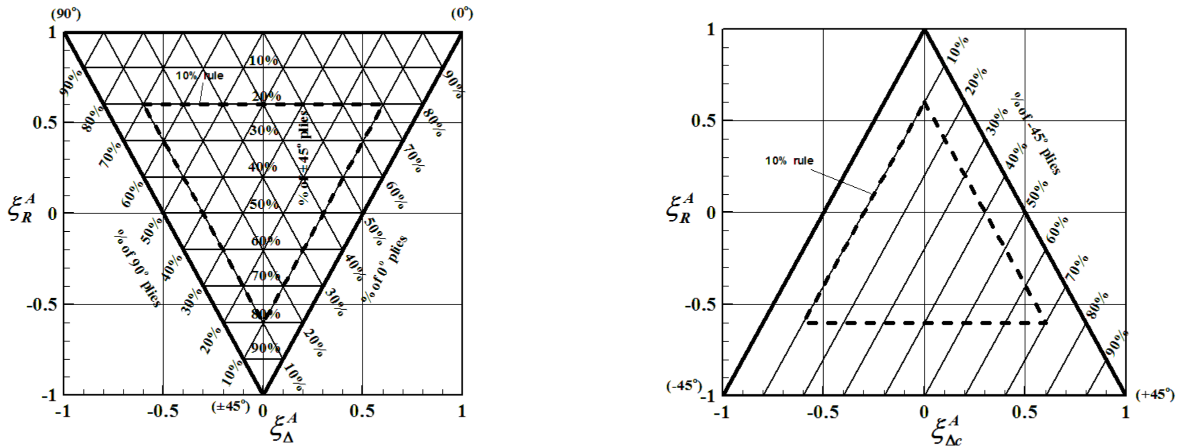
Lamination parameters ($\xi_\Delta^D, \xi_R^D, \xi_{\Delta c}^D$) = ($\xi_9$, ξ_{10}, ξ_{11}) are related to the associated **bending** stiffnesses through the following expression:

$$[\mathbf{D}] = \frac{H^3}{12}\begin{bmatrix} U_E + \xi_\Delta^D U_\Delta + \xi_R^D U_R & U_E - 2U_G - \xi_R^D U_R & \xi_{\Delta c}^D U_\Delta / 2 + \xi_{Rc}^D U_R \\ U_E - 2U_G - \xi_R^D U_R & U_E - \xi_\Delta^D U_\Delta + \xi_R^D U_R & \xi_{\Delta c}^D U_\Delta / 2 - \xi_{Rc}^D U_R \\ \xi_{\Delta c}^D U_\Delta / 2 + \xi_{Rc}^D U_R & \xi_{\Delta c}^D U_\Delta / 2 - \xi_{Rc}^D U_R & U_G - \xi_R^D U_R \end{bmatrix} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xs} \\ D_{xy} & D_{yy} & D_{ys} \\ D_{xs} & D_{ys} & D_{ss} \end{bmatrix} \quad (3)$$

Note that $\xi_{Rc}^D = 0$ for standard ply orientations, hence the [**D**] matrix is described by a **three dimensional lamination parameter coordinate** for all designs considered here.

Interpretation of Lamination Parameter Design Spaces: Ply percentages.

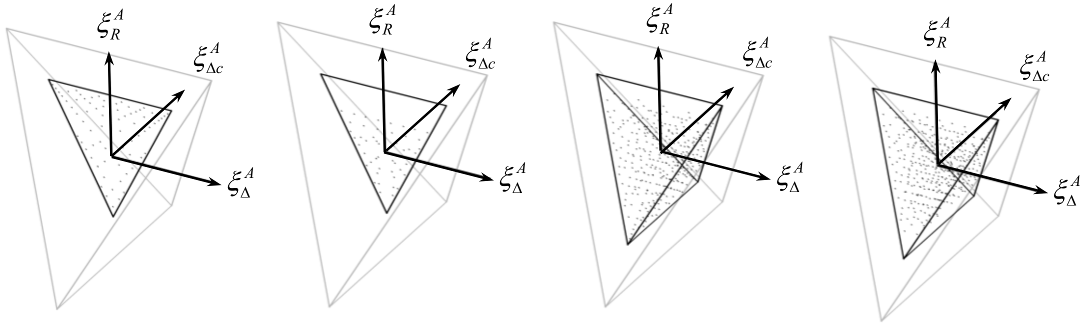
Lamination parameter coordinates can be illustrated as orthographic projections of ***Extensional Stiffness*** and related to **ply percentages**.



(a) (b)

Figure 1: Lamination parameter design space with ply percentage mapping for: (a) orthotropic stiffness (ξ_Δ^A, ξ_R^A) and; (b) anisotropic stiffness ($\xi_{\Delta c}^A$) relating to differing angle-ply percentages. The 10% design rule constraint is also illustrated.

Point clouds of **lamination parameter coordinates** from the databases can be illustrated as **isometric projections** for **extensional stiffness**:



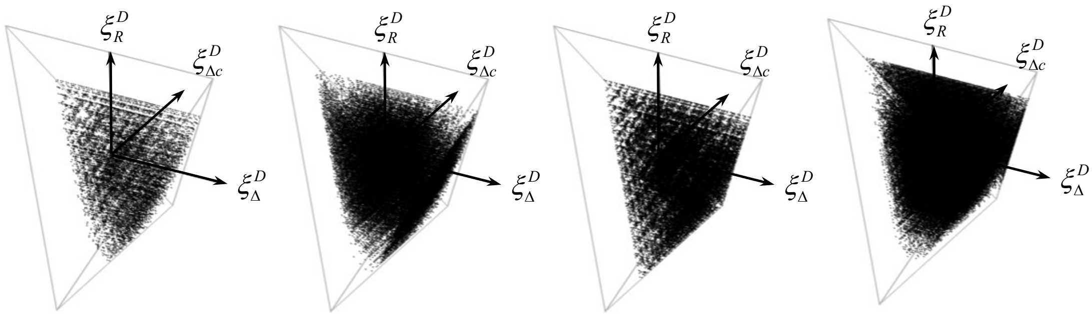
(a) (b) (c) (d)

Figure 2: Isometric view of lamination parameter design spaces for ***extensional stiffness***, **with 10% rule applied**, corresponding to:

(a) Symmetric and
(b) Non-symmetric *Bending-Twisting* coupled (balanced) laminates with up to 18 plies and;

(c) Symmetric and
(d) Non-symmetric *Extension-Shearing Bending-Twisting* coupled (unbalanced) laminates with up to 18 plies.

Point clouds of **lamination parameter coordinates** from the databases can be illustrated as **isometric projections** for **bending stiffness**:



(a) (b) (c) (d)

Figure 3 - Isometric view of lamination parameter design spaces for ***bending stiffness***, **with 10% rule applied**, corresponding to:

(a) Symmetric and
(b) Non-symmetric ***Bending-Twisting*** **coupled** (balanced) laminates with up to 18 plies and;

(c) Symmetric and
(d) Non-symmetric ***Extension-Shearing Bending-Twisting*** **coupled** (unbalanced) laminates with up to 18 plies.

For higher fidelity interpretation, the point clouds of **lamination parameter coordinates** can be illustrated as a three view orthographic projections of ***Extensional Stiffness***:

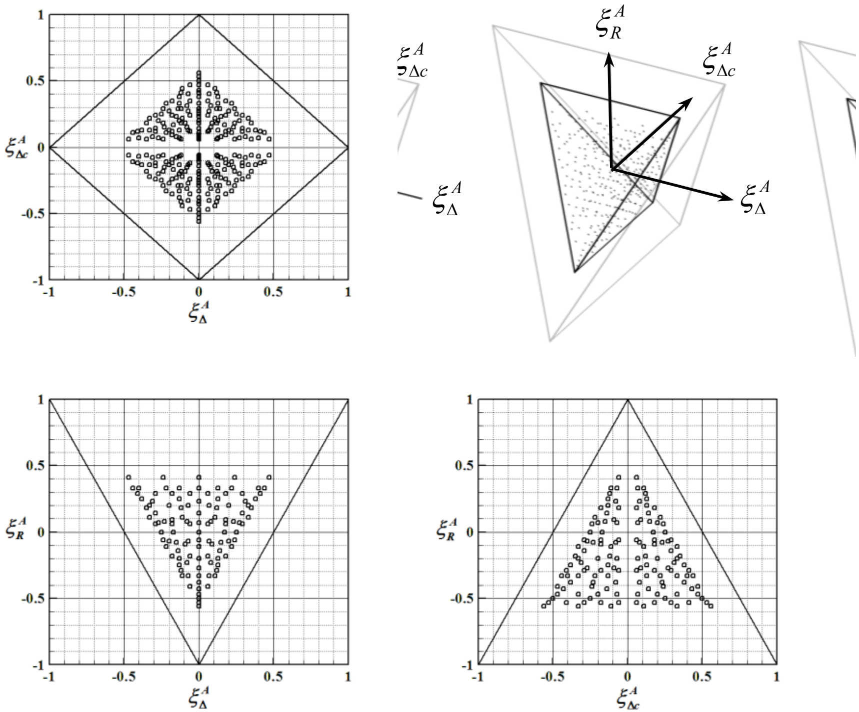


Figure 3: Lamination parameter design spaces for **symmetric** ***Extension-Shearing*** ***Bending-Twisting*** **coupled laminates** with $7 \le n \le 18$, with **10% rule and ply contiguity constraints (≤3) applied.**

For higher fidelity interpretation, the point clouds of **lamination parameter coordinates** can be illustrated as a three view orthographic projections of ***Extensional Stiffness***:

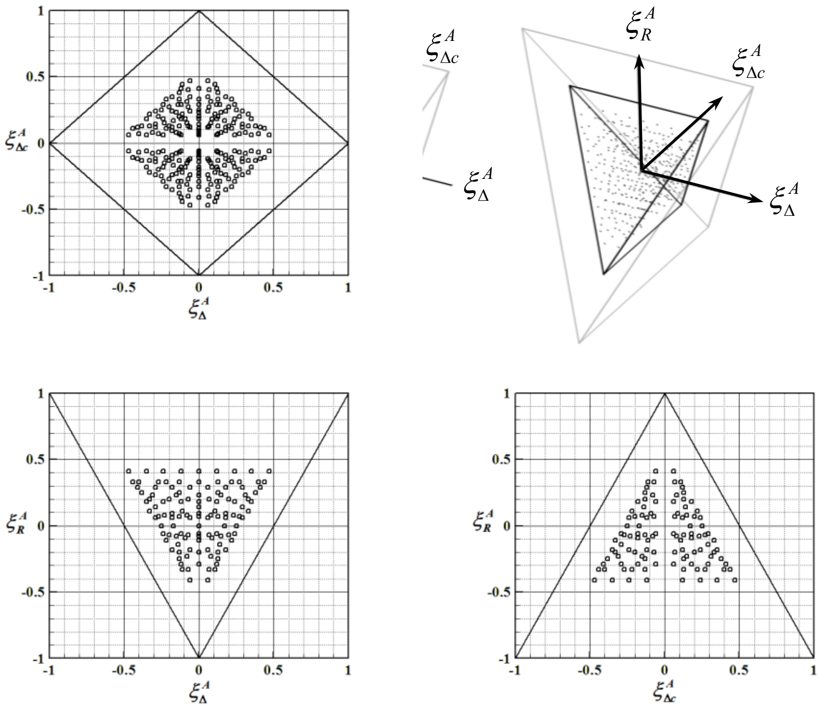


Figure 4: Lamination parameter design spaces for **<u>Non-symmetric</u>** ***Extension-Shearing*** ***Bending-Twisting*** **coupled laminates** with $7 \leq n \leq 18$, with **10% rule and ply contiguity constraints (≤3) applied.**

For higher fidelity interpretation, the point clouds of **lamination parameter coordinates** can be illustrated as a three view orthographic projections of ***Bending Stiffness***:

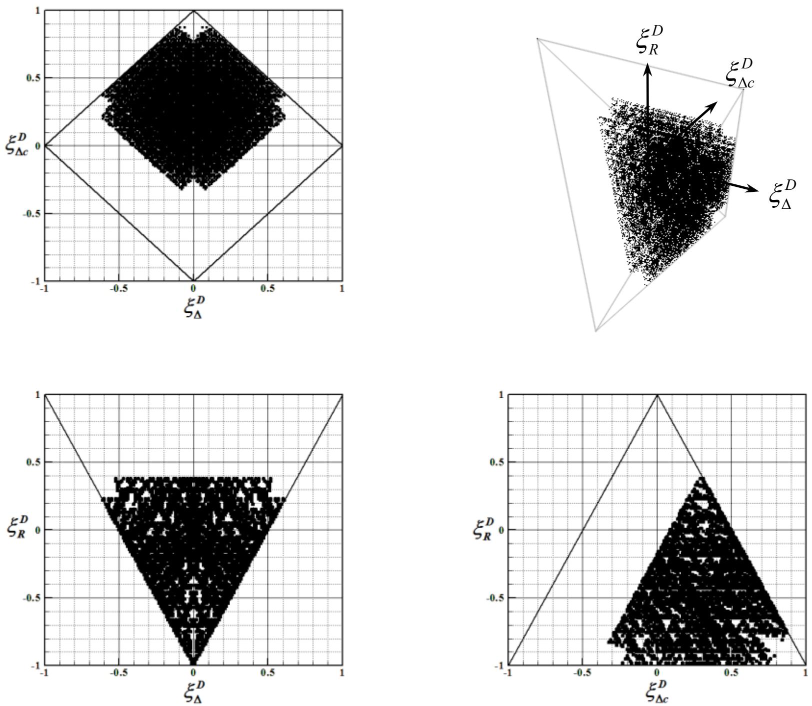


Figure 5: Lamination parameter design spaces for **<u>symmetric</u>** ***Extension-Shearing*** ***Bending-Twisting*** **coupled laminates** with $7 \le n \le 18$, with **10% rule and ply contiguity constraints (≤3) applied.**

For higher fidelity interpretation, the point clouds of **lamination parameter coordinates** can be illustrated as a three view orthographic projections of ***Bending Stiffness***:

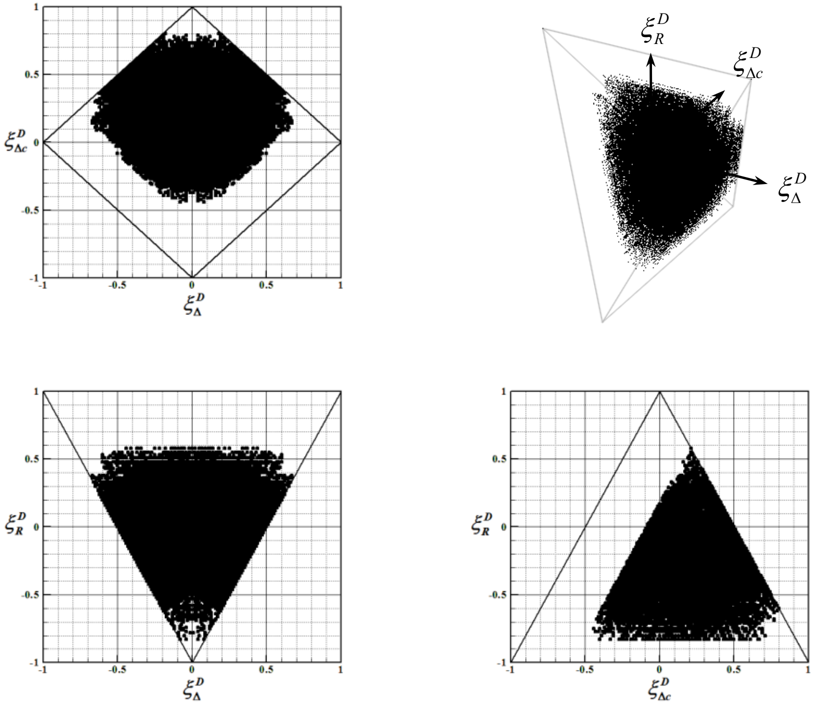


Figure 6: Lamination parameter design spaces for **<u>Non-symmetric</u>** ***Extension-Shearing*** ***Bending-Twisting*** **coupled laminates** with $7 \leq n \leq 18$, with **10% rule and ply contiguity constraints (≤3) applied.**

Table 1 – **Effect of ply contiguity constraints on the 10% design rule** design space for: (a) Symmetric and; (b) Non-symmetric ***Extension-Shearing Bending-Twisting*** **coupled laminates**.

	(a) **Symmetric** laminates				(b) **Non-symmetric** laminates			
n	1	≤2	**≤3**	**10%**	1	≤2	**≤3**	**10%**
7	2	-	**-**	**2**	-	-	**-**	**-**
8	-	-	**-**	**-**	-	-	**-**	**-**
9	26	40	**42**	**42**	4	8	**-**	**8**
10	-	34	**-**	**36**	-	-	**-**	**-**
11	94	150	**190**	**192**	8	38	**48**	**48**
12	-	214	**224**	**260**	8	32	**36**	**36**
13	382	934	**1,258**	**1,300**	146	916	**1,240**	**1,292**
14	-	1,114	**1,264**	**1,560**	36	412	**560**	**592**
15	1,380	4,796	**6,940**	**7,320**	924	14,212	**19,970**	**21,152**
16	-	5,104	**6,102**	**7,882**	266	5,554	**8,498**	**9,288**
17	4,720	21,840	**33,478**	**36,176**	6,582	165,022	**251,098**	**270,848**
18	-	22,016	**27,772**	**37,212**	1,896	62,632	**102,178**	**114,638**

Table 2 – **Effect of ply contiguity constraints on the 10% design rule** design space for: (a) Symmetric and; (b) Non-symmetric ***Bending-Twisting* coupled laminates**.

	(a) **Symmetric** laminates				(b) **Non-symmetric** laminates			
N	1	≤2	**≤3**	**10%**	1	≤2	**≤3**	**10%**
7	4	-	**-**	**4**	-	-	**-**	**-**
8	-	6	**-**	**6**	-	-	**-**	**-**
9	10	14	**18**	**18**	-	-	**-**	**-**
10	-	20	**-**	**24**	-	-	**-**	**-**
11	14	30	**44**	**48**	8	14	**16**	**16**
12	-	96	**104**	**128**	-	-	**-**	**-**
13	68	164	**242**	**260**	38	216	**272**	**276**
14	-	422	**-**	**534**	36	204	**220**	**224**
15	240	676	**980**	**1,080**	232	2,746	**3,628**	**3,734**
16	-	1,572	**1,790**	**2,302**	158	2,064	**2,734**	**2,868**
17	690	2,736	**4,184**	**4,612**	1,480	27,716	**39,258**	**41,142**
18	-	6,000	**7,142**	**9,324**	826	21,180	**31,568**	**34,154**

Closed form buckling equations for infinitely long plates in shear and compression.

An **exact closed form solution**, necessary to handle the vast number of database designs, can be used to assess the **compression buckling strength**:

$$N_{x,\infty} = \pi^2 \left[D_{xx} \left[\frac{1}{\lambda} \right]^2 + 2\left(D_{xy} + 2D_{ss}\right) \frac{1}{b^2} + D_{yy} \left[\frac{1}{b^4} \right] \lambda^2 \right] \tag{4}$$

…but only for uncoupled designs!

An **approximate closed form solution** can also be represented by a 2 dimensional, 4th order polynomial and can be solved against the exact closed form buckling solution of Eqn. (1) from equally spaced points across the lamination parameter design space (ξ_Δ^D , ξ_R^D):

$$\begin{aligned} k_\infty = {} & c_1 + c_2\xi_\Delta^D + c_3\xi_R^D + c_4\left(\xi_\Delta^D\right)^2 + c_5\left(\xi_R^D\right)^2 + c_6\xi_\Delta^D\xi_R^D + c_7\left(\xi_\Delta^D\right)^3 + c_8\left(\xi_R^D\right)^3 + c_9\xi_\Delta^D\left(\xi_R^D\right)^2 \\ & + c_{10}\left(\xi_\Delta^D\right)^2\xi_R^D + c_{11}\left(\xi_\Delta^D\right)^4 + c_{12}\left(\xi_R^D\right)^4 + c_{13}\xi_\Delta^D\left(\xi_R^D\right)^3 + c_{14}\left(\xi_\Delta^D\right)^2\left(\xi_R^D\right)^2 + c_{15}\left(\xi_\Delta^D\right)^3\xi_R^D \end{aligned} \tag{5}$$

For compression buckling, $k_\infty = k_{x,\infty}$ and is defined by:

$$k_{x,\infty} = \frac{N_{x,\infty} b^2}{\pi^2 D_{Iso}} \tag{6}$$

For IM7/8552 carbon-fiber/epoxy material….

Closed form buckling equations for infinitely long plates in shear and compression.

$$k_{x,\infty} = 4.000 - 1.049\xi_R^D - 1.217\left(\xi_\Delta^D\right)^2 + 0.340\xi_R^D\left(\xi_\Delta^D\right)^2 - 0.360\left(\xi_\Delta^D\right)^4 - 0.034\left(\xi_R^D\right)^2\left(\xi_\Delta^D\right)^2 \quad (7)$$

For **uncoupled laminates in shear (or *Bending-Twisting* coupled laminates in compression or shear)**, an **exact infinite strip analysis**[3] is used to generate buckling factors from which the polynomial coefficients of Eqn. (2) can be solved, where $k_\infty = k_{s,\infty}$ and is defined by:

$$k_{s,\infty} = \frac{N_{s,\infty} b^2}{\pi^2 D_{Iso}} \quad (8)$$

giving:

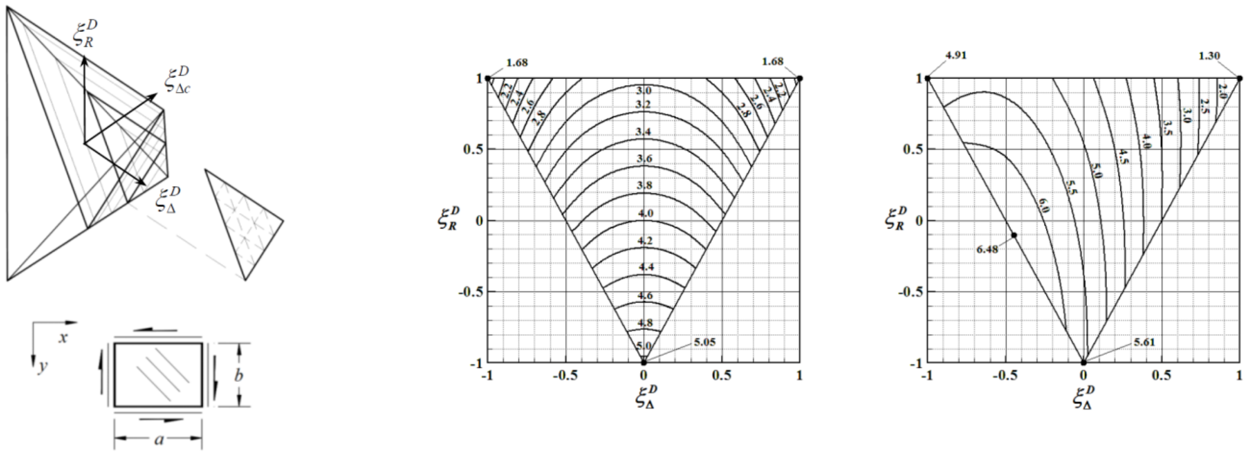
$$k_{s,\infty} = 5.336 - 2.914\xi_\Delta^D - 0.518\xi_R^D - 1.303\left(\xi_\Delta^D\right)^2 - 0.213\left(\xi_R^D\right)^2 + 1.048\xi_\Delta^D\xi_R^D - 0.236\left(\xi_\Delta^D\right)^3 + 0.031\left(\xi_R^D\right)^3 - 0.197\xi_\Delta^D\left(\xi_R^D\right)^2 + 0.405\left(\xi_\Delta^D\right)^2\xi_R^D - 0.443\left(\xi_\Delta^D\right)^4 - 0.001\left(\xi_R^D\right)^4 + 0.022\xi_\Delta^D\left(\xi_R^D\right)^3 - 0.185\left(\xi_\Delta^D\right)^2\left(\xi_R^D\right)^2 + 0.472\left(\xi_\Delta^D\right)^3\xi_v^D \quad (9)$$

These equations facilitate the mapping of buckling factor contours…..

[3] F. W. Williams, D. Kennedy, R. Butler and M. S. Anderson, VICONOPT: Program for exact vibration and buckling analysis or design of prismatic plate assemblies. *AIAA J.* 29, 1991, pp. 1927-1928.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on <u>cross-sections</u> through the design space

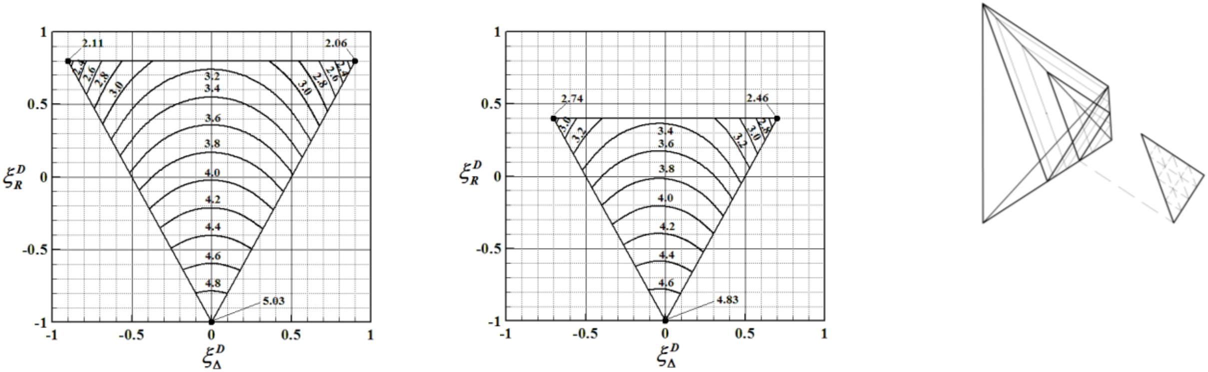


(a) (b) (c)

Figure 7: (a) Three-dimensional representation of the feasible design space indicating **the positions through which two dimensional cross-sections have been taken**. Positive shear load and positive fibre orientation are defined in the thumbnail sketch. Sections representing fully uncoupled laminates in bending, correspond to: **(b) compression buckling contours**, $k_{x,\infty}$ ($= N_x b^2/\pi^2 D_{Iso}$) and; **(c) positive/negative shear buckling contours**, $k_{s,\infty}$ ($= N_s b^2/\pi^2 D_{Iso}$).

Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on <u>cross-sections</u>

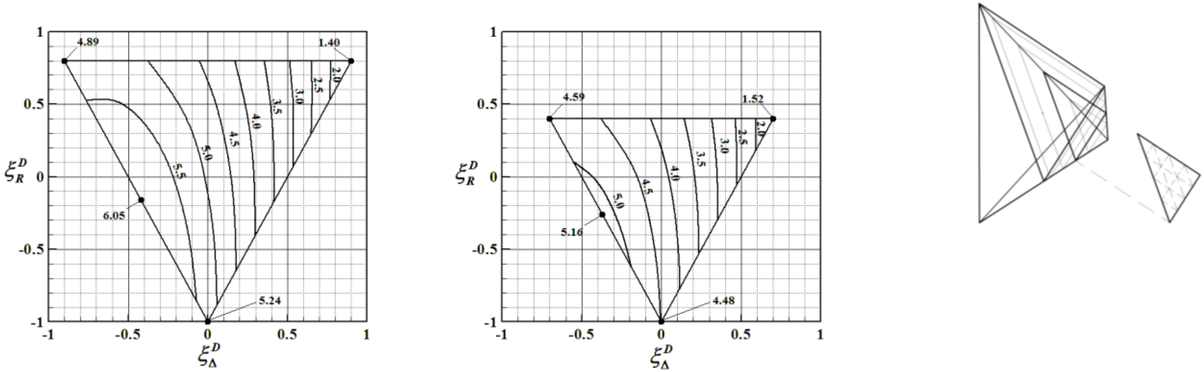


(a) (b)

Figure 8: **Compression buckling** factor contours, $k_{x,\infty}$ (= $N_s b^2/\pi^2 D_{Iso}$), for: (a) $\xi_{\Delta c}^D$ = 0.1 and; (b) $\xi_{\Delta c}^D$ = 0.3, representing *Bending-Twisting* coupled laminates.

Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on <u>cross-sections</u>

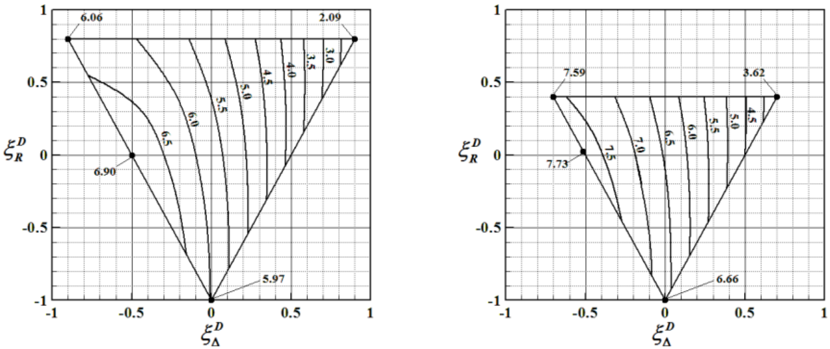


(a) (b)

Figure 9: **Positive shear buckling** factor contours, $k_{s,\infty}$ (= $N_s b^2/\pi^2 D_{Iso}$), for: (a) $\xi^D_{\Delta c}$ = 0.1 and; (b) $\xi^D_{\Delta c}$ = 0.3, representing *Bending-Twisting* coupled laminates.

Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on <u>cross-sections</u>



(a) (b)

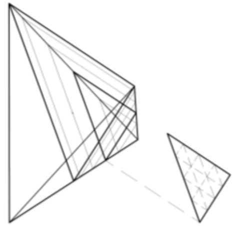
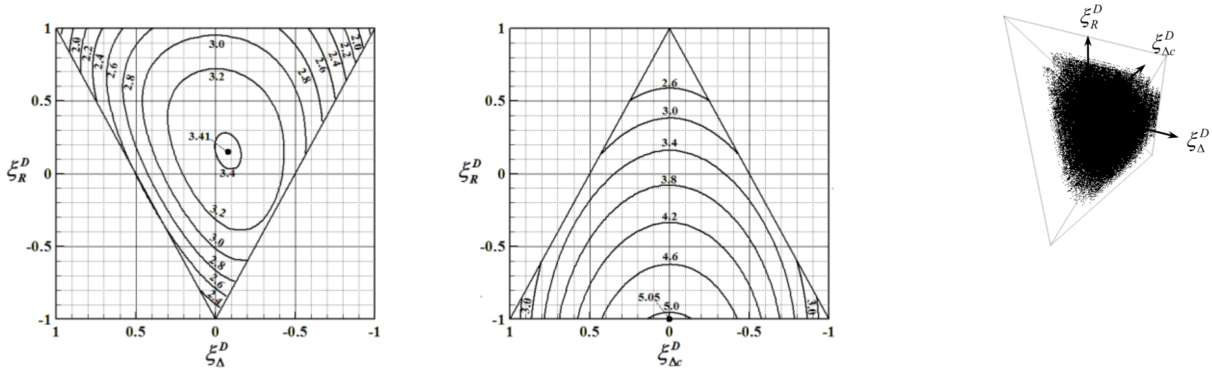


Figure 10: **Negative shear buckling** factor contours, $k_{s,\infty}$ (= $N_s b^2/\pi^2 D_{Iso}$), for: (a) $\xi_{\Delta c}^D$ = 0.1 and; (b) $\xi_{\Delta c}^D$ = 0.3, representing *Bending-Twisting* coupled laminates.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

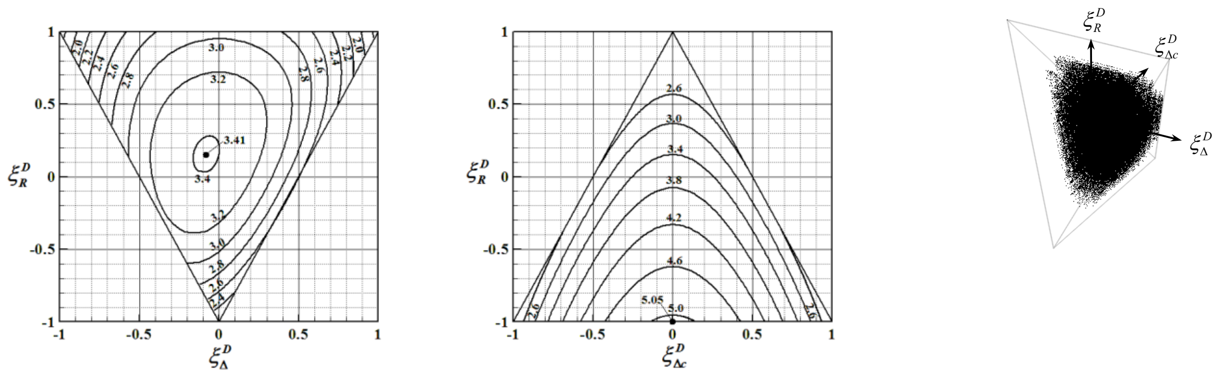


(a) (b)

Figure 11: Lamination parameter design space surface contours for **Compression buckling** factor, $k_{x,\infty}$ ($= N_x b^2/\pi^2 D_{Iso}$), corresponding to 3rd angle orthographic projections of: **(a) Rear (sloping) face** with; **(b) Left (sloping) face**.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on <u>surfaces</u> of the design space

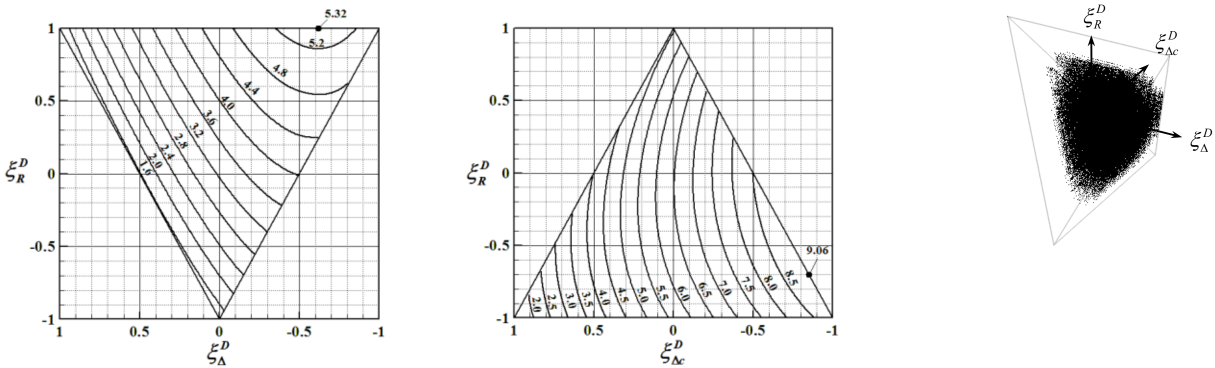


(c) (d)

Figure 11: Lamination parameter design space surface contours for **Compression buckling** factor, $k_{x,\infty}$ $(= N_x b^2/\pi^2 D_{Iso})$, corresponding to 3rd angle orthographic projections of: (c) **Front (sloping) face** and; **(d) Right (sloping) face**.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on <u>surfaces</u> of the design space

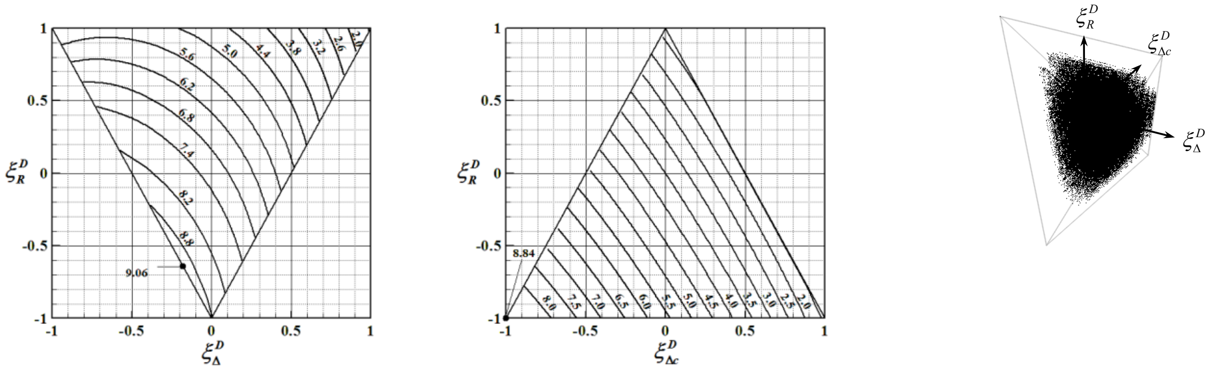


(a) (b)

Figure 12: Lamination parameter design space surface contours for **Positive Shear buckling** factor, $k_{s,\infty}$ $(= N_s b^2/\pi^2 D_{Iso})$, corresponding to 3rd angle orthographic projections of: **(a) Rear (sloping) face** with; **(b) Left (sloping) face**.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on <u>surfaces</u> of the design space



(c) (d)

Figure 12: Lamination parameter design space surface contours for **Positive Shear buckling** factor, $k_{s,\infty}$ (= $N_s b^2/\pi^2 D_{Iso}$), corresponding to 3rd angle orthographic projections of: (c) **Front (sloping) face** and; **(d) Right (sloping) face**.

The **global optimum** shear buckling factor corresponds to $\boldsymbol{k_{s,\infty}}$ **= 9.06** @ ($\xi_\Delta^D, \xi_R^D, \xi_{\Delta c}^D$) = (-0.18, -0.64, -0.82). A practical and near optimum design, [+45$_3$/90/+45/90/+45/−45/0]$_S$, leads to $\boldsymbol{k_{s,\infty}}$ **= 8.98**.

Closed form buckling equations for finite length plates in shear and compression.

An **exact closed form solution**, necessary to handle the vast number of database designs, can be used to assess the **compression buckling strength**:

$$N_x = \pi^2 \left[D_{xx} \left[\frac{m}{a} \right]^2 + 2\left(D_{xy} + 2D_{ss}\right)\frac{1}{b^2} + D_{yy} \left[\frac{1}{b^4} \right] \left[\frac{a}{m} \right]^2 \right] \tag{10}$$

…but only for uncoupled designs!

For orthotropic laminates, the following polynomial can be solved against the exact closed form buckling solution from equally spaced points across the lamination parameter design space:

$$\begin{aligned} k_x = {} & c_1 + c_2\xi_\Delta^D + c_3\xi_R^D + c_4\left(\xi_\Delta^D\right)^2 + c_5\left(\xi_R^D\right)^2 + c_6\xi_\Delta^D\xi_R^D + c_7\left(\xi_\Delta^D\right)^3 + c_8\left(\xi_R^D\right)^3 + c_9\xi_\Delta^D\left(\xi_R^D\right)^2 \\ & + c_{10}\left(\xi_\Delta^D\right)^2\xi_R^D + c_{11}\left(\xi_\Delta^D\right)^4 + c_{12}\left(\xi_R^D\right)^4 + c_{13}\xi_\Delta^D\left(\xi_R^D\right)^3 + c_{14}\left(\xi_\Delta^D\right)^2\left(\xi_R^D\right)^2 + c_{15}\left(\xi_\Delta^D\right)^3\xi_R^D \end{aligned} \tag{11}$$

In this case, k_x is defined by:

$$k_x = \frac{N_x b^2}{\pi^2 D_{Iso}} \tag{12}$$

By contrast to the infinite plate results investigated previously, mode changes now complicate the contour maps of finite length plates - *no longer continuous across the laminate design space*.

Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on cross-sections

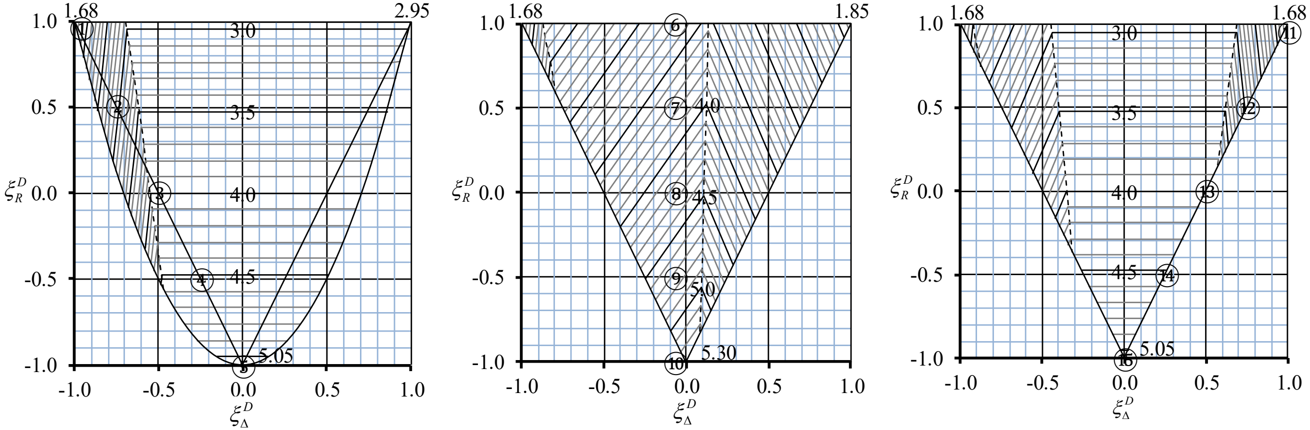


Figure 13: Compression buckling contours of uncoupled laminates (i.e. $\xi_{\Delta c}^D = 0.0$), k_x ($= N_x b^2/\pi^2 D_{Iso}$), for: (a) a/b = 1.0; (b) a/b = 1.5 and; (c) = a/b = 2.0.

Parabolic bounds[4], shown in (a) are applicable to non-standard ply angle combinations ±θ°, 0° and 90°.

[4] H. Fukunaga, H. Sekine, M. Sato and A. Lino, Buckling design of symmetrically laminated plates using lamination parameters, Computers and Structures, 57(4), 1995, pp. 643-649.

Interpretation of through Garland curves.

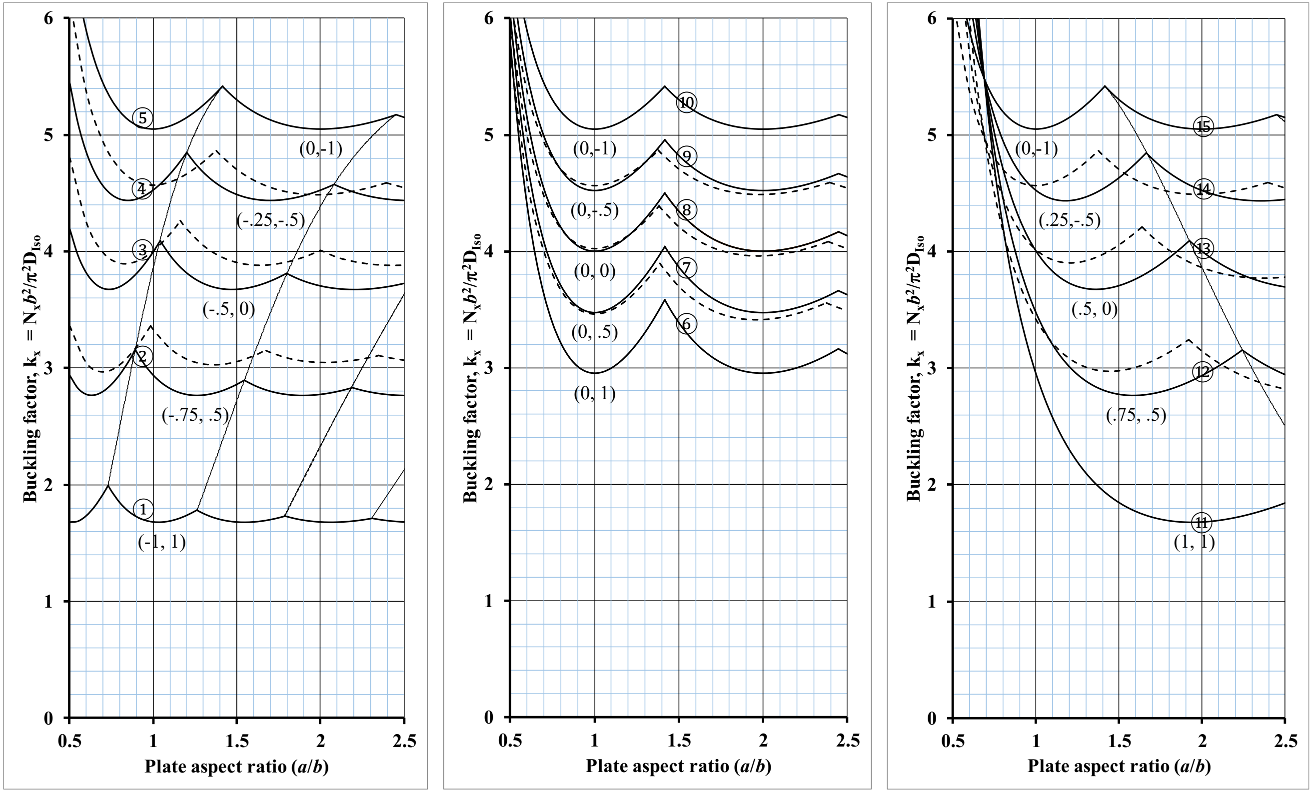
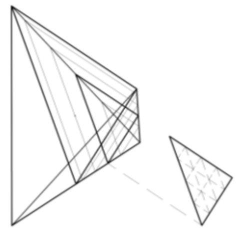


Figure 14: Garland curves for $\xi_{\Delta c}^{D}$ = 0 (solid lines) and 0.5 (broken lines).

Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on cross-sections



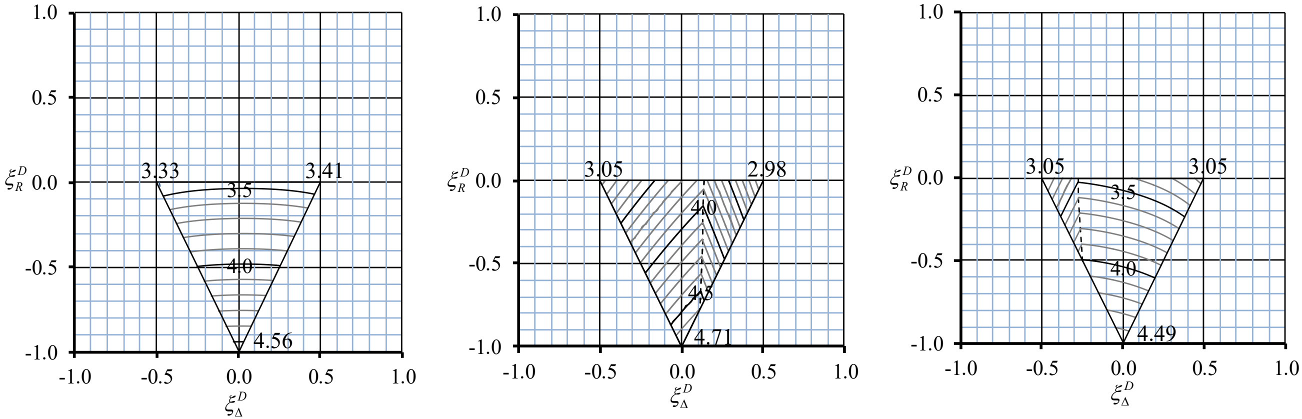


Figure 15: Compression buckling contours of *Bending-Twisting* coupled laminates (i.e. $\xi_{\Delta c}^D = 0.5$), k_x ($= N_x b^2/\pi^2 D_{Iso}$), for: (a) $a/b = 1.0$; (b) $a/$b = 1.5 and (c) $a/b = 2.0$.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

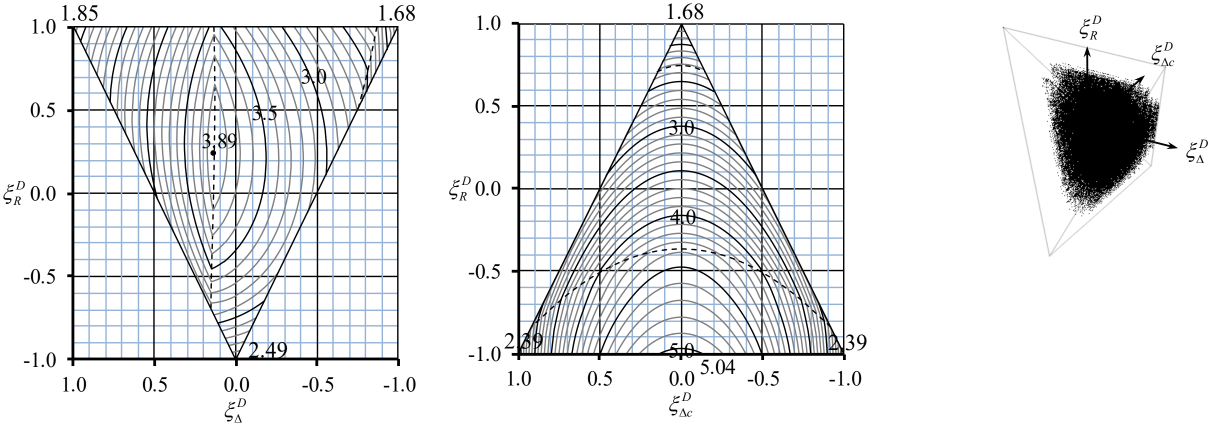


Figure 16: Lamination parameter design space surface contours for **Compression buckling** factor, k_x, $(= N_x b^2/\pi^2 D_{Iso})$, with aspect ratio $\boldsymbol{a/b} = \mathbf{1.5}$, corresponding to 3rd angle orthographic projections of: **(a) Rear (sloping) face** with; **(b) Left (sloping) face**.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on <u>surfaces</u> of the design space

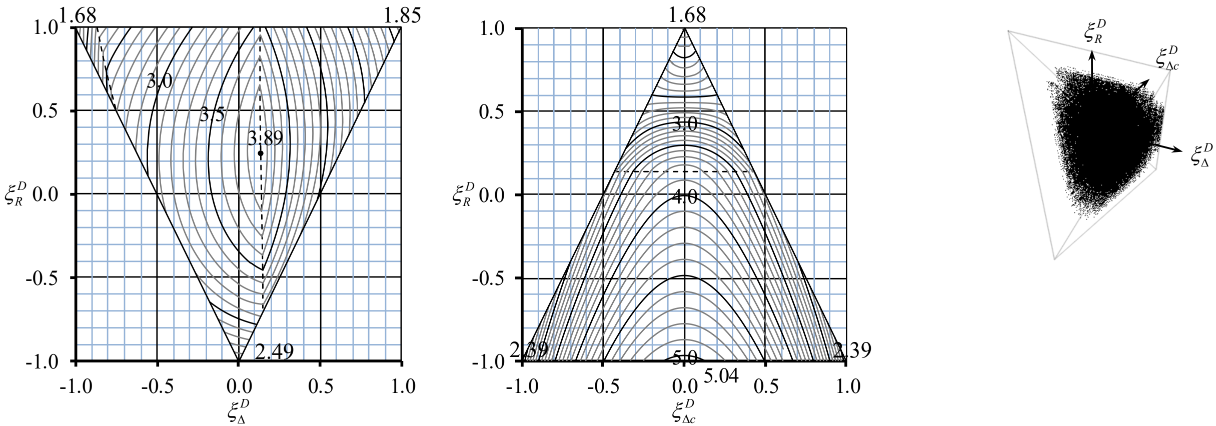


Figure 16: Lamination parameter design space surface contours for **Compression buckling** factor, k_x, $(= N_x b^2/\pi^2 D_{Iso})$, with aspect ratio ***a*/*b* = 1.5**, corresponding to 3rd angle orthographic projections of: (c) **Front (sloping) face** and; **(d) Right (sloping) face**.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on <u>surfaces</u> of the design space

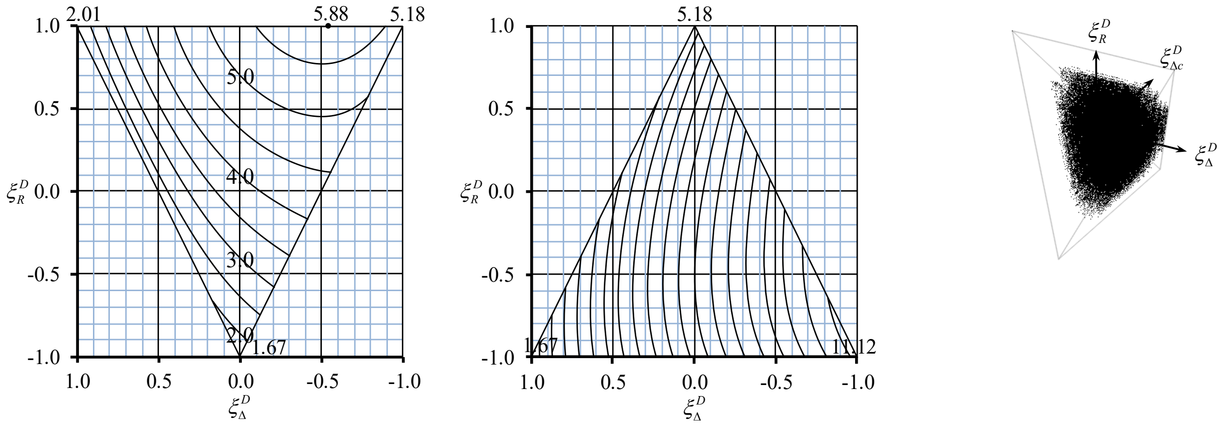


Figure 17: Lamination parameter design space surface contours for **Positive Shear buckling** factor, k_s, (= $N_s b^2/\pi^2 D_{Iso}$), with aspect ratio $\boldsymbol{a/b}$ **= 2.0**, corresponding to 3rd angle orthographic projections of: **(a) Rear (sloping) face** with; **(b) Left (sloping) face**.

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on <u>surfaces</u> of the design space

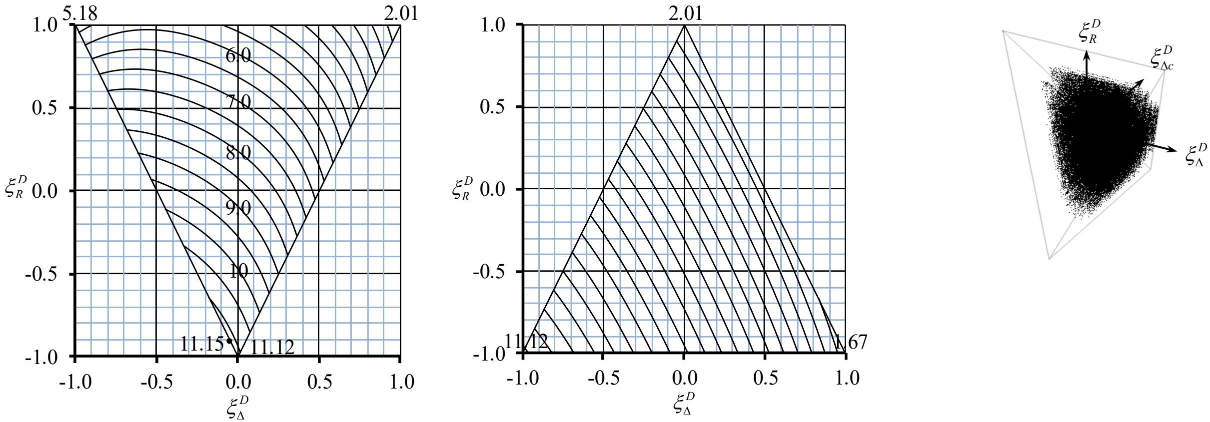


Figure 17: Lamination parameter design space surface contours for **Positive Shear buckling** factor, k_s, ($= N_s b^2/\pi^2 D_{Iso}$), with aspect ratio ***a*/*b* = 2.0**, corresponding to 3rd angle orthographic projections of: (c) **Front (sloping) face** and; **(d) Right (sloping) face**.

For a/b = 1.0 and 1.5, the global optimum is found at the bottom of feasible region ($\xi_\Delta^D, \xi_R^D, \xi_{\Delta c}^D$) = (0.0, -1.0, 1.0) as would be expect for compression buckling, but for higher aspect ratios, ***a*/*b* = 2.0**, $\boldsymbol{k_s}$ **= 11.15** @ ($\xi_\Delta^D, \xi_R^D, \xi_{\Delta c}^D$) = (-0.05, -0.9, 0.95)

Concluding Remarks

- **The reduced design space, resulting from the application of the 10% rule**, has been shown to virtually **match** the application of the common design constraint of **limiting the number of contiguous plies**, i.e. adjacent plies with the same orientation, **to a maximum of 3**.

  No significant impact has been **observed on the size of** the lamination **design space for bending stiffness** as a result of the combined effect of the 10% rule and limiting the number of contiguous plies to a maximum of 3.

- **New insights** have been given **for compression and shear buckling strength for infinitely long plates**, **through** the superposition of **contour maps** onto the lamination parameter design space **for** composite **laminates with *Bending-Twisting* coupling**.

- The **contour maps demonstrate the added complexity associated** with laminate selection from a design space in which compression buckling strength is a non-continuous function. This is due to **mode changes** that are dependent both on bending stiffness properties (or lamination parameter coordinate) and **plate aspect ratio**.

- The **contour maps demonstrate the degrading effect on the buckling strength** resulting from *Bending-Twisting* coupling by simple inspection. Inspection also reveals the **non-intuitive location for optimum shear buckling**.

Acknowledgements

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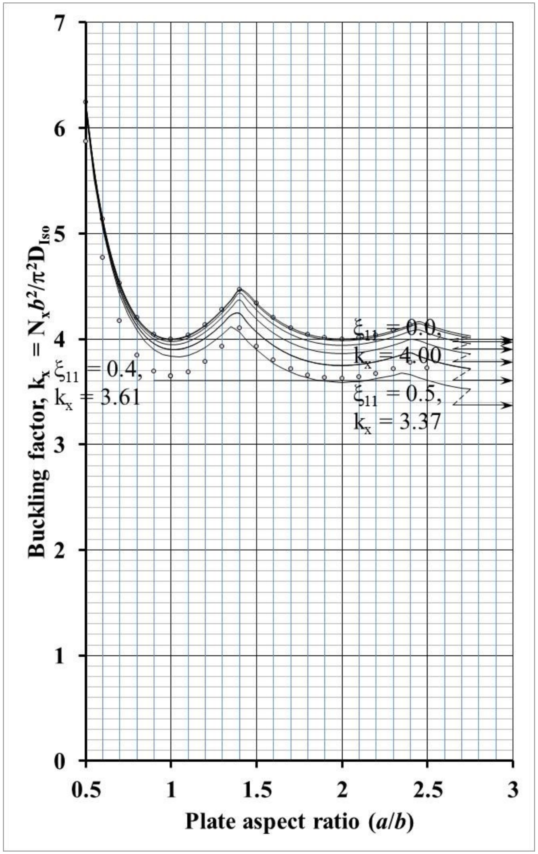
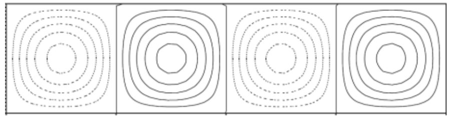
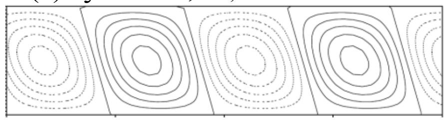
Additional interpretations through Garland curves for continuous vs finite length plates.





(a) $\xi_{11} = 0.0$, $k_{x,\infty} = 4.00$ and $\lambda = b$



(b) $\xi_{11} = 0.5$, $k_{x,\infty} = 3.37$ and $\lambda = (286/300)b$

Compression buckling mode shape comparisons for the infinitely long plate with simply supported edges, corresponding to Pseudo Quasi-Homogeneous Quasi-Isotropic laminates with: (a) $\xi_{11} = 0.0$ and (b) $\xi_{11} = 0.5$. Plate width b intervals are indicated along the panel edge.