Direction of Arrival Estimation using Root-Transformation Matrix Technique

Murdifi Muhammad[1, 2, a)], Minghui Li[1], Qammer H. Abbasi[1], Cindy Goh[1], Muhammad Imran[1]

[1] School of Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom
[2] Research and Development Department, RFNet Technologies Pte Ltd, Singapore 319319
[a)] Corresponding author: 2167422B@student.gla.ac.uk

***Abstract*—This paper presents a preliminary study of a novel polynomial-solving Direction of Arrival (DOA) estimator called Root-Transformation Matrix (root-T) technique which includes an investigation of its performance against current DOA algorithms such as root-MUSIC and improved-MUSIC. The main objective of this work is to conserve the performance of improved-MUSIC which achieves high DOA estimation accuracy and resolution while reducing the cost of computational complexity. It's shown that the Root-T performs better in low SNR with performance improvement of 86.7% and with closely-spaced signal condition with performance improvement of 96.8% as compared to root-MUSIC without degrading improved-MUSIC performance while reducing mean computational time by 49.5%.**

Keywords—DOA (Direction of Arrival), MUSIC, Root-MUSIC, ULA (Uniformed Linear Array)

I. Introduction

Past literatures have presented modification to the classical Multiple Signal Classification (MUSIC) DOA algorithm called improved-MUSIC to improve performance in a coherent signal environment with high resolution as compared to the classical MUSIC algorithm [1]-[3]. The problem with improved-MUSIC is that it is computationally intensive as it requires a scan of all possible angles to obtain pseudospectrum peaks to determine the signal's DOAs.

In this paper, we present a novel, but simple technique by introducing a polynomial-solving algorithm called the Root-Transformation Matrix (Root-T) technique by obtaining the roots of the improved MUSIC algorithm to obtain the DOAs which is a similar technique employed for root-MUSIC [5] for a Uniformed Linear Array (ULA). This conserve the performance of the improved-MUSIC technique while reducing computational complexity. We will observe its performance in low Signal-to-Noise Ratio (SNR), closely-spaced signals and its computational efficiency. In theory, this enables localization application such as low-cost beamforming smart antennas and Internet-of-Things (IoTs) devices. Although [6] had presented a comprehensive study on various state of the art DOA techniques based on a similar 4-ULA structure as presented in this paper, it can be too complex for lightweight implementation.

The rest of the paper is organised as follows. Section II presents the algorithm flowchart with brief explanation in each stage. Section III presents the simulation and performance results. Lastly, section IV presents a brief conclusion.

II. Algorithm Flowchart

Figure 1 presents the algorithm flowchart for the root-T technique. Note that $(.)^H$ denotes the Hermitian transpose of a matrix.

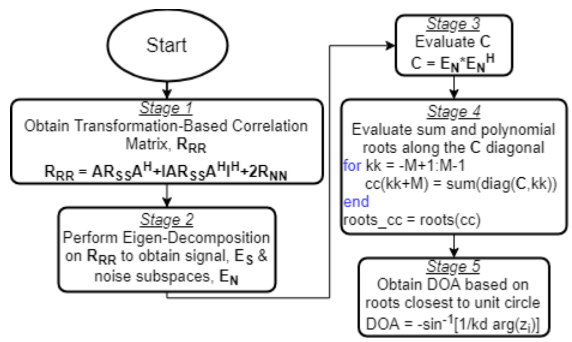

Fig. 1. Proposed Algorithm Flowchart

In stage 1, we obtain the transformation-based correlation matrix, $\mathbf{R_{RR}}$ as derived in [1]-[3], where $\mathbf{A}$ denotes the array steering matrix, $\mathbf{I}$ is the transformation diagonal identity matrix, $\mathbf{R_{SS}}$ is the signal correlation matrix and $\mathbf{R_{NN}}$ is the noise correlation matrix as derived in [3]. $\mathbf{R_{RR}}$ here is different than the correlation matrix used for root-MUSIC as it allows detection of signal in coherent environment [2]. Assuming an ergodic estimation process, we can approximate $\mathbf{R_{SS}}$ and $\mathbf{R_{NN}}$ using time-averaged correlation as derived in [4]. Stage 2 and 3 performs the eigen-decomposition of $\mathbf{R_{RR}}$ to obtain the noise subspace and subsequently multiply it by its Hermitian to obtain $\mathbf{C}$. From stage 4 onwards, we modified the algorithm by employing a polynomial solving technique unlike the improved-MUSIC technique which employs a pseudospectrum scan method. Stage 4 evaluates the sum across the diagonal of $\mathbf{C}$ and obtain the polynomial roots with length (-M+1) to (M-1), where M is the number of ULA antenna elements which significantly reduces computational costs. Roots closest to the unit circle corresponds to the DOAs of interest. Thus, stage 5 processes the estimated roots and converts it into DOAs as in [5].

III. Simulation Results and Discussion

For the simulation, we assume there are 2 narrowband signal impinging on a 4-element ULA of angle -10° and 10° at an operating carrier frequency of 5500 MHz. the antenna element spacing was set to half-wavelength and the number of k samples is 1024. We assume noise is an ideal AWGN. For simplicity, we only obtain DOA in the azimuth plane and assume elevation is 0°. For simulation III(A) and III(B), we ran 500 independent simulations with different noise process obtain the mean and Root Mean Squared Error (RMSE) values.

A. Low SNR Performance (SNR = -30 dB)

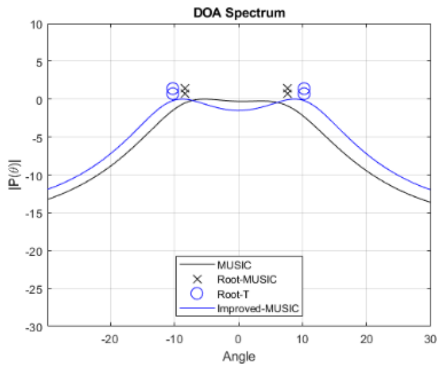

Fig. 2. Low SNR Performance

Fig. 2 presents mean performance in low SNR. At -30 dB, we observed that root-T achieves closer to the true DOA at -10.27° and 10.27° whereas root-MUSIC achieves -8.34° and 7.61° respectively. This presents an RMSE of 0.27 and 2.025 for root-T and root-MUSIC respectively. This presents an 86.7% performance increase over the root-MUSIC technique.

B. Closely-Spaced Signals (SNR = -20 dB)

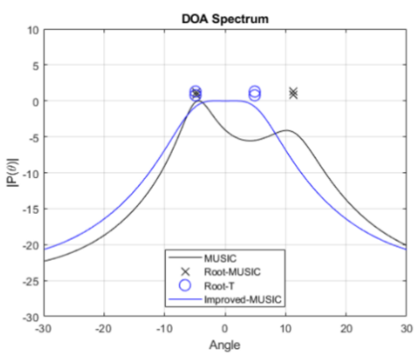

Fig. 2. Closely-Spaced Signal Performance

Fig. 3 presents mean performance of closely-spaced signals at -5° and 5°. We observed that the root-T performs better with a DOA estimation of -4.89° and 4.9° whereas root-MUSIC estimated the DOA at -4.73° and 11.29° respectively. This presents a RMSE of 0.105 and 3.28 for root-T and root-MUSIC respectively which is a 96.8% performance increase for root-T. We can also observe an asymmetrical detection for MUSIC because as signals converge towards each other, it leads to a coherency of signals. When signals are coherent, $\mathbf{R_{RR}}$ becomes singular as the rows are linear combinations of each other. Unlike an uncorrelated environment where $\mathbf{R_{RR}}$ is a diagonal matrix as its off-diagonal elements have no correlation. A singular $\mathbf{R_{RR}}$ causes a large error and degradation in statistical estimation [4]. Thus, [1]-[3] solves this problem by introducing the **I** matrix to decorrelate the incoming signals.

C. Computational Complexity

Both MUSIC and improved-MUSIC employs a spectral scan of all possible angles. For example, if the sweeping scan is from -90° to 90° with a resolution of 1° in the azimuth plane, there is a need to compute 181 iteration to determine the peaks in the pseudospectrum. Whereas the root-MUSIC and Root-T technique solves for polynomial roots along the diagonal with a maximum iteration of range -M+1:M-1 of the **C** matrix.

We've also observed the mean computational time for the various DOA techniques. With reference to Table 1 for a 1° resolution step, we observed that root-T is 49.5% faster than improved-MUSIC due to significantly lower computational iterations. Although root-T is 0.2 ms slower on over root-MUSIC, it is a trade-off to obtain better performance in low SNR and coherent signal situations.

Table 1: Algorithms Computational Runtime Comparison

Technique	**MUSIC**	**Improved-MUSIC**	**Root-MUSIC**	**Root-T**
Mean Time (milliseconds)	2.69	3.03	1.29	1.5

IV. Conclusion

The paper presents a preliminary study on a novel modification to the improved-MUSIC technique. From the results, we conclude that root-T maintains improved-MUSIC performance with lower computational complexity by 49.5%.

Acknowledgment

The authors would like to acknowledge and express appreciation to the Singapore Economic Development Board (EDB) and RFNet Technologies Pte Ltd for financing and providing a good environment and facilities to support the project.

References

[1] D. M. Vijayan and S. K. Menon, "Direction of arrival estimation in smart antenna for marine communication," 2016 International Conference on Communication and Signal Processing (ICCSP), Melmaruvathur, 2016, pp. 1535-1540.

[2] P. Gupta and S. P. Kar, "MUSIC and improved MUSIC algorithm to estimate direction of arrival," 2015 International Conference on Communications and Signal Processing (ICCSP), Melmaruvathur, 2015, pp. 0757-0761.

[3] Veerendra, M. Bakhar, R. M. Vani and P. V. Hunagund, "Implementation and optimization of modified MUSIC algorithm for high resolution DOA estimation," 2014 IEEE International Microwave and RF Conference (IMaRC), Bangalore, 2014, pp. 190-193.

[4] F. Gross, Smart antennas for wireless communications. New York: McGraw-Hill, 2005, pp. 176-178.

[5] B. D. Rao and K. V. S. Hari, "Performance analysis of Root-Music," in IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 37, no. 12, pp. 1939-1949, Dec. 1989

[6] Muhammad M., M. Li., Q. H. Abbasi, Goh C., Imran M. A., "Performance evaluation for direction of arrival estimation using 4-element linear array ," in press.