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On weak symmetries of Kenmotsu Manifolds with respect to quarter-symmetric metric connection

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Abstract

The aim of this paper is to study weakly symmetries of Kenmotsu manifolds with respect to quarter-symmetric metric connection. We investigate the properties of weakly symmetric, weakly Ricci-symmetric and weakly concircular Ricci-symmetric Kenmotsu manifolds with respect to quarter symmetric metric connection and obtain interesting results.

Keywords: Kenmotsu manifold; weakly symmetric manifold; weakly Ricci-symmetric manifold; weakly concircular Ricci-symmetric manifold; quarter-symmetric metric connection.

MSC: 53C15, 53C25, 53B05;

1. Introduction

In 1924, A. Friedman and J. A. Schouten ([8, 22]) introduced the notion of a semi-symmetric metric linear connection on a differentiable manifold. H.A. Hayden [10] defined a metric connection with torsion on a Riemannian manifold. In 1970, K. Yano [29] studied some curvature and derivational conditions for semi-symmetric connections in Riemannian manifolds. In 1975, S. Golab [9] initiated the study of quarter-symmetric linear connection on a differentiable manifold. A linear connection $\widetilde{\nabla}$ in an n-dimensional differentiable manifold is said to be a quarter-symmetric connection if its torsion T is of the form

$$T(X,Y) = \widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X - [X,Y] \\ = \eta(Y)\phi X - \eta(X)\phi Y, \tag{1.1}$$

where η is a 1-form and ϕ is a tensor of type (1, 1). In addition, a quarter-symmetric linear connection $\widetilde{\nabla}$ satisfies the condition

$$(\widetilde{\nabla}_X g)(Y,Z) = 0 \tag{1.2}$$

for all $X, Y, Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields of the manifold M, then $\widetilde{\nabla}$ is said to be a quarter-symmetric metric connection. If we replace ϕX by X and ϕY by Y in (1.1) then the connection is called a semi-symmetric metric connection [29]. In 1980, R.S. Mishra and S. N. Pandey [15] studied quarter-symmetric metric connection and in particular, Ricci quarter-symmetric metric connection on Riemannian, Sasakian and Kaehlerian manifolds. Note that a quarter-symmetric metric connection is a Hayden connection with the torsion tensor of the form (1.1). A studies on various types of quarter-symmetric metric connection and their properties included in ([1, 5, 18, 20, 21, 30]) and others.

On the other hand K. Kenmotsu [14] defined a type of contact metric manifold which is now a days called Kenmotsu manifold. It may be mentioned that a Kenmotsu manifold is not a Sasakian manifold.

The weakly symmetric and weakly Ricci-symmetric manifolds were defined by L. Tamássy and T. Q. Binh [26](1992, 1993) and studied by several authors (see [3, 4, 6, 13, 16, 19, 23, 24]. The weakly concircular Ricci symmetric manifolds were introduced by U. C. De and G. C. Ghosh (2005) [7] and these type of notion were studied with Kenmotsu structure in [11]. Many authors investigate these manifolds and their generalizations.

A non-flat Riemannian manifold $M(n > 2)$ is called a weakly symmetric if there exist 1-forms A, B, C, D and their curvature tensor R of type (0, 4) satisfies the condition

$$(\nabla_X R)(Y,Z,V) = A(X)R(Y,Z,V) + B(Y)R(X,Z,V) + C(Z)R(Y,X,V) \\ + D(V)R(Y,Z,X) + g(R(Y,Z,V),X)P \tag{1.3}$$

for all vector fields $X, Y, Z, V \in \chi(M)$, where A, B, C, D and P are not simultaneously zero and ∇ is the operator of covariant differentiation with respect to the Riemannian metric g. The 1-forms are called the associated 1-forms of the manifold.

A non-flat Riemannian manifold $M(n > 2)$ is called weakly Ricci-symmetric if there exist 1-forms α, β and γ and their Ricci tensor S of type (0, 2) satisfies the condition

$$(\nabla_X S)(Y,Z) = \alpha(X)S(Y,Z) + \beta(Y)S(X,Z) + \gamma(Z)S(Y,X) \tag{1.4}$$

for all vector fields $X, Y, Z \in \chi(M)$, where α, β and γ are not simultaneously zero.

A non-flat Riemannian manifold $M(n > 2)$ is called weakly concircular Ricci-symmetric manifold [7] if its concircular Ricci tensor P of type (0, 2) given by

$$P(Y,Z) = \sum_{i=1}^{n} \bar{C}(Y, e_i, e_i, Z) = S(Y,Z) - \frac{r}{n} g(Y,Z) \tag{1.5}$$

is not identically zero and satisfies the condition

$$(\nabla_X P)(Y,Z) = \alpha(X)P(Y,Z) + \beta(Y)P(X,Z) + \gamma(Z)P(Y,X), \tag{1.6}$$

where α, β and γ are associated 1-forms (not simultaneously zero). In equation (5.12), $\bar{C}$ denotes the concircular curvature tensor defined by [28]

$$\bar{C}(Y,U,V,Z) = R(Y,U,V,Z) - \frac{r}{n(n-1)}[g(U,V)g(Y,Z) - g(Y,V)g(U,Z)],$$

where r is the scalar curvature of the manifold.

The paper is organized as follows: In section 2, we give a brief account of Kenmotsu manifolds. In section 3 we give the relation between Levi-Civita connection ∇ and quarter-symmetric metric connection $\widetilde{\nabla}$ on a Kenmotsu manifold. Section 4 is devoted to the study of weakly symmetries of Kenmotsu manifolds with respect to quarter-symmetric metric connection $\widetilde{\nabla}$. It is shown that, in a weakly symmetric Kenmotsu manifold M $(n > 2)$ with respect to the connection $\widetilde{\nabla}$, the sum of associated 1-forms A, C and D is zero everywhere. In the last section, we study weakly Ricci-symmetric and weakly concircular Ricci-symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection $\widetilde{\nabla}$ in that we proved the sum of associated 1-forms α, β and γ is zero everywhere. Also, it is proved that, if the weakly Ricci symmetric Kenmotsu manifold with respect to the connection $\widetilde{\nabla}$ is Ricci-recurrent with respect to the connection $\widetilde{\nabla}$ then the associated 1-forms β and γ are in opposite directions. Finally, we consider weakly concircular Ricci-symmetric Kenmotsu manifold with respect to quarter-symmetric metric connection and prove that in such a manaifold, the sum of associated 1-forms is zero if the scalar curvature of the manifold is constant.

2. Kenmotsu manifolds

An $n(= 2m+1)$-dimensional differentiable manifold M is called an almost contact Riemannian manifold if either its structural group can be reduced to $U(n) \times 1$ or equivalently, there is an almost contact structure (ϕ, ξ, η) consisting of a (1,1) tensor field ϕ, a vector field ξ and a 1-form η satisfying

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \qquad \phi\xi = 0, \qquad \eta(\phi X) = 0, \tag{2.1}$$

Let g be a compatible Riemannian metric with (ϕ, ξ, η), that is

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{2.2}$$

or equivalently,

$$g(X,\phi Y) = -g(\phi X, Y) \quad and \quad g(X,\xi) = \eta(X) \tag{2.3}$$

for any vector fields X, Y on M [2].
An almost Kenmotsu manifold become a Kenmotsu manifold if

$$g(X,\phi Y) = d\eta(X,Y) \tag{2.4}$$

for all vector fields X, Y. If moreover

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.5}$$

$$(\nabla_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.6}$$

for any $X, Y \in \chi(M)$ then (M, ϕ, ξ, η, g) is called an almost Kenmotsu manifold. Here ∇ denotes the Riemannian connection of g. In a Kenmotsu manifold M the following relations hold [14]:

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

$$R(X,\xi)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.8}$$

$$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y), \tag{2.9}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{2.10}$$

$$S(\xi,\xi) = -(n-1), \tag{2.11}$$

for every vector fields X, Y on M where R and S are the Riemannian curvature tensor and the Ricci tensor with respect to LeviCivita connection, respectively.

3. Quarter symmetric metric connection on a Kenmotsu manifold

A quarter symmetric metric connection $\tilde{\nabla}$ on a Kenmotsu manifold is given by [25]

$$\tilde{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y. \tag{3.1}$$

A relation between the curvature tensor of M with respect to the quarter symmetric metric connection $\tilde{\nabla}$ and the Levi-Civita connection ∇ is given by [17, 25]

$$\begin{aligned}\tilde{R}(X,Y)Z = R(X,Y)Z &- 2d\eta(X,Y)\phi Z + [\eta(X)g(\phi Y,Z) - \eta(Y)g(\phi X,Z)]\xi \\ &+ [\eta(Y)\phi X - \eta(X)\phi Y]\eta(Z),\end{aligned} \tag{3.2}$$

where $\tilde{R}$ and R are the Riemannian curvatures of the connection $\tilde{\nabla}$ and ∇, respectively. From (3.2), it follows that

$$\tilde{S}(Y,Z) = S(Y,Z) - 2d\eta(\phi Z, Y) + g(\phi Y, Z) + \psi\eta(Y)\eta(Z), \tag{3.3}$$

where $\tilde{S}$ and S are the Ricci tensors of the connection $\tilde{\nabla}$ and ∇, respectively and $\psi = \sum_{i=1}^{n} g(\phi e_i, e_i) = Trace \;\; of \;\; \phi$. Contracting (3.3), we get

$$\tilde{r} = r + 2(n-1), \tag{3.4}$$

where $\tilde{r}$ and r are the scalar curvatures of the connection $\tilde{\nabla}$ and ∇, respectively. From (3.3) it is clear that in a Kenmotsu manifold the Ricci tensor with respect to the quarter-symmetric metric connection is not symmetric.

4. Weakly symmetric Kenmotsu manifolds admitting a quarter-symmetric metric connection

Analogous to the notions of weakly symmetric, weakly Ricci-symmetric and weakly concircular Ricci-symmetric Kenmotsu manifold with respect to Levi-Civita connection, in this section we define the notions of weakly symmetric, weakly Ricci-symmetric and weakly concircular Ricci-symmetric Kenmotsu manifodls with respect to quarter-symmetric metric connection. This notions have been studied by J. P. Jaiswal [12] in the context of Sasakian manifolds.

Definition 4.1. A Kenmotsu manifold $M(n > 2)$ is called weakly symmetric with respect to quarter-symmetric metric connection $\tilde{\nabla}$ if there exist 1-forms A, B, C and D and their curvature tensor $\tilde{R}$ satisfies the condition

$$\begin{aligned}(\tilde{\nabla}_X \tilde{R})(Y,Z,V) = A(X)\tilde{R}(Y,Z,V) + B(Y)\tilde{R}(X,Z,V) + C(Z)\tilde{R}(Y,X,V) \\ + D(V)\tilde{R}(Y,Z,X) + g(\tilde{R}(Y,Z,V),X)P,\end{aligned} \tag{4.1}$$

for all vector fields $X, Y, Z, V \in \chi(M)$.

Let M be a weakly symmetric Kenmotsu manifold with respect to the connection $\tilde{\nabla}$. So equation (4.1) holds. Contracting (4.1) over Y, we have

$$\begin{aligned}(\tilde{\nabla}_X \tilde{S})(Z,V) = A(X)\tilde{S}(Z,V) + B(\tilde{R}(X,Z,V)) + C(Z)\tilde{S}(X,V) \\ + D(V)\tilde{S}(X,Z) + E(\tilde{R}(X,V,Z))\end{aligned} \tag{4.2}$$

where E is defined by $E(X) = g(X, P)$. Replacing V with ξ in the above equation and then using the relations (2.7), (2.8),(2.10) and (3.3), we get

$$\begin{aligned}&(\tilde{\nabla}_X \tilde{S})(Z,\xi) \\ &= \{\psi - (n-1)\}\{A(X)\eta(Z) + C(Z)\eta(X)\} + \eta(X)\{B(Z) - B(\phi Z)\} \\ &\quad - \eta(Z)\{B(X) - B(\phi X)\} + D(\xi)\{S(X,Z) - 2d\eta(\phi Z, X) + g(\phi X, Z) \\ &\quad + \psi\eta(X)\eta(Z)\} + E(\xi)\{g(X,Z) - g(\phi X, Z)\} - \eta(Z)\{E(X) - E(\phi X)\}.\end{aligned} \tag{4.3}$$

We know that

$$(\tilde{\nabla}_X \tilde{S})(Z,\xi) = \tilde{\nabla}_X \tilde{S}(Z,\xi) - \tilde{S}(\tilde{\nabla}_X Z, \xi) - \tilde{S}(Z, \tilde{\nabla}_X \xi). \tag{4.4}$$

By making use of (2.3), (2.5), (2.9), (3.1) and (3.3) in (4.4) we have

$$(\tilde{\nabla}_X \tilde{S})(Z,\xi) = -S(X,Z) + 2d\eta(\phi Z, X) - g(\phi Z, X) + \{\psi - (n-1)\}g(X,Z) - \psi\eta(X)\eta(Z). \tag{4.5}$$

Applying (4.5) in (4.3), we obtain

$$\begin{aligned} &- S(X,Z) + 2d\eta(\phi Z,X) - g(\phi Z,X) + \{\psi - (n-1)\}g(X,Z) - \psi\eta(X)\eta(Z) \\ &= \{\psi - (n-1)\}\{A(X)\eta(Z) + C(Z)\eta(X)\} + \eta(X)\{B(Z) + B(\phi Z)\} \\ &\quad - \eta(Z)\{B(X) + B(\phi X)\} + D(\xi)\{S(X,Z) - 2d\eta(\phi Z,X) + g(\phi X,Z) \\ &\quad + \psi\eta(X)\eta(Z)\} + E(\xi)\{g(X,Z) - g(\phi X,Z)\} - \eta(Z)\{E(X) - E(\phi X)\}. \end{aligned} \tag{4.6}$$

Setting $X = Z = \xi$ in (4.6) and using (2.1) and (2.9), we find that

$$\{\psi - (n-1)\}\{A(\xi) + C(\xi) + D(\xi)\} = 0, \tag{4.7}$$

which implies that (since $n > 3$)

$$A(\xi) + C(\xi) + D(\xi) = 0 \tag{4.8}$$

holds on M.

Next, plugging Z with ξ in (4.2) and doing the calculations it can be shown that

$$\begin{aligned} &- S(X,V) + 2d\eta(\phi V,X) - g(\phi V,X) + \{\psi - (n-1)\}g(X,V) - \psi\eta(X)\eta(V) \\ &= \{\psi - (n-1)\}\{A(X)\eta(V) + D(V)\eta(X)\} + B(\xi)\{g(X,V) - g(\phi X,V)\} \\ &\quad - \eta(V)\{B(X) - B(\phi X)\} + \eta(X)\{E(V) - E(\phi V)\} - \eta(V)\{E(X) - E(\phi X)\} \\ &\quad + C(\xi)\{S(X,V) - 2d\eta(\phi V,X) + g(\phi X,V) + \psi\eta(X)\eta(V)\} \end{aligned} \tag{4.9}$$

Setting $V = \xi$ in (4.9) and then using the relations (2.1),(2.3)and (2.10) we get

$$\begin{aligned} &\{\psi - (n-1)\}A(X) - \{B(X) - B(\phi X)\} + \eta(X)B(\xi) \\ &\quad + \{\psi - (n-1)\}\eta(X)C(\xi) + \{\psi - (n-1)\}\eta(X)D(\xi) \\ &\quad - \{E(X) - E(\phi X)\} + \eta(X)E(\xi) = 0. \end{aligned} \tag{4.10}$$

Similarly, if we set $X = \xi$ in (4.9), we obtain

$$\begin{aligned} &\{\psi - (n-1)\}A(\xi)\eta(V) + \{\psi - (n-1)\}C(\xi)\eta(V) \\ &\quad + \{\psi - (n-1)\}D(V) - \eta(V)E(\xi) + \{E(V) - E(\phi V)\} = 0, \end{aligned} \tag{4.11}$$

Replacing V with X the above equation becomes

$$\begin{aligned} &\{\psi - (n-1)\}A(\xi)\eta(X) + \{\psi - (n-1)\}C(\xi)\eta(X) \\ &\quad + \{\psi - (n-1)\}D(X) - \eta(X)E(\xi) + \{E(X) - E(\phi X)\} = 0, \end{aligned} \tag{4.12}$$

Adding (4.10) and (4.12) and using the relation (4.8) we have

$$\begin{aligned}\{\psi-(n-1)\}\{A(X)+D(X)\}-\{B(X)-B(\phi X)\} \\ +\eta(X)B(\xi)+\{\psi-(n-1)\}C(\xi)\eta(X)=0.\end{aligned} \tag{4.13}$$

Now putting $X=\xi$ in the equation (4.6) and then using (2.1), (2.3) and (2.10) it follows that

$$\begin{aligned}\{\psi-(n-1)\}A(\xi)\eta(Z)-\eta(Z)B(\xi)+\{B(Z)-B(\phi Z)\} \\ +\{\psi-(n-1)\}C(Z)+\{\psi-(n-1)\}\eta(Z)D(\xi)=0.\end{aligned} \tag{4.14}$$

Replacing Z by X the above equation becomes

$$\begin{aligned}\{\psi-(n-1)\}A(\xi)\eta(X)-\eta(X)B(\xi)+\{B(X)-B(\phi X)\} \\ +\{\psi-(n-1)\}C(X)+\{\psi-(n-1)\}\eta(X)D(\xi)=0.\end{aligned} \tag{4.15}$$

Adding the equation (4.13) and (4.15) and using the relation (4.8) we get

$$\{\psi-(n-1)\}\{A(X)+C(X)+D(X)\}=0, \tag{4.16}$$

which implies that (since $n>3$)

$$A(X)+C(X)+D(X)=0,$$

for any X on M. Hence we are able to state the following:

Theorem 4.2. *In a weakly symmetric Kenmotsu manifold $M(n>2)$ with respect to quarter-symmetric metric connection, the sum of associated 1-forms A, C and D is zero everywhere.*

5. Weakly Ricci-symmetric Kenmotsu manifolds admitting a quarter-symmetric metric connection

Definition 5.1. A Kenmotsu manifold $M(n>2)$ is called weakly Ricci-symmetric with respect to quarter-symmetric metric connection if there exist 1-forms α,β and γ and their Ricci tensor $\tilde{S}$ of type (0, 2) satisfies the condition

$$(\tilde{\nabla}_X\tilde{S})(Y,Z)=\alpha(X)\tilde{S}(Y,Z)+\beta(Y)\tilde{S}(X,Z)+\gamma(Z)\tilde{S}(Y,X) \tag{5.1}$$

for all vector fields $X,Y,Z\in\chi(M)$.

Let us consider a weakly Ricci-symmetric Kenmotsu manifold with respect to the connection $\widetilde{\nabla}$. So by virtue of (5.1) yields for $Z=\xi$ that

$$(\tilde{\nabla}_X\tilde{S})(Y,\xi)=\alpha(X)\tilde{S}(Y,\xi)+\beta(Y)\tilde{S}(X,\xi)+\gamma(\xi)\tilde{S}(Y,X). \tag{5.2}$$

Equating the right hand sides of (4.5) and (5.2), it follows that

$$-S(X,Y)+2d\eta(\phi Y,X)-g(\phi Y,X)+\{\psi-(n-1)\}g(X,Y)-\psi\eta(X)\eta(Y) = \alpha(X)\tilde{S}(Y,\xi)+\beta(Y)\tilde{S}(X,\xi$$

Putting $X = Y = \xi$ in the above relation and then using the equations (2.1), (3.3) and (2.9) we get

$$\{\psi-(n-1)\}\{\alpha(\xi)+\beta(\xi)+\gamma(\xi)\} = 0.$$

which implies that (since $n > 3$)

$$\alpha(\xi)+\beta(\xi)+\gamma(\xi) = 0. \tag{5.3}$$

Next, taking $Y = \xi$ in equation (5.3) and then using relations (2.9), (3.3) and (5.3) we get

$$\alpha(X) = \alpha(\xi)\eta(X). \tag{5.4}$$

In a similar manner we can obtain

$$\beta(X) = \beta(\xi)\eta(X). \tag{5.5}$$

and

$$\gamma(X) = \gamma(\xi)\eta(X). \tag{5.6}$$

Adding (5.4), (5.5) and (5.6) and then using (5.3) we obtain

$$\alpha(X)+\beta(X)+\gamma(X) = 0, \tag{5.7}$$

for all vector field X on M. Thus, we state the following:

Theorem 5.2. *In a weakly Ricci-symmetric Kenmotsu manifold $M(n > 2)$ with respect to quarter-symmetric metric connection, the sum of associated 1-forms α, β and γ is zero everywhere.*

Definition 5.3. A weakly Ricci-symmetric Kenmotsu manifold $M(n > 2)$ with respect to quarter symmetric metric connection $\tilde{\nabla}$ is said to be Ricci-recurrent with respect to connection $\tilde{\nabla}$ if it satisfies the condition

$$(\tilde{\nabla}_X S)(Y,Z) = \alpha(X)S(Y,Z). \tag{5.8}$$

Suppose a weakly Ricci-symmetric Kenmotsu manifold with respect to quarter symmetric metric connection $\tilde{\nabla}$ is Ricci-recurrent with respect to the connection $\tilde{\nabla}$, then from (1.4) and definition (5.3), we have

$$\beta(Y)\tilde{S}(X,Z)+\gamma(Z)\tilde{S}(Y,X) = 0. \tag{5.9}$$

Putting $X = Y = Z = \xi$ in (5.9) and then using (3.3), we obtain

$$\beta(\xi)+\gamma(\xi) = 0 \tag{5.10}$$

for $\psi \neq (n-1)$. Putting $X = Y = \xi$ in (5.9), we get

$$\gamma(Z) = -\{\psi-(n-1)\}\beta(\xi)\eta(Z). \tag{5.11}$$

Similarly, we have

$$\beta(Z) = -\{\psi - (n-1)\}\gamma(\xi)\eta(Z).$$

Adding the above equation with (5.11)and using (5.10), we obtain

$$\beta(Z) + \gamma(Z) = 0.$$

for any vector field Z on M. So that β and γ are in opposite direction. Hence we state

Theorem 5.4. *If a weakly Ricci-symmetric Kenmotsu manifold $M(n > 2)$ with respect to quarter symmetric metric connection $\widetilde{\nabla}$ is Ricci-recurrent with respect to the connection $\widetilde{\nabla}$, then the 1-forms β and γ are in opposite direction.*

Definition 5.5. A Kenmotsu manifold $M(n > 2)$ is called weakly concircular Ricci-symmetric manifold with respect to quarter-symmetric metric connection $\widetilde{\nabla}$ if its concircular Ricci tensor $\widetilde{P}$ of type (0, 2) given by

$$\widetilde{P}(Y, Z) = \sum_{i=1}^{n} \widetilde{\widetilde{C}}(Y, e_i, e_i, Z) = \widetilde{S}(Y, Z) - \frac{\widetilde{r}}{n} g(Y, Z) \tag{5.12}$$

is not identically zero and satisfies the condition

$$(\nabla_X \widetilde{P})(Y, Z) = \alpha(X)\widetilde{P}(Y, Z) + \beta(Y)\widetilde{P}(X, Z) + \gamma(Z)\widetilde{P}(Y, X), \tag{5.13}$$

where α, β and γ are associated 1-forms (not simultaneously zero) and $\widetilde{\widetilde{C}}$ denotes the concircular curvature tensor with respect to the connection $\widetilde{\nabla}$.

Consider a weakly Concircular Ricci-symmetric Kenmotsu manifold $M(n > 2)$ with respect to the connection $\widetilde{\nabla}$, then the equation (5.13) holds on M.
In view of (5.12) and (5.13) yields

$$\begin{aligned}(\tilde{\nabla}_X \tilde{S})(Y, Z) - \frac{d\tilde{r}(X)}{n} g(Y, Z) = \alpha(X)[\tilde{S}(Y, Z) - \frac{\tilde{r}}{n} g(Y, Z)] \\ + \beta(Y)[\tilde{S}(X, Z) - \frac{\tilde{r}}{n} g(X, Z)] \\ + \gamma(Z)[\tilde{S}(X, Y) - \frac{\tilde{r}}{n} g(X, Y)].\end{aligned} \tag{5.14}$$

Setting $X = Y = Z = \xi$ in (5.14), we get the relation

$$\alpha(\xi) + \beta(\xi) + \gamma(\xi) = \frac{d\tilde{r}(\xi)}{[\tilde{r} - n\{\psi - (n-1)]\}} \tag{5.15}$$

Next, substituting X and Y by ξ in (5.14) and using (2.10) and (5.15), we obtain

$$\gamma(Z) = \gamma(\xi)\eta(Z), \qquad \tilde{r} - n\{\psi - (n-1)\} \neq 0. \tag{5.16}$$

Setting $X = Z = \xi$ in (5.14) and processing in a similar manner as above we get

$$\beta(Y) = \beta(\xi)\eta(Y), \qquad \tilde{r} - n\{\psi - (n-1)\} \neq 0. \tag{5.17}$$

Again, Taking $Y = Z = \xi$ in (5.14) and using (2.11) and (5.15), we get

$$\alpha(X) = \frac{d\tilde{r}(X)}{\tilde{r} - n\{\psi - (n-1)\}} + \big[\alpha(\xi) - \frac{d\tilde{r}(\xi)}{\tilde{r} - n\{\psi - (n-1)\}}\big]\eta(X), \tag{5.18}$$

provided $\tilde{r} - n\{\psi - (n-1)\} \neq 0$. Adding (5.16), (5.17) and (5.18) and using (3.4) and (5.15), we get

$$\alpha(X) + \beta(X) + \gamma(X) = \frac{d\tilde{r}(X)}{\tilde{r} - n\{\psi - (n-1)\}} = \frac{dr(X)}{\{r - n\psi + (n-1)(n+2)\}}$$

for any vector field X on M. This leads to the following:

Theorem 5.6. *In a weakly concircular Ricci-symmetric Kenmotsu manifold $M(n > 2)$ with respect to quarter symmetric metric connection $\widetilde{\nabla}$, the sum of the associated 1-forms is zero if the scalar curvature is constant and $\{r - n\psi + (n - 1)(n + 2)\} \neq 0$.*

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References

[1] Bagewadi. C. S., Prakasha. D. G. and Venkatesha, A study of Ricci quarter-symmetric metric connection on a Riemannian manifold, *Indian J. Math.*, Vol. 50(3)(2008), 607–615.

[2] Blair. D. E., Contact manifolds in Riemannian geometry, *Lecture Notes in mathematics, Springer-Verlag, Berlin, New-York*, Vol. 509 (1976).

[3] Demirbag. S. A., On weakly Ricci symmetric manifolds admitting a semi-symmetric metric connection, *Hacettepe J. Math & Stat.*, Vol. 41 (4)(2012), 507–513.

[4] De. U. C. and Bandyopadhyay. S., On weakly symmetric spaces, *Publ. Math. Debrecen*, Vol. 54 (1999), 377–381.

[5] De. U. C. and Sengupta. J., Quarter-symmetric metric connection on a Sasakian manifold, *Commun. Fac. Sci. Univ. Ank. Series*, A1, Vol. 49 (2000), 7–13.

[6] De. U. C., Shaikh. A. A. and Biswas.S., On weakly symmetric contact metric manifolds, *Tensor(N.S)*, Vol. 64(2)(2003), 170–175.

[7] De U. C. and Ghoash. G. C., On weakly concircular Ricci symmetric manifolds, *South East Asian J. Math & Math. Sci.*, Vol. 3(2)(2005), 9–15.

[8] Friedmann. A. and Schouten. J. C., Uber die Geometric der halbsymmetrischen Ubertragung, *Math. Zeitschr.*, Vol. 21 (1924), 211–223.

[9] Golab. S., On semi-symmetric and quarter-symmetric linear connections, *Tensor. N. S.*, Vol. 29 (1975), 293–301.

[10] Hayden. H. A., Subspaces of a space with torsion, *Proc. London Math. Soc.*, Vol. 34 (1932), 27–50.

[11] Hui. S. K., On weak concircular symmetries of Kenmotsu manifolds, *Acta Univ. Apulensis*, Vol. 26 (2011), 129–136.

[12] Jaiswal. J. P, The existence of weakly symmetric and weakly Ricci-symmetric Sasakian manifolds admitting a quarter-symmetric metric connection, *Acta Math. Hungar.*, Vol. 132 (4) (2011), 358–366.

[13] Jana. S. K. and Shaikh. A. A., On quasi-conformally flat weakly Ricci symmetric manifolds, *Acta Math. Hungar.*, Vol. 115 (3) (2007), 197–214.

[14] Kenmotsu. K., A class of almost Contact Riemannian manifolds, *Tohoku Math. J.*, Vol. 24 (1972), 93–103.

[15] Mishra. R. S. and Pandey. S. N., On quarter-symmetric metric F-connections, *Tensor, N.S.*, Vol. 34 (1980), 1–7.

[16] Özgür. C., On weakly symmetric Kenmotsu manifolds, *Diff. Geom.-Dyn. Syst.*, Vol. 8 (2006). 204–209.

[17] Prakasha. D. G., On ϕ-symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection, *Int. Electron. J. Geom.*, Vol. 4 (1) (2011), 88–96.

[18] Prakasha. D. G. and Taleshian. A, The structure of some classes of Sasakian manifolds with respect to the quarter symmetric metric connection, *Int. J. Open Problems Compt. Math.*, Vol. 3(5)(2010), 1–16.

[19] Prakasha. D. G., Hui. S. K. and Vikas. K, On weakly ϕ-Ricci symmetric Kenmotsu manifolds, *Int. J. Pure Appl. Math.*, Vol. 95 (4) (2014), 515–521.

[20] Rastogi. S. C., On quarter-symmetric metric connection, *C.R. Acad. Sci. Bulgar*, Vol. 31 (1978), 811–814.

[21] Rastogi. S. C., On quarter-symmetric metric connection, *Tensor*, Vol. 44(2) (1987), 133–141.

[22] Schouten. J. A., Ricci Calculus, *Springer*, (1954).

[23] Shaikh. A. A. and Hui. S. K., On weakly symmetries of trans-Sasakian manifolds, *Proc. Estonian Acad. Sci.*, Vol. 58 (4) (2009), 213–223.

[24] Shaikh. A. A. and Jana. S. K., On weakly symmetric Riemannian manifolds, *Publ. Math. Debrecen.*, Vol. 71 (2007), 27–41.

[25] Sular. S., Özgür. C. and De. U. C., Quarter-symmetric metric connection in a Kenmotsu manifold, *SUT J. Math.*, Vol. 44 (2) (2008), 297–306.

[26] Tamássy. L. and Binh. T. Q., On weakly symmetric and weakly projective symmetric Riemannian manifolds, *Colloq. Math. Soc. J. Bolyai.*, Vol. 56 (1992), 663–670.

[27] Tamássy. L. and Binh. T. Q., On weak symmetries of Einstein and Sasakian manifolds, *Tensor (N. S.)*, Vol. 53 (1993), 140–148.

[28] Yano. K., Concircular geometry I, concircular transformations, *Proc. Imp. Acad. Tokyo*, Vol. 16 (1940), 195–200.

[29] Yano. K., On semi-symmetric metric connections, *Rev. Roumaine Math. Pures Appl.*, Vol. 15 (1970), 1579–1586.

[30] Yano. K. and Imai. T., Quarter-symmetric metric connections and their curvature tensors, *Tensor, N.S.*, Vol. 38 (1982), 13–18.