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Vibrations of Circular Plates with Elastically Restrained Edge against Translation and Resting on Elastic Foundation

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Abstract: The present paper deals with exact solutions for the free vibration characteristics of thin circular plates elastically restrained against translation and resting on Winkler-type elastic foundation based on the classical plate theory. Parametric investigations are carried out for estimating the influence of edge restraint against translation and stiffness of the elastic foundation on the natural frequencies of circular plates. The elastic edge restraint against translation and the presence of elastic foundation has been found to have a profound influence on vibration characteristics of the circular plate undergoing free transverse vibrations. Computations are carried out for natural frequencies of vibrations for varying values of translational stiffness ratio and stiffness parameter of Winkler-type foundation. Results are presented for twelve modes of vibration both in tabular and graphical form for use in design. Extensive data is tabulated so that pertinent conclusions can be arrived at on the influence of translational edge restraint and the foundation stiffness ratio of the Winkler foundation on the natural frequencies of uniform isotropic circular plates.

Keywords: Plate, Frequency, Elastic edge, Translational stiffness, Elastic foundation.

اهتزازات الألواح الدائرية ذات الحافة المقيدة المرنة مقابل الانتقال والسكون على قاعدة مرنة

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الملخص: تتناول هذه الورقة الحلول الدقيقة لخصائص الاهتزاز الحر لألواح دائرية رقيقة مرنة ومقيدة مقابل الانتقال والسكون على قاعدة مرنة من نوع وينكلر بالاعتماد على نظرية اللوح التقليدية. وتم إجراء استقصاء بارامتري من أجل تقدير تأثير حافة مقيدة مقابل الانتقال والصلابة لقاعدة مرنة على الترددات الطبيعية لألواح دائرية. وتبين أن حافة المرونة المقيدة مقابل الانتقال وبوجود القاعدة المرنة لها تأثير شديد على خصائص اهتزاز اللوح الدائري الخاضع الى اهتزازات عرضية حرة. تم إجراء العمليات الحسابية للترددات الطبيعية للاهتزازات لقيم متنوعة من نسبة الصلابة الانتقالية ومتغير الصلابة لقاعدة نوع وينكلر. وقد تم عرض النتائج لأثنى عشر طريقة للاهتزاز سويًا على شكل جداول ورسوم بيانية وذلك لاستخدامها في التصميم. وتم جدولة بيانات واسعة النطاق بحيث أن الاستنتاجات ذات الصلة منها يمكن التوصل اليها لمعرفة تأثير الحافة المقيدة الانتقالية ونسبة الصلابة لقاعدة نوع وينكلر على الترددات الطبيعية للألواح الدائرية ذات الخواص الموحدة.

الكلمات المفتاحية: لوح، تردد، حافة مرنة، صلابة انتقالية، قاعدة مرنة

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Nomenclature

h	Thickness of a plate
a	Radius of a plate
ν	Poisson's ratio
E	Young's modulus
ρ	Density of a material
$W(r,\theta)$	Transverse deflection of the plate
D	Flexural rigidity of a plate
K_T	Translational spring stiffness
K_w	Stiffness of Winkler foundation
T	Translational spring stiffness ratio
ξ	Foundation stiffness ratio
ω_{mn}	Is the natural frequency of vibrations
λ_{mn}	Eigenvalue without foundation
$\lambda_{mn}{}^{*}$	Eigenvalue with Winkler foundation
m, n	Positive integers corresponding to the number of concentric circles and nodal diameters in each flexural mode

1. Introduction

Circular plates resting on elastic foundation have wide range of application in the static and dynamic design of linear/nonlinear vibration absorbers, dynamic exciters, telephone receiver diaphragms, computer discs, printed circuit boards etc. (Leissa 1969). Due to the essential use of vibration data in the computation of stresses in such structures, reliable prediction of vibration data is of great importance. In view of its importance in engineering design, the problem of vibration of circular plates on elastic foundation has attracted the focus and attention of many researchers.

Some of the recent studies have reestablished the efficiency of the classical approach in analyzing the vibrations of variety of structures. Circular plate problems allow for significant simplification in view of their symmetry but still many difficulties arise when the boundary conditions of the plate become complex involving linear and rotational restraints. A recent survey of literature shows that very few studies exist on the study of circular plates resting on elastic foundation. Wang and Wang (2003), who observed the switching between axisymmetric and asymmetric vibration modes, recently investigated the effect of internal elastic translational supports.

The vibration characteristics of plates resting on an elastic medium are different from those of the plates supported only on the boundary. Leissa (1993) discussed the vibration of a plate supported laterally by an elastic foundation. Leissa deduced that the effect of Winkler foundation merely increases the square of the natural frequency of the plate by a constant. Salari *et al*. (1987) speculated the same conclusion. Ascione and Grimaldi (1984) studied unilateral frictionless contact between a circular plate and a Winkler foundation using a vibrational formulation. Leissa (1969), who tabulated a frequency parameter for four vibration modes of simply supported circular plate with varying rotational stiffness, presented one of the earliest formulations of this problem. Kang and Kim (1996) presented an extensive review of the modal properties of the elastically restrained beams and plates.

Zheng and Zhou (1988) studied the large deflection of a circular plate resting on Winkler foundation. Ghosh (1997) studied the free and forced vibration of circular plates on Winkler foundation by exact analytical method. Chang and Wickert (2001), Kim *et al*. (2000) and Tseng and Wickert (1994) studied the dynamic characteristics of bolted flange connections involving circular plates displaying beating type of repeat frequencies and typical mode shapes of vibration. Bolted flange connections are practically the best examples for the elastically restrained boundary conditions of circular plates on partial or continuous Winkler type elastic foundation.

The most general soil model used in practical applications is the Winkler (1867) model in which the elastic medium below a structure is represented by a system of identical but mutually independent elastic linear springs. Recent investigations have reiterated the efficiency of the classical approach (Soedel 1993) in analyzing the behavior of structures under vibrations. There are other papers (Weisman 1970; Dempsey *et al*. 1984; Celep 1988) dealing with the study of plates on a Winkler foundation. In general, papers dealing with vibrating plates, shells and beams are concerned with the determination of eigenvalues and mode shapes (Leissa 1969).

A good number of studies are made by investigators (Wang and Lin 1996; Kim *et al*. 2001; Yayli *et al*. 2014) using the method of Fourier series for estimating the frequencies of beams with generally restrained end conditions including the effect of elastic soil foundation. The method includes use of Stoke's transformation in suitably modifying the complex boundary conditions. Very much similar to the dynamic stiffness matrix approach, the elements of the matrix involving infinite Fourier series are explicitly obtained in these studies. The determinant of this matrix for each case considered leads to the frequency equation and the same can be solved using well known numerical methods. The results obtained for various elastically restrained beam cases in these studies tallied well with those available in the literature establishing the efficiency of this method.

In view of the necessity of using complex combinations of rotational and translational springs at the boundary of the circular plate to suitably simulate the practical non-classical boundary connections being adopted in a wide range of industrial applications (Bhaskara and Kameswara 2009; Bhaskara and Kameswara 2010; Lokavarapu and Chellapilla 2013), the use of exact method of solution becomes imperative and hence the same is adopted in this paper. Even though the method adopted here is classical, the particular case of vibration of elastically restrained circular plate resting on elastic foundation considered here is not dealt with in the available literature.

Utilizing the classical plate theory, this paper deals with exact method of solution for the analysis of free transverse vibrations of thin circular plate that is elastically restrained against translation and resting on Winkler-type elastic foundation. For estimating the influence of edge restraint against translation and stiffness of the elastic foundation on the natural frequencies of circular plates, parametric investigations are carried out varying the values of elastic edge restraint stiffness against translation and the stiffness of the elastic foundation. The results obtained on natural frequencies of vibration clearly show that the vibration characteristics of the circular plate undergoing free transverse vibrations are found to have been profoundly influenced by these variations. Computations are carried out for natural frequencies of vibrations for varying values of translational stiffness ratio and stiffness parameter of Winkler-type foundation. Results presented for twelve modes of vibration both in tabular and graphical form are believed to be quite useful for designers in this area.

2. Mathematical formulation of the System

The considered elastic thin circular plate is supported on a Winkler foundation as shown in Fig. 1. In the classical plate theory (Leissa 1969), the following fourth order differential equation describes free flexural vibrations of a thin circular uniform plate.

$$D.\nabla^4 w(r,\theta,t) + \rho h \partial^2 w(r,\theta,t)/\partial t^2 = 0 \quad (1)$$

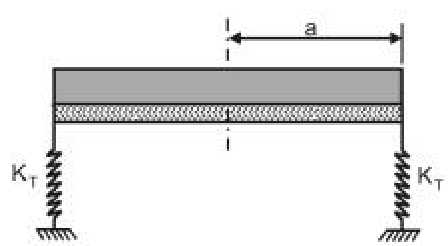

Figure 1. A thin circular plate with translational elastic edge restraint and supported on elastic foundation.

where $D = Eh^3/12(1-v^2)$ is the flexural rigidity of a plate and a,h,ρ,E,v are the plate's radiuses, thickness, density, Young's modulus and Poisson ratio respectively.

The homogeneous equation for Kirchhoff's plate on one parameter elastic foundation is given by the following equation.

$$D.\nabla^4 w(r,\theta,t) + K_w w(r,\theta,t) + \rho h \partial^2 w(r,\theta,t)/\partial t^2 = 0 \quad (2)$$

Displacement in equation (2) can be presented as a combination of spatial and time dependent components as follows;

Let $w(r,\theta,t) = W(r,\theta)e^{i\omega t}$ (3)

Now substitute the Eq. (3) in Eq. (2)

$$D.\nabla^4 W(r,\theta) + (K_w - \rho h\omega^2).W(r,\theta) = 0 \quad (4)$$

The solution of the equation takes the following form

$$W_{mn}(r,\theta) = A_{mn}\left[J_n\left(\frac{\lambda_{mn}r}{a}\right) + C_{mn}I_n\left(\frac{\lambda_{mn}r}{a}\right)\right] .\cos n\theta \;\; = 0,1,2,3\ldots; \; = 0,1,2,3, \text{ where} \quad (5)$$

where A_{mn} and C_{mn} are constants, J_n is Bessel function of the first kind of first order and I_n is modified Bessel function of the first kind of first order. Considering an elastically supported plate as shown in Fig. 1, boundary conditions can be formulated at $r = a$, in terms of translational stiffness (K_T) as follows:

$$M_r(a,\theta) = 0 \quad (6)$$

$$V_r(a,\theta) = -K_T.W(a,\theta) \quad (7)$$

where the Kelvin-Kirchhoff and bending moment are defined as follows

$$M_r(a,\theta) = -D.\left[\frac{\partial^2 W(a,\theta)}{\partial r^2} + v\left(\frac{1}{r}\frac{\partial W(a,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W(a,\theta)}{\partial \theta^2}\right)\right] \quad (8)$$

$$V_r(a,\theta) = -D.\left[\frac{\partial}{\partial r}\nabla^2 W(a,\theta) + (1-v)\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial^2 W(a,\theta)}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial W(a,\theta)}{\partial \theta}\right)\right] \quad (9)$$

By applying Eqs. (6) and (8), we obtain the following equation

$$\left[\frac{\partial^2 W(a,\theta)}{\partial r^2}+\nu\left(\frac{1}{r}\frac{\partial W(a,\theta)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 W(a,\theta)}{\partial\theta^2}\right)\right]=0 \qquad (10)$$

From Eqs. (5) and (10), we derive the following equation

$$C_{mn}=\frac{-Q_{mn}+\dfrac{2\nu}{\lambda_{mn}}P_{mn}+\left[2+\dfrac{4\nu n^2}{\lambda_{mn}{}^2}\right]J_n(\lambda_{mn})}{T_{mn}+\dfrac{2\nu}{\lambda_{mn}}S_{mn}+\left[2-\dfrac{4\nu n^2}{\lambda_{mn}{}^2}\right]I_n(\lambda_{mn})} \qquad (11)$$

where $P_{mn}=J_{n+1}(\lambda_{mn})-J_{n-1}(\lambda_{mn})$;
$Q_{mn}=J_{n+2}(\lambda_{mn})+J_{n-2}(\lambda_{mn})$;
$S_{mn}=I_{n+1}(\lambda_{mn})+I_{n-1}(\lambda_{mn})$;
$T_{mn}=I_{n+2}(\lambda_{mn})+I_{n-2}(\lambda_{mn})$;

From Eqs. (7) and (9), we get the following

$$-D.\left[\frac{\partial}{\partial r}\nabla^2 W(a,\theta)+(1-\nu)\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{r}\frac{\partial^2 W(a,\theta)}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial W(a,\theta)}{\partial\theta}\right)\right]=-K_T.W(a,\theta) \qquad (12)$$

From Eqs. (5) and (12), we derived the following equation

$$C_{mn}=\frac{R_{mn}-\dfrac{2Q_{mn}}{\lambda_{mn}}-\left[3+\dfrac{4+4(2-\nu)n^2}{\lambda_{mn}{}^2}\right]P_{mn}-\left[\dfrac{8(3-\nu)n^2}{\lambda^3}-\dfrac{4}{\lambda_{mn}}-\dfrac{8}{\lambda_{mn}{}^3}T\right]J_n(\lambda_{mn})}{U_{mn}+\dfrac{2T_{mn}}{\lambda_{mn}}+\left[3-\dfrac{4-4(2-\nu)n^2}{\lambda_{mn}{}^2}\right]S_{mn}+\left[\dfrac{8(3-\nu)n^2}{\lambda_{mn}{}^3}+\dfrac{4}{\lambda_{mn}}-\dfrac{8}{\lambda_{mn}{}^3}T\right]I_n(\lambda_{mn})} \qquad (13)$$

where, $T=\dfrac{a^3K_T}{D}$

$P_{mn}=J_{n+1}(\lambda_{mn})-J_{n-1}(\lambda_{mn})$; $Q_{mn}=J_{n+2}(\lambda_{mn})+J_{n-2}(\lambda_{mn})$; $R_{mn}=J_{n+3}(\lambda_{mn})-J_{n-3}(\lambda_{mn})$;
$S_{mn}=I_{n+1}(\lambda_{mn})+I_{n-1}(\lambda_{mn})$; $T_{mn}=I_{n+2}(\lambda_{mn})+I_{n-2}(\lambda_{mn})$; $U_{mn}=I_{n+3}(\lambda_{mn})+I_{n-3}(\lambda_{mn})$;

If $K_T\to\infty$ then this case becomes simply supported boundary condition as shown in Fig. 2. The frequency equation can be calculated from Eqs. (11) and (13), which allows determining eigenvalues λ_{mn}. The mode shape parameters C_{mn} can be determined corresponding to these eigenvalues by using either Eq. (11) or Eq. (13). The amplitude of each vibration mode in Eq. (5) is set by the normalization constant A_{mn} determined from the following condition.

$$\int_0^{2\pi}\int_0^{a} W_{mn}(r,\theta).W_{pq}(r,\theta)rdrd\theta=M_{mn}\delta_{mp}\delta_{nq} \qquad (14)$$

where, M_{mn} is a mass of the plate, $\delta_{mp}=\delta_{nq}=1$ if $m=p, n=q$ and $\delta_{mp}\delta_{nq}=0$ if $m\neq p$ or $n\neq q$.

The normalization constant A_{mn} can be derived using Eqs. (5) and (14) as given below:

$$A_{mn}=\left[\frac{1}{\pi a^2}.\int_0^{2\pi}\int_0^{a}\left(\left(J_n\left(\frac{\lambda_{mn}r}{a}\right)+C_{mn}.I_n\left(\frac{\lambda_{mn}r}{a}\right)\right).\cos n\theta\right)^2 rdrd\theta\right]^{-1} \qquad (15)$$

In Eq (4), ω_{mn} is the natural frequency of vibrations $=\left(\dfrac{\lambda_{mn}{}^2}{a^2}\right)\sqrt{\left(\dfrac{D}{\rho h}\right)}$ (16)

It is clear from the Eq. (16) the natural frequency of vibrations is dependent on the plate radius and eigenvalues.

From Eq. (16) we can express

$$\lambda_{mn}{}^4=\frac{\rho h a^4\omega_{mn}{}^2}{D} \qquad (17)$$

$$\lambda_{mn}^{*4} = \lambda_{mn}^{4} + \xi^{2} \tag{18}$$

where $\xi^{2} = \frac{K_w a^4}{D}$ (19)

$$\lambda_{mn}^{*} = \left[\lambda_{mn}^{4} + \xi^{2}\right]^{\frac{1}{4}} \tag{20}$$

where λ_{mn} is eigenvalue without foundation and λ_{mn}^{*} is eigenvalue with Winkler's foundation.

3. Solution

Using Matlab programming, computer software with symbolic capabilities, solves the above set of equations. The program determines eigenvalues (λ_{mn}^{*}), for a given range of boundary conditions. The boundary linear translational non-dimensional restraint parameter can be defined as follows:

$$T = \frac{K_T a^3}{D} \tag{21}$$

$$\xi^{2} = \frac{K_w a^4}{D} \tag{22}$$

The following are the input parameters to the program; (i) Translational stiffness ratio (T) (ii) Foundation ratio (ξ) (iii) Poisson ratio (ν) (iv) Upper bound for eigenvalues (N) (v) Suggested for eigenvalues (d) (vi) Number of mode shape parameters (n). The program finds eigenvalues λ_{mn}^{*} by using Matlab root finding function.

4. Results and Discussion

The code developed is used to determine eigenvalues of any set or range of translational and foundation constraints. This code also implanted for various plate materials by adjusting Poisson ratio. Such a wide range of results is not available in the literature yet. The eigenvalues for the plate edge, which is elastically restrained against translation and fully resting on the elastic foundation, at various values of the translational stiffness ratios, are computed and the results are given in Table 1. The effects of the translational stiffness ratios are plotted in Fig. 3. As seen from Fig. 3, eigenvalues increases with an increment in the translational stiffness ratio, and the plates become unstable in the region when the translational stiffness ratio exceeds a certain value. Twelve vibration modes are presented in Fig. 3. The smoothened stepped variation is observed in Fig. 3. The stepped region increases with increase in translational stiffness ratio and vibration modes. The location of the stepped region with respect to T changed gradually from the range of 0.01526 † [9.9997] ‡ – 5587.5316 [10.] to 16.62296 [14.6739] – 611824.96917 [16.75055].

The simply supported boundary conditions (Fig. 2) could be accounted for by setting ($K_T \to \infty$) shown in Fig.1. The translational edge supports becomes simply supported (or hinged) for very high values (close to infinity) of translational stiffness parameter i.e. $K_T \to \infty$. The frequency in this case is 2.23175 and this is in good agreement with the results published by Wang (2005). Another result considered for comparison is from Rao and Rao (2009) on study of the case of vibrations of elastically restrained circular plates supported on partial Winkler foundation. When the support position is full span which means that when b = 1, the case becomes a circular plate having full foundation support with elastically restrained edge against translation. For this case, the frequency is 2.1834 which is in good agreement with the frequency of 2.18341 obtained from the present study. Here †represents translational stiffness ratio and ‡ represents Eigen values throughout the text.

The eigenvalues at various values of the foundation stiffness ratios $\left[T = 100\,\&\,\nu = 0.33\right]$ are computed and the results are given in Table 2. The effects of the foundation stiffness ratio on eigenvalues are plotted in Fig. 4. As seen from Fig. 4, the eigenvalue increases with increase in the foundation stiffness ratio, and the plate becomes stiffer and stronger as the value of foundation stiffness becomes greater than 10^2. As seen from the Tables 1 and 2, the influence of foundation stiffness ratio on eigenvalue is relatively greater than the translation stiffness ratio in increasing the overall natural frequencies of the plate support system. As seen from Fig. 4, for all the modes considered here, up to a value of 10 the eigenvalues stay constant and beyond this value all the curves tend to converge to a constant eigenvalue as the foundation stiffness ratio increases up to 10^3. The convergence starts from 1.07897 [2.0325779] and continues up to a constant value of 9.63274 [13.84796].

Table 1. Eigenvalues for different Translational stiffness ratio for ξ =100 and ν =0.33.

Log10(T)	λ00	λ10	λ20	λ01	λ11	λ21	λ02	λ12	λ22	λ03	λ13	λ23
-3	10	10.0205	10.3517	10.10358	10.79568	12.46774	10.29701	11.43857	13.52984	10.63672	12.2505	14.67332
-2	10	10.02051	10.35171	10.10359	10.79568	12.46775	10.29702	11.43857	13.52985	10.63673	12.25051	14.67332
-1	10.00005	10.02061	10.3518	10.10368	10.79576	12.46779	10.29711	11.43864	13.52989	10.63682	12.25056	14.67335
0	10.00047	10.02162	10.35263	10.10462	10.79649	12.46827	10.29803	11.43927	13.53026	10.63769	12.25109	14.67365
1	10.003	10.03287	10.36125	10.11468	10.804	12.47302	10.30745	11.44568	13.53403	10.64649	12.25645	14.67667
2	10.00568	10.1199	10.46887	10.2319	10.89095	12.52351	10.42003	11.5759	13.57333	10.74585	12.31326	14.70772
3	10.00614	10.203	11.00421	10.49407	11.61576	13.1793	10.91134	12.27823	14.08459	11.41839	12.99719	15.0966
4	10.00619	10.21322	11.14468	10.53794	11.94459	14.15323	11.03916	12.88272	15.33948	11.70347	13.90786	16.53545
5	10.00619	10.21423	11.15725	10.54212	11.97273	14.2438	11.05093	12.93527	15.47979	11.72934	13.99516	16.74092
6	10.00619	10.21433	11.15849	10.54254	11.97546	14.25215	11.05209	12.94027	15.49231	11.73186	14.00329	16.75866
12	10.00619	10.21434	11.15862	10.54258	11.97576	14.25307	11.05222	12.94082	15.49369	11.73214	14.00418	16.7606

Table 2. Eigenvalues for different Foundation stiffness ratio for T =100 and ν =0.33.

Log10(ξ)	λ00	λ10	λ20	λ01	λ11	λ21	λ02	λ12	λ22	λ03	λ13	λ23
-3	2.18341	4.70075	6.69703	5.56683	7.98678	10.99197	6.50356	9.33292	12.43922	7.59879	10.67534	13.84974
-2	2.18341	4.70075	6.69703	5.56683	7.98678	10.99197	6.50356	9.33292	12.43922	7.59879	10.67534	13.84973
-1	2.18365	4.70077	6.69704	5.56684	7.98679	10.99197	6.50357	9.33292	12.43922	7.59879	10.67535	13.84974
0	2.20704	4.70315	6.69786	5.56827	7.98727	10.99216	6.50447	9.33323	12.43935	7.59936	10.67555	13.84983
1	3.3284	4.92488	6.77875	5.7064	8.03541	11.01074	6.5926	9.36352	12.45218	7.65514	10.69583	13.84914
2	10.00568	10.1199	10.46887	10.2319	10.89095	12.52351	10.42003	11.5159	13.57333	10.74585	12.31326	14.70772
3	31.62296	31.62664	31.63867	31.63037	31.6549	31.73756	31.63691	31.68259	31.81038	31.6491	31.72496	31.90972

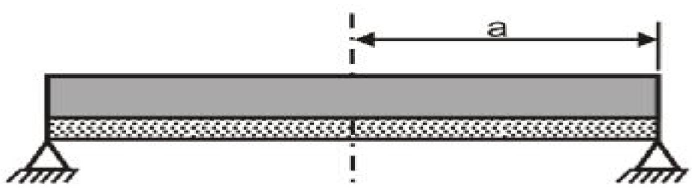

Figure 2. A simply supported thin circular plate resting on elastic foundation.

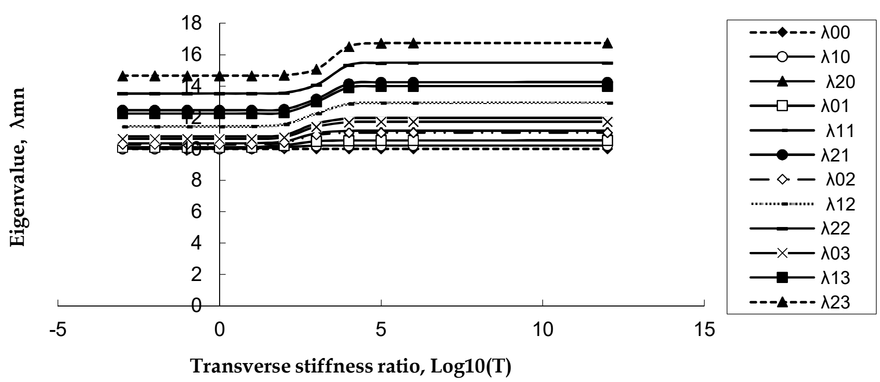

Figure 3. Effect of translational stiffness ratio ξ on eigenvalues, λ_{mn}.

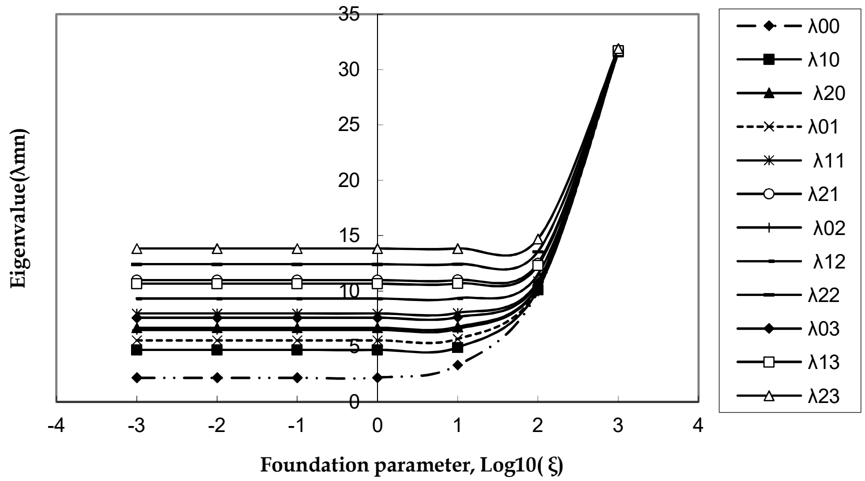

Figure 4. Effect of Foundation stiffness ratio, ξ on eigenvalues, λ_{mn}.

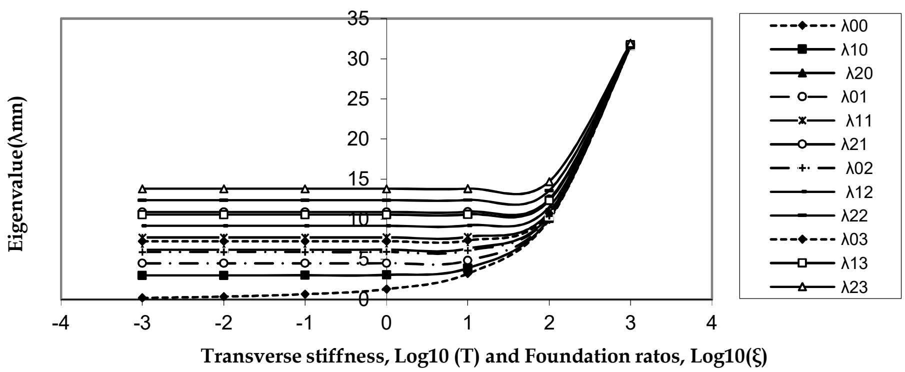

Figure 5. Effect of translational, T and foundation, ξ stiffness ratio son eigenvalues, λ_{mn}.

The eigenvalues at various values of the translational stiffness ratios and foundation stiffness ratios are computed and the results are given in Table 3. The effects of the translation and foundation stiffness ratios on eigenvalues are clearly observed in Fig. 5, eigenvalues increases with an increment in both the translational and foundation stiffness ratios. As observed from the Table 1 and 3, the influence of foundation stiffness ratio on eigenvalue is more predominant than that of translation stiffness ratio alone. As observed from Table 1, 2 and 3, in Table 3, lower eigenvalues are recorded for lower values of foundation and translation stiffness ratios together. As seen from Fig. 5, all the curves are stable up to certain region and beyond this all the curves tend to converge as the value of translation and foundation stiffness ratios increases.

The eigenvalues for different plate materials and various values of translational, foundation stiffness ratios are computed, and the results are given in Table 4. It was observed that for high ξ, eigenvalues are independent of Poisson ratio, as shown in Fig. 6. In addition, it was observed that for any value of T, eigenvalues are independent on Poisson ratio.

5. Conclusion

This paper deals with a method of computation of eigenvalues of axi-symmetric flexural vibrations of a circular plate with translational edge supports and resting on Winkler foundation using a specifically written MATLAB code. In this paper, the computed numerical results are presented in a tabular format to enable an estimating the accuracy of

Table 4. Eigenvalues for different Poisson ratios.

v	$T = \xi = 1000$	$T = 100, \xi = 10$	$T = 10, \xi = 1000$	$T = 1, \xi = 10$	$T = 50, \xi = 50$
0	31.62925	4.92456	10.03027	3.62553	7.28924
0.1	31.62929	4.92466	10.03111	3.64904	7.2896
0.2	31.62934	4.92476	10.0319	3.6708	7.28993
0.3	31.62938	4.92485	10.03265	3.69097	7.29025
0.4	31.62942	4.92494	10.03336	3.70974	7.29056
0.5	31.62946	4.92503	10.03404	3.72725	7.29085

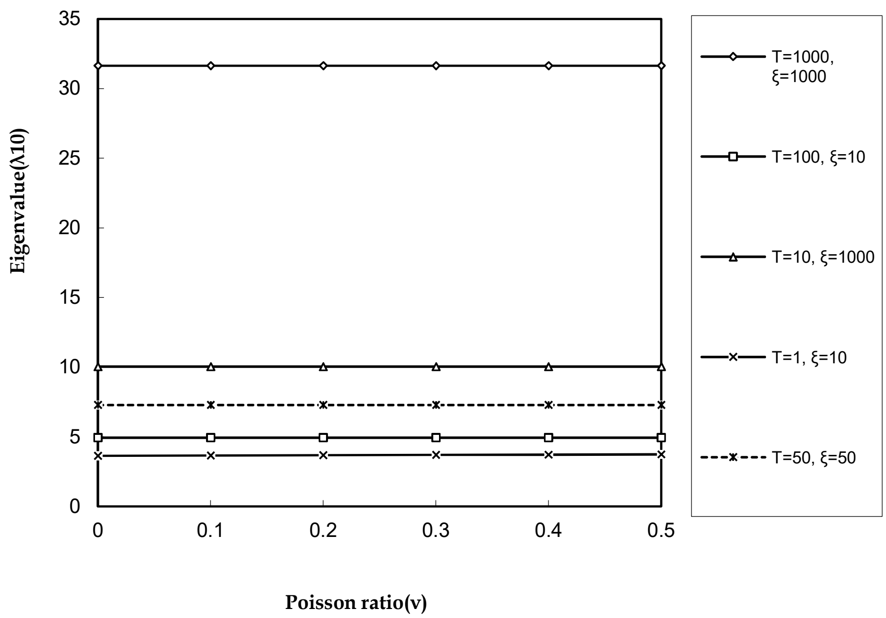

Figure 6. Effect of Poisson ratio, ν on eigenvalues, λ_{mn}.

approximate methods being used by other researchers for solving such problems. Two-dimensional plots of eigenvalues are drawn for a wide range of translational and foundation stiffness ratios facilitating their use in design. It has been observed that the eigenvalues remain constant without change only in a limited range of constraints (0 to 10) specific to each vibration mode and then steeply increase with increasing values of foundation ultimately converging to a constant value. It is also observed that the influence of foundation stiffness ratio on eigenvalues is more predominant than that of translational stiffness ratio. The effects of various parameters such as translational stiffness, foundation stiffness and Poisson ratio parameters on natural frequencies of the plate with elastic edge and resting on elastic foundation are studied in detail.

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