Journal of Mechatronics, Electrical Power, and Vehicular Technology 8 (2017) 22–32

Journal of Mechatronics, Electrical Power,
and Vehicular Technology

CrossMark

e-ISSN: 2088-6985
p-ISSN: 2087-3379

www.mevjournal.com

Performance comparison of consensus protocol and l-φ approach for formation control of multiple nonholonomic wheeled mobile robots

Ali Alouache*, Qinghe Wu

School of Automation, Beijing Institute of Technology
Haidian District 100081, Beijing, PR China

Received 21 January 2017; received in revised form 28 April 2017; accepted 3 May 2017

Abstract

This paper investigates formation control of multiple nonholonomic differential drive wheeled mobile robots (WMRs). Assume the communication between the mobile robots is possible where the leader mobile robot can share its state values to the follower mobile robots using the leader-follower notion. Two approaches are discussed for controlling a formation of nonholonomic WMRs. The first approach is consensus tracking based on graph theory concept, where the linear and angular velocity input of each follower are formulated using first order consensus protocol, such that the heading angle and velocity of the followers are synchronized to the corresponding values of the leader mobile robot. The second is l-φ approach (distance angle) that is developed based on Lyapunov analysis, where the linear and angular velocity inputs of each follower mobile robot are adjusted such that the followers keep a desired separation distance and deviation angle with respect to the leader robot, and the overall system is asymptotically stable.The aim of this paper is to compare the performances of the presented methods for controlling a formation of wheeled mobile robots with matlab simulations.

Keywords: nonholonomic WMR; the leader-follower structure; graph theory; consensus protocol; l-φ approach

I. Introduction

In the recent years there are a lot of interest to the design of mobile robots i.e. wheeled mobile robots (WMRs) [1, 2, 3, 4], due to their use in the societal and industrial applications. Unlike the majority of industrial robots that can move only in a specific workspace, mobile robots have the special feature of moving around freely within a predefined workspace to achieve their desired goals. This mobility capability makes mobile robots suitable for a large repertory of applications in structured and unstructured environments [5, 6, 7].

In certain time, the complexity of robot's tasks may increase, and a single mobile robot may not accomplish several tasks simultaneously or efficiently. To solve this problem, formations of these robots are called to work in parallel. There were many advantages when a team of mobile robots move in formation, such as increasing the efficiency, the accuracy, the robustness of the system to external effect, decreasing the system cost and increasing probability of success.

A group of robots can be used for accomplishing many tasks such as moving large awkward objects, terrain model acquisition, planetary exploration, surveillance applications [8, 9, 10, 11]. Formation of mobile robots control methods can be partitioned into three class approaches: virtual structure approach [12, 13], behavioral approach [14, 15] and the leader-follower approach [16, 17, 18].

In the leader-follower approach, one of the vehicles is designated as the leader, with the rest of the vehicles designated as followers. The basic idea is that the followers track the position and orientation of the leader with some prescribed (possibly time-varying) offset. There are numerous variations on the leader-follower topic including designating multiple leaders, forming a chain (vehicle tracks vehicle), and other tree topologies. There have been a number of works of leader-following mobile robotics. The leader-

---

* Corresponding Author. Tel: +86 188 0110 3264
E-mail address: alouache15@yahoo.fr

doi: https://dx.doi.org/10.14203/j.mev.2017.v8.22-32

Accreditation Number: (LIPI) 633/AU/P2MI-LIPI/03/2015 and (RISTEKDIKTI) 1/E/KPT/2015.

following technique based on the fuzzy logic approach is proposed for formation of wheeled mobile robots [18, 19].

Feedback linearization techniques are used to derive tracking control laws for nonholonomic robots that are used for leader-following. In addition, the authors used potential fields for obstacle avoidance [20]. A combination of a linear model predictive control and input-output feedback linearization is implementedon a team of WMRs in order to accomplish a formation task [21].

Controlling a formation that comprises large number robots poses some problems such as high communication load, high energy consumption and lack of robustness. Therefore, controlling some formation using graph theory is a solution [22] that increases the reliability of mathematical analysis, the effectiveness of realization, and reducing the power consumption with real robots [22].

This paper discusses two approaches for controlling a formation of multiple nonholonomic differential drive wheeled mobile robot based on the leader-follower structure. The first approach is consensus tracking based on graph theory concept [23]. Each WMR has single integrator nonlinear dynamics. The formation is described by a graph; each node of the graph represents a WMR, which is connected to its neighbours through an adjacency matrix. Each node also has some effects on its neighbours for sharing communication information hence the leader.

WMR can share its state information with the neighbor follower. Notice that the WMR receiving information about the input reference commands is named as the leader mobile robot and the other robots are follower robots. The linear and angular velocity inputs of each follower are formulated using first order consensus protocol, as well as the heading angle and velocity of the followers are synchronized to the corresponding values of the leader robot. The WMRs are synchronized to move off in formation with the same speed and directed orientation using consensus protocols. The separation distance and deviation angle between the leader and the follower robot motion are not controlled through the consensus protocol. The second approach is called l-φ (also called distance angle) which aims to control the desired distance and deviation angle between the leader and the follower robot. This approach is formulated based on Lyapunov analysis [24]. The linear and angular velocities of the follower are formulated such that the system is asymptotically stable in the sense of Lyapunov. In order to prescribe a formation maneuver, the leader's velocities commands are needed to be specified from the desired position and angle between the leader and the follower.

Related to the existing works on formation control of mobile robots based on the leader-follower structure, the main contribution of this paper is comparing the performances and the characteristics of consensus protocol with l-φ approach for controlling a formation of wheeled mobile robots using matlab simulations.

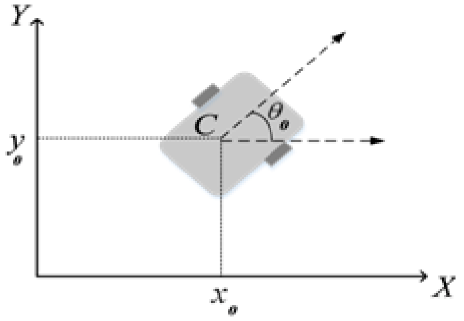

Figure 1. Wheeled mobile robot motion on the *X*-*Y* plane

II. WMR kinematic

Figure 1 displays a typical nonholonomic differential drive wheeled mobile robot moving on the X-Y plane with center of mass *C* and initial pose parameters. The WMR has two driving wheels mounted on the same axis and a free front wheel. The two driving wheels are derived to achieve both the orientation and translation pose. The derived nonlinear kinematic model which expresses the motion of the WMR is

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} cos\theta(t) & 0 \\ sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \tag{1}$$

The following $(x(t), y(t), \theta(t))^T$ is defined as the robot pose cartesian coordinates at instant time t, where $(x(t),y(t))$ represents the position of the mobile robot by the fixed cartesian coordinates, and the angle $\theta(t)$, orientation relatively to the X-axis. (x_0, y_0, θ_0) is the initial pose coordinates of the robot center of mass *C*. $v(t), w(t)$ and $\theta(t)$ are respectively the linear velocity, the angular velocity, and the heading angle of the robot. The kinematic model of Equation (1) describes the velocities of the vehicle but not the forces or torques that cause the velocity. The mechanical structure of the WMR is nonholonomic, it satisfies the following constraint.

$$\dot{x}(t)sin\theta(t) - \dot{y}(t)cos\theta(t) = 0 \tag{2}$$

This constraint means that the WMR cannot move in the direction of the wheel axis (i.e. Y).

III. Formation consensus tracking

The leader-follower concept can be modelled geometrically as shown in Figure 2. The robots are identical and their motion equations are given by Equation (1). The formation might have more robots, therefore this is only to define the symbol of the robots.

R_l and R_f denote the leader and the follower robot, respectively. According to the notation defined in Equation (2), the states and the inputs of R_l and R_f are denoted as (x_l, y_l, θ_l), (x_f, y_f, θ_f), (v_l, w_l), and (v_f, w_f), respectively.

A. Graph theory

Considering the formation of WMRs that is interconnected and able to share communication among robots, this communication network is modeled as a graph with directed edges corresponding to the allowed flow of information between the systems. The systems are modelled as the nodes in the graph that called agents.

A graph is a pair of $G=(V, E)$ with $V = \{v_1, v_2, \ldots v_N\}$ is a set of N nodes or vertices and E a set of edges or arcs. Elements of E are denoted as (v_i, v_j) which are termed an edge or arc from v_i to v_j, and represented as an arrow with tail at v_i and head at v_j.

It is assumed that the graph is simple by considering that $(v_i, v_i) \notin E, \forall i$ is no self-loops, and no multiple edges between the same pairs of nodes. Edge (v_i, v_j) is said to be out going to node v_i and incoming to v_j ; and node v_i is known as the major while v_j is the minor.

The in-degree of v_i is the number of edges having v_i as a head. The out-degree of a node v_i is the number of edges having v_i as a tail. The set of (in-) neighbors of a node v_i is $N_i = \{v_j : (v_j, v_i) \in E\}$, i.e., the set of nodes with edges incoming to v_i. The number of neighbors $|N_i|$ of node v_i is equal to its in-degree. In the case that teh in degree equals the out-degree for all nodes $v_i \in V$, then the graph is said to be balanced.

If $(v_i, v_j) \in E \Leftarrow (v_j, v_i) \in E, \forall i, j$, then the graph is said to be bidirectional, otherwise it is termed as directed graph or digraph, associate with each edge $(v_j, v_i) \in E$ a weight a_{ij} (note the order of the indices in this definition), assume that the non-zero weights are strictly positive. A graph is said to be undirected if $a_{ij} = a_{ji}, \forall i, j$, that is, if it is bidirectional and the weights of edges (v_i, v_j) and (v_j , v_i) are the same.

A directed path is a sequence of nodes $v_0, v_1, \ldots, v_r$, such that the $(v_i, v_{i+1}) \in E, i \in \{0,1, \ldots, r-1\}$. Node v_i is said to be connected to node v_j if there is a directed path from v_i to v_j. The distance from v_i to v_j is the length of the shortest path from v_i to v_j.

Graph G is said to be strongly connected if v_i, v_j are connected for all distinct nodes v_i, $v_j \in V$. For bidirectional and undirected graphs, if there is a directed path from v_i to v_j, then there is a directed path from v_j to v_i, and the qualifier is ‘strongly’ omitted.

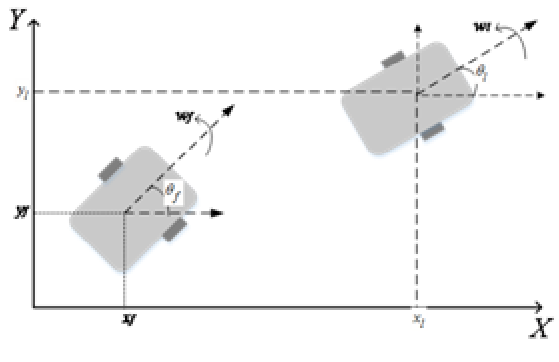

Figure 2. Multiple WMRs motion on the *X-Y* plane

A directed tree is a connected digraph where every node except one, called the root, has in-degree equal to other. A spanning tree of a digraph is a directed tree formed by graph edges that connects all the nodes of the graph.

A graph is said to have a spanning tree if a subset of the edges forms a directed tree. This is equivalent to say that all nodes in the graph are reachable from a single (root) node by following the edge arrows.

A graph may has multiple spanning trees. Define the root set or leader set of a graph as the set of nodes that are the roots of all spanning trees. If a graph is strongly connected, it contains at least one spanning tree. In fact, if a graph is strongly connected, then all nodes are root nodes.

B. Consensus tracking algorithm

Multi agent systems with the nodes of the graph have a scalar single integrator given by the following equation

$$\dot{x}_i = u_i \tag{3}$$

with $x_i, u_i \in R$.A as basic control design that play role as multi agent consensus tracking.

To implement the role as a multi agent consensus, a distributed control protocol that drives all states to the same values $x_i = x_j, \forall i, j$ has to be fulfilled. This value is known as a consensus value. The local control protocols for each agent *i* is given as follow

$$u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i). \tag{4}$$

with a_{ij} is the graph edge weights of the adjacency matrix $A_n \in R^{n \times n}$ associated with graph G at time t, x_i is the information state of the agent i and x_j the information of the corresponding neighbor j^{th} agent. This control distributed in that is only depends on the immediate neighbors N_i of node i in the graph topology.

remark 1: note that if these states are equal (or similar), this leads to zero i.e.$\dot{x}_i = u_i = 0$.

remark 2: setting $a_{ij}=0$ denotes the fact that the vehicle i cannot receive information from the vehicle j.

The local voting protocol of Equation (4) guarantees consensus of the multi agent single-integrator dynamics of Equation (3) if and only if the graph has a spanning tree. If the graph is strongly connected, then it has a spanning tree and consensus is reached.

C. Formation consensus protocol

The leader WMR in Figure 2 is moving with linear and angular velocity (v_l , w_l). The local voting protocol of Equation (4) is employed to derive the consensus protocols for formation consensus. The heading angle of the follower robot is synchronized to the leader by the following consensus protocol

$$\dot{\theta}_f = \theta_l - \theta_f \tag{5}$$

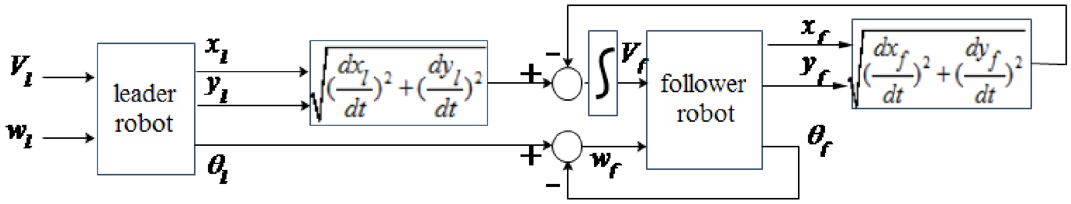

Figure 3. An overview of the leader follower motion consensus

Therefore, the angular velocity of the follower robot is given as follow

$$w_f = \theta_l - \theta_f. \tag{6}$$

Another consensus algorithm is applied on follower robot to synchronize the follower's velocity to the leader's speed

$$\dot{v}_f = v_l - v_f \tag{7}$$

From Equation (7), the linear velocity of the follower robot is given as follow

$$v_f = \int (v_l - v_f)dt \tag{8}$$

Figure 3 illustrates the general overview of the leader-follower control system, developed based on consensus protocol.

IV. Formation control using *l*-*φ* Approach

The leader-follower structure for a formation wheeled mobile robots *l*-*φ* approach is displayed in Figure 4. The different parameters of the *l*-*φ* approach (distance-angle) are indicated in Figure 4.

As shown in Figure 4, the variable *D* denotes the relative distance between the leader and the follower robot. The parameter *φ* indicated in Figure 4 denotes the bearing angle between the horizontal direction and the line connecting the leader and the follower.

α is the relative angle between the follower and the leader of mobile robot. The relative distance between the leader and the follower robot *D* is defined by the following equation

$$D = \sqrt{(x_l - x_f)^2 + (y_l - y_f)^2} \tag{9}$$

The angular position of the follower robot relative to the leader robot is defined by the following equations

$$\begin{cases} \alpha = \theta_f - \varphi \\ \beta = \theta_l - \varphi \end{cases} \tag{10}$$

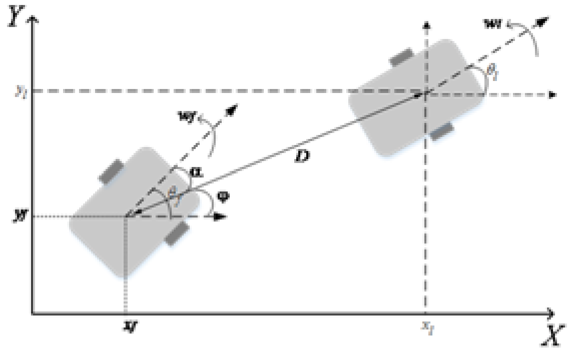

Figure 4. Parameter of the leader follower *l*-*φ* approach

The tracking errors between the desired values and the actual values of the robots are given this following equation

$$\begin{cases} e_D = D_d - D \\ e_\alpha = \alpha_d - \alpha \end{cases} \tag{11}$$

To derive the linear and angular velocity inputs of the follower robot, the following steps are carried out.

step 1: in this step the leader's motion is not considered, and the stability of the follower robot is proved. First of all, the time derivatives of the parameters *D* and *α* are given as follow

$$\begin{cases} \dot{D} = -v_f cos\alpha \\ \dot{\alpha} = -\left(\frac{v_f}{D}\right) sin\alpha \end{cases} \tag{12}$$

The Lyapunov candidate function is taken as follow

$$\begin{cases} V_1 = \frac{1}{2} e_D^2 \\ V_2 = \frac{1}{2} e_\alpha^2 \\ V_3 = V_1 + V_2 \end{cases} \tag{13}$$

Notice that the functions V_1 and V_2 are considered as the quadratic values of the tracking errors e_D and e_α respectively, and the function V_3 is the sum of the quadratic tracking errors. The time derivative of the function V_1 is given by the following equation

$$\dot{V}_1 = -e_D v_f cos\alpha \tag{14}$$

In order to comply the Lyapunov stability condition and to make $\dot{V}_1$ non-positive, it is obvious to choose v as follows

$$v_f = K_v e_D \cos\alpha \tag{15}$$

K_v is a positive gain. The time derivative of V_2 is given as follows

$$\dot{V}_2 = \alpha(w_f + \frac{v_f}{D} sin\alpha) \tag{16}$$

where w_f is chosen such that $\dot{V}_2$ is non-positive, and it is given as follows

$$w_f = -k_w\, \alpha - \frac{v_f}{D} sin\alpha \tag{17}$$

where k_w is a positive gain. The functions $\dot{V}_1$ of Equation (14) and $\dot{V}_2$ of Equation (16) are made non-positive yield that $\dot{V}_3$ is non-positive too and the system is asymptotically stable.

step 2: the motion of the leader robot is taken into consideration, and the stability of the leader-follower system is analyzed. Therefore the time derivatives of the parameters D and α become

$$\begin{cases} \dot{D} = v_l cos\beta - v_f cos\alpha \\ \dot{\alpha} = (\frac{v_l}{D}) sin\beta - (\frac{v_f}{D}) sin\alpha \end{cases} \tag{18}$$

The Laypunov function adopted here is the same form as to the Lyapunov candidate V_3 of step 1, thus V_3 is the quadratic sum of tracking errors. The time derivative of V_3 is carried out as in the same as step 1. To make the function $\dot{V}_3$ non-positive, the control laws are defined as follow

$$\begin{cases} v_f = v_l \frac{cos\beta}{cos\alpha} - K_v e_D \cos\alpha \\ w_f = -k_w\, \alpha - \frac{v}{D} sin\alpha + \frac{v_l}{D} sin\beta \end{cases} \tag{19}$$

where K_v and k_w are positive gains. It can be easily checked that v_f and w_f of Equation (19) create the non-positive function of $\dot{V}_3$ non-positive and the whole system is considered asymptotically stable.

Figure 5 gives a general overview of the control system developed by the leader-follower *l*-φ approach. It comprises the leader robot, the follower robot, and the Lyapunov based controller. As depicted in Figure 5, the inputs of the control system are as follow

- Leader's velocities *(*v_l*,* w_l*).*
- Desired distance D_d and desired deviation angle α_d of the follower with respect to the leader robot.

The leader-follower formation achieves the desired values D_d and α_d,when the tracking errors e_α and e_D converge to zero.

V. Simulation results

This section provides simulation examples for testing and comparing the performances of the presented approaches i.e. consensus protocol and the *l*-φ approach. In the forthcoming, it is considered that all the WMRs of the formation used in the examples are identical, where the motion of each WMR is expressed by the model of Equation (1).

A. Example 1

In this example, the consensus protocol approach are tested for a formation of five WMRs which comprises one leader WMR (L0) and four followers WMRs (F1, F2, F3, F4). At the initial instant t=0, the initial coordinates of the five WMRs poses on the X-Y plane are given as follows

- The leader L0 $(x_0(0), y_0(0), \theta_0(0)) = (0,2,\frac{\pi}{3})$
- The follower F1$(x_1(0), y_1(0), \theta_1(0)) = (2,2,0)$
- The follower F2$(x_2(0), y_2(0), \theta_2(0)) = (0,0,\pi)$
- The follower F3$(x_3(0), y_3(0), \theta_3(0)) = (1,0,\frac{-\pi}{2})$
- The follower F4$(x_4(0), y_4(0), \theta_4(0)) = (0,-1,\frac{\pi}{2})$

The communication topology of the formation of WMRs is described by the digraph of Figure 6 which has a spanning tree. The arrows between each of two WMRs of the formation indicate that the communication information is flowing in the arrow direction between the WMRs. The adjacency matrix which is describing the communication topology among the five WMRs of Figure 6 is given by the following matrix

$$A_{5\times5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

the information about the reference values is available only for the leader WMR L_0. The desired values for the reference velocity and the reference heading are taken as follows

$$\begin{cases} V_{ref} = 1m/s \\ \theta_{ref} = \frac{\pi}{4} rad \end{cases}$$

Using the reference values (V_{ref}, θ_{ref}), the consensus tracking for the formation is realized as illustrated by the control system of Figure 3. Figure 7 displays the results of synchronizing the headings of the robots to the reference value. Note that all the headings reach the same consensus value θ_{ref}.

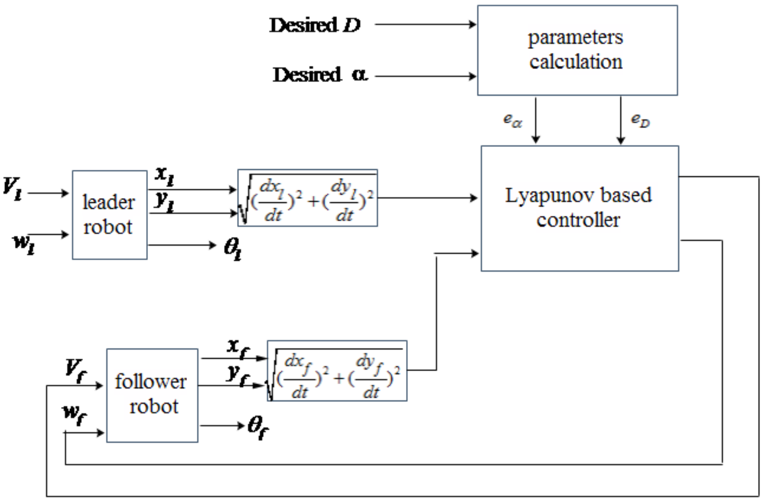

Figure 5. Controller overview of the leader follower *l*-φapproach

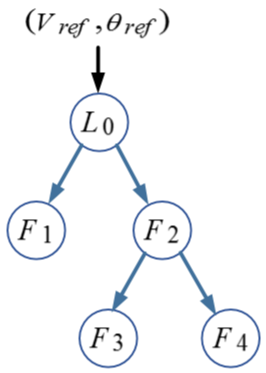

Figure 6. The tree-shaped graph for a network of four followers with a leader

Figure 8 illustrates the results of synchronizing all the velocities of the WMRs to the velocity reference value V_{ref}. Figure 9 shows the motion of the WMRs on the (X-Y) plane, each WMR starts from its corresponding initial pose, and all the robots converge simultaneously to the same heading value and move off in a formation together with same speed.

B. Example 2

In this example, the consensus protocol is compared with the *l*-φ approach. Consider a formation of wheeled mobile robots that comprises one leader and two follower mobile robots. At the initial instant t=0, considerate is considered that the coordinates of the robots as follow

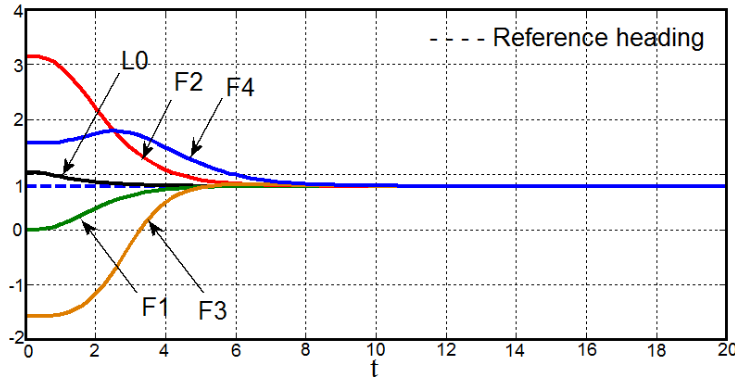

Figure 7. The consensus of the headings

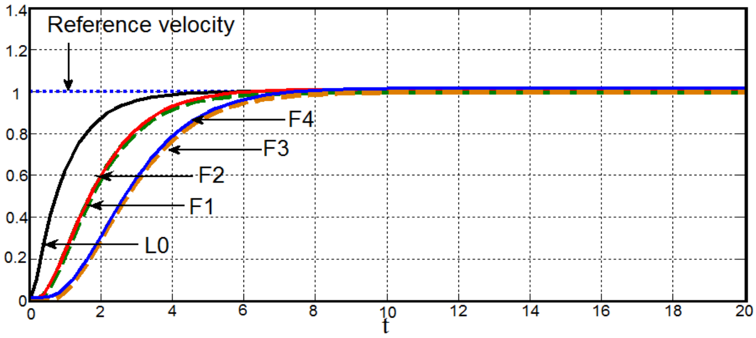

Figure 8. The consensus of the velocities

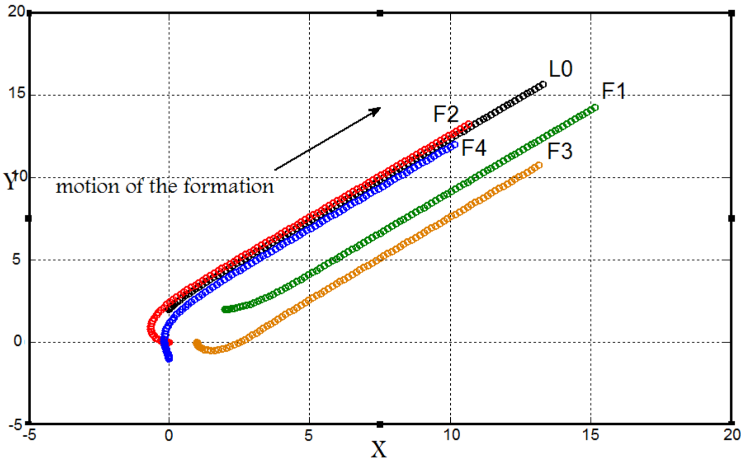

Figure 9. Motion consensus of the formation of WMRs in the *X*-*Y* plane

- The leaderRl $(x_l(0), y_l(0), \theta_l(0)) = (0,0,0)$
- The follower #1Rf1$(x_{f1}(0), y_{f1}(0), \theta_{f1}(0)) = (0,2,0)$
- The follower #2Rf2$(x_{f2}(0), y_{f2}(0), \theta_{f2}(0)) = (0,4,0)$

By a simple calculation, the initial distance between the robots R_l and R_{f1} is $D_1=2$. The initial distance between the robots R_l and R_{f2} is $D_2=4$. The input velocities of the leader mobile are shown in the Figure 10. Notice that the input linear and angular velocities are different from the previous example.

1) Consensus tracking

To realize formation of consensus tracking, the control system shown in Figure 3 is employed. The communication topology among the robots is given as indicated in Figure 11.

The arrows indicate that the information between the mobile robots is flowing in the arrow direction. Notice that the communication graph has a spanning tree. The adjacency matrix describing the graph of Figure 11 is given as follows

$$A_{3\times3} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Figure 12 shows the time histories of the linear velocity for the three mobile robots. Figure 13 displays the time histories for the angular velocity for the three mobile robots. Figure 14 displays the result of the headings consensus, note that the headings angles of the follower mobile robots are synchronized to the heading of the leader robot. Figure 15 shows the trajectories of the leader and the followers mobile robots on the (*X*-*Y*) plane, where all the mobile robots start from their corresponding initial poses.

Note that the three robots move off in formation simultaneously with the same heading angle and speed. The relative distances among the three mobile robots remain the same during the whole time of the simulation.

2) The leader-follower l-φ approach

It is desired that the distance and deviation angle between the leader R_l and the follower robot #1 R_{f1}, have the following values

$$\begin{cases} D_{d1} = 1 \\ \alpha_{d1} = 0.15\, rad \end{cases}$$

The desired distance and deviation angle between the leader R_l and the robot follower #2 R_{f2} are given as follow

$$\begin{cases} D_{d2} = 1.5 \\ \alpha_{d2} = 0.15\, rad \end{cases}$$

Figure 16 shows the time evolution of the distance between the robots, notice that both distances converge to the desired distances values D_{d1} and D_{d2}, respectively. The errors of the distances between the robots are depicted in Figure 17, where both errors convergeto zero. Figure 18 shows the time evolution of the deviation angles between the robots, both converge to the desired angles α_{d1} and α_{d2}. The angles errors between the mobile robots are depicted in Figure 19, where both errors converge to zero. Figure 20 displays the trajectories of the three mobile robots on the *X*-*Y* plane.

It is clear that the three mobile robots move forward in a formation and keeping the desired distance and angle with respect to the leader mobile robot. Comparing the results in Figure 15 and Figure 20, it can be concluded that the *l*-*φ* approach is controlling the distance and deviation angle between the robots compared to consensus protocol approach.

The followers' trajectories displayed in Figure 15 and Figure 20, demonstrated that in case of consensus protocol then the followers repeat simultaneously the same trajectory of the leader mobile robot; on the other hand, by using the *l*-*φ* approach the followers perform tracking of the leader's trajectory while maintaining the desired distance and deviation angle between the robots.

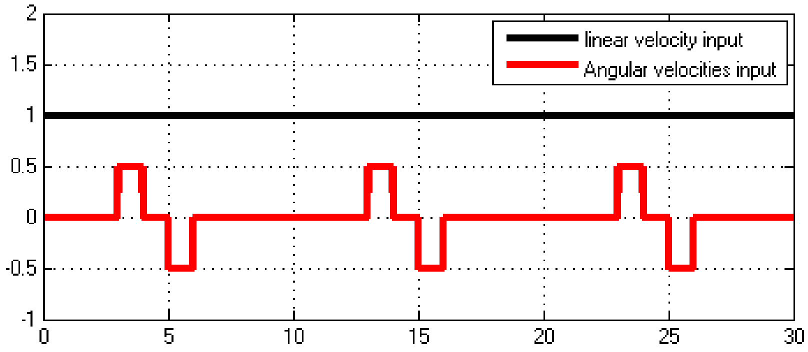

Figure 10. The input velocities for the leader mobile robot

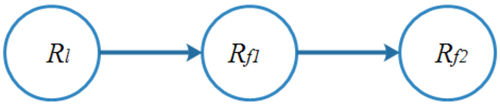

Figure 11. Communication topology among the mobile robot

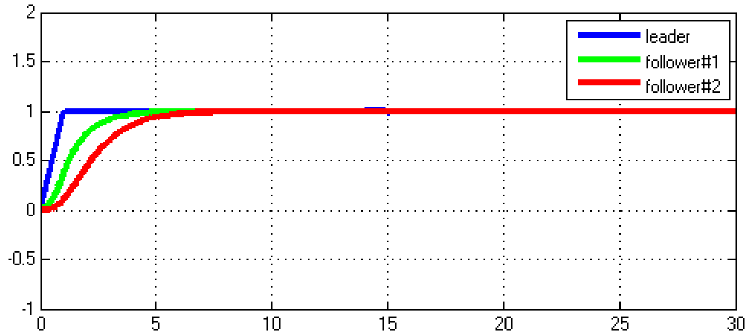

Figure 12. Time histories for the robots’ linear velocity

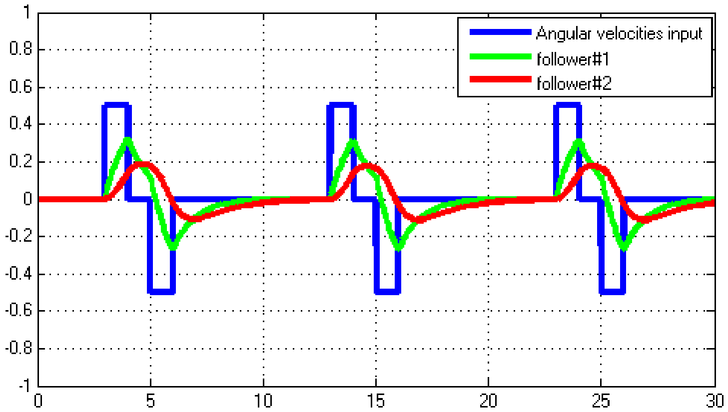

Figure 13. Time histories for the robots’ angular velocity

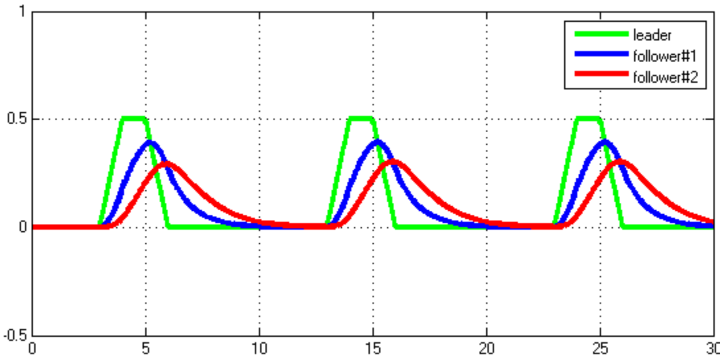

Figure 14. Headings consensus for the formation of mobile robots

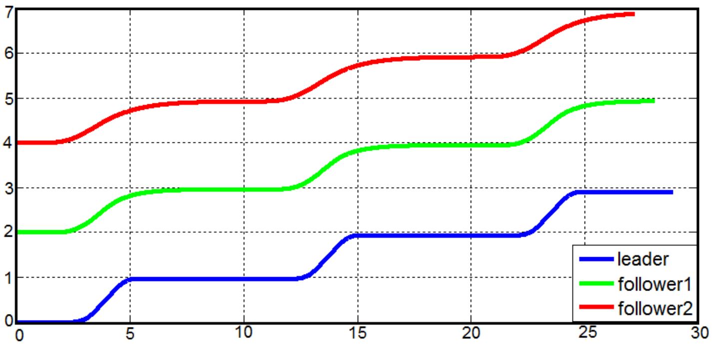

Figure 15. Mobile robots’ trajectories based on consensus protocol approach

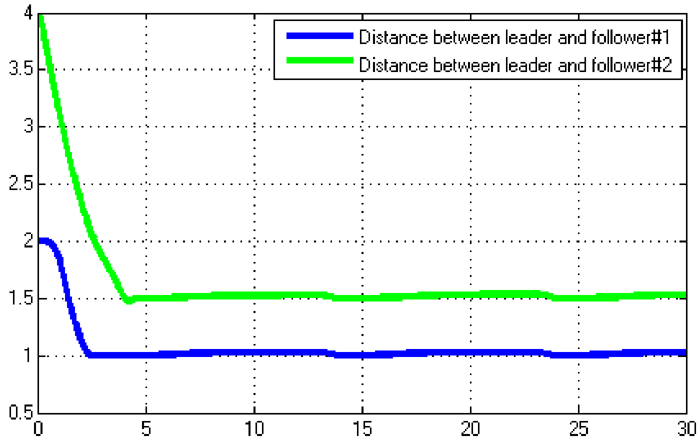

Figure 16. Time evolution of the distances between the Robots

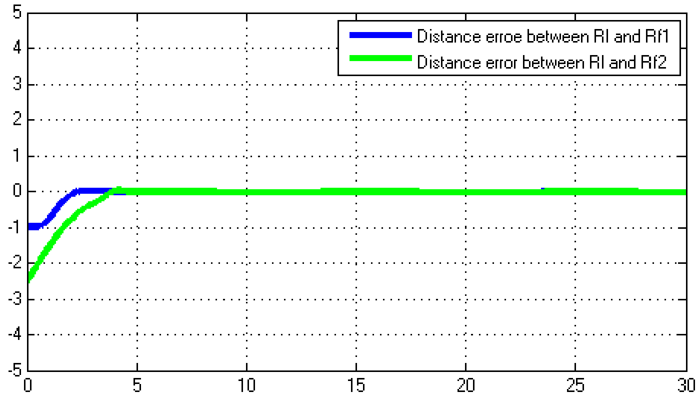

Figure 17. Distance errors between the mobile robots

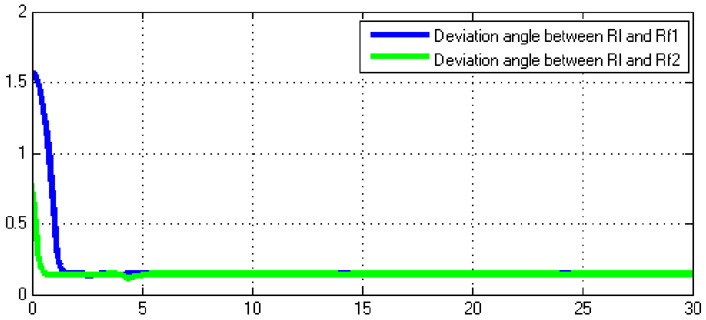

Figure 18. Time evolution of the deviation angles between the robots

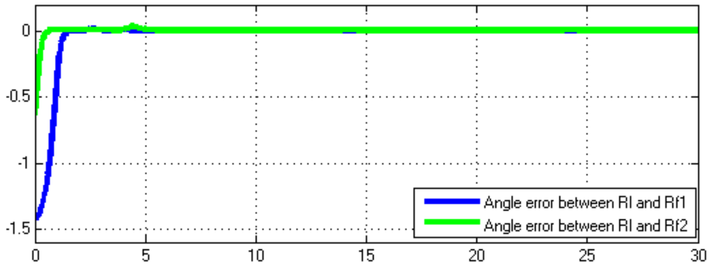

Figure 19. Errors of the angles between the mobile robot

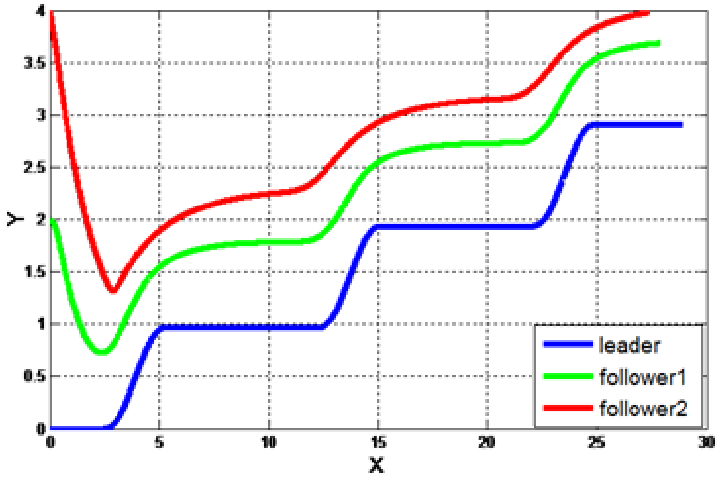

Figure 20. Robots'trajectories on the X-Y plane based on l-φ approach

Table 1.
Comparison of consensus and l-φ approach

	Consensus Approach	**l-φ Approach**
Principle	Algebraic approach (Graph theory)	Geometrical approach (Distance angle)
Pros	- Reliable Mathematical analysis, and fast convergence - Low Power Consumption	- Stability of the system, - Effective Convergence to the desired performances
Cons	- Communication problems in practice which cause the instability of the formation.	-Lack of robustness for dynamic changing the geometry of the formation. -High power consumption with large number of robots

From the results of example 1 and example 2, the main characteristics and performances of the consensus and l-φ approach can be summarized as given in Table 1. The comparison given in Table 1 is valid for a number of robots.

VI. Conclusion

In this paper we have compared the performances of two different approaches (i.e. consensus protocol and l-φ approach for formation control of multiple nonholonomic differential drive wheeled mobile robots based on the leader-follower structure.

Consensus protocol is developed based on graph theory concepts. A graph is used to represent the communication exchange between the robots. Each node of the graph represents a single robot, which is connected to its neighbours by an adjacency matrix, where each node has some effects on its neighbours for sharing communication information. The mobile robot that receives reference velocities commands is named the leader and others robots are the followers. The input velocities of each follower robot are formulated using first order consensus protocol.

It is shown that the consensus is achieved if the graph has a spanning tree. By using the consensus protocol, the heading angle and velocity of the follower robots are synchronized to the same values with the leader, and we have verified it by a simulation example using a formation of five WMRs. The l-φ approach is developed based on the Lyapunov theory, where the linear and angular velocity of the follower robots are adjusted such that the follower keeps a separation distance and deviation angle with respect to the leader, moreover the whole system is asymptotically stable in the sense of Lyapunov.

The effectiveness of the two methods are evaluated and compared by simulation examples. The simulation results demonstrated that the follower robots repeat simultaneously the trajectory of the leader when the consensus protocol is adopted. On the other hand, the follower robots perform trajectory tracking of the leader's trajectory using the l-φ approach, while maintaining a desired distance and angle between the mobile robots.

Consensus protocol approach is considered an advantageous because it is faster, and it consumes less power in real time applications. The l-φ approach is effective for controlling the follower robots to keep the desired separation distance and deviation angle relative to the leader mobile robot. In the future works it is necessary to develop both algorithms with obstacle avoidance.

Acknowledgement

This work was supported by the National Natural Science Foundation of China under Grant No. 61321002.

References

[1] Y. Inoue *et al.*, “Design of omnidirectional mobile robots with ACROBAT wheel mechanisms,” *IEEE Int. Conf. Intell. Robot. Syst.*, pp. 4852–4859, 2013.

[2] T. Jacobs *et al.*, “Design of wheel modules for non-holonomic, omnidirectional mobile robots in context of the emerging control problems,” in *ROBOTIK 2012; 7th German Conference on Robotics*, 2012.

[3] M. Lauria *et al.*, “Design and control of a four steered wheeled mobile robot,” , *IECON 2006-32nd*, Paris, pp. 4020–4025, 2006.

[4] Z. Zhang *et al.*, “Design and implementation of two-wheeled mobile robot by variable structure Sliding Mode Control,” in *2016 35th Chinese Control Conference (CCC)*, 2016, no. c, pp. 5869–5873.

[5] L. Pacheco and N. Luo, “Testing PID and MPC Performance for Mobile Robot Local Path-following,” *Int. J. Adv. Robot. Syst.*, p. 1, 2015.

[6] W.-Y. Lee *et al.*, “Mobile Robot Navigation Using Wireless Sensor Networks Without Localization Procedure,” *Wirel. Pers. Commun.*, vol. 62, no. 2, pp. 257–275, 2012.

[7] S. Hiroi and M. Niitsuma, “Building a Map including Moving Objects for Mobile Robot Navigattion in Living Environtment,” *Ieee*, pp. 1–2, 2015.

[8] Z. Yan *et al.*, “A survey and analysis of multi-robot coordination,” *Int. J. Adv. Robot. Syst.*, vol. 10, 2013.

[9] M. Defoort *et al.*, “Sliding-Mode Formation Control for Cooperative Autonomous Mobile Robots,” *IEEE Trans. Ind. Electron.*, vol. 55, no. 11, pp. 3944–3953, 2008.

[10] J. R. Oliveira *et al.*, “Integration of virtual pheromones for mapping / exploration of environments by using multiple robots,” pp. 835–840, 2014.

[11] R. Mendonc *et al.*, “A Cooperative Multi-Robot Team for the Surveillance of Shipwreck Survivors at Sea,” pp. 2–7, 2016.

[12] H. Su *et al.*, “Flocking of multi-agents with a virtual leader part II: With a virtual leader of varying velocity,” *Proc. IEEE Conf. Decis. Control*, vol. 54, no. 2, pp. 1429–1434, 2007.

[13] J. Ghommam *et al.*, “Formation path following control of unicycle-type mobile robots,” *Rob. Auton. Syst.*, vol. 58, no. 5, pp. 727–736, 2010.

[14] L. REN *et al.*, “Dynamic and Optimized Formation Switching for Multiple Mobile Robots in Obstacle Environments,” *Robot*, vol. 35, no. 5, p. 535, 2013.

[15] D. Xu *et al.*, “Behavior-based formation control of swarm robots,” *Math. Probl. Eng.*, vol. 2014, 2014.

[16] K. H. Kowdiki *et al.*, “Leader-follower formation control using artificial potential functions: A kinematic approach,” *Adv. Eng. Sci. Manag. (ICAESM), 2012 Int. Conf.*, pp. 500–505, 2012.

[17] J. Shao *et al.*, “Leader-Following Formation Control of Multiple Mobile Robots,” *Proc. 2005 IEEE Int. Symp. on, Mediterrean Conf. Control Autom. Intell. Control. 2005.*, no. Id, pp. 808–813, 2005.

[18] A. Bazoula *et al.*, “Formation Control of Multi-Robots via Fuzzy Logic Technique Mobile Robot Modeling Modeling of Leader-Follower Formation,” *Communications*, vol. III, no. May 2008, pp. 179–184, 2008.

[19] M. H. Amoozgar *et al.*, “A fuzzy logic-based formation controller for wheeled mobile robots,” *Ind. Robot An Int. J.*, vol. 38, pp. 269–281, 2011.

[20] J. Dong *et al.*, “Formation Control of Multirobot Based on I / O Feedback Linearization and Potential Function,” vol. 2014, pp. 1–7, 2014.

[21] M. A. Kamel and Y. Zhang, “Decentralized leader-follower formation control with obstacle avoidance of multiple unicycle mobile robots,” *2015 IEEE 28th Can. Conf. Electr. Comput. Eng.*, pp. 406–411, 2015.

[22] J. C. Barca *et al.*, “Controlling formations of robots with graph theory,” *Adv. Intell. Syst. Comput.*, vol. 194 AISC, no. VOL. 2, pp. 563–574, 2013.

[23] W. Ren and N. Sorensen, “Distributed coordination architecture for multi-robot formation control,” *Rob. Auton. Syst.*, vol. 56, no. 4, pp. 324–333, 2008.

[24] S. a. Panimadai Ramaswamy and S. N. Balakrishnan, “Formation control of car-like mobile robots: A Lyapunov function based approach,” *2008 Am. Control Conf.*, pp. 657–662, 2008.