Structural Pricing of XVA Metrics for Energy Commodities OTC Trades

Doctoral Thesis in Finance and Economics

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To my sister Stefania,
who has never stopped to believe in me.

Acknowledgments

First and foremost, I would like to start by thanking Professor Rosella Castellano for her patient supervision. For the opportunity I was given, I am very grateful to my tutor as well as Head of the Board of the Ph.D Program in Finance and Economics at Sapienza University of Rome, Professor Rita Laura D'Ecclesia. I am extremely grateful to all those who, in various ways, helped me during the realization of the present work through stimulating discussions, useful comments and precious suggestions. Among others I could not fail to mention Manuele Monti, Massimo Morini, Adamaria Perrotta, Andrew Green, Gianluca Fusai, Damiano Brigo, Roy Cerqueti Fang Fang, Edoardo Bonizzoni, Enzo Orsingher, Andrea Rodonò, Marco Bianchetti, Francesco Esposito and Andrea Pallavicini. I affectionately thank my dear post-graduate master colleagues and friends Valerio Restocchi and Gennaro Cibelli. A special thanks is addressed to my dear Ph.D colleagues and friends Salim Gadi, Stefano Vaccari, Roberto Giuzio, Marco Letta, Emy Zecca, Alessandro D'Ambrosio, Leonardo Bianchi, Stefano Filauro, Fiorenza Venturini and Rama Dasi Mariani for having spent two years encouraging each other at the Department of Statistics of Sapienza University. I am somehow grateful to the city of Rome for its incredible ability in making me feel at home. I enormously thank my parents for their love and unconditional support. Last but not least, I infinitely thank my girlfriend Oriana for her love and for always encouraging me to be ambitious. Without her, I could never obtain this award.

GENERAL INTRODUCTION

In recent years Counterparty Credit Risk (CCR) and Valuation Adjustments (XVA) dramatically changed both derivative pricing and risk management paradigms. Since the beginning of the global financial crisis, it became clear that neither high creditworthiness institutions could be considered default-free any more. CCR is the risk which economic agents face due to the possible default of their over the counter (OTC) counterparts occurring prior to the full compliance of contractual payments. Since no financial or corporate entity can be considered entirely default-free any more, CCR affects bilaterally OTC trades. XVA is the growing family of valuation adjustments due to CCR and other risks related to funding and collateral margining.

The scope of my Doctoral Thesis is to jointly address the topics of CCR and XVA through a common structural approach for default modelling. The work is composed by three linked Chapters. Within each of them, I provide a specific literature review of related arguments. In my Doctoral Thesis it is discussed a case study, accompanied by numerical analysis, based on the commodity asset class. In available Counterparty Credit Risk literature, reduced-form models have been widely preferred to structural ones because of their superior degree of analytical tractability. On the other side, structural models had typically struggled in calibrating non null CDS market implied default probabilities for very short maturities. Nevertheless, by the joint application of a suited underlying credit model and *state-of-the-art* numerical techniques, it is possible to replicate non zero short-term default probabilities and as a consequence, non zero short-term CCR valuation adjustments.

Chapter 1 relates to the literature of *pure* CCR valuation adjustments, since it is dedicated to the estimation of bilateral Counterparty Credit Risk metrics, known as Credit Valuation Adjustment (CVA) and Debit Valuation Adjustments (DVA). I inherit from Ballotta and Fusai (2015) the use of a first-passage type default model in which the bankruptcy is defined as the first passage time of the firm value from the level of a predetermined lower barrier. Unlike the cited Authors, I prefer to elect equity as convenient proxy for the firm creditworthiness because of its tradability. A pure jump Lèvy process obtained by brownian subordination is used to describe both the market and credit parts of the model. In other words, the economy is modelled through the Normal Inverse Gaussian (NIG) process proposed by Barndorff-Nielsen (1997). In the Chapter, I derived a simple model-independent formula representing the bilateral CCR adjustments. Unlike Burgard and Kjaer (2011b) and others, in my analysis no particular assumption for the hedging strategy is required. Such representation holds for both structural and reduced-form approaches of default modelling. In the numerical case study based on crude oil and natural gas, the Fourier Cosine Series (COS) method for European Option introduced by Fang and Oosterlee (2008) has been chosen for computing exposures at default (EAD). Furthermore, a contribution of the first Chapter is to extend the COS method for the estimation of EAD for Forward and Swap contracts. The COS method for CDS introduced by Fang et al. (2010) has been adopted for calibrating the credit part of the framework, including the default barriers. For the sake of calculating CVAs and DVAs upon a basket commodity derivatives negotiated between two defaultable counterparts, a first-to-default setting is addressed via joint Monte Carlo simulation of their equity dynamics.

Chapter 2 falls in the context of additional valuation adjustments literature as I tried to single out the economic role played by the other members of the XVA family, i.e. the Funding Valuation Adjustment (FVA) and Initial Margin Valuation Adjustment (MVA). The FVA, interpretable as the risk-neutral expectation of hedging financing costs, introduces a high degree of complexity, nonlinearity and recursivity. Nevertheless, I tried to simplify as possible the framework in order to understand whether FVA should affect fair valuation. In a reversed perspective, instead of adding funding related cash flows to those of the derivative payoff as Brigo and Pallavicini (2014) and others, I prefer to firstly understand the nature of the FVA term, retracing in some cases the analysis of Andersen et al. (2017). The MVA represents the risk-neutral expectation of Initial Margin (IM) financing costs and is asked by Central Clearing Counterparties (CCP) in order to mitigate additional risks due to extreme events. Strictly speaking, MVA is a relevant funding cost affecting both the micro point of view, in terms of pricing, and the macro point of view, in terms of systemic liquidity effects.

Chapter 3 configures as extension of my general structural model and relies on the inclusion of some additional features such as collateral and recovery risk. The effectiveness two different collateralization schemes has been compared in numerical experiments. A possibly original contribution of the third Chapter can be retrieved in the incorporation of stochastic recovery rates since, to my knowledge, stochastic recoveries has been scarcely investigated in the context of Counterparty Credit Risk. A valuable attempt aimed at estimating stochastic recovery rates for the Black and Cox (1976) model can be recovered in the work of Cohen and Costanzino (2017). However, their analysis is not applied to the pricing of CCR. In the Chapter, I propose a simple procedure to model state-dependent stochastic recovery rates, which reflects the severity of occurred defaults in terms of relative distance from the default barrier. It is underlined the relation between the underlying volatility and the expected value of stochastic recoveries. Moreover, I perform a sensitivity analysis of pure CCR adjustments with respect to credit and volatility shocks, highlighting some interesting asymptotic patterns. Finally, in the attempt of reconnecting CCR pricing to economic fundamentals and understand how much market and accounting information deviate, I propose an alternative default barrier calibration based on balance-sheet data.

1. LÈVY STRUCTURAL PRICING OF BILATERAL COUNTERPARTY CREDIT RISK (CCR)

ABSTRACT

The global financial crisis revealed that no economic entity can be considered *default-free* any more. Because of that, both banks and corporations have to deal with bilateral Counterparty Credit Risk (CCR) in their OTC derivatives trades. Such evidence implies the fair pricing of these risks, namely the Credit Valuation Adjustment (CVA) and its counterpart, the Debt Valuation Adjustment (DVA). Despite the more commonly used reduced-form approach, in this work the random default time is addressed via a structural approach à la Black and Cox (1976), so that the bankruptcy of a given firm is modelled as the first-passage time of its equity value from a predetermined lower barrier. As in Ballotta et al. (2015), I make use of a *time-changed* Lèvy process as underlying source of both market and credit risk. The main advantage of this setup relies on its superior capability to replicate non null short-term default probabilities, unlike pure diffusion models. Moreover, a numerical computation of the valuation adjustments for bilateral CCR in the context of energy commodities OTC derivatives contracts has been performed.

Keywords: Counterparty Credit Risk, Lèvy Processes, Jumps, First-passage Models, Fourier Pricing, COS, CVA, DVA, Energy Commodities.

1.1 Introduction

The Counteparty Credit Risk (CCR) is the potential risk for a given counterpart A to incur in losses caused by the default by its counterpart B before the expiry of the contract. The peculiarity of CCR with respect to usual Credit Risk relates to the stochastic nature of future exposures typical of derivatives payoffs. Conceptually, this type of risk refers mainly to over the counter (OTC) derivatives trades since, into regulated markets the presence of Clearing Houses prevents the occurring of big insolvencies[1]. The 2008 global financial crisis revealed that no economic entity can be supposed to be default-free any more. As a consequence both banks and corporations have to cope with the CCR in their OTC derivatives trades, i.e. Counterparty Credit Risk shows a *bilateral* nature.

The reason for my interest in the commodity asset class arises from some overview studies, for instance the one by ESMA (2017). In that paper, according to European Market Infrastructure Regulation (EMIR) weekly data on the size and structure of EU derivatives markets, which includes informations concerning financial and corporate entities, 305,685 different counterparts were reported to negotiate over the commodity asset class. This makes commodities the largest sector in terms of market participants. This is not surprising given to the widespread use of these contracts across many industries and types of counterparts, often non financials. The average notional amount of commodity trades is lower compared to other asset classes, reflecting the wide use of commodity derivatives also by small non-financial firms such as commodity producers or users managing their commodity price risk. Comparatively high level of concentration can be observed on the commodity sector, which in 2016 accounted for the 14% of the overall EU derivatives market, where many non-financial corporations interact with few large brokers.

In this study I make use of the approach suggested by Ballotta et al. (2015), who modelled the random default time within a structural approach a là Black and Cox (1976), meaning that the bankruptcy is defined as the first-passage time in which the firm value hits a predetermined down barrier. In their work, Ballotta et al. (2015), proposed a time-changed Levy process as underlying source of both market and credit risk. The inclusion of such a skewed, leptokurtic pure jump stochastic process is motivated by its superior capability to replicate non null short-term default probabilities compared to pure diffusion models. Indeed, that theoretical framework does not provide downward-biased short term default probabilities and as a consequence, downward-biased CCR metrics. While the characteristic function of Lèvy processes is always recoverable thanks to the well known Lèvy-Khintchine Representation, since most of times their probability density function (PDF) is not available in closed form, the numerical application of the present model requires the use of some Fourier-based pricing

[1] Actually under the Dodd-Frank act, Central Clearing Counterparts (CCPs) became mandatory also for all inter-dealers OTC trades executed post September 2016 (BCBS 2013). Nevertheless, the CCR is still a major concern for both financial and corporate firms operating in those markets.

methodologies.

1.1.1 *Brief literature review of structural CCR models and wrong-way risk*

The present subsection contains a brief review of the existing literature on structural models applied in the context of Counterparty Credit Risk and some insights of previous work on CCR relative to commodity derivatives. Furthermore, it is provided a discussion of default dependences as well as market-credit correlations in order to allow the introduction of wrong-way risk.

The distinction between structural and reduced-form models of default risk relates to how a *credit event* is modelled. A credit event is broadly defined both as the default of a given entity or as a noticeable deterioration in its creditworthiness impacting on its ability to pay back the entire market value of its outstanding monetary obligations.

While *structural models* aim at explaining the occurring of a credit event as the economic mechanism leading to the total assets of a given firm not to be sufficient to face its total liabilities, *reduced-form* models (also known as *intensity-based* models) configure a default as an unexpected event occurring according to some probabilistic law. In general, reduced-form models have been widely preferred to structural models particularly in the context of Counteparty Credit Risk because of their higher degree of analytical tractability, a desirable feature when dealing with very complex objects such as the CVA and the DVA.

Furthermore, structural models have been considered unsuitable or uncomfortable to be used for the computation of additional valuation adjustments such as the Funding Valuation Adjustment (FVA) and the Initial Margin Valuation Adjustment (MVA), among others. As regards the advantages of structural models, since they suggest default processes are entirely driven by the default-free market information, they are well suited to evaluate hybrid equity-credit products and to quantify Counterparty Credit Risk in equity derivatives. Moreover, when considering correlation among the entities involved and the underlying, structural models better describe the full structure of dependences without the necessity to impose exogenous *copula functions* otherwise necessary in intensity-based models.

Moving from the former work of Merton (1974) to more sophisticated classical first-passage type frameworks such as that of Black and Cox (1976), all structural models have typically assumed that the value of the firm follows a Geometric Brownian Motion (GBM). Among others, the lognormality assumption has been considered robust by KMV[2] empirical studies, see Brigo et al. (2013). Nevertheless, the lognormality assumption in classical structural models implies the impossibility to replicate non-null default probabilities for very short maturities. This is another crucial constraint in comparison with the competitive reduced-form approach. Last but not least, traditional first-passage type models for long time have appeared to be not flexible enough to properly calibrate the term structure of credit curves. This led to the introduction of other models

[2] The firm KMV is named after Kealhofer, McQuown and Vasicek, the founders of the company in 2002. It has then been sold to Moody's.

in which the volatility parameter is *time-dependent* and crucial to determine the time-varying level of *curved default barriers*. Those models can be viewed as extensions of the Black and Cox model. The joint objective of allowing for a more flexible setup while maintaining the possibility to derive closed-form solutions similar to the ones known in the context of Barrier Options pricing, has represented a technical challenge for long time. I refer to the work of Brigo et al. (2011), where the Authors apply the *Analytically-Tractable First-Passage (AT1F) model* and the *Scenario Barrier Time-Varying Volatility (SBTV) model* within a study based on Lehman Brothers default. While the first considers a deterministic barrier the latter, in order to take into account the uncertainty involved in balance sheet information, introduces different scenarios for the initial level of the default barrier.

In the Analytically-Tractable First-Passage (AT1F) model, the dynamics of the firm value is described by a Geometric Brownian Motion (GBM). The default barrier level is proportional to the firm's *leverage-ratio*[3] at the outset, the expected future value of the firm and dependent on the time-varying volatility parameter. Under such model, survival and default probabilities are still analytically recoverable in closed-form, see Brigo et al. (2011) for more details. In that model, the equation describing survival probabilities shows a one-to-one mapping between them and the time-dependent volatility levels. Being the other parameters fixed exogenously, volatility drives the theoretical fair CDS quotes produced by the model. Although the determination of the capital structure of the company is out of the scope of calibration procedure, which rather aims at matching risk-neutral default probabilities implied in the market, structural modelling might be interpreted as a tool to assess the economic coherence of calibration outputs.

Relying on a calibration to Lehman Brothers market data during the years of financial crisis which led to its bankruptcy, the Authors found that when the discount rate is deterministic the survival probabilities can be recovered in a *model-independent* way. They found a negligible relevance of the barrier level set at inception. Instead, crucial role in matching market CDS quotes was played by volatility, particularly for shortest maturities. The reason resides in the fact that pure diffusion models like the GBM would never generate defaults in the short-run (i.e. by touching the lower barrier) without assuming a very high level of volatility. The necessity to overstress volatility represents a severe drawback of modelling deterministic barriers. Moreover such a choice is arguable in the light of uncertainty upon the balance sheet, since the company might have hidden information within its accounting practices.

To overcome these drawbacks the same Authors introduced the Scenario Barrier Time-Varying Volatility (SBTV) model, in which the deterministic default barrier is replaced by a random variable determining different scenarios for the level of the default trigger. The assignment of a random state variable on the barrier level[4] aims at handling the uncertainty to collect all relevant

[3] This in turn can depend on the capital structure of the company, the level of outstanding debt and on the provision of safety covenants.

[4] In the cited paper the random variable can determine just 2 scenarios. The Authors argue

information for default modelling. In this framework, the much higher flexibility deriving from the possibility to vary both the barrier level given a certain scenario and the probability of occurrence of scenarios themselves, avoids the necessity to overstress the volatility parameters. They show how, in order to replicate different levels of quoted market CDS spreads, volatility does not need to vary as much as within the AT1P model. To sum up, the SBTV model provides more credible results from an economic point of view since it allows to recover a more stable term structure of volatilities and helps to overcome the problem of zero short-term credit spreads which affects the original formulation of the Black and Cox model.

A structural modelling of defaults applied to Counterparty Credit Risk can be found in the very technical work of Lipton and Savescu (2013), who investigated the case of CDS contracts affected by default risk. Considering the bilateral CCR embedded in a CDS contract led the Authors to build a three-dimensional extended structural model in order to compute the joint survival probabilities of the three defaultable entities involved in the deal (i.e. the protection buyer, the protection seller and the reference name). The Authors show a methodology to consistently compute survival probabilities and CVA/DVAs semi analytically by solving the pricing problem implied in the Green Function and making use of the Eigenvalues Expansion Method or the Method of Images, alternatively. The simplified *jump-free* version of model, might be used as a good benchmark for the more general jump diffusion case. For each firm, the Authors modelled their own *relative distance from default barrier* as stochastic processes described by correlated Brownian Motions. They confirm how neglecting jumps denotes the impossibility of obtaining a good fit of the short-term credit spread curve. They point out that the 3D extension of the structural model aimed at taking into account bilateral CCR, is crucial to recover the *symmetry* in the pricing problem, meaning that the buyer and seller can agree upon the trading price. By incorporating a set of indices of pairwise correlation, the Authors demonstrate that valuation adjustments for typical CDS contracts may be very large.

The contribution of Ballotta and Fusai (2015) is grounded in the use of a Lèvy process within a Merton-type default setting. They have investigated the impact of wrong and right-way risks through a multivariate factor model in which they decompose the risk drivers into idiosyncratic and systemic parts. After conditioning to the trajectory of the systemic risk factor, they compute exposures by means of the Fourier Cosine Expansion (COS) method and analyse the case of a Forward contract written on Brent crude oil. Their findings show the existence of an asymmetric impact of wrong and right-way risks on the size of CVA and DVA. Furthermore, in order to implement the consultation guidelines provided by the Basel Committee on Banking Supervision (BCBS) which suggest to take into account the volatility of CCR metrics, they build a a 95th-percentile confidence interval for CVAs. In their following work, Ballotta et al. (2015) while adopting the same multivariate pure jump Lèvy model as underlying

that the inclusion of additional possible scenarios does not increase significantly the quality of the calibration.

source of risk, rely on a Black and Cox-type modelling of defaults. The Authors show how to calibrate the model-implied survival probabilities to market CDS quotes trough an efficient numerical algorithm based on the Hilbert Transform Method. Moreover, they extend their previous work by considering both single and bilateral collateralization, showing that in such a case the complexity of the pricing problem increases considerably, since it embodies the evaluation of a package of Calendar Spread Option-type derivatives. Focusing on a Swap contract written on Brent crude oil, they demonstrate that by using the cited multivariate Lèvy model and applying *state-of-the-art* numerical techniques, even a structural model may represent a robust tool to fit short-term default probabilities. In the last part of the paper, they analyse the effects of Netting as a contractual provision to mitigate potential exposures and how the benefits of Netting interact with the structure of statistical dependences. Since the Netting case implies a Basket Option pricing problem, the Authors address the analysis by means of a comparison in terms of efficiency between Convolution and the Barakat Approximation.

Regardless the default modelling approach, a relevant topic which I consider worthy to be discussed attains to the structure of default dependences. A higher degree of detail is required in order to effectively introduce CCR modelling in presence of default dependences. A detailed treatment of structural default dynamics instead, represents the core of the present Chapter.

Default dependences as well as the correlation between market and credit risks originate the so called wrong-way risk and right-way risk. These affect significantly the picture across several asset classes. As it will be shown, in some cases wrong-way risk might exacerbate what is knows as *gap risk*, namely the occurrence of exposure jumps at default. In those cases, the derivative value upon the counterpart default would deviate significantly from that observed at the last collateral margining date. The existence of wrong-way risk and gap risk could make even full continuous collateralization schemes largely ineffective. As a consequence, material CVA risk would remain alive. Otherwise, as explained in Brigo et al. (2014), if exposure does not jump at the first-to-default event, continuous collateralization schemes perfectly mitigate CCR risk. Ensuring from wrong-way risk requires the *independence hypothesis*, in other words the default of economic agents are conditionally independent with respect to the reference market filtration. Unfortunately this is not the case for products such as Credit Default Swaps (CDS), whose value depends on the embedded stream of default and survival probabilities. As a consequence, if the default of the CDS reference entity is somehow correlated with that of the protection seller, the *pre-default* and *on-default* survival probability of the reference entity do not match any more. In such scenario even continuous collateralization schemes would be unable to fully eliminate counterparty credit risk. The Authors remark that this holds true for credit products such as CDS but not for others, for instance Interest Rate Swaps (IRS). In the IRS case in fact, continuously margining of collateral fully eliminates the CCR.

In their work, the Authors use a reduced-form credit model, in which the stochastic intensities of the counterparts involved in the transaction as well as the one of the CDS reference entity are defined as:

$$\lambda_t^i = y_t^i + \psi_t^i, \quad i \in \{B, C, E\} \tag{1.1}$$

where λ^i are the default intensities of the bank B, the counterpart C and the reference entity E, respectively. ψ^i are deterministic non negative shifts and y^i are driven by Cox-Ingersoll-Ross (CIR) square-root diffusion processes:

$$dy_t^i = \kappa^i(\mu^i - y_t^i)dt + \nu^i\sqrt{y_t^i}dW_t^i, \quad i \in \{B, C, E\} \tag{1.2}$$

In order to simplify the parametrization and focus on default correlation rather than spread correlation, the three independent Brownian Motions W^i are assumed to be independent under the risk-neutral measure. As I will refer in the following, the Authors correlate default processes by means of a Copula structure. In fact, they adopt a classical doubly stochastic Cox setting:

$$\tau_i = (\Lambda^i)^{-1}(\xi_i) \quad i \in \{B, C, E\}$$

where the cumulate intensity ξ_i up to default time τ_i, defined as $\Lambda^i(\tau) = \int_0^\tau \lambda_s ds$, is a standard exponential random variable. In other words, ξ_i represents a transformation of firm i default time τ_i. They than impose a trivariate Gaussian Copula function $C_R(u_B, u_E, u_C) \equiv \mathbb{Q}\{U_B < u_B, U_E < u_E, U_C < u_C\}$ on the uniform random variables $U_i \equiv 1 - \exp\{-\xi_i\}$ associated to the firms cumulated intensities ξ_i. The Gaussian Copula is parametrized via a correlation matrix R. By denoting $\Phi^t_{\mathrm{CIR},i}(x)$ the cumulative distribution function (CDF) of the cumulated shifted CIR process $\Lambda^i(t)$ evaluated at x and setting $\Upsilon(z) \equiv -\log(1 - \Phi(z))$ they obtain the copula model:

$$C_R(u_B, u_E, u_C) = \mathbb{E}^{\phi_R}[\Phi_{\mathrm{CIR},B}(\Upsilon(u_B))\Phi_{\mathrm{CIR},E}(\Upsilon(u_E))\Phi_{\mathrm{CIR},C}(\Upsilon(u_C))]$$

where ϕ_R denotes the density of the standard Gaussian vector (Z_B, Z_C, Z_E) and R is the correlation matrix.

In numerical part of their work, Brigo et al. (2014) calculate the CCR valuation adjustments on a 5y-CDS contract traded by two defaultable parties. They analyse three different credit risk levels (*low, mid, high*) and three scenarios for what concerns collateral: quarterly margining frequency, continuous margining or no collateralization at all. The analysis embraces both the cases of allowance and prohibition of rehypotecation. In the paper it is showed that, if the CDS protection buyer is largely in the money, its CVA component is expected to be relevant in all cases except that of zero default correlation, in which continuous collateralization is effective in mitigating CCR risk. On the contrary, as the correlation between the counterpart and the reference entity defaults increases, neither of them likely to go bankruptcy alone. The joint default of these two firms would cause a jump in bank exposure. That is the reason why continuous collateralization might be almost ineffective. The Authors shows that, as defaults dependence grows, the CVA under continuous collateral margining is similar in magnitude to that of the uncollaterized case. It takes place an instantaneous *contagion effect* which would drive the default probability of the survived entities jumping as the first credit event occurs. As default dependence increases, the term structure of on-default and pre-default survival probabilities diverge significantly. Such a potential instantaneous growth of the CDS value, jumping at default, would make it significantly higher than its value in correspondence of the last collateral margining date. That explains the root of collateral ineffectiveness under correlated defaults. The Authors conclude that in some cases the contagion is so relevant to modify CVA and DVA patterns, a distinctive feature of Copula frameworks to bear in mind when modelling default dependences. Nevertheless, Copula models are convenient in terms of easiness of simulation and capability to disentangle dependences blocks. CCR adjustments exhibit to be monotonic in correlation, which is responsible of a contagion effect invalidating the efficacy even of full collateralization. Acknowledging the existence of gap risk is the reason which led regulators, as it will be discussed, to introduce the additional layer of collateralization represented by Initial Margins.

Also in Brigo et al. (2018) joint stochasticity is assumed in order to introduce a structure of dependences between interest rates and credit dynamics in the case of estimating CCR adjustments for Interest Rate Swaps (IRS). The interest rate sector is modelled according to the Gaussian Shifted Two-factors short-rate process (G2++), while default intensities are modelled via the Shifted Square-root Diffusion CIR process in line with Brigo and Chourdakis (2009).

More in details, the instantaneous short-rate process under the risk-neutral measure is given by:

$$r_t = x_t + z_t + \varphi(t, \alpha), \quad r(0) = r_0 \tag{1.3}$$

where x and z are $\mathcal{F}_t$-adapted processes which satisfy:

$$\begin{aligned} dx_t &= -ax_t dt + \sigma dW_t^1, \quad x(0) = x_0 \\ dz_t &= -bz_t dt + \eta dW_t^2, \quad z(0) = z_0 \end{aligned} \tag{1.4}$$

(W_1, W_2) is a correlated two-dimensional Brownian Motion such that:

$$d\langle W_1, W_2 \rangle_t = \rho_{1,2} dt, \quad -1 \leq \rho_{1,2} \leq 1$$

They denote by $\alpha = [r_0, a, b, \sigma, \eta, \rho_{1,2}]$ the vector of positive parameters entering in φ. Interest rate models parameters are calibrated to market-observed zero coupon curves and swaptions volatilities.

As the credit part is concerned, the default dynamics of the two firms are modelled in a equivalently to the previously discussed article, i.e. according to equations 1.1 and 1.2. As a difference, Brigo et al. (2018) deal with wrong-way risk by means of diffusive correlation between interest rate factors x and z and intensity processes y^i, $i \in \{B, C\}$, so that instantaneous correlation is given by:

$$d\langle W_j, W_i \rangle_t = \rho_{i,j} dt, \quad j \in \{1, 2\}, \quad i \in \{B, C\}$$

In order to reduce the number of parameters, the Authors set homogeneous correlations when modelling the dependences between short-rate factors and default intensities, such that interest rates/credit-spreads correlations are:

$$\rho_{1,i} = \rho_{2,i} \equiv \bar{\rho}_i, \quad i \in \{B, C\} \tag{1.5}$$

As the previously discussed paper, the Authors prefer to model default correlation rather than spread correlation. In fact, a Gaussian Copula on default times is preferred to correlating default intensities. By applying some required transformations to random default times, they simulate the correlated survival indicators by sampling them from a bivariate standard normal distribution:

$$\mathbb{1}_{\{\tau^i > t\}} = \mathbb{1}_{\{U^i > \exp\{\Lambda^i(t)\}\}}, \quad U^i \equiv \Phi(z^i), \quad i \in \{B, C\}, \quad (z^B, z^C) \sim \mathcal{N}_2(\rho_G)$$

where Φ is the standard normal cumulative distribution function and $\mathcal{N}_2(\rho_G)$ is a bivariate standard normal distribution with correlation parameter ρ_G.

In the numerical part of the paper, Brigo et al. (2018) investigate the bilateral CCR valuation adjustments as a function the time between two consecutive collateral updates. Margining frequency is allowed to vary between one week up to six months. They consider both the cases of allowance rather than prohibition of rehypotecation. The Authors consider a 10-years IRS with payment frequency of one year for the fixed leg and of six months for the floating leg indexed to EURIBOR rates. They estimate BCCVA sensitivities with respect to collateral margining frequency. The calculation confirms that in the case of rehypotecation, CCR adjustments are exacerbated as a consequence of unsecured collateral posting. Surprisingly, under rehypotecaton the expected exposure might exceed the uncollateralized case. A remarkable contribution regards the analysis of the relation between margining frequency and the previously described credit spread/interest rate and default times correlations. The Authors found a positive impact in BCCVA of payer IRS deriving from correlating credit spreads and interest rates in the case. Interestingly, the effect of varying credit spread volatilities is smaller compared that due to credit spread and interest rates correlations. The impact of credit spread volatility on bilateral CCR adjustments seems be reversed according to the sign of correlation.

In Brigo and Pallavicini (2008) a similar setting is adopted in order to evaluate Counterparty Credit Risk for a plethora of interest rates derivatives, such as IRS, IRS portfolios, European and Bermudan Swaptions, Constant Maturity Swap (CMS) Spread Options and Contingent Credit Default Swaps (CCDS). In the paper they point out that, although CDS volatilities are not so liquid, they are typically higher compared to those obtainable by the CIR++ model. In order to match historical series of market CDS volatilities a solution might be the inclusion jumps. These would increase the volatility generated by the CIR++ model. For this reason, they make use of exponential jumps in order to build the JCIR++ model which they use for modelling the default intensity sector:

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dW_t + dJ_t(\zeta_1, \zeta_2) \tag{1.6}$$

where the jump part $J_t(\zeta_1, \zeta_2)$ is defined as:

$$J_t(\zeta_1, \zeta_2) \equiv \sum_{i=1}^{M_t(\zeta_1)} X_i(\zeta_2) \tag{1.7}$$

where M is a time-homogeneous Poisson process with intensity ζ_1, independent form W while X is exponentially distributed random variable with positive finite mean ζ_2, independent from M and W. The Authors decide to calibrate the diffusive part of the JCIR++ model to market CDS quotes and assume levels for the parameters ζ_1 and ζ_2 able reproduce realistic scenarios for CDS curves. In the numerical part of the work, Brigo and Pallavicini (2008) compute the payoff of several interest rates derivatives via joint Monte Carlo simulations of the stochastic factors x, z and y (see above). Among other payoffs, the Authors focus also on Contingent Credit Default Swaps (CCDS), which are contract paying upon the default of a reference entity the loss given default of a predetermined portfolio, if positive. Pricing the default leg of a CCDS issued on a given portfolio Π is equivalent to estimating the unilateral CVA of that portfolio. In literature, the mathematical shape of CCDS has made them to be considered elective instruments to hedge Counterparty Credit Risk. The Authors have found that the unilateral CVA decreases with correlation for receiver payoffs (IRS, IRS portfolios, European and Bermudan Swaptions) since, if default intensities increase, in presence of positive correlation the underlying interest rates increase making the mark-to-market of receiver contracts, upon which exposures at default are computed, being less likely in the money. On the other side, the CCR adjustment for payer payoffs increases with correlation. They conclude by arguing that the impact of CCR adjustments is relevant across many interest rate products they have analysed and that correlation patterns are relevant in turn. Nevertheless, they distinguish between the cases of low and high CDS implied default probabilities. In fact, when market implied PD are high, their contribution is predominant and the effect of other fine details is wiped out. On the contrary, when CDS implied PD are not extremely high, fine details such as the fine structure of factor dynamics and correlations in particular are more important.

A early work on Counterparty Credit Risk valuation applied to energy commodity trades in presence of correlation between market and credit risks is the one of Brigo and Bakkar (2009). The Authors focus on oil but much of their reasoning can be adapted to other commodities with similar characteristics in terms of storability, liquidity and seasonality. In their analysis of default-risky Forward and Swap contracts traded between an airline company and a very high credit quality bank, they exploit a reduced-form approach in the presence of correlation between credit spreads and the underlying commodity in order to quantify the unilateral CVA. The Authors firstly interpret the impact of CCR as the difference between Futures and Forward prices, since the firsts unlike the seconds, are subjected to margining procedures. They adopt the CIR++ framework for modelling stochastic hazard rates as in 1.2 and the Smith-Schwartz model for describing the dynamics of the underlying oil spot log price[5]:

$$\ln S_t = x_t + L_t + \varphi_t \tag{1.8}$$

[5] The existence itself of oil spot prices an assumption since they are not quoted on the market.

where under the risk-neutral measure:

$$\begin{aligned} dx_t &= -k_x x_t dt + \sigma_x dW_t^x \\ dL_t &= -\mu_L dt + \sigma_L dW_t^L, \quad d\langle W^x, W^L \rangle_t = \rho_{x,L} dt \end{aligned} \tag{1.9}$$

where φ is a deterministic shift used to calibrate oil futures quotes, x represent the short-run deviation whereas L is the backbone to the equilibrium price level in the long run. The dependence between market and credit risks is set as:

$$d\langle W_x, W_y \rangle_t = \rho_{x,y} dt, \quad d\langle W_L, W_y \rangle_t = \rho_{L,y} dt$$

In the light of the difficulties one incurs when trying to perform historical estimations of market correlations, they prefer to set $\rho_{x,y} = \rho_{L,y} = \bar{\rho}$.

Within their case study involving a defaultable airline company and an high credit quality bank, the Authors suppose that the airline is entering into the trade for risk management purposes. The Authors isolates the *receiver* case (right-way risk) from the *payer* case (wrong-way risk) and underline the prominent role of both correlation and volatilities. The adjustment they found ranges between roughly 1.5% and 6% depending on volatility and correlation levels. In the concluding remarks, they argue that the effective quantification of CCR cannot be carried out by predetermines multipliers, as it had been done in previous years.

1.1.2 Main contributions

The present work differs from Ballotta et al. (2015) by focusing on the equity rather then on the firm value, since the former has the appealing feature of being a tradable asset whose market value is observable. Another relevant contribution of the present Chapter pertains to the theoretical side. By applying some asset pricing principles, a general formula for the pricing of bilateral CCR Adjustments (i.e CVA and DVA) is derived. The solution is model-independent, in the sense it applies both to structural or reduced-form default modelling. Moreover, in my analysis no particular setting for the replication strategy is needed.

A numerical application has been performed in order to compute the price correction for Counterparty Credit Risk. In this work it has been applied the Fourier Cosine Series (COS) method introduced by Fang and Oosterlee (2008) since, as it was showed in the computational finance literature, it is more efficient compared to other Fourier-based methods such as the FFT (see Carr and Madan (1999)) or the CONV Method, see Lord et al. (2008). Both baseline *single-sided* and *double-sided* energy commodities contingent claims are taken into account. From the computational point of view, my contribution is to implement a hybrid numerical procedure which relies on the COS method for calibrating the relevant parameters sets including barriers and for computing Exposures-at-Defaults (EAD). A joint Monte Carlo simulation is used to experimentally address the dependence between the two firms implied by the so-called *first-to-default* problem. Furthermore, I extend the COS formula to Forwards and later to Swaps, interpreted as portfolio of Forwards with different maturities.

The Chapter is organized as follows: in Section 1.2 it is described the technical setup required for the analysis. In Section 1.3 I straightforwardly derive a model-independent additive formula for BVA pricing which does not require any specific assumptions about the hedging strategy. In Section 1.4 after the introduction of Fourier pricing, calibration results and absolute as well as relative sizes of the CVA/DVA metrics are discussed. Section 1.5 draws some concluding remarks.

1.2 *Model setup*

In a bilateral CCR perspective, let me introduce two defaultable parties involved in a OTC derivative deal[6] denoted by the investment bank B (the *dealer*) and its client C (which might be a corporate firm or another bank).

The market is modelled through a filtered probability space $(\Omega, \mathbb{Q}, \mathcal{G}_t)$ where Ω is the set of possible events, $\mathbb{Q}$ is some *risk-neutral* martingale measure and the global filtration is defined as $\mathcal{G}_t \equiv \mathcal{F}_t \vee \mathcal{H}_t$[7].

$\{\mathcal{F}_t\}_{0\leq t\leq T}$ is the reference filtration which contains all market information except of credit events up to time t while $\{\mathcal{H}_t\}_{0\leq t\leq T}$ is the $\sigma(\tau \wedge t)$-algebra generated by the default processes. τ is the *first-to-default* random time defined as $\tau \equiv \tau_B \wedge \tau_C$. It is noted that τ is a $\mathcal{G}$-stopping time and the above filtrations satisfy the usual conditions of completeness and right-continuity.

An infinitely divisible Lèvy process $\{X_t\}_{0\leq t\leq T}$ defined on the filtered probability space $(\Omega, \mathbb{Q}, \mathcal{G}_t)$ is a stochastic process with stationary and independent increments, whose distribution allows for non-zero skewness and excess kurtosis[8]. For the class of Lèvy processes, it is often impossible to determine the PDF analytically.

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[6] Typically counterparts trade a portfolio of deals, so that the eventual contractual provision of some *Netting scheme* would definitely mitigate the aggregate CCR risk of the portfolio.

[7] Actually, as well explained by Brigo et al. (2013) in Chapter 3, in the context of structural models, $\mathcal{F}$ and $\mathcal{G}$ coincide by construction since default processes are completely driven by default-free market information.

[8] Higher then second order moments describe the deviation from a Gaussian setting.

Nevertheless, their characteristic function is always available in closed form thanks to the well-known Lèvy-Khintchine representation:

$$\phi_X(u;t) = \mathbb{E}[e^{\imath u X_t}] = e^{t\varphi_X(u)}, \quad u \in \mathbb{R} \tag{1.10}$$

where $\varphi_X(u)$ is called *characteristic exponent* of parameter u with the process $\{X_t\}_{t\geq 0}$[9].

The Normal Inverse Gaussian (NIG) process introduced by Barndorff-Nielsen (1997) is a *pure jump* Lèvy process which can be obtained by subordinating a Geometric Brownian Motion (GBM) to an independent Inverse Gaussian (IV) process $IG(t)$. The Inverse Gaussian process, alternatively known as the distributional law of first-passage time of the Brownian Motion, is a *α-stable subordinator* with $\alpha = \frac{1}{2}$.

The NIG process, similarly to other ones which are obtainable by subordination, is said to be a *time-changed* Lèvy process: in other words, it is indexed to a "stochastic clock". Hence the NIG Process presents the following form:

$$X_t = \mu IG_t + \sigma W_{IG_t} \tag{1.12}$$

Building Lèvy processes by brownian subordination is particularly appealing from an economic point of view since the time-change might be interpreted as the switch from calendar time to *business time*. This implies to assume that asset prices are mainly driven by the relevant news, characterized by random arrival times and random impact on the market. Indeed, empirical evidence suggests that gaussianity seems to be recovered under such trading time. The characteristic exponent in the case of NIG process is:

$$\varphi_X(u) = \frac{1 - \sqrt{1 - 2\imath\theta\kappa + u^2\sigma^2\kappa}}{\kappa} \tag{1.13}$$

where u is the Fourier parameter, $\imath$ is the imaginary unit. As for the meaning of parameters is concerned, $\theta \in \mathbb{R}$ describes the sign of the skewness of the distribution, $\sigma > 0$ is the volatility parameter and $\kappa > 0$ controls the excess kurtosis of the distribution. Moreover, non Gaussian Lèvy processes have been largely applied in financial modelling because of their superior capability compared to that of pure diffusion models to represent the stylized behaviour observed in real markets. More specifically, Lèvy processes can replicate implied volatility surfaces without overstress model parameters and can accommodate

[9] In coherence with the pioneering contribution of Black and Scholes (1973), within the family of Lèvy processes the Geometric Brownian Motion (GBM) has been the most celebrated by both academic researchers and industrial practitioners. The characteristic exponent which uniquely defines the GBM is:

$$\varphi_X(u) = -\frac{u^2\sigma^2}{2} \tag{1.11}$$

where σ is the volatility parameter. Nevertheless, empirical evidence has shown that asset returns (i.e. log prices) are rarely described by a Gaussian distribution.

for jumps, see Cont and Tankov (2004). The presence of jumps in the path of risky assets implies that the market is in general incomplete[10].

In this framework the NIG process X_t is the relevant *risk driver* for the uncertain dynamics of the underlying asset S_t, whose price at time t under the risk neutral measure $\mathbb{Q}$ is:

$$S_t = S_0 e^{(r-q-\varphi_X(-i))t+X_t} \tag{1.14}$$

where $r > 0$ is the proxy for the risk-free rate (e.g. EURIBOR/EONIA) and $q > 0$ is the continuous dividend yield paid by the underlying stock. $\mu = r-q-\varphi_X(-i)$ is the *mean-correcting drift* needed to allow S_t to be an exponential martingale.

In figure 1.1 it is shown a simulated five years weekly path of equation 1.14, where I remind X_t is a NIG:

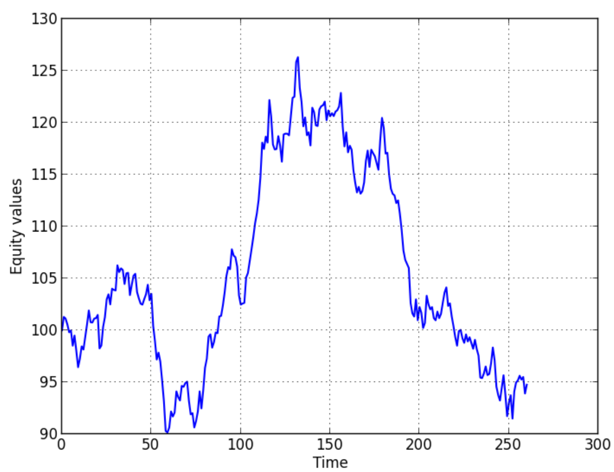

Fig. 1.1: Example of a five years weekly simulated path for a NIG process.

Coherently with the work of Ballotta and Fusai (2015), I further assume that the risk driver X_t can be disentangled as follows:

$$X_t = Y_t + aZ_t \tag{1.15}$$

where Y_t is a Lèvy process describing the *idiosyncratic* part or the risk, Z_t is an independent Lèvy process describing the *systemic* risk component and a is the regression coefficient relative to the systemic risk. This setup is robust from the economic perspective and allows, after conditioning to the path of the

[10] Market incompleteness means that it is not possible to replicate every uncertain derivative payoff in the market through a combination of elementary assets: hence, so that derivative claims are not redundant.

systemic risk factor Z_t, to describe the full structure of dependence between the two firms i and j simply through the index of linear pairwise correlation[11]:

$$\rho_{ij} = a_i a_j \frac{\mathbb{V}ar[Z(1)]}{\sqrt{\mathbb{V}ar[X^i(1)]\mathbb{V}ar[X^j(1)]}}$$

Within my structural approach the random default time τ is modelled a là Black and Cox (1976), i.e. as the *first-passage time* of the firm's equity at the level of a fixed default trigger barrier:

$$\tau_i = \inf\{t \in (0,T] : S_t^i \leq K^i\}, \quad i \in \{B, C\} \tag{1.16}$$

I assume that equation 1.14 describes also the dynamics of equity values of firms B and C in addition to those of the underlying assets. Then, replacing it in equation 1.16 it leads to:

$$\begin{aligned}\tau_i &= \inf\{t \in (0,T] : S_0^i e^{(r-q^i-\varphi_{X^i}(-i))t+X^i(t)} \leq K^i\} \qquad (1.17)\\ &= \inf\{t \in (0,T] : X_t^i \leq \log\left(\frac{K^i}{S_0^i}\right) - (r - q^i - \varphi_{X^i}(-i))t\}\\ &= \inf\left\{t \in (0,T] : Z_t^i \leq \frac{\log\left(\frac{K^i}{S_0^i}\right) - (r - q^i - \varphi_{X^i}(-i))t - Y_t}{a_i}\right\}\end{aligned}$$

The last two rearrangements highlight how, by disentangling the systemic and the idiosyncratic components of the risk, the barriers triggering credit events can be configured both as *time-dependent* and *stochastic*. Nevertheless, as I will explain later in details, setting a fixed barrier as denoted by equation 1.17 is convenient in terms of mathematical tractability[12].

1.3 Bilateral Counterparty Credit Risk (CCR) pricing

In this Section, I derive a representation of CCR risk metrics via a straightforward and model-independent framework. The result applies to both structural and reduced-form default modelling approaches. No assumption for the hedging of the *jump-to-default* risks is required. Furthermore, I provide an intuitive economic interpretation of such metrics which configure as quite complex exotic derivatives.

In presence of Counterparty Credit Risk it is needed to distinguish between $V(t,S)$, which denotes the price at time t of the *default-free* derivative contract and $\hat{V}(t,S,D_B,D_C)$, which denotes the price of the equivalent default-risky contingent claim. The state processes $\{D_t^i\}_{0\leq t\leq T}$ indicate the occurrence of the respective credit events:

$$D_t^i \equiv \mathbb{1}_{\{\tau_i \leq t\}}, \quad i \in \{B, C\}$$

[11] The structure of dependences which lead to wrong and right-way risk in the context of structural Lèvy models, has already been exhaustively investigated in previous literature.
[12] Simulating the stochastic case is destined to future research.

In the case of no default, at maturity time T the buyer will receive or make the contractual payment correspondent to the derivative payoff $\hat{\Phi}(S_T)$. The dealer will observe an opposite cash flow $-\hat{\Phi}(S_T)$.

According to the International Swaps and Derivatives Association (ISDA) 2002 Master Agreement upon default the surviving party, in the case of being *out of the money*, is obliged to pay all its debt. Conversely, in the case of being *in the money*, the surviving party can claim just a recovery fraction of her credit.

PROPOSITION 1. *Let me assume that the computation is based on the risk-free close-out, i.e. the Exposure at Default (EAD) denoted by ε_τ, equals to the mark-to-market of the default-free contract. In compliance with ISDA (2002) the following border conditions hold at the stopping times τ_C and τ_B:*

$$\begin{aligned} \hat{V}_{\tau_C} &= R_C(\varepsilon^+_{\tau_C}) - \varepsilon^-_{\tau_C} \\ \hat{V}_{\tau_B} &= \varepsilon^+_{\tau_B} - R_B(\varepsilon^-_{\tau_B}) \end{aligned} \tag{1.18}$$

then according to the Asset Pricing Theorem (APT), the price at time t of a counterparty-risky derivative claim $\hat{V}$ is:

$$\hat{V}_t = V_t - \underbrace{\mathbb{E}^{\mathbb{Q}}_t[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)(\varepsilon^+_{\tau_C} - R_C(\varepsilon^+_{\tau_C}))]}_{CVA} + \underbrace{\mathbb{E}^{\mathbb{Q}}_t[\mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)(\varepsilon^-_{\tau_C} - R_B(\varepsilon^-_{\tau_B}))]}_{DVA} \tag{1.19}$$

where V_t is the default-free value of the contract, D is the risk-free discount factor[13] *and R_C and R_B are respectively the client and the bank's recovery functions upon respective defaults.*

Proof. According to the APT the risk-neutral price of an asset is equal to the expected value of its discounted future payoff under some suitable pricing measure $\mathbb{Q}$.

In the case of a counterparty-risky derivative it is necessary to distinguish among possible scenarios of default or no default:

$$\hat{V}_t = \mathbb{E}^{\mathbb{Q}}_t[\mathbb{1}_{\{\tau>T\}} D(t,T)\hat{V}_T + \mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \tag{1.20}$$

where $\mathbb{1}_A$ denotes the indicator function of the event A. At expiry T the economic value of a derivative claim is simply its payoff, i.e. $\hat{V}_T = \hat{\Phi}(S_T)$. Since no premature default has occurred, it holds $\hat{\Phi}(S_T) = \Phi(S_T)$. By substituting this results in 1.20 I obtain:

$$\begin{aligned} \hat{V}_t &= \mathbb{E}^{\mathbb{Q}}_t[\mathbb{1}_{\{\tau>T\}} D(t,T)\hat{\Phi}(S_T) + \mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \\ &= \mathbb{E}^{\mathbb{Q}}_t[\mathbb{1}_{\{\tau>T\}} D(t,T)\Phi(S_T) + \mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \\ &= \mathbb{E}^{\mathbb{Q}}_t[(1-\mathbb{1}_{\{\tau\leq T\}}) D(t,T)\Phi(S_T) + \mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \\ &= V_t - \mathbb{E}^{\mathbb{Q}}_t[\mathbb{1}_{\{\tau\leq T\}} D(t,T)\Phi(S_T)] + \mathbb{E}^{\mathbb{Q}}_t[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \end{aligned}$$

[13] Under the independence hypothesis, the risk-free discount factor can be dragged out from the expectation and equals to the price of a zero-coupon bond, i.e. $D(0,t) = \mathbb{E}^{\mathbb{Q}}\left[\exp\left(-\int_0^t r_s ds\right)\right]$. r is proxied by the Eonia/EURIBOR rates.

Applying the Law of Iterated Expectation to the second term of the right-hand side it is then possible to exploit that the event $\mathbb{1}_{\{\tau\leq T\}}$ is $\mathcal{G}$-adapted while the derivative payoff $\Phi(S_T)$ is $\mathcal{F}$-adapted and independent from $\mathcal{H}$ it can be obtained:

$$\begin{aligned}
\hat{V}_t &= V_t - \mathbb{E}_t^{\mathbb{Q}}[\mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\{\tau\leq T\}} D(t,T)\Phi(S_T) \mid \mathcal{G}_\tau]] \\
&\quad + \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \\
&= V_t - \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau\leq T\}} D(t,\tau)\mathbb{E}^{\mathbb{Q}}[D(\tau,T)\Phi(S_T) \mid \mathcal{F}_\tau]] \\
&\quad + \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \\
&= V_t - \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau\leq T\}} D(t,\tau)V_\tau] \\
&\quad + \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \\
&= V_t - \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)V_{\tau_C}] \\
&\quad - \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)V_{\tau_B}] + \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)\hat{V}_{\tau_C} + \mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)\hat{V}_{\tau_B}] \\
&= V_t - \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)(V_{\tau_C} - \hat{V}_{\tau_C})] - \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)(V_{\tau_B} - \hat{V}_{\tau_B})]
\end{aligned}$$

Finally, substituting the ISDA border conditions 1.18 it can be showed:

$$\hat{V}_t = V_t - \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)(\varepsilon^+_{\tau_C} - R_C(\varepsilon^+_{\tau_C}))] + \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)(\varepsilon^-_{\tau_C} - R_B(\varepsilon^-_{\tau_B}))]$$

□

The **Credit Valuation Adjustment (CVA)** appears to be a call option with zero strike and random maturity issued on the uncollaterized exposure and represents the expected loss on banks' credits due to counterparty default risk. Conversely, the **Debt Valuation Adjustment (DVA)** appears to be a put option with zero strike and random maturity issued on the uncollateratized exposure and represents the expected gain on bank's debts due to its own default risk. This explains why many authors argue that Bilateral Counterparty Credit Risk adds two optionality levels to the derivative payoff.

1.4 *Numerical application of a hybrid Fourier - Monte Carlo procedure*

In Subsection 1.4.1 I first introduce the theoretical setup of the Fourier Cosine Series (COS) method for European Options and then in Subsection 1.4.2 it is described its recursive application for pricing single name CDS. While the first methodology has been used for computing Exposures at Default (EAD), the second has been required for calibrating the level of the default barriers. Subsection 1.4.3 provides the results for both calibrations and CCR metrics.

1.4.1 *Reviewing the COS method for European Options*

The main issue in solving pricing problems grounded on a time-changed Lèvy process such as the NIG is related to the unknown form of its conditional probability density function.

The main advantage of Fourier-based methods as pricing tool is related to the possibility to solve semi-analytically pricing problems within a broader class of underlying processes, i.e. those for which the characteristic function is known in closed form. This feature makes Fourier-based pricing often necessary in practical applications. State-of-the-art numerical integration techniques commonly rely on a transformation to the Fourier domain. Nevertheless, Fang and Oosterlee (2008) while introducing the Fourier Cosine Series (COS) method showed that, among the other Fourier-based pricing methodologies available in literature, their proposal guarantees higher efficiency, for instance in comparison to the Fast Fourier Transform (FFT) proposed by Carr and Madan (1999) or the Convolution Method (CONV), see Lord et al. (2008). In fact, quadrature rule based techniques do not provide the highest efficiency when solving Fourier transformed integrals since, as the integrands are highly oscillatory, a relatively fine grid has to be used in order to obtain satisfactory accuracy with the FFT. As the Authors explain in their work, Fourier Cosine expansions in the context of numerical integration represent an alternative with respect to methods based on the FFT. Other highly efficient techniques for pricing plain vanilla options can be retrieved in the Fast Gauss Transform (see Broadie and Yamamoto (2003)) and in the Double-Exponential Transformation (see Mori and Sugihara (2001)). The COS method can however handle more general dynamics for the underlying compared to these methodologies. In the paper of Fang and Oosterlee (2008) the derivation of the methodology has been accompanied by an error analysis. In several numerical experiments, the convergence rate of the COS method has shown to be exponential, while the computational complexity just is linear in the number of terms chosen in the Fourier Cosine series expansion.

I now introduce the method in detail since it has been applied in its original formulation for computing EAD and for the underlyings calibration. The recursive COS scheme for CDS, explained in next Subsection, has been applied for calibrating firms' parameters sets and the default barriers.

The COS method provides a different approach for computing inverse Fourier integrals, which consists in recovering the unknown conditional PDF through its cosine series expansion. This methodology is well suited for smooth densities (a propriety which holds for Lèvy processes) defined on a finite support[14].

As usual, the starting point is the risk-neutral valuation formula:

$$v(x,t) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[(y,T) \mid x] = e^{-r(T-t)} \int_{\mathbb{R}} v(y,T)f(y \mid x)dy \tag{1.21}$$

where $v(y,T)$ is the contract-specific payoff function and $f(y \mid x)$ is the conditional PDF of the underlying asset. Since in numerical applications would be pointless to compute an integral over a infinite domain, it is needed to truncate it to a finite one that properly approximates its infinite counterpart, i.e.:

$$v(x,t) \simeq e^{-r(T-t)} \int_a^b v(y,T)f(y \mid x)dy \tag{1.22}$$

being $\mid a \mid$ and $\mid b \mid$ large enough. As regards the choice of the truncation interval is concerned, the Authors suggest:

$$[a,b] = \left[c_1 - L\sqrt{c_2 + \sqrt{c4}} \quad , \quad c_1 + L\sqrt{c_2 + \sqrt{c4}}\right], \quad L = 10$$

where c_i is the i-th cumulant of $Y_T(y)$. The next step is to replace the integrand function with its cosine series expansion and in this regard the reader might feel useful to be reminded the following result.

Definition 1.4.1 (Fourier Cosine Series)**.** *Let $f : [a,b] \to \mathbb{R}$, its cosine series expansion is given by:*

$$f(x) = {\sum_{k=0}^{\infty}}' A_k \cos\left(k\pi \frac{x-a}{b-a}\right) \tag{1.23}$$

with

$$A_k(x) = \frac{2}{b-a} \int_a^b f(x) \cos\left(k\pi \frac{x-a}{b-a}\right) dx \tag{1.24}$$

where $\sum'$ indicates that the first term in the summation is halved. In order to apply this methodology to the derivatives pricing let me denote by F_k and V_k the *k-th* coefficients of $f(y \mid x)$ and $v(y,T)$, respectively. Applying this result in 1.22 I have:

$$v(x,t) = e^{-r(T-t)} \int_a^b v(y,T) {\sum_{k=0}^{\infty}}' F_k(x) \cos\left(k\pi \frac{y-a}{b-a}\right) dy$$

Then, interchanging the integration and summation operators it can be obtained:

[14] The usual Fourier series expansion is actually superior when a function is periodic, see Fang and Oosterlee (2008).

$$v(x,t) = e^{-r(T-t)} \sum_{k=0}^{\infty}{}' \int_a^b v(y,T) \cos\left(k\pi \frac{y-a}{b-a}\right) F_k(x) dy \tag{1.25}$$

Since by definition it holds:

$$V_k = \frac{2}{b-a} \int_a^b v(y,T) \cos\left(k\pi \frac{y-a}{b-a}\right) dy$$

by replacing this result in equation 1.25 I get:

$$v(x,t) = \frac{b-a}{2} e^{-r(T-t)} \sum_{k=0}^{\infty}{}' V_k F_k(x)$$

In the light of the rapid decay of Fourier coefficients, it is possible to truncate the series to the first N terms (N is typically set as some power of 2):

$$v(x,t) = \frac{b-a}{2} e^{-r(T-t)} \sum_{k=0}^{N-1}{}' V_k F_k(x)$$

F_k **coefficients.** As regards the computation of F_k coefficients of the conditional PDF, the idea is to approximate the finite integral in equation 1.24 through its infinite counterpart:

$$\begin{aligned}
F_k(x) =& \frac{2}{b-a} \int_a^b f(y \mid x) \cos\left(k\pi \frac{y-a}{b-a}\right) dy \\
\simeq& \tilde{F}_k(x) \\
=& \frac{2}{b-a} \int_{\mathbb{R}} f(y \mid x) \cos\left(k\pi \frac{y-a}{b-a}\right) dy \\
=& \frac{2}{b-a} \Re\left\{ \int_{\mathbb{R}} f(y \mid x) \left[\cos\left(k\pi \frac{y-a}{b-a}\right) + i \sin\left(k\pi \frac{y-a}{b-a}\right)\right] dy \right\} \\
=& \frac{2}{b-a} \Re\left\{ \int_{\mathbb{R}} f(y \mid x) e^{ik\pi \frac{y-a}{b-a}} dy \right\}
\end{aligned}$$

By the Euler's formula I have:

$$\begin{aligned}
F_k(x) =& \frac{2}{b-a} \Re\left\{ e^{-\frac{ik\pi a}{b-a}} \int_{\mathbb{R}} f(y \mid x) e^{i \frac{k\pi}{b-a} y} dy \right\} \\
=& \frac{2}{b-a} \Re\left\{ e^{-\frac{ik\pi a}{b-a}} \phi_{y|x}\left(\frac{k\pi}{b-a}\right) \right\}
\end{aligned}$$

where $\phi_{y|x}(u)$ is the characteristic function of parameter u of the model being used, which is obtainable in closed form. Focusing on log-strike prices, by some simple passages:

$$x \equiv \log\left(\frac{S_0}{K}\right) \quad \text{and} \quad y \equiv \log\left(\frac{S_T}{K}\right).$$

$$\begin{aligned}
\phi_{y|x}(u) =& \mathbb{E}\left[e^{iuy}\right] \\
=& \mathbb{E}\left[e^{iu\log\left(\frac{S_T}{K}\right)}\right] \\
=& \mathbb{E}\left[e^{iu\log\left(\frac{S_0 e^{\mu T + X_T}}{K}\right)}\right] \\
=& \mathbb{E}\left[e^{iu(x+\mu T + X_T)}\right] \\
=& e^{iu(x+\mu T)}\mathbb{E}\left[e^{iuX_T}\right] \\
=& e^{iu(x+\mu T)}\phi_{X_T^{NIG}}(u, T)
\end{aligned}$$

Therefore, the characteristic function of the variable $y \mid x$ as defined above is easily recoverable from the one of the NIG process. Being all the elements well specified, the pricing rule reads:

$$v(x,t) = e^{-r(T-t)} \sum_{k=0}^{N-1}{}' \Re\left\{e^{-\frac{ik\pi a}{b-a}}\phi_{y|x}\left(\frac{k\pi}{b-a}, T\right)\right\} V_k \tag{1.26}$$

Equation 1.26 represents the general pricing formula under the COS method. Finally, when evaluating a specific derivative claim it is necessary to compute its V_k coefficients.

V_k **coefficients.** The payoff at maturity of European-style options in log-strike prices, which in my model is required for the calculation of the exposures relevant for CVA and DVA, is:

$$v(y,T) = \left[\alpha K(e^y - 1)^+\right] \quad \text{with} \quad \alpha = \begin{cases} 1, & \text{for a Call} \\ -1, & \text{for a Put} \end{cases}$$

Following Fang and Oosterlee (2008), let me define the following functions:

$$\chi_k(c,d) \equiv \int_c^d e^y \cos\left(k\pi\frac{y-a}{b-a}\right) dy \tag{1.27}$$

and

$$\psi_k(c,d) \equiv \int_c^d \cos\left(k\pi\frac{y-a}{b-a}\right) dy \tag{1.28}$$

Above integrals can be solved analytically by basic calculus[15]. Their solutions are:

$$
\begin{aligned}
\chi_k(c,d) = {} & \frac{1}{1+\left(\frac{k\pi}{b-a}\right)^2}\left[\cos\left(k\pi\frac{d-a}{b-a}\right)e^d - \cos\left(k\pi\frac{c-a}{b-a}\right)e^c\right] \\
& + \frac{1}{1+\left(\frac{k\pi}{b-a}\right)^2}\frac{k\pi}{b-a}\left[\sin\left(k\pi\frac{d-a}{b-a}\right)e^d - \sin\left(k\pi\frac{c-a}{b-a}\right)e^c\right]
\end{aligned}
\tag{1.29}
$$

and

$$
\psi_k(c,d) = \begin{cases} \left[\sin\left(k\pi\frac{d-a}{b-a}\right) - \sin\left(k\pi\frac{c-a}{b-a}\right)\right]\frac{b-a}{k\pi} & k \neq 0 \\ (d-c) & k = 0 \end{cases}
\tag{1.30}
$$

In the case of a European-style Call option, it can be obtained:

$$
V_k^{call} = \frac{2}{b-a}\int_0^b K(e^y-1)\cos\left(k\pi\frac{y-a}{b-a}\right)dy = \frac{2}{b-a}K(\chi_k(0,b)-\psi_k(0,b))
\tag{1.31}
$$

Similarly, for a vanilla Put option I have:

$$
V_k^{put} = \frac{2}{b-a}\int_a^0 K(1-e^y)\cos\left(k\pi\frac{y-a}{b-a}\right)dy = \frac{2}{b-a}K(-\chi_k(a,0)+\psi_k(a,0))
\tag{1.32}
$$

1.4.2 Extending the COS pricing formula to Forward and Swap contracts

I argue that the COS formula for Call (Put) european options is immediately extensible to the pricing of a long (short) positions Forward trades by simply considering the whole integration interval $[a,b]$. The Fourier coefficients $V_k^{Fwd}$of a long position on a Forward are:

$$
V_k^{Fwd} = \frac{2}{b-a}\int_a^b K(e^y-1)\cos\left(k\pi\frac{y-a}{b-a}\right)dy = \frac{2}{b-a}K(\chi_k(a,b)-\psi_k(a,b))
\tag{1.33}
$$

Furthermore, by interpreting Swap contracts as portfolios composed by Forwards of different maturities makes straightforward their fair evaluation

[15] χ is obtained by integrating by parts twice. The integral for ψ is immediate.

through the COS method. The following formula provides the COS-computed value of a payer Swap contract with M payment dates:

$$v(x,t)^{Swap} = \sum_{m=1}^{M} e^{-r(T_m - t)} (T_{m+1} - T_m) \sum_{k=0}^{N-1}{}' \Re \left\{ e^{-\frac{ik\pi a}{b-a}} \phi_{y|x} \left(\frac{k\pi}{b-a}, t_m \right) \right\} V_k^{Fwd} \tag{1.34}$$

Figures 1.2 to 1.4 show the Exposures at Default (EAD) profiles over time computed by the COS method for long positions in analysed derivatives contracts. In the case of a Swap contract, the exposure profile over time presents a reverse U-shape, since it reaches its maximum around the 18th monitoring date and then it lowers as a consequence of the progressive decreasing number of remaining payments.

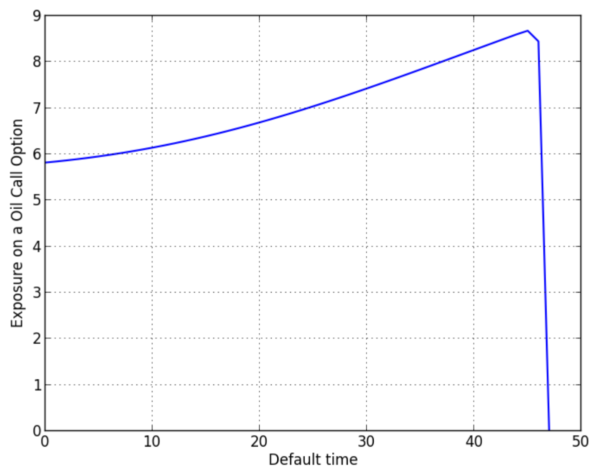

Fig. 1.2: Exposure at Default (EAD) over time for a European Call Option issued on crude oil computed by the COS formula. Weekly default monitoring dates are assumed (M = 48) within a time horizon of 1 year.

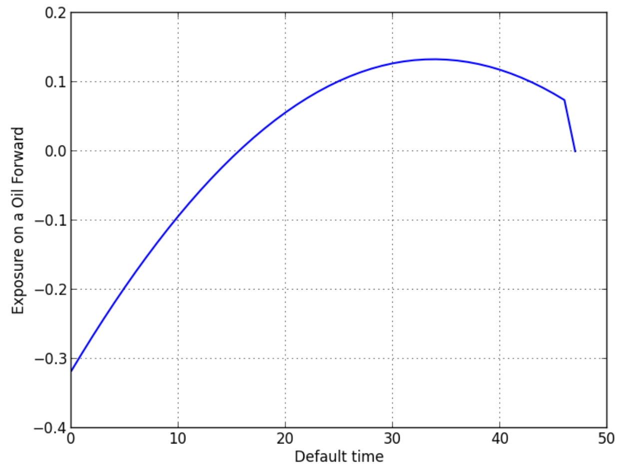

Fig. 1.3: Exposure at Default (EAD) over time for a Forward issued on crude oil computed by the COS formula. Weekly default monitoring dates are assumed (M = 48) within a time horizon of 1 year.

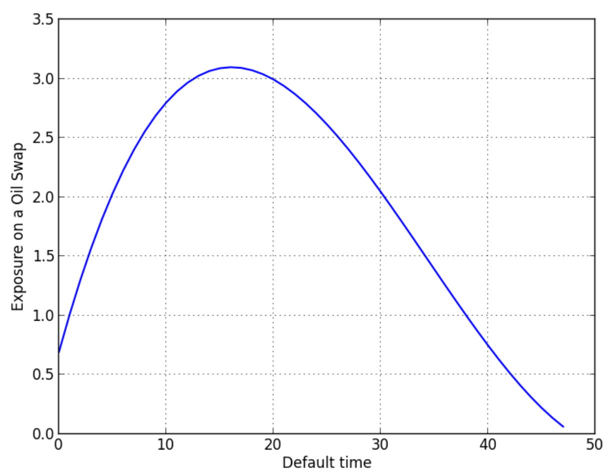

Fig. 1.4: Exposure at default over time (EAD) for a Swap issued on crude oil computed by the COS formula. Weekly default monitoring dates are assumed (M = 48) within a time horizon of 1 year.

1.4.3 Reviewing the COS method for CDS contracts

In order to simulate default processes within a structural approach, both credit drivers parameters and barriers levels have to be properly tuned. Fine tuning implies the possibility to reproduce survival and default probabilities coherently with the information available in the market. It is reasonable to believe that there is implicit information on creditworthiness in single name CDS contracts, since they basically provide to the buyer protection against the default of some reference entity. In these regards, the model parameters and the barrier levels will be retrieved in order to fit market-implied survival default probabilities and then reproduce the market structure of CDS spreads.

As described in equation 1.16, within a structural first-passage time modelling of default processes, the "credit event" is defined as:

$$\tau \equiv \inf\{t \in (0,T] : S_t \leq K\}$$

where K is a fraction of the firm value for the shareholders side at inception.

Focusing on $Y_t \equiv \log\left(\frac{S_t}{S_0}\right)$, the risk-neutral survival probability at time t $\mathbb{Q}\{\tau > t\}$ satisfies:

$$\begin{aligned}\mathbb{Q}\{\tau > t\} =& \mathbb{Q}\left\{Y_s > \log\left(\frac{K}{S_0}\right), \quad \text{for all} \quad 0 \leq s \leq t\right\} \\ =& \mathbb{Q}\left\{\min_{0\leq s\leq t} Y_s > \log\left(\frac{K}{S_0}\right)\right\} \\ =& \mathbb{E}^{\mathbb{Q}}\left[\mathbb{1}\left\{\min_{0\leq s\leq t} Y_s > \log\left(\frac{K}{S_0}\right)\right\}\right]\end{aligned} \tag{1.35}$$

Equation 1.35 corresponds to the price of an Binary Down-and-Out Barrier Option (BDOB) without discounting.

Let me define the reference value for the bankruptcy as $RV = \log\left(\frac{K}{S_0}\right)$ and suppose that, in the time interval $(0,T]$, there is a finite number of default monitoring dates identified in the grid $\mathcal{T} \equiv \{T_0, T_1, T_2, ..., T_M\}$, with $T_m = m\Delta T \quad (m = 0, 1, ..., M)$ and $\Delta T = T/M$, such that:

$$\mathbb{Q}\{\tau > T\} = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{1}_{\left\{Y_{T_1}\in[RV,\infty]\right\}}\mathbb{1}_{\left\{Y_{T_2}\in[RV,\infty]\right\}}\cdots\mathbb{1}_{\left\{Y_{T_M}\in[RV,\infty]\right\}}\right] \tag{1.36}$$

Equation 1.36 coincides with the pricing formula for Discrete Digital Options without discounting. This can be re-written in recursive form by exploiting the Markov propriety:

$$\begin{aligned}\mathbb{Q}\{\tau > T\} = \int_{RV}^{\infty} ... \int_{RV}^{\infty} ... \int_{RV}^{\infty} f_{Y_{T_M}|Y_{T_{M-1}}}(y_{T_M} \mid y_{T_{M-1}}) dy_{T_M} \\ .f_{Y_{T_m}|Y_{T_{m-1}}}(y_{T_m} \mid y_{T_{m-1}}) dy_{T_m} ... f_{Y_{T_1}|Y_{T_0}}(y_{T_1} \mid y_{T_0}) dy_{T_1}\end{aligned} \tag{1.37}$$

By defining:

$$p(y, T_M) \equiv 1 \tag{1.38}$$

and letting it correspond to the payoff at maturity of a Virtual Digital Option without discounting, it is possible to obtain the following recursive relation:

$$\begin{cases} p(y, T_m) = \int_{RV}^{\infty} f_{Y_{T_m+1}|Y_{T_m}}(y \mid x) p(y, T_{M+1}) dy, & m = M-1, ..., 2, 1, 0 \\ \mathbb{Q}\{\tau > T\} = p(y, T_0) \end{cases} \tag{1.39}$$

The computation of survival probabilities can be performed via a Backward Induction loop of the COS scheme. Recalling that the Fourier cosine series of $f_{Y_{T_M}|Y_{T_{M-1}}}(y \mid x)$ is:

$$f_{Y_{T_M}|Y_{T_{M-1}}}(y \mid x) = \frac{2}{b-a} {\sum_{k=0}^{N}}' \Re \left\{ e^{-ik\pi \frac{x-a}{b-a}} \phi_{y|x} \left(\frac{k\pi}{b-a}, \Delta T \right) \right\} \cos \left(k\pi \frac{y-a}{b-a} \right) \tag{1.40}$$

The *fair spread* at the starting date T_0 for a running[16] CDS with maturity T given the recovery rate R (conventionally fixed at 40% in standard CDS) is the one which makes equal the premium leg and the protection leg: i.e.:

$$s = \frac{(1-R)\int_0^T e^{-rs} \mathbb{Q}\{\tau > s\} ds}{\int_0^T e^{-rs} \mathbb{Q}\{\tau \leq s\} ds} \tag{1.41}$$

As expected, the CDS spread depends on a stream of survival and default probabilities. By assuming a constant discount rate, exploiting that the survival probability $\mathbb{Q}\{\tau > t\} = 1 - \mathbb{Q}\{\tau \leq t\}$ and integrating by parts, equation 1.41 can be simplified to:

$$s = (1-R) \left(\frac{1 - e^{-rT} \mathbb{Q}\{\tau > T\}}{\int_0^T e^{-rs} \mathbb{Q}\{\tau > s\} ds} - r \right) \tag{1.42}$$

Equation 1.42 can be discretized by applying the Composite Trapezoidal Rule, such that:

$$s_{trap} = (1-R) \left(\frac{1 - e^{-rT} \mathbb{Q}\{\tau > T\}}{\sum_{j=0}^{M} w_j e^{-rT_j} \mathbb{Q}\{\tau > T_j\} \Delta T} - r \right) \tag{1.43}$$

with $w_j = \frac{1}{2}$ for $j = 0$ and $j = M$ and $w_j = 1$ otherwise.

Hence, equation 1.43 has been used as theoretical fair CDS spread quote for my calibration purposes.

[16] Running means that no upfront is paid at inception.

1.4.4 *Numerical computation of bilateral CCR metrics via Monte Carlo: the first-to-default problem*

Concerning my numerical application, two firms have been chosen: BNP Paribas as a representative bank, and Enel, as representative corporation. Suppose that they negotiate OTC derivative claims issued on the most liquid energy commodity assets quoted in the NYMEX, i.e. crude oil and natural gas.

The underlyings are calibrated to the option prices provided by the Chicago Mercantile Exchange (CME). Within my numerical analysis, the discount factors are deterministic and calibrated by using the Hull-White 1 Factor model to the 1-year term structure of EURIBOR rates at various short maturities combined with the EONIA overnight rates[17].

Relying on the availability of data on quoted fair spreads from the iTraxx Series for the European CDS market, the calibration has been performed through the *minimization of a loss function* expressed in terms of Root Mean Square Error (RMSE) of the fair spread produced by my model, given a vector of benchmark securities:

$$[K^*, \kappa^*, \theta^*, \sigma^*] = \arg\min \sqrt{\sum_{\text{CDS}} \frac{(\text{market CDS spread - model CDS spread})^2}{\text{Number of benchmark CDS}}} \tag{1.44}$$

The chosen energy commodity underlyings are supposed to be non defaultable and calibrated to available option prices provided by the CME. For the purpose of calibrating the dynamics of the two firms, 5-years CDS quotes have been considered.

In the case of the recursive COS loop needed for CDS, the Fourier summation has been truncated to $N = 2^{10}$ terms in order to reach a satisfying level of accuracy. The truncation has been fixed to $N = 2^8$ terms in the case of European Options issued on crude oil and natural gas.

Calibration results					
NIG processes	Barrier	κ^*	θ^*	σ^*	RMSE
Enel	0.65033	0.23639	-0.20818	0.18073	1.5445
BNP Paribas	0.63839	0.24519	-0.20520	0.23570	1.1972
Crude Oil		0.86128	-0.14939	0.19686	0.5373
Natural Gas		0.87215	-0.13590	0.21107	0.4565

Tab. 1.1: Calibration results based on CDS data for corporates and on Options data for the underlying energy commodities. RMSEs are measured in basis points.

Nevertheless as shown by Table 1.1, the recursive application of the COS approximation scheme required in the case of CDS contracts provides a larger

[17] Source: https://www.emmi-benchmarks.eu/euribor-eonia-org/eonia-rates.html

RMSE relative calibration compared to those of options. I have found that the COS algorithm for CDS is very accurate, stable and efficient. In fact the calibration of each firm parameters set takes on average just 67.5 seconds at the cited level of accuracy. The algorithm has been coded in Python 2.7 on a MacBook Pro Retina (Late 2013) with 2,4 GHz Intel Core i5 ® processor and 8 GB 1600 MHz DDR3 RAM. This is due to the application of an efficient FFT algorithm to the recursive matrix-vector products, as shown by Fang et al. (2010). The algorithm displays flexibility as well since, among the set of possible starting values, it allows me to successfully calibrate around the observed level of historical market volatility. From a qualitative point of view, the calibration of both firms returns leptokuric and negatively skewed empirical distributions. Since Enel and BNP Paribas presented similar CDS spreads, the market-consistent parameters are similar as well. The only exception is relative to volatilities which matches quite well the afore mentioned historical average values. As regards the commodity assets, they also display quite fat left tails and negative skew. Two examples of simulated distributions relative to Enel and natural gas are displayed in figure 1.5.

Once the optimal vectors of parameters are recovered through the calibration procedure, the *first-to-default* problem and the computation of the CVA and DVA terms showed in the right-hand side of equation 1.19 can be performed via Monte Carlo simulation. My setting contemplates 10,000 simulated scenarios.

At this stage it shall be pointed out that, a joint Monte Carlo simulation of the two default processes is a reasonable choice in order to address the issue of statistical dependences, since in that instance the destiny of the two firms is intrinsically related. Indeed, the simulation breaks at the first default (i.e. whenever one of the two firms touches the down barrier) since, in a bilateral Counterparty Credit Risk setting, what happens after the first credit event is negligible. In this respect, my point is that previous works in which defaults are driven by independent point processes are basically not fully consistent with the goal to analyse the bilateral CCR.

Counterparty Credit Risk Valuation Adjustments						
Contract type	V_0	CVA	CVA/V_0	DVA	DVA/V_0	$\hat{V}_0$
ATM Call Oil	5.82103	0.81918	15275.381	0	0	4.93184
Forward Oil	-0.31708	0.00695	219.058	0.00393	124.097	-0.32009
Swap Oil	-1.37521	0.00031	2.238	0.01566	113.908	-1.35986
ATM Call Gas	0.41155	0.06156	1496.011	0	0	0.34998
Forward Gas	-0.02651	0.00010	38.305	0.00045	169.622	-0.02616
Swap Gas	-0.06716	0.00063	93.757	0.00061	90.210	-0.06718

Tab. 1.2: CVAs and DVAs for different derivatives claims. Relative valuation adjustments are expressed in basis points. At the money Call Options have been considered. Both forward and swap rates were set to their respective fair values.

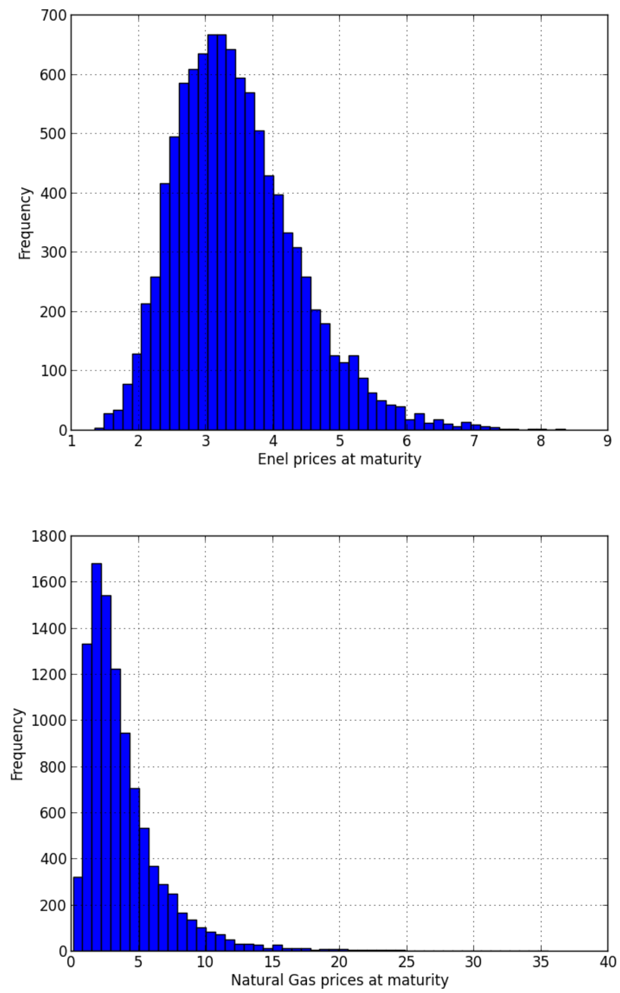

Fig. 1.5: Simulated distributions with calibrated parameters for Enel (top) and natural gas (bottom).

The above Table 1.2 shows the results for bilateral Counterparty Credit Risk adjustments computed trough the described hybrid Fourier-Monte Carlo methodology. Enel is assumed to be the computing counterpart being on the buyer side in all the transactions. I consider at the money derivative claims, so that also the Forwards and the Swaps are traded at their fair forward and swap rates, respectively. In accordance with previous literature, I have found that in the Monte Carlo simulation the volatility is the *key player* in determining credit events. Even if in my dataset the CDS spreads of BNP Paribas are slightly lower if compared to the ones of Enel (this evidence is reflected by the slightly higher level of the calibrated barrier for the corporation), since my calibration replicates the higher historical volatility registered for BNP, that financial firm is more likely to default first. This is amplified by the *first-to-default* setting, since even in simulations in which Enel is going to bankruptcy, BNP often defaults first. As previously stated, a potential late credit event of the corporate firm is basically not relevant. In order to reproduce this feature, as already mentioned, the simulation breaks at the first default. According to my results, the empirical first-to-default probabilities are stable roughly around 7%/8% for Enel and at 20%/22% for BNP Paribas.

As expected, I found that the CVA is considerably higher for one-sided payoff derivatives such as hypothetical European Call options negotiated over the counter[18]. In these cases the DVA is null, since the buyer is never going to be out-of the money when buying an option. Instead when dealing with two-sided payoff derivatives such ad Forward and Swap contracts, for Enel the DVA is sometimes higher than CVA. This reflects in the CCR adjusted prices, which are higher compared to the default-free one. In other words, when considering its *own default risk*[19] Enel sees the value of the contract become higher, given the positive probability of occurrence of a joint event of its own default and negative EAD.

1.5 Conclusions

In the present Chapter a Lèvy-based structural model has been applied to the computation of CVAs and DVAs for energy commodities derivative claims. In accordance with Ballotta et al. (2015) credit events are modelled as the first-passage time at a predetermined down barrier, albeit the present work differs since it is focused on the equity side of the firm value. Equity has been preferred since it is a tradable asset. The relevant risk drivers are NIG processes obtained by brownian subordination. By applying some asset pricing principles I derived a model-independent additive formula for bilateral Counterparty Credit Risk adjustments. The derivation does not require specific modelling assumptions on defaults mechanisms or particular hedging portfolios. My numerical application,

[18] European-style derivatives are generally traded into regulated markets. They have been here considered for the sake of simplicity in the exposition.

[19] This practise was recognized by IFRS 13 accounting principles and desirable in order to reach symmetric prices, so that the two negotiating parties can agree upon the value of the deal.

in the light of the non Gaussian underlying sources of risk, required the implementation of a hybrid Fourier-Monte Carlo approach in which both calibrations and exposures have been computed through the Fourier Cosine Series (COS) method and default processes have been simulated via Monte Carlo. As expected, in the Monte Carlo simulation the volatility still plays the key role in generating defaults. My numerical results show that, by considering counterparts with similar CDS spreads which negotiate OTC derivatives issued on commodity assets, the size of CVAs and DVAs range around 0,3% and 15%.

Next work will accomplish the need of incorporating the economic role played by Funding Valuation Adjustments (FVA) as well as addressing the issue of bilateral collateralization. In that regards, a particular focus is required for margining procedures under Central Clearing Counterparties (CCPs).

2. ON THE ECONOMIC ROLE OF FUNDING VALUATION ADJUSTMENTS (FVA) AND INITIAL MARGIN VALUATION ADJUSTMENS (MVA)

ABSTRACT

The global financial crisis revealed that no economic entity can be considered *default-free* any more, so that both banks and corporate firms have to cope with bilateral Counterparty Credit Risk (CCR) when negotiating OTC derivatives. Since the mainstream approach typically used in practical settings is to evaluate derivatives in terms of the cost of their respective hedging strategies, the pricing of CCR metrics implicitly relates to the way these strategies are financed. Within the numerical section of the present work, the valuation adjustments for CCR have been computed. Moreover, the role played by funding costs and their impact in widening bid-ask spreads have been assessed. A similar reasoning has been applied for the investigation of the cost of funding Initial Margins (IM), typically effective on top of Variation Margins (VM) when trading under Central Clearing Counterparties (CCPs). As the Initial Margin Valuation Adjustment (MVA) is concerned, it is here showed that, differently from what can happen for FVAs, no *offsetting effect* can materialize. As a consequence, in aggregate terms IMs can cause systemic liquidity effects. The computed XVA metrics are relative to energy commodities OTC derivative trades.

Keywords: XVA Metrics, Pricing, Hedging, Funding Costs, Funding Spread, FVA, Bid-Ask Spread, Transaction Costs, Central Clearing, MVA.

2.1 Introduction

The 2008 global financial crisis revealed that neither high creditworthiness institutions can be considered default-free, so that both financials and corporations have to cope with bilateral Counterparty Credit Risk (CCR) when trading OTC derivatives. From the operational point of view, the standard practice is to evaluate derivative claims in terms of the cost for their respective hedging strategies. As a consequence, both derivative prices and CCR metrics depend on how the hedging is financed. In the light of the exceptional growth of credit spreads it has been observed during the crisis, the impact of funding costs on the balance-sheets of major dealers has been massive.

From a regulatory side, it is still missing a unified standard framework to incorporate the costs faced for financing hedging strategies. Such framework will probably lack even in incoming times because of the the very high degree of complexity implied by funding modelling. Among industry participants, the first mover was J.P. Morgan Chase which, in its Q1 2014 public report announced that Funding Valuation Adjustments (FVA) had been accounted for the amount of US$ 1.5 billions. Many recognized that announcement as the first relevant move in financial industry, stimulating other dealers to align. Thereafter, some of the major consultancy firms such as EY and KPMG, publicly declared their acceptance of accounting for FVAs. Nevertheless, the debate on funding costs is still ongoing because of their asymmetry and non pure additivity. Their nature of valuation adjustment or profitability analysis tool has still to be clarified.

2.1.1 Brief literature review of the FVA debate, Netting and Initial Margins

In both academic and operational environments, the investigation of funding costs dramatically widened as a consequence of the global financial crisis. From the very beginning, it became clear that the inclusion of funding costs within pricing frameworks would have involved an unprecedented risk management challenge from the technical point of view. Since the publication of the work of Hull and White (2012), who raised some doubts regarding the theoretical foundations of adjusting derivatives prices for funding, the FVA debate became even more passionate. Dozens of follow-up papers followed to that article.

One among early relevant attempts to embed funding costs within a unified valuation framework can be recovered in Burgard and Kjaer (2011b). Within their derivation of a pricing formula, the Authors assumed bilateral default risk. Relying on a extended version of the Black and Scholes (1973) model, the Authors derived a PDE describing the adjusted price dynamics, which they solved by means of the Feynman-Kàc Theorem. The Authors proposed a hedging strategy through which the bank holds a usual delta position in the underlying and a appropriate amount of cash. Moreover, for the purpose of treating both the CCR and funding costs, the bank was assumed to hedge out the default risk by short selling the counterparty bonds (CVA risks) and by repurchasing back its own debt (DVA risks). The generalized PDE which they derived displayed to be flexible enough for being adapted to several possible scenarios, e.g. *risk-free* *close-outs* rather then *replacement close-outs* and no funding haircut[1] rather than non zero funding spread. In their work it is discussed the numerical computation of CCR metrics for a vanilla option. In the conclusive remarks, they claimed that possible extensions of their model could relate to the analysis of more complex exotic payoffs, netted portfolios, correlated defaults together with the allowance of stochastic interest and hazard rates.

In Burgard and Kjaer (2011a) it can be retrieved a first attempt to model balance-sheet implications of the choice among several attainable funding strategies. The Authors suggested two possible scenarios in which funding effects can be neglected. The first concerns the case in which the derivative can be posted as collateral and thenceforth, no haircut is charged by the external funder. The other one is relative to the case in which the dealer is able to strategically trade in its own bonds with different maturities.

In the work of Hull and White (2012), the Authors argued that FVAs should not be taken into account when evaluating derivatives. They suggested that the FVA is mainly arising from the cost faced for financing hedging strategies, but since the trading in hedging instruments is usually closed at market prices, these are null expected yield investments. For this reason, hedging should not affect derivatives valuation. Furthermore, according to the well known corporate finance principle expressed by the Modigliani-Miller Theorem, under some theoretical assumptions[2] funding should be considered apart from investments decisions. They concluded that, since funding does not reflect any economic value, if considered when making investment decisions it would lead to poor choices with respect to the shareholders profit maximization.

The papers of Crépey et al. (2013) and Brigo et al. (2015) among others, share the modelling approach of adding the cash flows relative to hedging funding and collateral margining to those of the derivative payoff when computing the adjusted derivatives prices. Taking into account the funding issue in pricing increases considerably calculation complexity. While the CVA causes nonlinearity in the payoff, the inclusion of funding costs makes the pricing operator itself to become nonlinear. Under such theoretical framework, as the current derivative price depends on the value of the FVA term in the future, which in turn depends on the future CCR adjusted price path, the pricing problem becomes recursive. For this reason, together with other Authors, they addressed the computation of FVAs via the Backward Stochastic Differential Equations (BSDEs) Theory. At first glance, they performed the numerical computation by discretizing the pricing problem through suitable BSDE schemes and than solved it by means of American Monte Carlo techniques. In Crépey et al. (2013) the pricing of CCR adjustments is analysed with respect to different collateral margining schemes and close-outs rules upon default. Moreover, they managed to disentangle the FVA term by isolating the liquidity from the credit component, claiming that

[1] No haircut applies when the derivative can be posted as collateral, so that the bank can borrow virtually at the risk-free rate.

[2] Despite this statement the Authors themselves admitted that there can realistically configure exceptions: for instance, in some countries tax benefits are recognized to debt financing so that it might be preferred to equity financing.

just the first one should be considered in pricing.

Brigo et al. (2015) similarly derived a general pricing formula in presence of replacement close-outs, collateral and funding risk by relying on a BSDE representation or on a semi-linear PDE setting, alternatively. Nevertheless, the Authors raised some concerns about the inclusion of asymmetric funding rates into pricing, pointing out the consequent inconsistency with respect to the Law of One Price. In these regards, they argued that it is common belief among market operators to consider Funding Valuation Adjustments as the main driver of bid-ask spreads in the years of financial crisis. Besides they raised concerns about the separability of risks in the light of the recursive nature of funding, whose asymmetry could also induce arbitrage opportunities. In their paper, it have been derived both discrete-time and continuous-time solutions. In the computational part of their work, it have been discussed a numerical case study extending Black and Scholes (1973) by using both backward and forward simulations and Least Squares Monte Carlo techniques. They claimed that the standard practice of removing nonlinearities by averaging asymmetric borrowing and lending rates and substituting replacement close-outs by risk-free close-outs causes a valuation error that they defined Nonlinearity Valuation Adjustment (NVA). They concluded that in general FVA is not a pure additive adjustment term as it has been commonly assumed by most market participants in simplified approaches. The Authors underlined that the introduction of funding costs in derivative valuation violates the bilateral nature of the deal price, posing doubts about the two counterpart's capability to close the trade. Indeed, they took leave of the reader with some comments about the distinction between price and value as well as the possibility to use FVAs as a cost analysis tool.

In Brigo et al. (2016) it is summarized the dialogue undertaken during the conference *Challenges in Derivatives Markets* held at Technische Universitat Munchen in March/April 2015. The point of Brigo was to underline the role of nonlinearities and to wonder whether in practical applications they should be embraced rather than be linearised. He argued that dealing with nonlinearities is too complex on daily basis, since they imply the need to repeat the evaluation at any relevant aggregation level. Furthermore, he raised provocative issues of *self-fulfilling prophecies* in pricing, occurring when the majority of agents is adopting wrong methodologies and other participants have to align in order to stay in the market. Fries started by remarking that, in the current financial market there could be *funding arbitrage* opportunities, as dealers with poor funding can swap against other ones with better funding in order to allow both to make a profit. According to him the new cutting edge in the current market landscape is related to Netting, since we can observe *portfolio effects* because of nonlinearities. Moreover, he stressed the issue of lack of data for calibration purposes. Hull explained that, being trained in finance, his view about FVA differs from the one of those trained in physics or mathematics. In corporate finance when investment opportunities are being evaluated, only the direct riskiness related to the investment is relevant, while the way the investment is financed is negligible. He remarked the difficulties one incurs when trying to disentangle default risk from other components, such as liquidity, from the overall funding cost. He also agreed with Brigo about possible threats of self-fulfilling prophecies in financial industry related to uniform pricing routines. He pointed out that regulators are concerned about pricing derivatives in terms of the mark-to-market rather than costs coverage and, since the regulatory trend is going toward the imposition of full central clearing collateralization schemes to most of OTC transactions, he expected that the FVA would not be an issue in few years. Sommers remarked that the main concern of accountants is the fair value, which in the case of financial derivatives is defined as the *exit price*. That concept is somehow related the self-fulfilling prophecies argument. In fact, if the majority of operators are taking into account some aspects in their evaluation systems, the price one would be proposed in the case of exiting from the trade would likely include those aspects. This still represents a puzzle, since the products interested by FVA are typically not so accessible on Bloomberg or other data providers. He insisted on the issue of portfolio effects, since the unit of account is not unique. For instance, the CVA is generally not referred to the single deal but to a given *netting set*. As far as the FVA is concerned, the unit of account is typically a given *funding set*, for instance all the trades denominated in a given currency. He concluded that, in the light of the Modigliani-Miller Theorem, the inclusion of the role of FVA in pricing would impact on the whole derivative pricing paradigm.

In their recent paper, which has gained notably interest among both academics and industry operators[3], Andersen et al. (2017), rather than reconnecting the FVA within a unified pricing framework, singled out in funding an incremental cost which impacts on the shareholders section of the balance-sheet and does not affect fair valuation. They highlighted that the conventional practice of accounting for FVAs as additive valuation adjustments had produced a theoretical inconsistency which could be quantified in a 6 US$ billions worth accounting mistake. They derived a pecking order of shareholders preferences about several funding strategies of new investments. Possible strategies range among financing new investments through debt issuing, rather than equity issuing or by withdrawing available cash from the balance-sheet. The Authors introduced a marginal theory for investment choices; firstly grounded in the use of a two-periods structural model, later extended to multi-periods settings and finally to a continuous-time equivalent reduced-form model. Such theory has then been applied to evaluate the economic convenience for the firm's shareholders in entering in additional swap trades. They put the basis for a fully consistent balance-sheet structural model, by which they showed that default events impact on both assets and liabilities sides. The Authors affirmed that FVAs do not affect derivative valuation but they are rather responsible of some widening in the bid-ask spread that dealers typically quote to their clients. They claimed that if traders operate according to the interest of firm's shareholders, they should not enter into swap trades at the fair price since, because of the incremental funding costs, the shareholders need to be compensated by the counterpart through a windfall equal at least to the sum of funding costs and the DVA

[3] see https://www.bloomberg.com/news/articles/2016-03-11/professor-to-wall-street-youre-doing-swaps-accounting-wrong

term[4]. In the light of the inefficient nature of OTC markets, characterized by opaqueness and searching costs, funding costs might be an incentive which can align traders' activity to shareholders value maximization. They affirmed that in the market it have been observed acquisitions of swap portfolios by dealers for which funding costs are less severe with respect to others. For this reason, the Authors suggested the establishment of XVA optimization desks able to quantify the impact on the equity side of additional funding and margining costs. Finally, in a numerical application the Authors provided the results of a calculation of XVA measures for plain vanilla Interest Rate Swaps (IRS).

As it will be pointed out later in the present Chapter, the topic of funding should be investigated in relation with the appropriate aggregation level. In practical settings, treasuries calculate funding requirements not focusing just on a single portfolio but in relation, for instance, to a given netting set denominated in given currency. Moreover, allowing for netted exposures leads to relevant implications for credit related credit related XVA terms. Among other references in previous literature, a significant study of netting can be found in Brigo and Masetti (2006), which analyse the unilateral CVA in the case of interest rate swaps (IRS) portfolios towards a single counterpart. In their work, they compare the effectiveness of several netting schemes in terms of resulting overall exposure. The Authors underline how incorporating netting agreements in the general CCR pricing framework increases considerably the degree of complexity as, upon a default event, CVA configures as an option term on the residual present value of the whole netted portfolio. This cannot be evaluated as a standard Swaption, so that one needs to resort to Monte Carlo simulation[5] or to derive some analytical approximations. By exploiting the independence hypothesis between interest rates and default times, approximated formulas have been derived in order to estimate on default exposures.

In the case an investor negotiate with a single counterpart several long and short positions on IRS with different tenors and maturities, overall cash flows to be paid by both the fixed and floating legs sum up at each resetting date. As a consequence, floating leg payments result in (positive and negative) multiplies of the underlying LIBOR and agreed contractual swap rates. In the case the IRS constituting a portfolio towards a single counterpart are possibly long or short, the portfolio evaluation becomes similar to that of a single IRS with different multiples of LIBOR and predetermined swap rates. In such cases the pricing can be computed via the Drift Freezing Technique which however, is reliable within some reasonable limits. In fact, in the case of allowing IRS trades in the portfolio to be both directions, the picture becomes even more interesting since, the overall residual present value of the IRS portfolio at the early default time can be seen as an option on the difference between two swap rates, each approximately lognormal. The Authors decide to evaluate such option by means of the Three Moment Matching procedure. In their numerical tests, the accuracy

[4] I here recall that the DVA term represents a wealth transfer from shareholders, who earn nothing upon default, to legacy bondholders.

[5] The simulation can be performed in the context of LIBOR Market Model or Swap Market Model, alternatively.

of both analytically approximated formulas is checked through a comparison against the results obtainable via Monte Carlo. The comparison of analytical approximations and Monte Carlo simulation has been carried out in relation to different netting schemes.

In their work, Brigo and Masetti (2006) consider a N-dimensional IRS portfolio with homogeneous resetting dates but differentiated start and maturity dates. Let T_a and T_b being the first start and the last maturity dates of the IRS portfolio, respectively. Then, for all $i \in [a+1, b]$:

$$\alpha_i = \beta_i \mid \sum_{j=1}^{N} A_i^j \phi_j \mid, \quad K_i = \beta_i \mid \sum_{j=1}^{N} A_i^j K_i^j \phi_j \mid \tag{2.1}$$

$$\chi_i = \text{sign}\left(\sum_{j=1}^{N} A_i^j \phi_j\right), \quad \psi_i = \text{sign}\left(\sum_{j=1}^{N} A_i^j K_i^j \phi_j\right) \tag{2.2}$$

where β_i is the year fraction between two consecutive resetting dates, $A_i^j > 0$ is the notional amount relative to the j-th IRS at the resetting date T_i, ϕ_j is the payer/receiver fixed rate flag which takes values in $\{-1, 1\}$, i.e. 1 for payer and -1 for receiver. K_i^j is the fixed rate to be paid in the j-th IRS at T_i reset time. Please note that χ_i may be different from ψ_i. The Authors denote by $L(T_{i-1}, T_i)$ the LIBOR rate in place between times T_{i-1} and T_i.

The discounted payoff of the IRS portfolio, evaluated at time $t < T_a$, can be written as:

$$\begin{aligned} \Pi_{\text{Pirs}}(t) &= \sum_{i=a+1}^{b} D(t, T_i)[\chi_i \alpha_i L(T_{i-1}, T_i) - \psi_i K_i] \\ &= \sum_{i=a+1}^{b} D(t, T_i)\chi_i[\alpha_i L(T_{i-1}, T_i) - \tilde{K}_i] \end{aligned} \tag{2.3}$$

where $\tilde{K}_i \equiv (\frac{\psi_i}{\chi_i})$ and α_i is called netting coefficient in front of the Libor rates in the total IRS portfolio towards a given counterpart while $\tilde{K}_i$ represent the cumulated fixed rate of the total portfolio of IRS to be exchanged at time T_i. The expected value at time t of a default-free IRS portfolio is known to be:

$$\mathbb{E}_t^{\mathbb{Q}}[\Pi_{\text{Pirs}}(t)] = \sum_{i=a+1}^{b} P(t, T_i)\chi_i[\alpha_i F_i(t) - \tilde{K}_i] \tag{2.4}$$

where $P(t, T_i)$ is the price at time t of the default-free zero coupon bond expiring in T_i and $F_i(t)$ is the LIBOR forward rate. It is remarked that each expectation in the sum can be easily computed by resorting to the related forward measure. The Authors then introduce CCR risk by considering unilateral CVA settings, so that:

$$\mathbb{E}_t^{\mathbb{Q}}[\Pi_{\text{Pirs}}^D(t)] = \mathbb{E}_t^{\mathbb{Q}}[\Pi_{\text{Pirs}}(t)] - \text{LGD}_C \mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau_C < T_b\}} D(t, \tau_C)(\varepsilon_{\tau_C})^+] \tag{2.5}$$

where $\varepsilon_{\tau_C} = \mathbb{E}^{\mathbb{Q}}_{\tau_C}[\Pi_{\text{Pirs}}(\tau_C)]$.

In these regards, they stress that the price of a default-free IRS portfolio is model-independent being forwards and discount curves sufficient in the evaluation, so that there is no need to postulate a dynamics for the term structure. However the inclusion of counterparty risk adds an optionality level which makes the evaluation model dependent even if the original payoff was model-independent.

Under the cited independence hypothesis between interest rates and the credit sector, the Authors simplify the pricing problem by assuming that the default is postponed to first time T_i following τ_C, so that the CVA can be rewritten as:

$$CVA = \text{LGD}_C \sum_{i=a+1}^{b} \mathbb{Q}\{\tau_C \in [T_{i-1}, T_i]\}\mathbb{E}^{\mathbb{Q}}_t[D(t, T_i)(\varepsilon_{T_i})^+] \tag{2.6}$$

where

$$\varepsilon_{T_i} = \sum_{k=i+1}^{b} P(T_i, T_k)\chi_k[\alpha_k F_k(T_i) - \tilde{K}_k]$$

Hence, the value of a CCR risky IRS portfolio has been decomposed into the sum of a swap with a non standard coefficients and a series of swaption prices weighted by a stream of default probabilities. It is than showed that, by some rearrangements, the swaption price in 2.6 can be rewritten in terms of the Black price of a option issued on the well defined underlying forward overall swap rate $\hat{S}_{i,b}(T_i)$:

In order to price such stream of swaptions the Authors make use of the Drift Freezing Technique (DF) which is similar to other approximated swaption pricing formulas available in literature. The technique consists in evaluating swaptions in terms of Black option prices issued on underlying IRS trades, see the original work of Brigo and Masetti (2006) for full technical details. The Authors expected this approximation working properly only in the cases all IRS in the portfolio had the same direction, i.e. all χ's are equal each other. In other words, this would mean that the overall swap rate would have the same sign in all scenarios so that the Brownian Motion could represent a reasonable approximation of its dynamics. Otherwise, in cases with mixed χ's (i.e.a portfolio with both long and short IRS) the robustness of this technique has to be carefully checked. Otherwise in order to properly handle the pricing, one might still apply the DT technique and resort on suited put-call parity arguments. However, the Authors have found the accuracy of DF being unsatisfactory in cases of mixed netting coefficients and since linear combinations of lognormal variables is no longer lognormal, they adopted the Three Moment Matching Technique (3MM), a method which takes into account an approximate estimation of the third moment of $\hat{S}_{i,b}(T_i)$. This is based in matching the first three moments of the underlying overall swap rate to those of an auxiliary shifted lognormal martingale process. Hence the Authors exploit the approximation of the swaption price in terms of the Black formula on the auxiliary process[6] .

[6] Full technical details are provided in the original work of Brigo and Masetti (2006)

In the numerical part of the paper, Authors assess the accuracy of described approximations in computing the expectation of $\mathbb{E}_t^{\mathbb{Q}}[D(t,T_i)(\varepsilon_{T_i})^+]$ through a comparison with the results obtainable by Monte Carlo simulation. They consider several test parameter sets which include instantaneous volatilities, correlations, forward rate curves and netting coefficients. The Authors analyse several possible compositions of the IRS portfolio, in which single trades can share the first payment date rather than the same last payment date. They investigate portfolios characterized by all single trades in the same direction as well as IRS portfolios with mixed χ's. Moreover, their analysis distinguishes among different possible levels of initial moneyness and netting schemes. As expected, the Authors found the Black like approximation (DF) working satisfyingly in cases netting coefficients have the same sign . Otherwise, in the case of mixed netting coefficients, DF is less accurate, especially for in the money and out of the money strikes. In general the more refined 3MM Black approximation outperforms the DF technique except for few circumstances. The Authors suggest that the degree of accuracy of approximated formulas is well suited for risk management purposes. In fact the computational time required to simulate scenarios for each risk factor is crucial so that an efficient and accurate approximation might be required in order to contain it.

The Authors conclude that the provision of netting schemes lowers considerably the valuation adjustment due to CCR. However without a correct implementation of netting schemes, the overall CVA adjustment would be equal to the sum of single CVAs, implying the multiple counting of the default impact of flows figuring in more than one IRS. Since it holds:

$$(\Pi_{IRS_1} + \Pi_{IRS_2} + ... + \Pi_{IRS_n})^+ \leq \Pi^+_{IRS_1} + \Pi^+_{IRS_2} + ... + \Pi^+_{IRS_N}$$

under netting agreements smaller losses upon default are expected with respect to the no netting case. In other words, netting is effecting to mitigate the valuation adjustment for counterparty default risk, i.e. the CVA.

Since in recent years regulators had pushed market participants to negotiate OTC trades under the jurisdiction of Central Clearing Counterparties (CCP), the final part of the present review is aimed at introducing the topic of initial margins and related implications. CCPs are commercial entities who interpose themselves between two counterparts by taking default risk and ensuring contractual payments even in case of credit events. If the default of a clearing member occurs, the deal is transferred to one among backup clearing members taking part to a competitive auction, so that slight residual CCR risk remains in place. Moreover, the inclusion of CCP into the picture does not fully eliminate counterparty risk since these can go bankruptcy themselves.

Initial Margins (IM) represent an additional layer of collateralization, required by CCPs, to those negotiating under their jurisdiction and are supposed to cover additional risks on top of traditional market and credit ones. These might be identified with deteriorating quality of collateral, wrong-way risk, potential additional losses arising during the margin period of risk as well as gap risks, namely the adverse variation in mark to market since the last collateral margining date. As it was discussed within the first Chapter review, some asset classes are highly affected by gap risk, for instance credit and CDS in particular.

Since IM represent additional collateral amounts segregated in CCP accounts, there configures a asymmetric collateral agreement. Initial margins are typically estimated by the CCP according to prevailing market conditions and the expected maturity of the portfolio. Moreover, CCP can adopt multipliers or require additional margins to absorb potential losses in case of default or downgrade of a clearing member and the consequent passage to a backup member. In accordance with ISDA (2013a), which defines the guidelines of Standard Collateral Support Annex (SCSA) agreements, the collateral posted to CCPs is called daily and is remunerated at overnight rates. On the other side, similarly to cleared trades, SCSA is pushing privately collateralized deals to embrace CCP-style initial margins. It can be stated that regulatory the trend evolves towards bilateral CSA collateralized deals resembling more an more to cleared trades. IM can be posted several time during the life of the deal as, for instance, deterioration of gap risk and related issues occur.

In the light of the massive impact on pricing, hedging and risk management related to the above discussed trend, Brigo and Pallavicini (2014) introduced a general approach for pricing derivative claims under CCP clearing including variation and initial margins. They point out that derivative prices depend on the choice of the investor and its counterpart to adopt a bilateral CSA rather than the interposition of a Central Clearing Counterparty. Furthermore they discuss numerical cases in the context of Interest Rate Swaps.

In the introductory section, the Authors argue that since the summer of 2007, as a consequence of the credit crunch, the term structure of forward rates as well as market quotes of zero coupon bonds started to violate standard non arbitrage relations. At first glance, that was partly due to the liquidity crisis affecting funding operations. Few months later it became clear that, as counterparty risk in the market started growing, the crisis was following a typical spiral pattern which might have caused a systemic break-down freezing credit lines. In such financial landscape, standard pricing models based on ideal risk-free rates, absence of credit risks and unrestricted access to funding instruments became inadequate. As in their previous works, the Authors stress that instead of a new and somehow *ad hoc* pricing theory, additional features characterizing the current financial environment can be introduced in terms of modified payoffs. Under suitable assumptions the Authors approximate the impact of hedging, funding and margining procedures in pricing equations in terms of modifications of discount factors and forward rates. The framework which they propose is fully arbitrage-free and is based on market observables. Moreover, they suggest that such modifications should be included also in the calibration and bootstrapping algorithms used to calculate model parameters. The framework proposed in Brigo and Pallavicini (2014) achieves a rigorous quasi separable decomposition of nonlinear interconnected risks into different valuation adjustments, which does not hold in general but is possible in some simplified settings. They argue that the mathematical nonlinearities arising from allowing for asymmetric borrowing and lending rates in the hedge of claims leading to deal dependent pricing measures and aggregation dependent valuations, hold for both CCP cleared and bilateral CSA trades.

In their work the Authors introduce a master pricing equation which takes into account all factors which affect the replication price of a derivative claim:

$$\hat{V}_t = \mathbb{E}^{\mathbb{Q}}[\Pi(t, T \wedge \tau) + \gamma(t, T \wedge \tau) + \varphi(t, T \wedge \tau) + \mathbb{1}_{\{t \leq \tau \leq T\}} D(t, \tau)\theta_\tau \mid \mathcal{G}_t] \quad (2.7)$$

where $\Pi(t, T \wedge \tau)$ is the sum of discounted payoff in the interval $(t, T \wedge \tau)$, $\gamma(t, T \wedge \tau)$ is the collateral in the interval $(t, T \wedge \tau)$, $\varphi(t, T \wedge \tau)$ represents the funding costs in the interval $(t, T \wedge \tau)$ while θ_τ is the replacement cost of the deal upon default, i.e. the close-out amount, expressed as function of the residual cash flows and collateral amounts, which originates the usual CVA/DVA terms. More in detail, the sum of cash flows indicated in the above equation can be written as:

$$\begin{aligned}
\hat{V}_t = &\int_t^T \mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\{u<\tau\}} D(t,u)(\Pi(u, u+du) + \mathbb{1}_{\{\tau \in du\}}\theta_u) \mid \mathcal{G}_t]du \\
&+ \int_t^T \mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\{u<\tau\}} D(t,u)((f_u - c_u)M_u + (r_u - f_u)\hat{V}_u - (r_u - h_u)H_u) \mid \mathcal{G}_t]du \\
&+ \int_t^T \mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\{u<\tau\}} D(t,u)((f_u^{N^C} - c_u)N_u^C + (f_u^{N^B} - c_u)N_u^B) \mid \mathcal{G}_t]du
\end{aligned} \quad (2.8)$$

where f_t is the funding rate at time t, c_t is the collateral accrual rate at time t, r_t is the risk-free rate at time t, h_t is the repo lending rate for hedging instruments at time t while $f_t^{N^C}$ and $f_t^{N^B}$ represent the funding rates for financing initial margins faced by the counterpart and the bank, respectively. The Authors distinguish between these from the funding rate f as they belong to different netting sets. Moreover, M_t is the variation margin at time t, H_t represents the value of hedging instruments at time t while N^C and N^C are the initial margins posted by the counterpart and the bank, respectively.

The above formula is an equation rather than a closed form since $\hat{V}$ appears also in the right-hand side and future rates can depend on it. As it is shown by equations 2.7 and 2.8, the derivative price is sensitive to the choice regarding funding and collateral margining agreements. In fact, the Authors stress that the replication price of the contract holding between two counterparts under bilateral CSA is different from an equivalent one cleared with a CCP. By the application of suitable technical assumptions (see the original work for full details), Brigo and Pallavicini (2014) show how to derive a Backward Stochastic Differential Equation (BSDE) representation of their master pricing equation:

$$d\hat{V}_t dt - f_t\hat{V}_t + \Pi(t, t+dt) + (f_t - c_t)M_t dt + (f_t^{N^C} - c_t)N_t^C dt + (f_t^{N^I} - c_t)dt = d\mathcal{M}_t^h \quad (2.9)$$

with the terminal condition set at time $\tau \wedge T$ as:

$$\hat{V}_{\tau \wedge T} = \mathbb{1}_{\{\tau < T\}}\theta_\tau \quad (2.10)$$

where $\mathcal{M}_t^h$ is a martingale under $\mathbb{Q}^h$, which is the probability measure equivalent to $\mathbb{Q}$, under which risky assets in the economy grow at the repo lending rate h (rather than the risk-free rate r). It appears clear that, through suitable *numeraire* changes, even if the evaluation started as the expectation under the risk neutral measure, the pricing equation can be rewritten in order to independent from the risk-free rate. This configures as an advantage since the risk-free rate is a fictitious instrumental variable not observable in real markets any more.

The resulting BSDE is nonlinear, since variation and initial margins as well as asymmetric funding, collateral and repo lending rates might depend on the future derivative adjusted value $\hat{V}$ itself and on its partial derivatives used for hedging purposes. As a consequence of nonlinearities, the valuation is no longer additive, namely the price of a portfolio does not equal to the sum of individual assets (so that one has to chose *ex ante* the relevant aggregation level) and the pricing measure is deal dependent since the valuation is performed under the $\mathbb{Q}^h$. The fundamental nonlinearity at the level of the pricing operator represents a further dramatic consequence with respect to that caused by CVA/DVA type terms, which introduce nonlinearity in the payoffs even of linear products.

However, the Authors point out that within particular conditions the fundamental nonlinearity introduced by funding can disappear. For instance by focusing on the case of interest rate derivatives, whose exposures do not jump at default, full collateralization schemes in which variation margins equal the residual mark to market, the adoption of collateralized hedging instruments traded at the collateral rate c_t (typically equal to the overnight rate e_t) together with suited modelling choices for credit spreads and liquidity basis, permit the achievement of a numerical scheme for solving the pricing BSDE in which the derivative price does not depend any more on future levels of the funding rate f and on future values of the derivative itself $\hat{V}$.

The Authors remind that the funding inclusive price is not separable in general in clear-cut components related to credit, debit and funding. Nevertheless under the assumptions formulated for the interest rates case, separability can be partially achieved. By the application of the Control-Variate Technique they obtain a decomposition in pieces of financial meaning. In these regards, the Initial Margin Valuation Adjustment (MVA), i.e. the expected cost faced for IM financing, equals to:

$$MVA = \int_t^T \mathbb{E}^{\mathbb{Q}^e}[D^{e+\lambda^C+\lambda^B}(t,u)({N^C}_u\ell_u^{N^C} + {N^B}_u\ell_u^{N^B}) \mid \mathcal{F}_t]du \tag{2.11}$$

where λ^B and λ^C are the default intensities of the bank and its counterpart while ℓ^{N^B} and ℓ^{N^C} are the liquidity basis charged by the respective treasuries for financing IM amounts. Separability can be achieved in the case of CCP cleared trades as well as for bilateral trades under SCSA. In order to reach such result the limitation attains to the need of dealing with $\mathcal{F}$-adapted close -out amount and assuming $\mathcal{F}$-conditional independence of default times. In fact, if these conditions are not met, it is not possible to switch the evaluation under the default-free market filtration.

Focusing on IRS, the Authors have compared the impact on derivative valuation of initial margins differentiating between the case of a bilateral collateralized trade and that of a CCP cleared one. While under CCP trading the additional layer of collateralization is asymmetric, in other words IM is posted only by the client towards the clearing member, in bilateral SCSA contracts both the two counterparts post VM as well as IM. Moreover, if bilateral the SCSA agreement allows the rehypotecation of initial margins, the client might exploit a funding benefit which otherwise, is not achievable under CCP clearing.

Initial margins are typically calculated as some market risk measure, such as historical Value-at-Risk (VaR) or Expected Shortfall (ES). According to their modelling framework, Brigo and Pallavicini (2014) suggest to be calculate the initial margin as Monte Carlo VaR, whose estimation under the physical probability measure is approximated by simulating under $\mathbb{Q}^h$:

$$N_t^C = \inf\{x \in \mathbb{R}^+ \mid \mathbb{Q}^{\mathrm{h}}\{\varepsilon_{\mathrm{t}+\delta} - \varepsilon_{\mathrm{t}} \leq \mathrm{x} \mid \mathcal{F}_{\mathrm{t}}\} \geq 1 - \alpha\} \tag{2.12}$$

and only for bilateral trades under SCSA:

$$N_t^B = \sup\{x \in \mathbb{R}^- \mid \mathbb{Q}^{\mathrm{h}}\{\varepsilon_{\mathrm{t}+\delta} - \varepsilon_{\mathrm{t}} \geq \mathrm{x} \mid \mathcal{F}_{\mathrm{t}}\} \geq 1 - \alpha\} \tag{2.13}$$

Furthermore in order to speed up the simulation, the Authors make use of the Moment Matching Technique to approximate the conditional expectations in above formulas as it follows:

$$N_t^C \simeq \Phi^{-1}_{0,\nu_t}(1 - \alpha) \tag{2.14}$$

where $\Phi_{\mu,\nu}$ is the cumulative distribution function of a normal distribution with mean μ and standard deviation ν, which are given by:

$$\mu_t = 0, \quad \nu_t^2 = \mathrm{Var}^h(\varepsilon_{t+\delta} \mid \mathcal{F}_t)$$

The Authors assume that in the bilateral SCSA case, the initial margin posted by bank is:

$$N_t^B \simeq -N_t^C \tag{2.15}$$

being zero in the CCP case.

In the numerical part of the paper, the Authors assess the impact of counterparty risk, funding, collateral and initial margin on the pricing of 10y-IRS and compare the cases of uncollateralized, bilateral SCSA and CCP cleared trades. They assume the absence of gap risks as IRS exposures should not jump at default, so that initial margins specifically cover adverse exposition movements during the margin period of risk. The Authors investigate the impact of correlation between market and credit risks, credit spreads volatilities and liquidity basis within a single curve framework[7], so that all relevant rates can be derived from overnight rates. Brigo and Pallavicini (2014) model interest rates

[7] The extension to multicurve setting is straightforward.

under the $\mathbb{Q}^e$ pricing measure by the two-factors shifted Hull & White model calibrated to swaptions volatilities:

$$e_t \equiv \varphi_t + \sum_{i=1}^{2} x_i \tag{2.16}$$

where φ_t is a time-dependent deterministic shift used to calibrate the term structure of zero coupon curves. The two factors dynamics are described by the following stochastic differential equation (SDE):

$$dx_t^i = -a^i x_t^i dt + \sigma^i dW_t^i, \quad x_0^i = 0 \tag{2.17}$$

where a^i and σ^i are positive constants and W^i are standard Brownian Motions. In their work, the Authors make use of reduced-form credit approach and model stochastic intensities λ of the counterparts involved in the transaction as shifted Cox-Ingersoll-Ross (CIR) square-root processes:

$$\lambda_t^i = y_t^i + \psi_t^i, \quad i \in \{B, C\} \tag{2.18}$$

where λ^i are the respective default intensities of the bank B and its counterparts C and ψ^i are deterministic non negative shifts. The diffusive component evolves as follows:

$$dy_t^i = \kappa^i(\mu^i - y_t^i)dt + \nu^i \sqrt{y_t^i} dW_t^i, \quad i \in \{B, C, E\} \tag{2.19}$$

Overnight rates and default intensities are then correlated via a correlation matrix of elements $\rho_{i,j}$ defined as:

$$d\langle W_i, W_j \rangle_t = \rho_{i,j} dt$$

In their numerical experiments, the Authors find that the impact of funding disappears when borrowing and lending rates equal each other, while in presence of asymmetric funding rates the difference between long and short quotes can be interpreted in terms of bid-ask spread widening. In the case of partial collateralization they observe residual CVA/DVA, due also to the lack of initial margins absorbing the losses possibly generated during the margin period or risk. They show that initial margins are effective in covering losses due to extreme events at the very high confidence levels suggested by ISDA and CCP, but they might be responsible of a significant increase of funding costs. Despite the use of very high confidence levels in computing initial margins, the Authors observe residual CVA and DVA in bilateral SCSA trades while in the CCP case the CVA is practically null.

According to 2013 Quantitative Impact Studies (QIS), the impact arising from the introduction of model based IM amounted to 0.7 trillion EUR. From the regulatory point of view, the consultation paper ISDA (2013b) aims at implementing the guidelines published by BCBS-IOSCO (2013). These are of extreme importance for practitioners since relate to the establishment of a common methodology for calculating the exchange of initial margins among cover entities. A shared methodology is fundamental for helping disputes resolution and allowing consistent regulatory compliance. In fact, if each covered entity were to apply its own internal model for initial margins, it would have to do reverse engineering in order to build the margin model that its counterpart was using and ensure the correctness of its margin call. Duplicating all counterparts margin models is infeasible beyond that probably impossible. For this reason the industry has developed the Standard Initial Margins Model (SIMM) for non cleared derivatives. The firsts insights of ISDA relate to the general criteria which a good model should satisfy. Desirable features for the SIMM would not being procyclical in order to avoid excess volatility of initial margins, being easy to replicate also in terms of operational costs, being transparent and appropriate in order to not underestimate risks in large portfolios.

As the general mathematical structure is concerned, the ISDA suggests to apply random shocks to all risk factors affecting a given portfolio Π. In these regards, a multi factor model should be build and in each scenario all risk factors have to be perturbed. A random shock $\{S_{j,k} \mid j = 1...J\} \in \mathcal{S}$, has to be applied to the k-th risk-factor in the portfolio. Then the loss in front of the random shock has to be calculated upon each scenario as the difference:

$$L_{j,k} = NPV(\Pi \mid S_{j,k}) - NPV(\Pi) \tag{2.20}$$

Then all losses have to be put together via the aggregation function $\mathcal{A}$. Possible choices for the aggregation function are Historical or Monte Carlo Value-at-Risk (VaR) rather than Expected Shortfall (ES), within a time horizon corresponding to a 10 days Margin Period of Risk. Hence, the SIMM prescribes to consider the worst potential loss at the confidence level of 99%. Risk factors have to be classified in the four asset classes currency/rates, equity, credit and commodity. Within each asset class i the initial margin IM_i can be computed in terms of the suggested risk measures. Finally, the overall initial margin equals to the sum of stand alone initial margins of single asset classes:

$$IM(\Pi) = \sum_{i=1}^{4} IM_i \tag{2.21}$$

Such a synthetic risk measure will correspond to the Initial Margin to be posted by involved entities. A preliminary analysis revealed that, in terms of IM consistency, Expected Shortfall (ES) is satisfying while VaR is not.

In order to make dealers able to easily provide quotes and speed up the computation, especially for nonlinear derivatives, a *sensitivity approach* is preferred instead of full revaluation, since the latter would require solution of complex equations. Portfolio greeks are precomputed as delta prices in front of one basis point risk factors shifts. Sensitivities are then multiplied for simulated shocks.

Recently the ISDA has published the update 2.1 of the SIMM, which however does not innovate meaningfully the prescribed overall methodology or greeks estimation but it rather suggests a recalibration of risk-weights and correlation among risk factors.

2.1.2 Main contributions

In the first Chapter, a general additive equation for the Bilateral Counterparty Credit Risk Adjustments (i.e CVA and DVA) has been derived. The solution is model-independent, meaning that the building blocks of my approach do not depend on how credit events are modelled. Indeed, the solution is equivalent to the result usually obtained in the context the reduced-form approaches. Furthermore, the derivation does not require any specific setup of the hedging portfolio.

The main contribution of the present Chapter is to extend the computation of the XVA metrics within the first-passage time framework introduced in Chapter 1 through the assessment of the role of funding and IM costs. The current analysis inherits the full structural setup built within Chapter 1.

To my knowledge, I differ from most of the previous research by investigating funding costs from a reversed perspective: instead of starting by the analysis by adding funding-related discounted cash flows into a unified valuation formula, I prefer to address the topic by trying to firstly understand the economic meaning of FVAs. This leads to deep consequences in the interpretation of the FVA term and represents a theoretical breaking point with respect to previous literature. From a technical side, the advantages of my approach consist in a set of simplifications which eliminates the recursion and, under additional assumptions, avoids the emerging of nonlinearities in the FVA pricing problem[8]. My line of reasoning retraces in some ways the approach of Andersen et al. (2017), meaning that the cost of funding appears to be outside of fair valuation principles but, it is rather responsible of some widening in the bid-ask spread that typically banks quote to their clients. In these regards, my improvement on top of previous work relies on precisely quantifying the size of the bid-ask spread widening throughout a straightforward approach. Another relevant contribution of my work is to extend the results of Burgard and Kjaer (2011b), who affirmed that the FVA effect disappears when the trading desk finances itself with no *haircut* with respect to the risk-free rate. Since in this context the distance between ask and bid prices is driven by the specular view of the moneyness, according to the present model I expect that FVAs should compensate in some of practical computations, i.e. when the funding policy is *symmetric*.

This implies that the impact on the bid-ask spread is null under symmetric funding policies such as, for instance, the Balance-Sheet Shrinkage, see Castagna (2014). Conversely, in the case of an asymmetric funding policy, the bid-ask spread do widen in a significant way. Moreover, within the current financial landscape, characterized by negative short term rates, BSS is even more convenient when compared with risk-free investing.

In the Chapter it also is described a numerical application aimed at computing the CCR price adjustments assessing also the role played by funding and CCP margining costs: all these risk metrics are commonly reconnected within the

[8] Nevertheless, FVA is not the only source on nonlinearity when dealing with the pricing of Counterparty Credit Risk.

family of valuation adjustment known as XVA[9]. The numerical results confirm my expectations and reveal significant magnitudes of FVAs and large bid-ask spread widening only in presence of asymmetric funding rates. As CCP margining costs are concerned, the impact in in terms of bid-ask spreads widening is more severe since no *offsetting effect* can take place as a consequence of symmetric funding policies.

The Chapter is organized as follows: in Section 2.2 I provide a recap of the building blocks of the structural model fully described in Chapter 1. In Section 2.3 I provide an economical interpretation of the Funding Valuation Adjustments and explain why my view is to exclude them from fair valuation. In Section 2.4 I decline more deeply the hint of Andersen et al. (2017) as regards the bid-ask spread widening and discuss its dependence on the funding policy which the trading desk puts in place. Section 2.5 describes the numerical procedure applied for the computation of the FVA terms and displays the results of the consequent bid-ask spread widening in several scenarios. Section 2.6 analyses the main implications of trading under Central Clearing Counterparties (CCP) and provides the numerical results for the MVA terms. Section 2.7 draws some concluding remarks.

2.2 Model setup

For the sake of self-consistency, in this Section I summarize the technical setup described in details in the first Chapter, since it applied the present analysis. In a bilateral CCR perspective, I introduce two defaultable counterparts negotiating some OTC derivative trades denoted by the investment bank B (the *dealer*) and its client C (which might be a corporate firm or another bank).

The market is modelled through a filtered probability space $(\Omega, \mathbb{Q}, \mathcal{G}_t)$ where Ω is the set of possible events, $\mathbb{Q}$ is some *risk-neutral* martingale measure and the enlarged filtration is defined as $\mathcal{G}_t \equiv \mathcal{F}_t \vee \mathcal{H}_t$[10].

$\{\mathcal{F}_t\}_{0 \leq t \leq T}$ is the reference filtration which contains all market information except of default events up to time t while $\{\mathcal{H}_t\}_{0 \leq t \leq T}$ is the $\sigma(\tau \wedge t)$-algebra generated by the defaults history. τ is the *first-to-default* random time defined as $\tau \equiv \tau_B \wedge \tau_C$. I is noted that τ is a $\mathcal{G}$-stopping time and the above filtrations satisfy the usual conditions of completeness and right-continuity.

An infinitely divisible Lèvy process $\{X_t\}_{0 \leq t \leq T}$ defined on the filtered probability space $(\Omega, \mathbb{Q}, \mathcal{G}_t)$ is a stochastic process with stationary and independent increments, whose distribution allows for non-zero skewness and excess kurtosis[11]. For the class of Lèvy processes, it is often impossible to determine the PDF analytically. Nevertheless, their characteristic function is always available in closed form thanks to the well-known Lèvy-Khintchine representation:

[9] In the acronym the "X" stands for a varying letter accommodating for several adjustments.
[10] Actually, as explained by Brigo et al. (2013) in Chapter 3, in the context of structural models, $\mathcal{F}$ and $\mathcal{G}$ coincide by construction since default processes are completely driven by default-free market information.
[11] Higher then second order moments describe the deviation from normality.

$$\phi_X(u;t) = \mathbb{E}[e^{\imath u X_t}] = e^{t\varphi_X(u)}, \quad u \in \mathbb{R} \tag{2.22}$$

where $\varphi_X(u)$ is the *characteristic exponent* of parameter u with the process $\{X_t\}_{t\geq 0}$

The Normal Inverse Gaussian (NIG) process is a *pure jump* Lèvy process which can be obtained by subordinating a Geometric Brownian Motion (GBM) to and independent Inverse Gaussian (IV) Process $IG(t)$. The Inverse Gaussian process, alternatively known as the distributional law of the first-passage time of the Brownian Motion, is a *α-stable subordinator* with $\alpha = \frac{1}{2}$.

The NIG process, similarly to other ones which are obtainable by subordination, is said to be a *time-changed* Lèvy process: in other words, it is indexed to a "stochastic clock". Hence the NIG Process presents the following form:

$$X_t = \mu IG_t + \sigma W_{IG_t} \tag{2.23}$$

Building Lèvy processes by brownian subordination is particularly appealing from an economic point of view, since the time-change might be interpreted as the switch from calendar time to *business time*. This implies to assume that asset prices mainly are driven by the relevant news, characterized by random arrival times and random impact on the market. Indeed, the empirical evidence suggests that gaussianity seems to be recovered under such trading time. The characteristic exponent in the case of a NIG process is:

$$\varphi_X(u) = \frac{1 - \sqrt{1 - 2\imath\theta\kappa + u^2\sigma^2\kappa}}{\kappa} \tag{2.24}$$

where u is the Fourier parameter, $\imath$ is the imaginary unit. As for the meaning of parameters is concerned, $\theta \in \mathbb{R}$ denotes the skewness of the process, $\sigma > 0$ is the volatility parameter and $\kappa > 0$ controls the excess kurtosis of the distribution.

Non Gaussian Lèvy processes have been largely applied in financial modelling because of their superior capability compared to that of pure diffusion models to represent the stylized behaviour observed in real markets. More specifically, Lèvy processes can replicate implied volatility surfaces without overstress model parameters and can accommodate for jumps, see Cont and Tankov (2004). The presence of jumps in the path of risky assets implies the market is in general incomplete[12].

In this framework the NIG process X_t is the relevant *risk driver* for the uncertain dynamics of the underlying asset S_t, whose price at time t under the risk-neutral measure $\mathbb{Q}$ is:

$$S_t = S_0 e^{(r-q-\varphi_X(-i))t+X_t} \tag{2.25}$$

where $r > 0$ is the proxy for the risk-free rate (e.g. EURIBOR/EONIA) and $q > 0$ is the continuous dividend yield paid by the underlying stock. $\mu =$

[12] Market incompleteness implies it is not possible to replicate every uncertain derivative payoff through a combination of elementary assets: hence, derivative claims are not redundant.

$r-q-\varphi_X(-i)$ is the *mean-correcting drift* needed in order to allow S_t to be an exponential martingale.

Within my structural approach the random default time τ is modelled a là Black and Cox (1976), hence as the *first-passage time* of the firm's equity at the level of a fixed default barrier:

$$\tau_i = \inf\{t \in (0,T] : S_t^i \leq K^i\}, \quad i \in \{B, C\} \tag{2.26}$$

I further assume equation 2.25 describes also the dynamics of equity values of firms B and C in addition to those of the underlying assets. Then, replacing it in equation 1.16 it leads to:

$$\tau_i = \inf\{t \in (0,T] : S_0^i e^{(r-q^i-\varphi_{X^i}(-i))t+X^i(t)} \leq K^i\}$$

Under Counterparty Credit Risk it is necessary to distinguish between $V(t,S)$, which denotes the economic value at time t of the *default-free* derivative contract, and $\hat{V}(t,S,D_B,D_C)$ which denotes the value of the correspondent counterparty risky claim. The state processes $\{D_t^i\}_{0\leq t\leq T}$ denote the occurrence of the respective credit events:

$$D_t^i \equiv \mathbb{1}_{\{\tau_i \leq t\}}, \quad i \in \{B, C\}$$

In the first Chapter, it has been derived a *model-independent* representation of the adjusted value of a default-risky derivative claim:

$$\hat{V}_t = V_t - \underbrace{\mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)(\varepsilon_{\tau_C}^+ - R_C(\varepsilon_{\tau_C}^+))]}_{CVA} + \underbrace{\mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)(\varepsilon_{\tau_C}^- - R_B(\varepsilon_{\tau_B}^-))]}_{DVA} \tag{2.27}$$

where $D(t,T)$ is a approximated risk-free discount factor, ε_t is the exposure computed at time t and R_C and R_B are respectively the client and the bank recovery functions upon respective defaults.

As in most practical application, I further suppose that both recovery functions take deterministic a values in $[0,1]$ and I adopt the risk-free close-outs convention, meaning that the EAD is equal to the default-free *mark-to-market* of the contract V_τ. Hence the previous equation becomes:

$$\hat{V}_t = V_t - \underbrace{\mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)(1-R_C)V_{\tau_C}^+]}_{CVA} + \underbrace{\mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)(1-R_B)V_{\tau_C}^-]}_{DVA} \tag{2.28}$$

2.3 Does the FVA impact on fair-valuation?

From a operational point of view, financial derivatives are priced in terms of the cost of their hedging strategies. Bearing in mind that risk-free funding is not available any more in the market, the economic valuation of derivative claims is depends on the policy by which hedging strategies are financed. In the light of the exceptional growth of credit spreads it has been observed during the crisis, the impact of funding costs on the balance-sheet of major dealers has been massive. Among others, J.P. Morgan Chase in its Q1 2014 public report declared Funding Valuation Adjustments (FVA) had been accounted for US$ 1.5 billions. Many experts recognized that announcement as the first relevant move in the financial industry, stimulating other dealers to align. Thereafter, some of the major consultancy firms such as EY and KPMG, publicly declared their acceptance for accounting FVAs. Nevertheless, the debate on funding costs is still ongoing because of their asymmetry, non linearity and non pure additivity. Their nature of valuation adjustment or profitability analysis tool has still to be clarified. So far, FVAs have been treated as additive adjustments on the fair value of OTC trades, similarly to what happens for CVA and DVA terms.

In line with the argument of Brigo et al. (2015), let me assume that the evaluating counterpart, which calculates the fair price of the default-risky deal $\hat{V}$ according to equation 2.28, intends to hedge its uncollaterized exposure[13] by running a *self-financing* portfolio aimed at protecting itself against both market and credit risks. Going back to the original work of Black and Scholes (1973), the hedging portfolio is typically composed by a set H of hedging instruments[14] and some amount β of cash:

$$\Pi_t = \hat{V}_t - C_t = H_t + \beta_t + \epsilon_t, \quad t \in [0, T] \tag{2.29}$$

where C is the cash collateral posted by the out of the money counterpart and ϵ is a composite *hedging error*[15] due to the incompleteness of the market. Holding a portfolio for hedging purposes requires the establishment of a *funding account* F sufficient to finance both the purchase of hedging instruments and to hold a cash position:

$$F_t = H_t + \beta_t = \hat{V}_t - C_t - \epsilon_t \tag{2.30}$$

In order to mitigate the overall default risk let me assume that rehypothecation[16] not is allowed, so that collateral cannot be used to reduce the amount of cash required in the funding account:

$$F_t = \hat{V}_t - \epsilon_t, \quad t \in [0, T] \tag{2.31}$$

Prior to an eventual rebalance of the portfolio, by the self-financing condition it must hold:

$$dF_t = d\hat{V}_t - d\epsilon_t, \quad t \in [0, T] \tag{2.32}$$

[13] The following computation attains to the seller case. Otherwise, in the buyer case a minus sign is required to obtain the effective hedge, i.e. $\Pi = -(\hat{V} - C)$.

[14] The "hedges" in the replicating portfolio must have the same cardinality of the sources of risk.

[15] Among other frictions characterizing a model with jumps, even if one is intended to hedge the DVA, is is impossible to replicate its own *jump-to-default risk*, since selling protection through a CDS written on own self is not allowed, nor credible.

[16] Rehypothecation is the contractual provision which allows to not keep the cash collateral "frozen" in a safe bank account and use it as collateral for other OTC positions against other counterparts.

let me define $\bar{\tau} \equiv \tau \wedge T$ the random default-adjusted expiry of the contract. By taking the risk-neutral expectation with respect to $\mathbb{Q}$ of the sum of time t-discounted cash flows in the continuous interval $[t, \bar{\tau}]$, it can be obtained:

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^{\bar{\tau}} D(t,s)dF_s \mid \mathcal{G}_t\right] = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\bar{\tau}} D(t,s)\tilde{f}_s\hat{V}_s ds \mid \mathcal{G}_t\right] - \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\bar{\tau}} D(t,s)d\epsilon_s \mid \mathcal{G}_t\right] \tag{2.33}$$

$$s > t$$

where

$$\tilde{f}_t \equiv f_t^{+}\mathbb{1}_{\{\hat{V}_t>0\}} + f_t^{-}\mathbb{1}_{\{\hat{V}_t<0\}}$$

The left-hand side of equation 2.33 corresponds to the definition of the FVA term. f^+ and f^- are respectively the unsecured borrowing and lending rates faced by the dealer. In real world transactions these are often asymmetric. In that case, there configures a complex nonlinear pricing problem which. Furthermore, under additional modelling assumptions the valuation becomes recursive. Nonlinear recursive pricing problems have been generally addressed via the Backward Stochastic Differential Equations Theory (BSDEs), see Crépey (2015a) and **?**, among others.

In order to shed light more intuitively on the economic meaning of funding costs[17], let me simplify the problem by assuming that the sign of the hedging error is not systematic over time. In such case the last term of equation 2.33 would collapse to zero:

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^{\bar{\tau}} D(t,s)d\epsilon_s \mid \mathcal{G}_t\right] \downarrow 0$$

By acknowledging the evidence of slightly negative risk-free rates characterizing the current financial landscape, for small r the discounted funding spread can be linearised as follows:

$$\lim_{r\uparrow 0} e^{-r_s(s-t)} f_s = \lim_{r\uparrow 0} f_s - r_s \tag{2.34}$$

Hence, the equation describing the cost of financing, commonly known as Funding Valuation Adjustment (FVA) becomes:

$$\begin{aligned} FVA &\simeq \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}}\left[(\tilde{f}_s - r_s)\hat{V}_s \mid \mathcal{G}_t\right] ds \\ &= \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}}\left[(f_s^{+} - r_s)\hat{V}_s^{+} - (f_s^{-} + r_s)\hat{V}_s^{-} \mid \mathcal{G}_t\right] ds \end{aligned} \tag{2.35}$$

[17] In a economic perspective they could be viewed as non-null transaction costs: in other words, these configure as a friction of the post-crisis financial market where no agent can borrow at the risk-free rate any more.

Please note that the Fubini's Theorem has been applied in order to interchange expectation and integration operators.

Within their outstanding contribution, Andersen et al. (2017) have formally derived a pecking order of preferences of firm's shareholders about several possible funding strategies. These range from financing OTC derivative trades by withdrawing available cash from the balance-sheet, to issue new unsecured debt or to issue new equity. Nevertheless, real market conditions could prevent the choice of using some of them. For instance in major dealers, derivative trading desks are typically separated entities from the treasury and hence, they are probably inhibited to take autonomous decisions regarding the use of existing cash in the balance-sheet. The same applies for equity issuing which moreover, can probably not be flexible enough for the purposes under discussion.

As the investing side is concerned, in previous FVA modelling any surplus of cash deriving from entering into derivative positions and their relative hedging, was assumed to be invested at the risk-free rate in order to not increase the overall risk in the portfolio. However this assumption might clash with reality since the current financial environment. In fact, the expansionary monetary policy trend held by both the Federal Reserve and the European Central Bank (ECB), led to negative short-term risk-free interest rates. In such case, investing at a below zero risk-free rate would reduce the P&L of the trade. It is not credible that such strategy would be followed by the trading desk. On the other hand, suppose traders holding a liquidity surplus can repurchase previously issued unsecured debt. This practise is known as the "Balance-Sheet Shrinkage" funding policy. Under BSS marginal funding benefits equal the marginal cost of new issued debt, so that borrowing and lending rates compensate each other, i.e. $f^- = -f^+$. Since f^+ can be easily inferable from current market quotations, for instance from the CDS market, the discounted funding rate could be interpreted as the spread of CDS quotes with respect to the risk-free rate. By recalling the discounted funding spread expressed in equation 2.34, it can be obtained:

$$\begin{aligned} \tilde{f}_t - r_t &= (f_t^+ - r_t)\mathbb{1}_{\{\hat{V}_t > 0\}} + (f_t^- + r_t)\mathbb{1}_{\{\hat{V}_t < 0\}} \\ &= (f_t^+ - r_t)\mathbb{1}_{\{\hat{V}_t > 0\}} + (-f_t^+ + r_t)\mathbb{1}_{\{\hat{V}_t < 0\}} \\ &= (f_t^+ - r_t)\mathbb{1}_{\{\hat{V}_t > 0\}} - (f_t^+ - r_t)\mathbb{1}_{\{\hat{V}_t < 0\}} \\ &= \underbrace{s_t^B \mathbb{1}_{\{\hat{V}_t > 0\}}}_{\text{discounted borrowing spread}} - \underbrace{s_t^B \mathbb{1}_{\{\hat{V}_t < 0\}}}_{\text{discounted lending spread}} \end{aligned} \tag{2.36}$$

Let me assume that s_t^B is the bank fair spread quoted in the CDS market at time t. Under the stated hypothesis, symmetric borrowing and lending rates would characterize the expected funding costs such that:

$$FVA = \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}} \left[s_s^B \left(\hat{V}_s^+ - \hat{V}_s^- \right) \mid \mathcal{G}_t \right] ds \tag{2.37}$$

This result justifies the following proposition.

PROPOSITION 2. *In the symmetric case, the* ***Funding Valuation Adjustment (FVA)****, which is defined as the risk neutral expectation of the costs faced for financing the hedging strategy, might be seen as the expected sum of discounted funding spreads weighted by the moneyness path of the default risk-adjusted derivative price* $\hat{V}$.

The valuation adjustment for the cost funding the hedging portfolio, is a cost which do impact on the P&L of the trade. Nevertheless it is *dealer-specific*, since it depends on the bank's excess cost for unsecured borrowing (which I showed to be its CDS spread) and on the choice regarding the way to finance the hedging strategy for the overall risk.

As concerns the positive part of the exposure, the inclusion of FVA in fair pricing would imply to make the client paying for both its default risk and the one of the bank (through the FVA charge). I retain that it pointless. Conversely, as regards the negative side of the exposure, accounting for the FVA benefit would make the dealer paying in proportion to:

- Its own default risk in the amount of $\mathbb{E}_t^{\mathbb{Q}}[\mathbb{1}_{\tau=\tau_B}(1-R_B)D(t,\tau_B)]$ through the DVA term
- Its excess cost for unsecured borrowing, i.e. its CDS credit spread s^B through the inclusion of FVA benefit term

These two elements describe basically the same effect, as they both measure the price of default risk of the bank with respect to a perfectly *default-free* counterpart. Because of that, there configures a high degree of *overlap*[18] with the DVA term when including the funding benefit in pricing models.

In the light of these arguments, to reconnect the FVA within the fair valuation principle seems hardly desirable in a theoretical perspective. It rather represents a friction affecting post-crisis financial markets which appears to be outside of fair pricing. From a theoretical point of view, my point is to consider desirable charging to the derivative price only the payoff-related adjustments due to default risk, i.e. the CVA and the DVA, since they modify the expected size of the payoff itself.

FVA and additional frictions-based or regulation-based adjustments instead, should not be included in the fair price. However, since they do impact into the internal valuation of economic convenience of derivatives trades, they would better be used as profitability analysis tool as it will be described in the following section.

[18] Some authors have pointed out how investing the FVA funding benefits in own bond repurchasing configures as a way for DVA hedging.

2.4 Bid-Ask spreads implications of FVAs

FVAs are well suited to be used as a *profitability analysis* tool by trading desks intended in entering into an OTC derivative trade. The reason can be find in Andersen et al. (2017) who claim that, despite funding costs do not affect derivative valuation, they are rather responsible of some widening in the bid-ask spread[19]. In these regards, my contribution aims at shedding light on the mechanism leading to the widening in a straightforward way.

In the case of selling the derivative to its counterpart, the dealer's P&L at inception equals to:

$$P\&L_{Sell} = U_{Ask} - \hat{V} - FVA_{Ask} \tag{2.38}$$

so that the minimum upfront or *entry price*, at which the trading desk would be willing to sell (i.e. ask price), would be:

$$U_{Ask} = \hat{V} + FVA_{Ask} \tag{2.39}$$

This occurs because the trading desk aims at least to be compensated for the sum of expected cash outflows correspondent to the liability $\hat{V}$ and incremental funding costs deriving from the entrance into the trade.

On the other hand, if the dealer would be intended to hold a long position into the trade, its P&L at inception would equal to:

$$P\&L_{Buy} = -U_{Bid} + \hat{V} - FVA_{Bid} \tag{2.40}$$

so that the maximum upfront at which the trader would be willing to buy (i.e. the bid price) is:

$$U_{Bid} = \hat{V} - FVA_{Bid} \tag{2.41}$$

This occurs because the trading desk sees the value which it gets by holding the derivative security $\hat{V}$, decreased by the additional funding cost.

I here stress that the funding costs faced by the dealer in case of going long rather than short in the OTC derivative trade are in general different, i.e. $FVA_{Bid} \neq FVA_{Ask}$. Therefore, the overall *widening of the bid-ask spread* caused by funding costs is:

$$U_{Ask} - U_{Bid} = FVA_{Ask} + FVA_{Bid} \tag{2.42}$$

In the next step, I quantify more precisely the amount by which the bid-ask spread widens when funding costs are incorporated within the analysis. Following my approach, under a symmetric funding strategy the impact would reflect the specular view of the moneyness of the contract relatively to the cases of assuming a long rather then short position in the derivative trade:

[19] In fact, the belief which FVA had driven bid-ask spreads during the past years of financial crisis is rather shared among market participants.

$$FVA_{Ask} + FVA_{Bid} = \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}} \left[s_s^B \left(\hat{V}_s^+ - \hat{V}_s^- \right) \mid \mathcal{G}_t \right] ds$$
$$+ \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}} \left[s_s^B \left(\hat{V}_s^- - \hat{V}_s^+ \right) \mid \mathcal{G}_t \right] ds \tag{2.43}$$

By simple rearrangements of equation 2.43 it can be intuitively obtained:

$$FVA_{Ask} + FVA_{Bid} = \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}} \left[s_s^B \left(\left(-\hat{V}_s^+ + \hat{V}_s^+ \right) - \left(\hat{V}_s^- - \hat{V}_s^- \right) \right) \mid \mathcal{G}_t \right] ds = 0 \tag{2.44}$$

In other words, under the cited assumptions it holds $FVA_{Bid} = -FVA_{Ask}$. In presence of symmetry between borrowing and lending spreads, the impact of funding on the bid-ask spread disappears via an *offsetting effect* due to specular view of the moneyness when selling rather than buying. The present result extends the one of Burgard and Kjaer (2011b) who postulated that the FVA, interpreted as an additive correction on the fair price, expires when the funding spread is null. My argument is that the bid-ask spread widening expires not only in the case of no funding spread, being sufficient that borrowing and funding spreads are symmetric as in the Balance-Sheet Shrinkage funding policy. Moreover, such policy would be particularly appealing in the current market landscape for dealers with a sufficiently liquid own bond market. To my knowledge, it misses in previous literature a comparative analysis regarding the efficiency of Balance-Sheet Shrinkage with respect to other funding policies based on equity repurchasing or on a combination of equity and bond repurchasing. Such analysis would configure as the extension of the pecking order of preferences of firms' shareholders of Andersen et al. (2017) to the investing side of funding cash flows. I point out that the present analysis applies to the impact of funding costs on bid-ask spreads and not to the P&L from the perspective of the *shareholders utility maximization*[20].

Conversely, bid-ask spreads widening is definitely not null when symmetric funding policies cannot be attained. For instance, suppose that the bank does not cope with a sufficiently liquid own bond market, meaning that it cannot invest available liquidity in repurchasing back its own debt at any point in time. In that scenario, negative rates would represent an extra cost for risk-free investing, configuring as the price of third party custodial service.

[20] In fact as explained by Andersen et al. (2017), the shareholders marginal utility from entering in a new trade amounts to:

$$V^* = \hat{V} - DVA - FVA \tag{2.45}$$

This occurs since, despite the DVA is nowadays well recognized by accounting principles in order to allow for symmetric trade prices, it does not involve any real benefit for the equity side of the firm upon default.

The overall widening of the Bid-Ask spread caused by the cost faced by the dealer for financing the hedges under an asymmetric funding policy amounts to:

$$\begin{aligned} U_{Ask} - U_{Bid} =& FVA_{Ask} + FVA_{Bid} \\ =& \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}} \left[s_s^B \left(\hat{V}_s^+ + \hat{V}_s^- \right) \mid \mathcal{G}_t \right] ds \end{aligned} \tag{2.46}$$

My line of reasoning can then be summarized in the following remark.

Remark 1 (**Bid-Ask spread widening**). *When the dealer faces an illiquid own bond market and decides to invest available liquidity at the risk-free rate, it incurs in instantaneous funding costs widening the bid-ask spread in the amount of its CDS spread. In the current negative rates environment, risk-free investing configures as an extra cost for the trading desk, interpretable as a custodial service price. Nevertheless funding can be used as a tool to evaluate the economic convenience of entering in derivative trades. In many cases, the optimal entry price is expected to be different from the fair value.*

In current market conditions, BSS funding might be assumed to be particularly appealing for dealers coping with sufficiently liquid own bond markets. Without loss of generality, the existence of a liquid own bond market can be supposed to be common knowledge among market participants. Because of that, clients are eased in making conjectures about dealers funding requirements. That might increase clients awareness when bargaining with the counterpart on the trade price. Charging funding costs to clients is never be fair from a valuation point of view and, especially in presence of a liquid own bond market, the dealer is not legitimate to quote an oversized bid-ask spread. Nevertheless, because of the scarce transparency and high searching costs observed OTC markets, corporations might be willing to give priority to their economic purposes to book the trade and pay an unfair bid-ask spread. Therefore as explained in Castagna (2014), the actual price at which the OTC trades are closed is influenced by the bargaining power of the two counterparts. As a consequence, upfronts do not necessary reflect fair prices, which in turn deviate from dealers' overall valuations, i.e. those affected by funding costs.

2.5 Numerical Results

Extending the case study based on the commodity asset class analysed in Chapter 1, the aim of the present numerical application is to quantify experimental magnitudes of FVA terms and bid-ask spreads widening. The exercise has been performed in order to verify whether numerical results confirm the theoretical statements built in previous Sections.

In this Section, I assume that just two possible funding policies are enforceable by the trading desk of the computing counterpart. In the first case, the trading desk can put in place the Balance-Sheet Shrinkage by investing the liquidity coming from the hedging strategy in repurchasing previously issued debt. In such scenario, the computation leads to what many have called Symmetric Funding Valuation Adjustment (SFVA), see Albanese et al. (2014). The second scenario concerns the case in which, for whatever reason, the trading desk decides to invest liquidity at the risk-free rate. In this case the calculation provides what the cited Authors have defined Asymmetric Funding Valuation Adjustment (AFVA).

Relying on my line of reasoning, I would expect a null impact on the bid-ask spread in the first scenario. Conversely, the second one it should configure a not negligible impact on the bid-ask spread. In order to run the computation, the idea is to discretize equation 2.37 in the time grid $\mathcal{T} \equiv \{T_0, T_1, T_2, ..., T_M\}$, where $t = T_0$, $T = T_M = M\Delta T \quad (m = 0, 1, ..., M)$ and $\Delta T = T/M$, such that:

$$FVA \simeq \sum_{m=0}^{M-1} \mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\{\tau_c > T_m, \tau_B > T_m\}} s^B_{T_m} \hat{V}_{T_m} \Delta T \mid \mathcal{G}_{T_0}] \tag{2.47}$$

A deterministic discount factor has been calibrated by means of the Hull and White 1 Factor model to the 1 year EURIBOR curve. The discounted funding spread of Enel has been set to the average of daily 5-years CDS quotes available in my dataset[21], i.e. $s^B_{T_m} = \bar{s}^B \; \forall m \in \{0, .., M\}$. By exploiting the calibration of parameters sets and default barriers performed in Chapter 1, joint survival probabilities have been pre-computed at every each time step of the grid via Monte Carlo simulations of structural defaults. This set of assumptions allows me pre-compute joint survival probabilities and drag them and the discounted funding spread outside the expectation operator[22]. The expected default risk-adjusted trade price is then obtained by the same hybrid Fourier-Monte Carlo algorithm built in Chapter 1.

$$FVA = \bar{s}^B \Delta T \sum_{m=0}^{M-1} \mathbb{Q}\{\tau_c > T_m, \tau_B > T_m\} \mathbb{E}^{\mathbb{Q}}[\hat{V}_{T_m} \mid \mathcal{F}_{T_0}] \tag{2.48}$$

[21] Source: iTraxx Series for the European CDS market.

[22] By factorizing I implicitly assume independence between default processes and discount factors.

Counterparty Risk and Funding Valuation Adjustments (XVA)						
Contract type	V_0	CVA/V_0	DVA/V_0	$\hat{V}_0$	FVA/V_0	B-A Spread
ATM Call Oil	5.821	15275.381	0	4.932	91.369	0
Forward Oil	- 0.032	219.058	124.097	-0.032	108.602	0
Swap Oil	-1.375	2.238	113.908	-1.360	88.914	0
ATM Call Gas	0.412	1496.011	0	0.350	91.642	0
Forward Gas	-0.027	38.305	169.622	-0.026	- 50.977	0
Swap Gas	-0.067	93.757	90.210	-0.067	107.757	0

Tab. 2.1: Case a) Balance-Sheet Shrinkage policy: Symmetric Funding Valuation Adjustments, bid-ask spread widening and magnitude comparison with respect to bilateral CCR adjustments. All relative adjustments are measured in basis points.

Counterparty Risk and Funding Valuation Adjustments (XVA)						
Contract type	V_0	CVA/V_0	DVA/V_0	$\hat{V}_0$	FVA/V_0	B-A Spread
ATM Call Oil	5.821	15275.381	0	4.932	91.345	99.804
Forward Oil	- 0.032	219.058	124.097	-0.032	-10.063	- 118.721
Swap Oil	-1.375	2.238	113.908	-1.360	88.845	97.074
ATM 1y Call Gas	0.412	1496.011	0	0.350	91.596	100.079
Forward Gas	-0.027	38.305	169.622	-0.026	4.720	55.687
Swap Gas	-0.067	93.757	90.210	-0.067	-9.982	-117.766

Tab. 2.2: Case b) Risk-free investing policy. Asymmetric Funding Valuation Adjustments, bid-ask spread widening and magnitude comparison with respect to bilateral CCR adjustments. All relative adjustments are measured in basis points.

It has been considered an ATM Call Option, both Forward and Swap contracts have been assumed to be negotiated at their respective fair forward and swap rates. The algorithm has been coded in Python 2.7 on a MacBook Pro Retina (Late 2013) with 2,4 GHz Intel Core i5 ® processor and 8 GB 1600 MHz DDR3 RAM. The computation of FVA metrics took roughly 50 minutes for each Call Option and Forward contract. The algorithm required roughly 3 hours and 30 minutes for calculating the FVA term for single Swap contracts. In both tables it is displayed the amount of funding costs faced in case of taking a short position in the trades, i.e. the FVA_{Ask}.

My numerical findings confirm a null impact on the bid-ask spread under a symmetric funding policy. On the contrary, a considerable effect takes place when the trading desk pursues an asymmetric funding policy. Except for the case of one-sided payoff derivatives such as Call Options, in which the CVA-type term is dominant, the average magnitude of FVAs is relevant and comparable to those of other XVA terms. In three cases, i.e. for the gas Forward in Table 2.1 (symmetric funding policy) and for the oil Forward and he gas Swap in Table 2.2 (asymmetric funding policy), the FVA terms present negative sign, meaning that they configure funding benefits. Unexpected results characterize the last two cases, since the calculation provides a negative overall impact of funding costs on the bid-ask spread. This occurs because in such cases the FVA involves an overall cash benefit when taking a back-to-back position in the trade. Nevertheless, since I do not expect to observe traders quoting negative bid-asks spread to their clients, the bid-ask widening should revert back to zero, allowing traders to somehow exploit a kind of *funding arbitrage*. Exploring the incidence as well as mechanisms leading traders to gain funding-related profits[23] will definitely be an interesting topic for future research.

Remark 2 (**Portfolio effects**). *As funding strategies are concerned, trading desks generally put them in place by focusing, for instance, on aggregate liquidity requirements of portfolios denominated in a given currency. In these regards, the nonlinear nature of funding costs is exacerbated, meaning that the sum of stand-alone funding effects at the netting sets level do not equal to the overall funding effect faced by the desk. As a consequence, within a real application I would raise some concerns regarding the reliability of a comparative analysis of the magnitudes of default risk adjustments with respect to those of FVA terms.*

2.6 Trading under Central Clearing Counterparties (CCP)

As previously introduced, under the Dodd-Frank Act, Central Clearing Counterparties (CCPs) became mandatory for all OTC inter dealer trades executed post September 2016 in the US market. In compliance with the EMIR/CRD4, similar dynamics has been observed in the European market. This involves the investigation of a new member of the XVA family called Initial Margin Valuation Adjustment (MVA).

[23] That are attainable because of the existence of zero floors for bid-ask spreads characterizing normal market conditions.

It is shared that one of the most effective mitigation practices for default risk is collateralization. For this reason, regulators are pushing OTC market participants to move their trading activity under the jurisdiction of CCPs, as it typically occurs for *listed* plain vanilla derivatives such as Futures and European-style Options. The aim of regulators leading to the establishment of compulsory CCP trading, is to ensure contractual payments of OTC transactions even in case of default and cover potential losses due to extreme events through an additional layer of collateralization on top of Variations Margins (VM), represented by the Initial Margin (IM). This is a cash amount, generally posted to a third custodial entity or to the CCP itself. The calculation of IM amounts is typically based on some kind of market risk measure, such as the Stressed Value-at-Risk (SVAR) or the Expected Shortfall (ES). An alternative is represented by outcome of suitable Stress Tests, see Green and Kenyon (2014).

In the light of the convenience of financial operators to collaterize exposures at netting set levels, I would suggest to prefer a coherent subadditive risk measure such as the Expected Shortfall as a consistent choice for the IM setting. In fact, the ES has the advantage on top of Value-at-Risk (VaR) of being more conservative. Moreover, it accounts also for diversification effects arising within the netting set.

In this Section it is assumed that cash collateral posting is financed through new debt issuing. As it has been already said, the existence of IM financing costs led to the birth of an additional member in the XVA family, the MVA. Despite I believe that MVA does not affect fair valuation as its "relative" FVA, I fear that its impact on bid-ask spreads might be even more severe compared to the one caused by FVAs. This explains why it is rather shared that the establishment of OTC central cleared trading will impact dramatically on the P&L of OTC trades. At aggregate level, as TABB Group estimated, the locking of around 2 USD trillions of dollars in CCPs cash accounts might generate systemic liquidity issues. Similar insights have been provided by 2013 QIS which reported an impact of 0.7 EUR trillions in the european market. For these reasons, especially for what concerns corporate firms involved in CCP trading, the efficiency of collateral management will become a more and more relevant topic in both academic and operative environments. Moreover, since CPPs can go burst themselves, Central Clearing poses additional Systemic Risk issues.

Roughly speaking, MVA is the expected cost for financing the Initial Margin (IM) asked to CCP members. Despite under particular circumstances the FVA can be mitigated or neglected at all, the IM effect on liquidity is always significant in terms relative magnitude to the fair value of the contract. Differently from what it might happen for FVAs, I am afraid that in the case of MVAs no *offsetting effects* can materialize. That is the theoretical reason for expecting that the locking of trillions of dollars of IM into CCPs accounts will cause a systemic liquidity effect. By incorporating initial margins within the analysis, additional costs impact on bid-ask spreads that dealers quote to their clients in the amounts of:

$$U_{Ask} - U_{Bid} = FVA_{Ask} + FVA_{Bid} + \underbrace{MVA_{Ask} + MVA_{Bid}}_{\text{IM iquidity Effect}} \tag{2.49}$$

As before, the cost of financing the IM in the case of buying differs from that in the case of selling, since the MVA_{Ask} and the MVA_{Bid} are computed as market risk measures on the opposite sides of the exposure profile:

$$\begin{aligned} MVA_{Ask} &= \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}} \left[s_{(B,s)} ES_{99\%}^{\text{Ask}} \mid \mathcal{G}_t \right] ds \\ MVA_{Bid} &= \int_t^{\bar{\tau}} \mathbb{E}^{\mathbb{Q}} \left[s_{(B,s)} ES_{99\%}^{\text{Bid}} \mid \mathcal{G}_t \right] ds \end{aligned} \tag{2.50}$$

For the sake of simplicity, in this work it is assumed that Initial Margins are kept fixed and computed at inception as ES risk measures on the single trades. Unlike the SIMM, I prefer to perform a *full revaluation* instead of a sensitivity based estimation. Formally:

$$ES_\alpha(\Phi(S_{th})) = \mathbb{E}^{\mathbb{Q}} \left[\Phi(S_{th}) \mid \Phi(S_{th}) \leq VaR_\alpha(\Phi(S_{th})) \right] = \frac{1}{\alpha} \int_0^\alpha VaR_\gamma(\Phi(S_{th})) d\gamma \tag{2.51}$$

where

$$VaR_\alpha(\Phi(S_{th})) = \inf\{x \in \mathbb{R}^+ \mid \mathbb{Q}\{\Phi(\mathrm{S_{th}}) + \mathrm{x} \leq 0\} \geq 1 - \alpha\} \tag{2.52}$$

I denote by VaR_α the Value-at-Risk at α confidence level, here set at the 99% (in line with SIMM), while th indicates the time-horizon, here equal one week Margin Period of Risk. Within my theoretical framework, since I am computing the risk measure for pricing purposes, the numerical calculation of the Expected Shortfall measures is performed via a Monte Carlo Simulation under the risk-neutral pricing probability measure $\mathbb{Q}$.

Initial Margins and MVA					
Contract type	V_0	IM_{Ask}	IM_{Bid}	MVA/V_0	B-A Spread
ATM 1y Call Oil	5.821	0.140	0	240.446	240.446
ATM 1y Call Gas	0.417	0.011	0	255.764	255.764

Tab. 2.3: MVA liquidity effect arising from the Initial Margins financing cost. All relative adjustments are measured in basis points.

As shown in Table 2.3, my expectations regarding MVA terms are confirmed: their contribution to the bid-ask spread widening is quite relevant. MVA components are larger than FVA components in terms of magnitude, and unfortunately "offsetting effect" can materialize under whatever funding policy undertaken by the trader. In fact, the symmetry of the funding policy does not matter in this context, since under CCP clearing[24] initial margins always configure as cash outflows. Table 2.3 displays the relative magnitude of MVA terms only for the Call Options since the value at inception of the analysed Forward and Swap contracts is virtually null. My numerical results show that the relative size of MVA as a fraction of the initial default-free derivative price reaches significant levels. Being Call Options one-sided derivatives, the IM required by the CCP is positive in the case of selling and null in the case of buying. According to my numerical results, the *liquidity effect* arising from financing the additional IM funding cost is material: its relative magnitude ranges around the 2.4%/2.55% of the initial fair value of the default-free derivative, making the MVA comparable or even bigger in some circumstances than other XVA family members.

From a theoretical point of view, my belief is that forcing market participants at trading under CCPs and posting Initial Margins grounded on market risk measures over unsecured exposures, poses the threat of a not negligible degree of overlap with the FVA term. In fact, assuming OTC trades are at least partially hedged (in the majority of cases they are fully hedged via back-to-back policies), some FVA costs have to be financed. In such scenario if the trader, in order to not record a loss on the deal, is forced to charge the counterpart at least the sum of FVAs and the MVAs via some bid-ask spread widening. In my opinion, a critical theoretical inconsistency might reside in that hedging could not be recognized when computing the initial margin through market risk measures. As a consequence, the counterpart is asked to pay two partially overlapping costs, the FVA and the MVA. In other words, the client would be asked to pay both the price of overall risk seen at the outset and the cost for hedging it out. The acknowledgement of such threat might represent a further signal of the uncontrolled behaviour which OTC derivatives prices might show under the current enlargement of the XVA family. The classical Law of the Unique Price risks to be broken in a not reversible way. In the light of these arguments, I expect that the topic of the FVA/MVA overlap could become more and more investigated by both the practitioners and academic research.

2.7 Conclusions

By starting the analysis of funding costs from a reversed perspective with respect to the usual one in literature, instead of including funding-related cash flows within the derivative payoff, I firstly aimed at understanding the economic role played by the FVA term. The FVA effect is dealer-specific, meaning that it is roughly proportional to the CDS spread of the computing party and dependent on the measure by which the dealer itself decides to hedge out the overall risk. The inclusion of the Funding Valuation Adjustments (FVA) in fair pricing would break the symmetrical nature of derivative prices. Furthermore, as the negative

[24] On the contrary, if IM is posted in the context of SCSA it might take place an *offsetting effect* similar to that observed for the FVA.

part of the FVA term is concerned (i.e. when funding provides positive cash flows), it involves some degree of overlap with the DVA term. The inclusion in pricing of FVA benefits would force the dealer to pay twice the price of its own default risk. As a consequence, I conclude that there is no theoretical reason for including funding costs within fair pricing principles.

Nevertheless, in a shareholders utility maximization perspective, FVA management can to be used as an internal tool of profitability analysis. I have found that funding costs are responsible of a widening in the bid-ask spread that dealers typically quote to their OTC clients. Such widening is exactly equal to the sum of the FVA costs faced in the cases of going long and short into the trade. One of the most relevant achievements of the present work, is to underline that under symmetric funding policies (for instance the Balance-Sheet Shrinkage), there configures an *offsetting effect* which drives the bid-ask spread widening to zero. Such a result extends the vanishment of funding effects introduced by Burgard and Kjaer (2011b) in the case of no haircut, to the more general scenario of symmetrical funding policies. Moreover in the current financial landscape, characterized by negative short-term nearly risk-free rates, BSS is probably the optimal choice for dealers coping with a liquid own bond market.

It has been performed a numerical application, which relies on the structural first-passage time default framework built in Chapter 1. The calculation involves a hybrid procedure in which the exposures upon defaults are computed by means of the Fourier Cosine Series (COS) method, while default processes are simulated via a joint Monte Carlo simulation. When the BSS is in place, numerical results confirm my theoretical expectations of null bid-ask spread widening. On the other hand, when symmetric funding policies are not attainable, the impact in the bid-ask spread is relevant and comparable in terms of magnitude, to other XVA terms. Assuming the existence of a liquid own bond market for the dealer as common knowledge among market participants increases the client awareness during the bargain with the counterpart. Nevertheless, in the reality the actual entry price of OTC derivative trades often depends on the bargaining power of the two counterparts.

Being discounted funding costs roughly proportional to CDS spreads, FVAs could represent a factor which raises markets efficiency: if the derivative contract represents a predominant asset and the bank A finances itself by paying the double of the credit spread of the bank B, it would incur in the double of the funding cost faced by the bank B. Although it seems not to be the case, in a frictionless and competitive market, the bank B would have an advantage in winning the trade. Otherwise, if the derivative represents a predominant liability, the opposite holds since funding benefits, interpreted as savings in terms of unsecured borrowing repayments, are larger for the bank A. However as previously stated, OTC markets are rather affected by frictions such as transaction and searching costs, opaqueness and, last but not least, market power. All these factors could prevent the achievement of a competitive equilibrium in which OTC trades are booked at the fair value.

In Section 2.6 it has been discussed the topic of the additional layer of collateralization represented by Initial Margins, requested when trading under

Central Clearing Counterparts (CCP). Within my numerical application, it has been shown how the Initial Margin Valuation Adjustments (MVA) *liquidity effect* on the bid-ask spread is quite relevant. I observed the MVA effect being bigger the one caused by the FVA, which conceptually is its closest relative within the XVA family. Moreover, since IM are computed as market risk measures, such as the Expected Shortfall, no *offsetting effect* can materialize under whatever funding policy carried out by the trader. Since the ES itself might not recognize the hedging, the counterpart could face the risk to be charged two potentially overlapping costs, the FVA and the MVA. In the light of the relevance of such argument, I expect that the investigation of the degree of overlap between funding and IM margining will attract the attention of regulators, practitioners and the academic researchers. Further developments on the topic might relate to the inclusion within the picture of the relationships between IM margining costs and Netting.

3. MODEL EXTENSIONS AND SENSITIVITY ANALYSIS

ABSTRACT

The aim of the present Chapter is to improve of the structural first-passage framework built in Chapter 1 along several directions as well as test its robustness. Since typically commodity trades are not clearable under Central Clearing Counterparts (CCPs), it is worthy to assess the effect of bilateral Collateral Support Annex (CSA) agreements on CVA/DVA metrics. Moreover I introduce within my CCR modelling, the impact of state-dependent stochastic recovery rates. Furthermore, in order to stress-test my framework, I investigate the effects on CCR measures of multiplicative shocks to the two major drivers in the game: credit and volatility. Finally I propose an alternative balance-sheet calibration based on hybrid market/accounting data which is well suited in the commodity context in the light of small and medium size of corporations usually operating in the EU commodity derivatives market for risk-management purposes.

Keywords: CCR Metrics, First-Passage Models, Energy Commodities, Collateral, Sensitivity Analysis, Stress Testing, Recovery Risk, Accounting Data.

3.1 Introduction

The aim of the present Chapter is to extend and stress test the first-passage structural Lèvy model built in Chapter 1 in order to allow for contractual provisions often agreed in practise, for instance bilateral collateral margining. Counterparts negotiating OTC derivative trades often agree on bilateral collateralization within the Collateral Support Annex (CSA) since, in many cases, it is the most effective way to mitigate the Counterparty Credit Risk (CCR). Such agreements might oblige the two counterparts to obey to standard rather than more exotic provisions. In particular, commodity transactions are practically not clearable under Central Clearing Counterparts (CPPs), so that they require tailored agreements between the parties involved in the trade. Nevertheless, although the effectiveness of the use of collateral in mitigating the CCR is conceptually convincing and supported by the empirical evidence, it presents the major drawback of being costly for the business. Especially for both medium and small corporate firms, posting the cash available from the balance-sheet to secure their derivative transactions might expose them to critical issues of *collateral management*. The cost for financing collateral margining interacts largely with other funding costs, as it partially offsets the cost for hedging unsecured exposures[1].

The Recovery Risk comes into the picture when acknowledging that real world recovery rates are stochastic themselves. Unfortunately, stochastic recovery rates are scarcely modellable risk drivers. This evidence derives from the lack of data on corporate recovery rates upon defaults. Recoveries are absolutely relevant in terms of relative weight for the calculation of the XVA metrics, since a deviation of the predicted recovery rate from the actual one upon default can compromise the accuracy even of a sophisticated model. Strangely, Recovery Risk has not been so far a popular topic within available CCR literature.

A stress test of my model is required in order to assess how much the valuation adjustments are sensitive to shocks impacting on the main underlying risk drivers, i.e. credit and volatility. Furthermore the analysis of CDS and Vega sensitivities, among others, could drive CCR hedging strategies. At first glance, within my theoretical framework CDS credit shocks impact on the calibration of the Normal Inverse Gaussian (NIG) processes as well as on that of the default barriers of the two firms. As a consequence, CDS credit shocks influence also the CVA and DVA measures computed through my hybrid Fourier-Monte Carlo framework. As volatility shocks are concerned, since they are exogenously set to the vectors of optimal parameters, they affect directly on the calculation of CVAs and DVAs.

My reason for assessing an alternative hybrid market-accounting calibration is in some way provocative. It derives from a well known risk in Economics, called *self-fulfilling prophecies*, which might apply also in the context of XVA pricing. On the other hand, relying on balance-sheet data might be necessary in the case of trading OTC derivative claims against counterparts lacking CDS quotations. Such scenario is often observed in the EU commodities market.

[1] See Chapter 2, Section 3.

3.1.1 Main contributions

I have extended my Monte Carlo framework by comparing the effectiveness of two different CSA collateralization schemes in terms of reduction of the "pure" CCR metrics, i.e. the CVA and the DVA. Another relevant contribution of the present Chapter consists in my attempt to model *state-dependent* stochastic recovery rates since, from a theoretical point of view, recoveries should be related to the severity of occurred defaults. For that purpose, I make use of a well known tool coming from the Recovery Risk literature: the Beta distribution, see Chen and Wang (2013). Furthermore, a Sensitivity Analysis has been performed in order to assess how the output of the model reacts to changes in the calibration coming from multiplicative shocks to the two major drivers in the game: credit and volatility. As I will show in the following, through the Sensitivity Analysis some interesting asymptotic behaviours of the CCR measures have emerged. My last contribution is the suggestion of an alternative choice for calibration benchmarks. My proposal consists in tuning the Normal Inverse Gaussian (NIG) parameters sets to the prices of liquid securities, such as European Options, while estimating the default barriers from publicly available balance-sheet data. My goal is to put light on whether accounting data provide different information on creditworthiness with respect to that implied in the CDS market.

The Chapter is organized as follows: in Section 3.2 I compare the effectiveness of two different CSA collateral schemes. Section 3.3 describes my modelling approach to state-dependent stochastic recovery rates. In Section 3.4 I perform a Sensitivity Analysis with respect to credit and volatility. Section 3.5 provides the results arising out of the alternative balance-sheet based calibration procedure. Section 3.6 draws the conclusive remarks of the present Chapter and suggests some ideas for future research on the topic.

3.2 The effects of CSA collateral agreements

It is widely shared that collateral is main driver for the purpose of mitigating CCR risk. Nevertheless collateral posting is costly and involves critical treasury issues similar to those found in the context of FVAs. Especially for corporate firms, funding the cash collateral required in order to secure OTC derivatives might be out of their scope given the non financial nature of their business.

The ISDA (2002) Master Agreement provides standard Collateral Support Annex (CSA) guidelines, which however might be customized by the parties involved in the trade. In this Section I set up a numerical study in order to compare the effectiveness of two different CSA schemes in mitigating the size of bilateral CCR metrics. In both cases I assume that the amount of collateral to be posted at each margining date is computed on the basis of the default-free value of the derivative V_{t-1} as in the previous step of the Monte Carlo simulation. Since in my calculation engine I configure 48 time steps per year, the Margin Period of Risk[2] equals to one week.

CSA scheme 1: Predetermined partial collateralization. As a first scheme I consider the case in which at any margining date, the cash collateral amount is set as a fraction α of the default-free value of the derivative V_{t-1} computed in the previous step of the simulation:

$$C_t = \alpha V_{t-1} \tag{3.1}$$

As a consequence, the actual unsecured exposure faced by the in the money counterpart at time t equals to:

$$\begin{aligned} \epsilon_t =& V_t - C_t \\ =& V_t - \alpha V_{t-1} \end{aligned} \tag{3.2}$$

My choice is to set $\alpha = 50\%$.

CSA scheme 2: Full collateralization above a fixed threshold. As a second scheme let me consider the case in which at any margining date, the cash collateral amount is such that there configures a cap, equal to a predetermined threshold T, to the maximum unsecured exposure faced by the in the money counterpart. In other words, I assume that if the value of the derivative V_{t-1} at the previous step in the Monte Carlo simulation is greater than the threshold, the excess exposure is fully collaterized:

$$C_t = (V_{t-1} - T)\mathbb{1}_{\{V_{t-1} > T\}} \tag{3.3}$$

[2] By definition, the Margin Period of Risk denotes the amount of time during which, even under full collateralization, the in the money counterpart faces the risk to incur in some losses upon default as a consequence of unfavorable prices movements meanwhile occurred.

As a consequence, the actual unsecured exposure faced by the in the money counterpart at time t equals to:

$$\begin{aligned}\epsilon_t =& V_t - C_t \\ =& V_t - (V_{t-1} - T)\mathbb{1}_{\{V_{t-1} > T\}}\end{aligned} \tag{3.4}$$

In the present numerical application I have set the threshold T to the 50% of the default-free value of the trade at the outset, i.e. $T = 0.5V_0$. No Minimum Transfer Amount (MTA) has been assumed.

The next step involved the recalculation of both the CVAs and DVAs for some trades in order to assess which one, among the two CSA schemes, is more effective in mitigating the CCR risk.

Counterparty Credit Risk Valuation Adjustments						
Contract type	V_0	CVA	CVA/V_0	DVA	DVA/V_0	$\hat{V}_0$
ATM Call Oil	5.82103	0.43051	739.578	0	0	5.39052
Forward Oil	-0.31709	0.00320	100.774	0.00549	173.255	-0.31479
Swap Oil	-1.37521	0.00021	1.550	0.02379	172.989	-1.35164

Tab. 3.1: CVAs and DVAs for different derivative claims issued on crude oil in presence of the CSA Scheme 1. Relative valuation adjustments are expressed in basis points.

Counterparty Credit Risk Valuation Adjustments						
Contract type	V_0	CVA	CVA/V_0	DVA	DVA/V_0	$\hat{V}_0$
ATM Call Oil	5.82103	0.36982	635.324	0	0	5.45121
Forward Oil	-0.31709	0.00407	128.366	0.00395	124.685	-0.31720
Swap Oil	-1.37521	0	0	0.01397	101.586	-1.36124

Tab. 3.2: CVAs and DVAs for different derivative claims issued on crude oil in presence of the CSA Scheme 2. Relative valuation adjustments are expressed in basis points

By looking at Table 3.1 and Table 3.2, it is possible to state that in general both the two analysed CSA schemes are effective in decreasing the CVAs significantly. As the European Option and the Swap are concerned, the CSA scheme 2 performs better than the competitor. On the contrary, my empirical findings display a larger reduction of the CVA of the Forward under the CSA scheme 1. On the DVA side, the CSA scheme 1 fails in mitigating the valuation adjustment for own default risk, while the CSA scheme 2 successes in reducing the DVA just in the case of the Swap.

In conclusion I have found that, under the usual number of Monte Carlo iterations (i.e 10,000), the introduction of the CSA scheme 2 causes an increase in the variance of both CVA and DVA results. In other words, CSA scheme 2 is responsible of a shrinkage of the stability of my results.

3.3 *Incorporating state-dependent stochastic recovery rates*

Although Recovery Risk should be quiet a debated topic in the context of CCR, since a deviation of the actual recovery rate upon default from the expected one can compromise the accuracy even of a sophisticated model, stochastic recoveries have been scarcely investigated in the CCR literature. Despite the scarcity of research on the topic, some insights can be found in Crépey (2015b) who modelled recovery rates as $\mathcal{G}$-adapted stochastic processes.

A quantitative approach for modelling stochastic recovery rates can start by assuming an explicit statistical distribution of the overall recovery rate upon the bankruptcy of the company. Actually, in real market conditions the recovery rate is not homogeneous among the claims issued by the defaulted company: more precisely, the holders of senior bonds or other secured instruments have priority in collecting their credits during the liquidation process. As a consequence, in comparison with junior creditors, they are likely to claim an higher fraction of the distressed assets of the defaulted company.

Among several distributions available in literature, the Beta distribution $\mathcal{B}$ has been widely preferred in order to model stochastic recoveries, since it is a continuous distribution taking values the interval $[0, 1]$[3] Furthermore, the Beta distribution allows for useful features such as skewness and excess kurtosis. The Beta distribution is specified by two non negative parameters α and β such that its probability density function (PDF) reads:

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathbb{B}(\alpha, \beta)} \tag{3.5}$$

where the Beta function $\mathbb{B}(\alpha, \beta)$ is defined as:

$$\mathbb{B}(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx$$

As concerns the location and scale parameters, in the case of a Beta random variable $X \sim \mathcal{B}(\alpha, \beta)$, they equal to:

$$\mathbb{E}[X] = \frac{\alpha}{\alpha+\beta} \tag{3.6}$$

and

$$\mathbb{V}ar[X] = \frac{\alpha\beta}{(\alpha+\beta)^2 + (\alpha+\beta+1)} \tag{3.7}$$

I aim to relate the forecast of stochastic recovery rates to the severity of credit events. In fact, my idea is to model the stochastic recovery as dependent on the relative Gaussian distance of the equity value upon default with respect to the default trigger K. Let me define the random variable $\xi_{\bar{\tau}} = f(S_{\bar{\tau}})$ as:

$$\xi_{\bar{\tau}} = \frac{\phi_{K,1}(S_{\bar{\tau}})}{\phi_{K,1}(K)} \tag{3.8}$$

[3] Such domain is consistent with admissible values for recovery rates.

where $\phi_{K,1}$ is the cumulative distribution function (CDF) of a Normal distribution with mean K and unitary variance.

The stochastic recovery rate is therefore defined as the inverse CDF of a Beta distribution $\mathcal{B}(\alpha^*, \beta^*)$ calculated in $\xi_{\bar{\tau}}$:

$$R_{\bar{\tau}} = B^{-1}_{(\alpha^*,\beta^*)}(\xi_{\bar{\tau}}) \tag{3.9}$$

This model setup makes the stochastic recovery *state-dependent*: in other words, the way the algorithm samples from the properly parametrized Beta distribution $\mathcal{B}(\alpha^*, \beta^*)$ depends on how much the equity value upon default is below the barrier.

In my numerical study, I have calibrated the parameters (α, β) in order to set up a prior Beta distribution $\mathcal{B}$ characterized by small variance and centered in the historical average of corporate recovery rates of 40%. As a consequence, the Beta optimal parameters are:

$$\alpha^* = 10.46464464, \quad \beta^* = 15.69696695$$

The model-independent formula for bilateral CCR adjustments built in Chapter 1 incorporating the Recovery Risk modifies to:

$$\begin{aligned} \hat{V}_t &= V_t \\ &- \underbrace{\mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)(1 - B^{-1}_{(\alpha^*,\beta^*)}(\xi^C_{\tau_C}))V^+_{\tau_C} \mid \mathcal{G}_t]}_{CVA} \\ &+ \underbrace{\mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)(1 - B^{-1}_{(\alpha^*,\beta^*)}(\xi^B_{\tau_B}))V^-_{\tau_B} \mid \mathcal{G}_t]}_{DVA} \end{aligned} \tag{3.10}$$

For the sake of self-consistency of the present Section, I remark that $\mathbb{E}^{\mathbb{Q}}$ denotes the risk-neutral expectation under some pricing measure $\mathbb{Q}$, conditional to the global filtration $\mathcal{G}_t$ which include all market information and default history up to time t. V_t denotes the default-free value of the contract and D is the risk-free discount factor[4]. The CVA and DVA acronyms stand respectively for Credit Valuation Adjustment and Debt Valuation Adjustment.

The below Table 3.3 shows the results for bilateral Counterparty Credit Risk adjustments computed trough the usual hybrid Fourier-Monte Carlo model, extended in order to allow for state-dependent recovery rates upon default. At the same time, in Figures 3.1 and 3.2 it is shown the degree of fitting of the stochastic recoveries experimental frequencies of both the firms with respect to the theoretical limit Beta distribution $\mathcal{B}_{(\alpha^*,\beta^*)}$. As it is shown in the two figures and by the descriptive statistics relative to case of the European Call Option issued on natural gas (listed in Table 3.4), the way how my algorithm samples from the Beta distribution is not centered in the theoretical mean but is *state-dependent*, as it is influenced by the actual severity of simulated defaults.

[4] Under the independence hypothesis, the risk-free discount factor can be dragged out from the expectation and equals to the price of a zero-coupon bond, i.e. $D(0,t) = \mathbb{E}^{\mathbb{Q}}\left[\exp\left(-\int_0^t r_s ds\right)\right]$. r is proxied by the Eonia/EURIBOR rates.

Counterparty Credit Risk Valuation Adjustments						
Contract type	V_0	CVA	CVA/V_0	DVA	DVA/V_0	$\hat{V}_0$
ATM Call Oil	5.82103	0.93075	1598.940	0	0	4.89028
ATM Call Gas	0.41155	0.06477	1573.876	0	0	0.34677

Tab. 3.3: CVAs and DVAs for the ATM European Call Options issued on crude oil and natural gas incorporating the *state-dependent* stochastic recovery rates. Relative valuation adjustments are expressed in basis points.

In fact as shown by Table 3.4, the average stochastic recoveries are 58% and 36% for Enel and BNP Paribas, respectively. I argue that these results are ultimately driven by the volatility, which is higher for the financial firm. For this reason, the relative Gaussian distance of the equity values upon default from the trigging barrier is on average larger for BNP Paribas. That clearly impacts also on the bilateral CCR metrics (and in particular on the CVA, since I am here considering one-sided payoff derivatives), which are higher if compared with those computed assuming deterministic recovery rates equal to the historical average of 40%, see Session 1.4.3. I can therefore conclude that, by neglecting the role of volatility and setting flat recoveries for all firms, in certain circumstances there configures some risk overestimation, as in the case of Enel. Conversely in other circumstances, there configures some risk underestimation, as in the case of BNP Paribas. The degree of *missing risk* can be severe and might vanish the accuracy even of a sophisticated model. For this reason even if one decides to adopt deterministic recoveries, I would suggest to efficiently manage the Recovery Risk by carrying out an *ex-ante* study of volatilities before setting static recovery rates.

Stochastic Recoveries Descriptive Statistics						
Firm	Min	Max	**Mean**	Variance	Skewness	Kurtosis
Enel	0.47762	0.78154	**0.58083**	0.00218	0.69119	0.59477
BNP Paribas	0.01008	0.72569	**0.36009**	0.01390	0.02283	-0.38029

Tab. 3.4: Descriptive statistics of the experimental recovery rates relative to the simulation for the computation of the CCR measures of the European Call Option issued on natural gas.

As a conclusive example, as shown in Figure 3.3, by assuming a negative 25% multiplicative shock on BNP Paribas volatility, the experimental frequencies of stochastic recovery rates of the financial firm fit quiet accurately the limit Beta distribution $\mathcal{B}_{(\alpha^*,\beta^*)}$. Such evidence suggests that a deterministic 40% recovery is appropriate in presence of that relaxed level of BNP volatility, and not for the real-world one. Nevertheless, in such a low volatility scenario, the CVA would reduce at just the 7.07% of the initial default-free value of the European Call Option issued on natural gas. Such a massive reduction is clearly not realistic. In the light of my experimental evidences it seems that state-dependent stochastic recoveries are characterized by some *Leverage Effect* with respect to volatility:

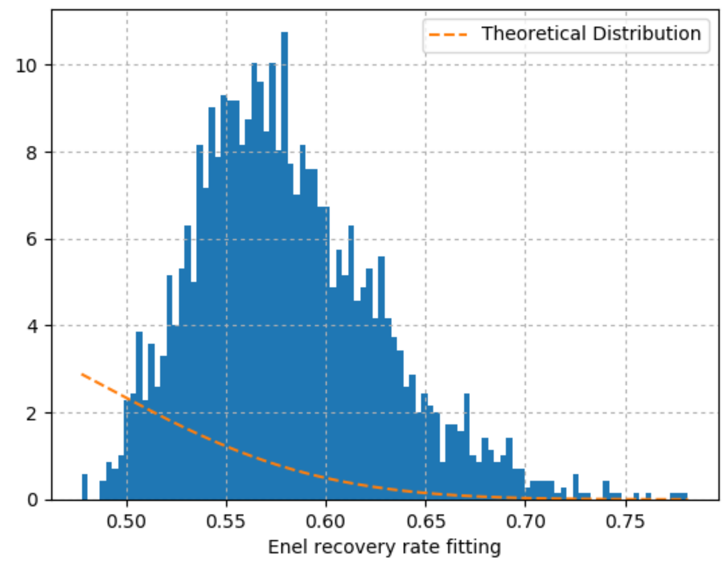

Fig. 3.1: Simulated distribution of Enel recovery rates under the baseline calibration.

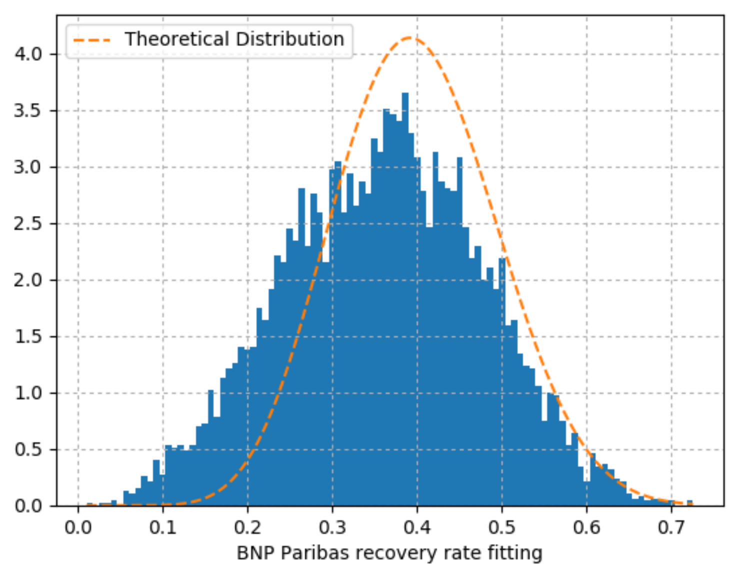

Fig. 3.2: Simulated distribution of BNP Paribas recovery rates under the baseline calibration.

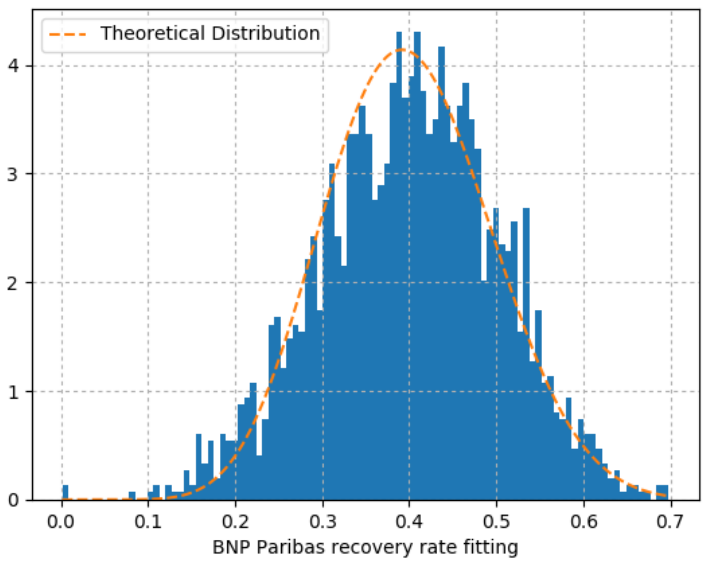

Fig. 3.3: Simulated distribution of BNP Paribas recovery rates in case of a negative 25% shock on volatility.

small mistakes in setting static recoveries imply large deviation from the actual implied volatilities.

3.4 Sensitivity Analysis on credit and volatility

In order to stress test my framework, the present Section describes a sensitivity analysis performed by applying multiplicative shocks to the two major drivers in the picture, i.e. credit and volatility. I aim to assess how my hybrid Fourier-Monte Carlo model reacts conditionally to variations of one relevant parameter, *ceteris paribus*.

As credit concerned, I assume parallel multiplicative shocks to the CDS quotes used as benchmarks in calibration of the Normal Inverse Gaussian (NIG) optimal parameters of both firms by means of the Fourier Cosine Series (COS) pricing formula for CDS explained in details in Subsection 1.4.2. The below Figures 3.4 and 3.5 display the behaviour of the several parameters with respect to variable CDS credit shocks on both Enel and BNP Paribas. The excess kurtosis parameters κ^i of the two NIG processes, together with the barrier level, appear to be those which react in a more predictable way, i.e. by increasing when the multiplicative shocks grow, and viceversa. This happens especially in the calibration of the financial firm parameters. Nevertheless, no calibration in general seems to be affected so sharply, as shown by the ambiguous paths drawn in the two Figures.

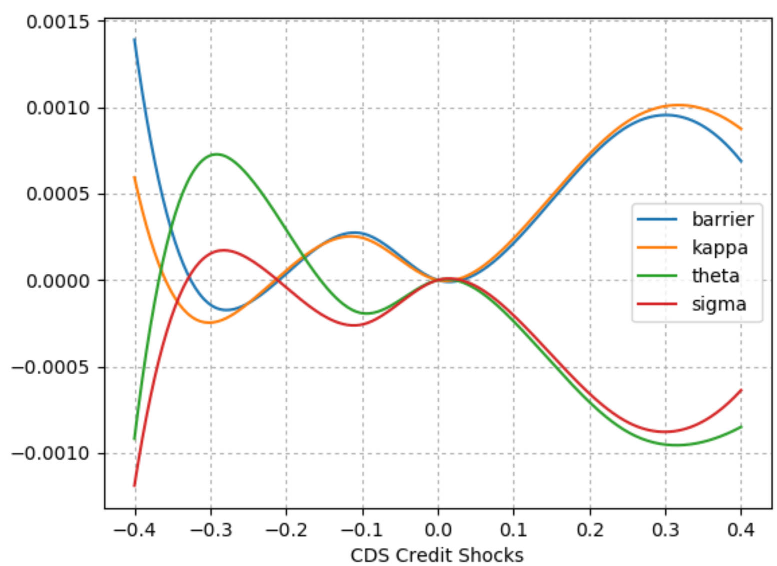

Fig. 3.4: Sensitivity to multiplicative CDS credit shocks of Enel calibrated parameters.

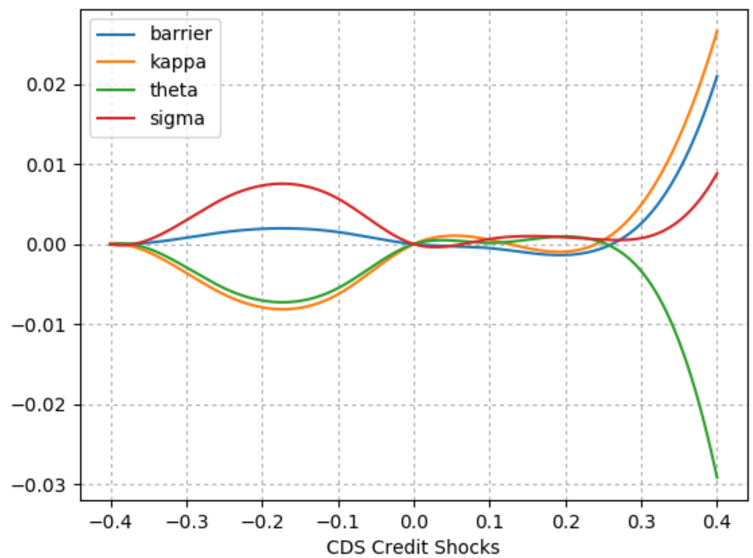

Fig. 3.5: Sensitivity to multiplicative CDS credit shocks of BNP Paribas calibrated parameters.

Sensitivity Analysis to CDS Credit Shocks						
Shock type	V_0	CVA	CVA/V_0	DVA	DVA/V_0	$\hat{V}_0$
-40% CDS Shock	5.82103	0.88492	1520.206	0	0	4.93611
-25% CDS Shock	5.82103	0.91272	1567.975	0	0	4.90831
-10% CDS Shock	5.82103	0.91312	1568.649	0	0	4.90792
+10% CDS Shock	5.82103	0.86501	1486.010	0	0	4.95602
+25% CDS Shock	5.82103	0.90137	1548.463	0	0	4.91967
+40% CDS Shock	5.82103	1.09750	1885.401	0	0	4.72353

Tab. 3.5: Sensitivity Analysis on the CVA of an ATM European Option issued on crude oil with respect to parallel CDS credit shocks to both firms. Relative valuation adjustments are expressed in basis points.

CVA Sensitivity Analysis to Volatility Shocks						
Shock	-40%	-25%	-10%	10%	25%	40%
CVA	0.01%	3.06%	3.10%	-2.29%	1.83%	19.38%

Tab. 3.6: CDS credit sensitivities with respect to the base scenario of the CVA of an ATM European Call Option issued on crude oil given parallel CDS shocks to both firms.

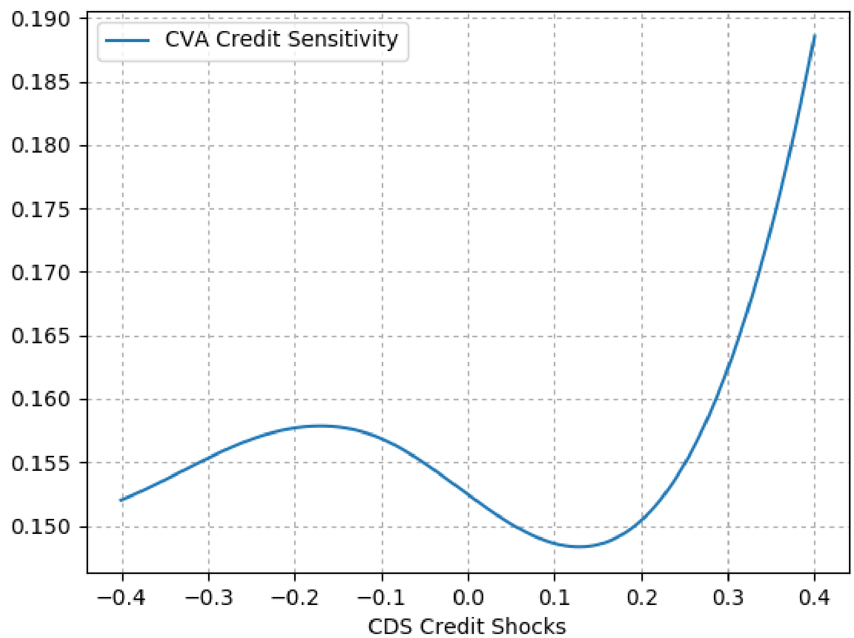

Fig. 3.6: Sensitivity of the Credit Valuation Adjustment (CVA) on an ATM European Call Option with respect to CDS credit shocks

Figure 3.6 shows that even if the sensitivity with respect to CDS credit shocks does follow an increasing trend, it is clear that the CVA on the European ATM Call Option on crude oil reacts significantly just in the positive 40% CDS credit shock case.

As volatility is concerned, through a similar procedure to that applied to credit, I have assumed multiplicative shocks impacting on both the σ^i parameters of the two firms calibrated in the base scenario. It can be argued that the impact of volatility shocks on the bilateral CCR metrics is in general significant in terms of magnitude. Nevertheless, volatility shocks fail in reaching the peak of 19.38% CVA sensitivity registered in correspondence of parallel 40% CDS shocks. Furthermore, the patterns of CDS and Vega sensitivities are significantly different. Interestingly, for both the CVA and DVA computed on a Forward contract issued on crude oil, Vega sensitivities exhibit an *asymmetric* behaviour, since they are much more reactive in the case of negative volatility shocks in comparison with positive ones. In my numerical simulation, while the two CCR measures shrink significantly as a consequence of negative volatility shocks, the Vega sensitivities of the two metrics seem to display a limited increase as a consequence of positive volatility shocks. Such experimental evidence might suggest the existence of an upper bound for both the bilateral CCR adjustments.

Sensitivity Analysis to Volatility Shocks						
Shock type	V_0	CVA	CVA/V_0	DVA	DVA/V_0	$\hat{V}_0$
-40% Vol Shock	-0.31709	0.00083	26.109	0.00113	35.728	-0.31678
-25% Vol Shock	-0.31709	0.00208	65.686	0.00212	66.867	-0.31705
-10% Vol Shock	-0.31709	0.00406	126.958	0.00297	93.635	0.31814
+10% Vol Shock	-0.31709	0.01017	320.731	0.00503	158.723	-0.32223
+25% Vol Shock	-0.31709	0.01478	466.005	0.00589	185.909	-0.32597
+40% Vol Shock	-0.31709	0.01991	627.813	0.00538	169.595	-0.33162

Tab. 3.7: Sensitivity Analysis of the CVA on a Forward contract issued on crude oil with respect to parallel volatility shocks to both firms. Relative valuation adjustments are expressed in basis points.

Vega Sensitivity Analysis to Volatility Shocks						
Shock	-40%	-25%	-10%	10%	25%	40%
CVA	-7.39%	-2.33%	-0.73%	0.32%	0.53%	0.65%
DVA	-2.47%	-0.86%	-0.33%	0.22%	0.33%	0.27%

Tab. 3.8: Vega sensitivities with respect to the base scenario of the CVA and DVA on a Forward contract issued on crude oil given parallel volatility shocks to both firms. Relative valuation adjustments are expressed in basis points.

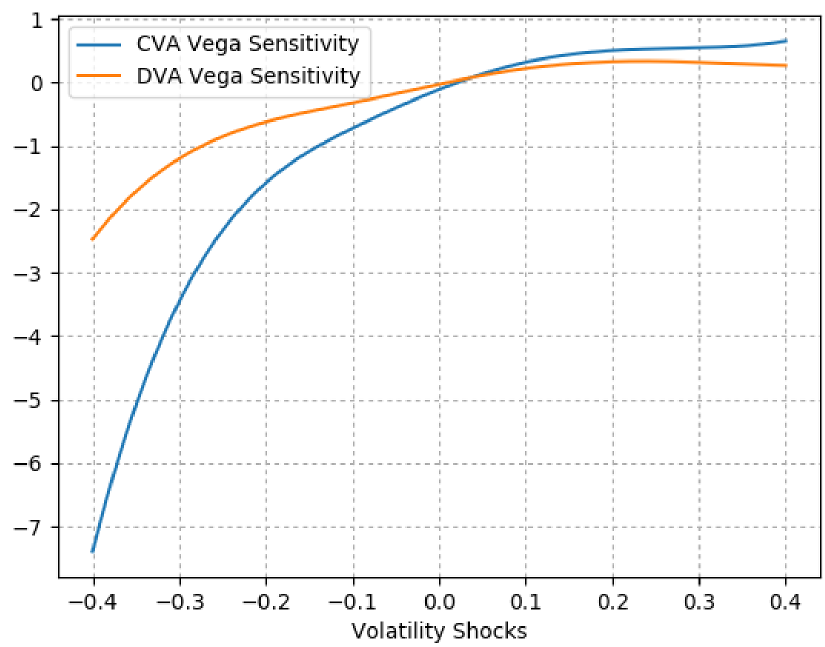

Fig. 3.7: Vega sensitivities of the Credit Valuation Adjustment (CVA) and Debt Valuation Adjustment (DVA) on a Forward contract issued on crude oil with respect to volatility shocks to both firms.

3.5 Exogenous balance-sheet based default barriers

In both industrial and academic application, survival and default probabilities are calibrated to those implied in the CDS market. Since such products provide protection against the default of a given reference entity, it is absolutely respectable to believe that CDS contain genuine information about the creditworthiness of a reference entity. CDS contracts are indeed strongly recommended by the Basel III regime to be used as a key tool for CCR mitigation.

As a drawback, it might exist the risk of making the output of different methodological approaches aligned to the same benchmarks. Therefore, even if a structural modelling of default events is adopted, there should be no reason to expect different results compared to those obtainable by a reduced-form model. The reason is that all those frameworks are calibrated in order to the key elements of the calculation, i.e. survival and default probabilities, to that implied in the market. Of course, this way of reasoning might be applied asymptotically to all financial applications and not just to the CCR. That would violate the shareable need to rely on a reference price through which to evaluate whether the output of a model is economically credible or not.

My point is that such issue is even worse in the case of CDS, since the quotes displayed by major data providers typically rely on well defined modelling choices, regarding the definition of credit events, hedging portfolios, discount curves. Moreover, CDS contracts are less accessible than plain vanilla derivatives because: 1) they are negotiated OTC 2) they are more complex in terms of conventions that might be customized in real transactions 3) they are poorly liquid for many maturities (except for the 5 years one) and last but not least, 4) they are not available for many reference names.

In the light of these arguments, especially within a structural approach, an alternative and economically sounding experiment could be calibrating model parameters by the use of some balance-sheet data albeit, how authoritatively argued by Brigo et al. (2011), these as well are affected by a high degree of uncertainty. One first attempt to rely on accounting data could be to access to the last published balance-sheets data[5] and exploit a particularly successful practical implementation of structural credit modelling, i.e. the KMV[6] approach. In a nutshell, the KMV model puts the barrier somewhere in between the face value of short term liabilities, and the face value of the total liabilities, arguing that the firm is strictly forced to service short term debt, but it can be more flexible in servicing long term repayments. Because of that within the KMV approach the default trigger is given by the sum of the full short term liabilities and the half of the long term liabilities. In the present Section, the KMV approach has been used for calibrating the barrier levels to balance-sheet data. NIG parameters instead, have been tuned to liquid option prices.

[5] Source: https://it.finance.yahoo.com

[6] The firm KMV is named after Kealhofer, McQuown and Vasicek, the founders of the company in 2002. It has then been sold to Moody's.

Alternative calibration results					
NIG processes	Barrier	κ^*	θ^*	σ^*	RMSE
Enel	0.27956	0.22369	-0.22369	0.20034	0.19305
BNP Paribas	0.76174	0.80464	-0.18519	0.14148	0.19305

Tab. 3.9: Calibration results based on the KMV approach for barrier levels and on European Call Options for NIG parameters.

As shown in Table 3.9, given the very high level of indebtedness of BNP Paribas emerging from its accounting data, the default trigger is set at about the 76% of its equity value at the outset. Albeit its level of volatility is lower in comparison with the one calibrated under the CDS-based case, I would expect that BNP is likely to default before Enel.

Alternative Counterparty Credit Risk Valuation Adjustments						
Contract type	V_0	CVA	CVA/V_0	DVA	DVA/V_0	$\hat{V}_0$
ATM Call Oil	5.82103	1.78212	3061.527	0	0	4.03890
Swap Gas	0.06716	0.00118	175.092	0	0	0.06598

Tab. 3.10: Recomputed CVAs and DVAs according to the alternative hybrid market/balance sheet calibration.

Table 3.10 displays the results for the recomputed CCR metrics relative to a subset of the initial basket of energy commodity derivatives. For both an ATM European Call Option issued on crude oil and a Forward contract issued on natural gas, the effect of incorporating the alternative KMV-type calibration is the avoidance of any kind of DVA benefit for Enel since the financial firm, in the light of its accounting data, defaults first in all the scenarios, making its experimental first-to-default probability within one year jumping to about 41%. As a consequence, CVA for Enel in both trades had roughly doubled.

The major issue of such a hybrid market-accounting calibration relates to the fact that securities are priced under a suitable risk-neutral probability measure, while accounting data are historical. This evidence makes them hardly comparable. In this context, ways to overcome such limitation might gain the interest of future research.

3.6 Conclusions

In the present Section, I draw some conclusive remarks regarding the several model extensions and the Sensitivity Analysis performed within the Chapter. In Section 3.2 it has been compared the impact of two different CSA schemes in mitigating the bilateral Counterparty Credit Risk metrics. Even if both the two schemes are effective in terms of CVA reduction, the full collateralization above a fixed threshold appears to be superior. On the contrary, the two schemes do not expose the same capability in reducing the DVA. Unfortunately I noticed that under the CSA scheme 2, at the usual number of Monte Carlo iterations (i.e. 10,000), the stability of results is lowered. Nevertheless being the computational time limited, it is sufficient to increase the number of simulations in order to reach the desired level of accuracy.

In order to investigate the Recovery Risk, in Section 3.3 it have been modelled state-dependent stochastic recovery rates. In other words, recovery rates have been related to the severity of defaults occurred, measured in terms of relative Gaussian distance. My experimental findings demonstrate that, even assuming a common Beta random variable centered in the historical average of corporate recovery rates as theoretical limit distribution, the sampling is significantly dependent on the volatility parameter of the analysed firm. As a consequence, in some cases assuming flat deterministic recoveries of 40% configures as a risk overestimation while in other cases it configures some degree of *missing risk*. Because of that, even if the use of deterministic recovery rates if preferred or necessary on daily basis, I would suggest to perform an *ex-ante* volatility assessment. In the light of my experimental evidences it seems that state-dependent stochastic recoveries are characterized by some *Leverage Effect* with respect to volatility: small mistakes in setting static recoveries imply large deviation from the actual implied volatilities.

For the purpose of stress testing my framework, in Section 3.4 it has been carried out a Sensitivity Analysis on the size bilateral CCR adjustments with respect to multiplicative shocks to the two major drivers in the picture, i.e. credit and volatility. In the first step, it have been applied both positive and negative shocks to listed CDS quotes of both the two firms. In the light of that, NIG parameters and barrier levels have been recalibrated by means of the COS formula for CDS. The impact on the overall calibration revealed to be quiet ambiguous except for the barrier levels and the excess kurtosis parameters κ^i, which, especially in the case of BNP Paribas, appeared to be more reactive to CDS credit shocks. The largest credit sensitivity has been registered in correspondence of quite high positive CDS shocks, which caused a 20% increase of the CVA on the European Call Option issued on crude oil. As volatility is concerned, in the case of a Forward contract issued on crude oil, my numerical analysis shows that in general both the CVA and DVA Vega sensitivities are significant. Interestingly, I noted an *asymmetric behaviour* of Vega sensitivities with respect to negative rather then positive volatility shocks. Both the CVA and the DVA displayed a large reduction in case of negative volatility shocks while they showed a limited growth in correspondence of positive ones. Such an evidence might suggest the existence of an empirical upper bound to the Vega sensitivities of the bilateral CCR adjustments.

Relatively to the alternative balance-sheet based KMV-type calibration proposed in Section 3.5, the results for CVAs are roughly doubled while the DVAs are null, since the financial firm is much more risky than the corporate firm according to the KMV approach. This huge difference in the creditworthiness announced by accounting data is not reflected in the CDS market and might be imputable to different accounting conventions used within different sectors of the economy. Another explanation might reside in the objective difficulty in evaluating some financial assets and liabilities. This limitation could make us doubters regarding the reliability of balance-sheet data but it still represents an initial attempt to relate XVAs numerical computations to economic fundamentals.

Further work have to be carried out in order to investigate more deeply topics of relevant interest for both the academic and industrial research such as the social cost of clearing OTC trades under CCPs, the issue of collateral management with a particular focus on non financial institutions, the impact on OTC derivatives valuation of the (K)apital Valuation Adjustment (KVA) and the role played by second order *cross-sensitivities* even in the context of CCR risk hedging.

GENERAL CONCLUSIONS

In Chapter 1, I derived a simple and general model-independent formula for bilateral CCR metrics, holding for both the cases of reduced-form or structural default modelling. The COS method for European Options has been extended to the pricing of Forward and Swaps. CCR metrics for both *one-sided* and *two-sided* payoffs have been computed through the application of a hybrid Fourier-Monte Carlo procedure. Model calibration and EAD computation have been performed via the COS method while the first-to-default event has been simulated via a joint Monte Carlo. The COS method displayed to be fast, accurate and flexible enough to easily calibrate model parameters around observed values of historical volatility. While volatility demonstrated to be the key driver in determining default events, the initial guess for the default trigger exhibited scarce relevance. The absolute size of CVA displayed to be larger in the case of European Options with respect to Forward and Swaps.

In Chapter 2, I concluded that there are no theoretical reasons to reconnect FVA within fair valuation principles. This insight derives from a simplified framework which displays that the FVA term is affected by dealer dependency. In fact, this depends on the expected cost for unsecured borrowing faced by the computing party and on its hedging policy itself. Moreover, if one includes funding in derivatives valuation, there configure the well known overlap between the DVA and funding benefits, making the computing counterpart paying twice the cost of its own default risk. More dramatically, the inclusion of FVA breaks the Law of Unique Price and poses serious doubts on the ability of the two parties to agree upon the price of the deal. However, OTC market are characterized by opaqueness, searching costs and market power so that trades are likely not to be closed according to fair values.

Funding costs can be rather interpreted as a friction affecting the current financial market in which agents face restrictions on the access to funding instruments. The FVA is responsible of some widening in the bid-ask spread that the dealer quotes to their clients. In the Chapter, I quantified the size of such widening as the sum of the expected funding costs faced by computing party in the cases of being long and short on the trade. Nevertheless, in the case the trader desk faces a liquid own bond market, it can adopt a symmetric funding strategy called Balance-Sheet Shrinkage which drives the bid-ask spread widening collapsing to zero. BSS is based on investing funding cash inflows in repurchasing previously issued debt. In the light of current negative nearly risk-free rates for short maturities, BSS appears to be quite appealing for those who cope with a liquid own bond market. Otherwise if a symmetric funding policy cannot be attained, the size of the bid-ask spread widening is significant. My work extends the results of Burgard and Kjaer (2011b) which argue that the FVA, interpreted as additive price correction, disappears in the case of no haircut (i.e. the trader can borrow at the risk-free rate) to a more general case. In fact, I believe that symmetric borrowing and lending rates suffice in eliminating the FVA related bid-ask spread widening through a *compensation effect*. In the case of MVA however, which can be thought as the closest relative of FVA, the compensation effect cannot materialize since the Initial Margins always configure as cash outflows asked by CCPs which need to be financed. As a consequence, even in case of a symmetric funding policy, an impact of MVA exists and is on average higher compared to that of FVA.

In Chapter 3, I have proposed a simple procedure to calculate state-dependent stochastic recovery rates which reflect the severity of occurred defaults. That is made possible by sampling from a properly calibrated Beta distribution. I defined a state random variable representing the relative Gaussian distance of occurred default from default barrier triggers. I have found a deep relation between volatility and the expected value of stochastic recoveries. Since average historical recovery rates displayed to be inconsistent with respect to calibrated levels of implied volatilities, I would suggest to perform an ex-ante study of volatility prior to approximate stochastic recoveries with fixed ones.

Within the Sensitivity Analysis, I showed that bilateral CCR metrics react significantly to both credit and volatility shocks. Interestingly, I singled out an *asymmetric behaviour* of bilateral CVA Vega sensitivity with respect to negative rather then positive volatility shocks. In fact, both the CVA and DVA displayed a large reduction in case of negative volatility shocks while they showed a limited growth in correspondence of positive ones. Such evidence might suggest the existence of an empirical upper bound of bilateral CCR adjustments responsiveness to volatility shocks.

In conclusion, as the regulatory measures aimed at mitigating counterparty risk among market participants will fully entry into force, I expect the residual size of pure CCR adjustments to decrease. On the other side, the counter effect might be the growth of IM and other margining costs widening bid-ask spreads. In presence of high competitiveness and constraints in quoting oversized bid-ask spreads to clients, I expect to observe smaller profitability for the trading business which might cause the acquisition of more and more market shares by a concentrated pool of few large dealers facing better average funding.

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