# Strategic formation and welfare effects of airline-high speed rail agreements

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**Abstract**

Policy makers encourage airline-high speed rail (HSR) cooperation to promote intermodal passenger transport. We study the strategic formation of airline-HSR partnerships (depending on sunk costs and firms' bargaining power) and their effects on consumer surplus and social welfare. We assume that airline-HSR agreements serve to offer a bundle of domestic HSR and international air services. In a capacity purchase (CP) agreement, the airline buys train seats to sell the bundle, whereas in a joint venture (JV) agreement firms create a distinct business unit. We find that both agreements increase traffic in the network, and thereby may not reduce congestion at hub airports. We provide antitrust authorities with a simple two-tier test for the CP agreement to improve consumer surplus. Contrary to airline-HSR mergers, the JV agreement benefits consumers independent of hub congestion and mode substitution. Simulation results show that, in case of cooperation, public agencies should prefer firms to create a JV, unless the related sunk costs are far greater than the costs of the CP agreement.

**Keywords:** Airline-high speed rail cooperation; Airport congestion; Capacity purchase agreements; Joint venture agreements; Competition policy

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## 1. Introduction

Over the last twenty years, there has been increasing evidence of cooperation between airlines and high-speed rail (HSR) operators, with many intermodal agreements signed worldwide. Most agreements relate to international connecting passengers.[1] Indeed, air transport and HSR services can be complements on long-haul routes served by connecting flights through a hub airport. If HSR is an effective substitute for either of these flights, then connecting passengers may combine air and HSR services. Airline-HSR agreements have received strong political support in Europe, where a major policy goal is promoting passenger transport intermodality (EC, 2006; Eurocontrol, 2005).

Congestion at hub airports is one of the main reasons for intermodal cooperation.[2] Airlines can divert part of the short-haul traffic to HSR, thereby making the relevant slots available for routes that are more profitable (Givoni and Banister, 2006).[3] In turn, HSR operators benefit from cooperation in that it increases their load factor and market share on short-haul routes (Givoni and Dobruszkes, 2013).

On the other hand, the impact of airline-HSR agreements on passengers' well-being is not clear-cut. Actually, intermodal cooperation increases product variety, but raises competition concerns as long as it involves coordinated pricing. Even with limited or no price coordination between transport operators, one should consider the role of congestion at hub airports. If a hub airport is capacity constrained then traffic volumes, and thereby consumer surplus, in each relevant market are affected by the airline's decision on how to allocate scarce slots among markets. Despite the relevance of

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[1] Examples of partnerships signed in Asia, US, and the EU include: *Air-Rail Service* between China Railway High-Speed and China Eastern Airlines at Shangai International Airport; *Acela Express* between Amtrak and United Airlines at Newark International Airport; *AIRail Service* between Deutsche Bahn and Lufthansa at Frankfurt Airport; *Rail&Fly Portugal* between Comboios de Portugal and TAP Portugal at Lisbon and Porto airports; *TGVAir* between SNCF and Air France at Paris Charles De Gaulle (CDG) Airport; and *AIR&RAIL* between Thalys and Air France at CDG Airport.

[2] Due to severe constraints to capacity expansion, airport slots are a scarce resource. The European Commission pursues the optimal allocation and use of slots to foster competition and improve quality of air transport services. In this framework, Avenali *et al*. (2015) study an incentive pricing mechanism to manage scarce capacity at congested hubs.

[3] For instance, in the case of the *AIRail Service* provided by Deutsche Bahn and Lufthansa, intermodal passengers can use either HSR or air services on the Frankfurt-Stuttgart route, but only HSR services on the Frankfurt-Cologne route.

airline-HSR agreements, competition authorities have so far devoted little attention to evaluating the effects of these agreements on passengers.

In this paper, we study the strategic formation of airline-HSR agreements, depending on the sunk costs necessary to make cooperation effective and on transport operators' bargaining power in negotiating agreements. Furthermore, we study the impact of airline-HSR agreements, first on traffic volumes in the transportation network and on the level of congestion at the hub airport, and then on consumer surplus and social welfare, depending on the hub capacity and on mode substitution between air and HSR services.

In doing so, we fill two important gaps in the academic literature (see Section 2 for a review). First, the literature considers a scenario of full-scale cooperation that resembles an airline-HSR merger, and ignores transport operators' incentives to join the alliance. Second, the literature finds that the welfare gains from an airline-HSR merger are driven by firms' profits rather than consumer surplus. This is not surprising insofar as a merger involves coordinated pricing in all relevant markets. In this framework, antitrust agencies would hardly approve the merger. In reality, airline-HSR cooperation does not involve a merger, which is hard to implement in practice (Xia *et al*., 2018), and we are not aware of any signed agreement that entails full coordination on the prices charged.

Thus, the issue at stake is, if price coordination in all markets is difficult to achieve, and would benefit firms to the detriment of consumers, are there any other forms of airline-HSR cooperation that largely improve consumer surplus, and thus remove antitrust concerns?

We propose two such forms of cooperation, a capacity purchase (hereafter, CP), or 'vertical' agreement, and a joint venture (hereafter, JV), or 'horizontal' agreement. In a CP agreement, the HSR operator sells seats on the train to the airline at a cost, and then the airline provides international connecting passengers with the combined airline-HSR service.[4] In a JV agreement, firms create a

[4] For instance, according to the *AIR&RAIL* agreement with Thalys, Air France forecasts and confirms traffic volumes to Thalys on an annual basis to book one or two carriages per journey (it can also book additional seats on an ad-hoc basis,

distinct business unit to offer the combined airline-HSR service in the connecting market. We assume that the combined transportation service is the only source of profit for the JV entity.

After signing an agreement, transport operators are able to offer a bundle of domestic HSR and international air services that passengers perceive as a substitute for the connecting flight. We posit that airlines and HSR operators have to undertake lumpy investments to make cooperation effective.[5] When signing the agreement, they have to share the related sunk costs. Clearly, an agreement is incentive-compatible as long as each transport operator achieves benefits in excess of the (relevant share of) sunk costs. In this sense, complementarity between transportation modes derives from compatibility, and compatibility is a strategic decision.

We summarize the main results as follows. First, we find that both CP and JV agreements increase traffic volumes in the connecting market and in the whole network. Although the airline substitutes some feeding flights for HSR rides in the connecting market, it uses the capacity made available at the hub airport to meet new demand. Thus, contrary to common wisdom, airline-HSR agreements do not necessarily reduce, but may even increase congestion at hub airports. We also find that such agreements may increase traffic in overlapping markets, thereby alleviating competition concerns.

Second, we find that intermodal agreements generally benefit consumers, even if hub airports are congested. This is in contrast to airline-HSR integration, which is almost exclusively in the firms' interest. More precisely, a CP agreement improves consumer surplus, unless transportation modes are very weak substitutes and the congested hub is of moderate size. In such a case, the HSR operator sets the wholesale price per train seat to induce the airline to allocate hub capacity to the multimodal

subject to availability on trains). Then, Air France handles the intermodal service, which is included in the Air France booking system, and is available to passengers who travel from/to Brussels-Midi Railways Station and CDG Airport.

[5] Eurocontrol (2005) lists some critical barriers to intermodal transport, such as costly investments for deploying infrastructures, a limited actors' willingness to coordinate or collaborate and, in some countries, a relatively poor passengers' perception of rail transport. Consider the mentioned *AIR&RAIL* agreement. Thalys has adapted the schedule of trains to match Air France departure/arrival timetables. It has also introduced dedicated luggage hold and integrated ticketing for Air France passengers at the Brussels check-in counter (see Eurocontrol, 2005, for further examples).

trip. Despite airline-HSR passengers may benefit from the agreement, there is a stronger negative effect on passengers in the remaining markets, where traffic volumes decrease. On the other hand, the JV agreement improves passenger surplus regardless of mode substitution and hub congestion. Compared to a CP agreement, it avoids double marginalization. Furthermore, in a CP agreement we still have some coordinated pricing, since the airline sets the prices of both transportation products in the connecting market, whereas in a JV agreement price coordination is removed at all, since distinct companies respectively provide the connecting flight and the combined airline-HSR service.

Third, we find that private and social interests on airline-HSR agreements are not aligned. Transport operators decide whether to cooperate depending on sunk costs and their bargaining power. On the one hand, they may not cooperate in the case where an agreement (of either type) improves consumer surplus. In this case, it may be optimal to subsidize cooperation. For instance, a publicly owned airport may participate in the infrastructure investment needed to make cooperation effective. On the other hand, firms may sign a CP agreement when it harms consumers, and possibly social welfare. In such a case, it is very doubtful that the agreement passes antitrust scrutiny.

In this framework, we propose a simple two-tier test that provides a sufficient condition for consumer surplus to be higher under the CP agreement. This test is a 'safe harbor' for a CP agreement: if it is satisfied, antitrust agencies should approve the agreement without need of costly investigations. The first step of the test consists in checking whether the total traffic volume in the overlapping market is higher with than without the agreement. If this is the case, then the agreement should be approved. If, instead, this is not the case, then the second step of the test controls that the wholesale price for a seat on the train (that the airline buys to offer the combined service in the connecting market) is lower than the HSR ticket price in the overlapping market.[6]

Finally, we extend the analysis so as to compare the CP and the JV agreements in terms of their effects on consumer surplus and social welfare (provided that they are incentive-compatible for transport

[6] For an extensive discussion on this point, see Section 4.2.

operators). Numerical simulations show that, from a policy perspective, the JV agreement should be preferred unless the related sunk costs are far greater than the costs of the CP agreement.

This paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model and the benchmark case of competition. Sections 4 and 5 respectively deal with CP and JV agreements. Section 6 compares these agreements. Section 7 concludes and points out directions for future work.

## *2.* Relevant literature

A burgeoning literature has investigated the case of airline-HSR competition, both theoretically (see e.g., Yang and Zhang, 2012; Jiang and Zhang, 2016) and empirically (see e.g., Fu *et al.*, 2014; Wan *et al.,* 2016). Conversely, the academic literature on airline-HSR cooperation is still sparse. It focuses on the welfare effects of cooperation in the extreme case of airline-HSR mergers. In this paper, we study different types of airline-HSR agreements that may be observed in practice, and we disentangle the impact of such agreements on consumer surplus, which is the main concern for antitrust agencies.

Jiang and Zhang (2014) find that airline-HSR integration improves social welfare when mode substitution in overlapping markets is sufficiently low (so that the adverse effect on competition is small), or the hub airport capacity is tight (so that integration may alleviate congestion). However, airline-HSR integration improves consumer surplus only in the special case where mode substitution is very low and the hub airport is not capacity constrained (so that passengers in the connecting market may benefit from the increased traffic due to the combined airline–HSR service).[7]

Xia and Zhang (2016) assume that air and HSR services are vertically differentiated. They find that after integration, when the hub capacity is tight, the airline withdraws from the market where it has less competitive advantage over HSR. They also find that airline-HSR integration is likely to improve welfare when the hub airport is capacity constrained. However, they are silent on consumer surplus.

[7] Jiang and Zhang (2014) build on a first attempt to evaluate the benefits of airline-HSR integration by Socorro and Viecens (2013), who however do not endogenize the airline's decision to allocate capacity to each route.

Jiang *et al.* (2017) investigate different cooperation schemes between a rail operator and a domestic and/or a foreign airline. Partners incur a fixed cost to cooperate and such cost is an increasing function of the cooperation level, which is endogenous. The authors find that the cooperation level is lower when both partnerships coexist than when only one partnership exists. They abstract away from congestion issues, network externalities, and the effects of partnerships on consumer surplus.

Airline-HSR agreements share some features with 'code-sharing' alliances between airlines, which have been widely investigated in the policy debate (ECA, 2003; ICAO, 2013) and in the academic literature (see e.g., Brueckner and Whalen, 2000; Brueckner, 2001; Jiang *et al.*, 2015). Generally, this literature assesses the welfare effects of the partnerships while ignoring the role of congestion at hub airports, which instead is an essential feature of our model.

Theory predicts that cooperation between airlines may yield anticompetitive effects in overlapping markets. Brueckner and Proost (2010) study the impact of a carve-out (that ban cooperation in such markets) on alliance partners, and find that it may be socially harmful when imposed on a JV alliance that fully exploits economies of traffic density (as a true merger). Hence, the welfare benefits of cooperation arise on the cost side. Instead, under an airline-HSR agreement, transport operators provide a differentiated product that raises traffic in the connecting market and in the whole network. Thus, even without cost efficiencies, the welfare benefits of cooperation arise on the demand side.

Some papers study revenue sharing agreements between airports and airlines, depending on whether airlines' services are substitutes or complements (Fu and Zhang, 2010).[8] Such agreements reduce social welfare when airports strategically share revenues with dominant airlines, thereby reducing airline competition. Different from our paper, these works consider non-stop flights between uncongested airports, while ignoring network effects.

[8] For extensive reviews on airport-airline agreements, see Fu *et al*. (2011) and D'Alfonso and Nastasi (2014).

An emerging field in industrial organization studies considers inter-firm bundling (Gans and King, 2006; Brito and Vasconcelos, 2015).[9] Typically, independent (single-product) firms coordinate to sell their products in bundle at a discount relative to the headline prices set for separate product selling. Nonetheless, these papers show that bundled discounts negatively affect consumers. We extend this literature by allowing firms to compete or cooperate in a hub-and-spoke transportation network where markets are interdependent if the hub airport is capacity constrained.

A further relevant strand concerns the formation and role of JVs for R&D investment (Kamien *et al.*, 1992; Yi and Shin, 2000). We share with this literature the idea that independent firms may have to coordinate decisions to achieve innovation (in our model, to introduce the new airline-HSR service). However, in traditional models of R&D JVs, firms collect profits only in those markets where they compete, which makes the case different from our framework.

## 3. The model

We consider the transportation network of three nodes (i.e., cities) illustrated in Figure 1. An airline, *a*, operates the short-haul (e.g., domestic) route between city O and city H. In the same market (also called overlapping market), a HSR transport operator, *r*, offers a direct ride. The airline also serves two long-haul (e.g., international) routes, that is, market HD with a direct connection and market OD (also called connecting market) with a one-stop trip via city H.[10]

*=== Insert Figure 1 about here ===*

[9] In turn, this field stems from the well-developed literature on product bundling by multi-product firms (Avenali *et al.*, 2013, discuss a list of relevant papers), and on product compatibility and systems competition, that is, competition between substitutes made of complements (see e.g., Matutes and Regibeau, 1988; Farrell *et al.*, 1998).

[10] We borrow from Jiang and Zhang (2014) the topology and market structure of the transportation network. An example of this setting is the following. Node O is Nantes, node H is Paris, and node D is Philipsburg (Saint Marteen). Air France is the monopoly airline in the single-leg routes from Nantes Atlantique Airport (NTE) to Paris CDG and from CDG to Princess Juliana International Airport (SXM), respectively, markets OH and HD in Figure 1. In addition, SNCF offers direct HSR rides from Gare de Nantes (QJZ) to CDG, that is, in market OH. Finally, passengers who travel from Nantes to Philipsburg (market OD) have two options available, both marketed by Air France: they can buy the *TGVAir* combined service from QJZ to SXM via CDG, or the connecting flight from NTE to SXM via CDG.

City H can serve as a multimodal hub, because there is a HSR station at the airport. In principle, passengers travelling from city O to city D could transfer from O to H by HSR and then fly from H to D. Given that market OD is covered by a single-carrier service, we assume that multimodal trips do not occur unless the airline and the HSR operator sign an intermodal agreement to offer a bundle of domestic HSR and international air services. This assumption is made for simplicity, but is not essential for the results (see the discussion in footnotes 14 and 20).

Let $M = \{OH, HD, OD\}$ be the set of markets $m$, and $T = \{A, R, AA, AR\}$ the set of transportation products $t$, where $t = A$ $(t = R)$ stands for a direct flight (HSR ride), $t = AA$ stands for a connecting flight via hub $H$, and $t = AR$ stands for the multimodal trip via hub $H$. Let $T_m$ be the subset of transportation products available in market $m$. Given the network in Figure 1, we have that $T_{OH} = \{A, R\}$, $T_{HD} = \{A\}$, and $T_{OD} = \{AA\}$ when the airline and the HSR do not cooperate, or $T_{OD} = \{AA, AR\}$ when they sign an agreement. Thus, there are at most five travel choices for passengers.

*3.1 Demand side*

We consider a representative passenger in each market $m \in M$. For simplicity, we assume that every representative passenger has the same strictly concave quadratic utility function:[11]

$$U_m(\boldsymbol{q}_m) = \sum_{t \in T_m} \alpha_m^t q_m^t - \frac{1}{2}\left(\sum_{t \in T_m} {q_m^t}^2 + 2\gamma \prod_{t \in T_m} q_m^t\right) \tag{1}$$

where $\boldsymbol{q}_m$ is the vector of travel volumes $q_m^t$ for transportation products $t \in T_m$, with $q_{OD}^{AR} = 0$ if transport operators do not sign an agreement to provide the combined air-rail product. Parameter $\alpha_m^t \geq 0$ measures the maximum willingness to pay (hereafter, wtp) for product $t \in T_m$. We assume that $\alpha_{OH}^t = \alpha_{HD}^t = \alpha$, whereas $\alpha_{OD}^t = 2\alpha$, with $\alpha > 0$. Thus, the maximum wtp for the long-distance travel from O to D is higher than the maximum wtp for direct trips from O to H or from H to D.[12]

[11] As is standard practice in the relevant literature, we thus follow Dixit (1979) and Singh and Vives (1984).

[12] This assumption implies that air passengers have the same maximum wtp per unit of hub capacity used by transportation products (products $A$ in markets OH and HD use one unit of capacity, while product $AA$ in market OD needs two such units). Qualitative findings are not affected if passengers have the same maximum wtp $\alpha$ in all markets, as in Jiang and

Parameter $\gamma$ $(0 < \gamma < 1)$ measures the degree of substitutability between transportation products, where the lower $\gamma$, the lower the substitutability. For simplicity, we assume that $\gamma$ is the same in all markets. Depending on $\gamma$, transportation modes can be (very) strong or (very) weak substitutes (see Figure 2 for a taxonomy of the critical values of $\gamma$ that drive the main results of the paper).

*=== Insert Figure 2 about here ===*

Let $p_m^t$ be the price for product $t \in T_m$. Solving the passenger's problem (i.e., maximizing (1) subject to the budget constraint), we can obtain the inverse demand curves in all markets:

$$\begin{aligned} p_{OH}^{A} &= \alpha - q_{OH}^{A} - \gamma\, q_{OH}^{R} \\ p_{OH}^{R} &= \alpha - q_{OH}^{R} - \gamma\, q_{OH}^{A} \\ p_{HD}^{A} &= \alpha - q_{HD}^{A} \\ p_{OD}^{AA} &= 2\alpha - q_{OD}^{AA} - \gamma\, q_{OD}^{AR} \\ p_{OD}^{AR} &= 2\alpha - q_{OD}^{AR} - \gamma q_{OD}^{AA} \end{aligned} \tag{2}$$

where product $AR$ can be purchased only in the case of an airline-HSR agreement. Thus, the consumer surplus of the representative passenger in market $m \in M$ can be written as:

$$CS_m(\boldsymbol{q}_m) = U_m(\boldsymbol{q}_m) - \sum_{t \in T_m} p_m^t q_m^t \tag{3}$$

where $q_{OD}^{AR} = 0$ if firms do not sign an agreement. Then, the total consumer surplus can be written as:

$$CS(\boldsymbol{q}) = \sum_{m \in M} CS_m(\boldsymbol{q}_m) \tag{4}$$

*3.2 Supply side*

We now turn to the supply side. We take the runway capacity $k > 0$ at hub H as exogenous. Parameter $k$ can be seen as the maximum number of flights that can be transferred via hub H in a given slot (or set of identical slots). Depending on air traffic, hub H may be capacity constrained.

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Zhang (2014). Indeed, numerical results show that our main findings do hold as long as $\alpha_{OD}^t > 0$ (these results are available from the authors upon request). However, the parameter region where we can compare the outcomes with and without an intermodal agreement is smaller in the case where $0 < \alpha_{OD}^t < \alpha$ than in the case where $\alpha_{OD}^t \geq \alpha$.

For simplicity, we assume that: (i) the size of vehicles is the same for each transportation mode in each market and (ii) the relation among passengers, seats, and flights/HSR rides is of the fixed proportions type (see e.g., Basso, 2008), such that the product between the size of vehicles and the load factor is constant for all services in all markets (we normalize such product to unity). Thus, prices per flight/HSR ride, per seat, and per traveler are equivalent.

Moreover, we abstract away from economies of traffic density and from cost asymmetry between transportation modes (see Section 7 for a discussion on this point). Thus, we assume that $C_m^t(q_m^t) = c_m^t q_m^t$ if $m = OH, HD$ and $C_{OD}^t(q_{OD}^t) = 2c_{OD}^t q_{OD}^t$, with $c_m^t = c$ (i.e., the cost per flight/HSR ride, per seat, or per traveler) is constant and, for simplicity, normalized to zero.[13]

We also assume that, if transport operators decide to cooperate, then they should jointly incur sunk costs $F$ ($F > 0$) for the agreement to become effective.[14] Thus, social welfare can be written as:

$$W(\boldsymbol{q}) = \sum_{m \in M} U_m(\boldsymbol{q}_m) - F \tag{5}$$

where $q_{OD}^{AR} = 0$ and $F = 0$ if transport operators do not sign an agreement.

Finally, we assume that transport operators are profit-maximizing firms that choose quantities. This assumption reflects that both the airport and the HSR platform capacities cannot be easily increased.

[13] Numerical results show that our main findings do hold in the case where $c_{OH}^A > c_{OH}^R = 0$ (and $c_{OD}^{AA} = 2c_{OH}^A > c_{OD}^{AR} = 2c_{OH}^R = 0$). These results are available from the authors upon request.

[14] As a baseline, transport operators should enable passengers to purchase a single ticket for the entire multimodal trip (i.e., to buy a bundle of domestic HSR and international air services). This requires operators to integrate their information technology and computer reservation systems. Moreover, it requires operators to coordinate schedules between air and HSR services, thereby taking the risk of possible delays on one segment of the journey, and providing passengers with proper warranties. Operators may also offer coordinated baggage handling (so that passengers should not care about baggage transfer at the intermediate stop), and/or supplementary services on HSR trains similar to those offered on short-haul flights (e.g., dining). In the absence of cooperation, a trip that uses the train from O to H and then the aircraft from H to D does not have these features. In principle, passengers could self-assemble the multimodal trip, but they would perceive it as an inferior substitute for the connecting flight, such that the two products would not compete head-to-head for traffic volumes in the connecting market. Thus, for simplicity, we assume that passengers may use a combined airline-HSR service only in the case where transport operators decide to cooperate (see also footnote 20).

Indeed, capacity adjustments are slower and more costly to implement than price adjustments, since the former require lumpy and irreversible investment.[15]

*3.3 Benchmark case: no agreement*

Consider first the benchmark case where transport operators do not sign an agreement. The airline is a monopoly in markets OD and HD, while the HSR operator and the airline compete in quantities in market OH. Thus, the HSR operator and the airline respectively solve:

$$max_{q_{OH}^{R}} \pi_r = p_{OH}^{R} q_{OH}^{R} \tag{6}$$

$$\begin{aligned} max_{q_{OH}^{A}, q_{HD}^{A}, q_{OD}^{AA}} \pi_a &= p_{OH}^{A} q_{OH}^{A} + p_{HD}^{A} q_{HD}^{A} + p_{OD}^{AA} q_{OD}^{AA} \\ s.t.\, q_{OH}^{A} + q_{HD}^{A} + 2q_{OD}^{AA} &\leq k \end{aligned} \tag{7}$$

In Appendix A.1, we find the vector $\boldsymbol{q^N} = \left(q_{OH}^{A,N}, q_{OH}^{R,N}, q_{HD}^{A,N}, q_{OD}^{AA,N}\right)$ that solves problem (6)-(7), where *N* stands for *no agreement*. For simplicity, we restrict the analysis to the case where $\boldsymbol{q^N} > 0$. Given that firms do not cooperate, we denote by $\Omega_{N,i}$ the constellation of parameters $(\alpha, \gamma, k)$ such that the hub capacity constraint is not strictly binding in equilibrium (subscript $i$ stands for *interior* solution), and by $\Omega_{N,b}$ the constellation of parameters $(\alpha, \gamma, k)$ such that the constraint is strictly binding (subscript *b* stands for *boundary* solution). We label the hub airport *congested* when $(\alpha, \gamma, k) \in \Omega_{N,b}$, and *not congested* otherwise.

## 4. Capacity purchase (CP) agreement

Under a CP agreement, the HSR operator agrees to sell seats on the train to the airline, which is in charge of the combined airline-HSR service in the connecting market. In this framework, while the airline and the HSR compete for short-haul trips in market OH, the airline is a multiproduct monopolist in market OD. We assume that, since the airline is responsible for the combined service,

[15] In this sense, we are taking a short-run rather than a long-run perspective. In doing so, we follow a number of relevant papers, such as Jiang and Zhang (2014), D'Alfonso *et al*. (2015, 2016), and Xia and Zhang (2016). It is well known that quantity competition reflects price competition with limited capacities (Kreps and Scheinkman, 1983).

then it cannot be forced by the HSR operator to book a predetermined number of seats on the train. Nonetheless, the HSR operator can affect the airline's decision on how many seats to book on the train by setting the price for each seat. Let $w$ be the (per ride, per seat, or per traveler) wholesale price charged by the HSR to the airline. Depending on $w$, the airline decides the number of seats to buy on the HSR train in order to provide the combined transportation service.[16]

The timing of the game is as follows. At stage one, transport operators decide whether to sign a CP agreement (thereby incurring sunk costs to make their services compatible) or not. In case of agreement, at stage two the HSR operator sets the wholesale price $w$. Finally, at stage three the airline and the HSR operator set quantities in relevant markets.[17] We solve the game backwards.

*4.1. Traffic volumes*

Assume that transport operators have signed a CP agreement to provide the combined transportation service in market OD. Then, at the third stage (for a given wholesale price $w$) the HSR operator and the airline respectively solve:

$$max_{q_{OH}^R} \pi_r = p_{OH}^R q_{OH}^R \tag{8}$$

$$\begin{gathered} max_{q_{OH}^A, q_{HD}^A, q_{OD}^{AA}, q_{OD}^{AR}} \pi_a = p_{OH}^A q_{OH}^A + p_{HD}^A q_{HD}^A + p_{OD}^{AA} q_{OD}^{AA} + (p_{OD}^{AR} - w) q_{OD}^{AR} \\ s.t.\, q_{OH}^A + q_{HD}^A + 2q_{OD}^{AA} + q_{OD}^{AR} \leq k \end{gathered} \tag{9}$$

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[16] This assumption is *also* made for analytical tractability. If the HSR operator decides the number of train seats (i.e., quantity, instead of price) that are made available for the combined transportation service, the airline is still free to decide how many seats to buy for providing the service, on condition that the number of reserved seats does not exceed the number of seats made available for that purpose. Hence, to ensure the feasibility of the airline's decision, we would have to introduce an additional constraint in our model. Furthermore, if the HSR operator sets the *maximum* quantity, and the airline can choose the *actual* quantity, then it would be very difficult (if ever possible) to find out the optimal wholesale price per train seat and thus determine the optimal profit for the HSR operator.

[17] The assumption that price is the decision variable at a given stage of the game, and quantity is the decision variable at a later stage is not rare in models of vertical relations. Consider the case where competing downstream firms need to have access to an essential input provided by an upstream firm to offer their services or products to end users. In this framework, several papers assume that downstream firms choose quantities depending on the input price set by the upstream firm or the regulator (for telecommunications, see Foros, 2004; Bourreau *et al*., 2014; Cambini and Silvestri, 2013; for the pharmaceutical industry, see Maskus and Chen, 2004; Matteucci and Reverberi, 2005, 2014, 2016).

In Appendix A.2, we find vector $\boldsymbol{q^{CP}}(w) = \left(q_{OH}^{A,CP}(w), q_{OH}^{R,CP}(w), q_{HD}^{A,CP}(w), q_{OD}^{AA,CP}(w), q_{OD}^{AR,CP}(w)\right)$ solving problem (8)-(9), where *CP* stands for *capacity purchase agreement*. For simplicity, we restrict the analysis to the case where $\boldsymbol{q^{CP}}(w) > 0$.

Let us study the impact of the wholesale price $w$ on passenger traffic in the transportation network. Assume first that the hub airport is not capacity constrained after the agreement is signed. We find that $w$ only affects passenger traffic in the connecting market. Intuitively, a higher $w$ reduces traffic volumes for the combined airline-HSR service, thereby increasing the number of connecting flights. Now assume that the hub airport is capacity constrained after the agreement. In such a case, markets are interdependent, and thereby the wholesale price affects traffic in all relevant markets. As expected, an increase in $w$ reduces traffic and increases price for the combined airline-HSR service (since we have $\partial q_{OD}^{AR,CP}(w)/\partial w < 0$ and $\partial p_{OD}^{AR,CP}(w)/\partial w > 0$). On the other hand, the effects of the increase in $w$ on pure air traffic volumes (both in the connecting market and in single-leg markets) depend on the substitutability between air and HSR services.

If transportation products are strong substitutes (i.e., if $\gamma > 0.5$), then an increase in price for the combined airline-HSR service (following an increase in $w$) causes a significant increase in demand for the connecting flight in market OD, while the relevant air ticket price is also increasing (i.e., $\partial p_{OD}^{AA,CP}(w)/\partial w > 0$). Under hub congestion, this prevents the airline from allocating capacity to single-leg markets. Therefore, the number of passengers carried by the airline decreases and air ticket prices increase in both markets OH and HD (since we have $\partial q_{OH}^{A,CP}(w)/\partial w < 0$, $\partial p_{OH}^{A,CP}(w)/\partial w > 0$, $\partial q_{HD}^{A,CP}(w)/\partial w < 0$, and $\partial p_{HD}^{A,CP}(w)/\partial w > 0$).

If, instead, transportation products are weak substitutes (i.e., if $\gamma < 0.5$), then an increase in price for the airline-HSR service (following an increase in $w$) *per se* does not imply a significant increase in demand for the connecting flight, and thus the relevant air ticket price decreases (that is, $\partial p_{OD}^{AA,CP}(w)/\partial w < 0$). This does not prevent the airline from allocating freed slots to single-leg markets. Therefore, the number of passengers carried by the airline increases and air ticket prices decrease in both markets OH and HD (since we have $\partial q_{OH}^{A,CP}(w)/\partial w > 0$ and $\partial p_{OH}^{A,CP}(w)/\partial w < 0$, as well as $\partial q_{HD}^{A,CP}(w)/\partial w > 0$ and $\partial p_{HD}^{A,CP}(w)/\partial w < 0$). Remark 1 summarizes the results.[18]

***Remark 1***. *Assume that the hub airport is capacity constrained after the CP agreement. In equilibrium, an increase in $w$ reduces traffic and increases price for the airline-HSR service. Moreover, if transportation products are strong substitutes ($\gamma > 0.5$) then as $w$ increases:*

*(i) both the number of passengers and the ticket price for the connecting flight increase;*

*(ii) the number of air passengers in markets OH and HD decreases, and air ticket prices increase.*

*Conversely, if transportation products are weak substitutes ($\gamma < 0.5$) then as $w$ increases:*

*(iii) the number of passengers of the connecting flight increases, and the air ticket price decreases;*

*(iv) the number of air passengers in markets OH and HD increases, and air ticket prices decrease.*

*4.2. Wholesale price*

At the second stage, the HSR operator sets the wholesale price per train seat, $w$, while anticipating the impact on quantities supplied:

$$max_w \, \pi_r = p_{OH}^{R,CP} q_{OH}^{R,CP}(w) + w q_{OD}^{AR,CP}(w) \tag{10}$$

Let $w^{CP}$ denote the solution to (10). By plugging $w^{CP}$ into $\boldsymbol{q^{CP}}(w)$, we obtain $\boldsymbol{q^{CP}} = \boldsymbol{q^{CP}}(w^{CP})$ (see Appendix A.2). Given that firms cooperate, we denote by $\Omega_{CP,i}$ the constellation of parameters $(\alpha, \gamma, k)$ such that the hub capacity constraint is not strictly binding in equilibrium and by $\Omega_{CP,b}$ the constellation of parameters $(\alpha, \gamma, k)$ such that the constraint is strictly binding. We label the hub airport *congested* after the CP agreement if $(\alpha, \gamma, k) \in \Omega_{CP,b}$, and *not congested* otherwise.

Let us study how the hub capacity affects the optimal wholesale price per train seat. For this purpose, we consider the more interesting case where the hub airport is capacity constrained (so that markets are interdependent). Assume that the hub capacity decreases, so that congestion rises. We find that, as long as transportation products are weak substitutes ($\gamma < 0.5$), the optimal wholesale price per

[18] For the sake of space, we omit the proofs of remarks and propositions. These are available from the authors on request.

train seat decreases (since $\partial w^{CP}/\partial k > 0$). This serves to induce the airline to use efficiently the hub capacity. From Remark 1, the airline-HSR traffic increases at the expense of the connecting flight (since $\partial q_{OD}^{AR,CP}(w)/\partial w < 0$ and $\partial q_{OD}^{AA,CP}(w)/\partial w > 0$). Moreover, the airline is transferring slots from single-leg markets to the connecting market. Indeed, the number of air passengers in both markets OH and HD decreases (since we have $\partial q_{OH}^{A,CP}(w)/\partial w > 0$ and $\partial q_{HD}^{A,CP}(w)/\partial w > 0$).[19] Remark 2 summarizes the results.

***Remark 2***. *Assume that the hub airport is capacity constrained after the CP agreement. In equilibrium, the wholesale price is increasing in the hub capacity (i.e., $\partial w^{CP}/\partial k > 0$) if and only if transportation products are weak substitutes (i.e., $\gamma < 0.5$).*

It is worth noting at this point that the wholesale price per train seat under the CP agreement can, in principle, be higher than the retail price per train seat in the single-leg market where the HSR operates (i.e., market OH). This may occur because of third-degree price discrimination. Indeed, the HSR operator can easily distinguish a business customer (such as the airline) from an end user (e.g. a leisure traveler), so that different prices can be targeted to those different classes of consumers. Moreover, the HSR operator can suitably use some legal restrictions (e.g., VATIN - VAT Identification Number) to prevent the airline from having access to the price targeted to end users.[20]

[19] If transportation products are strong substitutes, then a decrease in hub capacity (i.e., an increase in congestion) causes an increase in the wholesale price per train seat. From Remark 1, the traffic volume for the connecting flight increases at the expense of the airline-HSR service. Moreover, the number of air passengers in markets OH and HD decreases.

[20] This pricing strategy is also related to the fixed investment cost that transport operators have incurred at the first stage of the game to make cooperation effective. Generally, following an airline-HSR agreement passengers from O to D can choose among three options, that is, the connecting flight, the high-quality combined (i.e., bundled) airline-HSR service, and the low-quality self-assembled multimodal trip. In this framework, consumers have a higher wtp for the high-quality bundled service than for the low-quality self-assembled trip. Therefore, it is still possible that the wholesale price per train seat for the bundled service (and thereby the retail price for this service) is higher than the retail price for the HSR ticket in market OH. Indeed, because of the higher quality of the bundled service, the retail price for the bundle would be higher than the sum of the stand-alone prices for the HSR ticket in market OH and for the airline ticket in market HD. For these reasons, to simplify the analysis without affecting the qualitative results, we have assumed that the bundled product offered after an intermodal agreement is the only combined airline-HSR service that is available to passengers.

Let us now assess whether and when $w^{CP}$ is lower than $p_{OH}^{R,CP}$. Indeed, we show in Remark 6 that this fact has important welfare implications.

Consider first that, as $\gamma$ increases, transportation products in market OH are stronger substitutes, so that, *ceteris paribus*, the retail price per train seat declines (i.e., $\partial p_{OH}^{R,CP}/\partial\gamma < 0$). In turn, this means that, if $\gamma$ is sufficiently high, then $w^{CP}$ should be low enough for it to be lower than $p_{OH}^{R,CP}$.

If the hub airport is not capacity constrained after the agreement, then the wholesale price per train seat only affects traffic in the connecting market. We thus find the intuitive result that $w^{CP}$ is lower than $p_{OH}^{R,CP}$ if and only if transportation products are very strong substitutes. In principle, the airline has the incentive to serve market OD by means of the connecting flight instead of the combined service, because it avoids paying the wholesale price for a train seat (from Remark 1, we have $\partial q_{OD}^{AA,CP}(w)/\partial w > 0$ and $\partial q_{OD}^{AR,CP}(w)/\partial w < 0$). Note that this incentive is stronger when $\gamma$ is high, since the airline-HSR service does not significantly increase traffic in market OD when transportation products are very strong substitutes. Therefore, the HSR has to keep $w^{CP}$ as low as possible to promote the combined service.

Assume now that the hub airport is capacity constrained after the agreement. Since markets are interdependent, then the wholesale price per train seat affects the allocation of capacity at the hub across all markets. It follows from Remark 1 that, if transportation products are strong substitutes ($\gamma > 0.5$), then the HSR operator faces a trade-off in reducing $w^{CP}$. A low $w^{CP}$ induces the airline to increase supply of the combined service at the expense of connecting flights, thereby benefiting the HSR (indeed, $\partial q_{OD}^{AR,CP}(w)/\partial w < 0$ and $\partial q_{OD}^{AA,CP}(w)/\partial w > 0$). However, this in turn increases the number of flights and reduces the number of HSR rides in market OH, thereby harming the HSR (since $\partial q_{OH}^{A,CP}(w)/\partial w < 0$ and $\partial q_{OH}^{R,CP}(w)/\partial w > 0$). If transportation products are very strong substitutes or the hub capacity is sufficiently low, then the number of freed slots when connecting flights are reduced is relatively low. This implies that the HSR operator suffers a small loss in market

OH, so that the benefits from the traffic increase for the combined service in market OD do prevail. Remark 3 summarizes the results in case of congestion.

***Remark 3***. *Assume that the hub airport is capacity constrained after the CP agreement. In equilibrium, the wholesale price per train seat* $w^{CP}$ *is lower than the HSR ticket price* $p_{OH}^{R,CP}$ *in market OH if and only if transportation products are very strong substitutes (i.e.,* $\gamma > \hat{\gamma} \cong 0.63$*) or the hub capacity is sufficiently low (i.e.,* $k < k_1$*).*[21]

*4.3. Formation of the agreement*

At the first stage, transport operators decide whether to sign the CP agreement, thereby incurring sunk costs, or not. Let $\beta$ (respectively, $1-\beta$), $0 < \beta < 1$, be the fraction of sunk costs for the airline (HSR) in case of agreement. Proposition 1 shows that, as long as the fixed costs of the agreement are sufficiently low, the airline and the HSR operator can manage to share such costs so that they find it profitable to sign the agreement and provide the combined service in market OD.

***Proposition 1***. *Let the sunk costs of the agreement be sufficiently low, that is, let* $0 < F \leq F_1$*. Then, there exists* $\tilde{\beta}$ *(*$0 < \tilde{\beta} < 1$*) such that firms find it profitable to sign the CP agreement.*

The intuition is straightforward. For any given wholesale price per train seat, third-stage profits (given that the fixed costs of the agreement are sunk) are higher for both transport operators as long as they provide the combined airline-HSR service. Hence, they do not sign the CP agreement if and only if the fixed costs incurred to create the partnership are too high. In such a case, transport operators cannot agree on a cost sharing rule such that they both prefer to create the partnership.

*4.4. Comparison of results with and without the CP agreement*

In what follows, we compare the results in the presence and in the absence of the CP agreement, first in terms of traffic volumes in the transportation network and the degree of congestion at the hub airport, and then in terms of consumer surplus and social welfare.

[21] We relegate to Appendix A.4 the expressions of the critical values of $k$ that we find throughout the paper.

*4.4.1. Passenger traffic and hub congestion*

First, we assess the impact of the CP agreement between the airline and the HSR operator on passenger traffic in the transportation network. For each constellation of parameters, Table 1 shows the effects of the agreement on traffic volumes in each relevant market.

*=== Insert Table 1 about here ===*

Based on Table 1, we can state what follows.

***Remark 4***. *Compared to the case where firms do not cooperate, under a CP agreement between the airline and the HSR operator we have that:*

*(i)* *the total traffic in market OD increases, i.e.,* $q_{OD}^{AR,CP} + q_{OD}^{AA,CP} > q_{OD}^{AA,N}$*;*

*(ii)* *the total traffic in market OH may increase, i.e.,* $q_{OH}^{A,CP} + q_{OH}^{R,CP} > q_{OH}^{A,N} + q_{OH}^{R,N}$ *may hold;*

*(iii)* *the total traffic in the transportation network increases, i.e.,* $q_{OH}^{A,CP} + q_{OH}^{R,CP} + q_{HD}^{A,CP} + q_{OD}^{AR,CP} + q_{OD}^{AA,CP} > q_{OH}^{A,N} + q_{OH}^{R,N} + q_{HD}^{A,N} + q_{OD}^{AA,N}$.

The rationale for result (i) is that, despite under the CP agreement the airline substitutes some feeding flights for HSR rides in the connecting market (so that $q_{OD}^{AA,CP} < q_{OD}^{AA,N}$), the total traffic volume in that market increases due to product differentiation brought about the combined service.

Result (ii) offers new insights on the effects of airline-HSR agreements. It is widely argued that cooperation between airlines and HSR operators may raise some competition concerns, particularly in overlapping markets. Indeed, Jiang and Zhang (2014) find that airline-HSR integration reduces traffic in the single-leg market served by both transportation modes. We add to the literature as we find that the total traffic in market OH may increase under the CP agreement. We can draw from Table 1 that this occurs as long as the number of passengers carried by the airline in market OH increases after the agreement. In turn, this happens when the agreement alleviates traffic congestion at the hub airport (i.e., in the region $\Omega_{CP,i} \cap \Omega_{N,b}$) or when, given that the hub is capacity constrained (in the region $\Omega_{CP,b} \cap \Omega_{N,b}$), transportation products are strong substitutes ($\gamma > 0.5$).

Finally, result (iii) is driven by result (i) and by the fact that the airline may use the capacity made available at the hub airport after the CP agreement to simultaneously increase air traffic in market OH (result (ii)) and in market HD.

Let us now study the impact of the CP agreement on passenger traffic at the hub airport. One of the main arguments in support of airline-HSR agreements is that they alleviate traffic congestion at some major airports subject to capacity constraints. From Table 1, and from the results of Remark 4, we can find that this is not necessarily the case. Indeed, a CP agreement may not reduce, and sometimes may increase passenger traffic at the hub airport.

More specifically, this occurs when the hub airport is capacity constrained both before and after the agreement (in the region $\Omega_{CP,b} \cap \Omega_{N,b}$), and when the hub becomes capacity constrained after the agreement (in the region $\Omega_{CP,b} \cap \Omega_{N,i}$). On the other hand, the impact of the CP agreement is not clear-cut when the hub airport has spare capacity both before and after the agreement (i.e., in the region $\Omega_{CP,i} \cap \Omega_{N,i}$). We can prove that, in such a case, the CP agreement increases traffic flow at the hub (that is, $q_{OH}^{A,CP} + q_{HD}^{A,CP} + q_{OD}^{AR,CP} + 2q_{OD}^{AA,CP} > q_{OH}^{A,N} + q_{HD}^{A,N} + 2q_{OD}^{AA,N}$) as long as the air and HSR services are weak substitutes ($\gamma < 0.5$). Although the airline substitutes some feeding flights for feeding HSR rides in the connecting market after the agreement, thereby reducing connecting flight traffic at the hub, this effect is outweighed by the overall traffic increase in that market (since $2q_{OD}^{AA,N} \leq q_{OD}^{AR,CP} + 2q_{OD}^{AA,CP}$). Remark 5 summarizes the results.

***Remark 5***. *A CP agreement between the airline and the HSR operator does not necessarily reduce passenger traffic at the hub airport.*

*4.4.2. Consumer surplus and social welfare*

We now turn to study the effects of the CP agreement on consumer surplus and social welfare. First, let $CS(\boldsymbol{q^{CP}})$ be the total consumer surplus after the CP agreement is signed. Proposition 2 summarizes the results.

***Proposition 2.*** *Compared to the case where firms do not cooperate, under a CP agreement between the airline and the HSR operator we have that:*

*(i) if the hub airport is not congested, consumer surplus increases;*

*(ii) if the hub is congested, consumer surplus increases unless transportation products are very weak substitutes ($\gamma < \check{\gamma} \cong 0.33$ ) and the hub capacity is neither too small nor too large ($k_2 < k < k_3$).*

Proposition 2 shows that a CP agreement generally benefits consumers, even if the hub airport is congested. This is in stark contrast to the case of airline-HSR integration, where cooperation is mainly in the firms' interest, while consumer surplus increases only if the hub airport is not capacity constrained and mode substitution is very low (Jiang and Zhang, 2014).

Proposition 2 can be interpreted as follows. Consider first the case where the hub airport is not capacity constrained after the agreement. It follows from Table 1 (and thus from Remark 4) that consumer surplus is higher with than without the CP agreement. This result hinges on the (weak) increase in traffic in all relevant markets after the agreement. In turn, this traffic increase derives from product differentiation in the connecting market and, when the hub airport is congested before the agreement (i.e., in the region $\Omega_{CP,i} \cap \Omega_{N,b}$), from the opportunity to reallocate to single-leg markets (some of the) slots that are made available by reducing the number of connecting flights.

Now assume that the hub airport is capacity constrained after the agreement (i.e., consider region $\Omega_{CP,b}$). Generally, the impact of the agreement on consumer surplus in each market is consistent with the impact on passenger traffic in the same market. Thus, when traffic in a market increases (decreases) because of the agreement, consumer surplus in that market also increases (decreases).

Nonetheless, there is one notable exception. We can show that, if $\gamma < \tilde{\gamma} \cong 0.21$ and the hub capacity is not too small, consumer surplus in the connecting market decreases under the agreement, notwithstanding the increase in passenger traffic. This is because retail prices in market OD (where transportation products are very differentiated) are relatively high. Indeed, the airline prefers allocating slots to the connecting market, at the expense of single-leg markets, because of the higher passengers' wtp. Since consumer surplus in single-leg markets is lower under the agreement due to lower passenger traffic, then the total consumer surplus decreases under the agreement. When $\gamma > \tilde{\gamma} \cong 0.21$, the total consumer surplus may still decrease, specifically when the positive effect of the agreement in the connecting market is not enough to offset the negative effects in single-leg markets. This occurs when transportation products are very weak substitutes (i.e., $\gamma < \check{\gamma} \cong 0.33$) and the congested hub is of moderate size (i.e., $k_2 < k < k_3$).

Based on the results of Remark 3 and Proposition 2, we can derive a simple two-tier test that gives a sufficient condition for a CP agreement to improve consumer surplus. This test provides a 'safe harbor' for such agreements. If this test is satisfied then the agreement is compliant with competition law, and antitrust agencies should approve it without need of costly investigations.

The first step of the test consists in checking whether passenger traffic in the overlapping market increases under the CP agreement. If the total traffic volume in market OH is higher with than without the agreement, then the overall consumer surplus is also higher and the agreement should be approved. If traffic in the overlapping market decreases under the agreement, then the second step of the test requires checking that the wholesale price per train seat for the combined service is lower than the retail price per train seat in market OH (see the discussion of Remark 3). If this condition holds then the CP agreement improves consumer surplus and thus should be approved, otherwise further investigations are necessary. Remark 6 illustrates the proposed test.

***Remark 6***. *A simple sufficient condition for consumer surplus to be higher with than without the CP agreement is that traffic in the overlapping market increases under the agreement. If this condition does not hold, consumer surplus is still higher under the agreement if the wholesale price per train seat* $w^{CP}$ *is lower than the HSR ticket price* $p_{OH}^{R,CP}$ *in market OH.*

Let $W(\boldsymbol{q^{CP}})$ be the total social welfare after the CP agreement is signed. Proposition 3 studies the effect of the CP agreement on social welfare.

***Proposition 3***. *Assume that the sunk costs of the agreement are sufficiently low, that is,* $0 < F \leq F_2$. *Then, social welfare is higher with than without the CP agreement.*

We can combine the results of propositions 1 to 3 to draw some policy implications. From Proposition 2, a CP agreement reduces consumer surplus as long as $\gamma < \check{\gamma} \cong 0.33$ and $k_2 < k < k_3$. Let us compare the critical values of the sunk costs of the agreement for it to increase respectively firms' profits ($F_1$, from Proposition 1) and social welfare ($F_2$, from Proposition 3).

First, we find that $F_2 \leq F_1$ as long as $\gamma < \check{\gamma} \cong 0.33$ and $k_2 < k < k_3$. In this framework, it follows that: i) if $F \leq F_2$, then the agreement is both viable and welfare improving, but at the expense of consumers' well-being; ii) if $F_2 < F \leq F_1$, then the agreement is viable but consumer surplus and the overall social welfare decrease under the agreement. It is highly doubtful that, in both cases, the CP agreement will pass antitrust scrutiny.

In the remaining cases (i.e., when $\gamma > \check{\gamma} \cong 0.33$, or when $k < k_2$ or $k > k_3$), we find that $F_1 < F_2$. Therefore, we have that: i) if $F \leq F_1$, then the agreement is both viable and welfare improving, with a positive impact on consumer surplus; ii) if $F_1 < F \leq F_2$, then firms do not have incentives to cooperate, even though a CP agreement would increase social welfare and, more specifically, consumer surplus. It follows that, in the latter case, it may be optimal to subsidize cooperation. For instance, a publicly owned airport may consider participation in infrastructure investment that is required to make cooperation effective. Remark 7 follows from previous reasoning.

***Remark** 7. Depending on the sunk costs of the agreement, one of the following cases may occur for a CP agreement between an airline and a HSR operator:*

*(i) the agreement is viable, but it reduces consumer surplus and possibly social welfare;*

*(ii) the agreement is not viable, but it would improve consumer surplus and social welfare.*

*In case (i), the agreement should not pass antitrust scrutiny, while in case (ii) it might be subsidized.*

## 5. Joint venture (JV) agreement

In this section, we consider the case where the airline and the HSR operator may decide to form a JV to deliver the combined transportation service in the connecting market.

Under such an agreement, the airline and the HSR operator are supposed to create a distinct business unit that is in charge of the combined airline-HSR service, with a 'firewall' between that service and all other services provided in the network. This means that the JV is a sort of 'supervised' agreement where the only source of profit for the separate business unit is the airline-HSR service. Therefore, the JV (as a distinct organizational entity from transport operators) maximizes profit $\pi_{JV}(\boldsymbol{q}) = p_{OD}^{AR} q_{OD}^{AR}$, where *JV* stands for *joint venture agreement*.[22]

On the other hand, the airline (respectively, the HSR) maximizes profit from all other services it provides in the network, plus a fraction $\rho$ (respectively, $1-\rho$) of the JV entity's profit, with $0 < \rho < 1$. A possible interpretation is that the airline holds a percentage $\rho$ of the shares of the JV entity, while the HSR holds the remaining percentage. These percentages, in turn, are related to transport operators' bargaining power in negotiating the agreement.

The rationale for this setting of the JV agreement is reminiscent of imposing functional separation of bottleneck and competitive activities for dominant firms in the energy or telecommunications industries, and of imposing carve-outs on JVs formed in the case of alliances between airlines to grant airline antitrust immunity.[23]

[22] If firms create a JV then profits are maximized when they jointly set quantities to be provided in all relevant markets, as in the case of a merger (or integration). Since consumer surplus would typically decrease (Jiang and Zhang, 2014), antitrust agencies would hardly approve the agreement. Instead, the JV agreement passes antitrust scrutiny (while being incentive-compatible for firms) as long as the exclusive goal of cooperation is supplying the airline-HSR service in the connecting market. Indeed, we can prove that this is the most favourable case for consumers under the JV agreement, since it minimizes competition concerns.

[23] Functional separation (while being less intrusive than ownership separation) serves to ensure that the owner of the essential input (e.g.. the transport/transmission network in energy, and the local access network in telecommunications) offers bottleneck services on a strict non-discriminatory basis to all competing upstream/downstream firms, including the affiliated unit. Several papers (Avenali *et al*., 2014; Cremer and De Donder, 2013; Höffler and Kranz, 2011) model functional separation as the case where, when choosing their decision variables, one business unit (either the bottleneck owner or the affiliated unit) maximizes its own profit, while the other unit (respectively, either the affiliated unit or the bottleneck owner) maximizes the whole firm's profit. On the other hand, a carve-out on an airline alliance prohibits collaboration in hub-to-hub fare setting, while allowing cooperation in other markets (Brueckner and Proost, 2010).

The timing of the game is as follows. At stage one, transport operators decide whether to form the JV (thereby incurring sunk costs) or not. At stage two, the airline, the HSR and possibly the JV entity set quantities in relevant markets. We solve the game backwards.

*5.1. Traffic volumes*

Under the JV agreement, at the second stage of the game the HSR operator, the airline and the JV entity respectively solve:

$$max_{q_{OH}^{R}} \pi_r(\boldsymbol{q}) + (1-\rho)\pi_{JV}(\boldsymbol{q}) = p_{OH}^{R} q_{OH}^{R} + (1-\rho)\pi_{JV}(\boldsymbol{q}) \quad (11)$$

$$max_{q_{OH}^{A}, q_{HD}^{A}, q_{OD}^{AA}} \pi_a(\boldsymbol{q}) + \rho\, \pi_{JV}(\boldsymbol{q}) = p_{OH}^{A} q_{OH}^{A} + p_{HD}^{A} q_{HD}^{A} + p_{OD}^{AA} q_{OD}^{AA} + \rho\, \pi_{JV}(\boldsymbol{q})$$
$$s.t.\, q_{OH}^{A} + q_{HD}^{A} + 2q_{OD}^{AA} + q_{OD}^{AR} \leq k \quad (12)$$

$$max_{q_{OD}^{AR}} \; \pi_{JV}(\boldsymbol{q}) = p_{OD}^{AR} q_{OD}^{AR}$$
$$s.t.\, q_{OH}^{A} + q_{HD}^{A} + 2q_{OD}^{AA} + q_{OD}^{AR} \leq k \quad (13)$$

where $\pi_r$ and $\pi_a$ are as defined in (6) and (7). Let $\boldsymbol{q^{JV}} = \left(q_{OH}^{A,JV}, q_{OH}^{R,JV}, q_{HD}^{A,JV}, q_{OD}^{AA,JV}, q_{OD}^{AR,JV}\right)$ be the solution to (11)-(13), found in Appendix A.3. If firms cooperate, we denote by $\Omega_{JV,i}$ the constellation of parameters $(\alpha, \gamma, k, \rho, \mu)$ such that the hub capacity constraint is not strictly binding in equilibrium and by $\Omega_{JV,b}$ the constellation of parameters $(\alpha, \gamma, k, \rho, \mu)$ such that the constraint is strictly binding. We label the hub *congested* after the JV agreement if $(\alpha, \gamma, k, \rho, \mu) \in \Omega_{JV,b}$, and *not congested* otherwise. Since the airline and the JV entity share the same capacity constraint at the hub airport, then we have a Generalized Nash Equilibrium Problem (GNEP) with infinite Nash equilibria.[24]

*5.2. Formation of the agreement*

At the first stage, the airline and the HSR operator decide whether to form the JV, thereby incurring the relevant sunk costs, or not. For both firms to join the alliance, either firm's profit under the JV

[24] GNEPs arise quite naturally from standard Nash equilibrium problems when the players share some scarce resource (Facchinei and Kanzow, 2007), such as a communication link or an electrical transmission line. GNEPs have been increasingly applied to traffic and transportation systems, where the common resource of constrained capacity can be respectively railroad tracks (Bassanini *et al.*, 2002) or urban roads (Zhou *et al.*, 2005).

agreement must be higher than the profit under the outside option of no agreement. Formally, firm $o$ ($o = r, a$) decides to form the JV as long as the following condition holds:

$$\pi_o(\boldsymbol{q^{JV}}) + \rho_o(\pi_{JV}(\boldsymbol{q^{JV}}) - F) > \pi_o(\boldsymbol{q^N}) \quad (14)$$

where $\pi_o(\boldsymbol{q^N})$, with $o = r, a$, are as defined in (6) and (7), $\rho_a = \rho$, $\rho_r = 1 - \rho$, and the JV is formed as long as (14) holds for both firms.

In what follows, we assume that $\gamma > \bar{\gamma} \cong 0.19$.[25] Proposition 4 examines whether and when condition (14) holds for both firms, depending on the sunk costs of the agreement and firms' bargaining power.

***Proposition 4.*** *Assume that the sunk costs of the agreement are sufficiently low, that is, $F$ is such that $0 < F \leq F_3$. Then, we have that:*

*(i) if the hub airport is not congested before the agreement, there exists a critical value $\tilde{\rho}$ ($0 < \tilde{\rho} < 1$) such that the JV is formed if and only if $\rho > \tilde{\rho}$, that is, as long as the airline's share of the JV entity's profit is high enough;*

*(ii) if the hub airport is congested before the agreement, there exist two critical values $\tilde{\rho}_1$ ($0 < \tilde{\rho}_1 < 1$) and $\tilde{\rho}_2$ ($0 < \tilde{\rho}_2 < 1$) such that the JV is formed if and only if $\tilde{\rho}_1 \leq \rho \leq \tilde{\rho}_2$, that is, as long as neither firm collects a large share of the JV entity's profit.*

Intuitively, if the hub airport is not congested before the agreement then the supply of the combined transportation service in market OD does not affect the HSR's profit in market OH. Therefore, the HSR has the incentive to cooperate as long as forming the JV increases his profit relative to the outside option.[26] This depends on the sunk costs of forming the JV (that must be sufficiently low), but is independent of the HSR's bargaining power in negotiating the agreement. On the other hand,

[25] This assumption is not essential for the results, and is made solely to simplify exposition. Indeed, when $\gamma < \bar{\gamma} \cong 0.19$, whether the airline decides to form the partnership or not depends on the specific value of the hub capacity: if the latter is sufficiently low, then $\nexists \rho > 0$ such that $\pi_a(\boldsymbol{q^{JV}}) + \rho(\pi_J(\boldsymbol{q^{JV}}) - F) > \pi_a(\boldsymbol{q^N})$, that is, we cannot find an equilibrium of the JV game such that the airline decides to form the partnership.

[26] To avoid confusion, we refer here to the HSR operator as 'he', to the airline as 'she', and to the JV as 'it'.

the airline has the incentive to cooperate as long as her share $\rho$ of the JV entity's profit is high enough to outweigh the profit loss due to stronger competition in market OD.

If, instead, the hub airport is congested before the agreement then the airline (partially) allocates the slots released at the hub after the agreement to market OH. The implication is that the HSR has the incentive to cooperate as long as his share $1-\rho$ of the JV entity's profit is high enough to cover the loss of profit in market OH.

### *5.3. Comparison of results with and without the JV agreement*

Let us now compare the results with and without the JV agreement, first in terms of traffic volumes in the transportation network and the degree of congestion at the hub airport, and then in terms of consumer surplus and social welfare.

#### *5.3.1. Passenger traffic and hub congestion*

As to the impact of the JV agreement on traffic volumes in the network and at the hub airport, qualitative results are the same as with a CP agreement (see remarks 4 and 5).

#### *5.3.2. Consumer surplus and social welfare*

We now study the effects of the 'supervised' JV agreement on consumer surplus and social welfare. Consider first consumer surplus $CS(\boldsymbol{q}^{JV})$ after the JV is formed. Proposition 5 shows that the JV improves consumer surplus relative to the case of no agreement. This is mainly due to the positive effect of supplying the airline-HSR service in the connecting market, where competition is stronger as market structure changes from a monopoly to a duopoly. Even if the airline tends to soften competition in market OD as it manages to reap a higher profit from her share in the JV entity, the latter does not take account of the effect of its decisions on the airline's profit.

***Proposition 5.*** *Consumer surplus is higher under the JV agreement than in the case where the airline and the HSR operator are pure competitors.*

Consider now social welfare $W(\boldsymbol{q}^{JV})$ after the JV is formed. Given that firms decide to form the JV (see Proposition 4), we have shown in Proposition 5 that this improves consumer surplus relative to the case of no agreement. It easily follows that social welfare is also higher with than without the JV agreement. Proposition 6 immediately follows from previous reasoning.

***Proposition 6.*** *Assume that the sunk costs of the agreement are sufficiently low, that is,* $F$ *is such that* $0 < F \leq F_4$*. Then, social welfare is higher with than without the JV agreement.*

Since $F_3 < F_4$ may hold, then firms may not have sufficient incentives to form the JV, despite it would be welfare improving. In this case, it may be appropriate to subsidize the agreement.

## 6. Comparison of intermodal agreements

In this section, we compare the CP agreement and the JV agreement as to their effects on consumer surplus and social welfare, depending on the basic model parameters (hub capacity, mode substitution, and sunk costs of cooperation). In doing so, we take the point of view of a public agency (i.e. the antitrust authority, or the regulator) whose concern is identifying the agreement that delivers the highest consumer surplus and/or social welfare, given that it is incentive-compatible for firms.

Let us first compare the consumer surplus under the alternative airline-HSR agreements. It directly follows from Proposition 2 and Proposition 5 that the JV agreement benefits consumers more often than the CP agreement. Compared to the benchmark case of competition, consumer surplus is always higher under the JV agreement, whereas it may be lower under the CP agreement. The rationale is that a CP agreement entails some coordinated pricing, since the airline sets the prices of both transportation products in the connecting market, whereas in a JV agreement price coordination is removed at all, since distinct companies respectively provide the connecting flight and the combined airline-HSR service. Furthermore, under the CP agreement the ticket price for the airline-HSR service may be high because of double marginalization (since the airline has to purchase a HSR seat in the wholesale market to sell the combined transportation service to passengers).

We can make a further step in comparing the agreements by checking whether and when consumer surplus may nonetheless be higher under the CP agreement than under the JV agreement.

Assume first that the hub airport is not capacity constrained after the JV agreement, namely, consider the constellations of parameters $(\alpha, \gamma, k, \rho, \mu)$ that are included in the regions $(\Omega_{JV,i} \cap \Omega_{CP,i})$ and $(\Omega_{JV,i} \cap \Omega_{CP,b})$. Proposition 7 shows that, in this framework, consumer surplus is lower under the CP agreement than under the JV agreement. Thus, under the CP agreement consumers are harmed by the negative effects of (some) price coordination and of double marginalization.

***Proposition 7.*** *Assume that the hub airport is not capacity constrained after the JV agreement. Then, the JV agreement improves consumer surplus compared to the CP agreement.*

Now assume that the hub airport is capacity constrained after the JV agreement, namely, consider the parameter regions $(\Omega_{JV,b} \cap \Omega_{CP,i})$ and $(\Omega_{JV,b} \cap \Omega_{CP,b})$. In this framework, due to the highly non-linear nature of the problem at hand, we are not able to compare analytically intermodal agreements in terms of their effects on consumer surplus. Nonetheless, we can show that, even when the hub is capacity constrained, the consumer surplus is lower under the CP agreement than under the JV agreement (as in Proposition 7) in the specific case where $\rho = \gamma$. We can also check the robustness of this finding in the relevant parameter regions through a number of numerical simulations where we set the basic parameters $\rho$ and $\mu$ to some specific values.[27] Simulations yield that the consumer surplus is lower under the CP agreement than under the JV agreement.

Based on these findings, we can conjecture that the JV agreement should be preferred to the CP agreement in the light of maximizing the benefits to consumers. However, transport operators incur higher sunk costs to create a JV than to sign a CP agreement. Thus, to complete our analysis, it is necessary to compare the intermodal agreements in terms of social welfare.

Assume, for the moment, that the sunk costs of the agreements are the same. Thus, if firms choose to cooperate then they should jointly incur the sunk cost $F$ $(F > 0)$ for either agreement to become effective. Consider the more interesting situation where both agreements are viable and improve

[27] Specifically, we set $(\mu, \rho)$ so that $(\mu, \rho) \in \left\{\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}\right\} \text{x} \left\{\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}\right\}$.

consumer surplus as well as social welfare compared to the benchmark case of competition. As to the CP agreement, from propositions 1 to 3 this requires that $F \leq F_1 < F_2$ holds. As to the JV agreement, from propositions 4 to 6 we assume that $F \leq F_4 < F_3$ holds. With some algebra we obtain that, in this situation, social welfare is higher under the JV agreement than under the CP agreement. Thus, given that the sunk costs are the same the JV agreement should be preferred, since it is very likely to improve consumer surplus and always improves social welfare relative to the CP agreement.

Consider now the more realistic situation where the sunk costs under the JV agreement $F_{JV}$ are higher than the sunk costs under the CP agreement $F_{CP}$, that is, $F_{JV} > F_{CP} > 0$. In what follows, we compare the level of social welfare under the alternative agreements, depending on the degree of substitutability $\gamma$ between transportation products and the sunk costs of the JV agreement (relative to those of the CP agreement), for different values of the hub capacity $k$. For the sake of tractability, we carry out a number of simulations by setting some parameters to specific values, that is, $\mu = 1/2\,, c_R = c_A = c = 0, F_{CP} = 1/100\,, \alpha = 1, \rho = \beta = 8/10$.

Assume first that the hub airport is not capacity constrained after both agreements, namely, consider the parameter region $(\Omega_{JV,i} \cap \Omega_{CP,i})$. Figure 3 depicts the cases where social welfare is higher under the JV agreement, in the light gray areas, or under the CP agreement, in the dark gray areas, for three different values of the hub capacity $k$ (i.e., $30/10, 35/10\,, 40/10$). Note that a greater value of $k$ enlarges the feasible region for comparisons, given that in the white areas we cannot find the equilibrium of either or both games.

=== *Insert Figure 3 about here* ===

As expected, the sunk costs of cooperation play an important role in determining whether the JV agreement improves or reduces social welfare compared to the CP agreement. If the sunk cost to create a JV is comparable to the sunk cost to sign a CP agreement, so that $F_{JV} - F_{CP}$ is sufficiently low, then social welfare is higher under the JV agreement, independent of the hub capacity and mode substitution. Conversely, whenever $F_{JV}$ (and thereby $F_{JV} - F_{CP}$) is sufficiently high, social welfare is higher under the CP agreement. This finding occurs more frequently as long as both the hub capacity and mode substitution increase.

Now assume that the hub airport is capacity constrained after both agreements, namely, consider the parameter region $(\Omega_{JV,b} \cap \Omega_{CP,b})$. Figure 4 depicts the cases where social welfare is higher under the JV agreement, in the light gray areas, or under the CP agreement, in the dark gray areas (in the white areas, we cannot find the equilibrium of either or both games), for three values of the hub capacity $k$ (i.e., $15/10, 24/10, 28/10$).

=== *Insert Figure 4 about here* ===

The main qualitative results are the same as when the hub is not capacity constrained. Indeed, if $F_{JV}$ (and thereby $F_{JV} - F_{CP}$) is sufficiently low, then social welfare is higher under the JV agreement, independent of the hub capacity and mode substitution. Conversely, if $F_{JV}$ (and so $F_{JV} - F_{CP}$) is sufficiently high, then social welfare is higher under the CP agreement. This occurs more frequently as long as both the hub capacity and mode substitution increase (recall that, under the CP agreement, if the hub is capacity constrained consumer surplus increases as mode substitution increases).

## 7. Concluding remarks

Air transport and HSR have begun to act not only as simple competitors, but also as complementary modes. In recent years, several airline-HSR agreements have been signed worldwide. Cooperation may lead airlines to improve efficiency in slot allocation (particularly if hub airports are congested) and HSR operators to increase their market shares. Policy makers, especially in Europe, encourage cooperation as a means to abate transaction costs and promote intermodality in passenger transport.

In this paper, we have studied the effects of airline-HSR agreements on traffic volumes in the transportation network and on the level of congestion at hub airports. Then, we have assessed whether airline-HSR agreements improve consumer surplus and social welfare, depending on hub congestion and on mode substitution between services. Finally, we have considered firms' incentives to sign such agreements, depending on the relevant sunk costs and their bargaining power.

In our model, the airline-HSR agreement serves to offer a new combined transportation service in the connecting market, which bundles domestic HSR and international air services via the multimodal hub. Firms have to incur jointly some sunk costs to make cooperation effective (e.g., to deploy intermodal infrastructure and/or coordinate schedules). Under a CP agreement, the HSR operator sells train seats to the airline, which is in charge of the combined transportation service. Under a JV agreement, transport operators form a JV that provides only the new service.

We have found that airline-HSR agreements do not necessarily reduce, but may increase congestion at hub airports. This is because of the increase in traffic volumes in the connecting market and in the whole network. Despite the airline substitutes some feeding flights for feeding HSR rides in the connecting market, it uses the capacity made available at the hub airport to meet new demand. In contrast to the prevailing view, passenger traffic may even increase in the overlapping market.

Different from airline-HSR integration, intermodal agreements generally benefit passengers, even when the hub airport is congested. An exception is the CP agreement when transportation products are very weak substitutes and the congested hub is of moderate size. While airline-HSR passengers may still benefit from the agreement, this is not enough to outweigh the harm to all the remaining passengers in the network. On the other hand, the JV agreement definitely improves consumer surplus since it avoids double marginalization, while allowing for greater competition in the connecting market compared to a merger.

In some cases, firms should be encouraged to cooperate at all. This occurs if an agreement of either type is not incentive-compatible for firms when it improves consumer surplus. More specifically, under hub congestion, JV agreements are more likely to occur between companies with similar market power, such as air transport and HSR incumbents, than when one company (e.g., the HSR operator) is a dominant firm and the other company (e.g., the airline) faces strong competition. Thus, a publicly owned airport might incur part of the sunk costs needed to deploy the intermodal infrastructure, especially if firms have different market power.

On the other hand, operators may sign a CP agreement when it harms consumers, and possibly social welfare. Thus, antitrust agencies should not approve the agreement. In this framework, we have defined a simple two-tier test to check whether such an agreement improves consumer surplus. First, the test controls that the total traffic volume in the overlapping market is higher with than without the agreement; second, if this is not the case, the test controls that the wholesale price for a HSR seat is lower than the HSR ticket price in the overlapping market. If, at either step, this test is satisfied then antitrust agencies should avoid costly investigations and approve the agreement.

Finally, in an extension of our analysis we have found that, in case of cooperation, public agencies should generally prefer the JV agreement, since it is very likely to improve both consumer surplus and social welfare compared to the CP agreement, unless the sunk costs to create the JV are far greater than the costs of signing the CP agreement.

For analytical tractability, we have abstracted away from economies of traffic density and from cost asymmetry between transportation modes. We expect that the positive welfare effects of airline-HSR agreements are stronger if there are economies of traffic density. Indeed, airline-HSR agreements increase traffic volumes in the connecting market as well as in the whole network, and sometimes in the overlapping market. Airline-HSR agreements may also have a stronger positive impact on welfare in the case where the HSR incurs a lower unit operating cost than the airline. Indeed, following the agreement the airline substitutes some feeding flights for feeding HSR rides in the connecting market.

Future work could investigate whether the results obtained are robust to considering more complex network structures, stronger competition in transportation products and different types of agreements. For instance, transport operators' incentives and the welfare effects of agreements may be different if one changes the network structure to consider an additional single-leg route served by the HSR in monopoly, so that the airline and the HSR are symmetric in serving network passengers. One could also consider two distinct airline-HSR services competing in the connecting market.[28] Indeed, foreign airlines may face severe constraints on domestic routes. Therefore, by means of airline-HSR cooperation, foreign airlines can significantly increase their presence in domestic markets.

Furthermore, one could consider the design of partnership loyalty programs with joint participation of the airline and the HSR. Customers can earn points based on their use of either mode, and redeem rewards at any participating firm. An important question to ensure incentive-compatibility and assess the impact of these programs on passengers' well-being is how firms share the costs of rewarding.

Finally, one could consider the role played by hub airports in facilitating airline-HSR agreements, by assessing airports' incentives to co-invest with transport operators to make cooperation effective.

**Acknowledgments**

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[28] For example, this is the case for the Strasbourg-Paris-Dubai market, where *TGVAir*, a partnership between SNCF and Emirates, and *AIR&RAIL*, a partnership between SNCF and Air France, are available to passengers at the same time.

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**List of Figures**


***Figure 1.*** Structure of the network.

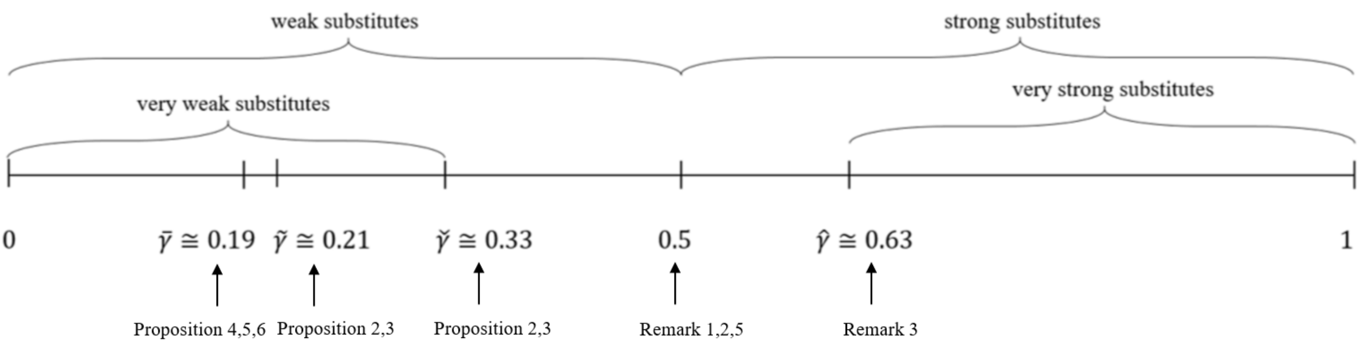
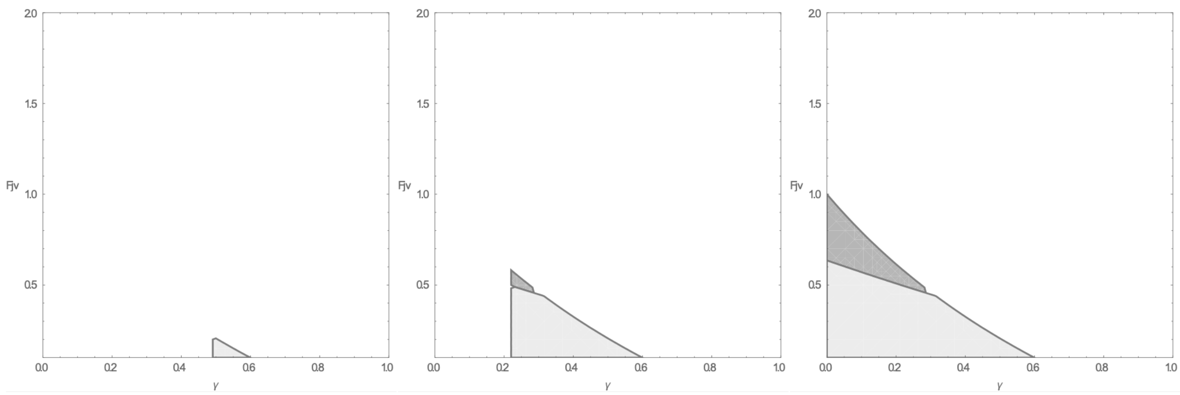
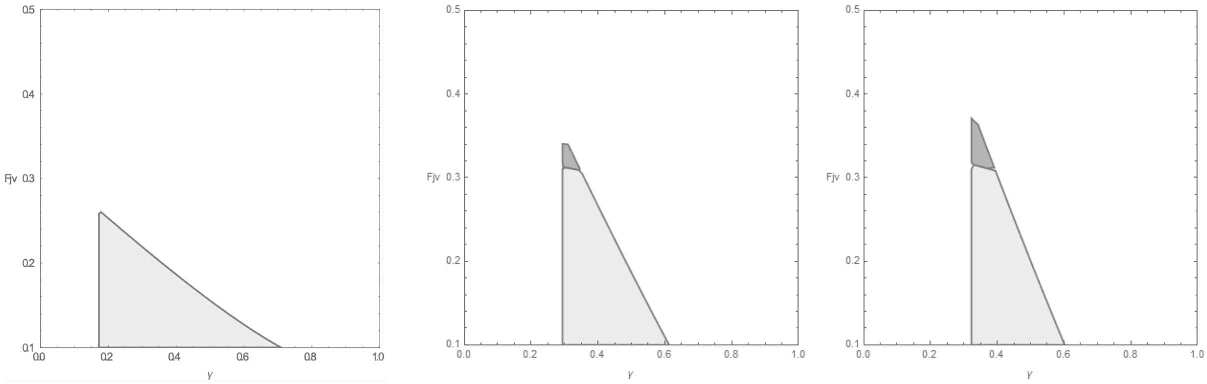



***Figure 2.*** A taxonomy of the degree of substitutability between air and HSR services.



$k = 30/10$ $\qquad$ $k = 35/10$ $\qquad$ $k = 40/10$

***Figure 3.*** Social welfare under the intermodal agreements when the hub is not capacity constrained.




$k = 15/10$ $k = 24/10$ $k = 28/10$

***Figure 4.*** Social welfare under the intermodal agreements when the hub is capacity constrained.

**List of Tables**

| | $\Omega_{CP,i} \cap \Omega_{N,i}$ | $\Omega_{CP,i} \cap \Omega_{N,b}$ | $\Omega_{CP,b} \cap \Omega_{N,b}$ | | $\Omega_{CP,b} \cap \Omega_{N,i}$ |
|---|---|---|---|---|---|
| | | | $\gamma < 0.5$ | $\gamma > 0.5$ | |
| $q_{OH}^{A,CP} - q_{OH}^{A,N}$, $q_{HD}^{A,CP} - q_{HD}^{A,N}$ | =0 | >0 | <0 | >0 | <0 |
| $q_{OH}^{R,CP} - q_{OH}^{R,N}$ | =0 | <0 | >0 | <0 | >0 |
| $q_{OH}^{A,CP} + q_{OH}^{R,CP} - q_{OH}^{A,N} - q_{OH}^{R,N}$ | =0 | >0 | <0 | >0 | <0 |
| $q_{OD}^{AA,CP} - q_{OD}^{AA,N}$ | <0 | <0 | <0 | <0 | <0 |
| $q_{OD}^{AA,CP} + q_{OD}^{AR,CP} - q_{OD}^{AA,N}$ | >0 | >0 | >0 | >0 | >0 |

**Table 1**. Effects of the CP agreement on quantities in relevant markets.

## Appendices

### *A.1 – No agreement*

Assume that firms do not sign an agreement. Let $\Omega_N = \{(\alpha, \gamma, k): \alpha > 0;\ 0 < \gamma < 1;\ k > 0\}$. Moreover, let $\widetilde{\Omega}_{N,i} = \{(\alpha, \gamma, k) \in \Omega_N: k > \frac{12\alpha+5\alpha\gamma}{4+2\gamma}\}$ be the subset of parameters for which the hub capacity constraint is not strictly binding in equilibrium (*interior* solution), and $\widetilde{\Omega}_{N,b} = \Omega_N \setminus \widetilde{\Omega}_{N,i} = \{(\alpha, \gamma, k) \in \Omega_N: k \leq \frac{12\alpha+5\alpha\gamma}{4+2\gamma}\}$ be the subset for which the constraint is strictly binding in equilibrium (*boundary* solution). The Karush-Kuhn-Tucker (KKT) optimality conditions yield the following solution to problem (6)-(7):[29]

$$q^N = \begin{pmatrix} q_{OH}^{A,N} \\ q_{HD}^{A,N} \\ q_{OD}^{AA,N} \\ q_{OH}^{R,N} \end{pmatrix} = \begin{cases} \begin{pmatrix} \dfrac{4k - 5\alpha\gamma}{24 - 5\gamma^2} \\ \dfrac{4k + \gamma(\alpha - \gamma k)}{24 - 5\gamma^2} \\ \dfrac{8k + 2\gamma(\alpha - \gamma k)}{24 - 5\gamma^2} \\ \dfrac{2(6\alpha - \gamma k)}{24 - 5\gamma^2} \end{pmatrix} & if\ k \leq \dfrac{12\alpha + 5\alpha\gamma}{4 + 2\gamma} \\ \begin{pmatrix} \dfrac{\alpha}{2 + \gamma} \\ \dfrac{\alpha}{2} \\ \alpha \\ \dfrac{\alpha}{2 + \gamma} \end{pmatrix} & if\ k > \dfrac{12\alpha + 5\alpha\gamma}{4 + 2\gamma} \end{cases}$$

We restrict attention to the constellations of parameters for which quantities are strictly positive, that is, to subsets $\Omega_{N,i} = \left\{(\alpha, \gamma, k) \in \widetilde{\Omega}_{N,i}: \boldsymbol{q^N} > 0\right\}$ and $\Omega_{N,b} = \left\{(\alpha, \gamma, k) \in \widetilde{\Omega}_{N,b}: \boldsymbol{q^N} > 0\right\}$ respectively.

### *A.2 – CP agreement*

Assume that firms sign a CP agreement to provide the airline-HSR service in market OD. Let $\Omega_{CP} = \{(\alpha, \gamma, k): \alpha > 0; 0 < \gamma < 1; k > 0\}$. The KKT optimality conditions yield the third-stage subgame equilibrium, $\boldsymbol{q^{CP}}(w)$, for problem (8)-(9). Let us denote by $\boldsymbol{q^{CP,i}}(w)$ and $\boldsymbol{q^{CP,b}}(w)$, respectively, the

[29] It is straightforward to prove that the decision problems solved in the appendices are strictly concave.

interior and the boundary solution in the subgame equilibrium at the third stage. In order to simplify the analysis in the case where $\boldsymbol{q}^{\boldsymbol{CP}} = \boldsymbol{q}^{\boldsymbol{CP},\boldsymbol{i}}(w)$, we restrict attention to the region of parameters $(\alpha, \gamma, k)$ where the capacity constraint is not active (i.e., $q_{OH}^{A,CP,i} + q_{HD}^{A,CP,i} + 2q_{OD}^{AA,CP,i} + q_{OD}^{AR,CP,i} < k$). Thus, the solution $\boldsymbol{q}^{\boldsymbol{CP}}(w)$ of problem (8)-(9) is as follows:

$$
\boldsymbol{q}^{\boldsymbol{CP}}(w)=\begin{pmatrix} q_{OH}^{A,CP}(w)\\ q_{HD}^{A,CP}(w)\\ q_{OD}^{AA,CP}(w)\\ q_{OD}^{AR,CP}(w)\\ q_{OH}^{R,CP}(w)\end{pmatrix}=\begin{cases}
\boldsymbol{q}^{\boldsymbol{CP,i}}(w)=\begin{pmatrix} \dfrac{\alpha}{2+\gamma}\\ \dfrac{\alpha}{2}\\ \dfrac{2\alpha-2\alpha\gamma+\gamma w}{2(1-\gamma^2)}\\ \dfrac{2\alpha-2\alpha\gamma-w}{2(1-\gamma^2)}\\ \dfrac{\alpha}{2+\gamma}\end{pmatrix} & k\geq 0\wedge\left(\left(0<\gamma<\frac{1}{2},\alpha>0,w\geq 0,w>\bar{w}\right)\vee\left(\gamma=\frac{1}{2},0<\alpha<\frac{10k}{29},w\geq 0\right)\vee\left(\frac{1}{2}<\gamma<1,\alpha>0,0\leq w<\bar{w}\right)\right)\\
\boldsymbol{q}^{\boldsymbol{CP,b}}(w)=\begin{pmatrix} \dfrac{\alpha\big(\gamma\big(-2+\gamma(4+\gamma)\big)-2\big)+2(2k+w)-4\gamma(\gamma k+w)}{2(28-16\gamma-14\gamma^2+4\gamma^3+\gamma^4)}\\ \dfrac{\alpha(\gamma(10+\gamma-4\gamma^2)-4)+(\gamma-2)(2+\gamma)(2(\gamma^2-1)k+(2\gamma-1)w)}{2(28-16\gamma-14\gamma^2+4\gamma^3+\gamma^4)}\\ \dfrac{\alpha(\gamma^3-8-4\gamma)+8w+(\gamma-2)(2(\gamma^2-4)k-\gamma(4+\gamma)w)}{2(28-16\gamma-14\gamma^2+4\gamma^3+\gamma^4)}\\ \dfrac{\alpha(24+(2-9\gamma)\gamma)+2(\gamma-2)(2+\gamma)(2\gamma-1)k+(5\gamma^2-24)w}{2(28-16\gamma-14\gamma^2+4\gamma^3+\gamma^4)}\\ \dfrac{\alpha(14-\gamma(7+6\gamma))+\gamma(2(\gamma^2-1)k+(2\gamma-1)w)}{(28-16\gamma-14\gamma^2+4\gamma^3+\gamma^4)}\end{pmatrix} & k>0\wedge\left(\left(0<\gamma<\frac{1}{2},\alpha>0,0\leq w<\bar{w}\right)\vee\left(\gamma=\frac{1}{2},\alpha>\frac{10k}{29},w\geq 0\right)\vee\left(\frac{1}{2}<\gamma<1,\alpha>0,w>\bar{w}\right)\right)
\end{cases}
$$

where $\overline{w} = (-16\alpha + 5\alpha\gamma + 10\alpha\gamma^2 + \alpha\gamma^3 + 4k + 2\gamma k - 4\gamma^2 k - 2\gamma^3 k)/(-2 + 3\gamma + 2\gamma^2)$.

At the second stage, solving problem (10), the HSR sets the wholesale price per train seat while anticipating that, depending on that price, the quantities supplied at the third stage may be constrained by the runway capacity at the hub airport. Thus, let $\widetilde{\Omega}_{CP,i} = \left\{(\alpha, \gamma, k) \in \Omega_{CP} : k > \frac{14\alpha+14\alpha\gamma+3\alpha\gamma^2}{4+6\gamma+2\gamma^2}\right\}$ and $\widetilde{\Omega}_{CP,b} = \left\{(\alpha, \gamma, k) \in \Omega_{CP} : 0 \leq k < \frac{720\alpha-168\alpha\gamma-584\alpha\gamma^2-5\alpha\gamma^3+82\alpha\gamma^4+10\alpha\gamma^5}{208+40\gamma-204\gamma^2-70\gamma^3+32\gamma^4+12\gamma^5}\right\}$. We obtain the following equilibrium values for the input charge:

$$w^{CP} = \begin{pmatrix} w^{CP,i} \\ w^{CP,b} \end{pmatrix}$$

$$= \begin{cases} \alpha - \alpha\gamma & if\left(\pi_R^i\left(\boldsymbol{q^{CP}}\left(w^{CP,i}\right)\right) > \pi_R^b\left(\boldsymbol{q^{CP}}\left(w^{CP,b}\right)\right) \wedge \widetilde{\Omega}_{CP,b} \wedge \widetilde{\Omega}_{CP,i}\right) \vee \left(!\,\widetilde{\Omega}_{CP,b} \wedge \widetilde{\Omega}_{CP,i}\right) \\ \frac{\alpha(\gamma(384+\gamma(480+\gamma(-180+\gamma(-110+\gamma(34+9\gamma)))))-672)}{2(-672+\gamma(384+\gamma(478+\gamma(-184+\gamma(-86+5\gamma(4+\gamma))))))} + \frac{2(2\gamma-1)(\gamma(64+\gamma(80+\gamma(-32+\gamma(-14+\gamma(4+\gamma)))))-112)k}{2(-672+\gamma(384+\gamma(478+\gamma(-184+\gamma(-86+5\gamma(4+\gamma))))))} & if\left(\pi_R^i\left(\boldsymbol{q^{CP}}\left(w^{CP,i}\right)\right) < \pi_R^b\left(\boldsymbol{q^{CP}}\left(w^{CP,b}\right)\right) \wedge \widetilde{\Omega}_{CP,b} \wedge \widetilde{\Omega}_{CP,i}\right) \vee \left(\widetilde{\Omega}_{CP,b} \wedge !\,\widetilde{\Omega}_{CP,i}\right) \end{cases}$$

We focus on the constellations of parameters for which quantities are strictly positive, that is, subsets $\Omega_{CP,i} = \left\{\left(\pi_R^i\left(\boldsymbol{q^{CP}}(w^{CP,i})\right) > \pi_R^b\left(\boldsymbol{q^{CP}}(w^{CP,b})\right) \wedge \widetilde{\Omega}_{CP,b} \wedge \widetilde{\Omega}_{CP,i}\right) \vee \left(!\,\widetilde{\Omega}_{CP,b} \wedge \widetilde{\Omega}_{CP,i}\right): \boldsymbol{q^{CP}} > 0\right\}$ and $\Omega_{CP,b} = \left\{\left(\pi_R^i\left(\boldsymbol{q^{CP}}(w^{CP,i})\right) < \pi_R^b\left(\boldsymbol{q^{CP}}(w^{CP,b})\right) \wedge \widetilde{\Omega}_{CP,b} \wedge \widetilde{\Omega}_{CP,i}\right) \vee \left(\widetilde{\Omega}_{CP,b} \wedge !\,\widetilde{\Omega}_{CP,i}\right): \boldsymbol{q^{CP}} > 0\right\}$, respectively.

***A.3 – JV agreement***

Assume that firms create a JV to provide the airline-HSR service in market OD. Since the JV entity and the airline share the same capacity constraint at the hub airport, then problem (11)-(13) gives rise to a GNEP (Facchinei and Kanzow, 2007). The KKT optimality conditions for problem (11)-(13) can be written as follows:

$$\begin{cases} \nabla_{q_{OH}^A}\left(\pi_a(\boldsymbol{q}) + \rho\,\pi_{JV}(\boldsymbol{q}) - \lambda_a(q_{OH}^A + q_{HD}^A + 2q_{OD}^{AA} + q_{OD}^{AR} - k)\right) = 0 \\ \nabla_{q_{OH}^R}\left(\pi_r(\boldsymbol{q}) + (1-\rho)\pi_{JV}(\boldsymbol{q}) - \lambda_{JV}(q_{OH}^A + q_{HD}^A + 2q_{OD}^{AA} + q_{OD}^{AR} - k)\right) = 0 \\ \nabla_{q_{OD}^{AR}}\pi_{JV}(\boldsymbol{q}) = 0 \\ \lambda_a(q_{OH}^A + q_{HD}^A + 2q_{OD}^{AA} + q_{OD}^{AR} - k) = 0 \\ \lambda_{JV}(q_{OH}^A + q_{HD}^A + 2q_{OD}^{AA} + q_{OD}^{AR} - k) = 0 \\ \lambda_a \geq 0, \lambda_{JV} \geq 0 \end{cases}$$

Note that, when the capacity constraint is binding, we have 7 unknowns and 6 equations. Therefore, problem (11)-(13) has $\infty^1$ Nash equilibria. For analytical tractability, we assume that $\lambda_{JV} = \mu\,\lambda_a$, with $0 < \mu \leq 1$. In doing so, we are postulating that: (i) the JV entity's benefit from a marginal increase in the hub capacity is proportional to the airline's benefit; and (ii) the airline benefits more than the JV entity from an increase in the hub capacity.

Let $\Omega_{JV} = \{(\alpha,\gamma,k,\rho,\mu): \alpha > 0; 0 < \gamma < 1; k > 0; 0 < \rho < 1; 0 < \mu \leq 1\}$. Moreover, let $\widetilde{\Omega}_{JV,i} = \{(\alpha,\gamma,k,\rho,\mu) \in \Omega_{JV}: k > \frac{\alpha(-64+4(-1+4\rho)\gamma+4(4+3\rho)\gamma^2+(1+\rho)\gamma^3)}{2(2+\gamma)(-4+(1+\rho)\gamma^2)}\}$ be the subset of parameters for which the hub capacity constraint is not strictly binding in equilibrium (*interior* solution), and $\widetilde{\Omega}_{JV,b} = \{(\alpha,\gamma,k,\rho,\mu) \in \Omega_{JV}: k \leq \frac{\alpha(-64+4(-1+4\rho)\gamma+4(4+3\rho)\gamma^2+(1+\rho)\gamma^3)}{2(2+\gamma)(-4+(1+\rho)\gamma^2)}\}$ be the subset for which the constraint is strictly binding in equilibrium (*boundary* solution). From the KKT optimality conditions, we obtain the solution $\boldsymbol{q}^{JV}$ to problem (11)-(13). We focus on the constellations of parameters for which quantities are strictly positive, that is, subsets $\Omega_{JV,i} = \left\{(\alpha, \gamma, k, \rho, \mu) \in \widetilde{\Omega}_{JV,i} \colon \boldsymbol{q}^{JV} > 0\right\}$ and $\Omega_{JV,b} = \left\{(\alpha, \gamma, k, \rho, \mu) \in \widetilde{\Omega}_{JV,b} \colon \boldsymbol{q}^{JV} > 0\right\}$ respectively.

$$
\boldsymbol{q^{JV}} = \begin{pmatrix} q_{OH}^{A,JV} \\ q_{HD}^{A,JV} \\ q_{OD}^{AA,JV} \\ q_{OD}^{AR,JV} \\ q_{OH}^{R,JV} \end{pmatrix}
= \begin{cases}
\begin{pmatrix} \dfrac{\alpha}{2+\gamma} \\ \dfrac{\gamma}{2} \\ \dfrac{2(2\alpha-\alpha\gamma-\alpha\rho\gamma)}{4-\gamma^2-\rho\gamma^2} \\ \dfrac{2(2\alpha-\alpha\gamma)}{4-\gamma^2-\rho\gamma^2} \\ \dfrac{\alpha}{2+\gamma} \end{pmatrix} & if\ \ k > \dfrac{\alpha(-64+4(-1+4\rho)\gamma+4(4+3\rho)\gamma^2+(1+\rho)\gamma^3)}{2(2+\gamma)(-4+(1+\rho)\gamma^2)} \\
\begin{pmatrix} \dfrac{-4(-4+(1+\rho)\gamma^2)k+\alpha(-16-4\gamma+16\rho\gamma+4\gamma^2+\gamma^3+\rho\gamma^3+4(-2+\gamma)(-1+\gamma+\rho\gamma)\mu)}{(1+\rho)\gamma^4+16(6+\mu)-4\gamma^2(7+2\rho+\mu)-16\gamma(1+\mu+\rho\mu)+4\gamma^3(1+\mu+\rho\mu)} \\ \dfrac{(\gamma^2-4)((1+\rho)\gamma^2-4)k+\alpha(8(\mu-2)-2\gamma^2(\mu-2)+(1+\rho)\gamma^3(2\mu-5)+4\gamma(5+4\rho-2(1+\rho)\mu))}{(1+\rho)\gamma^4+16(6+\mu)-4\gamma^2(7+2\rho+\mu)-16\gamma(1+\mu+\rho\mu)+4\gamma^3(1+\mu+\rho\mu)} \\ \dfrac{2\alpha(\gamma^2(4+\gamma)+\rho\gamma(\gamma^2-8)-4\gamma)-16-\alpha(\gamma-2)(8+8\gamma+\gamma^2+\rho\gamma(4+\gamma))\mu+2(\gamma^2-4)k((1+\rho)\gamma\mu-4)}{(1+\rho)\gamma^4+16(6+\mu)-4\gamma^2(7+2\rho+\mu)-16\gamma(1+\mu+\rho\mu)+4\gamma^3(1+\mu+\rho\mu)} \\ \dfrac{2(-2+\gamma)(-12\alpha(2+\gamma)+2(2+g)k(\gamma-\mu)+\alpha(12+5\gamma)\mu)}{(1+\rho)\gamma^4+16(6+\mu)-4\gamma^2(7+2\rho+\mu)-16\gamma(1+\mu+\rho\mu)+4\gamma^3(1+\mu+\rho\mu)} \\ \dfrac{2(-4+(1+\rho)\gamma^2)(-6\alpha+\gamma k)+4\alpha(-2+\gamma)(-1+\gamma+\rho\gamma)\mu}{(1+\rho)\gamma^4+16(6+\mu)-4\gamma^2(7+2\rho+\mu)-16\gamma(1+\mu+\rho\mu)+4\gamma^3(1+\mu+\rho\mu)} \end{pmatrix} & otherwise
\end{cases}
$$

***A.4 – Thresholds***

$$k_1 = \frac{\alpha\gamma(24 + 98\gamma - 97\gamma^2 - 68\gamma^3 + 34\gamma^4 + 9\gamma^5)}{-208 - 264\gamma + 328\gamma^2 - 30\gamma^3 - 62\gamma^4 + 10\gamma^5 + 4\gamma^6 - 432/(1+\gamma)}$$

$$k_2 = \frac{\alpha(387072 - 285696\gamma - 274176\gamma^2 + 149952\gamma^3 + 149088\gamma^4 - 65320\gamma^5 - 64180\gamma^6 + 21460\gamma^7 + 10910\gamma^8 - 2250\gamma^9 - 625\gamma^{10})}{2(193536 - 497664\gamma - 23424\gamma^2 + 601344\gamma^3 - 110912\gamma^4 - 266496\gamma^5 + 56124\gamma^6 + 52824\gamma^7 - 9470\gamma^8 - 4720\gamma^9 + 525\gamma^{10} + 150\gamma^{11})}$$

The expression of $k_3$ is available from the authors upon request.