Nonlinear Dynamics and Control in Macro/Micro-Mechanics: Some Computational Issues

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Abstract

Computational issues in the global dynamics of two systems in micro- and macro-mechanics, with different dimensionality, are addressed. Attention is focused on calculation of integrity measures, determination of saddle manifolds undergoing global bifurcations, implementation of a control procedure for delaying basins erosion, selection of 2D cross-sections of multidimensional basins of attraction for understanding the role of transient dynamics in the global scenario of coupled steady responses.

Keywords: global dynamics, invariant manifolds, basins of attraction, control, atomic force microscopy, thermoelastic coupling

1. Introduction

Nonlinear dynamics of systems/structures undergoing finite amplitude vibrations is a mature research area. Most of the advancements occurred in the last two decades within the engineering community have involved a variety of computational issues concerned with both the detection of local and global bifurcations governing the overall response scenario and the reliable characterization of complex dynamics. In view of further advancements, two main aspects are worth of mention, among others [1]: (i) the fundamental role played by global dynamics for a comprehensive description and understanding of the response scenario, with also important impacts on the evaluation of actual system safety; (ii) the on-going transition from the analysis of relatively simple models allowing deep understanding of a huge variety of complex phenomena, to the richer ones needed to reliably describe the nonlinear dynamic behaviour of actual multi- or infinite-dimensional systems, also possibly embedded in a multiphysics environment. This paper deals with some computational issues as occurring in the global nonlinear dynamics of two sample models in mechanics of different dimensionality.

2. Computational issues in global nonlinear dynamics

Basins of attraction of a nonlinear model of multi-well potential systems consist of the coexisting sets of initial conditions in the phase plane, whose ensuing trajectories end up to competing steady responses. Under variation of a meaningful control parameter (typically, the excitation amplitude or frequency), basins topological features are meaningfully modified, with possibly fractal behaviours entailing mutual erosion of competing basins and final settling of the whole dynamics onto a possibly undesired ‘escape’ solution, whose physical meaning depends on the system under consideration. Safety with respect to escape has to be evaluated in terms of system non-residual dynamical integrity [1], i.e. of the extent/compactness of the basin of a wanted response such to guarantee its adequate robustness with respect to the non-infinitesimal variations of initial conditions representative of the unavoidable disturbances/imperfections occurring in operating conditions. Within this overall framework, a number of operational steps can be identified, with the associated computational aspects.

(i) Selecting and comparatively calculating integrity measures reliably accounting for the evolution of fractal features of basins of attraction with the varying control parameter.

(ii) In the common case of a sharp, i.e. dangerous, fall down of the ensuing integrity profile, identifying the saddle, among the many possibly exhibited by the system in its complex evolution, whose invariant manifolds tangency triggers the fractal dynamics later on entailing the sharp erosion ending up to escape.

(iii) Implementing and optimizing a control procedure of the homo/heteroclinic intersection of the saddle(s) stable and unstable manifolds able to shift it towards higher values of the varying control parameter, with also possible beneficial effects in terms of delay of the sharp basin erosion and ensuing preservation of an acceptable residual integrity.

All of these items involve challenging computational aspects already for the single-degree-of-freedom nonlinear models often referred to for unveiling the basic response features of the underlying discrete systems or discretized structures. Yet, when moving to the higher-dimensional models to be used in the presence of internal resonances or of anyway coupled variables, further challenging issues come into play.

(iv) Constructing basins of attraction and reliably representing/ describing the multidimensional scenario of nonlinear dynamic response via properly selected 2D cross-sections.

(v) Generally pursuing global analyses and integrity evaluations in the actual multidimensional phase space, which is only possible through systematic and effective implementation of parallel computing techniques [2].

In the following, some of the mentioned issues are exemplarily illustrated with reference to two models of different dimensionality, yet suitable to catch some main aspects of the nonlinear dynamics of systems in micro- and macro-mechanics.

2.1. Global dynamics and control of an AFM model

A nonlinear reduced order model describing the single-mode finite dynamics of a noncontact Atomic Force Microscope (AFM) is given by

$$\ddot{x} + \alpha_1 x + \alpha_3 x^3 = -\Gamma_1 \big/ (1+x)^2 - \rho_1 \dot{x} + x \mu_1 U \omega^2 \sin(\omega t) \qquad (1)$$

where x is the microcantilever tip transverse displacement, Γ_1 the atomic interaction coefficient, and U, ω the amplitude and frequency of the parametric harmonic excitation.

Noncontact AFMs used for surface analysis in mechanics of materials and biotechnology must work in a regime of attractive interaction potential between the microcantilever tip and the sample such to avoid instability of the equilibrium configuration and the ensuing ‘jump to contact’, which is the physical manifestation of the unwanted dynamical ‘escape’.

Highly refined numerical investigations of the topological evolution of a number of saddles’ manifolds with increasing excitation amplitude [3] have allowed to identify in the phase plane of Fig. 1 the tangency between stable (black line) and unstable (red line) left manifolds of the in-well saddle S_1 as the homoclinic bifurcation responsible for penetration of tongues of the escape basin inside the potential well, with separation of the two competing basins and ensuing strong reduction of integrity of the safe basin encompassing both of them, as highlighted by the drop of the black curves of two different integrity measures (GIM and IF) in the erosion profile of Fig. 2.

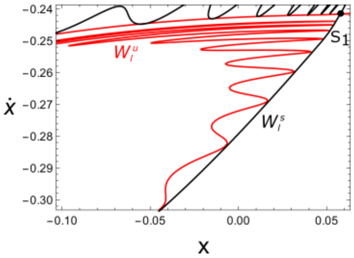

Figure 1: Manifolds tangency of a homoclinic bifurcation

Properly controlling the involved global bifurcations through the addition of a numerically optimized superharmonic component to the harmonic one allows to shift the fall down of the profiles to higher excitation amplitudes (red curves in Fig. 2), thus entailing a safer system dynamics.

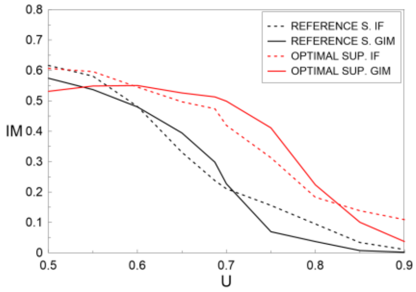

Figure 2: Integrity profiles without and with control

2.2. *Global dynamics of a thermoelastic plate*

A three-mode nonlinear reduced order model describing the thermomechanically coupled dynamics of a von Karman shear-indeformable laminate plate is given by

$$\begin{aligned}
&\ddot{W} + a_{12}\dot{W} + a_{13}W + a_{14}W^3 + a_{15}T_{R1} + a_{16}T_{R0}W + a_{17}\cos(t) = 0 \\
&\dot{T}_{R0} + a_{22}T_{R0} + a_{23}\alpha_1 T_\infty + a_{24}W\dot{W} = 0 \qquad (2) \\
&\dot{T}_{R1} + a_{32}T_{R1} + a_{33}\dot{W} = 0
\end{aligned}$$

where W is the plate mid-point transverse displacement, T_{R0} and T_{R1} are plate membrane and bending temperatures, T_∞ is the difference between reference and environmental temperatures.

Direct activation of the membrane thermal variable T_{R0} via a non-vanishing T_∞ value contributes to the system pretension via a parametrically varying increase of the linear mechanical stiffness, which is responsible for triggering buckling. However, the T_{R0} variable reaches its steady value very slowly. The meaningful role played by the lengthy transient thermal dynamics in the global scenario of system coupled steady response can be solely caught by jointly looking at properly selected cross-sections of the 4D basins of attraction. A partial, yet somehow enlightening, picture (Fig. 3) is provided by either one of the two $(T_{R0}, \dot{W})$ cross sections of the 4D basin obtained for trivial T_{R1} initial condition (i.c.) and $W(0)$ corresponding to the two buckled attractors, newly born with very small basins, at a non-null, yet nearly vanishing, value of the membrane temperature i.c. Below this value, the coupled cross-well response is monostable (leftmost gray zone in Fig. 3) since the axial load embedded in the linear mechanical stiffness is slightly lower than the value giving rise to buckled periodic solutions, and the gap between initial and steady value to be covered by the T_{R0} variable is too large; this can be verified by looking at the purely mechanical, and indeed totally gray, cross-section $(W, \dot{W})$ of the 4D basin for $T_{R0}(0) = T_{R1}(0) = 0.0$, here not reported. As the membrane temperature i.c. is slightly increased, a multistable behaviour occurs, with a pair of buckled solutions adding to the pre-buckling cross-well response. In Fig. 3, the blue basin of the considered buckled solution meaningfully expands along the velocity coordinate up to reaching its maximum extent as the membrane temperature i.c. coincides with the T_{R0} final steady value (≈ 0.9 in Fig. 3); correspondingly, it becomes the dominant compact basin over half of the (not shown) $(W, \dot{W})$ cross-section, where it competes with the basin of the dual buckled solution.

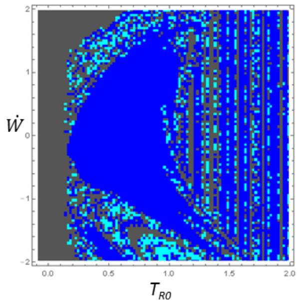

Figure 3: 2D cross-section of a 4D basin of attraction

References

[1] Rega, G. and Lenci, S., A global dynamics perspective for system safety from macro- to nanomechanics: Analysis, control and design engineering, *Appl. Mech. Rev.*, 67(5), 050802, 2015.

[2] Belardinelli, P. and Lenci, S., A first parallel programming approach in basins of attraction computation, *Int. J. Non-Lin. Mech.*, 80, pp. 76-81, 2016.

[3] Settimi, S. and Rega, G., Exploiting global dynamics of a noncontact atomic force microcantilever to enhance its dynamical robustness via numerical control, *Int. J. Bifurcation Chaos*, 26(7), 1630018, 2016.

[3] Saetta, E., Settimi, V. and Rega, G., Nonlinear vibrations of symmetric cross-ply laminates via thermomechanically coupled reduced order models, *Procedia Engineering, X International Conference on Structural Dynamics*, EURODYN 2017 (to appear).