SAPIENZA
UNIVERSITÀ DI ROMA

DOCTORAL SCHOOL OF ECONOMICS - DSE
SAPIENZA UNIVERSITY OF ROME

PHD DISSERTATION IN ECONOMICS AND FINANCE

The analysis of taxpayers behaviour and the shadow economy: theoretical approach and empirical evidence for European countries

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Academic Year 2017-2018

Acknowledgements

This dissertation would not have been possible without the contributions of many people who provided me with support throughout my graduate studies. I'm grateful to those who enabled me to move ahead and motivated me to accomplish this project.

First of all, I would like to express my great thanks to my Emy, who has taught me the love of learning. Thank you for your providing endless encouragement, especially when I'm down. Thank you for your patience, for sharing my dream, and for taking care of me during my journey.

I would like to express my deep thanks to my supervisor, Prof. Marco Valente from Department of Economics at University of L'Aquila, who has guided me to complete this dissertation. I have shared many ideas with him along the way.

I also wish to thank my tutor - member of the PhD board in Economics and Finance - Prof. Emma Galli, for all her helpful feedback and encouragement over the development of this dissertation.

I extend a special thanks to Prof. Roberto Dell'Anno from Department of Economics and Statistics at University of Salerno, who has provided me with great help in understanding the MIMIC approach.

I owe the biggest debt to my sponsor, the Sapienza University, with Prof. Rita Laura D'Ecclesia as person in charge, for providing me a scholarship to get the PhD degree in Economics and Finance. It would have been next to impossible to complete this project without its assistance. My deep thanks are also to Prof. Paolo Paesani and Dr. Maria Elena Perretti, who believed in me at the beginning of the "tale".

I would like to offer my sincerest thanks to my family, my father Laviero, my mother Maria and my brother Walter, and all friends - especially those of the XXX cycle as Alessandro, Leonardo, Marco, Paolo, Salim, Stefano F., Stefano V. and Vincenzo - who have supported me throughout of my academic journey and who have always provided me endless encouragement and motivated me to accomplish this work.

Last but not least, I'm also very grateful to all my colleagues at Cassa depositi e prestiti, for all moments that we have had to discuss, even if we have some differences and don't always agree, I want to thank you for being an open-minded people. I'm lucky to work with all of you.

GRAZIE!

Contents

Summary

In the last decades, the consequences of tax evasion and shadow economy have been evident, indeed the wealth is hidden in the black market and it is translated into missed revenues for central government.

Sometimes, the new type of approach is necessary to understand different phenomena and the reduction to single equation or formula is not sufficient to describe a single behaviour, of both consumer and producer.

As the recent Nobel Prize taught, to achieve a satisfying analysis, we need to approach to new experimental method, based on the expectations of audit and on the interactions among agents, analysed through pure simulations. Taking at the same time two different views and combining them, we should offer a new possibility of interaction among quantitative and qualitative methods, underlying the importance of both in a complete and exhaustive analysis.

This work, through some different chapters, attempts to debate the tax evasion and the underground economy in a wide perspective and from different points of view.

The second chapter, regards a collection of surveys of both tax evasion and shadow economy, gathering different models, aspects and solutions to these problems. In this part we analyse the problem of tax evasion and the shadow economy through different methodological approach. Secondly, we focus on Agent-based Model we are going to use in the following part, looking deepen to interactions among agents and creation of network and influences among them. Subsequently, analysing MIMIC model we will see the pros and cons of this model and how it investigates the problem of shadow economy in European countries. One of the most important problem about this topic is missing data because of underground economy's intrinsic nature is unobservable and we need some approximations to evaluate it.

The third part concerns the Agent-based Model and its application with different instruments. From one hand, we use an analytical model derived from Hokamp and Pickhardt (2010), whereas from another hand we use a qualitative model developed by *Lsd program* of Valente (2008) to analyse the same problem from different points of view and capturing different aspects.

The fourth chapter analyses the problem of shadow economy using a MIMIC Model developed by Dell'Anno (2007), compared to the Schneider (2015) estimation and evaluating the different results between two types of modelling.

Chapter 1

Introduction

The shadow economy is a broad concept. It has different names in the economic literature; underground, hidden, black, informal, unofficial, unreported, or unrecorded economy. It relates to the economic activities that are not included in the formal measurement of GDP. Increasing attention has been paid by many developed and developing countries in recent years to the shadow economy and its consequences. A recent study by Schneider (2015) overall the European countries between 2003 and 2014, found that the shadow economy has reached remarkable proportions, with an average value of 17.5% of official GDP of those countries. Almost all studies of the shadow economy phenomena find that the main cause of this phenomenon is a high tax and a high self employment rate.

The existence of the shadow economy creates a distortion in the market competition as a result of the unequal production situations between the producers in the formal economy compared to the producers in the shadow economy. This leads to significant distortions in official economic and social indicators. Most economists agree that there is a strong bidirectional causal relationship between the tax system and the size of shadow economy. The shadow economy reduces government revenues due to tax evasion, which, in turn, reduces the quality and quantity of public goods and services provided by governments. In order to cover its overall need for tax revenue, the government may raise tax rates.

The result is often an increase in the size of the shadow economy and more tax evasion. This can increase the distortions in the economy even more and can have serious distributional consequences. Furthermore, the shadow economy may be associated with underestimates of the true economic growth rate and the national income, and may overestimate the true unemployment rate. Policy makers will tend to base their decisions on inaccurate information, reducing the efficiency of public policy. For example, effective monetary and fiscal policy require a level of precision in the estimates of key statistics (such as: income, consumption, unemployment, etc.), and the existence of the shadow economy can distort these measures. Therefore, efforts should be made to estimate the size of the shadow economy.

1.1 Difficulties to estimate of the shadow economy

There is no unique definition for the shadow economy. According to Feige (1979), it "consists of those economic activities and the income derived from them that circumvent or otherwise elude government regulation, taxation, or observation".

Smith (1997) presents two alternative definitions of the shadow economy ranging from a narrow definition: "market based production of legal goods and services that escapes detection in the official estimates of GDP", to a broad definition; "market-and non-market-based production of goods and services, whether legal or illegal, that escapes detection in or is intentionally excluded from the official estimates of GDP".

While Schneider and Enste (2000) defines it as “all economic activities that contribute to value added and should be included in national income in terms of national accounting conventions, but are presently not registered by national measurement agencies”. Enste and Schneider (2002), concentrate in their definition on the “legal value-added creating activities which are not taxed or registered and where the largest part can be classified as “black” or clandestine labour”.

Due to the various definitions of the shadow economy and due to its unobserved and hidden nature, it is difficult to get accurate estimates for its size in the economy. Hence, as Schneider and Enste (2000) affirms; “doing research in this area can be considered a scientific passion for knowing the unknown”.

The following table, structured by Lippert and Walker (1997), shows the various classifications of shadow economic activities according to monetary and legal status of the activity.

Type of Activity	Monetary Transactions		Nonmonetary Transactions	
Illegal Activities	Trade in stolen goods; drug dealing and manufacturing; prostitution; gambling; smuggling; fraud.		Barter of drugs, stolen, or smuggled goods. Producing or growing drugs for own use. Theft for own use.	
	Tax Evasion	**Tax Avoidance**	**Tax Evasion**	**Tax Avoidance**
Legal Activities	Unreported income from self-employment. Wages, salaries, and assets from unreported work related to legal services and goods	Employee discounts fringe benefits.	Barter of legal services and goods.	All do-it-yourself work and neighbor help.

Figure 1.1: Types of shadow economy activities, Lippert and Walker, 1997.

Because it is impossible to estimate all aspects of the shadow economy (legal and illegal) due to the lack of information required for such an estimate, we will concentrate in this dissertation on the legal activities of the shadow economy that mainly relate to tax evasion and that can be estimated indirectly by using some macroeconomic indicators. Therefore, for the purpose of this study, we define the shadow economy as ‘all legal economic activities with monetary transactions that are subject to tax but are not reported to the tax authorities and to the statistical institutions in order to avoid the government regulations and evade paying tax on the sales and income result from this type of activities”. We will not take into consideration illegal activities that relate to criminal actions like stealing, robbery, drug dealing, etc.

We will also exclude the household economic activities which consist of all household services and production that are not sold in the market. The employee discounts and fringe benefits that are legally tax avoided will be excluded as well. By adopting this definition, we take into consideration, at least, the lower bound of the shadow economy.

1.1.1 Statement of the problem

The presence of shadow economy causes many difficulties in designing and conducting economic and social policies by the decision makers as those policies will depend on inaccurate economic indicators. Therefore, it is very important to estimate the size of the shadow economy in order to reduce the distortion in those policies.

Despite the growing interest in estimating the size of shadow economic activities in many countries, no attempt has been made to estimate the shadow economy, its consequences, and its policy implications particularly for the European economy.

The main statement of the problem in this dissertation is to estimate the annual size of the shadow economy in Europe during the period 1993-2015, in addition to analysing the economic consequences and the policy implications of this phenomenon, concentrating also on estimating the amount of the tax evasion, using a MIMIC model developed by Dell'Anno (2007), with an innovative two-step procedure based on a two different equation: the *measurement* and *structural* one.

1.2 The motivation behind choosing taxpayers behaviour

From the other hand, we analyse the tax evasion because it is strictly correlated with shadow economy, rather it is a predominant part of this phenomenon. In particular, we have chosen to study a new stream of taxpayers, based on their behaviours.

The chosen topic is motivated by the growing importance of tax compliance behaviour, especially when governments dealing with harsh budgetary cuts face the challenge of managing even less public funds collected through taxation. In order to achieve a desired level of compliance, authorities need to consider various factors which influence compliance and the way taxpayers answer to fiscal policy changes.

Taxation is a topic that almost every time generates controversies, exuberance and passions which often divide followers of the same political views and which, sometimes, bring closer partisans of very different economic doctrines. Nevertheless, modern society cannot survive without tax levies and especially without paying them, for as Franklin D. Roosevelt said, "taxes are the price we pay for civilized society".

The main categories of tax levies are income tax, ordinary tax and contribution. The difference between ordinary tax and income tax comes from the fact that, in the case of ordinary tax, there is an immediate benefit received from state authorities, whereas paying income tax does not grant one any immediate benefit. The difference between contribution and income tax is that the destination of the contribution is established right before it is even collected, but the income tax purpose is not known from the beginning. Income tax is distributed after the collecting process.

Among all tax levies, monetary income tax represents the oldest element in finance.

In addition to its main goal of raising financial resources for the state budget in order to finance public expenditures, income tax also influences the demand of goods and services. In other words, it has the role of economic leverage in stimulating or discouraging consumption of certain types of goods and services.

Depending on the criteria considered, income taxes can be: direct/indirect; real (objective)/personal (subjective); general consumption taxes /special consumption taxes; on income/wealth/consumption; central/local. As for collecting methods, there are fixe amount tax rates and percentage tax rates. The second category contains proportional, progressive and regressive tax rates. Fiscal policy is the instrument through which resources are collected for the state budget and expenditures are adjusted in order to monitor and influence national economy.

The main goals of the fiscal policy are: financing the state budget with resources necessary for accomplishing state objectives; adjusting the economy and boost economic growth.

As a main component of the fiscal system, tax authorities monitor taxpayers' economic activities, and their main goal is collecting tax levies with maximum efficiency (i.e., the lowest costs). The goal can be attained by creating a cooperative environment where all taxpayers comply with the tax law.

The taxpayer is the element within the fiscal system on which the fiscal policy and its changes are reflected. Either he is ordinary or professional, the taxpayer is obliged by law to transfer to the state budget a part of the value he creates through an activity.

When a taxpayer considers tax levies within a fiscal policy, two types of behaviour can emerge: compliance and non-compliance. According to Franzoni (2000), taxpayers have to meet simultaneously four conditions in order to say they are entirely compliant:

1. true reporting of the tax base;
2. correct computation of the liability;
3. timely filing of the return;
4. timely payment of the amounts due.

Any deviations from the aforementioned features results in non-compliance behaviour.

1.2.1 Statement of the problem

The factors which influence taxpayers' fiscal behavior have different nature:

- *economic:* audit probability, fines, tax rates, income;
- *political-legislative:* the complexity of fiscal legislation, instability of regulations, excessive regulation;
- *socio-psychological:* attitudes, norms, perceptions, motivational postures, trust in authorities, power of authorities;
- *tax morale:* intrinsic motivation of paying tax levies;
- *neurological:* physical states corresponding to emotions, hormones like oxytocin, serotonin.

The analysis of any type of economic behaviour cannot start without taking into consideration the theoretical model of *homo oeconomicus*, the prototype of the rational economic agent proposed in 1759 by Adam Smith in his work *Theory of Moral Sentiments*. Tax compliance models tackled in the present dissertation are economic and behavioural.

Economic models have as a starting point the classical model of income tax evasion developed by Allingham and Sandmo (1972), which assigns taxpayers the hypothesis of perfect rationality and states that, in order to increase tax compliance, the penalty should be applied to the amount of undeclared income. The shortcomings of the classical model were solved with Yitzhaki (1974) recommendation of applying the penalty to the evaded taxes. Nevertheless,Yitzhaki (1974) intervention generated unexpected results (i.e., compliance level increases along with increasing tax rate), contradictory with empirical results or economic reality.

Developed after extensive empirical studies and built on political and socio-psychological determinants of tax compliance behaviour (i.e., attitudes, norms, beliefs, sentiments, social and cultural features), behavioural models come to complete the portrait of the taxpayer who, until their appearance, was perceived exclusively through the feature of perfect rationality.

Finally, the necessity of a trust relationship between taxpayers and tax authorities, therefore creating a synergistic climate, has a practical side: implementing a system entirely based on enforcing taxpayers to pay liabilities is extremely expensive and cannot succeed unless "there is a tax agent under the bed of each taxpayer" Torgler (2005). In addition, many studies have shown the limits of fiscal systems based solely on enforcement (i.e. Frey and Feld (2002)). The shortcomings of implementing such systems are discussed also by Slemrod (1992) who states: "From the tax collection standpoint, it is extraordinarily expensive to arrange an enforcement regime so that, from a strict cost-benefit calculus, non-compliance does not appear attractive to many citizens. It follows that methods that reinforce and encourage taxpayers' devotion to their responsibilities as citizens play an important role in the tax collection process".

According to this purpose, we try to analyse a taxpayers behaviour, using a simulation improved by Valente (2008), to study the behaviour of taxpayers (and the peers) in front of changes of structure of taxes and the punishments in the case of non-compliance. Furthermore, we analyse also how it is reflected into economic system, analysing this research topic from analytical point of view, deriving from Hokamp and Pickhardt (2010).

1.3 Questions of the study

In this dissertation, we will investigate the following questions:

1. What are the consequences and policy implications if the Government approves a stronger punishment on tax evasion?

   We analyse the behavioural impact of the swing in tax policy from Government to try finding the tax evasion.

2. What are the behaviours of agents in front of the punishment change?

   We will use both "pure" Agent-based Model and analytical Agent-based model started by Hokamp and Pickhardt (2010) and modified it. In the first case we are going to use a simulation in economic environment with peers and the introduction (swinging in tax policies) of stronger punishment (i.e. the jail); on the other hand, we analyse the probabilities to evade in the case of stronger punishment inserting, to do that we modified the Hokamp and Pickhardt (2010) original model.

3. What is the size of the shadow economy in European countries?

   We will use MIMIC approach (using two steps procedure) to estimate the annual size of the shadow economy in European countries as percent of GDP during the period 1993-2015.

4. What are the main determinants of the shadow economy in Europe?

   We will check other possible causes of the shadow economy in Europe, including the degree of regulation in the economy, the unemployment rate and self-employment rate.

5. What are the consequences and the policy implications of the existence of the shadow economy in the European economy?

   We will analyse the macroeconomic impacts of the existence of the shadow on various sides of European economy, particularly: the consequences of financial crisis, the national accounts and fiscal policy. Furthermore, we expect a counter-cyclical relationship, because in recession some determinants increase their value and the underground economy increases its value as well.

Chapter 2

A survey of literature on tax evasion and the shadow economy[1]

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[1]This research chapter was developed within the framework of the PhD programme in Economics and Finance of Sapienza University.

Abstract

This chapter analyses a review of historical models useful to estimate tax evasion (or avoidance), using a different old study and combining the variables showing a new estimate method, adding also a new methodological approach with economics and econometrics results. The paper indeed, is organized as follows: Section 1 presents an introduction of shadow economy and tax evasion. Section 2 presents the main methodological models about the shadow economy, the its definition, the estimation methods and the newest empirical evidence. In Section 3 is present the problem of tax compliance, the theory conceal behind evasion and its definition, some estimation methods and the newest empirical evidence, focusing on Agent-based Model. We concentrate to look deepen two different methods, that will be used to evaluate shadow economy (starting from Dell'Anno (2007)) and tax evasion seen from simulation point of view by Valente et al. (2002).

2.1 An introduction on tax evasion and the shadow economy: what are these phenomena?

Activities associated with shadow economies affect the world life. Several governments attempt to control these activities through various measures such as prosecution, punishment, economic growth and sometimes the education. But trying to control this phenomenon, became really important allocating resources more effectively and efficiently, that leads to have more information about the extent of the shadow economy, its magnitude, who is engaged in these activities and their frequency.

Unfortunately, it is very difficult to get accurate information about shadow economy activities - just because hidden - including the goods and labour involved.

Several authors trying to measure the shadow economy face the difficulty of how to define it. One of the most common working definition is include into underground economy all currently unregistered economic activities that contribute to the officially calculated (or observed) Gross National Product.

Smith (1997) defines it as *"market-based production of goods and services, whether legal or illegal, that escapes detection in the official estimates of GDP."* To put it in another way, one of the broadest definitions of it includes *"those economic activities and the income derived from them that circumvent or otherwise avoid government regulation, taxation or observation"*.

It is obvious that a broad definition of the shadow economy includes unreported income from the production of legal goods and services, either from monetary or barter transactions – and so includes all productive economic activities that would generally be taxable by tax authorities.

The more specific definition of the shadow economy is that it includes all market-based legal production of goods and services that are deliberately concealed from public authorities for any of the following reasons:

1. to avoid payment of income, value added or other taxes;
2. to avoid payment of social security contributions;
3. to avoid having to meet certain legal labour market standards, such as minimum wages, maximum working hours, safety standards, etc.;
4. to avoid complying with certain administrative procedures, such as completing statistical questionnaires or other administrative forms.

At the same time, tax evasion occurs when individuals deliberately do not comply with their tax obligations. Tax revenue losses may cause serious damage to the proper functioning of the public sector, threatening its capacity to finance its basic expenses with damages for all community. Therefore, tax compliance is a major concern for all governments, which be careful with tax authorities and jurisprudence trying to fight this event.

As a complex phenomenon, tax compliance is composed by different perspectives. Indeed, taxpayers are influenced by many factors, including their disposition towards public institutions, the perceived fairness of the taxes, prevailing social norms, and the chances of non-compliance being detected and punished. The economic analysis of tax compliance behaviour of taxpayers can be fruitfully seen as the result of a rational calculus, a careful assessment of the costs and benefits of evasion (Smith (1997)). According the public purpose, economic perspective offers valuable insights that can be used to derive eligible policy measures.

The payment of tax is compulsory everywhere except in particular case, i.e. natural disasters when some people could be relieve to pay taxes; people pay their taxes out of a desire to do good for their governments, and the fear of consequences of non-payment, majority of people do not enjoy paying taxes, regardless of the benefits they may realize from taxation to the society generally and to themselves (Santoro and Fiorio (2011)) and they are often referred to as "free riders", where the classic example could be to take a bus without stamping ticket. Slemrod and Bakija (2001) noted that each citizen has a very strong incentive to ride free on the contributions of others because the own individual contribution is relatively immaterial to what one gets back from the government. For these reasons, paying taxes must became a legal responsibility for each citizen. In this way, most governments are approached with a great challenge in enforcing tax laws to induce people to pay their taxes.

Furthermore, only in legal terms there is a distinction between tax evasion and tax avoidance. As Slemrod and Yitzhaki (2002) notes, tax avoidance involves every attempt permitted by legal means in order to minimize ones tax burden. However, from economic point of view, and focusing on the consequences of both in terms of tax collection, tax evasion and tax avoidance are similar even if one is illegal and the other one is not.

In most countries there are many perfectly legal tax avoidance strategies for arranging financial affairs to minimize taxes. However, these strategies can often provide attractive alternatives for tax evasion and most taxpayers (i.e top companies in any sector) contract the services of tax consultants (i.e the Big Four - E&Y, Deloitte, PwC, KPMG) to avoid taxes. But most tax consultants tend to exploit uncertainty where it exists, hence engaging in tax evasion and this suggests that professional advice is strongly related with the practice of tax evasion. Taking actions against tax evasion without introducing corrective measurements against who encourage dodging, is inadequate to dilate the tax base.

Pyle (1989) describes non-compliance as a more neutral term than evasion since it does not assume that an inaccurate tax return is necessarily the result of an intention to defraud tax authorities, and it recognizes that the inaccuracy may only result in overpayment of taxes. However, in practice, it is difficult if not impossible to be certain of intent and as such the term non-compliance is often used.

Finally, several authors often use the term of non-compliance to characterize the intentional or unintentional failure of taxpayers to pay their taxes correctly, and here we will maintain the same meaning.

2.2 The problem of the shadow economy

2.2.1 Definition of shadow economy

The definition of shadow economy plays an important role in assessing its size. Having a clear definition, a number of ambiguities and controversies can be avoided.

In general, there are two types of shadow economic activities: illegal employment and the production of goods and services largely consumed within the household. The analysis is focused on both blocks, but tries - at the same time - to exclude criminal activities such as drug production, crime and human trafficking.

Thus, the focus goes on productive economic activities that would normally be included in the national accounts but which remain underground due to tax or regulatory burdens. Although such legal activities contribute to the value added of single country, they are not captured in the national accounts because they are produced in not conventional ways (e.g. by people without proper qualification or without a master certificate).

From the economic and social view, soft forms of illegal employment, such as moonlighting (e.g.construction work in private homes) and its contribution to aggregate value added can be assessed rather positively.

Determinant of shadow economy According to Frey and Pommerehne (1984), some variables may be considered the leading causes to an hidden economy, and these may be divided in two antithetic classes: the social and economic group. Belonging among social group the following determinants:

- *Intensity of Regulations*: increased intensity of regulations, for example labour market regulation, trade barriers, and labour restrictions for immigrants. is another important factor which reduces the freedom (of choice) for individuals engaged in the official economy;
- *Public Sector Services*: an increase of the shadow economy can lead to reduced state revenues which in turn reduce the quality and quantity of publicly provided goods and services.

- *Other Public Institutions*: the quality of public institutions is considered as another key factor of the development of the informal sector. In particular, corruption of bureaucracy and government officials seems to be associated with larger unofficial action, while a good measurement of law by securing property rights and contract enforceability, increases the benefits of being formal.

- *Tax Morale*: in addition to the incentives effects discussed before, the efficiency of the public sector has an indirect effect on the size of the shadow economy because it affects *tax morale*. Tax compliance is driven by a mental tax agreement that entails rights and obligations from taxpayers and citizens on the one hand, but also from the state and its tax authorities on the other hand. Citizen are more hardly disposed to pay their taxes frankly if they get worthy public services in exchange, but they are inclined even in cases when the main benefit of taxation does not captured, i.e. in the case redistributive policies.

Instead, the causal factors belong to economic group are the following:

- *Labour market conditions*: during working time, the higher opportunity costs of taking up additional work in the hidden economy reducing payment of taxes on the hand (whom demands labour) and maximize their wage (whom supplies labour);

- *Structural factors*: in some economic sectors (particularly those with low capital intensity), industries (e.g. handicraft) but also workers (e.g. foreign workers) in which a higher probability of working in the hidden economy can be assumed;

- *Tax and Social Security Contribution Burdens*: the tax and social security contribution burdens are among the intense causes for the existence of the shadow economy. The bigger difference between the total labour cost in the official economy and after-tax earnings (from work), is good incentive to reduce the tax wedge and work in the shadow economy.

2.2.2 Linkages among shadow, regulation, bureaucracy and inequality

Regulation and bureaucracy As Johnson et al. (1998) asserted, *"if the rules are clear on paper, but they have a great deal of discretion and implementation, this may lead an higher burden on business, more corruption and generally more unofficial economy."*

This idea led to three specific proposition about markets regulation:

1. the share of unofficial economy in GDP should be higher when there is more regulation because it can carry to viscous burocratic system;

2. the unofficial economy should be larger when there is a bigger tax burden on firms in the official sector;

3. a larger unofficial economy should be correlated with weaker publicly provided services, mean as measurement of corruption.

Inequality and shadow economy Dell'Anno (2001) specifies some different arguments about the relationship between inequality and shadow economy.

Indeed, in his point of view, exists economically significant relationship between the shadow and income inequality, although it may be concealed in the empirical analysis. However, when is present a statistically significant correlation, that is positive.

For instance, Valentini (2009) noted two crucial aspects that have been barely advised in the literature. First, since income inequality is measured using "declared" incomes, the bias of the inequality indexes may make these measures unreliable for comparisons among countries with different sizes of underground economy. Second, he argues that there are no reasons to suppose that a growth in unobserved income is uniform along income distribution. In particular, the sign of this correlation depends on the predominant nature of the shadow income, consequently, if the unobserved revenue is higher for the poorer than for the richer, we could have a positive relationship between the size of shadow economy and income inequality, or vice-versa.

2.2.3 Methodological approach behind the shadow: the Tanzi model

The idea behind Tanzi (1983) is to compute the shadow economy with Currency Demand Approach (CDA).

The demand of money of shadow economy is calculated as difference between the money demand into full model and the model with parameters equal to zero.

The model presents the money supply as dependent variable, while the variables of money demand for regular transactions are identified in the share of wages paid in cash, the interest rate on savings deposits and the average income per capita, that are independent variables or regressors. Tanzi (1983) has constructed an estimate of the money demand using the Feige (1979) model and compared it with the recorded money supply in the U.S. and suggested that one of the main factors to deter individuals from legally transacting in the U.S people. is to give away part of their transactions in the form of taxes.

Tanzi (1983) has built a model where the ratio of cash to non-cash money supply is generated by the ratio of personal income tax on total adjusted income, the ratio of legal cash remuneration on total personal income, the interest rate, and real income per capita:

$$\ln\left(\frac{C}{M_2}\right) = \beta_0 + \beta_1 \ln(T) + \beta_2 \ln\left(\frac{WS}{NI}\right) + \beta_3 \ln(R) + \beta_4 + \ln(Y) + \mu \tag{2.1}$$

In this model, the cash of money supply ratio $\left(\frac{C}{M_2}\right)$ is influenced by the personal income tax rates (T), the amount of cash wages to national income $\left(\frac{WS}{NI}\right)$, the annual interest rates (R) and income per capita (Y). Tanzi (1983) used (1) to estimate the relation between taxes and cash to money supply.

The amount of non-disclosed transactions (according to Tanzi (1983) it represents the size of the underground economy) is found by using the Fisher equation, where he assumes the velocity of "black money" to be similar to that of legal funds. Naturally, this estimation is only possible when the coefficient for tax rates is found to be significant and positive, and therefore, relationship (1) is crucial to the model.

Although presenting robust results, method of Tanzi (1983) was criticized by Enste and Schneider (2002) who argued that "*he assumes a base year for which there is no shadow economy without enlisting reasonable grounds for doing so, that there is more to tax evasion than the size of tax rates (i.e. tax morality, trust in the government etc)*". Further they argued that "*he does not take into account the usage of the dollar abroad as international currency and finally, that in the particular case of the US, increases in currency demand could also be due to an exogenous decreasing demand for deposits over the respective time period.*"

The model presented by Enste and Schneider (2002) is the following:

$$\begin{aligned}\ln\left(\frac{M_1}{M_3}\right) = \beta_0 + \beta_1 \ln(1+T_n) + \beta_2 \ln(1+GT) + \beta_3 \ln\left(\frac{WS}{Y}\right) + \beta_4 \ln(R) + \\ \beta_5 \ln\left(\frac{Y}{N}\right) + \beta_6 \ln\left(\frac{D}{C}\right) + \mu\end{aligned} \tag{2.2}$$

It showed the cash to money supply ratio $\left(\frac{M_1}{M_3}\right)$ depends on different tax rates (Tn), taxes collected by different forms of government (GT), the amount of income received in cash to personal income $\left(\frac{WS}{Y}\right)$, interest rates ($R$), income per capita $\left(\frac{Y}{N}\right)$ and the size of the captured drugs activities per unit of consumption $\left(\frac{D}{C}\right)$.

Assuming critical review step by step, it has been chosen the year 2002 as base year with no shadow economy since this was the year of the currency change in the other countries.

Addressing the second point of critique, one can group taxes according to the collecting authority (at a national and supranational level) and look at the separate impact. Moreover, it includes data on criminal activities and specify in more detail components of the underground economy: informal, illegal and criminal.

Furthermore, given the increasing expansion of electronic payments of transactions and services, supports by a period of non-violent economic development, it is reasonable to assume that the demand for deposits has significantly reduced between 2002 and 2009 in the Euro zone.

A revisited of Tanzi model: some ideas The fact that the Tanzi (1983) model turned out not to give a convincing explanation of nowadays underground economy, does not necessarily mean that the model is wrong anyway.

Unemployment might have a double impact on the shadow economy, both negative or positive. Indeed, the former case happens when the increased unemployment might lead to a crowding out of illegal low skilled workers instead of legal higher skilled workers, who might accept lower paid jobs. The latter case instead, occurs when unemployment represents an incentive for enterpriser to hire more illegal workers in order to save costs in arduous times that leads a positive relationship between unemployment and the shadow economy. Ferwerda et al. (2010) tried to use unemployment figures instead of tax rates in their revised Tanzi model.

Schneider and Enste (2000) and Blackburn et al. (2012) argue that the underground activities can be influenced by public policy and administration. Using some of these public policy and administration factors, they focused on adding to the tax rates used in Tanzi (1983), the amount of government expenditures in relation to GDP (evaluated as a share/percentage), police expenditures and judicial expenditures (measured in amount of dollars and amount of dollars per capita).

Furthermore, since the underground economy includes also criminal activities, the hidden could be largely influenced by those crimes that generate the most cash money. Making use of these crimes by using the total crime rate (measured in the amount of crimes and amount of crimes per capita), statistics on the amount of drugs use (as a proxy for the size of the drugs market), the amount of robberies, burglaries, stealing-thefts, vehicle-thefts, property crimes in general (all measured as a frequency) and corruption (measured as a perception index) they "*included corruption specifically because it could facilitate tax evasion*".

Another factor that could influence the shadow economy is literacy and inequality. Bank (2004) argues that illiteracy prevents people from opening bank accounts and forces them into the cash underground economy, using black payments or intractable. Bank (2004) used this indicator by taking the percentage that completed less than 5 years of elementary school and the percentage that completed high school as proxies for respectively illiteracy and literacy with checking about the robustness of gender and race.

The cash intensive part of the underground economy can be represent one of low skilled workers and poor people rather than one of rich tax evaders, who can probably find other – less cash intense – means for their activities, and inequality with income inequality statistics that can be calculated using a Lorenz-curve on income (the Gini-index of income inequality). World Bank has estimated the Tanzi (1983) model for United States by including at least one indicator for each of the arguments listed in the literature as key factors influencing the shadow economy.

The variables used are present in the following regression:

$$\Delta \ln\left(\frac{C_{domestic}}{M_{2domestic}}\right)_t = \Delta \ln(Y)_t + \Delta \ln\left(\frac{WS}{NI}\right)_t + \Delta \ln(R)_t + \Delta \ln(var\ of\ interest)_t \tag{2.3}$$

The results of the revisited Tanzi (1983) model are still looks very poor anyway, and they do not represent a good alternative for the tax rate used by Tanzi (1983) as an explanation for excess cash demand.

2.2.4 Critique and extension about Tanzi model

Relationship is not time robust Ferwerda et al. (2010) used the data of Tanzi (1983) disclosed in his paper and updated it accordingly until 2006. In order to delete structural breaks due to new datasets, they collected data from 1975 to 2006, while the data of Tanzi (1983) actually ended in 1980. The overlapping 5 years allowed them to assess the quality of their data set merger.

On this extended panel data they ran a *rolling regressions*. This method allows to run regression in a panel data for given time sub samples.

The regression used is the following:

$$\Delta \ln\left(\frac{C}{M_2}\right) = \beta_0 + \beta_1 \Delta \ln(T) + \beta_2 \Delta \ln\left(\frac{WS}{NI}\right) + \beta_3 \Delta \ln(R) + \beta_4 + \Delta \ln(Y) + \mu \tag{2.4}$$

Conclusion Tanzi (1983) built a model to estimate the size of the underground economy using taxes as a tool.

His model presented robust results at the time of the publication, although suffering from a unit root problem, were intuitively convincing. When corrected for unit roots, the Tanzi (1983) model still provided significant and correctly signed results.

However, a more fragmented analysis of the regression shows that its overall significance is motivated by only a relatively short period of time. Moreover, only the first time range between 1930 and 1960 in Tanzi (1983) dataset, seems to confirm the relationship between taxes and excess demand for cash money.

Moreover, the correction for a unit root that Ferwerda et al. (2010) have done using the data of Tanzi (1983) does not permit to distort his findings especially in the earlier part of his time sample, thus his results have been confirmed.

In the meantime, has been developed a new address path defined MIMIC or latent variable approach, where the shadow economy is evaluated as latent variable correlated to set of measurable indicators, and with several variables - considered the main determinant of underground economic activities - are used.

2.2.5 The estimation of unofficial economy

Although the issue of shadow economy has been investigated for a long time, the discussion regarding the "appropriate" methodology to assess its has not come to end yet. Ground on that, we can summarize three main methods of evaluation:

1. direct procedures at a micro level that aim at determining the size of the shadow economy at one particular point in time. An example is the survey method;

2. indirect procedures that make use of macroeconomic indicators in order to proxy the development of the shadow economy over time;

3. statistical models that use tools to estimate the shadow economy as an "unobserved" variable.

From macro-areas listed above, we can extract different approach to estimate underground economy, already largely used in the literature.

The *first method* implies to be careful at the difference in accounted incomes and expenses. If the latter is greater, then the difference must consist of undeclared income obtained through the underground economy.

The *second method* considers as an input factor for both (the underground and the legal economies), whereby changes in the labour force indicates the dynamics of shadow economy.

The *third method* developed by Kaufmann and Kaliberda (1996) and Lackó (1996) considers capital to be the main indicator of the underground economy, referring mainly to electricity consumption.

The *fourth method* is referred to MIMIC models bring together expected causes and effects (or indicators) of the underground sector and they are easily observed as opposed to the real underground economy. The underlying idea is that we can use measurements of both the causal and of the indicator variables, in a time frame to infer the movements of the intermediate/middle value: the unobserved size of the underground economy, which then is reported in percentage of GDP.

The *fifth method* is the monetarist approach developed by Feige (1979) and successively Tanzi (1983). Feige (1979) tried to estimate the size of the US economy from the perspective of payments and transactions.

There are three different reasons of why the public and government in general especially in developed countries, have interested in the hidden economy, especially in its increase:

- could be interpreted as a reaction to the overburdening of individuals and firm;
- may be caused by a rise in tax burden may lead to a fall in tax receipt;
- economic policy measures may be a wrong magnitude or direction for indicators of the state of economy, because this may be bias.

Furthermore, exist some indicators useful to estimate a shadow economy:

1. using well-designed surveys and samples based on voluntary replies;
2. tax auditing and other compliance methods. Three of the estimation methods currently used rely on the identification of residuals at the aggregate level, but the same are computed in quite different ways, as the
3. discrepancy between income and expenditures;
4. difference between officially measured and actual participation rates;
5. additional demand for currency and/or money actually observed compared to a situation in which there would be no hidden economy.

2.2.6 Empirical evidence: measurements of shadow

The Index Method measures the size of the shadow economy as a percentage of GDP. There are three common methods of measuring GDP: the output, expenditure and income approaches.

The Index of Putniņš and Sauka (2015) is based on the income approach, which calculates GDP as the sum of gross remuneration of employees (gross personal income) and gross operating income of firms (gross corporate income). Computation of the Index proceeds in three steps:

1. estimate the extent of under-reporting of employee remuneration and under-reporting of firm's operating income using the survey responses;
2. estimate each firm's shadow production as a weighted average of its under-reported employee remuneration and under-reported operating income, with the weights reflecting the proportions of employee remuneration and firm's operating income in the composition of GDP;
3. calculate a production-weighted average of shadow production across firms.

In the first step, under-reporting of firm $i's$ operating income, $UR_i^{OperatingIncome}$, is estimated directly from the corresponding survey question. Under-reporting of employee remuneration, however, consists of two components:

1. under-reporting of salaries, or "envelope wages";

2. unreported employees.

Combining the two components, firm $i's$ total unreported proportion of employee remuneration is:

$$UR_i^{Operating\ Income} = 1 - (1 - UR_i^{Salaries})(1 - UR_i^{Employees}) \tag{2.5}$$

In the second step, for each firm they constructed a weighted average of under-reported personal and under-reported corporate income, producing an estimate of the unreported (shadow) proportion of the firm's production (income):

$$Shadow\ proportion_i = \alpha_c UR_i^{Employee\ remunation} + (1 - \alpha_c) UR_i^{Operating\ income}) \tag{2.6}$$

In the third step we take a weighted average of under-reported production, *Shadow Proportion*, across a representative sample of firms in country c to arrive at the Shadow Economy Index for that country:

$$INDEX_c^{Shadow\ economy} = \sum_{i=1}^{N_c} W_i Shadow\ proportion \tag{2.7}$$

The weights, W_i, are the relative contribution of each firm to the country's GDP, which are approximated by the relative amount of wages paid by the firm. Similar to the second step, the weighting in this final average is important to allow the Shadow Economy Index to reflect a proportion of GDP.
As a final step, it follows the methodology of the World Economic Forum in their Global Competitiveness Report, it has been applied a weighted moving average of $INDEX_c^{Shadow\ economy}$ calculated on some surveys.

González-Fernández and González-Velasco (2015) analysed the shadow economy in Spain, estimating it. They used the Currency Demand Approach that is included in the monetary indirect estimation methods.

As already argued, this is one of the most commonly used approaches in the literature, along with the causal model or MIMIC. It was first presented by Cagan (1958) and Gutmann (1977). Tanzi (1983) in the opposite, has proposed (see above) an econometric model employing the Currency Demand Approach to estimate the shadow economy with integrating other operating variables that may affect the demand of cash that he obtained the regression model.

Salahodjaev (2015) instead presents the following model: the data on shadow economies is drawn from Schneider et al. (2010), who applied an MIMIC estimation approach to calculate the size of shadow economy relative to GDP for 162 countries over the period 1999–2007.

From another point of view, the main independent variable is average national IQ score could be seen as proxy for domestic intelligence. The data on IQ test results is byLynn and Vanhanen (2002). The dataset is an updated version of Schneider et al. (2010), and contains national IQ scores for 190 nations of the world. For countries with missing data, IQ scores were recovered based on school achievement results or provincial data from neighbouring regions with a similar culture. In line with existent studies on the determinants of shadow economies, Lynn and Vanhanen (2002) used a set of control variables (Dreher et al. (2009); Friedman et al. (2000); Singh et al. (2012)), namely GDP per capita and size of agricultural economy relative to GDP from WDI, democratic index from Freedom House, government effectiveness (Kaufman et al. (2003)), and fiscal burden from The Heritage Foundation.

They estimated the following econometric model:

$$SE_i = \alpha + \beta IQ_i + X'\lambda + \varepsilon_i \tag{2.8}$$

where SE is the size of informal economy in country i during 1999-2007, IQ is intelligence, X is a set of control variables suggested by empirical literature, and ε represents possible error term.

Concluding analysis, it has shown intelligence has a negative effect on the size of a shadow economy. The coefficient is significant and the size of informal economy relative to GDP decreases. The authors included the level of economic development, measured by GDP per capita, in the regression. Both intelligence and GDP per capita are significant and negatively related to shadow economy. The significance level for intelligence, measured by IQ scores, remains unaffected, which indicates that intelligence seems to be an important determinant of underground economy after controlling for the level of per capita wealth.

Kireenko and Nevzorova (2015) analysed effect of shadow economy on the quality of life. It was supposed that shadow economy effects positively on the level of life (as shadow income increased total revenues). But it can impact negatively on the quality of life. The relationship between growth of life level and quality of life depends from the level of the shadow economy. In this cases, the quality of life is the casual variable, which leads to the changes in the latent variable (in this case that is shadow economy).

The task has been to ascertain the reverse effect of the shadow economy on the quality of life. At the initial stage of their analysis, the original sample of 150 countries was divided into 5 groups, according to the shadow economy size. Than the average values of each indicator for each cluster was calculated. Next, was done a regression equation for each cluster and for general sample.

As a result of regression analysis, authors found that *life expectancy at birth* and *children in school rate* are significant for each cluster and for the whole sample. The level of life increases according to the shadow economy's growth and, in the same time, life quality indicators which reflects a long and healthy life and access to knowledge, decrease. The regression coefficient increases with the shadow economy growth and these results have demonstrated the correctness of hypothesis about relation between shadow economy and life quality, excluding income component.

Stankevičius and Vasiliauskaitė (2014), analysed the relationship between the tax burden and shadow economy's level in case of different economic development level within EU countries during 2005-2012. Empirical results of the influence the tax burden on the shadow economy are provided in the studies of Schneider (2012) and Johnson et al. (1998). Therefore, he concluded the European Union will be delicate and vulnerable to economic shocks and need more fiscal integration, demonstrated (see e.g. Sala-i Martin (1994), Jensen et al. (2012)).

Indeed, Schneider (2012) have proposed an increase in the European tax burden level in order to establish a horizontal fiscal equalisation mechanism. Further evidences conclude that the application of the tax system and the regulations by governments affects (an higher) the size of shadow economy, finding an unexpected negative correlation between the size of the unofficial economy and the tax rates. On the other hand Schneider (2011) found a positive correlation between the size of the shadow economy and the corporate tax burden. These findings show that it can be presumed to the overall conclusion that there is a large difference between the impact of either direct taxes or the corporate tax burden and indirect taxes, especially taxes on consumption and it means several attempts to transfer the tax burden from labour taxation to consumption which faced increased VAT (value added tax) rates.

Zaman and Goschin (2015) instead, have developed a new synthetic index of SE that includes three relevant indicators: shadow economy measured in Euro per inhabitant, shadow economy as percentage of GDP and shadow economy of each European member state as percentage of the total EU-28 shadow economy.

The synthetic index calculations reveal that Romania is not among the countries with very large shadow economy magnitude, and it is having low and stable shadow economy index values over 1999-2012, whereas providing empirical support to the hypothesis that shadow economy was connected to the Romanian economic development over same horizon time.

D'Hernoncourt and Méon (2012) reported a negative relationship between the size of shadow economy and generalized trust, in a sample of countries, both developed and developing. This relationship is robust controlling for a large set of economic, policy, and institutional variables, to change the estimates of shadow economy and the estimation period, suggesting the tax compliance effect of trust dominates its role as a substitute for the formal legal system.

However, D'Hernoncourt and Méon (2012) underlying the weight of legal and regulatory determinants on the shadow economy, implicitly suggesting that trust should be associated with a larger shadow sector.

Therefore, in order to evade taxes and regulations *informals* also have to renounce the benefits of law, which increases their transaction costs even if they cannot use the contract system, which is one of the main costs of informality that De Soto (1989) points out.

To sum up, the relationship between trust and the shadow economy may a priori be either non-existent, positive - if the trust is a substitute to the formal legal system - or negative -if the tax evasion mechanism dominates, given uncertainty.

Torgler and Schneider (2009) analysed a multivariate model to examine how tax morale of specific country and institutional quality affect the shadow economy. The literature strongly emphasizes the quantitative importance of these factors on the level of changes in the shadow economy. They found strong support that a higher tax morale and institutional quality lead to a smaller shadow economy.

It is a relevant issue to investigate whether differences in tax morale across countries are reflected in any diversity in real - or observed - behaviours in these countries.

According to Torgler and Schneider (2009), the control variable includes two additional variables, namely *Urban population* and *Unemployment*. A higher density of population in urban areas could increase anonymity and thus reduce loyalty towards the state; this may lead to a higher level of shadow economy. Moreover, the authors argued an higher unemployment rate may be correlated with a greater level of shadow economy. Individuals without an occupation have more leisure time at their disposal.

Furthermore Torgler and Schneider (2009) analysed the economic regulations – particularly labour regulation – can also affect the shadow economy. Stronger restrictions are a strong incentive to choose the exit option, as they reduce the freedom of action (Enste and Schneider (2002)). As a proxy they used the variable wage and prices developed by the Index of Economic Freedom provided by Heritage. Surprisingly, the coefficient was statistically significant in six specifications and the results are following: price and wage regulations are no reasons for firms to move into the unofficial economy. Moreover, higher GDP per capita is associated with a smaller shadow economy. GDP per capita is seen as a proxy for the level of development of a country.

In the end, tax morale and institutional quality are highly relevant in explaining the size of shadow economy.

Moreover, Alm et al. (2004) argue that the size of the underground economy can serve as a useful, if somewhat imperfect, measure of the extent of tax evasion, so that a negative correlation between the size of shadow economy and tax morale indicates individual revealed actions related to their attitudes about paying taxes.

The informal sector plays an important role not only in transition countries (as ex-URSS, Central and Eastern Europe, China, etc.), but also in developing countries. Employment in the informal sector seems to be a relevant income source for many people. As Tanzi and Zee (2000) has argued, it is realistic to assume that informal activities are more magnitude in developing than in developed countries, because it is easier to conduct underground activities and where the exemption levels for income and value added taxes are lower, social security taxes higher, and the obstacles to start activities in the formal economy also higher than in developed countries.

Torgler (2005) investigates the correlation between the size of shadow economy and tax morale in Latin America using the Latino barómetro. The key problem is that the government is not able to sufficiently make sure the property rights in those countries. On the other hand a combination of interventionism and bureaucracy is often observed. Thus, a situation of simultaneous "over-government" and "under-government" arises, as Eichenberger and Frey (2002) points out. The government and the administration have a strong discretionary power over the allocation of resources which enhances corruption. Thus, individual tax evasion can be seen as an "exit" option, a signal through which taxpayers can express their disagreement.

Friedman et al. (2000) show empirically that countries with more corruption have a higher share of unofficial economy. Moreover, they analysed the impact of institutional quality by using a large number of variables. Dreher and Schneider (2006) have also investigated the correlation between shadow economy and corruption. They observed the tendency that underground and corruption are substitutes in high-income countries, but complements in low-income countries.

The OLS method proposed by Friedman et al. (2000) is the following:

$$SHADOW_i = \alpha + \beta_1 CTRL_i + \beta_2 TAXM_i + \beta_3 INSTIT_i + \beta_4 REGION_i + \varepsilon_i \tag{2.9}$$

where i indexes the countries in the sample, $SHADOW_i$ denotes the level in each country of hidden economy as a percentage of official GDP, $TAXM_i$ the level of tax morale and $INSTIT_i$ are our indicators for institutional quality as described in the previous section. The regression also contains several control variables, $CTRL_i$, including factors such as government interventions, fiscal burden, wage and prices controls, logGDP per capita, the agriculture share of GDP, the unemployment rate and the share of urban population. $REGION_i$ are dummy variables that differentiate between developed, Asian, and developing or transition countries using a mean values cross-section data from 1990 to 1999.

Din et al. (2015), to examine the long-run as well as the short-run dynamics of the relationship between tourism and the shadow economy, have employed the popular error-correction model. Banerjee et al. (1998), has criticized the two-stage error-correction models of giving substantial small-sample bias compared to the one-step error-correction model, where the long-run relation is restricted to being homogeneous. In this study, following Yasar et al. (2006) and Bond et al. (1997) the generalized one-step error-correction model (ECM) is estimated using the system GMM estimator. They defined the following autoregressive distributed lag model:

$$\begin{aligned} receipts_{it} = \delta_1 receipts_{it-1} + \delta_2 receipts_{it-2} + \alpha_0 shadow_{it} + \alpha_1 shadow_{it-1} + \\ \alpha_2 shadow_{it-2} + \beta_0 governance_{it} + \beta_1 governance_{it-1} + \gamma_t + \mu_{it} \end{aligned} \tag{2.10}$$

where $\mu_{it} = \varepsilon_i + \tau_{it}$ with i cross-sectional units and t time units. *Receipts* is the measure of tourism sector proxy by the international tourism receipts; *shadow* is the size of the shadow economy; *governance* is the measure of safety of tourist destination countries. The time specific effect γ_{it} is included to capture aggregate shocks. Assuming fixed effect μ_{it} assumed two different effects: the first one is unobserved time-invariant (ε_i) and stochastic error term (τ_{it}), which varies across the time and cross section.

Following Banerjee et al. (1998), equation (13) can be transformed into the following one-step ECM equation that provides an explicit link between the short-run effects and long-run effects:

$$\begin{aligned} receipts_{it} = (\delta_1 - 1)\Delta receipts_{it-1} + (\alpha_0 + \alpha_1)\Delta shadow_{it-1} + \\ \pi(receipts_{it-2} - shadow_{it-2}) + \theta shadow_{it-2} + \\ \beta_0 \Delta governance_{it} + (\beta_0 + \beta_1) governance_{it-1} + \gamma_t + \mu_{it} \end{aligned} \tag{2.11}$$

To determine whether the instruments are valid in the system GMM approach, they applied the Hansen test, a test of over-identifying restrictions, to determine any correlation between instruments and errors.

A generalized one-step error-correction model is estimated using a system GMM estimator to obtain consistent and efficient estimates of the short-run and long-run relationships between tourism and shadow economy. Generally, it is possible to conclude that the temporary (short-run effect) and permanent (long-run effect) shocks of the shadow economy adversely impacted on the tourism sector.

Concluding, modellers asserted that there is long-run relationship between the shadow economy and the tourism sector worldwide, in other words, shadow economy and tourism are related worldwide. One policy implication of this study is that mitigating shadow economy can enhance the growth in the tourism sector. Tourism industry can also play a leading role in the alleviation of poverty particularly in the developing economies. Blanke and Chiesa (2013) reports that tourism sector has contributed to 9.5% of the global gross domestic product, 8.9% of the worldwide total employment and 5.4% of the global export in 2013.

2.2.7 Methods to estimate underground economy

It is possible using different approaches to assess the unofficial economy, each one with different characteristics.

Transaction approach Activities in both the official and underground economy require money to undertake the necessary transactions. If we follow theory of money quantitative, so exist a constant relationship between currency and transactions. Examining the ratio of total nominal GNP to total transactions, the hidden economy, evaluate on ground on GNP, can be derived residually by subtracting officially measured GNP from total GNP. This approach does not need to make any supposition about the type of currency used in the underground economy (except that it is not by barter), but following the quantity equation $Mv = pT$ (v = velocity of money, p = price level of transactions, T = volume of transactions), assumptions are required about v and about the relationship between the value of total transactions $p.\ T$ and nominal total GNP.

This approach has been propagated by Feige (1979) who applies it to the United States. The year 1939 is taken as the "base year" in which he assumed that there was no shadow economy and in which therefore the ratio of $p.\ T$ to nominal (official and total) GNP was "normal" (in his analysis it was equal 10.3).

Econometric approach The need to control is reversed into an effort made to identify the influences of these factors on demand in order to ensure that the "extra" currency can really be attributed to working of underground economy.

Assuming that the increasing burden of taxation leads people to take up additional currency, the goal is to econometrically estimate a stable relationship between currency and personal taxes.

Comparison of indicators approach Surveys tend to lead to comparatively small estimates of the hidden Economy. Tax auditing results in higher estimates of hidden incomes than voluntary responses to surveys.

The discrepancy approach yields estimates of similar size to the tax auditing method. This approach underestimates the true size if it is based on a comparison of the participation rate. Taking the italian case comparison with other countries, implicitly it assumes that there was no hidden economy in those other nations.

A latent variable approach Essentially, the MIMIC (Multiple Indicators Multiple Causes) model is specified as follows.

First, the scalar latent variable (in this case, the size of the hidden economy), denoted as η, is linearly determined by a set of observable causes $x' = (x_1, x_2, \ldots, x_q)$, subject to a scalar random error term,ζ , which is a $(px1)$ vector:

$$\eta = \gamma x + \zeta \tag{2.12}$$

where γ is a $(qx1)$ vector of parameters. This latent variable, in turn, linearly determines a set of observable endogenous indicators $y' = (y_1, y_2, \ldots, y_q)$, also subject to a scalar random error term, ε, which is a $(px1)$ vector,

$$y = \lambda\eta + \varepsilon \tag{2.13}$$

where λ is a $(px1)$ vector of parameters. It is assumed that λ and η are Normal and mutually uncorrelated.

Substituting (12) into (13), we obtain the reduced form relation connecting the observable variables and express them in a p-equation multivariate regression model,

$$y = \Pi x + z \tag{2.14}$$

The equation in (14) is restricted so that the regressor's coefficient matrix (X) has a rank of one and the error covariance matrix is similarly constrained.The first restriction is common in econometrics and typically arises in simultaneous equation models (SEMS) where the exclusion of certain exogenous variables from a structural equation implies that a certain portion of the reduced form coefficient matrix is short ranked. The second restriction derives by disturbance covariance matrix, that is singular.

In the case of hidden economy, the purpose of latent variable, if a value of it was known at some point in the sample, then the ordinal series could easily be converted into a cardinal time-path. In this way the estimate of hidden economy obtained could became a "benchmark" which would be used in the conversion and this has been the technique that has been applied in most studies.

The determinants of the MIMIC model estimation according to Dell'Anno (2007) to assess underground economy, are mainly six:

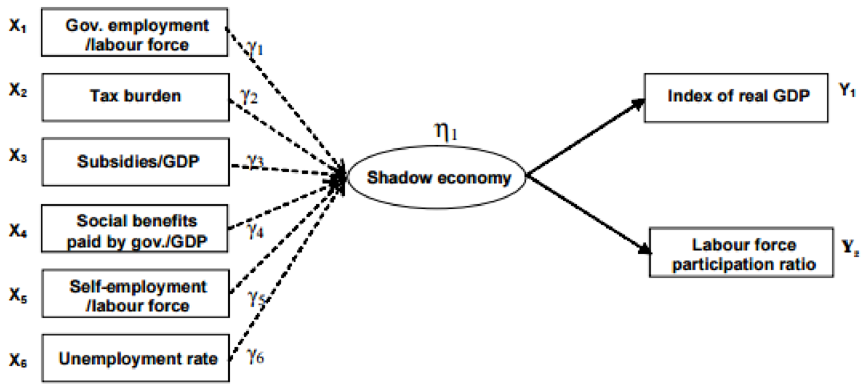


Figure 2.1: Dell'Anno MIMIC model, 2007.

2.3 The problem of tax evasion

2.3.1 Definition of tax evasion

The tax evasion may be defined a specific revenue deficiency, known as the "tax gap".

The outset from the tax gap is not equal to the amount of additional revenue that would be collected by stricter social control, that would significantly affect the economic scenario so the tax base would surely be altered. As a result, at least in theory, net revenue could even turn out to be smaller. Thus standard measures of tax gaps must be interpreted cautiously.

In economic terms, evasion problems originate in the fact that variables that define the tax base (incomes, sales, revenues, wealth, and so on) are often not "observable". An external observer cannot usually see the actual magnitude of a case-by-case's tax base, and hence cannot know his true tax liability. Sometimes this cognition can be obtained by means of costly audits, in other cases the tax base cannot be verified at all and taxpayers can take advantage of the imperfect information about their liability.

Another problem with the measuring of tax evasion relates to its proper boundary line within the noticeable set of the informal economy. No taxes are generally levied on transactions in the home and criminal sectors, which are usually beyond the ambit of authorities and official statistics and very difficult to include it. Hence, the extremities of evasion is often unresolvable from other illegal and unrecorded activities, because the boundary are not clearly defined.

Aggregate estimates of evasion must deal with all these problems, in addition to the classic problem of lack of direct data. Various estimation methods have been devised, some based on data collected by fiscal authorities, others based on data less reliable, derived from national accounts and surveys (see an example Feige (1979), Pyle (1989) and Tanzi and Shome (1994)).

The major implications concern the economic consequences of tax evasion are several, but the main question is how the set of policies that can be implemented are restricted. When taxes can be evaded, taxation will prove to be an fallible tool for pursuing government aims (e.g. the redistribution, the efficiency, or whatever), which will be only partly achieved. Indeed, effective taxation may turn regressive, as the more affluent usually have better opportunities to evade taxes. Furthermore, evasion may be powerfully deleterious to horizontal equity, leading to unequal distribution of opportunities to evade and of the willingness to seize them. This in turn may induce the production of inefficiency, because competition would be distorted by the unequal distribution of the tax effect among firms.

The adverse consequences of tax evasion are sometimes aggravated by laws, or even constitutions, drafted as if the tax base were observable, limiting the set of corrective instruments for governments. In order to evaluate the way in which non-compliance affects the actual tax payment of individuals, is necessary examining compliance of behaviour of taxpayers doing a theoretical model to predict how behaviour of taxpayers is affected, an example, by the relevant variables or by the neighbouring environment, where taxpayers interconnect themselves with other agents (other taxpayers - in quality of consumers or firms) whom affected their choices.

2.3.2 Theory behind evasion: some models

The shadow economy and tax evasion are not congruent, but activities in the shadow economy in most cases involve the evasion of direct or indirect taxes, such that the actors affecting tax evasion will most certainly also affect the shadow economy.

The modern use of economic tools for the analysis of tax compliance can be credited to Allingham and Sandmo (1972), who extended the influential work of Becker (1968) on law enforcement to taxation using modern risk theory.

According to Allingham and Sandmo (1972), tax compliance depends on its expected costs and benefits. The benefits of tax non-compliance result from the individual marginal tax rate and the true idiosyncratic income.

Kanniainen et al. (2004) incorporate many of these insights in their model of the shadow economy by also considering labour supply decisions.

A shortcoming of these analyses is the neglected endogeneity of tax morale and good governance. In contrast, Feld and Frey (2007) argued that tax compliance is the result of a complicated interaction between tax morale and deterrence measures.

The Becker model (1968) Analysis of Becker (1968) looked deep into the economic relevance of crimes. The tax evasion, he has renamed white collar crimes, are economically important activity.

Becker (1968) has developed his model subdividing it into five categories:

1. The relations between number of crimes (offenses) and the cost of offenses;
2. the number of offenses and the punishments meted out;
3. the number of offenses against the public expenditure on police and legal processes;
4. the number of convictions and the costs of imprisonments;

5. the number of offenses and the private expenditure on protection.

The elementary Becker (1968) model presents only legal and illegal sector. In this model, all potential criminals have a benefit of crime (denoted B) which is meant to include both the financial and any potential psychic benefits of crime. An individual committing crime faces costs from law-enforcement activities. If individuals are risk-neutral, these costs will just equal the probability of punishment (denoted p) times the costs of punishment. The costs of punishment come from the length of sentence and are denoted C. Thus, the net expected returns from crime equal $B - pC$. This return will compared with the returns from legal sector, which are denoted W. Thus, an individual will become a criminal:

$$B - pC > W \tag{2.15}$$

If everyone were identical, then they would choose either the legal or illegal sector. To create a realistic situation, Becker (1968) assumed that wages equal $W+w$, where w is distributed throughout the population with a cumulative distribution function $F(w)$ and a density $f(w)$. The marginal criminal is found so that $W = B-pC$, which means that $w* = B-pC-W$, where $w*$ indicates the idiosyncratic wage of the marginal criminal. The total number of criminals will be by cumulative distribution function $F(B - pC - W)$ which represents the total number of people whose legal returns are less than $W + w*$. Apparently, in reality there is heterogeneity in B (and possibly C as well), so that the liable of tax evasion are not rigorously the members of society with the lowest salary in the legal sector.

Standard differentiation tells us that the number of criminals rises as B rises, falls as p, C or W rises. These basic comparative statics are the central elements of the economics of crime and punishment. Reduction of crimes can occur through reducing the benefits of crime or raising the probability of being caught or the costs of punishment conditional upon being caught. Raising the wage in the legal sector is also predicted to reduce the number of criminals.

Moreover, the author used also welfare theorems and transferable pricing to describe the social losses from offenses and using elasticities of supply significantly affected by the optimality conditions.

From a social point of view, if there were no social loss from punishments, as with fines, the constant b will be equal to zero ($b = 0$) and the elasticity of supply would be irrelevant for optimality condition. If b is positive ($b > 1$), i.e. with imprisonment, the society would not be received nothing as compensation and would result a social loss.

Case A: The case of fines

A large use of fines requires knowledge of marginal gain and harm; the marginal gains is difficult to acquire, contrarily to damages.

The fines provide to compensate victims, in particular an optimal fine fully compensate individuals at the margin, re-establishing the status quo ex ante, and they do not worse off.

Anyway, Becker (1974) affirms fines as unfair, because they are proportional over income, in contrast with the target to minimize the social loss. He proposes that an optimal fair should be based on total damage done by offenders.

Case B: Non-monetary compensation

In some cases, became impossible to estimate the real harm, and the estimate would be affected by several errors. The crime is not different from any other economic activity, and with fines vanish the analytical differences in the computation.

Fines could be unfair to refund harm, but simpler to estimate without errors than non-monetary compensation.

Finally, the essay is concentrated on several advantages respect to punishments, i.e. they conserve resources, compensate society as well as punish offenders.

The Allingham - Sandmo model (1972) Allingham and Sandmo (1972) used a behaviour of taxpayer conform to the Von Neumann-Morgenstern utility approach under uncertainty. The taxpayer will choose how to maximize his utility:

$$E(U) = (1-p)U(W-\theta X) + pU(W-\theta X - \pi(W-X)) \tag{2.16}$$

$$Y = (W-\theta X) \tag{2.17}$$

$$Z = (W-\theta X - \pi(W-X)) \tag{2.18}$$

$$E(U) = (1-p)U(Y) + pU(Z) \tag{2.19}$$

where W is exogenous income - it is known by taxpayer but not by government, θ is constant rate which with tax levied, X is declared income by taxpayer, p represent the probability that taxpayer will be subject to investigate by authorities. $(W-X)$ represents undeclared income and π) is penalty rate od violation, higher to θ to prevent tax evasion. Analysing first and second order condition, it satisfies concavity of the utility function so, interior maximum exists; the FOC is the following:

$$-\theta(1-p)U'(Y) - (\theta-\pi)pU'(Z) = 0 \tag{2.20}$$

while the SOC is:

$$D = \theta^2(1-p)U'(Y) + (\theta-\pi)^2 pU'(Z) = 0 \tag{2.21}$$

In this analytic thinking the conditions for an interior maximum to exist are of particular importance. Obviously, it cannot be assumed a priori that $0 < X < W$, because should depend on the values of the parameters. Allingham and Sandmo (1972) evaluate expected utility at $X = 0$ and $X = W$.

The idea behind their work is a *comparative static result*, indeed with increasing up tax rates, two different effects subsist: income and substitution effect, that may lead a distinct tax compliance. The substitution effect could boost evasion because the marginal benefit of cheating goes up concurrently with tax rate whereas, the income effect tends to suppress evasion given agent risk aversion feeling him worse off. It concerns that the *net effect* is ambiguous.

An interested developing of this model is disposed by introduction of non-monetary factor, as s. It is an important factor about taxpayer whether or not to evade tax, because it concerns and adversely reputation into citizen's community; it may be represent in utility function and synthesis two different state of world (whether the evasion is detected s_0, or not s_1). Obviously, a missing detection gives a benefit for the agent, so $U(Y, s_o) > U(Y, s_1)$.

The focal point is the simplification done byAllingham and Sandmo (1972) about the $s - factor$; according to them, a change in s is insignificant compared to the effect of shift in income. In this way, the model can be seen as a good approximation result also for more complex model.

Introducing the Arrow - Pratt risk aversion measures, the authors indicated the absolute and the relative risk aversion function:

$$R_A(Y) = -\frac{U''(Y)}{U'(Y)} \tag{2.22}$$

and

$$R_R(Y) = -\frac{U''(Y)Y}{U'(Y)} \tag{2.23}$$

Allingham and Sandmo (1972) model assumes that absolute risk aversion is decreasing with income, while for the relative risk aversion the problem is more complicated, because they demonstrated the derivative was unequivocally positive just with $\pi \geq 1$ and in other case the sign of derivative depended by sign of π, taking equation (20) and making a derivation in W, they obtained:

$$\frac{\delta X}{\delta W} = -\frac{1}{D}\theta(1-p)U''(Y) + (\theta-\pi)pU''(Z) \tag{2.24}$$

Making a derivative of $\frac{\delta X/W}{\delta W}$ they obtained:

$$\frac{\delta X/W}{\delta W} = -\frac{1}{W^2}\frac{1}{D}\theta(1-p)U'(Y)[R_R(Y) - R_R(Z)] \tag{2.25}$$

Differentiating (20) with respect to θ, obtained:

$$\frac{\delta X}{\delta \theta} = -\frac{1}{D}X[\theta(1-p)U''(Y) + (\theta - \pi)pU''(Z) + \frac{1}{D}[(1-p)U'(Y) + pU'(Z)] \tag{2.26}$$

and substituting from (22) and (23), we obtained:

$$\frac{\delta X}{\delta \theta} = \underbrace{-\frac{1}{D}X\theta(1-p)U'(Y)[R_R(Y) - R_R(Z)]}_{(+/0/-)\ sub.\ effect} + \overbrace{\frac{1}{D}[(1-p)U'(Y) + pU'(Z)]}^{(-)\ inc.\ effect} \tag{2.27}$$

The second of two terms on the right is unambiguously negative. The first term is *positive*, (zero or negative) if the absolute risk aversion is *decreasing* (constant or increasing).

The economic meaning of this result is clear regard two terms in (27) that are the income effect and the substitution effect. The latter part is negative because an increase in the tax rate makes more profitable dodging taxes on the margin. The former instead, is positive because an increase tax rate makes the taxpayer less wealthy, reducing both Y e Z for any level of X, and this under decreasing absolute risk aversion, tends to reduce evasion and avoidance. They argued this sentence also respect the penalty rate π, from (20):

$$\frac{\delta X}{\delta \pi} = -\frac{1}{D}(W - X)(\theta - \pi)pU''(Z) - \frac{1}{D}pU'(Z) \tag{2.28}$$

and differentiating with respect to p to obtain:

$$\frac{\delta X}{\delta p} = \frac{1}{D}[-\theta U'(Y) + (\theta - \pi)pU'(Z)] \tag{2.29}$$

This derivative (29) is positive; it means an increase in the probability of detection will always lead to a larger income being declared.

Summing up, unequivocal result can be derived for two parameters of the model which are of particular interest for policy purposes, as the penalty rate and the probability of detection.

An innovation of Allingham and Sandmo (1972) model: Yitzhaki (1974) model Adding to Allingham and Sandmo (1972) model the intuition of Yitzhaki (1974), the same model became just income effect model; he showed when the penalty is imposed on the amount of evaded taxes, as under most current tax laws, the substitution effect vanishes and remaining income effect, taxpayer has incentive to cheat less. This results has induced some authors to leave behind the expected utility method analysing income tax evasion, but others have shown is not time to give up these types of studies yet.

An example, Gahramanov et al. (2009), shows is premature to dismiss the basic expected utility based formulation of the income tax evasion problem, assuming a higher tax rates can lead to lower tax evasion penalties as a special case of Yitzhaki (1974) model, an extended version of Allingham and Sandmo (1972) model.

Replacing the initial model, the problem of risk-averse taxpayer is to maximize his expected utility:

$$E(u) = (1-p)U(w - t(x)) + pU[w - t(x) - F(t(w) - t(x))] \tag{2.30}$$

choosing an optimal x.

In this model p represents the likelihood of getting caught, F is the fine rates ($F > 1$), and $t(x)$ is a general tax function, while zero income agent has no tax imposed like Yitzhaki (1974) model; the latter part $F(t(w) - t(x))$, is the total penalty payment.

Analysing the FOC,

$$\frac{\delta E(U)}{\delta x} = -(1-p)U'(Y)t'(x) - pU'(Z)(1-F)t'(x) = 0 \tag{2.31}$$

where $Y \equiv w - t(x)$, $Z \equiv w - t(x) - F(t(w) - t(x))$. Meanwhile, arranging the SOC we obtain:

$$D = (1-p)U''(Y)(t'(x))^2 + pU''(Z)(1-F)t'(x)) \tag{2.32}$$

it is always negative.

By combination of First Order Condition and Second Order Condition, the results obtained are the same of Yitzhaki (1974) model,

$$\frac{\delta x}{\delta F} = p\frac{t'(x)}{D}[U''(Z)(t(w) - t(x))(F-1) - U''(Z)] > 0 \tag{2.33}$$

and

$$\frac{\delta x}{\delta p} = -\frac{t'(x)}{D}[U'(Y) + U'(Z)(F-1] > 0 \tag{2.34}$$

To find out the relationship between the tax rate increase and declared income, he presented the tax function split in two different main part: the initial amount of declared income $t(x) + f(x)$ and the true income level $t(w) + v(w)$ where $f(x)$ and $v(w)$ are the income dependent shift function.

Changing from $f(x)$ and $v(w)$ to τx and $\varepsilon\tau w$ respectly, where τ is shift parameter. Gahramanov et al. (2009) argued two different case respect to constant ε.

Case 1: $\varepsilon = 1$

Differentiating (30) respect to τ and arranging, author obtains:

$$\left.\frac{\delta x}{\delta \tau}\right|_{\tau=0} = -\frac{t'(x)}{D}U'(Y)(1-p)\{\underbrace{x[R_A(Z) - R_A(Y)]}_{(+)} + \overbrace{F(w-x)R_A(Z)}^{(+)}\} \tag{2.35}$$

Both of these terms are positive.

Case 2: $\varepsilon < 1$

Differentiating (30) respect to τ and arranging author obtains:

$$\left.\frac{\delta x}{\delta \tau}\right|_{\tau=0} = -\frac{t'(x)}{D}U'(Y)(1-p)\{\underbrace{x[R_A(Z) - R_A(Y)]}_{(+)} + \overbrace{F(\varepsilon w-x)R_A(Z)}^{(+/-)}\} \tag{2.36}$$

In this case the equation is ambiguous. Indeed, an increase tax rate about optimum of taxpayer is associated with a smaller relative rise of endpoint tax rate bracket, then penalty payments decrease at the initial declared income, creating more incentives to tax evasion than before.

This kind of disincentive to undeclared real income is absent in first analyse because the tax rate shift ε is the same for any income level.

Allingham and Sandmo (1972) have represented an innovative paper about tax evasion or tax non-compliance. The particular method used over here it could be a suggestion for other topics, both theoretical and empirical.

Special Yitzhaki case A formulation of the problem of dishonest taxpayer is viewed as a special case of Yitzhaki (1974); the solution is counter intuitive and generated a puzzle that an higher tax rate encourage income declaration, resulting in opposite respect to common empirical evidence and economic intuition.

Some possible model extension In the literature have been developed several extension of the original Allingham and Sandmo (1972) model trying to achieve different useful results.

One of these possibilities was extended to the connections between incentives to avoid taxes and to supply work effort, or corporate saving against portfolio decisions; another kind of path was to insert the tax evasion within framework of optimal taxation theory. This latter assumed there was no evasion and the main conclusion has been the promotion of efficient allocation among resources, while taxes should be levied firstly on commodities with inelastic demand or supply.

Finally, several studies have evaluated the tax evasion and its amount through the Agent-based Model, capturing psychological approach in facing some specific behavioural rules and assessing the answer of heterogeneity multi-agent to these constraints.

2.3.3 The estimation of evasion

Methods to estimate tax evasion are evolved during time. The techniques may be subdivide into three different principal groups:

1. *Monetary method and shadow economy*, where the tax evasion is represented by synthetic indicators in the local economy, for instance as money in circulation or size of underground economy;
2. *National accounting*, the widespread method that compare declared taxes with cash paid in taxes;
3. *Comparison sample between declared taxes or observed taxes*, it is useful to determine the size of phenomenon when we are observing declared and effective taxes.

Two of the most important method (Bernardi and Franzoni (2004) and Besley et al. (2015)) have overtaken (or just tried it) the original Allingham and Sandmo (1972) model.

The Bernardi and Franzoni (2004) method critiques Allingham and Sandmo (1972) model because it is funded on not too much realistic hypothesis with target to examine the behaviour of taxpayer. But tax evasion could be push up also by individual and social motives and for this reason Besley et al. (2015) have built a simple dynamic model that incorporates these motives and their interaction.

Remembering the main assumptions of Allingham and Sandmo (1972) model, we can now deepen analysing the new proposed model:

- the agent is *rational* and would *maximize* his expected utility function;
- the agent is *risk averse*, otherwise he would evade total tax amount;
- the agent knows her income y, but the government knows only the declare income $y - e$, and this may be under *inspection with probability* p;
- exists the possibility *to pay fine* if the agent is discovered, $fine = ft(y - e)$.

The new Bernardi and Franzoni (2004) method adds a cost of inconvenience (in this specific case for instance, the inspection by controller) that leads to economic and time losses, defined as d.

$$E[U(e)] = \overbrace{(1-p)U(y - y(t-e))}^{Income\ without\ inspection} + \underbrace{pU(y - t(y-e)te - fte - d)}_{Income\ with\ inspection} \tag{2.37}$$

2.3.4 Measurement of evasion

Empirical attempts about to evaluate the amount of tax evasion are based on three approaches having most influences:

1. cross-section analysis;
2. time-series analysis;
3. controlled experiments.

First approach Clotfelter (1983) was the first to investigate how non-compliance responded to changes in the environment through *Tobit* model, where he found that non-compliance is strongly correlated to the marginal tax rate, that is robust conclusion.

Second approach Dubin et al. (1990) investigated the impact of audit rates and tax rates on paying taxes, using tax collections per return filed and returns filled per capita as reciprocal measure of non-compliance.

Third approach Used by Slemrod and Yitzhaki (2002), they found that for the middle-low taxpayers, treated with a kind of audit, produced a small but significant increase in reported income, which was larger for those with an higher possibility to evade.

From descriptive point of view of tax avoidance, Stiglitz (1986) distinguishes three basic principles of tax avoidance within an income tax: the postponement taxes, tax arbitrage across different tax brackets individuals and tax arbitrage across streams facing other tax treatment.

The Agent-based Model approach An Agent-based Model (ABM) is a simulated multi-agent system constructed with a particular aim: to capture key theoretical elements of some social or psychological process (see Hastie and Stasser (2000)) In such system, each agent typically represents an individual human acting according to a set of theoretically postulated behavioural rules (Hokamp and Pickhardt (2010)). These may involve simple heuristics or more complicated mechanisms that may require learning, constructing internal representations of the world, and so on. In an ABM, many simulated agents interact with each other and with a simulated environment over defined horizon period. This approach allows for the observation of large-scale consequences of the theoretical assumptions about agent behaviour when the same behaviours are carried out in the context of many other agents and iterated dynamically over an extended period of time (Pellizzari and Rizzi (2011)).

As reported in Allingham and Sandmo (1972) model, tax avoidance occurs only if the taxpayers image an increase of his expected income by taxes (Pickhardt and Seibold (2014)), including the awaited fines that he would pay if he is discovered. He should continue until the greater expectation is offset by the increased risk-bearing at the margin. Hence, the taxpayer increases both his exposure to risk and his expected income, creating an additional dead-weight loss for the community.

Recently, the literature has examined more general models about avoidance, with additional risk-bearing involved in tax evasion ; but the critical aspect is in infra-marginality of tax avoidance, where the costs depend not only by income effects, but also other aspects of behaviour (Besley et al. (2015)) that involve a different answer in the case of inquiry, an optimal progressiveness and optimal mix between income and consumption taxes.

Chapter 3

Different policies to fight tax evasion: Evidence from an Agent-based Model[1]

[1]I am grateful to Prof. Marco Valente for invaluable support during the whole study.
This research chapter was developed within the framework of the PhD programme in Economics and Finance of Sapienza University.

Abstract

The present chapter analyses the problem of tax evasion under different governmental policies, using an Agent-based Model with heterogeneous agents. We use the exponential utility function of Hokamp and Pickhardt (2010) model to represent the choice of producers and consumers included in a dynamic context with several other agents. We modify the utility function of maximizing a-type agents when they meet with stronger punishment and decide to evade analysing the effects. A novel aspect of our modelling is a "pure" Agent-based approach, based on expectations of audit and interactions among agents, finding some ethical norms - or stronger governmental policies - that lead to reduction of tax evasion. Finally, the model allows to define the impact of different governmental choices on tax evasion.

3.1 Introduction

In this paper we develop an Agent-based Model about large population of heterogeneous agents. From Hokamp and Pickhardt (2010), we refer about heterogeneity not only individually income levels but also different agent types, differentiating by their preferences on goods and leaving constant their risk preferences. One type of agent is in line with Allingham and Sandmo (1972), which is one of beginning paper about tax evasion.

Using firstly Allingham and Sandmo (1972), and modifying subsequently the model of Hokamp and Pickhardt (2010), we try to show that an interaction among heterogeneous agents in a dynamic context - considering different levels of punishment - affect the agents' decision and simultaneously the level of evasion in both quantitative and qualitative (it refers to simulations) Agent-based Model.

The paper is organized as follows: in the next section we describe briefly the literature behind tax evasion and Agent-based Modelling and its characteristics. Section 3 introduces our modified model with results about single agent. Section 4 shows scenario and results from different simulation, and finally we conclude.

3.2 Background

The tax evasion may be defined a specific revenue deficiency, known as the "tax gap."

The modern use of economic tools for the analysis of tax compliance can be credited to Allingham and Sandmo (1972), who extended the influential work of Becker (1968) on law enforcement to taxation using modern risk theory (Becker (1974)).

According to Allingham and Sandmo (1972), tax compliance depends on its expected costs and benefits. The benefits of tax non-compliance result from the individual marginal tax rate and the true idiosyncratic income.

The discussion about tax evasion has the classic problem of lack of direct data and various estimation methods are based on data collected by fiscal authorities, whereas others based on data derived from national accounts and surveys (see an example Feige (1979), Pyle (1989) and Tanzi and Shome (1994)). From an empirical point of view, these analyses present some shortcoming, as the neglected endogeneity of tax morale and good governance. In contrast, Feld and Frey (2007) argued that tax compliance is the result of complicated interaction between tax morale and deterrence measures. In this way, the Agent-based Models are enhanced their fame in economics since the early $'90s$ (see Tesfatsion and Judd (2006)).

Some of these interactions - based on Agent-based Model - have been defined by several authors (e.g see Newman (2003), Schwartz et al. (2002), Newman (2010), Wasserman and Faust (1994), Scott and Carrington (2011)) and applied in labour markets with (Montgomery (1991) and Granovetter (1995)).

These techniques have also been used to analyse unethical behaviour in organizations (Brass et al. (1998)), a concept useful for the shadow economy and detecting tax evaders. Usually, many Agent-based Models are represented in ring worlds or lattice structures, where an agent has a certain radius of neighbours (Andrei et al. (2014)).

Agent-based Modelling (ABM) has been a good candidate to be an alternative technique in modelling tax compliance and studying taxpayers behaviour. It has been becoming one of the most popular tool among the public finance researchers to give a simulation of real life taxpayers behaviour especially since the beginning of new millennium. Actually, the Agent-based Models are enhanced their fame in economics since the early '90s (see Tesfatsion and Judd (2006)).

An Agent-based Model is a simulated multi-agent system constructed with a particular aim: capturing key theoretical elements of some social or psychological process (see Hastie and Stasser (2000)) otherwise impossible to catch. In such system, each agent typically represents an individual human acting according to a set of theoretically postulated behavioural rules (Hokamp and Pickhardt (2010)). These may involve simple heuristics or more complicated mechanisms that may require learning and constructing an environment. In Agent-based Model, many simulated agents interact with each other and with a simulated environment over defined horizon period. This approach allows the observation of large-scale consequences on theoretical assumptions about behaviour of agents against many other agents into dynamic and extended period of time (Pellizzari and Rizzi (2011)).

As reported in Allingham and Sandmo (1972) model (see also Yitzhaki (1974)), tax avoidance occurs only if the taxpayers image an increase of his expected income by taxes (Pickhardt and Seibold (2014)), including the awaited fines that he would pay if he is discovered. He should continue until the greater expectation is offset by the increased risk-bearing at the margin. Hence, the taxpayer increases both his exposure to risk and his expected income, creating an additional dead-weight loss for the community.

Recently, the literature has examined more general models about avoidance, with additional risk-bearing involved in tax evasion; but the critical aspect are the costs depend not only by income effects, but also other aspects of behaviour (Besley et al. (2015)) that involve a different solution.

The idea behind the study of relationship and influences among different agent is originated with Simon (1991), when enunciated his theory of bounded rational, that led agents to act rationally on the basis of their perception about external environment and not only respect their objectively best response.

At the beginning, we can distinguish two different area: the economics area, where there are a number of Agent-based tax evasion models designed by economists (see Mittone and Patelli (2000), Davis et al. (2003), Korobow et al. (2007), Bloomquist (2004), Bloomquist (2006) and Bloomquist (2011)) and the second branch emerged from a new research area of physics called econophysics or sociophysics. Indeed, Schulz (2003) and Kulakowski and Nawojczyk (2008) are provided a overview of tha.

Tax evasion papers falling have generated some interesting results. For example Zaklan et al. (2008) find that even very small levels of enforcement are sufficient to establish almost full tax compliance while Zaklan et al. (2009) conclude that enforcement always works to enhance tax compliance.

In general, Agent-based tax evasion models that fall into the econophysics domain, for example papers by Lima and Zaklan (2008) and Lima (2010). The original model on ferro-magnetism was developed by the German physicist *Ernst Ising* in 1925 and Brush (1967) is extended it with a its generalization, that is a nowadays standard model in statistical mechanics. By economic point of view, the Agent-based tax evasion models grounds on the Ising model econophysics, is exploited to scrutinize cooperation among agents (see Zaklan et al. (2009)). Furthermore, levels of tax evasion in these type of models depend on two other element: the network structure of society and the tax enforcement mechanism. The former factor is enforced in alternative arrangement types, whereas the latter is modelled by two economic standard parameters, as the probability of audit and the rate of penalty (Bloomquist (2006)).

Instead agents can exist in one of two possible environmental status: evaders or not evaders. The influence of neighbours on a taxpayer changes the *social temperature* - derived from the fundamental measure of Ising model - which decreases or increases of stochastic subjective probability of taxpayers (Schulz (2003)). Concluding, these models have found that law enforcement has significant influence in directing a population towards tax compliance, even at low levels and despite of strong group influence.

Very important role is played by number of agents $j - type$ and periods susceptible to pay taxes t, parameters taking part of the model. According to Bloomquist (2006) and Zaklan et al. (2009), Agent-based tax evasion models designed by economists usually leading to an interval of 1000 agents, that represent a sufficient amount, whereas econophysics models have the use of about $1,000,000$ or more agents.

3.2.1 Focusing on Agent-based Model: a new experimental approach

Why using simulations The simulation should be assessed in terms of validation as performed in physical sciences. This kind of models, present specific difficulties, but they are supported by growing literature is starting to define how validation should be performed for this type of models Fagiolo et al. (2007), and authors are increasingly committing efforts to corroborate the reliability of their results Dosi et al. (2010), Valente (2012). Anyway, in many cases using attachment to reality is not the best way to evaluate a model, for several reasons.

As Valente argues, "*a model is a simplified representation of reality part, and therefore it expressly entails the loss of information in respect of the real system it is purported to represent and the authors have to be particularly keen in stressing the adherence of their models to reality as a natural way to assess the usefulness of a model. Even if in the "hardest", evidence-based, prototypical scientific discipline, adherence to reality (defining as an extension of validation) is a quite flexible criterion, which may appear as arbitrary and grounds on subjective preferences. Accepting this type of agents' decisions we are going against to diffused sentiment, but it also an independent manner to discuss further which aspects can be subjectively decided and which one are taken part of objective evidence.*"

According to Biggiero (2016) purpose, two different classes of phenomena have been defined for observers may to get interested into and provide an objective measure to admit or deny a certain statement. The first class includes a quantitative phenomena concerning events, respects two basic conditions:

- Phenomena defined a vector of quantitative variables;
- Phenomena observable in closed systems.

The second class relates to events - qualitative phenomena - is defined as follows:

- Phenomena that cannot be defined by quantitative variables only;
- Phenomena taking place in open system, that is, concerning systems that interact strictly with other group without possibility to isolate the characteristic of the system context.

Any relevant process concerning an economic system would never be fully represented by a single set of variables, but will always require different ones for other purposes.

Mainstream economics, based on the Chicago school, suggests a classic method inspired to standard mechanics, where, famously, correct predictions are all that matter to assess theoretical statement , where Friedman (1953) defines "*a common procedure to evaluate different arguments is based on the definition of knowledge as composed by explanations, whose subjective assessment can be based on such a objective basis to ensure that, even in cases of disagreement, the opposing parties should agree on what, exactly, they do disagree. Such explanations can be embodied into mathematical formulas or other formats.*"

The format of explanation to express a piece of knowledge is not necessarily the most convenient one, but have many advantages; indeed, it is possible to easily use for qualitative phenomena.

To assess simulation model results, most of modellers aim at showing that their results are compatible with those available from empirical evidence, i.e. they aim at validate their results. The nature of the evidence claimed to be replicated by the simulation data may vary, such as a given distribution function or of certain relations among the series produced (see, e.g., Dosi et al. (2010) and Ciarli et al. (2010)). The idea that validating a model results is a sufficient test for the assessment of a model is based on a core assumption: the unique relation between a model and its results.

Anyway, claiming that a model is correct because produces results indistinguishable from observed reality implies to state that any model differing from the one assumed would produce different results from those presented. In other words, that the only way to acquire those results is exploitation that model. This assumption is frequently satisfied by models for quantitative phenomena, in that it is sufficient to have a monotone function to ensure that the planned model is the unique path to use included data. However, this is not necessarily the case for many economic circumstances (or for qualitative phenomena).

The likeness between models results and reality is of little use, because of two, possibly contemporaneous, violations of the assumption:

- The same results may be produced by many different models;
- The same model may bring out very different results.

The issue of validation, applied blindly, leads to absurdities because it is based on the assumption that a model is a replication of reality. On the opposite, researchers model because they need to answer a specific questions. The difference is highly relevant but insidious.

Hence, standard economics, based on assumptions expressed as quantitative properties, relies on a methodological tool kit devised for disciplines, like physics, whose evidence consists of quantitative phenomena leaving apart some character that classical tool do not capture, just like the interaction and behavioural facets. According to this aim, is possible and useful join these methodologies and have the best property from both models.

Research oriented towards simulation models Simulation models developed for research purposes are no different from programs thought for other goals: they need to be judged according to the goals they are developed to reach. Scientific research is meant not to reproduce reality, though this is likely an intermediate goal: research is meant to develop and transmit knowledge. As Valente sustained above, "*knowledge can be universally represented as an explanations, and therefore we need to ensure that the means by which we implement a model permit to identify and evaluate the proposed explanations. For an explanation/knowledge implemented by means of a mathematical or logical theorems, this means to be able to assess step by step the proofs of the theorem.*"

According to scientific claims concerning dynamic phenomena, or even only non-linear dynamics, becomes vital the use of computer programs - which are able to representing the events that we believe are involved, and to go against the large scepticism concerning this tool yet, whereas the simulation models have continued to developing for long time.

With strengthening of these type of models, the computer programs are very important because can run very fast and produce large amounts of data, and indeed this is the main reasons they are generally produced. Similarly, researchers have largely exploited simulation models describing the code implementing the model and then reporting the results produced, but obviously in much more time. The calculator are able to investigating, supporting and diffusing scientific knowledge, that is necessary to clearly expose the intermediate steps. For scientific (and simulation) claims, these intermediate step concern qualitative phenomena which may involve statistically rare events, non linearities, innovation and other simulated cases whose importance and role is lost in the bare measurement.

Finally, computer simulation models have been increasingly adopted for research purposes following the dramatic larger use of computing power during last years. However, this kind of models ground on simulation among agents - also heterogeneous, are so far less successful than other inquiring tools to be largely accepted as instruments for theoretical analysis, but not for this reason less valid. Indeed, using the dichotomy between qualitative and quantitative phenomena - linking them developing a simulation model with empirical and theoretical economic model is achieved a more complete and complex analysis, born from combination of two different schools of thought that offering an interaction among similar quantitative and qualitative methods, underline the importance of both in a complete and exhaustive analysis.

The importance of social network Networks are defined as a set of items composed of vertices or nodes and several connections among them (edges or links) as Newman (2003) argued. These nodes can represent many things included: people, places, objects or even particles. The edges between these nodes can represent the linkage may be uni-directional or bi-directional, meaning the existence a one or two-sided correlation between nodes (see Schwartz et al. (2002)).

When we talk about networks, the main property where we must focusing is the centrality because assesses which nodes are the most central and critical (Newman (2010)), analysing the ideas based on betweenness centrality (measures the number of shortest pathes) and closeness centrality (evaluate the average distance from a *zeronodes* to all other connected nodes, see Wasserman and Faust (1994)).

The network theory is largely applied across the physical and social sciences, and social network analysis has been winning solution to study interactions among the individual, organizations, and society (Scott and Carrington (2011)) and labour markets (Montgomery (1991) and Granovetter (1995)).

These techniques have also been used to analyse unethical behaviour in organizations (Brass et al. (1998)), a concept useful for the shadow economy and detecting tax evaders. The structure of relevant taxpayer network is still not comprehended enough and researchers analysing the social structure of taxpayers use more simple types of networks in their models. Usually, many Agent-based Models are represented in ring worlds or lattice structures, where an agent has a certain radius of neighbours (Andrei et al. (2014)).

Much evidence has been found that networks are important to assess behaviour of taxpayer showing that the flow of information changes with the use of different network structure, especially in the dodging context.

Types of networks have been widely analysed in the literature during recent years. From these studies, we can get the main network model, which are shown below:

- No network consists of isolated nodes, i.e., there are no connections between entities within the set space;

- Von-Neumann neighbourhoods are a common structure in two-dimensional cellular automatic models. An agent with a Von-Neumann neighbourhood has four neighbours in the cardinal directions: one to the north, east, south, and west, creating a diamond-shaped; pattern on a graph (Weisstein (2012);

- Moore neighbourhoods are another common structure in two-dimensional cellular automata models. An agent with a Moore neighbourhood has eight neighbours in all of the cardinal and ordinal directions, forming a square-shaped pattern on a graph (Weisstein (2005));

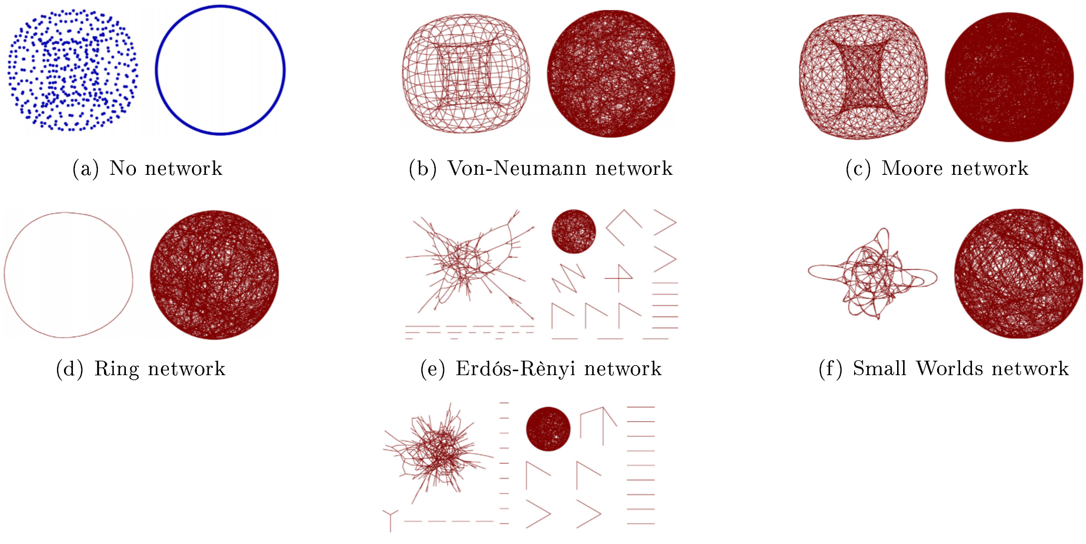
- Ring world networks are closed systems which are comprised of nodes that are connected to one node on either horizontal side (Boccaletti et al. (2006)) and the agent structure can be visualized as a helix bent around a circle to close in on itself;

- Erdós–Rènyi networks consist of vertices that are connected randomly. Extensions of this network have included connecting vertices with certain probabilities, or including a non-Poisson degree distribution (Boccaletti et al. (2006)). They are also known as *random graphs*;

- Small Worlds networks are usually generated on low-dimensional lattices. A fraction of links among nodes are broken and rewired with some probability to another node (Newman (2003)). The agents are viewed somehow "close" to each other. These networks are numerically large, decentralized, and highly united and clustered (Watts (1999)). They are also known as *Watts–Strogatz networks*;

- Power Law networks have a power law distribution of edges per node (Clauset et al. (2009)), where most nodes have only a few connections, but a small fraction of nodes are highly and disproportionately connected. This network is not able to represent single node (see Andriani and McKelvey (2005) and Barabási and Albert (1999)).

The graphic representation of network listed above, is the following:



(a) No network
(b) Von-Neumann network
(c) Moore network
(d) Ring network
(e) Erdós-Rènyi network
(f) Small Worlds network
(g) Power Law network

Figure 3.1: Types of networks

3.3 The model

The following Agent-based Model tries to show interactions among different agents from both demand and supply side. The model, modified in its substance and form, is based on Hokamp and Pickhardt (2010) model, where they analysed the evolution of tax evasion with different and alternative policies against heterogeneous agents. We using three different types of agents and government with control function:

- maximizing a-type agents;
- imitating b-type agents;
- ethical c-type agents.

We try to explain the dichotomy between qualitative (interactions among agents) and quantitative (amount of tax evasion) phenomena and we develop a simulation model with a base of empirical and theoretical model; putting together two different schools of economics and combining them we should offer a new possibility of interaction among quantitative and qualitative methods, underlying the importance of both in a complete and exhaustive analysis.

The following approach is based on exponential utility function, that remains a better way to assume a realistic audit probabilities and results respect the magnitude of tax evasion. The exponential utility function implies (Kirkwood (2004)) a constant absolute risk aversion (CARA), and obviously, it is a concave utility function.

The heterogeneity refers both on individual different income levels among the agents and three assorted types of agents, whereas we assume constant the risk aversion preferences.

With the following model, we change the original Hokamp and Pickhardt (2010) model adding a new variable (the strong punishment) leading to reduction of tax evasion, holding the interaction among agent typical of Hokamp and Pickhardt (2010) model, based in turn on Zaklan et al. (2009).

We distinguish the exponential utility function on based of punishment, and we use two different functions stand for lower penalty (e.g fines) and harder penalty (e.g. the imprisonment). The latter is viewed as discount of expected future incomes. The penalty rate π is always higher than tax rate θ ($\Longrightarrow \pi > \theta$), it occurs to give a disincentive for tax evasion in favour of taxes.

The number of agents $j - type$ and the number of tax relevant periods t are also parameters of the model. According to Bloomquist (2006) and Zaklan et al. (2009), Agent-based tax evasion models designed by economists usually deal with a range of just 1.000 agents.

3.3.1 A-type agents

Starting from Allingham and Sandmo (1972) and consequently Hokamp and Pickhardt (2010), we can analyse the utility function in case of fines (both consumers and firms):

$$E(U) = (1 - p_s)(1 - e^{-\lambda(W-\theta X)}) + p_s(1 - e^{-\lambda(W-\theta x-\pi(W-X)}) \tag{3.1}$$

Making derivative, concluding and re-arranging obtaining first and second order conditions, we obtain the lower and upper bound for A-type agents in the case of fines (see Hokamp and Pickhardt (2010)):

$$\frac{\theta}{\theta - (\theta - \pi)e^{\lambda\pi W}} < prob_s < \frac{\theta}{\pi} \tag{3.2}$$

Maintaining constant the level of risk aversion, we add a lump punishment (i.e. years of jail) translate into a poorer future wealth discounted for the imprisonment's years. We can describe this situation with addition of term $-\frac{\sum_{n=1}^{N} X}{(1+r)^n}$ in original utility function:

$$E(U) = (1 - p_s)(1 - e^{-\lambda(W-\theta X)}) + p_s(1 - e^{-\lambda\left(W-\theta x-\pi\left(W-X-\frac{\sum_{n=1}^{N} X}{(1+r)^n}\right)\right)}) \tag{3.3}$$

Adding this type (harder) of punishment, the Hokamp and Pickhardt (2010) original model is modified in the following way and applying the first and second order conditions in the case of imprisonment (see appendix A) we obtain:

$$\frac{\theta}{\left[\theta - (e^{\lambda\pi W})\left(\theta - \pi - \frac{\pi n}{(1+r)^n}\right)\right]} < prob_s < \frac{\theta}{\left[\theta - (e^{-\lambda\pi\omega})\left(\theta - \pi - \frac{\pi n}{(1+r)^n}\right)\right]} \tag{3.4}$$

and the inner solution is obtained setting first order condition and re-arranging:

$$\frac{\delta E(U)}{\delta X} = 0 \tag{3.5}$$

Making derivatives, we obtain the following inner solution:

$$X = \frac{1}{\left[\lambda\pi\left(1 - \frac{n}{(1+r)^n}\right)\right]}\left[\lambda\pi W - log\left(-\frac{\theta(1-p_s)}{C \cdot p_s}\right)\right] \tag{3.6}$$

The p_s is still the subjective probability of audit and it is equal to objective probability to audit of government. The interval of p_s became larger when the years of prison rise up according to stronger punishment, indeed the lower bound of interval goes down (leading to zero), while the upper bound goes up in appreciable way. In this case, the subjective probability represents the incentive to evade or not and, and more the interval cover the sample space of probabilities, more the agent will be discouraged evading taxes. That is, given their risk aversion due to CARA utility function

According to our purpose, agents declaring their entire income also in front of very low subjective probability of audit, when the years of prison rise up.

At the same level of risk for agents, the enhancement of penalty leads to lower level of subjective probability of audit; it involves that agents are encouraged to declare their entire income also when the subjective probability of audit is really small, close to zero in its lower bound as in the case of Hokamp and Pickhardt (2010).

Finally, the most interesting result is interval of subjective probability becomes larger in presence of harder punishment (i.e. jail), instead of presence of smoother punishment as illustrated in table 1,2,3,4 and *figure 2* shown below. The points on the blue line (*figure 2-b*, upper bound of penalty rate in the case of fines represent an incentive to evade for agents despite the high penalty rate assumed.[2] Instead in the case of fines, the penalty rate rises up, the agent could have an incentive to evade because of the range of subjective probability is smaller than case of imprisonment, and also in this case it leads to zero bound.

Obviously, the result is achieved at the same level of risk averse, whereas if we use an augmented risk aversion for agents, this condition should lead to larger interval both in the case of fines and jail.

Subjective probability changes with lower punishment - Lower Bound												
Period	0.25	0.5	1	2	3	4	5	6	7	8	9	10
Penalty rate	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99	1
Tax rate	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Sub audit prob	**1**	**1**	**0.3**	**0.12**	**0.051**	**0.024**	**0.011**	**0.01**	**0.009**	**0.006**	**0.003**	**0.000**

Table (3.1): Lower bound of subjective probability of audit in the case of fines

Subjective probability changes with lower punishment - Upper Bound												
Period	0.25	0.5	1	2	3	4	5	6	7	8	9	10
Penalty rate	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99	1
Tax rate	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Sub audit prob	**1**	**1**	**0.66**	**0.5**	**0.4**	**0.33**	**0.28**	**0.24**	**0.2**	**0.16**	**0.12**	**0.08**

Table (3.2): Upper bound of subjective probability of audit in the case of fines

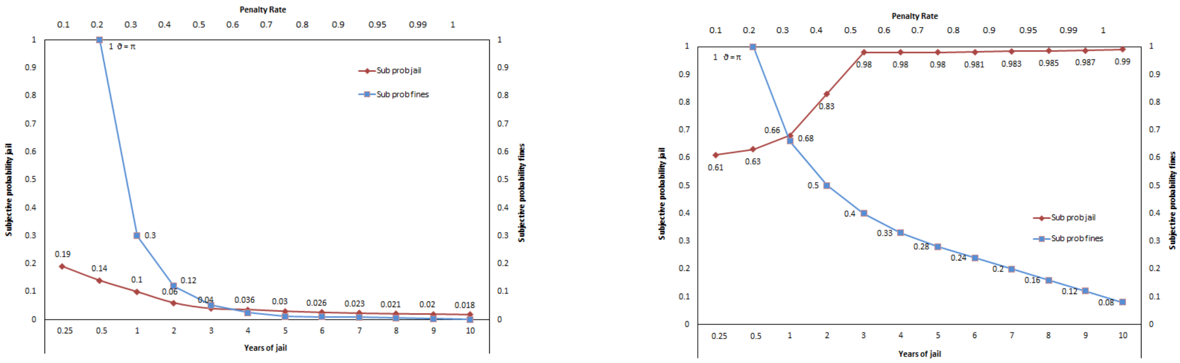
Subjective probability changes with higher punishment - Lower Bound												
Period	0.25	0.5	1	2	3	4	5	6	7	8	9	10
Year of jail	0.3	0.6	1	2	3	4	5	6	7	8	9	10
Tax rate	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Sub audit prob	**0.19**	**0.14**	**0.1**	**0.06**	**0.04**	**0.036**	**0.03**	**0.026**	**0.023**	**0.021**	**0.02**	**0.018**

Table (3.3): Lower bound of subjective probability of audit in the case of imprisonment

Subjective probability changes with higher punishment - Upper Bound												
Period	0.25	0.5	1	2	3	4	5	6	7	8	9	10
Year of jail	0.3	0.6	1	2	3	4	5	6	7	8	9	10
Tax rate	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Sub audit prob	**0.61**	**0.63**	**0.68**	**0.83**	**0.98**	**0.98**	**0.98**	**0.981**	**0.983**	**0.985**	**0.987**	**0.99**

Table (3.4): Upper bound of subjective probability of audit in the case of imprisonment

[2]The idea behind is that the fine is too much high and it will be impossible to pay it, so I can decide to evade taxes today.



(a) `LowerBound` (b) `UpperBound`

Figure (3.2): Comparison between subjective probabilities of fines and imprisonment

3.3.2 B-type agents

According to Hokamp and Pickhardt (2010), the following empirical example would give us a classical quantitative case to analyse the interaction among agents (both consumers and firms) based on p signal. The succeeding formula describes interactions among consumers:

$$X_{c,t} = \frac{1}{v} \sum_{n=c-v}^{c-1} \frac{X_{b,t-1}}{W_{b,t-1}} W_{c,t} \cdot p_{f,t} \tag{3.7}$$

while the next delineates interactions among consumers:

$$X_{f,t} = \frac{1}{g} \sum_{m=f-g}^{f-1} \frac{p_{h,t-1}}{W_{h,t-1}} W_{f,t} \tag{3.8}$$

where:

- v and g are social networks size (Zaklan, 2009), respectively for consumers and firms;
- h and b represents respectively the others firms and other consumers present into environment where they are situated, that is the market;
- t is the instant time, while the $t-1$ represents the first lag;
- $X_{i,t}$ and $X_{f,t}$ represents the declared income (for consumers) and the sales volume (for firms) only if tax evasion (on average) is observed within the social network agent whom belong;
- p is the price observed both consumers and firm from demand and supply side;
- W represents the *effective wealth* of consumers (demand side) and firms (supply side) hidden to the government.

According to Hokamp and Pickhardt (2010), if the tax evasion is observed, the following conditions are must fulfilled after agents first check:

CONSUMERS:

$$\frac{1}{v}\sum_{n=c-v}^{c-1}\frac{ATPI_{b,t-1}}{W_{b,t-1}} > 1-\theta_{t-1} \tag{3.9}$$

FIRMS:

$$\frac{1}{g}\sum_{m=f-g}^{f-1}\frac{ATPI_{h,t-1}}{W_{h,t-1}} > 1-\theta_{t-1} \tag{3.10}$$

where $ATPI$ is after tax and penalty income both consumers and firm. Whether the double conditions above do not hold, the agents decide to declare their true income, or in other words, try to imitate the *ethical agents*.

3.3.3 C-type agents

Finally, there are *ethical agents*, who decide do not evade and they declare in any case their entire income (both consumers and firms):

$X = W$ or $X - W = 0$, true in the case of no tax evasion.

Simplifying the notation, the X represents the sales for firms, while in the case of consumers the X corresponds to the whole declared income for each type of agent, which is equal to gained income.

3.3.4 The government

In the market (the environment), the government[3] is present with monitoring tasks and he has risk-neutral utility function (principal-agent theory) because of different and copious portfolio choices. Also in this case, government (the principal) should be try to maximize its utility function, and it would discover dodgers (the agents) who present risk-aversion utility function (Jensen and Meckling (1976)):

$$E[U(G)] = \alpha V(1-c_1) + (1-\alpha)V(-c_2) \tag{3.11}$$

where:

- $(1-c_1)$ is the audit success;
- $(-c_2)$ is an unsuccessful audit (see as $(0-c_2)$).

The maximization problem is restricted by participation constraint when the agent is not evader *(ne)*:

$$s.t. \qquad \alpha(ne)U(Pr_1) + (1-\alpha(ne))U(Pr_2) - \theta \geq \overline{U} \tag{3.12}$$

where:

- (Pr_1) is the premium paid by government in the case of evasion, but the agent is not dodger.
- (Pr_2) is the premium paid by government when the it did not discover evasion and the agent is not dodger.
- $(Pr_1) > (Pr_2)$ because the (Pr_1) is *premium ad hoc*[4], while (Pr_2) is a *converted premium.*[5]

[3] We assume a welfare government indicating the will to increase welfare of its citizens.
[4] It is an incentive against tax evasion, available only for single agent.
[5] It is a collective premium (as a distribution of better public services) for all agents.

3.4 Simulation scenario

We model an economy based on the monopolistic competition, in which there is free entry for suppliers - they are characterized by several small firms (Stiglitz (1986), Dixit and Stiglitz (1993)) and the products are differentiated for the quality and recognisable through the prices (Chamberlin (1961)). Different quality levels represent competition (Saviotti and Pyka (2004)) and firms are heterogeneous with respect to the product, competing on the basis of price (Soete and Verspagen (1993), , Lorentz and Savona (2009)). We assume the level of price as "signal" of a possible tax evasion, where variation of price reflects an opposite fluctuation of demand.

Consumption decisions are responsible for firm activity and there is a probabilistic choice to purchase a product by consumers (Oliva et al. (1995)) observing its price. If consumer decides to purchase good, he only knows that a lower price may denote either a worse quality or an attempt of producer to evade, given the presence of cognitive limitations and the limited information on market structure (Simon (1957), Nelson (1970)).

In the opposite, Stiglitz (1969) highlights the importance of information for price stability. As Valente (2012) argued, the consumers often have few information on product details, and primarily they seek a product with definite characteristics - useful for their level of utility - and subsequently look at minimum price of these goods.

Supposing a classic supply economic law, each supplier chooses one product and he assigns it to one consumer, having a population of $f \in \{1, 2, ..., F\}$ firms, who want to maximize their profits equal to[6]:

$$\Pi = \bar{V}_t[(\pi \cdot g + E) - \Phi \cdot f] - (1 - \Phi) \cdot L \tag{3.13}$$

We leave out the analysis of entire set of production costs: wages, capital and other costs on investments, while considering only the evading costs, as fines and imprisonment. In our model, firms are eligible to product and have a sustainable costs structure, according *Neo-Schumpeterian Simulation Models* (see Chiaromonte and Dosi (1993), Dosi et al. (1994), , Nelson and Winter (2009). In this way they are able to be placed in the market.

We assume the current expected sales as a convex combination of past expectations, in line with the share of demand faced by a firm Y_{t-1}:

$$Y_t^e = (1 - \alpha)Y_{t-1}^e + \alpha Y_{t-1} \tag{3.14}$$

The formula shows a slow adaptation in sales expectations as an outcome of agents' conservative behaviour. In order to cover unexpected changes in demand, firms maintain a constant level of production (see Ciarli et al. (2010) and Caliari et al. (2017)).

We can outline each single product as a set of several characteristics (m) (see Ciarli and Valente (2005)), but for simplicity they are reduced only two: the price (i_p) and the quality (i_q). Furthermore, each product satisfies consumer needs, as it occurs in *Lancasterian*, (Lancaster (1966a) and Lancaster (1966b)) and *post-Lancasterian* (Saviotti and Metcalfe (1984)) approach in the consumption theory.

Finally, each firm can observe - through normal random function - up to ten firms connected.

As in the case of supply, from demand side we leave out the disposable income and we focus only on the consumer behaviour. We model bounded rational consumption behaviour inspired by the literature on experimental psychology, which has the properties of empirically observed behaviour (Gigerenzer et al. (1997) and Gigerenzer et al. (2001)). To find the junction with economic literature we can draw lexicographic preferences[7] that have been developed to arrange consumers' purchasing decisions (Valente (1999)). The model implements independent consumer choices (see Ciarli et al. (2010)), assigned to each firms using the

[6]It has been suppress the index of firms, but is implicit that each equation is replicated for each firm.

[7]Lexicographic preferences or lexicographic orderings describe comparative preferences where an agent prefers any amount of one good (X) respect to another good (Y). Precisely, if offered several bundles of goods, the agent will choose the bundle that offers more X, without interest on bundle Y. Only when there is a tie between bundles regard the quantity of X,

perceived values on the price and quality of goods; the mechanism works through normal random function with specific parameters. Several repetition of the same choice (*t-times*) in the simulation, guarantees random choice and random product evaluation of firms by consumers that decide to buy only best product for them (on the basis of highest quality or lowest price (Ciarli and Valente (2005))).

We have a population of $c \in \{1, 2, ..., C\}$ consumers and their ultimate goal is maximize the utility deriving from the choice of product composed by the quality combined with the difference of unpaid tax and extra gain for supplier, translated into lower purchase price:

$$U = b + E \cdot (g - t)$$

The choice of consumers is probabilistic, depending on the acceptance of tax cheating firms by the consumer and on the utility of the product offered by different sellers (in the code the equation register also the selling of firms):

$$X_t = (1 - \alpha)X_{t-1} + \alpha U$$

Finally, each consumer can observe - through normal random function - up to fifty consumers connected.

We are interested in tax evasion and fear to evade when the strong punishment occurs and we investigate how they move when the policies change. The policies are introduced by central government, and each alteration has a different effect on its utility and total controls carried out. The utility of government is measured by fines, that are a monetary size. In this case, when imprisonment is introduced in the environment, we expect a decreasing in its level. Unluckily, we do not able to capture with this model an increasing of government utility through greater revenues by taxes paid.

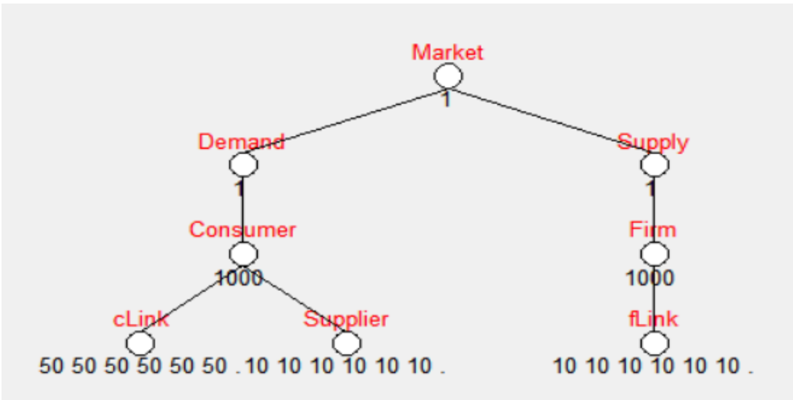


Figure (3.3): Simulation model, 2017.

The aim of the analysis is to check whether an introduction of stronger punishment can lead to reduce the level of tax evasion on aggregate level. In order to consider the microeconomic effects, we rely on an initialization with a sufficient heterogeneity across firms and consumers[8]. The economy represented in the following simulation is composed of $f = \{1, 2, ..., 1000\}$ firms and $c = \{1, 2, ..., 1000\}$ in the market - controlled by government - and we assume that the number of firms does not change over time.

Firms initially differ only with respect to the quality of good and final price, whereas they are identical with respect to all other initial conditions. They are interested to maximize their profits, and when they

agent starts to compare the number of units of Y across bundles. With lexicographic preferences, the utility of definite goods is infinitesimal in relation to others (Sen (1970)). Furthermore, lexicographic orderings are complete, transitive, strongly monotone and strictly concave. Nevertheless, there is no utility function that can represent this type of relation of preferences, but the assumption of continuous is crucial to ensure the existence of a utility function (Mas-Colell et al. (1995)).

[8]The list of main variables and parameters is available in the table Appendix D.

face a fine as penalty, the gain from evasion is higher for them. In the opposite case (imprisonment), they are in front of an higher fear to evade and they are encouraged to declare their entire earnings. In the code this situation is translated in decrease in profits resulted by illicit activities, with consequent reduction of evasion.

From demand side, consumers can obtain the maximization of utility with their choices of consumption. Indeed, they will choose good with the lowest price with the same quality. The consumers support a trade-off: choosing (in probabilistic way) a lowest price good and risking to be discovered as evaders, or pay higher price incurring in a cost.

Finally, we investigate efficiency (inefficiency) of government (in the sense of credibility), the influence of other peers (against a higher value of direct previous experience) along the fluctuation of tax avoidance and the fear to evade for agents when some policies are used rather than others.

The following results discussed are averages obtained over 10 simulation runs with different random properties for 120 time-steps each one. As Ciarli et al. (2008) showed, it is a adequate number of simulation to verify the series volatility due to random cases. Some graphs are represented in logarithm to give a clearer view.

3.4.1 The simulations

Simulation 0 The simulation "zero" (*figure 4*) represents an high inefficiency of government in the control. The *ProbControl* is equal to 0.01 while the *CostControl* is 0.02, so there is no high importance to expectations of being controlled by government *AMemControlled* and previous personal experience *WeightPersMemory* very low, both equal to 0.9 (it means a greater weight of peers) in the supply side. On the demand side, the utility of consumer (deriving from the last product purchased) and the probability to accept a seller as evader remain constant, whereas we play on the consumer previous personal experience *WeightDirExp*, here equals to 0.5. The standard deviation used to quantify the amount of dispersion, it becomes flatter during long run and the diffusion from the means is greater within evading rather than fear to evade.

Given the government inefficiency (the range of control is $3 - 117$), the evading is close to 0.6 and the fear to evade is always below that. This simulation is useless to fight tax evasion. Furthermore, the government inefficiency is measured by low level of its utility (in average it is equal to 11.2).

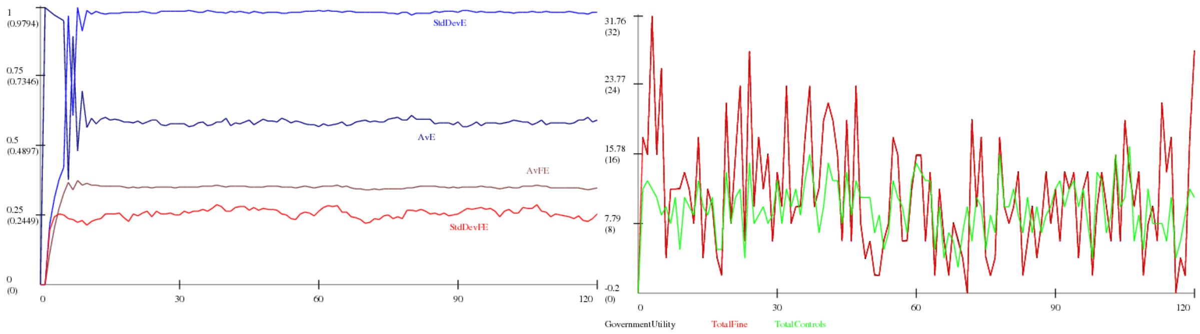


Figure (3.4): The government utility versus tax evasion and fear evading - Sim 0

Simulation 1 With this simulation, the government has greater reliability about his role, and the probability of control is risen up to 0.1 (10%). Greater credibility is translated into more controls (between 77 and 121). Moreover, a larger number of controls and embitterment of penalties[9] lead to tax evasion (0.474) below fear evading (0.520). The government is become efficient and his utility is higher than previous case, even if the cost control is much higher (*figure 5*).

When the probability to control rise up to 10%, also the fear to evade rises up. We expect a decreasing of effective average evading. So, when central government became more credible in its final scope, the average evading tends to decrease. According to Sim.0, the standard deviation becomes flatter during long run, but within the series of *FearEvading* we observe a very low deviation within this simulation.

On the right graph, we can see the utility of government is risen up around 10% (11.2 versus 110.3) according with the increase of fines inflicted, originated by controls carried out by central government. The controls (situated in range 77 − 121) are risen up around 10% in average while the penalty based on fines are increased more than 10% (11.42 versus 120.3).

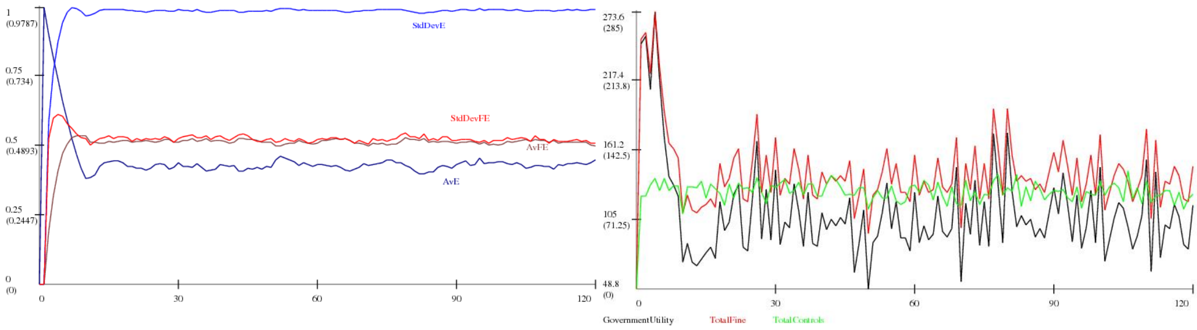


Figure (3.5): The government utility versus tax evasion and fear evading - Sim 1

Simulation 2 Starting from the previous simulation, we decrease *AMemControlled* and previous personal experience *WeightPersMemory* for supplier, both equal to 0.2. On the demand side, we use an higher level of personal experience *WeightDirExp*, here equals to 0.2. A value lower of 0.5 of *WeightPersMemory* and *WeightDirExp* gives a less weight to peers on the agent's choices respect to previous experience, both in the supply and demand side.

When the influence of peers is missed (*figure 6*), to fight tax evasion is necessary increasing up the level of credibility of controller (here it equals to 0.5 with a cost 0.6) [10] that is central government. In this case, analysis of results seems to be very jagged. Here the standard deviation reflects the randomization of series, observing an higher dispersion for tax evasion whereas this feature is cancelled with the fear to evade, overlapping themselves.

Finally, the utility of government is higher than previous simulation, given the greater probability to undergo a check. In this case, it is overlapped to total amount of fines. Unlucky, the situation is almost unrealistic because the government never has an high efficiency to carry out controls.

[9] Unitfine is risen to 3.0, but also the gain from tax evasion has been made more attractive.

[10] The importance of previous experience (where the decision to evade or not is almost total independent and different for each agent) compared to influence of peers (i.e. the external environment), does not permit to forecast the level of tax evasion, depending on case by case. Anyway, in this case the level of tax evasion is around 0.355, an half of fear to evade.

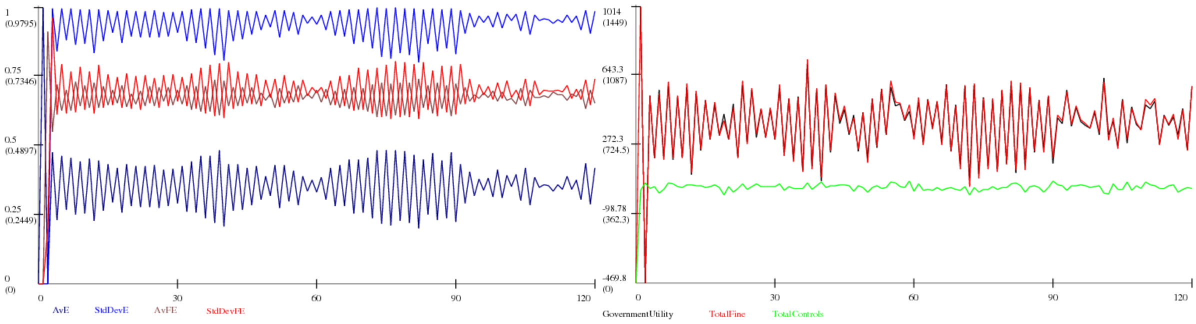


Figure (3.6): The government utility versus tax evasion and fear evading - Sim 2

Simulation 3 (*Figure 6*) represents how the choices change in the environment where is provided with a stronger punishment. We assume a lump punishment equals to 10.0, which are the years of imprisonment for the cheaters. The probability of inspection is 0.5 with a cost of control equals to 0.9, ceteris paribus in accordance with simulation 2.

Despite the randomization compared to previous simulation, we can observe a greater intensity of *AvE* and *AvFE*. It depends on a lower gain evasion, which combined with an higher personal weight experience[11] makes these variables more unstable. According to previous simulation, here the standard deviation reflects in a heavier way the randomization of both of series, reducing in some cases the scattering also between *AvE* and *StdDevE*. However, the average of tax evasion is lower in this case (0.273 against 0.355 in the case of fines), which would seems to confirm our starting idea.

In conclusion, the previous experience has a smoothing role about the routine to evade, whereas the possibility of observe other agents and give them greater importance - consequently imitate them - lead to an higher - lower - level of average evading (In an environment with high - low - tax evasion)[12].

As we know,the utility of government depends on majority part by fines; in fact fines may represent a refund for the government in presence of evasion. With the introduction of lump punishment there is a missing revenue in its utility function. When the tax evasion is punished by imprisonment, the government utility is very low with the lower influences from other peers, the decision to evade (average evading) is almost total independent and different for each agent and it has an opposite correlation with the evading fear. Consequently, also utility of government is very jagged, despite of a greater range of control (459-536, in average 501).

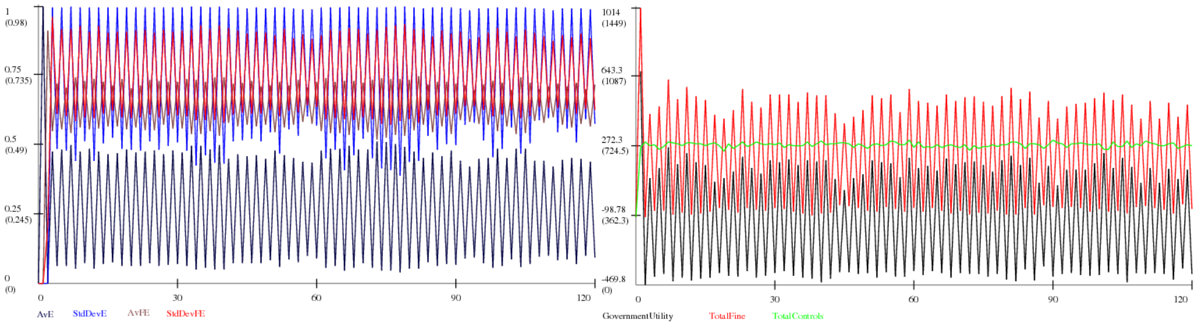


Figure (3.7): The government utility versus tax evasion and fear evading - Sim 3

[11] Remembering that the decision to evade or not is almost total independent and different for each agent.
[12] Summarizing, the agent is driven by the idea that *"if he can do it, I can do it myself too"*.

Simulation 4 We increase *AMemControlled* and previous personal experience *WeightPersMemory* for supplier, both equal to 0.9. On the demand side, we use a lower level of consumer previous personal experience *WeightDirExp*, in this case is equal to 0.5, ceteris paribus. Furthermore, we decrease *ProbControl* to 0.1 with a cost of control equals to 0.5.

In opposite to previous simulation, the agents tend to uniform their behaviour when the adaptation of being controlled and the previous experience is lesser important than external agents' environment in the evading choices for single agent, despite of low-level efficiency of controller.

We can observe that the series randomization derives exclusively from the previous experience about tax evasion, that leads to opposite behaviour in the next time step (both demand and supply side) within the economic environment.

In case of tax evasion becomes more important - and therefore makes the system more stable - the influence of external environment with respect to the past experience of the agent . In fact, if the agent is inclined to evade taxes, he will do so if he is in a favourable environment where everybody do it, otherwise he will not. Furthermore, if the agent has already suffered a check or punishment for evading, he will be less inclined to evade taxes again. This situation is represented in *figure 8*. Furthermore, the standard deviation becomes flatter during long run, but the dispersion in this case is negative for the fear whereas it is positive for the evading, both in more dramatically way.

In this simulation, the imprisonment is useful to fight tax evasion (here is equal to 0.229 in average), finding an other confirm that a stronger penalty can lead to an abatement of tax evasion with a greater fear to evade.

On the right side of *figure 8*, we observe very low level of utility for the government. It is typical because of the utility follows the amount of fines whereas in this case they are substituted by years of jail. The utility is equal to 72.49 against 82.50 in the case of fines while the range of controls is included between 77 and 121 (100.1 in average).

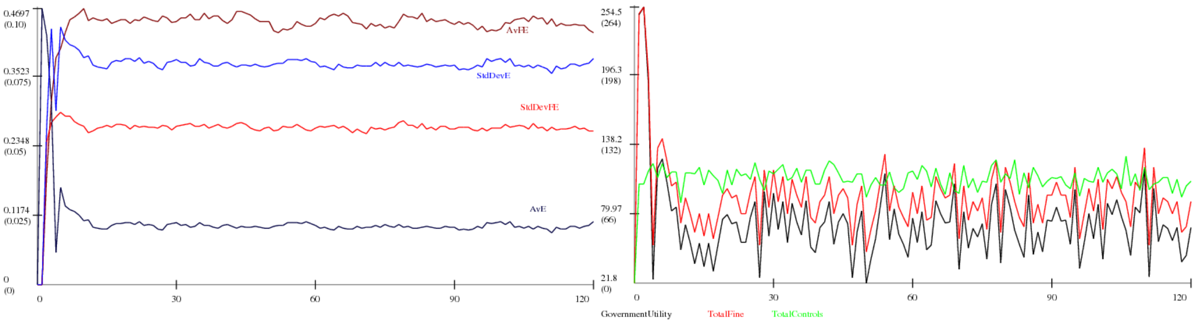


Figure (3.8): The government utility versus tax evasion and fear evading - Sim 4

Simulation 5 We measure (*figure 9*) how the choices change with an introduction of exogenous shock during running time: years of imprisonment as punishment and the parameter, at time step 31st, increases from 0.0 to 10.0.

The probability of inspection is 0.1 with a cost of control equals to 0.5, while we set *AMemControlled* and previous personal experience *WeightPersMemory* for supplier, both equal to 0.9. On the demand side, we use a level of consumer previous personal experience *WeightDirExp* equals to 0.5.

We can observe a structural break both in average evading and government utility (*see figure 9*). After the shock, we see an abatement of tax evasion around 30%. This lower level of tax evasion seems confirming the idea that stronger punishment could lead to fight tax evasion and consequently to decrease it. Note that the fear to evade seems decreasing with the insertion of stronger penalty. This weird trend is justified by the code in *Lsd*, where the *FearEvading* depends only by the level of fear during period $t-1$ and the sales at time t, weighted for the level of tax evasion in that moment. Sales remain stable, but after the introduction of *LumpPunishment* they are recorded in legal transaction whereas the tax evasion level is decreased. About standard deviation we can observe that it becomes flatter during long run and they suffer in equal measure of the exogenous shock, drastically decreasing the dispersion from the mean.

Moreover (see the right side of *figure 9*) where at time step $31st$ is present a structural break both in government utility and amount of inflicted fines), we have seen that a lump sum punishment decrease a government utility level because of the revenues are lesser than in the case of fines; but in a wider context we must consider that a greater tax revenues much more compensate the less revenue from the lack of fines inflicted.

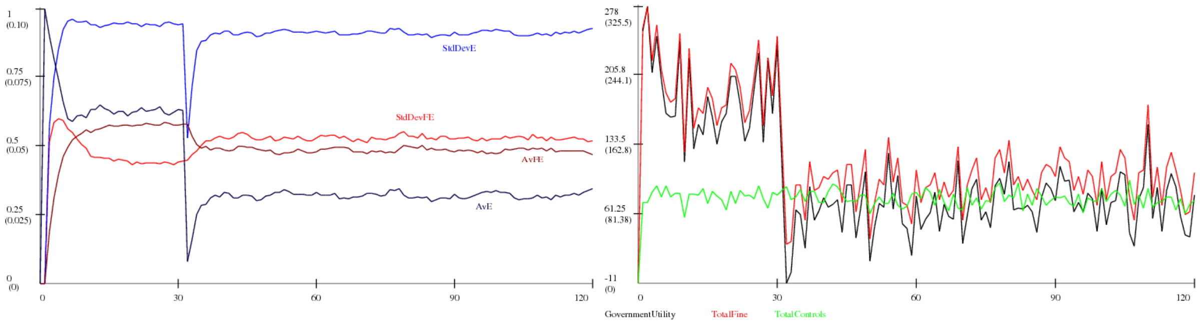
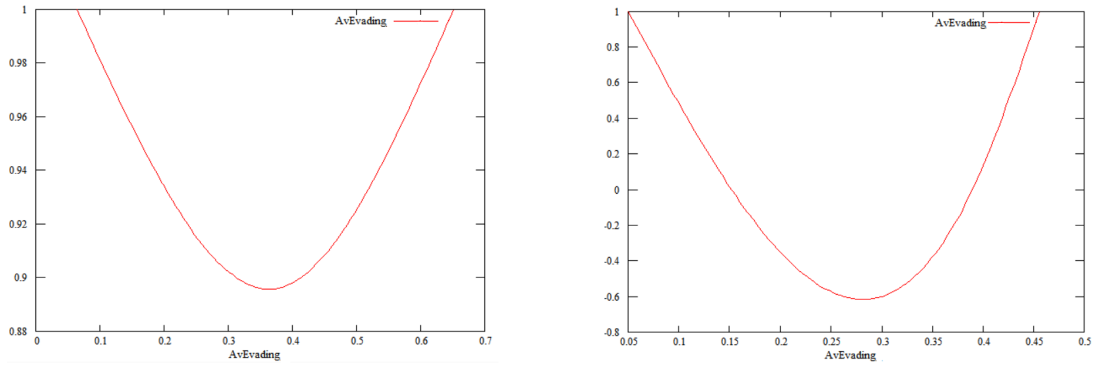


Figure (3.9): The government utility versus tax evasion and fear evading - Sim 5

Cross section simulations In this analysis is clear how scale of evasion and fear evading is completely different when we introduce a lump sum punishment in the environment.

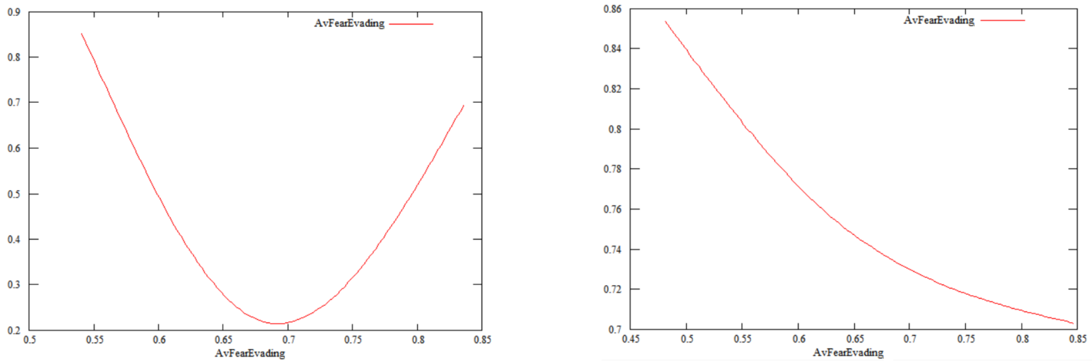
The *figure 10* shows two different time steps: time $29th$ and time $70th$. The first graph represents the time before introduction of stronger penalty where scale is higher (between 0.9 and 1) representing the situation before lump sum punishment. The second graph provides a softer scale of evading, indicating a strong reduction of tax evasion. All steps represent convex function in this case.



(a) `AvEva29` (b) `AvEva70`

Figure (3.10): Cross section analysis about tax evasion

Figure 11 instead shows the same results of the previous figure, but considering fear evading: in this case the first graph have a larger scale (it indicates a smaller fear when agent does not pay taxes, at time



(a) `AvFear25` (b) `AvFear120`

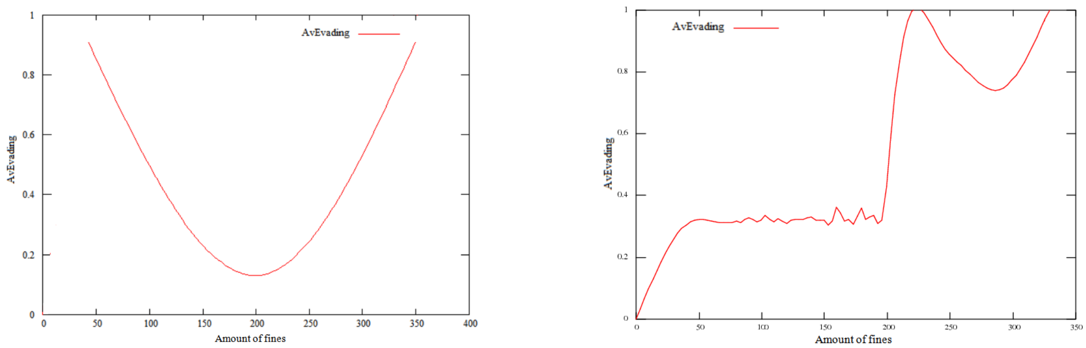
Figure (3.11): Cross section analysis about fear to evade

step 25*th*) whereas, after time step 31*th*, the scale becomes closer and it represents an higher fear evading for agents. The interval - with a stronger punishment - is included between 0.86 and 0.7 (time step 120*th*).

Before introduction of lump sum punishment we can observe a lower level of fear evading (around 0.2) in line with our hypotheses. In the case of imprisonment (e.g the exemplary punishment) the level of fear evading rises up to 0.7, having a strong increment of that.

It seems to confirm our starting hypotheses in accordance with quantitative model previous examined.

In the *figure 12* we show the path evolution of two type of punishment. With respect to fines, we observe a constant path of *AvE* around 0.3, until to an amount of fines equals to 200. After this threshold, the average of evading is climbed up towards maximum value. In line with the idea of agent[13] after a certain threshold, the high-peak fines loses their anti-evasive role and it becomes an incentive to evade taxes rather combating them. The function represents on the left side of *figure 12* assume a strictly convex trend for this type of penalty.



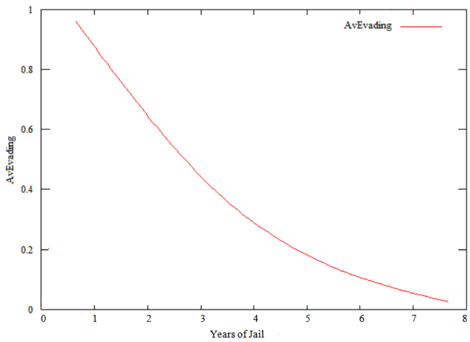
(a) `finevsevading` (b) `finevsevading`

Figure (3.12): Analysis about tax evasion in the case of fines

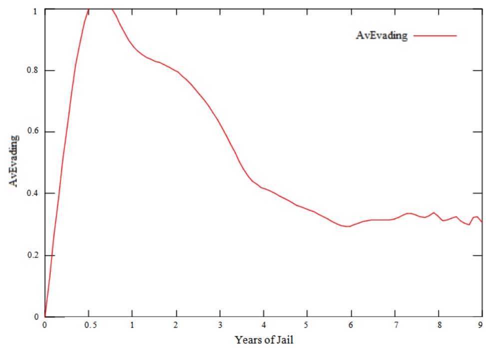
In the opposite (*figure 13*), when a stronger punishment is included in the economic environment, its trend is defined as decreasing, in line with the starting idea that an tightening of penalty lead to an

[13]The agent in some cases is driven by the idea that fine could be so high that it will be impossible to pay it, so he is pushed to evade taxes.

abatement of tax evasion. The function represents on the left side of *figure 13* assume a decreasing trend for this type of penalty.



(a) `jailvsevading`



(b) `jailvsevading`

Figure (3.13): Analysis about tax evasion in the case of imprisonment

3.5 Concluding remarks

In this paper we have simulated income tax evasion in a society of heterogeneous agents, where heterogeneity refers to different behavioural types of agents, but with the same risk aversion.

The simulation results presented widely confirm the findings of previous theoretical, experimental on Agent-based studies (e.g. Allingham and Sandmo (1972); Andrei et al. (2014); Alm et al. (2004). In particular, raising the audit probability or the tax rate on undeclared income would lead - *ceteris paribus* - to less tax evasion and vice versa.

Whether we introduce a stronger punishment, this can lead to lower tax evasion ratio, especially in the case of high weight of the external environment, indicating by larger influence of peers. At the same time we found that previous direct experience is relevant, but only in the case of an high government's efficiency in controlling. Indeed, only an high weight of personal memory lead to results fully random.

Besides, we stressed the efficiency and efficacy of government that is reflected on subjective probability to be caught: an higher (lower) value lead to lower (higher) tax evasion ratio and vice versa in expected way.

Final results from "pure" simulations (Ciarli and Valente (2005)), seems to confirm the quantitative results based on subjective probabilities of each agent, analysing various situations through different simulations. Indeed, when a stronger penalty is included in the economic setting there is a reduction of tax evasion assessed between 10% and 30%.

These results corroborate our thesis and then agents tend to declare their whole income when imprisonment is included in the economic system.

We have shown as quantitative and qualitative analysis, nonetheless they are two different models, being useful to achieve the same results completing each others. Indeed, we have demonstrated with quantitative one that sometimes the analysis is truncated and a qualitative investigation could strengthen the former inquiry.

Finally, being difficult to improve the situation of central government, this work suggests that an embittering of laws reduces the level of tax evasion independently of the type of external environment. Otherwise, to fight tax evasion is necessary improving the efficiency of central government, but this is very difficult to implement in the process.

Appendix A - Derivation of maximization of A-type agents

If we include a stronger punishment in Hokamp and Pickhardt (2010) model - in our case the years of imprisonment - the utility function change as follows:

$$E(U) = (1-p_s)(1-e^{-\lambda(W-\theta X)}) + p_s(1-e^{-\lambda\left(W-\theta x-\pi\left(W-X-\frac{\sum_{n=1}^{N} X}{(1+r)^n}\right)\right)}) \tag{3.15}$$

$$\frac{\delta E(U)}{\delta X} = (1-p_s)(-e^{-\lambda(W-\theta X)})\lambda\theta + p_s\big(-e^{-\lambda\left(W-\theta X-\pi(W-X\frac{\sum_{n=1}^{N} X}{(1+r)^n})\right)}\big)\left(\lambda\theta - \lambda\pi - \frac{\lambda\pi n}{(1+r)^n}\right) \tag{3.16}$$

Starting again from Allingham and Sandmo (1972) where they found two constraints and taking the first-order derivative with respect to X, we can obtain the necessary and sufficient condition for an inner solution, at the limits of income ($X = 0$ and $X = W$):

$$\left.\frac{\delta E(U)}{\delta X}\right|_{X=0} > 0 \tag{3.17}$$

$$(1-p_s)(-e^{-\lambda W})\theta + p_s(-e^{-\lambda W+\lambda\pi W})\left(\theta - \pi - \frac{\pi n}{(1+r)^n}\right) > 0 \tag{3.18}$$

$$-\theta(1-p_s)(e^{-\lambda W}) - \theta p_s(e^{-\lambda W+\pi W}) + \pi p_s(e^{-\lambda W+\lambda\pi W}) + \left(\frac{\pi n}{(1+r)^n}\right)p_s(e^{-\lambda W+\lambda\pi W}) > 0 \tag{3.19}$$

$$p_s(e^{-\lambda W})\left[\theta - \theta e^{\lambda\pi W} + \pi e^{\lambda\pi W} + \frac{\pi n}{(1+r)^n}\right] > \theta(e^{-\lambda W}) \tag{3.20}$$

$$prob_s > \frac{\theta}{\left[\theta - \theta e^{\lambda\pi W} + \pi e^{\lambda\pi W} + \frac{\pi n}{(1+r)^n}\right]} \tag{3.21}$$

and re-arranging the same terms:

$$prob_s > \frac{\theta}{\left[\theta - (e^{\lambda\pi W})\left(\theta - \pi - \frac{\pi n}{(1+r)^n}\right)\right]} \tag{3.22}$$

Instead, the upper bound is the following:

$$\left.\frac{\delta E(U)}{\delta X}\right|_{X=W} < 0 \tag{3.23}$$

$$-\theta(1-p_s)(e^{-\lambda W+\lambda\theta W}) - p_s\big(e^{-\lambda W+\lambda\theta W-\lambda\pi\frac{\sum_{n=1}^{N} W}{(1+r)^n}}\big)\left(\theta-\pi-\frac{\pi n}{(1+r)^n}\right) < 0 \tag{3.24}$$

$$-\theta(e^{-\lambda W+\lambda\theta W}) + \theta p_s(e^{-\lambda W+\lambda\theta W}) - p_s\big(e^{-\lambda W+\lambda\theta W}e^{-\lambda\pi\frac{\sum_{n=1}^{N} W}{(1+r)^n}}\big)\left(\theta-\pi-\frac{\pi n}{(1+r)^n}\right) < 0 \tag{3.25}$$

To simplify the computation, we can set $\frac{\sum_{n=1}^{N} W}{(1+r)^n} = \omega$ and we are going to arrange:

$$p_s(e^{-\lambda W+\lambda\theta W})\left[\theta-(e^{-\lambda\pi\omega})\left(\theta-\pi-\frac{\pi n}{(1+r)^n}\right)\right] < \theta(e^{-\lambda W+\lambda\theta W}) \tag{3.26}$$

$$prob_s < \frac{\theta}{\left[\theta-(e^{-\lambda\pi\omega})\left(\theta-\pi-\frac{\pi n}{(1+r)^n}\right)\right]} \tag{3.27}$$

Finally, we obtain again the necessary and sufficient condition for an inner solution, given our specification:

$$\frac{\theta}{\left[\theta-(e^{\lambda\pi W})\left(\theta-\pi-\frac{\pi n}{(1+r)^n}\right)\right]} < prob_s < \frac{\theta}{\left[\theta-(e^{-\lambda\pi\omega})\left(\theta-\pi-\frac{\pi n}{(1+r)^n}\right)\right]} \tag{3.28}$$

The interval of p_s (remember, that is a subjective probability of audit and it is equal to objective probability of government) when the years of prison rise up according to stronger punishment, leads to an enhanced interval of audit, because the lower bound of this interval goes down (leading to zero), while the upper bound goes up in appreciable way.

According to our purpose, this can lead to agents declaring their entire income also in front of very low subjective probability of audit and when the years of prison grow up, the "scissors" of subjective probability of audit became larger.

At the same level of risk for agents, the enhancement of penalty leads to lower level of subjective probability of audit; it involves agents are encouraged to declare their entire income also when the subjective probability of audit is really small, close to zero lower bound (Hokamp and Pickhardt (2010)).

The most interesting result is that interval of subjective probability becomes larger in presence of harder punishment (i.e. the imprisonment), instead of presence of smoother punishment. Obviously, this result is achieved at the same level of risk averse. Whereas when the agent is more risk averse, this leads to larger interval both in the case of fines and jail; that is a confirmation of our research idea.

The inner solution is obtained again setting first order condition and re-arranging:

$$\frac{\delta E(U)}{\delta X} = 0 \tag{3.29}$$

$$(1-p_s)(-e^{-\lambda(W-\theta X)})\lambda\theta + p_s\big(-e^{-\lambda\left(W-\theta X-\pi\left(W-X-\frac{\sum_{n=1}^{N} X}{(1+r)^n}\right)\right)}\big)\left(\lambda\theta-\lambda\pi-\frac{\lambda\pi n}{(1+r)^n}\right)=0 \tag{3.30}$$

To simplify the items, we can set $\left(\lambda\theta-\lambda\pi-\frac{\lambda\pi n}{(1+r)^n}\right)$ as constant C, and simplifying for λ, we obtain:

$$\frac{\delta E(U)}{\delta X}=(1-p_s)(-e^{-\lambda(W-\theta X)})\theta + C\cdot p_s\big(-e^{-\lambda\left(W-\theta X-\pi\left(W-X-\frac{\sum_{n=1}^{N} X}{(1+r)^n}\right)\right)}\big)=0 \tag{3.31}$$

$$=-\theta(1-p_s)[(e^{-\lambda(W-\theta X)}]-C\cdot p_s\big(e^{-\lambda(W-\theta X)})(e^{\lambda\pi\left(W-X-\frac{\sum_{n=1}^{N} X}{(1+r)^n}\right)}\big)=0 \tag{3.32}$$

Isolating the term $e^{-\lambda(W-\theta X)}$ and note it does not have solution for 0,

$$=(e^{-\lambda(W-\theta X)}[-\theta(1-p_s)-C\cdot p_s\big(e^{\lambda\pi\left(W-X-\frac{\sum_{n=1}^{N} X}{(1+r)^n}\right)}\big)=0 \tag{3.33}$$

we can solve only this part:

$$-\theta(1-p_s)-C\cdot p_s\big(e^{\lambda\pi\left(W-X-\frac{\sum_{n=1}^{N} X}{(1+r)^n}\right)}\big)=0 \tag{3.34}$$

$$e^{\lambda\pi\left(W-X-\frac{\sum_{n=1}^{N} X}{(1+r)^n}\right)}=-\frac{\theta(1-p_s)}{C\cdot p_s} \tag{3.35}$$

$$\lambda\pi W-\lambda\pi X-\frac{n\cdot X}{(1+r)^n}=log\left(-\frac{\theta(1-p_s)}{C\cdot p_s}\right) \tag{3.36}$$

$$\lambda\pi X-\frac{n\cdot X}{(1+r)^n}=\lambda\pi W-log\left(-\frac{\theta(1-p_s)}{C\cdot p_s}\right) \tag{3.37}$$

$$X\left[\lambda\pi\left(1-\frac{n}{(1+r)^n}\right)\right]=\lambda\pi W-log\left(-\frac{\theta(1-p_s)}{C\cdot p_s}\right) \tag{3.38}$$

$$X=\frac{1}{\left[\lambda\pi\left(1-\frac{n}{(1+r)^n}\right)\right]}\left[\lambda\pi W-log\left(-\frac{\theta(1-p_s)}{C\cdot p_s}\right)\right] \tag{3.39}$$

Remember that $C<0$ because the penalty rate π is higher than tax rate θ, so $\left(\lambda\theta-\lambda\pi-\frac{\lambda\pi n}{(1+r)^n}\right)<0$ and $log\left(-\frac{\theta(1-p_s)}{C\cdot p_s}\right)\big]$ becomes positive.

Appendix B - Additional results of simulation

Simulation 0					
Market	Value	**Supply**	Value	**Demand**	Value
ProbControl	0.01	Unitax	1.5	aUtility	0.9
CostControl	0.02	Unitfine	2	aProbEvasion	0.9
		UnitGainEvasion	1.5	WeightDirExp	0.5
		LumpPunishment	0.0		
		AMemControlled	0.9		
		WeightPersMemory	0.9		

Simulation 0 - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
AvEvading	0.604	0.007	0.38	1	0.087
FearEvading	0.337	0.002	0	0.367	0.040
StDevEvading	0.932	0.0021	0.979	1	0.117
StDevFearEvading	0.248	0.0007	0	0.282	0.028
GovernmentUtility	11.2	45.38	-0.2	31.76	6.765
TotalFines	11.42	45.83	0	32	6.795
TotalControls	9.858	9.022	3	117	3.016

Table (3.5): Simulation 0: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months.

Simulation 1					
Market	Value	**Supply**	Value	**Demand**	Value
ProbControl	0.1	Unitax	2.0	aUtility	0.9
CostControl	0.1	Unitfine	3.0	aProbEvasion	0.9
		UnitGainEvasion	2.5	WeightDirExp	0.5
		LumpPunishment	0.0		
		AMemControlled	0.9		
		WeightPersMemory	0.9		

Simulation 1 - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
AvEvading	0.363	0.010	0.306	1	0.101
FearEvading	0.478	0.003	0	0.518	0.0655
StDevEvading	0.917	0.003	0.980	1	0.093
StDevFearEvading	0.529	0.002	0	0.600	0.050
GovernmentUtility	110.3	1379	48.8	273.6	37.28
TotalFines	120.3	1395	57	285	37.51
TotalControls	100.1	78.29	77	121	8.885

Table (3.6): Simulation 1: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months.

Simulation 2					
Market	Value	**Supply**	Value	**Demand**	Value
ProbControl	0.5	Unitax	2.0	aUtility	0.9
CostControl	0.9	Unitfine	3.0	aProbEvasion	0.9
		UnitGainEvasion	2.5	WeightDirExp	0.2
		LumpPunishment	0.0		
		AMemControlled	0.2		
		WeightPersMemory	0.2		

Simulation 2 - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
Evading	0.355	0.011	0	1	0.108
FearEvading	0.658	0.006	0	0.897	0.0.074
StDevEvading	0.905	0.016	0	0.979	0.130
StDevFearEvading	0.678	0.010	0	0.943	0.101
GovernmentUtility	390.7	3.178e+004	-469.8	1014	179
TotalFines	842.1	3.211e+004	0	1449	179.9
TotalControls	501.5	235.4	459	536	15.41

Table (3.7): Simulation 2: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months.

Simulation 3					
Market	Value	**Supply**	Value	**Demand**	Value
ProbControl	0.5	Unitax	3.0	aUtility	0.9
CostControl	0.9	Unitfine	3.0	aProbEvasion	0.9
		UnitGainEvasion	3.0	WeightDirExp	0.2
		LumpPunishment	10.0		
		AMemControlled	0.2		
		WeightPersMemory	0.2		

Simulation 3 - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
AvEvading	0.273	0.045	0	1	0.213
FearEvading	0.627	0.010	0	0.897	0.103
StDevEvading	0.721	0.068	0	0.98	0.263
StDevFearEvading	0.735	0.029	0	0.944	0.171
GovernmentUtility	-24.75	1.51e+005	-469.8	1014	390.2
TotalFines	426.6	1.512e+005	0	1449	390.4
TotalControls	501	235.4	459	536	15.41

Table (3.8): Simulation 3: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months.

Simulation 4					
Market	Value	**Supply**	Value	**Demand**	Value
ProbControl	0.1	Unitax	3.0	aUtility	0.9
CostControl	0.5	Unitfine	3.5	aProbEvasion	0.9
		UnitGainEvasion	3.0	WeightDirExp	0.9
		LumpPunishment	10.0		
		AMemControlled	0.9		
		WeightPersMemory	0.9		

Simulation 4 - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
AvEvading	0.229	0.011	0.117	1	0.105
FearEvading	0.440	0.002	0	0.469	0.049
StDevEvading	0.794	0.006	0	0.935	0.081
StDevFearEvading	0.566	0.003	0	0.623	0.053
GovernmentUtility	72.49	1135	21.8	254.5	33.83
TotalFines	82.5	1146	30	264	33.99
TotalControls	100.1	78.29	77	121	8.885

Table (3.9): Simulation 4: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months.

Simulation 5 - inserting LumpPunishment at time 31					
Market	Value	**Supply**	Value	**Demand**	Value
ProbControl	0.1	Unitax	3.0	aUtility	0.9
CostControl	0.5	Unitfine	3.5	aProbEvasion	0.9
		UnitGainEvasion	3.0/2.5	WeightDirExp	0.5
		LumpPunishment	0.0/10.0		
		AMemControlled	0.9		
		WeightPersMemory	0.9		

Simulation 5 - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
AvEvading	0.403	0.025	0.079	1	0.160
FearEvading	0.496	0.004	0	0.587	0.068
StDevEvading	0.903	0.009	0	0.963	0.100
StDevFearEvading	0.39	0.033	0	0.620	0.182
GovernmentUtility	98.65	3499	-11	278	59.4
TotalFines	148.7	3598	45.5	325.5	60.23
TotalControls	100.1	78.29	77	121	8.885

Table (3.10): Simulation 5: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months.

Simulation $c.6 - c.7 - c.8$ - inserting LumpPunishment at time 30					
Market	Value	**Supply**	Value	**Demand**	Value
ProbControl	0.1	Unitax	3.0	aUtility	0.9
CostControl	0.5	Unitfine	3.5	aProbEvasion	0.9
		UnitGainEvasion	3.0/2.5	WeightDirExp	0.9
		LumpPunishment	0.0/10.0		
		AMemControlled	0.9		
		WeightPersMemory	0.9		

Table (3.11): Simulation $c.6 - c.7 - c.8$: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months, SEEDS=200 − 5000 − 10000

Simulation $c.6$ - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
AvEvading	0.399	0.025	0.074	1	0.16
FearEvading	0.496	0.004	0	0.602	0.059
StDevEvading	0.902	0.009	0	0.956	0.099
StDevFearEvading	0.512	0.003	0	0.602	0.059
GovernmentUtility	96.43	3660	-10.5	388.5	60.75
TotalFines	146.2	3818	35	444.5	62.05
TotalControls	99.56	82.16	79	120	9.102

Table (3.12): Simulation $c.6$: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months, SEEDS=200

Simulation $c.7$ - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
AvEvading	0.493	0.012	0.263	1	0.112
FearEvading	0.512	0.004	0	0.580	0.065
StDevEvading	0.943	0.009	0	0.966	0.095
StDevFearEvading	0.453	0.005	0	0.578	0.068
GovernmentUtility	98.22	3458	-12	368	59.05
TotalFines	147.6	3502	38.5	416.5	59.43
TotalControls	98.79	61.18	76	118	7.855

Table (3.13): Simulation $c.7$: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months, SEEDS=5000

Simulation $c.8$ - Time series descriptive statistics					
	Average	Variance	Min	Max	Sigma
AvEvading	0.402	0.025	0.079	1	0.159
FearEvading	0.495	0.004	0	0.579	0.066
StDevEvading	0.903	0.009	0	0.957	0.098
StDevFearEvading	0.510	0.003	0	0.603	0.059
GovernmentUtility	101.2	4416	-46.5	428	66.73
TotalFines	151	4574	7	486.5	67.92
TotalControls	99.74	88.01	76	121	9.421

Table (3.14): Simulation $c.8$: firms=1000, consumers=1000, firms'links=10, consumers'links=50, time steps=120 months, SEEDS=10000

Appendix C - Robustness check

Simulation c.6-c.7-c8 The following simulations derive from simulation 5 where we introduced an exogenous shock in the market (lump sum punishment at time step 31st is increased from 0.0 to 10.0) and we have the same results for the government.

The structural break is evident (*see figure 14*, *figure 15* and *figure 16*) with the introduction of lump sum punishment: the fear evading decreases, and in these cases we have used different initial *seeds*[14] equal to 200, 5000 and 10000 respectively.

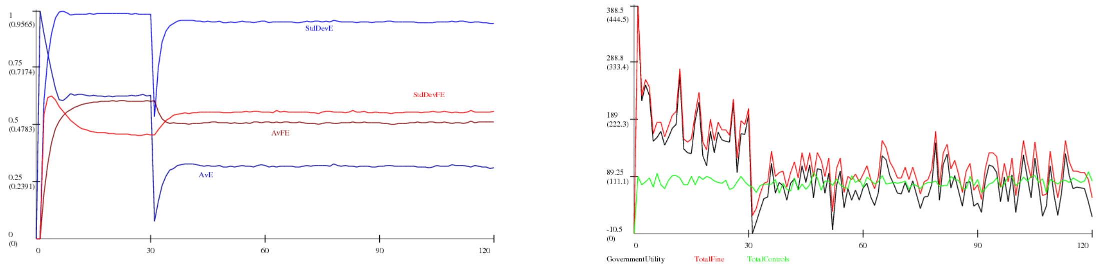


Figure (3.14): The government utility versus evading and evading fear - Sim *c*.6 - SEEDs = 200

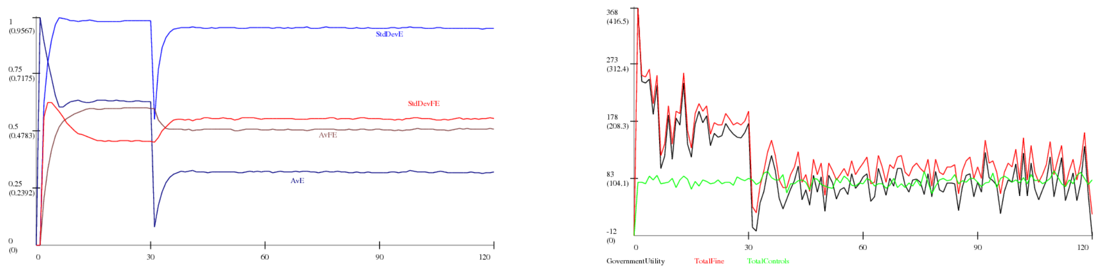


Figure (3.15): The government utility versus evading and evading fear - Sim *c*.7 - SEEDs = 5000

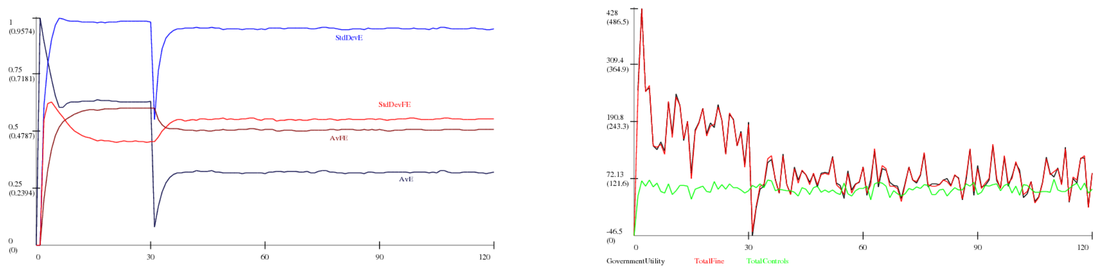


Figure (3.16): The government utility versus evading and evading fear - Sim *c*.8 - SEEDs = 10000

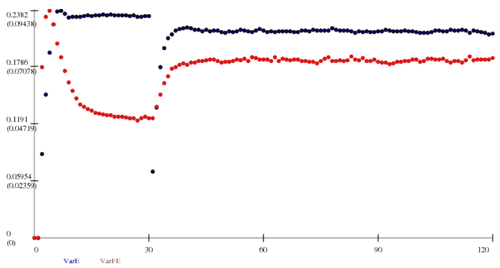
Lsd[15] (Valente et al. (2002), Andersen and Valente (2003)) is based on standard system of generating

[14] Many of the simulation programs make use of a pseudo-random number generator that requires a seed. This option allow us to specify a different seed for the random number generator and it is important for the repeatability of the results. Changing this value, we will get a different stream of random numbers, otherwise each simulation should have an unique indentifier associated with that.

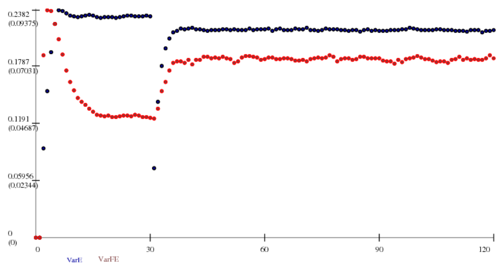
[15] *http : //www.labsimdev.org*

Variance of pseudo-random number for fear to evade and average evading					
	Average	Variance	Min	Max	Sigma
VarFE200	0.0691	0.0001	0	0.0944	0.0114
VarFE5000	0.0690	0.0001	0	0.0937	0.0115
VarFE10000	0.0688	0.0001	0	0.0947	0.0115
VarE200	0.2146	0.0008	0	0.2382	0.0297
VarE5000	0.2151	0.0008	0	0.2382	0.0292
VarE10000	0.2150	0.0008	0	0.2386	0.0293

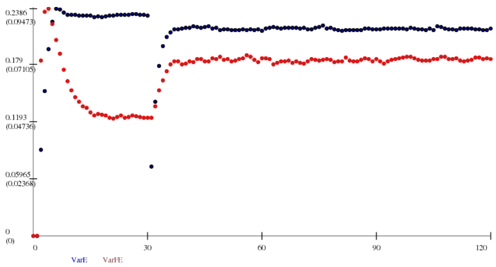
Table (3.15): Analysis of variance with SEEDS=200, 5000, 10000



(a) `VarSEEDs=200`



(b) `VarSEEDs=5000`



(c) `VarSEEDs=10000`

Figure (3.17): Analysis of variance

pseudo-random number[16]. Changing seed usually involves modifying results of simulation completely. Programming languages cannot produce genuine random values, for this reason we need to use pseudo-random number generators which are a deterministic functions (Valente (2014)) that provide sequences of values whose statistical properties are identical to a random series; the seed is useful to catalogue random series and replicating exactly the pseudo-random events.

Most frequently the model is run only once, but *Lsd* allows to run also batteries of simulations with the same configuration and different *seeds* to test the robustness of result; in this specific case simulation is run with three different *seeds* (200, 5000 and 10000) with the same result of simulation 5, while the variance becomes flatter with introduction of exogenous shock (see *figure 17*). Before shock variance moved in a range between 0.09 and 0.05 in each of three cases (results in appendix B). This confirms the robustness (Valente (2008)) and goodness of our results (*see figure 14, figure 15* and *figure 16*).

[16]A pseudo-random number is random number sequence in reality but not in computer language because the values are computed using a deterministic algorithm and probability plays no real role. It may be initialized from an arbitrary initial state using a specific seed state. It will always produce the same sequence when initialized with that starting state, otherwise, if we try to change the initial seed state, we will obtain a different sequence of pseudo-random numbers.

Appendix D - The syntax in Lsd - LabSimDev

In order to test our ideas, we built a model composed by several independents and heterogeneous agents representing supply and demand side in front of crucial decision: being tax evaders or not. The supplier may decide if become an evader offering a deterministic type of good, while the consumer may be an accomplice if he decided to buy a good from dodgers. The simulation time scale represents months, and we simulate along ten years (120 months). Agents sell and purchase goods according to common pattern and idiosyncratic scheme for each agent. The following syntax (in alphabetical order) is sorted according to use in *Lsd* (Valente (2008)).

- `aMemControlled`

`aMemControlled` measures the speed of updating of the indicator used to track the expectations of producers of being controlled by government after happened purchase. It is indicated as a and is included in `Fear Evading`

- `aUtility`

The parameter α measures the speed of updating of consumer welfare, tracking the last utility from product bought. Used in the equation for `Consumer Welfare`.

- `AvSales`

`AvSales` indicates a moving average of sales depending on the sales (indicate as V_t) of last period. It appears as $\bar{V}_t$ in `Fear Evading` and `Evading`, whereas the sales V_t are present in equation `Profit`.

- `Consumer Welfare`

Supposing a classic economic law, each consumer chooses one product and he assigns it to one producer. This choice is probabilistic and it depends on acceptance of tax cheating firms by the consumer and it depends on utility of the product offered. The equation register also the selling of firms:

$$X_t = (1 - \alpha)X_{t-1} + \alpha U$$

- `CostEvading`

It is a statistic that registers the cost from evading. It is indicate as $E(C_E)$ and it appears in equation `Evading`.

- `Cost Control`

Cost control (c) is an initial value that indicates the cost of control paid by government.

• `Evading`

The variable E signals if the taxes are paid or not. To compute it, we use an estimation of the gain from paying taxes (the *cost*) against not paying them (the *gain*) and the value is discounted by the observed risk to being caught:

$$E(C_E) = P_t \cdot [L + \bar{V}_{t+1} \cdot g]$$

$$E(G_E) = (1 - P_t)\bar{V}_{t+1} + (g + \pi) - E(C_E)$$

If *Evading* is flagged ($E = 1$), it signals the cheating of firms, otherwise ($E = 0$) it signals the ethic choice of firms. This situation is reported in two different results:

$$W_{g_{t+1}} = \begin{cases} if\ G_E(1 - P_t) > C_E \Longrightarrow E = 1 \\ if\ G_E(1 - P_t) < C_E \Longrightarrow E = 0 \end{cases}$$

• `Fear Evading`

This variable P_t measures the probability to increase fear evading when we arrange the previous fear (time step $t - 1$) with the sales of this period weighted with level of evasion:

$$P_t = (\mu)P_{t-1}a + (1 - \mu)\left[\frac{\bar{V}_t}{E_t}\right]$$

• `Fined`

In this case the cheater has been discovered and he must pay a fine. The parameter is also present in `Government Utility`.

• `GainEvading`

It is a statistic that registers the gain from evading. It is indicate as $E(G_E)$ and it appears in equation `Evading`.

• `Government Utility`

This variable $W_{g_{t+1}}$ measures the utility for the government when trading between consumer and producer is already completed:

$$W_{g_{t+1}} = \begin{cases} if\ \varepsilon S_t = 1 \Longrightarrow \Phi = 1 \\ if\ \varepsilon S_t = 0 \Longrightarrow \Phi = 0 \end{cases}$$

if government found a crime, we can distinguish in two other cases, whereas if not government has a cost from controls:

$$\begin{cases} case\ \Phi = 1 \Longrightarrow \begin{cases} if\ \Phi = 1\ and\ Evasion > Threshold \Rightarrow \Phi = M \\ if\ \Phi = 1\ and\ Evasion < Threshold \Rightarrow \Phi = L \end{cases} \\ case\ \Phi = 0 \Longrightarrow F - C \end{cases}$$

- `Lump Punishment`

The `Lump Punishment` (L) represents the extra cost to pay by the firm that is discovered cheating. It is independent from the sales of firm but it depends on the magnitude of cheating (if the cheat is above the threshold). It appears in `Profit`.

- `ProbAcceptEvasion`

This variable represents the probability to include dodging producer within a set of potential purchase and it is a slow approximation to the average probability of evading from its own previous experience, in adding with experience of other consumers.

- `Profit`

The profits (Π) represent total gains for each producer. They are proportional to sales and, if firm is dodger, profits rise up also with sharing of unpaid taxes. In case of caught, firm must pay a fine. This fine is computed on single sale. Additionally, in case of larger evasion (greater than threshold) is added a lump punishment (i.e. the jail) exacerbates the penalty:

$$\Pi = \bar{V}_t[(\pi \cdot g + E) - \Phi \cdot f] - (1 - \Phi) \cdot L$$

- `ProbControl`

`ProbControl` (ε) is an initial value that indicates the probability to suffer controls by government.

- `Punished`

`Punished` (Φ) records an evading when a firm suffered a control and it has cheated, It appears in `Government Utility` and `Profit`.

- `Shopping`

The `Shopping` specifies that a round of purchase is done at *time t*. After purchase, the government may control whether the supplier has evaded and punishes the consumer with a fine. It appears in `Government Utility` and it is specified as S_t.

- `Total Controls`

Total controls (γ) is an initial value that indicates the number of controls done by government.

- `Total Costs`

Total costs indicates the total costs of controls and it is represented by cost control multiplied number of controls.

- `Unit Fine`

This parameter (f) represents the fine paid by firm if it discovered that is cheating.

- `Unit Profit`

This parameter means a unit of profit generated by single sale in the case of ethical agents. It appears in equations `Profit` as π.

- `Unit Tax`

The `Unit tax` represents in the environment the level of tax that lawful firms must pay for each sale. It appears as t in equations `Utility` and `Profit`.

- `Utility`

Variable *Utility* (U) represents the level of utility for consumers. It comes from the choice of product composed by basic utility (indicated as b) that is the intrinsic quality of product, and in the case of tax evasion is combined with the difference of unpaid tax and extra gain for supplier:

$$U = b + E \cdot (g - t)$$

- `WeightPersonalMemory`

This parameter is specified as μ and indicates the relevance of direct experience versus the observed levels of peers concerning the expectation of being caught if cheating. It appears in `Fear Evading`.

- `WeightDirectExperience`

`WeightDirectExperience` represents the relevance of personal experience for consumers versus the observed nature state of other consumers on accepting the purchases from evading firm.

Chapter 4

A measurement of shadow economy in European countries: Evidence from MIMIC Model [1]

[1] I am grateful to Prof. Roberto Dell'Anno for invaluable support during the whole study.
This research chapter was developed within the framework of the PhD programme in Economics and Finance of Sapienza University.

Abstract

The following chapter analyses the problem of shadow economy using a MIMIC model developed by Jöreskog and Goldberger (1975). According to Dell'Anno (2001), this method is valid to estimate shadow economy when there is a particular attention on some techniques (e.g see Ferwerda et al. (2010) and the unit root test detected), trying to have a comparison with the method by Schneider (2015). Confirming the reliability of results using the different and alternative benchmark two-steps MIMIC already proposed by Dell'Anno (2007).

4.1 Introduction

One of the most common definition of the shadow economy includes all market-based legal production of goods and services that are deliberately concealed from public authorities for any of the following reasons:

1. to avoid payment of income, value added or other taxes;
2. to avoid payment of social security contributions;
3. to avoid having to meet certain legal labour market standards, such as minimum wages, maximum working hours, safety standards, etc.;
4. to avoid complying with certain administrative procedures and high level of regulation.

Several activities associated with shadow economies are affected the world life and governments' attempt to control these throughout various measures such as prosecution, punishment, economic growth and sometimes the education (Schneider and Enste (2000)). To control this phenomenon, became really important the allocation of resources.

Unfortunately, it is very difficult to get accurate information about shadow economy activities - just because hidden - including both goods and labour market.

Several authors trying to measure the shadow economy face the difficulty of how to define it. One of the most common working definition is include into underground economy all currently unregistered economic activities that contribute to the officially calculated (or observed) Gross Domestic Product, leading to a decrease of its magnitude.

On the other hand, the shadow economy creates an extra added value that can be spent in the official economy. Schneider and Enste (2000) for instance state that at least two thirds of the income earned in the shadow economy is immediately spent in the official economy, thus having a positive effect on the latter. As Smith (1997) points out, in a world of minimum wages, high payroll taxes and limits on hours worked, the underground economy may enable some individuals to be employed who would otherwise be unemployed, enable other individuals to increase their incomes by holding second jobs, and provide services that would otherwise be unavailable. Irregular activities may add a dynamic element to an economy and increase competition in some sectors, while underground production may improve the distribution of income in society (see Dell'Anno (2015) and Dell'Anno and Amendola (2015)).

Hence, it is clear that the shadow economy not only has negative effects on the economic system but also generates positive ones. These potentially positive aspects of shadow economy should be considered in the planning of policies, as the main aim of the policy maker should be to adopt economic policies which drive the shadow activities towards the regular economy rather than easily fight them. The knowledge of the size, sector distribution, dynamics and determination of the main causes of the shadow economy are necessary conditions for adopting a coherent plan of economic policies.

4.2 Background

4.2.1 Definition of shadow economy

The definition of shadow economy (Tanzi (1983), Frey and Pommerehne (1984) and Schneider and Enste (2000)) plays an important role in assessing its size. Having a clear definition, an high number of ambiguities and controversies can be avoided.

In general, there are two types of shadow economic activities: illegal employment and the production of goods and services largely consumed within the household. The analysis is focused on both blocks, but tries - at the same time - to exclude criminal activities such as drug production, crime and human trafficking.

Thus, the focus goes on productive economic activities that would normally be included in the national accounts but which remain underground due to tax or regulatory burdens. Although such legal activities contribute to the value added of single country, they are not captured in the national accounts because they are produced in not conventional ways (e.g. by people without proper qualification or without a master certificate).

From the economic and social view, soft forms of illegal employment, such as moonlighting (e.g.construction work in private homes) and its contribution to aggregate value added can be assessed rather positively.

Determinant of shadow economy According to Frey and Pommerehne (1984), some variables may be considered the leading causes to an hidden economy, and these may be divided in two antithetic classes: the social and economic group. Belong to social group the following determinants:

- *Intensity of Regulations*: increased intensity of regulations (Johnson et al. (1998)), for example labour market regulation, trade barriers, and labour restrictions for immigrants. is another important factor which reduces the freedom (of choice) for individuals engaged in the official economy.
- *Public Sector Services*: an increase of the shadow economy can lead to reduced state revenues which in turn reduce the quality and quantity of publicly provided goods and services (see Feige (1979) and Blackburn et al. (2012)).
- *Other Public Institutions*: the quality of public institutions is considered as another key factor of the development of the informal sector. In particular, corruption of bureaucracy (Smith (1997)) and government officials seems to be associated with larger unofficial action, while a good measurement of law by securing property rights and contract enforceability, increases the benefits of being formal.
- *Tax Morale*: in addition to the incentives effects discussed before, the efficiency of the public sector has an indirect effect on the size of the shadow economy because it affects *tax morale*. Tax compliance (Santoro and Fiorio (2011) and Slemrod and Bakija (2001)) is driven by a mental tax agreement that entails rights and obligations from taxpayers and citizens on the one hand, but also from the state and its tax authorities on the other hand. Citizen are more hardly disposed to pay their taxes frankly if they get worthy public services in exchange, but they are inclined even in cases when the main benefit of taxation does not captured, i.e. in the case redistributive policies (Pyle (1989)).

Instead, the causal factors belong to economic group are:

- *Labour market conditions*. As Schneider (2011) and Schneider (2012) have shown, during working time the higher opportunity costs of taking up additional work in the hidden economy reducing payment of taxes on the hand (who demands labour) and maximize their wage (who supplies labour);
- *Structural factors*. In some economic sectors (particularly those with low capital intensity), industries (e.g. handicraft) but also workers (e.g. foreign workers) in which a higher probability of working in the hidden economy can be assumed see (Kaufmann and Kaliberda (1996) and Lackó (1996));
- *Tax and Social Security Contribution Burdens*: the tax and social security contribution burdens are among the intense causes for the existence of the shadow economy as Tanzi and Zee (2000) argued. The bigger difference between the total labour cost in the official economy and after-tax earnings (from work), is good incentive to reduce the tax wedge and work in the shadow economy.

4.2.2 Structural Equation Model (SEM): a latent variable approach

A latent variable or MIMIC approach A latent variable approach, in some specific cases called MIMIC (Multiple Indicators Multiple Causes) model (Hayduk (1987)), is based on two different equation (structural and measurement) that are linked throughout the latent variable. This method is defined also two-step estimation, or Structural Equation Model (SEM), because are needed two different steps to reach the final result.

MIMIC models are a class of models positing latent, unrecorded, common causes. The causal structure consists of three components: a set of latent variables (not observed), a set of inputs (observed variables that act as causes of latent variables), and a set of outputs (observed variables that are caused by latent variables). MIMIC models are often used in cases where there are believed to be unobserved variables acting as causes on either some observed variables (i.e., outputs) or other latent variables. The applications of MIMIC models are widespread, ranging from economics, to psychology, and even public health. Advocates of MIMIC models emphasize their usefulness in simultaneously assessing multiple dimensions of complex social issues. Finally, MIMIC models are usually specified a priori. That is, an investigator assumes she knows all of the relevant causal relations and the only function of the data is to enable estimation of parameters and to confirm the hypothesis by statistical tests Essentially, the MIMIC (Multiple Indicators Multiple Causes) model is specified as follows.

First, the scalar latent variable (in this case, the size of the hidden economy), denoted as η, is linearly determined by a set of observable causes $x' = (x_1, x_2, \dots, x_q)$, subject to a scalar random error term,ζ , which is a $(px1)$ vector:

$$\eta = \gamma x + \zeta \tag{4.1}$$

where γ is a $(qx1)$ vector of parameters. This latent variable, in turn, linearly determines a set of observable endogenous indicators $y' = (y_1, y_2, \dots, y_q)$, also subject to a scalar random error term, ε, which is a $(px1)$ vector,

$$y = \lambda\eta + \varepsilon \tag{4.2}$$

where λ is a $(px1)$ vector of parameters. It is assumed that λ and η are Normal and mutually uncorrelated.

Substituting (5) into (6), we obtain the reduced form relation connecting the observable variables and express them in a p-equation multivariate regression model,

$$y = \Pi x + z \tag{4.3}$$

The equation in (7) is restricted so that the regressor's coefficient matrix (X) has a rank of one and the error covariance matrix is similarly constrained.

The first restriction is common in econometrics and typically arises in simultaneous equation models (SEMs) where the exclusion of certain exogenous variables from a structural equation implies that a certain portion of the reduced form coefficient matrix is short ranked.

The second restriction derives by disturbance covariance matrix, that is singular.

In the case of hidden economy, the purpose of latent variable, if a value of it was known at some point in the sample, then the ordinal series could easily be converted into a cardinal time-path. In this way the estimate of hidden economy obtained could became a "benchmark" which would be used in the conversion and this has been the technique that has been applied in most studies.

In comparison to other statistical methods, SEMs/MIMIC models offer several **advantages** for the estimation of shadow economic activities.

According to Tedds and Giles (2002), the MIMIC approach is a wider approach than most other competing methods, since it allows one to take multiple indicator and causal variables into consideration at the same time. Moreover, this approach is quite flexible, allowing one to vary the choice of causal and indicator variables according to the particular features of the shadow economic activity studied, the period in question, and the availability of data. SEMs/MIMIC models lead to formal estimation and testing procedures, such as those based on the method of maximum likelihood. These procedures are well known and are generally "optimal" if the sample is sufficiently large (Tedds and Giles (2002)).

Schneider and Enste (2000) emphasize that these models lead to some progress in estimation techniques for the size and development of the shadow economy, because this methodology allows wide flexibility in its application. Therefore, they consider it potentially superior to other estimation methods. argues that, when compared to other methods, SEMs/MIMIC models do not need restrictive assumptions to operate.

Similarly, Thomas (1992) argues that the only real constraint of this approach is not in its conceptual structure, but in the choice of variables. These positive aspects of the SEM approach in general and the MIMIC model in particular do not only apply in its application to the shadow economy, but to all informal economic activities. This means that the MIMIC procedure relies on a broad definition of the shadow economy.

Criticism of the MIMIC model Of course this method has its limitations, too, which are identified in the literature. The three most important points of criticism focus on the model's implementation, the sample used, and the reliability of the estimates:

1. The most frequent objection is around the meaning of the latent variable (e.g. Helberger and Knepel (1988) and Dell'Anno (2004)). The confirmatory rather than exploratory nature of this approach means that one is more likely to determine whether a certain model is valid than to "find" a suitable model. Therefore, it is possible that the specified model includes potential definitions or informal economic activities other than those studied. For example, it is difficult for a researcher to ensure that traditional crime activities such as drug dealing are completely excluded from analysis of the shadow economy. This criticism, which is probably the most common in the literature, remains difficult to overcome as it goes back to the theoretical assumptions behind the choice of variables and empirical limitations on data availability.

2. Helberger and Knepel (1988) argue that SEM/MIMIC model estimations lead to unstable coefficients with respect to changes in the sample size and alternative model specifications. Dell'Anno (2004) shows, however, that instability disappears asymptotically as the sample size increases. Another issue is the application of SEMs to time series data because only simple analytical tools such as q- and stemleaf plots are available to analyze the properties of the residuals (Dell'Anno (2004)).

3. Criticism is also made with respect to the benchmarking procedure used to derive "real world" figures of shadow economic activities (Breusch et al. (2005)). As the latent variable and its unit of measurement are not observed, SEMs only provide a set of estimated coefficients from which one can calculate an index that shows the dynamics of the unobservable variable. Application of the so-called calibration or benchmarking procedure, regardless which one is used, requires experimentation, and a comparison of the calibrated values in a wide academic debate. Unfortunately, at this stage of research it is not clear which benchmarking method is the best or the most reliable.

The economic literature using SEMs is well aware of these limitations. It acknowledges that it is not an easy task to apply this methodology to an economic dataset, but also argues that this does not mean one should abandon the SEM approach. On the contrary, following an interdisciplinary approach to economics, SEMs are valuable tools for economic analysis, particularly when studying the shadow economy. Moreover, the objections mentioned should be considered incentives for further research in this field rather than as a reason to abandon the method. Again going back to the definition of the shadow economy, the MIMIC estimation provides upper-bound macro value added figures, including mostly legally-bought material.

4.3 The model

In this work, we use a special case of structural equation modelling (SEM) defined as MIMIC (Multiple Indicators Multiple Causes) developed by Jöreskog and Goldberger (1975).

The hidden economy (η) is linearly determined using a set of observable exogenous causes $x_1, x_2, x_3, x_4, x_5, x_6$ with an error ξ:

$$\eta = \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + \gamma_4 x_4 + \gamma_5 x_5 + \gamma_6 x_6 + \xi \tag{4.4}$$

The latent variable - i.e the shadow economy η - determines a set of observable endogenous indicators y_1 and y_2 linearly, with a disturbance of ε_1 and ε_2:

$$y_1 = \lambda_1 \eta + \varepsilon_1 \qquad y_2 = \lambda_2 \eta + \varepsilon_2 \tag{4.5}$$

Both structural and measurement disturbance are normally distributed, independent and have the expectation value equal to zero.

The shadow economy as a percentage of GDP is calculated by converting the index of shadow economy estimated by the structural model (equation 7). As Breusch et al. (2005) highlights, several benchmarking procedures exist that estimate the size of the shadow economy by MIMIC outputs (Giles (1999)).

Unfortunately, at this stage of research on the Model approach, which benchmark method should be applied is not definite yet. In the following, we apply an already suggested two-step procedure by Dell'Anno (2007).

STEP I

Measurement equation:

$$\frac{GDP_t - GDP_{t-1}}{GDP_{2010}} = \delta_1 - \frac{\overline{\eta}_t - \overline{\eta}_{t-1}}{GDP_{2010}} \tag{4.6}$$

Structural equation[2]:

$$\frac{\tilde{\eta}_t}{GDP_{2010}} = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 \tag{4.7}$$

x_1=unemployment rate
x_2=self employment rate
x_3=tax marginal rate
x_4=index of regulation

STEP II

$$\frac{\tilde{\eta}_t}{GDP_{2010}} \frac{\tilde{\eta}_{2010}}{GDP_{2010}} \frac{GDP_{2010}}{\tilde{\eta}_{2010}} \frac{GDP_{2010}}{GDP_t} = \frac{\hat{\eta}_t}{GDP_t} \tag{4.8}$$

$\frac{\tilde{\eta}_t}{GDP_{2010}}$ = structural equation
$\frac{\tilde{\eta}_{2010}}{GDP_{2010}}$ = exogeneous estimate
$\frac{GDP_{2010}}{\tilde{\eta}_{2010}}$ = structual equation (2010)
$\frac{GDP_{2010}}{GDP_t}$ = convert index to GDP in base year

[2]The intercept is not included because it is not statistically significant.

$\frac{\hat{\eta}_t}{GDP_t}$ = estimated shadow economy % of GDP

We study the effect of changes in the black labour market have on the participation ratio could be biased by the growing female participation in the workforce trough a MIMIC 4-1-2 with *Female LFPR*.

When the variables are not (multivariately) normally distributed, then it is possible for maximum likelihood estimators to produce biased standard errors and an ill-behaved "chi-square" test of overall model fit. To avoid this type of problem, we transform the variables in order to obtain a better approximate to multinormality and it happens using a first difference (FD) modification (see Juodis (2016)).

4.3.1 The causes of MIMIC - Variables of structural equation

Data come from OECD database and the years used are 1970-2015. To have a conformity with the Schneider (2015) work, only the unofficial economy for last twenty-three years is calculated. Furthermore, the countries take into account are twenty-two of EU-OECD considered as developed countries.

Unemployment rate The unemployment rate is composed by two different part: one portion is classified as unemployed because they are components of the official labour force, the other portion is composed of retired people, minors and housewives who are not part of the official workforce. Furthermore, there are people who have both an official and unofficial job at the same time because of they are not reported (Tanzi and Zee (2000)). In this sense, the official unemployment rate could be weakly correlated with the shadow economy.

Self employment rate The rate of self-employment is considered as a determinant of the shadow economy, as in Dell'Anno (2007). A significant diffusion of this kind of jobs (e.g different professional person in various sectors) and self-employed, compared to the total workforce, increments the potential number of opportunities to conceal income and other gains from the controllers.

Dell'Anno (2001) and Alañón and Gómez-Antonio (2005), have found a considerable correlation between self-employment and the shadow economy evading taxes by deducting some of their private consumption as business disbursal or other kind of benefits.

Social benefit This variable includes all subsidies from central government received by the unemployed or low-income people. This kind of incentive should have a uncertain effects on the shadow economy. They increase the cost of being irregular, because they do not have some type of working protection or pension contributions paid. At the same time, they are an incentive to participate and remain in the irregular market, by reducing the willingness of the unemployed to work and providing incentives to under-declare official income in order to receive undue social benefits.

Tax marginal rate In the literature, the most popular determinants of shadow economy are tax rates. The common hypothesis is that an increase of the tax marginal rate is a strong incentive to evade taxes and increasing the level of shadow economy.

Index of regulation This variable is introduced in order to take into account both the level of economic freedom and as an index of the presence of the public sector in the economy. About this, the economic surveys on the relationship between the shadow economy and regulation is contrasting. Several studies support a decreasing function of the public sector in the economy with market opening to competition, avoiding an over bureaucratisation and a potential increasing level of corruption. Indeed, the more regulated economies lead to an increase in the hidden activities. Furthermore, this presence of the public sector in the market could be justified through an elaborated taxation, implying distortions in the allocation of resources between private businesses (more efficient) and public institutions (less efficient) (Dell'Anno (2007)). According to these bullets, we would expect a negative sign between the shadow economy and the index of regulation.

Government expenditure on labour They are current unrequited payments that government units make to enterprises to provide employment support.

This type of subsidies have contradictory effects on the shadow economy. In one hand, an increase in subsidies raises the costs to be irregular - But, on the other hand, it introduces distortions to competition and, by altering the net tax burden of enterprises and decreasing the cost of labour having only a short term targets.

4.3.2 The indicators of MIMIC - Variables of measurement equation

Two variables are used as indicators of the latent variable: the real (official) gross domestic product index (2010=100) and the labour force participation rate, to measure the development of the shadow economy.

Real GDP index2010 This indicator is decisive in the problem of identification as well as for the theoretical consequences it implies, mainly because it is chosen as reference variable. Being the latent variable is measureless, we must set it in measurement model. We fix the coefficient of this reference variable equal to a *nonzero* value, using two alternatives (+1 or -1)after a normalization, making the estimated coefficients more easily comparable. Following Dell'Anno (2007), we use a scheme that determines the sign of coefficient of scale through a *reductio ad absurdum*. As in the MIMIC model the vector of structural coefficients is proportional to the coefficient of scale, when the sign of lambda is changed, the structural parameters of the causes change from positive to negative (and viceversa) keeping the same absolute values. Moreover, Schneider and Bajada (2005) shows a negative sign for transition and developing countries and a positive relationship for developed ones. We are going to use lambda +1 following Schneider and Enste (2000) because of our starting database.

Labour Force Participation Rate The labour force participation rate is calculated as the ratio of the total labour force (LF) and the population of working age (15-64 years old).

According to Giles (1999) a decline or a low level in this rate may reflect a movement of the workforce from the measured economy into hidden activities. By including this variable as an indicator, it is possible to check through empirical observation if there is a stream of resources between registered and hidden economy. The fact that changes in participation reflect variations in the shadow economy, or vice versa, is uncertain and conflicting hypothesis must be considered.

4.4 Estimates of MIMIC

The causes are measured with the same unit of measurement (percentage points), the coefficients (see Table 3 in appendix) are directly comparable in order to evaluate relative weight to explain the dynamics of shadow economy. Beginning from MIMIC 6-1-2 and deleting non-significant paths, we consider MIMIC 4-1-2 as the best model. All variables are tested for unit root (and they converged), and both indicators and causes are stationary.

Following Dell'Anno (2007), we use a strategy that determines the sign of coefficient of scale through a *reductio ad absurdum*: "As in the MIMIC model the vector of structural coefficients is proportional to the coefficient of scale, when the sign of lambda is changed, the structural parameters beta of the causes change from positive to negative (and viceversa) keeping the same absolute values. According with this property, if the signs of the estimated coefficients that link the underground economy and its causes are completely divergent from well-known theories and empirical studies in one case (e.g., lambda = +1), then the hypothesis supporting the opposite sign for the relationships between shadow economy and reference variable should be accepted as more rational and so we take the absolute values of all coefficient".

In particular, the MIMIC estimation reveals that the main causes of shadow economy are self employment, the index of regulation, the tax marginal rate and the unemployment rate.

Several empirical evidence corroborate that unregistered activities are only partly undertaken by labour force. According to Bajada and Schneider (2005), the labour force participation rate cannot be affected by underground economy if the surveys is undertaken during weekends when the employees are not active in measured sectors. Meanwhile, as suggested Dell'Anno (2007), the last four decades the labour force has hardly mutated. the increasing of female participation in the work force could lead to be biased the participation rate in the shadow economy. As we show in the last column, the bias is present but it is very low due to characteristics of employees. According to this result, we compute shadow economy using only the total labour force rate. Finally, it has been shown tha people prefer to remain in regular labour market rather move into underground economy (Dell'Anno (2001), Schneider and Enste (2000), Schneider (2012)).

Regarding the significance of MIMIC we found some significant variables whereas others not. Concerning the variables about labour market (unemployment rate, self employment rate and government expenditure on labour) the first two do not have a great influence on the underground economy, while the government expenditure on labour is not significant from MIMIC 5-1-2 and we delete it in the final structural equation model.

The tax marginal rate and the presence of the State in the market, measured by index of regulation level are always statistically.

The social benefit - that is all subsidies paid to unemployed or low income people - has not statistical significant from MIMIC 6-1-2 and the theories about that could be not corroborated.

Analysing the model from the indicators, we have used three different variables as real GDP index 2010, the total labour force participation rate and the female labour force participation rate to analyse different structure of labour market as supposed Dell'Anno (2007).

The relationship between shadow economy and growth rate of GDP is positive, according Schneider and Bajada (2005), Dell'Anno (2007) and Schneider et al. (2010).

Also the state between black economy and LFPR and FemLFPR is positive, but in the first case this relation has a greater magnitude, confirming the change in labour market structure and its structural composition. Indeed, if we analyse only the female part of labour force, this could have a problem of identification on unmeasured variable implying an under estimation of black economy.

Table 4.1: Regression MIMIC table and goodness of fit

	(1) MIMIC 6-1-2	(2) MIMIC 5-1-2	(3) MIMIC 4-1-2	(4) MIMIC 4-1-2 †
SE				
unemployrate	0.00996**	0.00969**	0.0101***	0.00905***
	(0.00313)	(0.00313)	(0.00318)	(0.00285)
selfempl	0.0989***	0.110***	0.102**	0.0915**
	(0.0358)	(0.0355)	(0.0357)	(0.0336)
indexregul	0.0370***	0.0397**	0.0369**	0.0373**
	(0.0136)	(0.0135)	(0.0155)	(0.0136)
taxmargrate	0.0199**	0.0211**	0.0201***	0.0212***
	(0.00437)	(0.00443)	(0.00413)	(0.00391)
socialbenefit	0.000318			
	(0.000311)			
governmentunemp	0.00820	0.0103*		
	(0.00555)	(0.00542)		
realgdp2010				
SE	1	1	1	1
	(.)	(.)	(.)	(.)
_cons	0.997***	0.999***	0.993***	1.004***
	(0.0326)	(0.0331)	(0.0305)	(0.0262)
lfpr				
SE	7.445***	7.434***	7.796***	
	(1.579)	(1.650)	(1.656)	
_cons	0.540**	0.560**	0.520**	
	(0.227)	(0.230)	(0.232)	
lfprfem				
SE				0.198***
				(0.0389)
_cons				0.0162***
				(0.00510)

Standard errors in parentheses * $p < .1$, ** $p < .05$, *** $p < .01$

Fit statistic	(1) MIMIC 6-1-2	(2) MIMIC 5-1-2	(3) MIMIC 4-1-2	(4) MIMIC 4-1-2 †
Discrepancy				
χ^2_{ms} **	6.876 (5)*	2.458 (4)*	1.007 (3)*	0.394 (3)*
$p > \chi^2$	0.230	0.652	0.800	0.941
χ^2_{bs} ***	41.807 (13)*	36.166 (11)*	29.541 (9)*	50.493 (9)*
$p > \chi^2$	0.000	0.000	0.001	0.001
Population error				
RMSEA⋆	0.042	0.024	0.000	0.000
90% CI, lower bound	0.000	0.000	0.000	0.000
upper bound	0.111	0.083	0.070	0.022
pclose‡	0.495	0.826	0.902	0.974
Baseline comparison				
CFI⋆⋆	0.935	1.000	1.000	1.000
TLI⋆⋆⋆	0.831	1.169	1.291	1.425
Size of residuals				
SRMR	0.019	0.017	0.020	0.012
CD ‡‡	0.253	0.261	0.234	0.285

† *Female LFPR as indicator* § Means |z-statistic|>1.96

* *Degrees of freedom* ** *Model vs saturated* *** *Model vs baseline*

⋆ *RMSEA approximation* → $p < 0.05$

⋆⋆ *Comparison fit index* ⋆⋆⋆ *Tucker – Lewis index*

‡ *Probability RMSEA* ⟸ 0.05 ‡‡ *Coefficient of determination*

Table 4.2: Shadow Economy estimated by MIMIC model

Year	93/95	95/97	97/99	99/00	01/03	03/05	05/07	07/09	09/10	10/11	11/13	14/15
MIMIC model	21.36%	21.50%	20.65%	17.20%	12.99%	10.59%	10.06%	11.87%	14.85%	13.04%	12.10%	10.71%
Schneider 2015	16.11%	16.24%	19.85%	20.00%	19.70%	18.90%	17.43%	17.18%	16.13%	16.57%	15.82%	15.08%

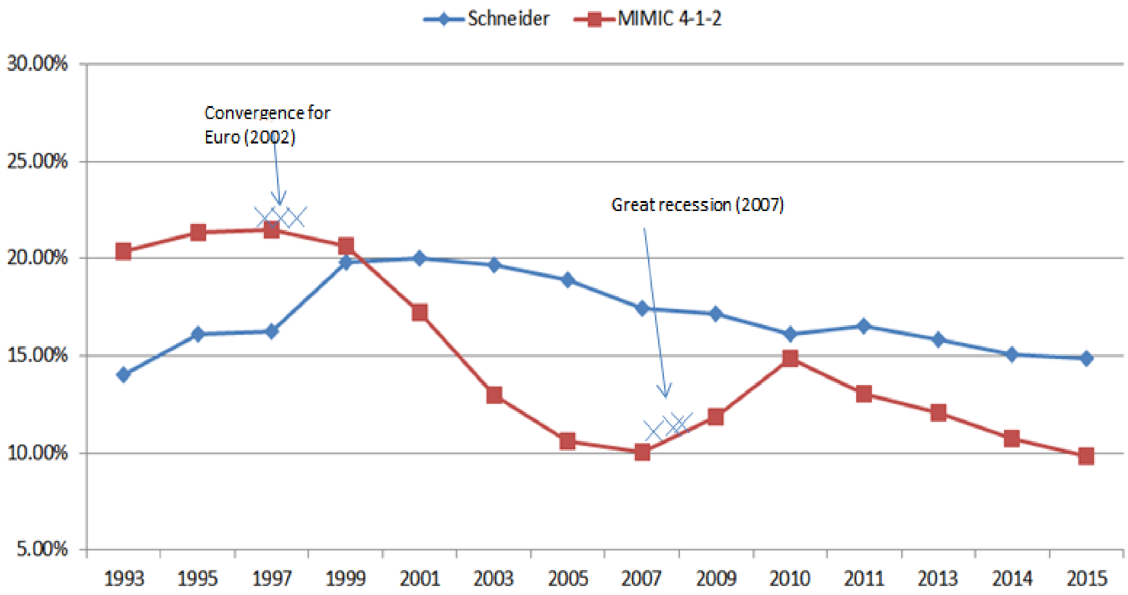


Figure (4.1): MIMIC model compared with Schneider (2015)

The result of this work suggest:

- The range of shadow economy among EU-OECD countries is included between 21̃% and 10̃% during last two decades;

- We observe two different structural point break, the first at the beginning of new millennium when countries had to converge toward Maastricht's parameters due to entry in Euro area, whereas the second one after the great recession (2007) where we have found an increasing in shadow economy.

These results are partially concord with Bovi and Dell'Anno (2010) and Schneider and Enste (2000); indeed, we observe the same trend of Schneider (2015) except in the case of structural break during the great recession of 2007.

As Dell'Anno (2001) and Dell'Anno (2007) have shown, the strong dependence of outcomes depends by the own choice of the coefficient of scale, in this case. In particular, if the parameter of scale is chosen equal to $+1$ (instead of -1) following Schneider and Bajada (2005), the estimated shadow economy became the opposite respect to negative case. It means that the signs of underground economy depends on the personal choice of researcher fellow.

4.5 Summary and comments

In order to estimate not only the relative size of the parameters but also their levels, it is necessary to fix a scale for the unobservable latent variable. A convenient way to determine the relative magnitude of the variables is to set the coefficient of one of the measurement model's indicator variables to non-zero. As we already affirmed, the coefficient is set equal to $+1$. We now summarize our findings from the estimations.

First, we can confirm our theoretical considerations in large part: indeed, we cannot confirm the our expectations only for *index regulation* where we expected negative instead it has positive correlation. The empirical results show that regulations could be one of the main causes for the size of the shadow economy. Deregulation is a promising alternative to the reduction of tax rates and tax and social security contributions burden, which is often quite difficult due to the necessity to of financing public goods and services as well as the social security system. Less regulations, fewer guidelines and more efficient bureaucracy do not increase the budget deficit, but the freedom of choice for people, so that shadow economy is less attractive. Finally, the results show that deregulation can help to reduce the size of the shadow economy considerably (Enste (2010)).

Second, for *unemployment rate* and *self employment rate* we confirm respectively a weakly and considerable correlation as our hypothesis suggested. With regard to unemployment, it turns out that the effect is weakly positive. That is, the income effect exceeds the substitution effect. This finding is supported by the positive relation between the shadow economy and official GDP, suggesting that the two are complements rather than substitutes (Buehn and Schneider (2008)). In the second case, it is very common for self-employees to evade taxes working (i.e. without issue an invoice) and increasing their private consumption.

Third, *tax marginal rate* is important cause of the size of the shadow economy. This finding confirm our hypothesis as well as previous empirical findings. As expected, the relation between tax marginal rate and the shadow economy is positive, i.e. a tax marginal rate encourage people to evade taxes and creating more informal economy (Tanzi and Zee (2000)).

Last but not least, *social benefit* and *government expenditure on labour* are the poorest causes creating underground economy in our model. The first one is not significant in MIMIC 6-1-2, while the second cause is not significant as well, but in this case in MIMIC 5-1-2.

4.6 Concluding remarks

The paper estimates the European shadow economy from 1993 to 2015 by applying the MIMIC approach.

As declared in Dell'Anno (2007) there are several interest related to the quality of shadow economy estimation (the problem of endogeneity, the relationship between indicators and causes chosen, the issue of calibration through an exogenous computation of shadow economy in the base year, etc.) every choice may lead to different size of unofficial economy.

Using twenty-three years (1993-2015) in which the assumption of multivariate normality on differenced causes is confirmed by empirical literature on macro approach estimation on shadow economy (see Schneider (2015)), it should underestimate the reliability of results and the path in this case is confirmed.

This research topic involves a trade-off between statistical and economic circumstances. To find an economically valid definition of the latent variable for shadow economy, the MIMIC specification has to admit institutional and taxation variables - available only annually - reducing sample size and the statistical dependability of the results, also due to difficult of official statistics to catch some peculiarity of hidden economy.

According the best MIMIC result (i.e. 4-1-2, four observed causes and two observed indicators), the magnitude of unofficial economy respect to real GDP is limited between 21% to 10% of official GDP.

The trend of underground economy follows the econometric result of Schneider (2015), confirming a convergence in the last years of twenty century for EU-OECD countries following Maastricht's parameters in order to entry into Euro area while the results are in diminution from 2010 to 2015 after they reached local peak during the great recession of 2007. Therefore, it is possible to hypothesize the counter-cyclical trend of the unofficial economy compared to the economic cycle.

The results suggest - in order to reduce the EU-OECD countries shadow economy - that political agenda has to focus on fiscal policy (i.e. government spending, taxes and regulation) and macroeconomic policy (i.e. unemployment); indeed, after the crisis, there is a need of fiscal and labour policy reform to improve the market (to face another financial crisis which is poured into a real crisis) and make it come out the majority part of unofficial economy.

Appendix A - Table of estimation

We want to be able to compare the resulting inference of these different approaches. It is difficult to compare different t-statistics with different degrees of freedom. Therefore, for each of the different inference approaches we convert to the probability of the contrast being greater than zero. This provides us with a measure which we can compare directly between the different approaches. We represent this probability as a z-statistic by ensuring that the area under one tail of a standardised (zero mean and standard deviation of one) Normal distribution corresponds to that probability. This is a complementing and exhaustive analysis of causes and indicator by Dell'Anno (2007).

Table 4.3: Regression MIMIC table - z-statistic

	(1) MIMIC 6-1-2	(2) MIMIC 5-1-2	(3) MIMIC 4-1-2	(4) MIMIC 4-1-2 †
SE				
unemployrate	2.88§	2.89§	2.99§	3.27§
	(0.00313)	(0.00313)	(0.00318)	(0.00285)
selfempl	2.81§	3.08§	2.91§	2.88§
	(0.0358)	(0.0355)	(0.0357)	(0.0336)
indexregul	3.06§	3.26§	2.61§	3.12§
	(0.0136)	(0.0135)	(0.0155)	(0.0136)
taxmargrate	5.26§	5.44§	5.90§	6.10§
	(0.00437)	(0.00443)	(0.00413)	(0.00391)
socialbenefit	0.96			
	(0.000311)			
governmentunemp	1.51	1.92		
	(0.00555)	(0.00542)		
realgdp2010				
SE	6.41	6.44	6.67	6.65
	(.)	(.)	(.)	(.)
_cons	18.70	18.68	18.84	19.27
	(0.0326)	(0.0331)	(0.0305)	(0.0262)
lfpr				
SE	6.66	6.42	7.17	
	(1.579)	(1.650)	(1.656)	
_cons	2.35	2.41	2.26	
	(0.227)	(0.230)	(0.232)	
lfprfem				
SE				7.24
				(0.0389)
_cons				3.07
				(0.00510)

† *Female LFPR as indicator*
Standard errors in parentheses
§ Means |z-statistic|>1.96

Table 4.4: MIMIC - table of variance

	(1) MIMIC 6-1-2	(2) MIMIC 5-1-2	(3) MIMIC 4-1-2	(4) MIMIC 4-1-2 †
var(e.realgdp2010)				
_cons	0.00669***	0.00677***	0.00674***	0.00707***
	(0.000753)	(0.000765)	(0.000732)	(0.000675)
var(e.lfpr)				
_cons	0.276***	0.294***	0.296***	
	(0.0420)	(0.0451)	(0.0459)	
var(e.SE)				
_cons	0.00127**	0.00130**	0.00138***	0.00100**
	(0.000518)	(0.000524)	(0.000526)	(0.000394)
var(e.lfprfem)				
_cons				0.000106***
				(0.0000198)
N	213	213	233	233

† *Female LFPR as indicator*
Standard errors in parentheses
* $p < .1$, ** $p < .05$, *** $p < .01$

Appendix B - Unit root tests

Table 4.5: Unit root tests - indicators

	(1) ADF 2 lags	(2) ADF 4 lags	(3) ADF 5 lags	(4) PP 2 lags
Real GDP index2010				
Inverse chi-squared (P)	56.77	70.27	50.37	166.21
	(0.093)	(0.007)	(0.236)	(0.000)
Inverse normal (Z)	-0.94	-1.90	-0.56	-1.47
	(0.172)	(0.029)	(0.289)	(0.000)
Modified inv. chi-squared (Pm)	1.36	2.80	0.68	13.03
	(0.086)	(0.003)	(0.248)	(0.000)
LFPR				
Inverse chi-squared (P)	35.31	49.18	41.55	34.74
	(0.913)	(0.425)	(0.733)	(0.924)
Inverse normal (Z)	1.40	0.24	1.33	1.07
	(0.920)	(0.594)	(0.908)	(0.858)
Modified inv. chi-squared (Pm)	-1.29	0.12	-0.66	-1.35
	(0.902)	(0.452)	(0.745)	(0.912)

P-value in parentheses

Table 4.6: Unit root test - Causes

	(1) ADF 2 lags	(2) ADF 4 lags	(3) ADF 5 lags	(4) PP 2 lags
Unemployment rate				
Inverse chi-squared (P)	23.38	82.22	24.36	28.67
	(0.997)	(0.0008)	(0.996)	(0.978)
Inverse normal (Z)	2.96	1.21	4.68	2.05
	(0.998)	(0.887)	(1.000)	(0.980)
Modified inv. chi-squared (Pm)	-2.36	3.77	-2.25	-1.81
	(0.990)	(0.0001)	(0.998)	(0.964)
Self employment				
Inverse chi-squared (P)	14.96	18.38	34.58	23.63
	(1.0000)	(0.999)	(0.844)	(0.997)
Inverse normal (Z)	4.28	4.04	2.26	3.54
	(1.0000)	(1.000)	(0.988)	(0.999)
Modified inv. chi-squared (Pm)	-3.23	-2.73	-1.00	-2.33
	(0.994)	(0.996)	(0.842)	(0.990)
Index of regulation				
Inverse chi-squared (P)	7.57	8.28	12.31	20.92
	(1.000)	(1.000)	(1.000)	(0.998)
Inverse normal (Z)	3.94	4.53	4.02	1.88
	(1.000)	(1.000)	(1.000)	(0.970)
Modified inv. chi-squared (Pm)	-3.88	-3.54	-2.95	-2.46
	(0.999)	(0.999)	(0.998)	(0.993)
Tax marginal rate				
Inverse chi-squared (P)	137.65	43.59	86.63	102.91
	(0.000)	(0.487)	(0.000)	(0.000)
Inverse normal (Z)	-0. 466	1.00	0.31	0.36
	(0.320)	(0.940)	(0.617)	(0.639)
Modified inv. chi-squared (Pm)	9.98	-0.04	4.54	6.28
	(0.000)	(0.52)	(0.000)	(0.000)
Social Benefit				
Inverse chi-squared (P)	30.94	22.94	23.84	1290.45
	(0.956)	(0.998)	(0.997)	(0.000)
Inverse normal (Z)	2.12	3.73	4.29	-32.93
	(0.983)	(0.999)	(1.000)	(0.000)
Modified inv. chi-squared (Pm)	-1.57	3.76	-2.31	129.74
	(0.942)	(0.9919)	(1.000)	(0.000)
Govern. uneployment				
Inverse chi-squared (P)	53.78	27.28	41.82	146.84
	(0.200)	(0.977)	(0.565)	(0.000)
Inverse normal (Z)	-0.61	1.94	0.99	-2.22
	(0.269)	(0.973)	(0.841)	(0.131)
Modified inv. chi-squared (Pm)	0.81	-1.78	-0.23	10.51
	(0.208)	(0.962)	(0.592)	(0.000)

P-value in parentheses

Chapter 5

Conclusions

The present Ph.D. thesis is developed on two dimensions, the first one concerning the theoretical framework, focusing on terminology and principles of the relationship between taxpayers and tax authorities. The second one concerns the shadow economy that represents economic activities with monetary transactions that are subject to taxes but are not reported to the tax authorities or to institutions in order to avoid regulations and evade paying taxes on the sales and income resulting from these activities.

For all these reasons, in the chapter 3 we analysed the basic features of taxpayer's profile and the peers by discussing the economic, socio-psychological, neurological determinants and basic tax behaviour models from both the perspectives of economic and socio-psychological rationality.

The simulation results presented widely confirm the findings of previous theoretical, experimental on Agent-based studies (e.g. Allingham and Sandmo (1972); Andrei et al. (2014); Alm et al. (2004). In particular, raising the audit probability or the tax rate on undeclared income would lead - *ceteris paribus* - to less tax evasion and vice versa.

The final results from "pure" simulations (Ciarli and Valente (2005)), seems to confirm the quantitative results based on subjective probabilities of each agent, analysing various situations through different simulations. Indeed, when a stronger penalty is included in the economic setting there is a reduction of tax evasion assessed between 10% and 30%.

In the chapter 4, the dissertation estimates the annual size of the shadow economy in European countries for period 1993-2015 using the multiple indicators multiple causes (MIMIC) approach. The MIMIC approach is a special case of structural equation models (SEMs), and it is used to estimate the annual size of the shadow economy in Europe during the period of study. This approach uses multiple observed variables as causes and indicators of the shadow economy, which is considered itself a latent variable.

According to the MIMIC approach, the causal variables for the shadow economy in European countries are found to be: the self employment rate, the unemployment rate, the extent of government regulation and the tax marginal rate. The growth rate of real GDP and the labour force participation ratio are found to be indicators of the shadow economy in Europe. In this case the shadow economy in selected countries is included between 20% (1999-2000) and 15% (2014-2015).

Finally, the dissertation has shown - on the one hand - as quantitative and qualitative analysis, nonetheless they are two different models, being useful to achieve the same results completing each others. Being difficult to improve the situation of central government, this work suggests that an embittering of laws reduces the level of tax evasion independently of the type of external environment. Otherwise, to fight tax evasion is necessary improving the efficiency of central government, but this is very difficult to implement in the process. On the other hand, this dissertation has illustrated the significance of shadow economy in the European economy; its absolute value has highlighting an anti-cyclic trend overtime and it represents a significant percent of official GDP based on the results of the methodology that we used to estimate it.

Motto: Research is never completed… Around the corner lurks another possibility of interview, another book to read, a document to verify.

Catherine Drinker Bowen

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