Hindawi
Advances in High Energy Physics
Volume 2017, Article ID 7656389, 10 pages
https://doi.org/10.1155/2017/7656389

Research Article

Energy-Momentum for a Charged Nonsingular Black Hole Solution with a Nonlinear Mass Function

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Received 5 July 2017; Revised 20 September 2017; Accepted 21 November 2017; Published 24 December 2017

Academic Editor: Elias C. Vagenas

The energy-momentum of a new four-dimensional, charged, spherically symmetric, and nonsingular black hole solution constructed in the context of general relativity coupled to a theory of nonlinear electrodynamics is investigated, whereby the nonlinear mass function is inspired by the probability density function of the continuous logistic distribution. The energy and momentum distributions are calculated by use of the Einstein, Landau-Lifshitz, Weinberg, and Møller energy-momentum complexes. In all these prescriptions, it is found that the energy distribution depends on the mass M and the charge q of the black hole, an additional parameter β coming from the gravitational background considered, and the radial coordinate r. Further, the Landau-Lifshitz and Weinberg prescriptions yield the same result for the energy, while, in all the aforesaid prescriptions, all the momenta vanish. We also focus on the study of the limiting behavior of the energy for different values of the radial coordinate, the parameter β, and the charge q. Finally, it is pointed out that, for $r \to \infty$ and $q = 0$, all the energy-momentum complexes yield the same expression for the energy distribution as in the case of the Schwarzschild black hole solution.

1. Introduction

The problem of the energy-momentum localization in general relativity has been investigated over the years by using various and different powerful tools such as superenergy tensors [1–4], quasilocal expressions [5–9], and the mostly known pseudotensorial energy-momentum complexes introduced by Einstein [10, 11], Landau and Lifshitz [12], Papapetrou [13], Bergmann and Thomson [14], Møller [15], Weinberg [16], and Qadir and Sharif [17].

As it is well-known, the main difficulty which arises consists in developing a properly defined expression for the energy density of the gravitational background. Until today, no generally accepted meaningful definition for the energy of the gravitational field has been established. However, despite this difficulty, many physically reasonable results have been obtained by applying the aforesaid definitions for the energy-momentum localization. At this point, one cannot but notice the existing agreement between the pseudotensorial prescriptions and the quasilocal mass definition elaborated by Penrose [18] and further developed by Tod [19].

Although the dependence on the coordinate system continues to be the main "weakness" of these tools, a number of physically interesting results have been obtained for gravitating systems in $(3+1)$, $(2+1)$, and $(1+1)$ space-time dimensions by using the energy-momentum complexes [20–50].

In fact, the Møller energy-momentum complex is the only computational tool independent of coordinates. In the context of other pseudotensorial prescriptions, in order to calculate the energy and momentum distributions, one introduces Schwarzschild Cartesian and Kerr-Schild coordinates.

An alternative option for avoiding the problem of the coordinate system dependence is provided by the teleparallel theory of gravity [51, 52], whereby one notices the considerable similarity of results obtained by this approach with results achieved by using the energy-momentum complexes [53–57].

Finally, closing this short introduction to the topic of the energy-momentum localization, it is necessary to point out the broadness of the ongoing attempts in order to define properly and, actually, rehabilitate the concept of the energy-momentum complex [58–61].

The outline of the present paper is the following. In Section 2, we introduce the new static and charged, spherically symmetric, nonsingular black hole solution under study. Section 3 is devoted to the presentations of the Einstein, Landau-Lifshitz, Weinberg, and Møller energy-momentum complexes used for the calculations. In Section 4, the computations of the energy and momentum distributions are presented. In the discussion given in Section 5, we comment on our results and explore some limiting and particular cases. We have used geometrized units ($c = G = 1$), while the signature is $(+, -, -, -)$. The calculations for the Einstein, Landau-Lifshitz, and Weinberg energy-momentum complexes are performed by use of the Schwarzschild Cartesian coordinates. Greek indices range from 0 to 3, while Latin indices run from 1 to 3.

2. The New Charged Nonsingular Black Hole Solution with a Nonlinear Mass Function

The determination of nonsingular black hole solutions by coupling gravity to nonlinear electrodynamics has attracted interest long ago (for a review, see, e.g., [62] for spherically symmetric solutions or [63] and references therein for charged axisymmetric solutions). Recently, Balart and Vagenas [64] constructed a number of new charged, nonsingular, and spherically symmetric, four-dimensional black hole solutions with a nonlinear electrodynamics source. Indeed, start with the static and spherically symmetric space-time geometry described by the line element:

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad (1)$$

with the metric function

$$f(r) = 1 - \frac{2M}{r}\left[\frac{\sigma(\beta r)}{\sigma_\infty}\right]^\beta, \quad (2)$$

where the distribution function $\sigma(\beta r)$ depends on the mass M, the charge q, the radial coordinate r, and the parameter $\beta \in \mathbb{R}^+$, and $\sigma_\infty = \sigma(r \to \infty)$ is a normalization factor. Thus one has a nonlinear mass function of the following form:

$$m(r) = M\left[\frac{\sigma(\beta r)}{\sigma_\infty}\right]^\beta, \quad (3)$$

which, at infinity, becomes M. It is shown in [64] that for specific distribution functions $\sigma(r)$ the curvature invariants (R, $R_{\mu\nu}R^{\mu\nu}$, $R_{\kappa\lambda\mu\nu}R^{\kappa\lambda\mu\nu}$) and the associated nonlinear electric field are nonsingular everywhere.

Based on these results, we have already studied [65] the problem of the localization of energy for a function $\sigma(r)$ resembling the form of the Fermi-Dirac distribution. Here, following up that work, we adopt another distribution function given in [64] that is inspired by the form of the probability density function of the continuous logistic distribution [66], such that

$$f(r) = 1 - \frac{2M}{r}\left\{\frac{4\exp\left(-\sqrt{2q^2/\beta M r}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2}\right\}^\beta, \quad (4)$$

$$\beta \in \mathbb{R}^+$$

with the nonlinear mass function given by (3).

The associated nonsingular space-time exhibits two horizons, while the nonlinear and nonsingular electric field that asymptotically goes to q/r^2 reads now

$$E(r) = \frac{q}{8r^2}\left(\operatorname{sech}\sqrt{\frac{q^2}{2\beta M r}}\right)^{2(1+\beta)} \times \left[(1+\beta) - \beta\cosh\sqrt{\frac{2q^2}{\beta M r}} + 7\sqrt{\frac{\beta M r}{2q^2}}\sinh\sqrt{\frac{2q^2}{\beta M r}}\right]. \quad (5)$$

When $\beta \to 0$, the metric function (4) takes the form $f(r) = 1 - 2M/r$; that is, we get the Schwarzschild black hole geometry.

Thus, in what follows, we are going to investigate the problem of energy-momentum localization for a charged and nonsingular black hole solution with the space-time described by (1), (4) and the nonlinear mass function:

$$m(r) = M\left\{\frac{4\exp\left(-\sqrt{2q^2/\beta M r}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2}\right\}^\beta. \quad (6)$$

3. Einstein, Landau-Lifshitz, Weinberg, and Møller Energy-Momentum Complexes

The definition of the Einstein energy-momentum complex [10, 11] for a (3 + 1)-dimensional gravitating system is given by

$$\theta_\nu^\mu = \frac{1}{16\pi}h^{\mu\lambda}_{\nu,\lambda}, \quad (7)$$

where the von Freud superpotentials $h_{\nu}^{\mu\lambda}$ are given as

$$h_{\nu}^{\mu\lambda} = \frac{1}{\sqrt{-g}} g_{\nu\sigma} \left[-g\left(g^{\mu\sigma} g^{\lambda\kappa} - g^{\lambda\sigma} g^{\mu\kappa}\right)\right]_{,\kappa} \quad (8)$$

and satisfy the required antisymmetric property

$$h_{\nu}^{\mu\lambda} = -h_{\nu}^{\lambda\mu}. \quad (9)$$

The components θ_0^0 and θ_i^0 correspond to the energy and the momentum densities, respectively. In the Einstein prescription, the local conservation law holds:

$$\theta_{\nu,\mu}^{\mu} = 0. \quad (10)$$

Thus, the energy and the momenta can be computed by

$$P_{\nu} = \iiint \theta_{\nu}^{0} dx^1 dx^2 dx^3. \quad (11)$$

Applying Gauss' theorem, the energy-momentum is

$$P_{\nu} = \frac{1}{16\pi} \iint h_{\nu}^{0i} n_i dS, \quad (12)$$

where n_i represents the outward unit normal vector on the surface dS.

The Landau-Lifshitz energy-momentum complex [12] is defined as

$$L^{\mu\nu} = \frac{1}{16\pi} S_{,\rho\sigma}^{\mu\nu\rho\sigma}, \quad (13)$$

where the Landau-Lifshitz superpotentials are given by

$$S^{\mu\nu\rho\sigma} = -g\left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}\right). \quad (14)$$

The L^{00} and L^{0i} components represent the energy and the momentum densities, respectively. In the Landau-Lifshitz prescription, the local conservation law reads

$$L_{,\nu}^{\mu\nu} = 0. \quad (15)$$

By integrating $L^{\mu\nu}$ over the 3-space, one obtains for the energy-momentum:

$$P^{\mu} = \iiint L^{\mu 0} dx^1 dx^2 dx^3. \quad (16)$$

By using Gauss' theorem, we have

$$P^{\mu} = \frac{1}{16\pi} \iint S_{,\nu}^{\mu 0 i \nu} n_i dS = \frac{1}{16\pi} \iint U^{\mu 0 i} n_i dS. \quad (17)$$

The Weinberg energy-momentum complex [16] is given by the following expression:

$$W^{\mu\nu} = \frac{1}{16\pi} D_{,\lambda}^{\lambda\mu\nu}, \quad (18)$$

where $D^{\lambda\mu\nu}$ are the corresponding superpotentials:

$$D^{\lambda\mu\nu} = \frac{\partial h_{\kappa}^{\kappa}}{\partial x_{\lambda}} \eta^{\mu\nu} - \frac{\partial h_{\kappa}^{\kappa}}{\partial x_{\mu}} \eta^{\lambda\nu} - \frac{\partial h^{\kappa\lambda}}{\partial x^{\kappa}} \eta^{\mu\nu} + \frac{\partial h^{\kappa\mu}}{\partial x^{\kappa}} \eta^{\lambda\nu} + \frac{\partial h^{\lambda\nu}}{\partial x_{\mu}} - \frac{\partial h^{\mu\nu}}{\partial x_{\lambda}}, \quad (19)$$

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}.$$

Here the W^{00} and W^{0i} components correspond to the energy and the momentum densities, respectively. In the Weinberg prescription, the local conservation law reads

$$W_{,\nu}^{\mu\nu} = 0. \quad (20)$$

The integration of $W^{\mu\nu}$ over the 3-space yields for the energy-momentum:

$$P^{\mu} = \iiint W^{\mu 0} dx^1 dx^2 dx^3. \quad (21)$$

Applying Gauss' theorem and integrating over the surface of a sphere of radius r, one obtains for the energy-momentum distribution the following expression:

$$P^{\mu} = \frac{1}{16\pi} \iint D^{i 0 \mu} n_i dS. \quad (22)$$

The Møller energy-momentum complex [15] is given by the following expression:

$$\mathcal{J}_{\nu}^{\mu} = \frac{1}{8\pi} M_{\nu,\lambda}^{\mu\lambda}, \quad (23)$$

where the Møller superpotentials $M_{\nu}^{\mu\lambda}$ are

$$M_{\nu}^{\mu\lambda} = \sqrt{-g}\left(\frac{\partial g_{\nu\sigma}}{\partial x^{\kappa}} - \frac{\partial g_{\nu\kappa}}{\partial x^{\sigma}}\right) g^{\mu\kappa} g^{\lambda\sigma} \quad (24)$$

and satisfy the necessary antisymmetric property:

$$M_{\nu}^{\mu\lambda} = -M_{\nu}^{\lambda\mu}. \quad (25)$$

Møller's energy-momentum complex also satisfies the local conservation law

$$\frac{\partial \mathcal{J}_{\nu}^{\mu}}{\partial x^{\mu}} = 0, \quad (26)$$

with $\mathcal{J}_0^0$ and $\mathcal{J}_i^0$ representing the energy and the momentum densities, respectively. In the Møller prescription, the energy-momentum is given by

$$P_{\nu} = \iiint \mathcal{J}_{\nu}^{0} dx^1 dx^2 dx^3. \quad (27)$$

With the aid of Gauss' theorem, one gets

$$P_{\nu} = \frac{1}{8\pi} \iint M_{\nu}^{0i} n_i dS. \quad (28)$$

4. Energy and Momentum Distributions for the New Charged Nonsingular Black Hole Solution

In order to calculate the energy and momenta by using the Einstein energy-momentum complex, it is required to transform the metric given by the line element (1) in Schwarzschild Cartesian coordinates. We obtain the line element in the following form:

$$ds^2 = f(r) dt^2 - \left(dx^2 + dy^2 + dz^2\right) - \frac{f^{-1}(r) - 1}{r^2} (x dx + y dy + z dz)^2. \quad (29)$$

The components of the superpotential h_{ν}^{0i} in Schwarzschild Cartesian coordinates are

$$\begin{aligned} h_1^{01} &= h_1^{02} = h_1^{03} = 0, \\ h_2^{01} &= h_2^{02} = h_2^{03} = 0, \\ h_3^{01} &= h_3^{02} = h_3^{03} = 0. \end{aligned} \tag{30}$$

The remaining, nonvanishing, components of the superpotentials in the Einstein prescription are as follows:

$$\begin{aligned} h_0^{01} &= \frac{2x}{r^2}\frac{2M}{r}\left\{\frac{4\exp\left(-\sqrt{2q^2/\beta M r}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2}\right\}^{\beta}, \\ h_0^{02} &= \frac{2y}{r^2}\frac{2M}{r}\left\{\frac{4\exp\left(-\sqrt{2q^2/\beta M r}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2}\right\}^{\beta}, \\ h_0^{03} &= \frac{2z}{r^2}\frac{2M}{r}\left\{\frac{4\exp\left(-\sqrt{2q^2/\beta M r}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2}\right\}^{\beta}. \end{aligned} \tag{31}$$

From the line element (29), the expression (12), and the superpotentials (31), we obtain for the energy distribution in the Einstein prescription (see Figure 1) the following:

$$E_{\mathrm{E}} = M\left\{\frac{4\exp\left(-\sqrt{2q^2/\beta M r}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2}\right\}^{\beta}. \tag{32}$$

By using (12) and (30), we find that all the momenta vanish:

$$P_x = P_y = P_z = 0. \tag{33}$$

In order to apply the Landau-Lifshitz energy-momentum complex, we use the $U^{\mu 0i}$ quantities defined in (17) and we find the following nonvanishing components:

$$\begin{aligned} U^{001} &= \frac{2x}{r^2}\frac{(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}, \\ U^{002} &= \frac{2y}{r^2}\frac{(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}, \\ U^{003} &= \frac{2z}{r^2}\frac{(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}. \end{aligned} \tag{34}$$

Now, replacing (34) in (17), we obtain the energy distribution in the Landau-Lifshitz prescription:

$$E_{\mathrm{LL}} = \frac{M\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}, \tag{35}$$

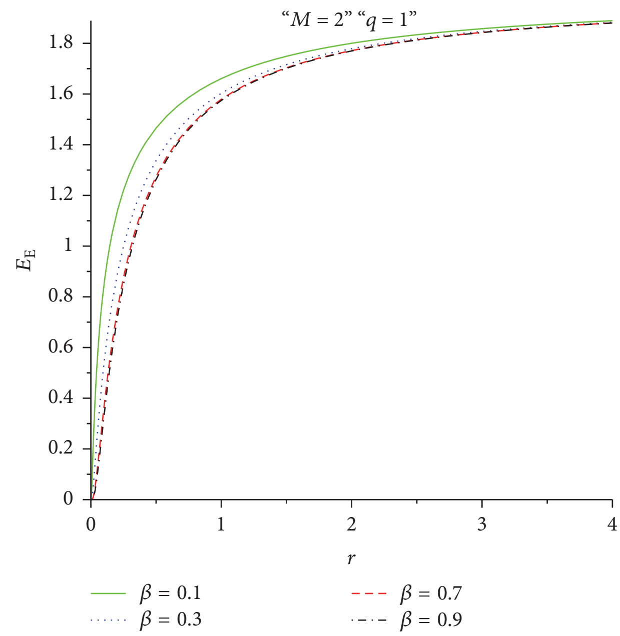

FIGURE 1: Energy distribution obtained in the Einstein prescription for different values of β.

while all the momenta vanish:

$$P^x = P^y = P^z = 0. \tag{36}$$

In the Weinberg prescription, the nonvanishing superpotential components are as follows:

$$\begin{aligned}
D^{100} &= \frac{2x}{r^2}\frac{(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}},\\
D^{200} &= \frac{2y}{r^2}\frac{(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}},\\
D^{300} &= \frac{2z}{r^2}\frac{(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}.
\end{aligned} \tag{37}$$

Substituting the expressions obtained in (37) into (22), we get for the energy distribution inside a 2-sphere of radius r the following expression:

$$E_W = \frac{M\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta M r}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta M r}\right)\right]^2\right\}^{\beta}}. \tag{38}$$

Table 1: Energy distributions calculated by use of the energy-momentum complexes of Einstein, Landau-Lifshitz, Weinberg, and Møller.

Prescription	Energy
Einstein	$E_{\mathrm{E}} = M\left\{\frac{4\exp\left(-\sqrt{2q^2/\beta Mr}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)\right]^2}\right\}^{\beta}$
Landau-Lifshitz	$E_{\mathrm{LL}} = \frac{M\left\{4\exp\left(-\sqrt{2q^2/\beta Mr}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta Mr}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)\right]^2\right\}^{\beta}}$
Weinberg	$E_{\mathrm{W}} = \frac{M\left\{4\exp\left(-\sqrt{2q^2/\beta Mr}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)\right]^2\right\}^{\beta}}{1-(2M/r)\left\{4\exp\left(-\sqrt{2q^2/\beta Mr}\right)/\left[1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)\right]^2\right\}^{\beta}}.$
Møller	$E_{\mathrm{M}} = M\left[\frac{4\exp\left(-\sqrt{2q^2/\beta Mr}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)\right]^2}\right]^{\beta} \times \left\{1-\sqrt{\frac{q^2\beta}{2Mr}}\left[\frac{1-\exp\left(-\sqrt{2q^2/\beta Mr}\right)}{1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)}\right]\right\}$

In the Weinberg prescription, all the momenta vanish:

$$P^x = P^y = P^z = 0. \tag{39}$$

One can see from (35) and (38) that the energy in the Landau-Lifshitz prescription is identical with the energy in the Weinberg prescription.

Finally, in the Møller prescription, we find only one nonvanishing superpotential:

$$M_0^{01} = 2M\sin\theta\left[\frac{4\exp\left(-\sqrt{2q^2/\beta Mr}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)\right]^2}\right]^{\beta}\left\{1 - \frac{q^2}{Mr^3\sqrt{2q^2/\beta Mr}}\left[\frac{1-\exp\left(-\sqrt{2q^2/\beta Mr}\right)}{1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)}\right]\right\}. \tag{40}$$

Using the above expression for the superpotential and with the aid of the metric coefficient (4) and the expression for energy given by (28), we obtain the energy distribution in the Møller prescription (see Figure 2):

$$E_{\mathrm{M}} = M\left[\frac{4\exp\left(-\sqrt{2q^2/\beta Mr}\right)}{\left[1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)\right]^2}\right]^{\beta} \cdot \left\{1-\sqrt{\frac{q^2\beta}{2Mr}}\left[\frac{1-\exp\left(-\sqrt{2q^2/\beta Mr}\right)}{1+\exp\left(-\sqrt{2q^2/\beta Mr}\right)}\right]\right\}, \tag{41}$$

while all the momenta vanish:

$$P_r = P_\theta = P_\phi = 0. \tag{42}$$

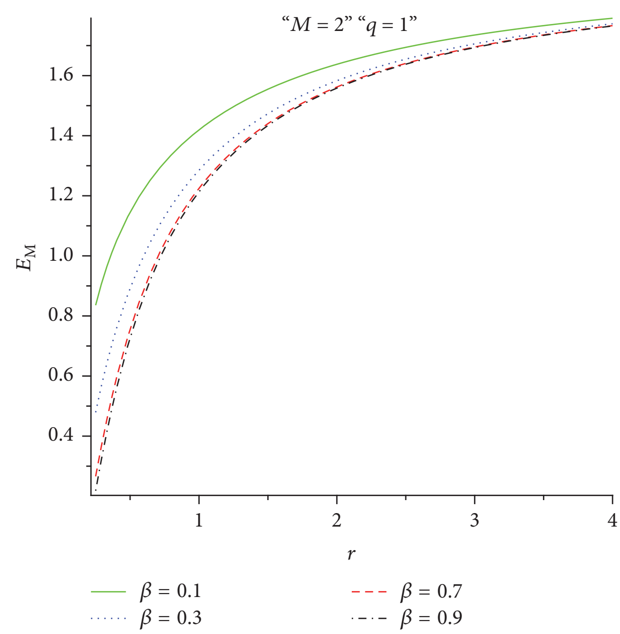

Figure 2: Energy distribution obtained in the Møller prescription for different values of β.

5. Discussion

Following up our previous results [65], we have studied the problem of energy-momentum localization for a new charged, nonsingular and spherically symmetric, static black hole solution in (3 + 1) dimensions with a nonlinear mass function recently constructed by Balart and Vagenas [64]. To this purpose, we have applied the Einstein, Landau-Lifshitz, Weinberg, and Møller pseudotensorial prescriptions. The calculations have shown that, in all the four prescriptions, the momenta vanish, while the energy depends (see Table 1) on

Table 2: Limiting cases for the energy.

Prescription	Energy for $r \to \infty$	Energy for $q = 0$	Energy for $\beta \to 0$	Energy for $\beta \to \infty$
Einstein	M	M	M	$M \exp\left(-\dfrac{q^2}{2Mr}\right)$
Landau-Lifshitz	M	$\dfrac{M}{1-2M/r}$	$\dfrac{M}{1-2M/r}$	$\dfrac{M\exp(-q^2/2Mr)}{1-(2M/r)\exp(-q^2/2Mr)}$
Weinberg	M	$\dfrac{M}{1-2M/r}$	$\dfrac{M}{1-2M/r}$	$\dfrac{M\exp(-q^2/2Mr)}{1-(2M/r)\exp(-q^2/2Mr)}$
Møller	M	M	M	$\left[M-\dfrac{q^2}{2r}\right]\exp\left(-\dfrac{q^2}{2Mr}\right)$

the black hole's mass and charge, the radial coordinate, and a parameter $\beta \in \mathbb{R}^+$ that is a scaling factor of r (in essence a dilation factor, as it is always positive) inspired by the form of the logistic distribution and marks the space-time geometry considered. In fact, for each value of β, one can numerically determine the values of the two existing horizon radii (an inner and an outer) obtained from the metric function. It is pointed out that the Landau-Lifshitz and the Weinberg prescriptions yield the same energy distribution.

We have also examined the limiting behavior of the energy in the cases $r \to \infty$, $q = 0$, $\beta \to 0$, and $\beta \to \infty$. For $\beta \to 0$, the metric function $f(r)$ becomes $1 - 2M/r$, while, for $\beta \to \infty$, $f(r)$ becomes $1-(2M/r)[\exp(-q^2/2Mr)]$. The corresponding energies are presented in Table 2, where one can see that for $q = 0$ as well as at infinity the Einstein and Møller prescriptions yield the same result which is also obtained for the classical Schwarzschild black hole solution, namely, the ADM mass M. In the case of the Landau-Lifshitz and Weinberg prescriptions, the energy equals the ADM mass M at infinity $r \to \infty$, while, in the chargeless case $q = 0$, these two energy-momentum complexes give for energy the expression $M/(1 - 2M/r)$. These results coincide with those obtained by Virbhadra's approach in Schwarzschild Cartesian coordinates [67]. As it is pointed out at the end of Section 2, for $\beta \to 0$ we get the classical Schwarzschild black hole metric. Hence, the corresponding energies given in Table 2 for $\beta \to 0$ are those obtained in the four prescriptions for the Schwarzschild black hole. It is noteworthy that for $\beta \to \infty$ a factor of 2 appears in the exponential in all four prescriptions. In fact, the same dependence on this factor of 2 is obtained for the Einstein and Møller energies in [65], where a different metric function $f(r)$ is considered.

Of particular interest is the behavior of the energy near the origin, namely, for $r \to 0$. In the case of the Einstein prescription, the energy tends to zero (see Figure 3). However, the energy obtained by the application of the Landau-Lifshitz, the Weinberg and the Møller energy-momentum complexes shows a rather pathological behavior. In the first two cases the energy is jumping between infinitely positive and negative values at some radial distances in the range $0 < r < 5$ for different values of the parameter β, while for significantly larger values of r ($r > 50$) the energy falls off rapidly to a constant value for various values of the parameter β. The energy calculated by the Møller prescription becomes clearly negative in the range $0 < r < 0.2$ for different values of the parameter β, while the position where the energy retains a positive value and then keeps increasing monotonically is shifted nearer to the origin as β becomes smaller (see Figure 4). This strange behavior of the energy distribution may enhance the argumentation (see, e.g., [67]) according to which the Einstein energy-momentum complex is indeed a more reliable tool for the study of the gravitational energy-momentum localization as it yields physically meaningful results. Thus, it may gain a preference among the different pseudotensorial energy-momentum complexes.

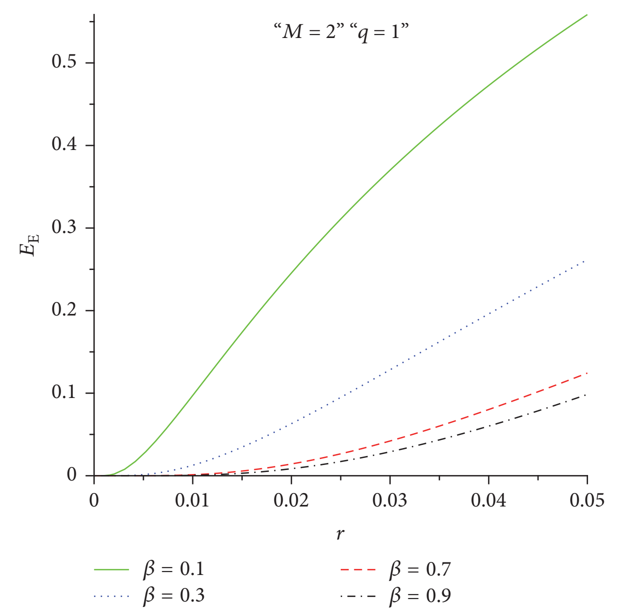

Figure 3: Energy distribution near zero obtained in the Einstein prescription for different values of β.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper. Also, there are no conflicts of interest regarding the received funding mentioned in Acknowledgments.

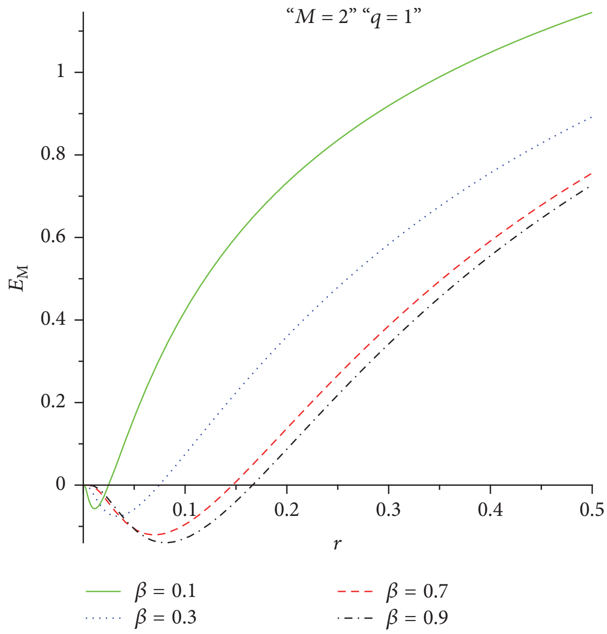

FIGURE 4: Energy distribution near zero obtained in the Møller prescription for different values of β.

Acknowledgments

Farook Rahaman is grateful to the Inter-University Centre for Astronomy and Astrophysics (IUCAA), India, for providing Associateship Programme. Farook Rahaman and Sayeedul Islam acknowledge DST, Government of India, for providing financial support under the SERB and INSPIRE Programme.

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