

**INVENTORY POLICY PLANNING
FOR SPARE PARTS AND ITS
APPLICATION IN THE
HEAVY-DUTY TRUCK AND BUS INDUSTRY**

by

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**A thesis report submitted to the Faculty of Engineering, University of the Witwatersrand,
Johannesburg, in fulfilment of the requirements for the degree of
Master of Science in Engineering**

Promoted by Mr K Sandrok

November 1994

ACKNOWLEDGEMENT

I wish to record my gratitude and sincere appreciation to all those who provided encouragement and assistance in the compilation of this thesis:

Mr K Sandrok, who promoted this thesis, my deep gratitude for his valuable time, constructive guidance, constant encouragement, and competent leadership over the past three years.

Prof D Lubinski of Applied Maths at Wits for his valuable assistance in developing the application in the Excel software program for calculating the cost of loss of sales according to the Gamma and Poisson distribution.

A special word of thanks to Dr D J C Jackson for the time he spent reading through the thesis, and the valuable comments he made.

Mr F Pitz, Managing Director of the Central Parts Department (CPD), who encouraged and supported this research project in his company which is one of the largest stockholders of spare parts.

Dr J Boer and my colleagues at the NPI for their encouragement, advice and permission to use NPI facilities when required. In particular R Lloyd, K Greeff, L Jurriaanse and S von der Plaats, my sincere appreciation.

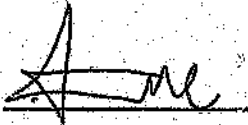
Above all, I record my deepest gratitude to my wife, Rachel, for her constant encouragement and support during my continuous studying and self-enrichment, and to my children and family who endured so much so patiently.

DECLARATION

I, Simon (Shimon) Azran, hereby declare that this thesis, entitled *Inventory Policy planning for spare parts and its application in the heavy duty truck and bus industry*, is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of a complete reference list.

This thesis has not previously been submitted for degree purposes to any university.

Simon Azran



22/12/94

ABSTRACT

Inventories are produced, used (e.g. for raw materials, supplies, spare parts, and so forth) or distributed by every organisation. Moreover, inventories represent a major investment from the perspectives of both individual firms and entire national economies. In addition, enormous costs are incurred in the planning, scheduling, control and actual carrying out of replenishment-(procurement) related activities.

Interest in the subject of inventory management is constantly increasing, yet Silver and Peterson⁽¹⁾ (Preface) found that "although inventory management has been studied in considerable depth from a theoretical perspective, yet, those of us who, through consulting work, come into close contact with managerial decision procedures in this area are repeatedly surprised to find how limited, and ad hoc, many of the existing decision systems actually are. The rate at which theory has been developed has far outstripped the rate at which decision practices of firms have been successfully upgraded. A major gap has existed between the theoretical solutions, on the one hand, and the real world problems, on the other".

Inventory control is the science-based art of ensuring that just enough inventory (or stock) is held by an organisation to meet both its internal and external demand commitments economically. There can be disadvantages in holding either too much or too little inventory. Therefore, inventory control is primarily concerned with obtaining the correct inventory with compromise between these two extremes.

The control and maintenance of inventories is a problem common to all enterprises in any sector of a given economy. The primary aim of this study is to identify what the inventory policy of a company should be to secure a reduction in inventory-related costs while maintaining a high level of customer service.

Lewis⁽²⁾ defines two basic types of inventory policy. Those in which decisions concerning replenishment are based on the level of inventory held, are known as "fixed-order quantity models" or "re-order level policies" and those in which such decisions are made on a time basis are known as "fixed-time period models" or "re-order cycle policies". According to Nadder⁽³⁾ (7, 11) the basic distinction between fixed-order quantity models and fixed-time period models is that the former are "event-triggered" while the latter are "time-triggered". That is, a fixed-order quantity model initiates an order when the "event" of reaching a specified re-order level occurs. This event may take place at any time, depending on the demand for the items considered. In contrast, the fixed-time period model is limited to placing orders at the end of a predetermined time period; hence, the passage of time alone "triggers" the model.

In this thesis, we shall discuss both classical inventory models and heuristic models. We shall also conduct an investigation into the factors affecting high levels of inventory - mainly lead times (supplier and internal lead times) in relation to spare parts in the heavy-duty truck and bus industry. The thesis also suggests guidelines for controlling stock of these types of commodities in a practical environment. This will be done by either researching the existing inventory models or developing new inventory models or a combination of both, the intention being not to look for absolute optimisation, but rather to achieve significant improvements over current operations.

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LIST OF SYMBOLS, VARIABLES AND ABBREVIATIONS

Current inventory system

- SKU = Stock-keeping unit
- BO = Back-order quantities (units)
- SH = Stock in hand (units)
- SO = Stock on order (units)
- ST = Stock in transit
- LTs = Supplier lead time (weeks)
- ETc = Internal company lead time (weeks)
- LT = Total lead time (weeks)
- NAD = New average demand of SKU (units)
- SOS = Suggested order size of SKU (units)
- Y = Average usage for the last six months (units)
- X = New monthly average (excluded from this calculation is all the monthly demand that exceeds twice the monthly average)
- MAD = Mean average deviation
- Sigma = $1,25 \times \text{MAD}$

(When distribution of demand during the replenishment lead time is normally distributed Sigma =)

LIST OF SYMBOLS AND ABBREVIATIONS FOR THE LITERATURE SURVEY AND PROPOSED INVENTORY MODELS

V	= unit variable cost in unit
r	= inventory carrying charges in R/R/year
A	= order cost (order)
D	= demand rate in units/year
X_i	= demand in month i (units)
x	= monthly average demand (units) for last 12 months
L_s	= supplier lead time (weeks)
R	= review period (weeks)
$L + R$	= replenishment lead time (weeks)
k	= safety factor
SS	= safety stock (units)
X_L	= forecast (or expected) demand over supplier lead time
X_R	= forecast (or expected) demand over review period
X_{L+R}	= forecast (or expected) demand over replenishment lead time (units)
STD_{L+R}	= standard deviation of errors of forecasts over replenishment lead time (units)
ROP	= re-order point (units)
OHQ	= on-hand quantity (units)
MOHQ	= maximum on-hand quantity (units)
ROQ	= re-order quantity (units)
AOHQ	= average on-hand quantity
$Pu \geq (k)$	= probability that a unit normal (mean 0, STD 1) variable takes on a value of k or larger
$Gu \geq (k)$	= special function of the unit normal (mean 0, STD 1) variable
Z	= inventory position

LIST OF SYMBOLS AND ABBREVIATIONS FOR THE LITERATURE SURVEY AND PROPOSED INVENTORY MODELS

PAGE

C_n	= Coefficient of variation of distribution of order size
C_n	= Coefficient of variation of distribution of number of orders
C_L	= Coefficient of variation of distribution of replenishment lead time
C_d	= Coefficient of variation of demand during replenishment lead time
B_1	= specified fixed cost per stockout occasion
B_2	= specified fractional charge per unit short
B_3	= specified fractional charge per unit short per unit time
P_1	= specified probability of no stockout per replenishment cycle
P_2	= specified fraction of demand to be satisfied routinely from shelf
P_3	= specified ready rate or fraction of time during which the net stock is positive
TBS	= specified average time between stockout occasions

1. INTRODUCTION

1.1 OVERVIEW

The South African based company used as the model in this thesis is a subsidiary of one of the leading German companies in the heavy-duty truck and bus industry. The company's South African facilities comprise an assembly plant located in Durban and warehouses throughout the country, plus a head office in Johannesburg.

The company distributes locally assembled trucks and buses as well as spare parts to bus and truck owners.

The plant sources components for production from overseas (Germany) in the form of CKD kits, as well as from local suppliers. The CKD kits are ordered through the company's head office, whereas local components are ordered directly by the assembly plant.

Inventory items held by the assembly plant are characterised as "items with dependent demand", i.e. if the demand for an end product (say bus or truck) is known, then the demand for component items is also known. In contrast, inventory items held by the Central Parts Department (CPD) and the branches (spare parts) are characterised as "items with independent demand", i.e. the demand for one item is not dependent on the demand for another item.

This study concerns itself with the inventory policy that is appropriate for independent demand situations. Control of the inventory of items with dependent demand is beyond the scope of this investigation.

1.1.1 THE INVENTORY SYSTEM

There are great differences between existing inventory systems. They differ in size and complexity, in the types of items they carry, in the costs associated with operating the system, in the nature of the stochastic processes associated with the system, and in the nature of the information available to decision makers at any given moment. All these differences can be considered to reflect variations in the structure of the inventory system. These variations can have an important bearing on the choice of operating doctrine that should be used in controlling the system. By an operating doctrine, we simply mean the rule which tells us when to order and how much to order.

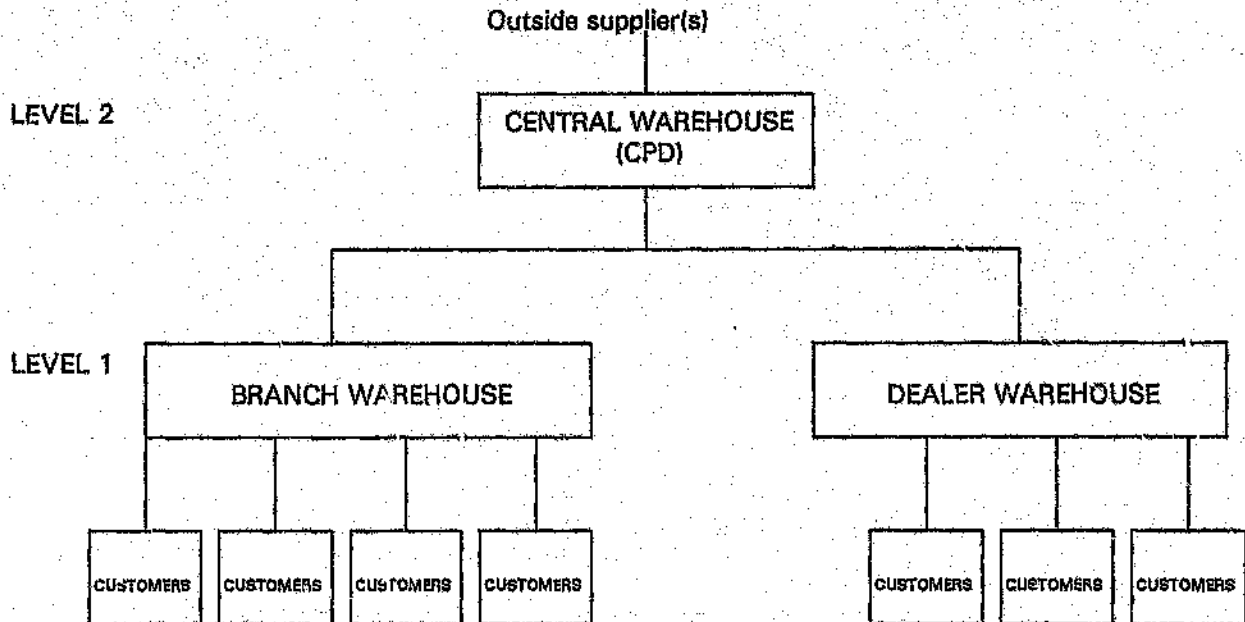
The inventory system in this organisation has a "multi-echelon structure". This means that an item may be stocked in the inventory system at many physical locations. When there is more than a single stocking point, there exists the possibility for many forms of interaction between the stocking points.

In our study the form of interaction involved is one stocking point (CPD) which serves as a warehouse for other stocking points. This leads to what is called a "multi-echelon inventory system". The type of multi-echelon system is illustrated in Figure 1. The arrows indicate the normal pattern for the flow of goods through the

system. This might be referred to as a two-echelon system since there are two levels. In the system shown, customer demands occur only at the stocking points on level 1. These stocking points have their stocks replenished by shipments from warehouses at level 2, which in turn receive replenishments for their stock from the suppliers. Figure 1 represents only one type of multi-echelon system. In other cases, customer demands might occur at all levels, or stocking points at any level might not only receive shipments from the next highest level (CPD) but might also get replenishments from any higher level (from the source supplier). It is also permissible, on occasion, to redistribute of stocks among various stocking points between branches. However, branches rarely distribute among themselves.

FIGURE 1

MULTI-ECHELON INVENTORY SYSTEM



The CPD currently carries approximately 25 560 stock-keeping units (SKUs) to supply twelve branches and six dealers with spare parts as required for maintenance purposes, emergency repairs, overhaul purposes and current services throughout South Africa. Each branch carries between 2 500 and 4 000 SKUs, mainly fast moving items, and each branch sources its parts from the CPD only, when the CPD does not supply parts to customers directly.

1.2 THE PROBLEM

The inventory under investigation comprises 25 560 SKUs with an annual average stock value of R49,9 million, of which approximately R36,2 million are carried by the CPD and R13,6 million by the branches.

At the single-item level, the inventory management objective can be formulated as minimising the total inventory-related cost per unit of time with respect to such restrictions as demand and desired service level. To measure the performance of an inventory management and control system, attention is usually shifted from individual items to aggregates. Allen⁽⁴⁾ indicates that costs, especially ordering and shortage costs, are no longer as widely used to measure performance as is the value of the inventory. The most common inventory measure, the inventory turnover (IT) rate, is normally used to measure inventory system performance. The rate can be defined as follows:

$$IT = \frac{\text{annual usage (R)}}{\text{average inventory investment (R)}} \quad \text{or} \quad IT = \frac{\text{annual usage (units)}}{\text{average inventory (units)}} = \frac{67,7}{49,9}$$

The inventory turnover rate in this case is 1,36 times per year, where

- annual usage (R) = annual sales at cost R67,7 million
- average annual inventory investment is R49,9 million.

Obsolete stock consists of 2 300 SKUs, with 9% of the total number of SKUs in the inventory valued at R3,7 million, of which 1 500 SKUs with a value of R2,5 million were scrapped from the inventory, and 800 SKUs with a value of R1,2 million were sent back to Germany. The inventory turnover rate after excluding the obsolete stock suggests that

$$IT = \frac{67,7}{46,2} = 1,46 \text{ times per year.}$$

The turnover rate tells how many times, on average, the inventory is filled up and emptied during a year in order to satisfy the demand during the same period. If a company can reduce its inventory and still maintain the same sales level, the turnover rate will increase. An increased number of turns is thereby a good trend, whereas a decreasing rate can cause alarm and trigger extra managerial attention.

There is, however, an inherent danger in relying too much on the turnover ratio as a goal variable. There are several reasons for this. One is that single items, or groups of items, might have actual turnover rates that differ, and should differ, substantially from the set goal. It may therefore be bad policy to stipulate the same rate for all materials. Another reason is that the inventory turnover rate is only a surrogate measure, indirectly related to the holding cost through the inventory investment in the denominator. The measure completely disregards other inventory-related costs, such as the ordering cost (increased cost of ordering, because of increased turnover rate) and the cost of being out of stock.

In this case, however, the inventory turnover ratio of 1,46 which is still considered to be low since it suggests an average of 7,8 months until a spare part is sold, where the suppliers' lead times vary between 1 month and 5 months, with a few exceptions of 9 months.

Other problem areas were identified when the inventory records were analysed:

- SKUs out of stock (unfulfilled orders) for November 1992 were found to amount to 2 200 SKUs. This is 12,5% of the total SKUs ordered.
 - The spare parts industry for heavy-duty trucks have common parts (mainly fast movers) that can be sold by competitors, and therefore running out of stock of those parts represents a potential loss of sales.
 - For the non-common spare parts and unique parts, running out of stock could lead to the loss of sales of the final product, mainly trucks and buses because of the low service level to owners.
- A high rate of obsolete stock which amounted to 10,2% of the total SKUs and 7,9% of the stock value.
- An over-stock situation where 81,7% of the total SKUs comprising 64% of the stock value, are slow movers and dead stock items, of which 38,1% of the SKUs with a value of R18,7 million are slow movers, and 43,7% of the SKUs with a value of R16,5 million are dead stock items. (The definitions for slow movers and dead stock items are defined in Section 3.2.)

1.3

OBJECTIVE

The main objective of the thesis (and therefore the main objective to solve the problems indicated above) is to minimise inventory investments and to maximise customer service levels by introducing and implementing an effective inventory control policy.

The proposed inventory policy should seek answers to the following two fundamental questions:

- When should the inventory be replenished?
- How much should be added to the inventory, or how much should be ordered of each item. Were the time and quantity elements are the variables that are subjected to control in the inventory system?

The proposed inventory system should also provide the management and stock control personnel with the information required to maintain and measure performance.

1.4

METHODOLOGY

The investigation took the following form:

- Evaluation of the current inventory system
 - Characteristics of truck and bus spare parts

- Analysis of the stock (ABC analysis)
- Defining fast, medium and slow-moving spare parts
- Study and evaluation of current inventory policy, simulating with chosen sample.
- o Investigation into important factors for inventory decisions
 - Cost factors
 - Replenishment lead time reduction.
- o Literature survey of inventory control systems
 - Type of inventory control systems
 - Establishment of safety stock
 - Studying available inventory models.
- o Development and evaluation of alternatives
 - Programming inventory control models
 - Simulating inventory control models on the computer, using data from a chosen sample
 - Analysis of the results for the trial run
 - Advantages and disadvantages of various models.
- o Selection and recommendations
 - Choosing the best methods from the models under investigation to control fast, medium and slow-moving items.

2. EVALUATION OF CURRENT INVENTORY SYSTEM

2.1 CHARACTERISTICS OF TRUCK AND BUS SPARE PARTS⁽⁴⁾

When we talk about truck and bus spare parts (or replacement parts), we are really talking about any commodity with particular supply and demand characteristics.

In a paper presented at the Operational Research Society of South Africa (ORSSA) conference (1979) these characteristics were identified as follows:

2.1.1 SUPPLY

The following are characteristics of supply:

- The item can be bought from a limited number of suppliers
- The lead time is highly variable and can range from off-the-shelf service to nine months. A typical lead time would be between three and five months
- Lead times are more dependent on the queue of items waiting for processing than on actual manufacturing times
- The set-up cost is often a substantial proportion of the final selling price
- The unit price varies over a wide range
- Items are normally manufactured in batches in a jobbing environment.

2.1.2 DEMAND

The following are characteristics of demand:

- The item is not a raw material or in the main stream of the production process. In other words the demand, in the short term, is not related to sales
- The demand (excluding fast moving items) tends to be unpredictable and erratic due to the following factors:
 - Demand dependent on infrequent random (truck or bus) breakdown
 - Demand affected by the fads of a particular maintenance supervisor
 - Demand reduced by the use of reconditioned spares
 - Demand affected by optional substitution or technical obsolescence
 - Demand affected by changes in the policy for planned maintenance
 - Demand affected by shifting to new truck and bus models.
- In the event of a breakdown, urgent orders can be expedited or alternative sources of supply can be exploited
- There tends to be a long pipe line via agencies between the source of supply and the ultimate user.

The above characteristics are not relevant to truck and bus fast moving items.

2.2 ABC CLASSIFICATION

Silver and Peterson⁽¹⁾ (p67) made the following statement:

"Many existing inventory management systems can be significantly improved on by simply adopting decision rules that do not treat all SKUs or all categories of aggregate inventory investment equivalently."

The task of the CPD management team is to plan and control the stockholding level of approximately 20 123 SKUs (after excluding 2 300 SKUs, part of which were scrapped, and a certain number were sent back to Germany, and after approximately 3 000 old SKU records were deleted). However, the relative importance of the items is not fixed, and therefore it is necessary to differentiate between them and to apply more management resources to those items which have a high relative importance. The method used is the ABC inventory management approach. The theory behind the ABC approach is that a small group of SKUs normally generate most of the turnover of a company.

From a control viewpoint, the importance of an item can be defined in terms of its annual usage value. This is calculated by multiplying the quantity used in a year by the unit cost of the item. The rank value of annual usage is the common factor used for categorising inventory items.

The company Management Information System (MIS) generates a monthly ABC inventory analysis report. Table 1 shows the different inventory categories as a percentage of the total SKU value and usage range.

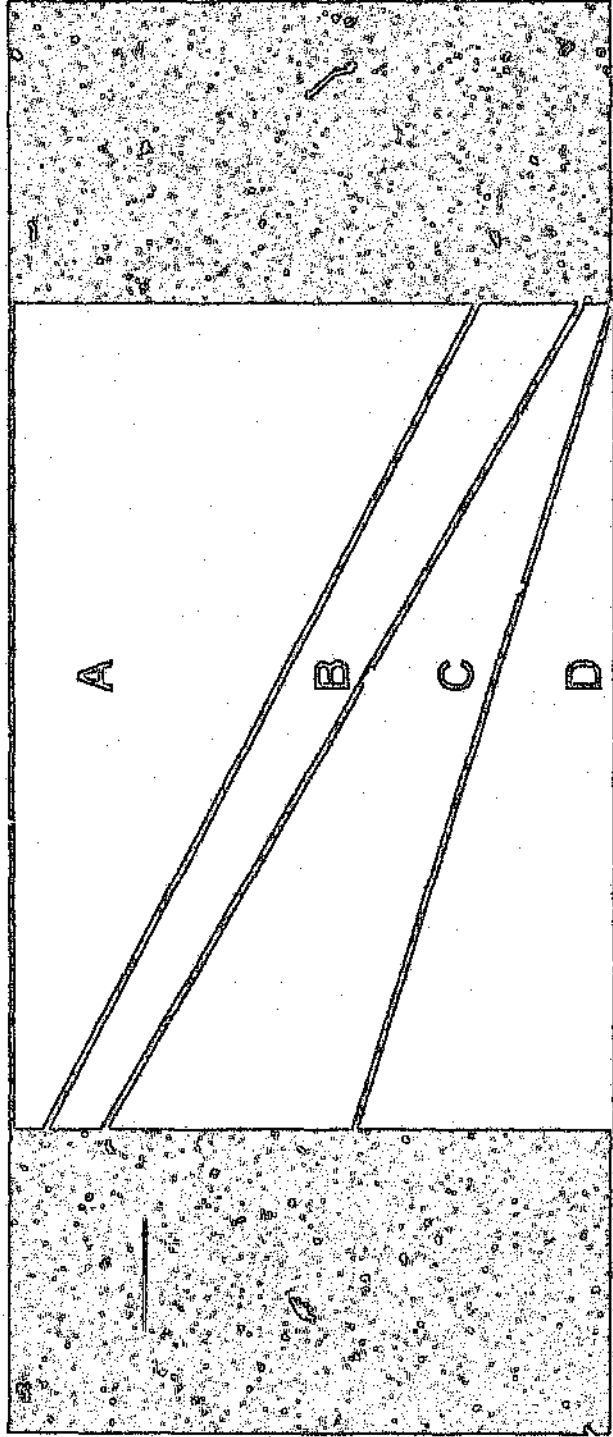
TABLE 1
ABC INVENTORY SUMMARY

CATEGORY	AVERAGE USAGE		NUMBER OF LINE ITEMS	STOCK VALUE (R1.000)	USAGE VALUE (R1 000)	TURN-OVER RATE	MONTHS STOCK	PERCENTAGE OF TOTAL		
	LOWER LIMIT (R)	UPPER LIMIT (R)						LINE ITEMS	STOCK VALUE	USAGE VALUE
A	14 435	NONE	977	17 385	53 000	3,05	4	5%	35%	78%
B	2 009	14 436	1 831	9 351	11 300	1,21	10	9%	19%	17%
C	0	2 000	8 601	7 620	3 400	0,45	27	43%	15%	5%
DEAD	-	-	8 714	15 564	0	0	Infinite	43%	31%	0%
TOTAL			20 123	49 900	67 %			100%	100%	100%

A pictorial representation of the ABC inventory is presented in Diagram 1

DIAGRAM 1 ABC INVENTORY ANALYSIS

CLASS



977 (5 %)
1831 (9 %)

8601 (43 %)

8714 (43 %)

R 53 M (78 %)

R 11,3 M (17 %)

R 3,4 M (5 %)

NUMBER OF
ITEMS

ANNUAL USAGE
VALUE

2.3 ABC CLASSIFICATION BY THE SKU MOVEMENT CATEGORY

Another way to analyse the inventory is by classifying the inventory items according to importance and according to the level of usage, namely the item movement within the stock.

The different categories of movement are as follows:

CATEGORY	CODE	DEFINITION
Fast	F	More than 250 units sold nationally per year, with movement (sales or demand) every month
Medium	M	More than 50 but less than 250 units sold nationally per year, with movement (sales or demand) every month
Continuous	C	Less than 50 units sold nationally per year, with movement (sales or demand) every month
Lumpy	L	More than 50 units sold nationally per year, but <u>without</u> movement (sales or demand) in every month
Slow	S	Less than 50 units sold nationally per year, but <u>without</u> movement (sales or demand) in every month
Dead	D	No movement (sales or demand) in the last 6 months
New	New	From creation until 12 months after first receipt or first sale

Table 2 shows the different movement categories, the number of SKUs in each category, the percentage of total SKUs and their stock value within the different classes and movement categories. Diagram 2 and 3 illustrate the ABC inventory analysis by movement categories.

TABLE 2
ABC INVENTORY ANALYSIS
BY STOCK ITEM MOVEMENT CATEGORY

CLASS	CLASS A				CLASS B				CLASS C				TOTAL SKU			
	NUMBER OF SKU	PER CENT SKU	STOCK VALUE		NUMBER OF SKU	PER CENT SKU	STOCK VALUE		NUMBER OF SKU	PER CENT SKU	STOCK VALUE		NUMBER OF SKU	PER CENT SKU	STOCK VALUE	
			(R1 000)	PER CENT			(R1 000)	PER CENT			(R1 000)	PER CENT			(R1 000)	PER CENT
HIGHEST	408	41,6	4 511	26,1	336	16,4	641	6,9	285	1,7	180	0,8	1 037	6,2	5 332	13,7
MEDIUM	228	23,4	2 72		301	16,4	513	5,7	345	2,0	188	0,8	875	4,3	3 733	8,2
CONTINUOUS	58	3,9	58	0,4	63	3,4	202	2,2	67	0,4	44	0,2	189	0,9	533	2,0
BUMPY	113	11,6	2 800	16,7	240	13,1	1 217	13,0	782	4,4	595	2,6	1 115	5,5	4 712	11,1
LOW	183	18,7	6 595	37,8	824	45,0	5 600	59,9	6 651	38,4	6 585	28,4	7 958	38,1	18 760	36,7
HEAD	8	0,8	51	0,3	87	3,7	875	9,4	8 714	50,3	15 529	67,1	8 799	43,7	18 458	25,3
NEW	0	0	0	0	0	0	0	0	481	2,8	22	0,1	481	2,4	22	0
TOTAL	877	100	17 365	100	1 831	100	8 351	100	17 215	100	23 134	100	20 123	100	48 950	100

CLASS A: 80% OF TOTAL SALES
CLASS B: 15% OF TOTAL SALES
CLASS C: 5% OF TOTAL SALES

DIAGRAM 2 ABC INVENTORY ANALYSIS by movement category and value of SKU's

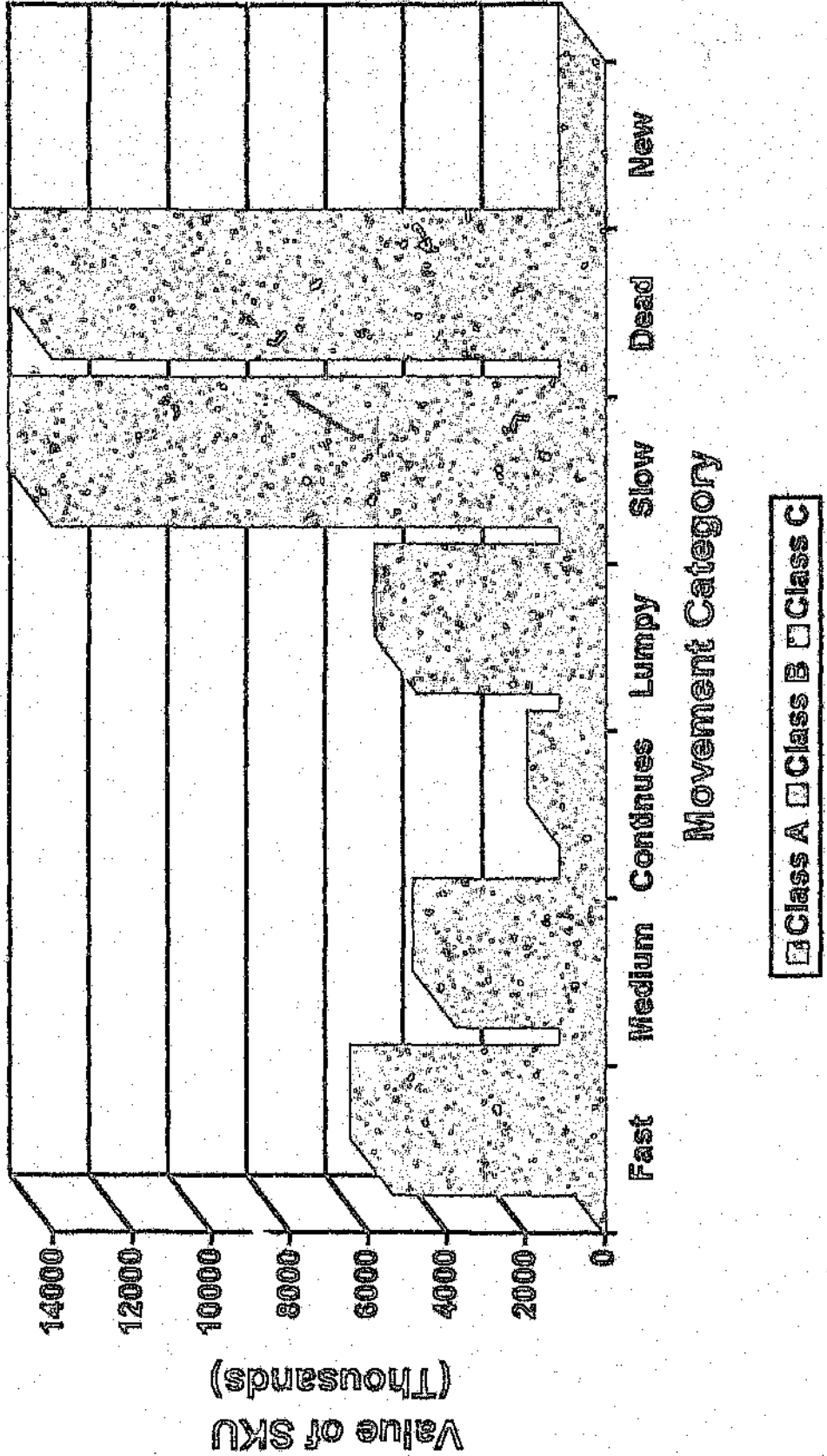


DIAGRAM 2 ABC INVENTORY ANALYSIS

by movement category and value of SKU's

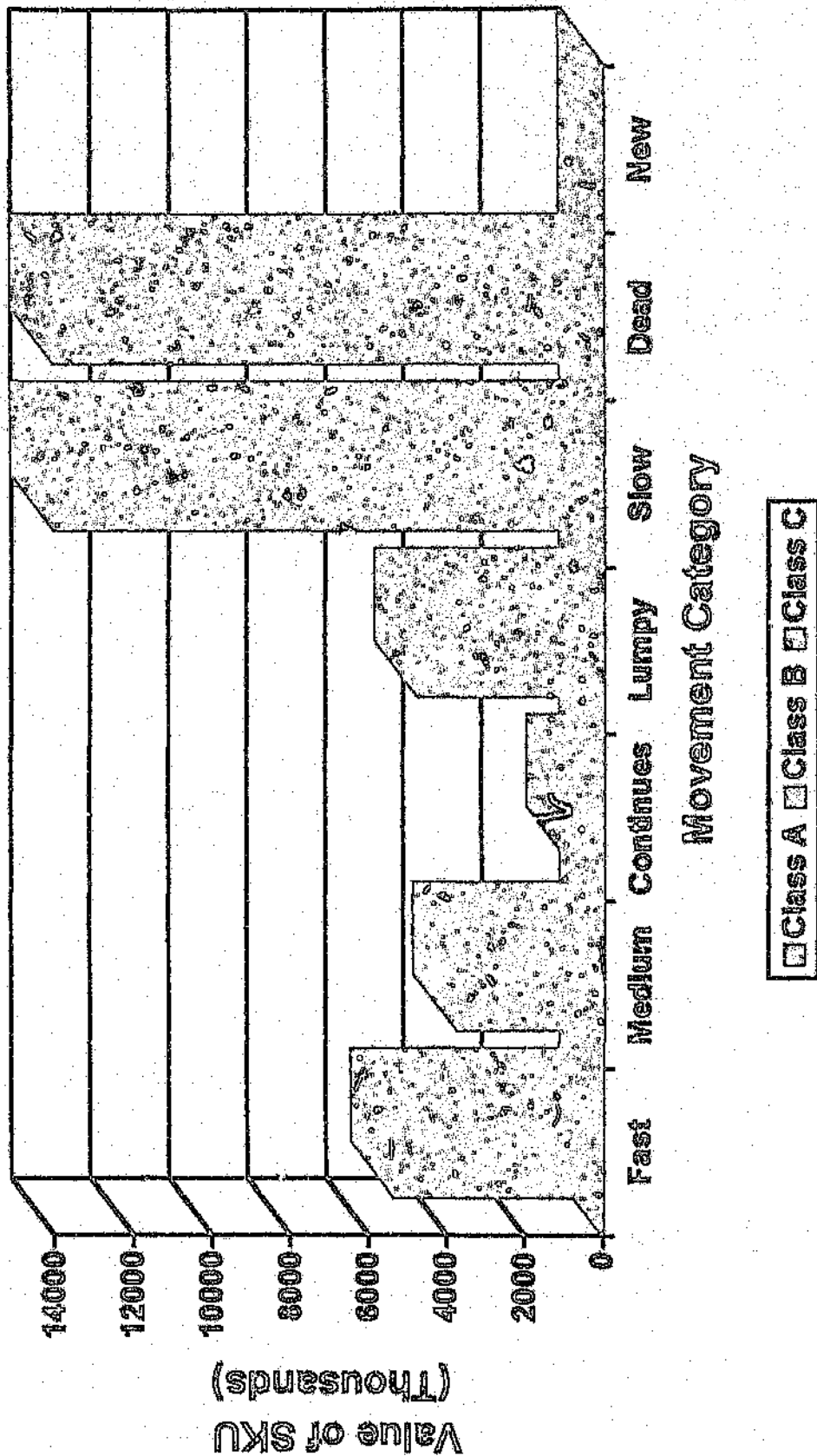


DIAGRAM 3 ABC INVENTORY ANALYSIS

by movement category and number of SKU's

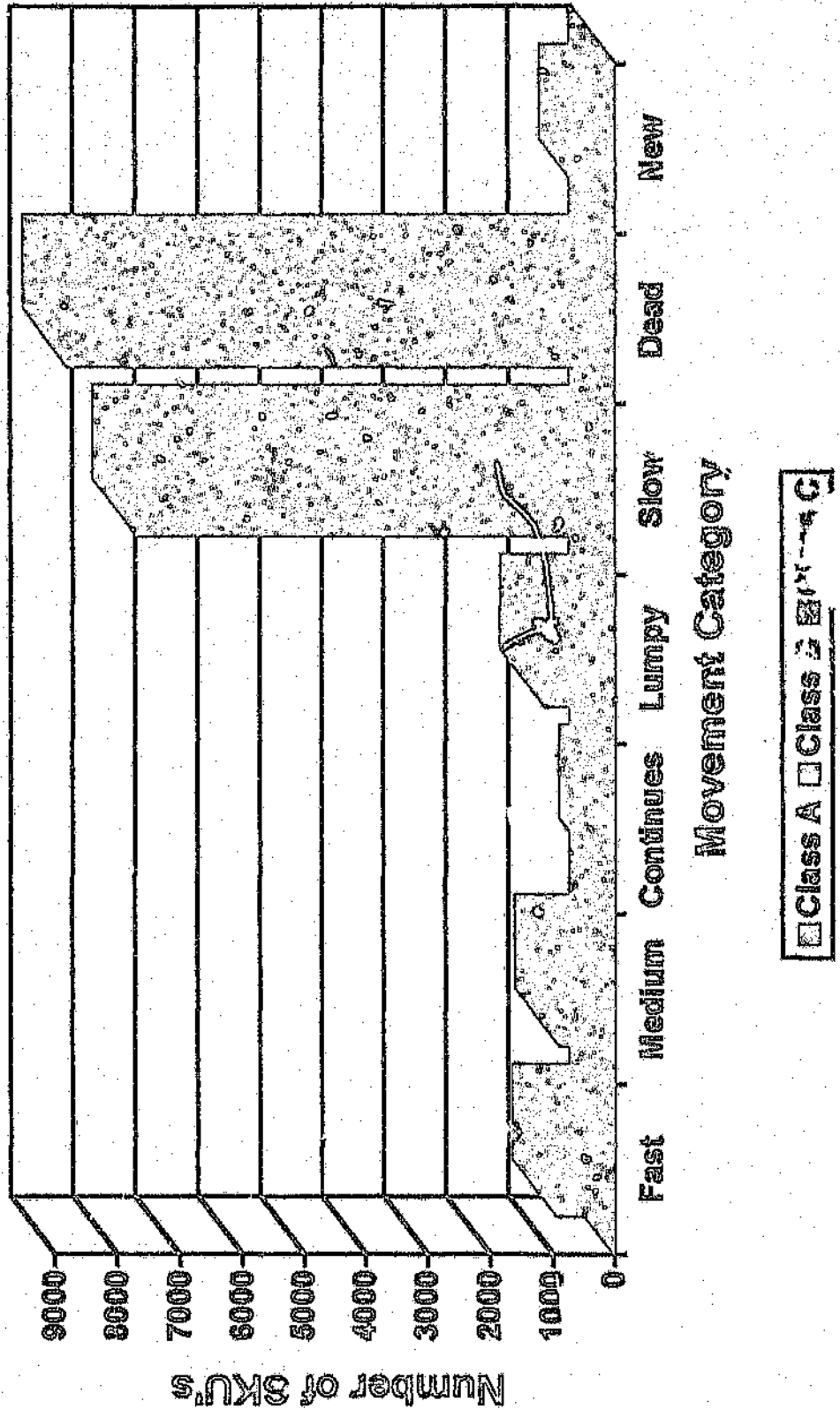


Table 2 and Diagrams 2 and 3 reveal the following findings:

- Approximately 44% of the total SKUs (25% of the stock value) are classified as dead stock, where the majority of those SKUs belonging to class C accounts for 93% of total dead stock (24% of the total stock value). The above figures also indicate that 24% of the stock value do not contribute to sales.
- Thirty eight percent of the total SKUs, at 39% of stock value, are slow movers. Among these SKUs (1 003 SKUs) 29% are from Class A and Class B, which reflects their high value.
- A substantial reduction of total inventory value can be expected from nearly each moving category (fast, medium, etc.) and especially from the B and C classes.

2.4 CURRENT INVENTORY POLICY

2.4.1 STOCK REPLENISHMENT

A suggested order report is generated by the Management Information System (MIS) once a month, normally at the end of the month. The report contains a list of items per supplier with the suggested order quantity. It is split into two categories, local and overseas suppliers, and is passed to the inventory manager for evaluation. This activity takes approximately three to four weeks and entails the following steps:

- Manual checking of the trends in demand, i.e. abnormal or peak demand, during the last six months
- Checking whether the suggested order quantity is in line with the minimum supplier order quantity
- Evaluating the suggested order quantity from an economic order quantity point of view, i.e. order quantities which will entitle the company to a discount.

The revised suggested order quantity report is then passed back for entry into the computer. A new suggested order report is generated which is sent for evaluation regarding cost and reasonableness. After it has been authorised, orders are placed with the suppliers - most by post, but some by telefax or courier.

2.4.2 SUGGESTED ORDER SIZE OF SKUs

All stock records are kept on the computer inventory system, and for each SKU the following records are stored:

- Parts number
- Description
- Movement category
- Unit cost

- Supplier lead time (one figure for all suppliers' items)
- Vendor/supplier
- Classification
- Quantity in hand
- Quantity on order
- Quantity allocated (back orders quantity)
- Rolling movement (the last 12 months usage history).

The computer system currently calculates the suggested order size (SOS), for each SKU according to the following equation:

SOS of SKUs (m) =

$$\frac{\text{NAD} \times (\text{LT}_c + \text{LT}_s) + \text{BO} - \text{SH} - \text{SO} - \text{ST}}{4}$$

where

SOS = suggested order size of SKU_(m) (units)

NAD = new average Demand of SKU_(m) (units)

LT_c = internal (company) lead time (weeks)

LT_s = supplier lead time (weeks)

BO = back order quantities (units)

SH = stock in hand (units)

SO = stock on order (units)

ST = stock in transit (units).

Internal (company) lead time is determined by the company management to be ten weeks, made up as follows:

- Order compilation - 3 weeks
- Order receiving/binning and data capturing - 3 weeks
- Safety factor for supplier lead time - 4 weeks.

Supplier lead time is determined by the supplier. The figure of four weeks is based on the assumption that there are four weeks in the month.

• **Compute the New Average Demand (NAD)**

The NAD of SKUs (m) is currently computed by means of the following steps :

$$Y = \frac{1}{6} \sum_{i=1}^6 Y_i$$

Where

Y = average usage of SKUs (m) for the last six months

Y_i = usage of SKUs (m) in month i

- Check irregularities in demand during the last six months according to the criteria. If Y_i > 2 Y, extract the monthly average from the new average to be calculated

- Calculate the new average X:

$$X = \frac{1}{n} \sum_{i=1}^n X_i$$

Where

If Y_i < 2 Y then X_i = Y_i

n = Number of months that monthly usage did not exceed twice the monthly average

• **Compute the Mean Average Deviation (MAD)**

$$MAD = \frac{\sum_{i=1}^n |X - X_i|}{n}$$

Where

|X - X_i| = the absolute value obtained by subtracting the monthly usage from the monthly average

- Compute the deviation (sigma) which should be added to the new average (X):

$$\text{Sigma} = 1,25 \times MAD$$

Where

A correcting factor of 1,25 is allowed to adjust the MAD to the value of the Standard Deviation of Sample (STDS), assuming the demand is normally distributed.

• **Compute the New Average Demand (NAD)**

$$NAD = X + \text{Sigma}$$

Figures 1 and 2 present flow charts for the current method of computing the suggested order size.

2.5 PROBLEM AREAS IN CURRENT INVENTORY MODEL

The following problems were identified in the current inventory model:

The use of six readings (the last six months' average demands) to calculate the suggested order size is not suitable in the spare parts environment, for the following reasons:

- Long lead times of a large portion of the items (ranging between 2 to 5 months)
- Forty four per cent of SKUs are dead stock items, and it is statistically impossible to assess the NAD on the last six months' readings only
- The same applied
 - In dealing with SKUs that have slow and lumpy demand patterns. Six readings are not statistically sufficient to draw conclusions about the NAD.
- The computer program rounds off
 - Six months' average demand
 - Mean Average Deviation (mad)
 - Sigma
 - $\frac{nad^*}{4}$

causing a cumulative error in computing the NAD*.

2.6 CURRENT INVENTORY MODEL SIMULATION WITH VARIOUS DATA INPUTS

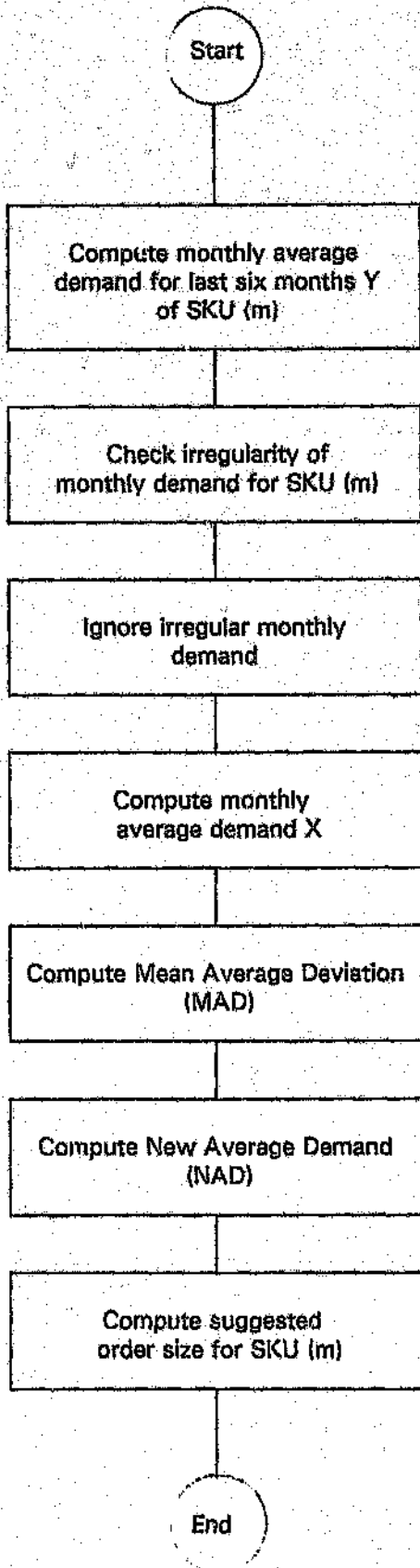
The approximation of Sigma used by the current inventory model to measure the standard deviation of the sample does not always correlate with the standard deviation.

A simulation containing the last six months' readings of 120 SKUs was run on the computer with the current inventory model. The values and variables, together with their definitions, which were calculated are as follows:

Variables defined and values calculated in the simulation

- Y_i = demand in month i
- Y = last six months' average demand (i.e. $Y = \frac{1}{6} \sum y_i$)
- Y_r = round (Y)

FIGURE 2
CURRENT INVENTORY MODEL
PROCEDURE FOR COMPUTING SUGGESTED ORDER SIZE
FOR SKU (m)



CURRENT INVENTORY MODEL PROCEDURE FOR COMPUTING THE NEW AVERAGE DEMAND(NAD)

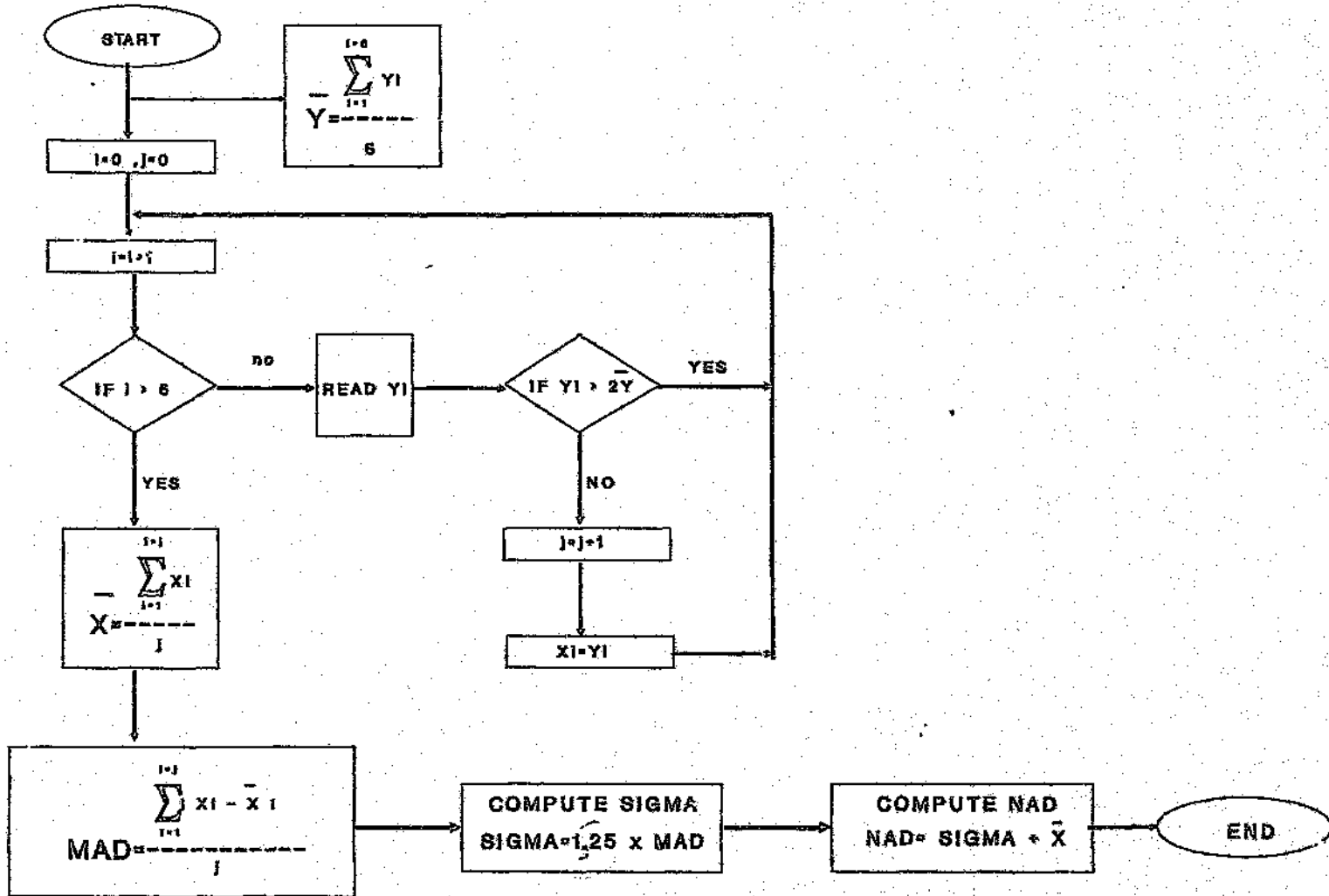


FIGURE 3

◦ mad^* = mean Average Deviation

$$\text{mad} = \frac{\sum |Y_i - \bar{Y}|}{n} = \frac{\sum |Y_i - \bar{Y}|}{6}$$

◦ MAD^* = round (mad)

◦ X_i = monthly average demand that is less than twice the average

$$X_i = Y_i \text{ if } Y_i < 2\bar{Y}$$

◦ \bar{X} = average of the months in which the monthly average demand did not exceed twice the last six months' average monthly demand

$$\bar{X} = \frac{\sum X_i}{n}$$

n = the number of months where monthly usage did not exceed twice the last six months' monthly average.

◦ X_r = round (\bar{X})

Deviation from the average

◦ STDS = the standard deviation of sample

$$\text{STDS} = \frac{1}{n-1} \sum (Y_i - \bar{Y})^2$$

In this case for six months

$$\text{STDS} = \frac{1}{5} \sum (Y_i - \bar{Y})^2$$

◦ σ = $\text{MAD} \times 1,25$

◦ σ_r = $\text{MAD} \times 1,25$

Sigma = round (σ_r)

New average demand

◦ nad = $\bar{X} + \sigma$

◦ nad_r = $X_r + \sigma_r$

◦ NAD_r = $X_r + \text{sigma}$

◦ NAD_s = $\bar{Y} + \text{STDS}$

* All variables used with capital letters represent the rounded figures of the original variables.

Suggested order size (SOS)

- $SOS(nad) = (nad \times LT)/4$
- $SOS(nad_r) = (nad_r \times LT)/4$
- $SOS(NAD_r) = (NAD_r \times LT)/4$
- $SOS(NAD_s) = (NAD_s \times LT)/4$

Where

- $LT =$ company and supplier lead times

All SOS figures are rounded to calculate the relevant SOS, which is used with the relevant nad.

The lead time used in this example is 22 weeks:

$$LT = 22 \text{ weeks}$$

It was necessary to analyse the suitability of the current inventory model to predict what the order size per SKU (m) should be and to quantify the error generated by rounding the variables computed during calculation of the NAD, as well as the closeness of the approximation (Σ) to the standard deviation of the sample (STDS). The current inventory model was developed on the computer and data were entered as follows:

- Data with random item demand patterns, where the last six months' average is fixed and the coefficient of variance ranges between 0 and 2,4. (The coefficient of variance is determined as the ratio between the standard deviation of the sample and the average of the sample.)
- Data with random item demand patterns are categorised by ranging the coefficient of variances, as follows:
 - Range 1 - Low coefficient of variances, range between 0 and 0,5
 - Range 2 - Medium coefficient of variance, range between 0,5 and 1,0
 - Range 3 - High coefficient of variance, range between 1,0 and 2,1.

The input data, the calculation of variables and the correlation analysis can be found in Appendices 1 to 4.

2.7 CORRELATION ANALYSIS⁽⁶⁾

Levin⁽⁶⁾ (518) defines correlation analysis as the statistical tool that can be used to describe the degree to which one variable is linearly related to another.

The measures describing the correlation between two variables are

- the coefficient of determination

- the coefficient of correlation.

The coefficient of determination is the primary way in which we can measure the extent, or strength, of the association that exists between two variables, X and Y.

It is reasonable to express the variation of the Y values around the regression line with this equation:

$$\text{Variation of the Y values around the regression line} = \sum(Y_i - \hat{Y})^2 \text{ where: } \hat{Y} = ax.$$

The second variation, that of the Y values around their own mean, is determined by this equation:

$$\text{Variation of the Y values around their own mean} = \sum(Y - \bar{Y})^2.$$

One minus the ratio between these two variations is the sample coefficient of determination, which is symbolised by r^2 :

$$\text{Sample coefficient of determination } r^2 = 1 - \frac{\sum(Y - \hat{Y})^2}{\sum(Y - \bar{Y})^2}$$

- The coefficient of determination

The coefficient of determination r^2 as defined by the above equation is a measure of the degree of linear association between X and Y. The intuitive interpretation of r^2 is as follows:

When $r^2 = 1$, there is perfect correlation or the regression line is a perfect estimator.

- When $r^2 = 0$, there is no correlation.

- $0 < r^2 < 1$. When r^2 is close to 1, this indicates a strong correlation between X and Y, while when r^2 is near 0 there is little correlation between these two variables.

- The coefficient of correlation

The coefficient of correlation is used to describe how well one variable is explained by another and is expressed as the square root of the coefficient of determination:

$$r = \sqrt{r^2}.$$

The sign \pm of r indicates the direction of the relationship between the two variables X and Y.

For our investigation of the extent of fit of the current inventory model to different item pattern demands, we checked the correlation between the variables contained in the current inventory model.

The correlation analysis carried out was based on the assumption that there is a linear relationship between the variables. (This assumption is appropriate for items that have pattern demand characterised by a low coefficient of variance.)

The regression line function is in the form of

$$Y = AX + B$$

where

A = X coefficient

B = constant

X = independent variable

Y = dependent variable.

For example, if we test the correlation between two variables, STDS and MAD calculated from a given stream of numbers, the dependent variable is the statistical measure of the sample standard deviation expressed as

$$\text{STDS} = \left[\frac{1}{n-1} \sum (x_i - \bar{x})^2 \right]^{1/2}$$

and the independent variable is the Mean Average Deviation (MAD) expressed as

$$\text{MAD} = \frac{1}{n} \sum |x_i - \bar{x}|$$

The regression line is in the form of

$$\text{STDS} = A \times \text{MAD} + B$$

where the regression analysis will indicate the extent or strength of the association between the variables STDS and MAD by calculating the value of r^2 .

Secondly, if there is a strong correlation between the two variables, the parameter A will indicate the correcting factor between STDS and MAD.

The standard error of Y is the estimated standard error of the Y (STDS) values and represents the deviation of the observed (MAD) values from the values of the linear combination.

Table 3 presents the correlation analysis summary of the current inventory variables computed with the approximation Sigma and using the irregularity checking criteria ($\sqrt{Y_i} < 2\bar{Y}$ ignore Y_i) compared with the variables using the Standard Deviation of Sample (STDS), while Diagrams 4 to 6 are pictorial representations of the correlation analysis for the various input data.

The detailed breakdown of the pictorial representation for a low, medium and high coefficient* of variance are to be found in Appendices 5 to 13.

From the correlation analysis summarised in Table 3 and from the charts shown in Diagrams 4 to 6 (and Appendices 5 to 13), the following findings are reported:

- The correlation between the current inventory variables calculated with Sigma, and those calculated with STDS, is as follows:
 - High when the coefficient of variance is low (range 0 to 0,5)
 - High when the coefficient of variance is medium (range 0,5 to 1,0). However, there is an error with a correcting factor (A in the regression line) of 6% and medium fluctuation (around 9%) of the values calculated with Sigma, compared with those calculated with STDS
 - High to medium when the coefficient of variance ranges between 1 and 2. However, the correcting factor A is 21% higher than the one in use and the fluctuation of the current variables in use (computed with Sigma) is around 15% compared with those calculated with STDS.
- Using the data input of the last six months, the average demand is fixed and the coefficient of variance ranges between 0 and 2,4. Low correlation and high fluctuation are indicated among the variables calculated with sigma and those calculated with STDS.

The conclusion to be drawn from the above findings is that the current inventory model is inadequate when dealing with items that are subject to an erratic type of demand and for slow-moving items.

3. IMPORTANT FACTORS FOR INVENTORY DECISIONS

Through empirical studies and deductive mathematical modelling, Silver and Peterson⁽¹⁾ (61) have identified a number of important factors with respect to inventory decisions. These are discussed below.

3.1 COST FACTORS

Prior to my investigation, there was no measurement of the cost involved in the holding of stock and ordering of commodities. It was therefore considered important to determine the real costs involved in ordering and holding stock for future use in the proposed inventory models. A number of factors would be relevant.

3.1.1 UNIT VALUE OR UNIT VARIABLE COST

The unit value of an item (denoted by the symbol V) is expressed in rands per unit (including freight) paid to the supplier. The unit value is important for two reasons: Firstly, the total acquisition costs per year clearly depend on its value, and secondly, the cost of carrying items in inventory depends on V .

TABLE 3
CORRELATION ANALYSIS SUMMARY

THE DEMAND COEFFICIENT OF VARIANCE RANGE $Z = STDS/\gamma$	CORRELATION VARIABLES	REGRESSION LINE FUNCTION	CORRECTION FACTOR ERROR PERCENTAGE	STANDARD ERROR OF DEPENDENT VARIABLE PERCENTAGE	CORRELATION OF DETERMINATION R^2 Squared	
LOW COEFFICIENT OF VARIANCE 0.0 - 0.5	<u>DEVIATION</u>					
	STDS, mod	STDS=A x mod +B STDS=1.33 x mod + 0	8.4	±5.9	0.9560	
	STDS, SIGMA	STDS=A x SIGMA +B STDS=0.98 x SIGMA + 0	-2.0	±13.2	0.9777	
	<u>New Average Demand (NAD)</u>					
	NADs, mod	NADs=A x mod +B NADs=1.01 x mod + 0	1.0	±1.2	0.9998	
	NADs, NADr	NADs=A x NADr +B NADs=1 x NADr + 0	0.0	±2.8	0.9986	
	<u>Suggested Order Size (SOS)</u>					
	SOS(NADs), SOS(mod)	SOS(NADs)=A x SOS(mod) +B SOS(NADs)=1.01 x SOS(mod) + 0	1.0	±1.1	0.9996	
	SOS(NADs), SOS(NADr)	SOS(NADs)=A x SOS(NADr) +B SOS(NADs)=1 x SOS(NADr) - 1	0.0	±5.0	0.9877	
	MEDIUM COEFFICIENT OF VARIANCE 0.5 - 1.0	<u>DEVIATION</u>				
		STDS, mod	STDS=A x mod +B STDS=0.86 x mod +4.7	-31.2	±16.6	0.8834
		STDS, SIGMA	STDS=A x SIGMA +B STDS=0.70 x SIGMA + 4.5	-30.0	±17.1	0.8760
<u>New Average Demand (NAD)</u>						
NADs, mod		NADs=A x mod +B NADs=1.05 x mod + 4	5.0	±6.7	0.9653	
NADs, NADr		NADs=A x NADr +B NADs=1.08 x NADr + 4	6.0	±8.9	0.9637	
	<u>Suggested Order Size (SOS)</u>					
	SOS(NADs), SOS(mod)	SOS(NADs)=A x SOS(mod) +B SOS(NADs)=1.05 x SOS(mod) + 22	5.0	±8.7	0.9654	
	SOS(NADs), SOS(NADr)	SOS(NADs)=A x SOS(NADr) +B SOS(NADs)=1.08 x SOS(NADr) + 22	5.0	±8.7	0.9638	
	HIGH COEFFICIENT OF VARIANCE 1.0 - 2.0	<u>DEVIATION</u>				
		STDS, mod	STDS=A x mod +B STDS=1.18 x mod + 4	-9.6	±8.7	0.7588
		STDS, SIGMA	STDS=A x SIGMA +B STDS=0.84 x SIGMA + 4.2	-6.0	±10.0	0.7487
<u>New Average Demand (NAD)</u>						
NADs, mod		NADs=A x mod +B NADs=1.21 x mod + 7.8	21.0	±14.9	0.8314	
NADs, NADr		NADs=A x NADr +B NADs=1.20 x NADr + 7.8	20.0	±15.1	0.8254	
	<u>Suggested Order Size (SOS)</u>					
	SOS(NADs), SOS(mod)	SOS(NADs)=A x SOS(mod) +B SOS(NADs)=1.21 x SOS(mod) + 43	21.0	±14.9	0.8316	
	SOS(NADs), SOS(NADr)	SOS(NADs)=A x SOS(NADr) +B SOS(NADs)=1.21 x SOS(NADr) + 42	21.0	±15.1	0.8254	
	RANDOM COEFFICIENT OF VARIANCE 0.0 - 2.4	<u>DEVIATION</u>				
		STDS, mod	STDS=A x mod +B STDS=1.05 x mod + 1.84	-18.0	±35.8	0.7074
		STDS, SIGMA	STDS=A x SIGMA +B STDS=0.81 x SIGMA + 1.92	-19.0	±37.8	0.6966
<u>New Average Demand (NAD)</u>						
NADs, mod		NADs=A x mod +B NADs=1.01 x mod + 4.93	1.0	±24.9	0.3489	
NADs, NADr		NADs=A x NADr +B NADs=0.97 x NADr + 5.79	-3.0	±25.3	0.3326	
LAST SIX MONTHS AVERAGE FIXED	<u>Suggested Order Size (SOS)</u>					
	SOS(NADs), SOS(mod)	SOS(NADs)=A x SOS(mod) +B SOS(NADs)=1.01 x SOS(mod) + 25.4	1.0	±24.9	0.3478	
	SOS(NADs), SOS(NADr)	SOS(NADs)=A x SOS(NADr) +B SOS(NADs)=0.97 x SOS(NADr) + 31.8	-3.0	±25.3	0.3317	

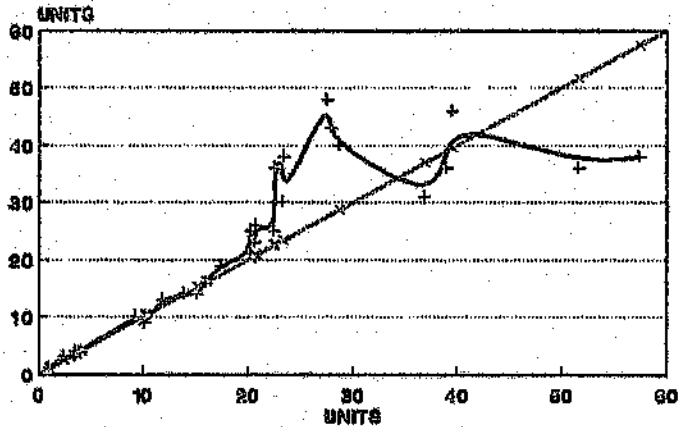
DIAGRAM 4

STANDARD DEVIATION COMPARISON

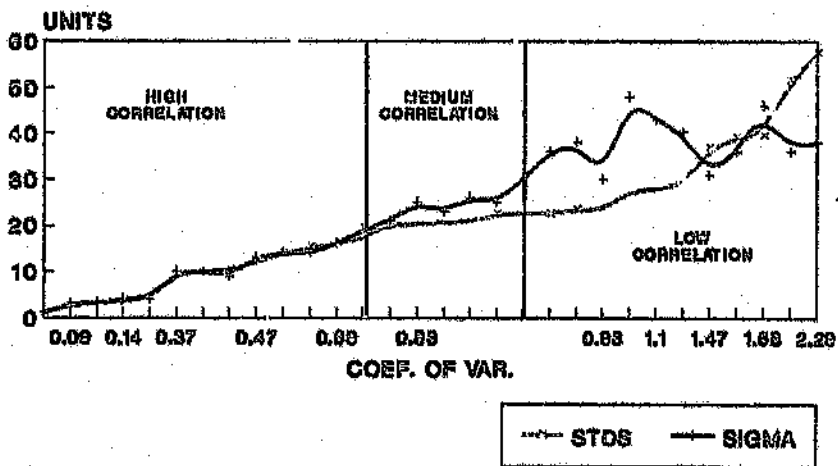
ITEM DEMAND PATTERN : RANDOM

WITH LAST SIX MONTHS AVERAGE FIXED

SIGMA FLUCTUATION AROUND STDS LINE



STDS AND SIGMA CORRELATION AS A FUNCTION OF THE COEF. OF VAR.



SIGMA = MEAN AVERAGE DEVIATION x 1.25
STDS = STANDARD DEVIATION OF SAMPLE

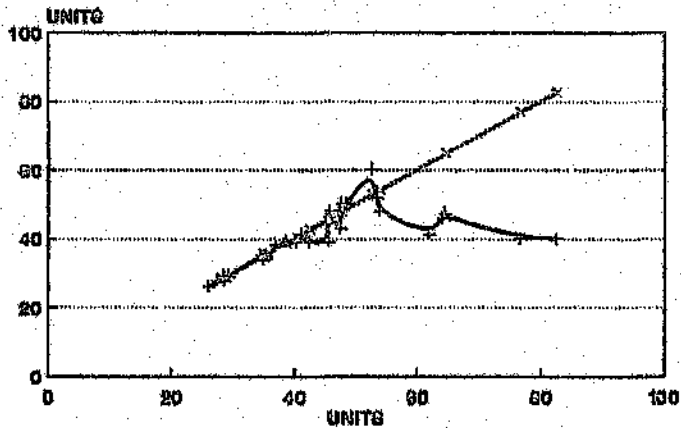
DIAGRAM 5

NEW AVERAGE DEMAND COMPARISON

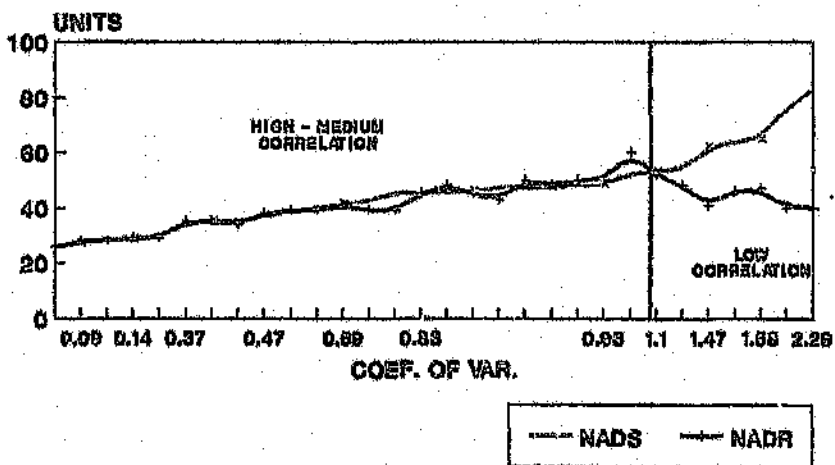
ITEM DEMAND PATTERN : RANDOM

WITH LAST SIX MONTHS AVERAGE FIXED

NADR FLUCTUATION AROUND NADS LINE



NADS AND NADR CORRELATION AS A FUNCTION OF THE COEF. OF VAR.



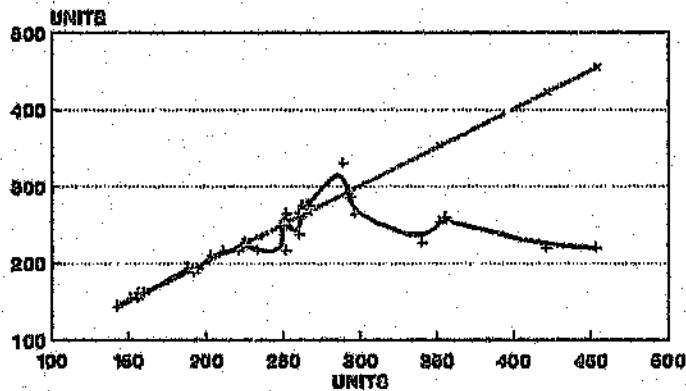
NADR = NEW AVERAGE DEMAND CALCULATED WITH SIGMA
NADS = NEW AVERAGE DEMAND CALCULATED WITH STDS

DIAGRAM 6

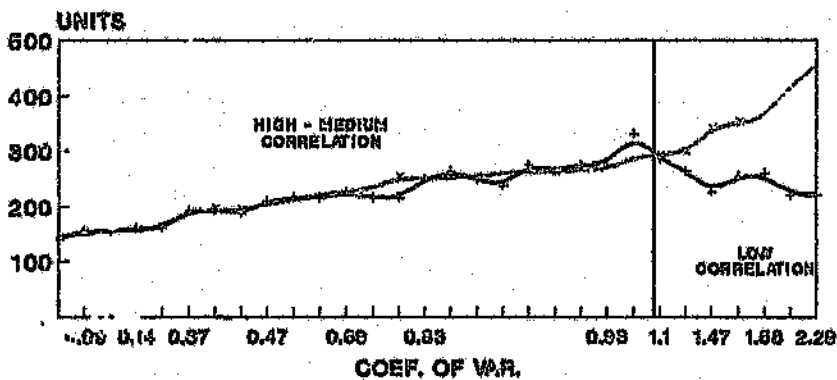
SUGGESTED ORDER SIZE COMPARISON

ITEM DEMAND PATTERN : RANDOM
WITH LAST SIX MONTHS AVERAGE FIXED

SOS(NADR) FLUCTUATION AROUND
SOS(NADS) LINE



SOS(NADS) AND SOS(NADR) CORRELATION
AS A FUNCTION OF THE COEF. OF VAR.



— SOS(NADS) + SOS(NADR)

SOS(NADR)-SUGGESTED ORDER SIZE CALCULATED WITH SIGMA
SOS(NADS)-SUGGESTED ORDER SIZE CALCULATED WITH STDS

3.1.2 COST OF CARRYING ITEMS IN INVENTORY

The cost of carrying items in inventory includes the opportunity cost of the money invested, the expenses incurred in running a warehouse, the costs of special storage requirements, deterioration of stock, obsolescence, insurance, and taxes. The most common formula for costing is

$$\text{Carrying costs per year} = \text{AOHQ} \times V \times r$$

where AOHQ is the average inventory on hand in units (hence AOHQ x V is the average inventory expressed in rands) and where r is the carrying charge, i.e. the cost in rands of carrying one rand's worth of inventory for one year.

Carrying costs can be regarded as a top management policy variable that can be changed from time to time, in line with changes in the environment. A detailed cost structure put to management to enable them to make better decisions concerning the real costs. It was agreed that fixed costs are to some extent not relevant to the number of orders or the level of stock, because if there were no inventories and no orders, obviously there would be no need for a procurement department and therefore all costs (fixed and variable) would be eliminated.

As far as variable costs are concerned, it was agreed that the percentage of variable costs linked to stock levels and to the number of orders placed, is 80%.

The current costs of holding inventories (carrying charges r) are as follows:

- Fixed costs	R1 426 000 per annum
- Variable costs	R1 475 000 per annum
- Variable costs subject to stock levels (80%)	R1 180 000

° Holding cost as a percentage of average stock value

- Variable costs	$\frac{1,18}{49,9} \times 100 =$	2,4%
- Obsolete stock (budget percentage)		2,5%
- Insurance		1,0%
- Deterioration (budget percentage)		2,5%
- Cost of capital		14,5%
- TOTAL	22,9% say	23,0%

3.1.3 ORDERING COST

The symbol A denotes the fixed cost (independent of the extent of the replenishment) associated with a replenishment. This ordering cost includes the cost of order forms, postage, telephone calls, authorisation, typing of orders, receiving, inspection, following up on unexpected situations, and dealing with vendor invoices. The cost of administrative time includes the buyer's time for order preparation and the system control costs (computer, software, maintenance, etc).

The costs involved in ordering and holding inventory are detailed in Appendix 14.

Once the costs involved in ordering and holding inventories are known, the problem transfer to what proportion of those costs related to stock levels and what proportion to the number of orders placed. Two ways of looking at the ordering cost in relation to the number of orders placed are shown in Diagram 7. In reality there is no linear relationship between the number of orders placed and the ordering cost - only a certain reduction in the number of orders will bring about a reduction in the ordering cost. The connection between these two factors should in fact be represented by a series of steps rather than a straight line.

° Ordering cost

- Total fixed costs	R337 000 per annum
- Total variable costs	R559 000 per annum
- Variable costs subject to the volume of orders (80%)	R442 600 per annum
- Number of orders placed October 1992 to September 1993	4 666
- Average cost per order	R96

The average cost of R96 per order is misleading since the order size varies in a big range between 1 SKU per order to more than 1 000 SKUs per order.

Diagram 8 shows the order size distribution for bulk orders (placed once a month) and daily VOR (Vehicle of the Road) orders. Diagram 9 indicates the order cost per SKU as a function of the order size for a current buyer utilisation of 45% and the expected order cost per SKU as 100% of buyer utilisation.

Appendices 15 and 16 contain a detailed breakdown of the information concerning buyer utilisation as well as the information shown in Diagrams 7 and 8.

3.1.4 DISCUSSION - DETERMINING REALISTIC COSTS

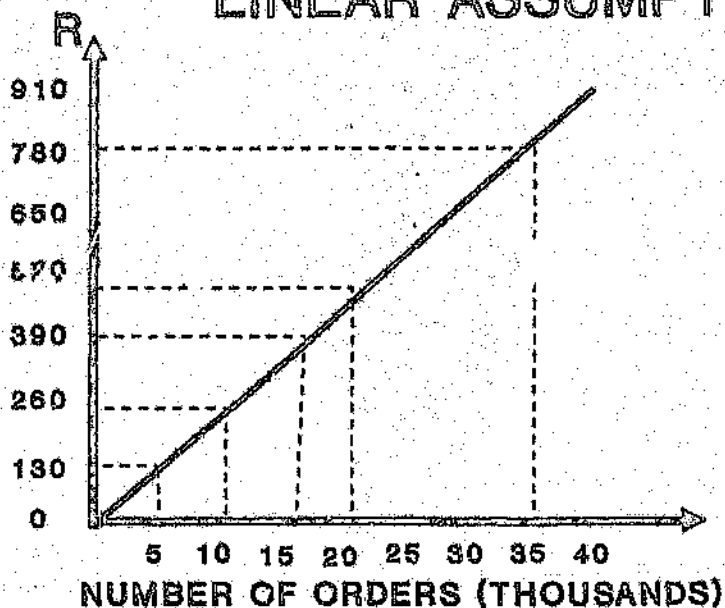
Most inventory models offer optimal solutions so long as the conditions of the system meet the constraints of the model. While this is easy to state, it is difficult to achieve. Obtaining actual figures for carrying, shortage and especially order cost, is difficult. Part of the problem firstly occurs because the accounting data are averages, whereas the cost required is the marginal cost. Secondly there is the difficulty of assigning the exact cost to the order (various order sizes), and thirdly there is no longer a direct relationship between the number of orders and the cost of the orders. In other words, reducing the number of orders will not necessarily produce a reduction in the cost of orders (see Diagram 7). Fourthly it is very difficult in practice to implement various order costs for various SKUs.

DIAGRAM 7

NUMBER OF ORDERS PLACED

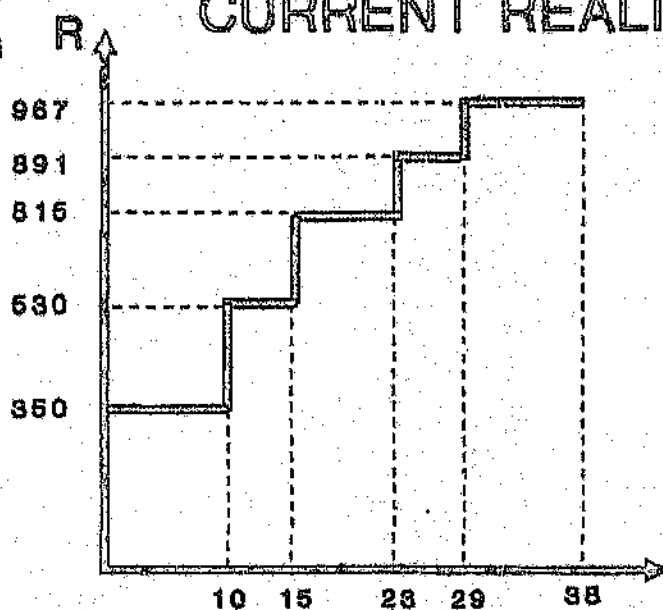
ORDERING COST
(R 1000)

LINEAR ASSUMPTION



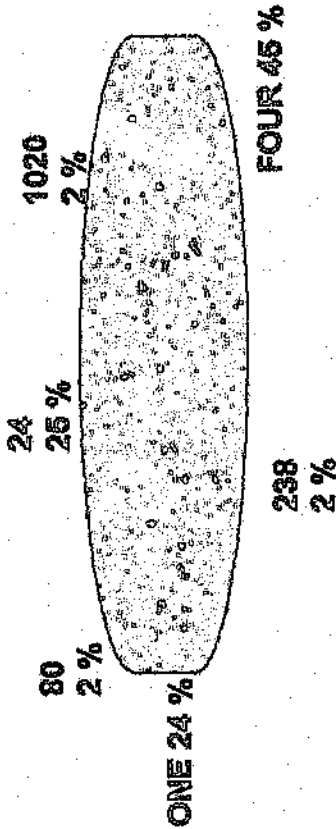
ORDERING COST
(R 1000)

CURRENT REALITY

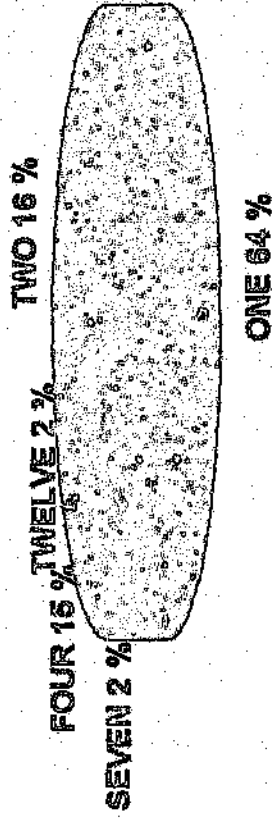


NUMBER OF ORDERS (THOUSANDS)

DIAGRAM 8
ORDER SIZE DISTRIBUTION ANALYSIS
NUMBER OF SKUS PER ORDER
FEBRUARY 1992 - MARCH 1993



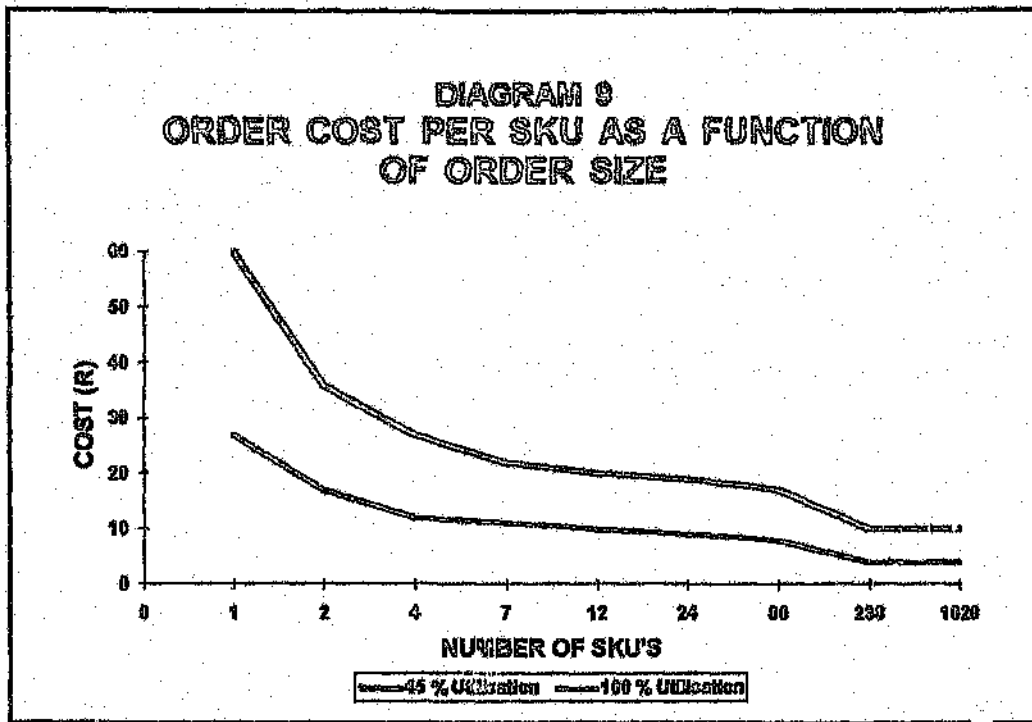
DAILY, VOR ORDERS



751 ORDERS

3916 ORDERS

DIAGRAM 9



To illustrate the cost effects of changing ordering cost A, Table 4 presents the computed order quantity for an SKU with an annual demand of 1 000 units, a unit cost of R50, a carrying cost of 23% per annum, and various order costs per SKU as indicated in Diagram 9. The model used for the computation was the simple EOQ model without safety stock. If we assume that the average order cost is R26, then the optimal order quantity is 67,2 units. If order costs of R7 (a quarter as much) and R60 (more than twice the average) are used, the variable cost would be computed between -48% and 52%.

The total error range in using a quarter of and more than double the cost, makes a total annual difference of R773 in the average order cost. The total cost, however, varies less than 1%, ranging from 0,73% low to 0,79% high.

The above results with the general assumption that inventory models normally involve quadratic equations, suggest that even significant errors often have less effect than one might expect. These comments are not meant to cause the reader to reduce emphasis on computing order quantities, but rather to recognise where inventory costs come from and where the emphasis must be placed in order to reduce costs significantly.

Clearly then, if a company seeks to make significant reductions in the total cost of inventory, it must cut order set-up times or reduce the existing level of inventory through major lead time reductions. No less important is the reduction of forecast errors via better modelling.

The approach recommended therefore is to conduct the inventory analysis in terms of inventory investment and workload, rather than in terms of order cost (see details in section 4.4).

Lead time reduction and inventory models are discussed in Sections 3.2.2 and 5.

3.1.6 SHORTAGE COST

The costs involved in establishing the shortage cost are all the costs incurred when stockouts take place.

The main shortage cost includes the following:

- The attendant costs of expediting and rescheduling. Higher rates are paid for overtime or premium is paid to subcontractors (agents) in order to expedite emergency orders.
- The costs incurred when parts have to be air-freighted from overseas suppliers instead of normal sea-freight shipment.
- The cost incurred when parts manufactured in South Africa have to be imported from the parent company at a higher unit cost.
- The costs incurred due to loss of sales, especially of parts supplied by competitors are higher than the profit loss on the parts that are out of stock, since the customer might cancel the whole order.

In addition to the measurable costs there are some nebulous costs that cannot be measured in rands and cents. For example, the loss of goodwill and reputation and the loss of market share both play a major role when deciding on the company's strategic planning, and on the inventory control system in particular.

From the customer's point of view, shortage costs depend mainly on time lost for production in the event of a breakdown.

The cost involved in loss of truck/bus running time is approximately R3 500 per day.

The criteria for establishing shortage costs are as follows:

- Additional costs incurred for freight

For overseas suppliers these costs are on average 15% of the SKU cost, but for local suppliers these costs are minor.

- For expediting the manufacturing process, there is an additional cost of between 10% and 20% of the SKUs value, i.e. an average of 15%. For overseas suppliers there are no charges.
- For loss of sale it is recommended that 10% of the unit cost for those SKUs supplied by the competitors be added, i.e. a total of 25% of the unit cost (which is the average gross profit margin on those SKUs). For other SKUs the total shortage cost will remain the same (15%).

TABLE 4
COST EFFECT OF CHANGING ORDER COST A

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
ANNUAL DEMAND D	ORDER COST A	UNIT COST C	HOLDING RATE I	UNIT HOLDING COST $r=IC$	ORDER QUANTITY Q	ORDERING COST $(D \times A)/Q$	HOLDING COST $(Q \times r)/2$	TOTAL VARIABLE COST $(6+7)$	TOTAL ANNUAL COST $(8)+D \times C$	CHANGE IN VARIABLE COST	CHANGE IN TOTAL COST
1000	7	50	0.23	11.5	34.9	201	201	402	50402	-47.99	-0.73
1000	14	50	0.23	11.5	49.3	284	283	567	50567	-26.55	-0.41
1000	16	50	0.23	11.5	52.8	303	304	607	50607	-21.47	-0.33
1000	18	50	0.23	11.5	56	321	322	643	50643	-16.82	-0.26
1000	21	50	0.23	11.5	60.4	348	347	695	50695	-10.09	-0.15
1000	26	50	0.23	11.5	67.2	387	386	773	50773	0	0
1000	36	50	0.23	11.5	79.1	455	455	910	50910	17.72	0.27
1000	40	50	0.23	11.5	83.4	480	480	960	50960	24.19	0.37
1000	45	50	0.23	11.5	88.5	508	509	1017	51017	31.57	0.48
1000	60	50	0.23	11.5	102.2	587	588	1175	51175	52.01	0.79

3.2 REPLENISHMENT LEAD TIME

3.2.1 OVERVIEW

A stockout can only occur during periods when the inventory on hand is low. The decision as to when an order should be placed will always depend on how low the inventory level should be allowed to fall, so that the expected number of SKUs demanded during a replenishment lead time will not give rise to a stockout more often than a specified number of times. Replenishment lead time is defined as the time that elapses from the moment it is decided to place an order until the ordered items are physically on the shelf to meet customer demands. The symbol *LT* will be used to denote replenishment lead time. As suggested by Silver and Peterson⁽¹⁾ (65), it is convenient to think of the lead time as being made up of four distinct components:

- **Administrative time at the stocking point (order preparation time)**

This is the time that elapses from the moment when it is decided to place the order until the order is actually transmitted from the stocking points.

This stage includes the suggested order size evaluation and the order compilation. For bulk orders it takes from 3 to 4 weeks. Daily orders are compiled during an average of two days, and Vehicle of the Road (VOR) orders take one day.

- **Transit time to the supplier**

This may be negligible if the order is placed electronically or by telephone, but transit time can be several days if a mailing system is used.

For bulk orders the company uses the post. In the RSA this takes up to one week and overseas (mainly Germany), one to two weeks.

- **Supplier lead time**

This time constitutes the primary variable component. Its duration is materially influenced by the supplier's stock situation when the order arrives. It also includes the transit time back to the stocking point.

As already mentioned, the company currently uses the lead time determined by the supplier for computing the suggested order size.

- **Time from order receipt until the stock is available on the shelf - Receiving time.**

This time is often neglected when it should not be. Contributing factors include inspection and catalogueing.

Company policy allocates three weeks for receiving goods, with the following activities:

- Checking and generating a goods received voucher (GRV)

- Binning
- Data capturing.

The records analysis for the period January 1993 to November 1993 concerning the above activities for bulk orders is summarised in Table 5.

TABLE 5
SUMMARY OF INTERNAL LEAD TIME ACTIVITIES

ACTIVITY	AVERAGE TIME (DAYS)	STANDARD DEVIATION (DAYS)	TOTAL
Receiving and binning	3,2	2,95	6,2
Data capturing by computer	4,9	2,6	7,5

The above findings indicate that the current lead time allocated to the above activities by management exceeds the actual average lead time plus the standard deviation by 33%.

The data-capturing actual lead time is too long and is not acceptable in this type of environment. The above table suggests that entering the data in the computer is more time-consuming than the physical receiving and binning of the parts.

The cumulative frequency and frequency distribution of lead times for the receiving/binning and data-capturing activities are represented in Diagrams 10 and 11.

3.2.2 REPLENISHMENT LEAD TIME REDUCTION

As stated under 3.1.4, the inventory cost could be significantly reduced through lead time reduction. The replenishment lead time is combined from the company (internal) lead time L_c and the supplier lead time L_s .

3.2.2.1 INTERNAL (COMPANY) LEAD TIME

The company lead time L_c comprises

- order compilation - 3 weeks
- order receiving/binning and data capture - 3 weeks
- review period - 4 weeks.

It is recommended that management reduce the internal lead time as follows:

◦ **Order completion - 1 week**

A saving of two weeks on the time allowed for this activity can be achieved through implementation of the following recommendations:

Split the report between the inventory clerks/buyers according to supplier importance (number of line items and value of stock supplied)

The suggested order report is not generated at once (beginning of the month) but during the week per supplier according to buyers' workload, the supply scheduling (flight, trucks) and arrangements with suppliers.

◦ **Receiving/binning and data-capturing lead time - 1 week**

Analysis of the company records for the last two years indicated a one-week lead time.

The aim should be to reduce the data-capturing lead time to less than one week for bulk orders by improving administration flows and procedures. Currently 90% of the orders received are binned and the data captured on the same day.

The introduction of a bar code system could further reduce the time devoted to data-capturing and binning.

3.2.2.2 SUPPLIERS' LEAD TIMES

To measure supplier lead times, the following should be carried out:

- Define ordering (transmit) date per SKU
- Define receiving date per SKU.

The supplier's lead time is therefore the receiving date minus the ordering date per SKU. Since the company held records of orders for a large number of SKUs per order, it was unfortunately impossible to quantify the actual lead time per SKU per supplier. Therefore the following steps were carried out to calculate the supplier lead time per order:

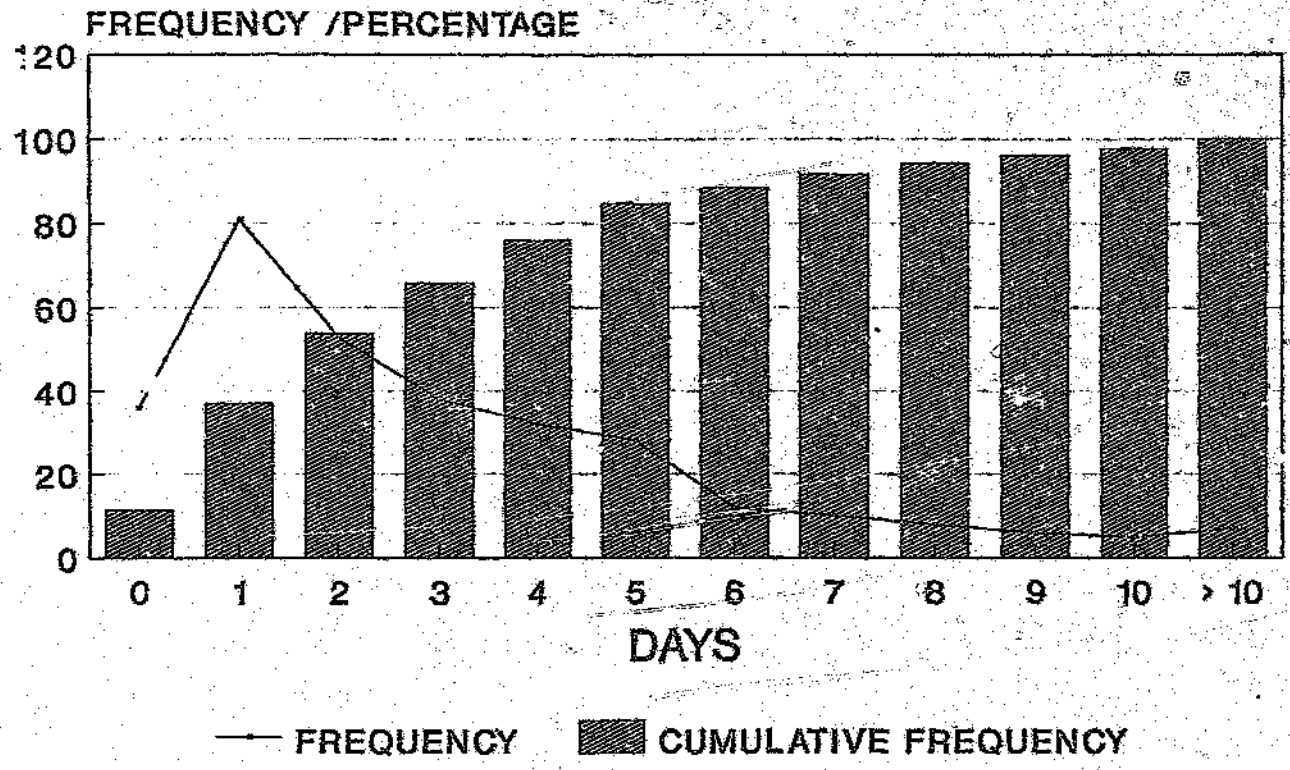
A study was carried out on the lead times of the main suppliers of the company which, according to the orders placed during the last two years, supplied 80% of the line items and 85% of the total stock value.

- The supplier lead time data were entered in the computer in the inventory section. For each order, the transmitted date to the supplier and the receiving date were recorded. The supplier lead time was then calculated as follows:

Supplier lead time = receiving date - order transmitted date to the supplier for the first two deliveries made to the company under the same order number.

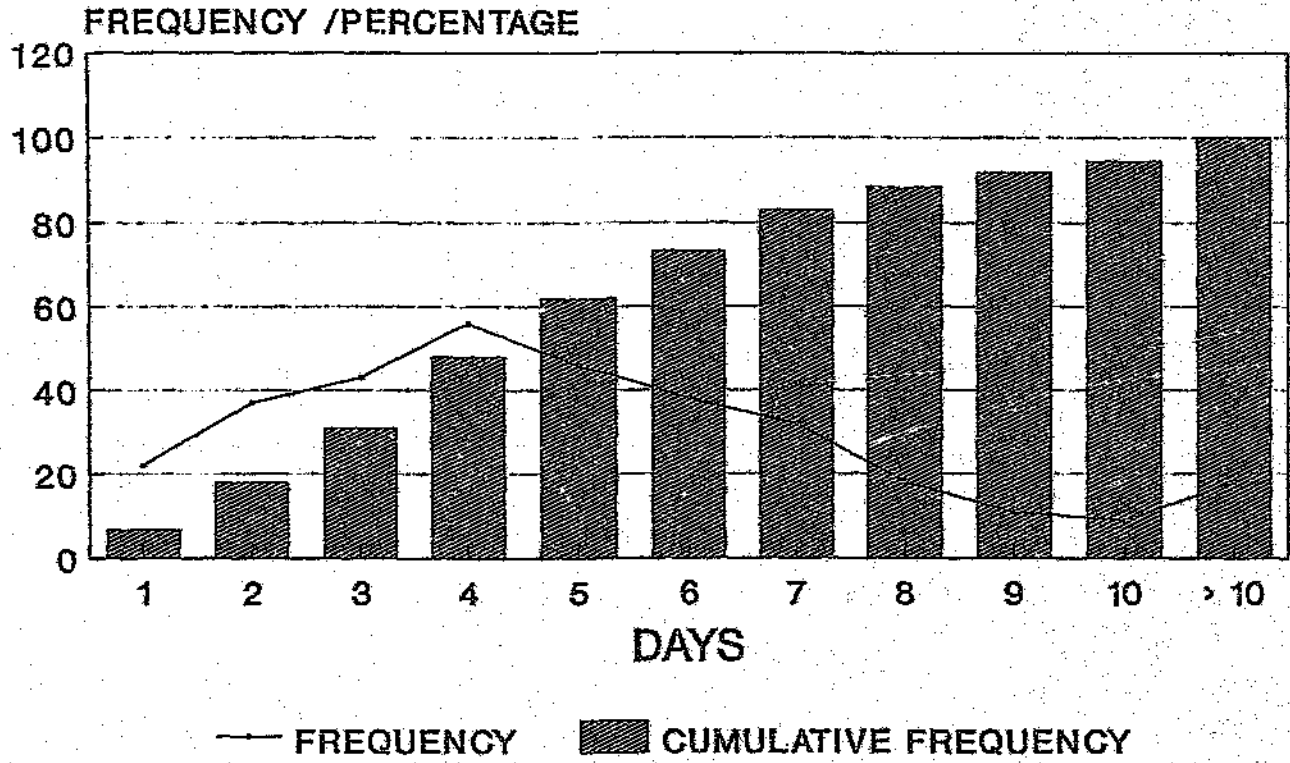
The average and the standard deviations were calculated for each delivery.

LEAD TIME DISTRIBUTION



BINNING

LEAD TIME DISTRIBUTION



DATA-CAPTURING

DIAGRAM 11

The suggested supplier lead time was calculated as follows:

$$\text{Suggested supplier lead time} = (\text{average first delivery lead time} + \sigma) \times 0,75 + (\text{average second delivery lead time} + \sigma) \times 0,25.$$

Where

σ = STD if $\text{STD} \leq 0,3$ of average delivery lead time

σ = $0,3$ of average delivery lead time if $\text{STD} >$ average delivery lead time.

The following points should be noted:

- The above formula assumed availability of the 100% items within two shipments.
- Abnormal lead times were excluded from the stream of data on which the average and the STD were calculated.

Table 5 summarises the average supplier lead times, the standard deviation and the suggested supplier lead times for the main local and overseas suppliers.

TABLE 6

SUGGESTED LEAD TIMES FOR USE IN PROPOSED INVENTORY MODELS

SUPPLIER	ACTUAL LEAD TIME (WEEKS)	PROPOSED LEAD TIME (WEEKS)	SAVINGS (WEEKS)
MAN	24	20	4
ADE	14	7	7
ASTAS	23	29	-6
DIESEL ELECTRIC	16	9	7
SACHS	36	33	3
KARL SCHMIDT	22	19	3
BALTMAN INTERNATIONAL	22	19	3
KNORR BREMSE	20	14	6
WABCO	16	13	3
PROPOWER	24	12	12
FAG	26	17	9
ZF OF SA	14	7	7
RFS	18	12	6
GLYCO	22	16	6
ELERING GASKETS	22	19	3
KILBER	18	11	7
C & J	16	9	7
REINZ	22	11	11
DONALDSON	18	13	6
BOSAL	22	13	9
GOETZE	22	11	11
OTHER	20	17	3

The above table indicates the following:

- Substantial savings on supplier lead times can be achieved.

- The company's slowest supplier is ASTAS, whose lead time is far longer than originally promised (43% longer). Moreover, the deviation on this lead time is extensive, which indicates low reliability.
- Substantial savings are possible on the lead times for the suppliers Goetze (a reduction of 50%) because they shipped by air freight and long lead times were allocated to them.

The quantified savings of the lead time reduction are discussed in Section 6. With reference to the lead time study results summarised in Table 5, it was recommended to management that the company take the following steps in the short to medium term:

- Use the suggested suppliers' lead times indicated by the study in the proposed inventory model
- Monitor each order for each supplier as installed on the computer in the inventory department
- Check the availability level of line items in each shipment
- Update the suggested lead times once a year according to actual supplier performance
- Investigate any irregularities in the lead times of the main suppliers, should these occur.

In the long term the supplier lead time should be monitored per SKU. The following data should be collected and processed as follows:

- Supplier lead time
 - Average (days)
 - Average demand during lead time
 - Standard deviation of demand during lead time.

This way of processing the data is very important for future use in the proposed inventory models.

3.3 DEMAND PATTERNS

Naddor⁽²¹⁾ regards demand component as the most important of the properties of an inventory system.

Inventories are kept so that demands may be met, orders filled, and requirements satisfied. Generally, demands cannot be controlled directly, and in many cases they cannot even be controlled indirectly. Demands usually depend on decisions by people outside the company with inventory problems.

Although demands themselves are generally not controllable, their properties may be studied. The following questions regarding demand may be asked:

- When do customers place their orders?
- What is the order size?
- Do we have information for future requirements?

The following are important factors/properties relating to the demand pattern for an item:

- Variability
- Seasonality
- Ability to forecast
- The stage in the life cycle of the product/item.

The above properties are significant in the calculation of the inventory parameters and therefore they affect stock levels.

The demand for SKUs and the selection of an appropriate inventory model are discussed in Section 4.3.5.

4. INVENTORY CONTROL SYSTEMS - LITERATURE SURVEY

4.1 DIFFERENT DEFINITIONS OF STOCK LEVEL

It is useful to categorise inventories conceptually as follows:

- On-hand stock

This is stock that is physically on the shelf. It can never be negative. This quantity is relevant in determining whether a particular customer demand can be satisfied directly from the shelf.

- Net stock = (on hand) - (back orders)

This quantity can become negative (if there are back orders). It is used in some mathematical derivations and is also a component of the following important definition.

- Inventory position (sometimes also called the available stock)

The inventory position is defined by the equation

Inventory position = (on hand) + (on order) - (back orders) - (committed).

On-order stock is that stock which has been requisitioned but not yet received by the stocking point under consideration. The inclusion of the "committed" quantity in the above equation is based on not being able to borrow from such stock for other purposes in the short run. If a commitment is made farther than a replenishment lead time in advance of use, borrowing may be possible. The inventory position is a key quantity in deciding on when to replenish.

- **Safety stock**

The safety (or buffer) stock is defined as the average level of the net stock just before replenishment. If it was planned to just run out, on average, at the moment when the replenishment arrived, the safety stock would be zero. A positive safety stock provides a cushion or buffer against greater-than-average demand during the effective replenishment lead time. The numerical value of the safety stock depends on what happens to demands when there is a stockout.

4.2 CONTINUOUS VERSUS PERIODIC REVIEW

The fundamental purpose of a replenishment control system is to provide answers to the following three questions:

- How often should the inventory status be determined?
- When should a replenishment order be placed?
- How large should the replenishment order be?

The answer to the question How often should the inventory status be determined?, specifies the review interval (T). This is the time that elapses between two consecutive moments at which it is known what the stock level is. An extreme case is where there is continuous review, i.e. the stock status, is always known.

With periodic review, as the name implies, the stock status is determined only every R time units. Between reviews there may be considerable uncertainty as to the value of the stock level.

4.2.1 ADVANTAGES AND DISADVANTAGES OF CONTINUOUS AND PERIODIC REVIEW SYSTEMS

Items may be produced on the same piece of equipment, purchased from the same supplier, or shipped in the same transportation mode. In any of these situations coordination of replenishments may be attractive. In that case periodic review is particularly appealing in that all items in a coordinated group can be given the same review interval (for example, all items purchased from a particular supplier might be scheduled for review every week). Periodic review also allows a reasonable prediction of the level of the workload on the staff involved in issuing replenishment orders. In contrast, under continuous review, a replenishment decision can be made at practically any time; hence the load is less predictable. A rhythmic rather than a random pattern usually appeals to staff.

Another disadvantage of continuous review is that it is generally more expensive in terms of reviewing costs and reviewing errors. This is particularly true for fast-moving items where there are many transactions per unit of time. Moreover, for extremely slow-moving items, very little costs are incurred by continuous reviews because updates are only made when a transaction occurs. On the other hand there is the anomalous condition that periodic review may be more effective than continuous review in detecting spoilage (or pilferage) of such slow-moving items, in that periodic review forces an occasional review of the situation, whereas in transactions recording no automatic review will take place without a transaction occurring.

The major advantage of continuous review is that, to provide the same level of customer service, it requires less safety stock (hence lower carrying costs) than does periodic review. This is because the period over which safety protection is required is longer under periodic review (the stock level has the opportunity to drop appreciably between reviews without any reordering action being possible in the interim).

The current inventory system employed by the company is a periodic review system with a review period of four weeks.

The reasons for the use of the periodic review system (apart from the periodic review system advantages outlined above) are:

- A long time for evaluating and compiling orders - approximately 4 weeks
- Some suppliers accept only monthly orders.

4.3 TYPES OF INVENTORY CONTROL SYSTEMS

There are a number of possible control systems. The four most common ones and a brief outline of the advantages and disadvantages of each of the systems are discussed below:

4.3.1 ORDER-POINT, ORDER-QUANTITY (s, Q) SYSTEM

This system involves continuous review (i.e. $R = 0$). A fixed quantity Q is ordered whenever the inventory position drops to the reorder point s or lower. The inventory position, and not the net stock is used to trigger an order, because it includes the on-order stock, not yet received from the supplier. In contrast, if the net stock was used for ordering purposes another order might unnecessarily be placed today while a large shipment was due in tomorrow. To use the system, it may be necessary to adjust Q upward so that it is appreciably greater than the average demand during a lead time.

This is a simple system, particularly in the two-bin form, for the stock clerk to understand. A fixed order quantity also has advantages in terms of less likelihood of error and also predictability of production requirements on the part of the supplier. One disadvantage of an (s, Q) is that in its unmodified form it may not be able to cope effectively with the situation where individual transactions are of appreciable magnitude. In particular, if the transaction that triggers the replenishment in an (s, Q) system is big enough, a replenishment of size Q will not even raise the inventory position above the reorder point.

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4.3.2 ORDER-POINT, ORDER-UP-TO-LEVEL (s, S) SYSTEM

This system again involves continuous review and a replenishment is made whenever the inventory position drops to the order point s or lower. However, in contrast to the (s, Q) system, here a variable replenishment quantity is used, with enough being ordered to raise the inventory position to the order-up-to-level S . If all demand transactions are unit-sized, the two systems are identical because the replenishment requisition will always be made when the inventory position is exactly at s i.e. in this case $S = s + Q$. As soon as transactions are bigger than unit size, the replenishment quantity in the (s, S) system becomes variable. Figure 4 illustrates the difference in the behaviour of the two systems. The (s, S) system is frequently referred to as a min-max system because the inventory position, except for a possible momentary drop below the reorder point, is always between a minimum value of s and a maximum value of S .

The best (s, S) system can be shown to have total costs of replenishment, carrying inventory, and shortage no greater than those of the best (s, Q) system. However, the computational effort to find the best (s, S) pair is prohibitive, except perhaps where we are dealing with an item where the potential savings in the aforementioned costs are appreciable (i.e. an A item). A possible disadvantage of the (s, S) system is the danger of errors in requisitioning, among others operations, caused by the variable order quantity.

4.3.3 PERIODIC-REVIEW, ORDER-UP-TO-LEVEL (R, S) SYSTEM

This system is also known as a replenishment cycle system. The control procedure is that at every R units of time (i.e. at each review) enough is ordered to raise the inventory position to the level S . A typical behaviour of a periodic versus a continuous system is shown in Figure 5.

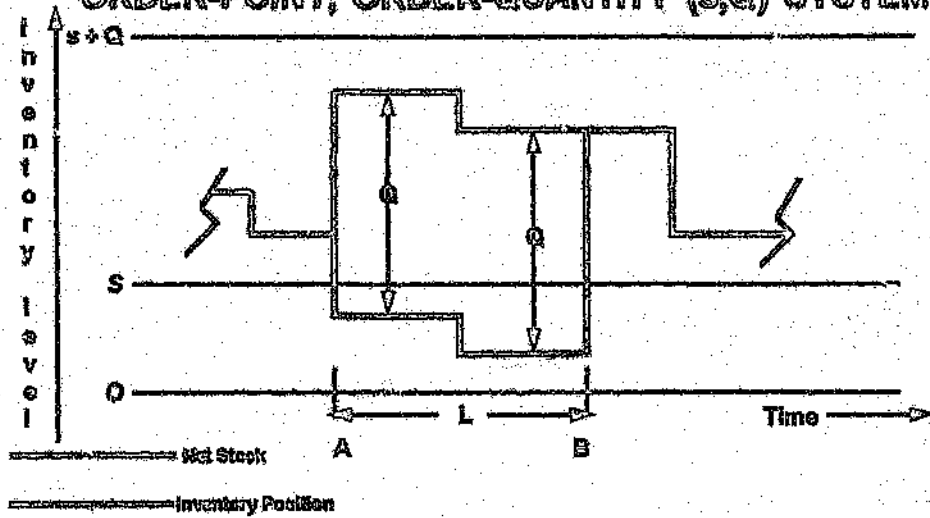
Because of the periodic review property, this system is much preferred to order point systems in terms of coordinating the replenishments of related items. In addition, the (R, S) system offers a regular opportunity (every R units of time) to adjust the order-up-to-level S , a desirable property if the demand pattern is changing with time. The main disadvantage of the (R, S) system is that the carrying costs are higher here than in continuous review systems.

4.3.4 (R, s, S) SYSTEMS

This is a combination of the (s, S) and (R, S) systems. The idea is to check the inventory position every R units of time. If it is at or below the reorder point s , enough is ordered to raise it to S . If the position is above s , nothing is done until the next review. The (s, S) system is the special case where $R = 0$, and the (R, S) is the special case where $s = S - 1$. Alternatively one can think of the (R, s, S) system as a periodic version of the (s, S) system. Also, the (R, S) situation can be viewed as a periodic implementation of (s, S) with $s = S - 1$.

FIGURE 4
CONTINUOUS REVIEW SYSTEMS

ORDER-POINT, ORDER-QUANTITY (s,Q) SYSTEM



ORDER-POINT, ORDER-UP-TO-LEVEL (s,S) SYSTEM

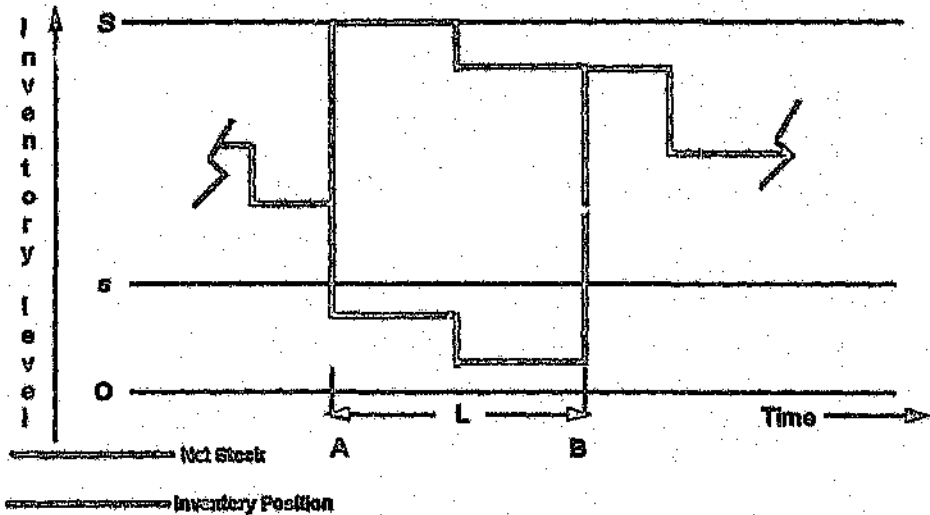
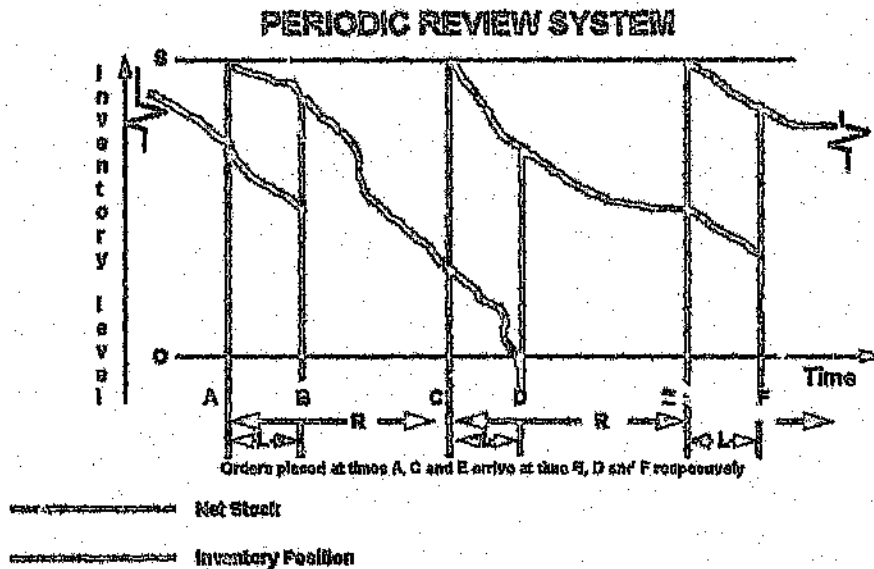
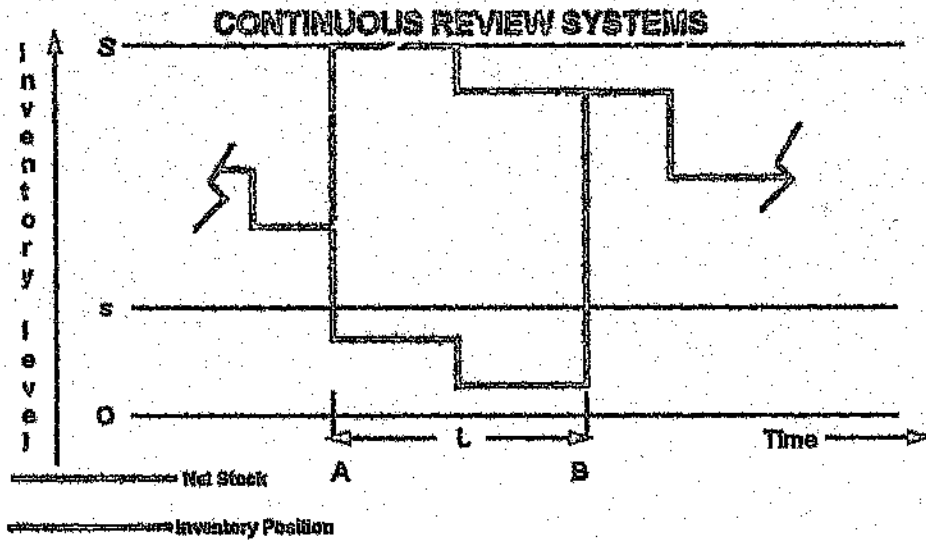


FIGURE 5
INVENTORY REVIEWING SYSTEMS:
PERIODIC vs CONTINUOUS



It has been shown that, with quite general assumptions concerning the demand pattern and the cost factors involved, the best (R, s, S) system produces a lower total of replenishment, carrying, and shortage costs than does any other form of system. This system is more difficult for a clerk to understand than some of the previously mentioned systems. (R, s, S) systems are found in practice where R is selected largely for convenience.

4.4 SELECTION OF APPROPRIATE INVENTORY CONTROL SYSTEM - DISCUSSION

The appropriate inventory control system for the type of operation of the company is a periodic review system of the order-up-to-level (R, S) type. This system is advocated because it holds the following advantages:

- A periodic review system allows coordination of SKUs that share the same supplier and the same transportation mode by setting the same review interval. Thus employing the (R, S) system offers the following benefits:
 - Ease of scheduling - coordinated handling of a vendor group of items facilitates the scheduling of buyers' time and the receiving (and inspection) workload
 - Savings on ordering costs - extensive lists of items are placed on the same order
 - Savings on unit transportation costs.
- The (R, S) system is familiar to buyers and intuitively understood. In fact, by and large managers and purchasers alike tend to think and deal in terms of vendors or suppliers rather than individual SKUs.
- Not all the current company suppliers are geared to handle orders on a continuous basis.
- Continuous review (applicable to fast-moving items) is more expensive to run, and periodic review offers savings in review costs.
- Periodic review is more effective than continuous review when dealing with slow-moving items, from the point of view of detecting spoilage (or pilferage). This is because a periodic review system forces an occasional review of the situation, whereas with the transaction recording employed in continuous review, no review takes place unless a transaction occurs.

The main disadvantage of a periodic review system is the increase in average inventory levels. In order to minimize this, it is suggested that the current review period of four weeks be reduced to two weeks. Figure 6 shows that with a two-weekly review system, the effect of holding more stock and the danger of stockouts are very minor when the replenishment lead time is quite long (more than eight weeks), which is the case with the majority of line items.

The notation to be used is: s = Re order point; Q = Re order quantity; S = Order-up-to-level (maximum on hand quantity)

Cutting the review period to two weeks is expected to result in the following:

- The number of bulk orders will be doubled. Therefore there will be 1 502 orders per annum instead of the 751 currently compiled.
- The number of daily and VOR orders will be reduced by 25%. Therefore $75\% \times 3\ 916 = 2\ 937$ orders, or in other words only 2 937 orders, will be compiled.

Moreover, a further reduction is expected to occur with better modelling (proposed inventory models).

- The current bulk order size (number of SKUs per order) distribution will remain the same.
- The handling of non-bulk orders will require an additional receiver when more than 4 000 orders are received per annum.
- Buyer utilisation on 60% of the available buyer time will be around 71%.

The expected buyer utilisation calculation for a two-weekly review system is presented in Appendix 17.

Since the variable costs attributable to manpower resources will not increase (mainly due to the under-utilisation of the manpower resources), no increase in order cost is expected.

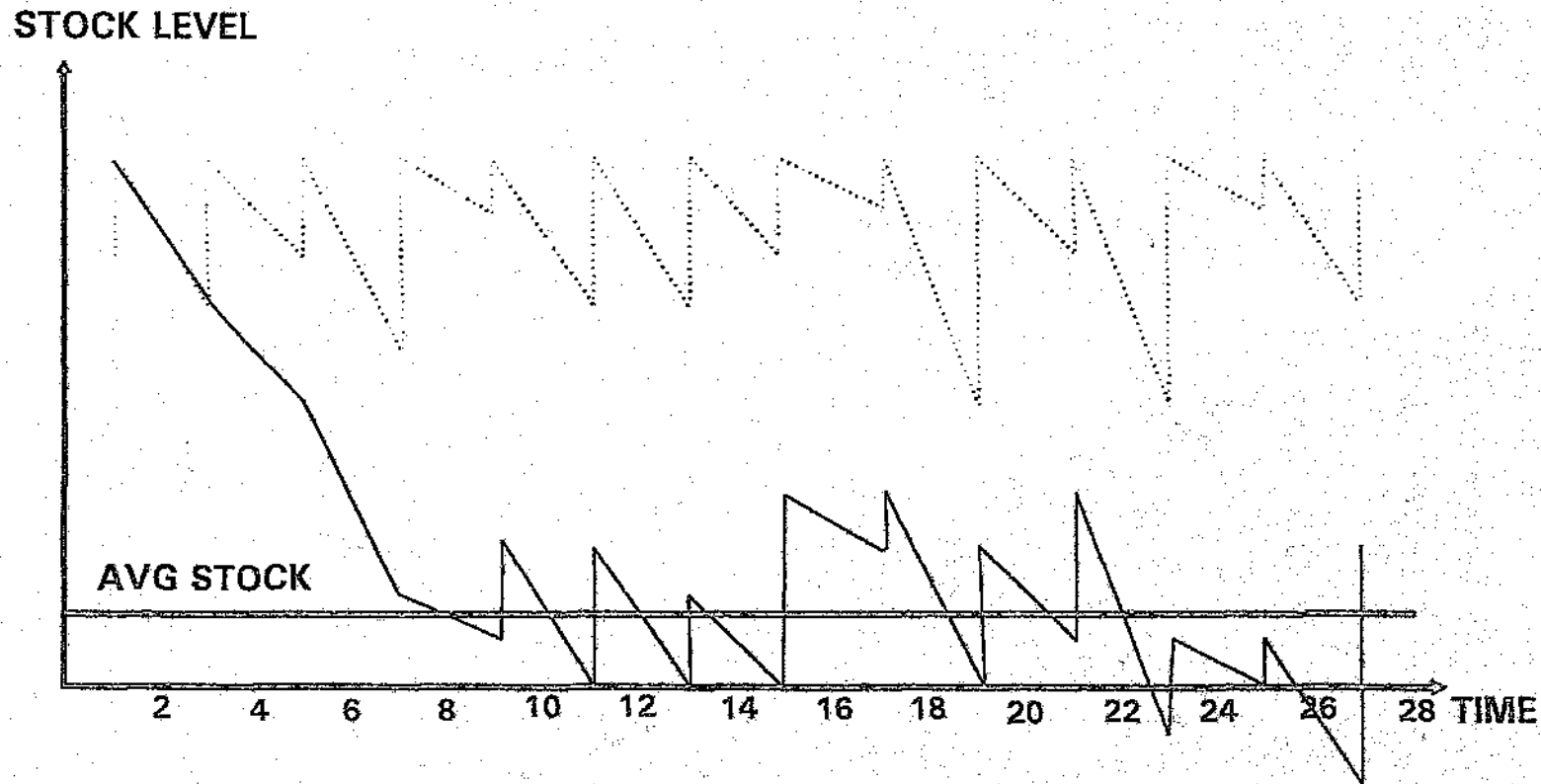
Other variable costs (transport, phones, mail, etc.) are expected to remain the same or to decrease since the number of daily and VOR orders will drop.

Moreover, the marginal cost of selecting a continuous review system will require an additional buyer at an annual cost of R70 000 while the marginal carrying charges of the periodic review system (R = 2 weeks) amount to only R99 000 per annum. The above lower cost favour the periodic review system. This is also indicated in a paper presented by Donaldson⁽¹⁸⁾ in which he states that when the supplier lead time is longer than the review period, the periodic review system generates the lowest total costs.

Diagram 12 depicts ordering costs as a function of the number of orders placed with the proposed two-weekly review system. The proposed overall company lead time is therefore three weeks. A detailed breakdown of the cost involved is presented in Appendices 18 and 19.

The recommendations set out above will result in an internal lead time reduction of seven weeks. This does not mean that the total replenishment lead time will be reduced by seven weeks, since three to four weeks of the ten weeks' company lead times currently in use are built in to cover for variability in demand patterns and supplier lead times. This variability is addressed in the proposed inventory models.

PERIODIC REVIEW SYSTEM REVIEW PERIOD VS REPLENISHMENT LEAD TIME



TYPICAL BEHAVIOUR OF PERIODIC REVIEW SYSTEM
WHEN REVIEW PERIOD SMALLER THAN REPLENISHMENT LEAD TIME

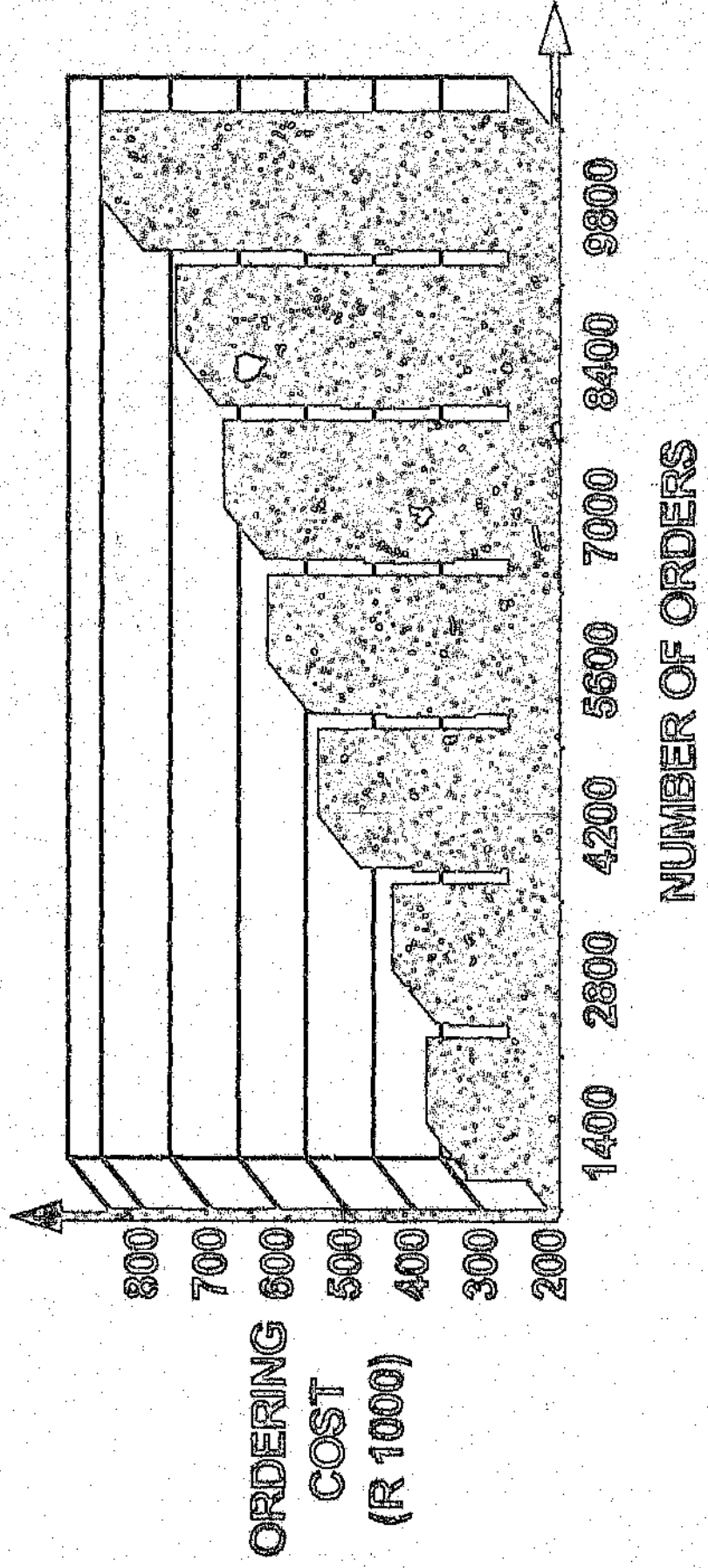
REVIEW PERIOD - 2 WEEKS
REPLENISHMENT LEAD TIME - 8 WEEKS

..... INVENTORY POSITION
— NET STOCK

FIGURE 6

DIAGRAM 12

COST OF PLACING ORDERS VERSUS NUMBER OF ORDERS PLACED PROPOSED TWO WEEKLY REVIEW



4.5 SKU MOVEMENT CATEGORY CLASSIFICATION AND ITS APPROPRIATE DEMAND FREQUENCY DISTRIBUTION

Lumpy or sporadic demand patterns with highly skewed distributions are common in parts and supplies types of stockholdings, and much of available inventory control methodology is not appropriate for such items.

The following are two of the basic problems in inventory control:

- The classification of SKUs into movement categories namely, slow-moving, fast-moving (smooth demand), and sporadic/lumpy demand patterns
- Finding a frequency distribution that adequately represents the frequency distribution of demand.

4.5.1 SKU CLASSIFICATION

An interesting approach presented by Williams⁽⁸⁾ based on an idea by Donaldson is called variance partition, used to classify SKUs into movement categories. The variance partition attempts to split the variance of demand during the replenishment lead time (var (X_L)) into its constituent causal parts. The variance partition equation is as follows:

$$\text{Var}(X_L) = X^2 L \text{Var}(n) + nL \text{Var}(x) + n^2 x^2 \text{Var}(L) \text{ which could also, imprecisely, be expressed as}$$

$$\text{Var}(X_L) = \text{variance due to } n + \text{variance due to } x + \text{variance due to } L$$

Where

n = average number of orders arriving in successive units of time (week, month)

$\text{Var}(n)$ = variance of n

x = average order size

$\text{Var}(x)$ = variance of x

L = average lead time

$\text{Var}(L)$ = variance of L

The above equation is made dimensionless by writing it as

$$C^2_{X_L} = \frac{C^2_n}{L} + \frac{C^2_x}{nL} + C^2_L$$

where

C_x = coefficient of variation in the distribution of order size

C_n = coefficient of variation in the distribution of the number of orders

C_L = coefficient of variation in the distribution of lead times

C_{x_L} = coefficient of variation in the distribution of demand during the replenishment lead time.

Assume the number of demands (orders) per unit time (n) is Poisson with mean μ . Then the above equation can be expressed as

$$C_{x_L}^2 = \frac{1}{\mu_L} + \frac{C_x^2}{\mu_L} + C_L^2$$

If the lead time is assumed to be constant, so that $\text{var}(L) = 0$, this simplifies to

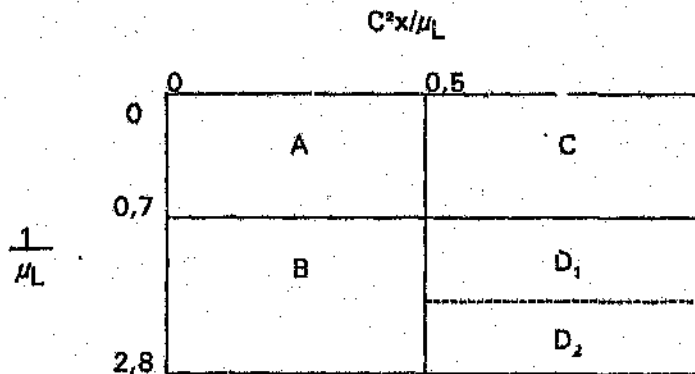
$$C_{x_L}^2 = \frac{1}{\mu_L} + \frac{C_x^2}{\mu_L}$$

The SKU can now be classified by the size of these two components, which are both dimensionless:

$\frac{1}{\mu_L}$ = represents the average number of lead times between demands

C_x^2/μ_L = this ratio expresses the "lumpiness" of the demand.

Williams⁽⁹⁾ indicated in his paper that when these parameters were evaluated for each of the SKUs, it was observed that the "non-sporadic" products tended to have low parameters, the "low-sporadic" products had a higher $1/\mu_L$ but still a low C_x^2/μ_L , and the "high-sporadic" products, while being more scattered, generally appeared to have higher parameters. This two-dimensional measurement suggested a classification into five categories:



where:

category A = "low-sporadicity" SKUs or SKUs with a "smooth" demand, normally fast-moving items

category B = a high $1/\mu_L$ (mean number of lead times between demands), corresponding to slow-moving SKUs

categories C and D₁ = SKUs with frequent demands of widely varying sizes

category D₂ = highly sporadic SKUs namely SKUs with very few demands but with high size magnitude. The parameter ranges suggested by Williams for the different categories are as follows:

CATEGORIES	PARAMETER RANGE	CLASS CATEGORY
D ₂	$\frac{1}{\mu_L} > 2,8,$ $\frac{C_x^2}{\mu_L} > 0,5$	Sporadic
B	$\frac{1}{\mu_L} > 0,7$ $\frac{C_x^2}{\mu_L} < 0,5$	Slow-moving
A, C, D ₁	$\frac{1}{\mu_L} < 0,7$ $\frac{C_x^2}{\mu_L} < 0,5$	Smooth

Some other selected definitions to classify SKUs into movement categories offered by the literature, are listed below:

◦ **Fast-moving SKUs**

Archibald, Silver and Peterson⁽¹⁷⁾ suggest that SKUs be classified into fast-moving category when the demand during the replenishment lead time (X_{L+R}) is greater than 10 units, while Heyvaert and Hurt⁽¹⁸⁾ suggest that X_{L+R} should be greater than 15 units.

◦ **Slow-moving SKUs**

Other definitions offered at the Operational Research Society of South Africa (ORSSA) Conference of 1979:

- No usage for the last 12 to 24 months
- The on-hand quantity is greater than twice the annual demand
- The reorder quantity is 1 and $X_{L+R} = 1$
- Monthly usage of less than two units
- $X_{L+R} < 5$ units
- $0,4 < X_{L+R} < 10$
- $0,3 < X_{L+R} < 15$

4.5.2 SELECTION OF RULE TO CLASSIFY SKUs INTO MOVEMENT CATEGORIES - DISCUSSION

The author proposes to use the average demand during the replenishment lead time (X_{L+R}) as a criterion to classify items into movement categories for the following reasons:

- A large number of inventory models use the average demand and standard deviation of forecast error during the replenishment lead time to calculate the control parameters, namely the reorder point, the reorder quantity and safety stocks.
- X_{L+R} ranges are adequate criteria to classify items into movement categories (especially when dealing with slow-moving items) based on extensive testing carried out by Archibald⁽¹⁷⁾.

The chosen ranges of average demand during the replenishment lead time, (in units) to classify SKUs into movement categories, are as follows:

- Fast-moving SKUs $X_{L+R} > 10$
- Slow-moving SKUs $0,4 < X_{L+R} \leq 10$
- Very slow-moving SKUs $X_{L+R} \leq 0,4$

The sporadic nature of demand and the frequency of demand during the replenishment lead time are discussed below.

4.6 FREQUENCY DISTRIBUTION OF DEMAND DURING REPLENISHMENT LEAD TIME

One of the basic problems in inventory control is to find a frequency distribution that will adequately represent the observed frequency distribution of demand for an item.

As indicated by Burgin⁽¹⁸⁾ in the field of inventory control of finished goods, it was found that the observed frequency distributions of demand have the following general characteristics:

- They exist only for non-negative values of demand
- As the mean demand of items increases, the observed distributions change from
 - monotonic, decreasing to
 - unimodal distributions heavily skewed to the right, and finally to
 - normal type distributions (truncated at zero).

A statistical frequency distribution with these general characteristics therefore has to be found.

A survey of the literature on inventory control shows that, of the continuous frequency distributions, the two that are mostly used are the normal and negative exponential, and the Poisson for the discrete frequency distribution.

The properties of some of the frequency distribution functions and their adequacy to represent the demand during the replenishment lead time are discussed below.

4.6.1 NORMAL DISTRIBUTION

Normal distribution is undoubtedly the most important single probability distribution in decision rules of production planning and inventory management, as well as in general usage, of probability (particularly in the area of applied statistics). The properties of normal distribution, particularly those needed for the decision rules presented in Section 4.6, are discussed below.

◦ Probability density function

The probability density function (pdf) of a normal variable x , with mean x_0 and standard deviation σ_x is denoted by

$$f_x(x_0) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp[-(x_0 - x)^2 / 2\sigma_x^2] \quad -\infty < x_0 < \infty \quad (4.6.1)$$

A typical sketch is shown in Figure 7A. This is the familiar bell-shaped curve. As the standard deviation σ_x decreases, the distribution tightens up around the mean value x .

◦ Moments

The mean $E(x)$ and standard deviation σ_x are also presented by the average \bar{x} and standard deviation STD of the data set:

$$\bar{X} = \sum x_i / n \quad (4.6.2)$$

$$\text{STD} = [\sum (x_i - \bar{x})^2 / n]^{1/2} \quad (4.6.3)$$

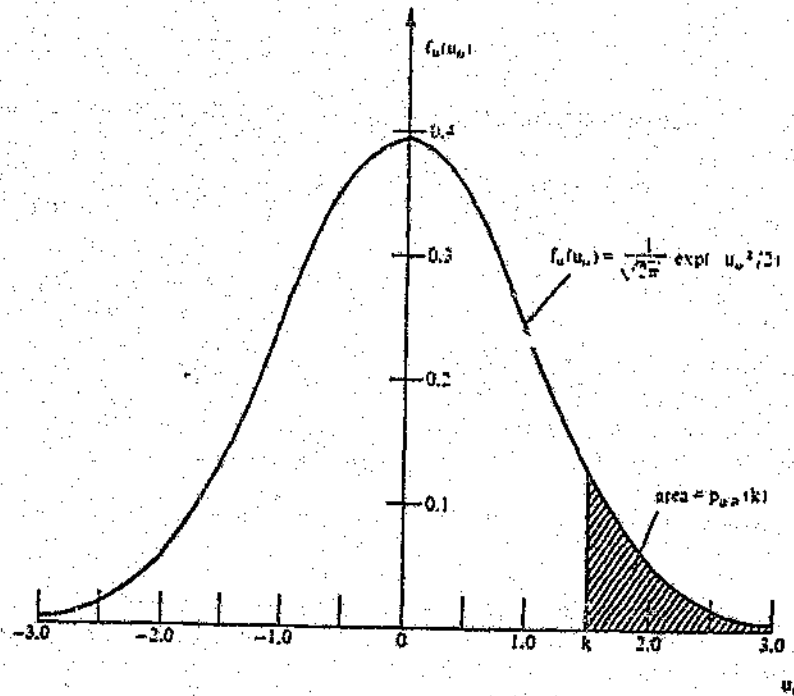
A very special case of normal distribution is the one where the mean value is 0 and the standard deviation is 1. We denote this variable by u :

$$f_u(u_0) = \frac{1}{\sqrt{2\pi}} \exp(-u_0^2 / 2) \quad -\infty < u_0 < \infty \quad (4.6.4)$$

with $E(u)$ or $\mu = 0$ and $\sigma = 1$. This pdf is shown graphically in Figure 7A. A quantity of frequent interest is the probability that u is at least as large as a certain value k :

$$\begin{aligned} p_u \geq (k) &= \text{prob}(u \geq k) = \int_k^{\infty} f_u(u_0) du_0 \\ &= \frac{1}{\sqrt{2\pi}} \int_k^{\infty} \exp(-u^2 / 2) du_0 \end{aligned} \quad (4.6.5)$$

FIGURE 7A
THE UNIT (OR STANDARD) NORMAL DISTRIBUTION



There is no indefinite integral for $\exp(-x^2/2)dx$.

Therefore, Equation 4.6.5 has to be numerically integrated. (The result represents the hatched area in Figure 7A). $p(u \geq k)$ has been tabulated for a range of k values. Only positive values of k are shown in Appendix 21. If $p(u \geq k)$ is needed for a negative argument, it is clear from the symmetry of Figure 7A that

$$p(u \geq -k) = 1 - p(u \geq k)$$

By differentiating both sides of 4.6.5 with respect to k , we obtain

$$dP_u(k)/dk = -f_u(k) \tag{4.6.6}$$

A second quantity of interest relative to production inventory decision rules, denoted by $G_u(k)$ is given by

$$G_u(k) = \int_k^{\infty} (u_0 - k)f_u(u_0)du_0 \tag{4.6.7}$$

Using a special property of the unit normal distribution, namely that

$$\int_k^{\infty} u_0 f_u(u_0)du_0 = f_u(k) \tag{4.6.8}$$

* $p_u \ge (k)$ = Replace by $p(u \geq k)$

* $p_u \ge (-k)$ = Replace by $p(u \geq -k)$

equation 4.6.8 can be expressed as

$$G_u(k) = f_u(k) - kp(u \geq k) \tag{4.6.9}$$

Appendix 21 also shows $G_u(k)$ as a function of k . It can also be shown that

$$G_u(-k) = G_u(k) + k \tag{4.6.10}$$

Differentiating both sides of Equation 4.6.7 with respect to k , and again using the aforementioned theorem, the result is

$$dG_u(k)/dk = -p_u(k) \tag{4.6.11}$$

The disadvantage of the normal distribution with regard to the desired characteristics mentioned above are:

- It is defined for $-\infty \leq x \leq \infty$. The concept of negative demand is unrealistic in the case of inventory control - although a possible interpretation would be customer returns. Truncating the distribution at zero to remove this difficulty greatly increases the mathematical complexity in subsequent work
- Because it is symmetrical, it is only adequate for representing the demand of very fast-moving items.

Silver and Peterson⁽¹⁾ indicate that it is adequate to use the normal distribution to represent demand during the replenishment lead time when the coefficient of variance of the demand during lead time (coef_{L+R}) is low. Empirically they recommend using the normal distribution when $\text{coef}_{L+R} < 0,5$.

4.6.2 POISSON DISTRIBUTION

- Probability mass function

A Poisson variable is an example of a discrete random variable. Its probability mass function given by

$$P_x(x_0) = \text{Prob}(x = X_0) = \frac{a^{x_0}}{x_0!} e^{-a} \quad x_0 = 0, 1, 2 \dots \tag{4.6.12}$$

where a is the single parameter of the distribution. That is the parameter a is the mean of the distribution while the standard deviation is given by the relation

$$\sigma_x = \sqrt{a}$$

Empirically the Poisson distribution has been found to provide a reasonable fit for the distribution of demand for stock items having low average demand rate. For slow-moving items, it is important to be able to deal with discrete units and the discrete nature of the Poisson distribution provides the tool to deal with such items as shown above. The Poisson distribution has but a single parameter, namely the average demand in the context of inventory. Once the average demand during a replenishment lead time (X_{L+R}) is specified, a value of the standard deviation of forecast error (STD_{L+R}), follows from the Poisson relation

$$\text{STD}_{L+R} = \sqrt{X_{L+R}}$$

Therefore, the Poisson distribution is appropriate to use only when the actually observed STD_{L+R} for the item under consideration is quite close to X_{L+R} . An operational definition of quite close is within 10% of X_{L+R} .

• Probability of stockout

The probability (P) of not running out of stock with reorder level s is

$$P = \sum_{j=0}^{j=s} e^{-x_0} \cdot X_0^j / j! \quad j=0,1,2,3 \quad (4.6.13)$$

• Mean potential loss of sales

For reorder level s the potential loss of sales LS is given by

$$LS = \sum_{j=s}^{\infty} (j-s) \cdot P_x(j)$$

multiplying Equation 4.6.12 by j and dividing by X_0 we get

$$\begin{aligned} jP_x(j) &= j e^{-x_0} \cdot X_0^j / j! = \\ &= e^{-x_0} \cdot X_0^j / (j-1)! = \\ &= X_0 \left[e^{-x_0} \cdot X_0^{j-1} / (j-1)! \right] = \\ &= X_0 \cdot P_x(j-1) \\ LS &= \sum_{j=s}^{\infty} jP_x(j) - s \sum_{j=s}^{\infty} P_x(j) = \\ &= \sum_{j=s}^{\infty} X_0 P_x(j-1) - s \sum_{j=s}^{\infty} P_x(j) = \\ &= X_0 \sum_{j=s}^{\infty} P_x(j-1) - s \sum_{j=s}^{\infty} P_x(j) = \\ &= X_0 \sum_{j=s-1}^{\infty} P_x(j) - s \sum_{j=s}^{\infty} P_x(j) = \\ &= X_0 \left(1 - \sum_{j=0}^{s-2} P_x(j) \right) - s \left(1 - \sum_{j=0}^{s-1} P_x(j) \right) \end{aligned} \quad (4.6.14)$$

The author was assisted by Prof D Lubinsky from Applied Maths at the University of the Witwatersrand to develop Equation 4.6.14 above, and Equation 4.6.29 for the Gamma distribution.

4.6.3 COMPOUND POISSON

Feeney and Sherbrooke⁽¹⁰⁾, Ward⁽¹¹⁾ (see below), Mitchell, Rappiid and Faulkner⁽²¹⁾, as well as Silver and Archibald⁽¹²⁾ tested the assumption of compound Poisson distribution of demand during the replenishment lead time. The Compound Poisson distribution is the obvious generalisation of the simple Poisson (in the Poisson case a customer always demands exactly one unit, but in the compound Poisson case multiple demands are possible. The distribution of the time is identical between batches for the compound Poisson and between demands for the simple Poisson).

Ward⁽¹¹⁾ presents a simple, easily used regression model in his paper, derived from the compound Poisson distribution to calculate order points for lumpy items from knowledge of demand parameters and the desired service level.

The compound Poisson is a two-parameter distribution. With a demand arrival rate of μ demands per unit of time, the average number of demands during an interval L is μL . Each demand upon arrival consists of one or more units of the item, the amount being given by the geometric distribution with parameter p . The average batch size of the demand magnitude is $1/(1 - p)$ units. The properties of the resulting compound Poisson distribution of demand over an interval of length L are summarised as follows:

- Mean value: $m = \mu L / (1 - p)$
- Variance: $v = \mu L (1 + p) / (1 - p)^2$
- Coefficient of variation: $C = (1 + p) / \mu L)^{1/2}$
- Ratio of variance to mean: $C^2 m = (1 + p) / (1 - p)$

where: $\mu > 0$, and $0 \leq p < 1$

The probability R_n that n units are demanded in a time interval of length (L) is shown in Equation (4.6.16):

$$R_0 = e^{-\mu L} \tag{4.6.15}$$

$$R_n = (1 - p)^{\mu L} \sum_{i=1}^{i=n} i p^{i-1} R_{n-i} / n \tag{4.6.16}$$

The parameters of the compound Poisson model can be estimated from the mean and variance of the observations and they are shown in Equations 4.6.17 and 4.6.18.

$$p = (C^2m - 1)/(C^2m + 1) \quad (4.6.17)$$

$$\mu = 2m/L(C^2m + 1) \quad (4.6.18)$$

These parameters can be used to produce a Compound Poisson distribution, giving the probabilities of demand occurrences during replenishment lead time L.

In addition to the probability distribution R_n , Ward⁽¹¹⁾ defines several related quantities as follows:

$$K_n = \sum_{j=n+1}^{\infty} R_j = 1 - \sum_{j=0}^{n} R_j \quad n = 0, 1, 2, 3 \dots \quad (4.6.19)$$

K_n = the cumulative probability that the demand during lead time (L) is greater than n.

For example, given a reorder point of n units, the probability of a stockout in each lead time is K_n as determined from the lead time demand distribution. This measure ignores whether the possible shortage is one unit only or many.

An alternate service criterion considers the amount by which available stock may be short of filling requests during the replenishment lead time.

$$S_n = \sum_{j=n+1}^{\infty} (j - n)R_j \quad (4.6.20)$$

The above equation can be rewritten as

$$S_n = m - n(1 - R_0) + \sum_{j=1}^{n-1} (n - j)R_j \quad n = 1, 2, 3, \dots \quad (4.6.21)$$

and $S_0 = m$

where

S_n is the expected value of demand in excess of n, or equivalently, the average amount short each lead time for an order point of n units.

As indicated above by many researchers, it is adequate to use the compound Poisson to describe the frequency distribution of demand when dealing with lumpy items. However, the author could not test its applicability for the current inventory system since the magnitude of individual demand and the frequency were not recorded.

4.6.4 GAMMA DISTRIBUTION

As stated by Burgin⁽¹⁹⁾ the "gamma distribution is not encountered as frequently as one would expect", but its general characteristics are as follows:

- It is defined only for non-negative values.
- It ranges - according to the values of a parameter of the distribution called the modulus - from a monotonic decreasing function, through unimodal distributions skewed to the right, to normal type distributions (in fact, it includes the negative exponential and Weibull distributions as a particular case and tends to normality as the modulus tends to infinity).
- It is easily convolvable.
- The probability integral of the distribution is well tabulated.
- The distribution is generally mathematically tractable in its inventory control applications.

According to Burgin⁽¹⁹⁾ the reasons why it is not being used more widely include the following:

- Unawareness of its existence - it does not hold such a central place in statistical theory as does the normal distribution.
- The mathematical equation of the distribution includes an unfamiliar function in the denominator (this may suggest a highly abstract distribution having little application in reality).
- The properties of the distribution are somewhat scattered in statistical literature.
- Difficulty of calculation, especially in computer-operated systems of inventory control.

However, the author believes that today, with easy accessibility to modern technology, the calculation of the controlling parameters (ROP, ROQ, MOHQ) of any inventory system according to the gamma distribution is far less difficult than it was a few years ago (some software packages offer probability functions that include the gamma distribution).

◦ Definition of the gamma distribution

A continuous variable x which is distributed with probability density

$$f(x) = \alpha^k x^{k-1} e^{-\alpha x} / \Gamma(k) \quad 0 \leq x \leq \infty, \alpha > 0, k > 0, \quad (4.6.22)$$

is said to be a gamma variate and its frequency distribution a gamma distribution. It arises as a k -fold convolution of the negative exponential distribution $\alpha e^{-\alpha x}$.

The term $\Gamma(k)$ in the denominator is the complete gamma function defined as

$$\Gamma(k) = \int_0^{\infty} \alpha^k x^{k-1} e^{-\alpha x} dx$$

ensuring that $\int_0^{\infty} f(x) dx = 1$. If k is an integer then $\Gamma(k) = (k-1)!$

The parameter α of the distribution is a scale parameter, while k is known as the modulus and determines the shape of the distribution.

For $0 < k \leq 1$ the distribution is monotonic decreasing (or J-shaped). For $k = 1$ in particular, the distribution is the familiar negative exponential. For $k > 1$ the distribution is unimodal (with the mode at $x = (k-1)/\alpha$) and skewed to the right. As k approaches ∞ the distribution tends to normality. The shape of the distribution for different values of k is shown in Figure 7B.

◦ Moments of the gamma distribution

The mean μ of the distribution is

$$\mu = \int_0^{\infty} x f(x) dx = \int_0^{\infty} \frac{\alpha^k x^{k-1} e^{-\alpha x}}{\Gamma(k)} dx \quad (4.6.23)$$

$$= \frac{\kappa}{\alpha} \int_0^{\infty} \frac{(\alpha x)^{\kappa-1} e^{-\alpha x}}{\kappa \cdot \Gamma(\kappa)} \alpha dx = \frac{\kappa}{\alpha} \quad (4.6.24)$$

The variance σ^2 of the distribution is similarly obtained giving

$$\sigma^2 = \kappa/\alpha^2 \quad (4.6.25)$$

The fitting of a gamma distribution to observed demand data is as follows.

◦ Computation

The mean μ and the variance σ^2 from the data are determined.

Then from 4.6.24 and 4.6.25 the modulus and scale parameter can be determined with ease:

$$\kappa = \mu^2/\sigma^2, \quad \alpha = \mu/\sigma^2$$

The data will be that of demand per unit of time, e.g. daily, weekly, or monthly. In classical inventory control theory the distribution of demand in a lead time is required.

For a fixed lead time L time units, if the demand/unit time is gamma distributed modulus k , then the distribution of demand in the fixed lead time is gamma distributed modulus Lk . In other words, given the demand/unit time

$$f(x) = \frac{\alpha^k \gamma^{k-1} \cdot e^{-\alpha x}}{\Gamma(k)}$$

then the distribution of demand in fixed L is

$$f(x/L) = \frac{\alpha^{Lk} \gamma^{Lk-1} e^{-\alpha x}}{\Gamma(Lk)} \quad (4.6.26)$$

• Probability of stockout

The possibility of not running out of stock with reorder levels is

$$P = \int_0^s f(x) dx = \int_0^s \frac{\alpha^k \cdot \gamma^{k-1} \cdot e^{-\alpha x}}{\Gamma(k)} \alpha x \quad (4.6.27)$$

• Mean of potential lost sales

For a reorder level s the potential lost sales LS is given by

$$LS = \int_s^\infty (x-s)f(x)dx = \int_s^\infty (x-s) \frac{\alpha^k \cdot \gamma^{k-1} \cdot e^{-\alpha x}}{\Gamma(k)} dx \quad (4.6.28)$$

The author was assisted by Prof D Lubinsky of Applied Maths at the University of the Witwatersrand to develop the loss of sale application for the use in the software package (Excel). Equation 4.6.28 is solved as follows:

Lets $f(x) = F_{\alpha,k}(x)$

Multiplying Equation 4.6.27 by x gives

$$xf(x) = \frac{\alpha^k \cdot \gamma^k \cdot e^{-\alpha x}}{\Gamma(k)}$$

Multiplying the denominator and the numerator by $\Gamma(k+1)$ and by α

$$\begin{aligned} \text{gives} &= \frac{\alpha^{k+1} \cdot \gamma^{k+1} \cdot e^{-\alpha x}}{\Gamma(k+1)} \cdot \frac{\Gamma(k+1)}{\Gamma(k)} \cdot \frac{1}{\alpha} \\ &= F_{\alpha, k+1}(x) \cdot \frac{k}{\alpha} \end{aligned}$$

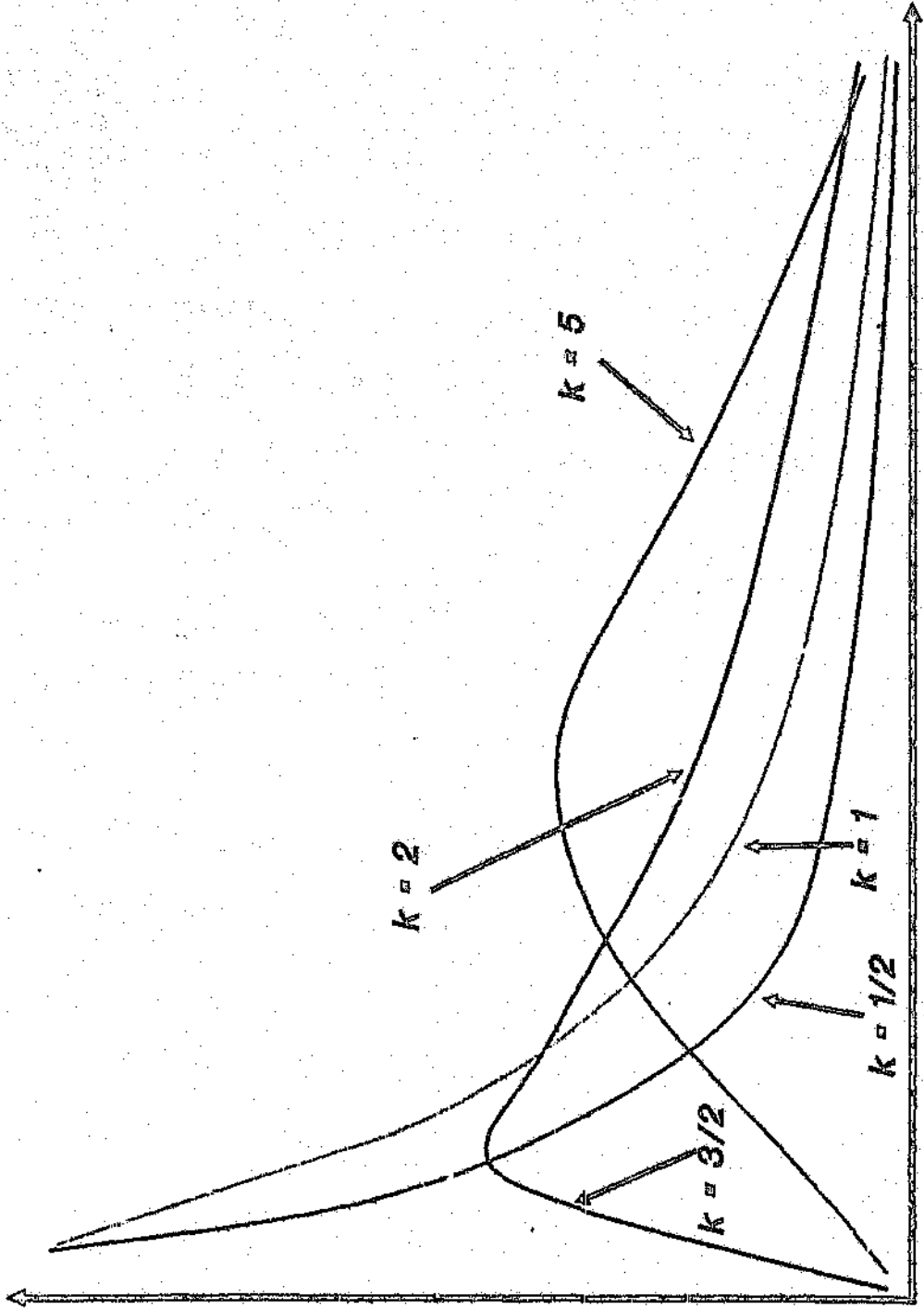
Equation 4.6.28 can now be written as follows:

$$LS = \frac{k}{\alpha} \int_s^\infty F_{\alpha, k+1}(x) dx - s \int_s^\infty F_{\alpha, k}(x) dx \quad (4.6.29)$$

$$= \frac{k}{\alpha} \left(1 - \int_0^s F_{\alpha, k+1}(x) dx \right) - s \left(1 - \int_0^s F_{\alpha, k}(x) dx \right)$$

FIGURE 7B

THE SHAPE OF THE GAMMA DISTRIBUTION
FOR LOW VALUES OF THE MODULUS k



In summary, for demand during replenishment lead time, many researchers have found the gamma distribution appealing and providing empirical support. In the latter case vein Hayya⁽²⁰⁾ found that the gamma distribution provided the best goodness-of-fit model for demand during the replenishment lead time in a sample of 50 spare parts used by the United States Air Force.

4.6.5 OTHER FREQUENCY DISTRIBUTIONS

The above distributions are of course not the only ones mentioned in the field of inventory control. The Log normal is, for example, occasionally referred to, but suffers particularly on account of intractability and convolution difficulty.

The negative exponential distribution is defined only for non-negative values and is monotonic decreasing. It therefore meets the general requirements for presenting the demand of slow-moving items. However, for medium- and fast-moving items it is not a tenable distribution.

Tadikamalla⁽²²⁾ shows that the Weibull distribution can effectively be used to approximate the lead time demand. Several cases of distribution demand coupled with distributed lead time, e.g. when lead time and demand distributions are Gamma and Poisson or Normal and Gamma distributions respectively. Bagchi and Hayya⁽²³⁾ derived an expression for the density function of demand during lead time, when the lead time is distributed Erlang and the demand unit Normal.

4.7 GENERAL APPROACH FOR ESTABLISHING ORDER-UP-TO-LEVEL S IN A PERIODIC-REVIEW (R, S) SYSTEM

Recall that in an (R,S) system, every R units of time a replenishment order is placed of sufficient magnitude to raise the inventory position to the order-up-to-level S:

• Review Interval (R)

In computing the value of S, we predetermined the value of R. The nature of (R, S) control is such that a replenishment order is placed every R units of time. Determination of R is equivalent to determination of an economic order quantity expressed as a time supply. In this case the review period R is two weeks, and the reorder quantity (ROQ) is equal to

$$(R + L_0) D / 50$$

where D = annual demand (units)

L_0 = internal lead time for goods received and data capture (one week).

$$\text{Therefore ROQ} = \frac{3}{50} \times D$$

The key period over which protection is required, is for the duration $R + L$, instead of just a supplier lead time L_0 (recall $L = L_0 + L_1$).

4.7.1 COMMON ASSUMPTIONS AND NOTATION

The assumptions and notation defined by Silver and Peterson¹¹ (292 to 293) include the following:

- 1) Although demand is probabilistic the *average* demand rate changes very little with time.
- 2) There is a negligible chance of no demand between reviews. Consequently a replenishment order is placed at every review (for fast-moving items).
- 3) If two or more replenishment orders for the same item are simultaneously outstanding, they must be received in the same order in which they were placed, i.e. crossing of orders is not permitted. A special case is where the replenishment lead time is a constant.
- 4) Unit shortage costs (explicit or implicit) are so high that a practical operating procedure will always keep the average level of back orders negligibly small compared to the average level of on-hand stock.
- *5) Forecast errors have a normal distribution with no bias (i.e. the average error is zero) and a *known* standard deviation σ_{R+L} for forecasts over a period of duration $R + L$. In fact, there is only an estimated STD_{L+R} .
- 6) The value of R is assumed predetermined.
- 7) The costs of the control system do not depend on the specific value of S used.

$$*G_u(k) = \int_k^{\infty} (u_0 - k) \frac{1}{\sqrt{2\pi}} \exp(-u^2/2) du_0, \text{ a special function of the unit normal (mean 0, standard deviation 1) variable}$$

k = safety factor

L = replenishment lead time, in weeks

* $P(u \geq k)$ = probability that a unit normal (mean 0, standard deviation 1) variable takes on a value of k or larger

D = demand rate, in units/year

r = inventory carrying charge, in $R/R/yr = 0, 23$

R = prespecified review interval = 2 weeks

S = order-up-to-level, in units

SS = safety stock, in units

v = unit variable cost, in $R/unit$

X_{L+R} = forecast (for expected) demand over a review interval plus a replenishment lead time, in units

*Relevant when SKU demand pattern is normally distributed

STD_{R+L} = standard deviation of error of forecasts over a review interval plus a replenishment lead time, in units

◦ Order-up-to-level

The order-up-to-level S or maximum on-hand quantity (MOHQ) is expressed as follows:

$$S = X_{L+R} + (\text{safety stock}) \quad (4.7.1)$$

and

$$\text{safety stock} = k \times STD_{L+R} \quad (4.7.2)$$

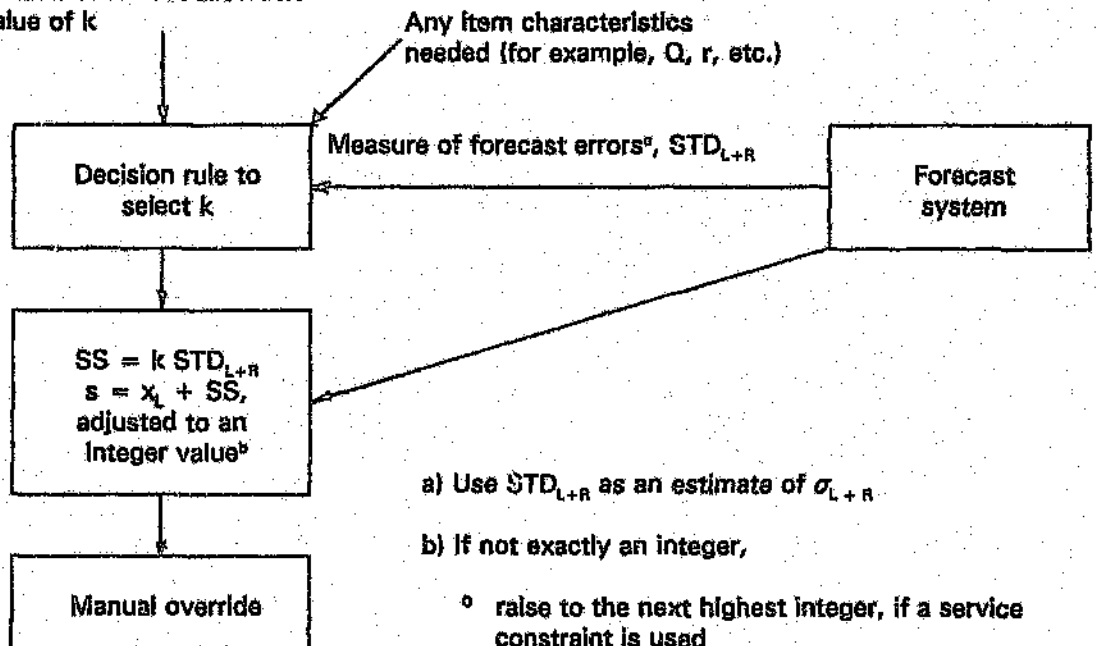
where k is known as the safety factor. Determination of a k value leads directly to a value of S through use of these two relations.

A general logic used in computing the appropriate value of S (via k) as proposed by Silver and Peterson⁽¹⁾ (272), is illustrated in Figure 8. Of particular note is the manual override. The user has the option to adjust the reorder point to reflect factors not included in the model.

FIGURE 8

GENERAL DECISION LOGIC USED IN COMPUTING THE VALUE OF S

Implicit or explicit specification of value of relevant policy variable; also specification of lowest allowable value of k



- a) Use STD_{L+R} as an estimate of σ_{L+R} .
- b) If not exactly an integer,
 - raise to the next highest integer, if a service constraint is used
 - Round to the nearest integer, if a shortage cost is used, except raise to the next highest integer if the lowest allowable k is being used.

4.7.2 SAFETY STOCK, PROBABILITY OF STOCKOUT AND EXPECTED SHORTAGE PER REPLENISHMENT CYCLE

Because of assumption 2 above (section 4.7.1) we have

$$(\text{number of reviews per year}) = 1/R$$

and

$$(\text{number of replenishment orders placed per year}) = 1/R \quad (4.7.3)$$

A stockout can occur in a particular cycle if the total demand in an interval of length $R+L$ exceeds the order-up-to-level S .

If the demand (x) in $R+L$ has a probability density function $f_x(x_0)$ defined so that

$$f_x(x_0)dx_0 = \text{prob} \{ \text{Total demand in } R+L \text{ lies between } x_0 \text{ and } x_0 + dx_0 \}$$

then the above reasoning leads to the following three important results:

- Safety stock (SS) = E (net stock just before an order arrives)

$$= \int_0^{\infty} (s - x_0) f_x(x_0) dx_0$$

that is

$$SS = S - x_{R+L} \quad (4.7.4)$$

- Prob {stockout in a replenishment cycle}

$$= \text{Prob} \{x \geq S\}$$

$$= \int_S^{\infty} f_x(x_0) dx_0 \quad (4.7.5)$$

the probability that the total demand during a review interval plus lead time is at least as extensive as the order-up-to-level.

- Expected Shortage Per Replenishment Cycle, ESPRC

$$ESPRC = \int_S^{\infty} (x_0 - S) f_x(x_0) dx_0 \quad (4.7.6)$$

E (on-hand (OH) stock just before a replenishment arrives)

$$= \text{safety stock SS}$$

$$= S - x_{R+L}$$

Because there is a replenishment every R units of time, the average size of a replenishment is DR . Therefore

$$E \text{ (OH just after a replenishment has arrived)} \cong S - x_{R+L} + DR$$

Now, the mean rate of demand is constant with time. On average, the OH level is halfway between these two extremes. Therefore

$$E(OH) = S - x_{R+L} + DR/2 \quad (4.7.7)$$

It is convenient to set

$$SS = k \times STD_{L+R} \quad (4.7.8)$$

Up to this point the results hold for any probability distribution of lead time demand (or forecast errors). To proceed further one has to specify the particular distribution. It can be shown that if forecast errors are normally distributed, then Equation 4.7.4 and 4.7.5 simplify to

$$\text{Prob \{stockout in a replenishment cycle\}} = P(u \geq k) \quad (4.7.9)$$

$$\text{and ESPRC} = G_u(k) STD_{L+R} \quad (4.7.10)$$

The safety factor derivations are diverse based on the particular shortage cost or service level used (see the various methods to set safety stocks in Section 4.8).

4.8 CRITERIA FOR ESTABLISHING SAFETY STOCKS

The most important parameter used to establish the re-order point s within any inventory model, is the determination of the safety stock level. There are five possible methods to set safety stock levels, suggested by Silver and Peterson⁽¹⁾ (263):

4.8.1 SAFETY STOCKS ESTABLISHED THROUGH THE USE OF A COMMON FACTOR

The following are the two most frequently used factors:

◦ Equal time supplies

The safety stock of a broad group of (if not all) items in an inventory population is set to be equal to the time supply. For example, re-order any item when its inventory position minus the forecast lead time demand drops to a two-month supply or lower. This approach is seriously in error because it fails to take account of the difference in uncertainty of forecasts from item to item. The policy variable here is the common number of time periods of supply.

◦ Equal safety factors

The safety stock SS is defined as the product of two factors, as follows:

$$SS = k \times STD_{L+R} \quad (4.8.1)$$

$$ROP = X_L + SS \quad (4.8.2)$$

where

ROP = re-order point

k = safety factor

STD_{L+R} = standard deviation of the replenishment lead time $L + R$

X_L = the demand during supplier lead time

The *equal safety factors* approach uses a common value k for a broad range of items.

4.8.2 SAFETY STOCK BASED ON THE COSTING OF SHORTAGES

This approach involves specifying (explicitly or implicitly) a way of costing a shortage once certain characteristics of the shortage are known. The safety stock of an item is then established to keep the expected overall shortage and carrying costs as low as possible.

Silver and Peterson⁽¹⁾ (277 to 280) present three illustrative cases:

4.8.2.1 SPECIFIED FIXED COST B_s PER STOCKOUT OCCASION

Here, it is assumed that the only cost associated with a stockout occasion is a fixed value B_s , independent of the magnitude or duration of the stockout. One possible interpretation would be the cost of expediting an action to avert an impending stockout.

◦ The rule

Step 1: Calculate the value T

$$T = \frac{DB_s}{\sqrt{2\pi} Q \cdot v \cdot \text{STD}_{L+R} \cdot r} \quad (4.8.3)$$

where

The re-order quantity Q is predetermined and equal to three weeks' supply $3D/50$ while the other variables are as defined previously.

Step 2: If $T < 1$ then $k = 1,00$ where $1,00$ is the lowest allowable value of k .

$$\text{If } T > 1 \text{ then } k = 2\sqrt{\ln(T)} \quad (4.8.4)$$

Step 3: Re-order point (ROP) = $X_L + K \text{STD}_{L+R}$

◦ Discussion

It is seen from Equation 4.8.3 that k decreases as STD_{L+R} or v goes up. Intuitively the behaviour of k with v makes sense under the assumed stockout costing mechanism. If there is only a fixed cost *per stockout occasion which is the same for all items*, it makes sense to allocate a greater proportional safety stock to the less expensive items where an adequate level of protection is achieved with relatively little investment. Furthermore it can be shown that k decreases as Dv increases. This means that higher safety factors are provided to the slower-moving items.

The appearance of Q in the decision rule for finding the safety factor k indicates that a change in Q will affect the required value of the safety stock. For B items this turns out to be a reasonable approximation.

There is no solution to Equation 4.8.4 when $T < 1$. In such a situation, the best solution is to set the lowest allowable value for k .

4.8.2.2 SPECIFIED FRACTIONAL CHARGE B_2 PER UNIT SHORT

Here it is assumed that a fraction B_2 of unit value is charged per unit short, i.e. the cost per unit short of item i is $B_2 v_i$, the unit variable cost of the item.

◦ The rule

Step 1

$$\text{Is } \frac{Qr}{DB_2} > 1 \quad (4.8.5)$$

Again Q has been predetermined and is equal to three weeks supply $3D/50$.

The equality could be written as

$$(3Dr/50 DB_2) > 1$$

Or

$$(3r/50B_2) > 1 \quad (4.8.6)$$

If yes, go to step 2.

If no, continue as follows. Select k to satisfy

$$P(u \geq k) = 3r/50B_2 \quad (4.8.7)$$

If the use of Equation 4.8.7 gives a k value of lower than the minimum allowable safety factor specified, then go to Step 2. Otherwise, move to Step 3.

Step 2: Set k at its lowest allowable value, $k = 1,00$

Step 3: Re-order point

$$ROP = \bar{X}_L + k \text{ STD}_{L+R}$$

◦ Discussion

As could be expected, safety factor k increases as B_2/r increases. Moreover, with the use of economic order quantities, it is found that larger safety factors are given to the faster-moving items, all other things being equal.

The $p(u \geq k)$ value represents a probability. Hence there is no solution to Equation 4.8.7 when the right-hand side of the equation exceeds one, i.e. when Equation 4.8.6 is met. Therefore, the lowest permissible value of the safety factor ($k = 1,00$) should be used.

4.8.2.3 SPECIFIED FRACTIONAL CHARGE B_3 PER UNIT SHORT PER UNIT TIME

The assumption here is that there is a charge B_3 rand per unit short per unit time. An example would be when the item under consideration was a spare part and each unit short results in a truck being idle for the duration of the shortage.

• The Rule

Step 1: Select the safety factor k that meets

$$G_u(k) = \frac{Q}{STD_{L+R}} (B_s + r) \tag{4.8.8}$$

where Q has been predetermined, and equal to $3D/50$,

and B_s and r are in R/R unit time.

Set $Q = 3D/50$, e.g. 4.8.8 can be written as follows:

$$G_u(k) = \frac{3D}{50} \frac{r}{STD_{L+R} B_s + r} \tag{4.8.9}$$

Step 3: Re-order point

$$ROP = X_L + k STD_{L+R}$$

4.8.3 SAFETY STOCKS BASED ON SERVICE CONSIDERATIONS

Recognising the severe difficulties associated with costing shortages, an alternative approach is to introduce a control parameter known as the *service level*. This becomes a constraint in establishing the safety stock of an item. For example, the carrying costs of an item could be minimised, subject to satisfying, routinely from stock, 95 per cent of all demands.

The following are among the more common measures of service.

4.8.3.1 SPECIFIED PROBABILITY P_1 OF NO STOCKOUT PER REPLENISHMENT CYCLE

Equivalently, this is the fraction of cycles in which a stockout does not occur. A stockout is defined as an occasion when the on-hand stock drops to the zero level. Using a common P_1 , across a group of items is equivalent to using a common safety factor k .

If management has specified the probability of no stockout in a cycle not to be lower than P_1 (conversely, the probability of a stockout should be no higher than $1 - P_1$), then we have the following simple decision rule:

• The rule

Step 1: Select the safety factor k to meet

$$P(u \geq k) = 1 - P_1 \tag{4.8.10}$$

where

$$P(u \geq k) = \text{Prob \{unit normal variable (mean 0, standard deviation 1) takes on a value of } k \text{ or greater\}}$$

GENERAL NOTE: Probability function $P_u(k)$ is the probability that variable u takes on the values k . This can also be written as $P(u = k)$. However, I would suggest $P(u \geq k)$ is clearer than $P_{u \geq}(k)$.

◦ The Rule

Step 1: Select the safety factor k that meets

$$G_u(k) = Qr / \text{STD}_{L+R} (B_s + r) \tag{4.8.8}$$

where Q has been predetermined, and equal to $3D/50$,

and B_s and r are in R/R unit time.

Set $Q = 3D/50$, e.g. 4.8.8 can be written as follows:

$$G_u(k) = \frac{3D}{50} \frac{r}{\text{STD}_{L+R} (B_s + r)} \tag{4.8.9}$$

Step 3: Re-order point

$$\text{ROP} = X_L + k \text{STD}_{L+R}$$

4.8.3 SAFETY STOCKS BASED ON SERVICE CONSIDERATIONS

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◦ The rule

Step 1: Select the safety factor k subject

$$P(u \geq k) = 1 - P_1 \tag{4.8.10}$$

where

$$P(u \geq k) = \text{Prob \{unit normal variable (mean 0, standard deviation 1) takes on a value of } k \text{ or greater\}}$$

GENERAL NOTE: Probability function $P_u(k)$ is the probability that variable u takes on the values k . This can also be written as $P(u = k)$. However, I would suggest $P(u \geq k)$ is clear than $P_{us}(k)$.

Step 2: Safety stock $SS' = k \text{ STD}_{L+R}$

Step 3: Re order point

$$RQP = X_L + SS$$

• Discussion

From Equation 4.8.10 it is seen that the safety factor k depends on the value of P_1 . All items for which the same service level, P_1 , is required, will have identical values of the safety factor k . There is therefore an equivalence between using a specified value of k and a specified value of P_1 .

From Equation 4.8.10 $p(u \geq k)$ decreases as the desired service level P_1 goes up, whereas k increases as $p(u \geq k)$ decreases. Therefore, we have the desirable behaviour of the safety factor namely k increases as the required service level increases, which is ideal.

Because of the discrete nature of the re-order point, it is unlikely that the exact level of service desired can be provided since this usually requires a non-integer value of ROP. Therefore a non-integer value of ROP found in Equation 4.8.10 is rounded up to the next higher integer, with the predicted service level then being slightly higher than required.

The fact that the safety factor does not depend on any individual item characteristics may require a re-examination of the meaning of service. Consider two items, the first being replenished twenty times a year, the other once a year. If both are given the same safety factor so that both have a probability of 0,10 of stockout per replenishment cycle, then $20 \times (0,10)$ or two stockouts per year could be expected for the first item and only 1 stockout every ten years (0,1 per year) for the second item. Therefore the same service on these two items would in fact not be the same.

4.8.3.2 SPECIFIED FRACTION P_2 OF DEMAND TO BE SATISFIED ROUTINELY FROM THE SHELF (I.E. NOT LOST OR BACK-ORDERED)

This service measure can be used when a significant portion of the replenishment lead time is unalterable (for example, dealing with items for which the major part of the lead time is the transit time by rail). A generalisation is that the fraction P_2 must be met within a specified time interval rather than instantaneously. It can also be shown that use of the B_2 shortage costing measure leads to a decision rule equivalent to that for the P_2 service measure, where the equivalence is given by the relation

$$P_2 = \frac{B_2}{B_2 + r}$$

with r , as before, being the carrying charge.

• The rule

Step 1: Select the safety factor k that meets

$$G_u(k) = \frac{Q}{\text{STD}_{L+R}} (1 - P_2) \quad (4.8.11)$$

where Q has been predetermined and is equal to $3D/50$.

The other relevant variables are defined in Section 4.6.1. If k is less than the lowest allowable value, then $k = 1,00$.

◦ **Step 2: Re-order point**

$$ROP = X_L + k \text{ STD}_{L+R}$$

◦ **Discussion**

It would be reasonable to expect that the required safety stock would increase if

Q decreased (more opportunities for stockouts)

STD_{L+R} increased (higher uncertainty of forecasts)

P_2 increased (better service desired). If any of the above-mentioned changes take place, $G_u(k)$ decreases, but a decrease in $G_u(k)$ implies an increase in k , i.e. exactly the desired result.

In addition, with this decision rule the faster-moving items get higher safety factors than the slower-moving items.

The P_2 value is usually quite close to unity. Therefore the value of the safety factor is little influenced by whether the model assumes complete back-ordering or complete lost sales (or any mix of these two extremes).

In the case of completely lost sales, Equation 4.7.11 is written as

$$G_u(k) = Q(1 - P_2)/\text{STD}_{L+R} P_2 \quad (4.8.11)$$

4.8.3.3 SPECIFIED READY RATE P_3

The ready rate is the fraction of time during which the net stock is positive, i.e. there is some stock on the shelf. The ready rate finds common application in the case of equipment used for emergency purposes. Under Poisson demand, this measure is equivalent to the P_2 measure.

4.8.3.4 SPECIFIED AVERAGE TIME BETWEEN STOCKOUT OCCASIONS (TBS)

TBS represents the desired average number of stockout occasions per year. If each stockout occasion is dealt with by way of an expediting action, a specific TBS value can be selected to produce a tolerable number of expediting actions.

◦ **The rule**

Step 1: Is

$$Q/D(\text{TBS}) > 1 \quad (4.8.12)$$

where Q has been predetermined and is equal to $3D/50$.

If yes, go to Step 2.

If no, continue with the following. Select the safety factor k to meet

$$P(u \geq k) = Q/D(TBS) \quad (4.8.13)$$

When using $Q = 3D/50$ Equation 4.8.13 can be written as

$$P(u \geq k) = 3/50(TBS) \quad (4.8.14)$$

If the resulting k is lower than the minimum allowable value specified ($k = 1.00$) go to Step 2. Otherwise, move to Step 3.

Step 2: Set k at its lowest allowable value

Step 3: Re-order point

$$ROP = X_L + k STD_{L+R}$$

◦ Discussion

A comparison carried out by Silver and Peterson⁽¹⁾ (287) of Equations 4.8.7 and 4.8.13 reveals that there is an equivalence between using the B_2 costing method and using the TBS service measure.

Specifically, we have, for equivalence, that

$$TBS = B_2/r$$

4.8.4 SAFETY STOCKS BASED ON AGGREGATE CONSIDERATION

The idea of this general approach is to establish the safety stocks of individual items, using a given available budget, to provide the best possible *aggregate* service across a population of items. Equivalently, individual safety stocks are selected to keep the overall investment in stocks as low as possible while meeting a desired aggregate service level.

Two common aggregate considerations:

- Allocation of a given total safety stock among items to minimize the expected total stockout occasions per year (ETSOPY)

Allocating a fixed total safety stock among several items to minimize the expected total number of stockout occasions per year leads to a decision rule for selecting the safety factor for each item, which is identical to that obtained by assuming a value (the same for all items) of the fixed cost B_1 and then selecting the safety factor to keep the total of carrying and stockout costs as low as possible. (The latter approach was discussed in Section 4.7.2.) This allocation interpretation in some cases appeal more to management.

- Allocation of a given total safety stock among items to minimize the expected total value of shortages per year

Allocating a fixed total safety stock among a group of items to minimise the expected total value (in rands) of shortages per year leads to a decision rule for selecting the safety factor which is identical to either

- the one obtained by assuming a value (the same for all items) of the factor B_s and then selecting the safety factor for each item to minimise the total of carrying and shortage costs, or
- the one obtained by specifying the same average time (TBS) between stockout occasions for every item in the group.

The aggregate view of allocating a limited resource may be considerably more appealing to management than the micro-detail of attempting to explicitly ascertain a value from cost considerations or the somewhat arbitrary specification of a TBS value.

4.8.5 TIME INCREMENT CONTINGENCY FACTOR (TICF)

A significant phenomenon becomes apparent when some of the classic statistical safety stock approaches described above are reviewed. The calculated safety stock is a fixed quantity applied to all future periods. When forecasts of future demand tend upward or downward, the fixed quantity does not react to this change. On the upside, therefore, the safety stock does not increase in proportion to the demand, thus providing the potential of inadequate coverage of demand variability. On the downside, failure to react can create exposure to excess inventory, especially in the case of a product approaching the end of its product life cycle.

To counteract these potential problems, Krupp²⁴⁾ expresses statistical variance of demand in units of time rather than quantity by converting the deviation to a decimal factor of the incremental forecast per period. Krupp²⁴⁾ distinguishes this approach from classical applications, by designating the resulting technique the time increment contingency factor (TICF). The TICF is derived by incorporating the additional step of dividing each deviation for a given period by the average usage (or forecast) for that period. Expressing the concept mathematically, the equation is

$$TICF_n = \sum_{i=1}^n \left| \frac{u_i - x_i}{u_i} \right| / n = \sum_{i=1}^n \left| 1 - \frac{x_i}{u_i} \right| / n \quad (4.8.15)$$

The TICF is then used to determine the safety stock to be planned for each month, based on future forecasts:

$$SAFETY STOCK_t = k \times (TICF_n \times u_{t+1}) \quad (4.8.16)$$

where

- u_i = average demand or forecast in period i
- x_i = actual demand in period i
- $| |$ = absolute value
- n = total number of occurrences considered.

The calculations are based on a 12-month rolling history, with n fixed at 12. u_{t+1} is used to ensure that the safety stock existing at the end of u_t is planned to support the following month's forecast.

Therefore, as future forecast demand levels change, the planned safety stock will change proportionately.

Krupp²⁴ demonstrated in his paper that the TICF concept provides a better tool for planned safety stock in view of the fact that it is self-adjusting in relation to future trends, yet retains all the ingredients of classic statistical safety stock techniques.

4.8.5.1 FORECAST ERROR TRACKING SIGNAL

One of the basic assumptions of statistical techniques which has been criticised occasionally is that the population of demands centres on the mean average of the demands. In a forecast environment, however, the forecast demand is often not consistent with the mean average, particularly where trend is involved. The bias of the forecast from the actual mean is expressed mathematically by using the forecast error tracking signal (FETS).

The FETS for the TICF would be

$$FETS_n = \frac{\sum_{i=1}^n |1 - \frac{x_i}{u_i}|}{n} \quad (4.8.16)$$

Interpretation of the FETS:

- FETS = 0 optimum - total population of deviations centred around the forecast
- FETS < 0 indicates a skew where actual demands are greater than forecast
- when FETS = -1 indicates that in all cases where a deviation exists, actual demand is greater than forecast
- FETS > 0 indicates a skew where actual demands are fewer than forecast
- FETS = 1 indicates that in all deviations forecasts are greater than actual demand.

4.8.5.2 COMPENSATING FOR OVEROPTIMISTIC FORECASTING

When FETS > 0, it may be desirable to suppress the level of safety stock which the applied statistical technique would yield. If the FETS is less than 1.0, an indication exists that some portion of the actual demands exceeds the forecast. To abandon safety stock totally is therefore inappropriate. One alternative is to consider only those instances where demand exceeds forecast for the calculation. While this is a potentially viable alternative, this course of action carries the disadvantage of reducing the total population of variances on which the calculation is based and assigns disproportionate importance to the reduced population. A second heuristic alternative is a suppression factor which comes into play only when the FETS is greater than zero. This is used as a multiplier for the calculated safety stock. One such suppression factor provides for a straight-line reduction of safety stock

$$w = 1 - FETS \quad (FETS > 0)$$

while a second model provides accelerated suppression

$$w = 1 - \sqrt{FETS} \quad (FETS > 0)$$

Figure 9
Straight Line Versus Accelerated
Suppression Factors

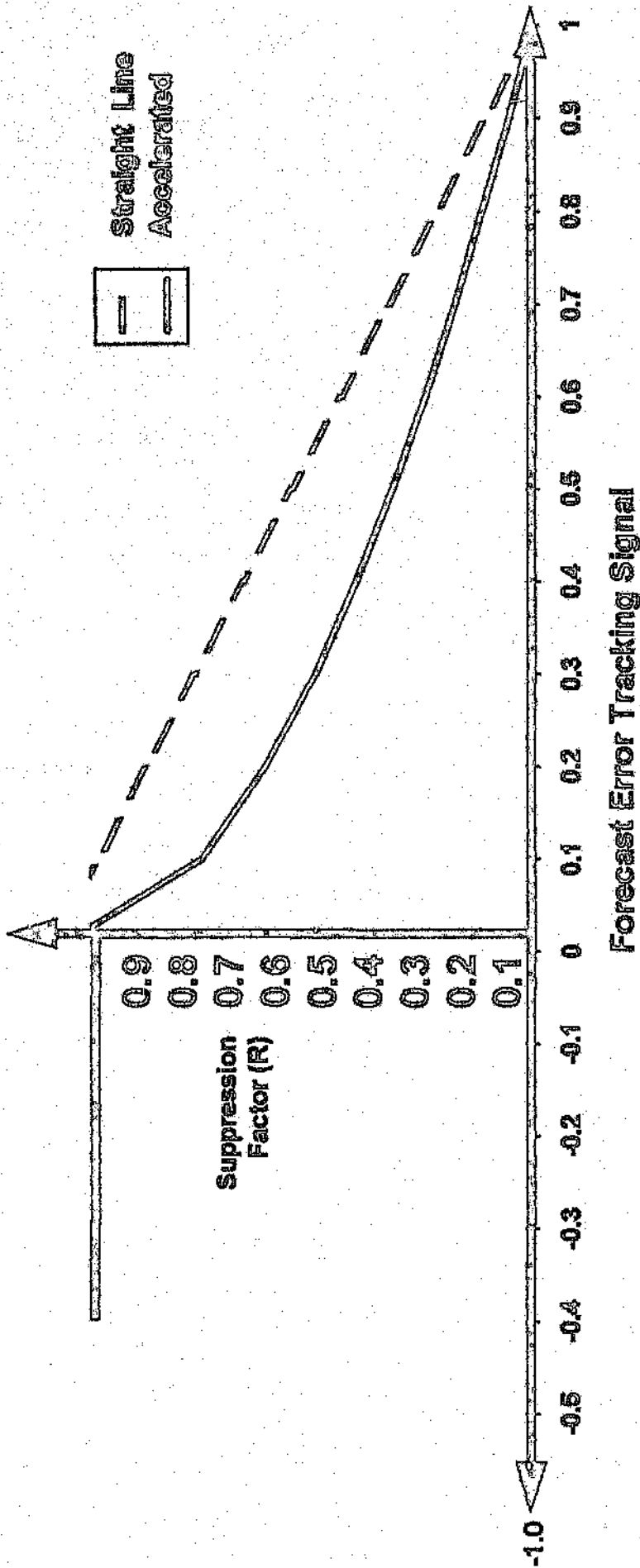


Figure 9 is a graphic presentation of the impact of straight-line versus accelerated suppression. The suppression factor is used as a compensating factor in safety stock calculations for overoptimistic forecasting, as in the equation

$$SS_t = w \times k \times (TICF_n \times V_{t+1}) \quad (4.8.17)$$

4.8.5.3 DETERMINING THE SERVICE MULTIPLIER k

The service multiplier k was referred to above. In some cases, particularly where high-value inventory is involved, the use of such an arbitrary heuristic approach may be deemed inappropriate. Optimisation of k based on financial return and derived by more scientific methods often presents a better alternative.

This approach involves the application of marginal return analysis to determine the optimum k factor.

The mathematical model describing the relationship between the k factor and recouped lost sales is non-linear, i.e. elasticity changes along the curve.

A graphic example of a mathematical model relating lost sales to the applied k factor is presented in Figure 10A: The curve follows a non-linear trend typical of the relationship:

$$\text{Lost sales recouped} = ak - bk^2$$

where a, b are constants, $a > b$

while k is defined as a linear function. The point at which the two curves cross defines the level at which the safety stock investment equals the loss of sales recouped.

The objective is to consider *bottom-line* return. Krupp relates the recouped lost *profit* to the *cost of carrying* the safety stock level relative to the applied k factor. This analysis defines the true cost/benefit return which can be expected from carrying safety stock inventory. A graphic representation of this new relationship is presented in Figure 10B. The data lines are described by the generalised mathematical relationships

$$\text{Lost profit recouped} = p(ak - bk^2)$$

where p = profit margin (expressed as a decimal)

and

$$\text{cost of carrying inventory} = r \times \text{safety stock (in rands)}$$

where r = inventory-carrying cost factor

With the relationship between recouped lost profit and cost of carrying safety stock defined, an optimum k factor for each class of items can be determined on the basis strategic general management considerations. If the purpose of the safety stock is to maximise service without incurring financial penalties, k would be defined by the break-even point, i.e. the point at which recouped profit equals the cost of carrying the safety stock level required. If the purpose is to maximise profit (without consideration to service), then k would be defined by the point (prior to break-even) at which the maximum differential exists between the two curves.

FIGURE 10A
SAFETY STOCK INVENTORY VERSUS
RECOUPED LOST OF SALES

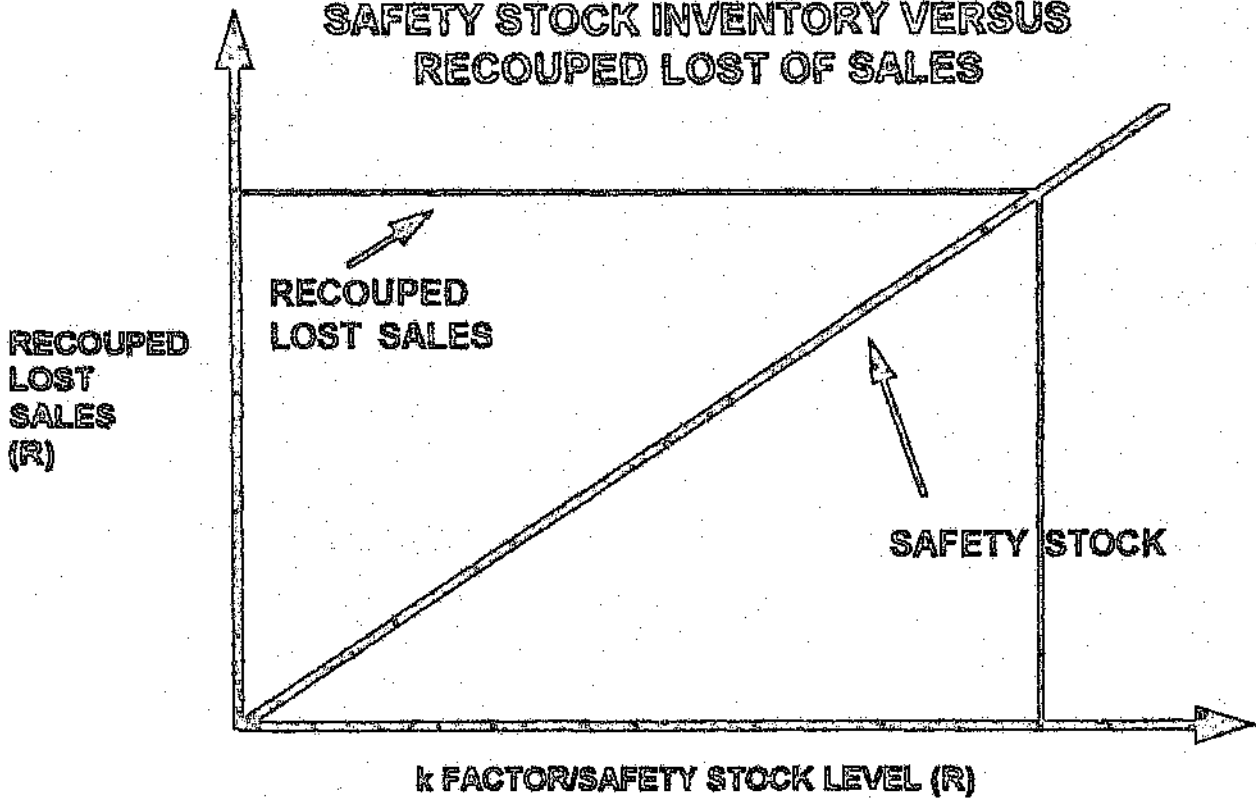
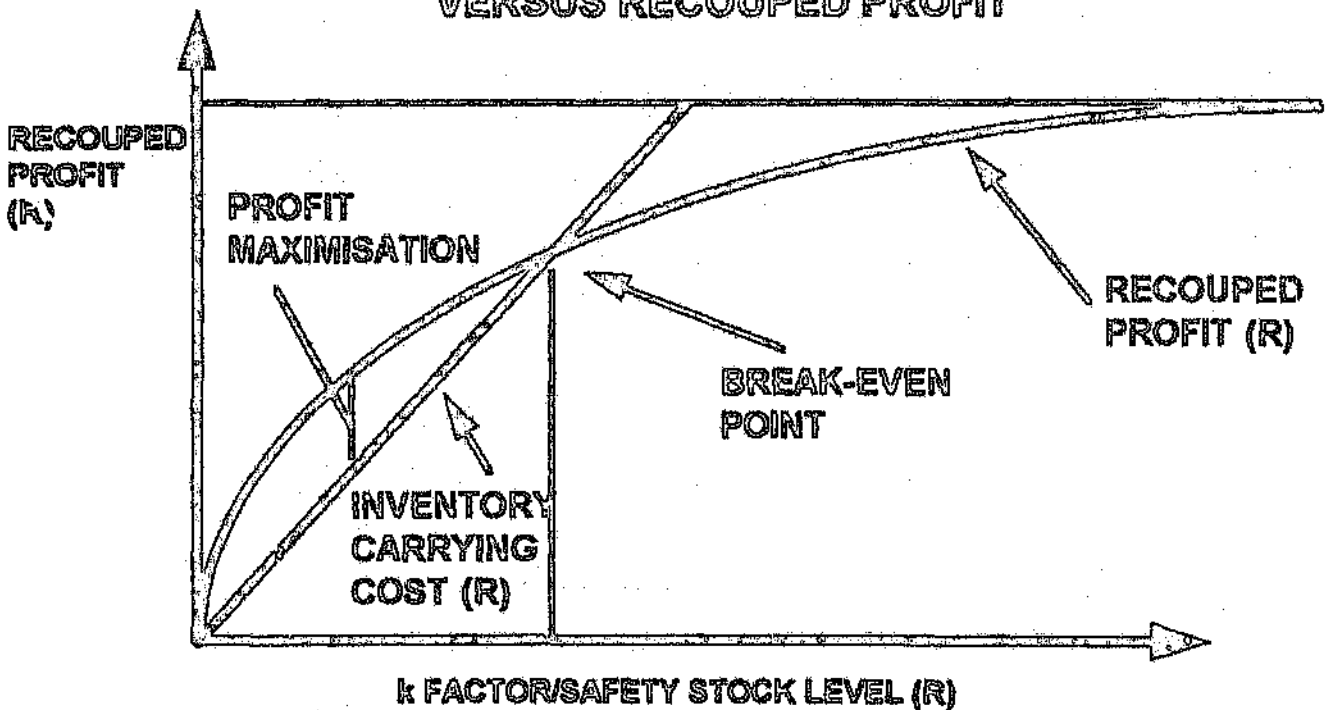


FIGURE 10B
COST OF CARRYING SAFETY STOCK INVENTORY
VERSUS RECOUPED PROFIT



4.8.6 SAFETY STOCK CONCLUSIONS

Safety stock is necessary evil in many environments. The safety stock of the company investigated was no exception. However, the creative approaches discussed above can minimise the exposure to inventory imbalances which will contribute to the bottom line.

In Section 6.2 exchange curves of safety stock are provided for some of the rules discussed above. These curves will facilitate decisions on the rules to be selected to determine safety stock levels.

4.9 DISCUSSION OF WHETHER TO STOCK AN ITEM OR NOT

One of the major problems concerned with slow-moving items, particularly when the item is a spare part, is the decision on whether or not to stock the spare part. Shorrock⁽¹⁴⁾ indicates that this aspect of stock control is the most neglected one.

In the inventory under investigation, 8 789 stock-keeping units (44% of total stock) account for R8,2 million in value (25% of total stock value) had shown no movement for at least 12 months.

The risk of adding an item to stock that might be declared obsolete or that might become a very slow-mover at the current carrying cost of 0,23 R/R/year, should be investigated very carefully.

There are essentially two points to be considered:

- o Whether or not the item should be stocked to back up a particular maintenance activity or service to fleet owners
- o Whether or not it would be cheaper to make an economic quantity and to hold it in stock rather than order directly against each requirement.

Some papers on the subject are discussed below.

According to Shorrock⁽¹⁴⁾ the worth of stocking depends essentially on how many requirements or calls per annum are made for the item. If more than a critical number of calls are received, it will be cheaper to stock the component.

Shorrock⁽¹⁴⁾ shows that the critical number of calls for an item is given by the formula

$$\text{Critical number of calls per annum} = \frac{Dvr}{2A}$$

where:

D - annual demand

V - unit cost

r - carrying charges percentage

A - order cost.

A part is worth stocking if the number of calls for it per year exceeds the critical number of calls.

Shorrock⁽¹²⁴⁾ provides tables to simplify the calculation when using a manual system. The table and example of how to use them are presented in Appendix 20.

Mitchell⁽¹²⁵⁾, in his studies on engineering spares stocks, particularly high-value slow-moving parts, relates his decision to a cost function, aiming to find the value of S (stock level) that minimises the cost function. Mitchell developed a diagrammatic method to find the optimum S. The curves $CO = C1$ and $C1 = C2$ (Figure 11) are illustrated in a graph of B/v (shortage cost/unit cost) against T (average time between demands). For parts whose values of B/v and T correspond to a point below $CO = C1$ (line-H of Figure 11), no spares should be held. For those lying between H and the curve $C1 = C2$, corresponding with the appropriate average lead time, one spare should be held. Holding three slow-moving spares has not been recommended in any situation.

Bartakke⁽¹²⁶⁾ offers a rule to draw management's attention when they have to decide whether to stock or not to stock:

$$X_L < 5 \text{ AND } v > R500$$

where

X_L = average demand during lead time

v = unit price.

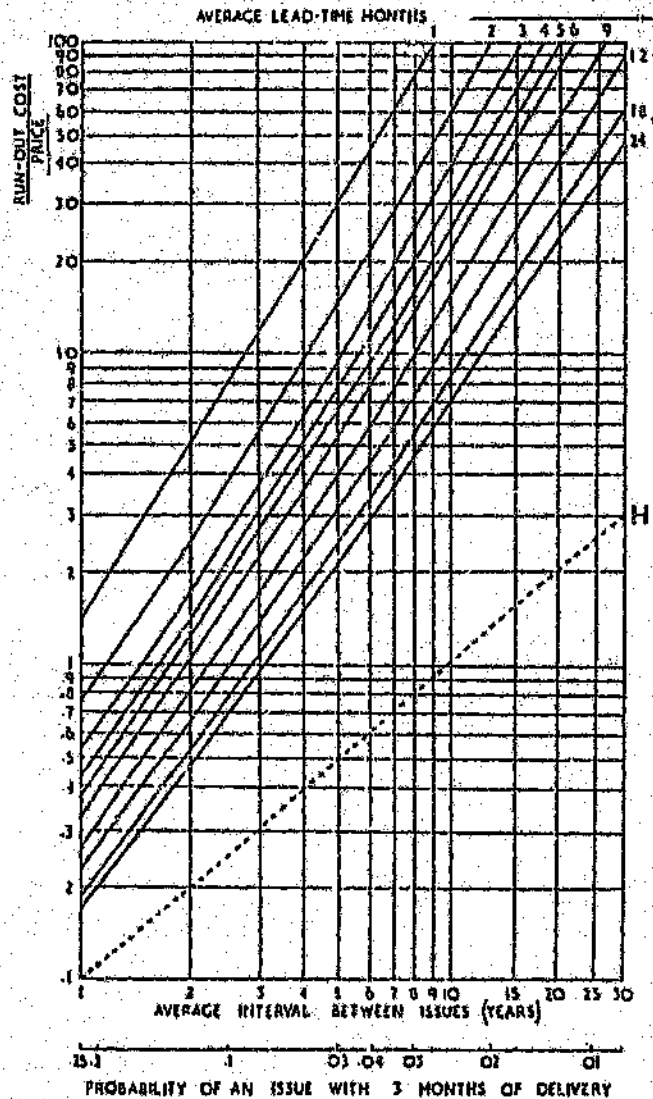
Silver and Peterson⁽¹¹⁾ (330 to 335) developed indifference curves for the re-order point under the B_2 cost measures (fractional change B_2 per unit short). In the special case of very slow-moving expensive items, it is decided not to stock when the proposed re-order point $s = 0$. (See detailed discussion of the model in Section 5.3.2 under procedure 1.)

A simple rule based on work by Popp⁽¹²⁸⁾, is presented below. This rule is valid under the following assumptions:

- The unit variable cost is the same for stocking and nonstocking
- The fixed set-up cost is the same for stocking and nonstocking
- In deriving the decision rule to decide on whether to stock the item or not, the order quantity is allowed to be a non-integer (of course, if the item was actually stocked and demands were in integer units, an integer value would be used for the order quantity)
- The replenishment lead time is negligible and consequently there is no back-ordering cost.

Some of the above assumptions are very restrictive, especially the last one. However the author believes that even when these assumptions are violated, the rule should still be useful as a guideline.

FIGURE 11
CHART FOR ON DECIDING STOCK POLICY FOR SPARE PARTS



o The decision rule

Do *not* stock the item if *either* of the following two conditions holds (otherwise stock the item):

$$c_s > A/E(i) \quad (4.9.1)$$

or

$$E(t)vr = (E(i)/2A) [A/E(i) - C_s]^2 \quad (4.9.2)$$

where

c_s = system cost in rands per unit time, of stocking the item

A = fixed set-up cost, in rands, associated with a replenishment

$E(i)$ = expected (or average) interval (or time) between demand transactions

$E(t)$ = expected (or average) size of a demand transaction in units

v = unit variable cost of the item, in R/unit

r = carrying charge, in R/R/unit time.

An example and a graphical representation of the above decision rule are presented in Appendix 22.

Croston⁽²⁹⁾ has developed a decision rule, including a graphical aid, for the case of a periodic-review, order-up-to-level (R, S) system. His model assumes a negligible replenishment lead time, at most one demand transaction in each review interval, and a normal distribution of transaction sizes. His paper reveals that it is always uneconomical to keep stock of less than the average demand size for any spare parts.

o In conclusion

Several models for dealing with stocking/non-stocking decision rules have been discussed. It is obvious that there are no hard and fast rules about whether to stock an item or not. Moreover, several other factors that have a bearing on the decision are not taken into account in the above models. Their applicability is therefore limited. A more elaborate manual method with relevant factors that influence the decision on whether to stock (and the extent of the inventory) or not to stock, is discussed in Section 5.3.6.

5. DEVELOPMENT AND SELECTION OF MODELS USED IN THE PROPOSED INVENTORY CONTROL SYSTEM

5.1 GUIDING PRINCIPLES

A problem diagnosis was presented in Section 2. In Section 3 important factors inventory decisions were discussed, with special attention to replenishment lead time reduction. Section 4 presented a wide range of selected inventory models from the literature.

The strategy employed in the selection and development of models to be used in the proposed inventory control system was directed by the following guiding principles:

- A mathematical model permits the selection of values in a limited set of variables so that some reasonable measure of effectiveness can be attained.
- The models used can be fitted into the environment in which the company operates. The selected/developed models to aid decision-making incorporate most of the important factors.
- The measures of effectiveness used in the models are consistent with company objectives.
- The required managerial time for maintenance of the models and their use for the decision-making, suit the current company staff complement (buyers and stock controllers). The system control cost is therefore not increased.
- Most of the proposed models make use of analytical decision rules and can therefore be implemented by using formulas, tables and graphs.

5.2 APPROPRIATE INVENTORY CONTROL SYSTEM

Recall from Section 4.4 that the inventory control system which was selected is the (R, S) type. The guiding principles in this selection is the system's applicability to the company's type of operation, and other benefits such as lower costs due to better coordination of transport mode and staff utilisation (thus reducing ordering cost). There are other non-quantifiable benefits such as improved communication with suppliers. The main disadvantage of this system is the higher carrying charges compared to continuous review systems. However, these had been minimised by introducing a reduced review period of two weeks instead of four weeks as before. (See the detailed discussion on this issue in Section 4.4.)

5.3 PROPOSED INVENTORY CONTROL SYSTEM

The proposed inventory control system described in this section is a hybrid, i.e. a mathematical and manual procedure to establish the inventory control parameters. The properties of the proposed inventory control system are presented below.

5.3.1 MAIN PROCEDURE - SKUs DEMAND PATTERN CLASSIFICATION

Some of the methods presented in Section 4.5 to classify SKUs are used in the main procedure below. Some of the classification methods had been rejected mainly because of a physical constraint, namely the way the current system captures transactions. This method differs from the data entry required by these models. For this reason other criteria that have been tested empirically by several researchers were used. Refer to Sections 4.5.1 and 4.5.2 for a detailed discussion of SKU classification methods.

The main procedure algorithm uses the following variables:

- x = monthly average demand (units) for last 12 months
- STD = monthly standard deviation of demand (units) for last 12 months
- X_{L+R} = forecast (or expected) demand over a replenishment lead time (units)
- Coef $_{L+R}$ = coefficient of variance over a replenishment lead time

where $\text{Coef}_{L+R} = \text{STD}_{L+R} / X_{L+R}$

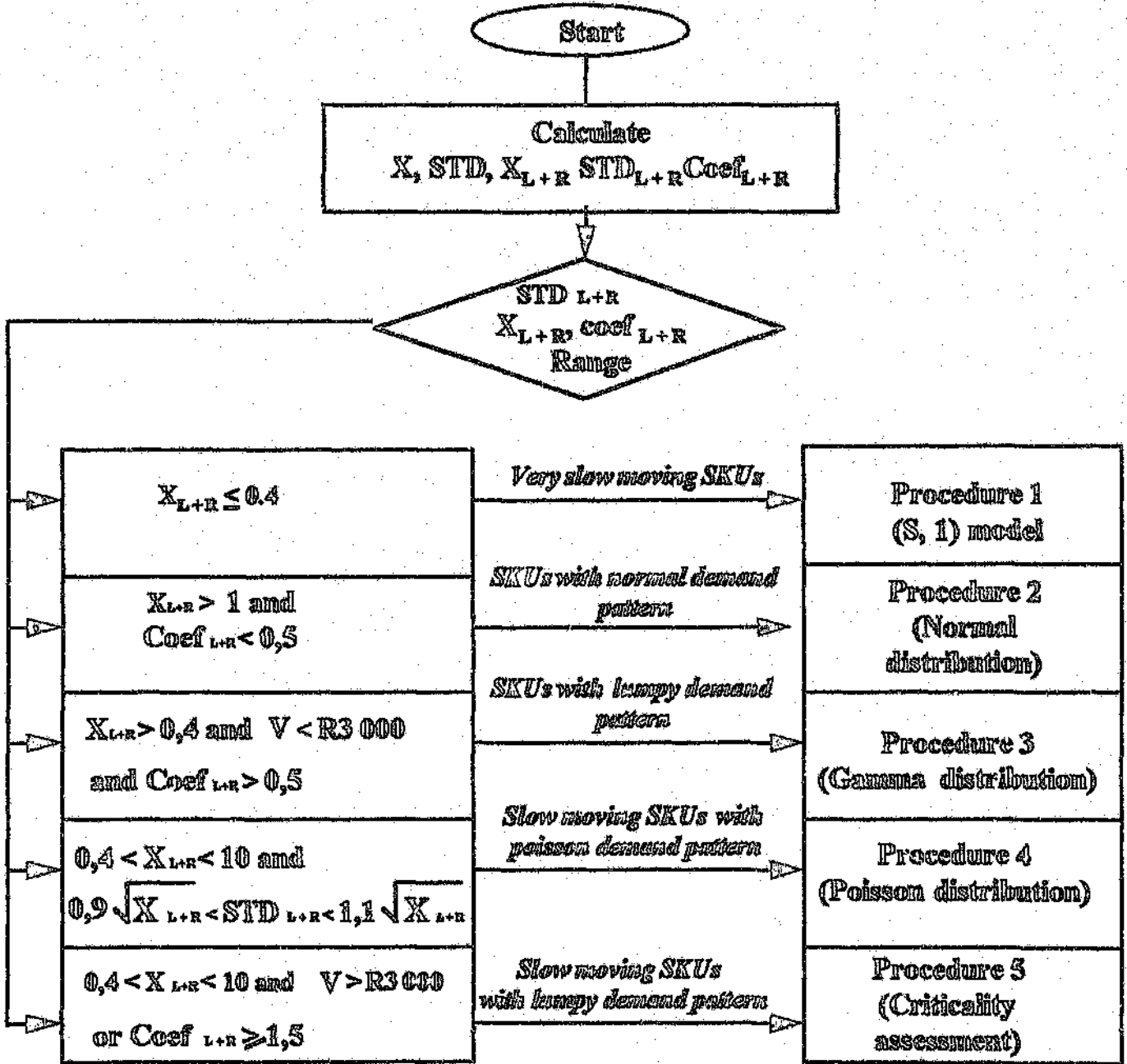
The above variables are calculated on the basis of the last 12 months' demand. The value range of X_{L+R} and coef_{L+R} will determine the procedure to be used for the calculation of the control parameters, i.e. required safety stock and the re-order point. As mentioned in Section 4.7, the re-order quantity is predetermined and equal to three weeks' supply.

The main procedure represents a summary of the methodology for the SKU demand pattern classification which will trigger the selection of the appropriate procedure. As can be seen from the flow diagram of the main procedure, the different ranges of X_{L+R} and coef_{L+R} trigger different procedures:

- $X_{L+R} \leq 0,4$ when the SKU demand during the replenishment lead time is less than or equal to 0,4 units, and the SKU is classified as a very slow-moving item. The SKU control parameters will be determined in procedure 1.
- SKUs that fulfil both conditions $X_{L+R} > 10$ and $\text{coef}_{L+R} \leq 0,5$ are classified as SKUs with a normal distribution of demand (Procedure 2)
- $X_{L+R} > 0,4$ and $0,5 \leq \text{coef}_{L+R} \leq 5$
SKUs that fulfil these conditions are classified as items that have an erratic lumpy demand pattern (procedure 3).
- $0,4 < X_{L+R} \leq 10$ and $\sqrt{X_{L+R}} = \text{STD}_{L+R}$
($\sqrt{X_{L+R}}$ is in the range of $\pm 10\%$ of the STD_{L+R} value). These items have a Poisson demand pattern (Procedure 4).
- $X_{L+R} < 0,4$ and $\text{coef}_{L+R} \geq 1,5$ and unit cost $> R3\ 000$. These are very expensive slow-moving items, with a very erratic/lumpy demand pattern. (procedure 5).

MAIN PROCEDURE

Proposed inventory models - SKU's demand pattern classification



X_{L+R} Average demand
 STD_{L+R} Standard deviation of demand
 $Coef_{L+R}$ Coefficient of variation of demand
 $L+R$ Replenishment lead time
 V Unit price (R/unit)

5.3.2 PROCEDURE 1 - INVENTORY MODEL FOR VERY SLOW-MOVING SKUs

Items with extremely low sales may be minor variations of faster-moving products. In this case it could be attractive to eliminate the slow-moving special versions. Of course, decisions of this type as well as those where there is no substitute product, go beyond the field of inventory management. Marketing considerations, including customer relations (for example, the slow-mover may be carried as a service item for an important customer), are obviously relevant. Furthermore, the appropriate course of action may not be to discontinue selling the item, but rather to make or buy it to order as opposed to stocking it, an issue addressed in detail in Section 4.8.

In view of the decision to satisfy customer demands for a given SKU, the question that now arises is *Should we make a special purchase from the supplier to satisfy each individual customer-demand transaction or should we purchase to stock?* Normally a low demand rate favours nonstocking but there are a number of other factors that also influence the decision, and these factors are discussed in procedure 5.

Procedure 1 below describes the methodology used to establish control variables. This procedure is activated when the unit cost exceeds R3 000 or when the SKU is a critical item to a fleet owner (Procedure 5-manual procedure).

If the unit cost is less than R3 000 and if the item is not critical to a fleet owner, model (s, 1) is used. In this model S is the maximum on-hand quantity and $S = 1$. The re-order quantity is also one unit. The re-order point is zero units, derived from the relationship $ROP = S - ROQ$.

5.3.3 PROCEDURE 2 - INVENTORY MODEL FOR SKUs WITH NORMAL DISTRIBUTION OF DEMAND

Section 4.7 presents a wide choice of criteria for establishing safety stocks. The choice of a criterion is a strategic decision, directly involving senior management. Procedure 2 was designed to facilitate the selection of the criterion and the implied associated decision rule. Exchange curves that will be helpful in this regard derive from this procedure and are presented in section 6.

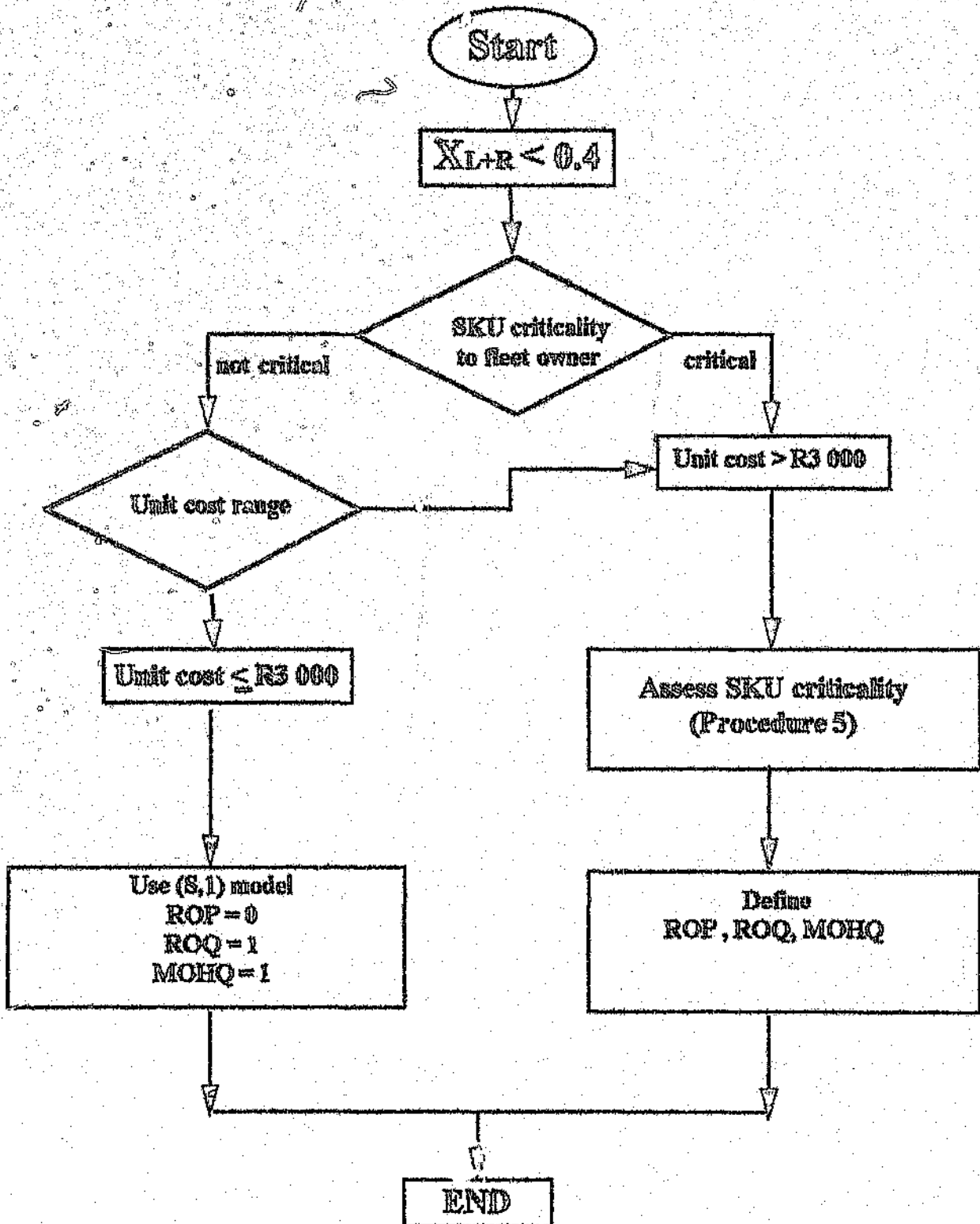
Procedure 2 below deals with all the SKUs that have a smooth demand pattern. These SKUs are characterised by a low coefficient of variation (coef_v is less than 0.5). Items with an average demand during a lead time that is equal or greater than 10 units, are classified as fast-moving items.

The following steps are described in Procedure 2 below:

- ° Specify the decision rule to select safety factor k.
- ° Compute the relevant k value in accordance with the selected decision rule. (Refer to Section 4.7 for a detailed explanation of the formulas presented in this procedure.)
- ° If the computed value k is lower than the lowest allowable value ($k = 1,00$) or cannot be computed from the formula, the value of k is selected as $k = 1,00$.

PROCEDURE 1

Proposed inventory model for very slow moving SKUs



ROP = re-order point
ROQ = re-order quantity
MOHQ = maximum on-hand quantity
 X_{L+R} = average demand during

The values of $G_u(k)$, $P_u(k)$ are obtained from k , or can be looked up in a table. A table of the unit normal distribution is presented in Appendix 21.

◦ When the value of the safety factor k has been ascertained, the following quantities are computed:

- Maximum on-hand quantity (MOHQ) = $X_{L+R} + k \text{STD}_{L+R}$
- Average on-hand quantity (AOHQ) = $X_R/2 + \text{SS}$
- Safety stock (SS) = $k \text{STD}_{L+R}$
- Re-order point (ROP) = $X_L + \text{SS}$

$$\text{Expected shortage per replenishment cycle (ESPRC)} = \text{STD}_{L+R} G_u(k)$$

The value of $G_u(k)$, $P_u(k)$ are obtained from k , or can be looked up in a table. A table of the unit normal distribution is presented in Appendix 21.

◦ When the value of the safety factor k has been ascertained, the following quantities are computed:

- Maximum on-hand quantity (MOHQ) = $X_{L+R} + k \text{STD}_{L+R}$
- Average on-hand quantity (AOHQ) = $X_R + \text{SS}$
- Safety stock (SS) = $k \text{STD}_{L+R}$
- Re-order point (ROP) = $X_L + \text{SS}$

$$\text{Expected shortage per replenishment cycle (ESPRC)} = \text{STD}_{L+R} G_u(k)$$

5.3.4 PROCEDURE 3 - INVENTORY MODEL FOR SKUs WITH ERRATIC/LUMPY DEMAND PATTERN

Recall from Section 4.5.3.4 many researchers have found the Gamma distribution to be appropriate for describing the demand pattern during lead time, with intuitive appeal and empirical support.

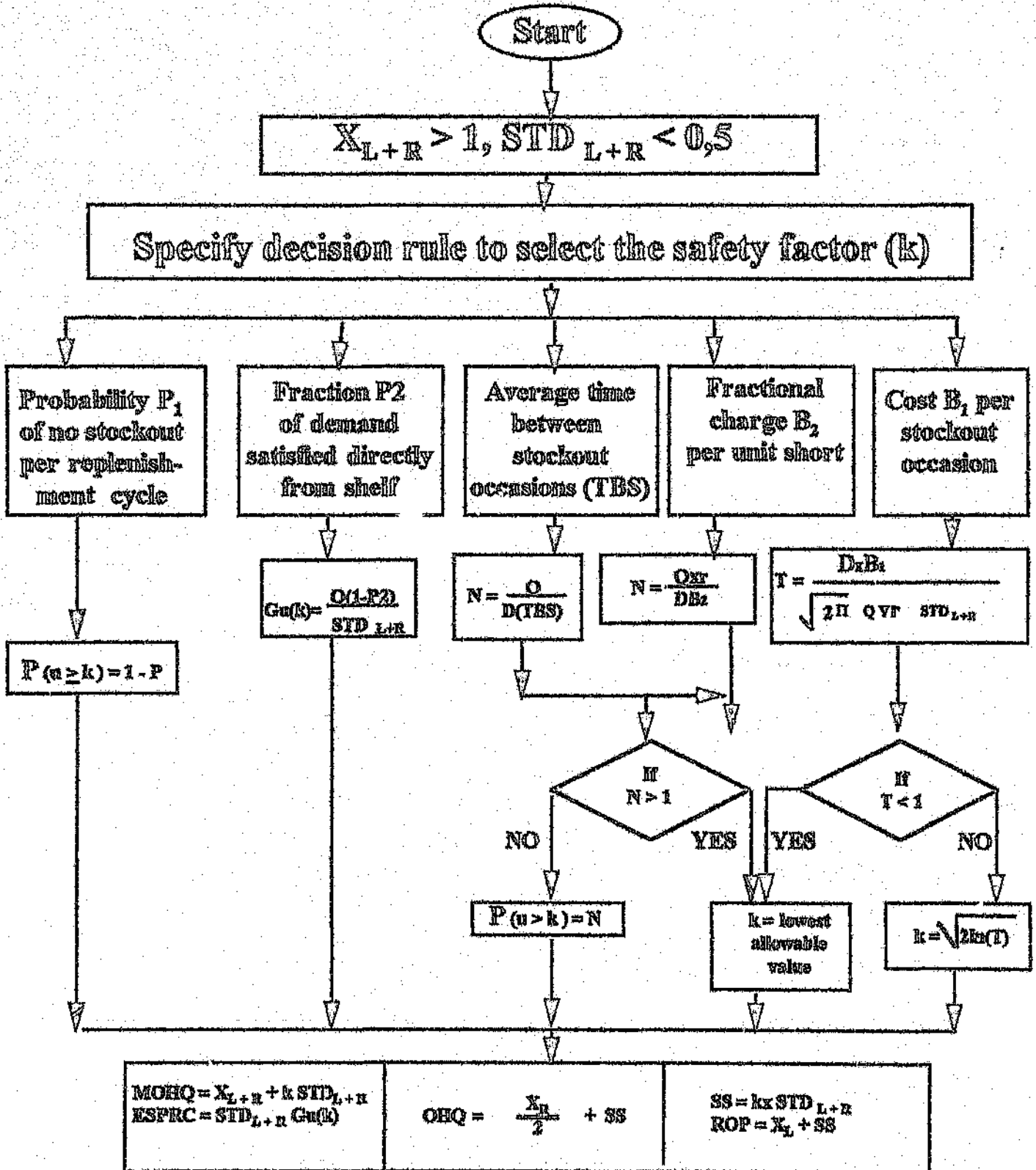
In Procedure 3 below the Gamma distribution is used to calculate the relevant quantities of SKUs with an erratic/lumpy demand pattern, or for slow-moving items. The following criteria were used to select the items to be discussed under this procedure:

- The demand during replenishment lead time is greater than 0,4 units ($X_{L+R} > 0,4$).
- The coefficient of variation of demand during lead time is greater than 0,5 ($\text{coef}_{L+R} > 0,5$).
- The unit cost is less than R3 000 ($v < \text{R3 000}$).

This procedure uses one decision rule only to calculate the quantities below.

PROCEDURE 2

Inventory model for SKUs with normal distribution of demand



SS = safety stock
 ESPRC = expected stockout per

End

The decision maker should specify the probability P_1 of no stockout per replenishment cycle. When P_1 has been established, the Gamma distribution parameters are calculated from the data as follows:

$$\alpha = X_{L+R} / \text{STD}_{L+R}^2$$

$$k = (X_{L+R} / \text{STD}_{L+R})^2$$

With the Gamma distribution function parameters α , k and P the following quantities can be calculated:

- Maximum on-hand quantities S (MOHQ) = Gamma invert (P_1 , k , α). The Gamma invert is a built-in function of the software package EXCEL. This function returns the inverse of the Gamma cumulative distribution function.

- Safety stock (SS) = MOHQ - X_{L+R}

This relationship is derived from Equation 4.6.4 in Section 4.6.2.

- Average on-hand quantity (AOHQ) = MOHQ - X_{L+R} + $X_R/2$

This relationship derives from Equation 4.6.7 in Section 4.6.2.

- Re-order point (ROP) = X_L + SS,

- Expected shortage per replenishment cycle (ESPRC) =

$$k/\alpha [1 - \text{Gamma Dist}(s, k+1, \alpha, T)] - s[1 - \text{Gamma Dist}(s, k, \alpha, T)].$$

Gamma Dist returns the Gamma distribution function where

- s (ROP) - is the value at which we want to evaluate the distribution.

- k , α parameters of the distribution, T - returns the cumulative distribution function.

5.3.5 PROCEDURE 4 - INVENTORY MODEL FOR SKUs WITH POISSON DEMAND DISTRIBUTION (SLOW-MOVING SKUs)

As indicated by many researchers (refer to Section 4.5.3) it is appropriate to use the Poisson distribution to represent demand during the replenishment lead time under the following two conditions:

- Demand during lead time ranges between 0,4 units and 10 units

$$(0,4 < X_{L+R} \leq 10).$$

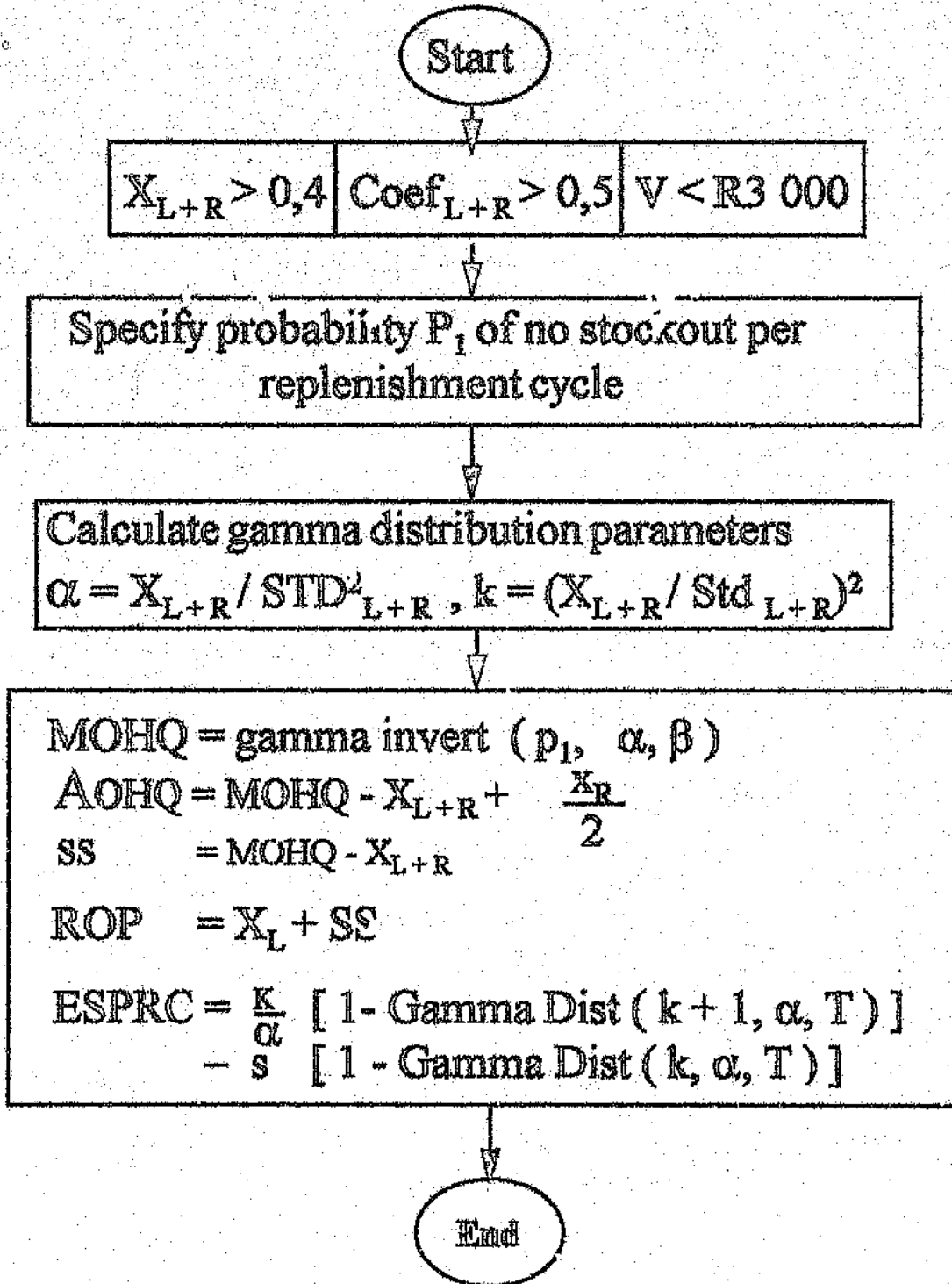
- The standard deviation of demand during the replenishment lead time ranges in $\pm 10\%$ of the demand during the replenishment lead time

$$(0,9 \sqrt{X_{L+R}} \leq \text{STD}_{L+R} \leq 1,1 \sqrt{X_{L+R}})$$

when the demand and the standard deviation of demand fulfil the above conditions, Procedure 4 is applicable.

PROCEDURE 3

Inventory model for SKUs with gamma distribution of demand (erratic / lumpy demand pattern or slow moving items)



- Three decision rules are offered to the decision maker for cost evaluation:

Cost B_1 per stockout occasion

It is assumed that the cost B_1 is incurred only if the demand during the lead time exceeds the re-order point s . The expected total relevant costs (ETRC(s)) per unit time associated with using a re-order point s are

$$ETRC(s) = vr \sum_{j=s}^{\infty} (s-j) P_{DL=j} + DB_1 P_{DL>s}/Q \quad (5.2.1)$$

where

$P_{DL=j}$ = probability that total demand during lead time is j

and $P_{DL=j} = \exp(-\lambda t) (\lambda t)^j / j!$

$P_{DL>s}$ = probability that demand during the lead time exceeds re-order point s

and $P_{DL>s} = \sum_{j=s+1}^{\infty} P_{DL=j}$

Equating $ETRC(s) = ETRC(s+1)$ for indifference between s and $s+1$ leads to

$$P_{DL=s+1}/P_{DL=s} = Qvr/DB_1$$

As indicated in Procedure 4, the value of the re-order point (s) is determined by way of the following steps:

Step 1: Calculate $T = Qvr/DB_1$ (5.2.2)

Step 2: The calculated value T corresponds with value s in a table designed according to Equation 5.2.2 above. (See Appendix 23).

ROP = $(s)T$

Step 3: Calculate all other control parameters

- maximum on-hand quantity

$$MOHQ = ROP + Q$$

(Q is predetermined and equal to $3D/50$)

- Safety stock $SS = MOHQ - X_{L+R}$

- Average on-hand quantity

$$AOHQ = MOHQ - X_{L+R} + X_R/2$$

- Expected total relevant cost per lead time $ETRC(s)$ is calculated according to Equation 5.2.1 above.

- Fractional charge B_2 per unit short and re-order quantity $Q=1$

The expected shortage costs per lead time C_s :

$$C_s = (\text{cost per shortage}) \times (\text{expected demand per unit time}) \times \text{prob \{a demand is not met\}} =$$

where:

$$C_s = B_2 VDP_{(DL \geq S)} \quad (5.2.3)$$

B_2 is defined above.

DV = annual demand value (R)

$P_{(DL \geq S)}$ = probability that demand is not met

S = Maximum on hand quantity.

The quantities MOHQ, SS, AOHQ are calculated as above.

The expected total relevant costs per replenishment lead time as a function of S are

$$ETRC(S) = AOHQvr + C_s.$$

For the proposed control system (R, S), the model under the fractional charge B_2 per unit short, an order for one unit is placed when the inventory position drops to level S - 1.

$$ETRC(S) = vr \sum_{j=0}^{S-1} (S-j) P(DL=j) + B_2 v X_{L+R} P(DL \geq S)$$

A convenient solution, as suggested by Silver and Peterson⁽¹⁾ (360), is the use of indifference curves. These are obtained by equation $ETRC(S)$ to $ETRC(S+1)$ for a given value of S.

After simplification, this gives

$$\frac{P(DL=S)}{P(DL \leq S)} = \frac{r}{X_{L+R} B_2}$$

- Probability P_1 of no stockout per replenishment cycle

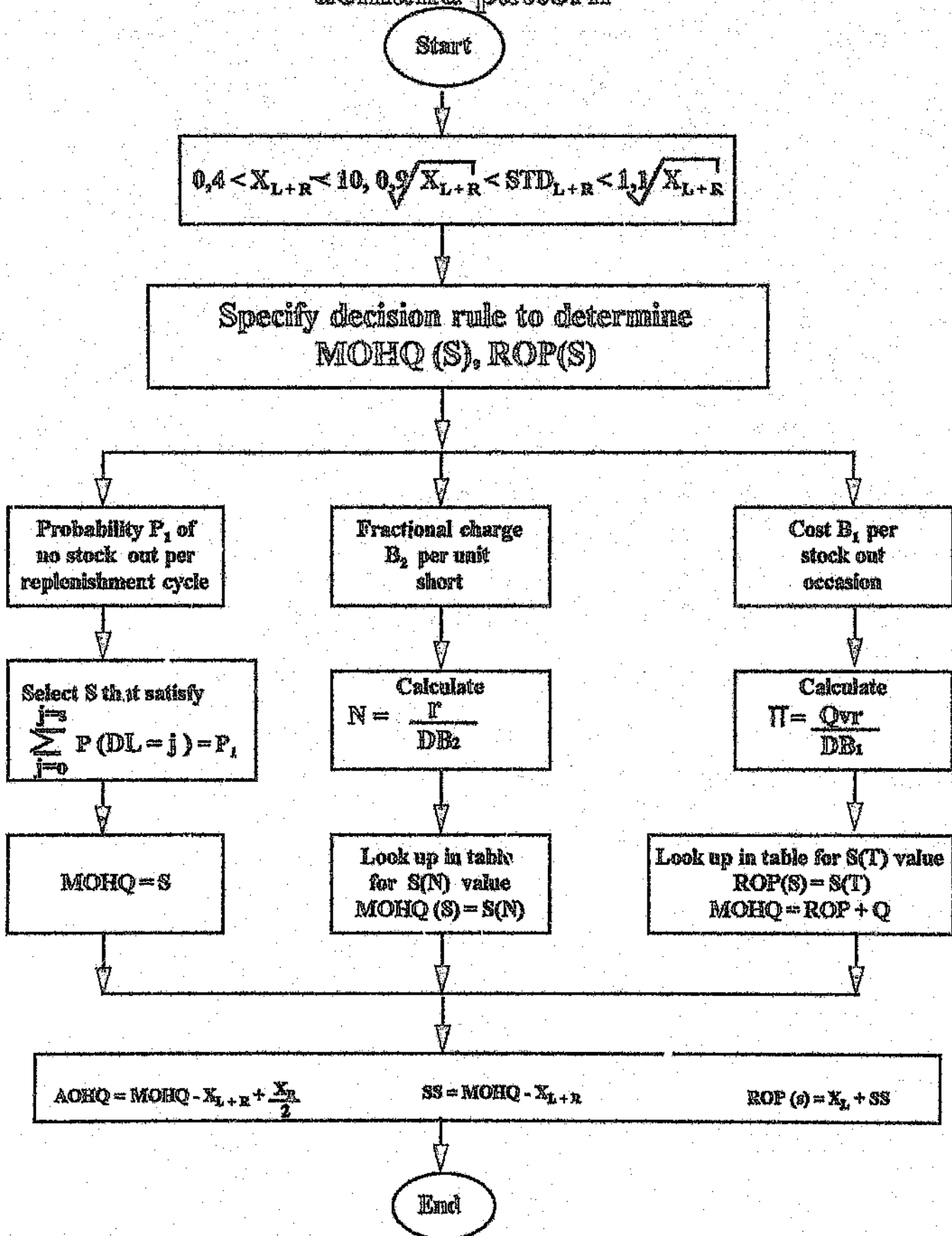
With the Poisson distribution function parameters X_{L+R} and P_1 , the following quantities can be calculated

- maximum on-hand quantity (MOHQ) from look up table (see Appendix 24)
- safety stock (SS), average on hand quantity (AOHQ)

Re-order point (ROP) calculated from the relation described above

PROCEDURE 4

slow moving SKU's with poisson demand pattern



- Expected shortage per replenishment lead time (or mean potential lost of sales) (ESPRC) =

$$ESPRC = X_{L+R} \left(1 - \sum_{j=0}^{s-2} P(DL=j) \right) - S \left(1 - \sum_{j=0}^{s-1} P(DL=j) \right)$$

(See our derivation in Section 4.6.2)

$$ESPRC = X_{L+R} (1 - \text{Poisson}(s-2, X_{L+R}, T)) - S (1 - \text{Poisson}(s-1, X_{L+R}, T))$$

Poisson (X, mean) returns the Poisson probability distribution

where

X = the number of events (S in our case)

Mean = the expected numeric value (X_{L+R})

T = cumulative Poisson probability

5.3.6 PROCEDURE 5 - CRITICAL ASSESSMENT OF SPARES

As discussed in Section 2 under ABC inventory analysis, as many as 82% of SKUs to the value of R35,3 million are classified as slow-moving/dead stock. This group of items is the main contributor to the high investment tied up in inventory.

The normal method of assessing spares stockholding requirement is either a "gut feel" estimate with spares held by default, or some form of scientific judgment (as discussed in the above proposed models) is made on the basis of statistical analysis of one or two variables. The "gut feel" approach tends to lead to a stocking situation for safety's sake, and the latter approach also has a number of problems.

- Detailed statistical data on the spares demand pattern are required. Normally such data do not exist due to the slow movement of these items.
- If some statistical data do exist, a high level of skill is required to apply the appropriate statistical analysis methods.
- The statistical methods selected in most cases do not address all of the factors that should be considered for decisions on stocking versus nonstocking and the quantity that should be stocked.

Procedure 5 is based on a situation appraisal technique and uses logistical decision-making intelligence to establish the criticality of the spare parts to fleet owners and other users by leading the stock controller through a series of questions, answers and choices.

This procedure ensures that the stock controller considers most of the factors that contribute to the decision on whether to stock or not to stock.

The following factors are involved in decision-making:

- Planned use of the item
- Alternatives for the item/common part
- Sources for supply
- Downtime to fleet owner
- Predictability and probability of failure
- Lead time
- Cost involved (of sales negative risk)
- Where item is used in new truck/bus or in truck/bus that is going to phase out
- Usage history if it exists.

An important advantage in considering a large number of factors is that less scientific accuracy is required because the importance of any single factor becomes relatively less. In view of the above circumstances a pre-set structure of questions and range of answers to elicit the required information from the stock controller were designed as described below.

PROCEDURE 5
SPARES CRITICALITY ASSESSMENT PROCEDURE
APPLICATION FOR ALTERATION TO CONTROL FIGURES

GENERAL

ASSESSMENT DATE

STOCK NUMBER

ITEM NAME

ITEM DETAILS

STOCK ON HAND

UNIT VALUE (R)

LEAD TIME (WEEKS)

ANNUAL USAGE (UNITS)

NUMBER INSTALLED (NATIONAL FLEET SIZE)

AVERAGE LIFE (MONTHS/YEARS)

SUPPLIER

1.A Is the item made to order? What is the lead time (days)?

1.B Is the item supplied from stock? What is the lead time (days)?

(Answer Yes or No, and number of days)

1.C When had the supplier lead time been verified with the supplier? Can it be shortened? If the last order to the supplier for this item had been replaced more than 18 months before the assessment date, the buyer should verify the lead time with the supplier.

PROBABILITY OF FAILURE

2. Is the item likely to fail in service?

2.A Failures are common with this type of item

2.B Item is subjected to noticeable wear and tear

2.C Item is robust and not subjected to undue stress

(Answer 2A, 2B or 2C)

PROCEDURE 5 (continued)

PREDICTABILITY OF FAILURE

3. Is failure predictable?

- 3.A Very difficult or impossible - no visible or detectable symptoms
- 3.B Moderately difficult - requires special access/expertise
- 3.C Relatively easy - item shows obvious symptoms of decay

(Answer 3A, 3B or 3C)

KEY INTERPRETATION OF ANSWERS TO QUESTIONS 2 AND 3

		3A x 2C (2)	3B x 2C (3)	3C x 2C (4)
PROBABILITY OF FAILURE	2C			
	2B	3A x 2B (2)	3B x 2B (3)	3C x 2B (4)
	2A	2A x 3A (1)	2A x 3B (2)	2A x 3C (2)
		3A	3B	3C

PREDICTABILITY OF FAILURE

- 1) Item is likely to fail and the failure is hard to predict. Therefore it is likely to be in the company's interests to hold spares in stock.
- 2) Reduce stock levels if possible.
- 3) Question in more detail. Does the company really need this spare?
- 4) Item is not likely to fail and if it did, it would be easy to predict that failure. Under these circumstances spares availability may be coordinated. Therefore it may not be in the company's interests to hold spares in stock.

NOTE: This question/answer system offers a guide to the criticality of items. Decisions have to be taken as to the quantities that should be stocked.

PROCEDURE 5 (continued)

BACK-UP AND ALTERNATIVES

4. Does the truck/bus containing the item have a back-up?
5. Is there an alternative to the item?
6. Does the item have seasonal/planned maintenance usage? If so, when is planned maintenance/seasonal demand due?
7. Is the item repairable when item failure occurs? If so, what is the repair lead time?
8. Will the truck/bus containing the item be phased out soon?
9. Will failure of the item stop the truck/bus?
10. What is the expected downtime when failure occurs (days)?

If the answers to questions 4 to 8 are yes, the criticality of the item is reduced. Therefore a minimum number of items should be held for the case where $X_{L+R} < 0,4$.

Re-order point ROP = 0

Re-order quantity ROQ = 1

If $X_{L+R} > 0,4$ the ROP should be established according to Procedure 3. However if the item is to be phased out soon, the ROP should take into account very high carrying charges, e.g. 40% to 50%. The ROQ will be established in consultation with the main customers (big fleet owners).

◦ Smoothing of demand

When demand is lumpy, the purchaser should check if high readings (demands) can be eliminated due to major one off maintenance activity by the fleet owner or a major technological change which is not going to be repeated. This activity will reduce the standard deviation of demand during the lead time, which will have an effect on the ROP and the ROQ values.

6. CURRENT AND PROPOSED INVENTORY MODEL SIMULATION

6.1 SAMPLE SELECTED FOR SIMULATION

The sample that was selected for the simulation of the current and proposed inventory models was taken from the current inventory file. The SKUs that were selected have the characteristics of the type of demand discussed in Section 4.5.3. The selected sample size was 188 SKUs with recorded usage for the past 12 months. The annual usage value of the chosen sample amounted to R8,34 million. The SKUs sampled procured both locally and overseas with varying lead times ranging between 7 and 35 weeks.

The following information was available for the 188 items:

- Part number
- Description
- Supplier lead time
- Unit cost
- Rolling movement (monthly usage over a continuous period of 12 months).

6.2 CURRENT INVENTORY MODEL SIMULATION RESULTS

The current inventory model described in Section 2.4 had been programmed by the author on an Excel spreadsheet.

The current inventory model excludes all monthly readings in excess of twice the monthly average, to calculate the new monthly average. The number of SKUs with irregular readings, to be excluded in the calculation of the new average monthly demand, are distributed as follows:

NUMBER OF SKUs	NUMBER OF IRREGULAR READINGS	% OF TOTAL
31	0	16,5
35	1	18,6
31	2	16,5
26	3	13,8
22	4	11,7
43	5	22,9
Total	188	100

The above distribution indicates that only 16,5% of the SKUs in the selected sample can be classified as items having a smooth demand pattern according to the current classification criteria. Moreover, the incidence where three or more readings are excluded from the new monthly average is approximately 45% of the total SKUs sampled. That means that the control parameters (NAD, MAD) are based on three readings only, which is statistically meaningless.

The quantities below were calculated in accordance with the current inventory model, using the criteria to check irregularity in demand and without using it.

TABLE 7
SIMULATION SUMMARY RESULTS
CURRENT INVENTORY MODEL

QUANTITY	LAST SIX MONTHS READINGS IRREGULARITY CRITERIA		LAST TWELVE MONTHS' READINGS (R1 000)
	INCLUDE (R1 000)	EXCLUDE (R1 000)	
TMOHV	859,0	6 432,0	6 409,0
TSSV	-1 381,0	4 102,0	4 079,0
TAOHV	-1 163,0	4 110,0	4 088,0
ETVSPY	-1 715,0	959,1	940,0
ETSOPY	2 700	161	153
TOTAL COST*	1 715	1 719,1	1 701,1

The shortage cost is calculated at 25% of the expected total value short per year (ETVSPY) and the carrying charges at a rate of 23% of the total average on hand value (TAOHV).

The above quantities were calculated in accordance with the following equations:

◦ Total maximum on hand stock value (TMOHSV) = TMOHV = $\sum V_i MOHQ_i$

$$MOHQ_i = NAD_i (LT_{oi} + LT_{oi})/4$$

where

NAD, LT_{oi} , LT_{oi} as defined in Section 2.4.2

LT_{oi} = company lead time is ten weeks as it is currently.

◦ Total Safety stock value (TSSV):

$$TSSV = \sum V_i SS_i$$

$$SS_i = kSTD_{L+R}$$

The safety stock value calculated here assumed a normal distribution of demand with a probability $P_1 = 0,95$ of no stockout per replenishment cycle for the selected sample. The current inventory model assumes a normal distribution of demand.

◦ Total average on-hand stock value (TAOHSV):

$$TAOHSV = \sum V_i (MOHQ_i - X_{L+R,oi} + X_{R,oi}/2)$$

- Expected total value short per year (TEVSPY):

$$ETVSPY = \sum V_i \cdot ESPRC_i$$

$$ESPRC_i = G_i(k) \cdot STD_{L_i + R_{iH}}$$

where $r = 0,23$ R/R/annum

B_i = fractional charge $B_i = 25\%$ of unit cost

- Total cost = carrying charges per annum + TEVSPY x h

where h = shortage rate

All other variables are as defined in Section 4.7.

The quantities TAOHV, TSSV, TEVSPY cannot be quantified directly from the current inventory model and since the current model assumes a normal distribution of demand, the quantities for this distribution as defined in Sections 4.6.1 and 4.7.2 were used.

The results for the above quantities show that when the irregularity criteria is used, the current inventory formula generates negative on-hand stock, and since negative on-hand stock is not possible (this is not a case of back-ordering) this indicates incidence of stockout. This suggests that buyers are in fact overriding the stock quantities generated by the current formula using the irregularity criteria. For purposes of comparison the author calculated the above quantities by not using the irregularity criteria. It is clear that the only cost that applies when using the irregularity criteria is the shortage cost and the total expected shortage value per annum is R1 715 million. On the other hand, when use of this criteria is avoided, the costs of both shortage and the carrying charge costs were R1 719 million per annum. The expected short total value short per year is 11,5% of total usage value.

The results of the simulation using the current inventory model for the selected sample are presented in Appendix 23.

6.3 PROPOSED INVENTORY MODELS SIMULATION

6.3.1 SKU CLASSIFICATION

The number of SKUs in the various SKU movement categories according to the proposed classification rule discussed in Section 4.5.2 and as shown in Procedure 1, is as follows:

MOVEMENT CATEGORY	No. OF SKUs	% OF TOTAL
Normal demand pattern - slow-moving	15	8,0
- fast-moving	65	34,6
Lumpy/erratic demand pattern - slow-moving	59	31,4
- fast-moving	14	7,5
Slow-moving SKUs with Poisson demand pattern	28	14,8
Very slow-moving/very erratic demand pattern manual override	7	3,7
Total	188	

Eleven SKUs from the selected sample can be classified as SKUs having both Normal and Poisson demand patterns during the replenishment lead time. Fast-moving SKUs are those items with demand of 10 units or more during replenishment lead time, while the slow-moving SKUs have a demand ranging between one and ten units.

6.3.2 EXCHANGE CURVES INVOLVING SAFETY STOCKS FOR (R, S) SYSTEM

As was discussed under "Guiding principles" in Section 5.1, management should be primarily concerned with aggregate measures of effectiveness. In this section exchange curves showing the aggregate consequences, in terms of total safety stock (in rands) versus aggregate service will be developed. The exchange curves that will be developed, will represent the particular decision rule for establishing safety stocks when it is used across the population of items. As indicated in Section 4.8, there is a policy variable for each type of decision rule. Management's selection of an aggregate operation point implies a value of the policy variable. The use of this implicit value in the corresponding individual-item decision rule will then lead to the desired aggregate consequences.

6.3.2.1 DERIVATION OF THE SAFETY STOCK EXCHANGE CURVES

In all cases, for a given value of the relevant policy variable, to sound the corresponding point on each exchange curve the following three quantities are computed for each item i

- Safety stock (in rands) of item $i = SS_i v_i$ (6.1)

where $SS_i = MOHQ - X_{L,R}$

- Expected stockout occasions per year = $P_i D_i/Q_i$ for item i (6.2)

where:

$D_i/Q_i =$ number of replenishment per year for item i

$P_i =$ Probability of stockout in replenishment lead time for item i

$$P_i = \int_0^s f(x) dx \quad (6.3)$$

- Expected value short per year = v_i ESPRC D_i/Q_i

where:

$$ESPRC = \int_s^{\infty} (x-s)f(x) dx$$

Equation 6.2 follows from multiplying the expected number of replenishment cycles per year (D_i/Q_i) by the probability of a stockout per cycle. Similarly Equation 6.3 is a result of the product of the expected number of replenishment cycles per year and the expected shortage per replenishment cycle expressed in rands.

Now, each of the three quantities (Equation 6.1, 6.2 and 6.3) is summed across all items:

$$\text{Total safety stock in rands} = \sum_i SS_i v_i \quad (6.4)$$

$$\text{Expected total stockout occasions per year (ETSOPY)} = \quad (6.5)$$

$$= \sum_i P_i D_i / Q_i$$

$$\text{Expected total value short per year (ETVSPY)} = \quad (6.6)$$

$$= \sum_i ESPRC_i v_i D_i / Q_i$$

The results of Equations 6.4 and 6.5 give a point on the exchange curves of safety stock versus expected stockout occasions and those of Equations 6.4 and 6.6 give a point on the exchange curves of safety stock versus expected value short for the particular safety stock decision rule. Repeating the whole process for several values of the policy variables generates a set of points for each one of the four curves. (See Figures 12 to 13 in Section 6.3.2.2, and Figure 14 in Section 6.3.5.)

6.3.3 SIMULATION RESULTS: SKUs WITH NORMAL DISTRIBUTION OF DEMAND

According to the proposed classification rule, 80 SKUs from the chosen sample belong to the category of SKUs having a normal distribution of demand during the replenishment lead time. The annual usage of those SKUs is 4 944 million (59% of the total usage value of the chosen sample). For five of the methods presented in Section 4.8, a simulation run was completed with various policy variables. A detailed breakdown of the simulation run is provided for the five decision rules with the following policy variable:

$$P_1 = P_2 = 0,95, \text{ TBS} = 2 \text{ years}, B_2 = 20\%, B_1 = R150.$$

See Appendix 24.

Tables 8 to 12 below summarise the results of the simulation run of five of the decision rules discussed in Section 4.8. Another simulation run was completed for the current company lead time of ten weeks and for the proposed company lead time of three weeks. The total costs comprise two components, the holding costs (H_c) $H_c = r \text{TAOHV}$ and the shortage costs (S_c) $S_c = h \text{ETVSPY}$. The ordering costs are excluded from the total costs since they are considered to be fixed costs that have had to be added to the H_c and the S_c . The ordering costs are fixed and are not affected by changing the policy variable values.

TABLE 8

SIMULATION SUMMARY RESULTS

SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN

PROBABILITY (P1) OF NO STOCKOUT PER REPLENISHMENT CYCLE

POLICY VARIABLE P1	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	ETSOPY Occas- ions	TOTAL COSTS (R1 000) SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
0.700	1865.1	240.7	366.4	1485	402.9	111.7	385.9	460.2
0.750	1933.9	309.6	455.3	1163.7	335.8	279.3	337.5	395.6
0.800	2018.9	394.5	540.3	843.3	268.8	250.8	292.9	335.1
0.850	2100.1	475.7	621.4	606.2	201.5	233.9	264.2	294.5
0.900	2212.6	586.2	733.9	369.4	134.3	224.2	242.7	261.1
0.930	2301.7	677.3	823.1	241.6	94.0	225.5	237.6	249.7
0.950	2379.3	754.9	900.7	163.0	67.2	231.8	239.8	247.9
0.970	2487.6	863.2	1008.0	90.7	40.3	245.7	250.2	254.7
0.990	2692.1	1067.7	1513.4	26.4	13.4	283.1	284.4	285.7
0.995	2806.6	1182.2	1327.9	12.3	6.7	307.3	307.9	308.5
COMPANY LEAD TIME = 10 WEEKS								
0.700	2592.3	287.7	433.4	1242.5	359.2	286.1	348.2	410.3
0.750	2674.6	370.0	516.8	873.5	296.4	264.7	313.3	362.0
0.800	2776.2	471.6	617.3	705.4	235.1	247.8	283.1	318.4
0.850	2873.2	568.6	714.4	507.1	175.2	240.4	265.7	291.1
0.900	3007.6	703.1	848.8	309.0	116.1	241.6	257.0	272.5
0.930	3114.2	809.6	951.9	202.1	81.3	249.3	259.4	269.5
0.950	3206.9	902.4	1048.1	136.4	57.7	261.5	268.3	275.2
0.970	3336.4	1031.8	1177.6	75.8	34.5	282.2	286.0	289.8
0.990	3580.8	1276.3	1422.0	22.1	11.5	330.4	331.5	332.6
0.995	3717.7	1413.1	1558.9	10.3	5.7	360.1	360.6	361.1
DEVIATION FROM PROPOSED COMPANY LEAD TIME								
0.700	727.2	47.0	47.0	-242.8	-43.7	-25.6	-37.7	-49.9
0.750	740.7	60.5	60.5	-190.2	-39.4	-14.6	-24.1	-33.6
0.800	757.3	77.1	77.1	-137.8	-33.5	-2.9	-9.8	-16.7
0.850	773.1	92.9	92.9	-99.1	-26.2	6.5	1.6	-3.4
0.900	785.1	114.9	114.9	-60.4	-18.2	17.4	14.4	11.3
0.930	812.5	132.3	128.9	-39.5	-12.7	23.7	21.7	19.8
0.950	827.7	147.5	147.5	-26.6	-8.4	29.9	28.6	27.3
0.970	848.8	168.6	168.6	-14.8	-5.8	36.6	35.8	35.1
0.990	888.8	208.6	208.6	-4.3	-2.0	47.3	47.1	46.9
0.995	911.1	231.0	231.0	-2.0	-1.0	52.8	52.7	52.6

TABLE 9
SIMULATION SUMMARY RESULTS
SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN
FRACTION (P2) OF DEMAND SATISFIED DIRECTLY FROM SHELF

POLICY VARIABLE	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	ETSOPY SKUs	TOTAL COSTS (R1 000)		
						SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
0.700	1840.5	285.4	427.7	1454.7	1785.0	316.6	389.3	462.0
0.750	1874.5	319.1	461.7	1212.2	1487.5	288.0	348.8	409.2
0.800	1921.1	365.7	508.3	969.8	1190.0	262.4	310.9	359.4
0.850	1985.4	430.0	572.6	727.3	892.5	240.8	277.2	313.5
0.900	2079.4	524.1	666.7	484.9	595.0	226.1	250.3	274.6
0.930	2155.9	600.5	743.1	339.4	416.5	221.8	238.8	255.8
0.950	2226.7	671.3	813.9	242.4	297.5	223.6	235.7	247.8
0.970	2327.7	772.3	915.0	146.5	178.5	232.3	239.5	246.8
0.980	2527.3	972.0	1114.6	48.5	59.5	263.6	266.1	268.5
0.995	2656.4	1101.0	1243.6	24.2	29.8	289.7	290.9	292.1
COMPANY LEAD TIME = 10 WEEKS								
0.900	2999.1	694.5	840.9	495.6	65	267.6	292.4	317.2
0.930	3093.5	789.0	934.7	348.9	38	267.0	284.4	301.7
0.950	3176.2	871.6	1017.4	247.8	30	271.2	263.5	295.9
DEVIATION FROM PROPOSED COMPANY LEAD TIME								
0.900	919.7	170.5	173.6	10.7	-530.0	41.5	42.1	42.6
0.930	937.7	188.5	191.6	7.5	-378.5	45.2	45.6	45.9
0.950	949.5	200.3	203.4	5.3	-287.5	47.6	47.9	48.1

TABLE 10

SIMULATION SUMMARY RESULTS
SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN
AVERAGE TIME BETWEEN STOCKOUT (TBS) OCCASIONS

POLICY VARIABLE (TBS)	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	ETSOPY Occasions	TOTAL COSTS (R1 000) SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
0.50	2175.3	550.9	696.7	437.7	155.0	225.9	247.8	269.7
0.75	2285.1	660.7	806.4	261.8	103.3	224.8	237.8	250.9
1.00	2357.9	733.5	879.2	185.7	77.5	230.1	239.4	248.6
1.25	2402.5	778.1	923.8	146.2	62.0	234.4	241.7	249.0
1.50	2456.0	831.6	977.3	108.6	51.7	241.1	246.5	251.9
1.75	2487.6	863.2	1008.9	90.8	44.3	245.7	250.2	254.8
2.00	2523.0	899.5	1045.3	73.7	38.7	251.5	255.1	258.8
2.25	2523.9	899.5	1045.3	71.9	34.4	251.2	254.8	258.4
2.50	2566.9	942.6	1088.3	57.9	31.0	259.0	261.9	264.8
3.00	2620.3	995.0	1141.7	41.5	25.8	268.8	270.9	273.0
3.50	2620.3	995.0	1141.7	40.3	22.1	268.6	270.7	272.7
4.00	2692.1	1067.7	1213.4	28.9	19.4	283.1	284.5	285.8
4.50	2692.1	1067.7	1213.4	28.9	17.0	283.1	284.5	285.8
5.00	2692.1	1067.7	1213.4	26.9	15.0	283.1	284.5	285.8
COMPANY LEAD TIME = 10 WEEKS								
1.25	3234.7	930.1	1075.9	174.8	15.0	273.7	282.4	291.1
1.50	3298.6	994.0	1139.6	129.8	15.0	281.6	288.1	294.6
2.00	3379.8	1075.3	1221.0	88.0	15.0	284.0	298.4	302.8
DEVIATION FROM PROPOSED COMPANY LEAD TIME								
1.25	832.2	152.0	152.0	28.6	-47.0	39.2	40.7	42.1
1.50	842.8	162.5	162.5	21.2	-36.7	40.6	41.6	42.7
2.00	855.9	175.7	175.7	14.4	-23.7	42.6	43.3	44.0

TABLE 11

SIMULATION SUMMARY RESULTS
SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN
COST (B1) PER STOCKOUT OCCASION

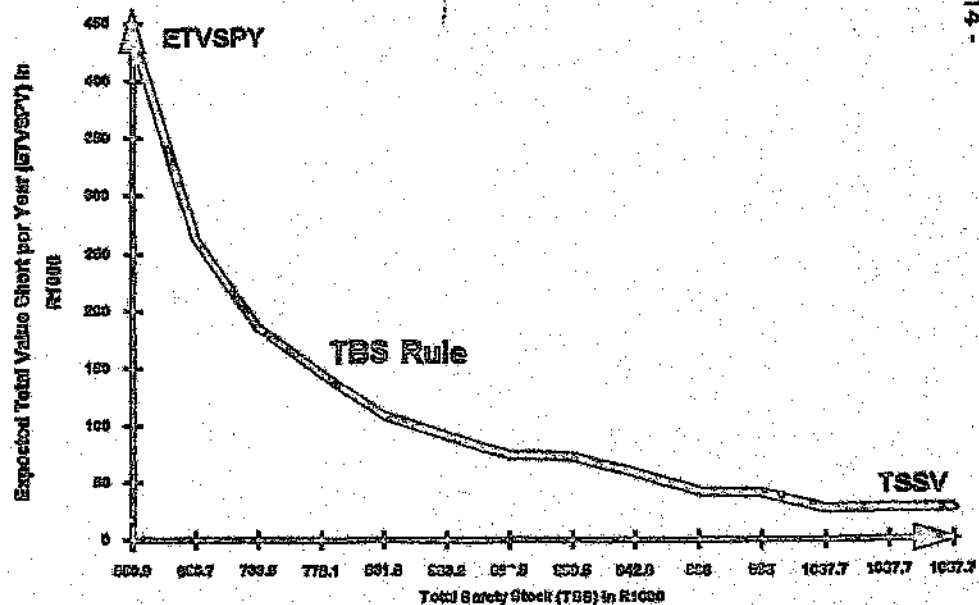
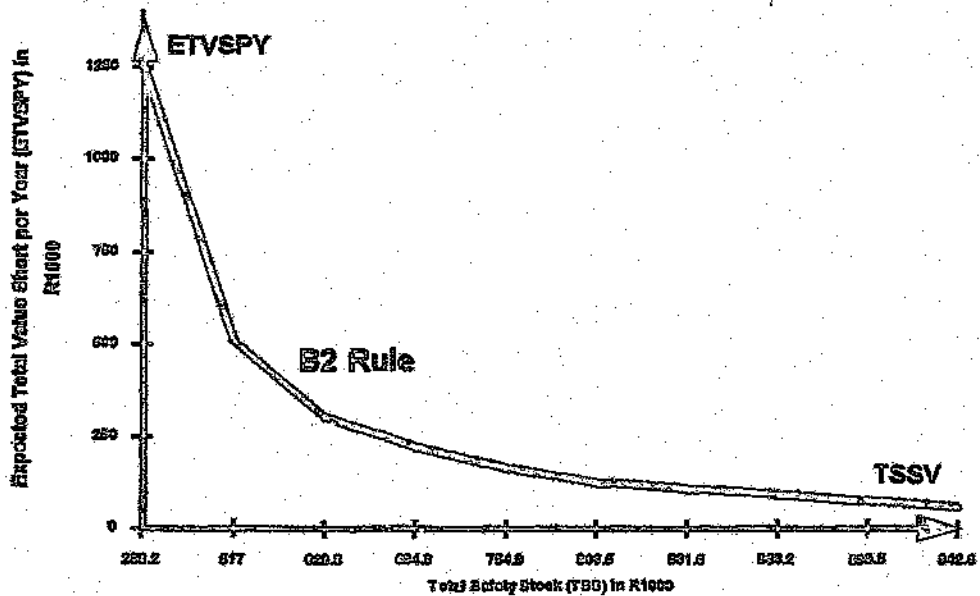
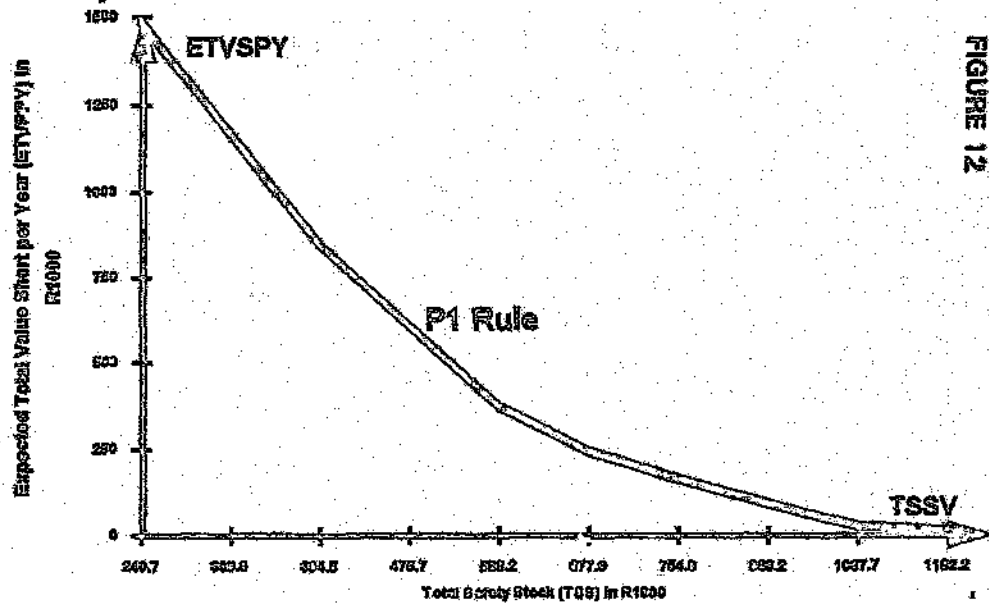
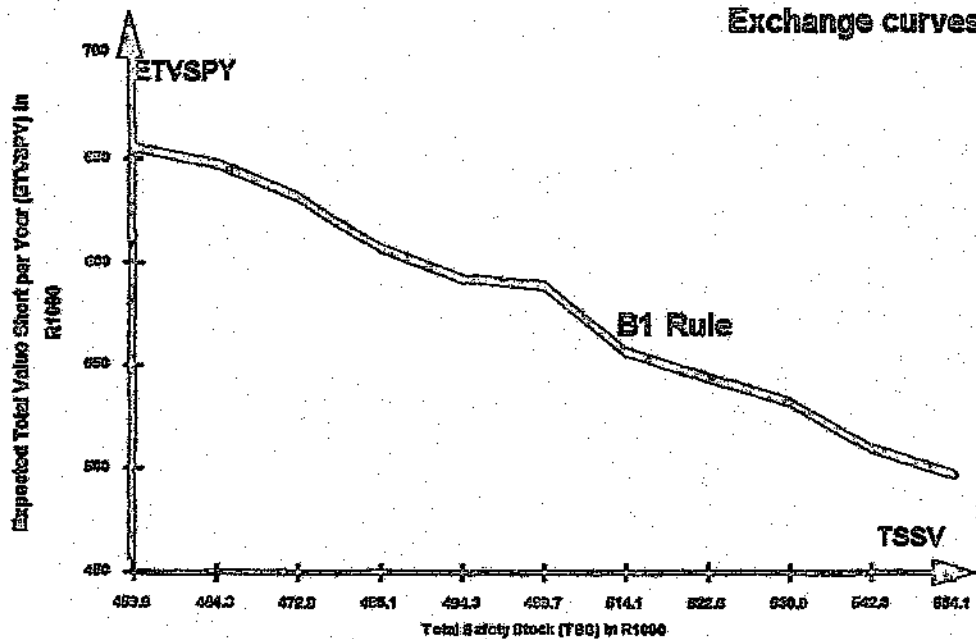
POLICY VARIABLE B1	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	ETSOPY Occas- ions	TOTAL COSTS (R1 000) SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
50.0	2084.3	459.9	605.6	655.4	210.5	237.6	270.4	303.1
100.0	2088.7	464.3	610.1	647.2	194.9	237.4	269.8	302.1
150.0	2097.1	472.8	618.5	632.1	177.8	237.1	268.7	300.3
200.0	2109.5	485.1	630.9	606.8	160.2	236.1	266.5	296.8
250.0	2118.6	494.3	640.0	592.0	148.6	236.0	265.6	295.2
300.0	2124.1	499.7	645.5	588.5	141.1	236.7	266.2	295.6
350.0	2138.5	514.1	659.9	556.4	130.1	235.2	263.1	290.9
400.0	2148.9	522.6	668.3	543.7	123.1	235.3	262.5	289.6
450.0	2155.3	530.9	676.7	531.9	117.5	235.4	262.0	288.6
500.0	2166.7	542.3	688.0	509.5	110.8	234.7	260.2	285.6
600.0	2178.4	554.1	699.8	496.9	102.9	235.5	260.3	285.2
COMPANY LEAD TIME = 10 WEEKS								
100.0	2856.4	551.8	697.6	780.7	202.6	277.6	316.6	355.6
150.0	2862.7	558.1	703.9	770.1	188.3	277.4	315.9	354.4
200.0	2876.6	572.1	717.8	739.1	170.2	276.0	312.9	349.9
DEVIATION FROM PROPOSED COMPANY LEAD TIME								
100.0	767.7	87.5	87.5	133.5	7.7	40.1	46.8	53.5
150.0	765.6	85.4	85.4	138.1	10.5	40.3	47.3	54.2
200.0	767.1	86.9	86.9	132.3	9.9	39.8	46.6	53.1

TABLE 12

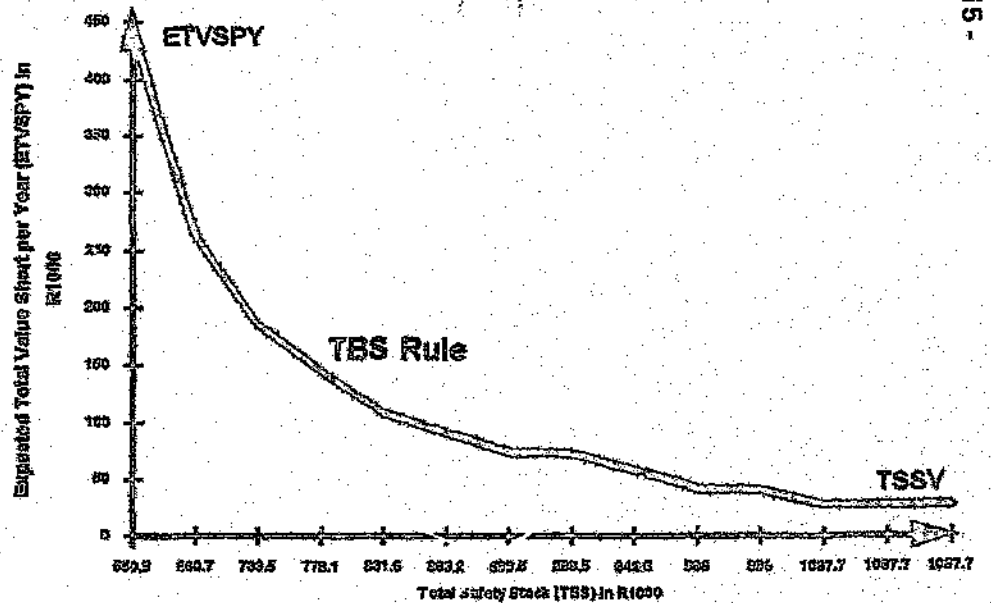
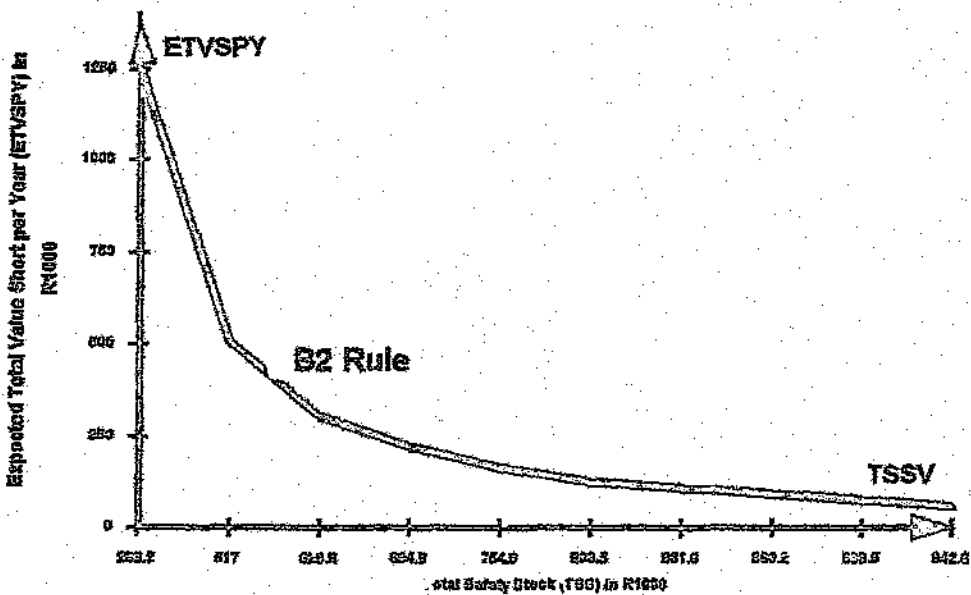
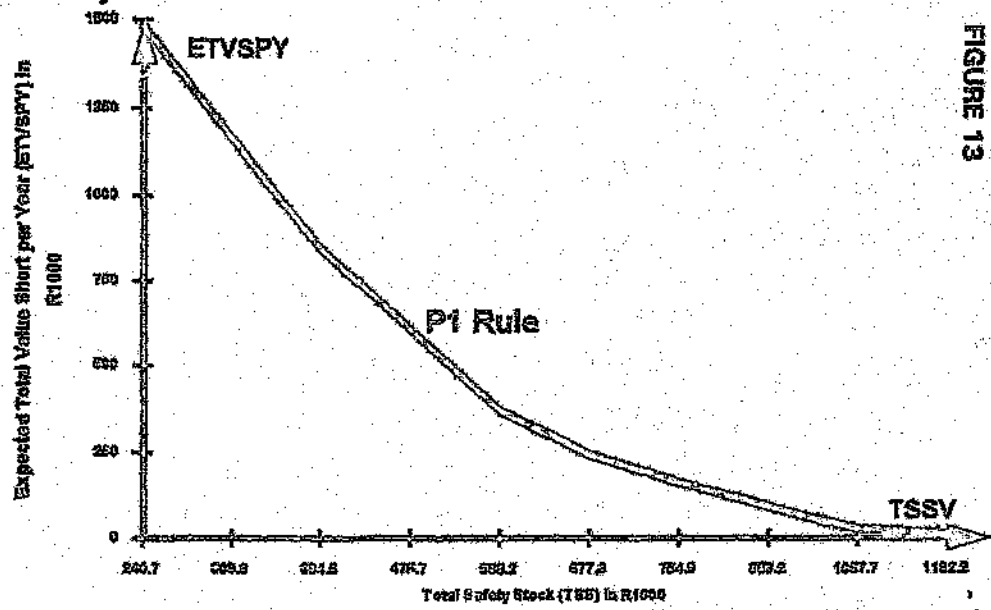
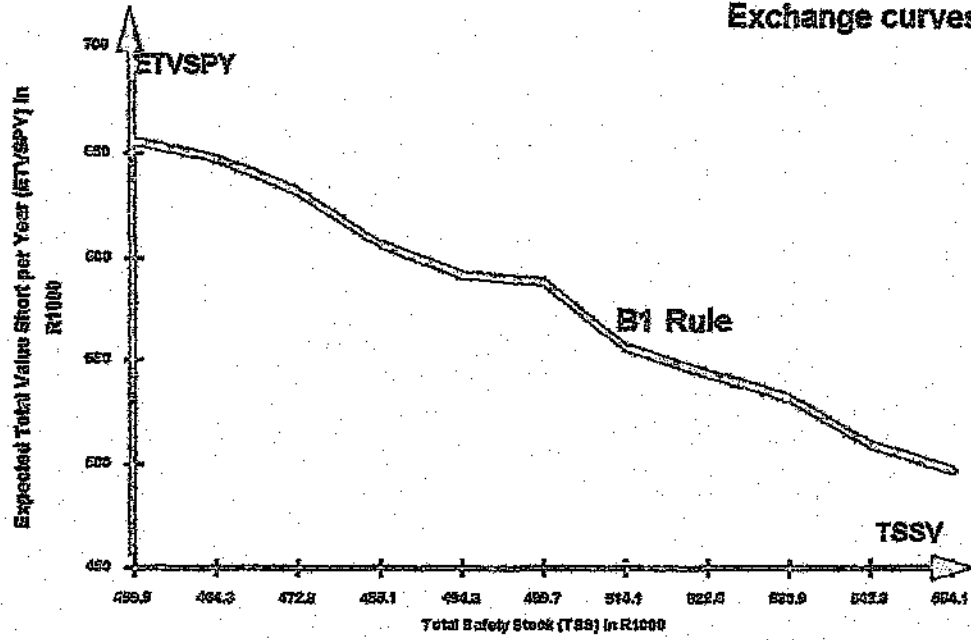
SIMULATION SUMMARY RESULTS
SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN
FRACTIONAL CHARGE (B2) PER UNIT SHORT

POLICY VARIABLE B2	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	ETSOPY Occasions	TOTAL COSTS (R1 000) SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
0.05	1912.6	288.2	434.0	1257.1	356.4	288.4	351.2	414.1
0.10	2141.3	517.0	662.7	508.0	178.2	228.6	254.0	279.4
0.15	2254.2	629.8	775.5	304.1	118.8	224.0	239.2	254.4
0.20	2319.3	694.9	840.6	221.4	89.1	226.6	237.6	248.7
0.25	2379.3	754.9	900.7	163.0	71.3	231.6	239.8	247.9
0.30	2427.9	803.5	949.2	126.0	59.4	237.2	243.5	249.8
0.35	2456.0	831.6	977.3	108.1	50.9	241.0	246.4	251.8
0.40	2487.6	863.2	1008.9	90.7	44.6	245.7	252.2	254.7
0.50	2523.9	899.5	1045.3	73.7	35.6	251.5	255.2	258.8
0.60	2566.9	942.6	1088.3	57.3	29.7	258.9	261.8	264.6
COMPANY LEAD TIME = 10 WEEKS								
0.20	3135.2	830.7	976.4	264.7	51.0	264.3	277.5	290.7
0.25	3206.9	902.4	1048.1	194.9	36.0	270.3	280.0	289.8
0.30	3265.0	960.5	1106.2	150.6	28.0	277.0	284.5	292.1
DEVIATION FROM PROPOSED COMPANY LEAD TIME								
0.20	815.9	135.8	135.8	43.3	-38.1	37.7	39.9	42.0
0.25	827.7	147.5	147.5	31.8	-35.3	38.7	40.3	41.9
0.30	837.1	157.0	157.0	24.6	-31.4	39.8	41.0	42.3

SKU's with normal distribution of demand
Exchange curves of safety stock vs ETVSPY



SKU's with normal distribution of demand
Exchange curves of safety stock vs ETVSPY



The tables above reveal the following:

- The expected total value short per year (ETVSPY) is lower for the current company lead time of ten weeks for lower values of the policy variables P_1 and P_2 (between 0,7 and 0,86). That can be explained by the high safety stocks given by the excessive company lead time of ten weeks.
- As the required policy variable goes up, the ETVSPY and the total safety stock value (TSSV) are lower for the proposed company lead time of three weeks.
- The total cost for the various decision rules and the policy variables operating range is between R224 000 and R296 000 per annum.
- The minimum total cost is obtained for the following decision rules and the following values of the policy variables:
 - P_1 rule : $P_1 = 0,90$, $h = 15\%$
 - P_2 rule : $P_2 = 0,93$, $h = 15\%$
 - TBS rule : TBS = 0,75 years, $h = 15\%$
 - B_1 rule : $B_1 = R500$, $h = 15\%$
 - B_2 rule : $B_2 = 15\%$ of SKU value, $h = 15\%$

As can be seen, all the minimum costs are achieved for the lower rate of shortage (h) of 15%. That can be explained by the higher rate of carrying charges ($r = 23\%$) which dominates the total costs rather than the lower shortage rate ($h = 15\%$), which has less influence on the total costs.

As the shortage rate (h) goes up, total minimum costs are achieved for higher policy variables, which provide higher safety stocks due to the increase of shortage rate (h) compared to charges rate (r).

- The policy variable range presented under company lead time of ten weeks is more appealing. The decision rule that provides the minimum total cost under this policy variable range is the P_2 rule, with $P_2 = 0,93$, TSSV = R0,600 million, TAOHV = R0,743 million, ETVSPY - R0,339 million and a total cost of R0,221 million per annum.

When comparing total costs for the various decision rules and for the range of policy variables, the total costs are lower between 7% to 21% (depending on the selected decision rule) when using the proposed company lead time of three weeks.

- The total cost when the current inventory model is used for the SKUs classified as normally distributed, is R575 300 per annum. The total cost reduction compared to the proposed decision rules ranges between 52% and 62% of the total current costs.

6.3.4 SIMULATION RESULTS: SKUs WITH GAMMA DISTRIBUTION OF DEMAND

Seventy-two SKUs of the chosen sample were classified as SKUs that have a Gamma distribution of demand pattern during the replenishment lead time. The total annual usage value is R2,759 million (33% of the total annual usage sampled).

A simulation run was completed for the probability P_1 of no stockout per replenishment cycle. Table 13 summarises the simulation run results for policy variable P_1 , ranging between 0,7 to 0,995 for the proposed company lead time of three weeks and the current company lead time of ten weeks, and for the different shortage rates (15%, 20%, 25%).

Table 13 illustrates the following performance of the Gamma distribution model:

- The total cost when shortage rates of 15% and 20% are used, are 5,5% to 7,5% lower when using the P_1 decision rule for the proposed company lead time of three weeks compared to the current company lead time of ten weeks.
- When dealing with a higher shortage rate ($h = 25\%$), the Gamma model using a company lead time of ten weeks (current) offers a lower total cost for the P_1 range: up to 0,75 compared to the proposed company lead time of three weeks. That is due to the excessive safety stocks for the higher company lead time (an extra seven weeks). However, it is insignificant since it is not the desired company policy variable operating range (P_1 between 0,9 to 0,97).
- The minimum total costs for the proposed company lead time of three weeks and the various shortage rates, are as follows:

$$h = 15\% \quad P_1 = 0,75 \quad TC = R139\ 600 \text{ pa}$$

$$h = 20\% \quad P_1 = 0,80 \quad TC = R164\ 500 \text{ pa}$$

$$h = 25\% \quad P_1 = 0,85 \quad TC = R183\ 100 \text{ pa}$$

All the above P_1 values are out of the company's desired operating range. Therefore the company's desired minimum total costs for the various shortage rates are achieved for the policy variable $P_1 = 0,9$, with the minimum total costs of

$$h = 15\% \quad TC = R164\ 600 \text{ pa}$$

$$h = 20\% \quad TC = R176\ 400 \text{ pa}$$

$$h = 25\% \quad TC = R188\ 200 \text{ pa.}$$

A detailed breakdown of the simulation run results for the policy variable $P_1 = 0,95$ is to be found in Appendix 25.

6.3.5 SIMULATION SUMMARY RESULTS - SKUs WITH POISSON DEMAND PATTERN

This category consists of 28 SKUs (15% of total number of SKUs) with an annual usage value of R637 695 (7,6% of the total annual usage value sampled).

The three decision rules are presented in Procedure 4. A simulation run was completed for the policy variables P_1 , B_1 , and B_2 , and for various range values. The simulation results for the various decision rules are summarised in Tables 14, 15 and 16.

TABLE 13

SIMULATION SUMMARY RESULTS
SKUs WITH GAMMA DISTRIBUTION OF DEMAND PATTERN
PROBABILITY (P1) OF NO STOCKOUT PER REPLENISHMENT CYCLE

POLICY VARIABLE P1	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	E. SOPY Occas- ions	TOTAL COSTS (R1 000)		
						SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
0.700	529.4	82.7	154.6	708.0	372.3	141.7	177.1	212.5
0.750	596.9	150.2	222.1	590.0	310.3	139.6	169.1	198.6
0.800	679.5	232.8	304.7	472.0	248.2	140.9	164.5	188.1
0.850	786.1	339.4	411.2	354.0	186.2	147.7	165.4	183.1
0.900	936.5	489.8	561.7	236.0	124.1	164.6	176.4	186.2
0.930	1069.0	622.3	694.2	165.2	86.9	184.4	192.7	201.0
0.950	1194.2	747.5	819.3	118.0	62.1	206.1	212.0	217.9
0.970	1384.4	937.7	1009.6	70.8	37.2	242.8	246.4	249.9
0.990	1794.1	1347.4	1419.3	23.6	12.4	330.0	331.2	332.3
0.995	2052.8	1606.1	1677.9	11.8	6.2	387.7	388.3	388.9
COMPANY LEAD TIME = 10 WEEKS								
0.700	911.2	146.9	426.3	398.6	322.0	158.3	178.2	198.2
0.750	999.2	234.8	448.9	377.3	247.0	159.8	178.7	197.6
0.800	1104.5	340.1	486.3	323.1	186.0	160.3	176.5	192.6
0.850	1237.2	472.8	550.5	254.1	123.0	164.7	177.4	190.1
0.900	1419.9	655.5	663.5	176.3	85.0	179.0	187.9	196.7
0.930	1577.4	813.1	778.4	126.4	60.0	198.0	204.3	210.6
0.950	1723.9	959.6	896.0	91.9	37.0	219.9	224.5	229.1
0.970	1943.2	1178.9	1086.2	56.3	29.0	258.3	261.1	263.9
0.990	2405.4	1641.1	1521.1	19.4	10.0	352.8	353.7	354.7
0.995	2692.2	1927.9	1804.0	9.8	6.0	416.4	416.9	417.4
DEVIATION FROM PROPOSED COMPANY LEAD TIME								
0.700	381.8	64.2	273.7	-309.4	-50.3	16.5	1.1	-14.4
0.750	402.3	84.6	226.8	-212.7	-63.3	20.3	9.6	-1.0
0.800	424.9	107.3	181.6	-148.9	-62.2	19.4	12.0	4.5
0.850	451.1	133.5	139.2	-99.9	-63.2	17.0	12.0	7.0
0.900	483.4	165.8	101.8	-59.7	-39.1	14.5	11.5	8.5
0.930	508.4	190.8	84.2	-38.8	-26.9	13.6	11.6	9.7
0.950	529.7	212.1	76.7	-26.1	-25.1	13.7	12.4	11.1
0.970	558.8	241.2	76.6	-14.5	-8.2	15.5	14.7	14.0
0.990	611.3	293.7	101.8	-4.2	-2.4	22.8	22.6	22.4
0.995	639.4	321.8	126.1	-2.0	-0.2	28.7	28.6	28.5

TABLE 14

SIMULATION SUMMARY RESULTS

SKUs WITH POISSON DISTRIBUTION OF DEMAND PATTERN

PROBABILITY (P1) OF NO STOCKOUT PER REPLENISHMENT CYCLE

POLICY VARIABLE P1	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ET/SPY (R1000)	ETSOPY Occas- ions	TOTAL COSTS (R1 000)		
						SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
0.700	140.5	30.3	49.9	357.7	245	65.1	83.0	100.9
0.750	139.4	29.2	48.7	235.5	175	49.5	62.3	75.1
0.800	153.7	43.4	63.0	211.7	145	46.2	56.8	67.4
0.850	170.1	59.9	79.5	150.4	103	40.8	48.4	55.9
0.900	181.2	71.0	90.5	103.7	71	38.4	41.6	46.7
0.930	191.3	81.1	100.7	73.0	50	34.1	37.8	41.4
0.950	213.5	103.3	122.9	52.6	36	36.1	38.8	41.4
0.970	223.4	113.2	132.7	29.2	20	34.9	36.4	37.8
0.990	269.3	159.1	178.6	7.3	5	42.2	42.5	42.9
COMPANY LEAD TIME = 10 WEEKS								
0.700	195.4	42.2	69.4	254.0	174	54.1	66.8	79.5
0.750	218.2	45.7	76.3	207.3	142	48.6	59.0	69.4
0.800	234.9	66.4	96.3	143.1	98	43.6	50.8	57.9
0.850	240.5	84.7	112.4	103.7	71	41.4	46.6	51.8
0.900	271.5	98.1	128.9	83.2	57	42.1	46.3	50.4
0.930	282.0	119.5	148.4	29.2	20	38.5	40.0	41.4
0.950	289.3	140.0	166.5	21.9	15	41.6	42.7	43.8
0.970	306.4	155.2	182.1	16.1	11	44.3	45.1	45.9
0.990	358.8	211.9	238.0	5.5	4	55.6	55.8	56.1
DEVIATION FROM PROPOSED COMPANY LEAD TIME								
0.700	54.9	11.8	19.5	-103.7	-71.0	-11.1	-16.3	-21.4
0.750	78.8	16.5	27.5	-48.2	-33.0	-0.9	-3.3	-5.7
0.800	81.3	23.0	33.3	-68.6	-47.0	-2.6	-6.1	-9.5
0.850	70.4	24.8	32.9	-46.7	-32.0	0.6	-1.8	-4.1
0.900	90.3	27.1	38.3	-20.4	-14.0	5.8	4.7	3.7
0.930	90.6	38.4	47.7	-43.8	-30.0	4.4	2.2	0.0
0.950	75.8	36.7	43.6	-30.7	-21.0	5.4	3.9	2.4
0.970	83.0	42.0	49.3	-13.1	-9.0	9.4	8.7	8.1
0.990	89.5	52.9	59.4	-1.8	-1.2	13.4	13.3	13.2

TABLE 15

SIMULATION SUMMARY RESULTS
SKUs WITH POISSON DISTRIBUTION OF DEMAND PATTERN
FRACTIONAL CHARGE (B2) PER UNIT SHORT

POLICY VARIABLE B2	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	ETSOPY Occas- ions	TOTAL COSTS (R1 000)		
						SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
0.05	117.5	26.8	7.3	385.3	263.9	59.5	78.7	98.0
0.10	153.5	62.9	43.3	152.1	104.1	32.8	40.4	48.0
0.15	190.2	99.6	80.0	82.3	56.4	30.8	34.9	39.0
0.20	196.2	105.6	86.0	58.6	40.1	28.6	31.5	34.4
0.25	215.1	124.4	104.9	41.8	28.7	30.4	32.5	34.6
0.3	223.5	132.9	113.3	30.4	20.8	30.6	32.1	33.7
0.35	232.5	141.8	122.3	19.6	13.4	31.1	32.1	33.0
0.40	236.7	146.0	126.5	16.8	11.5	31.6	32.5	33.3
0.50	236.7	146.0	126.5	16.8	11.5	31.6	32.5	33.3

TABLE 16

SIMULATION SUMMARY RESULTS
SKUs WITH POISSON DISTRIBUTION OF DEMAND PATTERN
COST (B1) PER STOCKOUT OCCASION

POLICY VARIABLE B2	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	ETSOPY Occas- ions	TOTAL COSTS (R1 000)		
						SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
50.0	126.0	15.8	35.4	219.0	150.0	41.0	51.9	62.9
100.0	149.7	39.5	59.0	99.4	68.1	28.5	33.5	38.4
150.0	167.6	57.4	76.9	65.7	43.0	27.5	30.8	34.1
200.0	178.1	67.8	87.4	50.2	34.4	27.6	30.1	32.6
300.0	191.3	81.1	100.6	30.1	20.6	27.7	29.2	30.7
400.0	213.6	103.6	123.2	25.4	17.4	32.1	33.4	34.7

The performance of the following decision rules is indicated in Tables 14, 15 and 16.

- The minimum total costs are realised for the high policy variable P_1 and various shortage rates in a comparison of the proposed and current company lead times.
- Again the lower costs for lower values of P_1 (between 0,7 and 0,85) are achieved when the current company lead time of ten weeks is used. Again this can be ascribed to the high safety stock provided by the excessive lead time.
- The minimum total costs for the three different decision rules are achieved when the B_1 rule is used. The total costs for the company's desired policy variable operating range are less than the total cost of the other decision rules in the range of 8% to 24%, depending on the decision rule and the policy variable that are selected.

6.3.5 SIMULATION SUMMARY RESULTS: COMBINED MODELS

In Section 5 the author offered the use of hybrids of different models to accommodate the various demand patterns.

Table 17 summarises the simulation results of the five inventory models proposed in Procedures one to five. The simulation was carried out for the probability P_1 of no stockout per replenishment cycle, for the current and proposed company lead times, and for various values of the policy variable P_1 .

The following performance of the combined model is indicated with reference to the above table:

- Lower total costs are achieved when the current company lead time of ten weeks is used for lower values of policy variable P_1 (ranging between 0,7 to 0,85) and for the different shortage rates.
- The minimum total costs are realised with the desired company policy variable range (P_1 between 0,9 and 0,97):

$$h_1 = 15\% \quad P_1 = 0,9 \quad TC = R449\,900 \text{ pa}$$

$$h_2 = 20\% \quad P_1 = 0,9 \quad TC = R493\,100 \text{ pa}$$

$$h_3 = 25\% \quad P_1 = 0,93 \quad TC = R530\,600 \text{ pa}$$

The total costs for this desired company policy operating range (P_1) are between 9% and 13% lower when the combined models are compared and when the proposed company lead time (three weeks) is compared with the current company lead time (ten weeks). The tables with the summarised simulation results (8 to 17) are comparing the proposed inventory models with the proposed and current company lead times. They therefore indicate the effect of the excessive current company lead time of ten weeks, using the proposed inventory models. Table 18 summarises the results of the simulation run, using the current inventory model and the proposed combined models for the desired policy variable operating range (P_1 between 0,9 and 0,97).

TABLE 17

**SIMULATION SUMMARY RESULTS
COMBINED MODELS**

PROBABILITY (P1) OF NO STOCKOUT PER REPLENISHMENT CYCLE

POLICY VARIABLE P1	TMOHV (R1000)	TSSV (R1000)	TAOHV (R1000)	ETVSPY (R1000)	ETSOPY Occas- ions	TOTAL COSTS (R1 000) SHORTAGE RATES (h)		
						15%	20%	25%
COMPANY LEAD TIME = 3 WEEKS								
0.700	2593.1	442.2	714.8	2411.1	1033.2	526.1	646.6	767.2
0.750	2693.4	542.5	815.2	2024.2	834.0	491.1	592.3	693.5
0.800	2836.9	686.0	958.7	1637.3	674.8	466.1	548.0	629.8
0.850	3024.3	873.4	1146.1	1250.4	503.6	451.2	513.7	576.2
0.900	3271.2	1120.2	1392.9	863.5	342.4	449.9	493.1	536.3
0.930	3498.9	1348.0	1620.6	731.4	243.9	467.5	499.0	530.6
0.950	3717.1	1566.2	1838.8	615.6	178.2	494.4	518.3	542.1
0.970	4018.2	1867.3	2140.0	522.9	110.5	540.5	556.6	572.7
0.990	4673.4	2522.5	2795.1	467.1	43.8	668.0	676.3	684.7
0.995	5061.1	2910.2	3182.9	431.1	30.9	751.7	758.3	764.8
COMPANY LEAD TIME = 10 WEEKS								
0.700	3822.9	542.8	787.3	1815.0	854.2	453.3	544.1	634.8
0.750	4016.0	716.6	937.0	1495.2	694.4	439.8	514.5	589.3
0.800	4239.6	944.2	1136.0	1157.5	528.1	434.9	492.8	550.7
0.850	4474.9	1192.3	1353.1	877.1	378.2	447.8	486.6	530.5
0.900	4823.1	1522.8	1657.0	578.8	267.1	467.8	496.9	525.8
0.930	5097.6	1808.4	1918.5	413.5	170.3	503.3	523.9	544.6
0.950	5344.2	2068.1	2166.5	307.6	118.7	544.4	559.8	575.2
0.970	5710.0	2432.0	2517.7	203.6	83.5	609.6	619.8	630.0
0.990	6469.0	3195.4	3268.9	105.3	34.5	767.6	772.9	778.2
0.995	6892.7	3584.5	3651.7	84.3	24.7	852.5	856.7	861.0
DEVIATION FROM PROPOSED COMPANY LEAD TIME								
0.700	1229.8	100.6	72.4	-596.0	-179.0	-72.7	-102.6	-132.4
0.750	1322.5	174.1	121.8	-529.0	-139.6	-51.3	-77.8	-104.2
0.800	1402.6	258.2	177.3	-479.8	-146.7	-31.2	-55.2	-79.2
0.850	1450.6	318.9	207.0	-373.3	-125.4	-8.4	-27.0	-45.7
0.900	1551.9	402.6	264.1	-284.8	-75.3	18.0	3.8	-10.5
0.930	1598.7	460.4	297.8	-217.9	-73.6	35.8	24.9	14.0
0.950	1627.1	501.9	327.6	-169.0	-59.5	50.0	41.6	33.1
0.970	1691.8	564.7	377.7	-118.3	-27.0	69.1	63.2	57.3
0.990	1795.6	672.9	473.1	-61.8	-9.4	99.7	96.6	93.5
0.995	1831.6	674.3	468.8	-46.8	-6.2	100.8	98.5	96.1

TABLE 13

TOTAL SAMPLED STOCK SIMULATION SUMMARY RESULTS

INVENTORY MODEL	P1	TOTAL COSTS(R1000)					DEVIATION FROM PROPOSED COMBINED MODELS					TOTAL COSTS(R1000)					
		TMOHV (R1000)	TSSV (R1000)	TAORV (R1000)	ETVSPY (R1000)	ETSOPY Occas- ions	SHORTAGE RATES (h)			TMOHV (R1000)	TSSV (R1000)	TAORV (R1000)	ETVSPY (R1000)	ETSOPY Occas- ions	SHORTAGE RATES (h)		
							15%	20%	25%						15%	20%	25%
CURRENT MODEL																	
LAST SIX MONTHS' READINGS :																	
IRREGULATORY CRITERIA INCLUDED		859.0	-1381.0	-1183.0	-1715.0	2700.0	1715.0	1715.0	1715.0	-2858.1	-2947.2	-3001.8	-2101.6	2521.8	788.6	764.8	748.9
IRREGULATORY CRITERIA EXCLUDED		6432.0	4102.0	4110.0	959.1	161.0	1623.2	1671.2	1719.1	2714.9	2535.8	2271.2	442.5	-17.2	696.8	720.9	745.1
LAST TWELVE MONTHS' READINGS		6409.0	4079.0	4088.0	940.0	153.0	1615.1	1682.1	1709.1	2691.9	2512.8	2149.2	463.4	-25.2	688.7	711.8	735.0
ASSUMING TOTAL STOCK HAS NORMAL DISTRIBUTION OF DEMAND DURING REPLENISHMENT LEAD TIME	0.900	3543.3	1254.3	1494.7	767.8	319.8	833.1	972.5	1011.9	272.1	134.1	101.8	-75.7	-22.8	51.2	47.4	43.7
	0.930	3733.4	1444.5	1684.9	517.2	223.7	936.0	931.7	987.5	234.5	96.5	64.2	-118.2	-20.2	36.5	30.7	24.9
	0.950	3898.9	1609.9	1850.3	347.6	159.8	948.9	966.3	983.7	181.8	43.6	11.5	-129.0	-18.4	22.5	16.2	9.8
	0.970	4125.8	1840.9	2081.3	193.3	95.9	978.9	988.5	998.2	111.6	-28.4	-58.7	-128.6	-14.6	8.4	0.0	-6.5
ASSUMING TOTAL STOCK HAS GAMMA DISTRIBUTION OF DEMAND DURING REPLENISHMENT LEAD TIME	0.900	3503.1	1262.7	1528.2	2109.9	306.0	1122.2	1227.7	1333.2	231.9	142.5	135.3	1246.4	-36.4	240.3	302.6	364.9
	0.930	3798.7	1558.4	1821.8	1463.5	214.2	1092.8	1165.9	1239.1	297.8	206.4	201.2	832.1	-29.7	193.3	234.9	276.5
	0.950	4069.3	1828.9	2094.3	1037.8	153.0	1091.6	1143.5	1195.4	362.2	262.7	255.5	561.2	-25.2	165.2	193.2	221.3
	0.970	4476.5	2236.1	2501.5	617.0	91.8	1122.1	1153.0	1183.8	458.2	368.8	361.6	295.1	-18.7	149.7	164.4	179.2
PROPOSED COMBINED MODELS	0.900	3271.2	1120.2	1382.9	863.6	342.4	881.9	925.1	988.2	-	-	-	-	-	-	-	-
	0.930	3498.9	1348.0	1620.6	631.4	243.9	899.5	931.0	982.6	-	-	-	-	-	-	-	-
	0.950	3717.1	1566.2	1838.8	476.6	178.2	926.4	950.2	974.1	-	-	-	-	-	-	-	-
	0.970	4018.2	1867.3	2140.0	321.9	110.5	972.5	988.6	1004.7	-	-	-	-	-	-	-	-

A simulation run using the normal and gamma distributions for the total sampled stock was carried out for reference and purposes of comparison.

- The current inventory model simulation run was carried out with the variable MAD, NAD and SOS calculated as follows:
 - According to the last six months' readings, using the irregularity criteria
 - According to the last six months' readings, excluding the irregularity criteria
 - According to the last twelve months' readings.

The simulation run was carried out using the current company lead time of ten weeks.

Simulation runs for all other models used the proposed company lead time of three weeks.

Table 18 indicates the following performances/variations for the different models:

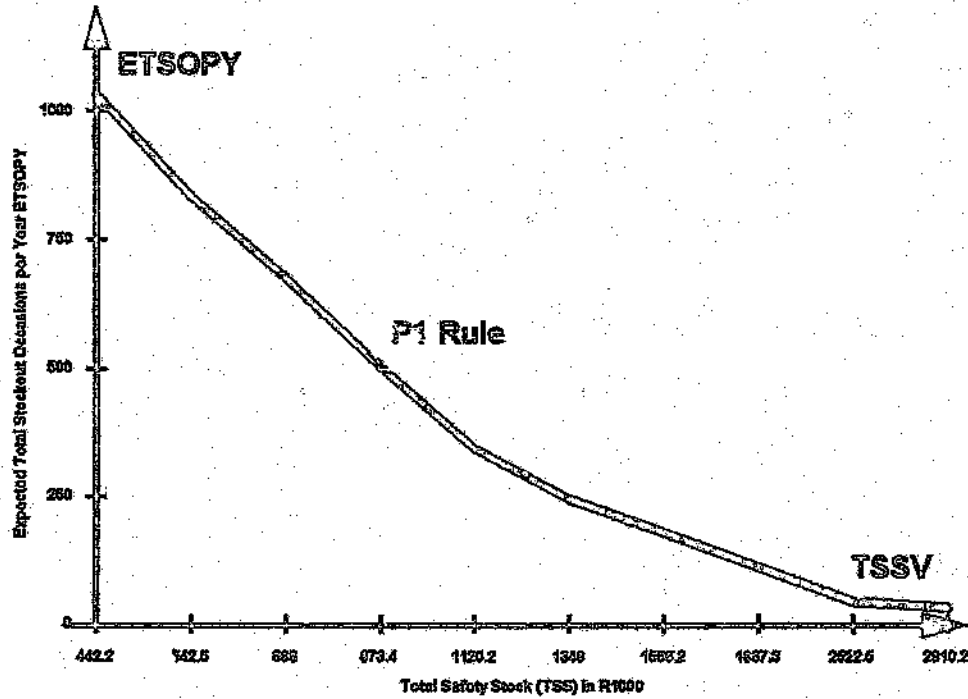
- The current inventory model, using the last six months' readings and the irregularity criteria, offers a higher average total shortage of R1 715 million, and negative safety stocks and average on-hand stock. This clearly indicates the inapplicability of this model in this environment, as well as the possible overriding by buyers of the current control parameters (ROP, ROQ) of the system.
- Using the last six months' readings or twelve months' readings offers a high stock on hand. However, the low expected stockout occasions and shortages are mainly due to the high safety stocks provided by the current inventory model and the excessive company lead time of ten weeks.
- The combined models offered the lowest total costs in a comparison with the current inventory model, the Gamma and the normal distribution models, for the various shortage rates.

The total cost of the combined models for policy variable $P_1 = 0,95$ is 76% lower than the current inventory model, 22% lower than the model assuming Gamma distribution of demand for all SKUs, and 1% lower than the model assuming all SKUs have a normal distribution of demand.

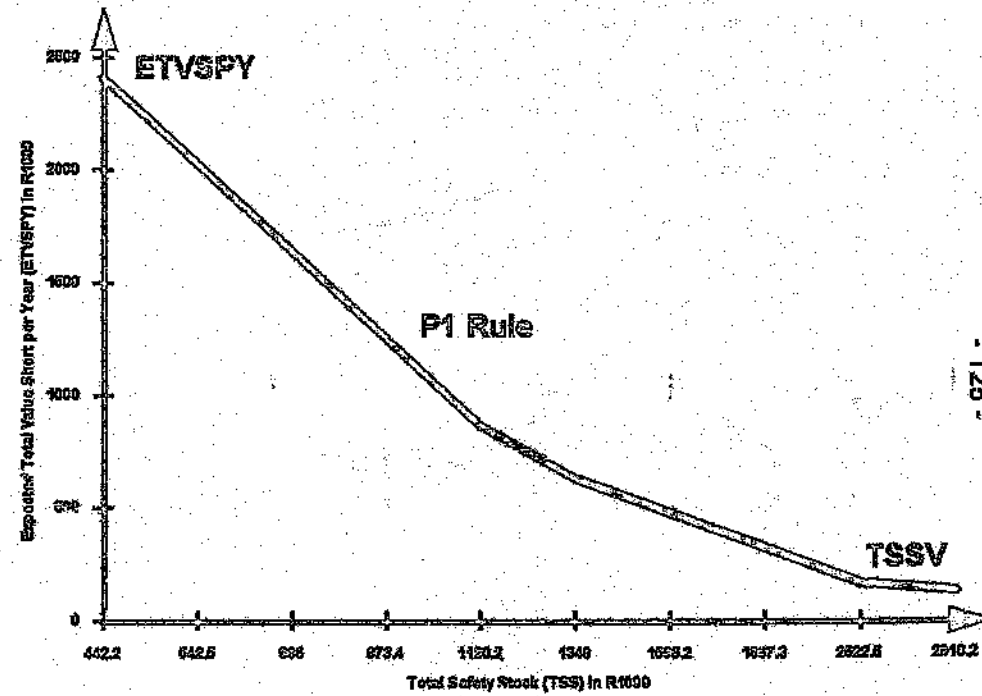
Exchange curves for the total SKUs sampled and for the P_1 decision rule is presented in Figure 15 below.

Total SKU's: Exchange curves for Probability P1 of no stockout per replenishment cycle

Safety Stock vs ETSOPY



Safety Stock vs ETVSPY



7. CONCLUSION, BENEFITS AND FUTURE DIRECTION FOR RESEARCH

7.1 CONCLUSIONS

As indicated in Section 2, the current inventory model is not adequate for use in the environment in which the company is operating. Models existing in the literature have been surveyed and those that can be applied in the environment in which the company is operating have been tested.

The proposed models also have been presented in a manner in which management can make better decisions on the required inventory policy through a comparison between the different inventory models and various policy variables.

The steps taken to minimise stock levels and to increase spare parts availability follow three dimensions:

- Lead time reduction - Intensive study has been carried out to analyse the current company lead time structure and the validity of current supplier lead time in use. The results of the study are detailed in Section 3.2.2.
- Modelling - The proposed models represent an improvement over the current inventory model from the following two aspects.
 - Usage of more readings (last twelve months readings instead of six readings) which has greater statistical validity
 - Usage of various probability distribution functions to describe the various demand patterns.

The effect of the above mentioned aspects are:

- Reducing the forecast error which affects the level of the safety stock required and therefore minimising holding costs (less stock is required).
- Reduction of the incident for stock out thus reducing the shortage costs.

Management attention should be directed to the following points concerning the proposed inventory models:

- Some of the proposed inventory models are elaborated with certain complexity on one hand, however with significant benefits on the other hand (see quantified benefits below).

The following is thus required:

- Staff training (buyers and stock controllers)
- Strong computational power to run the models and a sensitivity analysis. The implementation of the proposed inventory models can be supported by today's technology.
- Reordering frequency - changing current company reordering from four weeks to two weeks, reduce the average on hand quantity less safety stock by half.

As already indicated in Section 4.4 there is no effect on ordering cost due to the low staff utilisation.

7.2 BENEFITS

The financial benefits to be derived from implementing the proposed inventory models are as follows:

- Extrapolating the stock reduction achieved on the chosen sample onto the total stock will yield stock reduction of 30% or R15,16 million of total current stock on hand. This will yield savings in carrying charges of R3,49 million per annum, at current carrying charge rate of 23%.

These savings could be ascribed to the above three routes taken for stock reduction and increase of availability.

- Lead time reduction - 35% of total savings or R1,22 million per annum.
- Better modelling - 50% of total savings or R1,75 million per annum.
- Reducing reordering frequency from four weeks to two weeks - 15% of total savings of R0,52 million per annum.

Other benefits:

- Improved stock control
- Increased stock availability
- Reduction/elimination of obsolete stock in the future.

7.3 FUTURE DIRECTION FOR RESEARCH

7.3.1 MODELS FOR NEW SKUs

All proposed models discussed above (except the model in Procedure 5) assume that there is sufficient historical data, in order to establish the control parameters. SKUs in our environment that have insufficient historical data are the spare parts for new vehicles. Future research should address the development of new models to establish the inventory control parameters.

7.3.2 MODELS FOR SKUs HAVING TERMINAL DEMAND

Models that are nearing the end of their life cycle should be researched or developed. In Section 4.8.5 the Time increment Contingency Factor (TICF) method has been surveyed. The applicability and the effectiveness of this method to handle this type of SKUs should be simulated.

7.3.3 ASSURANCE SPARE PARTS

Procedure 5 in Section 5 has been developed to assist management to establish the control parameters for those SKUs that have no or very little demand. This procedure has been tested on very small number of SKUs (seven of the chosen sample). Future research should test its effectiveness on a wider scale.

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CURRENT INVENTORY VARIABLES CALCULATIONS

ITEM DEMAND PATTERN : RANDOM WITH LAST SIX MONTHS AVERAGE FIXED

COEFFICIENT OF VARIANCE RANGE : 0.0 - 2.4

MONTHLY READINGS Yr	MONTHLY AVERAGE					SM/ASM	v/Y	DEVIATION FROM AVERAGE					NEW AVERAGE DEMAND				SUGGESTED ORDER SIZE (LT=22W)				
	- Y	- Yr	- X	- Yr	Ym			STDS	mod	MAD	σ	or	SIGMA	mod	modr	NADr	NADs	SOS mod	SOS modr	SOS NADr	SOS NADs
1	25	25	25	25	25	SM	0.24	0.89	0.67	1	0.84	1.25	1	25.84	26.3	26	25.89	142	144	143	142
2	25	25	25	25	25	SM	0.39	2.37	2	2	2.5	2.5	3	27.5	27.5	28	27.37	151	151	154	151
3	25	25	25	25	25	SM	0.14	3.41	2.33	2	2.91	2.5	3	27.81	27.5	28	28.41	154	151	154	156
4	25	25	25	25	25	SM	0.14	3.41	3	3	3.75	3.75	4	28.75	28.8	29	29.41	158	158	160	156
5	25	25	25	25	25	SM	0.16	4	2.67	3	3.34	3.75	4	28.34	28.8	29	29	156	158	160	160
6	25	25	25	25	25	SM	0.37	9.23	8.33	8	10.41	10	10	35.41	35	35	34.23	185	193	193	158
7	25	25	25	25	27	ASM	0.41	10.2	8	8	10	10	10	35	35	35	35.2	193	193	193	194
8	24.67	25	24.67	25	22.5	ASM	0.41	10.15	7.33	7	9.15	8.75	9	33.53	33.8	34	34.82	186	186	187	192
9	25	25	25	25	25	SM	0.47	11.93	10	10	12.5	12.5	13	37.5	37.5	38	38.53	205	205	209	203
10	24.5	25	24.5	25	19.5	ASM	0.57	13.9	11.17	11	13.96	13.75	14	38.46	38.8	39	38.4	212	213	215	211
11	25	25	25	25	27.5	ASM	0.6	15.1	10.67	11	13.34	13.75	14	38.34	38.8	39	40.1	211	213	215	221
12	25	25	25	25	19	ASM	0.64	15.97	13.33	13	16.66	16.25	16	41.66	41.3	41	40.87	228	227	226	229
13	25	25	20	20	17.5	ASM	0.69	17.31	15.2	15	19	18.75	19	39.3	39.3	39	42.31	215	213	215	233
14	25.33	25	18.4	18	15	ASM	0.8	20.23	16.8	17	21	21.25	21	39.3	39.3	39	45.56	217	216	215	251
15	25	25	19.8	20	20.5	ASM	0.81	20.17	19.5	20	24.5	25	25	44.5	45	45	45.17	244	248	249	248
16	25	25	25	25	25	SM	0.82	20.6	18.33	18	22.91	22.5	23	47.91	47.5	48	45.6	264	261	264	251
17	25	25	19.2	19	22	ASM	0.83	20.74	21.2	21	26.5	26.25	26	45.7	45.3	45	45.74	251	241	248	252
18	25	25	18.2	18	15.5	ASM	0.89	22.36	19.6	20	24.5	25	25	42.7	43	43	47.36	235	237	237	260
19	25	25	20	20	25	SM	0.9	22.58	24.4	24	30.5	30	30	50.5	50	50	47.58	278	275	275	262
20	24.83	25	11.5	12	21	ASM	0.91	22.49	28.25	29	36.56	36.25	36	49.06	48.3	48	47.32	264	265	264	260
21	25	25	11.5	12	21	ASM	0.93	23.32	29.5	30	38.88	37.5	38	48.38	49.5	50	48.32	266	272	275	266
22	25	25	19.6	20	25	SM	0.93	23.27	24	24	30	30	30	49.6	50	50	48.27	273	275	275	265
23	25	25	12	12	24	SM	1.1	27.41	37.5	38	46.88	41.5	48	58.88	59.5	60	52.41	324	327	330	288
24	25.33	25	8.5	9	17	ASM	1.1	27.81	33.5	34	41.88	42.5	43	50.38	51.5	52	53.14	277	283	286	292
25	25.17	25	7.5	8	13	ASM	1.14	28.62	31.75	32	39.68	40	40	47.19	48	48	53.79	260	264	264	296
26	25	25	10.4	10	14	ASM	1.47	36.79	25.2	25	31.5	31.25	31	41.9	41.3	41	61.79	230	227	226	340
27	25	25	10.4	10	6	ASM	1.56	38.88	29.2	29	36.5	36.25	36	46.9	46.3	46	63.88	258	254	253	351
28	25	25	1	1	2	ASM	1.58	39.49	37	37	46.25	46.25	46	47.25	47.3	47	64.49	260	260	259	355
29	25	25	4	4	5.5	ASM	2.06	51.61	29.2	29	34.5	36.25	36	40.5	40.3	40	76.61	223	221	220	421
30	25	25	1.6	2	7	ASM	2.29	57.35	30	30	37.5	37.5	38	39.1	39.5	40	82.35	215	217	220	453

CORRELATION ANALYSIS SUMMARY

ITEM DEMAND PATTERN : RANDOM WITH LAST SIX MONTHS AVERAGE FIXED

COEFFICIENT OF VARIANCE RANGE : 0.0 - 2.4

V_A
STDEsq = $\sigma^2 + b$

Regression Output:		
Constant	1.8448	
Std Err of Y Est	7.6302	36.8
R Squared	0.7074	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.028
Std Err of Coef. 0.125

V_B
STDEsq = $\sigma^2 + b$

Regression Output:		
Constant	1.846	
Std Err of Y Est	7.63076	36.8
R Squared	0.70736	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 0.972
Std Err of Coef. 0.1

V_C
STDEsq = $\sigma^2 + b$

Regression Output:		
Constant	2.0285	
Std Err of Y Est	7.7272	37.3
R Squared	0.6999	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 0.8139
Std Err of Coef. 0.1007

V_D
STDEsq = $\sigma^2 + b$

Regression Output:		
Constant	1.92853	
Std Err of Y Est	7.770694	37.5
R Squared	0.692576	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 0.8145
Std Err of Coef. 0.101591

V_E
NADsq = $\sigma^2 + b$

Regression Output:		
Constant	4.5247	
Std Err of Y Est	11.404	24.9
R Squared	0.3489	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.008
Std Err of Coef. 0.26

V_F
NADsq = $\sigma^2 + b$

Regression Output:		
Constant	5.4958	
Std Err of Y Est	11.4771	25.3
R Squared	0.34048	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 0.982
Std Err of Coef. 0.258

V_G
NADsq = $\sigma^2 + b$

Regression Output:		
Constant	5.7851	
Std Err of Y Est	11.545	25.3
R Squared	0.3376	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 0.9722
Std Err of Coef. 0.2605

V_H
SOS(NADs) = $\sigma^2 \text{SOS}(nAD) + b$

Regression Output:		
Constant	25.368	
Std Err of Y Est	62.75	24.9
R Squared	0.3478	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.005
Std Err of Coef. 0.26

V_I
SOS(NADs) = $\sigma^2 \text{SOS}(nAD) + b$

Regression Output:		
Constant	30.9038	
Std Err of Y Est	63.1548	25.3
R Squared	0.33894	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 0.979
Std Err of Coef. 0.258

V_J
SOS(NADs) = $\sigma^2 \text{SOS}(nAD) + b$

Regression Output:		
Constant	21.766	
Std Err of Y Est	63.499	25.3
R Squared	0.3317	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 0.9716
Std Err of Coef. 0.2606

CURRENT INVENTORY MODEL SIMULATION

ITEM DEMAND PATTERN : LOW COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 0.0 - 0.5

STREAM N	1	2	3	4	5	E	SEMS/r	1r	2r	3r	1	2	3	4	5	6
1	42	43	42	42	43	41	0.02	42	44	42	42	43	42	42	43	41
2	39	39	39	39	39	39	0.03	39	40	39	39	39	39	39	39	39
3	15	14	15	14	14	14	0.04	14	15	14	15	14	15	14	14	14
4	50	55	57	56	50	58	0.06	56	112	56	60	55	57	56	50	55
5	24	22	21	20	23	21	0.07	22	44	22	24	22	21	20	23	21
6	30	31	34	29	32	28	0.07	31	32	31	30	31	34	29	32	28
7	14	15	12	3	11	15	0.08	15	25	13	14	13	12	15	11	13
8	25	22	22	26	26	21	0.1	24	48	24	25	22	22	26	26	21
9	25	28	24	20	21	21	0.11	23	48	23	25	26	24	20	21	21
10	12	7	10	8	9	8	0.13	9	18	8	10	7	10	8	9	8
11	7	7	7	6	5	5	0.16	6	12	6	7	7	7	6	5	5
12	14	15	12	10	10	15	0.17	12	24	12	14	13	12	10	10	15
13	2	2	2	2	2	2	0.19	2	4	2	2	2	2	2	2	2
14	12	13	8	11	7	10	0.21	10	20	10	12	13	9	11	7	10
15	20	25	26	14	25	19	0.23	22	44	22	20	25	26	14	25	19
16	4	3	3	2	4	4	0.24	3	5	3	4	3	3	2	4	3
17	31	31	17	16	23	24	0.25	22	44	22	31	31	17	16	23	24
18	7	11	15	15	9	10	0.25	11	22	11	7	11	15	15	9	10
19	38	40	40	23	21	40	0.28	38	72	38	38	40	40	23	21	40
20	24	18	15	17	10	14	0.28	16	32	16	24	18	15	17	10	14
21	25	33	45	20	28	27	0.3	30	60	30	25	33	45	20	28	27
22	50	36	39	40	17	40	0.32	35	70	35	50	36	39	40	17	40
23	5	10	11	5	8	8	0.34	8	16	8	9	10	11	5	8	8
24	26	20	10	16	11	12	0.39	16	32	16	26	20	10	16	11	12
25	13	8	17	25	28	15	0.4	16	28	16	13	8	17	25	28	15
26	27	7	20	18	14	26	0.42	23	46	23	27	7	20	18	14	26
27	50	32	47	27	11	30	0.43	33	66	33	50	32	47	27	11	30
28	24	7	13	21	10	24	0.44	19	38	19	24	7	13	21	10	24
29	10	7	8	16	21	8	0.45	12	24	12	10	7	8	16	21	8
30	3	11	9	7	5	9	0.48	7	14	7	3	11	9	7	5	9

STREAM N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	41	39	14	50	23	22	11	21	20	7	5	10	2	7	14	2	16	7	21	10	20	17	5	10	6	7	11	2	7
2	42	26	14	55	21	29	12	22	21	8	6	10	2	8	19	2	17	9	25	14	23	29	8	11	13	10	27	13	7
3	42	30	14	56	21	30	13	23	21	9	6	12	2	10	20	3	21	10	38	15	27	35	8	12	15	20	30	19	8
4	42	30	14	57	22	31	13	25	24	9	7	13	2	11	25	3	23	11	40	17	28	40	9	16	17	27	32	21	10
5	43	21	16	58	23	32	13	26	25	10	7	14	2	12	25	4	24	13	46	18	33	43	10	20	23	38	47	24	16
6	43	21	16	59	24	32	14	26	25	10	7	15	3	13	28	4	31	15	45	24	45	50	11	26	28	34	50	24	16
7m	42	20	14	56.5	21.5	30.5	13	23.5	22.5	9	6.5	12.5	2	10.5	22.5	3	22	10.5	38	16	28	37.5	8.5	14	16	23.5	31	20	13.5
8m	42	20	14	56.5	21.5	30.5	13	23.5	22.5	9	6.5	12.5	2	10.5	22.5	3	22	10.5	38	16	28	37.5	8.5	14	16	23.5	31	20	13.5

CURRENT INVENTORY VARIABLES CALCULATIONS

ITEM DEMAND PATTERN : LOW COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 0.0 - 0.5

MONTHLY READINGS Yr	MONTHLY AVERAGE					SW/ASM	STDS/Y	DST	DEVIATION FROM AVERAGE					NEW AVERAGE DEMAND				SUGGESTED ORDER SIZE (LT=22W)				
	Y	Yr	X	Yr	Ym				STDS	mod	MAD	σ	σ^2	SGMA	mad	modr	MADr	NADs	SOS mod	SOS modr	SOS MADr	SOS NADs
1	42.17	42	42.17	42	42	SM	0.02	R	0.75	0.5	1	0.63	1.25	1	42.8	43.25	43	42.92	235	238	237	236
2	20.33	20	20.33	20	20	SM	0.03	R	0.52	0.33	0	0.41	0	0	20.7	20	20	20.85	114	110	110	115
3	14.33	14	14.33	14	14	SM	0.04	R	0.52	0.33	0	0.41	0	0	14.7	14	14	14.85	81	77	77	82
4	56	56	56	56	56.5	SM	0.06	R	3.41	2.33	2	2.81	2.5	3	58.9	58.5	59	59.41	324	322	325	327
5	21.83	22	21.83	22	21.5	SM	0.07	R	1.47	1.17	1	1.46	1.25	1	23.3	23.25	23	23.3	128	128	127	128
6	30.67	31	30.67	31	30.5	SM	0.07	R	2.16	1.67	2	2.09	2.5	3	32.8	33.5	34	32.83	189	184	187	181
7	12.67	13	12.67	13	13	SM	0.08	R	1.05	0.67	1	0.84	1.25	1	13.5	14.25	14	13.7	74	78	77	75
8	23.67	24	23.67	24	23.5	SM	0.1	R	2.25	2	2	2.5	2.5	3	26.2	25.5	27	25.92	144	146	149	143
9	22.83	23	22.83	23	22.5	SM	0.11	R	2.48	2.17	2	2.71	2.5	3	25.5	25.5	26	25.31	140	140	145	139
10	8.83	9	8.83	9	9	SM	0.13	R	1.17	0.83	1	1.04	1.25	1	9.87	10.25	10	10	54	56	55	55
11	6.17	6	6.17	6	6.5	ASM	0.15	R	0.98	0.83	1	1.04	1.25	1	7.21	7.25	7	7.15	40	40	39	39
12	12.33	12	12.33	12	12.5	SM	0.17	R	2.07	1.67	2	2.09	2.5	3	14.4	14.5	15	14.4	79	80	83	79
13	2.17	2	2.17	2	2	ASM	0.19	R	0.41	0.17	0	0.21	0	0	2.38	2	2	2.58	13	11	11	14
14	10.33	10	10.33	10	10.5	SM	0.21	R	2.16	1.67	2	2.09	2.5	3	12.4	12.5	13	12.49	58	69	72	69
15	21.83	22	21.83	22	22.5	SM	0.23	R	5.12	4.17	4	5.21	5	5	27	27	27	26.95	149	149	149	148
16	3.17	3	3.17	3	3	ASM	0.24	R	0.75	0.5	1	0.53	1.25	1	3.8	4.25	4	3.92	21	23	22	22
17	22	22	22	22	22	SM	0.25	R	5.44	4	4	5	5	5	27	27	27	27.44	149	149	149	151
18	10.83	11	10.83	11	10.5	SM	0.26	R	2.66	2.17	2	2.71	2.5	3	13.5	13.5	14	13.69	74	74	77	75
19	36	36	36	36	39	ASM	0.28	R	9.94	7.67	8	9.59	10	10	45.5	46	46	45.84	251	253	253	253
20	15.33	16	15.33	16	16	SM	0.29	R	4.68	3.33	3	4.16	3.75	4	29.5	19.75	20	21.01	113	109	110	116
21	29.5	30	29.5	30	29	ASM	0.3	R	8.85	6.5	7	8.13	8.75	9	37.6	38.75	39	38.35	207	213	215	211
22	35.17	35	35.17	35	37.5	ASM	0.32	R	11.27	8.17	8	10.21	10	10	45.4	45	45	46.44	250	248	248	255
23	8.17	8	8.17	8	8.5	SM	0.34	R	2.79	1.83	2	2.29	2.5	3	10.5	10.5	11	10.96	58	58	61	60
24	15.83	16	15.83	16	14	ASM	0.39	R	8.21	4.83	5	8.04	6.25	6	21.8	22.25	22	22.04	120	122	121	121
25	17.5	18	17.5	18	16	ASM	0.4	R	6.92	5.5	6	6.88	7.5	8	24.4	23.5	26	24.42	134	140	143	134
26	22.5	23	22.5	23	23.5	SM	0.42	R	9.4	7.17	7	8.96	8.75	9	31.5	31.75	32	31.9	173	175	176	175
27	32.83	33	32.83	33	31	ASM	0.43	R	14.25	10.5	11	13.13	13.75	14	46	46.75	47	47.08	253	257	259	259
28	19.17	19	19.17	19	20	SM	0.44	R	8.4	6.17	6	7.71	7.5	8	26.9	26.5	27	27.37	148	146	148	152
29	12	12	12	12	9.5	ASM	0.45	R	5.37	4.33	4	5.41	5	5	17.4	17	17	17.37	96	94	94	96
30	7	7	7	7	8	ASM	0.48	R	3.35	2.67	3	3.34	3.75	4	10.3	10.25	11	10.35	57	59	61	57

APPENDIX 2 (continued)

CORRELATION ANALYSIS SUMMARY

ITEM DEMAND PATTERN : LOW COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 0.0 - 0.5

\sqrt{a}
STDSes = mad + b

Regression Output:

Constant	-0.028	
Std Err of Y Est	0.2498	5.9
R Squared	0.9956	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.33
Std Err of Coef. 0.02

\sqrt{b}
STDSes = a + b

Regression Output:

Constant	-0.0251	
Std Err of Y Est	0.24579	5.9
R Squared	0.99555	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.057
Std Err of Coef. 0.013

\sqrt{c}
STDSes = ar + b

Regression Output:

Constant	-0.02	
Std Err of Y Est	0.4795	11.3
R Squared	0.9836	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.04155
Std Err of Coef. 0.02541

\sqrt{d}
STDSes = SCMA + b

Regression Output:

Constant	-0.083	
Std Err of Y Est	0.5589	13.2
R Squared	0.9777	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.0195
Std Err of Coef. 0.0251

\sqrt{e}
NADs = a + mad + b

Regression Output:

Constant	-0.055	
Std Err of Y Est	0.2943	1.2
R Squared	0.9996	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.01
Std Err of Coef. 0.00

\sqrt{f}
NADs = a * mad + b

Regression Output:

Constant	-0.0209	
Std Err of Y Est	0.51075	2.5
R Squared	0.9982	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.005
Std Err of Coef. 0.008

\sqrt{g}
NADs = a * MAD + b

Regression Output:

Constant	-0.07	
Std Err of Y Est	0.6805	2.8
R Squared	0.9978	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.00173
Std Err of Coef. 0.00896

\sqrt{h}
SOS(NADs) = a * SOS(mad) + b

Regression Output:

Constant	-0.26	
Std Err of Y Est	1.5022	1.1
R Squared	0.9996	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.01
Std Err of Coef. 0.00

\sqrt{i}
SOS(NADs) = a * SOS(mad) + b

Regression Output:

Constant	-0.0267	
Std Err of Y Est	3.27205	2.5
R Squared	0.99829	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.005
Std Err of Coef. 0.008

\sqrt{j}
SOS(NADs) = a * SOS(MAD) + b

Regression Output:

Constant	-0.558	
Std Err of Y Est	3.7747	2.9
R Squared	0.9977	
No. of Observations	30	
Degrees of Freedom	28	

X Coefficient(s) 1.00119
Std Err of Coef. 0.00903

CURRENT INVENTORY MODEL SIMULATION

ITEM DEMAND PATTERN : MEDIUM COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 0.5 - 1.0

STREAM #	1	2	3	4	5	6	STDS/Y	7	8	9	10	11	12	13	14	15	16
1	12	8	3	7	4	5	0.5	7	14	7	12	8	3	7	4	5	
2	50	35	12	40	17	40	0.51	34	15	34	50	35	12	40	17	40	
3	10	45	23	57	29	27	0.55	37	74	37	10	43	23	57	29	27	
4	10	7	9	23	21	9	0.61	15	25	15	10	7	9	23	21	9	
5	50	4	47	27	19	30	0.55	25	55	28	50	4	47	27	19	30	
6	13	8	17	35	22	4	0.57	15	35	18	13	8	17	35	22	4	
7	55	50	45	20	2	57	0.58	49	54	39	55	50	45	20	2	57	
8	2	45	40	25	21	9	0.72	25	39	25	2	45	40	25	21	9	
9	27	7	50	41	34	25	0.74	35	75	37	27	7	50	41	34	25	
10	9	10	25	3	1	31	0.77	15	30	11	9	10	25	3	1	31	
11	31	21	17	51	23	31	0.78	35	72	25	31	21	17	51	23	31	
12	9	59	39	59	21	21	0.78	35	75	25	9	9	30	59	21	21	
13	3	11	23	7	3	9	0.3	9	15	7	3	11	7	3	9	9	
14	1	35	21	22	2	40	0.81	20	40	10	1	35	21	22	2	40	
15	12	0	5	19	4	23	0.84	11	22	5	12	0	5	19	4	23	
16	8	3	19	2	12	31	0.85	15	25	9	8	3	19	2	12	31	
17	14	47	10	20	10	20	0.85	29	55	20	14	47	10	20	10	20	
18	10	7	23	3	0	1	0.85	9	15	5	10	7	23	3	0	1	
19	30	50	100	4	100	0	0.85	49	55	23	30	50	100	4	100	0	
20	24	57	15	41	10	3	0.85	27	54	19	24	24	57	15	41	10	
21	55	12	1	44	39	21	0.89	4	55	23	55	12	1	44	39	21	
22	50	5	20	10	10	10	0.9	25	52	19	50	5	20	10	10	10	
23	3	32	7	53	23	16	0.91	24	48	15	3	32	7	53	23	16	
24	14	0	20	55	5	50	0.93	25	50	11	14	0	20	55	5	50	
25	8	9	24	34	1	5	0.94	14	25	9	8	9	24	34	1	5	
26	20	59	28	14	55	3	0.95	37	74	25	20	59	28	14	55	3	
27	2	2	42	17	17	12	0.98	15	30	10	2	2	42	17	17	12	
28	0	7	13	28	55	31	0.95	10	50	21	0	7	13	28	55	31	
29	25	14	3	55	1	50	0.95	37	74	25	25	14	3	55	1	50	
30	5	29	25	5	70	11	0.99	25	50	15	5	29	25	5	70	11	

STREAM #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	
1	3	12	10	5	4	4	5	7	5	17	8	3	7	0	2	10	1	4	3	1	5	3	0	1	3	2	0	1	1	
2	4	17	23	7	11	9	20	9	27	8	21	21	3	7	4	5	10	3	0	10	12	10	2	5	14	2	7	10	1	
3	5	35	27	9	27	13	45	21	28	9	23	21	7	21	5	14	7	35	15	21	10	18	14	8	20	12	13	25	3	
4	7	40	43	10	38	17	58	28	34	10	31	30	9	22	12	20	9	59	24	30	29	25	20	9	25	17	31	31	5	
5	8	49	59	21	47	25	67	40	41	25	34	50	11	25	18	18	47	10	109	41	44	50	32	50	24	55	17	55	65	6
6	12	50	57	23	55	25	95	49	50	31	51	59	23	40	25	31	70	25	100	57	55	50	53	55	34	55	42	70	50	10
Tot	5	37.5	35	9.5	28.5	15	50.5	24.5	31	9.5	27	28.5	8	21.5	9	10.5	17	0	40	19.5	25.5	15	19.5	17	5.5	24	14.5	22	30	10
	5	37.5	35	9.5	28.5	15	50.5	24.5	31	9.5	27	28.5	8	21.5	9	10.5	17	0	40	19.5	25.5	15	19.5	17	5.5	24	14.5	22	30	10

CURRENT INVENTORY VARIABLES CALCULATIONS

ITEM DEMAND PATTERN : MEDIUM COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 0.5 - 1.0

MONTHLY READINGS Y	MONTHLY AVERAGE					SM/ASM	STDS/Y	JST	DEVIATION FROM AVERAGE					NEW AVERAGE DEMAND				SUGGESTED ORDER SIZE (LT=22W)				
	Y	Yr	X	Yr	Ym				STDS	mod	MAD	σ	σr	σGMA	mod	modr	NAOr	NAOs	SOS mod	SOS modr	SOS NAOr	SOS NAOs
1	6.5	7	6.5	7	6	ASM	0.5		3.27	2.5	3	3.13	3.75	4	9.53	10.75	11	9.77	53	59	61	61
2	34	34	34	34	37.5	ASM	0.51	R	17.45	13	13	16.25	16.25	16	50.3	50.25	50	51.45	276	276	275	283
3	36.67	37	36.67	37	35	SM	0.56	R	20.64	16.67	17	20.84	21.25	21	57.5	58.25	58	57.31	316	320	319	315
4	12.5	13	12.5	13	9.5	ASM	0.61	R	7.58	6.5	7	8.13	8.75	9	20.6	21.75	22	20.08	113	120	121	110
5	28.17	28	28.17	28	28.5	SM	0.66	R	18.52	14.17	14	17.71	17.5	18	45.9	45.5	46	46.69	252	250	253	257
6	17.67	18	17.67	18	15	ASM	0.67	R	11.76	9.33	9	11.66	11.25	11	29.3	29.25	29	29.43	161	161	160	162
7	48.5	49	48.5	49	50.5	SM	0.68	R	33.36	32.6	33	40.75	41.25	41	79.4	80.25	80	81.66	436	441	440	450
8	24.83	25	24.83	25	24.5	SM	0.72	R	17.95	14.17	14	17.71	17.5	18	42.5	42.5	43	42.78	234	234	237	235
9	37.83	38	37.83	38	31	ASM	0.74	R	27.97	21	21	26.25	26.25	26	53.7	53.25	53	55.8	295	293	292	362
10	14.5	15	14.5	15	9.5	ASM	0.77	R	11.22	9.8	10	12.25	12.5	13	23.5	23.5	24	25.72	129	129	132	141
11	35.67	36	35.67	36	27	ASM	0.78	R	27.67	18.4	18	17.71	17.5	18	47.6	47.5	48	48.34	262	261	264	369
12	38.33	38	38.33	38	25.5	ASM	0.79	R	30.22	25.6	26	32	32.5	33	60.2	60.5	61	68.55	331	333	336	377
13	9.53	9	9.53	9	8	ASM	0.8	R	7.42	6	6	7.5	7.5	8	14.1	14.5	15	16.79	78	80	83	92
14	20.33	20	20.33	20	21.5	ASM	0.81	R	16.4	16.8	17	21	21.25	21	37.4	37.25	37	36.73	206	205	204	202
15	10.67	11	10.67	11	9	ASM	0.84	R	8.98	8.8	9	11	11.25	11	19.2	19.25	19	19.65	106	106	105	108
16	12.67	13	12.67	13	10.5	ASM	0.86	R	10.93	9.6	10	12	12.5	13	21	21.5	22	23.6	116	116	121	130
17	28.5	29	28.5	29	17	ASM	0.86	R	24.62	20.6	21	25.75	26.25	26	46	46.25	46	53.12	253	254	253	292
18	8.83	9	8.83	9	8	ASM	0.88	R	7.76	6.6	7	8.25	8.75	8	14.3	14.75	15	16.59	78	81	83	91
19	48.53	49	48.53	49	40	ASM	0.88	R	42.87	55.25	55	69.06	68.75	69	92.3	91.75	92	91.7	508	505	505	504
20	26.67	27	26.67	27	19.5	ASM	0.89	R	23.72	20.8	21	26	26.25	26	44.6	45.25	45	50.39	245	249	248	277
21	33.5	34	33.5	34	29.5	ASM	0.99	R	29.66	26.6	27	33.25	33.75	34	56.5	56.75	57	63.36	310	312	314	349
22	25.83	26	25.83	26	15	ASM	0.9	R	23.33	21	21	26.25	26.25	26	45.3	45.25	45	49.16	249	249	248	270
23	24	24	24	24	19.5	ASM	0.91	R	21.82	18.4	18	23	22.5	23	39.2	38.5	39	45.82	216	212	215	252
24	24.67	25	24.67	25	17	ASM	0.93	R	23	27.5	28	34.38	35	35	44.9	46	46	47.67	247	253	253	262
25	13.5	14	13.5	14	8.5	ASM	0.94	R	12.72	10.6	11	13.25	13.75	14	22.7	22.75	23	26.22	125	125	127	144
26	35.67	37	35.67	37	24	ASM	0.95	R	34.7	29.2	29	36.5	36.25	36	61.3	61.25	61	71.37	337	337	336	393
27	15.33	15	15.33	15	14.5	ASM	0.96	R	14.72	12.8	13	16	16.25	16	26	26.25	26	30.05	143	144	143	165
28	29.5	30	29.5	30	21.4	ASM	0.96	R	28.26	27.4	27	34.25	33.75	34	55.7	54.75	55	57.76	306	301	303	318
29	36.67	37	36.67	37	30	ASM	0.96	R	35.3	32	32	40	40	40	66	66	66	71.97	363	353	363	396
30	24.83	25	24.83	25	19.5	ASM	0.99	R	24.52	21	21	26.25	26.25	26	42.1	42.25	42	49.36	231	232	231	271

CORRELATION ANALYSIS SUMMARY

ITEM DEMAND PATTERN : MEDIUM COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 0.5 - 1.0

VA
STDSca = a * mod + b

Regression Output:

Constant	4.7048	
Std Err of Y Est	3.4209	16.6
R Squared	0.8834	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	0.85	
Std Err of Coef.	0.06	

VB
STDSca = a * r + b

Regression Output:

Constant	4.70422	
Std Err of Y Est	3.42112	16.6
R Squared	0.88336	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	0.8599	
Std Err of Coef.	0.0600	

VC
STDSca = a * rr + b

Regression Output:

Constant	4.5505	
Std Err of Y Est	3.5006	17
R Squared	0.8779	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	0.69108	
Std Err of Coef.	0.04671	

VD
STDSca = a * SOWA + b

Regression Output:

Constant	4.454	
Std Err of Y Est	3.5272	17.1
R Squared	0.876	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	0.6927	
Std Err of Coef.	0.0493	

VE
NACS = a * mod + b

Regression Output:

Constant	1.2792	
Std Err of Y Est	3.9954	8.7
R Squared	0.8653	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	1.05	
Std Err of Coef.	0.04	

V
NACS = a * mod + b

Regression Output:

Constant	1.26987	
Std Err of Y Est	4.10247	8.9
R Squared	0.86351	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	1.057	
Std Err of Coef.	0.039	

VG
NACS = a * NACr + b

Regression Output:

Constant	1.0232	
Std Err of Y Est	4.0903	8.9
R Squared	0.8637	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	1.06051	
Std Err of Coef.	0.03889	

VI
SOS(NACr) = a * SOS(mod) + b

Regression Output:

Constant	9.325	
Std Err of Y Est	21.958	8.7
R Squared	0.8654	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	1.05	
Std Err of Coef.	0.04	

V
SOS(NACr) = a * SOS(mod) + b

Regression Output:

Constant	6.91524	
Std Err of Y Est	22.5163	8.9
R Squared	0.86341	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	1.057	
Std Err of Coef.	0.039	

VJ
SOS(NACr) = a * SOS(NACr) + b

Regression Output:

Constant	5.0674	
Std Err of Y Est	22.469	8.9
R Squared	0.8638	
No. of Observations	30	
Degrees of Freedom	28	
X Coefficient(s)	1.06134	
Std Err of Coef.	0.03886	

CURRENT INVENTORY MODEL SIMULATION

ITEM DEMAND PATTERN : HIGH COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 1.0 - 2.0

STREAM #	1	2	3	4	5	6	STDS/Y	1	2	3	4	5	6		
1	110	4	47	27	11	55	1	38	76	24	4	47	27	11	55
2	87	5	12	40	50	0	124	54	88	23	3	12	40	50	0
3	60	5	20	10	20	10	127	30	80	18	0	20	10	20	10
4	1	47	155	0	10	70	1.3	18	78	26	1	47	0	10	70
5	1	22	7	83	128	16	1.55	42	84	24	3	22	7	83	16
6	8	174	45	68	1	21	1.9	83	126	20	8	174	45	68	1
7	8	29	124	1	73	11	1.2	42	84	25	8	29	1	73	11
8	27	7	158	41	0	28	1.23	43	85	24	27	7	158	41	0
9	25	24	5	55	1	168	1.20	50	100	26	25	24	5	55	1
10	0	83	0	20	5	74	1.29	23	66	6	0	0	20	5	74
11	10	45	0	129	22	27	1.3	53	106	28	10	45	0	129	22
12	15	2	37	51	3	131	1.21	42	84	7	10	2	37	51	3
13	8	0	80	34	1	5	1.54	16	32	10	8	0	80	34	1
14	25	0	1	0	26	21	1.25	24	48	12	0	1	0	26	21
15	14	0	20	126	6	50	1.42	82	164	14	0	20	126	6	50
16	2	2	42	17	0	100	1.44	54	108	13	2	2	42	17	0
17	0	23	0	41	10	3	1.46	48	96	11	0	0	41	10	3
18	13	0	37	112	29	0	1.48	24	48	12	13	0	37	112	29
19	0	59	26	14	156	3	1.48	10	20	21	0	59	26	14	156
20	10	7	23	80	0	1	1.51	23	46	6	10	7	23	80	0
21	1	0	23	0	0	9	1.56	6	12	2	1	0	23	0	9
22	2	0	200	4	100	8	1.58	53	106	23	2	0	4	100	8
23	10	7	1	38	21	2	1.58	24	48	8	10	7	1	38	21
24	0	3	52	2	0	43	1.61	23	46	10	0	3	52	2	0
25	12	0	0	19	4	59	1.61	23	46	8	12	0	0	19	4
26	2	117	0	76	5	9	1.63	27	54	9	2	117	0	76	5
27	0	7	1	122	0	21	1.77	25	50	10	0	7	1	122	0
28	1	135	0	1	1	40	1.8	10	20	3	1	135	0	1	40
29	12	0	0	27	0	5	1.82	17	34	3	12	0	0	27	0
30	8	0	26	3	1	231	2.04	45	90	8	8	0	26	3	231

STREAM #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	4	0	0	0	3	3	1	0	1	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	
2	11	3	10	1	7	8	6	2	3	0	2	1	0	3	2	0	0	3	1	0	2	4	0	0	2	0	0	1	1	
3	27	12	19	10	16	23	11	27	26	5	7	10	5	14	2	3	15	14	7	0	4	7	0	0	8	5	7	1	1	
4	10	40	20	47	32	45	20	28	34	20	4	8	21	20	17	10	17	25	18	3	9	15	3	12	8	13	2	2	2	
5	47	50	30	70	33	58	28	41	65	74	50	61	34	35	53	42	41	28	38	23	8	103	31	43	19	28	31	40	10	
6	110	57	80	105	128	171	158	138	168	50	180	181	60	85	156	100	83	110	156	20	33	200	10	82	88	117	160	12	10	
Yes	26.5	26	15	25.5	24	33	20	27.5	33	12.5	35	13.5	6.5	11	17	6.5	6.5	15	21	8.5	1.5	6.5	6.5	7.5	9	7	10	15	1.5	
	28.5	26	15	26.5	24	33	20	27.5	30	12.5	35	13.5	6.5	11	17	6.5	6.5	15	21	8.5	1.5	6.5	6.5	7.5	9	7	10	15	1.5	

CURRENT INVENTORY VARIABLES CALCULATIONS

ITEM DEMAND PATTERN : HIGH COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 1.0 - 2.0

MONTHLY REORDER Yr	MONTHLY AVERAGE					SM/ ASM	STDS/Y	ZST	DEVIATION FROM AVERAGE					NEW AVERAGE DEMAND				SUGGESTED ORDER SIZE (LT=22W)				
	- Y	- Yr	- Y	- Yr	- Yr				STDS	mod	MAD	σ	σ^2	SCMA	mod	modr	NADR	NADr	SOS mod	SOS modr	SOS NADR	SOS NADr
1	38.17	38	23.8	24	28.5	ASM	1	R	38.5	30.2	30	37.75	37.5	38	51.6	61.5	62	76.47	339	328	341	421
2	33.67	34	23	23	25	ASM	1.04	R	34.99	34.4	34	43	42.5	43	56	65.5	66	68.66	363	360	363	378
3	30	30	19	10	15	ASM	1.07	R	32.25	35	35	43.75	43.75	44	53.8	53.75	54	62.25	256	256	297	342
4	38.23	39	25.6	26	28.5	ASM	1.1	R	42.86	42.2	42	52.75	52.5	53	78.4	78.5	79	81.69	431	432	435	449
5	41.5	42	24.2	24	24	ASM	1.15	R	47.66	39.4	39	48.25	48.75	49	73.5	72.75	73	85.16	404	400	402	490
6	53.17	53	29.2	29	33	ASM	1.19	R	63.48	50.8	51	63.25	63.75	64	92.5	92.75	93	116.55	508	510	512	642
7	42.17	42	25	25	20	ASM	1.2	R	50.59	43.4	43	54.25	53.75	54	79.3	78.75	79	92.76	436	433	435	510
8	40.33	40	20.5	21	27	ASM	1.25	R	50.6	37.2	37	46.5	46.25	46	67.1	67.25	67	90.93	359	370	369	500
9	49.67	50	26	26	30	ASM	1.26	R	62.64	47.6	48	59.5	60	60	85.5	86	86	112.31	470	473	473	618
10	32.63	33	6.25	6	12.5	ASM	1.29	R	42.51	46.75	47	58.44	58.75	59	84.7	84.75	85	75.34	356	356	359	414
11	53.17	53	26	26	35	ASM	1.3	R	69.19	48.4	49	61.75	61.25	61	87.8	87.25	87	122.35	483	480	479	673
12	41.83	42	7.25	7	13.5	ASM	1.32	R	55.38	58.25	58	72.81	72.5	73	90.1	79.5	80	97.31	440	437	440	535
13	18	18	9.5	10	5.5	ASM	1.34	R	24.09	20	20	25	25	25	34.5	35	35	42.09	190	193	193	231
14	24.17	24	12	12	11	ASM	1.39	R	33.49	28.6	29	35.75	36.25	36	47.8	48.25	48	57.65	263	265	264	317
15	41.33	41	18.4	18	17	ASM	1.42	R	58.73	40.8	41	51	51.25	51	69.4	69.25	69	100.26	352	361	360	530
16	27.17	27	12.5	13	9.5	ASM	1.44	R	39.07	31	31	38.75	38.75	39	51.4	51.75	52	66.24	282	255	286	364
17	22.83	23	10.8	11	6.5	ASM	1.45	R	33.32	26.6	27	33.25	33.75	34	44.1	44.75	45	56.15	242	246	248	309
18	28	28	11.6	12	15	ASM	1.48	R	41.56	29.9	29	36	36.25	36	47.6	48.25	48	59.56	262	265	264	363
19	50	50	20.8	21	21	ASM	1.49	R	74.68	53.2	53	65.5	66.25	66	87.3	87.25	87	124.68	480	480	479	666
20	70.17	70	8.2	8	8.5	ASM	1.51	R	50.46	21	21	26.25	26.25	26	34.5	34.25	34	50.63	189	188	187	278
21	5.83	6	2.4	2	1.5	ASM	1.56	R	9.11	7	7	8.75	8.75	9	11.2	10.75	11	14.94	61	59	61	82
22	52.5	53	23	23	8.5	ASM	1.56	R	81.93	66.2	66	82.75	82.5	83	106	105.5	106	134.43	582	580	563	738
23	23.67	24	8.6	9	8.5	ASM	1.59	R	37.53	23.6	24	29.5	30	30	38.1	39	39	61.2	210	215	215	337
24	23.33	23	9.8	10	2.5	ASM	1.61	R	37.57	30	30	37.5	37.5	39	47.1	47.5	48	60.9	259	261	264	335
25	23.33	23	8.2	8	9	ASM	1.61	R	37.66	24	24	30	30	30	38.2	38	38	60.89	210	209	209	335
26	26.83	27	5.9	9	7	ASM	1.69	R	45.31	29.4	29	36.75	36.25	36	45.6	45.25	45	72.14	251	249	248	397
27	35.17	35	10.2	10	10	ASM	1.77	R	62.22	39.4	39	49.25	48.75	49	59.5	58.75	59	97.39	327	323	325	536
28	29.83	30	3.8	8	1.5	ASM	1.8	R	53.83	37.8	38	47.25	47.5	48	56.1	56.5	57	83.66	308	311	314	460
29	17.33	17	3.4	3	2.5	ASM	1.99	R	34.45	20.8	21	28	26.25	26	28.4	29.25	29	51.78	162	161	160	285
30	45	45	7.6	8	6	ASM	2.04	R	91.63	52.4	52	65.5	65	65	73.3	73	73	136.63	403	402	402	751

CORRELATION ANALYSIS SUMMARY

ITEM DEMAND PATTERN : HIGH COEFFICIENT OF VARIANCE

COEFFICIENT OF VARIANCE RANGE : 1.0 - 2.0

VA
 $STDSig = a + b$

Regression Output	
Constant	4.0907
Std Err of Y Est	5.8514 15.7
R Squared	0.7588
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	1.18
Std Err of Coef.	0.13

VB
 $STDSig = c + b$

Regression Output	
Constant	4.0907
Std Err of Y Est	5.8514 15.7
R Squared	0.75879
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	0.945
Std Err of Coef.	0.101

VC
 $STDSig = cr + b$

Regression Output	
Constant	3.9377
Std Err of Y Est	6.5954 18.8
R Squared	0.7564
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	0.9485
Std Err of Coef.	0.10168

VD
 $STDSig = SGM + b$

Regression Output	
Constant	4.2721
Std Err of Y Est	9.5542 19.1
R Squared	0.7487
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	0.5412
Std Err of Coef.	0.103

VE
 $NAD = a + b$

Regression Output	
Constant	7.5019
Std Err of Y Est	12.025 14.9
R Squared	0.8314
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	1.21
Std Err of Coef.	0.1

VF
 $NAD = a + b$

Regression Output	
Constant	7.5297
Std Err of Y Est	12.0217 14.9
R Squared	0.8295
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	1.215
Std Err of Coef.	0.104

VG
 $NAD = a + b$

Regression Output	
Constant	7.52
Std Err of Y Est	12.254 15.1
R Squared	0.825
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	1.2085
Std Err of Coef.	0.1052

VA
 $SOS(NAD) = SOS(NAD) + b$

Regression Output	
Constant	41.838
Std Err of Y Est	55.15 14.9
R Squared	0.8315
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	1.21
Std Err of Coef.	0.1

VB
 $SOS(NAD) = SOS(NAD) + b$

Regression Output	
Constant	41.0813
Std Err of Y Est	55.2639 14.9
R Squared	0.83016
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	1.217
Std Err of Coef.	0.104

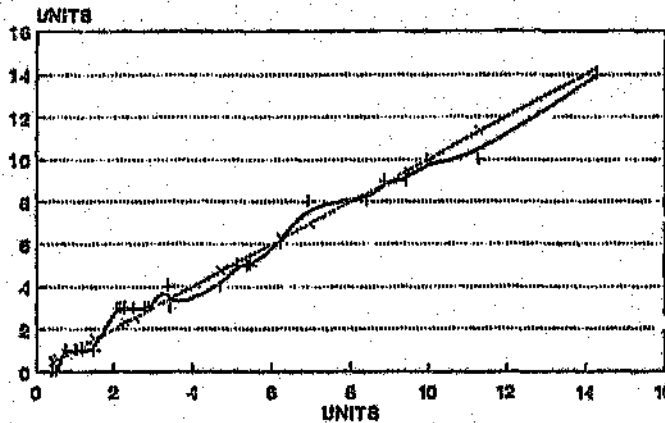
VC
 $SOS(NAD) = SOS(NAD) + b$

Regression Output	
Constant	42.426
Std Err of Y Est	67.32 15.1
R Squared	0.8254
No. of Observations	30
Degrees of Freedom	28
X Coefficient(s)	1.20811
Std Err of Coef.	0.10508

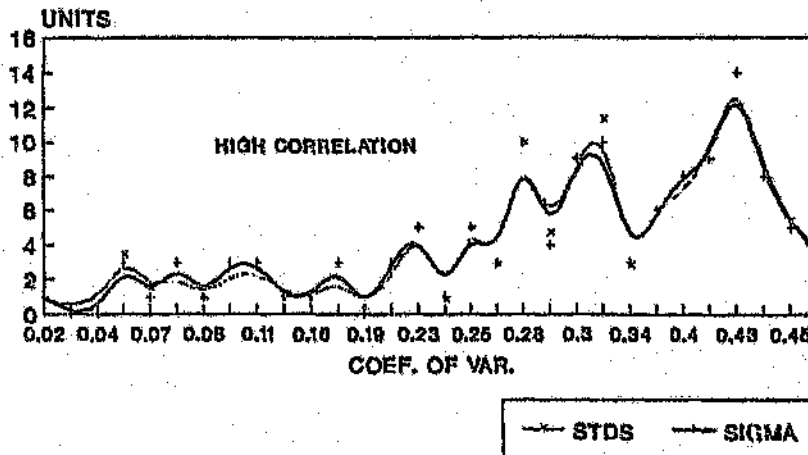
STANDARD DEVIATION COMPARISON

ITEM DEMAND PATTERN : WITH LOW COEFFICIENT OF VARIANCE - RANGE 0.0 - 0.5

SIGMA FLUCTUATION AROUND STDS LINE



**STDS AND SIGMA CORRELATION AS
A FUNCTION OF THE COEF. OF VAR.**

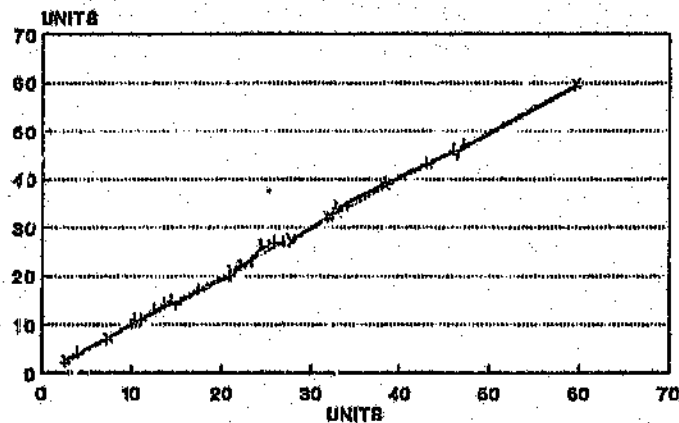


SIGMA = MEAN AVERAGE DEVIATION x 1.25
 STDS = STANDARD DEVIATION OF SAMPLE

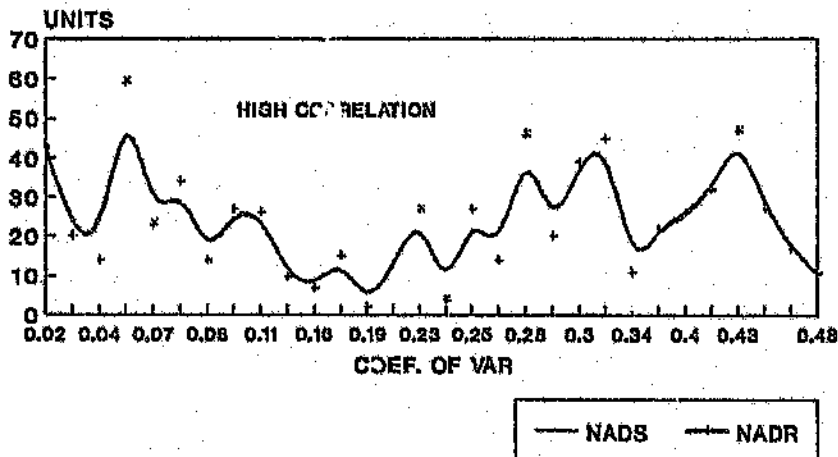
NEW AVERAGE DEMAND COMPARISON

ITEM DEMAND PATTERN : WITH LOW COEFFICIENT OF VARIANCE - RANGE 0.0 - 0.5

NADR FLUCTUATION AROUND NADS LINE



NADS AND NADR CORRELATION AS A FUNCTION OF THE COEF. OF VAR.

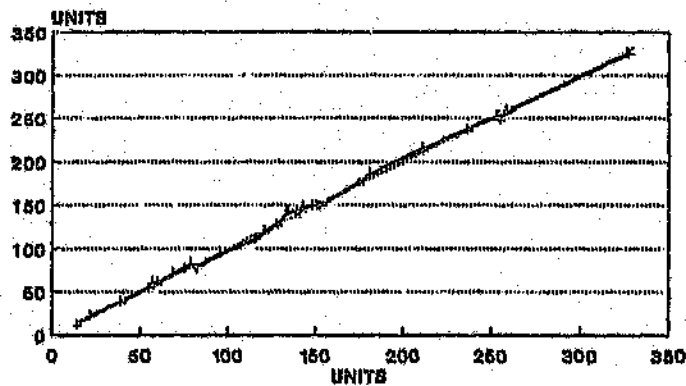


NADR - NEW AVERAGE DEMAND CALCULATED WITH SIGMA
NADS - NEW AVERAGE DEMAND CALCULATED WITH STDS

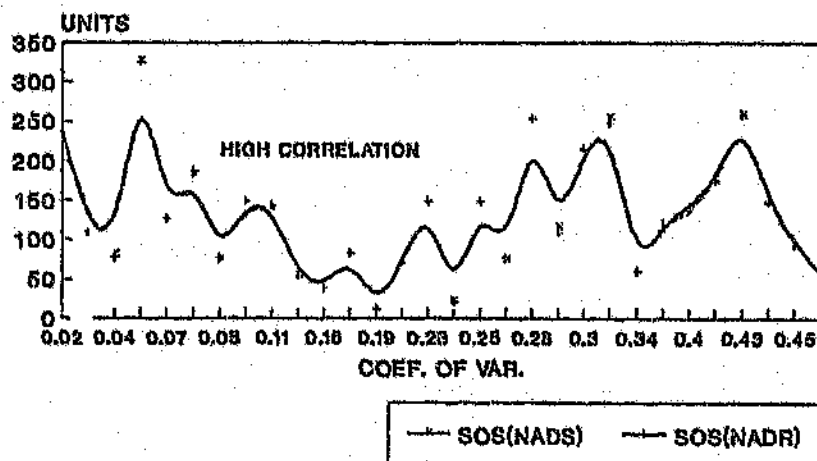
SUGGESTED ORDER SIZE COMPARISON

ITEM DEMAND PATTERN : WITH LOW COEFFICIENT OF VARIANCE - RANGE 0.0 - 0.5

**SOS(NADR) FLUCTUATION AROUND
SOS(NADS) LINE**



**SOS(NADS) AND SOS(NADR) CORRELATION
AS A FUNCTION OF THE COEF. OF VAR.**

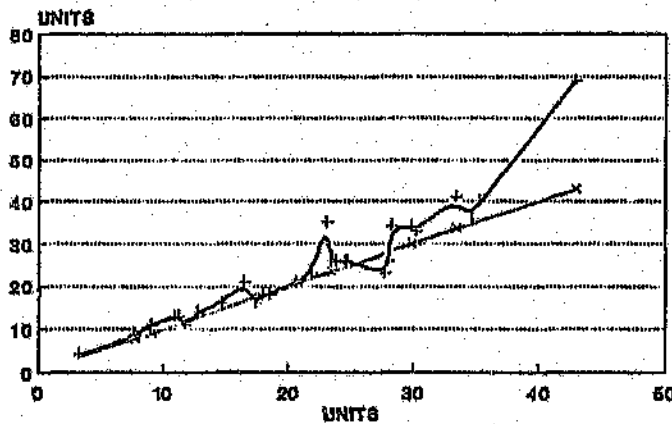


SOS(NADR)-SUGGESTED ORDER SIZE CALCULATED WITH SIGMA
SOS(NADS)-SUGGESTED ORDER SIZE CALCULATED WITH STDS

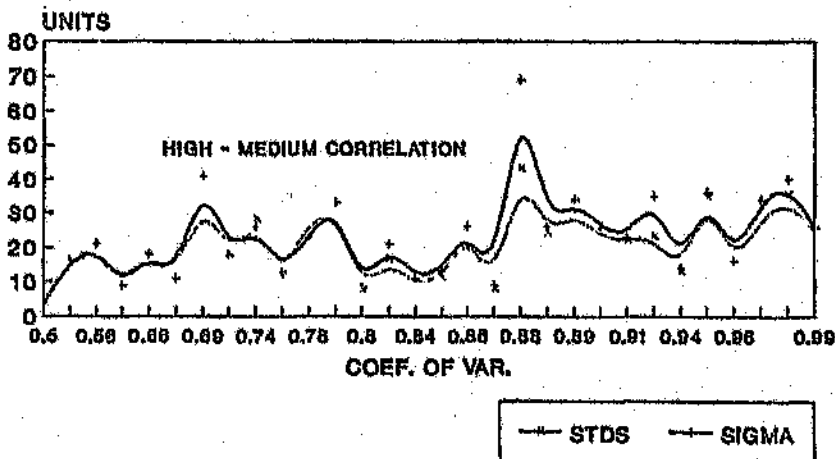
STANDARD DEVIATION COMPARISON

ITEM DEMAND PATTERN : RANDOM WITH MEDIUM COEFFICIENT OF VARIANCE - RANGE 0.5 - 1.0

SIGMA FLUCTUATION AROUND STDS LINE



STDS AND SIGMA CORRELATION AS A FUNCTION OF THE COEF. OF VAR.

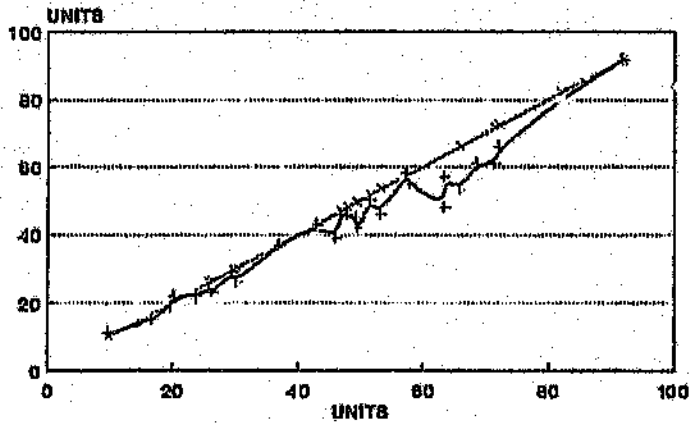


SIGMA = MEAN AVERAGE DEVIATION x 1.25
STDS = STANDARD DEVIATION OF SAMPLE

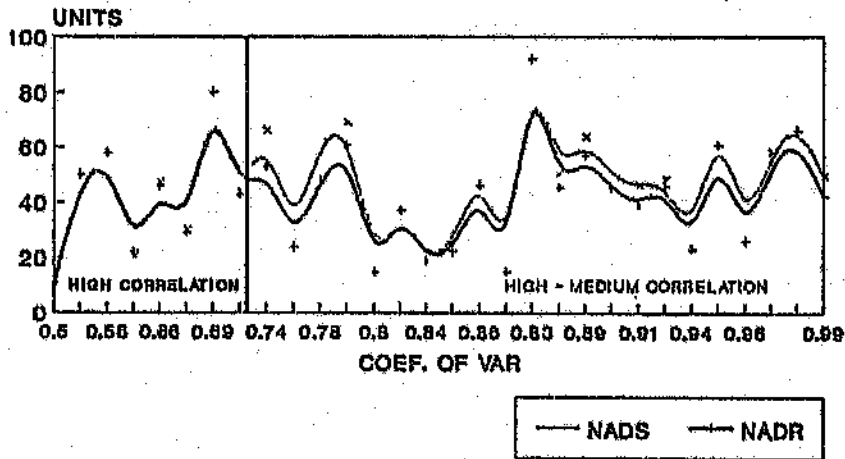
NEW AVERAGE DEMAND COMPARISON

ITEM DEMAND PATTERN : RANDOM WITH MEDIUM COEFFICIENT OF VARIANCE - RANGE 0.5 - 1.0

NADR FLUCTUATION AROUND NADS LINE



NADS AND NADR CORRELATION AS A FUNCTION OF THE COEF. OF VAR.

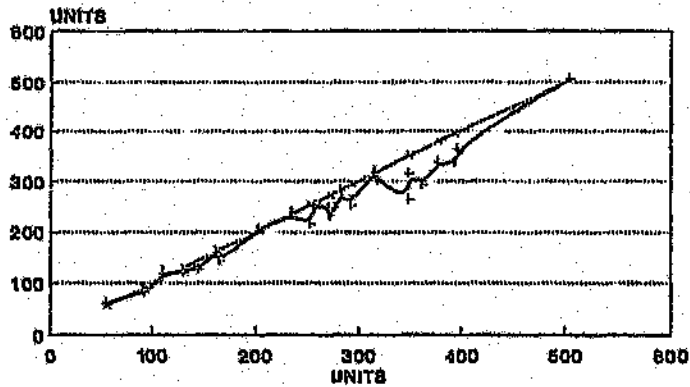


NADR = NEW AVERAGE DEMAND CALCULATED WITH SIGMA
NADS = NEW AVERAGE DEMAND CALCULATED WITH STDS

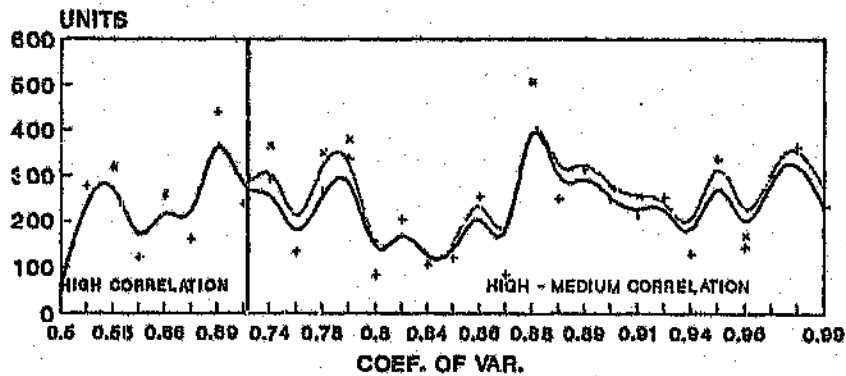
SUGGESTED ORDER SIZE COMPARISON

ITEM DEMAND PATTERN : RANDOM WITH MEDIUM COEFFICIENT OF VARIANCE - RANGE 0.5 - 1.0

SOS(NADR) FLUCTUATION AROUND SOS(NADS) LINE



SOS(NADS) AND SOS(NADR) CORRELATION AS A FUNCTION OF THE COEF. OF VAR.



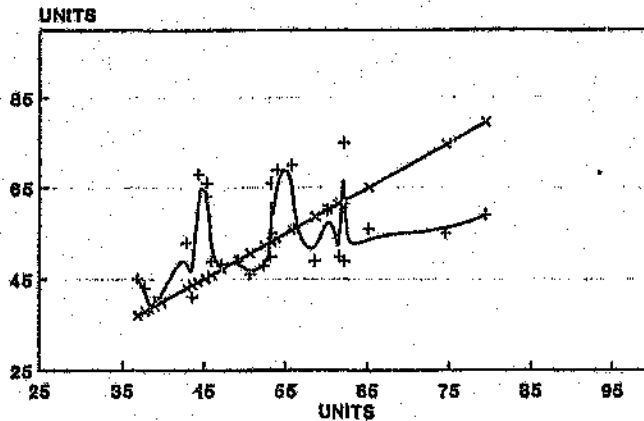
—*— SOS(NADS) — SOS(NADR)

SOS(NADR)-SUGGESTED ORDER SIZE CALCULATED WITH SIGMA
 SOS(NADS)-SUGGESTED ORDER SIZE CALCULATED WITH STDS

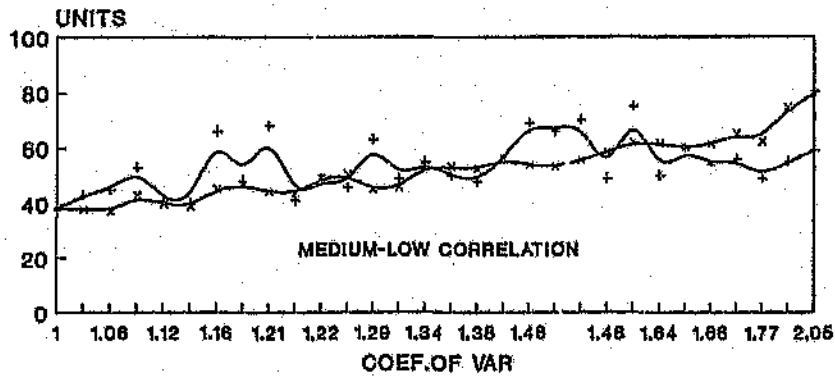
STANDARD DEVIATION COMPARISON

ITEM DEMAND PATTERN : RANDOM WITH HIGH COEFFICIENT OF VARIANCE-RANGE 1.0 -2.1

SIGMA FLUCTUATION AROUND STDS LINE



STDS AND SIGMA CORRELATION AS A FUNCTION OF THE COEF. OF VAR.



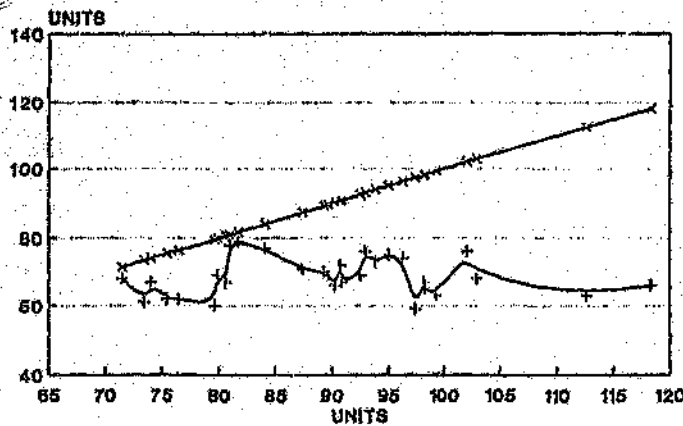
—	STDS	- - -	SIGMA
---	------	-------	-------

SIGMA = MEAN AVERAGE DEVIATION x 1.25
 STDS = STANDARD DEVIATION OF SAMPLE

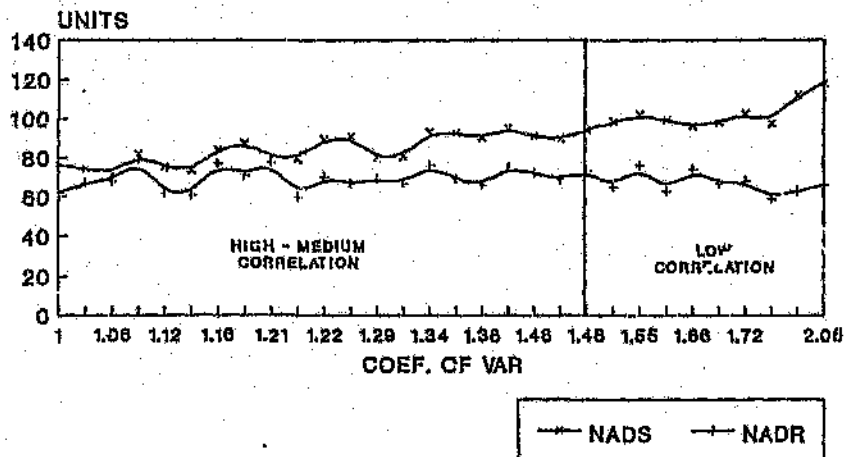
NEW AVERAGE DEMAND COMPARISON

ITEM DEMAND PATTERN : RANDOM WITH HIGH COEFFICIENT OF VARIANCE - RANGE 1.0 - 2.1

NADR FLUCTUATION AROUND NADS LINE



NADS AND NADR CORRELATION AS A FUNCTION OF THE COEF. OF VAR.

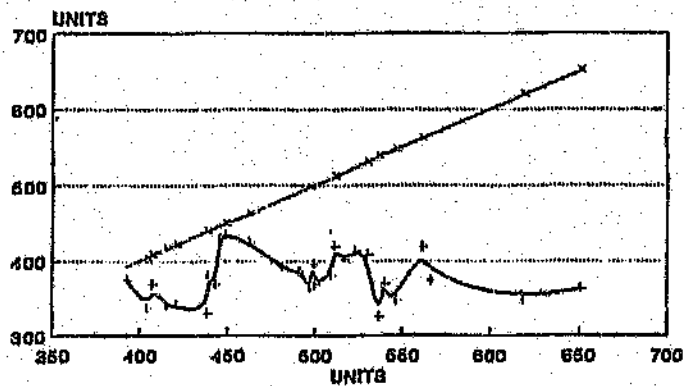


NADR - NEW AVERAGE DEMAND CALCULATED WITH SIGMA
NADS - NEW AVERAGE DEMAND CALCULATED WITH STDS

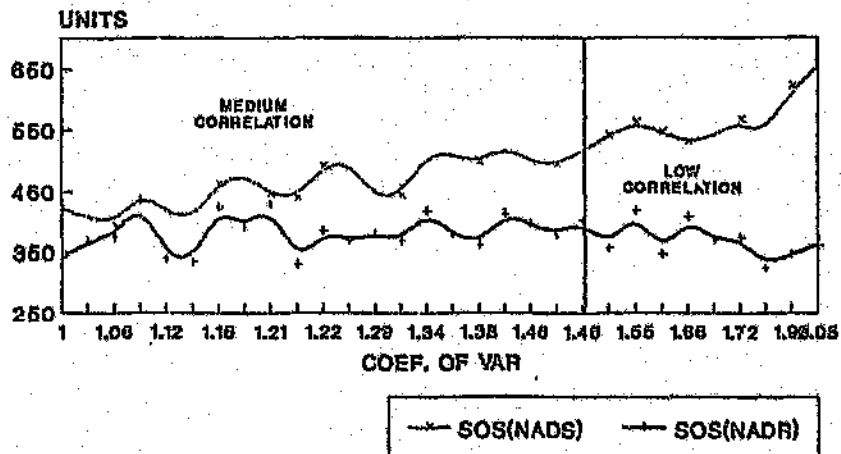
SUGGESTED ORDER SIZE COMPARISON

ITEM DEMAND PATTERN : RANDOM WITH HIGH COEFFICIENT OF VARIANCE - RANGE: 1.0 - 2.1

SOS(NADR) FLUCTUATION AROUND SOS(NADS) LINE



SOS(NADS) AND SOS(NADR) CORRELATION AS A FUNCTION OF THE COEF. OF VAR.



SOS(NADR)-SUGGESTED ORDER SIZE CALCULATED WITH SIGMA
 SOS(NADS)-SUGGESTED ORDER SIZE CALCULATED WITH STDS

APPENDIX 14

A/C NUMBER	DESCRIPTION	ORDER COST			WAREHOUSE COST		
		FIXED	VAR.	TOTAL	FIXED	VAR.	TOTAL
001-5102-111	RAILAGE & CARTAGE - CUSTOMER			0			0
001-5102-112	PACKING & STRIPPING - CUSTOMER						0
001-5102-113	HANDLING CHARGES		72,000	72,000			0
001-5102-114	EXCESS FREIGHT CHARGES		24,000	24,000			0
001-5102-116	M90 DELIVERY VEHICLE - CUSTOMER			0			0
	<i>CFP</i>						
001-5202-201	DIRECT SALARIES & WAGES - SELLING			0			0
001-5202-203	INDIRECT SALARIES & WAGES	150,000	153,000	303,172	400,000	894,013	1,294,013
001-5202-204	INDIRECT OVERTIME		22,000	22,000		47,000	47,000
001-5202-205	RENT PAID ETC.			0			0
001-5202-207	BONUS PROVISION	12,700	12,700	25,430	33,200	74,630	107,830
001-5202-208	FRINGES & CONTRIBUTIONS	19,300	19,300	38,757	50,000	114,340	164,340
001-5202-210	STAFF RELOCATION			0			0
001-5202-213	TEMPORARY STAFF/PERF INC			0		12,000	12,000
001-5202-216	STAFF RECRUITMNT		6,000	6,000			0
001-5202-219	LEAVE PAY ON RESIGNATION		6,000	6,000		6,000	6,000
001-5202-220	COMMUTATION OF LEAVE	2,000	2,000	4,000	2,000	2,000	4,000
001-5202-223	SERVERENCE PAY	14,000		0			0
001-5202-228	CAR ALLOWANCES	14,000	14,000	28,000	28,000		28,000
001-5302-301	MOTOR VEHICLE EXP.		23,339	23,339	23,339	23,000	46,339
001-5302-302	ENTERTAINMENT			0			0
001-5302-303	TRAVEL & SUBSISTENCE			0			0
001-5302-304	F/F & C/C MAINTENANCE	900	900	1,800	900	900	1,800
001-5302-305	PROTECTIVE CLOTHING				1,600	8,000	9,600
001-5302-306	TRAINING & CONFERENCES	1,000		1,000	1,000		1,000
001-5302-307	CO. CAR LEASE AGREEMENTS			0	180,000	60,000	240,000
001-5302-308	ADVERTISING & PROMOTIONS			0			0
001-5302-309	HANDOUTS			0			0
001-5302-310	HIRE OF EQUIPMENT			0		12,000	12,000
001-5302-314	P & M MAINTENANCE			0		84,000	84,000
001-5302-316	OFF. MACH. MAINTENANCE	6,000		6,000			0
001-5302-344	LEASED EQUIPMENT	6,000		6,000	6,000		6,000

APPENDIX 14 (continued)

A/C NUMBER	DESCRIPTION	ORDER COST				WAREHOUSE COST		
		FIXED	VAR.	TOTAL		FIXED	VAR.	TOTAL
001-5402-401	MAINTENANCE & REPAIRS					60,000	60,000	
001-5402-402	POWER, WATER & LIGHTS		20,000	20,000		80,000	80,000	
001-5402-404	PROPERTY RENT PD.	50,000		50,000	400,000		400,000	
0001-5402-405	SECURITY			0	171,000		171,000	
001-5402-406	GROUND MAINTENANCE	2,000		2,000	2,000		2,000	
001-5602-607	INTEREST PAID - PARTS - STD					4,320,000	4,320,000	
001-5702-708	POSTAGES, TEL. EX, TELEGRAM		3,200	3,200		1,000	1,000	
001-5702-709	STAFF SERVICES & REFRESH.	5,000	5,000	10,000	11,000	24,000	35,000	
001-5702-710	SUBSCRIPTIONS							
001-5702-712	PRINTING & STATIONARY	5,000	5,000	10,000	20,000	20,000	40,000	
001-5702-714	DEPRECIATION	25,000		25,000	60,000		60,000	
001-5702-717	TELEPHONE	24,000	36,000	60,000	16,000	4,000	20,000	
	TOTAL:	336,900	403,100	740,000	1,426,040	5,848,870	7,274,910	

**ORDER COMPILATION TIME AS A
FUNCTION OF THE ORDER SIZE**

ORDER SIZE (NUMBER OF SKUs)	COMPILATION TIME	ORDER SIZE (NUMBER OF SKUs)	COMPILATION TIME
1	12,9	17	59,3
2	15,8	18	62,2
3	18,7	19	65,10
4	21,6	20	68,0
5	24,5	21	70,9
6	27,4	22	73,8
7	30,3	23	76,70
8	33,2	24	79,6
9	36,1	25	82,5
10	39,0	26	85,4
11	41,9	27	88,3
12	44,8	28	91,20
13	47,70	29	94,10
14	50,60	30	97,0
15	53,50	31	99,9
16	56,40	32	102,8

BULK ORDER

Order size evaluation per SKU	1,2 minutes
Data capturing pre SKU	0,5 minutes
Follow up/expediting per SKU	0,5 minutes
Total	2,2 minutes
Contingency allowance 30%	<u>0,7 minutes</u>
Total per SKU	2,9 minutes

**TOTAL TIME SPENT ON ORDER COMPILATION
AND THE ORDER COST FOR THE PERIOD
FEBRUARY 1992 TO MARCH 1993**

AVERAGE SKUs/ ORDER	NUMBER OF ORDERS	COMPILATION TIME/ORDER MINUTES	TOTAL TIME PER MINUTE	TOTAL TIME PER HOUR	% OF VARIABLE COST	ORDER COST (R)
1	2 696	12,9	34 655	578	29	60,00
2	627	15,8	9 907	165	8	71,00
4	926	21,8	19 988	333	17	103,00
7	79	30,3	2 394	40	2	142,00
12	118	44,8	5 286	88	4,4	209,00
24	188	79,6	14 964	250	12,5	373,00
80	15	242,0	3 640	61	3	1 120,00
238	15	367,0	5 516	92	4,6	1 644,00
1 020	15	1 540,0	23 110	385	19	7 093,00
TOTAL	4 869			1 992		

For orders having more than 100 SKUs per order compilation time per SKU

- Available time for order compilation 60% of annual available time.
230 day x 8 hours x 60% = 1 104 hours/per annum
Since there are 4 buyers
4 x 1 104 = 4 416 hours/annum

- Total time spent on order placing 1 992 hours/annum

∴ Buyers utilisation = $\frac{1\ 992}{4\ 416} \times 100 = 45\%$

- Each Buyer can compile 2 585 orders/annum according to the current mix
1 104 available hours for order compilation
0,43 hours - average order compilation according to the current mix.

$\frac{1\ 104}{0,43} = 2\ 585$ orders/annum

APPENDIX 17

EXPECTED BUYER UTILISATION

Expected time that will be spend on order compilation according to weekly inventory review

NUMBER OF SKUs/ORDER	NUMBER OF ORDERS	COMPILATION TIME PER ORDER MINUTE	TOTAL TIME MINUTES	TOTAL TIME HOURS
1	1 360	12,9	17 544	293
2	160	15,8	2 528	42
4	1 500	21,6	32 400	540
7	20	30,3	606	10
12	30	44,8	1 344	23
24	750	79,6	22 200	370
80	52	242,0	12 584	210
238	52	367,0	19 084	318
1 020	52	1 540,0	80 080	1 335
TOTAL	3 976			3 141

The expected buyer utilisatio or 80% available time will be:

$$= \frac{3\,141}{4\,416} \times 100 = 71\%$$

- o Average time for order compilation according to the proposed two weekly review 0,79 hours/order.
Thus possible number of orders to be compiled Per annum per buyer is $\frac{1\,104}{0,79} = 1\,397$ orders

VARIABLE COSTS AFFECTING NUMBER OF ORDERS

NUMBER OF BUYERS	NUMBER OF RECEIVERS	CREDITORS	NUMBER OF ORDERS/ ANNUM CAPACITY	COST PER ANNUM (R)				OTHER VARIABLE COSTS (R)	TOTAL (R)
				BUYERS	RECEIVERS	CREDITORS	SUB TOTAL		
1	1	1	1 400	50 000	20 000	35 000	105 000	215 000	320 000
2	2	1	2 800	85 000	40 000	35 000	155 000	215 000	370 000
3	3	2	4 200	120 000	60 000	70 000	250 000	230 000	480 000
4	4	3	5 600	155 000	80 000	102 000	335 000	230 000	555 000
5	4	3	7 000	190 000	80 000	100 000	370 000	250 000	620 000
6	5	3	8 400	240 000	100 000	100 000	440 000	250 000	690 000
7	6	4	9 800	275 000	12 000	135 000	530 000	270 000	800 000

Notes: 1 Buyer has $240 \times 8 = 1\,920$ available hours per annum

2 40% of annual available time is devoted to market research and meeting with users and suppliers

APPENDIX 19

**AVERAGE ORDER COST PER SKU
FEBRUARY 1992 TO MARCH 1993**

ORDER SIZE SKUs/ORDER	NUMBER OF ORDERS	NUMBER OF SKUs REPLENISH	% FROM TOTAL SKUs	ORDER COST AT (R) BUYER UTILISATION	
				45%	100%
1	2 686	2 686	7,8	60	27
2	627	1 254	3,7	71	32
4	926	3 704	10,8	103	47
7	79	553	1,6	142	64
12	118	1 416	4,1	209	94
24	188	4 512	13,2	379	168
80	15	1 200	3,7	1 120	504
238	15	3 570	10,7	1 644	740
1 020	15	15 300	44,4	7 093	3 192
TOTAL		34 195			

Average order cost per SKU)
45% buyer utilisation: R16,00
100% buyer utilisation R7,20

In other words the order cost per SKU with current buyer complement with increased number of orders placed according to current order size mix from 4 400 orders per annum to 9 700 orders per annum will not result with an increase in order cost, therefore the recommended order cost per SKU is R16.

TABLE 1
 TABLE 5 OF INDEX LETTERS (FOR USE WITH TABLE 2)

RATIO OF SET-UP COST TO UNIT COST (a/c)	MARGINAL INVENTORY CHARGE P.A. (I)				RATIO OF SET-UP COST TO UNIT COST (a/c)	MARGINAL INVENTORY CHARGE P.A. (II)			
	10-11%	12-14%	15-18%	19-23%		10-11%	12-14%	15-18%	19-23%
Under					1,0+	L	L	K	K
0,006					1,3+	M	L	L	K
0,006+					1,6+	M	M	L	L
0,008+					2,0+	N	M	M	L
0,01+					2,5+	N	N	M	M
0,013+					3,0+	O	N	N	M
0,016+					4,0+	O	O	N	N
0,02+	A	A	A	A	5,0+	P	O	O	N
0,026+	A	A	A	A	6,0+	P	P	O	O
0,03+	B	B	A	A	8,0+	Q	Q	P	P
0,04+	B	B	A	A	10+	Q	Q	P	P
0,05+	C	B	B	A	13+	R	Q	Q	P
0,06+	C	C	B	B	16+	R	R	Q	Q
0,08+	D	C	C	B	20+	S	R	R	Q
0,19+	D	D	C	C	25+	S	S	R	R
0,13+	E	D	D	C	30+	T	S	S	R
0,16+	E	E	D	D	40+	T	T	S	S
0,29+	F	E	E	D	50+	U	T	T	S
0,23+	F	F	E	E	60+	U	U	T	T
0,3+	G	F	F	E	80+	V	U	U	T
0,4+	G	G	F	F	100+	V	V	U	U
0,5+	H	G	G	F	130+	W	V	V	U
0,6+	H	H	G	G	160+	W	W	V	V
0,8+	I	H	H	G	200+	X	W	W	V
	I	I	H	H	300+	X	X	X	W
	J	I	I	H	400+	Y	X	X	X
	J	J	I	I	500+	Y	Y	X	X
	K	J	J	J	600+	Z	Y	Y	Y
	K	K	J	J	800+	Z	Z	Y	Y
	L	K	K	J		Z	Z	Z	Y

TABLE 2 TABLE FOR DETERMINING IF A PART IS WORTH STOCKING

Annual demand (A)	Index letter																										Annual demand (A)
	A & B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z		
1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
4-6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4-6
7-9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7-9
10-15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10-15
16-24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16-24
25-39	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	25-39
40-59	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40-59
60-99	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	60-99
100-159	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	100-159
160-249	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	160-249
250-399	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	250-399
400-599	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	400-599
600-999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	600-999
1000-1599	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1000-1599
1600-2499	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1600-2499
2500-3999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2500-3999
4000-5999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4000-5999
6000-9999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6000-9999
10,000-15,999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10,000-15,999
16,000-24,999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16,000-24,999
25,000-39,999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	25,000-39,999
40,000-59,999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40,000-59,999
60,000-99,999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	60,000-99,999

A part is worth stocking if the number of calls for it per year exceeds the number shown. Stocking is never economic if no number is given.

Table 2 is based on a full inventory charge (I_f) equal to twice the marginal inventory charge (I) and hence (1 + I_f/I) = 3, and

$$\text{critical calls p.a.} = 3 \sqrt{\left(\frac{\text{Annual demand}}{2} \times \frac{\text{Marginal inventory charge p.a.} \times \text{Unit cost}}{\text{Set-up cost per order}} \right)}$$

if order quantity = EOQ.

Or

$$\text{critical calls p.a.} = 10 + \frac{\text{Annual demand}}{10} \times \frac{\text{Marginal inventory charge p.a.} \times \text{Unit cost}}{\text{Set-up cost per order}}$$

FOR DETERMINING IF A PART IS WORTH STOCKING

APPENDIX 20 (continued)

APPENDIX 20 (continued)

Example using Table 1 and 2

- a) Calculate the ratio of set-up cost (S) over unit cost (C) for the component
- b) Read off the index letter corresponding to the relevant charge p.a. from Table 5
e.g. $A/V = 100 / 5 = 20\%$
index letter = U.
- c) ✓ Read off the critical number of calls p.a. from Table 2
e.g. index letter = U, annual demand = 100
critical calls = 1
(i.e. if more than one call p.a. is expected it will be cheaper to make for stock).

THE NORMAL PROBABILITY DISTRIBUTION
SOME FUNCTIONS OF THE UNIT NORMAL DISTRIBUTION

TABLE 1 (CONTINUED)

k	$f_z(k)$	$\rho_z(k)$	$G_z(k)$	$G_z(-k)$	k	k	$f_z(k)$	$\rho_z(k)$	$G_z(k)$	$G_z(-k)$	k
0.82	0.2850	0.2061	0.1160	0.9360	0.82	1.22	0.1895	0.1112	0.05384	1.2738	1.22
0.83	0.2827	0.2033	0.1140	0.9440	0.83	1.23	0.1872	0.1093	0.05274	1.2827	1.23
0.84	0.2803	0.2005	0.1120	0.9520	0.84	1.24	0.1849	0.1075	0.05165	1.2917	1.24
0.85	0.2780	0.1977	0.1100	0.9600	0.85	1.25	0.1826	0.1056	0.05059	1.3006	1.25
0.86	0.2755	0.1949	0.1080	0.9680	0.86	1.26	0.1804	0.1038	0.04954	1.3095	1.26
0.87	0.2732	0.1922	0.1061	0.9761	0.87	1.27	0.1781	0.1020	0.04851	1.3185	1.27
0.88	0.2709	0.1894	0.1042	0.9842	0.88	1.28	0.1758	0.1003	0.04750	1.3275	1.28
0.89	0.2685	0.1867	0.1023	0.9923	0.89	1.29	0.1736	0.09853	0.04650	1.3365	1.29
0.90	0.2661	0.1841	0.1004	1.0004	0.90	1.30	0.1714	0.09680	0.04553	1.3455	1.30
0.91	0.2637	0.1814	0.09850	1.0086	0.91	1.31	0.1691	0.09510	0.04457	1.3545	1.31
0.92	0.2613	0.1788	0.09660	1.0168	0.92	1.32	0.1669	0.09342	0.04363	1.3636	1.32
0.93	0.2589	0.1762	0.09483	1.0250	0.93	1.33	0.1647	0.09178	0.04270	1.3727	1.33
0.94	0.2565	0.1736	0.09308	1.0333	0.94	1.34	0.1626	0.09012	0.04179	1.3818	1.34
0.95	0.2541	0.1711	0.09136	1.0416	0.95	1.35	0.1604	0.08851	0.04090	1.3909	1.35
0.96	0.2516	0.1685	0.08966	1.0499	0.96	1.36	0.1582	0.08692	0.04002	1.4000	1.36
0.97	0.2492	0.1660	0.08810	1.0582	0.97	1.37	0.1561	0.08534	0.03916	1.4092	1.37
0.98	0.2468	0.1635	0.08654	1.0665	0.98	1.38	0.1539	0.08379	0.03831	1.4183	1.38
0.99	0.2444	0.1611	0.08491	1.0749	0.99	1.39	0.1518	0.08226	0.03749	1.4275	1.39
1.00	0.2420	0.1587	0.08332	1.0833	1.00	1.40	0.1497	0.08076	0.03667	1.4357	1.40
1.01	0.2395	0.1562	0.08174	1.0917	1.01	1.41	0.1476	0.07927	0.03587	1.4459	1.41
1.02	0.2371	0.1538	0.08019	1.1002	1.02	1.42	0.1455	0.07780	0.03508	1.4551	1.42
1.03	0.2347	0.1515	0.07866	1.1087	1.03	1.43	0.1435	0.07636	0.03431	1.4643	1.43
1.04	0.2323	0.1492	0.07716	1.1172	1.04	1.44	0.1415	0.07493	0.03356	1.4736	1.44
1.05	0.2299	0.1469	0.07568	1.1257	1.05	1.45	0.1394	0.07353	0.03281	1.4828	1.45
1.06	0.2275	0.1446	0.07422	1.1342	1.06	1.46	0.1374	0.07215	0.03208	1.4921	1.46
1.07	0.2251	0.1423	0.07279	1.1428	1.07	1.47	0.1354	0.07078	0.03137	1.5014	1.47
1.08	0.2227	0.1401	0.07138	1.1514	1.08	1.48	0.1334	0.06944	0.03067	1.5107	1.48
1.09	0.2203	0.1379	0.06999	1.1600	1.09	1.49	0.1315	0.06811	0.02998	1.5200	1.49
1.10	0.2179	0.1357	0.06862	1.1686	1.10	1.50	0.1295	0.06681	0.02931	1.5293	1.50
1.11	0.2155	0.1335	0.06727	1.1773	1.11	1.51	0.1276	0.06552	0.02865	1.5386	1.51
1.12	0.2131	0.1314	0.06595	1.1859	1.12	1.52	0.1257	0.06426	0.02800	1.5480	1.52
1.13	0.2107	0.1292	0.06465	1.1946	1.13	1.53	0.1238	0.06301	0.02736	1.5574	1.53
1.14	0.2083	0.1271	0.06336	1.2034	1.14	1.54	0.1219	0.06178	0.02674	1.5667	1.54
1.15	0.2059	0.1251	0.06210	1.2121	1.15	1.55	0.1200	0.06057	0.02612	1.5761	1.55
1.16	0.2036	0.1230	0.06086	1.2209	1.16	1.56	0.1182	0.05938	0.02552	1.5855	1.56
1.17	0.2012	0.1210	0.05964	1.2296	1.17	1.57	0.1163	0.05821	0.02494	1.5949	1.57
1.18	0.1989	0.1190	0.05844	1.2384	1.18	1.58	0.1145	0.05705	0.02436	1.6044	1.58
1.19	0.1965	0.1170	0.05726	1.2473	1.19	1.59	0.1127	0.05592	0.02380	1.6138	1.59
1.20	0.1942	0.1151	0.05610	1.2561	1.20	1.60	0.1109	0.05480	0.02324	1.6232	1.60
1.21	0.1919	0.1131	0.05496	1.2650	1.21	1.61	0.1092	0.05370	0.02270	1.6327	1.61
						1.62	0.1074	0.05262	0.02217	1.6422	1.62

(continued)

THE NORMAL PROBABILITY DISTRIBUTION
SOME FUNCTIONS OF THE UNIT NORMAL DISTRIBUTION

TABLE 1 (CONTINUED)

k	$f_z(k)$	$p_z(k)$	$G_z(k)$	$G_z(-k)$	k	k	$f_z(k)$	$p_z(k)$	$G_z(k)$	$G_z(-k)$	k
1.63	0.1057	0.05155	0.02165	1.6516	1.63	2.02	0.0519	0.02169	0.008046	2.0280	2.02
1.64	0.1040	0.05050	0.02114	1.6611	1.64	2.03	0.0508	0.02118	0.007832	2.0378	2.03
1.65	0.1023	0.04947	0.02064	1.6706	1.65	2.04	0.0498	0.02068	0.007623	2.0476	2.04
1.66	0.1006	0.04846	0.02015	1.6801	1.66	2.05	0.0488	0.02018	0.007418	2.0574	2.05
1.67	0.0989	0.04746	0.01967	1.6897	1.67	2.06	0.0478	0.01970	0.007219	2.0672	2.06
1.68	0.0973	0.04648	0.01920	1.6992	1.68	2.07	0.0468	0.01923	0.007024	2.0770	2.07
1.69	0.0957	0.04551	0.01874	1.7087	1.69	2.08	0.0459	0.01876	0.006835	2.0868	2.08
1.70	0.0940	0.04457	0.01829	1.7183	1.70	2.09	0.0449	0.01831	0.006649	2.0966	2.09
1.71	0.0925	0.04363	0.01785	1.7278	1.71	2.10	0.0440	0.01786	0.006468	2.1065	2.10
1.72	0.0909	0.04272	0.01742	1.7374	1.72	2.11	0.0431	0.01743	0.006292	2.1163	2.11
1.73	0.0893	0.04182	0.01699	1.7470	1.73	2.12	0.0422	0.01700	0.006120	2.1261	2.12
1.74	0.0878	0.04093	0.01658	1.7566	1.74	2.13	0.0413	0.01659	0.005952	2.1360	2.13
1.75	0.0863	0.04006	0.01617	1.7662	1.75	2.14	0.0404	0.01618	0.005788	2.1458	2.14
1.76	0.0848	0.03920	0.01578	1.7758	1.76	2.15	0.0396	0.01578	0.005628	2.1556	2.15
1.77	0.0833	0.03836	0.01539	1.7854	1.77	2.16	0.0387	0.01539	0.005472	2.1655	2.16
1.78	0.0818	0.03754	0.01501	1.7950	1.78	2.17	0.0379	0.01500	0.005320	2.1753	2.17
1.79	0.0803	0.03673	0.01464	1.8046	1.79	2.18	0.0371	0.01463	0.005172	2.1852	2.18
1.80	0.0789	0.03593	0.01428	1.8143	1.80	2.19	0.0363	0.01426	0.005028	2.1950	2.19
1.81	0.0775	0.03515	0.01392	1.8239	1.81	2.20	0.0355	0.01390	0.004887	2.2049	2.20
1.82	0.0761	0.03438	0.01357	1.8336	1.82	2.21	0.0347	0.01355	0.004750	2.2147	2.21
1.83	0.0748	0.03362	0.01323	1.8432	1.83	2.22	0.0339	0.01321	0.004616	2.2246	2.22
1.84	0.0734	0.03288	0.01290	1.8529	1.84	2.23	0.0332	0.01287	0.004486	2.2345	2.23
1.85	0.0721	0.03216	0.01257	1.8625	1.85	2.24	0.0325	0.01255	0.004358	2.2444	2.24
1.86	0.0707	0.03144	0.01226	1.8723	1.86	2.25	0.0317	0.01222	0.004235	2.2542	2.25
1.87	0.0694	0.03074	0.01195	1.8819	1.87	2.26	0.0310	0.01191	0.004114	2.2641	2.26
1.88	0.0681	0.03005	0.01164	1.8916	1.88	2.27	0.0303	0.01160	0.003996	2.2740	2.27
1.89	0.0669	0.02938	0.01134	1.9013	1.89	2.28	0.0297	0.01130	0.003981	2.2839	2.28
1.90	0.0656	0.02872	0.01105	1.9111	1.90	2.29	0.0290	0.01101	0.003970	2.2938	2.29
1.91	0.0644	0.02807	0.01077	1.9209	1.91	2.30	0.0283	0.01072	0.003962	2.3037	2.30
1.92	0.0632	0.02743	0.01049	1.9305	1.92	2.31	0.0277	0.01044	0.003956	2.3136	2.31
1.93	0.0620	0.02680	0.01022	1.9402	1.93	2.32	0.0270	0.01017	0.003953	2.3235	2.32
1.94	0.0608	0.02619	0.009957	1.9500	1.94	2.33	0.0264	0.009903	0.003952	2.3334	2.33
1.95	0.0596	0.02559	0.009698	1.9597	1.95	2.34	0.0258	0.009642	0.003955	2.3433	2.34
1.96	0.0584	0.02500	0.009445	1.9694	1.96	2.35	0.0252	0.009387	0.003959	2.3532	2.35
1.97	0.0573	0.02442	0.009199	1.9792	1.97	2.36	0.0246	0.009137	0.003967	2.3631	2.36
1.98	0.0562	0.02385	0.008957	1.9890	1.98	2.37	0.0241	0.008894	0.003977	2.3730	2.37
1.99	0.0551	0.02330	0.008721	1.9987	1.99	2.38	0.0235	0.008656	0.003989	2.3829	2.38
2.00	0.0540	0.02275	0.008491	2.0085	2.00	2.39	0.0229	0.008424	0.003994	2.3928	2.39
2.01	0.0529	0.02222	0.008266	2.0183	2.01	2.40	0.0224	0.008198	0.0039720	2.4027	2.40
						2.41	0.0219	0.007976	0.002540	2.4126	2.41
						2.42	0.0213	0.007760	0.002561	2.4226	2.42

(continued)

THE NORMAL PROBABILITY DISTRIBUTION
SOME FUNCTIONS OF THE UNIT NORMAL DISTRIBUTION

TABLE 1 (CONTINUED)

k	$f_p(k)$	$p_p(k)$	$G_p(k)$	$G_p(-k)$	k	k	$f_p(k)$	$p_p(k)$	$G_p(k)$	$G_p(-k)$	k
2.43	0.0208	0.007549	0.002404	2.4325	2.43	2.82	0.0075	0.002491	0.0007115	2.8207	2.82
2.44	0.0203	0.007344	0.002310	2.4424	2.44	2.83	0.0073	0.002327	0.0006879	2.8307	2.83
2.45	0.0198	0.007143	0.002237	2.4523	2.45	2.84	0.0071	0.002256	0.0006650	2.8407	2.84
2.46	0.0194	0.006947	0.002167	2.4623	2.46	2.85	0.0069	0.002186	0.0006428	2.8506	2.85
2.47	0.0189	0.006756	0.002109	2.4722	2.47	2.86	0.0067	0.002118	0.0006213	2.8606	2.86
2.48	0.0184	0.006569	0.002052	2.4821	2.48	2.87	0.0065	0.002052	0.0006004	2.8706	2.87
2.49	0.0180	0.006387	0.002007	2.4921	2.49	2.88	0.0063	0.001988	0.0005802	2.8805	2.88
2.50	0.0175	0.006210	0.002000	2.5020	2.50	2.89	0.0061	0.001926	0.0005606	2.8906	2.89
2.51	0.0171	0.006037	0.001943	2.5119	2.51	2.90	0.0060	0.001866	0.0005417	2.9005	2.90
2.52	0.0167	0.005868	0.001883	2.5219	2.52	2.91	0.0058	0.001807	0.0005233	2.9105	2.91
2.53	0.0163	0.005703	0.001826	2.5318	2.53	2.92	0.0056	0.001750	0.0005055	2.9205	2.92
2.54	0.0158	0.005543	0.001769	2.5418	2.54	2.93	0.0055	0.001695	0.0004883	2.9305	2.93
2.55	0.0154	0.005386	0.001715	2.5517	2.55	2.94	0.0053	0.001641	0.0004716	2.9405	2.94
2.56	0.0151	0.005234	0.001652	2.5617	2.56	2.95	0.0051	0.001589	0.0004555	2.9505	2.95
2.57	0.0147	0.005085	0.001593	2.5716	2.57	2.96	0.0050	0.001538	0.0004398	2.9604	2.96
2.58	0.0143	0.004940	0.001536	2.5816	2.58	2.97	0.0048	0.001489	0.0004247	2.9704	2.97
2.59	0.0139	0.004799	0.001481	2.5915	2.59	2.98	0.0047	0.001441	0.0004101	2.9804	2.98
2.60	0.0135	0.004661	0.001428	2.6015	2.60	2.99	0.0046	0.001395	0.0003959	2.9904	2.99
2.61	0.0132	0.004527	0.001377	2.6114	2.61	3.00	0.0044	0.001350	0.0003822	3.0004	3.00
2.62	0.0129	0.004396	0.001328	2.6214	2.62	3.01	0.0043	0.001306	0.0003689	3.0104	3.01
2.63	0.0126	0.004269	0.001280	2.6313	2.63	3.02	0.0042	0.001264	0.0003560	3.0204	3.02
2.64	0.0122	0.004145	0.001234	2.6413	2.64	3.03	0.0040	0.001223	0.0003436	3.0303	3.03
2.65	0.0119	0.004025	0.001190	2.6512	2.65	3.04	0.0039	0.001183	0.0003316	3.0403	3.04
2.66	0.0116	0.003907	0.001147	2.6612	2.66	3.05	0.0038	0.001144	0.0003199	3.0503	3.05
2.67	0.0113	0.003793	0.001106	2.6712	2.67	3.06	0.0037	0.001107	0.0003087	3.0603	3.06
2.68	0.0110	0.003681	0.001067	2.6811	2.68	3.07	0.0036	0.001070	0.0002978	3.0703	3.07
2.69	0.0107	0.003573	0.001029	2.6911	2.69	3.08	0.0035	0.001035	0.0002873	3.0803	3.08
2.70	0.0104	0.003467	0.001000	2.7011	2.70	3.09	0.0034	0.001001	0.0002771	3.0903	3.09
2.71	0.0101	0.003364	0.000962	2.7110	2.71	3.10	0.0033	0.0009676	0.0002672	3.1003	3.10
2.72	0.0099	0.003264	0.000928	2.7210	2.72	3.11	0.0032	0.0009354	0.0002577	3.1103	3.11
2.73	0.0096	0.003167	0.000896	2.7310	2.73	3.12	0.0031	0.0009040	0.0002485	3.1202	3.12
2.74	0.0093	0.003072	0.000865	2.7409	2.74	3.13	0.0030	0.0008740	0.0002396	3.1302	3.13
2.75	0.0091	0.002980	0.000836	2.7509	2.75	3.14	0.0029	0.0008447	0.0002311	3.1402	3.14
2.76	0.0088	0.002890	0.000808	2.7609	2.76	3.15	0.0028	0.0008164	0.0002227	3.1502	3.15
2.77	0.0086	0.002803	0.000781	2.7708	2.77	3.16	0.0027	0.0007888	0.0002147	3.1602	3.16
2.78	0.0084	0.002718	0.000756	2.7808	2.78	3.17	0.0026	0.0007622	0.0002070	3.1702	3.17
2.79	0.0081	0.002635	0.000732	2.7908	2.79	3.18	0.0025	0.0007364	0.0001995	3.1802	3.18
2.80	0.0079	0.002555	0.000711	2.8008	2.80	3.19	0.0025	0.0007114	0.0001922	3.1902	3.19
2.81	0.0077	0.002477	0.000691	2.8107	2.81	3.20	0.0024	0.0006871	0.0001852	3.2002	3.20
						3.21	0.0023	0.0006637	0.0001785	3.2102	3.21
						3.22	0.0022	0.0006410	0.0001720	3.2202	3.22

(continued)

THE NORMAL PROBABILITY DISTRIBUTION
SOME FUNCTIONS OF THE UNIT NORMAL DISTRIBUTION

TABLE 1 (CONTINUED)

k	$f_z(k)$	$p_w(k)$	$G_z(k)$	$G_w(k)$	k	k	$f_z(k)$	$p_w(k)$	$G_z(k)$	$G_w(k)$	k
3.23	0.0022	0.0006190	0.0001657	3.2302	3.23	3.62	0.0006	0.0001473	0.00003605	3.6200	3.62
3.24	0.0021	0.0005976	0.0001596	3.2402	3.24	3.63	0.0005	0.0001417	0.00003460	3.6300	3.63
3.25	0.0020	0.0005770	0.0001537	3.2502	3.25	3.64	0.0005	0.0001363	0.00003321	3.6400	3.64
3.26	0.0020	0.0005571	0.0001480	3.2601	3.26	3.65	0.0005	0.0001311	0.00003188	3.6500	3.65
3.27	0.0019	0.0005377	0.0001426	3.2701	3.27	3.66	0.0005	0.0001261	0.00003059	3.6600	3.66
3.28	0.0018	0.0005190	0.0001373	3.2801	3.28	3.67	0.0005	0.0001213	0.00002935	3.6700	3.67
3.29	0.0018	0.0005009	0.0001322	3.2901	3.29	3.68	0.0005	0.0001166	0.00002816	3.6800	3.68
3.30	0.0017	0.0004834	0.0001273	3.3001	3.30	3.69	0.0004	0.0001121	0.00002702	3.6900	3.69
3.31	0.0017	0.0004665	0.0001225	3.3101	3.31	3.70	0.0004	0.0001078	0.00002592	3.7000	3.70
3.32	0.0016	0.0004501	0.0001179	3.3201	3.32	3.71	0.0004	0.0001036	0.00002486	3.7100	3.71
3.33	0.0016	0.0004342	0.0001135	3.3301	3.33	3.72	0.0004	0.00009962	0.00002385	3.7200	3.72
3.34	0.0015	0.0004189	0.0001093	3.3401	3.34	3.73	0.0004	0.00009574	0.00002287	3.7300	3.73
3.35	0.0015	0.0004041	0.0001051	3.3501	3.35	3.74	0.0004	0.00009201	0.00002193	3.7400	3.74
3.36	0.0014	0.0003897	0.0001012	3.3601	3.36	3.75	0.0004	0.00008842	0.00002103	3.7500	3.75
3.37	0.0014	0.0003758	0.00009734	3.3701	3.37	3.76	0.0003	0.00008496	0.00002016	3.7600	3.76
3.38	0.0013	0.0003624	0.00009365	3.3801	3.38	3.77	0.0003	0.00008162	0.00001933	3.7700	3.77
3.39	0.0013	0.0003495	0.00009009	3.3901	3.39	3.78	0.0003	0.00007841	0.00001853	3.7800	3.78
3.40	0.0012	0.0003369	0.00008666	3.4001	3.40	3.79	0.0003	0.00007532	0.00001776	3.7900	3.79
3.41	0.0012	0.0003248	0.00008335	3.4101	3.41	3.80	0.0003	0.00007235	0.00001702	3.8000	3.80
3.42	0.0012	0.0003131	0.00008016	3.4201	3.42	3.81	0.0003	0.00006948	0.00001632	3.8100	3.81
3.43	0.0011	0.0003018	0.00007709	3.4301	3.43	3.82	0.0003	0.00006673	0.00001563	3.8200	3.82
3.44	0.0011	0.0002909	0.00007413	3.4401	3.44	3.83	0.0003	0.00006407	0.00001498	3.8300	3.83
3.45	0.0010	0.0002803	0.00007127	3.4501	3.45	3.84	0.0003	0.00006152	0.00001435	3.8400	3.84
3.46	0.0010	0.0002701	0.00006852	3.4601	3.46	3.85	0.0002	0.00005906	0.00001375	3.8500	3.85
3.47	0.0010	0.0002602	0.00006587	3.4701	3.47	3.86	0.0002	0.00005669	0.00001317	3.8600	3.86
3.48	0.0009	0.0002507	0.00006331	3.4801	3.48	3.87	0.0002	0.00005542	0.00001262	3.8700	3.87
3.49	0.0009	0.0002415	0.00006085	3.4901	3.49	3.88	0.0002	0.00005423	0.00001209	3.8800	3.88
3.50	0.0009	0.0002326	0.00005848	3.5001	3.50	3.89	0.0002	0.00005312	0.00001157	3.8900	3.89
3.51	0.0008	0.0002241	0.00005620	3.5101	3.51	3.90	0.0002	0.00005208	0.00001108	3.9000	3.90
3.52	0.0008	0.0002158	0.00005400	3.5201	3.52	3.91	0.0002	0.00005115	0.00001061	3.9100	3.91
3.53	0.0008	0.0002078	0.00005188	3.5301	3.53	3.92	0.0002	0.00005034	0.00001015	3.9200	3.92
3.54	0.0008	0.0002001	0.00004984	3.5400	3.54	3.93	0.0002	0.00004964	0.000009723	3.9300	3.93
3.55	0.0007	0.0001926	0.00004788	3.5500	3.55	3.94	0.0002	0.00004904	0.000009307	3.9400	3.94
3.56	0.0007	0.0001854	0.00004599	3.5600	3.56	3.95	0.0002	0.00004853	0.000008908	3.9500	3.95
3.57	0.0007	0.0001785	0.00004417	3.5700	3.57	3.96	0.0002	0.00004811	0.000008525	3.9600	3.96
3.58	0.0007	0.0001718	0.00004242	3.5800	3.58	3.97	0.0002	0.00004776	0.000008158	3.9700	3.97
3.59	0.0006	0.0001653	0.00004073	3.5900	3.59	3.98	0.0001	0.00004746	0.000007806	3.9800	3.98
3.60	0.0006	0.0001591	0.00003911	3.6000	3.60	3.99	0.0001	0.00004720	0.000007469	3.9900	3.99
3.61	0.0006	0.0001531	0.00003755	3.6100	3.61	4.00	0.0001	0.00004703	0.000007145	4.0000	4.00

(continued)

THE NORMAL PROBABILITY DISTRIBUTION
SOME FUNCTIONS OF THE UNIT NORMAL DISTRIBUTION

TABLE 1

h	$f_{1,h}$	$F_{1,h}$	$G_{1,h}$	$G_{2,h}$	k	k	$f_{1,k}$	$F_{1,k}$	$G_{1,k}$	$G_{2,k}$	k
0.00	0.3989	0.5000	0.3989	0.3989	0.00	0.40	0.3683	0.3446	0.2304	0.6304	0.10
0.01	0.3989	0.4960	0.3949	0.4040	0.01	0.41	0.3668	0.3409	0.2270	0.6370	0.41
0.02	0.3989	0.4920	0.3890	0.4090	0.02	0.42	0.3653	0.3372	0.2236	0.6436	0.42
0.03	0.3988	0.4880	0.3841	0.4141	0.03	0.43	0.3637	0.3335	0.2203	0.6503	0.43
0.04	0.3986	0.4840	0.3793	0.4193	0.04	0.44	0.3621	0.3300	0.2169	0.6569	0.44
0.05	0.3984	0.4801	0.3744	0.4244	0.05	0.45	0.3605	0.3264	0.2137	0.6637	0.45
0.06	0.3982	0.4761	0.3697	0.4297	0.06	0.46	0.3589	0.3228	0.2104	0.6704	0.46
0.07	0.3980	0.4721	0.3649	0.4349	0.07	0.47	0.3572	0.3192	0.2072	0.6772	0.47
0.08	0.3977	0.4681	0.3602	0.4402	0.08	0.48	0.3555	0.3156	0.2040	0.6840	0.48
0.09	0.3973	0.4641	0.3555	0.4458	0.09	0.49	0.3538	0.3121	0.2009	0.6909	0.49
0.10	0.3970	0.4602	0.3509	0.4509	0.10	0.50	0.3521	0.3085	0.1978	0.6978	0.50
0.11	0.3965	0.4562	0.3462	0.4562	0.11	0.51	0.3503	0.3050	0.1947	0.7047	0.51
0.12	0.3961	0.4522	0.3418	0.4618	0.12	0.52	0.3485	0.3015	0.1917	0.7117	0.52
0.13	0.3956	0.4483	0.3373	0.4673	0.13	0.53	0.3467	0.2981	0.1887	0.7187	0.53
0.14	0.3951	0.4443	0.3328	0.4728	0.14	0.54	0.3448	0.2948	0.1857	0.7257	0.54
0.15	0.3945	0.4404	0.3284	0.4784	0.15	0.55	0.3429	0.2912	0.1828	0.7328	0.55
0.16	0.3939	0.4364	0.3240	0.4840	0.16	0.56	0.3410	0.2877	0.1799	0.7399	0.56
0.17	0.3932	0.4325	0.3197	0.4897	0.17	0.57	0.3391	0.2843	0.1771	0.7471	0.57
0.18	0.3925	0.4286	0.3154	0.4954	0.18	0.58	0.3372	0.2810	0.1742	0.7542	0.58
0.19	0.3918	0.4247	0.3111	0.5011	0.19	0.59	0.3352	0.2776	0.1714	0.7614	0.59
0.20	0.3910	0.4207	0.3069	0.5069	0.20	0.60	0.3332	0.2743	0.1687	0.7687	0.60
0.21	0.3902	0.4168	0.3027	0.5127	0.21	0.61	0.3312	0.2709	0.1659	0.7759	0.61
0.22	0.3894	0.4129	0.2985	0.5186	0.22	0.62	0.3292	0.2676	0.1633	0.7833	0.62
0.23	0.3885	0.4090	0.2944	0.5244	0.23	0.63	0.3271	0.2643	0.1606	0.7906	0.63
0.24	0.3876	0.4052	0.2904	0.5304	0.24	0.64	0.3251	0.2611	0.1580	0.7980	0.64
0.25	0.3867	0.4013	0.2863	0.5363	0.25	0.65	0.3230	0.2578	0.1554	0.8054	0.65
0.26	0.3857	0.3974	0.2824	0.5424	0.26	0.66	0.3209	0.2546	0.1528	0.8128	0.66
0.27	0.3847	0.3936	0.2784	0.5484	0.27	0.67	0.3187	0.2514	0.1503	0.8203	0.67
0.28	0.3836	0.3897	0.2745	0.5545	0.28	0.68	0.3165	0.2483	0.1478	0.8278	0.68
0.29	0.3825	0.3859	0.2706	0.5606	0.29	0.69	0.3144	0.2451	0.1453	0.8353	0.69
0.30	0.3814	0.3821	0.2668	0.5668	0.30	0.70	0.3123	0.2420	0.1429	0.8429	0.70
0.31	0.3802	0.3783	0.2630	0.5730	0.31	0.71	0.3101	0.2389	0.1405	0.8505	0.71
0.32	0.3790	0.3745	0.2592	0.5792	0.32	0.72	0.3079	0.2358	0.1381	0.8581	0.72
0.33	0.3778	0.3707	0.2555	0.5855	0.33	0.73	0.3056	0.2327	0.1358	0.8658	0.73
0.34	0.3765	0.3669	0.2518	0.5918	0.34	0.74	0.3034	0.2297	0.1334	0.8734	0.74
0.35	0.3752	0.3632	0.2481	0.5981	0.35	0.75	0.3011	0.2266	0.1312	0.8812	0.75
0.36	0.3739	0.3594	0.2445	0.6045	0.36	0.76	0.2989	0.2236	0.1289	0.8889	0.76
0.37	0.3725	0.3557	0.2409	0.6109	0.37	0.77	0.2966	0.2206	0.1267	0.8967	0.77
0.38	0.3712	0.3520	0.2374	0.6174	0.38	0.78	0.2943	0.2177	0.1245	0.9045	0.78
0.39	0.3697	0.3483	0.2339	0.6239	0.39	0.79	0.2920	0.2148	0.1223	0.9123	0.79
						0.80	0.2897	0.2119	0.1202	0.9202	0.80
						0.81	0.2874	0.2090	0.1181	0.9281	0.81

(continued)

STOCKING VERSUS NON-STOCKING A GRAPHICAL AID FOLLOWS POPP MODEL

A graphical representation of the decision rule discussed under section 4.8 below. For a given c_s value we use the associated curve. If the $E(t)vr$ and $A/E(i)$ values are such that the corresponding point falls to the left or above the curve, the item should not be stocked. If, on the other hand, the point falls to the right or below the curve, then the item should be stocked.

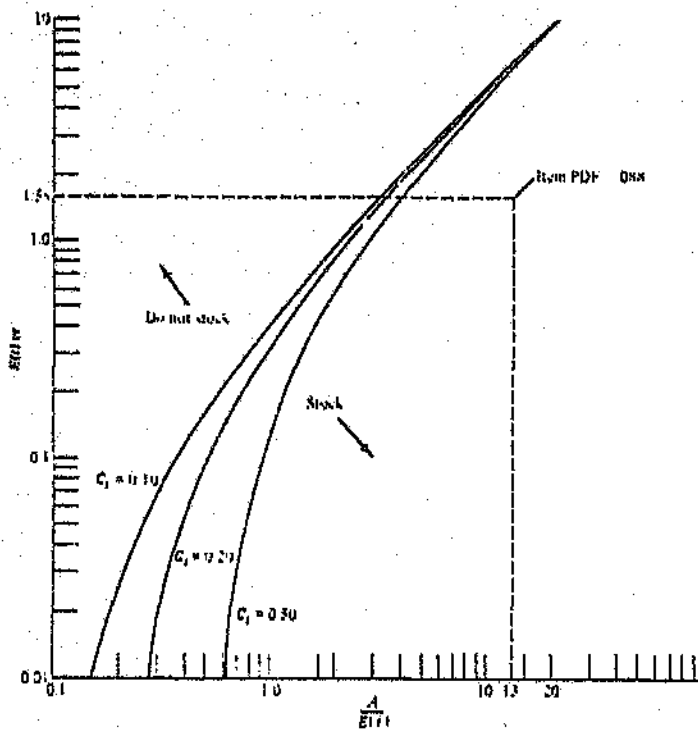


FIGURE 9.1 A Rule for the Stock-No Stock Decision

The behaviour rule. As the setup cost A goes up or as the expected time between transactions decreases we move to the region where stocking is preferred. Considering the vertical axis, as v or r increase, we tend to the nonstocking situation; it becomes too expensive to carry the item in inventory. It is seen that as c_s increases we are less likely to prefer stocking of the item.

Numerical illustration

Relevant parameter values for an item have been estimated as follows:

v = R4,70/roll

$E(t)$ = 1,4 rolls

$E(i)$ = 10 weeks or 10/52 year

$$A = R2.50$$

$$r = 0.24 \text{ R/R/yr}$$

$$C_o = 0.20 \text{ R/yr}$$

We have

$$E(t/yr) = (1.4)(4.70)(0.24) = 1.58 \text{ R/yr}$$

$$A/E(t) = 2.50/10/52 = 13.00 \text{ R/yr}$$

Above Figure reveals that the item should indeed be purchased for stock.

IPP-A (SM-D2)

PROBABILITY OF 95% SATISFING D **CURRENT INVENTORY MODEL**

(LAST SIX MONTHS STATISTICS)

NO	PART-NO	PRIC (R)	USING IRREGULATORY CHECKING CRITERIA						EXCESS STOCK			
			AVG	MAD	NAD	MOH	MOHQ	SS	SS	SS	AOHQ	AOHQ
			UNIT	UNIT	UNIT	(R)	UNIT (R)	NEGATIV	UNITS	(R)	(R)	(R)

SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DE (R)

1	181081300020	2110	0.67	0.44	1.27	7	15473	3	6506	0	4.61	9729	2751	0
2	MA0877200393	1350	0.67	0.89	1.78	6	8400	-3	-3776	-3776	-0.57	-76	-13411	-13411
3	181625304115	393	0.83	0.56	1.53	9	3328	4	1547	0	6.17	2241	647	0
4	SE181322010098	328	0.50	0.67	1.33	8	2624	3	1016	0	4.76	1562	44	0
5	CC158151020503	368	1.17	0.83	2.21	11	4063	2	843	0	5.05	1859	113	0
6	SB031882342131	1788	0.83	0.83	1.88	19	33525	11	19909	0	13.48	24099	18502	0
7	SE031861646133	687	0.67	0.67	1.50	15	13306	10	8802	0	11.80	10465	7639	0
8	CD181322070009	450	0.50	0.67	1.33	8	3600	-1	-666	-666	0.19	84	-2828	-2828
9	181839030049	187	0.87	0.67	1.50	9	1683	0	-29	-29	1.72	322	-941	-941
10	SC181358040006	510	0.33	0.44	0.89	5	2720	-3	-1615	-1615	-2.06	-1048	-4169	-4169
11	SD031882209304	750	0.33	0.44	0.89	9	6667	1	955	0	2.38	1788	-2310	-2310
12	181523158118	403	0.50	0.50	1.13	7	2720	4	1798	0	5.87	2365	1608	0
13	LD0582275	201	0.00	0.00	0.00	0	0	-8	-1844	-1844	-9.17	-1844	-3550	-3550
14	SD0158541010502	680	1.17	0.83	2.21	10	6559	7	4465	0	9.52	6286	4332	0
15	SE181307258020	233	0.83	0.83	1.88	10	2294	1	349	0	3.84	895	-824	-824
16	MA151051006133	608	0.00	0.00	0.00	0	0	-8	-4895	-4895	-8.08	-4895	-9724	-9724
17	MD030501308844	144	0.00	0.00	0.00	0	0	-7	-1066	-1066	-7.40	-1066	-2150	-2150
18	181152026786	248	0.67	0.67	1.50	9	2232	6	1502	0	7.93	1967	1294	0

SLOW MOVING SKUs WITH ERATIC (LUMPY) DEMAND PA

GAMMA DISTRIBUTION OF DEMAND

19	181825104146	1155	0.33	0.56	1.03	6	7123	-1	-807	-807	0.59	677	-6851	-6851
20	SD0158541010501	691	0.83	0.56	1.53	7	4338	1	868	0	3.28	2073	-1377	-1377
21	SG158152020513	120	0.50	0.50	1.13	6	675	1	135	0	2.53	304	-208	-208
22	SE0216775	248	0.33	0.56	1.03	5	1138	-3	-736	-736	-1.71	-420	-2301	-2301
23	SD181434900544	404	1.00	1.00	2.25	10	4091	6	1954	0	7.65	3090	968	0
24	SF181444008014	208	0.50	0.67	1.33	6	1236	1	234	0	2.80	577	-435	-435
25	1818111165219	265	0.50	0.67	1.33	8	2120	1	214	0	2.47	656	-1192	-1192
26	SE181811518135	696	1.00	0.33	1.42	7	5177	5	3490	0	6.79	4723	2991	0
27	SE02K1845	328	0.83	0.83	1.88	8	2751	5	1509	0	6.97	2273	982	0
28	181806200000	156	0.33	0.44	0.89	5	832	-2	-239	-239	-0.42	-66	-1156	-1156
29	SE0181514010024	182	0.33	0.44	0.89	3	566	-1	-144	-144	0.32	58	-772	-772
30	SA870160883	2000	0.67	0.89	1.78	6	12444	1	1406	0	2.93	5850	-7567	-7567
31	SD181823206069	528	0.83	0.83	1.88	10	5178	5	2770	0	7.61	4003	1313	0
32	SG05890759910	163	0.83	0.56	1.53	7	1121	6	810	0	6.88	1122	757	0
33	SB181391056728	1603	0.83	0.56	1.53	5	7959	4	5740	0	5.49	8801	5956	0
34	181122018245	488	0.33	0.44	0.89	5	2587	3	1635	0	4.48	2174	1025	0
35	MC873170080	245	0.67	0.89	1.78	6	1524	-2	-388	-388	0.64	156	-2241	-2241
36	MA877200121	1475	0.67	0.89	1.78	6	9178	0	-353	-353	1.98	2925	-9213	-9213
37	CA865010200	1190	0.50	0.67	1.33	5	6347	-1	-1686	-1686	0.25	277	-9554	-9554
38	181301010087	1180	0.50	0.50	1.13	7	7985	5	6036	0	6.52	7696	554	0
39	LA8781010724	1130	0.33	0.56	1.03	4	4065	-3	-3085	-3085	-1.44	-1633	-10731	-10731
40	SD31297302019	351	0.17	0.28	0.51	3	1082	-4	-1328	-1328	-3.14	-1102	-3900	-3900
41	SC878110138	650	0.67	0.67	1.50	6	4044	3	1944	0	5.21	3389	622	0
42	SE0181621818013	285	0.50	0.67	1.33	6	1710	0	82	0	1.96	557	-1469	-1469
43	SF181434908022	235	0.83	0.83	1.88	8	1983	5	1237	0	7.61	1788	851	0
44	SF0216721	220	0.50	0.67	1.33	6	1320	1	250	0	2.80	616	-745	-745
45	SF5FH3120014	380	0.33	0.56	1.03	4	1562	2	576	0	2.80	1084	-195	-195
46	SE158181018500	194	0.83	0.83	1.88	8	1455	3	548	0	5.17	1003	-188	-188
47	181626007183	2380	0.33	0.56	1.03	6	14677	3	7674	0	4.51	10732	1934	0
48	SE181504101279	683	0.17	0.28	0.51	3	1762	0	4	0	0.65	423	-1916	-1916
49	SF181352010035	222	0.00	0.00	0.00	0	0	-8	-1669	-1669	-7.52	-1669	-3895	-3895
50	SF181434906021	235	0.17	0.28	0.51	2	543	-1	-153	-153	-0.01	-2	-999	-999
51	SE0216717	474	0.67	0.67	1.50	7	3200	5	2197	0	6.51	3086	1670	0
52	SE887721610	302	0.00	0.00	0.00	0	0	-8	-2428	-2428	-8.04	-2428	-5718	-5718
53	LC877 2028	291	0.00	0.00	0.00	0	0	-8	-2390	-2390	-8.21	-2390	-6040	-6040
54	181081016318	2450	0.83	0.83	1.88	11	27563	9	21155	0	10.98	26897	17769	0
55	SG30780118007	443	0.33	0.44	0.89	5	2363	4	1928	0	5.46	2420	1730	0
56	SG877830232	310	0.67	0.44	1.22	4	1326	3	1076	0	5.00	1549	1157	0
57	SA870150984	2110	0.83	0.83	1.88	7	13847	4	8734	0	6.48	13679	5676	0
58	LA877002086	635	0.33	0.44	0.89	3	1976	-5	-2982	-2982	-3.59	-2277	-10403	-10403
59	181301010119	1430	0.17	0.28	0.51	3	4409	-4	-5876	-5876	-3.47	-4957	-20835	-20835
60	SA181434006264	996	0.17	0.28	0.51	2	2303	-7	-6757	-6757	-6.14	-6117	-20771	-20771

PROBABILITY OF 95% SATISFYING D CURRENT INVENTORY MODEL

(LAST SIX MONTHS STATISTICS)

NO	PART-NO	PRC (R)	USING IRREGULARITY CHECKING CRITERIA					EXCESS STOCK						
			AVG UNIT	MAD UNIT	NAD UNIT	MOH UNIT	MOHQ (R)	SS UNIT	SS (R)	SS NEGATIV	AOHQ UNITS	AOHQ (R)	(R)	NEGATIVE (R)
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DE (R)														
61	SD02A5105	989	0.00	0.00	0.00	0	0	-2	-2092	-2092	-2.12	-2092	-5661	-5661
62	181061018155	3200	0.33	0.56	1.03	6	19733	1	2995	0	2.22	7106	-20027	-20027
63	LA685D10010	817	0.50	0.67	1.33	5	3291	-4	-2369	-2369	-2.17	-1341	-11328	-11328
64	SD181432208052	793	0.67	0.89	1.78	5	4934	4	3226	0	6.29	4988	1767	0
65	SF030501002032	153	0.17	0.28	0.51	2	275	0	49	0	0.96	147	-274	-274
66	SA895000183	20100	0.00	0.00	0.00	0	0	-3	-62233	-62233	-3.10	-62233	-177612	-177612
67	SG873350230	235	0.50	0.67	1.33	5	1097	4	907	0	5.53	1299	927	0
68	SC0158122048504	1107	0.67	0.44	1.22	4	4736	3	2947	0	4.19	4639	1465	0
69	SG188328105502	113	0.33	0.44	0.89	4	402	1	128	0	2.24	254	-232	-232
70	SG181952020038	171	0.17	0.28	0.51	3	527	-1	-144	-144	-0.20	-34	-1181	-1181
71	115102509080	106	0.00	0.00	0.00	0	0	-4	-416	-416	-3.92	-416	-1127	-1127
72	SG0181511016264	160	0.67	0.89	1.78	8	1280	7	1077	0	8.95	1432	1070	0
73	SF897100081	470	0.50	0.67	1.33	5	2193	4	1687	0	5.26	2471	1395	0
74	SF0220368	272	0.00	0.00	0.00	0	0	-1	-403	-403	-1.48	-403	-1230	-1230
75	SA820050225300	7062	0.33	0.56	1.03	4	25404	0	1637	0	1.52	10710	-46451	-46451
76	SA865000210	565	0.00	0.00	0.00	0	0	-8	-4694	-4694	-8.31	-4694	-15862	-15862
SLOW MOVING SKUs WITH POISSON DISTRIBUTION OF D														
77	SA870001851	11300	0.83	0.56	1.53	5	60424	4	49776	0	6.31	71355	50348	0
78	CO0158417025501	380	0.33	0.66	1.03	5	1758	-4	-1458	-1458	-2.55	-970	-3070	-3070
79	SC181907156051	552	0.67	0.89	1.78	12	5379	2	933	0	3.91	2160	-1316	-1316
80	18104408271	1960	0.33	0.44	0.89	5	10453	-2	-3644	-3644	-0.75	-1466	-11668	-11668
81	CA875000941	2040	0.67	0.67	1.50	5	10710	0	0	0	1.88	3825	-6209	-6209
82	CE5D4222	166	0.50	0.67	1.33	5	885	0	23	0	1.81	300	-504	-504
83	SC181504108051	1901	0.50	0.50	1.13	6	7684	2	2780	0	3.54	4810	125	0
84	181826007179	1180	0.50	0.67	1.33	8	9280	-2	-2476	-2476	-0.47	-543	-7914	-7914
85	SC5FTG180558	560	0.67	0.67	1.50	6	3360	2	1228	0	4.07	2278	60	0
86	SF887503620	402	0.67	0.67	1.50	10	3920	6	2451	0	7.97	3204	1791	0
87	SA877100140	1820	0.33	0.44	0.89	3	5662	-1	-1443	-1443	0.32	579	-7110	-7110
88	SF1219884	345	0.33	0.44	0.89	4	1380	1	358	0	2.15	742	-329	-329
89	CA870150582	2050	0.67	0.67	1.50	5	10763	2	4415	0	4.03	8259	991	0
90	151023050009	730	0.67	0.67	1.50	9	6570	6	4422	0	7.93	5791	3542	0
91	181081300028	2050	0.67	0.67	1.50	9	18450	5	11078	0	7.28	14922	7620	0
92	SED590345110	313	0.67	0.67	1.50	7	2113	3	1053	0	5.24	1640	486	0
93	SD878000074	660	0.83	0.56	1.53	5	3529	3	2196	0	5.24	3457	1716	0
94	181482006135	4050	0.50	0.67	1.33	8	32400	6	23132	0	7.38	29882	18605	0
95	SA872000355	1880	0.50	0.67	1.33	5	8680	3	4674	0	4.18	7774	1852	0
96	181381090014	216	0.50	0.67	1.33	8	1728	6	1375	0	8.03	1735	1233	0
97	181152025385	180	0.33	0.44	0.89	5	960	4	725	0	5.14	925	615	0
98	SA870000085	6050	0.17	0.28	0.51	2	10882	0	1923	0	0.96	5809	-9811	-9811
99	181424015230	287	0.17	0.28	0.51	3	885	2	510	0	2.42	694	52	0
100	SC877200477	2408	0.50	0.67	1.33	5	11237	4	9617	0	5.66	13630	9918	0
101	CA031888999270	4351	0.50	0.67	1.33	13	58013	-3	-11017	-11017	-0.87	-3765	-32793	-32793
102	ME181434900021	159	0.50	0.67	1.33	6	954	-4	-694	-694	-2.70	-429	-1532	-1532
FAST MOVING SKUs WITH NORMAL DISTRIBUTION OF DE														
103	181152100058	113	0.17	0.28	0.51	3	348	-83	-9367	-9367	-82.26	-9295	-13119	-13119
104	151012010298	236	0.17	.28	0.51	3	798	-524	-123729	-123729	-523.63	-123577	-172722	-172722
105	FA181501100144	241	0.00	0.00	0.00	0	0	-177	-42620	-42620	-176.85	-42620	-62359	-62359
106	S9031861584137	797	0.33	0.56	1.03	10	8191	-10	-7994	-7994	-8.75	-6970	-13121	-13121
107	FA181501100226	178	0.00	0.00	0.00	0	0	-304	-54153	-54153	-304.23	-54153	-78413	-78413
108	MA181356085001	602	0.00	0.00	0.00	0	0	-50	-30112	-30112	-50.02	-30112	-43511	-43511
109	181255200087	49	0.00	0.00	0.00	0	0	-518	-25358	-25358	-517.52	-25358	-36711	-36711
110	FA0151025030813	35	0.00	0.00	0.00	0	0	-383	-13390	-13390	-382.66	-13390	-19313	-19313
111	FA0864050821	350	0.17	0.28	0.51	2	809	-73	-25696	-25696	-72.78	-25472	-38749	-38749
112	MA181356100024	973	0.00	0.00	0.00	0	0	-37	-35945	-35945	-36.94	-35945	-52592	-52592
113	MA031882331003	1391	0.33	0.56	1.03	10	14296	-107	-149012	-149012	-105.84	-147225	-218020	-218020
114	151024108468	10	0.00	0.00	0.00	0	0	-904	-9040	-9040	-904.04	-9040	-13466	-13466
115	151068010116	208	0.00	0.00	0.00	0	0	-69	-14280	-14280	-68.65	-14280	-21313	-21313
116	1818150182	82	0.33	0.56	1.03	6	506	-15	-1264	-1264	-14.13	-1158	-2039	-2039
117	FA181307258026	465	0.17	0.28	0.51	2	1075	-58	-26762	-26762	-56.91	-26463	-41716	-41716
118	MA031881486234	948	0.00	0.00	0.00	0	0	-83	-78812	-78812	-83.13	-78812	-115177	-115177
119	18181104020	680	0.67	0.89	1.78	11	7040	-1	-512	-512	1.46	955	-2947	-2947

PROBABILITY OF 95% S.

**CURRENT INVENTORY MODEL
(LAST SIX MONTHS STATISTICS)**

NO	PART-NO	PRIC (R)	USING IRREGULARITY CHECKING CRITERIA						EXCESS STOCK					
			AVG UNIT	MAD UNIT	NAD UNIT	MOH UNIT	MOHQ (R)	SS UNIT	SS (R)	AOHQ UNITS	AOHQ (R)	(R)	NEGATIVE (R)	
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DE (R)														
120	151018047023	109	0.00	0.00	0.00	0	0	-118	-12900	-12900	-118.35	-12900	-19464	-19464
121	106562790215	83	0.00	0.00	0.00	0	0	-328	-30495	-30495	-327.90	-30495	-46067	-46067
122	181960200235	27	0.00	0.00	0.00	0	0	-289	-7794	-7794	-288.67	-7794	-11920	-11920
123	181394018011	7	0.00	0.00	0.00	0	0	-102	-29070	-29070	-102.00	-29070	-44593	-44593
124	181182025521	192	0.00	0.00	0.00	0	0	-49	-9478	-9478	-49.37	-9478	-14542	-14542
125	181303050182	705	0.33	0.56	1.03	6	4348	-12	-8559	-8559	-10.86	-7654	-14694	-14694
126	FA0151025017300	624	0.33	0.56	1.03	7	4169	-245	-152923	-152923	-243.78	-152122	-238339	-238339
127	FA0106560432243R	14	0.00	0.00	0.00	0	0	-480	-6726	-6726	-480.46	-6726	-11036	-11036
128	FA0897840172	55	0.33	0.56	1.03	5	254	-224	-12311	-12311	-222.55	-12240	-20093	-20093
129	181443010148	429	0.33	0.56	1.03	6	2646	-8	-3526	-3526	-6.93	-2974	-6576	-6576
130	181305800029	41	0.00	0.00	0.00	0	0	-94	-3847	-3847	-93.83	-3847	-6103	-6103
131	CDQ31861488133	771	0.50	0.87	1.33	13	10280	0	5	0	1.67	1290	-4371	-4371
132	15104106488	10	0.00	0.00	0.00	0	0	-411	-4108	-4108	-410.77	-4108	-6570	-6570
133	LD30750132032	218	0.17	0.28	0.51	3	672	-13	-2891	-2891	-12.62	-2751	-4878	-4878
134	181466106333	340	0.50	0.87	1.33	8	2720	-6	-2060	-2060	-4.39	-1493	-4389	-4389
135	181415018012	480	0.00	0.00	0.00	0	0	-117	-56335	-56335	-117.37	-56335	-90211	-90211
136	18144018136	865	0.00	0.00	0.00	0	0	-14	-11877	-11877	-13.73	-11877	-19224	-19224
137	151011116439	58	0.00	0.00	0.00	0	0	-71	-3974	-3974	-70.96	-3974	-6456	-6456
138	FA181442056010	289	0.17	0.28	0.51	2	520	-128	-36984	-36984	-127.33	-38798	-64643	-64643
139	151011116489	58	0.00	0.00	0.00	0	0	-55	-3069	-3069	-54.81	-3069	-5009	-5009
140	FA1815011002227	191	0.00	0.00	0.00	0	0	-55	-9892	-9892	-54.65	-9892	-16270	-16270
141	151044109117	109	0.00	0.00	0.00	0	0	-60	-6557	-6557	-60.15	-6557	-10941	-10941
142	181418130008	1555	0.17	0.28	0.51	3	4795	-15	-22657	-22657	-13.93	-21658	-38806	-38806
143	181117018057	148	0.33	0.56	1.03	6	913	-42	-6200	-6200	-40.61	-6010	-10538	-10538
144	181837308149	51	0.00	0.00	0.00	0	0	-483	-24609	-24609	-482.54	-24609	-40340	-40340
145	181351896316	1540	0.00	0.00	0.00	0	0	-25	-38767	-38767	-25.17	-38767	-63728	-63728
146	181417158003	726	0.00	0.00	0.00	0	0	-65	-46930	-46930	-64.73	-46930	-77257	-77257
147	SB031861861031	728	0.17	0.28	0.51	5	3741	-10	-7347	-7347	-9.45	-6879	-13523	-13523
148	181637010010	439	0.57	0.89	1.78	11	4683	-4	-1919	-1919	-2.15	-944	-5374	-5374
149	151065008228	490	0.00	0.00	0.00	0	0	-22	-10573	-10573	-21.58	-10573	-17761	-17761
150	MA877200529	1376	0.00	0.00	0.00	0	0	-16	-22397	-22397	-16.28	-22397	-40699	-40699
151	181864200231	68	0.17	0.28	0.51	3	204	-21	-1393	-1393	-20.47	-1351	-2457	-2457
152	CC181358110008	544	0.33	0.56	1.03	6	3355	-4	-2159	-2159	-2.68	-1460	-5279	-5279
153	FA031250307686	110	0.00	0.00	0.00	0	0	-47	-5123	-5123	-46.58	-5123	-9445	-9445
154	181323010272	278	0.00	0.00	0.00	0	0	-15	-4151	-4151	-15.04	-4151	-7168	-7168
155	SD031861748101	731	0.50	0.50	1.13	11	8224	-2	-1518	-1518	-0.67	-490	-7118	-7118
156	151303010322	1095	0.33	0.56	1.03	6	6753	-11	-11505	-11505	-9.22	-10098	-23597	-23597
157	151024018141	391	0.33	0.44	0.89	5	1925	-19	-6690	-6690	-17.42	-6289	-12709	-12709
158	104274059260	100	0.00	0.00	0.00	0	0	-49	-4904	-4904	-49.04	-4904	-8642	-8642
159	MA873000066	1420	0.33	0.56	1.03	4	5108	-17	-23756	-23756	-15.44	-21932	-48143	-48143
160	SC031882320032	1533	0.33	0.44	0.89	9	13627	-2	-2912	-2912	-0.79	-1209	-13239	-13239
161	181437018187	584	0.83	0.83	1.88	11	6345	-1	-477	-477	1.50	845	-4486	-4486
162	FA181335112002	217	0.00	0.00	0.00	0	0	-168	-36464	-36464	-168.04	-36464	-65867	-65867
163	MA181307256031	771	0.17	0.28	0.51	2	1783	-14	-10775	-10775	-13.33	-10280	-21085	-21085
164	LA02K1947	592	0.00	0.00	0.00	0	0	-17	-9768	-9768	-16.50	-9768	-18207	-18207
165	LA02K1948	602	0.00	0.00	0.00	0	0	-13	-7895	-7895	-13.12	-7895	-14789	-14789
166	FA0106580722422R	27	0.00	0.00	0.00	0	0	-234	-8318	-8318	-234.00	-8318	-12118	-12118
167	ME181521506005	218	0.00	0.00	0.00	0	0	-17	-3756	-3756	-17.23	-3756	-7210	-7210

**FAST MOVING SKUs WITH ERATIC (LUMPY) DEMAND PAT
GAMMA DISTRIBUTION OF DEMAND**

168	FA030750131003	68	0.00	0.00	0.00	0	0	-178	-12074	-12074	-177.58	-12074	-24955	-24955
169	LB887721800	274	0.00	0.00	0.00	0	0	-26	-7208	-7208	-26.31	-7208	-14061	-14061
170	MA106328780001	183	0.00	0.00	0.00	0	0	-28	-4608	-4608	-28.27	-4608	-9635	-9635
171	181322050077	507	0.50	0.50	1.13	7	3422	-16	-8346	-8346	-15.06	-7633	-19149	-19149
172	MA878000171	985	0.17	0.28	0.51	2	1772	-12	-11488	-11488	-11.02	-10855	-25710	-25710
173	LA033151119001	237	0.67	0.67	1.50	15	3555	-74	-17502	-17502	-71.97	-17057	-37659	-37659
174	LA181434008815	1114	0.33	0.56	1.03	5	5152	-10	-11344	-11344	-8.90	-9912	-27646	-27646
175	MA897811545	825	0.00	0.00	0.00	0	0	-17	-13771	-13771	-16.69	-13771	-30164	-30164
176	MA871080024	330	0.17	0.28	0.51	2	594	-30	-9979	-9979	-29.50	-9767	-23172	-23172
177	LB887721820	301	0.00	0.00	0.00	0	0	-29	-8579	-8579	-28.50	-8579	-18572	-18572
178	SD687721830	348	0.17	0.28	0.51	3	1162	-12	-4305	-4305	-11.73	-4082	-14078	-14078

PROBABILITY OF 85% SATISFYING D CURRENT INVENTORY MODEL

(LAST SIX MONTHS STATISTICS)

NO	PART-NO	PRIC USING IRREGULATORY CHECKING CRITERIA										EXCESS STOCK		
		(R)	AVG	MAD	NAD	MOH	MOHQ	SS	SS	SS	AOHQ	AOHQ	(R)	NEGATIVE
		UNIT	UNIT	UNIT	UNIT	(R)	UNIT	(R)	NEGATIV	UNITS	(R)	(R)	(R)	
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DE (R)														
179	SF101391150140	164	0.50	0.50	1.13	7	1107	-5	-877	-877	-3.94	-646	-4316	-4316
180	LA0693298C0	203	0.00	0.00	0.00	0	0	-227	-46136	-46136	-227.27	-46136	-142008	-142008
181	LA156463020508	585	0.00	0.00	0.00	0	0	-21	-7833	-7833	-21.48	-7833	-38245	-38245
SKUs WITH VERY ERATIC (LUMPY) DEMAND PATTERN										0	0.00			
MANUAL OVERRIDE										0	0.00			
182	EG896720129	575	0.17	0.28	0.51	2	1034	2	879	0	2.17	1249	712	0
183	EG0191322050125	152	0.50	0.67	1.33	5	2576	4	2353	0	5.93	3273	2457	0
184	SA897003632	17100	0.50	0.87	1.33	5	79800	4	70592	0	5.79	99092	71708	0
185	1R1463020180	20	0.00	0.00	0.00	0	0	-1	-708	-708	-0.98	-708	-2867	-2867
186	SA893000151	20100	0.00	0.00	0.00	0	0	-1	-27058	-27058	-1.35	-27058	-112624	-112624
187	SF21103201920	170	0.38	0.56	1.03	4	1331	2	733	0	3.27	121	-1294	-1294
188	SF211003210000	134	0.00	0.00	0.00	0	0	-5	-649	-649	-4.85	-649	-2686	-2686
TOTAL							859280	-1381057	-1714840	78	-3017219	-3283610		
SLOW MOVING SKUs (GAMMA)							859280	-1381057	-1714840	-1163078	-3007493	-3274850		
SLOW MOVING SKUs (POISSON)														
FAST MOVING SKUs (GAMMA)														
FAST MOVING SKUs (NORMAL)														
MANUAL OVERRIDE														

PROBABILITY OF 95% SATISFYING DEMAND

CURRENT INVENTORY MODEL
(LAST SIX MONTHS STATISTICS)

NO	PART-NO	PRIC	MONTHLY DEMAND													ANNUAL										EXCESS STOCK																						
			(R)	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	DEMAN	SD	AVG	MAX	MOB	COHQ	SS	SS	AQHQ	AQHQ																							
																									UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)	UNITS (R)
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																																																
162	FA101335112002	217	0	70	5	01	44	10	05	05	0	5	50	514	31.00	49.00	87.75	627	114281	358.8	77788	408	88511	59108																								
163	MA181307250031	771	17	0	2	4	8	3	12	7	0	1	4	5	77	2.50	5.33	8.98	40	31091	28.0	18523	31	24231	13426																							
164	LA12K1047	582	2	11	4	5	9	15	10	6	0	7	0	78	3.68	5.33	8.78	44	26048	27.8	16280	35	20720	12281																								
165	LA12K1048	602	0	4	8	3	7	11	3	5	5	0	12	0	62	3.17	4.17	8.13	57	22011	23.4	14115	20	17704	10310																							
166	FA10133507224220	27	82	243	55	87	128	220	14	142	141	0	148	78	1352	53.17	85.50	158.21	638	17037	398.8	10788	529	14279	8479																							
167	ME10152190005	218	2	0	3	4	4	15	3	0	15	0	4	4	84	3.33	5.00	8.17	49	10401	30.0	8735	37	8078	4623																							

FAST MOVING SKUs WITH ERATIC (LUMPY) DEMAND PATTERN
GAMMA DISTRIBUTION OF DEMAND

168	FA1013350111003	68	44	148	32	154	243	174	128	19	120	3	156	101	1318	51.11	87.87	151.58	830	35970	352.8	23998	480	32821	18740
169	LA101434003015	274	12	0	12	7	0	18	15	0	0	0	3	5	72	4.11	3.83	8.87	38	18830	32.8	8771	39	10888	3818
170	MA181307250031	163	7	23	25	31	31	29	31	11	10	0	7	5	210	8.89	10.87	19.28	87	10930	38.2	8380	58	9881	4254
171	LA12K1047	307	0	0	1	12	18	5	17	1	1	1	0	0	71	4.58	3.33	8.03	54	27463	31.0	15884	38	18158	7840
172	MA181307250031	985	3	8	18	6	10	10	6	8	21	1	5	5	100	4.44	7.83	13.39	47	48160	33.4	32889	43	42370	27515
173	LA1013351119001	237	0	54	5	0	25	8	1	40	2	0	4	1	140	10.87	1.00	21.33	213	50980	124.6	29503	138	32884	12882
174	LA101434003015	1144	2	4	2	12	8	8	10	18	8	2	3	3	79	5.17	8.83	13.39	60	88831	45.0	50135	52	57633	39500
175	MA181307250031	825	3	12	15	12	2	11	27	5	20	9	7	10	124	8.00	11.50	21.50	75	82081	80.8	48340	70	58147	41753
176	MA181307250031	330	13	17	35	11	10	13	1	33	81	11	12	25	238	15.89	23.17	43.03	181	48987	118.8	38124	141	48878	33272
177	LA12K1047	301	1	6	2	3	4	20	24	5	0	0	0	0	70	7.87	7.00	18.58	108	32448	78.3	23887	87	28124	16130
178	LA12K1047	348	0	0	0	0	3	13	28	0	0	0	1	0	43	7.17	4.50	13.48	87	30443	71.8	24975	76	28414	18416
179	FA1013351119001	184	1	0	0	0	0	4	8	1	25	0	1	1	37	6.11	5.33	12.97	78	12788	65.7	10781	89	11384	7885
180	LA1013351119001	203	8	0	0	0	0	10	8	111	498	0	8	24	822	122.87	101.00	254.33	1833	33893	888.8	289457	1488	301588	205728
181	LA1013351119001	385	0	0	0	1	0	0	7	0	0	0	0	0	124	1.94	1.17	3.60	14	8282	-7.1	-2582	5	1770	-28641

SKUs WITH VERY ERATIC (LUMPY) DEMAND PATTERN
MANUAL OVERRIDE

182	FA1013351119001	575	0	0	0	0	0	1	0	0	1	0	0	0	2	0.28	0.17	0.51	2	1034	1.8	879	2	880	453
183	SG0181322050125	552	0	0	0	0	0	0	0	1	2	0	0	0	3	0.87	0.50	1.38	3	2576	4.3	2353	5	2512	1888
184	FA1013351119001	17109	0	0	0	0	0	1	0	2	0	0	0	0	4	0.87	0.50	1.38	5	78000	4.1	70592	5	77189	48732
185	LA12K1047	720	0	0	0	0	0	0	0	0	3	0	0	0	7	0.83	0.50	1.54	8	8880	0.3	8554	0	8182	4001
186	FA1013351119001	20100	2	0	1	0	1	0	0	0	0	0	0	0	10	1.87	1.00	3.08	11	218813	9.4	188958	10	208182	123610
187	FA1013351119001	370	0	0	0	0	0	0	10	2	0	0	0	0	12	2.87	2.00	5.33	19	8801	17.1	8508	18	8736	4233
188	FA1013351119001	134	0	0	0	0	0	0	14	18	4	0	0	0	35	8.87	8.00	14.33	58	8722	45.3	8073	48	8537	4500

TOTAL
SLOW MOVING SKUs (GAMMA)
SLOW MOVING SKUs (POISSON)
FAST MOVING SKUs (GAMMA)
FAST MOVING SKUs (NORMAL)
MANUAL OVERRIDE

ITEMS

7

$F_1 = 0.72$
 $F_2 = 0.70$
6431522
6431522
4181184
5177807
4993136
3138893
448456
1485458

PROBABILITY OF 95% SATISFYING DEMAND

CURRENT INVENTORY MODEL
(LAST SIX MONTHS STATISTICS)

NO	PART-NO	PRIC	MONTHLY DEMAND												ANNUAL											
			(R)	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	DEMAN	MAX	AVG	MAX	LOH	MOHQ	SS	SS	ADHQ	ADHQ	EXCESS
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																										
102	FA191335112002	217	0	70	5	91	44	10	85	89	85	0	8	60	514	31.00	49.00	87.75	827	114261	350.8	77788	408	88511	(R)	59108
103	MA1013072580031	771	17	6	2	4	8	3	12	7	8	1	4	5	77	2.50	5.83	8.88	40	31091	24.0	18523	31	24231		13426
104	LA02K1947	592	2	11	4	5	9	15	10	8	9	0	7	0	78	3.58	5.33	9.78	44	28048	27.5	18280	35	20728		12281
105	LA02K1948	802	6	4	8	3	7	11	3	5	5	0	12	0	82	3.17	4.17	8.13	37	22011	23.0	14115	29	17704		10918
106	FA019830723422R	87	92	243	55	97	128	225	88	142	141	0	148	16	1382	58.17	83.80	158.21	633	17037	388.8	10789	529	14279		8479
107	ME101521508005	218	2	6	3	4	4	15	3	4	15	0	4	4	84	3.33	5.00	9.17	48	10481	30.8	6735	37	8078		4823
FAST MOVING SKUs WITH ERATIC (LUMPY) DEMAND PATTERN																										
GAMMA DISTRIBUTION OF DEMAND																										
169	FA030796131003	89	44	149	32	154	243	175	128	19	120	3	155	101	1310	51.11	87.87	151.58	830	36070	352.8	23996	480	32821		19740
168	LB897721600	274	12	0	12	7	0	18	15	0	0	0	3	5	72	4.11	3.83	8.97	68	18920	32.0	8771	39	10968		3816
170	MA108323790001	183	7	23	25	31	31	28	31	11	10	0	7	5	210	8.89	10.87	19.28	87	10988	39.2	6390	59	8991		4694
171	1B122050077	607	8	0	1	12	16	5	17	1	1	0	0	0	71	4.58	3.33	8.03	64	27403	31.0	15694	38	19156		7640
172	MA87000071	885	3	8	18	6	10	10	6	0	21	1	8	5	108	4.44	7.83	13.38	47	48150	33.4	32889	43	42370		27515
173	LA03315118001	237	0	54	3	0	25	8	1	45	2	0	4	1	140	10.67	8.00	21.33	213	50880	124.5	28503	138	32694		12092
174	LA181434088915	1114	2	4	2	12	0	9	10	10	16	0	2	3	70	5.17	6.83	13.28	80	86831	45.0	5013	52	57833		39900
175	MA87811545	825	3	12	15	12	2	11	27	5	20	0	7	10	121	8.00	11.50	21.50	75	62881	68.8	48310	70	58147		41733
176	MA871600024	330	13	17	35	11	19	13	1	33	61	11	12	21	233	15.89	23.17	43.03	151	49997	118.8	38124	141	48876		33272
177	LB897721820	301	1	8	2	3	4	20	28	5	0	0	0	8	70	7.87	7.00	16.58	103	32486	78.3	23887	87	28124		18130
178	SD897721810	348	0	0	0	0	3	13	28	0	0	0	1	0	43	7.17	4.50	13.48	87	30443	71.9	24975	76	28414		18418
179	SP181291150140	184	1	0	0	0	0	4	8	1	23	0	1	1	37	8.11	5.33	12.97	78	12765	69.7	10761	89	11364		7695
180	LA089227800	203	8	0	0	0	0	10	5	111	450	0	6	34	822	122.87	101.00	254.33	1653	33688	688.8	288457	1488	301598		205728
181	LA159453020303	385	0	0	0	117	0	0	7	0	0	0	0	0	124	1.84	1.17	3.80	14	5282	-7.1	-2582	5	1770		-28841
SKUs WITH VERY ERATIC (LUMPY) DEMAND PATTERN																										
MANUAL OVERRIDE																										
182	SP804729128	575	0	0	0	0	0	1	0	0	1	0	0	0	2	0.28	0.17	0.51	2	1084	1.3	879	2	890		453
183	SG018132290125	552	3	0	0	0	0	0	0	1	2	0	0	0	3	0.97	0.50	1.33	5	2876	4.3	2353	5	2512		1888
184	SA80700332	17100	0	0	0	0	0	1	1	0	2	0	0	0	4	0.87	0.50	1.32	6	7899	4.1	70592	5	77169		49782
185	1B1453820180	728	0	0	0	0	0	0	0	0	3	0	0	0	3	0.83	0.50	1.54	9	6980	8.3	5854	9	8182		4901
186	SA808000151	20100	2	0	1	0	1	0	0	0	0	0	0	0	10	1.67	1.00	3.08	11	218013	8.4	188855	10	208182		123816
187	SP21103281820	376	0	3	0	0	0	0	10	2	0	0	0	0	12	2.87	2.00	5.83	10	6987	17.1	6309	18	8738		333
188	SP211003210000	134	0	0	0	0	0	0	14	18	4	0	0	0	38	8.87	8.00	14.33	50	8722	48.3	6073	49	8597		4500

TOTA ITEMS
SLOW MOVING SKUs (GAMMA)
SLOW MOVING SKUs (POISSON)
FAST MOVING SKUs (GAMMA)
FAST MOVING SKUs (NORMAL)
MANUAL OVERRIDE 7

P. = 0.02
P. = 0.02
643152
643152
4181184
5177807
643152
643152
3138985
4485458
1485458

PROBABILITY OF 95% SATISFYING DEMAND

CURRENT INVENTORY MODEL
(LAST SIX MONTHS STATISTICS)

NO	PART-NO	PRIC	MONTHLY DEMAND												ANNUAL										EXCESS STOCK		
			(R)	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	DEMAN	MAX	AVG	NAD	SDH	ISOHQ	SS	SE	AOHQ		AORQ	(R)
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																											
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																											
1	18109130020	2110	0	0	1	2	1	2	3	0	1	1	1	13	0.61	1.17	1.93	12	24441	7.3	15473	9	18111	11133			
2	MA087720035	1350	2	8	7	14	9	8	5	5	3	2	4	67	1.17	3.50	4.95	17	23420	9.3	11252	15	19849	7314			
3	181635304115	383	1	0	1	0	1	0	4	2	3	1	1	15	1.00	2.00	3.25	20	7078	14.8	5288	18	5622	4228			
4	8E181322010050	355	3	0	3	1	0	2	3	1	2	0	0	15	1.00	1.00	2.25	14	4429	8.8	2820	10	3293	1775			
5	CC158151020503	383	3	2	4	3	6	3	4	2	3	1	2	38	0.78	2.33	3.31	17	8002	7.8	2862	11	4101	2355			
6	3B03892342131	1783	1	0	2	2	1	1	1	2	0	0	2	12	0.93	0.93	1.88	18	33525	11.1	19809	12	21972	14434			
7	5E231881848133	887	0	1	0	1	1	1	1	2	0	0	0	8	0.67	0.67	1.50	19	13305	9.8	3802	11	9484	6856			
8	CD181322070008	450	7	1	2	3	2	2	3	3	3	0	2	28	2.00	2.00	3.25	20	8775	10.0	4509	13	5763	2652			
9	18183930040	187	0	3	4	2	4	1	1	5	5	0	2	28	1.78	2.33	4.56	27	5111	18.2	3400	21	3803	2840			
10	SC181358040008	510	0	2	3	1	2	1	1	5	4	0	4	28	1.83	2.50	4.79	26	14803	20.3	10328	23	11603	8482			
11	SD031882200304	750	1	0	1	1	1	2	0	1	0	0	4	12	1.00	1.00	2.25	23	18875	14.8	11163	18	12029	7930			
12	181523158116	403	1	1	1	0	0	1	1	0	1	0	1	7	0.50	0.50	1.13	7	2720	4.8	1788	5	2089	1312			
13	LD059275	201	0	3	5	4	9	7	5	0	3	0	3	53	1.78	2.67	4.89	20	3031	10.4	2087	15	3111	1405			
14	SD015841010902	680	0	1	1	1	2	4	2	3	2	0	2	15	0.78	1.87	2.84	12	7835	8.7	3743	10	8225	4741			
15	SE181307296920	213	0	1	3	1	8	8	2	2	4	0	5	31	1.44	2.33	4.14	22	6093	13.4	3118	16	3813	2094			
16	MA151051008132	808	8	8	5	3	8	3	3	3	10	0	8	60	2.56	4.17	7.38	28	15813	17.7	10718	23	14214	9388			
17	MD03050130384	144	0	0	0	1	1	4	0	0	5	0	0	55	2.33	5.00	7.92	22	3950	20.3	2824	28	3685	2801			
18	18152025788	248	2	1	0	1	1	0	2	0	1	0	1	9	0.87	0.87	1.50	8	2232	8.1	1502	7	17	1044			
SLOW MOVING SKUs WITH ERATIC (LUMPY) DEMAND PATTERN																											
GAMMA DISTRIBUTION OF DEMAND																											
19	181828104148	1153	0	3	1	3	4	0	3	0	3	0	1	21	1.33	1.33	3.00	18	20769	11.1	12860	13	15183	7685			
20	500158541010501	831	3	3	2	8	1	1	8	1	5	1	1	28	1.89	2.67	5.03	23	14278	17.1	10808	20	12383	8934			
21	8015015200051	120	1	0	2	3	0	2	1	4	3	0	1	11	1.22	1.67	3.19	16	1617	11.5	1377	13	1584	1073			
22	SE021675	246	1	3	3	2	2	5	4	2	9	0	5	30	2.67	3.33	6.67	30	7380	22.4	5507	26	6356	4477			
23	SD181424220344	404	5	3	1	0	3	2	2	8	2	0	2	25	1.22	1.83	3.38	15	8111	9.8	3974	12	4945	2823			
24	SF181434800314	293	2	2	0	1	5	3	1	3	4	0	2	23	1.33	1.87	3.38	15	3460	10.1	2088	12	2543	1532			
25	101812108218	385	0	1	2	0	2	4	4	2	6	0	0	22	1.69	2.17	4.53	27	7129	30.0	5293	22	8254	4006			
26	SS181521810135	695	2	0	0	1	0	0	2	1	1	0	1	9	0.93	1.00	1.42	7	8177	6.0	3450	6	4082	2381			
27	SE02K1945	325	4	3	1	0	1	1	3	2	1	0	2	18	1.00	1.33	2.58	12	3780	7.8	2545	10	3113	1821			
28	181902800020	158	1	2	1	2	7	3	0	1	1	0	3	21	0.83	0.83	1.88	11	1758	4.4	824	6	989	91			
29	SE181814810024	182	0	4	3	2	0	2	4	1	3	0	3	29	1.33	2.00	3.67	13	2338	8.8	1625	12	2133	1908			
30	SA970190383	2000	8	0	4	4	8	4	0	2	0	0	5	41	2.00	2.50	5.00	18	35009	12.8	23862	19	31846	18428			
31	SD181523206908	528	3	1	0	0	4	4	1	0	2	0	2	17	0.63	0.83	1.88	18	5170	5.3	2770	7	3830	940			
32	SG05880758810	183	0	1	1	0	0	2	0	1	2	0	1	9	0.99	0.83	1.58	7	1121	3.0	810	6	951	587			
33	8B181361050728	1603	0	1	1	1	2	2	1	1	0	0	2	12	0.56	0.83	1.53	6	7858	3.8	5740	5	7599	4744			
34	181122015745	485	2	1	0	1	0	0	1	1	0	0	0	8	0.44	0.33	0.88	5	2897	3.4	1839	4	1815	786			
35	NC02170088	245	2	2	8	5	7	12	4	4	12	0	2	58	2.67	4.00	7.33	20	6288	17.9	4375	23	5742	3345			
36	MA07710121	1475	3	10	3	5	2	4	2	3	4	0	3	48	2.90	3.50	6.93	23	34282	18.7	24871	21	31478	16340			
37	CA658010200	1180	1	1	3	1	2	5	8	8	6	0	1	39	3.33	4.33	8.50	34	40460	27.3	32428	31	35890	27038			
38	181391010057	1180	1	0	0	1	0	0	1	0	1	0	1	5	0.50	0.50	1.13	7	7885	5.1	6038	6	6503	4454			
39	LA97010724	1130	8	8	7	0	8	0	0	8	0	0	2	47	2.67	3.33	6.67	23	26387	17.8	18217	22	24324	15228			
40	SD31287302018	381	0	2	0	2	1	0	0	7	0	3	1	21	2.33	2.67	5.58	34	11758	28.8	9349	29	10056	7280			
41	SC9701010130	650	2	2	3	0	1	2	2	5	5	0	2	34	1.78	2.33	4.56	18	10884	12.7	6264	15	6764	6997			
42	SE18191521516013	265	2	1	0	1	4	8	0	2	3	0	1	27	1.50	1.83	3.71	17	4736	11.0	3128	14	3888	1642			
43	SF181434800022	235	1	0	0	3	2	4	0	0	1	0	2	15	0.83	0.83	1.88	0	1883	5.3	1237	7	1578	630			
44	SF0216721	220	5	1	0	4	1	0	0	8	3	0	2	23	1.67	2.00	4.08	18	4843	13.8	2872	16	3459	2097			
45	SE2708120014	380	0	1	1	0	1	1	3	0	3	0	3	15	1.22	1.63	3.36	13	5109	10.8	4122	12	4870	3412			
46	SE156191018900	184	2	2	1	3	8	1	5	1	2	0	0	27	1.38	1.67	3.33	13	2887	8.7	1680	11	2184	993			
47	181628007183	2380	1	0	0	2	1	0	0	1	2	3	0	9	1.11	0.83	2.22	13	31733	16.4	24731	11	28780	17893			
48	SE181504108278	883	1	0	1	1	0	3	1	3	0	0	0	10	0.89	0.67	1.78	9	6895	6.8	4337	8	4864	2825			
49	SF181528220035	223	3	0	2	0	0	5	0	0	0	0	0	23	2.89	2.17	5.78	38	7886	27.1	8027	28	6518	4292			
50	SF191434800021	235	3	0	0	2	3	1	0	0	4	1	0	14	1.11	0.93	2.22	10	3360	7.0	1854	8	1970	972			
51	SE0216717	474	1	0	0	2	0	0	3	0	2	0	0	10	0.89	1.17	2.28	10	4639	8.1	3858	9	4312	2688			

PROBABILITY OF 95% SATISFYING DEMAND

CURRENT INVENTORY MODEL
(LAST SIX MONTHS STATISTICS)

NO	PART-NO	PRIC	MONTHLY DEMAND												ANNUAL												ADHQ	ADHQ	EXCESS
			(R)	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	DEMAN	MAJ	AVG	MAJ	MOH	MOHQ	SS	SS	UNITS (R)	UNITS (R)	UNITS (R)			
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																													
52	5E8977210	302	2	0	0	7	7	0	0	0	0	0	0	0	0	22	1.67	1.00	3.08	20	6053	12.0	3625	14	4284	974			
53	LC977 2028	291	5	1	0	10	0	8	12	8	15	0	0	0	0	81	5.50	5.50	12.38	43	12004	36.1	10214	41	11821	6271			
54	181C31010310	2450	0	0	0	0	0	0	0	0	0	0	0	0	0	8	1.00	1.33	2.58	10	37075	12.8	31567	14	33452	24324			
55	SG30750118007	443	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0.44	0.33	0.89	6	2303	4.4	1928	6	2058	1365			
56	5627730232	310	0	1	0	1	0	0	0	0	0	0	0	0	0	4	0.44	0.67	1.22	4	1326	3.8	1078	4	1255	863			
57	SA070150984	2110	0	0	0	3	2	4	1	2	4	0	2	0	0	18	1.17	1.50	2.89	10	21847	7.9	16735	10	20387	12383			
58	LA077002085	835	5	1	14	0	10	5	1	1	14	0	0	0	0	80	4.22	3.67	8.54	31	18378	23.5	14921	20	18482	10338			
59	181301010119	1430	0	0	10	1	0	0	0	0	0	0	0	0	0	22	1.11	0.83	2.22	13	18087	6.1	8782	8	11807	4071			
60	SA191434000294	896	0	1	0	10	1	5	14	11	1	0	0	0	0	43	5.44	4.33	11.14	30	48925	41.0	40895	45	44983	20320			
61	8092A5108	939	0	0	0	0	2	2	3	0	0	0	0	0	0	10	1.33	1.00	2.67	12	11868	9.8	8778	11	10727	7158			
62	181081010165	3202	0	0	0	0	0	2	8	4	2	0	0	0	0	16	2.44	2.33	5.39	32	103467	27.1	88728	29	81651	64518			
63	LAR5010019	817	0	1	1	18	3	0	13	3	1	0	2	0	0	53	3.83	3.50	8.29	33	30484	24.0	14804	29	17848	7881			
64	SD10143220082	783	0	0	2	0	0	0	0	2	5	0	4	2	0	15	1.58	2.17	4.11	14	11410	12.2	8702	14	10922	7701			
65	SF03050102032	153	1	1	0	1	1	2	0	1	4	0	0	0	0	11	1.11	0.83	2.22	8	1100	6.3	863	7	1125	704			
66	SAR8800103	20100	0	0	0	0	0	5	5	5	5	3	0	0	0	23	2.00	3.00	5.50	19	388926	18.2	324892	15	388144	253765			
67	SG67300230	235	1	0	1	1	0	0	1	0	2	0	0	0	0	6	0.67	0.50	1.33	5	1087	3.8	907	4	1042	871			
68	SC0150122048504	1097	0	0	2	4	0	2	1	0	1	0	0	0	0	12	0.44	0.67	1.22	4	4738	2.7	2947	4	4225	1051			
69	SG158326165502	113	0	3	0	0	0	0	3	1	5	1	0	0	0	14	1.28	1.17	2.76	11	1240	8.6	875	10	1128	842			
70	8-181352029036	171	0	0	1	0	0	4	0	0	1	0	0	0	0	12	1.81	1.17	3.18	18	3283	15.2	2592	18	2780	1842			
71	115103500000	106	0	0	0	0	1	1	0	0	4	0	0	0	0	12	1.11	0.67	2.03	12	1307	8.4	891	10	1014	903			
72	SG01819101010284	160	1	0	0	1	0	0	0	2	0	2	0	0	0	5	0.89	0.67	1.78	8	1280	6.7	1777	7	1169	507			
73	SF887102081	470	0	0	1	0	0	1	1	3	2	0	0	0	0	8	1.00	1.00	2.25	0	3701	6.8	3185	8	3557	2481			
74	SF0220389	272	2	0	0	2	0	2	0	3	0	0	0	0	0	7	0.83	0.50	1.54	7	1887	5.5	1454	6	1697	840			
75	SA520050225200	7002	8	0	0	0	0	10	0	7	2	0	0	0	0	25	2.00	1.50	4.00	14	83889	10.8	75102	13	92078	34917			
76	SA003000210	585	0	0	0	0	10	15	0	0	25	0	0	0	0	48	0.39	3.83	11.82	47	20712	39.0	22018	44	24628	13457			
SLOW MOVING SKUs WITH POISSON DISTRIBUTION OF DEMAND																													
77	SA270001651	11300	0	0	0	0	0	2	1	0	1	0	1	2	0	7	0.58	0.83	1.53	5	80424	4.4	49778	5	57381	38374			
78	CC0185417029501	306	2	2	5	5	8	4	3	4	4	0	3	2	0	40	1.11	2.87	4.08	10	0038	9.8	3720	14	5181	3081			
79	SC181307186081	852	5	3	2	0	3	4	3	2	0	3	2	0	0	27	1.11	1.87	3.08	20	10983	10.0	5518	13	6951	3478			
80	13104406271	1860	0	3	2	2	0	4	3	1	0	0	3	1	0	22	1.17	1.83	3.29	20	30716	12.0	24813	15	28759	18557			
81	CA07500041	2040	5	5	2	2	3	3	1	4	7	1	2	1	0	39	1.83	3.17	5.48	18	38873	13.0	28263	18	35813	25878			
82	CE504222	168	4	2	2	4	1	1	2	5	0	0	1	0	0	30	1.87	2.67	4.75	16	3154	13.0	2282	17	2771	1987			
83	SC181804108251	1301	2	1	1	1	3	0	1	1	0	0	0	1	0	14	0.50	0.50	1.13	6	7684	2.1	2750	3	4532	46			
84	10162000778	1160	3	2	1	5	2	2	5	2	0	1	3	1	0	31	1.67	2.67	4.75	28	39880	18.4	21304	21	24782	17391			
85	SC5FTG160558	500	0	2	1	3	1	4	3	4	2	0	1	1	0	22	1.17	1.83	3.29	13	7373	8.4	6241	11	6428	4208			
86	SF867500820	402	0	0	2	2	0	2	1	1	2	0	0	0	0	10	0.67	0.67	1.50	13	3820	6.1	2451	7	2837	1424			
87	SA077102140	1020	2	3	7	2	2	4	1	5	4	0	3	1	0	38	1.83	2.50	4.11	17	30523	12.9	23418	16	28483	20804			
88	SF0216884	345	1	1	2	1	3	1	1	1	3	0	0	0	0	14	0.83	0.83	1.82	8	2911	5.5	1889	7	2354	1284			
89	CA020150882	2030	1	3	2	1	3	1	1	5	2	0	1	3	0	23	1.33	2.00	3.67	13	28308	0.7	18961	12	24495	17227			
90	151023050809	730	2	0	0	0	2	1	1	1	0	0	2	0	0	9	0.67	0.67	1.50	8	6870	6.1	4422	7	5054	2805			
91	181061300029	2050	1	2	1	0	3	0	0	0	1	1	0	2	0	11	0.67	0.67	1.50	0	18450	5.8	11078	8	13240	5845			
92	SE0500345110	313	4	2	0	1	1	1	0	3	2	0	1	1	0	18	0.89	1.17	2.28	10	3203	8.8	2149	8	2630	1477			
93	SD878000074	690	2	0	3	1	1	0	1	1	2	0	1	3	0	15	0.75	1.33	2.31	8	5328	0.1	3883	7	4845	3204			
94	181462006135	4050	1	0	1	0	2	0	0	0	1	0	2	0	0	7	0.87	0.50	1.33	8	32680	5.7	23132	6	25898	14581			
95	SA072000555	1830	2	1	0	1	2	0	4	0	3	2	0	1	0	18	1.33	1.67	3.33	12	21789	8.5	17884	11	20555	14833			
96	181381080014	216	0	0	1	1	0	0	2	0	1	0	0	0	0	5	0.67	0.50	1.33	0	1729	6.4	1375	7	1479	977			
97	18115202385	180	0	0	1	1	0	0	0	0	1	1	0	0	0	4	0.44	0.33	0.88	5	830	4.8	725	4	784	484			
98	SA570000085	8050	0	2	1	0	2	2	1	0	3	0	0	0	0	11	0.89	0.67	1.78	6	37844	4.7	28888	6	35085	10484			
99	101424015230	287	0	1	0	2	0	0	2	0	1	0	0	0	0	4	0.28	0.17	0.51	3	585	1.0	510	2	620	22			
100	SC077200477	2403	0	0	0	1	1	0	2	1	0	0	0	0	0	5	0.67	0.50	1.33	5	11237	4.0	8817	4	10774	7083			
101	CA03128888220	4351	3	3	3	1	2	3	1	0	4	0	3	2	0	25	1.33	1.67	3.33	33	145033	17.5	76003	20	89482	57434			
102																													

PROBABILITY OF 85% SATISFING DEMAND

CURRENT INVENTORY MODEL
(LAST SIX MONTHS STATISTICS)

NO	PART-NO	PRIC	MONTHLY DEMAND												ANNUAL												AOKQ	AOKQ	EXCESS STOCK									
			(R)	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	DEMAN	HAD	AVG	NAD	MOH	MOHQ	SS	RS	MOH	MOHQ	SS				RS	UNITS (R)	UNITS (R)	UNITS (R)					
SLOW MOVING SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																																						
106	FA181501100144	247	58	72	74	04	75	77	89	71	109	13	77	51	835	21.87	64.50	91.58	412	08322	238.3	58702	318	73075	58336													
108	SB031891584137	797	3	5	2	2	2	1	2	4	6	0	3	3	32	1.22	2.83	4.39	44	34758	23.3	18573	25	21025	14874													
107	FA181501100228	178	82	124	103	78	110	140	81	88	148	38	88	84	1130	28.56	83.83	114.03	888	108558	284.4	52408	403	71746	47488													
108	MA181358285001	602	16	14	20	18	11	9	19	8	13	0	18	11	153	4.83	11.17	17.21	103	62167	53.2	32046	86	40801	27502													
109	181255200897	48	81	118	165	137	138	195	103	122	210	3	124	89	1983	51.22	124.67	183.69	1132	55476	814.8	30118	787	37578	26223													
110	FA08151025030613	35	86	85	125	93	09	184	113	44	78	53	56	118	1047	25.87	78.67	108.75	797	24741	324.3	11351	425	14875	8852													
111	FA0806250821	360	32	28	33	40	28	37	35	30	41	1	32	23	358	10.00	27.00	39.50	178	82213	102.0	36707	138	47755	34478													
112	MA181358100024	973	15	10	11	10	12	12	14	4	10	0	9	8	113	3.83	7.17	11.98	72	88813	34.6	33888	43	44440	27763													
113	MA031882331003	1391	6	13	13	13	20	28	40	18	15	5	2	11	14	185	4.67	10.60	16.33	193	227187	45.9	63888	84	88832	17857												
114	181024105848	10	290	273	208	230	227	418	233	285	478	5	258	224	3134	93.00	247.00	383.25	1886	18079	88888	10838	1385	13852	9526													
115	181888010116	208	22	20	28	37	25	28	30	12	19	0	17	10	219	4.67	11.33	17.17	103	21424	34.3	7144	55	11344	4311													
116	1818150162	82	10	7	9	5	8	7	8	3	2	0	8	8	66	2.50	4.17	7.29	44	3888	22.2	1818	29	2339	1458													
117	FA18135725028	485	28	32	33	21	24	32	20	17	28	1	14	31	283	8.00	18.67	28.67	329	58885	68.1	32148	86	44801	28548													
118	MA031881488234	948	5	12	8	23	28	18	5	12	0	18	9	131	3.50	6.83	11.21	112	168255	20.9	27443	42	38385	3019														
119	18181104920	660	5	3	1	3	4	3	4	5	2	0	2	3	35	1.33	2.67	4.33	28	17160	14.6	9808	18	11829	7828													
120	181818047023	109	36	23	28	28	38	51	38	37	48	0	23	13	382	12.83	24.83	40.88	245	28732	128.9	13833	162	17827	11083													
121	1085827290215	83	74	87	83	88	58	183	53	87	106	6	101	197	1003	39.67	81.67	131.25	78	73238	458.6	42742	558	61712	39139													
122	181890290825	27	106	80	34	68	84	84	35	77	95	0	152	38	883	41.83	85.17	118.46	711	18579	422.1	11388	507	13888	9562													
123	18138491801	205	48	25	28	41	18	43	28	0	25	0	28	28	312	10.33	20.00	32.82	188	5828	93.8	27218	128	35788	20244													
124	18115025621	192	21	6	13	13	1	12	6	13	18	4	18	23	194	5.67	14.17	21.84	128	24240	78.9	14762	91	17550	12485													
125	18130050182	705	2	4	7	7	3	4	2	8	7	0	5	7	58	1.77	4.83	8.08	48	33888	28.8	21051	53	24547	17807													
126	FA08151025017300	624	48	41	22	48	45	82	87	38	58	2	128	78	888	34.17	67.17	108.88	714	44583	482.4	285581	529	328801	243684													
127	FA01088500321438	74	288	188	283	248	202	382	273	390	187	8	294	184	2778	88.00	211.00	321.00	1284	17079	893.5	11250	1070	14888	10877													
128	FA08087804172	55	84	83	101	100	82	178	140	87	91	2	101	83	1080	30.78	80.87	118.14	538	28487	387.7	18821	412	22833	24780													
128	181443010148	428	7	2	3	4	5	1	5	3	2	0	8	8	44	2.00	3.67	6.17	37	18873	22.6	9702	27	11517	7915													
130	18130564006	41	40	28	13	38	12	48	15	21	31	0	18	19	287	6.66	17.33	25.83	153	6280	88.3	2433	87	3884	1308													
131	CO031881488133	771	3	3	2	1	4	3	0	4	1	0	2	0	21	1.22	1.17	2.69	27	28774	13.6	10488	16	12056	6395													
132	18180410848	10	68	188	89	74	77	282	153	116	234	5	188	88	1424	62.00	125.33	202.83	1118	11188	794.8	7048	842	8417	5955													
133	LO08780732032	218	8	2	5	5	1	5	7	5	8	1	5	9	80	2.50	4.83	7.53	45	884	28.0	6283	34	7331	5204													
134	181488188333	389	8	5	3	2	4	5	9	3	2	0	4	1	45	2.00	3.00	5.50	33	11220	18.0	8440	23	7846	4850													
135	181415016012	480	14	42	32	37	51	28	23	25	70	0	15	22	389	14.72	25.83	44.24	390	127400	148.1	71085	183	87834	53758													
136	18144016138	885	5	3	0	1	7	4	4	8	3	0	8	8	42	1.44	3.67	5.47	33	28401	9.1	18524	23	20017	12870													
137	18101116436	58	88	28	14	12	8	42	20	19	31	0	40	21	246	9.11	21.83	33.22	183	18232	111.8	6258	135	7583	5101													
138	FA181443086010	288	88	110	108	89	57	88	82	91	122	1	102	81	888	30.11	78.17	113.81	389	118114	280.6	77811	361	104388	78554													
138	18101118848	58	13	18	14	13	5	28	38	14	17	0	22	12	188	8.17	16.83	27.04	148	8328	93.8	5260	112	6283	4343													
140	FA1815011002227	181	22	18	18	13	4	30	38	28	22	0	11	7	203	10.00	18.00	28.50	180	27088	95.0	17180	114	20723	14345													
141	1813544100117	109	12	13	18	13	14	11	38	11	28	0	10	10	18	11.00	17.50	31.25	188	20438	127.3	13881	145	15888	11725													
142	181416130808	1855	2	4	10	4	4	4	5	1	8	0	5	8	54	2.58	4.33	7.83	46	70234	27.5	42782	33	50858	33708													
143	181517816057	148	9	12	18	11	9	13	10	2	28	0	8	22	147																							

MANORMAL.XLS

SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																							
NO	PART-NO	PRICE (R)	MONTHLY DEMAND												COEF STDH+ DD+	ANNUAL UNITS	DEMAND USAGE (R)	LEAD TIME					ROQ
			X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12				WEEKS	TOTAL WEEKS	UNIT UNITS	STDH+ UNITS		
1	181152102058	113	20	13	31	24	10	28	27	25	27	1	24	26	0.19	283	29719	14	17	86.0	15.91	16	
2	151012010296	238	150	121	86	164	132	154	124	101	110	1	180	85	0.19	1444	340784	16	19	527.6	109.56	83	
3	FA181501100144	241	86	72	74	64	76	77	69	71	106	13	77	51	0.20	830	201478	8	11	176.8	35.02	48	
4	SB031861584137	797	3	6	2	2	2	1	2	4	8	0	3	3	0.20	32	25504	30	33	20.3	4.12	2	
5	FA181501100226	178	92	124	103	78	110	140	81	86	148	18	86	64	0.21	1130	201140	11	14	304.2	62.65	56	
6	MA181266085001	802	16	14	20	16	11	9	18	8	13	0	16	11	0.22	153	82106	14	17	50.0	10.81	9	
7	181235200037	49	81	118	166	137	138	195	193	122	216	3	124	86	0.22	1083	77667	14	17	517.5	112.74	81	
8	FA0151025030813	35	66	86	125	63	63	184	113	44	79	63	56	115	0.22	1047	36845	16	19	382.6	84.26	60	
9	FA0884050821	350	32	28	33	40	26	37	35	30	41	1	32	23	0.22	356	125300	6	11	75.7	15.73	21	
10	MA181356100024	873	15	10	11	10	12	12	14	4	10	0	9	6	0.23	113	109949	14	17	36.9	8.89	7	
11	CAD31886999270	4351	3	3	3	1	2	3	1	0	4	0	3	2	0.23	25	106776	30.0	33	16.9	3.61	1	
12	MA031882331003	1381	8	13	16	20	26	40	16	18	5	2	11	14	0.2*	185	257395	30	33	117.4	27.61	11	
13	151024108466	10	298	273	206	230	227	418	283	285	476	5	269	224	0.2**	3134	31340	12	15	904.0	213.49	181	
14	151065010116	208	22	20	29	17	26	25	10	12	19	0	17	10	0.26	210	43680	14	17	88.7	18.82	12	
15	1816150162	82	10	7	6	5	8	7	8	3	2	0	6	5	0.26	66	5412	14	17	21.5	4.36	4	
16	FA181307258026	485	29	32	33	21	24	32	23	17	26	1	14	31	0.26	283	134595	8	11	88.9	14.93	16	
17	MA031861486234	548	8	12	8	23	26	16	6	0	1	0	10	9	0.26	131	124188	30	33	83.1	20.98	8	
18	151018047023	109	36	23	28	26	39	61	36	37	40	0	23	13	0.26	362	39468	14	17	118.3	30.17	21	
19	108562780715	83	74	67	63	98	58	163	53	87	105	6	101	167	0.26	1003	83279	14	17	327.9	63.95	68	
20	CC168151020503	368	3	2	4	3	6	3		2	3	1	2	2	0.26	35	12680	20	13	8.8	2.26	2	
21	18161104020	660	6	3	1	3	4	3	4	8	2	0	2	3	0.26	35	23100	14	17	11.4	2.97	2	
22	181560000236	27	196	89	64	66	84	84	38	77	95	0	152	38	0.27	803	23841	14	17	283.7	77.16	51	
23	18139003011	285	46	25	20	41	15	43	3	9	28	0	29	28	0.27	312	88920	14	17	192.0	27.58	18	
24	181150000021	182	21	6	13	12	1	12	1	13	18	4	18	23	0.27	161	28992	14	17	48.4	13.35	9	
25	181300050182	705	2	4	7	7	3	4	2	8	7	0	5	7	0.28	58	39480	14	17	16.3	6.07	3	
26	FA0151025017300	624	48	41	22	48	45	82	87	39	65	2	129	78	0.28	888	428938	16	18	251.8	71.68	40	
27	FA01065804322436	14	266	189	263	248	202	382	273	390	197	8	234	164	0.20	2776	38884	6	9	480.6	138.02	160	
28	FA0897840172	55	84	63	101	100	82	176	140	67	91	2	101	83	0.30	1080	69400	8	11	226.5	87.65	62	
29	181448010146	429	7	2	3	4	6	1	5	3	2	0	6	6	0.30	44	18876	14	17	14.3	4.32	3	
30	181308000009	41	49	28	13	36	12	46	15	21	31	0	18	18	0.30	287	11767	14	17	83.8	28.33	17	
31	18104108463	10	88	186	89	74	77	202	169	116	234	6	169	69	0.30	1424	14240	12	15	410.8	124.34	82	
32	CD031861485133	771	3	1	2	1	4	3	0		1	0	2	0	0.31	21	18191	30	33	13.3	4.08	1	
33	SB031882342131	1788	1	0	2	2	1	1	1	2	0	0	2	3	0.31	12	21466	30	33	7.6	2.88	1	
34	LD9C750132032	218	6	2	6	5	1	3	7	5	8	1	6	6	0.31	50	10900	14	17	16.3	5.04	3	
35	181415016012	480	14	42	32	37	51	28	23	25	70	0	16	22	0.31	359	172320	14	17	117.4	36.60	21	
36	181465105333	340	6	5	3	2	4	5	8	3	2	0	4	1	0.31	43	14620	14	17	14.1	4.41	2	
37	15101116433	86	18	20	14	12	8	42	20	19	31	0	40	21	0.32	248	13776	12	15	71.0	22.56	14	
38	FA181441356010	289	88	110	108	96	67	86	82	91	120	1	102	61	0.32	984	278598	4	7	129.8	41.54	56	
39	18144016126	866	5	3	0	1	7	4	4	5	4	0	3	6	0.32	42	38330	14	17	13.7	4.41	2	
40	15101116489	86	13	16	14	13	5	28	32	14	17	0	22	12	0.32	180	10640	12	15	54.8	17.67	11	
41	151044100117	103	12	13	16	13	14	11	39	11	28	0	10	18	0.32	184	20068	14	17	80.2	19.48	11	
42	1814161300C	1598	2	4	10	4	4	4	6	1	8	0	6	6	0.33	54	83970	14	17	17.7	5.74	3	
43	FA1815011002222	181	22	19	19	13	4	30	20	22	0	11	1	1	0.33	203	38743	11	14	54.7	17.80	12	
44	181117016057	148	8	12	18	11	9	11	18	2	29	0	8	22	0.33	147	21766	14	17	48.1	15.97	8	

SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																									
NO	PART-NO	PRICE (R)	MONTHLY DEMAND												COEF STDM K1	ANNUAL UNITS	DEMAND			LEAD TIME			ROQ (C)		
			X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12			STDM K1	USAGE (R)	SUPP WEEKS	TOTAL WEEKS	X+ UNITS	STDM UNITS			
43	1E1637306149	51	44	114	155	189	181	238	185	203	189	4	17	21	0.33	476	75278	14	17	482.5	161.12	85			
48	8S081261851031	728	0	0	2	0	2	3	5	3	8	0	3	1	0.34	24	17472	30	33	15.2	6.11	1			
47	181361886316	1540	1	3	11	10	10	4	4	10	13	0	7	4	0.34	77	118580	14	17	25.2	8.48	4			
48	181417156003	725	12	23	29	29	28	29	18	6	8	0	0	18	0.34	188	143580	14	17	64.7	21.89	11			
49	8E931861646133	827	0	1	0	1	1	1	1	2	0	0	0	1	0.35	8	7088	30	33	5.1	1.79	0			
50	181637010010	439	6	2	3	7	6	6	8	0	6	0	2	2	0.35	46	20194	14	17	15.0	6.31	3			
51	151069005228	490	1	7	4	3	7	5	11	4	14	0	5	5	0.36	68	32340	14	17	21.6	7.74	4			
52	CD181322070008	450	7	1	2	3	2	2	3	3	3	0	2	1	0.36	29	13050	14	17	9.6	3.41	2			
53	MA877200329	1375	1	7	18	9	7	10	7	17	10	12	9	14	0.37	121	166378	4	7	16.3	5.98	7			
54	181894200231	68	15	7	8	6	2	6	9	6	19	0	1	4	0.37	74	4884	14	17	24.2	8.88	4			
55	CC181368110008	644	4	1	2	3	2	7	3	3	0	0	3	2	0.37	31	18854	14	17	10.1	3.71	2			
56	SC181356040006	519	0	2	3	1	2	3	1	5	4	0	4	1	0.38	28	13280	14	17	8.6	3.24	2			
57	FA031250307896	110	37	28	20	46	43	38	34	28	25	0	14	15	0.38	348	38580	4	7	46.6	17.78	20			
58	SD031861743101	731	1	4	2	0	5	1	1	1	5	0	1	0	0.38	21	16351	30	33	13.3	5.13	1			
59	181323010272	276	1	9	4	5	5	4	8	6	0	0	0	6	0.39	46	12688	14	17	15.0	5.82	3			
60	181539030049	187	0	0	4	2	4	1	1	6	5	0	2	1	0.39	28	8228	14	17	9.2	3.60	2			
61	181303010322	1095	5	6	1	11	1	2	2	4	8	0	5	5	0.39	51	53845	14	17	16.7	6.58	3			
62	151024016141	361	11	13	7	2	2	12	9	1	10	0	5	1	0.40	75	28353	14	17	23.9	9.50	4			
63	SD031882209304	750	1	0	1	1	1	2	0	1	0	0	4	1	0.41	12	9000	30	33	7.6	3.10	1			
64	184274059260	100	11	9	41	4	10	12	8	10	18	0	18	11	0.41	150	15000	11	17	48.0	20.03	8			
65	SC031882320032	1533	0	0	2	1	1	2	1	4	3	0	1	0	0.41	17	26081	30	33	16.8	3.46	1			
65	181437016187	584	4	4	1	8	3	5	1	6	0	0	2	2	0.42	37	20888	14	17	12.1	5.08	2			
67	MA873000086	1420	14	16	17	11	9	8	6	14	29	2	11	14	0.42	151	214420	4	7	20.3	8.55	8			
68	FA181335112602	217	0	70	5	91	44	10	85	89	65	0	6	80	0.44	514	111538	14	17	168.0	73.13	30			
69	MA181307268031	771	17	8	2	4	8	3	12	7	6	1	4	5	0.44	77	68957	8	11	16.3	7.16	4			
70	LA02K1947	592	2	11	4	5	9	15	10	6	9	0	7	0	0.44	78	48178	8	11	16.5	7.28	5			
71	181523168116	403	1	1	1	0	0	1	1	0	1	0	1	0	0.44	7	2821	14	17	2.3	1.02	0			
72	LA02K1948	602	9	4	6	3	7	11	3	5	5	0	12	0	0.46	62	37324	8	11	13.1	5.86	4			
73	FA0106850722422R	27	92	243	55	57	129	229	85	142	141	0	148	16	0.46	1352	38504	8	9	234.0	105.65	78			
74	LD0592275	201	9	3	5	4	8	7	5	0	3	0	3	5	0.46	63	10953	6	9	9.2	4.22	3			
75	MA151051006133	605	8	8	5	3	8	3	3	3	10	0	5	3	0.47	60	38380	4	7	8.1	3.78	3			
76	SE181307266020	239	0	1	3	1	6	8	2	2	4	0	6	1	0.47	31	7223	11	14	8.3	3.93	2			
77	SD0156541010502	880	0	1	1	1	2	0	2	3	2	0	2	1	0.48	16	9900	8	11	3.2	1.53	1			
78	MD030601306844	144	8	8	6	1	1	4	8	6	8	0	8	8	0.48	23	7920	4	7	7.4	3.60	3			
79	ME181821808005	218	2	8	3	4	4	15	3	4	15	0	4	1	0.48	64	13952	11	14	17.2	8.48	4			
80	181152029788	246	2	1	0	1	1	0	2	0	1	0	1	0	0.51	9	2232	14	17	2.9	1.49	1			
																	562703								
total																									

SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																																									
PROBABILITY (P1=0.95) OF NO STOCKOUT										FRACTION (P2=0.95) OF DEMAND										AVERAGE TIME BETWEEN(TBS=2yr)																					
PER REPLENISHMENT CYCLE										SATISFIED DIRECTLY FROM SHELF										STOCKOUT OCCASIONS																					
MOHQ	MOHV	ROP	SS	SSV	EVSPY	AOHQ	AOHV	EOHQ	EOHV	ROP	SS	SSV	EVSPY	AOHQ	AOHV	MOHQ	MOHV	ROP	SS	SSV	EVSPY	AOHQ	AOHV																		
units	(R)	units	units	(R)	(R)	units	(R)	units	(R)	units	units	(R)	(R)	units	(R)	units	(R)	units	units	(R)	(R)	units	(R)																		
112	12673	87	28.17	2958	37.6	33.8	3814.9	-106	12020	91	20.39	2304.3	1457.37	27.98	3161.6	117	13240	102	31.19	3524.17	105.46	38.77	4361.45																		
694	163709	610	166.06	39191	497.8	207.7	49021.6	666	157212	683	136.54	32696.0	16711.62	180.19	42525.3	726	171216	642	197.98	46699.25	1397.43	239.53	56529.55																		
234	58504	188	57.61	13984	176.4	81.7	19895.5	215	51931	187	38.83	8310.8	9890.07	62.76	15122.4	249	59188	197	68.64	16543.62	495.05	92.76	22395.23																		
27	21584	26	6.77	5399	68.8	7.7	6134.2	27	21584	29	6.77	5398.5	1260.68	7.70	6134.2	28	22618	27	8.07	6432.76	192.49	8.89	7168.45																		
408	72593	342	103.37	18400	239.7	136.0	24202.5	383	68176	318	73.78	14023.2	8863.60	111.39	18925.3	427	78076	382	123.18	21825.37	656.10	155.77	27727.48																		
89	40821	59	17.78	10708	136.0	22.2	13366.9	65	39049	56	14.84	8933.9	4516.74	19.28	11390.8	71	42872	62	21.20	12760.54	361.65	25.61	15417.46																		
703	34445	612	185.44	9987	115.4	23.1	11324.2	672	32839	581	154.70	7680.6	3803.77	200.87	8818.8	738	38186	647	220.97	10827.45	324.00	268.93	13084.98																		
521	18240	481	134.60	4851	61.6	168.8	9907.9	504	17635	443	121.29	4245.3	1797.01	161.50	6302.4	548	19170	487	165.15	5780.13	172.96	195.35	6837.29																		
103	38138	83	27.52	8533	122.4	27.8	13247.1	85	33387	78	19.66	6981.0	6144.62	28.97	10485.6	108	37984	88	32.79	11478.04	343.47	43.12	15082.48																		
51	49378	44	13.81	13433	170.6	17.1	18604.8	49	47420	42	11.79	11475.0	6397.73	16.05	14646.6	53	61251	47	18.45	16006.62	478.98	19.71	19178.23																		
22	94840	20	5.93	25809	327.8	6.7	28947.0	22	95633	21	6.11	28802.3	5334.16	8.84	29740.1	23	99784	21	7.07	30763.55	520.27	7.79	33891.29																		
163	228483	192	46.42	63174	802.4	50.8	70597.4	164	228424	154	46.81	65115.5	12919.31	62.16	72598.7	172	23855	161	54.12	76276.67	2262.56	56.40	82698.79																		
1265	12551	1074	351.10	3511	44.5	441.5	4415.0	1180	11903	1009	288.19	2881.9	1695.87	376.69	3766.9	1322	13224	1142	418.36	4183.68	125.19	503.76	5087.82																		
88	20095	84	27.87	5755	73.1	33.7	7015.3	93	19317	81	24.22	5036.9	2142.00	30.27	6288.9	102	21138	90	32.97	6667.65	205.21	33.03	8117.85																		
30	2492	27	8.81	722	9.2	10.7	878.5	29	2402	29	7.71	632.2	265.40	9.91	788.4	32	2630	28	10.50	880.81	26.76	12.40	1616.93																		
84	39269	68	24.68	11421	146.1	32.7	16219.7	78	36354	82	18.31	8516.2	6453.22	28.45	12312.2	89	41448	73	28.27	13806.82	407.23	37.43	17404.68																		
118	111493	110	34.47	32681	416.1	38.3	35283.2	120	113568	112	36.69	34788.6	6089.99	40.47	38368.0	124	117763	117	41.08	38941.84	1166.29	44.86	42925.98																		
168	18308	147	49.62	6409	88.7	60.1	6548.8	162	17683	141	43.43	4733.6	1834.95	63.87	5871.6	177	19346	157	59.13	6444.80	192.86	69.97	7683.11																		
468	43336	408	138.68	12841	163.1	167.0	15832.0	449	41735	391	120.84	11238.3	4874.26	149.77	13828.0	492	48798	436	164.83	15301.28	467.68	193.46	17892.00																		
12	4590	10	3.72	1370	17.4	4.7	1741.8	12	4512	10	2.97	1091.7	631.62	3.98	1463.2	13	4693	11	4.44	1632.26	48.66	5.46	2004.10																		
16	10777	14	4.89	3225	41.0	6.9	3891.3	16	10374	14	4.28	2822.4	1132.78	6.29	3488.7	17	11395	15	5.82	3842.73	114.89	8.83	508.08																		
418	11222	365	126.96	3428	43.5	152.4	4116.6	403	10870	352	113.81	3076.5	1188.13	1382.88	3763.2	440	11679	359	151.28	4084.51	122.23	178.75	4772.23																		
147	41968	129	45.38	12918	184.1	54.3	16482.0	143	40660	126	40.67	11580.1	4360.50	49.67	14196.1	158	44482	138	54.01	15392.48	480.81	63.01	17857.48																		
71	13693	83	21.85	4215	63.6	26.3	3051.1	89	13260	80	18.70	3781.6	1421.72	24.05	4617.9	76	14500	67	26.16	6022.28	160.29	30.61	6868.66																		
27	16790	23	8.34	5883	74.7	10.0	7021.7	28	18185	23	7.49	5782.2	1936.74	9.19	6417.0	29	18917	26	9.94	7098.84	208.78	11.56	8148.68																		
370	230877	390	117.82	73585	894.7	197.8	85986.7	363	226047	323	111.47	59554.8	21083.40	131.34	81956.8	392	244773	363	140.52	87841.47	2623.79	160.39	100083.47																		
707	9905	547	227.02	3178	40.4	5.1	4259.4	646	9046	486	165.87	2319.4	1895.83	246.76	3440.6	701	10514	591	270.81	3787.18	113.33	350.58	4809.28																		
340	16885	277	111.27	8120	77.7	142.4	7633.6	317	17442	265	88.68	4876.3	2812.88	113.61	6588.8	361	19888	289	132.89	7282.47	218.22	163.74	9008.94																		
21	8219	19	7.10	3048	38.7	8.4	3592.1	21	8976	18	6.54	2805.3	826.85	7.81	3349.6	23	9802	20	8.48	3931.42	108.67	9.73	4176.82																		
149	5758	124	46.61	1811	24.3	54.9	2260.4	137	5806	120	42.80	1759.0	677.84	51.18	2098.4	148	6124	133	55.54	2276.96	68.14	69.81	2818.88																		
615	6153	833	204.52	2045	26.0	246.6	2456.0	584	8943	612	183.80	1836.0	898.31	224.88	2246.8	654	6545	672	243.70	2437.00	72.82	284.78	2947.77																		
20	16453	19	6.72	6178	68.8	7.3	5845.2	21	16978	20	7.40	6704.1	789.88	8.00	8171.1	21	16446	20	8.00	6170.13	184.84	8.61	8637.17																		
11	20614	11	3.88	6897	87.6	4.2	7518.2	12	21214	11	4.25	7987.8	1062.17	4.60	6218.7	12	21835	12	4.50	8216.57	245.93	4.94	9887.49																		
28	6370	22	8.29	1807	22.9	9.7	2121.0	24	6226	21	7.63	1882.9	634.52	8.07	1977.4	25	6716	23	8.67	2152.62	84.41	11.32	2467.04																		
177	89150	187	60.03	28816	366.0	70.4	33788.8	174	83572	163	68.74	27237.0	8490.31	67.10	32207.7	189	89871	188	71.53	34335.16	1027.46	81.89	38904.86																		
21	7246	19	7.28	2466	81.3	8.6	2888.1	21	7111	18	6.85	2331.3	716.84	8.10	2768.0	23	7718	20	6.64	2838.68	87.84	9.88	3360.91																		
108	6062	94	37.11	2076	26.4	44.2	2475.8	104	5839	90	33.20	1884.7	675.95	40.38	2262.0	118	6480	101	44.22	2478.40	74.10	41.32	2673.79																		
198	67250	142	68.33	19747	280.8	96.1	27783.1	177	51026	121	48.79	19022.5	15931.82	74.80	21688.0	211	81039	166	61.42	29926.63	704.10	109.23	31666.06																		
21	16157	19	7.28	6280	79.8	8.5	7327.5	21	17813	18	6.85	6935.9	1781.67	8.07	6983.8	22	19390	20	8.68	7482.53	233.91	9.86	8930.82																		
84	4687	73	29.07	1628	20.7	34.6	1935.0	81	4630	70	26.08	1460.7	621.77	31.58	1787.8	88	5009	76	34.64	1838.94	68.05	40.12	2246.86																		
92	10061	82	32.08	3495	44.4	37.4	4073.1	90	9880	80	30.30	3303.2	883.52	35.81	3881.7	98	10721	86	38.20	4163.98	124.80	43.61	4742.63																		
27	42131	24	9.44	14679	186.8	11.0	17101.4	27	41327	23	8.82	13976.3	4117.78	10.48	16287.5	29	44945	26	11.28	17491.34	523.41	12.81	19913.65																		
84	18193	72	29.29	5301	67.3	38.1	6360.6	81	14848	69	26.26	4786.0	1891.82	32.13	6816.9	90	16201	78	34.90	6916.37	189.01	40.76	7376.26																		
74	11001	68	26.27	3688	49.4	30.5	4515.6	73	10788	64	24.83	3675.1	1068.88	28.07	4202.7	79	11748	71	31.30	4652.92	138.84	35.64	6260.60																		
748	88128	662	268.03	13518	171.7	307.6	15687.8	733	37386	648	285.81	12778.1	3891.42	283.88	14947.5	798	40716	713	316.60	16105.89	481.95	358.37	18277.11																		
24	17208	22	8.41	6120	77.7	9.1	6824.5	25	18088	23	9.61	6998.4	856.80	19.31	7602.4	25	18381	24	10.02	7293.01	218.24	10.71	1777.01																		
39	69240	35	13.94	21474	272.8	18.2	24594.6	38	69064	34	13.16	20297.9	6814.88	16.40	23718.4	42	64354	37	16.82	25687.68	788.68	18.84	29008.25																		

SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																							
PROBABILITY (P1=0.95) OF NO STOCKOUT PER REPLENISHMENT CYCLE										FRACTION (P2=0.95) OF DEMAND SATISFIED DIRECTLY FROM SHELF						AVERAGE TIME BETWEEN (TBS=2yr) STOCKOUT OCCASIONS							
MOHQ	MOHV	ROP	SS	SSV	EVSPY	AOHQ	AOHV	MOHQ	MOHV	ROP	SS	SSV	EVSPY	AOHQ	AOHV	MOHQ	MOHV	ROP	SS	SSV	EVSPY	AOHQ	AOHV
units	(R)	units	units	(R)	(R)	units	(R)	units	(R)	units	units	(R)	(R)	units	(R)	units	(R)	units	units	(R)	(R)	units	(R)
101	73034	89	35.01	26106	331.6	41.7	30245.5	99	71605	87	34.03	24675.0	7039.47	39.76	26816.9	108	78036	96	42.90	31105.57	630.21	48.62	35246.44
8	7117	8	2.95	2613	53.2	3.2	2818.0	8	7491	8	3.37	2998.2	347.98	3.60	3182.9	9	7617	8	3.51	3113.94	93.18	3.74	3318.64
24	10437	21	8.74	3636	48.7	10.1	4418.1	24	10328	21	8.49	3726.8	890.28	9.52	4509.3	26	11172	23	10.41	4570.40	138.76	11.74	5162.92
34	16808	30	12.73	6235	79.2	14.6	7168.4	34	16831	30	12.36	6038.6	1686.50	14.27	6891.5	37	16003	33	15.16	7430.03	222.34	17.07	8382.32
15	6794	18	6.62	2527	32.1	6.5	2803.7	15	6722	13	5.46	2455.6	636.85	6.29	2832.0	18	7278	14	6.69	3011.42	90.11	7.53	3387.87
26	35858	19	9.78	13461	171.0	13.3	16260.6	23	32220	16	7.14	9823.6	8153.77	10.63	14622.9	28	38437	21	11.67	18040.10	478.99	15.16	20639.38
39	2509	34	14.57	862	12.2	16.7	1102.6	38	2534	34	14.16	934.6	233.50	16.39	1076.0	42	2743	37	17.37	1148.16	34.30	19.50	1267.05
16	8835	14	6.11	3322	42.2	7.0	3898.4	16	8741	14	6.83	3227.7	826.68	6.83	3714.2	17	9472	16	7.28	3958.38	116.45	8.17	4444.85
14	7054	12	5.33	2719	34.5	8.1	3101.7	14	7054	12	5.33	2719.2	650.26	6.08	3101.7	15	7676	13	6.35	3245.08	96.96	7.10	3622.58
76	8337	26	29.22	3214	40.8	39.2	4311.9	88	7489	48	21.32	2345.0	1866.40	31.30	3443.3	81	8953	81	34.82	3828.69	114.60	44.90	4927.58
22	16908	21	8.43	8168	78.3	9.0	8898.7	23	16783	22	9.64	7050.3	762.79	10.26	7483.2	23	17089	22	10.05	7347.12	219.86	10.88	7788.93
25	6793	22	9.57	2643	33.6	10.9	3008.8	25	6793	22	9.57	2642.5	622.58	10.90	3008.8	26	7289	24	11.41	3148.77	84.22	12.74	3816.00
15	2820	13	6.93	1103	14.1	6.7	1289.5	15	2820	13	6.93	1106.4	286.77	6.74	1268.5	16	3033	15	7.06	1320.77	39.52	7.87	1471.81
27	30108	25	10.82	11851	150.5	12.3	13462.3	27	30108	25	10.82	11851.4	2738.56	12.29	13482.3	30	32379	27	12.90	14121.79	422.86	14.37	15732.71
39	14257	36	15.63	6642	71.7	17.7	6402.2	39	14257	36	15.63	6642.1	1282.31	17.73	6402.2	42	15338	38	18.82	6723.91	201.18	20.73	7483.10
13	8939	12	5.10	3827	48.6	6.4	4088.9	14	10272	13	6.08	4560.5	441.36	6.43	4820.1	14	10272	13	6.08	4680.48	136.47	6.43	4820.08
82	8190	73	32.66	3295	41.8	37.3	3727.6	62	8199	73	32.65	3284.9	735.56	37.28	3727.6	68	8930	80	39.26	3926.14	117.49	43.69	4356.84
18	27782	17	7.33	11243	142.8	7.8	11894.7	20	28936	18	8.74	13396.8	1277.99	9.23	14148.5	20	28936	19	8.74	13396.77	400.89	9.23	14148.53
20	11537	18	8.38	4714	59.9	9.4	5316.3	20	11537	18	8.38	4714.4	1023.33	8.48	5316.3	22	12440	20	9.85	5517.61	188.10	11.03	6219.47
34	48927	28	14.09	18953	263.8	18.4	26148.2	31	44078	22	10.71	10214.1	10914.83	15.07	21389.3	37	62592	28	18.75	23767.40	711.82	21.11	28972.59
288	82688	209	120.29	28104	331.6	195.1	28321.2	292	63370	282	123.93	26806.9	6469.85	138.82	30123.4	311	67569	282	143.34	31104.52	930.77	158.17	34321.86
28	21623	24	11.78	8064	115.1	14.0	10776.7	27	20691	23	10.66	6132.5	2811.27	12.77	6845.1	30	23359	26	14.07	10800.82	323.20	16.23	12513.13
28	16853	24	11.97	7085	80.0	14.2	8416.8	28	16290	23	11.02	6521.7	2264.40	13.27	7853.7	31	18210	26	14.28	8442.11	262.62	16.51	8774.11
4	1696	4	1.67	674	6.6	1.9	765.1	4	1617	4	1.72	694.4	188.34	1.93	776.8	4	1726	4	1.89	802.79	24.02	2.19	684.16
23	3695	19	9.63	5789	73.7	11.4	8875.8	22	18284	18	8.67	5338.1	1830.31	10.66	6414.8	25	14808	21	11.48	8910.07	265.78	13.27	7986.72
409	450	331	178.28	4732	60.1	214.3	6785.1	384	10360	306	148.72	4042.3	1780.10	188.72	6095.3	443	11957	365	208.84	6638.09	168.73	247.84	6691.69
19	3239	13	6.84	1395	17.7	8.6	1702.0	16	3064	12	6.07	1220.6	622.41	7.60	1527.9	17	3606	14	8.27	1661.94	49.73	9.60	1969.24
14	8663	11	6.22	3768	47.9	7.9	4817.1	18	7897	10	4.95	3002.5	1783.04	6.69	4691.3	19	9365	12	7.41	4480.16	134.36	9.14	5538.00
15	3450	13	6.46	1606	19.1	7.4	1714.0	15	3450	13	6.46	1605.6	354.20	7.36	1714.0	16	3729	14	7.70	1784.08	63.69	8.59	2002.44
6	3758	6	2.62	1664	21.1	3.0	1948.4	6	3697	5	2.38	1572.7	488.48	2.82	1886.6	6	4077	5	3.00	1882.67	58.33	3.44	2288.15
13	1919	10	6.92	853	10.8	7.5	1081.4	12	1781	9	4.83	685.2	388.38	6.41	823.7	14	2082	11	7.08	1018.32	30.41	8.64	1244.78
31	6798	27	13.95	3041	38.9	16.8	3443.9	31	6798	27	13.95	3041.5	884.18	15.60	3443.9	34	7320	30	16.62	3824.12	108.46	18.47	4326.69
6	1337	5	2.45	607	7.7	2.7	671.3	6	1376	5	2.64	648.0	109.46	2.88	710.3	6	1453	5	2.92	723.17	21.64	3.18	787.56
	2379289			54901	9588		900652		2322293			687906	247777		843658		2623907			899519	28817		1045278

SKUs WITH NORMAL DISTRIBUTION OF DEMAND PATTERN																
FRACTIONAL CHARGE (B2=0.2)								COST (B1=R150) PER STOCKOUT								
PER UNIT SHORT								OCCASION								
MOHQ	MOHV	ROP	SS	SSV	EVSPY	A.OHQ	A.OHV	MOHQ	MOHV	ROP	SS	SSV	EVSPY	A.OHQ	A.OHV	ESPY
units	(R)	units	units	(R)	(R)	units	(R)	units	(R)	units	units	(R)	(R)	units	(R)	
114	12864	89	27.86	3147.88	493.55	35.44	4005.18	114	12829.8	88	27.8	3113.7	585.308	35.1	3971.0	0.8
704	168230	821	178.75	41712.94	8540.07	218.40	51543.24	629	146343.8	545	101.0	23828.8	34134.8	142.6	33858.9	2.7
238	57387	190	61.32	14777.00	2316.85	85.43	20588.81	219	52782.5	171	42.2	10172.5	8044.23	68.3	15884.3	2.0
28	21931	28	7.21	5745.90	300.89	8.13	8481.58	27	21288.7	26	8.4	6081.4	1683.3	7.3	5917.1	1.1
414	73737	349	110.02	19584.29	3070.57	142.62	25386.40	372	68297.4	307	68.0	12104.3	13565.3	100.6	17603.4	2.4
89	41510	80	18.83	11398.04	1787.07	25.55	14054.94	64	38630.5	55	14.2	8518.9	5288.98	18.8	11175.8	1.7
715	35030	624	197.38	8671.38	1516.35	243.04	11908.89	672	32825.7	561	154.4	7567.2	3918.54	200.1	8804.7	1.5
530	18552	470	147.51	5162.98	808.48	177.71	6220.02	518	18098.2	455	138.3	4888.7	1285.32	163.5	5723.8	1.0
105	36758	84	29.28	10252.47	1807.46	38.02	13698.89	88	34402.0	78	22.8	7886.2	4153.77	32.9	11510.6	1.5
52	60242	48	14.88	14287.52	2241.87	17.85	17489.12	47	45898.5	41	10.2	8853.6	7783.21	13.5	13125.2	2.0
22	98500	21	6.31	27469.84	4305.83	7.03	30807.58	19	9948.3	18	5.5	14318.1	28883.1	4.0	17453.8	3.1
168	230548	155	48.34	87239.01	10542.23	53.68	74682.13	145	201716.0	134	21.8	38407.2	54975.1	32.9	45830.4	2.7
1278	12777	1087	373.69	3739.68	585.80	484.08	4840.91	1283	12829.5	1082	358.9	3689.1	758.142	449.9	4483.1	0.8
98	20408	88	28.45	6128.60	902.42	35.51	7385.80	84	18824.4	82	25.7	6344.4	1887.93	31.8	8504.4	1.1
31	2538	27	9.36	768.90	120.25	11.28	825.01	33	2691.4	29	11.2	822.1	54.8271	13.1	1078.3	0.3
86	39883	70	26.14	12156.58	1808.84	34.30	16881.57	79	39790.1	63	19.2	8912.7	5988.28	27.8	12708.7	1.7
120	113995	112	38.87	34783.65	5453.64	40.47	38893.00	104	88890.2	97	21.0	18888.8	28439.4	24.7	23450.8	2.7
171	18858	150	52.81	5758.75	802.58	63.28	8894.88	165	17888.8	144	48.7	5089.0	1588.28	57.1	6227.2	1.1
475	44183	417	148.98	13687.47	2142.89	175.89	18358.21	432	40153.3	374	103.9	9858.2	7048.58	132.8	12348.0	1.8
13	4878	11	3.98	1458.24	228.63	4.97	1829.78	13	4836.9	11	4.4	1618.9	184.524	5.4	1688.5	0.5
17	10984	15	5.20	3432.43	538.18	6.21	4088.77	17	10897.8	14	5.1	3943.9	818.405	6.1	4012.2	0.8
424	11443	373	135.13	3648.39	572.02	160.60	4338.11	419	13131.1	388	130.3	3518.9	740.188	165.8	4266.6	0.8
150	42819	132	48.24	13748.95	2185.88	57.24	18913.85	138	38788.9	118	34.0	8898.9	7091.81	43.0	12281.8	1.9
73	13964	84	23.38	4488.02	703.35	27.72	5822.33	71	13845.2	62	21.7	4187.1	1018.79	28.1	5003.4	0.9
27	18188	24	8.88	6281.38	881.70	10.50	7400.21	28	18344.1	23	7.7	5437.2	1725.33	8.3	6578.0	1.1
377	235411	338	125.81	78318.28	12278.48	145.38	90721.28	323	201888.3	284	71.7	44738.3	81034.4	91.8	57138.3	2.7
722	10109	562	241.83	3382.80	530.38	821.71	4503.68	717	10032.2	558	236.1	3305.7	607.494	316.2	4428.8	0.8
347	18078	285	118.43	8513.82	1021.28	148.68	8227.28	330	18173.2	288	102.0	8807.8	1858.44	133.1	7321.3	1.2
22	9415	19	7.58	3243.88	508.57	8.83	3788.18	22	9363.4	19	7.4	3182.4	582.508	8.7	3738.9	0.8
148	5881	127	48.61	2083.83	318.88	57.88	2373.28	148	6000.2	130	52.5	2153.3	273.582	80.8	2482.7	0.8
628	6284	648	217.68	2176.78	341.28	258.78	2837.68	639	6388.5	657	228.2	2281.8	282.812	288.3	2832.5	0.9
20	15786	18	7.15	5511.31	884.10	7.76	5978.38	20	19191.2	18	8.4	4918.2	1362.72	7.0	5383.2	1.0
12	20857	11	4.11	7841.03	1180.88	4.45	7889.98	11	19787.7	10	3.4	6151.4	2384.52	3.8	8770.3	1.3
25	5488	22	8.82	1822.77	301.47	10.28	2237.19	26	5615.8	23	9.4	2052.3	258.648	10.8	2366.7	0.6
181	87004	161	83.89	30689.05	4898.52	74.25	35839.82	148	71223.8	128	31.0	14888.2	33248.9	41.4	19889.0	3.4
22	7405	19	7.72	2825.08	411.58	8.98	3046.81	22	7455.7	19	7.8	2674.1	411.58	9.1	3085.8	0.7
110	8188	98	33.50	2211.98	348.81	48.80	2689.37	112	6287.0	98	41.3	2313.1	297.547	48.4	2710.5	0.8
202	58521	147	72.72	21017.25	3285.24	100.53	29083.87	173	50085.5	118	43.8	12582.1	18858.3	71.4	20828.8	2.8
21	18581	19	7.73	6883.59	1047.80	8.94	7731.57	20	17598.4	18	6.8	5721.3	2009.48	7.8	6789.3	1.2
88	4802	75	35.84	1732.80	271.68	36.42	2039.73	88	4846.1	77	33.5	1878.9	185.501	39.0	2163.8	0.5
84	10276	84	34.12	3718.38	583.15	38.43	4287.82	89	10132.0	82	32.8	3575.3	764.535	38.1	4163.8	0.8
28	48075	25	10.05	15823.71	2448.80	11.81	18045.92	24	87888.8	21	6.8	10547.1	8957.42	8.3	12888.3	2.0

SKUS WITH GAMMA DISTRIBUTION OF DEMAND PATTERN

NO	PART-NO	PRICE (₹)	MONTHLY DEMAND													AVG UNITS	STD UNITS	COEF	ANNUAL DEMAND UNITS	LEAD TIME				LIT		PARAMETERS	
			X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	SUPP WEEKS					TOTAL WEEKS	XI+r UNITS	STDI+r UNITS	COEF	alpha	beta		
FAST MOVING SKUs WITH GAMMA DISTRIBUTION OF DEMAND																											
81	FA030750131003	68	44	146	32	154	243	174	128	19	120	3	155	101	109.9	69.35	0.63	1319	4	7	177.8	91.74	0.52	3.75	47.40		
82	LB887721600	274	12	0	12	7	0	18	15	0	0	0	3	5	8.0	6.38	1.06	72	16	19	26.3	13.80	0.53	3.58	7.34		
83	MA106326750001	163	7	23	25	31	31	29	31	11	10	0	7	5	17.5	11.35	0.85	210	4	7	28.3	15.02	0.53	3.54	7.98		
84	191322050077	507	8	8	1	12	16	5	17	1	1	1	0	0	5.9	6.08	1.03	71	14	17	23.2	12.52	0.54	3.43	6.76		
85	MA878000171	985	3	6	18	6	10	10	6	8	21	1	6	5	8.3	5.99	0.67	100	4	7	13.5	7.39	0.55	3.32	4.06		
86	LA033151119001	237	0	54	3	0	26	9	1	40	2	0	4	1	11.7	17.49	1.50	140	30	33	88.8	50.24	0.57	3.13	28.40		
87	LA181454006815	1114	2	4	2	12	0	9	10	10	16	0	2	3	5.8	5.08	0.87	70	8	11	14.8	8.42	0.57	3.09	4.79		
88	MA897811545	825	3	12	15	12	2	11	27	5	20	0	7	10	10.3	7.46	0.72	124	4	7	16.7	9.87	0.59	2.86	5.64		
89	MA871060024	330	13	17	35	11	10	13	1	33	61	11	12	21	19.8	15.46	0.78	238	4	7	32.0	20.46	0.64	2.45	13.08		
90	LB887721620	301	1	6	2	3	4	20	29	5	0	0	0	8	6.5	8.81	1.32	78	16	19	28.5	16.76	0.66	2.31	12.35		
91	SD887721630	346	0	0	0	0	3	13	26	0	0	0	1	0	3.6	7.64	2.13	43	16	19	15.7	16.66	1.06	0.89	17.88		
92	SF181391150140	164	1	0	0	0	0	4	6	1	23	0	1	1	3.1	6.26	2.03	37	14	17	12.1	12.91	1.07	0.88	13.79		
93	LA0693298CD	203	6	0	0	0	0	10	6	111	459	0	6	24	51.8	126.34	2.44	622	16	19	227.3	275.35	1.21	0.68	333.61		
94	LA158453020508	365	0	0	0	117	0	0	7	0	0	0	0	0	10.3	32.22	3.12	124	6	9	21.5	48.33	2.25	0.20	108.83		
SLOW MOVING SKUs WITH GAMMA DISTRIBUTION OF DEMAND																											
95	181152025788	248	2	1	0	1	1	0	2	0	1	0	1	0	0.8	0.72	0.96	9	14	17	2.9	1.49	0.51	3.91	0.75		
96	SD181434900544	404	5	3	1	0	3	2	2	5	2	0	0	2	2.1	1.66	0.79	25	8	11	5.3	2.75	0.52	3.71	1.43		
97	SE181322010059	328	3	0	3	1	0	2	3	1	2	0	0	0	1.3	1.23	0.99	15	14	17	4.9	2.54	0.52	3.72	1.32		
98	121625104146	1155	0	3	1	5	4	0	2	0	3	0	3	0	1.8	1.74	0.99	21	14	17	6.9	3.58	0.52	3.67	1.87		
99	SD0156541010501	631	3	3	2	0	1	1	6	1	5	1	2	1	2.2	1.72	0.80	26	8	11	5.5	2.86	0.52	3.70	1.49		
100	SF181434906014	208	2	2	0	1	5	3	1	3	4	0	2	0	1.9	1.55	0.81	23	8	11	4.9	2.57	0.53	3.57	1.36		
101	SE0216775	246	1	3	3	2	2	5	4	2	9	0	5	0	3.0	2.42	0.81	36	8	11	7.6	4.01	0.53	3.62	2.11		
102	181612105219	265	0	1	2	0	2	4	4	2	6	0	0	1	1.8	1.86	1.02	22	14	17	7.2	3.84	0.53	3.51	2.05		
103	181906200000	156	1	2	1	2	7	3	0	1	1	0	3	0	1.8	1.86	1.07	21	14	17	6.9	3.87	0.56	3.15	2.18		
104	SE181521616135	698	2	0	0	1	0	0	2	1	1	0	1	1	0.8	0.72	0.96	9	11	14	2.4	1.35	0.56	3.22	0.75		
105	SE0181514010024	182	6	4	3	2	0	2	4	1	3	0	3	1	2.4	1.71	0.71	29	4	7	3.9	2.26	0.58	2.99	1.30		
106	SAB70150883	2000	8	0	4	4	6	4	0	2	6	0	5	2	3.4	2.53	0.74	41	4	7	5.5	3.35	0.61	2.72	2.03		
107	SD181523206099	526	3	1	0	0	4	4	1	0	2	0	0	2	1.4	1.50	1.06	17	11	14	4.2	2.80	0.61	2.67	1.72		
108	SB181391056728	1603	0	1	1	1	2	2	1	1	0	0	2	1	1.0	0.71	0.71	12	3	6	1.4	0.67	0.63	2.56	0.54		
109	SG05690759610	163	0	1	1	0	0	2	0	1	2	0	1	1	0.6	0.72	0.96	9	8	11	1.9	1.20	0.63	2.53	0.75		
110	MC873170080	245	2	2	6	5	7	12	4	4	12	0	2	2	4.8	3.72	0.77	58	4	7	7.8	4.92	0.63	2.52	3.09		
111	181301010087	1180	1	0	0	1	0	0	1	0	1	0	1	0	0.4	0.49	1.18	5	14	17	1.6	1.02	0.62	2.59	0.63		
112	CA885010200	1190	1	1	3	1	2	5	8	9	6	0	1	2	3.3	2.89	0.89	39	6	9	6.8	4.34	0.64	2.42	2.78		
113	MA877200121	1475	3	10	3	5	2	4	2	3	11	0	3	2	4.0	3.14	0.78	48	4	7	6.5	4.15	0.64	2.43	2.66		
114	LA897810724	1130	8	8	7	0	6	0	0	6	6	0	6	2	3.9	3.07	0.78	47	4	7	6.3	4.06	0.64	2.43	2.60		

SKUS WITH GAMMA DISTRIBUTION OF DEMAND PATTERN

NO	PART-NO	PRICE (R)	MONTHLY DEMAND												AVG UNITS	STD UNITS	COEF	ANNUAL DEMAND UNITS	LEAD TIME			LIT COEF	PARAMETERS		
			X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12					SUPP WEEKS	TOTAL WEEKS	Xi+r UNITS		STDi+r UNITS	alfa	beta
115	SD31297302019	351	0	2	0	2	1	0	0	5	7	0	3	1	1.8	2.17	1.24	21	14	17	6.9	4.46	0.65	2.37	2.90
116	SE0181521515013	285	2	1	0	1	4	8	0	2	5	0	1	3	2.3	2.31	1.03	27	8	11	5.7	3.84	0.97	2.22	2.58
117	SF181434908022	235	1	0	0	3	2	4	0	0	1	0	2	2	1.3	1.30	1.04	15	8	11	3.2	2.15	0.88	2.17	1.48
118	SC878110139	650	2	2	3	0	1	2	2	5	5	0	2	0	2.0	1.63	0.82	24	4	7	3.2	2.16	0.67	2.24	1.44
119	SF0216721	220	5	1	0	4	1	0	0	6	3	0	2	1	1.9	2.02	1.05	23	8	11	4.9	3.35	0.68	2.11	2.30
120	SE156191015500	194	2	2	1	3	8	1	5	1	2	0	2	0	2.3	2.17	0.88	27	6	9	4.7	3.25	0.69	2.07	2.26
121	181626007183	2380	1	0	0	2	1	0	0	2	3	0	0	0	0.8	1.01	1.35	9	14	17	2.9	2.08	0.71	2.00	1.47
122	SE181504106279	653	1	0	1	1	0	3	1	3	0	0	0	0	0.8	1.07	1.28	10	11	14	2.7	2.00	0.74	1.82	1.48
123	181152025385	180	0	0	1	1	0	0	0	1	1	0	0	0	0.3	0.47	1.41	4	14	17	1.3	0.97	0.74	1.81	0.72
124	SA872000355	1860	2	1	0	1	2	0	4	0	3	2	0	1	1.3	1.25	0.94	18	4	7	2.2	1.65	0.77	1.70	1.26
125	SF181352020035	222	3	0	2	0	0	5	0	0	8	0	0	4	1.9	2.75	1.44	23	14	17	7.5	5.67	0.75	1.76	4.28
126	SF181434908021	235	3	0	0	2	3	1	0	0	4	1	0	0	1.2	1.40	1.20	14	8	11	3.0	2.33	0.79	1.82	1.83
127	SE687721610	302	2	0	0	7	7	0	0	0	0	0	0	8	1.8	2.95	1.55	22	16	19	8.0	6.22	0.77	1.87	4.81
128	SE0216717	474	1	0	0	2	0	0	3	1	2	0	1	0	0.8	0.98	1.18	10	8	11	2.1	1.84	0.77	1.67	1.26
129	LC877 2028	281	5	1	6	10	0	6	12	6	15	0	0	0	5.1	4.94	0.97	61	4	7	8.2	6.54	0.80	1.58	5.20
130	18106101631A	2450	0	0	0	0	0	0	1	3	2	0	0	2	0.7	1.03	1.54	8	14	17	2.6	2.12	0.81	1.52	1.72
131	SG877830232	310	0	1	0	1	0	0	0	1	1	0	1	1	0.5	0.50	1.00	6	4	7	0.8	0.66	0.82	1.49	0.54
132	SA870150964	2110	0	0	0	3	2	4	1	2	4	0	2	0	1.5	1.50	1.00	18	4	7	2.4	1.98	0.82	1.49	1.63
133	LA877002086	635	6	1	14	0	10	5	1	1	14	0	6	0	4.8	5.10	1.05	58	4	7	7.8	6.74	0.86	1.34	5.82
134	1301010119	1430	0	0	10	1	6	0	0	0	4	0	0	1	1.8	3.08	1.68	22	14	17	7.2	6.34	0.86	1.28	5.60
135	30750118007	443	0	0	0	0	0	1	1	0	1	0	0	0	0.3	0.43	1.73	3	14	17	1.0	0.89	0.91	1.21	0.91
136	SA181434008264	998	0	1	0	10	1	5	14	11	1	0	0	0	3.6	4.92	1.37	43	8	11	8.1	8.17	0.90	1.24	7.33
137	SA870000085	6950	0	2	1	0	2	2	1	0	3	0	0	0	0.9	1.04	1.13	11	4	7	1.5	1.37	0.93	1.16	1.27
138	181061016155	3200	0	0	0	0	0	2	8	4	2	0	0	0	1.3	2.36	1.77	16	14	17	5.2	4.86	0.83	1.16	4.51
139	SC0156122045504	1107	0	0	2	4	0	2	1	0	1	0	1	1	1.0	1.15	1.15	12	4	7	1.6	1.53	0.95	1.12	1.44
140	SD02A5106	989	0	0	0	0	2	2	3	3	0	0	0	0	0.8	1.21	1.46	10	8	11	2.1	2.01	0.95	1.11	1.91
141	LA865010019	617	9	1	1	18	3	0	15	3	1	0	2	0	4.4	5.92	1.34	53	6	9	9.2	8.88	0.97	1.07	8.60
142	SG0181511016264	180	1	0	0	1	0	0	2	0	2	0	0	0	0.5	0.76	1.53	6	8	11	1.3	1.27	1.00	1.00	1.20
143	SG156328105502	113	0	3	0	1	0	3	1	5	1	0	0	0	1.2	1.57	1.35	14	6	9	2.4	2.39	0.97	1.08	2.30
144	SF030501002032	163	1	1	0	1	1	2	0	1	4	0	0	0	0.9	1.11	1.22	11	4	7	1.5	1.47	1.00	1.01	1.47
145	SG181352020036	171	0	0	1	0	0	4	0	0	1	0	6	0	1.0	1.87	1.87	12	14	17	3.9	3.86	0.98	1.03	3.79
146	1151065080090	108	0	0	0	6	1	1	0	0	4	0	0	0	1.0	1.87	1.87	12	14	17	3.9	3.86	0.98	1.03	3.79
147	181424015230	287	0	1	0	2	0	0	0	0	1	0	0	0	0.3	0.82	1.87	4	14	17	1.3	1.29	0.98	1.03	1.28
148	SD181432206052	793	0	0	2	0	0	1	0	2	5	0	4	2	1.3	1.65	1.24	16	4	7	2.2	2.18	1.01	0.97	2.21
149	SA898000193	20100	0	0	0	0	0	5	5	5	5	3	0	0	1.9	2.33	1.21	23	4	7	3.1	3.08	0.99	1.01	3.08
150	SF897100091	470	0	0	1	0	0	1	1	3	2	0	0	0	0.7	0.94	1.41	8	4	7	1.1	1.25	1.16	0.75	1.44
151	SF0220389	272	2	0	0	6	2	0	3	0	0	0	0	0	0.6	1.04	1.78	7	8	11	1.5	1.72	1.16	0.74	2.00
152	SA820050225300	7062	8	0	0	0	0	10	0	7	2	0	0	0	2.1	3.38	1.82	25	4	7	3.4	4.47	1.33	0.57	5.93
153	SA865000210	565	0	0	0	0	10	15	0	0	23	0	0	0	4.0	7.43	1.86	48	8	9	6.3	11.14	1.34	0.58	14.94

SLOW MOVING SKUs WITH POISSON DISTRIBUTION OF DEMAND

NO	PART-NO	PRICE (R)	MONTHLY DEMAND												ANNUAL DEMAND UNITS	LEAD TIME			LIT	TOTAL USAGE VALUE	ROQ		
			X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12		SUPP WEEKS	TOTAL WEEKS	Xi+r				STDI+r	COEF
1	SC877290477	2408	3	0	0	0	1	1	0	2	1	0	0	0	5	4	7	0.7	0.65	1.28	12040	1	
2	SG873380230	235	1	0	1	1	0	0	1	0	2	0	0	0	6	4	7	0.8	0.65	1.06	1410	1	
3	SA870001851	11300	0	0	0	0	0	2	1	0	1	0	1	0	7	4	7	0.9	1.00	1.07	79100	1	
4	SD878000074	660	2	0	3	1	1	0	1	1	2	0	1	3	15	4	7	2.0	1.34	0.68	9900	1	
5	181381090014	216	0	0	1	1	0	0	2	0	1	0	0	0	5	14	17	1.6	1.32	0.81	1080	1	
6	181122015245	485	2	1	0	1	0	0	1	1	0	0	0	0	6	14	17	2.0	1.33	0.68	2910	1	
7	SE5FHG120014	360	0	1	1	0	1	1	3	0	3	0	3	2	15	6	9	2.8	1.75	0.67	5700	1	
8	CA870150582	2050	1	3	2	1	3	1	1	5	2	0	1	3	23	4	7	3.1	1.75	0.56	47150	1	
9	181462006135	4050	1	0	1	0	2	0	0	0	1	0	2	0	7	14	17	2.3	1.57	0.68	28350	1	
10	SF0219694	345	1	1	2	1	3	1	1	1	3	0	0	0	14	8	11	3.0	1.84	0.55	4830	1	
11	SA877100140	1820	2	3	1	2	2	4	1	5	5	0	3	1	26	4	7	3.9	2.08	0.53	52780	1	
12	151023050009	730	2	0	0	0	2	1	1	1	0	0	0	2	9	14	17	2.9	1.71	0.58	6570	1	
13	SED590345110	313	4	2	0	1	1	1	0	3	2	0	1	1	18	8	11	3.4	1.95	0.58	5008	1	
14	SC5FTG180558	560	0	2	1	3	1	4	3	4	2	0	1	1	22	9	9	3.8	2.02	0.53	12320	1	
15	SE02K1945	326	4	3	1	0	1	1	3	2	1	0	2	0	16	8	11	3.8	2.09	0.55	5868	1	
16	SC18150410805	1301	2	1	1	1	3	3	1	1	0	0	0	1	14	11	14	3.8	1.84	0.49	18214	1	
17	181081300026	2050	1	2	1	0	3	0	0	1	1	0	2	0	11	14	17	3.6	1.97	0.65	22550	1	
18	CA875000841	2040	5	5	2	2	3	3	1	4	7	1	2	4	39	4	7	5.3	2.30	0.44	79560	2	
19	SF687603620	402	0	0	2	2	0	2	1	1	2	0	0	0	10	18	19	3.7	1.98	0.54	4020	1	
20	CE5D4222	166	4	2	2	4	1	1	2	5	3	0	1	5	30	6	9	6.2	2.41	0.48	4980	1	
21	SG15615202051	120	1	0	2	3	0	2	1	4	3	0	1	1	18	10	13	4.5	2.27	0.50	2160	1	
22	181625304115	363	1	0	1	0	1	0	4	2	3	1	1	1	15	14	17	4.9	2.40	0.49	5445	1	
23	15104408271	1980	0	3	2	2	0	4	3	1	3	0	3	1	22	14	17	7.2	2.77	0.39	43120	1	
24	CCD1564170255	380	2	2	5	5	6	4	3	4	4	0	3	2	40	8	11	8.6	2.65	0.31	15200	2	
25	ME18143490032	159	5	6	8	5	4	4	7	5	2	0	4	1	49	8	11	10.4	3.35	0.32	7791	2	
26	SC18130715605	552	5	3	2	0	3	4	3	0	2	0	3	2	27	16	19	9.9	3.34	0.34	14904	1	
27	181628007179	1160	3	2	1	5	2	2	5	5	2	0	1	3	31	14	17	10.1	3.31	0.33	35960	1	
28	CA03188899927	4351	3	3	3	1	2	3	1	0	4	0	3	2	25	30	33	15.9	3.61	0.23	108775	1	
																					637695		

SKUs WITH POISSON DISTRIBUTION OF DEMAND PATTERN

PROBABILITY (P1=0.95) OF NO STOCKOUT								FRACTIONAL CHARGE (B2=20%)						COST (B1=R150) PER STOCKOUT OCCASION									
PER REPLENISHMENT CYC' E								PER UNIT SHORT								ROQ = 1							
MOHQ	MOHV	AOHQ	AOHV	SS	SS	ROP	ETSOPY	MOHQ	MOHQ	OHQ	OHQ	SS	SS	ETSOPY	ROP	MOHQ	MOHV	AOHQ	AOHV	SS	SS	ETSOPY	
UNITS	VALUE	UNITS	VALUE	UNITS	VALUE	UNITS	CAS- IONS	UNITS	VALUE	UNITS	VALUE	UNITS	VALUE	CAS- IONS	UNITS	UNITS	VALUE	UNITS	VALUE	UNITS	VALUE	CAS- IONS	
2	4818	1.83	4399	1.3	3195	1	1.90	2	7224	2.83	8807.2	2.3	5603	0.03	1	2	4818	1.83	4399	1.3	3195	1.90	
2	470	1.69	398	1.2	290	1	2.52	2	705	2.69	632.7	2.2	515	0.07	2	4	940	3.69	868	3.2	750	0.01	
3	33900	2.56	28902	2.1	23252	2	0.13	2	33900	2.56	28901.9	2.1	23252	0.13	0	2	22600	1.56	17602	1.1	11952	3.19	
4	2640	2.48	1637	2.0	1307	3	1.17	4	3300	3.48	2297.3	3.0	1987	0.35	4	6	3960	4.48	2957	4.0	2627	0.09	
4	864	2.87	619	2.4	511	3	0.17	3	1080	3.87	834.9	3.4	727	0.04	3	6	1296	4.87	1051	4.4	943	0.01	
4	1940	2.54	1231	2.0	989	3	0.42	3	1940	2.54	1231.2	2.0	989	0.42	3	6	2910	4.54	2201	4.0	1956	3.03	
5	1900	2.90	1103	2.4	913	4	1.10	5	2280	3.90	1483.5	3.4	1293	0.36	5	8	3040	5.90	2243	5.4	2053	0.03	
6	12300	3.40	6978	2.9	5933	4	1.36	6	14350	4.40	9027.9	3.9	8003	0.48	4	7	14350	4.40	9028	3.9	8003	0.48	
5	20250	3.21	13007	2.7	10982	4	0.29	4	20250	3.21	13006.7	2.7	10982	0.29	1	4	16200	2.21	8957	1.7	6932	0.67	
6	2070	3.54	1221	3.0	1048	5	0.67	6	2415	4.54	1595.8	4.0	1393	0.22	6	8	2760	5.54	1811	5.0	1738	0.07	
7	12740	3.60	6545	3.1	5635	5	2.16	8	16380	5.60	10185.0	5.1	8275	0.30	6	8	14580	4.60	8365	4.1	7455	0.84	
6	4380	3.56	2597	3.1	2232	5	0.41	5	4380	3.56	2597.1	3.1	2232	0.41	4	7	5110	4.56	3327	4.1	2982	0.14	
7	2191	4.12	1288	3.6	1132	6	0.55	6	2191	4.12	1288.1	3.6	1132	0.55	7	9	2817	6.12	1814	5.6	1758	0.06	
7	3820	3.88	2068	3.2	1788	5	1.44	7	4480	4.69	2627.7	4.2	2348	0.55	7	9	5040	5.69	3188	5.2	2908	0.18	
7	2282	3.69	1234	3.2	1041	6	1.18	7	2608	4.69	1529.7	4.2	1367	0.45	7	9	2934	5.69	1856	5.2	1593	0.16	
7	9107	3.73	4854	3.2	4203	6	0.87	7	10408	4.73	8154.7	4.2	5504	0.33	5	8	10408	4.73	6155	4.2	5304	0.33	
7	14350	3.90	8003	3.4	6978	6	0.53	8	14350	3.90	8002.9	3.4	6978	0.53	4	6	12300	2.90	5953	2.4	4928	1.33	
9	18360	4.75	9690	3.8	7650	6	1.43	10	22440	6.75	13770.0	5.8	11730	0.24	6	9	18360	4.75	9690	3.8	7650	1.43	
7	2814	3.85	1548	3.3	1345	6	0.52	6	2814	3.85	1548.2	3.3	1345	0.52	6	9	3618	5.85	2350	5.3	2149	0.06	
9	1494	4.31	715	3.8	632	7	2.05	10	1826	6.31	1047.1	5.8	894	0.34	11	13	2158	8.31	1379	7.8	1286	0.04	
8	980	4.00	420	3.5	420	7	1.22	8	1080	5.00	600.0	4.5	540	0.49	9	12	1440	8.00	950	7.5	900	0.02	
8	3257	4.60	1666	4.1	1487	8	0.72	8	3257	4.60	1666.4	4.1	1487	0.72	8	11	3693	6.60	2394	6.1	2213	0.11	
12	23520	5.31	10403	4.8	9423	10	1.33	11	25480	6.31	12363.1	5.8	11383	0.62	9	11	21580	4.31	8443	3.8	7463	2.71	
13	4940	5.54	2105	4.5	1725	10	2.02	14	5700	7.54	2864.8	6.5	2485	0.49	13	15	5700	7.54	2885	6.5	2485	0.49	
15	2385	5.63	896	4.6	737	12	3.37	16	2703	7.63	1213.9	6.6	1055	0.98	17	19	3021	9.63	1532	8.8	1373	0.23	
15	8280	5.63	3110	5.1	2834	13	2.50	15	9384	7.63	4214.3	7.1	3938	0.65	15	17	9384	7.63	4214	7.1	3938	0.65	
15	17400	5.37	6224	4.9	5844	13	3.57	15	16720	7.37	8543.3	6.9	7864	0.98	14	16	18560	6.37	7384	5.9	6804	1.91	
20	87020	4.63	20185	4.1	17890	19	8.57	20	91371	5.63	24516.2	5.1	22341	5.46	19	21	91371	5.63	24516	5.1	22341	5.46	
	300560		143056		121325		44		328026		170522		148791		17		213635		123186		103631		17
213540	103336	122891			36																		

TOTAL STOCK

SIMULATION SUMMARY RESULTS :NORMAL DISTREBUTION

PROBABILITY (P1) OF NO STOCKOUT PER REPLENISHMENT CYCLE

L_{Tc} =3 weeks

L_{Tc} =10 weeks

P1	L _{Tc} =3 weeks						L _{Tc} =10 weeks					
	TMOHV (R1,000)	TSSV (R1,000)	TAOHV #####	ETVSPY (R1,000)	ETSOPY occassio	TOTAL COST (R1000)	TMOHV (R1,000)	TSSV (R1000)	TAOHV (R1,000)	ETVSPY (R1000)	ETSOP occassio	TOTAL COST (R1000)
0.700	2802.2	513.3	753.7	3167.6	959	965.2	4055.8	645.0	885.4	2520.6	763	833.8
0.750	2949.1	660.2	900.6	2481.8	799	827.6	4240.4	829.6	1070.0	1974.8	636	739.8
0.800	3130.3	841.4	1081.8	1798.4	639	698.4	4468.1	1057.4	1297.7	1431.1	509	656.2
0.850	3303.4	1014.4	1254.8	1292.8	479	611.8	4685.6	1274.8	1515.2	1028.7	381	605.7
0.900	3543.3	1254.3	1494.7	787.8	320	540.7	4987.1	1576.3	1816.7	626.8	254	574.6
0.930	3733.4	1444.5	1684.9	515.2	224	516.3	5226.0	1815.3	2055.7	409.9	178	575.3
0.950	3896.9	1609.9	1850.3	347.6	160	512.5	5434.0	2023.2	2263.6	276.6	127	589.8
0.970	4129.8	1840.9	2081.3	193.3	96	527.0	5724.2	2313.4	2553.8	153.8	76	625.8
0.990	4565.9	2277.0	2517.4	56.4	32	593.1	6272.2	2861.5	3101.9	44.9	25	724.6
0.995	4810.1	2521.2	2761.5	26.3	16	641.7	6579.1	3168.4	3408.7	20.9	13	789.2

TOTAL STOCK

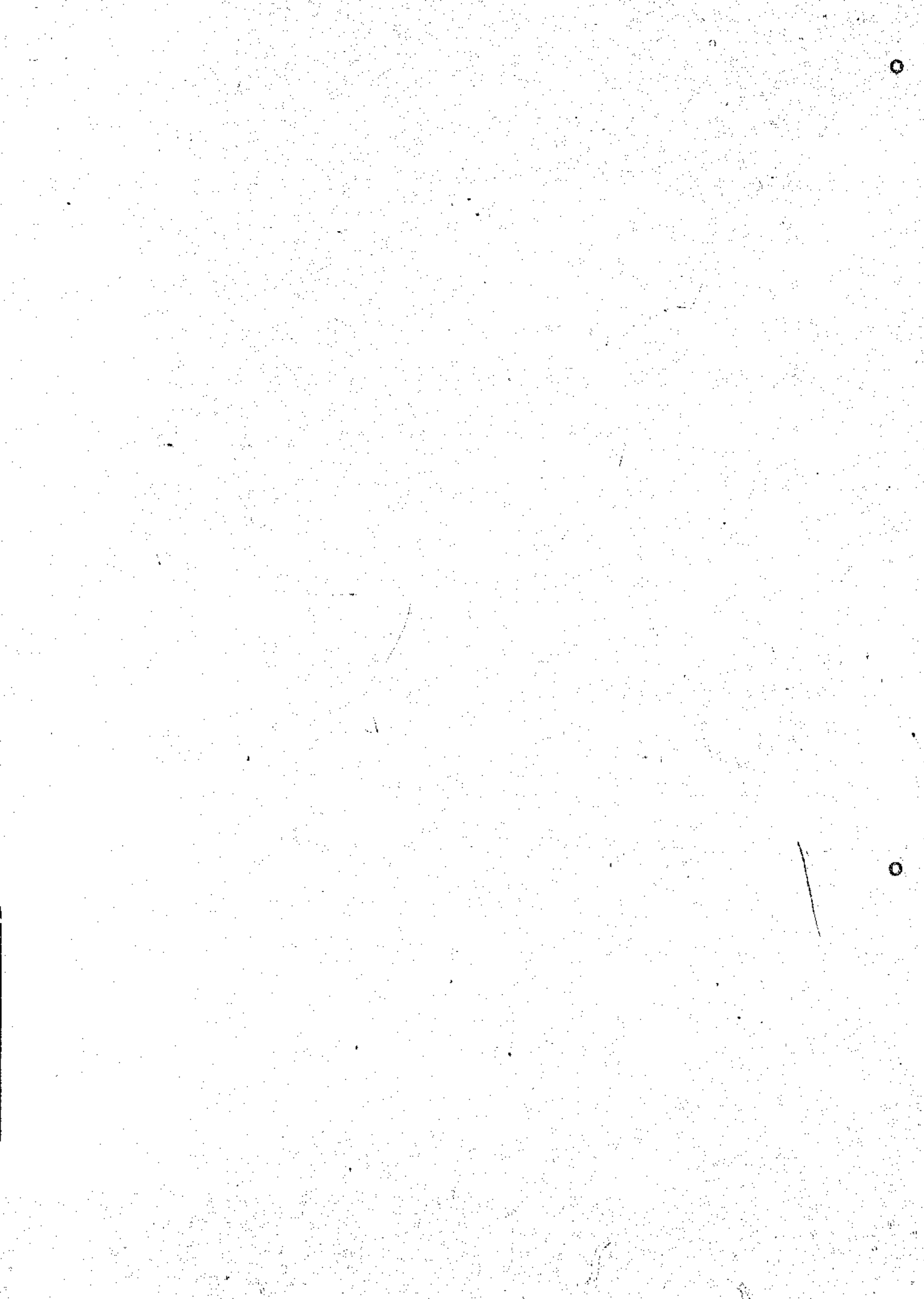
SIMULATION SUMMARY RESULTS : GAMMA DISTREBUTION

PROBABILITY (P1) OF NO STOCKOUT PER REPLENISHMENT CYCLE

LTc =3 weeks

LTc =10 weeks

P1	TMOHV	TSSV	TAOHV	ETVSPY	ETSOPY	TOTAL	TMOHV	TSSV	TAOHV	ETVSPY	ETSOPY	TOTAL
	(R1,000)	(R1,000)	(R1,000)	(R1,000)	occassions	COST (R1000)	(R1000)	(R1000)	(R1000)	(R1000)	occassion	COST (R1000)
0.700	2548.5	308.2	573.8	6625.0	918.0	1667.2	3808.8	445.76	1406.9	4580.3	609.00	1405.2
0.750	2714.8	474.4	739.9	5462.7	765.0	1441.4	4018.1	655.09	1460.2	3956.2	517.00	1275.0
0.800	2912.9	672.5	938.0	4323.2	612.0	1228.8	4264.0	900.95	1552.0	3227.1	458.00	1130.2
0.850	3161.7	921.3	1186.8	3205.6	459.0	1034.0	4566.0	1204.96	1703.3	2451.1	410.00	989.6
0.900	3503.1	1262.7	1528.2	2109.9	306.0	867.3	4977.8	1614.79	1961.4	1649.9	265.00	869.8
0.930	3796.7	1556.4	1821.8	1463.5	214.2	792.1	5324.7	1961.67	2217.4	1161.1	201.00	821.4
0.950	4069.3	1828.9	2094.3	1037.8	153.0	762.3	5642.9	2279.82	2475.0	832.5	176.00	810.2
0.970	4476.5	2236.1	2501.5	617.0	91.8	767.3	6112.5	2749.47	2884.7	501.8	112.00	836.6
0.990	5332.4	3092.0	3357.4	202.6	30.6	886.0	7082.5	3719.42	3799.9	168.4	34.00	988.7
0.995	5862.0	3621.7	3887.1	100.6	15.3	994.9	7673.8	4310.78	4384.1	84.5	12.00	1115.5



Author: Azran Simon.

Name of thesis: Inventory policy planning for spare parts and its application in the heavy-duty truck and bus industry.

PUBLISHER:

University of the Witwatersrand, Johannesburg

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