

# A METHODOLOGY FOR IMPLEMENTING THE ANALYTICAL HIERARCHY PROCESS TO DECISION-MAKING IN MINING

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A research report submitted to the Faculty of Engineering and the Built Environment,  
University of the Witwatersrand, in fulfilment for the degree of Master of Science in  
Engineering

Johannesburg 2015.

## **Declaration**

I declare that this research report is my own unaided work. It is being submitted to the Degree of Master of Science in Engineering to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any University.

Signature

Date

## **Abstract**

The Analytic Hierarchy Process (AHP) is a Multi Criteria Decision-Making (MCDM) tool, which has gained wide acceptance in all disciplines in science and engineering. Although it has been used in mining engineering applications, it is only recently gaining significant momentum in the mining industry. Given its simplicity, it may seem surprising that it has not received wide acceptance, but this is probably due to a lack of both publicity and a user-friendly methodology. This report introduces a simple methodology that can be employed by anyone who possesses basic knowledge of arithmetic and spreadsheets, without having to know or understand fully the mathematics that the process is based on.

## **Acknowledgements**

I wish to thank the University of the Witwatersrand and, in particular, Professor T.R. Stacey for his mentorship, not only in respect of this study but also throughout my mining career.

I wish to acknowledge the valuable contributions made by WorleyParsons RSA who sponsored the study financially and with other resources in the form of time and opportunities to conduct case studies in real time as part of my work.

I am grateful to my colleagues for their valuable input.

Lastly, but mostly, I wish to express my sincere appreciation of my wife, Henrietta, for her love, patience, understanding and support.

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# Chapter 1 Introduction

This chapter serves as an introduction to Multi Criteria Decision Analysis (MCDA) in general and specifically to one of its many methods namely the Analytic Hierarchy Process (AHP). It introduces the research objectives and key deliverables and motivation for the research. The chapter concludes with a discussion on the organisation of the research report.

Concepts introduced in this chapter are:

- Multi Criteria Decision Analysis ; and
- Analytic Hierarchy Process.

## 1.1 Multi Criteria Decision Analysis

Multi Criteria Decision Analysis (MCDA) is concerned with facilitating decision-making when many criteria have to be considered to arrive at optimal choices from amongst a collection of alternatives. One of the objectives of MCDA is to provide documented support for the decision makers that have to make these choices. A unique optimal choice is rare and often the judgments of several participants are required to adequately differentiate between alternatives (Anon, 2015).

The discipline of MCDA has various names such as Multi Criteria Decision Making (MCDM) or Multi Attribute Decision Making (MADM) and some authors distinguish between these on the basis of small differences (Yavuz, 2015). In this research report MCDA will be used.

According to the United States Environmental Protection Agency (EPA) (Linkov and Steevens, 2008), effective decision-making requires an explicit structure for judges to collectively consider many factors that are relevant to evaluating several alternatives. Integrating such diverse information with respect to one or more objectives demands a systematic and understandable framework (Linkov and Steevens, 2008). Such a framework is described in the British Government manual entitled “Multi Criteria

Analysis: a manual” (Dodgson et al., 2009). It describes Multi Criterion Analysis (MCA) and recognises MCDA as a form thereof.

In this British manual, MCDA is described as both an approach and a set of techniques, with the objective of providing an overall ordering of alternatives, from most to least preferred. It is a way of considering both quantifiable as well as intangible objectives and criteria and of breaking down complex problems into more manageable elements. Once individual elements have been considered they are reassembled to present a coherent overall picture to decision makers (Dodgson et al., 2009). In short, MCDA overcomes the limitations of some less structured methods (Linkov and Steevens, 2008).

Optimization approaches to MCDA employ numerical scores to communicate the merit of alternatives on a single scale. The relative performances of alternatives, with respect to individual criteria, are evaluated and then aggregated into an overall score. Individual scores may be simply summed or averaged, or a weighting mechanism can be used to favour some criteria more heavily than others (Linkov and Steevens, 2008).

There are a number of MCDA methodologies, examples of which are: MAUT – **M**ulti **A**tttribute **U**tility **T**heory; ELECTRE - **E**Limination **E**t **C**hoix **T**raduisant la **R**Ealité (ELimination and Choice Expressing REality) and PROMETHEE - **P**reference **R**anking **O**rganization **M**ETHod for **E**nrichment of **E**valuations. Descriptions of these methods fall outside the scope of this report, but they all share similar steps of organization and the construction of a decision matrix. They differ, however, in the manner of synthesis. Some techniques rank alternatives, while others identify a single optimal alternative. Some provide an incomplete ranking and yet others differentiate between acceptable and unacceptable alternatives (Musingwini, 2010a; Fülöp, 2005).

## 1.2 The Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) is a MCDA tool that has proven to simplify complex decision analyses because it allows for the quantification of subjective criteria to be synthesised together with qualitative criteria in a simple, powerful and structured manner (Yavuz, 2015). In addition, the consistency of judgements on the criteria can

be measured (Saaty, 2012). In short, the method provides for the recording and documenting of “gut feel” or “engineering intuition”.

Although AHP has been utilised in mining, its use has been limited and mainly conducted by consultants or academics. In order for the system to be applied as common practice in mining, a user-friendly tool or methodology is required. Such a tool was used by Owusu-Mensah and Musingwini, (2011) to solve a mining transportation system selection problem. This tool, however, does not appear to be available anymore, perhaps highlighting a need for in-house skills of the technology.

### **1.3 Motivation for this Research**

Some shortcomings that conventional decision-making processes suffer from are as follows:

- Subjective judgement is required for many decisions in mining where information is sparse (Yavuz, 2015; Musingwini, 2010a). Sometimes decisions are based on the judgement of the most dominant participant in the decision-making process (Kahneman, 2011). Such decisions are not necessarily the best, optimum or even the correct ones. Sometimes the decisions are overturned or overruled by the next generation dominant party, again based on subjective judgement;
- Group subjective judgements enhance the probability of arriving at the best decision as opposed to the most favoured. Subjective judgements, however, are not necessarily quantified and are inherently prone to inconsistencies (Kahneman, 2011; Vick, 2002); and
- Minutes of meetings usually record decisions but seldom show how those decisions have been arrived at. As a result, the paths leading to the decisions are not recorded and become obscured, leaving nothing for later generation decision makers to refer back to or build upon.

The Analytic Hierarchy Process provides a simple and elegant solution to address the above and other decision-making challenges because of its structure and analytical approach.

Although many commercial software packages are available for AHP (Anon., 2015a, Decision Lens Anon., 2015b, MakeItRational Anon., 2015c), a thorough understanding of the AHP and a simple, basic methodology are better than the indiscriminate use of sophisticated software. Since only rudimentary mathematical knowledge is required to understand and implement the process there is no need for anything more than spreadsheet software such as Excel ©, from Microsoft ™.

#### **1.4 Aim of this Research Report and Problem Definition**

In order to reduce requirements for specialist analyst consultants to facilitate, complicate and, sometimes, obfuscate decisions, this report attempts to provide a methodology that will be easy to implement and facilitate the active participation of stakeholders to make good decisions in the work place collectively.

#### **1.5 Key Deliverables**

This research proposes to:

- Show that AHP can be applied effectively to decision-making in the mining industry; and
- Provide guidelines to facilitate the use of AHP to make it more accessible to decision makers in the mining industry.

#### **1.6 Research Objective**

The objective of the research is to present a user-friendly methodology for the application of the Analytic Hierarchy Process to decision-making in mining.

#### **1.7 Organisation of the Research Report**

**Chapter 1** serves as the report introduction.

The literature survey in **Chapter 2** provides background to the subject of AHP. It reviews the significance of AHP as a MCDA tool in general as well as in the mining sector. It briefly touches on the philosophy and processes of decision-making and

inconsistency of subjective judgments in a group context. Practical generic examples, as well as some pertaining specifically to mining related decisions, are briefly examined.

In **Chapter 3**, the fundamental principles of the AHP are comprehensively described. Those mathematical principles that are required to understand the process are discussed superficially but in adequate detail.

**Chapter 4** deals with the application of Microsoft Excel, which is introduced in the light of illustrative examples, to take care of the mathematics involved with the AHP. This chapter also further expands on the concepts discussed in Chapter 3.

In **Chapter 5** a few relevant case studies and examples of the application of AHP and their results are discussed.

The intention of **Chapter 6** is to provide a quick reference guide to users. The process is summarised step wise in a table together with relevant sections and pages as cross references.

**Chapter 7** deals with conclusions drawn during the study and of this research report.

## Chapter 2 Literature Review

In Chapter 1 MCDA in general and the AHP in particular were introduced and this research work was motivated. Key deliverables were spelt out and the chapter ended with an explanation of the organisation and structure of the report.

Chapter 2 elaborates on MCDA in mining engineering and introduces the AHP, as one of the more widely used MCDA tools, with a focus on its global significance and its potential for application in the mining industry. The concepts of decision-making, subjective judgment and inconsistency of judgments are introduced and some examples of successful application of the AHP, with emphasis on mining engineering and risk management, are cited.

### 2.1 The Application of MCDA in Mining Engineering

Musingwini, (2010a) discussed the complexities that mining engineers are faced with when confronted with decisions. He considered the potential for the application of MCDA to decision-making in the minerals industry and this view is also expressed by other authors on the subject (Australian Mining, n.d.; Vieira, 2003; Yavuz, 2015; Ataei et al., 2008; Jang and Topal, 2014a).

According to Musingwini, (2010a), MCDA techniques are appropriate tools for analysing complex decision-making problems in the mineral industry because they have two unique desirable features that match the level of complexity of the problems to be solved:

- firstly, the techniques can be applied to solve problems that have both quantitative and qualitative data; and
- secondly, the structure of MCDA techniques provides a decision-making platform that promotes participation and collaboration among stakeholders from different disciplines with different objectives and with different expectations (Kiker et al., 2005).

According to Musingwini, (2010 a), the main areas of application may include:

- mine planning;
- equipment or technology selection;
- location of a mining facility and
- strategy formulation.

Musingwini, (2010a) concluded that “...most decision-making in mine planning and equipment selection are multi-criteria in nature and can be solved using MCDA techniques. The successful application of MCDA techniques has helped to eliminate ‘gut-feel’ and empirical decisions from being made due to limited or uncertain information by laying out a systematic, logical and transparent decision-making process that is defensible and repeatable by other decision-makers. This is of paramount importance in today’s modern world characterised by strong demands for good corporate governance. There is now a gradual recognition of and application of MCDA techniques in the minerals industry, particularly the AHP technique. ”

Whether the process eliminates ‘gut-feel’ is, however, debatable. In fact, one of the strongest arguments for MCDA is that it allows for the quantification of subjective judgements derived from experience.

## **2.2 Introduction to AHP**

Thomas Saaty, who holds a Ph.D in mathematics from Yale University, developed the AHP, which is one of the more widely used MCDA tools. Saaty’s work at the Arms Control and Disarmament Agency in Washington and world events that took place during his tenure as Professor at the Wharton School of the University of Pennsylvania from 1969-1979, served as inspiration for the development of the AHP (Saaty and Saaty, 2003). The AHP technique has gained wide spread application by the international science and engineering communities as a robust and flexible MCDA tool for dealing with complex decision problems.



### 2.2.1 Critique of the AHP

Some criticism has been levelled against aspects of AHP. Rank reversal (a phenomenon by which the relative priorities of items can change if new items are added or some removed) is the most notable of these. Another criticism is that the number of required matrices can quickly become many and this makes the process prohibitive for large problems. These objections are essentially only of academic interest as the final usefulness of the analysis is still the responsibility of the decision makers. The AHP, like any other modelling tool, facilitates decision-making by humans; it does not make decisions for them. Forman, (1993) objectively addressed some of these criticisms.

### 2.2.2 Qualitative and Quantitative Criteria

The main advantage of AHP is its ability to deal with complex and ill-structured problems, which cannot usually be handled through rigorous mathematical models. AHP's power and popularity as a decision-making tool stems from its simplicity, ease of use, flexibility and intuitive appeal. In addition, it has the ability to mix qualitative and quantitative criteria in the same decision framework. Ataei et al., (2008) stated that *"the AHP is a tool that can be used for analyzing different kinds of social, political, economic and technological problems, and it uses both qualitative and quantitative variables"*. Shen, Muduli and Barve, (n.d. ) cited, as one of the advantages of AHP, *"...its ability to quantify both the experts' objective and subjective judgments in order to make a trade-off and to determine priority weights"*. Benítez, et al. (2011) stated that a system like the AHP is essential for *"... when tangible and intangible factors need to be considered within the same pool"*. Subramanian and Ramanathan, (2012) found through their research that *"... a significant number of AHP applications are found when problems require considerations of both quantitative and qualitative factors"* and Franek and Kresta, (2014) asserted that *"One of the most prominent features of AHP methodology is to evaluate quantitative as well as qualitative criteria and alternatives on the same preference scale"*.

The unique scale that the AHP uses and the inherent pairwise comparison, facilitate the direct comparison of concrete and subjective criteria. This allows for the judgement of their comparative weight with respect to the objective under consideration.

### 2.2.3 Consistency of Judgements

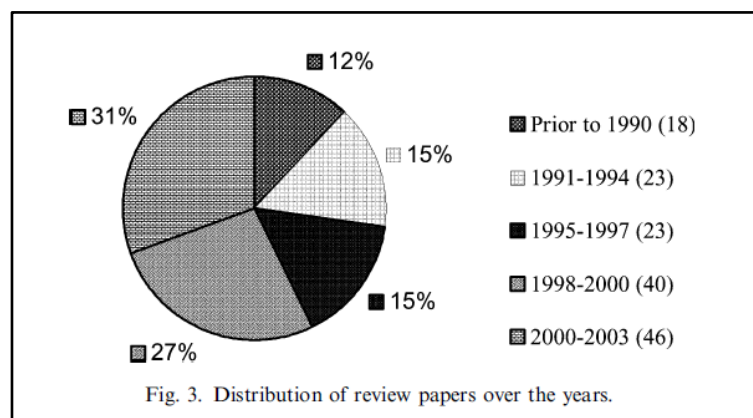
A feature of the AHP that distinguishes it from other techniques is its ability to test the consistency of judgement of participants throughout the process and the opportunity to reconsider judgements until acceptable consistency is achieved (Alonso and Lamata, 2006; Benítez et al., 2011). This feature provides for considerable confidence in the judgements, but also highlights inconsistencies, which, through discussion and debate, leads to better understanding of the decision problem at hand.

## 2.3 The Significance of the Analytic Hierarchy Process (AHP) in Industry

The AHP was first applied to planning with the Sudan Transport Study, followed by another application to one of the largest beer industries in Mexico. Since that time, the process has been used widely in several countries to set priorities, carry out cost-benefit analysis and allocate resources. According to Subramanian and Ramanathan, (2012), there were more than 1300 papers and 100 doctoral dissertations on the applications of AHP at the time of publishing their paper.

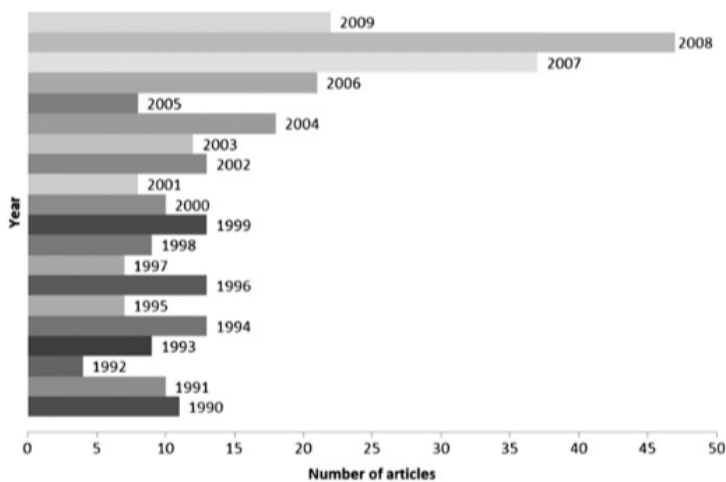
### 2.3.1 Growth in the Application of the AHP

Vaidya and Kumar, (2006), illustrated the growth of the use of AHP in their chart replicated in Figure 2-1. They conclude that the AHP is a flexible and popular multi-criteria decision-making tool.



**Figure 2-1: Distribution of Review Papers over the Years, after Vaidya and Kumar, (2006)**

Subramanian and Ramanathan, (2012), reviewed the applications of AHP in operations management over the period from 1990 to 2009 and their work highlights the growth of AHP and its increasing significance in all industries. They illustrate this by their graph replicated in Figure 2-2 which shows the number of articles on AHP per year. One of their conclusions is that there are important gaps in the application of AHP and that there is potential for extending its use to many more sectors. One such sector is the mining industry.



**Fig. 2.** Distribution of articles in various years.

**Figure 2-2: Distribution of Articles in Various Years, after Subramanian and Ramanathan, (2012)**

In a similar study, covering 2005 to 2009, Sipahi and Timor, (2010) selected 232 from 600 papers published in academic journals for their review of literature pertaining to the AHP. They reported that the use of the AHP technique has continued to increase exponentially over the period of their study.

### 2.3.2 AHP in Mining Engineering

Several AHP analyses have been undertaken in the field of mining engineering. For example, Sivakumar, Kannan and Murugesan, (n. d.), used AHP to select vendors for outsourcing mining services. Owusu-Mensah and Musingwini, (2011), use AHP to select an optimal ore transport system for Obuasi mine in Ghana. Badri et al., (2013),

applied it to risk management for underground mining in Quebec. Ataei, Shahsavany and Mikaeil, (2013), combined AHP with Monte Carlo analyses for mining method selection, and Yavuz, (2015), applied AHP to mining equipment selection.

Kluge and Malan, (2011) described the AHP as a recommended decision-making tool in the mining industry. They suggested that AHP is particularly valuable when complex problems are analysed by teams, especially for projects that involve human perceptions and judgments and have long-term repercussions and high stakes, attributes that typify large mining engineering projects.

Musingwini and Minnitt, (2008) suggested several potential applications of the AHP in the minerals industry including examples such as:

- performance evaluation of line managers for promotion;
- performance evaluation of operating shafts;
- ranking of projects competing for funding;
- measuring company performance on mining score card in meeting the requirements of the Mining Charter;
- comparison of different ore haulage systems and
- The evaluation of different support systems, (Yavuz et al., 2008).

It is clear that AHP application is gaining momentum and there is increasing opportunity for AHP to find widespread application within mining engineering.

## 2.4 Decision-making

It seems that people intuitively know what a decision is, however an accurate definition is quite elusive. A quick visit to the internet through Google illustrates the truth of this statement. From the somewhat less helpful “ *the act or process of deciding*” (“Decision | Define Decision at Dictionary.com,” n.d.) to more obscure definitions such as: “...bring to a resolution in the mind as a result of consideration” (“decision - definition of decision by The Free Dictionary,” n.d.) and a seemingly sensible view, at first sight, such as

*“...the thought process of selecting a logical choice from the available options”* (“What is decision making? definition and meaning,” n.d.). None of these definitions accurately and concisely defines a decision independently of equally elusive concepts such as “resolution” and “logical”.

In an article on the website *Virtual Salt*, Harris, (2012), defined decision-making as “... *identifying and choosing alternatives based on the values and preferences of the decision maker*”(also cited by Fülöp, 2005). Harris suggested a second definition for decision-making: “*Decision-making is the process of sufficiently reducing uncertainty and doubt about alternatives to allow a reasonable choice to be made from among them*” and argued that uncertainty is reduced rather than eliminated.

This implies that decision-making involves alternatives with attributes that require judgement by the decision maker against some values. Judgement, by its very nature, is not easily directly quantifiable. According to Kahneman, (2011): “...*humans are incorrigibly inconsistent in making summary judgements of complex information. When asked to evaluate the same information twice, they frequently give different answers.*” Judgment is also subject to numerous biases, as comprehensively described by Kahneman, (2011) and also discussed by Vick, (2002) as heuristics and biases. This inevitably leads to different and sometimes conflicting judgements in a group context. It is clear that a facility to check consistency and correct judgements is indispensable.

Saaty, (2012), on the other hand, argued that whereas decision-making was once thought to be an art, it has now become a science. He stated that the most significant test of a scientific theory is its success in predicting outcomes correctly. To do this well, decisions can be decomposed into separate structures involving benefits, costs, opportunities and risks and then, by combining the separate outcomes, the best decision can be reached. The AHP provides a framework within which judgements are scientifically tempered.

## **2.5 The Anatomy of a Decision**

For the purposes of this report, a decision is defined as a selection, from a set of alternatives, based on the extent to which their attributes satisfy the criteria that are

demanded by the objective and scope. Assuming that this is a reasonable postulate, one can deduce that the process of making a decision involves at least three components:

- An objective together with an appropriate scope;
- A number of alternatives, the attributes of which have the potential to satisfy the conditions required to achieve the objective; and
- A set of criteria against which to measure those attributes.

### 2.5.1 Objective and Scope

Formulating the objective and scope of any study deserves considerable attention since this is the singular question for which the study must provide answers. Appropriate allocation of time and resources is required for the objective and scope to be defined accurately and precisely. Ill-defined objectives and scopes are very often at the heart of the wasteful application of expensive resources and effort to find precise answers to the wrong questions. It is often quoted that “it is better to be roughly right than precisely wrong”(“A quote by John Maynard Keynes”, n.d.).

The objective and scope have to be formulated through consensus by all interested parties, including the ultimate decision makers (the client to be served by the process), members of the management team, subject matter experts and other role players (“Subject Matter Experts” are those who know much about the subject and they could include university professors and machine operators alike). Saaty, (2013a) said that: *“The object of planning is not to produce plans for others to use but to engage the users in their formulation and application. Effective planning cannot be done for individuals or organizations; it must be done by them”*. This sentiment is echoed by Ilbury and Sunter, (2011):*“The secret to successful strategy for an uncertain future out of our control is not to leave it in the hands of a single person. Not even a small group for that matter. It is rather, on a broad basis, to involve those who are expected to implement that strategy.*

The objective must be specific and concise, for example: “find the most suitable excavation method for a tunnel” or “identify the most suitable mining method for a given orebody”. The scope, on the other hand, should contain as much information as possible to define the context within which the decision is to be made. This may contain geographical, geological or geotechnical limitations, resource availability and many other parameters.

To elucidate these concepts, the following example is presented:

To reach a certain travel destination (the objective), one could walk, or run or swim or one could get there by car, train, boat or plane. It is immediately clear that the attributes of these modes of transport are not equal. It is also evident that more information is required regarding the destination and the point of departure, such as the distance that separate them, whether the destination is land or water locked and whether there are time constraints. This information provides the scope of the objective. As Ilbury and Sunter, (2011), put it, “*Context is where the whole game begins*”.

Within this context, some alternatives will be more attractive than others. A few will be entirely inappropriate and will require no further consideration.

If the point of departure and the destination in the above example are 100km or so apart then one can immediately eliminate the flight and boat alternatives, unless the destination is a small island that can only be reached by boat, or swimming (possibilities). The thought of covering 100km by foot or swimming, while possible, does not sound feasible (potential), unless there are no time constraints and ample rest points are available (possibilities). If one is required to reach the destination quickly and there are roads one could go by car, unless the roads are in such poor condition that off road vehicles are more suitable. If there are different routes with better roads, then going by car might still be quicker, even if the route is longer, and one gets the comfort of a car as a bonus.

The above example serves to illustrate how complicated a simple decision can become and how easy it is to become entrapped in complexity if the objective and scope are not

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well defined. Of course, it is a trivial example and, with the correct information and one's experience, one can reach a decision almost immediately. If, on the other hand the contributions towards the desired outcomes are complex, the attributes of the alternatives are varied and several stakeholders with varied backgrounds, experiences and expectations are involved, effective decisions require a structured framework.

According to Saaty, (2012), *“What we need is not a more complicated way of thinking, since it is difficult enough to do simple thinking. Rather, we need to view our problems in an organized but complex framework that allows for interaction and interdependence among factors and still enables us to think about them in a simple way. This new way of thinking should be accessible to all without straining our innate capabilities.”*

### 2.5.2 Alternatives or Options

The options or alternatives have to satisfy the criteria in order to achieve the objective. Alternatives may be different types of heavy equipment, shaft locations, risk mitigation measures, conflict resolution possibilities and so forth.

In the example of Section 2.5.1, several alternative modes of transport were identified. The major categories, land, water or air, could have several sub-alternatives and elimination from consideration for any one of these, automatically eliminates the sub-alternatives. To illustrate, if the destination were land locked, then any mode of transport that depends on water is eliminated and requires no further consideration, however, many possibilities are still available by land or air.

### 2.5.3 Criteria

The extent to which the alternatives meet the objectives is measured against the requirements defined in the scope. These requirements are the criteria. A criterion may be defined as a standard, rule, or test to which a judgment of compliance can be based. As a measurement, it requires a scale against which to determine the degree of compliance.

If it is a requirement that the 100km be reached within an hour, then the appropriate alternative will be neither hiking nor biking. If, on the other hand, it is more important for

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the passengers to be relatively fresh and relaxed, then a longer route in a luxury car might be more appropriate. It should be noted that a significant concept in decision-making is introduced here: the one of relative importance of criteria. In many decision-making tools, this concept is ignored. Effective decisions, however, not only depend on the criteria, but also on their relative importance, or weight, with respect to the objective.

#### 2.5.4 Group Decision

Several authors agree that group decision-making is beneficial:

- Ilbury and Sunter, (2011), in the context of business decisions, are of the opinion that business organisations need to draw more and more on the expertise of those within the organisation. They stated that “...*all planning must be intensely participative or it fails*”;
- Saaty, (2012), said that brainstorming and sharing ideas and insights often lead to a more complete representation and understanding of the issues under discussion; and
- Vick, (2002), summed it up rather eloquently as: “ *For a decision to remain a decision it cannot be instructed. Instead it must be informed...*”

In a seemingly contradictory perspective, Kahneman, (2011), who received the Nobel Prize in 2002 for his work on judgement and decision-making, believes that the standard practice of open discussion gives too much weight to the opinions of those who speak early and assertively, causing others to line up behind them. He observes that when many judgements are averaged, the average tends to be quite accurate but, because of the ease with which judgements are influenced, this is only true when the judgements are independent and the errors of the judges uncorrelated. He asserts that in order to derive the most useful information from multiple sources of evidence, the sources should be made independent. The AHP facilitates discussion and debate, which tends to neutralise, to an extent, the concerns raised by Kahneman.

It is clear that group decision-making is not as straightforward as getting a group of people together and getting a good decision out of it. Preparation and structure seem to be essential elements of the process.

Decision makers, nevertheless, wish to be coherent when making decisions, but when faced with a relatively large number of alternatives (six or more is considered large in this context), judgement becomes less consistent. The principle of transitivity, for example, supposes that if A is preferred to B, and B to C, then A should be preferred to C (Saaty, 2008; Saaty, 1999). If, on the other hand, A is judged better than E, but not as good as C, but C is better than D although, A is not quite as good as D etc., then coherency becomes a problem.

According to Saaty, (2008), two important issues in group decision-making are:

1. how to aggregate judgements by individuals in a group into a single, representative judgement for the entire group; and
2. how to construct a group choice from individual choices.

In practical terms, the AHP helps individuals and groups to achieve reasonably coherent preferences within the frame of the problem at hand. Once coherent preferences are established, decisions can be taken with more confidence (Dodgson et al., 2009).

### 2.5.5 Consistency

When confronted with choices, experts rely on the information at hand, together with knowledge and experience accumulated over a number of years, to reach a decision. Subjectivity is inevitable, perhaps indispensable. Vick, (2002), for instance, studied engineering judgment and he strongly advocated for subjective probability in design to complement theory and analysis in the face of uncertainty. Barford, (2007), also gave a number of reasons for inconsistency in judgments. The phenomenon of subjective judgement is as amorphous and elusive as human behaviour in general and it would seem that “consistent subjective judgment” is a contradicting term. In a group context, even if the individuals share the same body of knowledge about the subject matter, their

experiences will certainly differ and consistent judgement among them is very unlikely. Decision makers should be aware of this inherent tendency towards inconsistent judgements and its effect on decisions. A decision-making tool with the ability to measure this inconsistency of judgments and allow judges to reconsider their evaluations collectively, will help to overcome many of the biases that afflict subjective judgement.

The capability of the AHP to measure consistency distinguishes it from other techniques. This is indeed what makes it an attractive method ,Saaty and Kearns, 2013a; (Saaty, 2012); Alonso and Lamata, 2006; Benítez et al.,2011; Vargas, 2013; Franek and Kresta, 2014).

### 2.5.6 Forced Consistency

Saaty, (2012) recommended the iteration of measurement and reconsideration of judgements to derive a satisfactory level of consistency. He also warned against the temptation of mechanically forcing consistency. If consistency in judgment cannot be achieved through discussion and debate, then the decision should be postponed until better information becomes available.

Similarly, Vick, (2002), questioned the merits of theory and analysis to produce consistent results at the expense of subjective human judgment based on experience and expertise. He asserted: *“But consistency is indifferent to truth. One can be entirely consistent and still be entirely wrong...”* .

## 2.6 Practical Applications of AHP

### 2.6.1 Literature Overview

The application of the AHP has been as varied as its use in many disciplines. Vaidya and Kumar, (2006), discussed this varied and diverse nature of applications of AHP and according to them it is one of the most widely used MCDA tools. They briefly described the process and critically reviewed 27 papers selected out of 150 reference papers. Their reviews were classified into several categories including selection, evaluation and

allocation. They observed that AHP is predominantly used in engineering disciplines, for personal purposes and in social sciences.

In their overview of applications of the AHP, Sipahi and Timor, (2010), found that the use of the AHP increased exponentially over the period of their research (2005 to 2009). They found that the AHP has received wide acceptance in manufacturing, where it has been applied to supplier selection, supply chain evaluation, location selection, system selection and strategy evaluation. The AHP has also been used extensively in environmental management, agriculture and power and energy management. Other industries in which AHP is receiving increasing attention are transportation, construction, healthcare, education, logistics, e-business, IT, research and development, telecommunications, finance and banking, urban management, defence and military, government, marketing, tourism and leisure, archaeology, auditing, and mining.

Subramanian and Ramanathan, (2012), presented a comprehensive listing of AHP applications in Operations Management from 291 peer reviewed journal articles published over the period 1990 to 2009. They categorised the applications of the AHP into five broad categories:

- Operations strategy;
- Process and production design;
- Planning and scheduling resources;
- Project management and
- Managing the supply chain.

These categories show a correspondence to large mining engineering project components.

## 2.6.2 Mining

Some applications specific to mining include the selection of a plant location (Ataei, 2005), green supply chain management (Shen et al., n.d.), and green vendor evaluation

and selection (Sivakumar et al., n.d.). Selected applications of AHP specific to mining and in the field of risk assessment were reviewed in this report in Sections 2.6.2.1 to 2.6.2.8 below.

### 2.6.2.1 Evaluation of Mining Method Efficiency

Musingwini and Minnitt, (2008), used the AHP to evaluate the efficiency of different mining methods employed in South African platinum mines. They chose the AHP as a ranking method for the following reasons:

1. When compared with other MCDA techniques, the AHP can detect inconsistent judgements and provide an estimate of the degree of inconsistency in the judgements;
2. The AHP is supported by easy-to-use commercially available software packages; and
3. The AHP can rank alternatives in the order of their effectiveness when conflicting objectives or criteria have to be met.

### 2.6.2.2 Mining Method Selection

Ataei et al.,(2008) used AHP to select a suitable mining method from among six for a bauxite mine in Iran. The paper gives a systematic account of the AHP and arrives at an optimal mining method. The authors concluded that: *“Unlike the traditional approach to mining method selection, AHP makes it possible to select the best method in a more scientific manner that preserves integrity and objectivity. The model is transparent and easy to comprehend and apply by the decision maker. For selecting a mining method, the AHP model is unique in its identification of multiple attributes, minimal data requirement, and minimal time consumption.”*

Ataei also co-authored with Naghadehi et al., (2009), on the application of fuzzy AHP to mine method selection at the same mine and later combined AHP and the Monte Carlo simulation method to mining method selection to estimate the degree of importance of criteria as a means of enhancing human judgements (Ataei et al., 2013).

Alpay and Yavuz, (2009), discussed the development of a computer program for underground mining method selection based on the AHP, to select an underground mining method based on spatial, geologic and hydrologic, economic and environmental considerations. Each of these main categories was further broken down into a number of sub-categories and the mining alternatives included sub level stoping, sub level caving, long wall mining and room and pillar mining. Yavuz also published an article on mining equipment selection (Yavuz, 2015).

Karimnia and Bagloo, (2015), used AHP to select the appropriate mining method for a salt mine in Iran. A number of geo-mechanical considerations are incorporated among the 10 criteria used to select the most appropriate mining method from among four alternatives.

#### 2.6.2.3 Layout Selection

Abdalla et al., (2013), applied AHP to the problem of longwall layout selection. They gave a systematic account of the process through which it was found that *in situ* stress was the most important criterion. The initial inconsistency was found to be unacceptable and it was required to review the subjective judgements. Ultimately, the layout found to be most desirable was shown to be the one most favourably oriented with respect to *in situ* stress. The authors concluded that AHP could be successfully applied to the appropriate selection of longwall layouts. They made the point that new information or critical factors can be readily incorporated into the AHP model and the AHP requires less data and time to reach a decision when compared with traditional techniques of selection.

#### 2.6.2.4 Support Design Selection

Yavuz et al., (2008), used AHP to select the optimum support design for the main haulage road in Western Lignite Corporation (WLC) Tuncbilek colliery. They used displacement estimates from numerical models as one of the selection criteria. The other selection criteria were the factor of safety of the support, cost, labour and applicability of support. Among 19 alternatives, one support system was found to be

most appropriate thus illustrating that the AHP can be successfully applied to support system selection.

#### 2.6.2.5 Risk Management Frameworks

Dey, (2010), described the development of a risk management framework in which the AHP is combined with a risk map to manage risk on both project and implementation levels. He applied the methodology to a 1500km oil pipeline in India to demonstrate its effectiveness. The author concluded that the collaborative approach of the AHP facilitated group decisions and this was very effective for the management of risk across project, work package and activity levels. He found that buy-in from all the stakeholders fostered team spirit and that the AHP was instrumental in reaching consensus over controversial issues. The author suggested that the framework can be applied to any complex project to help manage risk throughout the project life cycle.

Aminbakhsh et al., (2013), suggested a framework for risk management in the construction industry and Badri et al., (2013), suggested such a framework for underground mining. Both articles describe the use of existing risk analysis techniques and the application of the AHP to evaluate the relative weights of various hazards.

#### 2.6.2.6 Preparedness Activities

Manca and Brambilla, (2011), developed a methodology based on the AHP to evaluate the effectiveness of preparedness activities and emergency response in case of accidents in road tunnels. They chose to use AHP because it allows for quantitative comparisons of variables that otherwise lack parity. The methodology was applied in a case study on a transnational road tunnel between Italy and France. They found that the main aspects to consider for accidents in road-tunnels were contextual, physical and organisational attributes and that the physical and organisational attributes were of similar importance while the contextual attributes were less significant. In the case study, they found that it was possible to identify the most important factors that affect the emergency response system as well as the most efficient enhancements to the system performance. One of the conclusions was that the methodology (which is

essentially the AHP) is useful for both sensitivity analyses as well as for monitoring the risk management dynamics.

#### 2.6.2.7 Road Construction Projects

Zayed et al., (2008), proposed a risk index to evaluate the level of risk for road construction projects. They based their work on four road construction projects in China. They distinguished between company and project risk categories. In the company risk category, they identified financial, political, cultural and market related risks while the project risk category included technology, resources, design, contractual and legal issues. Their risk index was the product of the weight of each risk factor as determined by the AHP and risk effect factors. The risk effect factors were subjective judgements on performance of projects with respect to the risk factors. The judges were industry experts in top management positions of the four projects that were evaluated. The authors suggest that their risk index can facilitate decisions on projects to pursue.

#### 2.6.2.8 Tunnel Boring Machines

Hyun et al., (2015), carried out a risk analysis associated with tunnel boring machines. They applied a fault tree analysis to estimate the probability of occurrence of risk factors and AHP to evaluate the impact of these risk factors. The product of these two was then assigned risk levels according to a standard risk matrix.



## 2.7 Summary of Chapter 2

The literature reviewed in this chapter served to illustrate the present status on the use of MCDA in mining engineering. AHP was briefly introduced as one of the more widely used MCDA tools, with a focus on its global significance and its potential for application in the mining industry. The concepts of decision-making, subjective judgment and inconsistency of judgments were introduced and some examples of successful application of the AHP, with emphasis on mining engineering and risk management, were cited.

Concepts that were discussed in this chapter include the following:

- MCDA in mining engineering
- AHP
  - Global significance
  - Potential for application in the mining industry
- Concepts
  - Decision-making
  - Subjective judgment
  - Inconsistency
- Examples of successful application of the AHP

The fundamental principles of AHP are explained in Chapter 3 where AHP is defined and the background of its development and application is discussed, as are its components and processes. The mathematical basis for the process, which gives it the scientific edge, is explored. Chapter 3 serves as the foundation for the objective of this research report, namely to provide a user-friendly methodology for the application of the AHP in mining engineering.

## Chapter 3 Understanding the fundamentals of AHP

The literature review in Chapter 2 provided insight into the present status on the use of MCDA in mining engineering and briefly introduced the AHP as one of the more widely used MCDA tools, with a focus on its global significance and its potential for application in the mining industry. The concepts of decision-making, subjective judgment and inconsistency of judgments were introduced and some examples of successful application of the AHP, with emphasis on mining engineering and risk management, were cited.

In this chapter, the fundamental principles of AHP are explained. AHP is defined and the background of its development and application is discussed. The components and processes of the AHP are explained. The mathematical basis for the process, which gives it the scientific edge, is explored.

The chapter serves as the foundation for the objective of the research report. The objective is to provide a user-friendly methodology for the application of the AHP in mining engineering.

### 3.1 The Analytic Hierarchy Process

#### 3.1.1 Definition and Background

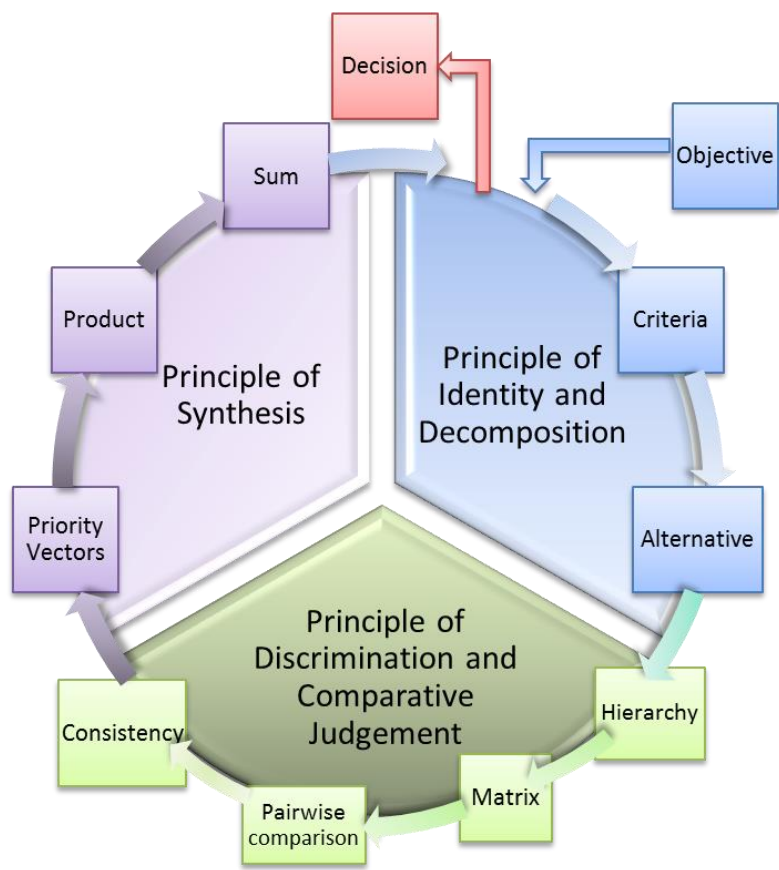
Saaty, (2013a), described the Analytic Hierarchy Process as follows: *“The Analytic Hierarchy Process is a systematic procedure for representing the elements of any problem, hierarchically. It organizes the basic rationality by breaking down a problem into its smaller and smaller constituent parts and then guides decision makers through a series of pairwise comparison judgments (which are documented and can be re-examined) to express the relative strength or intensity of impact of the elements in the hierarchy.”*

#### 3.1.2 Overview

The AHP consists of three principles according to (Saaty and Kearns, 2013a). The three principles are the principle of identity and decomposition, the principle of

discrimination and comparative judgement and the principle of synthesis. These principles and their associated processes are illustrated in Figure 3-1.

The process starts with the identification and briefing of the participants. Their first task is to define the objective and scope, identify key criteria and identify a list of alternatives. The construction of a hierarchy follows. The criteria are compared pairwise with respect to the objective to determine their relative importance, or priorities. The consistency of the judgements is tested. Each alternative is then ranked against the criterion.



**Figure 3-1: Wheel Diagram to illustrate the AHP**

It is instructive to compare this model to the strategic conversation model proposed by Ilbury and Sunter, (2011), in their approach to draw analogy between strategic planning and game playing (Figure 3-2). Their model consists of two phases, “Defining the game” and “Playing the game”, and each phase contains five elements. The parallels between this model and the AHP are drawn in Table 3-1 to illustrate the potential use of AHP in strategic planning. The parallels run much deeper than shown here but a more detailed comparison falls outside the scope of this research. What is important to note is that AHP has great potential to complement the conversation model if it is used in any or all of its steps.



**Figure 3-2: The Conversation Model developed by Ilbury and Sunter, (2011)**

**Table 3-1: Comparison of the AHP with the Strategic Conversation Model by Ilbury and Sunter, (2011)**

Phase	Conversation Model	AHP
Defining the game	Scope of the game	Define the objective and scope
	Players	Define the stakeholders
	Rules of the game	Construct the hierarchy, determine the criteria and identify the alternatives
	Key uncertainties	
	Scenarios	Pairwise comparison
SWOT		
Playing the game	Options	Ranked priorities and measured consistencies
	Decisions	
	Measurable outcomes	Documented, repeatable and presentable decision model
	Meaning of winning	

### 3.1.3 The AHP Steps

The major steps required for the AHP are outlined in Sections 3.1.3.1 to 3.1.3.7 below:

#### 3.1.3.1 Identify the Participants, their Objectives and their Policies

The participants are analogous to the players that Ilbury and Sunter, (2011) described in their Conversation Model. Some people will be involved in facilitating the AHP and others will be responsible for implementing the decisions. Some will be impacted by the decision and others will impose constraints.

It is intuitively known that the objectives of stakeholders regarding a decision are not necessarily aligned with the objectives of the decision makers. Ilbury and Sunter, (2011) stated that: *“Even more fascinating is that, on a personal level, an individual’s ‘purpose’ as measured by actual behaviour may differ from his or her expressed purpose”*. Moreover, both an individual’s actual purpose and his or her

expressed purpose can change according to changing circumstances and/or perceptions. Cognisance must be taken of the potential impact that the objectives and attitudes of the participants might have on the outcome of the decision-making process. It is here where the built-in function of the AHP to compute consistency of judgement becomes relevant.

Understanding the policies of the participants is important, whether or not they form part of the analysis. If it is a customer's policy, whether official or not, to accept incentives for awarding contracts, the decision makers must know so that they can decide whether this is in accordance with their own. In some environments, the policies of stakeholders could be critical to the outcome of the process.

### 3.1.3.2 Identify the Overall Objective

In determining the overall objective of the process, due consideration must also be given to the context around the required decision. In fact, the conversation model that Ilbury and Sunter, (2011), advocated does not even have a separate element to represent "objective" but the model starts with "scope".

### 3.1.3.3 Identify Criteria that must be met to achieve the Objective

The purpose of the AHP is to match alternative solutions to the overall objectives. This is achieved through the process of determining the relative priorities of criteria and ranking the alternatives against the criteria.

This activity of identifying and defining criteria must consider the contribution of as many participants as possible. Each person involved in the process brings a unique set of experience and perceptions. Dialogue and debate ensure that the broadest possible range of preferences and perspectives is captured.

The AHP deals very effectively with quantified criteria and can deal effectively with criteria that are intangible or unquantifiable. Efficient fund allocation, increased resources and more efficient deployment of personnel are examples of tangible criteria as they can be measured directly. Examples of intangible benefits are

economic security and company image. The AHP also facilitates the direct comparison of tangible and intangible factors.

#### 3.1.3.4 Identify Alternatives

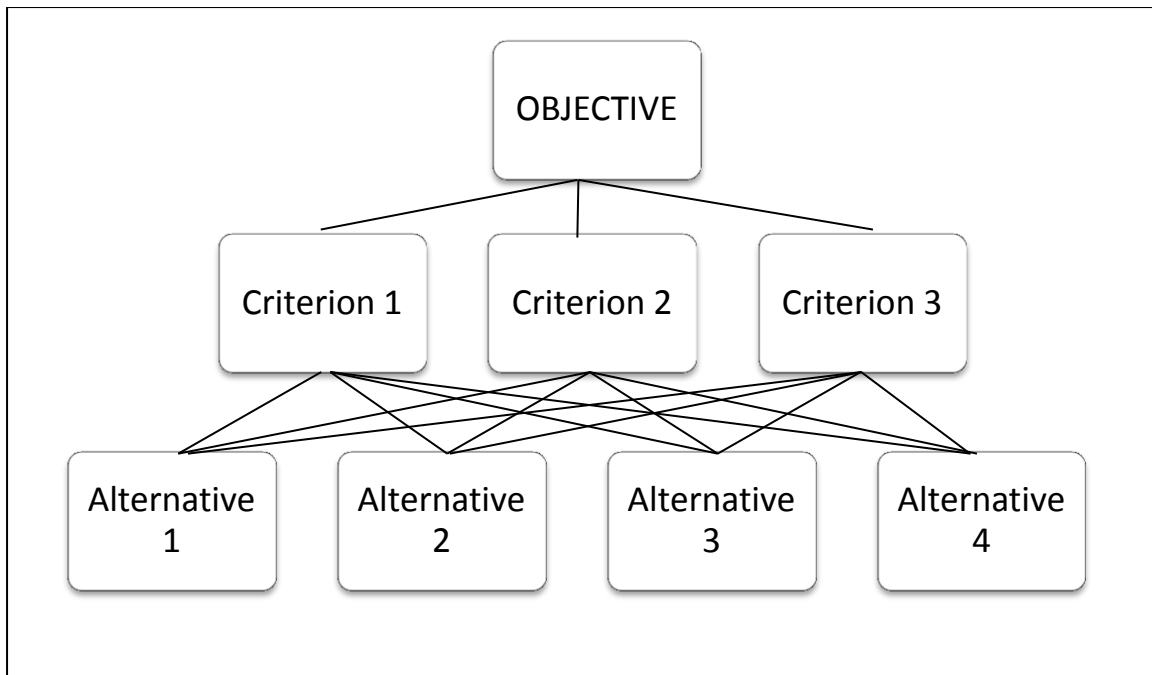
Normally, the alternatives are on the lowest level of the hierarchy. These items ultimately have to be matched with the overall objective. Examples include alternative products or services, alternative suppliers of products or services, alternative methods of production, alternative candidates for a position, outsourcing or insourcing, etc.

#### 3.1.3.5 Construct the Hierarchy

One of the first steps in the application of the AHP is to construct a hierarchy. It is customary to use the dominance hierarchy, which means that an item in the top most level dominates the items in the level below, which in turn are dominant over the subsequent lower levels. This type of hierarchy can also be viewed as a pyramid structure and is similar to organograms widely used to represent organisational structures.

The hierarchy is used to break down complex systems into their constituent parts. This is done in accordance with essential inter-relationships.

The top level of the hierarchy contains one element, the overall objective and focus of the activity. The next levels down consist of sets of criteria and sub-criteria, if any, and the lowest level contains the alternatives. These concepts are illustrated in Figure 3-3.



**Figure 3-3: A simple Hierarchy**

The number of levels that a hierarchy may comprise of is theoretically unlimited, however, in practice it should be kept to three or four to minimise the computation time. When the elements on a level cannot be compared readily, a new level can be created with sub-elements to refine the resolution. The number of elements on a level should ideally not exceed seven. This is based on the theory that the human brain can only deal with seven plus or minus two chunks of information in short term memory according to a study by Millar, (1955).

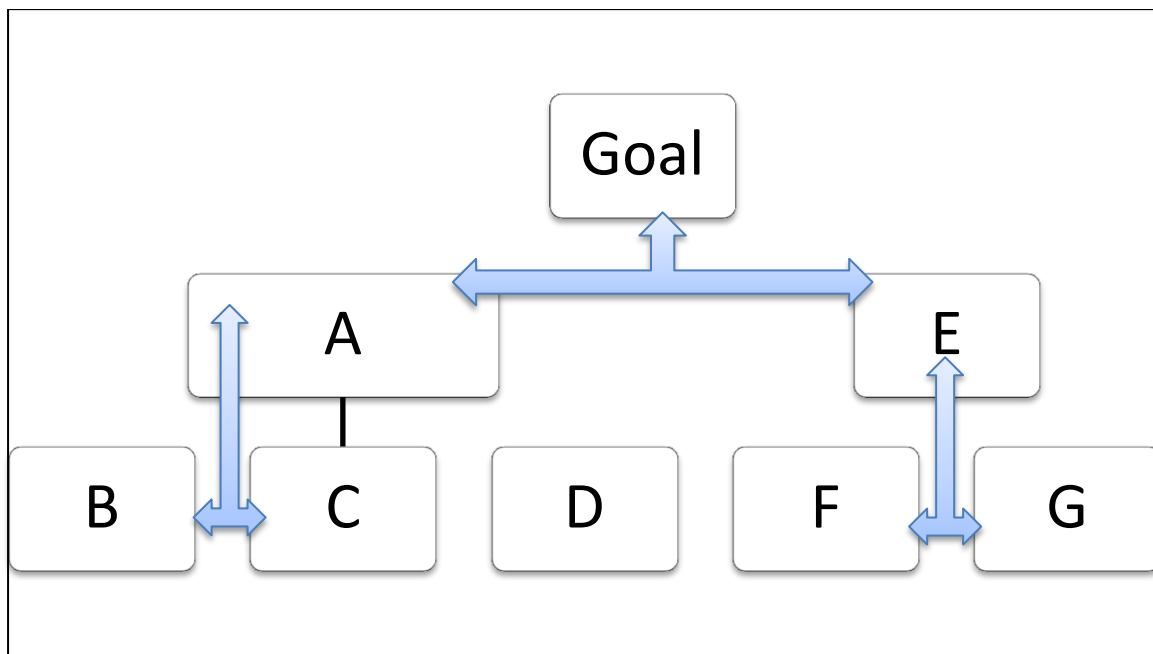
In AHP, the analysis always follows the hierarchy in a top down approach, working from the objective, down the hierarchy to the alternatives. Criteria will be compared with each other in respect of the objective; sub-criteria will be compared with each other in respect of the criteria in the level directly above, and so on, until on the lowest level, elements (alternatives) are compared with each other in respect of the elements (criteria) in the second lowest level. This top down approach is essential and its importance cannot be over-emphasised.

In this report, for the sake of brevity, the illustrative hierarchies are kept to three, or at the most, four levels, as the process is the same for each pair of levels.



### 3.1.3.6 Pairwise Comparison

One of the unique and attractive features of the AHP is the process of pairwise comparisons. On each level, one element is evaluated against one other element on the same level to judge the relative priorities of those elements in terms of a common element on the level above. With reference to Figure 3-4, A and E are the main criteria and are evaluated with respect to the objective. The elements B, C and D are on a lower level and A is the common element in the level above them. Here B will be evaluated against C to judge its relative contribution to A. B will also be evaluated against D and finally C is evaluated against D. The evaluations are plotted in a pairwise comparison matrix (Table 3-2).



**Figure 3-4: Example of a Hierarchy to illustrate Pairwise Comparison**

A similar matrix will be constructed for item E (in respect of which items F and G will be evaluated) and any other elements on that same level. A separate comparison matrix can also be constructed for each or some of the elements B, C, D, F and G depending on the presence of elements in levels below them. The meaning and derivation of weight entries in the matrix in Table 3-2 are explained in subsequent Sections.

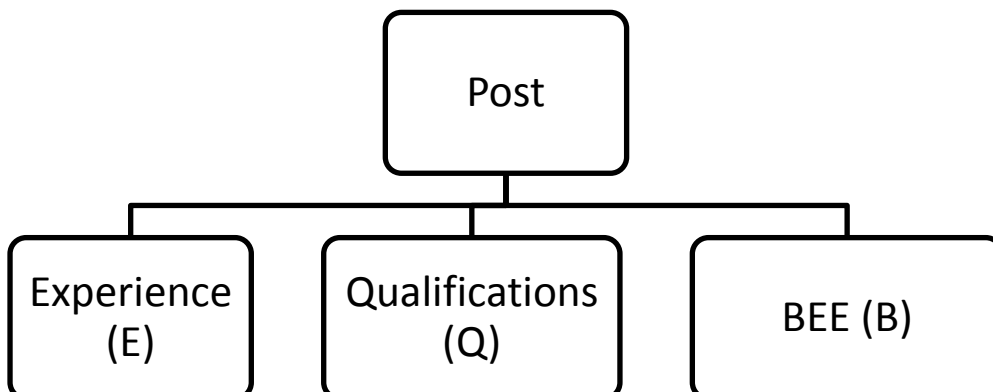
**Table 3-2: Pairwise Comparison Matrix**

A	B	C	D
B	1	9	3
C	1/9	1	1/7
D	1/3	7	1

**3.1.3.6.1 Constructing the Matrix**

The top row and the first column of the matrix contain the names of the elements. This way a square matrix is necessarily formed. The top left corner of the matrix can be used for the name of the element in the upper level in respect of which the elements in the matrix are compared. The reader should not be concerned with the meaning of the numbers in Table 3-2 at this stage as it is all fully explained in this section.

One comparison matrix is constructed for every element that is linked to two or more elements on the next lower level. For example, assume that a human resources manager wishes to select the best candidate for a given post. The selection will be based on the qualifications, experience and Black Economic Empowerment (BEE) requirements. The elements can then be arranged in a hierarchy as shown in Figure 3-5.



**Figure 3-5: Hierarchy to score for the Post**

Now the comparison matrix may be constructed as indicated in Table 3-3. By definition, the matrix must always be square; that is the rows and columns are equal in number or the matrix is of order  $n \times n$ .

**Table 3-3: Prepared Comparison Matrix to prioritise the Selection Criteria for the Post**

Post	E	Q	B
E			
Q			
B			

When comparing the elements, the row (horizontal) is compared with the column (vertical) and the weight is entered at the junction of the two. When the two elements are equal in weight with respect to the element in the upper level, then in terms of the scaling system used for AHP (described next), the value of the weight is one. A direct result of this is that the diagonals of the matrices always contain 1's, because when an element is compared to itself, the weight is equal. The comparison matrix now looks like that in Table 3-4.

**Table 3-4: Completing the Diagonal of the Comparison Matrix**

Post	E	Q	B
E	1		
Q		1	
B			1

### 3.1.3.6.2 *The Comparison Scale*

In the pairwise comparison process, the element in each **row** is compared with the elements in each **column**, one by one, in respect of a common element in the next higher level (entered in the top left corner of the matrix). Two questions require answers:

- Is the importance of the element in the row greater or less than that of the element in the column? (Does it carry more or less weight or does it matter more or less?); and
- How much more or less important is it? (By how much does it matter more or less?).

For the second question, a value is assigned according to the Fundamental Scale of the AHP for pairwise comparisons given by Saaty, (2012), as presented in Table 3-5 and universally applied by practitioners of the AHP. Note that the scale spans from one to nine in intervals of two. Intervals in between (that is two, four, six and eight), can be used if the quantification of the judgment requires finer resolution.

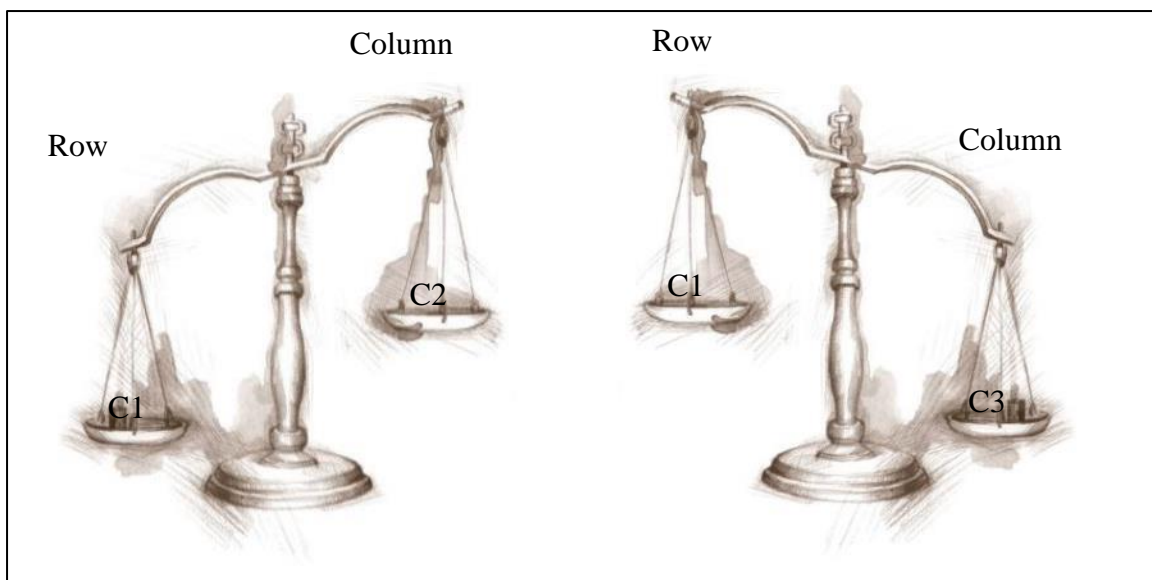
**Table 3-5: Fundamental Scale for Pairwise Comparison, (Saaty, 2012)**

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favour one activity over another
5	Strong Importance	Experience and judgment strongly favour one activity over another
7	Very strong or demonstrated importance	An activity is favoured very strongly over another; its dominance is demonstrated in practice
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation

The pairwise comparison process is quite simple, but leads to errors most often because it is very easy to confuse the logic and assign the incorrect value. It is worthwhile to grasp this concept thoroughly and it is recommended that convention be observed. It is useful to think of the process in terms of balance scales. The left

hand balance contains the row criterion while the right hand balance holds the column criterion. This concept is illustrated in Figure 3-6.

Here, the left hand picture depicts that criterion C1 weighs more than criterion C2, so in the comparison matrix, at the junction of row C1 and column C2, a whole number will be inserted. Considering the right hand picture, it is seen that criterion C1 in the row weighs less than criterion C3 in the column. In the comparison matrix, at the junction of this pair, the number will be a fraction because the criterion in the row weighs less than the criterion in the column. The relative value of the row compared to the column is entered in the matrix.



**Figure 3-6: Illustrating the Concept of Evaluating the Row against the Column with respect to the Element in the Upper Next Level.**

If C1 is judged to be 9 times as important as C2, then, logically, C2 is 1/9 times as important as C1. The implication of this is that pairs below the diagonal of the comparison matrix are the reciprocals of the corresponding pairs above the diagonal (refer to Table 3-6).

Returning to the example of selecting a candidate for a post (see 3.1.3.6.1 above), assume that the post dictates that experience (E) matters more than qualifications

(Q), but that BEE (B) matters most. The corresponding comparison matrix might then look like that in Table 3-6, with the following explanation based on Table 3-5:

- The entry in (E:Q) is 3, signifying that experience is slightly favoured over qualifications. It follows that qualifications matter slightly less than experience so the entry in (Q:E) is the reciprocal, 1/3;
- Experience is considered much less important than BEE requirements, so the entry at (E:B) is 1/7. The reciprocal condition is that BEE is favoured very strongly over experience and the score in (B:E) is 7; and
- BEE is absolutely more important than qualifications and the entry in (Q:B) is 1/9. The reciprocal is true when comparing BEE to qualifications and the value in (B:Q) is 9.

**Table 3-6: The completed Comparison Matrix for the Post**

Post	E	Q	B
E	1	3	1/7
Q	1/3	1	1/9
B	7	9	1

During the process of pairwise comparisons, inconsistencies often occur because of input errors and inconsistent judgement. The AHP provides for measuring such inconsistencies so that the judgements might be reconsidered or errors might be corrected. The issue of consistency is more comprehensively discussed later in Section 3.1.4.2.

Pairwise comparison is carried out for the criteria with respect to the overall objective once; thereafter, if there are sub-criteria, these are compared to one another in pairwise fashion with respect to each criterion and so on down to the lowest level of the hierarchy. Here the alternatives are compared to one another in pairwise fashion with respect to the (sub) criteria in the level above. For the sake of clarity and brevity, examples in this report are restricted to three-tiered hierarchies; the objective, the criteria and the alternatives.

Once the pairwise comparisons have been carried out, there will be a set of matrices. This will comprise one matrix for the criteria with respect to the overall objective (Table 3-6) and a number of matrices, one for each criterion in respect of which the alternatives are ranked. Assume that for the example above there are two candidates, C1 and C2. Three comparison matrices will then be constructed as shown in Table 3-7, Table 3-8 and, to evaluate the candidates with respect to the criteria.

**Table 3-7: The completed Comparison Matrix for Experience**

E	C1	C2
C1	1	5
C2	1/5	1

**Table 3-8: The completed Comparison Matrix for Qualifications**

Q	C1	C2
C1	1	1/7
C2	7	1

**Table 3-9: The completed Comparison Matrix for BEE**

B	C1	C2
C1	1	7
C2	1/7	1

### 3.1.3.7 Synthesising

The final step in AHP is to synthesise the results in order to obtain the overall ranking of the alternatives with respect to the goal; that is to what extent each alternative succeeds in satisfying the overall goal according to the criteria. At this stage of the process there ought to be one matrix for the criteria with respect to the overall objective and one matrix for each of the criteria in respect of which the

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alternatives have been evaluated. It should be apparent that the final evaluation would be a function that brings these matrices together. That function is called synthesising and involves summing the products of the priority vectors obtained for the matrices. These concepts are clarified in following sections of this report.

### 3.1.4 The Maths

It is not the intention of this report to provide an in-depth explanation of the mathematics that drive the AHP. This subject is comprehensively dealt with by Saaty, (2013b). A basic understanding is, however, necessary in order to appreciate how the ranking is derived and how consistency is measured.

The basic process is as follows:

- Construct the comparison matrix through pairwise comparison of the criteria;
- Determine the priority vector (eigenvector); and
- Calculate the consistency ratio.

#### 3.1.4.1 Priority Vector

The **priority vector** of the matrix provides the ranking of the compared elements. The priority vector is the same as the **eigenvector**.

Two methods of determining the eigenvector for the special case matrices used in AHP are dealt with in this report. Both methods require some knowledge of how to manipulate matrices.

The two methods are the approximate solution and exact solution. These methods are explained in the next two Sections.

##### *3.1.4.1.1 Approximate Solution (Saaty, (2012))*

This is the simplest of the methods and is most commonly used. It gives an approximate solution that is close to the exact solution in a nearly consistent matrix.

This approximate method involves three steps:



1. Sum the values in each column of the comparison matrix;
2. Normalise the judgements by dividing each element of the comparison matrix by the sum obtained in step 1. Enter the normalised values into corresponding places in a new matrix, the normalised matrix; and
3. Calculate the average of the values in each row of the normalised matrix to derive the priority vector.

This sequence is repeated for each of the matrices constructed for the analysis. The “candidate for a post” example in section 3.1.3.6 above will serve to illustrate these concepts. The comparison matrix for the criteria in that example is replicated in Table 3-10 after the completion of Step 1, i.e. the summation of the values in each column.

**Table 3-10: Comparison Matrix with summed Columns for the Criteria to choose the “best candidate”**

Post	E	Q	B
E	1	3	1/7
Q	1/3	1	1/9
B	7	9	1
Sum	8.3333	13	1.2540

Table 3-11 shows the normalised matrix. This is obtained by dividing each entry in the comparison matrix by the sum in the corresponding column. So, in the normalised matrix:

- the value in (E:E), 0.1200, is obtained by dividing 1 by 8.3333;
- the value in (E:Q), 0.2308, is obtained by dividing 3 by 13; and
- The value in (E:B), 0.1139 is obtained by dividing 1/7 by 1.2540.

The same procedure is used to obtain the values in the second and third rows. For example, the value in (Q:Q), 0.0769, is obtained by dividing 1 by 13. Note that the column sums in the normalised matrix is equal to one. If it is not, there is an error.

**Table 3-11: Normalised Matrix**

Post	E	Q	B
E	0.1200	0.2308	0.1139
Q	0.0400	0.0769	0.0886
B	0.8400	0.6923	0.7975
Sum	1	1	1

In the third step, the priority vector is obtained by averaging the values in each row as illustrated in Table 3-12. For example, the value in the first row and the priority column (0.1549) is obtained by averaging (E:E), (E:Q) and (E:B).

**Table 3-12: Determining the Priority Vector.**

Post	E	Q	B	Priority (Averages)
E	0.1200	0.2308	0.1139	0.1549
Q	0.0400	0.0769	0.0886	0.0685
B	0.8400	0.6923	0.7975	0.7766
Sum	1	1	1	

From Table 3-12 it can now be determined that BEE is most important at almost 78 per cent, experience scores second highest at approximately 15 per cent and the category of qualifications has the lowest score of about 7 per cent. The criteria can now be ranked in order, and in addition, the margin by which one criterion outranks another can be determined. In other words, BEE is in excess of 5 times more preferable to experience, which, in turn, is more preferable to qualifications by a margin greater than 2.

#### *3.1.4.1.2 Exact Solution (Saaty, 2012).*

This method requires that the comparison matrix be squared iteratively and the eigenvector (priority vector) be computed until the difference between the eigenvector of two successive iterations is smaller than a pre-defined number (or even 0). It is often worked until there is only small difference in the fourth decimal.

This method sounds simple but is quite complex if it has to be done manually. Squaring a matrix is not merely multiplying each element of the matrix by itself. That only squares each element of the matrix. Instead, the matrix is multiplied by itself (or another identical matrix). The normal rules for matrix multiplication apply. The elements in the new matrix (product matrix) are the sums of the products of each column of the first matrix and each row of the second matrix. Table 3-13 serves as illustration. Matrix B is identical to Matrix A and Matrix C is the product matrix. The value in (A7) is 3.00 and is obtained by  $(A1 \times A4) + (B1 \times A5) + (C1 \times A6)$ . Similarly, the value in C7, which is 21, is obtained by  $(A1 \times C4) + (B1 \times C5) + (C1 \times C6)$ .

**Table 3-13: Matrix Multiplication**

	Matrix A			Matrix B			Matrix C		
	1	2	3	4	5	6	7	8	9
A	1.00	0.33	0.33	1.00	0.33	0.33	3.00	2.33	0.73
B	3.00	1.00	0.20	3.00	1.00	0.20	6.60	3.00	1.40
C	3.00	5.00	1.00	3.00	5.00	1.00	21.00	11.00	3.00

It is clear that this method is much more tedious than the approximate method. When considering that this might have to be repeated several times, until the priority vectors have converged, this method becomes daunting. Fortunately, however, one does not have to do it manually and with Microsoft® Excel it is uncomplicated as will be illustrated in Chapter 4.

#### 3.1.4.2 Consistency

The best description of consistency of judgements is given by Saaty, (2012) as follows:

*“The consistency is perfect if all the judgments relate to each other in a perfect way. If you say that you prefer spring three times more to summer and that you prefer summer twice more to winter, then when you give the judgment comparing your preference of spring to winter it should be 6 and not anything else. The greater your deviation from 6, the greater your inconsistency. This observation applies to relations among all the judgments given. We would have perfect consistency, then, if all the relations checked out correctly. As we will see, there is a rather simple way of verifying inconsistency and how much it deviates from perfect consistency. There is also a good way for interpreting what inconsistency means in practical terms. When we are revising judgments, this method is useful and necessary. ”*

The following sections describe the verification and interpretation of consistency. The consistency of a matrix is measured against the principal eigenvalue of the matrix. The principal eigenvalue of a consistent positive reciprocal matrix is equal to the sum of the diagonal elements of the matrix. In AHP, the diagonals are always

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equal to 1, so the principal eigenvalue is equal to the number of elements on the diagonal if, and only if, the matrix is consistent.

In the real world, the judgements are always inconsistent. Saaty's philosophy on this is that a measure of inconsistency is a good thing and forced consistency without considering the judgement values will deprive judges of the opportunity to reconsider and change their minds and learn through the process (Saaty, 2008). The principal eigenvalue of an inconsistent matrix can be obtained by computing the products of the sums of the columns (obtained in Step 2) of the comparison matrix and the elements of the priority vector (the averages obtained in Step 3).

In the example above (refer to Table 3-10), the columns in Step 2 add up to 8.333, 13 and 1.2540 respectively for E, Q and B. The averages of the rows are 0.1549, 0.0685 and 0.7766 respectively for E, Q and B. The principal eigenvalue (denoted by  $\lambda_{\max}$ ) is now obtained as follows:

$\lambda_{\max} = (8.333 \times 0.1549) + (13 \times 0.0685) + (1.2540 \times 0.7766) = 3.1553$ . The principal eigenvalue for a consistent matrix in this example is equal to 3 (the sum of the diagonal values). The judgements are therefore not consistent.

The fact that the judgements are not consistent is no cause for concern since that is already expected. However, the measure of consistency requires consideration. If the inconsistency is large, then there might be a computational error or the judgements are unacceptably inconsistent.

In order to obtain a measure of the inconsistency, the concept of a consistency index ( $C_i$ ) and random consistency index ( $R_i$ ) is introduced. The theory is that if the consistency index,  $C_i$ , of any given reciprocal matrix is close to the consistency index,  $R_i$ , of a large number of random reciprocal matrices with the same number of elements, then the given matrix is as inconsistent as the average of the random matrices, in other words, random. The consistency index of a matrix is determined by Eq 3-1:

$$C_i = \frac{\lambda_{\max} - n}{n - 1} \quad \text{Eq 3-1}$$

Where:  $\lambda_{\max}$  is the principal eigenvalue and

$n$  is the number of elements in the diagonal

The  $R_i$  is obtained from a table that was compiled by Saaty and Kearns, (2013a), widely published and replicated in Table 3-14.

**Table 3-14: Random Consistency Index Table after Saaty, (2012)**

Size of matrix	1	2	3	4	5	6	7	8	9	10
$R_i$	0.00	0.00	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Finally, the consistency ratio,  $C_r$ , is obtained by Eq. 3-2:

$$C_r = \frac{C_i}{R_i} \quad \text{Eq. 3-2}$$

Saaty recommended that  $C_r$  should not be more than 0.10 or 10 per cent. This good guideline should be adhered to as far as possible, however, in practice it is often quite difficult to obtain such consistency, especially when information or experience upon which judgements are based is lacking.

To improve consistency when it is higher than 0.1, the input data must first be assessed for obvious input errors. Then the participants should review their judgements and make revisions where necessary after debate and discussions among the participants.

Finally, if the consistency ratio is still too high, the impact on the decision should be assessed. There might be a fundamental error in the construction of the matrix, the logic or the way in which questions are interpreted. If the ranking is not likely to change, or if the differences in priorities are relatively large, the effort of further improvement might not justify the benefits.

To illustrate the above, returning to the example under evaluation, it was established that the principal eigenvalue of the comparison matrix was 3.1553. From Eq. 1:

$$C_i = \frac{3.1553 - 3}{3 - 1} = 0.07765$$

and from Eq. 2 and Table 3-14:

$$C_r = \frac{0.07765}{0.52} = 0.1493$$

That is, almost 15 per cent, and some further investigation is warranted.

In addition to measuring consistency, there are also ways of determining the most inconsistent pair of judgements in order to guide the process of improving consistency. This is done by constructing a matrix:

$$\epsilon_{ij} = a_{ij} \frac{w_j}{w_i},$$

This simply means that a new matrix is formed by multiplying the elements of the judgement matrix with the ratio of the  $j^{\text{th}}$  element to the  $i^{\text{th}}$  element of the priority vector. The most inconsistent pair is then indicated by the highest value in the matrix. This concept is explained, with the aid of an example, in Section 4.2.

### 3.1.4.3 Synthesis

Once a priority vector has been determined for each one of the matrices in the analysis the process of synthesising the information is carried out. Recall that there will be a priority vector for the criteria in respect of the overall objective and one priority vector for each criterion in respect of which the alternatives are ranked. Priority vectors have been calculated for the matrices of alternatives in the example of selecting a candidate for a post in the preceding sections (refer to Table 3-7, Table 3-8 and Table 3-9). These are listed in Table 3-15 in columns C1 and C2. The priority vector from Table 3-12 for the criteria is listed in columns E,Q and B. (The reason for the layout of Table 3-15 has to do with matrix multiplication and will become apparent in Chapter 4). To arrive at the final score for each candidate the values for E,Q and B are multiplied by the value obtained by each candidate for each criterion and the products are summed; so  $((15.49\% \times 83\%) + (6.85\% \times 13\%) + (77.66\% \times 88\%)) = 82\%$  for C1 and  $((15.49\% \times 17\%) + (6.85\% \times 88\%) + (77.66\% \times 13\%)) = 18\%$  for C2. C1 is therefore to be the preferred candidate, despite his/her lack of qualifications because she/he scored well for both experience and BEE status.

**Table 3-15: Synthesis of results for selecting a candidate**

Criteria			Alternatives	
E	Q	B	C1	C2
15.49%	6.85%	77.66%		
<b>E</b>			83%	17%
<b>Q</b>			13%	88%
<b>B</b>			88%	13%
		Score	82%	18%



## 3.2 Summary of Chapter 3

This chapter described the fundamental principles of AHP and detailed its components and processes. The mathematical basis for the process was also briefly discussed. Concepts covered include:

- AHP
- Definition and background
- Overview
  - The AHP steps
    - Participants
    - Objective
    - Criteria
    - Alternatives
    - Pairwise comparison
    - Synthesising
- The Maths
  - Comparison matrix
  - Priority vector
    - Approximate solution
    - Exact solution
    - Consistency
    - Synthesis

Chapter 4 deals with the use of spreadsheets for practical application of the AHP. The chapter deals with the practical execution of the AHP and is the heart of the methodology for its practical application.

# Chapter 4 Employing Microsoft® Excel to execute AHP

Chapter 3 described the fundamental principles of AHP and detailed its components and processes. The mathematical basis for the process was also briefly discussed.

In Chapter 4 the practical application of AHP using Microsoft® Excel is explained in detail using an example. The chapter deals with every component of the AHP and its practical execution and, whereas Chapter 3 forms the theoretical foundation of the AHP, this chapter is the heart of the methodology for its practical application. To improve readability, calculations are presented in Appendix A.

## 4.1 Using Microsoft® Excel

The methodology proposed in this research report relies heavily on Microsoft® Excel, and in particular, functions that relate to arrays and methods to create and manipulate arrays.

### 4.1.1 Creating and manipulating an Array

In Excel an array is a collection of arranged items. The arrangement can be one dimensional or two dimensional. A one dimensional array can consist of a single row (horizontal array), or a single column (vertical array). A two dimensional array consists of multiple rows and columns.

Any formula that can be applied to a single cell can also be applied to an array. The difference is that the array formula is entered with the combination Ctrl-Shift-Enter instead of just Enter. For example, a hauling tally schedule is shown in Table 4-1. To calculate the tonnes hauled, select cells C5:C13 and type the following formula in the formula bar, but **do not** hit ENTER

=B5:B13\*B3

Now press the **CTRL** and **SHIFT** keys simultaneously and while holding them, hit **ENTER**.

Excel surrounded the formula with braces, (**{ }**), and the formula was placed and calculated in each cell of the selected range. This was an example of calculating multiple values in a vertical array from another vertical array and a single value (B3). (Note that the nomenclature of Excel is used to maintain consistency with literature and help utilities, hence braces rather than brackets).

Now select the cell B14 (next to Average) and in the formula bar type the following formula:

```
=SUM(B5:B13/COUNT(A5:A13))
```

in addition, hit **Ctrl-Shift-Enter** to calculate the average number of trucks hauled. This is an example of using the array function to calculate a single value from two vertical arrays (B5:B13 and A5:A13). To check that this is correct, enter the Excel built-in Average function in some other cell.

**Table 4-1: Example of the use of Array Functions: Hauling Tally**

	A	B	C
1	Trucks per span	8	
2	Tons per truck	14	
3	Tons per span:	112	
4	Tip	Spans	Tons
5	1	17	1904
6	2	24	2688
7	3	22	2464
8	4	19	2128
9	5	25	2800
10	6	19	2128
11	7	21	2352
12	8	20	2240
13	9	18	2016
14	Average	20.56	

#### 4.1.2 Rules for Arrays (“Guidelines and examples of array formulas”, n.d.)

- 1 The primary rule for creating an array formula is to press CTRL+SHIFT+ENTER whenever an array formula is to be created or edited. The rule applies to both single-cell and multi-cell formulae.
- 2 One must select the range of cells to hold the results *before* entering the formula.
- 3 One cannot change the contents of an individual cell in an array formula.
- 4 One can move or delete an entire array formula, but cannot move or delete part of it. In other words, to shrink an array formula, first delete the existing formula and then start over.
- 5 One cannot insert blank cells into or delete cells from a multi-cell array formula.
- 6 To delete an array formula, select the entire formula (for example, **=C2:C11\*D2:D11**), press DELETE, and then press CTRL+SHIFT+ENTER.

#### 4.1.3 Excel Functions specific to Matrices

In Excel matrices are stored in arrays. The version of Excel used for this research report contains three standard functions for matrix calculations. They are MDETERM, which returns the determinant of a matrix, MINVERSE, which returns the inverse of a matrix and MMULT, which returns the matrix product of two matrices. MDETERM and MINVERSE are not used in this report but MMULT is employed extensively.

##### 4.1.3.1 Matrix Multiplication

In Section 3.1.4.1.2, it was shown how tedious it is to multiply one matrix with another. First the element in each row of one matrix must be multiplied with the element in the corresponding column of the second matrix and then the whole lot has to be summed. The MMULT function in Excel makes this a trivial task as illustrated in Table 4-2. The values in the horizontal array A1 to D1 are to be

multiplied by the values in the vertical array F1 to F4. The result is placed in G1. To do this, select G1 then in the formula bar type the formula

```
=MMULT(A1:D1,F1:F4)
```

and hit ENTER. Note that it is not necessary to use the Ctrl-Shift-Enter combination because the result is in a single cell.

It is important to observe that the number of columns in one array must be equal to the number of rows in the other. To illustrate the effect of this enter the following formula in G2:

```
=MMULT(F1:F4,A1:D1)
```

The result is 2, not quite what was expected. The reason for this is that the first array (F1:F4) has only one column, so the resultant array also has only 1 column and the value is the product of F1 (1) and A1 (2). This principle applies to both one dimensional arrays (vectors) as well as two dimensional arrays (matrices).

**Table 4-2: Illustrating the MMULT Function on One Dimensional Arrays**

	A	B	C	D	E	F	G
1	2	4	6	8		1	128
2						3	2
3						7	
4						9	

When using two dimensional arrays, the resultant array must have as many rows as the first array and as many columns as the second array. So a 3x3 array multiplied

by a 5X3 array will have dimensions of 5X3. For AHP this is not important because only square matrices are used.

#### 4.1.4 Squaring a Matrix

To illustrate the use of the MMULT function to square a matrix, refer back to Table 3-13. It will be recalled that Matrix B was identical to Matrix A. The distinction was made for the sake of clarity of the explanation and it is not necessary to construct Matrix B. In order to raise a matrix to the power of 2, consider Table 4-3. The first step is to select the cells E1:G3 and then in the formula bar type in the following formula:

=MMULT(A1:C3,A1:C3).

Because the results will be stored in an array it is necessary to use the Ctrl-Shift-Enter key combination. The result in the range E1:G3 is the square of the matrix stored in the range A1:C3.

**Table 4-3: Squaring a Matrix**

	A	B	C	D	E	F	G
1	1	1/3	1/3		2.980	2.310	0.726
2	3	1	1/5		6.600	2.990	1.390
3	3	5	1		21.000	10.990	2.990

## 4.2 Illustrative Example

To illustrate all the foregoing and the application thereof to AHP, a simple case study is presented next using the approximate solution. (Refer to section 3.1.4.1.1). For the sake of clarity, a generic day-to-day example was used that is not mining related,

but with which most readers should easily identify. To facilitate reading, the actual operations are presented in the Appendix A and referenced appropriately in the text that follows.

#### 4.2.1 Objective, Scope, and Alternatives

A second year student decides that it will be more economical for her to live with her parents and travel to campus and back in her own car. In order to get the best car for this purpose she uses AHP to evaluate the alternatives based on the following criteria:

- Performance (P)
- Economy (E)
- Comfort (C)

The alternative vehicles that she considers are as follows:

- A luxury car of the German kind (Mer)

It has all the features one can dream of and a few more. It consumes petrol at a rate of 12 litres per 100 kilometres and has a service plan for three years. It has a high rate of acceleration and a high top speed.

- A small city car (SCC)

It comes standard with basic features such as an air conditioner and a radio. The service plan is 1 year and it can get 100 kilometres on 10 litres of petrol. The acceleration and top speed are adequate.

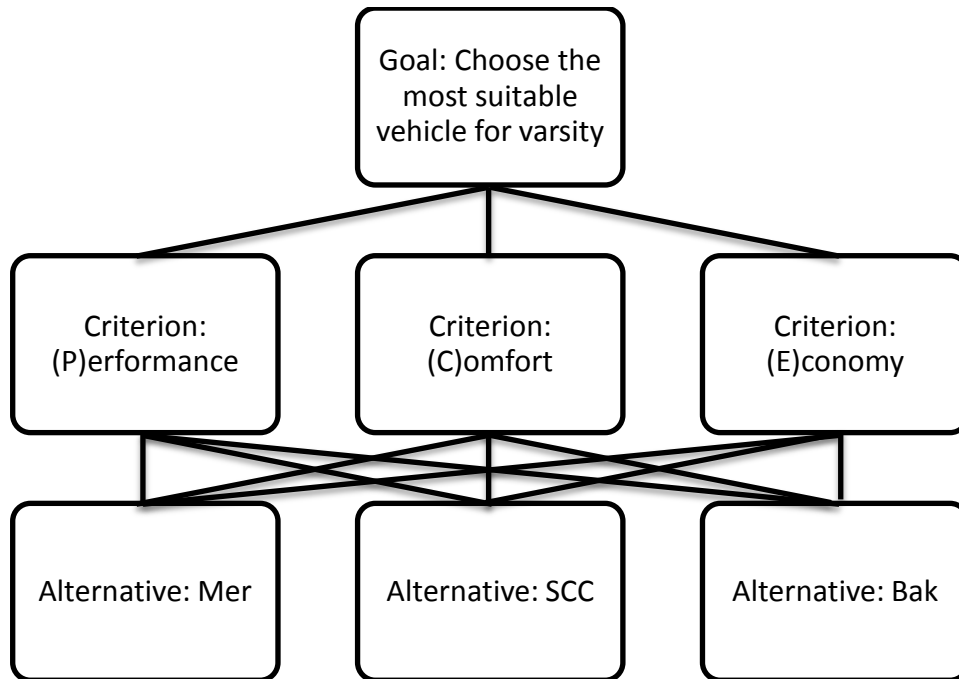
- A small utility vehicle (Bak)

Nothing is automatic. It has neither an air conditioner nor a radio. There is no service plan and it uses about 10 litres of petrol for every 100 kilometres. Acceleration and top speed are not exciting.



## 4.2.2 Constructing the Hierarchy

The hierarchy for this problem is indicated in Figure 4-1. The goal is on the top most level and on the next level are the identified criteria. These are followed on the lowest level by the alternatives.



**Figure 4-1: Hierarchy for choosing the Best Vehicle.**

## 4.2.3 Pairwise Comparison

The AHP requires that the hierarchy be worked from top to bottom. The first thing that the student has to do is compare the criteria to each other in a pairwise manner and judge their relative importance with respect to the objective. She uses Excel to enter the array depicted in Figure A-2 and then she enters 1's on the diagonal in B2, C3 and D4 because each criterion weighs exactly 1 when compared to itself. In order to have fractions shown in the comparison matrix, use the format cell function as shown in Figure A-2.

The student judges that performance is very much less important to her than comfort and she enters 1/7 in C2 at the junction of (P:C) (refer to the scale in Table 3-5). Note, the row weighs less than the column and so a fraction is entered. (Refer to

Figure 3-6). The reciprocal value, 7, must be entered in the corresponding cell below the diagonal, in other words at the junction of (C, P) in cell B3.

The student next makes the judgement that economy is absolutely more important than performance and, because the row weighs less than the column, enters the fraction 1/9 at the junction of (P:E) in Cell D2. In cell B4, at the junction of (E:P) she enters the reciprocal 9.

She finally judges that economy absolutely outweighs comfort. The value at the junction of (C:E) in cell D3 is a fraction, because the row weighs less than the column and so she enters 1/9. In cell C4, she enters 9 and completes her pairwise comparison matrix for the criteria. The completed comparison matrix for the criteria is shown in Figure 4-2.

A1		fx Best Car				
	A	B	C	D	E	
1	Best Car	P	C	E		
2	P	1	1/7	1/9		
3	C	7	1	1/9		
4	E	9	9	1		
5						

**Figure 4-2: Completed Comparison Matrix for Criteria**

#### 4.2.4 Normalising the Matrix

The next step is to normalise the matrix by dividing each value in every column by the sum of the values in that column according to the procedure outlined in A.3

##### Normalising the Matrix.

Having completed the comparison matrix for the criteria, the student now constructs the normalised matrix. This is done by creating a new array equal in size to the comparison matrix (i.e. 3 by 3). The value in each cell of the new array is obtained by dividing the value in the corresponding cell of the comparison matrix by the sum of that column.

The normalised matrix is shown in cells F2 to H4 of Figure 4-3. This matrix now contains the values of each element of the comparison matrix divided by the sum of the column in which the element appears. So  $1/17 = 0.058824$  (B2/B5). Likewise  $9/10.143 = 0.887324$  (C4/C5).

F2		fx {=B2:D4/B5:D5}							
	A	B	C	D	E	F	G	H	I
1	Best Car	P	C	E					
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	
3	C	7	1	1/9		0.411765	0.098592	0.090909	
4	E	9	9	1		0.529412	0.887324	0.818182	
5		17.000	10.143	1.222					
6									

**Figure 4-3: Normalised Matrix completed**

#### 4.2.5 Prioritising

The priority vector (eigenvector), that is the relative importance of the criteria, is now calculated by averaging the rows in the normalised matrix as explained in A.4 Prioritising and shown in Figure 4-4.

I4		fx {=AVERAGE(F4:H4)}								
	A	B	C	D	E	F	G	H	I	J
1	Best Car	P	C	E						PV
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	0.054606	
3	C	7	1	1/9		0.411765	0.098592	0.090909	0.200422	
4	E	9	9	1		0.529412	0.887324	0.818182	0.744973	
5		17.000	10.143	1.222						
6										

**Figure 4-4: Average the Rows**

As can be seen in Figure 4-4, the priority vector shows not only which criteria are most and least important, but also how much more important one is relative to another. In other words, Economy is the overriding consideration and carries almost 75 per cent of the total weight, while comfort is 4 times more important than performance (20% vs. 5%). This priority vector will later be used to evaluate and find

the most appropriate vehicle, but first the consistency of the student's judgement must be tested.

#### 4.2.6 Test for Consistency

As explained in the maths part of this report (Section 3.1.4), the principal eigenvalue is used as a measure of the consistency. The closer this value is to the number of elements in the diagonal of the matrix, the more consistent the judgements. How to determine the principal eigenvector is explained in A.5 Test for Consistency.

The Consistency Index is determined through Eq 3-1. The way it can be done in Excel is shown in Figure 4-5, in the formula bar. The Consistency Ratio is determined by dividing the Consistency Index by the Random Consistency index, which is 0.52 for a 3 by 3 matrix according to Table 3-14.

The calculated Consistency Ratio in this example comes to 0.84, clearly way above the recommended 0.1. Means to improve the Consistency Ratio is discussed next.

#### 4.2.7 Improving the Consistency

The most common cause for poor consistency of judgement is an error during the input where the reciprocal value is entered in the incorrect cell, not observing convention that the row is weighed against the column.

Since only four values have are being considered, it can be seen that there are no obvious input errors. Obviously, the larger the matrix, the more difficult it will be to spot errors, but there are ways of determining the most inconsistent pairs, which simplifies this task and will be discussed later.

The next most common error is the accuracy of the priority vector and  $\lambda_{\max}$ . Recall that in a previous discussion the approximate method was used. Using the exact method (an example of which will follow),  $\lambda_{\max}$  was found to be 3.4356, the Consistency Index was 0.2178 and the Consistency Ratio was 0.4189. This

represents an almost 50% improvement in the consistency, but is still too far above the acceptable limit of 0.1

The third area to look for improved consistency is to inspect the judgements carefully again and see if there are errors in the logic. Looking at Figure 4-5, one can notice that cells B4 and C4 have the same value, 9. In other words, economy is 9 times more important than performance and it is 9 times more important than comfort. It should logically follow that comfort and performance are equally important, yet the judgment was that comfort was 7 times more important than performance. In the light of this, the equality in cells B4 and C4 cannot be. After careful consideration and adjustment of the values in the comparison matrix (Appendix 0) a consistency ratio of 0.04 was finally reached. Before accepting the matrix, the student (judge) has to be satisfied that this is an accurate reflection of her revised judgement. She finds it reasonable to expect that C:E is somewhere between 1/9 and 1/5 and accepts the matrix.

K3											
fx =(K2-COUNT(F2:H2))/(COUNT(F2:H2)-1)											
	A	B	C	D	E	F	G	H	I	J	K
1	Best Car	P	C	E					PV		
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	0.054606	$\lambda_{max}$	3.871669
3	C	7	1	1/9		0.411765	0.098592	0.090909	0.200422	Ci	0.435834
4	E	9	9	1		0.529412	0.887324	0.818182	0.744973		
5		17.000	10.143	1.222							
6											
7											

**Figure 4-5: Determine the Consistency Index**

#### 4.2.8 Prioritising the Options with respect to the Criteria

Having determined the priority vector, the student now has a good idea of what are the most and least important features to look for in the vehicles she is considering (this vector might be thought of as the “what-does-it-matter” vector as it will determine how much weight is assigned to the attributes of the alternatives). In order to achieve the objective of selecting the best car, each vehicle must now be prioritised with respect to each criterion. This is the third level of the hierarchy

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(Figure 4-1 and Figure A-1) where the cars are compared to one another with respect to Performance, Comfort and Economy respectively. For each criterion, a comparison matrix must be created and the judgments made. The priority vectors must be calculated and the consistency must be checked. All this is detailed in Appendix 0, where the exact method is illustrated as well. The construction of a useful “Consistency Indicator” is also described.

#### 4.2.9 Final Analysis (Synthesising)

In the final step, the score of each alternative in terms of the objective is derived by summing the products of the score for each attribute and the priority of each criterion with respect to the objective. In other words, the score of each attribute of each alternative is multiplied by how much that attribute matters and the products are added together for the final score. To achieve this for the example under consideration, a final matrix is constructed in which the priority of each alternative is tabulated against each criterion. The final score for each alternative is found with Excel’s MMULT function as illustrated in Figure 4-6 . For example, the result for Mer is obtained by the following:  $0.731*0.0599+0.705*0.1897+0.072*0.7504=0.2316$ .

	A	B	C	D	E	F	G	H
33		<b>P</b>	<b>C</b>	<b>E</b>		<b>Crit</b>	<b>Score</b>	
34	<b>Mer</b>	0.731	0.705	0.072		<b>P</b> 0.0599	23.1%	
35	<b>SCC</b>	0.188	0.211	0.649		<b>C</b> 0.1897	53.8%	
36	<b>Bak</b>	0.081	0.084	0.279		<b>E</b> 0.7504	23.0%	
37								

**Figure 4-6: Synthesising**

In the final analysis then, the student can confidently choose SCC as the best vehicle for commuting to and from campus. Note that in this case, the difference between Mer and Bak is very small, however, considering the very high score for SCC, the interpretation is that they are both poor choices. Mer is eliminated because it scores low in Economy, which carries most weight and Bak because it scores low in all three criteria, but particularly in performance and comfort. SCC

scores relatively low in the criteria that are less important and high in the one that is most important.

#### 4.2.10 Improving Consistency by determining the most inconsistent Pairs of Values

In small matrices, it is comparatively easy to spot mistakes or work through the judgements quickly with the judges to improve consistency. If matrices become larger, (probably anything larger than 4 by 4), the task becomes arduous and time consuming. It is recommended that matrices be restricted in size to about 7 to 9 because it was found that this is the number of objects, on average, that can be stored easily in the short term memory of humans, (Miller, 1956).

To find the most inconsistent pair of pairwise comparisons requires that a matrix be constructed, the elements of which are the product of the elements of the comparison matrix and the ratio of the elements of the  $j^{\text{th}}$  and  $i^{\text{th}}$  elements of the priority vector. This is achieved in three steps:

**Construct a quotient matrix, with the same number of elements as the comparison matrix. The elements of this matrix are derived as shown in**

1. Table 4-4. (Note that the columns and the rows each contain the elements of the priority vector);
2. Multiply the elements of the comparison matrix with the elements of the quotient matrix to derive the consistency indicator matrix; and
3. Find the largest number in the consistency index indicator. This value is indicative of the most inconsistent pair of judgements. The corresponding judgement in the original comparison matrix can now be adjusted with the approval of the original judges.

**Table 4-4: Deriving a Quotient Matrix**

$w_j/w_i$	0.37	0.17	0.26	0.20
0.37	0.37/0.37	0.17/0.37	0.26/0.37	0.20/0.37
0.17	0.37/0.17	0.17/0.17	0.26/0.17	0.20/0.17
0.26	0.37/0.26	0.17/0.26	0.26/0.26	0.20/0.26
0.20	0.37/0.20	0.17/0.20	0.26/0.20	0.20/0.20

#### 4.2.10.1 Illustrative Example for “Consistency Indicator”

It is rather difficult to spot the highest numbers in a matrix, especially when it is large. This process can be automated to the extent that the most inconsistent values are highlighted through Excel’s conditional formatting feature. To see how to use this method of indicating the most inconsistent pair, which will be termed the consistency indicator, refer to Appendix 0.

### 4.3 Summary of Chapter 4

This chapter deals with the use of spreadsheets for practical application of AHP. The application of Microsoft® Excel is explained in detail using an illustrative example. To improve readability, calculations are presented in the Appendix A. Concepts dealt with in this chapter include:

- Arrays in Excel
- Squaring a matrix
- Objective, scope, alternatives
- Hierarchy construction
- Pairwise comparison
- Normalising



- Prioritising
- Consistency
- Synthesis

Chapter 5 discusses examples and case studies of the application of AHP to demonstrate its potential value to the mining industry. Each case study has been selected with the aim of highlighting aspects that enhanced the decision-making process.

## **Chapter 5 Case Studies**

Chapter 4 dealt with the practical execution of the processes of AHP with the use of Microsoft ® Excel in an illustrative example. Every component of the AHP and its practical execution was discussed. Chapter 4 described the practical implementation of the methodology for AHP and, together with Chapter 3, it forms the kernel of this research report.

Chapter 5 deals with three case studies and an example of the application of AHP to demonstrate its potential value to the mining industry. Two case studies were summarised from literature and a third was an original case in which the author was personally involved. Each case study has been selected with the aim of highlighting aspects that enhanced the decision-making process. The chapter is concluded with an example of the potential application of the AHP to fault tree analyses.

### **5.1 Case studies from Literature**

This section deals with two case studies taken from literature. It illustrates the potential for application of AHP in the mining industry.

In the first case study the AHP was used to determine the optimum level spacing and raise spacing in a platinum mine. Whereas literature on the application of the AHP to selection studies is abundant, it has not been used as frequently as an optimisation tool and this case study illustrates its effectiveness as such.

The second case study deals with the selection of a longwall layout from among a few different ones. The case study was selected to serve as an example of the use of the approximate method of AHP, prioritisation and measures to improve consistency. The case study also illustrates the importance of the objective and scope of a problem through the assertion that the decision would have been different if the operations were in another location where surface restriction regulations are more stringent.

### 5.1.1 Case study 1: Optimisation of Level and Raise Spacing

This case study is based on the work carried out by Musingwini, (2010b), as part of a PhD thesis, which dealt with the optimisation of level and raise spacing in a platinum mine in the Bushveld Igneous Complex.

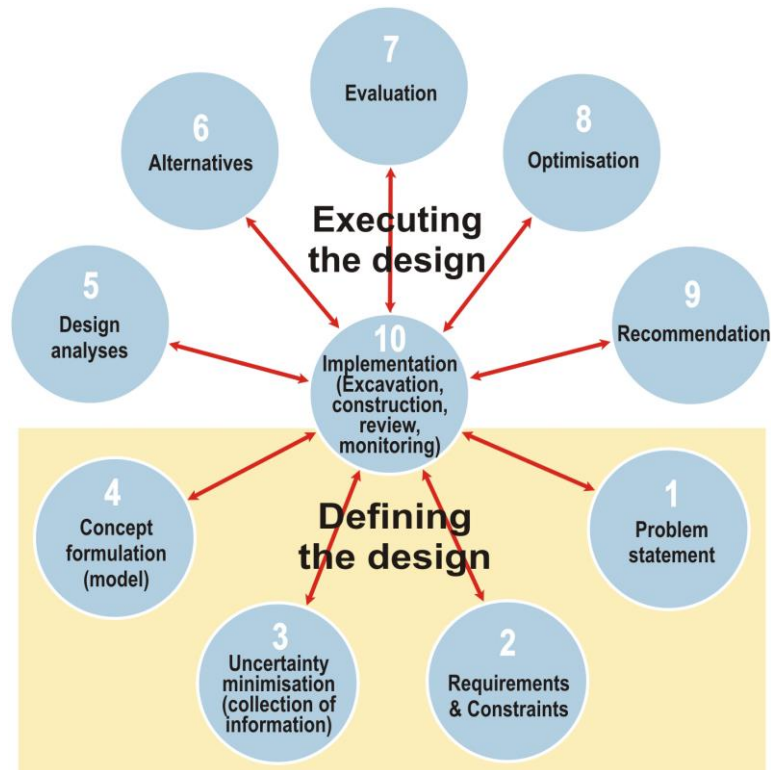
This case study was chosen for discussion for the following reasons:

- It demonstrates how the AHP can be beneficial in mining applications that require the consideration of multiple criteria and can have any of a number of outcomes;
- It demonstrates the potential of AHP to be applied to optimisation studies. Whereas literature on the application of the AHP to selection studies is abundant, its use as an optimisation tool is less frequent.. This case study illustrates its effectiveness as such;
- It shows that the AHP can be fully integrated into planning and design processes; and
- It demonstrates an unusual but very effective graphical representation of the result of the AHP.

Musingwini's research centred on the question of whether there was an optimal range of level and raise spacings for platinum mines in the Budhveld Igneous Complex. His research led him to conclude that the problem should be solved using MCDA techniques and he chose to use the AHP for reasons as set out in his thesis.

Musingwini followed the principles of the design wheel proposed by Stacey, (2009). Stacey drew parallels between the conversation model for strategic planning by Ilbury and Sunter, (2011), (see Figure 3-2), and the rock engineering design principles proposed by Bieniawski, (1993) and constructed a generic engineering design wheel to reflect these. This design wheel is replicated in Figure 5-1 .

This was not documented by Musingwini, but the similarity between these design principles and the description of the AHP steps in Section 3.1.3 and Figure 3-1 is evident.



**Figure 5-1: Engineering circle or wheel of design, (Stacey, 2009)**

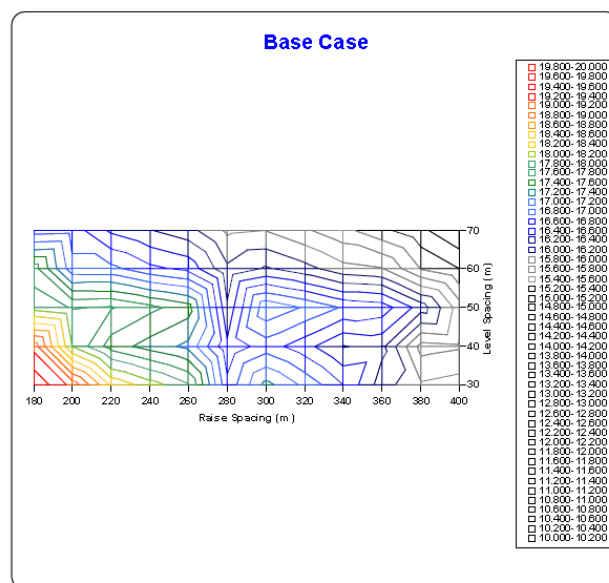
Musingwini used the AHP in the seventh and eighth spokes of the wheel of design to evaluate different level and raise spacing and to identify an optimal range. AHP was chosen because of its advantages over other MCDA techniques and the fact that it was gradually gaining recognition in the minerals industry. The main advantages that Musingwini saw in the AHP over other techniques were its ease of use, the ability to rank conflicting criteria in terms of importance and the built-in feature to measure consistency in judgement.

Musingwini undertook a survey on a number of identified decision criteria to establish the weighting attached to each. The weights were applied to the scores that each of the 15 layouts obtained for every optimisation criterion. The layouts were developed

according to planning parameters applicable to the orebody being studied and the optimisation parameters were derived through literature surveys. The layouts were ranked according to their relative performance with respect to the decision criteria and finally a sensitivity analysis was carried out to check the effect of inconsistencies in judgment on the optimal raise and level spacing. The steps followed by Musingwini, which are in line with the discussions in the preceding chapters of this research report (Refer to Chapters 3 and 4), were as follows:

- 1 Enter pairwise weighting of criteria from respondents into a judgment matrix;
- 2 Aggregate the pair-wise responses;
- 3 Normalise the pairwise comparisons;
- 4 Calculate the average aggregate weight of each criterion;
- 5 Estimate the consistency of judgements; and
- 6 Apply weights to layout scores to obtain relative priorities.

A novel approach by Musingwini was to use Excel to make a contour plot of the layout priorities obtained in a raise-spacing/level-spacing plotting space as shown in Figure 5-2 which he used to read off the optimal raise and level spacing according to the score of the layouts.



**Figure 5-2: Excel Surface Contour Plot of Scores after Musingwini, (2010b)**

The sensitivity analysis (the results of which were similar to the base case, implying that the sensitivity to judgments with respect to NPV was low) was carried out for three cases:

- All criteria have a relative weight of 1;
- Increasing the importance of NPV by 10 per cent; and
- Decreasing the NPV by 10 per cent.

It would be inappropriate to list all the conclusions of Musingwini's research since it entailed much more than AHP in deriving them, however, Musingwini's research showed that an optimal range of level and raise spacings could be obtained and that the AHP can be instrumental in such studies. This, if applied in the mining industry, should bring about enormous financial benefits.

The most important conclusions from this case study for this research are as follows:

- The AHP is a preferred method for problems with multiple criteria multiple outcomes;
- AHP can be effectively applied to optimisation studies in the mining industry;
- The AHP can be fully integrated into planning and design processes such as the design wheel published by Stacey; and
- Results from the AHP can be integrated effectively with any graphical representation of choice.

### 5.1.2 Case study 2: Long Wall Layout Selection using the AHP (Abdalla et al., 2013)

This study was aimed at the selection of optimal panel orientation for longwall coal mining operations in Queensland, considering factors such as geology, geography, geotechnical and economy.

The selection of this work as a case study was based on the following:

- The absence of a well-defined and considered objective and scope for this case study demonstrates the importance of this element of the AHP;
- The case study shows that, although a very important aspect, it is not necessary to achieve the stringent level of consistency that the literature recommends. A satisfactory level of inconsistency can be agreed upon by the role players;
- The case study illustrates how a large number of criteria (fourteen in this case) can make it very difficult to reach consistency; and
- The case study illuminates the repeatability of the AHP through the independent assessment of the judgements.

**Note:** it is not necessarily repeatable for different panels of judges. It is unlikely that different judges will have exactly the same disposition and make exactly the same judgments; however, if the judges are thoroughly conversant with the subject matter the final ranking should not be significantly different.

The definition of the objective and scope of the analysis is an aspect of AHP which is often overlooked and many times not afforded the required attention to detail. As a consequence too many (or too few) criteria may be identified. This appears to have happened with the case study under consideration. In this case study, so many factors relating to longwall layouts were identified that the authors had to limit them. Excluding criteria at this early stage of an analysis based on quantity may compromise the analysis in terms of quality since the priority of the criteria have not yet been established.

According to accepted AHP practice, the number of elements on any particular level of the hierarchy should be 7 plus or minus 2. (See Vargas, 2013). That means that ideally the number of criteria should not exceed nine. In this study, 14 criteria were considered and this extraordinarily large number created difficulty in achieving good judgement consistency. A well-defined objective and scope would almost certainly have eliminated some of the criteria, but, failing this, a better approach would have

been to limit the number of criteria by means of clustering. Clustering means that related criteria are grouped together as sub-criteria of one parent criterion. In this way, extra levels are introduced to the hierarchy while the number of elements per level is reduced.

It was inferred by the researcher that the objective of the study was to find the best longwall layout, however, this is not evident from the paper. The authors limited the criteria to the following:

- Depth of cover;
- Seam inclination;
- Coal quality;
- Gad (Gross air dried) make; (Note that this is “Gas Make in the original table by the authors, Figure 5-3).
- Roof and floor strata;
- Geological structures;
- In situ stress;
- Multiple seam mining;
- Surface restrictions;
- Surface subsidence;
- Access to reserve;
- Reserve losses due to layout;
- Seam thickness; and
- Roof cavability.

In a workshop involving experts in longwall mine planning and design, the judges assigned the relative importance of the factors according to the fundamental judgement scale (see Table 3-5). The comparison matrix is shown in Figure 5-3.



	Depth of Cover	Seam Inclination	Coal Quality	Gas Make	Roof and Floor Strata	Geological Structures	In-situ Stress	Multiple Seam Mining	Surface Restrictions	Surface Subsidence	Access to reserve	Reserve losses due to the layout	Seam Thickness	Roof Cavability
Depth of Cover	1.00	7.00	0.13	1.00	1.00	3.00	0.11	7.00	8.00	8.00	1.00	0.33	2.00	2.00
Seam Inclination	0.14	1.00	0.11	0.13	0.13	0.33	0.11	1.00	0.25	1.00	0.14	0.11	0.14	0.13
Coal Quality	8.00	9.00	1.00	9.00	0.50	3.00	1.00	8.00	5.00	8.00	4.00	3.00	6.00	6.00
Gas Make	1.00	8.00	0.11	1.00	0.17	0.33	0.11	5.00	0.25	6.00	0.14	0.14	0.33	0.20
Roof and Floor Strata	1.00	8.00	2.00	6.00	1.00	6.00	0.50	6.00	8.00	8.00	4.00	1.00	1.00	1.00
Geological Structure	0.33	3.00	0.33	3.00	0.17	1.00	0.25	7.00	8.00	9.00	0.50	1.00	1.00	1.00
In-situ Stress	9.00	9.00	1.00	9.00	2.00	4.00	1.00	9.00	9.00	9.00	7.00	3.00	8.00	5.00
Multiple Seam Mining	0.14	1.00	0.13	0.20	0.17	0.14	0.11	1.00	5.00	6.00	0.33	0.14	0.13	0.17
Surface Restrictions	0.13	4.00	0.20	4.00	0.13	0.13	0.11	0.20	1.00	1.00	0.20	0.13	0.14	0.14
Surface Subsidence	0.13	1.00	0.13	0.17	0.13	0.11	0.11	0.17	1.00	1.00	0.13	0.11	0.13	0.13
Access to Reserve	1.00	7.00	0.25	7.00	0.25	2.00	0.14	3.00	5.00	7.69	1.00	0.33	5.00	2.00
Reserve Losses (due to the layout)	3.00	9.00	0.33	7.00	1.00	1.00	0.33	7.00	8.00	9.00	3.00	1.00	7.00	5.00
Seam Thickness	0.50	7.00	0.17	3.00	1.00	1.00	0.13	8.00	7.00	8.00	0.20	0.14	1.00	0.50
Roof Cavability	0.50	8.00	0.17	5.00	1.00	1.00	0.20	6.00	7.00	8.00	0.50	0.20	2.00	1.00
Sum	25.87	82.00	6.05	55.49	8.63	23.05	4.22	68.37	72.5	89.69	22.15	10.64	33.87	24.26

**Figure 5-3: Pairwise Comparison Matrix after Abdalla et al., (2013)**

After constructing the comparison matrix, the priority vector was obtained through the approximate method of prioritization in AHP (refer to 3.1.4.1.1). This involved three steps as follows:

- 1 Summation of each column of the comparison matrix;
- 2 Division of each element of the matrix by the sum of the column it belonged to (normalisation); and
- 3 Calculation of the normalised principal eigenvector, or the priority vector, by averaging the rows.

The normalised matrix and resulting priority vector is shown in Figure 5-4 from which it was concluded that *in-situ* stress was the most important factor. Surface restrictions were considered one of the least important factors but the authors highlighted the fact that judgements could be influenced by external factors such as location. Their study dealt with locations in central Queensland, where, apparently, restrictions on the impact of mining on surface are lower than in New South Wales. If the locations were there, surface restrictions and surface subsidence factors might have been assigned higher priorities. This observation by the authors in their study serves well in this research to demonstrate the importance of the objective and scope when applying the AHP.

	Depth of Cover	Seam Inclination	Coal Quality	Gad Make	Roof and Floor Strata	Geological Structures	In-Situ Stress	Multiple Seam Mining	Surface Restrictions	Surface Subsidence	Access to Reserve	Reserve losses due to the Layout	Seam Thickness	Roof Cavability	Sum	Priority Vector
Depth of Cover	0.04	0.09	0.02	0.02	0.12	0.13	0.03	0.10	0.11	0.09	0.05	0.03	0.06	0.08	0.96	7%
Seam Inclination	0.01	0.01	0.02	0.00	0.02	0.01	0.03	0.01	0.00	0.01	0.01	0.01	0.00	0.01	0.15	1%
Coal Quality	0.30	0.11	0.17	0.16	0.06	0.13	0.24	0.12	0.07	0.09	0.18	0.28	0.18	0.25	2.33	17%
Gad Make	0.04	0.09	0.02	0.02	0.02	0.01	0.03	0.07	0.00	0.07	0.01	0.01	0.01	0.01	0.41	3%
Roof and Floor Strata	0.04	0.09	0.33	0.11	0.12	0.26	0.12	0.09	0.10	0.09	0.18	0.09	0.03	0.04	1.69	12%
Geological Structures	0.01	0.04	0.06	0.05	0.02	0.04	0.06	0.10	0.11	0.10	0.02	0.09	0.03	0.04	0.78	6%
In-Situ Stress	0.35	0.11	0.17	0.16	0.23	0.17	0.24	0.13	0.13	0.10	0.32	0.28	0.24	0.21	2.83	20%
Multiple Seam Mining	0.01	0.01	0.02	0.00	0.02	0.01	0.03	0.01	0.07	0.07	0.01	0.01	0.00	0.01	0.28	2%
Surface Restrictions	0.00	0.05	0.03	0.07	0.02	0.01	0.03	0.00	0.01	0.01	0.01	0.01	0.00	0.01	0.27	2%
Surface Subsidence	0.00	0.01	0.02	0.00	0.01	0.00	0.03	0.00	0.01	0.01	0.01	0.01	0.00	0.01	0.14	1%
Access to Reserve	0.04	0.09	0.04	0.13	0.03	0.09	0.03	0.04	0.07	0.09	0.05	0.03	0.15	0.08	0.95	7%
Reserve losses due to the Layout	0.12	0.11	0.06	0.13	0.12	0.04	0.08	0.10	0.11	0.10	0.14	0.09	0.21	0.21	1.60	11%
Seam Thickness	0.02	0.09	0.03	0.05	0.12	0.04	0.03	0.12	0.10	0.09	0.01	0.01	0.03	0.02	0.75	5%
Roof Cavability	0.02	0.10	0.03	0.09	0.12	0.04	0.05	0.09	0.10	0.09	0.02	0.02	0.06	0.04	0.86	6%
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	14.00	100%

**Figure 5-4: Normalised Matrix after Abdalla et al., (2013)**

By summing the products of the priority vector and the columns of the comparison matrix, the  $\lambda_{max}$  was calculated to be 18. The consistency index (Ci) was calculated according to Eq 3-1, repeated below, as discussed in Section 3.1.4, and came to 0.32.

$$Ci = \frac{\lambda_{max} - n}{n - 1} ;$$

The random consistency index for a 14 by 14 matrix is quoted by the authors as 1.57 and the ratio of the Consistency index and the Random index, that is the Consistency Ratio, came to 0.2, which is double the recommended 0.1!

The authors then used the Consistency Indicator technique (refer to A.13 Illustrative Example for “Consistency Indicator”) to enable them to detect the most inconsistent pair in the comparison matrix and went back to the judges for approval of adjusting their judgments. This was done iteratively until a consistency ratio was achieved that was considered acceptable by the panel of judges. The final consistency ratio agreed upon was 0.13.

This case study was carried out using the approximate method. As an exercise for this research report, the information from the case study by Abdalla et al., (2013) was used to attempt to improve the consistency by first applying the exact method of prioritising rather than the approximate method and then to apply the Consistency Indicator technique to the results.

The best consistency that could be achieved was 0.12. The adjusted matrix obtained is compared to the adjusted matrix of Abdalla et al., (2013) in Figure 5-5. There are differences in the pairs of judgements that were adjusted. This is to be expected as the authors of the case study had the benefit of the insight of the original judges. The final priorities were however similar, and more importantly the ranking was the same. This exercise highlighted again the importance of avoiding mechanical adjustment of the judgements as that could potentially lead to incorrect decisions. It also illustrates the repeatability of the AHP since the data was used independently by the author and results were significantly similar.

	Depth of Cover	Seam Inclination	Coal Quality	Gas Make	Roof and Floor Strata	Geological Structures	In-situ Stress	Multiple Seam Mining	Surface Restrictions	Surface Subsidence	Access to Reserve	Reserve losses (due to the layout)	Seam Thickness	Roof Cavability	Priority Vector
Depth of Cover	1.00	7.00	0.13	1.00	1.00	1.00	0.11	3.00	5.00	8.00	1.00	0.33	2.00	1.00	6%
Seam Inclination	0.14	1.00	0.11	0.13	0.13	0.33	0.11	0.25	0.25	1.00	0.14	0.11	0.14	0.13	1%
Coal Quality	8.00	9.00	1.00	9.00	0.50	3.00	1.00	8.00	5.00	8.00	4.00	2.00	4.00	6.00	17%
Gas Make	1.00	8.00	0.11	1.00	0.17	0.33	0.11	5.00	0.25	6.00	0.14	0.14	0.33	0.20	3%
Roof and Floor Strata	1.00	8.00	2.00	6.00	1.00	2.00	0.50	6.00	8.00	8.00	3.00	1.00	1.00	1.00	11%
Geological structure	1.00	3.00	0.33	3.00	0.50	1.00	0.25	3.00	6.00	9.00	0.50	0.50	1.00	1.00	6%
In-situ Stress	9.00	9.00	1.00	9.00	2.00	4.00	1.00	9.00	9.00	9.00	7.00	3.00	5.00	5.00	21%
Multiple Seam Mining	0.20	4.00	0.13	0.20	0.17	0.20	0.11	1.00	2.00	2.00	0.33	0.14	0.13	0.17	2%
Surface Restrictions	0.20	4.00	0.20	4.00	0.13	0.17	0.11	0.50	1.00	1.00	0.20	0.13	0.14	0.14	2%
Surface Subsidence	0.13	1.00	0.13	0.17	0.13	0.11	0.11	0.50	1.00	1.00	0.13	0.11	0.13	0.13	1%
Access to Reserve	1.00	7.00	0.25	7.00	0.33	2.00	0.14	3.00	5.00	7.69	1.00	0.33	1.00	2.00	7%
Reserve Losses (due to the layout)	3.00	9.00	0.50	7.00	1.00	2.00	0.33	7.00	8.00	9.00	3.00	1.00	4.00	1.00	11%
Seam Thickness	0.50	7.00	0.17	3.00	1.00	1.00	0.13	8.00	7.00	8.00	0.20	0.14	1.00	0.50	6%
Roof Cavability	1.00	8.00	0.17	5.00	1.00	1.00	0.20	6.00	7.00	8.00	0.50	1.00	2.00	1.00	7%

	Depth of Cover	Seam Inclination	Coal Quality	Gas Make	Roof and Floor Strata	Geological Structures	In-Situ Stress	Multiple Seam Mining	Surface Restrictions	Surface Subsidence	Access to Reserve	Reserve losses due to the layout	Seam Thickness	Roof Cavability	Priority Vector
Depth of Cover	1.00	7.00	0.13	1.00	1.00	3.00	0.11	7.00	8.00	8.00	1.00	0.33	2.00	2.00	7%
Seam Inclination	0.14	1.00	0.11	0.13	0.13	0.33	0.11	1.00	0.25	1.00	0.14	0.11	0.14	0.13	1%
Coal Quality	7.69	9.00	1.00	9.00	0.11	3.00	1.00	8.00	5.00	8.00	4.00	3.00	6.00	6.00	18%
Gas Make	1.00	7.69	0.11	1.00	0.17	0.33	0.11	5.00	0.11	6.00	0.11	0.14	0.33	0.20	3%
Roof and Floor Strata	1.00	7.69	2.00	6.00	1.00	6.00	0.50	6.00	7.00	8.00	4.00	1.00	1.00	1.00	12%
Geological Structures	0.33	3.00	0.33	3.00	0.17	1.00	0.25	7.00	8.00	9.00	0.50	1.00	1.00	1.00	5%
In-Situ Stress	9.09	9.00	1.00	9.00	2.00	4.00	1.00	9.00	9.00	9.00	7.00	3.00	8.00	5.00	22%
Multiple Seam Mining	0.14	1.00	0.13	0.20	0.17	0.14	0.11	1.00	2.00	2.00	0.33	0.14	0.13	0.17	1%
Surface Restrictions	0.13	4.00	0.20	4.00	0.14	0.13	0.11	0.20	1.00	1.00	0.20	0.13	0.14	0.14	2%
Surface Subsidence	0.13	1.00	0.13	0.17	0.13	0.11	0.11	0.50	1.00	1.00	0.13	0.11	0.11	0.13	1%
Access to Reserve	1.00	7.00	0.25	7.00	0.25	2.00	0.14	3.03	5.00	7.69	1.00	0.33	1.00	1.00	6%
Reserve losses due to the Layout	3.03	9.00	0.33	7.14	1.00	1.00	0.33	7.00	7.69	9.00	3.03	1.00	7.00	5.00	12%
Seam Thickness	0.50	7.00	0.17	3.00	1.00	1.00	0.13	8.00	7.00	7.69	0.20	0.14	1.00	0.50	5%
Roof Cavability	0.50	8.00	0.17	5.00	1.00	1.00	0.20	6.00	7.00	7.69	1.00	0.20	2.00	1.00	6%

**Figure 5-5: Comparison of Adjusted Judgment Matrices**

For this research, the most important conclusions from the case study are as follows:

- A poorly defined objective and scope for the analysis could lead to disproportionately large number of criteria and consequently too many hierarchies;
- A large number of criteria (14 in this case) can make it very difficult to reach consistency;
- Criteria should be clustered, thereby increasing the number of levels of the hierarchy in order to limit the number of elements per level. This improves understanding and consistency;

- A stringent adherence to the requirement of a consistency ratio less than 10 per cent is not necessary if the level of inconsistency is agreeable to the role players and appropriate for the level of study; and
- The independent analysis by the researcher in an attempt to improve consistency illuminates the repeatability of the AHP through the independent re-assessment of the judgements.

## 5.2 Original Case Study and Example

### 5.2.1 Case study 3: Mining Method Selection for a Mine in Africa

This case study describes the application of AHP by the researcher, to confirm or refute that an appropriate mining method has been selected for a pre-economic assessment of the underground mining portion of a green field deposit in North Africa.

The case serves to illustrate the following:

- The use of the AHP in an adaptation of a mining method selection study to serve as a confirmational study;
- The empowerment of judges to halt the analysis when it is deemed by them to have served its purpose;
  - Some of the judgements were very inconsistent. Ideally, these inconsistencies should be resolved, however, given the level of the study and the lack of available information, the analysis was deemed to have served its purpose, which was to confirm that the Step-Room-and-Pillar, and Drift- and-Fill mining methods were the most suitable for this deposit;
  - The AHP analysis could be different when assessing deeper mining areas as additional criteria related to the geomechanical properties of the rock mass and the in-situ stress regime would have to be considered. At a pre-economic assessment level of study, which is

equivalent to a concept study, this information was, as is usual, not available. Since an analysis would have been based on assumptions any way, and given that the initial focus would be on the shallower areas for early production, the judges considered that the identified selection criteria would be adequate to differentiate among the identified mining methods; and

- The relative importance of any given criterion does not necessarily qualify it as a good differentiator.

The study centred on a very large tabular copper deposit. The orebody dips gently and uniformly at less than 25 degrees and the thickness is fairly uniform between 3m and 6m. A large scale high production rate, mechanised mining method is required. Initial mining will target relatively shallow and easily accessible ore reserves from surface with declines which allow for a quick production ramp-up and early payback. A number of mining methods were identified with preference for Stepped Room and Pillar (SR&P) mining in the shallower area (less than 500m deep) and Drift and Fill (D&F) mining in the deeper areas.

A panel of experienced judges included two mining engineers that were very familiar with the mining methods considered, an electrical/mechanical engineer, who was also the project manager, a cost accountant and a senior mine planner. The researcher facilitated the analysis and also provided input pertaining to rock mechanics where required. The judges identified the mining method selection criteria reflected in Table 5-1.

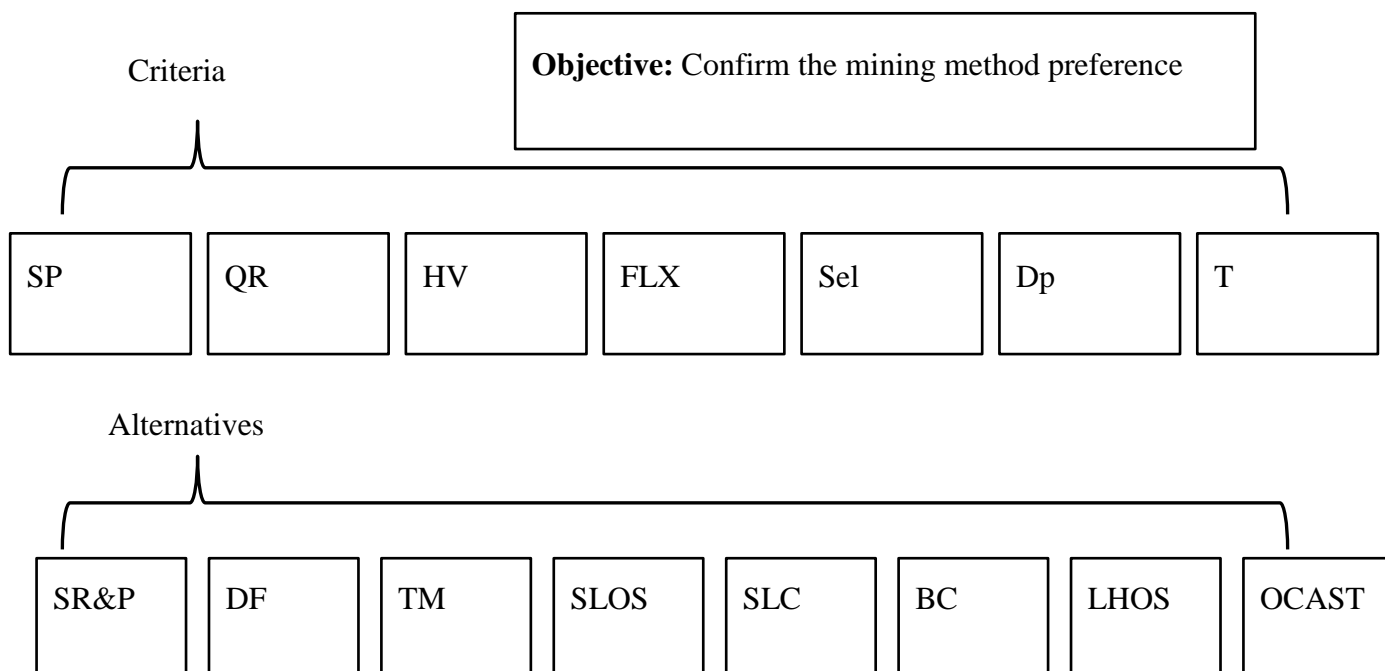
**Table 5-1: Mining Method Selection Criteria**

Criterion	Explanation
Short time to production (SP)	It is a requirement that the mining method will allow quick access to the orebody with early production commencement and a steep ramp-up profile.
Quick ROI (QR)	The stakeholders expect a quick return on investment.
High Volumes (HV)	The mining method should be capable of producing at least 3Mt of ore per year
Flexibility (FLX)	Although the orebody is uniform in most respects, mining flexibility is a requirement to sustain the high production rate.
Selectivity (SEL)	Especially during the initial phases of mining, it is essential that high grade ore be mined. The mining method should allow for the selection of such
Dip (Dp)	The mining method should be amenable to a consistent, relatively shallow dip
Thickness (T)	The mining method should be amenable to a uniform thickness of between 3m and 6m.

The mining methods that were considered are listed in Table 5-2. The first step in the AHP was to identify the objective and construct the hierarchy. After deliberation among the judges, it was agreed that the mining methods that are amenable to the criteria were already evident and the objective of the exercise was defined as **“To confirm or refute the appropriateness of the preferred mining methods”**. The AHP analysis would demonstrate adequately that the process of mining method selection has been thought through and engineered appropriately. The hierarchy that was constructed is shown in Figure 5-6.

**Table 5-2: Considered Mining Methods**

Abbreviation	Mining Method
SR&P	Step Room & Pillar mining
DF	Drift and Fill mining
TM	Trough Mining
SLOS	Sub-Level Open Stopping
BC	Block Caving
SLC	Sub-Level-Caving
LHOS	Long Hole open Stopping
OCAST	Open Cast



**Figure 5-6: The Mining Method Selection Hierarchy for a Mine in Africa**



The approximate method of analysis was used to evaluate the criteria with respect to the objective and to evaluate the alternatives with respect to the criteria. The pairwise comparison matrix for the analysis is shown in Figure 5-7. Also shown is the priority vector and the consistency ratio, which, at 3 per cent is acceptable. It can be seen that the dip and thickness of the ore-body were the most important considerations while the high volume criterion, which initially appeared to be of utmost importance, had the lowest priority. Seen in context, this does not mean that the high volumes criterion was the least important, but rather that it was not a good differentiator. This subtlety might only have surfaced after much more effort if the AHP was not applied. This point also demonstrates that fixed criteria do not necessarily require consideration: all the mining methods considered were capable of producing 3Mt/a of ore. For best performance of the AHP, criteria should not be fixed. For instance, a requirement from a client that the IRR must be at least 20 per cent is a very poor criterion because it automatically excludes many factors which might have been considered to derive an optimal IRR. More appropriately the criterion ought to be “the best IRR”.

The geometry of the ore body was highlighted in this analysis as the biggest differentiator. At first this might appear odd, however, the judges correctly argued that if the mining method is not appropriate to the orebody, then most of the other criteria could not be realised. For instance, in this analysis, the judges deemed the dip of the orebody to be five times more important than a quick return. The argument that supports this judgement is that if the incorrect mining method were to be applied a quick return could not be realised. The final synthesis table is also shown in Figure 5-7.

Criteria Ranking w.r.t. Goal		SP	QR	HV	FLX	SEL	Dp	T
Short time to production	SP	1	1/3	3	1/3	1	1/4	1/4
Quick ROI	QR	3	1	5	2	2	1/5	1/5
High Volumes	HV	1/3	1/5	1	1/3	1/4	1/4	1/4
Flexibility	FLX	1/3	1/2	3	1	1	1/3	1
Selectivity	SEL	1	1/2	4	1	1	1/2	3
Dip	Dp	4	5	4	3	2	1	1
Thickness	T	4	5	4	1	1/3	1	1
		<b>13 2/3</b>	<b>12 1/2</b>	<b>24</b>	<b>8 2/3</b>	<b>7 4/7</b>	<b>3 1/2</b>	<b>6 5/7</b>

Criterion	Priority (%)
SP	7.76%
QR	17.65%
HV	3.74%
FLX	8.85%
SEL	11.14%
Dp	29.19%
T	21.68%

**Table 5-3: Final Synthesis**

	SP	QR	HV	FLX	SEL	Dp	T	Score
SR&P	21%	2%	32%	32%	31%	35%	37%	31%
DF	21%	4%	32%	32%	31%	35%	19%	28%
TM	14%	6%	7%	7%	12%	12%	12%	11%
SLOS	12%	18%	9%	9%	12%	4%	9%	9%
SLC	12%	32%	7%	7%	2%	4%	9%	7%
BC	3%	6%	2%	2%	2%	4%	3%	3%
LHOS	12%	47%	10%	10%	12%	4%	9%	9%
CR	<b>2.3%</b>	<b>10.3%</b>	<b>51%</b>	<b>4.8%</b>	<b>12.1%</b>	<b>16.1%</b>	<b>3.2%</b>	

**Figure 5-7: The Pairwise Comparison Matrix and Final Synthesis for the Mine in Africa Analysis**

The consistency ratio for each of the criteria is listed in Table 5-4. Several consistency ratios exceed 10 per cent. Despite the inconsistencies, the judges were satisfied that the analysis had achieved its objective. The inconsistencies were ascribed to insufficient understanding of the subject matter due to lack of information, which is normal for such a high-level study. Using the exact method of calculating priorities might well have improved the inconsistencies; however, the judges did not deem it justifiable to spend more resources to effect these refinements since the preferred mining methods were shown to be the most suitable by far. The exception to this was the very high inconsistency with respect to volumes, which demonstrates the difficulty that the judges had in reconciling the expectation that this ought to be of utmost importance with the fact that it was not an important criterion. With the realisation that high volumes was a very important criterion, but a poor differentiator,

the judges decided that improving this inconsistency would not change the outcome of the analysis and it was therefore not worthy of the required resources.

If the differential between mining methods had been smaller, it would have been necessary to investigate the inconsistencies more thoroughly, and ideally sensitivity analyses would have been conducted. In such a case, the analysis would have been repeated, but the high volumes criterion would have been excluded.

As a separate exercise by the researcher, the criteria matrix was re-evaluated with the exclusion of high volumes. Table 5-5 compares the results with the original analysis and shows that the difference is negligible.

**Table 5-4: Consistency ratios for the selection criteria**

Criterion	Consistency Ratio
Short time to production	2.3%
Quick return on investment	10.3%
High volumes	51.4%
Flexibility	4.8%
Selectivity	12.1%
Dip	16.1%
Thickness	3.2%

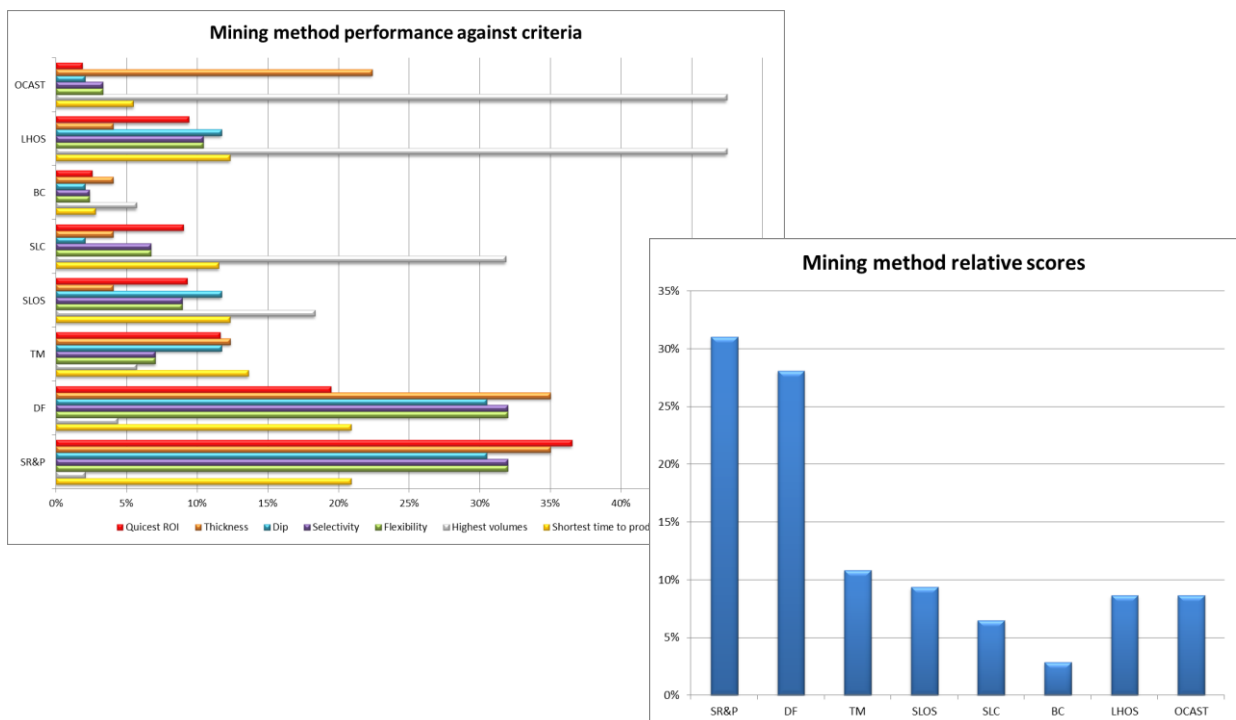
**Table 5-5: Comparison of original with re-evaluated priorities**

Criterion	Original PV	Re-evaluated PV
Short time to production	7.76%	6.30%
Quick return on investment	17.65%	14.67%
High volumes	3.74%	
Flexibility	8.85%	9.25%
Selectivity	11.14%	15.83%
Dip	29.19%	28.90%
Thickness	21.68%	21.39%
Consistency Ratio	3%	2%

Figure 5-8 shows two examples of the presentation of the results, which the decision makers found useful for interpretation.

The one graph shows how the decision criteria relate to the mining methods. This is viewed in the context of how well one mining method is suited to any given criteria as compared to another mining method. In this graph it can be seen that although some mining methods are very well suited to some of the criteria, their performance in respect of the rest are very poor. For example, opencast mining can deliver very high volumes, but it performs poorly in all the other criteria. On the other hand, Step-Room-and-Pillar and Drift-and-Fill score consistently high in the criteria that were found to be most important, namely the dip and thickness of the ore body and the quickest return on investment.

The other graph shows the overall relative performance of the mining methods and informs the initial objective, namely to confirm that the two preferred mining methods were indeed the most suitable to the ore body.



**Figure 5-8: Examples of Presentation of Results**

The analysis confirmed that the Step Room and Pillar and Drift and Fill mining methods were best suited to the orebody by considerable margins. These were also the preferred mining methods.

Although this analysis was not a selection exercise *per se*, it was beneficial in the sense that it confirmed in a scientific and methodical way that the two preferred mining methods were the most suited to the circumstances. This afforded a high level of confidence to the project mining engineers and provided them with a means to present a persuasive and documented argument.

This case study demonstrated the following:

- The AHP can be used effectively to confirm design decisions already made (This is a form of what is known in AHP as backward planning; that is, given a desired future outcome, what present actions and decisions are required);
- The objective and scope of the analysis should be well defined, considered and understood to prevent wasteful application of resources in refining results that are not beneficial; and
- The relative importance of any given criterion does not necessarily qualify it as a good differentiator. This proved to be the case with high volumes in this study. Another example of such a criterion is safety. Since safety is an overriding consideration for any mining activity, it cannot be used as a differentiator. If a mining method is inherently incapable of being safe it is automatically disqualified from further consideration.

#### 5.2.2 Case study 4: Example of a Fault Tree Application

This is not truly a case study but an illustration of how the AHP could be employed in conjunction with fault tree techniques to find the root causes of a fall of ground incident. The case is based on the rock engineering experience and the collective, composite memory of a great number of fall of ground incident investigations and

enquiries into fatal fall of ground accidents on the part of author over a period of more than 30 years.

Part of a typical fault tree for a fall of ground incident is shown in Figure 5-9. This fault tree can be used directly for an AHP hierarchy.

First, some basic causes were identified based on in loco inspections and preliminary enquiries. The basic causes could come from a prompt list or flow from a discussion or workshop among stakeholders. For the sake of brevity, it was assumed that three basic causes were identified by the investigators into this hypothetical case. These three causes were as follows:

- Discontinuous rock walls (DC);
- Deficient support (SD); and
- Inadequate barring (IB)

A pairwise comparison was carried out for the three identified basic causes with respect to the fall of ground incident and the consistency was measured and adjusted. The results are shown in Table 5-6. The determined priorities with respect to the root (Fall of Ground Incident) showed that support deficiency was the main basic cause with 63% contribution, followed by inadequate barring (26%) and the smallest contributor was discontinuous rock (11%). The consistency ratio for the analysis was just more than 5 per cent, which is acceptable.

**Table 5-6: Pairwise comparison of root causes for the fault tree analysis of a fall of ground**

Pairwise Comparison					Normalised Matrix			Priority Vector	Consistency analysis	
	D	SD	IB		DC	SD	IB	PV	$\lambda_{max}$	3.0554
DC	1	1/5	1/3	D	0.111	0.130	0.076	0.1062	Ci	0.028
SD	5	1	3	S	0.555	0.652	0.692	0.6333	Ri	0.52
IB	3	1/3	1	IB	0.333	0.217	0.230	0.2605	Ci	5.3%
Su	9	1	4		1.000	1.000	1.000			

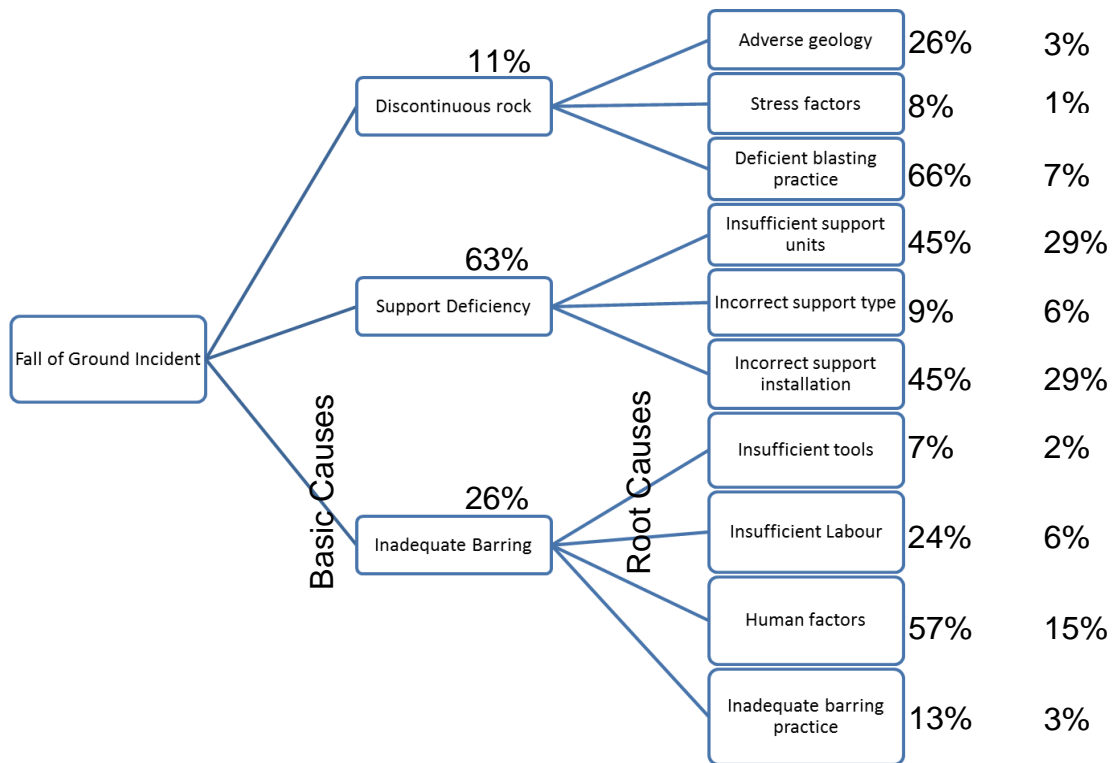
Next, root causes were compared in a pairwise manner with respect to each basic cause and consistency measured and adjusted. The priorities for each of the root causes (for example geology, incorrect support type, etc.) were determined with respect to each basic cause on the level above. This is summarised in Table 5-7 which also indicates the consistency ratio for each of the root causes. According to the analysis, the highest scoring root cause of discontinuous rock was deficient blasting practice. Insufficient support units and incorrect support installation together contributed 90 per cent to support deficiency and human factors made a 57 per cent contribution to inadequate barring.

Figure 5-9 shows the completed fault tree up to this level of the investigation. The percentages shown to the far right in Figure 5-9 were obtained by the product of the contribution of each basic cause to the incident and the contribution of each root cause to each basic cause. For example the overall priority for Human Factors is  $26\% \times 57\% = 15\%$ .

**Table 5-7: Pairwise comparison of root causes for a fall of ground investigation**

DC	Geo	Stress	Blast		DC	Geo	Stress	Blast	PV	λmax	3.0489	
Geo	1	4	1/3		Geo	0.2353	0.3333	0.2258	0.2648	Ci	0.024	
Stress	1/4	1	1/7		Stress	0.0588	0.0833	0.0968	0.0796	Ri	0.52	
Blast	3	7	1		Blast	0.7059	0.5833	0.6774	0.6555	Ci	4.7%	
	4 1/4	12	1 1/2			1.0000	1.0000	1.0000				
SD	Ins	IncT	Incl		SD	Ins	IncT	Incl	PV	λmax	3.000	
Ins	1	5	1		Ins	0.4545	0.4545	0.4545	0.4545	Ci	0.000	
IncT	1/5	1	1/5		IncT	0.0909	0.0909	0.0909	0.0909	Ri	0.520	
Incl	1	5	1		Incl	0.4545	0.4545	0.4545	0.4545	Ci	0%	
	2 1/5	11	2 1/5			1	1	1				
IB	Tools	Lab	HF	Proc	IB	Tools	Lab	HF	Proc	PV	λmax	4.17878
Tools	1	1/3	1/7	1/3	Tools	0.0714	0.0714	0.0852	0.0357	0.0659	Ci	0.0596
Lab	3	1	1/3	3	Lab	0.2143	0.2143	0.1989	0.3214	0.2372	Ri	0.89
HF	7	3	1	5	HF	0.5000	0.6429	0.5966	0.5357	0.5688	Ci	6.7%
Proc	3	1/3	1/5	1	Proc	0.2143	0.0714	0.1193	0.1071	0.1280		
	14	4 2/3	1 2/3	9 1/3		1	1	1	1			





**Figure 5-9: Partial Fault Tree for a Fall of Ground Incident**

The analysis in this example shows that the root causes of the fall of ground incident are “insufficient support” and “incorrect support installation”. In addition, human factors may have contributed to inadequate barrering which is the second most likely root cause.

The root cause analysis could now continue with these three branches of the fault tree, the other root causes having been excluded because of their relatively low contributions. Each of the three remaining elements could now be deconstructed further and AHP applied to every stage. For example, insufficient support may have basic causes such as insufficient supplies and deficient support standards. In turn, insufficient supplies may be ascribed to inadequate stock control and late delivery and so on. Human factors might have branched into elements such as poor supervision and inadequate training. At each level, the adjudged contributions are prioritised with the aid of AHP and the lowest causes are continually discarded until the root causes are established. It should be noted that, once the root causes have been established, AHP can again be used to determine the

most appropriate ameliorating measures (e.g. training, improved stock control and delivery systems etc.) to prevent future incidents.

The significance of this type of analysis is that the root causes of an incident can be accurately identified in a transparent and repeatable fashion, greatly reducing the emotional element and/or mechanical application invariably associated with this type of analysis. A record of the analysis is automatically provided and accordingly, the AHP can be applied to find the most effective ameliorating measures.

Although this was not truly a case study, it clearly demonstrated how the AHP could be combined with fault tree analyses to find the true root causes of any incident.

### **5.3 Conclusions and Insights gained from the presented case studies**

The following list serves as a summary of the conclusions and insights gained from the case studies presented in this chapter:

- The AHP is an effective MCDA method for mining applications;
- AHP can be effectively applied to optimisation studies in the mining industry;
- The AHP can be fully integrated into planning and design processes such as the design wheel published by Stacey, (2009);
- Results from the AHP can be integrated effectively with any graphical representation of choice;
- A poorly defined objective and scope for an AHP analysis could lead to disproportionately large numbers of criteria and consequently too many hierarchies;
- A large number of criteria (14 in one case study) can make it very difficult to reach consistency;

- When there are many criteria, they should be clustered, thereby increasing the number of levels of the hierarchy in order to limit the number of elements per level. This improves understanding and consistency;
- A stringent adherence to the requirement of a consistency ratio less than 10 per cent is not necessary if the level of inconsistency is agreeable to the role players and appropriate for the level of study. It is of critical importance, however, that such agreement be based on a thorough understanding of the reasons for and the impact of such inconsistencies;
- The AHP is a repeatable methodology. That is, given the same set of circumstances, similar rankings and priorities and hence conclusions and decisions should be reached. Repeatability also means that a repetition of the analyses based on better information and improved understanding should yield better results;
- By applying forward and backward planning, the AHP can be used effectively to confirm design decisions already made;
- The objective and scope of an analysis should be well defined, considered and understood to prevent wasteful application of resources in refining results that are not beneficial; and
- The relative importance of any given criterion does not necessarily qualify it as a good differentiator. This is particular to criteria that are ubiquitous to all the choices. Examples are high volumes and safety.

## 5.4 Summary of Chapter 5

Chapter 5 dealt with examples and case studies of the application of AHP in order to demonstrate its potential value to the mining industry. It was illustrated how AHP was applied in the following cases:

- A Fault Tree analysis;
- Level and raise spacing optimisation;
- Long wall Layout Selection; and
- Mining Method selection.

Conclusions and insights gained from the case studies were presented to close the chapter.

# Chapter 6 Summary of the Methodology of the AHP

Chapter 5 dealt with examples and case studies of the application of AHP in order to demonstrate its potential value to the mining industry. It was illustrated how AHP could be applied in fault tree analyses, selection of various mining layouts and mining method selection. Conclusions and insights gained from the case studies were presented to close that chapter.

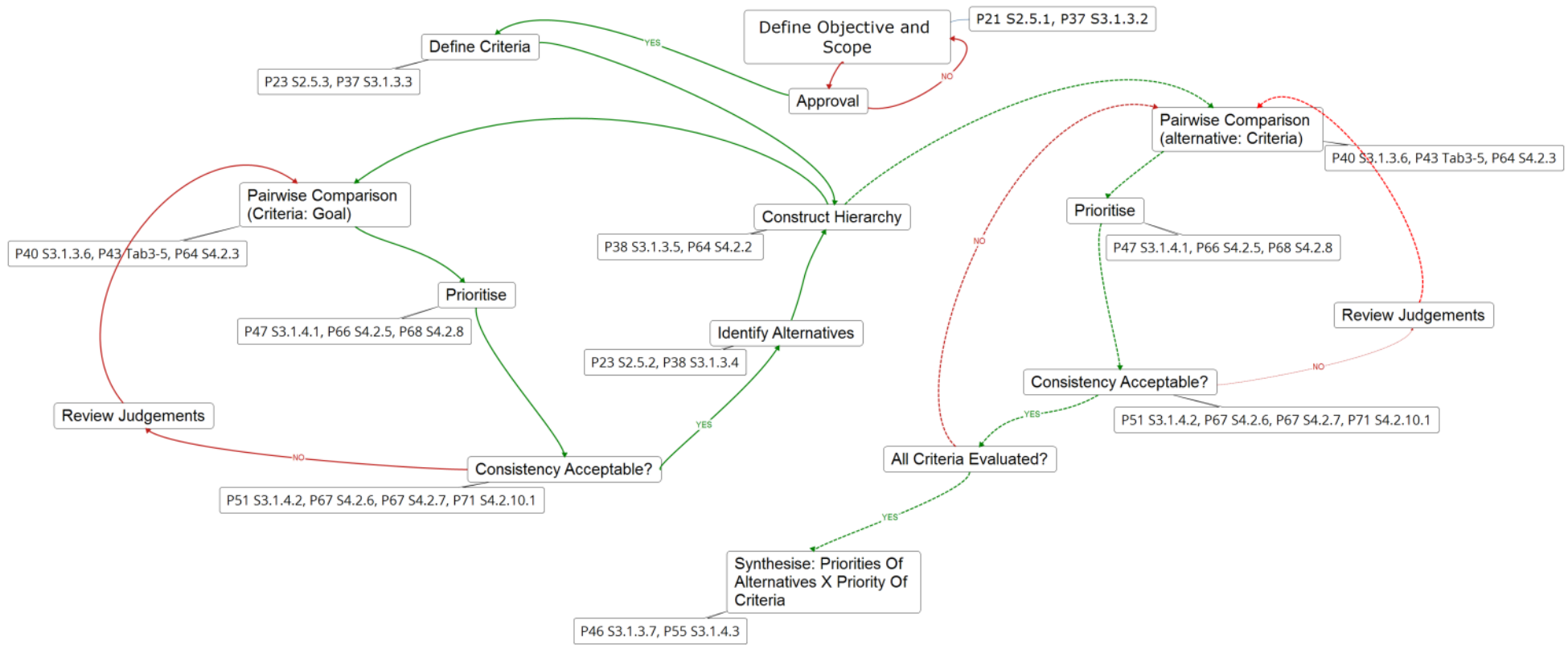
Chapter 6 summarises the methodology described in the report and is intended to serve as a quick reference guide for carrying out AHP assessments. A generic flowchart is given to help as a guide to the process.

## 6.1 An Outline of the Steps

The AHP starts with the laying out the elements of a problem as a hierarchy. Next, paired comparisons are made among the elements of a level as required by the criteria of the next higher level. These comparisons give rise to priorities and finally, through synthesis, to overall priorities. Consistency of judgement is measured for the paired comparisons throughout the process (Saaty, 2012). The basic steps of the process are outlined below:

1. Define the problem: Identify the objective and scope, criteria and alternatives;
2. Structure the hierarchy;
3. Obtain judgements and carry out pairwise comparisons;
4. Obtain priorities and measure consistencies; and
5. Synthesise the priorities.

To facilitate visualisation of the process, a generic flow diagram is presented in Figure 6-1. The relevant page number and section where the discussion of the concepts can be found are also shown. An index is also provided in this chapter rather than at the end of the report to facilitate finding discussions on important and relevant concepts.



**Figure 6-1: Generic Flow Diagram of the AHP Process**

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## Chapter 7 Conclusions

This research report shows that to deal with the complexities inherent in decisions in mining, a structured approach is required, according to which the substance of the decision can be broken down into its components. Several less complex decisions are aggregated to reach an optimal solution. This process is widely known as Multi Criteria Decision-Making.

It is shown, through extensive literature research that the Analytic Hierarchy Process has become a significant decision-making tool that has gained exponential popularity over the last three decades in all sectors, from government, the military, all industries and agriculture. Its application has also increased in the mining industry.



The widespread use of AHP could be ascribed to its simplicity, its capability of simplifying complex problems, the inherent capability that it provides to compare tangible and intangible criteria on the same scale and the means for evaluating the consistency of subjective judgements. Another one of its strengths is the facility to determine the relative importance among decision criteria, a feature that is not prominent in other ranking systems. It is further shown that the application of AHP is within the grasp of anyone with access to spreadsheet software and that it is not in the sole domain of consultants and academics.

The report systematically describes each step in the process with simple examples using Microsoft Excel. The Excel functions used are fully described. A selection of case studies is presented to show the effectiveness of the process and a summary is given with cross references to examples that facilitate the use of the process. It is also shown that the AHP can be applied effectively to decision-making in the mining industry and guidelines are provided to facilitate such applications.

# Appendix A Illustrative Example

## A.1 Constructing the Hierarchy

The hierarchy for the problem in Chapter 4 is indicated in Figure A-1.

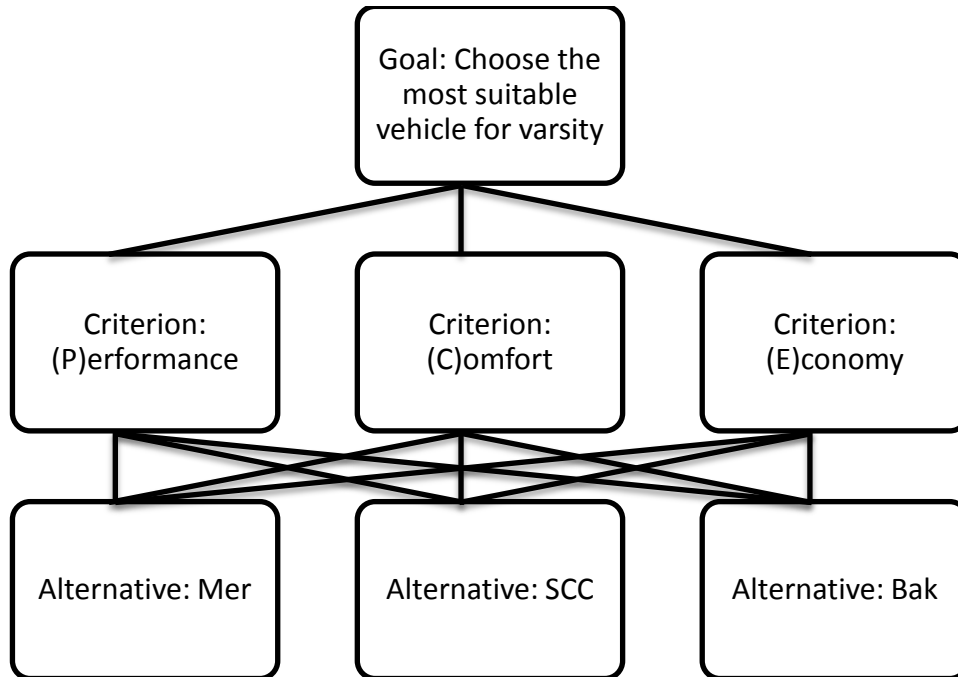


Figure A-1: Hierarchy for choosing the Best Vehicle

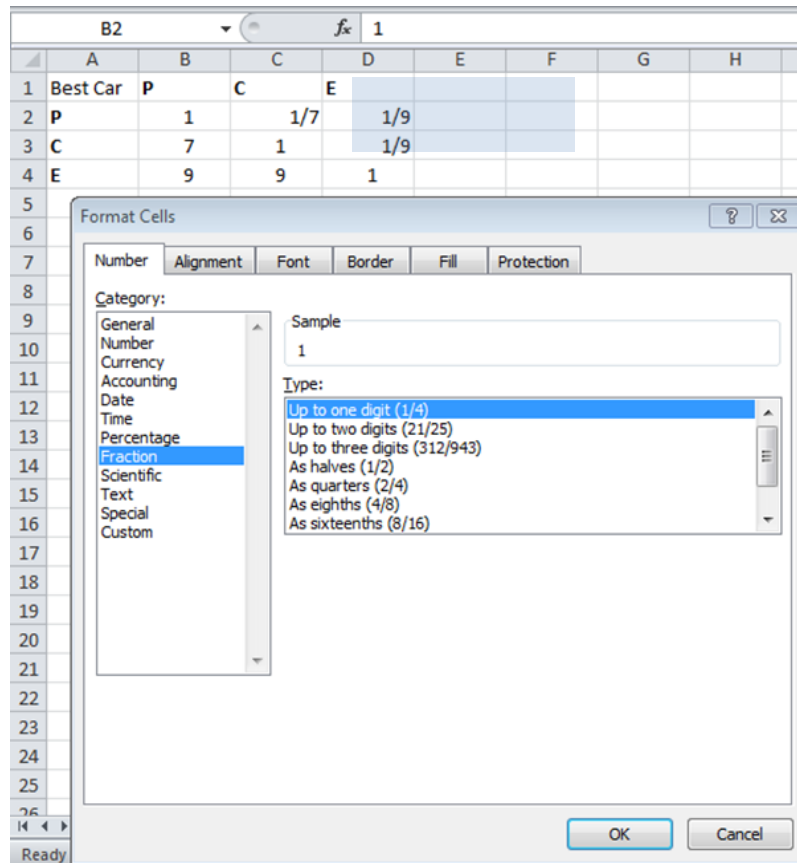
## A.2 Pairwise Comparison

Figure A-2 illustrates the preparation for a Pairwise Comparison

	A1	fx Best Car			
	A	B	C	D	E
1	Best Car	P	C	E	
2	P	1			
3	C		1		
4	E			1	
5					

Figure A-2: A prepared Array for Pairwise Comparison

Figure A-3 shows how to format cells to display fractions.



**Figure A-3: Formatting Cells to display Fractions**

The completed comparison matrix for the criteria of Chapter 4 is shown in Figure A-4.

A1		Best Car			
	A	B	C	D	E
1	Best Car	P	C	E	
2	P	1	1/7	1/9	
3	C	7	1	1/9	
4	E	9	9	1	
5					

**Figure A-4: Completed Comparison Matrix for Criteria**

### A.3 Normalising the Matrix

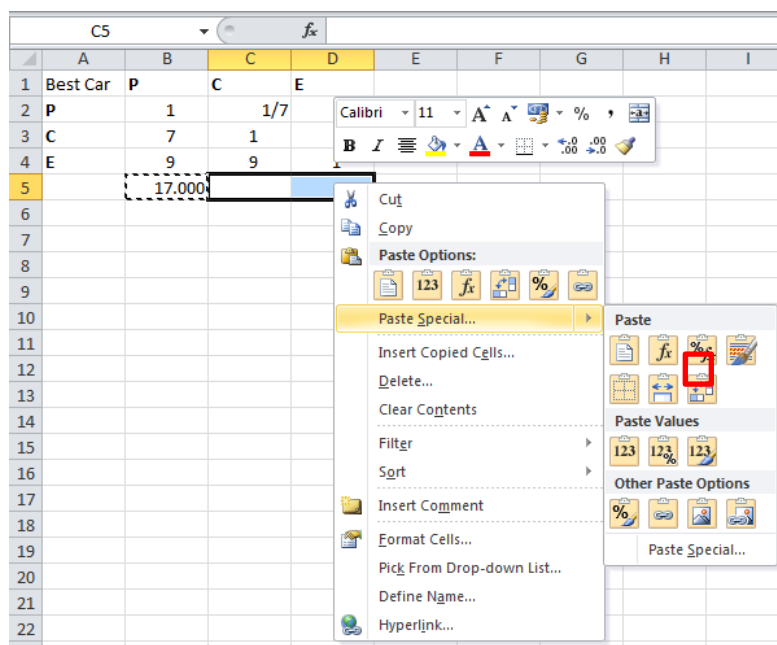
To normalise the matrix, divide each value in every column by the sum of the values in that column.

Enter Excel's sum formula in cell B5 as shown in Figure A-5.

- Then copy that formula into cells C5 and D5 using the paste function as shown in Figure A-6

	A	B	C	D	E
1	Best Car	P	C	E	
2	P	1	1/7	1/9	
3	C	7	1	1/9	
4	E	9	9	1	
5		=sum(B2:B4)			
6					

**Figure A-5: Sum the Columns**



**Figure A-6: Copy and paste the Formula**

The results are shown in cells B5, C5 and D5 in Figure A-7

A1		fx				Best Car
	A	B	C	D	E	
1	Best Car	P	C	E		
2	P	1	1/7	1/9		
3	C	7	1	1/9		
4	E	9	9	1		
5		17.000	10.143	1.222		
6						

**Figure A-7: The Comparison Matrix ready for Normalisation**

The normalised matrix is now constructed by creating a new array equal in size to the comparison matrix (i.e. 3 by 3). The value in each cell of the new array is obtained by dividing the value in the corresponding cell of the comparison matrix by the sum of that column. The simplest and quickest way to do this normalisation is to make use of an array function. Proceed as follows:

- select the cell range F2:H4 as shown in Figure A-8;
- Type the equals sign (=) as shown in Figure A-9;
- Next, select the comparison matrix range B2:E4 as shown in Figure A-10;
- Type the divide operator (/) and;
- then select the sums of the columns in the comparison matrix, which are cells B4:E4 as shown in Figure A-11; and
- Now press and hold the Ctrl and Shift keys and hit Enter.

F2		fx							
	A	B	C	D	E	F	G	H	I
1	Best Car	P	C	E					
2	P	1	1/7	1/9					
3	C	7	1	1/9					
4	E	9	9	1					
5		17.000	10.143	1.222					
6									

**Figure A-8: Select the Array for the Normalised Matrix**

CODE									
	A	B	C	D	E	F	G	H	I
1	Best Car	P	C	E					
2	P	1	1/7	1/9		=			
3	C	7	1	1/9					
4	E	9	9	1					
5		17.000	10.143	1.222					
6									

Figure A-9: Type “=”

CODE									
	A	B	C	D	E	F	G	H	I
1	Best Car	P	C	E					
2	P	1	1/7	1/9		=B2:D4			
3	C	7	1	1/9					
4	E	9	9	1					
5		17.000	10.143	1.222					
6									

Figure A-10: Select the Corresponding Cells in the Comparison Matrix.

CODE									
	A	B	C	D	E	F	G	H	I
1	Best Car	P	C	E					
2	P	1	1/7	1/9		=B2:D4/B5:D5			
3	C	7	1	1/9					
4	E	9	9	1					
5		17.000	10.143	1.222					
6									

Figure A-11: Divide by the Sums

The normalised matrix appears in cells F2 to H4 as shown in Figure A-12. This matrix now contains the values of each element of the comparison matrix divided by the sum of the column in which the element appears. So  $1/17 = 0.058824$  (B2/B5). Likewise  $9/10.143 = 0.887324$  (C4/C5).

		F2			fx {=B2:D4/B5:D5}				
	A	B	C	D	E	F	G	H	I
1	Best Car	P	C	E					
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	
3	C	7	1	1/9		0.411765	0.098592	0.090909	
4	E	9	9	1		0.529412	0.887324	0.818182	
5		17.000	10.143	1.222					
6									

Figure A-12: Normalised Matrix completed

#### A.4 Prioritising

To calculate the priority vector enter Excel's built-in "Average" formula in cell I2 and then copy and paste the formula into cells I3 and I4 (Figure A-13). Be sure to paste the formula, not the value.

		I4			fx =AVERAGE(F4:H4)					
	A	B	C	D	E	F	G	H	I	J
1	Best Car	P	C	E					PV	
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	0.054606	
3	C	7	1	1/9		0.411765	0.098592	0.090909	0.200422	
4	E	9	9	1		0.529412	0.887324	0.818182	0.744973	
5		17.000	10.143	1.222						
6										

Figure A-13: Average the Rows

#### A.5 Test for Consistency

The principal eigenvalue is used as a measure of the consistency. The closer this value is to the number of elements in the diagonal of the matrix, the more consistent the judgements. To calculate the principal eigenvector, the MMULT function is used to multiply the sums of the columns in the comparison matrix to the averages of the rows of the normalised matrix. This process is illustrated in Figure A-14.

CODE											
=mmult(B5:D5,I2:I4)											
	A	B	C	D	E	F	G	H	I	J	K
1	Best Car	P	C	E					PV		
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	0.054606	Lambda	=mmult(B5:D5,I2:I4)
3	C	7	1	1/9		0.411765	0.098592	0.090909	0.200422		
4	E	9	9	1		0.529412	0.887324	0.818182	0.744973		
5		17.000	10.143	1.222							

K2											
=MMULT(B5:D5,I2:I4)											
	A	B	C	D	E	F	G	H	I	J	K
1	Best Car	P	C	E					PV		
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	0.054606	Lambda	3.871669
3	C	7	1	1/9		0.411765	0.098592	0.090909	0.200422		
4	E	9	9	1		0.529412	0.887324	0.818182	0.744973		
5		17.000	10.143	1.222							
6											
7											

**Figure A-14: Determine the Principal Eigenvalue**

The Consistency Index is determined through Eq 3-1. The way it can be done in Excel is shown in Figure A-15, in the formula bar. The count function returns the number of values in a horizontal or vertical array. Finally, the Consistency Ratio is determined by dividing the Consistency Index by the Random Consistency index, which is 0.52 for a 3 by 3 matrix according to Table 3-14. For the example, the consistency ratio so calculated comes to 0.84 (Figure A-16)

K3											
=(K2-COUNT(F2:H2))/(COUNT(F2:H2)-1)											
	A	B	C	D	E	F	G	H	I	J	K
1	Best Car	P	C	E					PV		
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	0.054606	$\lambda_{max}$	3.871669
3	C	7	1	1/9		0.411765	0.098592	0.090909	0.200422	Ci	0.435834
4	E	9	9	1		0.529412	0.887324	0.818182	0.744973		
5		17.000	10.143	1.222							
6											
7											

**Figure A-15: Determine the Consistency Index**

K5											
=K3/K4											
	A	B	C	D	E	F	G	H	I	J	K
1	Best Car	P	C	E					PV		
2	P	1	1/7	1/9		0.058824	0.014085	0.090909	0.054606	$\lambda_{max}$	3.871669
3	C	7	1	1/9		0.411765	0.098592	0.090909	0.200422	Ci	0.435834
4	E	9	9	1		0.529412	0.887324	0.818182	0.744973	Ri	0.52
5		17.000	10.143	1.222						CR	0.838143
6											

**Figure A-16: First calculation of consistency ratio**



Inspecting the principal eigenvalue (or  $\lambda_{\max}$ ) immediately reveals that it is closer to 4 than 3 (recall that for a consistent reciprocal square matrix, the principal eigenvalue is equal to the number of elements on the diagonal).

## A.6 Improving the Consistency

In the preceding comparison matrices, cells B4 and C4 contain the same value, 9. In other words, economy is 9 times more important than performance and it is 9 times more important than comfort. It should logically follow that comfort and performance are equally important, yet the judgment was that comfort was 7 times more important than performance. The equality in cells B4 and C4 is therefore not logical. The comparison of economy and comfort must be adjusted to achieve improved consistency. The student first adjusts the value in C2 to 1/5, which brings the consistency ratio down to 0.340 using the exact method. She then adjusts the value in D3 to 1/7, which results in a consistency ratio of 0.04. Before accepting the matrix, the judges (in this example the student) has to be satisfied that this is an accurate reflection of their judgement. If P:C=1/5 and P:E=1/9, is it reasonable to expect that C:E is somewhere between 1/9 and 1/5? The answer is yes, and so the matrix can be accepted.

## A.7 Prioritising the Options with respect to the Criteria

The third level of the hierarchy (Figure A-17) is next considered and the vehicles are compared to each other with respect to Performance, Comfort and Economy. The prepared matrix to compare the vehicles with respect to Performance is shown in Figure A-18.

	A	B	C	D
7	P	Mer	SCC	Bak
8	Mer	1		
9	SCC		1	
10	Bak			1
11				
12				

**Figure A-17: Comparison Matrix for Performance**

Considering, for argument sake, speed as a measure of performance then from the scope Mer is capable of good performance, SCC is not quite that fast but is adequate and Bak can be consider as slow. The student judges that the Mer outperforms the SCC by 5 and the Bak by 7. The row weighs more than the column so the whole numbers are entered in cells C8 and D8 as illustrated in Figure A-19.

	A	B	C	D
7	P	Mer	SCC	Bak
8	Mer	1	5	7
9	SCC		1	
10	Bak			1
11				

**Figure A-18: Comparing Mer to SCC and Bak with respect to Performance**

Instead of typing in the reciprocals cells B9 and B10, Excel can be used to calculate the reciprocals. First format the cells of the array to display fractions and then select cells B9:B10 and type this:

=1/transpose(

Select cells C8:D8 and close the brackets like in Figure A-19.

	A	B	C	D	E	F
7	P	Mer	SCC	Bak		
8	Mer	1	5	7		
9	SCC	=1/transpose(C8:D8)				
10	Bak			1		
11						

**Figure A-19: Using the Transpose Function in an Array to calculate the Reciprocals**

Now hold the Ctrl and Shift keys and hit Enter. The matrix should look like the one in Figure A-20. Notice the braces around the formula in the formula bar that Excel inserts automatically to indicate that this is an array function. In small arrays like the ones used in this example, it seems trivial to bother with this reciprocal-transpose function, however it goes a long way towards automating calculations in larger arrays. When adjustments have to be made, only the values above the diagonal need to be changed and all the other calculations will follow automatically.

		B9				
		fx {=1/TRANSPOSE(C8:D8)}				
	A	B	C	D	E	F
7	P	Mer	SCC	Bak		
8	Mer	1	5	7		
9	SCC	1/5	1			
10	Bak	1/7		1		

**Figure A-20: Completing the Array Formula for the Reciprocal Values.**

Returning to the example, the student judges that SCC is at least 3 times better in performance than Bak. She enters 3 in cell D9 and a function, =1/D9 in cell C10, once again to automate adjustments should these be required. The completed comparison matrix for Performance is shown in Figure A-21.

		C10			
		fx =1/D9			
	A	B	C	D	
7	P	Mer	SCC	Bak	
8	Mer	1	5	7	
9	SCC	1/5	1	3	
10	Bak	1/7	1/3	1	
11					

**Figure A-21: Completed Comparison Matrix in respect of Performance.**

Notice that, having learned from the comparison matrix for the criteria; the student is now more moderate in her judgments.

The exact method will be illustrated next and used for the rest of the example. This method involves squaring the comparison matrix successively until the differences in eigenvector (priority vector) values are below a predetermined limit; usually when there is no difference in the fourth decimal.

The first step is to sum each row of the comparison matrix. Then the sum of the sums is computed as illustrated in Figure A-22.

H30					
	A	B	C	D	E
7	P	Mer	SCC	Bak	
8	Mer	1	5	7	13
9	SCC	1/5	1	3	4 1/5
10	Bak	1/7	1/3	1	1 1/2
11					18 2/3

B8+C8+D8

B9+C9+D9

B10+C10+D10

E8+E9+E10

**Figure A-22: Summing up the Comparison Matrix**

The sum of each row (E8:E10) is then divided by the sum of the sums (E11) to find the priority vector as in Figure A-23. Note that an array formula was once again used as indicated by the curly braces in the formula bar.

F8						
	A	B	C	D	E	F
7	P	Mer	SCC	Bak		PV1
8	Mer	1	5	7	13	0.6961
9	SCC	1/5	1	3	4 1/5	0.2249
10	Bak	1/7	1/3	1	1 1/2	0.0790
11					18 2/3	

**Figure A-23: Find the Priority Vector**

The sequence of activities that follows has to be carried out only once, thereafter copying and pasting will do the work.

## A.9 Squaring the Matrix

Select cells B13:D15 and type

=mmult(

in the formula bar as shown in Figure A-24.

	CODE	B	C	D	E
7	P	Mer	SCC	Bak	
8	Mer	1	5	7	13
9	SCC	1/5	1	3	4 1/5
10	Bak	1/7	1/3	1	1 1/2
11					18 2/3
12	P	Mer	SCC	Bak	
13	Mer	=mmult(			
14	SCC	MMULT(array1, array2)			
15	Bak				

Figure A-24: Preparing to square

Then select the cells B8:D10 of the comparison matrix and type a comma as shown in Figure A-25.

	CODE	B	C	D	E
7	P	Mer	SCC	Bak	
8	Mer	1	5	7	13
9	SCC	1/5	1	3	4 1/5
10	Bak	1/7	1/3	1	1 1/2
11					18 2/3
12	P	Mer	SCC	Bak	
13	Mer	=mmult(B8:D10,			
14	SCC	MMULT(array1, array2)			
15	Bak				

Figure A-25: Selecting the first Array

After the comma, select the same array again, that is cells B8:D10, close the brackets and press Ctrl-Shift-Enter to tell Excel that it is an array formula. The result should look like Figure A-26.

		B13					fx
		{=MMULT(B8:D10,B8:D10)}					
	A	B	C	D	E	F	
7	P	Mer	SCC	Bak		PV1	
8	Mer	1	5	7	13	0.6961	
9	SCC	1/5	1	3	4 1/5	0.2249	
10	Bak	1/7	1/3	1	1 1/2	0.0790	
11					18 2/3		
12	P	Mer	SCC	Bak			
13	Mer	3.0000	12.3333	29.0000			
14	SCC	0.8286	3.0000	7.4000			
15	Bak	0.3524	1.3810	3.0000			

**Figure A-26: Completed Matrix Squaring**

Copy the sums and priority vector formulae from the comparison matrix to the same relative position for the squared matrix. To do this select cells E8:F11, press Ctrl and “c” simultaneously to copy the selected region to the clipboard, then place the cursor in cell E13 and press Ctrl and “v” simultaneously. Ctrl-c copies selected cells to the clipboard and Ctrl-v pastes anything from the clipboard into selected cells.

The spreadsheet should now look like Figure A-27.

		E16					fx
		=SUM(E13:E15)					
	A	B	C	D	E	F	
7	P	Mer	SCC	Bak		PV1	
8	Mer	1	5	7	13	0.6961	
9	SCC	1/5	1	3	4 1/5	0.2249	
10	Bak	1/7	1/3	1	1 1/2	0.0790	
11					18 2/3		
12	P	Mer	SCC	Bak			
13	Mer	3.0000	12.3333	29.0000	44.33333	0.7353	
14	SCC	0.8286	3.0000	7.4000	11.22857	0.1862	
15	Bak	0.3524	1.3810	3.0000	4.733333	0.0785	
16					60.29524		

### Figure A-27: Copied and pasted the Sum and Priority Vector Formulae

The hard work has now been done and all that remains is to copy and paste the formulae until the priority vector for the last matrix is identical to the priority vector of the second last matrix in the fourth decimal.

Proceed as follows:

- 1 Select cells A12:A16 and press Ctrl-c; and
- 2 Select cell A17 and press Ctrl-v. Compare the priority vector with the previous one and if it is not the same in the fourth decimal repeat 1.(Now copy cells A17:F21 into cell A22 and so on).

This process will normally require 3 to 5 iterations. Figure A-28 shows the completed cycle for this example.

	A	B	C	D	E	F
7	P	Mer	SCC	Bak		PV1
8	Mer	1	5	7	13	0.6961
9	SCC	1/5	1	3	4 1/5	0.2249
10	Bak	1/7	1/3	1	1 1/2	0.0790
11					18 2/3	
12	P	Mer	SCC	Bak		
13	Mer	3.0000	12.3333	29.0000	44.33333	0.7353
14	SCC	0.8286	3.0000	7.4000	11.22857	0.1862
15	Bak	0.3524	1.3810	3.0000	4.733333	0.0785
16					60.29524	
17	P	Mer	SCC	Bak		
18	Mer	29.4381	114.0476	265.2667	408.7524	0.7305
19	SCC	7.5790	29.4381	68.4286	105.4457	0.1885
20	Bak	3.2585	12.6317	29.4381	45.32834	0.0810
21					559.5264	
22	P	Mer	SCC	Bak		
23	Mer	2595.346	10065.47	23422.01	36082.82	0.7306
24	SCC	669.2002	2595.346	6039.282	9303.829	0.1884
25	Bak	287.5849	1115.334	2595.346	3998.265	0.0810
26					49384.92	
27	P	Mer	SCC	Bak		
28	Mer	20207451	78370111	1.82E+08	2.81E+08	0.7306
29	SCC	5210418	20207451	47022067	72439936	0.1884
30	Bak	2239146	8684031	20207451	31130628	0.0810
31					3.85E+08	

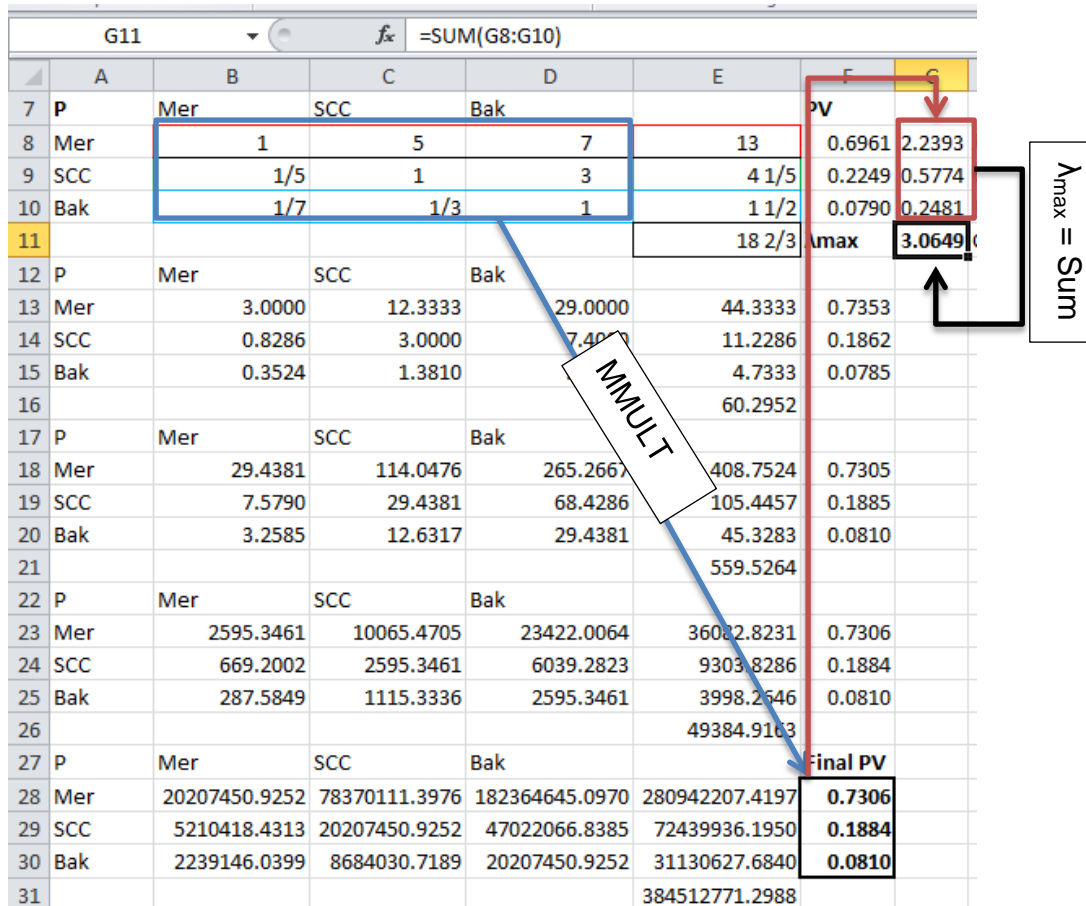
Final Priority Vector

**Figure A-28: Completed Priority Vector calculated according to the Exact Method.**

Calculating the priorities for the alternative vehicles with respect to the other two criteria is left to the reader as an exercise. It should not take longer than a few seconds to prepare the arrays by copying and pasting the whole block (A7:F31) to a new location, say H7. If all the formulae have been entered correctly according to the preceding procedures, then entering the judgements above the diagonal for each of the remaining criteria is all that is required.

### A.10 Checking for Consistency

In the exact method,  $\lambda_{\max}$  is calculated slightly differently. Here the comparison matrix is multiplied by the priority vector to arrive at a new column vector  $\eta$ . The sum of the elements of  $\eta$  is  $\lambda_{\max}$ . This is illustrated in Figure A-29



**Figure A-29: Calculating  $\lambda_{\max}$  in the Exact Method.**



The Consistency Index, Random Consistency Index and Consistency Ratio are derived in exactly the same manner as before. The completed matrix for the performance criterion is indicated in Figure A-30 which shows an acceptable Consistency Ratio of 0.0624. The completed matrices for Comfort and Economy are shown in Figure A-31 and Figure A-32.

	A	B	C	D	E	F	G	H	I
7	P	Mer	SCC	Bak					
8	Mer	1	5	7	13	0.6961	2.2393	$\lambda_{max}$	3.0649
9	SCC	1/5	1	3	4 1/5	0.2249	0.5774	Ci	0.0324
10	Bak	1/7	1/3	1	1 1/2	0.0790	0.2481	Ri	0.5200
11					18 2/3	$\lambda_{max}$	3.0649	CR	0.0624
12	P	Mer	SCC	Bak					
13	Mer	3.0000	12.3333	29.0000	44.3333	0.7353			
14	SCC	0.8286	3.0000	7.4000	11.2286	0.1862			
15	Bak	0.3524	1.3810	3.0000	4.7333	0.0785			
16					60.2952				
17	P	Mer	SCC	Bak					
18	Mer	29.4381	114.0476	265.2667	408.7524	0.7305			
19	SCC	7.5790	29.4381	68.4286	105.4457	0.1885			
20	Bak	3.2585	12.6317	29.4381	45.3283	0.0810			
21					559.5264				
22	P	Mer	SCC	Bak					
23	Mer	2595.3461	10065.4705	23422.0064	36082.8231	0.7306			
24	SCC	669.2002	2595.3461	6039.2823	9303.8286	0.1884			
25	Bak	287.5849	1115.3336	2595.3461	3998.2646	0.0810			
26					49384.9163				
27	P	Mer	SCC	Bak					
28	Mer	20207450.9252	78370111.3976	182364645.0970	280942207.4197	0.7306			
29	SCC	5210418.4313	20207450.9252	47022066.8385	72439936.1950	0.1884			
30	Bak	2239146.0399	8684030.7189	20207450.9252	31130627.6840	0.0810			
31					384512771.2988				

Figure A-30: Completed Matrix for the Performance Criterion.

J	K	L	M	N	O	P	Q	R
C	Mer	SCC	Bak		PV	$\eta$		
Mer	1	4	7	12.0000	0.6770	2.1376244	$\lambda_{max}$	3.0324
SCC	1/4	1	3	4.2500	0.2398	0.6395863	Ci	0.0162
Bak	1/7	1/3	1	1.4762	0.0833	0.2551559	Ri	0.5200
				17.7262	$\lambda_{max}$	3.0324	CR	0.0311
C	Mer	SCC	Bak					
Mer	3.0000	10.3333	26.0000	39.3333	0.7072			
SCC	0.9286	3.0000	7.7500	11.6786	0.2100			
Bak	0.3690	1.2381	3.0000	4.6071	0.0828			
				55.6190				
C	Mer	SCC	Bak					
Mer	28.1905	94.1905	236.0833	358.4643	0.7049			
SCC	8.4315	28.1905	70.6429	107.2649	0.2109			
Bak	3.3639	11.2421	28.1905	42.7965	0.0842			
				508.5257				
C	Mer	SCC	Bak					
Mer	2383.0459	7964.6126	19964.4875	30312.1460	0.7049			
SCC	713.0174	2383.0459	5973.4594	9069.5228	0.2109			
Bak	284.4504	950.6899	2383.0459	3618.1862	0.0841			
				42999.8550				
C	Mer	SCC	Bak		Final PV			
Mer	17036722.7371	56940111.2778	142728871.1428	216705705.1578	0.7049			
SCC	5097459.6837	17036722.7371	42705083.4583	64839265.8792	0.2109			
Bak	2033575.4028	6796612.9116	17036722.7371	25866911.0515	0.0841			
				307411882.0884				

Figure A-31: Completed Matrix for the Comfort Criterion

S	T	U	V	W	X	Y	Z	AA
E	Mer	SCC	Bak		PV	$\eta$		
Mer	1	1/7	1/5	1.3429	0.0719	0.220449	$\lambda_{max}$	3.0649
SCC	7	1	3	11.0000	0.5890	1.989474	Ci	0.0324
Bak	5	1/3	1	6.3333	0.3391	0.854964	Ri	0.5200
				18.6762	$\lambda_{max}$	3.0649	CR	0.0624
C	Mer	SCC	Bak					
Mer	3.0000	0.3524	0.8286	4.1810	0.0693			
SCC	29.0000	3.0000	7.4000	39.4000	0.6535			
Bak	12.3333	1.3810	3.0000	16.7143	0.2772			
				60.2952				
C	Mer	SCC	Bak					
Mer	29.4381	3.2585	7.5790	40.2756	0.0720			
SCC	265.2667	29.4381	68.4286	363.1333	0.6490			
Bak	114.0476	12.6317	29.4381	156.1175	0.2790			
				559.5264				
C	Mer	SCC	Bak					
Mer	2595.3461	287.5849	669.2002	3552.1312	0.0719			
SCC	23422.0064	2595.3461	6039.2823	32056.6349	0.6491			
Bak	10065.4705	1115.3336	2595.3461	13776.1503	0.2790			
				49384.9163				
C	Mer	SCC	Bak		Final PV			
Mer	20207450.9252	2239146.0399	5210418.4313	27657015.3964	0.0719			
SCC	182364645.0970	20207450.9252	47022066.8385	249594162.8607	0.6491			
Bak	78370111.3976	8684030.7189	20207450.9252	107261593.0416	0.2790			
				384512771.2988				

Figure A-32: Completed Matrix for the Economy Criterion

## A.11 Final Analysis (Synthesising)

In the final step, the score of each alternative in terms of the objective is derived by summing the products of the score for each attribute and the priority of each criterion with respect to the objective. In other words, the score of each attribute of each alternative is multiplied by how much that attribute matters and the products are added together for the final score. To achieve this for the example, construct a final matrix in which the priority of each alternative is tabulated against each criterion. The final score for each alternative is found with Excel's MMULT function as illustrated in Figure A-33. For example, the result for Mer is obtained by the following:  $0.731 \times 0.0599 + 0.705 \times 0.1897 + 0.072 \times 0.7504 = 0.2316$ .

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H
33		P	C	E		Crit	Score	
34	Mer	0.731	0.705	0.072		P	0.0599	23.1%
35	SCC	0.188	0.211	0.649		C	0.1897	53.8%
36	Bak	0.081	0.084	0.279		E	0.7504	23.0%
37								

The formula bar shows: `=MMULT(B34:D34,F34:F36)`

Figure A-33: Synthesising

In the final analysis then, the student can confidently choose SCC as the best vehicle for commuting to and from campus. Note that in this case, the difference between Mer and Bak is very small, however, considering the high score for SCC, the interpretation is that they are both poor choices.

## A.12 Improving Consistency by determining the most inconsistent Pairs of Values

In small matrices, it is comparatively easy to spot mistakes or work through the judgements quickly with the judges to improve consistency. If matrices become larger, (probably anything larger than 4 by 4), the task becomes arduous and time consuming. It is recommended that matrices be restricted in size to about 7 to 9. (Miller, 1956)

To find the most inconsistent pair of pairwise comparisons requires that a matrix be constructed, the elements of which are the product of the elements of the comparison matrix and the ratio of the elements of the  $j^{\text{th}}$  and  $i^{\text{th}}$  elements of the priority vector. This is achieved in three steps:

1. Construct a quotient matrix, with the same number of elements as the comparison matrix. The elements of this matrix are derived as shown in Table 7-1. (Note that the columns and the rows each contain the elements of the priority vector);
2. Multiply the elements of the comparison matrix with the elements of the quotient matrix to derive the consistency indicator matrix; and
3. Find the largest number in the consistency index indicator. This value is indicative of the most inconsistent pair of judgements. The corresponding judgement in the original comparison matrix can now be adjusted with the approval of the original judges.

**Table 7-1: Deriving a Quotient Matrix**

$w_j/w_i$	0.37	0.17	0.26	0.20
0.37	0.37/0.37	0.17/0.37	0.26/0.37	0.20/0.37
0.17	0.37/0.17	0.17/0.17	0.26/0.17	0.20/0.17
0.26	0.37/0.26	0.17/0.26	0.26/0.26	0.20/0.26
0.20	0.37/0.20	0.17/0.20	0.26/0.20	0.20/0.20

### **A. 13 Illustrative Example for “Consistency Indicator”**

To illustrate how to use this method of indicating the most inconsistent pair, which will be termed the consistency indicator, assume a comparison matrix as shown in Figure A-34. The priority vector for this matrix, using the exact method is shown in Figure A-35 together with the consistency ratio, which is almost 13 per cent.

	A	B	C	D	E	F	G	H
1		C1	C2	C3	C4	C5	C6	Sums
2	C1	1	5	3	5	3	2	19.000
3	C2	1/5	1	3	3	3	6	16.200
4	C3	1/3	1/3	1	1/2	1	3	6.167
5	C4	1/5	1/3	2	1	1/2	2	6.033
6	C5	1/3	1/3	1	2	1	2	6.667
7	C6	1/2	1/6	1/3	1/2	1/2	1	3.000
8	Sum	2.567	7.167	10.333	12.000	9.000	16.000	57.067

**Figure A-34: Comparison Matrix for Consistency Indicator Example**

PV	$\eta$		
0.407	2.772	$\lambda_{max}$	6.809
0.233	1.587	$C_i$	0.162
0.096	0.656	$R_i$	1.250
0.092	0.626	CR	0.129
0.107	0.730		
0.064	0.438		
1.000	6.809		

**Figure A-35: Priority Vector of Consistency Indicator Example**

In order to build the consistency indicator, the first step is to construct the Quotient Matrix. In an appropriate blank space on a spreadsheet, select a row of blank cells corresponding to the number of cells in a row of the comparison matrix. Type

=TRANSPOSE(

and select the priority vector, then close the brackets and type Shift-Ctrl-Enter. For the example above, this looks like in Figure A-36. The values of the priority vector are now in a row and comprise the  $j$  elements. Note that this could have been typed in instead of using the transpose formula, but if any changes were to be made to the priority vector, the new values will again have to be entered manually.

J	K	L	M	N	O	P
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064

**Figure A-36: Starting the Quotient Matrix**

The next step is to enter the i elements. Select a column immediately to the left of the j elements and one row down as, indicated in Figure A-37. Type “=” and select the priority vector again. Then type Shift-Ctrl-Enter to arrive at a layout as shown in Figure A-38. Make sure that the j elements and the corresponding i elements are identical.

J	K	L	M	N	O	P
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064

**Figure A-37: Selecting the Space for i-elements**

J	K	L	M	N	O	P
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064
0.407						
0.233						
0.096						
0.092						
0.107						
0.064						

**Figure A-38: i Elements completed**

Populate the matrix with the quotients of  $w_j/w_i$  as follows:

- Select the block of cells directly below the j-elements and to the right of the i-elements. See Figure A-39;
- Now type “=”;
- Next, select the j-elements (top row);
- Type the division operator (/);
- Select the i-elements (left column). This is illustrated in Figure A-40; and
- Finally, type Shift-Ctrl-Enter to complete the calculation as shown in Figure A-41.

Note that the MMULT function is not used here because the elements of the two vectors are divided individually.

J	K	L	M	N	O	P
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064
0.407						
0.233						
0.096						
0.092						
0.107						
0.064						

**Figure A-39: Selecting the Cells for the Quotients**

J	K	L	M	N	O	P
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064
0.407	=K1:P1/J2:J7					
0.233						
0.096						
0.092						
0.107						
0.064						

**Figure A-40: Calculating the Quotients**

J	K	L	M	N	O	P
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064
0.407	1.000	0.573	0.237	0.226	0.263	0.158
0.233	1.746	1.000	0.413	0.394	0.460	0.276
0.096	4.224	2.419	1.000	0.954	1.112	0.668
0.092	4.426	2.535	1.048	1.000	1.165	0.700
0.107	3.799	2.176	0.899	0.858	1.000	0.601
0.064	6.321	3.620	1.496	1.428	1.664	1.000

**Figure A-41: Completed Quotient Matrix**

To construct the consistency indicator matrix, the elements of the judgement matrix are multiplied with the elements of the quotient matrix. Again, this is not a matrix function so MMULT is not used. First, select an area of cells equal in number to the judgement matrix. (It helps if such area is highlighted in some way, either by coloured cells or by placing a boundary around it, and to copy the criteria names appropriately outside the boundary) as shown in Figure A-42. Then type the equals sign, select the elements of the judgement matrix, type the multiplication operator (\*) and type Shift-Ctrl-Enter to end up with the consistency indicator matrix as shown in Figure A-43 and Figure A-44.



J	K	L	M	N	O	P	
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064	
0.407	1.000	0.573	0.237	0.226	0.263	0.158	
0.233	1.746	1.000	0.413	0.394	0.460	0.276	
0.096	4.224	2.419	1.000	0.954	1.112	0.668	
0.092	4.426	2.535	1.048	1.000	1.165	0.700	
0.107	3.799	2.176	0.899	0.858	1.000	0.601	
0.064	6.321	3.620	1.496	1.428	1.664	1.000	
	C1	C2	C3	C4	C5	C6	
C1							
C2							
C3							
C4							
C5							
C6							

Figure A-42: Selecting Cells for the Consistency Indicator Matrix

J	K	L	M	N	O	P	
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064	
0.407	1.000	0.573	0.237	0.226	0.263	0.158	
0.233	1.746	1.000	0.413	0.394	0.460	0.276	
0.096	4.224	2.419	1.000	0.954	1.112	0.668	
0.092	4.426	2.535	1.048	1.000	1.165	0.700	
0.107	3.799	2.176	0.899	0.858	1.000	0.601	
0.064	6.321	3.620	1.496	1.428	1.664	1.000	
	C1	C2	C3	C4	C5	C6	
C1	=B2:G7*K2:P7						
C2							
C3							
C4							
C5							
C6							

Figure A-43: Finalising the Consistency Indicator Matrix

J	K	L	M	N	O	P
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064
0.407	1.000	0.573	0.237	0.226	0.263	0.158
0.233	1.746	1.000	0.413	0.394	0.460	0.276
0.096	4.224	2.419	1.000	0.954	1.112	0.668
0.092	4.426	2.535	1.048	1.000	1.165	0.700
0.107	3.799	2.176	0.899	0.858	1.000	0.601
0.064	6.321	3.620	1.496	1.428	1.664	1.000
	C1	C2	C3	C4	C5	C6
C1	1.000	2.864	0.710	1.130	0.790	0.316
C2	0.349	1.000	1.240	1.183	1.379	1.657
C3	1.408	0.806	1.000	0.477	1.112	2.005
C4	0.885	0.845	2.096	1.000	0.583	1.401
C5	1.266	0.725	0.899	1.716	1.000	1.202
C6	3.161	0.603	0.499	0.714	0.832	1.000

**Figure A-44: The completed Consistency Indicator Matrix**

The most inconsistent judgment pair is now indicated by the largest value in the consistency indicator matrix. It can be seen that finding the largest value is not particularly easy and it should be evident that this will be more difficult with even larger matrices. The Excel conditional formatting feature can be used to overcome this challenge. To use this, first select the Consistency Indicator Matrix and then select "Conditional Formatting" from the Styles group in the Ribbon as indicated in Figure A-45 and Figure A-46. One can experiment with the various alternatives, but in this report the Top/Bottom rules alternatives will be used to highlight the three highest values as shown in Figure A-47

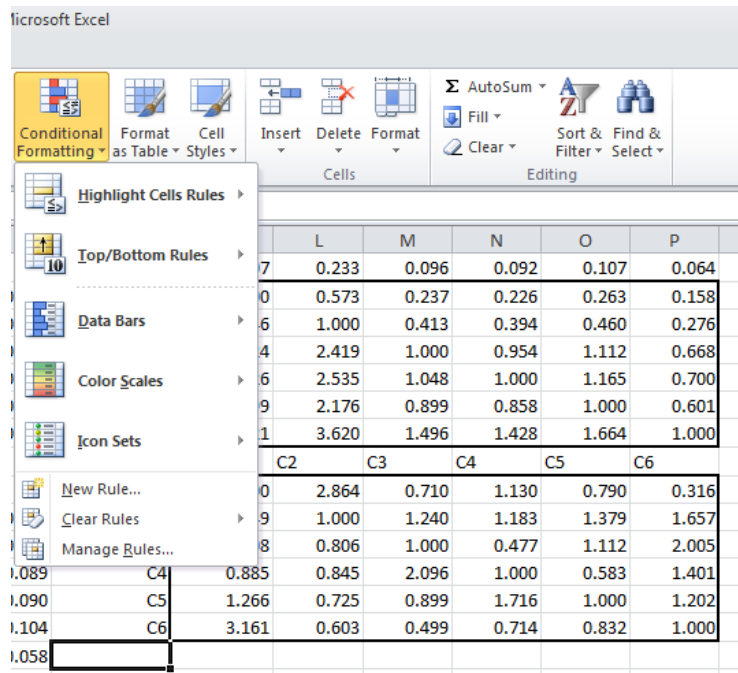


Figure A-45: Selecting the Conditional Formatting Feature.

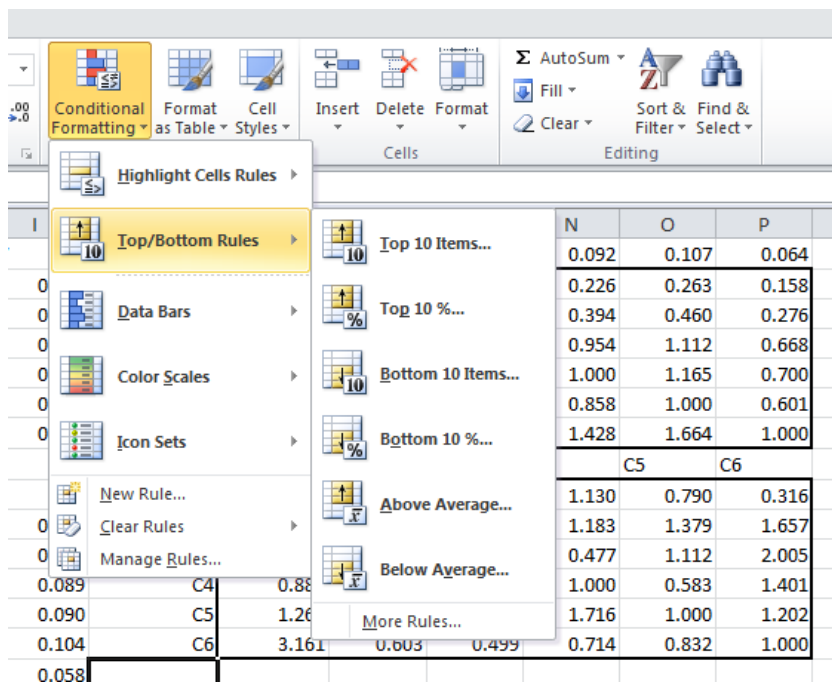


Figure A-46: Towards highlighting the highest Values.

J	K	L	M	N	O	P
wj/wi	0.407	0.233	0.096	0.092	0.107	0.064
0.407	1.000	0.573	0.237	0.226	0.263	0.158
0.233	1.746	1.000	0.413	0.394	0.460	0.276
0.096	4.224	2.419	1.000	0.954	1.112	0.668
0.092	4.426	2.535	1.048	1.000	1.165	0.700
0.107	3.799	2.176	0.899	0.858	1.000	0.601
0.064	6.321	3.620	1.496	1.428	1.664	1.000
	C1	C2	C3	C4	C5	C6
C1	1.000	2.864	0.710	1.130	0.790	0.316
C2	0.349	1.000	1.240	1.183	1.379	1.657
C3	1.408	0.806	1.000	0.477	1.112	2.005
C4	0.885	0.845	2.096	1.000	0.583	1.401
C5	1.266	0.725	0.899	1.716	1.000	1.202
C6	3.161	0.603	0.499	0.714	0.832	1.000

**Figure A-47: Highest Values highlighted**

This matrix can now be used to investigate and find ways of improving consistency. The first course of action taken in this example was to investigate the three pairs C1:C6, C1:C2 and C4:C3 for reciprocal error and it was found that C1:C2 should have been 1/5 and not 5. Once this was corrected, the Consistency Ratio dropped from almost 13 per cent to just over 10 per cent. The consistency indicator changed too as shown in Figure A-48, in which the pair C1:C2 is again flagged as the highest value. After consultation with the judges, the judgment was adjusted to 1/3, which resulted in a Consistency Ratio of 8.1 per cent. The judgement matrix was now accepted and approved by the judges as the Consistency Ratio was below the required 10 per cent.

	C1	C2	C3	C4	C5	C6
C1	1.000	0.369	1.259	2.107	1.433	0.493
C2	2.712	1.000	0.683	0.686	0.777	0.803
C3	0.794	1.464	1.000	0.502	1.138	1.763
C4	0.475	1.458	1.992	1.000	0.567	1.171
C5	0.698	1.287	0.879	1.765	1.000	1.033
C6	2.027	1.245	0.567	0.854	0.968	1.000

**Figure A-48: Consistency Indicator after first pass Adjustments.**

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