A QUALITATIVE ACTION RESEARCH STUDY INTRODUCING A METACOGNITIVE FRAMEWORK FOR TEACHING PREPARATION AND ANALYSIS OF ITS EFFICACY

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DECLARATION

I declare that ***'A qualitative action research study introducing a metacognitive framework for teaching preparation and analysis of its efficacy'*** is my own work, except where indicated, and that it has not been submitted for any degree at any other university.

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Abstract

This qualitative study sets out to examine a teacher's metacognitive preparation for and reflection on the mathematical content and pedagogy of a short series of Grade Six Probability lessons, with their contribution to the development of a general framework for lesson preparation from a metacognitive perspective. An action research methodology is followed in which teaching episodes are collaboratively assessed by a panel of three colleagues. These fora serve to enlighten the author of their observations after the preparation and teaching of a series of Probability lessons, which also capture audio data for deep reflections and dialogue post teaching and fora. The analysis of the data will serve as the foundation for the development of a metacognitive framework that teachers can use in their preparation for and reflection of lessons that will pay particular attention to higher cognitive engagement of teachers with content and facilitating lesson delivery, always striving for them to be of higher quality. The efficacy of this framework for the teacher-researcher's teaching practice is also examined.

Key words: metacognitive framework, metacognition, constructivism, action research, critical incidents

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I
Introduction

“Metacognition is multifaceted in that it includes knowledge about strategies, tasks, and the self as well as the skills to evaluate strategies” (Flavell, 1978 in Carr & Biddlecomb, 1998, p. 69). This study sought to develop a metacognitive framework and examine its value for improving teaching practice through a classroom investigation. The developed framework comprises of a series of steps that the teacher-researcher (and potentially any teacher) can use to carry out lesson preparation and reflection, metacognitively, on a series of lessons. It seems appropriate, before continuing, to outline briefly what is meant by metacognition and more appropriately for this study, what it means to do something metacognitively. Cognition concerns mental faculties and processes and as an underlying concept for metacognition, a formal definition seems appropriate: ‘cognition’ is “the mental action or process of acquiring knowledge and understanding through thought, experience, and the senses” (Oxford Dictionaries, 2015). Metacognition is what a person understands about intellectual activities that are necessary for comprehension and growth in knowledge (Flavell, 1979). Cognitive strategies are used to make progress intellectually but metacognitive strategies are used to monitor intellectual progress, a difference between cognition and metacognition made explicitly clear by Flavell (1979). Therefore to do something metacognitively, in the context of this study, can be understood as intensely focusing on and monitoring one’s own thoughts and ideas when planning a lesson and subsequently assessing that lesson with careful reflection of key moments from the lesson. Flavell (1979) discusses the four key components of metacognition, namely metacognitive knowledge, metacognitive experiences, goals and actions. These components will be discussed in further detail so as to provide a sound base for this research. The typical focus in the metacognitive field of research has been to apply metacognitive components to engaging with and solving mathematical objects. In an attempt to engage in metacognition and simultaneously interrogate teaching strategies, rather than mathematical objects, the emergence of a framework for aiding teaching preparation for teachers to use, though still accommodated within the metacognitive field, was an important aim.

This study used the aforementioned framework based in an exploration into the link between critical incidents and teaching preparation and reflection. The research strived to be exploratory by using an action research approach together with metacognitive components in preparation for teaching probability lessons to a Grade Six South African class. While developing the framework through teaching practice, the study concurrently focused on its examination and some application.

1.1. Ontological framework of the study

I grew up with a love for mathematics and I always found new concepts relatively easy to grasp and apply consistently. Most of my experience with mathematics in school entailed learning something, copying it, seeing it repeated and then applying it to different cases. I was taught rules and formulae for finding solutions to mathematical problems and set procedures were presented to me and my classmates to simply be learnt off by heart and then used in multiple instances. This approach was the case throughout school, with exception for one high school teacher who attempted to challenge me to understand mathematics and how it worked. A system of learning developed that only incorporated knowing the rules specific to a problem and then using those rules often enough in similar problems so that not much understanding was required. For me, the word 'Mathematics' became synonymous with 'finding answers'.

I took compulsory mathematics courses in an Education degree and applied the same memorisation system to solve most of the problems presented. Later on I picked up an optional course, Mathematics Modelling, which required problem solving language to discuss and then solve the tasks given. We were asked to solve problems verbally which compelled me to understand the problems and use mathematical language. The lack of direction in the course was frustrating and I struggled initially however the lack of guidance was intended to trigger trial and error methods, forcing deeper thinking. Attempting to find patterns and develop an understanding of new problems was difficult and out of my comfort zone. Up to that point I had been a student of procedures, however I was not given a set of rules or formulae to follow, as the course was structured to challenge me to think conceptually. I gradually adjusted to this new way of thinking and after some time realised the benefit and importance of the ability to understand concepts in depth and apply knowledge to a variety of situations.

Schoenfeld (1987) highlights that often because of the way they have been taught, students don't have positive beliefs or experiences with regards to Mathematics. I experienced this first hand as although I had always enjoyed mathematics, many of my classmates throughout school did not. As a new teacher I wanted to investigate the process of teaching and learning, discover how the process of teaching could affect emotional responses and find a way to teach that would encourage learners to enjoy the subject and look forward to learning. While I knew that this question – how to teach mathematics so that it is understood well and enjoyed – has been asked countless times and the answer evaded many teachers and researchers, I felt hopeful that further investigation on my part could at least serve to improve my own teaching practice in some way.

My attempt to discover the connection between teaching and learning led to the design of my Honours research project. My supervisor introduced me to Metacognition and I was immediately fascinated with how thinking about and writing down my own thoughts could help me to realise how well, or poorly, I actually understood a concept. Zohar (1999) proposes that even though there is recognition of the role metacognitive processing plays in the success of students, limited research has been conducted to explore the explicit awareness that teachers have of their own metacognitive skills (and effect of those skills) on their teaching process. "…teachers' declarative metacognitive knowledge of thinking skills must be consolidated before they can use it effectively for instruction" (Zohar, 1999, p. 427). In an attempt to fill this gap in the literature, of the lack teachers' explicit awareness of their own metacognition, my Honours research project sought to discover the level of my conceptual understanding and compare it to that of three other educators using metacognition. "Metacognitive knowledge also develops from children's interactions with peers and adults" (Carr & Biddlecomb, 1998, p. 71). Part of this project involved using my supervisor as a sounding board to discuss ideas which developed my awareness of metacognition, and its value, over time.

Apart from realising how little I actually knew compared to how much I thought I knew, the completion of my research project simply fuelled more questions and I gravitated into an academic space – realising how great the need is for research and the need to fill the gaps that I noticed. Zoghby and van Jaarsveld (2013) revealed:

> "My recommendations would be for more teachers to use the tool of metacognition in order to discover their understanding (or lack thereof) of certain mathematical concepts, in an attempt to deepen that understanding to attain a teaching method that is both knowledgeable and understandable" (p. 67).

Zoghby and van Jaarsveld (2013) saw the need for using metacognition in preparation for teaching and realised how easily a teacher could go into the classroom thinking (perhaps mistakenly) that they knew enough about a topic to teach it well. In much of the existing body of literature, the focus is on students being taught how to be more metacognitive so that they can think more effectively and learn more efficiently. However, the key role of the teacher is to teach students how to become more metacognitive but not necessarily use metacognition themselves (discussed in further detail in the context of this study). "A necessary condition for teaching students to be metacognitive is a pedagogical understanding of metacognition" (Wilson & Bai, 2010, p. 270). In order to become more metacognitive, I read much literature in an attempt to understand in a fully academic sense what metacognition and its relevance to education is. Definitions of metacognition up to this point have focused on the interrogation of mathematical objects and probing the understanding and processing of a mathematical problem. I wanted to focus rather on examining my thoughts in preparation for a lesson, attempting to recognise how much I already understood and still needed to learn, and thereafter how to deliver the lesson successfully. I realised that investigating my mathematical thinking in preparation for a lesson started taking the form of a metacognitive framework that could potentially be developed for use by teachers. Consequently for this dissertation, I sought to develop a framework consisting of a set of steps that ideally would become a tool for teachers to be more metacognitive in their teaching preparation and reflection, through the intense interrogation of their understanding.

As the researcher and author of this Masters dissertation (and in following an action research methodology) I was closely involved with the research process which occurred in my classroom. In addition to being the sole researcher, I taught the lessons to be used for the purposes of this study and therefore will refer to myself from here on as the 'teacher-researcher', a term used in a number of action research and practitioner-based studies (Blair, 2010; Doerr & Tinto, 2000; Feldman & Minstrell, 2000; Ferrance, 2000; Gray & Campbell-Evans, 2002; McNiff & Whitehead, 2002; Opie, 2004).

1.2. Context of the study

The annual National Diagnostic report of the South African Department of Basic Education (2014) on the National Senior Certificate (NSC) mathematics examination gives educators much to consider. It communicates that in several of the mathematics content areas there is a "poor understanding of the basics and foundational competencies" and "candidates lack the ability to respond to complex and higher-order questions that require a deeper understanding" (p. 135). In addition to this the report points out that a cause for concern is the "literacy levels of many candidates". Many candidates were subjected to "knowledge and routine-type questions" and they "struggled with concepts in the curriculum that required deeper conceptual understanding". Questions were presented to them that necessitated interpreting information, or required reasoning skills and these "presented challenges to most". It was apparent that numerous candidates found it difficult to respond to concepts that had been examined previously but were asked in another way. "This suggests that the 'stimulus-response' method makes up much of the teaching strategy" (p. 135-136). The report also suggests that the general problem resides with the quality of teachers and not the pupils. This may cause some to question the quality of mathematics education in South Africa and subsequently provokes the question 'What could teachers be doing differently?'.

This state of affairs is not something remedied easily or quickly, or necessarily at all, and much research is necessary to investigate the role of the teacher in student learning. If part of the problem does reside with the quality of teachers, then surely the quality of teachers needs to be investigated. "...to enhance learning to the fullest, learners need to become aware of themselves as self-regulatory organisms who can consciously and deliberately achieve specific goals" (Kluwe, 1982, as cited in Hacker, 1998, p. 20). If the 'learner' in this case is the teacher, then to enhance teaching ability to the fullest, the teacher needs to become aware of themselves as self-monitoring individuals who can achieve specific goals with knowledge and purpose. The aforementioned study (Zoghby & van Jaarsveld, 2013) quickly led the author to acknowledge their lack of understanding in many basic concepts. While this is not necessarily applicable to all teachers, one of the recommendations was that teachers use metacognition to discover their level of understanding of a concept before attempting to teach it (Zoghby & van Jaarsveld, 2013).

Multiple researchers in the metacognitive field make a distinction between two major constituents of metacognition, which are different to Flavell's components: "*knowledge of cognition* and *regulation of cognition*" (Schraw, 1998, p. 113). Schraw (1998) and Jacobs and Paris (1987) both highlight three essential skills needed for regulating cognition: namely planning, monitoring and evaluation. Flavell (1979) discusses planning and evaluation as requirements for novel situations and groups them under metacognitive experiences, one of the four components of metacognition he introduced. Following either of these characterisations, one can conclude that after the teacher discovers how well they understand a concept, the next logical step would be to regulate their teaching preparation or implementation by improving that understanding and attempting to teach concepts better. This process is continuous and every time the teacher has 'finished' a lesson, further regulation and evaluation would be necessary for planning how to develop those students further or how to teach it better to the next group of students.

Further review of the literature on metacognition, and an attempt to implement this recommendation, led to the discovery of a gap in the literature and identified the need for a teaching preparation framework. To begin with, as mentioned above, most metacognitive research has been conducted using children or students as the subjects of studies (see references to specific researchers below) and there is not much research on adults learning to be metacognitive thinkers, even though results from the studies conducted with children could possibly be adapted for adults to use. Secondly, a metacognitive approach to solving a problem that is not mathematical – such as planning a lesson carefully, being aware during teaching a lesson or improving that teaching – is missing from the body of research. Thirdly, due to this gap in research for teachers to be metacognitive thinkers for their own teaching preparation, instead of teaching students how to be metacognitive, there is also no framework which exists in the body of knowledge that teachers can use to prepare for lessons metacognitively. A number of researchers have conducted experiments and studies to find how best to teach students how to be metacognitive, to measure children's level of metacognitive knowledge or experiences, to analyse the use of metacognition and self-regulation in students and how students use metacognitive strategies to solve mathematical problems (Carr & Biddlecomb, 1998; Brown, 1977; Brown 1981; Schoenfeld, 1987; Wilson & Bai, 2010; Hacker, 1998). Fox & Riconscente (2008) refer to Jean Piaget's work and his theories with regard to children and the application of metacognition. Flavell (1979) did much research on metacognition investigating metacognitive components displayed by "preschool and elementary school children" (p. 906) as the participants of his research. However none of these studies focused on the teacher and how the teacher can learn to be more metacognitive, self-regulate knowledge and apply knowledge or evaluate their own teaching practice.

In light of this gap in the literature, a need arose to address it with the development of a practical metacognitive framework that would serve the purpose of guiding teachers in how to structure their lesson preparation and reflection practice. This framework needed to include interrogation of language and teaching methods for fostering deeper understanding of concepts to be taught and better preparedness for teaching. The development of this framework, its effectiveness and where it could be placed in the literature was necessary and provided purpose and structure for this research.

1.3. Purpose of the study

The purpose of the study was to design a metacognitive framework that could be used by the teacher-researcher for lesson preparation, thereafter testing its effectiveness. The aim was to create a tool that would aid the teacher-researcher in having more carefully thought out lesson preparation and only include lesson content that is fully understood. The reflection process after lessons needed to be well structured with a straightforward method that could be followed easily. This reflection is necessary for the teacher-researcher to be thorough in examining her own teaching practice and knowing where to make improvements. The study was carried out to address the gap in the literature by developing a practical metacognitive framework, with a carefully thought out design having the potential to be developed in future research and thereafter used as an effective teaching preparation tool by any teacher. Its aim would be to guide any teacher on how to structure their lesson preparation and reflection with the use of metacognitive strategies. The development of this framework, its efficacy in the preparation for the lessons of the teacher-researcher and where the framework could be located in the literature is presented.

1.4. Significance of the study and potential research contribution

It is the hope that this research will add successfully to the body of knowledge on metacognition and be helpful to teachers who require a tool that will aid lesson preparation, and aid in exploring a deeper understanding of how to prepare oneself mindfully for teaching lesson content. In addition, the framework was structured so as to assist teachers in focusing on deeper lesson reflection in order to recognise their successes and failures, thereafter attempting to change and/or improve their teaching strategies.

This study was defined by several key attributes which evolved (due to its qualitative nature) as the study progressed. The research was undertaken from a constructivist epistemology – both radical and social. Aspects in the field of metacognition that are relevant to this study have been summarised and critiqued, and a rationale for the direction of the research has been detailed. An interpretive analysis tool (Hatch, 2002) was used for data analysis and its steps then used for the purpose of structuring Chapter 4 to present the results of the analysis.

Epistemology is about how we understand and come to acquire knowledge. The assumptions that underlie action research are: "The object of the enquiry is the 'I'; Knowledge is uncertain; Knowledge creation is a collaborative process" (McNiff, 2005, p. 1). As mentioned above, the methodology of action research was adopted. "Action research is a name given to a particular way of researching your own learning. It is a practical way of looking at your practice in order to check whether it is as you feel it should be" (McNiff & Whitehead, 2002, p. 15). The research intended to develop a framework for aiding metacognitive lesson preparation in order to assist effective teaching, which if successful would make a significant contribution to research and to teaching.

1.5. Research aims

The aim of this study was to develop a metacognitive framework that the teacher-researcher could use to improve her own teaching practice. It sought to achieve this by using metacognition in the preparation and reflection of a series of Probability lessons with a Grade Six class. An aim of the study was to give guidance through a developed framework, as to the practice of using metacognitive processing in preparation for and in reflection of lessons. Key developments of the framework were discovered through identification of the link between the preparation process and critical incidents in the classroom. Metacognitive strategies adopted by the teacher-researcher played a major role in this preparation process. The data collected from each lesson and the discussions that took place subsequent to each lesson were used to underpin the planning for the following lesson, in collaboration with metacognitive strategies adopted by the teacher-researcher. The effectiveness of the framework for the teacher-researcher's own practice was also examined and is reported on.

1.6. Research question

The research aims assisted in shaping the following research question:

> *Does a developed framework for metacognitive lesson preparation and reflection have the potential to facilitate effective teaching practice?*

This question, while it stands as a research question, also reflects an important research goal, which is that of developing a framework that could be used by the teacher-researcher to be more metacognitive in her lesson preparation and reflection. The effectiveness is examined from a personal action research perspective. In addition to this, a minor research goal indicated is that the framework developed should have the potential to facilitate effective teaching practice for teachers other than the teacher-researcher. This could only be determined by further research.

1.7. Research Design

This brief summary of the methodology will be supplemented in greater detail in the third chapter of this dissertation, however a concise outline of the participants, instrumentation and procedure of the study is necessary to present the reader with an overview of the research design.

The twenty-six students who took part in the lessons taught by the teacher-researcher were participants in this study. Their involvement in the series of lessons was central to the analysis procedure in their responses and in the interaction they had with each other and the teacher-researcher. The participants were selected deliberately, and as the students of the teacher-researcher the collection of data for the study could be carried out in the teacher-researcher's classroom during the normal school timetable. The location of the study was therefore also indirectly selected as it was where the teacher-researcher's taught. There were three direct observers who, having experience in the mathematics education field, were able to give advice and direction to the teacher-researcher after observing the lessons taught. In order to accomplish this, they were recorded and their input analysed and interpreted. The teacher-researcher was also a participant in this study by preparing and teaching the lessons used for analysis and development of the framework. Their own metacognitive processes in the preparation for and reflection of these lessons were also fundamental to the development of the framework.

The instrumentations to collect data included a journal that the teacher-researcher used to document preparation and reflection notes (before, during and after lessons), a recording device used to record the lessons and subsequent discussion fora, forms which were used to collect the demographics of the observers (see Appendices 1, 2 and 3) and an observation tool that the observers used while monitoring the lessons.

The procedure that was followed included the teacher-researcher attempting to use metacognitive components in preparing for a lesson. The lesson was then taught, with three direct observers watching and taking notes. During the lesson the teacher-researcher was able to make a few brief notes on the teaching process and how the lesson was progressing. After the lesson the teacher-researcher reflected on how the lesson had progressed, noting problems or successes that arose and identifying what could be changed for the following lesson. This was followed by a discussion forum where the three observers critically discussed the lesson with the teacher-researcher about what was carried out well or what could be improved for the next lesson. The teacher-researcher's plan for the following lesson was also discussed, with ideas given for how to teach it in a way that benefited the students the most. This procedure was then repeated with the teacher-researcher then taking their own observations into account, while considering the direction given by the observers, in order to plan for the next lesson. Metacognitive questioning occurred repetitively in preparation for and reflection of these lessons throughout this process. After the series of three lessons and three discussion fora took place, the teacher-researcher started analysis of these data to begin developing a working framework. From the beginning of the preparation and reflection process the framework had already begun to take shape.

1.8. Theoretical framework

The research is situated in a radical and social constructivist theoretical framework to accommodate the personal metacognitive interrogation of the teacher-researcher and the social aspect of the teacher-researcher's interaction with the direct observers. The research is also positioned within the framework of metacognition which involves the monitoring of an expansive range of cognitive activities and occurs through the actions of and interactions between the four main components mentioned above (Flavell, 1979). The metacognitive interrogation of the teacher-researcher was key to the development of a framework that could assist other teachers in being metacognitive in their own planning for and reflection of lessons. The metacognitive processing of the teacher-researcher in preparation for and reflection of a series of lessons was investigated. It is for this reason that the research design selected for use in this study was qualitative and necessitated the use of an interpretive data analysis tool. A reflective cyclical process, which was vital to the research process, occurred and included the preparation for lessons, reflections during and after lessons, essential changes made and planning for the following lesson based on reflections and necessary adjustments. An action research methodology was therefore deemed appropriate and thus adopted for the teacher-researcher to examine their own teaching practices for the development of the framework.

1.9. Assumptions, limitations and scope

An assumption that was made before the research commenced was that the participants would answer questions truthfully, taking part in the lessons and interacting positively with the teacher-researcher. Another assumption regarding the students was that they might feel a little uncomfortable with the presence of the observers in the classroom and/or knowing that they were being recorded, even though they had given their permission. This presence of outsiders might have led to unnatural behaviour in the lesson, such as not participating as they would have in a typical lesson. Another assumption made was that the observers would be truthful and give accurate feedback on the lesson based on their observation and personal experience, in an attempt to be critical, yet constructive and supportive, for the purpose of the study and the teacher-researcher's development of the framework.

Some limitations that were expected before the research commenced included the teacher-researcher's uncertainty in the lesson times that could be used for the data collection and how the lessons would follow each other (as the timetable for the students was already set and based on the schedule of other teachers and students). In addition to this the teacher-researcher knew that the time after the lesson taught, needed for the discussion forum, would be difficult to organise due to a full timetable with lessons mostly one after another throughout the day. A lesson ahead of break would have been ideal so that the break time could be used for the discussion forum but this was not always available in the timetable. A limitation linked to this was the possibility of the three direct observers (one or all) not being able to come at the appointed time (fixed in the school timetable) for the lessons for observation.

Another limitation considered prior to the research commencing was the bias of the teacher-researcher. As the teacher taking part in an action research practice in their own classroom, the researcher for the study and author of this dissertation, bias was considered to be a limitation. The attempt to compensate for this, making the process as fair as possible by including observers in the process and having multiple triangulation methods, are discussed in detail under data analysis procedures. This detail also includes how the triangulation methods were used in an attempt to avoid perceptual misrepresentations which were considered another limitation.

The twenty-six student participants, three direct observers and one teacher-researcher participant brought the total count of participants to thirty. The geographical location of the study was at the private primary school where the teacher-researcher was employed with students, as participants, who were being taught mathematics as a school subject by the teacher-researcher that year. A series of three lessons, each followed by a discussion forum with the direct observers, took place within the guidelines of the timetable of the school and in the time constraints for typical lessons.

1.10. Organisation of the dissertation

Chapter 2 includes a clear outline of the literature on constructivism, the epistemology of the study, and the value it added by underpinning the study. A comprehensive account of the literature on metacognition, and the gap in the literature that this research aimed to fill is also presented. Specific metacognitive strategies that can be found in the body of literature are reviewed followed by the description of purposeful development of the metacognitive framework that this study sought to present. An informative overview of critical incidents, how they can be identified and examined and why they were essential for this study, follows. A brief introduction to Systems 1 and 2 and their link to metacognition is provided. The findings and data analysis of the study are summarised in Chapter 4 through the employment of the steps that make up the Interpretive Analysis method, as its structure (Hatch, 2002). Synthesis of these data leads to the content of Chapter 5 – the proposal of the metacognitive framework for improving teaching practice and discussion of its efficacy for the teacher-researcher, found during the progression of the study. Chapter 6 concludes this dissertation, reviewing the research aims and outlining the findings and framework of the study. Limitations, generalisations and implications are discussed followed by recommendations for teachers and for the direction of further research.

II

Literature Review

Situating the research in the chosen school was a deliberate design choice, as the motivation for and context of the research in this particular school, rested on action research literature related to solving problems within a relevant context. Some of this literature has already been drawn on in the above section under the significance of the study. Another meaningful piece of research is metacognition related to teaching and learning, more specifically in mathematics education. This study was designed in a way that the body of research in this field together with the teacher-researcher's teaching practice, could support the design of a framework intended for the use of other teachers in their teaching preparation. This study also drew on value from the mathematics education field of Probability as documented in the curriculum (South African CAPS) for Grade 6 (Department of Basic Education, 2011) and other sources that offered mathematical knowledge on Probability.

This chapter reviews the literature that informed the study and attention has been paid to a number of key bodies of research on mathematics education upon which this study rested. In proposing qualitative research, the teacher-researcher acknowledged the possibility of the study uncovering other areas of literature in need of exploration as the study progressed. Some new terms and areas of literature for the teacher-researcher (System 1 and 2) that were found to be relevant during the research process, are also reflected on.

2.1. Constructivism

A rudimentary constructivist principle is that "…learning is not a passive receiving of ready-made knowledge but a process of construction in which the students themselves have to be primary actors" (Von Glasersfeld, 1995, p. 1). Jean Piaget carried out his research with this assumption, that understanding and knowledge is created and/or constructed by each person and that understanding can correspond with the understanding of others through social collaboration (Von Glasersfeld, 1997). As Von Glasersfeld (1997) points out, Piaget theorised that we create and construct our own knowledge (which was consistent with what Albert Einstein had already written) and developed two levels of 'adaptation'. Creating systems that are able to avoid obstructions is the first, practical level, where being adaptable in nature means being able to change to survive. The second level is conceptual, and adaptation means attaining a state of equilibrium whereby internal conflicts are circumvented. Carrying on from this belief, it is imperative to understand how knowledge develops and so the definition of an operation becomes necessary. Piaget (1964/1997) defines it thus: "An operation is an interiorised action” and “…a particular type of action which makes up logical structures” (p. 20). An operation is always linked to another operation, is never separate from other operations and forms part of a whole structure. These structures, otherwise referred to as schemas, are building blocks of knowledge and have the function of forming the knowledge base. Piaget (1964/1997) named the stages of development of these schemas as sensory-motor (pre-verbal), pre-operational, concrete operational, and formal or hypothetic-deductive operational. Processes that facilitate the shift from one stage to the next include assimilation, accommodation and equilibrium. Maturation, experience of the physical environment, social transmission and equilibration (self-regulation) are four factors that Jean Piaget used to explain the movement between structures.

Von Glasersfeld (1997) reminds us of Jean Piaget’s reiteration that knowledge cannot be, and is not, how the world is seen by all but rather a collection of representations and intellectual ideas of what is experienced. Knowledge is created through the experience of reality and is not reality itself and we examine how individuals construct reality through their experience for “…all science is the product of a thinking mind’s conceptualization” (Von Glasersfeld, 1997, p. 5). Specifically in the field of mathematics, we are focused on how mathematical realities are formed through experiences resulting in our own construction of knowledge and understanding. Steffe and Kieren (1994) confirm that as constructivist mathematics education researchers, we aim to study the formation of mathematical concepts and operations throught which children arrange their experiences. Von Glasersfeld (1990; 1997) reminds us that Jean Piaget’s teaching of Constructivism emphasised that insight into the world never just materialises but is always as a result of action.

Even though Constructivism is the framework within which this study will reside, it is to be noted that there were two parts to the research. One was radical constructivism and the other, social constructivism. Radical constructivism supported the practice of the teacher-researcher interrogating her own thoughts and social constructivism was the basis for the teacher-researcher thinking with the influence of observing classroom activities and engaging in discussion fora with educator colleagues.

Von Glasersfeld (1990) outlines the basic principles of radical constructivism, the main one being that knowledge is not received passively through either senses or any method of communication but rather built up by the cognisant individual. Principles of cognition are also described: “the function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability” and “cognition serves the subject’s organisation of the experiential world, not the discovery of an objective ontological reality” (Von Glasersfeld, 1990, p. 23). Radical constructivism accounts for the interrogation by the teacher-researcher of experiences in the classroom and creating knowledge unaided. In addition, “...radical constructivism holds that the only instruction or information a human knower can possibly receive from 'nature' or 'reality' is negative" (Von Glasersfeld, 1996, p. 19). The world, 'nature' and 'reality' cannot tell individuals what to think or believe or know, no matter how many different concepts or theories are presented and explained to us. We constantly construct and modify our own thought processes (Von Glasersfeld, 1996). Brown (1997) concurs with this, pointing out the radical constructivist assertion that "the student constructs his own knowledge as opposed to receiving it 'ready made'..." (p. 29), whereby no knowledge is simply given and accepted but always constructed by the individual unaided. Thought processes are flexible and can change based on experiences and the process of thinking helps us to systematise input received from the world. This was the foundation for the process that the teacher-researcher experienced by having time to think and reflect on lessons in a private reflective space.

An element of social constructivism appeared whereby the teacher-researcher constructed knowledge about good teaching practices and what to avoid in the classroom as a result of social settings – informal feedback from students (being aware of negative responses from students during the lessons) and discussions with other educators who have directly observed the lessons. "Vygotsky is stressing a close and complex relationship between external social processes and internal psychological ones" (Meadows, 2004, p. 169). 'Internalisation' is an important concept that aids in the construction of awareness through social interactions. The social interaction that the teacher-researcher experienced with the students and the direct-observers, aided in constructing knowledge which was then internalised, changing from external social processes into the psychological. "The initial influence of constructivist thought stemmed mainly from Piaget's cognitive-development psychology rather than from his epistemology" (Steffe & Kieren, 1994, p. 711). At a later stage, the study of the construction of mathematical concepts and operations was guided by the genetic epistemology rather than cognitive-development psychology. The characteristics of constructivism, as discussed, can be seen throughout the study.

2.2. Metacognition

The term 'metacognition', which was coined about four decades ago by John Flavell, is expressed as "one's knowledge concerning one's own cognitive processes and outcomes or anything related to them" (Flavell, 1976, p. 232) and has been a comprehensively researched topic ever since. Even before that, the underpinnings of metacognitive research had already been accomplished by others which Flavell (1979) acknowledged. These include subjects such as "oral communication of information, oral persuasion, oral comprehension, reading comprehension, writing, language acquisition, attention, memory, problem solving, social cognition and various types of self-control and self-instruction" (p. 906). These topics were researched thoroughly by certain scholars such as Brown (1977; 1981). Hacker

(1998) refers to most of these early investigations of metacognition as explanatory, because they attempted to explain the typical patterns in what children know about memory, specifically the processes that deal with storing and recovering information thoughtfully and with intention.

In addition to the above research, in their theories of how children think, the works of William James, Lev Vygotsky and Jean Piaget include processes that are regarded as metacognitive. Fox and Riconscente (2008) highlight that multiple researchers have noted that the ideas in William James' literature came earlier and triggered an increase in theories of metacognition and self-regulation. In a certain manner, James practiced what is now defined as metacognition, encouraged self-regulation and explored in the realm of self-investigations. He detailed 'introspective observation' as consciously considering and conveying one's own thoughts and expressed 'thinking about thinking' – a loose definition of metacognition (Fox & Riconscente, 2008).

Fox and Riconscente (2008) point out again that a number of researchers noted the relevance of Lev Vygotsky's work for metacognition and self-regulation. "We call the internal reconstruction of an external operation *internalisation*" (Vygotsky, 1978, p. 56). Vygotsky (1978) conjectured that in order to establish meaning, children rely on the responses from knowledgeable others who are initially obligated to monitor the child's progress, set their goals, plan activities and allocate attention. Over time the obligation for these decision-making practices becomes the child's, whose capability gradually increases to result in the ability to regulate their own cognitive events. This shift from regulations conducted by the knowledgeable other to self-regulation could be regarded as development in one's metacognition. Fox and Riconscente (2008) point out that Vygotsky conceptualised metacognition and self-regulation as entwined notions.

Jean Piaget's research on human development has also been examined and in commenting on this research, Fox and Riconscente (2008) reference a number of scholars who have found links between Piaget's inquiries and modern works on metacognition and self-regulation. "Arrival at metacognitive thought involves transforming the child's social and intellectual epistemic egocentrism into the adult's decentered, relativistic, and socialized thought" (Fox & Riconscente, 2008, p. 379). Piaget posited that knowledge of one's own knowledge develops from a conscious awareness of self and the ability to communicate and learn naturally from interaction with peers. Metacognition is prompted when children are encouraged to reflect on their own thinking. Palincsar and Brown (1984) built upon Piaget's theory and came to discover that through collaboration, children can examine their own understanding and new skills can be developed. Flavell (1979) himself discussed metacognitive roots in Piaget's work "You can observe relationships among goals, means, metacognitive experiences, and task outcomes and—Piagetian fashion—assimilate these observations to your existing metacognitive knowledge and accommodate the knowledge to the observations" (p. 908).

Over time Flavell's model of metacognition and cognitive monitoring developed. He addressed at length a person's ability to control an extensive range of cognitive activities that transpired through the occurrences and collaboration of four phenomena: metacognitive knowledge, metacognitive experiences, goals (tasks) and actions (strategies). Flavell (1979) defined *metacognitive knowledge* as the piece of a person's accumulated knowledge of the world that entails people as thinking beings who have varied intellectual "tasks, goals, actions, and experiences" (p. 907). *Metacognitive experiences* include the awareness of any intellectual or emotional encounters that occur with and relate to any activity of the mind. Flavell (1979) gave the example of someone experiencing abruptly the knowledge that they did not comprehend something that was just spoken by another person. *Goals*, otherwise known as tasks, refer to the aims of an intellectual process and *actions* (or strategies) refer to the thoughts or other actions exercised to attain them.

Since Flavell's (1976) coining of metacognition, a vast number of scholars have contributed to his work either echoing his notions or finding other processes that can be regarded as metacognitive or significant to self-regulation. Hacker (1998) also mentions that the field of metacognition began to grow so quickly that classification systems were necessary for the purpose of categorising, examining and assessing the vast metacognitive studies. Some of the more well-known contributors to the field are Kluwe (1982), Schoenfeld (1987; 1992), Paris and Winograd (1990), Schneider (1985), Brown (1977; 1981; 1987) and Brown and Campione (1996). In addition to scholars mentioned above, other contributors to the field include Carr and Biddlecomb (1998), especially to the field of mathematics, Cavanaugh and Perlmutter (1982), Butler and Winne (1995), Mevarech and Kramarski (1997), Jacobson (1998), Zimmermann (2002) and Boekaerts and Corno (2005).

Three modes of application of metacognition and their importance

Schoenfeld (1987) wrote a response to a challenge by other researchers on the importance and validity of metacognition, outlining three modes of application of cognitive behaviour that are associated but discrete and have been the main focus of metacognitive research. The first is what one knows about one's own cognitive processes and how accurate one is in describing one's own thinking. The second focus is on control or self-regulation and how well one is able to monitor the process of solving problems (for example) and how well one uses information from one's observations to guide decision-making when problem solving. The third mode of application is on beliefs and intuitions. "What ideas about mathematics do you bring to your work in mathematics, and how does that shape the way that you do mathematics?" (Schoenfeld, 1987, p. 190).

Most of the research that has direct consequences for mathematics educators focuses on the second and third grouping and so Schoenfeld (1987) briefly sums up the first category. Schoenfeld (1987) advises that the second aspect of metacognition, self-regulation, can be reflected on as a management issue and this requires looking at how well time and effort is managed, as one works on complex tasks. To manage one's cognition well, it is important to ensure that the following happens as the problem is solved: understand the problem before attempting to solve it; plan a strategy; monitor how well the process is going (to finding a solution), deciding if the strategy must be changed; and determine a time limit for each step and the whole problem. Schoenfeld (1987) goes on to explain that self-regulation is not about what one knows but how one uses it that matters. Good strategy choice (which can only be accomplished by understanding the problem well and then planning and monitoring the solution) is very important and often the solution to a problem comes down to what strategy one decides to use and not how much one knows. Schoenfeld (1987) also highlights a number of occasions where students knew the mathematics that would help them find a solution to the problem they had been given. However, because they did not stop to ask themselves if the strategy they had selected was helping them solve the problem, or look for a different way to find a solution, they could not solve it. "This is an all too typical example of the disastrous consequences of an absence of self-regulation" (p. 193)

Schoenfeld (1987) then gives a clear summary of information on the third grouping, beliefs and intuitions, and how they "…like self-awareness and self-regulation, are important determinants of students' mathematical behaviour" (p. 198). Students may have presumptions or biases and misunderstandings about mathematics which teachers need to be aware of and not just expect "empty containers waiting to be filled with knowledge" (p. 195). Schoenfeld (1987) asserts that as a result (mainly) of what they are taught, students develop incorrect beliefs about all that makes up mathematics and this has a negative consequence on their mathematical behaviour. "Many students come to believe that school mathematics consists of mastering formal procedures that are completely divorced from real life, from discovery, and from problem solving" (Schoenfeld, 1987, p. 197). Some other students have the belief that all problems can be answered in ten minutes or less, that setting out solutions for mathematics is more important than the accuracy of content and that only prodigies are adept at discovering mathematics.

These beliefs, Schoenfeld (1987) emphasises, have undesirable results such as students abandoning problems that they can't solve very quickly, students worrying about how their work looks rather than whether they understand what they've written and students who become submissive users of mathematics. They believe that their best would be "accepting and memorizing what is handed to them without attempting to make sense of it on their own" (p. 198). Paying attention to this requires a better understanding of metacognition and a knowledge of how to transfer this understanding to students. Schoenfeld (1987) asserts that this would assist them to be aware of their thinking, learn how to monitor their progress regularly and have confident beliefs about understanding and discovering mathematics. If students can be taught to stop regularly during problem solving and ask themselves questions about their progress and why they are doing something or whether a different method would work better, they would be able to use the knowledge they have gained and not spend so much time "pursuing wild mathematical geese" (Schoenfeld, 1987, p. 193).

Schoenfeld (1987) also outlines clearly the value that the different aspects of metacognition have for student exploration and interest in mathematics. What follows here are specific metacognitive strategies (to develop students as metacognitive thinkers) that exist in the literature, beginning with a general method to increase metacognition in the classroom and then the details of (only) two metacognitive strategies that can be used specifically for mathematics problem solving. The strategies that come after these are metacognitive, however they have been designed for the use of improving text and reading comprehension (rather than use in mathematics problem solving).

Increasing metacognition in classroom settings

Schraw (1998) discusses the importance of increasing the use of metacognition (which can be used over a variety of domains) in the classroom. The methods he recommends correlate to the first three modes of application of metacognition outlined by Schoenfeld (1987). Firstly he suggests that the teacher encourage each student to have an overall cognisance of the importance of metacognition. This can be achieved by the teacher modelling cognitive and metacognitive skills, by other students modelling these skills as well (which in some situations is more effective than the teacher's modelling) and also by having prolonged repetition and reflection. This ultimately aids students in understanding the difference between cognition and metacognition which assists in self-regulation.

A second method that Schraw (1998) highlights to increase metacognition is improving the students' knowledge of cognition which includes "three subcomponents; declarative, procedural, and conditional knowledge" (p. 119). Schraw (1998) follows Jacobs and Paris (1987) in using these three subcomponents to define metacognition, a definition also referred to by a number of other researchers (Schoenfeld, 1992; Zohar, 1999; Wilson & Bai, 2010). Schraw (1998) mentions that personal use of a strategy evaluation matrix (SEM) in the classroom, has aided students in improving their knowledge of cognition.

A SEM presents a description of a variety of strategies with how, when and why to use each one. The students are asked to select one to focus on for an extended period of time to practice solving different problems with each strategy. Students are also given some time each week to reflect individually, or by sharing their thoughts with other students, about when and where to use a specific strategy.

A third way to increase the use of metacognition is to teach the students how to improve the regulation of their cognition and one method that Schraw (1998) has found effective is the use of a regulatory checklist (RC) which provides "an overarching heuristic that facilitates the regulation of cognition" (p. 120). An example of an RC details three classifications (planning, monitoring and evaluating) with a set of questions for each one that students can use as a checklist while attempting to solve a problem. The checklist does not have to be explicit but can involve similar steps in the problem solving process.

The final key to promoting metacognitive awareness is cultivating a favourable environment (Schraw, 1998). A suggestion made is for teachers to put the focus on students' increasing their existing level of performance, to reward the students for increased effort and persistence and to encourage the use of strategies. Schraw (1998) links this suggestion to one of his previous studies and believes that the culmination of these may create a "mastery environment". Through the promotion of mastery in the classroom "students may acquire a broader repertoire of strategies, may be more likely to use strategies, and acquire more metacognitive knowledge about regulating strategy use" (p. 122).

IMPROVE

In the metacognitive body of knowledge that exists, very little research has been carried out on developing metacognitive strategies for solving mathematical problems. One strategy that has been presented is called 'IMPROVE' (Mevarech & Kramarski, 1997) and it was only proposed as a strategy for solving mathematical problems in 1997. Since then, there is no clear evidence that other strategies or methods (specific for solving mathematical problems) have been proposed.

Mevarech and Kramarski (1997) discussed the need for a tool that could enhance mathematical reasoning. They had reviewed prior research and discovered that one of the primary difficulties for students working in small groups was that the students were unable to purposefully monitor and thereafter adjust cognitive processes used in cooperative problem-solving. Mevarech and Kramarski (1997) designed and suggested a metacognitive instructional method for teaching mathematics in the classroom using the acronym IMPROVE to embody the teaching steps. These are "Introducing new concepts, Metacognitive questioning, Practicing, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment" (p. 369).

Mevarech and Kramarski (1997) detail how this method was carried out in the classroom. The whole class was introduced to the new concepts by the teacher using a question-answering technique. The students then began to work in small groups and were given three kinds of metacognitive questions (comprehension, strategic and connection) to ask and answer each other on. These questions were designed as a "series of individual hand-held strategy prompt cards" (Mevarech & Kramarski, 1997, p. 374), for the purpose of aiding students in their awareness of the problem-solving process and to assist them in self-regulation. They then attempted to solve the problem using the questions and until there was consensus, the group discussed the problem and made repeated attempts to solve it. If none of the students knew how to solve the problem they would ask the teacher for help. If they all agreed on a solution, it was written down with their mathematical working and some metacognitive processing thoughts. At the end of the lesson, the teacher revised the central concepts of the lesson with the whole class. For common difficulties that were noticed by the teacher, supplementary clarification was given to the whole class. Mevarech and Kramarski (1997) also discuss the importance of the teacher working with each group for a focused period of time and taking a turn in answering the metacognitive questions and modelling how to use them. Feedback-corrective-enrichment involved giving the students a formative assessment at the end of each unit focusing on the main ideas taught. Those who achieved eighty percent or more were given enrichment activities to do on the same work and the other students were given corrective activities to complete. The teacher could choose to work with either group as they completed these corrective or enrichment activities. At the end of this session, the students who did the corrective activities completed a corresponding form of the formative assessment in order to see whether they had obtained mastery in that unit.

Mevarech and Kramarski (1997) present a number of important components to this method that include enabling the students to acquire strategies and metacognitive processing and also learning in collaborating teams of four (each having different prior knowledge: one high, two middle, and one low-achieving student). They also discuss the importance of providing a "feedback-corrective-environment" (p. 369) that focuses on different levels of cognitive processes. In a very basic sense these steps represent what the teacher could theoretically use for introducing a new concept and guiding the students in their use of metacognitive strategies to increase the likelihood of understanding concepts and mastering tasks (Mevarech & Kramarski, 1997).

Think-aloud strategy

This strategy is concisely explained by van Someren, Barnard and Sandberg (1994), "To summarize: the subject is asked to talk aloud, while solving a problem and this request is repeated if necessary during the problem-solving process thus encouraging the subject to tell what he or she is thinking" (p. 26). The think-aloud method, as described by these researchers, is intended for use by psychologists and other social scientists. However it has been applied to various teaching settings and it is asserted that "Problem-solving is the cognitive process to which the think aloud method is applied most frequently" (van Someren et al., 1994, p. 8). Two mathematical problem solving protocols are given as examples for using the think-aloud strategy to monitor cognitive processing. Problem solving means finding the solution to a question for which one doesn't have an immediate answer (van Someren et al., 1994). This could be because it is not available straight from memory but must be constructed from material in memory or acquired from the situation (information given in the problem or asking for extra information). Another reason for not knowing the answer immediately could be that finding the answer needs exploration of possible answers that aren't instantly obvious solutions to the problem. "Most problem solving involves a combination of these two types of reasoning: constructing solutions and constructing justifications of these solutions" (van Someren et al., 1994, p. 8).

Metacognitive strategies for dealing with non-mathematical problems

A number of metacognitive strategies, which guide students in dealing with reading comprehension or text analysis, has been reviewed and they are summarised below.

The think-aloud strategy detailed above for mathematical problem solving has mostly been researched for text comprehension and self-monitoring skills. One of the studies that highlighted an important question when using this strategy is noted here. Baumann, Seifert-Kessell and Jones (1992) conducted a study on the effect that think-aloud training has on the comprehension monitoring skills of elementary students. Their description and introduction of the think-aloud strategy to the students serves as a straightforward and useful introduction to those who wish to understand it. "One technique that has been used to evaluate comprehension monitoring abilities is the oral think-aloud procedure" (Baumann et al., 1992, p. 144). In teaching the students how to think aloud, the study aimed to stimulate the comprehension monitoring and self-regulation skills of the students and then aid the students in gaining control over these facilities. Ten lessons on think-alouds were carried out and a brief note is made here on two of the lessons that stood out for using the think-aloud strategy in the classroom. Lesson one's aim was to teach the students about self-questioning as the first step in learning to monitor their comprehension. Lesson three introduced the think-aloud strategy to the students and the key here seemed to be the question "Is the story making sense?" (Baumann et al., 1992, p. 153). The idea of thinking aloud as they read was introduced to them thus. They were taught to read brief segments of the text, stop regularly (saying out loud what they were thinking about the story), ask 'Is this making sense?' and then verbalise whether the text was making sense to them or not (Baumann et al., 1992). The research led to two conclusions about self-monitoring: teacher-led instruction that is explicit is effective in developing students' comprehension monitoring abilities; and teachers need to engage students in some way in a collaborative manner with selections of text “to promote comprehension monitoring behaviour” (p. 164).

PLAE (Preplan, List, Activate, and Evaluate) is an exam preparation study technique that summarises five processes students can use for controlling the strategy they select and how to regulate their development (Nist & Simpson, 1989). First, students need to learn how to set objectives, distribute resources and design an idea for how to integrate applicable strategies and practice over time. Secondly, students need to have a collection of strategies for different tasks and/or texts that they will come across as there is not one standard way to study. Next, students need to learn how to choose the most suitable strategies based on the features of the text or task and also their own studying preferences/strengths. Fourthly, students need to learn how to initiate and check their plan and make any necessary changes. Lastly, students need to learn how to assess the success or failure of their plan and the objectives they set so that they can plan accordingly for future situations (Nist & Simpson, 1989). The use of PLAE puts students through self-monitoring activities of “…Preplanning or defining tasks and goals, listing or selecting the strategies they will employ and constructing task-specific study plans, activating or implementing the plan and using appropriate fix-up strategies, and evaluating the plan's effectiveness once they receive feedback” (Nist & Simpson, 1989, p. 183)

AIM is a metacognitive strategy developed for determining what the author’s intended message (AIM) is and for supporting students to work out the key concepts of any given text independently and in a significant way (Jacobowitz, 1990). “To determine the effectiveness of the strategy, a study was conducted involving 48 undergraduates enrolled in six college reading and study skill improvement courses” (Jacobowitz, 1990, p. 623). Its purpose, metacognitively, is “…to promote self-knowledge, task knowledge, and self-monitoring by the student” (p. 622). It is all-inclusive, combining “…the various skills related to establishment of purpose, prereading, activation of background knowledge, prediction, determination of text organization, and critical evaluation into one study strategy” (p. 622) and is not material specific but can be applied to any length of any text choice (Jacobowitz, 1990). AIM has been designed with specific questions that assist the reader in working out the main idea or message of the author. The questions apply to any text/content and the reader can ask them before, during and after reading a piece of text. Jacobowitz (1990) suggests that it is beneficial to write down brief answers to the questions, some of which may be more or less useful depending on the text being read.

K-W-L is a teaching method that has been designed for the purpose of guiding students in applying metacognitive strategies when reading text (Ogle, 1986). K-W-L has been named thus for three straightforward thinking stages: “accessing what I *Know*, determining what I *Want* to learn, and recalling what I did *Learn* as a result of reading” (p. 565). The first two stages involve a discussion between the teacher and the students after which the students fill in the worksheet during the thinking-reading process (created for the purpose of assisting group work and also to help make the stages more concrete for the students). For the first stage where the students are attempting to retrieve what they know, it is suggested that they brainstorm as a whole class with the teacher recording their ideas. The students each make suggestions or give information which encourages them to bring their own experience or context to the lesson (Ogle, 1986). The second part of students retrieving what they know involves the teacher prompting them to construct more general categories around the topic that’s about to be read. This first stage is likely to elicit disagreement between the students on the topic, or questions that they have after realising they don’t know something and a reason to read the text emerges for each student which leads into the second stage. The students can now answer ‘What do I want to learn?’ as the discussion has led them to discovering gaps in their knowledge and the teacher’s role here is essential in guiding them to write down which information they want clarified the most (the students can write these down individually, thereby actively constructing their own learning). “In this way each student develops a personal commitment that will guide the reading” (Ogle, 1986, p. 567) and they can begin to read the text. In the third stage the students can fill out the ‘What did I learn?’ section. This can be done as they read or directly after finishing the text and they can then check if the text answered all the questions they wrote down beforehand. Dialogue as a whole class follows this individual answering session and clarifications are given by the teacher.

Self-questioning and prediction are two cognitive strategies that have been combined for use when reading texts for improving comprehension (Nolan, 1991). Self-questioning with prediction (SQWP) comprises firstly of the teacher modelling for students how to identify the main idea of a text and then change it into a question to be answered. The second part is to model how to predict what the author might portray thereafter in the passage which compels the student to really attempt to understand what they read so that they can predict what might happen next. By devising their own assumption they are also motivated to discover if they are correct. Additional practice using self-questioning and then prediction with different texts is helpful so that the students can learn how to apply the strategies competently (Nolan, 1991).

SQ3R (Survey, Question, Read, Recite, Review) is a student-directed study method that is focused on analysis of text and completion of assignments (Huber, 2004). A number of variants of SQ3R were developed by different researchers (Huber, 2004) and most articles surrounding this method are based on modifications or applications of the method. However Johns and McNamara (1980) feel that “More research should be done to determine if SQ3R is a superior reading/study technique". There is a general lack of research conducted on this method and it has not been proved to be a more effective strategy than others for aiding study skills or reading comprehension (Huber, 2004; Johns & McNamara, 1980).

What I Know sheets serve the purpose of modelling metacognitive strategies in the content area of the classroom to find particular information in a specific text. The teacher needs to be an active part of this process and model the type of thinking and questioning that ultimately the students need to acquire (Heller, 1986). The *What I Know sheets* include the precise topic to be read, the aim of reading the text (asked as a question) and three columns guiding the students as to which information they need to think about or look for in the text (Heller, 1986). The first column is "What I already know" (p. 418) and contains information that the student already knows before reading the content. The second column is "What I now know" (p. 418) which is filled in after the student has read the content. All the new concepts, information and/or definitions that the student thinks they understand well and that relate to the aim of reading the text can be included here. The third column is "What I don't know" (p. 418) which is where the student can write down concepts or information that they find confusing and don't understand, perhaps that which makes it difficult to address the aim of reading the text. Heller (1986) strongly advises that the teacher ensure the aim of reading the text is not instantly apparent in the text and that the students will have to think with reason in order to identify and address it.

One strong strategy has been proposed by Call (1991) as a metacognitive study strategy and is a combination of SQ3R and *What I know sheets*. The SQ3R process is written in the What I Know format and the teacher begins with a pre-reading activity as follows. The first two steps of *Survey* (SQ3R) are followed by skimming over chapter headings and subheadings for a mental picture and activating prior knowledge. Vocabulary words and graphic aids can then be checked quickly. Call (1991) suggests that the first column on the *What I know sheets* (What I already know) can be filled in with students freely writing down their prior knowledge and concepts remembered on the topic. This can be reread quickly and any additional ideas that come to mind should be noted. The students will then be asked to share what they have written down with the class. Still applying the *Survey* step, the students should read introductory and summary paragraphs and immediately fill in the second column of the *What I know sheets* (What I now know) using the brief information that they have read and input from the class discussion. The final pre-reading activity is for the students to follow the last step of *Survey* and the second stage of SQ3R (*Question*) by glancing over the study questions and writing in the third column of the *What I know sheets* (what I don't know) some content or self-produced questions, leaving space for answers (Call, 1991). The students can then *Read* the full text and *Recite* what they have read by putting it into their own words, writing answers in the third column of the *What I know sheets*. The students then move to the last stage of SQ3R by *Reviewing* what they have learnt and writing a summary on the back of their *What I know sheets*. A follow-up then completes the process with the whole class working together again and the teacher can "…have the students generate questions and answers to clarify concepts still not clear" (Call, 1991, p. 52). Practice in using these two strategies can result in one useful, successful strategy that will enhance comprehension in reading (Call, 1991).

The above metacognitive strategies have all been recommended for aiding reading comprehension/study techniques and activation of schema is important in the development of any metacognitive strategy. "Children activate schema, which is important when developing metacognitive strategies, and by reflecting on prior experiences, children develop metacognitive thinking" (Israel, 2007, p. 9). Even at preschool level children are developing metacognitive strategies through asking questions and striving to make sense of the world. The questions that children ask may sometimes seem out of place however they are attempting to place the story, passage or text within the context of their own experience or information assembly (Israel, 2007). It is therefore important to keep asking students to explain their thinking and assist them in developing connections between new information and existing knowledge/schema. With this in mind, it makes sense for Israel (2007) to assert that this questioning can continue throughout schooling and gradually over time students will begin to question themselves without needing to be prompted.

2.3. Purposeful development of a metacognitive framework in this study

There is a gap in the field of metacognition that neither previous nor current research has addressed. This study sought to develop and then assess the effectiveness of a framework for metacognitive analysis that is more qualitative in nature. It needs to be clear that this is not a type of reflection that occurs metacognitively which is why the term 'metacognitive reflection' has not been adopted. It is specifically a part of the branch of metacognition that is more reflective in nature – use of language and focusing on how to present a lesson to a class. This framework consists of a set of steps that has been developed through the analysis of the data collected. The data of particular importance for developing the framework were the journal notes that the teacher-researcher formulated through the reflective metacognitive process. The data collected from the classroom of the lessons and discussion fora on those lessons were essential for the reflection between lessons so that decisions could be made on what needed to be changed. In addition, identifying the link between teacher preparation and occurrences in the classroom aided in development of the set of steps that teachers could take in their preparation for a lesson, which may affect classroom discussion and progress in students' understanding. The framework that this study developed was designed to guide teachers to use metacognitive strategies for lesson preparation so that they could be fully immersed in the concepts and be able to teach them effectively.

There have been studies conducted for applying metacognitive strategies when reading (Othman, 2010). Much research has been conducted for the purpose of teaching learners how to think and learn metacognitively through the teacher's modelling of metacognitive strategies for the learners to follow (Jacobson, 1998; Carr & Biddlecomb, 1998; Schneider & Artelt, 2010). The gap in the field of metacognition, highlighted by Wilson and Bai (2010), includes teachers' lack of awareness of metacognition and also the absence of thinking, talking and writing about their thinking processes. A study by Zohar (1999) investigated what in-service teachers know about metacognition, concluding that their knowledge was unsatisfactory for teaching purposes. In the field of metacognition there is an abundance of knowledge that explains how teachers can instruct students in the use of metacognitive skills and what students and teachers know about metacognition. However what is lacking is what makes this study relevant – a metacognitive approach that a teacher can use to prepare effectively for teaching a lesson. This research discusses how teachers can help students to be effective thinkers and learners but makes no mention of how a teacher can be an effective thinker in order to teach well. This study was conducted to design and test a metacognitive framework that could aid teachers in lesson preparation. Fogarty (1994) outlines three distinct phases that form the process of metacognition in order to be successful thinkers: devise a strategy before attempting a task, monitor understanding of the task and evaluate thinking after completing the task. In the analysis process, the development and design of the metacognitive framework is outlined clearly in an attempt to follow the three phases that Fogarty (1994) summarised.

2.4. Critical Incidents

The critical incident technique comprises a collection of techniques for the purpose of gathering direct observations of human behaviour that are useful for solving practical problems and developing extensive psychological philosophies (Flanagan, 1954). These techniques have a distinctive importance and need to meet criteria that have been delineated methodically. Flanagan (1954) defines an incident as "any observable human activity that is sufficiently complete in itself to permit inferences and predictions to be made about the person performing the act" (p. 327). For an incident to be critical, it should occur in a setting where the function or goal looks fairly clear to the observer. Its consequences should also be adequately certain so that there is little uncertainty concerning its effect (Flanagan, 1954).

“Incidents happen, but critical incidents are produced by the way we look at a situation: a critical incident is an interpretation of the significance of an event” (Tripp, 1993, p. 8). In a similar way, when assuming something is a critical incident, a judgement is made and the basis of that judgement is the importance attached to what that incident signifies. Tripp (1993) discusses how critical incidents are not objects but rather are produced and form a vital part of reflecting on lesson practice. Tripp (1993) also highlights that reflection is fundamental to developing professional judgement as teachers and critical incidents are helpful to that reflection process. They enable us “to move beyond our everyday ‘working’ way of looking at things” (p. 13), so that we are not dependent upon our existing world view but can change our cognisance through intentionally attempting to view our teaching practice in novel ways. Being aware is something we actively do for ourselves, but over time we might be able to structure our awareness so that we don’t always consciously control it.

Tripp (1993) defines ‘awareness’ as that which we notice about our practice and are aware of. He defines ‘problematic’ as the basic system that constrains and enables what we intentionally and reflexively choose to deal with. Critical incidents are very useful for fostering a growing understanding of and control over professional judgement, and thus over practice. Tripp (1993) also points out that “Interpretation is important because we act according to what we think things mean” and “…is also the process by which we render incidents into critical incidents” (p. 24). This has vast significance for this study as the teacher-researcher decided which incidents to form into critical incidents that were then used to make decisions about changing the structure of a lesson and in follow-up lesson preparation. Interpretation is deemed appropriate by Tripp (1993) for creating and analysing critical incidents, as it links to an interpretive sociology which “aims to produce micro-analytic accounts of the everyday in terms of how participants receive, perceive, create and negotiate their ‘reality’, which is precisely what one does in the analysis of critical incidents” (p. 29). Tripp (1993) discusses how most critical incidents commence with a specific account of an occurrence or thought. Detail is an important trait of describing incidents because when dealing with making sense of critical incidents in broader perspectives, there are unavoidable generalisations that have to be managed. To record critical incidents of quality that will be useful, two rudimentary principles should be met. Incidents should be

a) detailed
b) methodically collected and stored (Tripp, 1993).

Unless we do something with being aware of the things that have meaning for us, Tripp (1993) points out, there is little practical value in them. Flanagan’s (1954) description of critical incidents corresponded to this in that he discusses the necessity of having incidents that will be useful in solving a problem in some way or aiding in developing theory. In the same way, creating a critical incident before evaluating the incident, does not avoid but rather postpones action and reflection. Tripp (2003) suggests the Diagnostic Teaching Cycle (represented in Figure 2.1 below) as a method that can assist teachers in using critical incidents from the classroom. This occurs through following a kind of action research cycle (the major underlying methodology of this research) which is used for more than just reviewing, reflecting and changing action after a critical incident is noted. This approach alternates between action in the research place and interpretation in the research literature. “Reflection is centred on a series of incidents, each of which is explained prior to action, the explanation then being used to inform the response to what happened” (Tripp, 1993, p. 31). A series of incidents which are kept aside are described and analysed. A plan is designed for further action and then a method for evaluation and interpretation of that action. Tripp (1993) also comments on the constant switching between action in the classroom and interpretation of events, a cyclical form which is essential in any action research procedure. The series of incidents are reflected on and explained (which informs the response that will take place) before a decision is made on the course of action to follow.

Figure 2.1: Diagnostic Teaching Cycle (Tripp, 2003, p. 32)

PHASE	ACTION
Interpretation of needs	Observe situation → Create Critical incidents ↓
Meeting of needs	Implement Response ← Plan Response ↓
Review strategy and re-assess needs	Effects of response / **Observe** / New needs → Create Critical incidents ↓ etc

This study aimed to develop a metacognitive framework with a step-by-step procedure that would aid teachers in lesson preparation for a series of lessons. The teacher-researcher (author of this study, researcher and teacher using the action research approach) decided to follow an action research process in her own classroom and use the lesson events and preparation and reflection activities as an underlying factor in developing the framework. One principal factor that occurred in the lesson events was the presence of incidents that could be created into critical incidents for use in assessing how the students reacted to the teacher-researcher. These incidents helped to identify how the teacher-researcher's metacognitive processing aided in lesson preparation and reflection (deciding what changes needed to be made for follow-up lessons).

It is important to note that Figure 2.1 highlights how needs were interpreted, met and the strategy reviewed so that needs could be reassessed, outlining the process by which critical incidents are created (Tripp, 2003). The initial step of *observing the situation* occurred when, in the classroom, the teacher-researcher was aware of interactions between the students, interactions between the students and herself, what was spoken, what was misunderstood and what was learned. This was supported by the audio-recording of the lesson which, when transcribed, validated the observations of the teacher-researcher (which could not have all been accurately recorded to memory while in the classroom). The teacher-researcher pinpointed various incidents that were of interest and categorised them according to 'teacher-researcher's explanations', 'exploration/development of the students (group or individual)' and 'lack of understanding/too complicated'.

The following step, to *create critical incidents*, took place when particularly interesting incidents, that reflected something from the teacher-researcher's metacognitive preparations or reflections (or lack thereof) or from the discussion fora, were extracted. A *planned response* for each one was drawn up so that the *plan could be implemented* in the following lesson. Some incidents that were especially interesting were marked in the critical incident file for further analysis to be conducted at a later stage (written up in Chapter 4). Once again, the teacher-researcher *observed* the situation, *effects of the response* were noted and *new needs* were identified. After the lesson the teacher-researcher reviewed the strategy, reassessed needs and new critical incidents were created. A critical incident could be created that centred on the same 'issue' or 'concern' and if this was the case then a response was planned, implemented and assessed repeatedly until the teacher-researcher felt that the issue or concern had been adequately addressed. The 'solution' of the issue or concern may very well be the creation of a different critical incident that the teacher-researcher felt was important to be created. This cyclical process took place a number of times (with ten particular critical incidents selected) over the period of the three lessons taught and discussed. These critical incidents are analysed in further detail in Chapter 4 as they aid in the analysis and synthesis of data for this study, leading to development of the framework and discussion of its efficacy.

2.5. Systems 1 and 2

Schoenfeld (1987) sets the scene for introducing System 1 and System 2 as he discusses a scenario following the progress of three subjects in their attempt to solve a problem (working on it for twenty minutes). They were observed in their efforts and the stages they went through to solve the problem;

and the time spent at each stage and their management and sequencing skills were charted on a bar graph. The different stages included read, analyse, explore, plan, implement and verify and it was also noted if they stopped at any given time to comment on their progress. The first two subjects were novice Mathematics students registered for one of Schoenfeld's problem solving courses. The third subject was a mathematician who had not worked with geometry for a number of years but was an expert problem solver. The key point here is how different the graphs analysing the progress of the students looked from that of the mathematician's. Schoenfeld (1987) knew that both the students understood enough mathematics to solve the problem without difficulty (and one of the students had solved a similar problem correctly in an examination the previous week) however the problem was given to them out of context and their approach let them down. The students read the problem, made the correct speculation and then attempted to solve it. They made a few mistakes and then got caught up in their calculations. Their graph shows them being involved in only two stages: that of *reading* (for about a minute) and then in the *explore* stage for the next nineteen minutes, still unable to solve the problem when their time ran out, as Schoenfeld (1987) explains.

> "The students had spent twenty minutes on a wild goose chase. They had ample opportunity to stop during that time and ask themselves 'Is this getting us anywhere? Should we try something else?' but they didn't. And as long as they didn't, they were guaranteed not to solve the problem" (p. 193).

The mathematician's graph, on the other hand, provided an interesting distinction whereby he engaged in all six stages throughout the problem solving process. There were two parts to the problem which is clear in the graph when after *reading, analysing, planning, implementing* and then *verifying* the first solution, he proceeded to go back to the *analysis* stage, then *explored, planned, implemented* and *verified* the second solution (all quite evenly spaced and before the twenty minutes was up). There were also thirteen points marked off on his graph which represented episodes where he stopped the solving process to basically question himself on how he was doing and whether he needed to change direction or keep following that path. "The mathematician spent the vast majority of his time *thinking* rather than *doing*" (Schoenfeld, 1987, p. 194). The two students did not take one moment to pause, assess their progress and decide whether or not they should try something different. "…the difference between the mathematician's success and the students' failure cannot be attributed to a difference in knowledge of subject matter" (p. 195). The students started off with a clear advantage having used the same mathematics very recently to successfully solve a problem whereas the mathematician had not seen or worked with the geometry in years. The students knew all the procedures but the mathematician did not and had to figure them out for himself. “What made the difference was how the problem solvers made use of what they did know” (p. 195).

This lengthy summary was given to illustrate a point so that System 1 and System 2 could be introduced. These terms were originally suggested by psychologists Keith Stanovich and Richard West (Stanovich & West, 2000) and adapted by Kahneman (2011). Systems 1 and 2 are both approaches to making decisions but Kahneman (2011) typifies them as completely different thinking systems. “System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control” (Kahneman, 2011, p. 20). System 1 has proficiencies which include innate skills – we are ready to observe the world around us and recognise objects and focus our attention on things. Skills like reading and understanding social situations are also learned by System 1. Mental actions that are entirely instinctive and unconscious are managed by System 1 and its knowledge is stored and accessed without being planned for or worked at. “System 2 allocates attention to the effortful mental activities that demand it, including complex computations. The operations of System 2 are often associated with the subjective experience of agency, choice, and concentration” (Kahneman, 2011, p. 20). System 2 is the system that forms thoughts in a methodical series of stages and sometimes also takes over from System 1, superseding its carefree impulses and organisations. System 2 has highly varied processes which all necessitate attention and are disturbed when that attention is diverted.

We turn now to examine the connection between Systems 1 and 2 and the scenario described above by Schoenfeld (1987). The two mathematics students did not put any time into analysing or planning how to solve the problem given to them and after a brief reading of the problem went straight into exploring its solutions, using System 1 to immediately and automatically solve something that they thought didn’t need further analysis. “System 2 has some ability to change the way System 1 works, by programming the normally automatic functions of attention and memory” (Kahneman, 2011, p. 23). System 2 could have programmed System 1 to carry on with following procedures and rules that the students had decided were necessary to solve the problem however they did not put System 2 to full use by pausing to allow for more careful consideration of their method and realise that they were going in circles. The mathematician used his System 2 throughout the whole process, regularly pausing to assess his progress and changing direction if need be. Even in the moments that System 2 might have programmed System 1 to follow procedures and rules to solve equations, he went back to System 2 to check his thinking and make sure he was on the right track and if not, to change direction. He was willing to test and reject more ideas and this was his System 2 taking control and making sure that as soon as he realised an idea was not working, it was changed. When Schoenfeld (1987) said that the mathematician did more thinking than doing, we can apply the above theory by saying that the mathematician used his System 2 (careful thinking and consideration) more than his System 1 (doing system).

Teachers have a background, a history of experience, a number of variables that shape how they teach and how they relate to students in the classroom. This could lead very easily to falling into the trap of following automatic processes. With hours of preparation and teaching and marking and other responsibilities at school, the time and effort required for thinking about the process becomes more difficult and less appealing. This could cause the teacher to switch on System 1 mode of thinking for teaching processes, handling students in the same way, teaching repetitively, making quick judgements about students or quality of teaching preparation. System 2 being used as the thinking system could occur less often, which could lead to automatic reactions to classroom scenarios and a lack of reflection on lessons, on why a student didn't work (or did) and why a student is failing or unhappy in class. The link between System 1 and 2 and teaching preparation and reflection will be discussed in further detail in the analysis chapter.

"By action research, we mean teachers researching their own practice of teaching" (Feldman & Minstrell, 2000, p. 432). This study aimed to be an in-depth examination of the teacher-researcher's personal mathematics cognition in preparation and reflection of lessons, for the ultimate purpose of suggesting an effective metacognitive framework that teachers can use to metacognitively prepare for and reflect on their lessons. The efficacy of this framework for the teacher-researcher has also been examined, forming a major part of this research. The above bodies of literature are essential for the study and were drawn from for the purpose of informing the qualitative research design of the study and methodology. The details of the overall research design and its justifications follow.

III

Key elements of the research design

This section describes the action research methodology that was used for the study, in addition to the method of qualitative data analysis. The sample for the study is described clearly along with the setting of the study. The data analysis procedures which include the design for data collection methods, measures, ethical considerations and the reliability and validity of the study are outlined. Thereafter, the tools and approaches used for analysis, incorporating the research paradigm and triangulation procedures, are discussed.

3.1. Methodology: Action Research

Lewin (1946) coined the term ‘action research’, describing it as “research which will help the practitioner” (p. 34) and that research needed for social practice is a type of action-research that makes comparisons across conditions and effects of social action. Action research is also a type of research that results in social action. While Lewin (1946) introduced the idea of action research and used some related terms, he never methodically articulated its principles. His works, however, were used by other researchers to develop action research giving it more definition and structure, becoming what it is today in the field of knowledge.

In Part 1 of his book, Sagor (2000) gave the title: “Action Research: A Methodology for Refining Teaching”, highlighting the importance he places on using this methodology in order to refine one’s teaching. There are a number of synonyms for ‘refine’ and the Oxford Dictionaries (2015) lists them thus: improve, perfect, polish (up), hone, temper, fine-tune, elaborate, touch up, revise, edit, copy-edit; and the informal, tweak. In selecting an Action Research methodology the teacher-researcher inherently chose a method to aid the process, in part, of researching how to use their personal teaching practice and the implementation of metacognitive components, in order to develop a framework that would have the potential to be developed and tested for use by any teacher. “Action research is always relevant to the participants. In action research, relevance is guaranteed because the focus of each research project is determined by the researchers, who are also the primary consumers of the findings” (Sagor, 2000, p. 13). This study transpired as a result of witnessing how engaging in metacognitive practice aids understanding, as discussed in the first chapter. The issue of particular interest to the teacher-researcher is metacognition and its role in increasing understanding of concepts and in effective preparation for teaching. This study sought to thoughtfully develop a metacognitive framework and highlight its potential usefulness in the preparation and reflection of lessons.

Sagor (2000) gives a concise definition of action research that appears in the materials used by the Institute for the Study of Inquiry in Education – "a disciplined process of inquiry conducted *by* and *for* those taking the action. The primary reason for engaging in action research is to assist the 'actor' in improving and/or refining his or her actions." (p. 11). Action research has the positive outcome of empowering those who decide on using it as a course of enquiry, often due to the relevance of its focus for the teacher-researcher. The teacher-researcher determines the focus of the study and therefore acquires the most out of its findings. This study sought to help the teacher-researcher become a more effective educator and develop a framework for assisting effective teaching in other classrooms. "And the practice of action research *becomes entwined with other practices* whenever it aims to understand those other practices, to change the way they are done or to change the ways people relate to each other in them" (Kemmis, 2010, p. 420). The use of action research becomes interwoven with the discovery of other concepts, ideas or practices. In this study, exploring the teacher-researcher's use of a metacognitive tool for the improvement of teaching practice could only be investigated to its fullest extent when a concerted effort, with confidence in its effectiveness, was put into the action research process. The teacher-researcher carefully selected the methodology with this in mind.

In defining action research with the teacher as the researcher, Feldman and Minstrell (2000) point out that the focus should be on the research and on its aims, which should include improving the teaching and learning in the classroom; and having a better awareness of the researcher's education setting. Two specific action research aims that are also appropriate for this study included: "Generating knowledge about teaching and learning" and "Increasing understanding in teaching practice" (this refers to the understanding on the part of the teacher-researcher; however, the progress of the students' understanding is important as well). These aims are simply two of the numerous products that can result from an action research process. These results, as discussed by Feldman and Minstrell (2000), complement each other in the third which is referred to as "Improvements in teaching and learning" (p. 433).

"…it is helpful to commit to a time line and a process for completing the work of data collection" (Sagor, 2000, p. 92). Nothing in the current body of knowledge suggests a fixed or minimum time period that would ultimately define whether a process followed an action research methodology or not, therefore making the three lessons with three follow-up discussion fora valid components of an action research study. Mertler (2012) provides a number of reasons why one should engage in an action research process and one of these is that "action research is very timely; it can start now—or whenever you are ready—and provides immediate results" (p. 23). A large number of researchers discuss action research studies carried over a period of time or within a time limit but do not specify how long or short this is required to be for it to be classified as action research (McNiff, 1995; Sagor, 2000; Feldman & Minstrell, 2000; Ferrance, 2000; McNiff &Whitehead, 2002; Brydon-Miller, Greenwood & Maguire, 2003; Pine, 2009). More specifically McNiff and Whitehead (2002) suggest, as part of the advice given for what to do or not do when conducting action research, that a teacher-researcher should decide on a realistic timeline, however the length of this timeline is not specified. The process for this action research study was brief and intensive as opposed to prolonged, however as can be seen in the following particulars, all the stages necessary, in order to follow an action research methodology, were present.

Sagor (2000) gives a detailed explanation of the cyclical seven-step process that shapes the action research structure within which the teacher-researcher is immersed. These seven steps which form the basis of the methodology that will be used in this study are:

> " 1. Selecting a focus
> 2. Clarifying theories
> 3. Identifying research questions
> 4. Collecting data
> 5. Analysing data
> 6. Reporting results
> 7. Taking informed action " (p. 12)

Each of these steps, discussed in detail, will follow here according to Sagor's (2000) descriptions but with specificity to this study. The **focus was selected** by the teacher-researcher through serious contemplation about what component of their teaching practice (including preparation for and reflection of lessons) they wanted to investigate.

The **clarification of theory** took place when the teacher-researcher identified the views and theoretical perspectives related to the focus of the study. After a detailed history was laid out and much thought went into which direction the study should take, the research problem was identified and a meaningful **research question** began to form. Over the research process, and after more deliberation and metacognitive questioning, the final research question became more definitive (Sagor, 2000).

A valuable **collection of data** served in producing sound instructional decisions and it was therefore essential to design a successful data collection strategy as Sagor (2000) advises. The validity and reliability of the data collected is discussed and the conclusions drawn in the analysis align with the distinctive characteristics of the school (specifically the Mathematics department). The process of triangulation, defined further on in data analysis procedures, was used for the purpose of enhancing the validity and reliability of the findings. The design of data collection is detailed further on.

The **analysis of data** did not require complex statistical computations due to the qualitative nature of the study. The teacher-researcher needed to carefully examine the collected data and look for patterns and what critical incidents were evident, and also a reason as to why certain developments occurred. The crucial part of this process was for the teacher-researcher to better understand, and therefore define, the use of metacognition in preparation for and reflection of teaching, how it improved personal teaching practice and the development of a framework that could be used to help other teachers be metacognitive in their lesson preparation (Sagor, 2000).

The data analysis and final chapters comprise a **report on the analysis and findings** of the study, thereby "making a contribution to a collective knowledge base regarding teaching and learning" (Sagor, 2000, p. 14). Making the **next decision** after evaluation of the findings was the last (but vital) step in the action research process and often part of lesson preparation and reflection for teachers. Although much of teaching is a trial and error process, action research seems to aid researchers in discovering mistakes in their teaching practice, aiding in learning how to avoid and/or solve them, which was an essential part of this study.

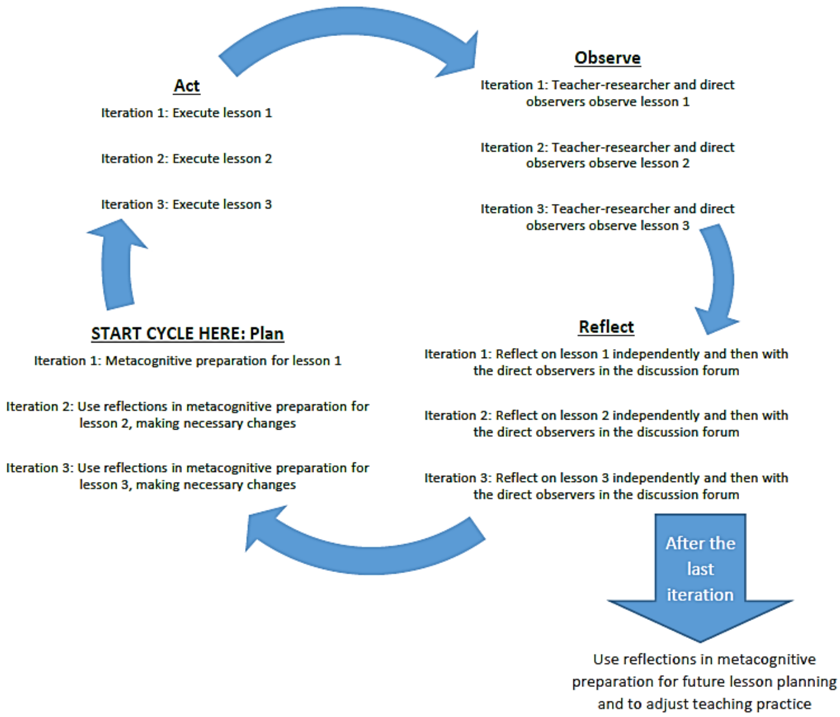
Something that Sagor (2000) did not reflect on or even mention was the cyclical nature of action research which is key to this study and emphasised in many action research articles, journals and books (Doerr & Tinto, 2000; Feldman & Minstrell, 2000; Ferrance, 2000; McNiff & Whitehead, 2002; Opie, 2004; McNiff, 2005; Pine, 2009; Blair, 2010; Mertler, 2012). In using an action research model, this study strove to display the cyclical nature of action research through the iteration in the data collection stage of two steps, collecting and analysing data. In the first session, data were collected and it was in the preparation for the second session that analysis of those data took place. Data were collected again in the second session and analysis of those data, in conjunction with the initial data, took place to prepare for the third session. After the third session's data were collected, they were analysed. Throughout these procedures, notes were written in a journal to keep a record of thought processes and as part of the metacognitive approach to lesson preparation. The final analysis of all data came much later and is detailed in the fourth chapter of this study.

Planning, acting, observing and reflecting as a set of spiral steps is generally understood as the basic theory of action research (Doerr & Tinto, 2000; Feldman & Minstrell, 2000; Ferrance, 2000; McNiff & Whitehead, 2002; Opie, 2004; McNiff, 2005; Pine, 2009; Blair, 2010; Mertler, 2012). Multiple models of action research have been suggested in this field of research however most of them have the same key structure of a cyclical progression which begins with identifying a problem and then planning and implementing change. This cyclical activity is summarised very neatly by Pine (2009):

> "The spiral of steps or cycles consisted of a basic cycle of activities: identifying a general idea, engaging in reconnaissance, making a general plan, developing the first action step, implementing the first action step, evaluating, and revising the general plan. From this basic cycle, the researchers then spiral into a second cycle of activities: developing the second action step, implementing, evaluating, revising the general plan, developing the third action step, implementing, evaluating, and so on continuing into a third, fourth, fifth cycle of activities" (p. 40).

Reflection and modification of the process is repetitive in order to mould and further the progression towards a viable solution (Doerr & Tinto, 2000; Ferrance, 2000; McNiff & Whitehead, 2002; Opie, 2004; McNiff, 2005; Pine, 2009; Blair, 2010; Mertler, 2012). This fluidity of movement throughout the process, reflecting and changing course where necessary, linked very well with the metacognitive activity of reflecting deeply and regulating thinking activity, changing plans for lessons where necessary. See a cyclical representation of the action research process specific to this study illustrated below.

Figure 3.1: Cyclical action research process specific to this study



3.2. Methodology rationale

Feldman & Minstrell (2000) stated that "the goal of action research is greater understanding that can be linked to improved practice" (p. 432). This was explicitly concurrent with the teacher-researcher's search for improving their personal understanding in order to better their teaching practice. Using this research design allowed the teacher-researcher to carefully select an issue that was of interest while playing multiple roles in the research process. The teacher-researcher was closely involved in the research process for the purpose of developing and then examining the framework and its usefulness. In order to examine teaching preparation in depth, it was necessary to highlight incidents in a familiar classroom, thus an action research approach was appropriate. Multiple discussions took place between the teacher-researcher and the Head of the Mathematics Department about finding solutions to teaching with understanding and constantly striving to improve teaching practice, before the study began. These dialogues motivated an action research approach, which included the colleague in question in the process and findings of the study (as an observer). "The methodology of action research always is situated in a particular context or setting and is directed toward actions to be taken by teachers and other members of the community of practice, often in a collaborative role with university researchers" (Doerr & Tinto, 2000, p. 411).

The study took place in the teacher-researcher's typical school environment and classroom and thus made the research relevant to the students. The teacher-researcher was related to the research field, influenced others, was influenced by others and brought personal values (wanting to improve teaching practice and student understanding) to the study, which made choosing the context for the research straightforward. These were highlighted by Doerr and Tinto (2000) as necessary traits for action research. The lessons were designed and taught in a way that sought practical solutions for improving teaching practice. Doerr and Tinto (2000) highlight that action research serves more to grow knowledge and solve issues that have been identified, than to necessarily reorganise entire educational theories. However, it can lead to an extensive transformation in the process of learning and relationships in the field as "it leads practitioners and researchers to mutually redefine their roles and to share their knowledge with the larger community of practitioners" (p. 426).

3.3. Research procedure: Qualitative

This study was qualitative in nature by examining authentic actions of the participants in a typical lesson and attempting to decipher and understand different experiences/incidents and the meaning that the participants brought to them (Hallberg, 2008). These experiences were refined to a smaller sample and analysed in depth. For the purpose of this study these incidents were referred to as 'critical incidents'. Incidents refer to any occurrence in the classroom but it was when they were selected for analysis that they were made 'critical', as detailed further below (Tripp, 2003). It was imperative as a qualitative study that the teacher-researcher shared the knowledge gained from the study with others. This sharing of knowledge can happen after the examination takes place and any necessary changes are made to the content.

The process involved the analysis of qualitative data including journaling, field notes, samples of a student's work from a lesson, observations, discussion fora and analysis of audio-recordings of the lessons. In a qualitative study, particularly, the research is influenced by the teacher-researcher who plays a central role in the research process. The study involved the development of a metacognitive framework through its effect on teaching probability to a Grade Six class, and was carried out with integrity so it was made a meaningful learning experience for all involved in the research process. "In a qualitative study the researcher is striving for closeness and is listening attentively to people concerned, without directing the narratives or the interpretation of those through his/her preconceptions" (Hallberg, 2008, p. 66).

3.4. Sample

"Classroom/school studies are teachers' explorations of practice-based issues using data based on observation, interview, and document collection involving individual or collaborative work" (Pine, 2009, p. 51). The action research investigation was a classroom study and due to the nature of the methodology, the setting of the study was the teacher-researcher's typical environment with a familiar class. The only change was having the direct observers present in the classroom and the students involved as participants in the research (all of whom knew and consented to be part of the lessons for research purposes). The student participants were selected because of their familiarity with the teacher-researcher and the standard routine in the classroom was kept in place in order to make the participants feel comfortable. The teacher-researcher was also a participant of the study, as it involved an investigation into their personal reflective metacognition. For the selection of direct observers to participate in the study, three educators with teaching experience and a passion for teaching Mathematics in a way that encourages understanding and enthusiasm by the children were asked to participate. Their biographical details were also collected (with permission) and are published as part of this study in Appendices 1, 2 and 3. These include questions about their view on Mathematics and how it should be taught and experienced by the students in the classroom.

McNiff (1995, p. 22) discusses the importance of having 'critical friends' or a 'validation group' as part of the action research process. A critical friend, also known as a critical colleague or learning partner, is someone whose views are valued by the teacher-researcher and "who is able to critique your work and help you see it in a new light" (McNiff, 1995, p. 22). Critical appraisal is crucial to help evaluate the quality of the research. One or two people can be asked, at the beginning of the research, to be critical friends. A validation group (made up of four to ten people) may or may not include a critical friend. The participants in the validation group could be gathered from one's professional circle and normally agree to meet with the teacher-researcher from time to time, to receive progress reports on the research and inspect the data. "Although they might not be entirely familiar with your research, they would be able to make professional judgements about the validity of your report, and would offer critical feedback" (McNiff, 1995, p. 18). While the teacher-researcher would be wise to pay attention to any advice offered, they are not obligated to act on it. There was no validation group that took part in this study; however, three direct observers who played a role as participants in the study might be referred to as critical colleagues. They spent time critically reviewing parts of the process and their feedback was vital for data collection and analysis. Their involvement is detailed below.

The participants that were used as part of a subsample to provide in-depth information were selected after critical incidents from the lessons were identified. These students are referred to by a number for anonymity and it is clear which students were part of the subsample as they have been included in the analysis of final critical incidents. The teacher-researcher was aware of processes that took place during the lessons and looked for opportunities where there was learning with understanding (or lack thereof). The teacher-researcher was also aware of the students (and their contributions) who were mentioned by the direct observers during the discussion fora. These critical incidents are discussed in detail in the fourth chapter and examined with reference to the metacognitive framework designed by the teacher-researcher in preparation for the lessons.

3.5. Data Collection Procedures

The methodological strategies chosen were used to ensure high-quality data were collected. One session consisted of a lesson preparation carried out by the teacher-researcher, a sixty minute lesson, a thirty to sixty minute discussion forum with three direct observers who had observed the aforementioned lesson, and a lesson reflection carried out by the teacher-researcher subsequent to the lesson and discussion fora. Three of these sessions took place and formed the data for this study. The teacher-researcher observed each lesson informally while teaching and the three sessions were audio-recorded. The teacher-researcher also took notes during the discussion fora (and some notes were taken during lessons when possible and if necessary). "Journals are teachers' written accounts of classroom life over time, including records of observations, analyses of experiences, and reflections and interpretations of practices" (Pine, 2009, p. 51). In the role of the teacher-researcher, I was also well-known to the student participants (as their teacher for a number of months) and made use of my rapport with them to advantage the study, as Blair (2010) suggests, which allowed for relaxed participants who seemed to feel more disposed to offering complete, truthful answers.

The time period for data collection fell over a six-day period, but actual data collection occurred on three separate days. One session (see the description of one session above) was conducted each day. Further reflection by the teacher-researcher was carried out in between these days and sessions. There was a data collection and analysis overlap and so the reflection process was constant, causing many changes in direction of thought. Any changes that took place in the focus of this study or planning of lessons are discussed in detail in the fourth chapter with the analysis. The lessons and discussion fora which were audio-recorded were also transcribed and any critical incidents or points that needed to be referred to were recorded and quoted directly. Parts of these transcriptions have been referred to and quoted from as part of the write-up for this dissertation. The direct observers were asked to sit in on all three sessions and were given observation sheets to fill in during the lessons. These observations sheets were regarded as helpful by the observers to collect their thoughts. They were, however, unnecessary to include as appendices since the observers gave full feedback during the discussion fora (referring to these observation sheets) and all the feedback correlates to the transcriptions of those sessions. The observers were also asked during the discussion fora on other thoughts that they had about how the lesson had progressed or how the teacher-researcher had approached the content. The discussion fora were also audio-recorded, during which the observation sheets and all other pertinent aspects of the lessons were discussed. Notes were made of the critical incidents and are discussed in further detail in the analysis chapter. The teacher-researcher took journal notes during preparation for the lessons, during the discussion fora and for reflection on lessons in an attempt to have all parts of the action research process recorded.

The teacher-researcher was responsible for the data collection. Maxwell (1996) highlights that "…qualitative researchers have long recognised that in this field, the researcher is the instrument of the research" (p. 37), however it should not be disregarded that the effect of 'bias' must be removed from the research strategy. Preparation for lessons was completed; lessons were taught and then reflected on afterwards with the direct observers, after which individual reflection by the teacher-researcher occurred. The closeness of the topic to the teacher-researcher is evident in the detail in the first chapter and due to the interest displayed, all potential bias and assumptions about the phenomenon in question were addressed there.

Before any preparation went into collecting data for the research, approval from the WITS School of Education Ethics Committee was obtained (see Appendix 4), after which the following steps took place.

There were twenty-six participants in the study and an additional three direct observers, also participants in the study. As each of the twenty-six participants were minors, they and their parents/guardians (in addition to the participants) were each given a participant information sheet (see Appendices 5 and 6) which informed them as to what their involvement in the study would be and what they would be required to do. They were then invited to join the study and, once they had thought it over, were given informed consent forms (see Appendices 7 and 8). The participants and their parents/guardians were required to sign their consent forms if they were interested in participating, which all twenty-six (with parents/guardians) did. The three direct observers were also given a participant information sheet (see Appendix 9) specific to their differing role, in addition to a consent form (see Appendix 10), which they signed to show their interest in participating.

Confidentiality and anonymity were ensured by the use of pseudonyms throughout the data collection and analysis. The observation sheets were also labelled with Observer 1, 2 and 3 which became the pseudonyms for the analysis and write-up process. The teacher-researcher alone knows the identity of the observers and participants and matched the data from the audio-recordings according to pseudonyms. All research data was scanned into the teacher-researcher's personal computer and saved in a secure location (which is password protected). Transcriptions were completed by the teacher-researcher onto her personal laptop. In the analysis process the teacher-researcher worked alone, therefore the data were not seen or heard by any other individual and all raw data would be destroyed within 3-5 years.

"…different procedures and methods such as triangulation, respondent validation, clear detailing of methods of data collection and analysis, reflexivity and fair dealing, detailed reports and sampling techniques are all means to improve validation of the qualitative study" (Qazi, 2011, p. 13). How triangulation was used to ensure validity in this study is detailed below, while methods of collecting data and how they were analysed is also covered extensively in this chapter. Detailed reports and sampling techniques have been carefully reported on. All knowledge was produced by the combination of input from the respective participants and direct observers. Due to this array of different perspectives and opinions which included those of the teacher-researcher, the data gathered had to be carefully analysed to ensure objectivity. In addition, due to much of the data being drawn from the teacher-researcher's personal metacognitive reflection, there was a need for corroboration of evidence. This corroboration was sought in triangulation which was the main purpose for the inclusion of the direct observers. The sample was not random as it was made up of a select few: student participants chosen for their familiarity with the teacher-researcher and direct observers selected for their passion for mathematics and experience in the field of education. It was appropriate to not have more than three direct observers in an attempt to keep the natural setting in the classroom.

None of the participants withdrew from the study and no replacements were needed. Planning for lessons took into account the possibility of participants being absent due to unforeseen circumstances (so that there were no gaps in the data collected). The direct observers were asked to give their hard copy observation sheets to the teacher-researcher immediately after the discussion fora or soon afterwards. No other person had access to the data before they reached the teacher-researcher and no other person saw it thereafter. All audio-recording was conducted using the teacher-researcher's personal equipment and no other individual was part of the analysis process. The data, therefore, are reliable and valid as there were not any opportunities for it to be tampered with.

Qazi (2011) discusses how the researcher should use credibility, dependability and confirmability to inform the research. "The term credibility (vs. internal validity) refers to developing internal consistency and showing the readers the way by which rigor is maintained in the research" (Qazi, 2011, p. 14). Credibility in a qualitative research study can be generated through a lengthy arrangement, reflection and participant checklists. Dependability suggests that the study is consistent with time, researchers and analysis techniques. The three lessons were uniform in duration, the researcher was the teacher-researcher, and author of this study, throughout the process and the analysis techniques throughout never varied. The dependability of this study was also preserved through discussion with peer educators (rather than peer researchers) who also gave further input during the analysis stages on data validity. Confirmability "addresses that the researcher should focus on the situation and beliefs of those that are being researched rather than his presupposition and beliefs" (Qazi, 2011, p. 14). This direction of focus is accurate for the study in that the situation and beliefs of the teacher-researcher were being researched, in addition to the students, hence it was necessary to incorporate them. What makes this valid is that while the teacher-researcher focused on herself in addition to the students, as that formed an integral part of the study, she had to be careful not to make assumptions. Potential bias and assumptions were addressed in the introductory chapter.

3.6. Data Analysis Procedures

In the fourth chapter the collected data was processed to convey to others what has been learned in this study. In processing the data, there was an attempt to breakdown and scrutinise components in a manner that allowed the teacher-researcher to identify patterns, ideas, find associations and develop explanations for findings as noted by Hatch (2002). "Data analysis is a systematic search for meaning" (Hatch, 2002, p. 148). It is important to note that "*all* research is interpretative in that it can only offer an interpretation, not an exact replica, of the world" (Opie, 2004, p. 18), however this framework is strongly interpretive by seeking to give meaning to the data by producing explanations of situations after making sense of the data. The teacher-researcher constructed an interpretation of the data, attempting to understand the social incidents being studied, and followed the "Interpretive analysis" model as detailed below in Figure 3.2.

Figure 3.2: Interpretive Analysis Tool (Hatch, 2002, p. 181)

1. Read the data for a sense of the whole
2. Review impressions previously recorded in research journals and/or bracketed in protocols and record these in memos
3. Read the data, identify impressions, and record impressions in memos
4. Study memos for salient interpretations
5. Reread data, coding places where interpretations are supported or challenged
6. Write a draft summary
7. Review interpretations with participants
8. Write a revised summary and identify excerpts that support interpretations

There are two parts to the data analyses process which follow the stages of an Interpretive Analysis procedure. The first part of data analyses covered the reflective metacognitive process that the teacher-researcher engaged with, in preparation for and reflection of lessons. The metacognitive process is a type of analysis itself, analysis of the whole process, key factors from the lesson and comments by the direct-observers. The second part of data analysis was to investigate critical incidents that occurred during the lessons and relate them back to the reflective metacognitive process that transpired, which may or may not have accounted for the incident. It was not necessary to adapt any of the above steps to interpret the collected data. The only point to be noted is that not all the steps taken by the teacher-researcher is evident in the analysis chapter. For step one where reading the data for a sense of the whole is listed, the teacher-researcher could clarify what data were read for a sense of the whole. For identifying or reviewing impressions of the data, the teacher-researcher included some notes or memos in Chapter 4 so that the impressions which were often repeated were not verbose in the write-up of the analysis. Some impressions or notes recorded in the journal that were deemed unnecessary or superfluous by the teacher-researcher were excluded entirely. The draft summary that was step six in the Interpretive Analysis procedure was not written out in Chapter 4 as it would have been redundant information once the revised summary was written. The draft summary step therefore includes significant thought processes or how a conclusion was reached in the development of the framework (which was then not repeated further on).

Triangulation "involves the use of multiple independent sources of data to establish the truth and accuracy of a claim" (Sagor, 2000, p. 89). Multiple sources of data were used and more than one perspective given (by using three direct observers), in order to triangulate the data. Triangulation in research is when information is collected from a varied scope of individuals and settings, using an array of different methods and finding different sources of data (Maxwell, 1996; Mathison, 1988).

The phenomenon studied were the critical incidents and the outcome of using metacognitive phenomena in the preparation for and reflection of teaching three Grade Six Probability lessons. The teaching style of the teacher-researcher, and the responses from the students, has been reviewed in depth from multiple angles in the fourth chapter. These necessitated direct observation by three participants, discussion fora with the direct observers and the teacher-researcher, audio-recording of the lessons and the discussion fora and informal observation by the teacher-researcher of the lessons' progression. Feldman & Minstrell (2000) describe how "triangulation consists of collecting data that represent several views of the same situation" (p. 436). The measures described above were taken purposefully to avoid perceptual misrepresentation, which was a limitation that had been considered before the study commenced.

Yeasmin and Rahman (2012) define triangulation as a way to enhance validity by integrating numerous perspectives and methods. While there is the possibility of triangulating using two or more theories and/or data sources, the combination of two or more methods and investigators with a single common phenomenon was specific to this study. Investigator triangulation also occurred through the presence of different investigators with the same qualitative method in the three direct observers, and the teacher-researcher, who observed the same lessons. Methodological triangulation was also present through the use of multiple qualitative methods, which include observation sheets, discussion fora and reflective metacognition.

For the second lesson, the class was divided into four groups and one of those groups was engaged with the teacher-researcher and sitting at the front of the classroom. The other groups worked individually, in pairs or as a whole group on activities that the teacher-researcher had prepared for them. For ease of discussion the group working with the teacher-researcher is referred to as the 'front group'.

3.7. Mathematical content: Probability

Grade Six content requirements – Method in context of curriculum

The Curriculum provided by the Department of Basic Education (DBE) was used to delineate the depth of content required when teaching Probability to Grade Six students.

In the DBE (2011) curriculum documents the content was outlined. The table below represents the general and specific focus of the content area of data handling, and also probability, for Intermediate Phase (Grades Four, Five and Six). This delineation was significant for clarifying what content should be taught, as the questions asked and language used by the teacher-researcher needed to be at a level that the students would be able to grasp; the skills outlined below would be helpful in determining that. Including difficult content was a flaw in the teacher-researcher's practice, as described later in the fifth chapter. Data handling skills are valid for understanding probability and the students should learn how to ask questions and find answers in order to describe the events and experiments that they carry out. They develop specific skills of gathering, classifying, expressing, examining, deducing and conveying the data gathered from carrying out various experiments. Development of these skills all take place before 'calculating' probability even begins to feature. The students need to learn how to apply these skills to multiple situations, only one of which being probability. Learning how to gather and interpret data is important and comes before producing multiple numerical answers, for example in the calculating of theoretical probability.

Table 3.1: General and specific focus of Data Handling (DBE, 2011, p. 11)

MATHEMATICS CONTENT KNOWLEDGE		
Content area	General content focus	Intermediate Phase specific content focus
Data handling	Data handling involves asking questions and finding answers in order to describe events and the social, technological and economic environment. Through the study of data handling, the learner develops the skills to collect, organize, represent, analyze, interpret and report data. • The study of probability enables the learner to develop skills and techniques for making informed predictions, and describing randomness and uncertainty. It develops awareness that --different situations have different probabilities of occurring --for many situations, there are a finite number of different possible outcomes.	• Learners should focus on all the skills that enable them to move from collecting data to reporting on data. • Learners should be exposed to: -- a variety of contexts for collecting and interpreting data -- a range of questions that are posed and answered related to data • Learners should begin to analyse data critically through exposure to some factors that impact on data such as from whom, when and where data is collected. • The focus of probability is to perform repeated events in order to list, count and predict outcomes. • Learners are not expected to calculate the probability of events occurring

It should be noted that the focus of the probability topic at this stage should be for the students to execute events repeatedly for the purpose of registering, counting and predicting outcomes. Students are also not required to determine the probability of events occurring. However the use of language on making predictions for outcomes and talking about the likelihood of certain events, lends itself to a situation where students might start questioning what the probability of an event actually is in terms of 'number'.

In Section 2 of the DBE (2011) document, the specification of content is outlined which demonstrates how the teacher should progress in concepts and skills for the Intermediate Phase for each Content

Area. Probability has been selected here to show the progression from grade to grade and the concepts and skills expected to be taught.

Table 3.2: Specification of Content (DBE, 2011, p. 31)

TOPICS	GRADE 4	GRADE 5	GRADE 6
5.4 Probability	Probability experiments • Perform simple repeated events and list possible outcomes for experiments such as: -- tossing a coin -- rolling a die	Probability experiments • Perform simple repeated events and list possible outcomes for experiments such as: -- tossing a coin -- rolling a die -- spinning a spinner • Count and compare the frequency of actual outcomes for a series of trials up to 20 trials	Probability experiments • Perform simple repeated events and list possible outcomes for experiments such as: -- tossing a coin -- rolling a die -- spinning a spinner • Count and compare the frequency of actual outcomes for a series of trials up to 50 trials

The teacher-researcher gathered information informally about prior knowledge on probability from the students themselves and from their previous mathematics teachers. Generally it was suggested that probability was not covered in great detail (if at all) in previous years. Experimentation with typical instruments (die, cards, spinners, etc.) had been introduced. An extensive investigation with carrying out different experiments had not been conducted and the students were not familiar with important probability vocabulary. As suggested in the above table, tossing a coin and rolling a die were useful experiments to carry out and discuss, however another three were added (drawing numbers from a group of number cards, pulling out coloured balls from a tin and drawing cards from a pack of playing cards). The teacher-researcher decided to give the students a series of thirty trials to carry out for the purpose of collecting sufficient data, but also keeping in mind time constraints.

Clarification of Content is detailed below from Section 3 of the DBE (2011) document and provides teachers with guidance as to how the development of concepts should be attended to, with suggested order and pacing of topics. The content area was divided into separate topics with Probability being one of the topics that falls under Data Handling. “Teachers may choose to sequence and pace the contents differently from the recommendations in this section. However, cognisance should be taken of the relative weighting and number of teaching hours of the content areas for this phase” (DBE, 2011, p. 32).

Table 3.3: Clarification of Content (DBE, 2011, p. 289)

CONTENT AREA	TOPICS	CONCEPTS AND SKILLS	SOME CLARIFICATIONS OR TEACHING GUIDELINES	DURATION (in hours)
DATA HANDLING	5.1 Probability	Perform simple repeated events and list possible outcomes for events such as: • tossing a coin • rolling a die • spinning a spinner Count and compare the frequency of actual outcomes for a series of trials: • Up to 50 trials	Performing simple repeated events Learners need to perform experiments by tossing a coin, rolling a die or spinning a spinner. Doing experiments with a coin is easier than with a die because the coin can only have two outcomes (heads or tails), while rolling the die can have 6 outcomes (numbers 1 - 6). The spinner can have any number of outcomes, depending on the number of divisions made on the spinner. Learners must first list the possible outcomes before doing the experiments. They should learn how to record the results of their experiments in a table using tally marks. Learners then count how many times heads or tails, or each number, or colour on a spinner, occurs in 20 trials. If learners do this in groups, the results from all the groups can be collated. They can then compare the number of outcomes that occur as the number of trials increase.	2 hours

Even though DBE (2011) discusses the ease of flipping a coin over rolling a die, due to the fewer number of possible outcomes, the students were asked to roll a die and flip a coin three times each, recording the outcomes. They were also asked to think about and try predict how many different outcomes they think there are, beforehand. The teacher-researcher decided that after discussing the possible outcomes for one coin flip or one die throw, the students could explore how the number of possible outcomes increased vastly when carrying out the same investigation multiple times for one experiment. Tally tables were used to record outcomes (see Appendices 17-20) and the teacher-researcher made sure to have a tally table with data filled, as a reminder for the students on how to use tally tables. The students recorded the results of their own experiments themselves and the teacher-researcher ensured that they were working in groups of two or three initially, and later groups of four, to warrant that every student was involved in either carrying out experiments or recording results throughout the lesson. The results from different groups were collated during the first lesson onto Microsoft Excel and displayed on the board to compare the number of outcomes that occurred as the trials were increased. As theoretical probability and how to calculate actual probability had not been introduced, difficulty in discussing this increase arose and is detailed further on in incident 4, Appendix 11.

Teacher-researcher's content preparation

Measuring the chances of an event can be conveyed as a ratio and "this measurement of chance in terms of a ratio is one definition of *probability*" (Johnson, 1963, p. 7). Multiple discussions about probability with the students ensued in the classroom and no matter what detail of mathematical content was discussed, it all narrowed down to one main concept: that probability is about measuring the chance or likelihood of a specific thing happening.

A more classic definition of probability is provided by de Laplace (1814/1951).

> "The theory of chance consists in reducing all the events of the same kind to a certain number of cases equally possible, that is to say, to such as we may be equally undecided about in regard to their existence, and in determining the number of cases favourable to the event whose probability is sought. The ratio of this number to that of all the cases possible is the measure of this probability, which is thus simply a fraction whose numerator is the number of favourable cases and whose denominator is the number of all the cases possible" (p. 6).

This basically amounts to the equation Hume (1966, p. 130) provides that can be used to calculate probability for an event E:

$$P\ (E) = \frac{number\ of\ outcomes\ in\ E}{total\ number\ of\ outcomes}$$

Understanding this equation, and the circumstances that provoke its existence, is the key in trying to figure out probability and more importantly in this study, the key to guide students in figuring out probability.

Ormell (1968) comments that in addition to this definition, focusing awareness on the description of the event is helpful for solving probability problems. Students should also be made fully aware that probabilities lie in the range $0 \leq p \leq 1$ where *p* is the probability. This point is also valid for the study as the teacher-researcher was not certain if this concept would be too complex for Grade Six students and so while it was introduced carefully, very little time was spent discussing it. The teacher-researcher engaged with much use of probability language regarding this concept, whereby when *p = 0*, the chance or likelihood of that event occurring was 'not at all' or 'impossible' and some examples were given to the class. The teacher-researcher asked which number has zero chance of being pulled out of a bag that only contained the numbers one to ten. The students were initially unsure in their replies, but grew more confident and responded with any number that they knew wasn't in the bag (see incident 22 in Appendix 11). The teacher-researcher also asked what the chance is of pulling a black paper (rolled up) out of a sample and the students immediately asked if there were any black papers in the sample, answering that if not, it would be impossible (see incident 32 in Appendix 11). The teacher-researcher then made the link between the language, the event and the number by saying "so there is zero chance that I can pull out the number one hundred from this bag?" and the students agreed. The teacher-researcher introduced *p=1* in the same way by showing that if the likelihood of an experiment was 'definite' or the event 'will happen' then it can be measured by saying the probability was equal to 1.

The teacher-researcher also pointed out to the students that everything in between zero and one represents the other probabilities ranging from highly unlikely (closer to zero) to highly likely (closer to one).

Ormell (1968) makes a brief note about the philosophy of simple probability statements. It is seemingly impossible to express the frequencies that are suggested by simple probability statements. If the frequencies are from previous trials then one cannot be certain that they will be duplicated exactly in future trials. If the frequencies are in the next few or many or infinite number of trials to be carried out, it is not feasible to fairly assume what their limits will be. In fact, Ormell (1968) points out that “Von Mises, the chief modern proponent of the frequency theory, was forced to the conclusion that the frequencies implied by simple probability statements are those which would obtain in an infinite series of trials” (p. 153). This is an important point for what was introduced to the students in the classroom because when the link between experimental and theoretical probability was discussed, the teacher-researcher wanted them to understand that the greater the number of experiments that were carried out, the greater the probability tends towards the ‘theoretical answer’. Or rather, an infinite number of trials would result in the implied frequencies of theoretical probabilities. Ormell (1968) taking this into account adds that according to *“a priori”* theory of probability, probability statements reflect and express a “symmetric distribution of ignorance” (p. 153) and that frequency facts must always be considered before deciding that two possibilities have an equal likelihood of occurring. For the education of probability at school level according to the content covered by the DBE (2011) and for the purposes of this study, which was based on an action research exploration in a classroom, basic probability statements according to the system of acquiring theoretical probability frequencies were applied.

Carrying out an episode like tossing a coin or drawing a coloured ball out of a box, can be described as an *experiment*. Each time the experiment is carried out, it can be described as a *trial* of the experiment. The result of carrying out the experiment is called an *outcome* and which outcome will occur from a specific trial is simply a *matter of chance* (Hume, 1966). “The theory of (mathematical) probability is concerned with experiments in which the occurrence of the outcomes is governed by chance alone” (p. 128).

Adler (1963) discusses a straightforward experiment that results in chance events, namely rolling a die. When the die stops moving one of the numbers (1, 2, 3, 4, 5 or 6) will face upwards. If the question of which numbers is facing upwards is asked, there are six possible solutions each of which are a possible outcome of this experiment. Adler (1963) defines a *sample space* for the experiment as “the set of numbers {1, 2, 3, 4, 5, 6} each of which designates one of these possible outcomes" (p. 16). There is also the possibility of asking whether the upward facing number is an odd or even number and then there are only two possible answers, therefore only two possible outcomes to this question. The words {odd, even} form a different set of outcomes and this is another possible sample space for the experiment of rolling one die. Depending on the question asked about the upward facing number, the different sample spaces for the same experiment are revealed. Adler (1963) also notes that it is not necessary to physically roll a die in order to determine the possible outcomes of rolling one. The possible outcomes can be listed, relating to the specific question of likelihood asked, by merely envisaging that a die has been rolled, which would make it a conceptual experiment. This listing leads to a definition namely:

> "*A sample space for a real or conceptual experiment is a set of symbols designating possible outcomes of the experiment* where each possible outcome is a possible answer to some specific question, and where the result of any performance of the experiment corresponds to one and only one member of the set" (Adler, 1963, p. 16).

The difference between real and conceptual experiments were not discussed with the students, however it was illustrated subtly by conducting real experiments in the class and through the teacher-researcher's questioning of different hypothetical scenarios without actually carrying out the experiment. The students were able to give a list of possible outcomes even without physically seeing or touching the sample described.

A more carefully selected sample space, Adler (1963) points out, enables the experimenter to answer more questions about the experiment in retrospect, for example having {1, 2, 3, 4, 5, 6} as the sample space instead of {even, odd} allows the experimenter to know which number resulted for each experiment, and whether it was even or odd (instead of only knowing if the result was even or odd). "The finer the classification of possible outcomes in a sample space, the more useful that sample space will be" (Adler, 1963, p. 17). With regards to this point, the teacher-researcher selected the classification of possible outcomes in a sample space for the students and ensured that they were useful for analyses of the experiments (ensuring that students could answer all the questions that had been prepared for them, with focused attention).

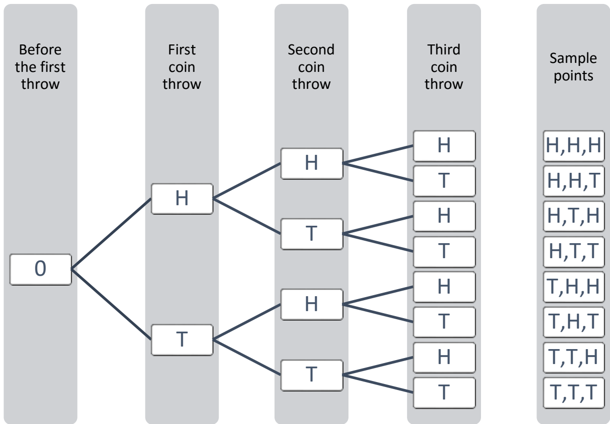
Adler (1963) also discusses an experiment where there will only be two possible outcomes, namely tossing a coin and "if we use *H* to stand for head and T to stand for tail, the set {H, T} is a sample space for the experiment" (p. 18). If the coin lands standing up or leaning against something, the throw is not counted and the coin should be thrown again. Adler (1963) also discusses infinite sample spaces where the possibility of an outcome in a certain position may occur at a very late, even indefinite space, however only finite sample spaces were used in the experiments conducted with the students in the classroom.

An event takes place when one of the outcomes from a specific sample set matching a specific question occurs (Adler, 1963). Therefore if rolling a die and looking for an event whereby the upward facing number is even, then the event occurs if the number 2, 4 or 6 faces up on rolling the die. Hume (1966) defines an event as occurring in a trial if any one of the outcomes in the event occurs. Instead of referring to the set of specific events as a subset of the set of sample space, as Adler (1963) describes, it was explained more simply to the students by referring to, and even defining, an event as a 'collection of outcomes', closer to how Hume (1966) describes it. While events are actually a collection of outcomes (matching the question asked) the students were not introduced to subsets of sample spaces, therefore the outcomes in the original sample space were what caused the event to occur. The experiments were planned so that the outcomes were always from the subset and therefore a collection of experiments and their subsequent outcomes were referred to as events (defined with the students as a 'collection of outcomes'). The formula detailed above, $P(E) = \frac{number\ of\ outcomes\ in\ E}{total\ number\ of\ outcomes}$, was used with the students in determining probability once sample space had been introduced, however it was not dwelt on. While DBE (2011) commented that students are not expected to calculate probability, discussing the amalgamation of the events from different groups and how an increase in trials results in a 'stabilisation' of sorts of the outcomes (tending towards actual probability of the event). This provoked interest in theoretical probability and why the different outcomes all seem to start appearing a similar number of times (see incident 24 in Appendix 11). This was the reasoning behind introducing how to determine probability but this was not covered in too much detail or for too long.

Hume (1966) also comments that "the notion of *equally likely* outcomes…is actually rather complicated" and involves balanced coins and dice in addition to the awareness of fair throws or draws. In a real experiment one can never be completely convinced that the perfect circumstances necessary for ensuring equally likely outcomes, have been attained. In terms of *mathematics*, equally likely outcomes can be assumed "in a particular experiment with n outcomes, the probability of each outcome is $\frac{1}{n}$" (Hume, 1966, p. 133). Other concepts that Hume (1966) discusses such as sample points (elements of the samples space which are the outcomes) and simple events (an event comprising only one sample point), were not introduced to the students at this level.

Another important piece of knowledge that is necessary to highlight is the role that tree diagrams play in the field of probability. Tree diagrams are basically useful tools that contain a sequence of routes to keep track of all possible outcomes (Wisner, 1973) and "in probability, the objects of the counting process are normally outcomes, and keeping track of outcomes can sometimes be tricky" (p. 236). Tree diagrams enable the monitoring of one thought at a time, and enable the recording of that thought. Wisner (1973) also points out that in problems that are too complicated, the tree diagram has so many branches that it does not simplify the counting process. A number of researchers have used tree diagrams in their explanations for multiple coin throws (Adler, 1963; Hume, 1966; Ormell, 1968; Wisner, 1973). The tree diagrams have been examined carefully and an adaptation of them is presented below in Figure 3.3. The tree diagram of three coin throws reveals a total of eight possible outcomes (sample points) in the sample space.

Figure 3.3: Tree diagram representing experiment of throwing three coins



It was initially planned for this diagram to be used in the third probability lesson (see original lesson plan above) until a discussion took place that made the teacher-researcher realise it was too abstract and complicated for the students. One of the critical incidents, discussed in detail in the analysis chapter, reveals a few students (one in particular) thinking of and deciding that using tree diagrams would be a more efficient way to solve a more difficult problem rather than trying to imagine all the possible outcomes. This occurred without influence from the teacher-researcher or even any mention of tree diagrams and their use in probability. It concerned the experiment of throwing a die three times in a row and student 7, who persevered in trying to find a solution even redrawing the tree diagram, came to the correct answer (see incident 40 in Appendix 11).

Final content planned for/taught to students (lesson preparation)

"The goal of the theory of probability is inherently paradoxical: to draw certain conclusions about uncertain events" (Adler, 1963, p. 13). It is for this reason that students may find Probability a difficult topic to grasp – they need to give definite answers about events that are indeterminate and have multiple outcomes. The events needed to be set out clearly so that some certainty about some of its aspects was ensured.

The teacher-researcher in preparing for the lessons used parts of lesson ideas on Probability from Houghton Mifflin Company (n.d.) and decided to include the following content:

- Important definitions to understand (outcome, event, probability, experiment, sample space)
- Using the correct probability language consistently with the students
- The difference between experimental and theoretical probability
- Using tables to record and interpret data and combining those data from different students (and even different classes)
- The range of probability ($0 \leq p \leq 1$) where p is the probability is too complex but it is useful to explain why fractions are used to describe probability
- Equation: $Probability\ of\ event\ A\ = \frac{number\ of\ total\ outcomes\ in\ event\ A}{total\ number\ of\ outcomes}$
- Probability can be written as common fractions, decimal fractions or percentages
- Important to understand that the larger the number of experiments, the higher the likelihood of getting close to theoretical probability is (could discuss with the students how this is possible to see once everyone's data is put into the table together)
- Looking at specific experiments:
    1. Discussing possible outcomes that there are in an event
    2. Discussing likelihood of each outcome happening
    3. Seeing which outcomes happen

Preparation for Lesson 1:

- Children sit in pairs and each pair has a group of cards with numbers 1-10
- Do 30 probability trials (which shows experimental probability, and then discuss informally)
- Discuss theoretical probability
- 'Let's do an experiment'. One person in the pair draw out one number without looking. 'What did you get?' (go around the classroom getting the results from different students)
- 'Let's do another experiment'. Now a different person in the group draw out one number without looking. 'What is your result?' (gather different results from a number of students)
- 'Let's do an experiment'. The next person in the group draw out one number without looking. 'What is the outcome?'

- Write **Outcome:** on the board and then ask 'What do you think an outcome is?' wait for the students to reply 'an answer', 'the number you get', 'what we pulled out' and see if any students remember you repeating 'What is your result? What was your result? What was the result this time?'
- Once students agree and formulate the definition together, write **Outcome: the result of an experiment.**

- 'Ok now we're going to do 30 experiments and write all the outcomes in a table' (Give table on the board, numbers 1-10 and tally outcome, and ask them to write on paper and record their outcomes for each experiment)
- Give 5 minutes for 15 experiments, one student draws a number and the other student records it, then they switch roles.
- 'Look at your outcomes and let's share a few with the rest of the class'
- 'In student A's event, 3 was the highest number of outcomes. In Student M's event, the lowest number of outcomes was 9. Here are my events'
- Display your events on the screen from the laptop.
- 'What is an event?'
- Write **Event:** on the board and then ask 'What do you think an event is?' wait for the students to reply 'something that happens', 'all your answers', 'the results you get' and see if any students remember you repeating 'Let's look at student A's event. In student G's event, the lowest number of outcomes was 6'
- Once students agree and formulate the definition together, write **Events: a collection of outcomes.**
- Ask questions like:
    1. 'Because of what we can see here, which number seems to have the highest likelihood of being the next outcome?'
    2. 'Look at student H's table. Which number has the highest likelihood of being pulled out next?'
    3. 'Look at student Y's table. Which number has the highest probability of being chosen next?'
    4. 'Look at student V's table. Which number has the lowest probability of being chosen? Which number has no chance of being pulled out? Which other number has no probability of being pulled out of the box? How many others have no probability of being pulled out?'
- Write **Probability of an event:** on the board and then ask 'What do you think the probability of an event is?' wait for the students to reply 'likelihood of something happening', 'if the event will happen', 'when something happens' and see if any students remember you repeating 'what is the likelihood or probability of getting a 6 here', 'what is the probability of student H getting a 4'

- Once students agree and formulate the definition together, write **Probability of an event: how likely an event is to occur/likelihood of the event happening**
- Review the definitions with the class, making sure they understand and remember.

<u>**Original Preparation for Lesson 2:**</u>

- 'I have a bag and ten different coloured balls with three of them being quite dark colours, black and brown and green (dark), and seven of them being light colours like yellow, orange, pink, blue (light), red, white, silver'. Show students the different coloured balls and then put them back into the bag.
- 'If I reach into the bag without looking and pick one out, what colour do you think it will be?' Students will probably say that it could be any of the colours, since they are all equally likely.
- 'Do you think we are more likely to pick a dark colour or a light colour?' Students should say that it is more likely that a light colour will be picked since there are seven light colours but only three dark colours.
- 'Let's look at the event of reaching into the bag without looking and pulling out either a red or black ball. How many balls are there in total? How many of those balls match what we are looking for (red or black)? So the probability of reaching into the bag and pulling out a red or black ball is $\frac{2}{10}$ or $\frac{1}{5}$.'
- 'The different colours are a sample space for choosing a ball from the bag, because it lists all the possible outcomes. Let's look at some more events.'
- 'What is the probability of pulling out a black, brown, blue, yellow, orange, pink, green, red, white or silver ball? Explain.' Students should say that the probability is 1, since the event includes all possible outcomes.
- 'What is the probability of selecting a ball that will be silver, pink, green, orange or blue? Explain.' Students will probably say that there are five outcomes that match those colours so the probability is $\frac{5}{10}$ or $\frac{1}{2}$.
- 'What is the probability of pulling out a ball that is a light colour?' Since there are seven outcomes that could be a light colour, the probability is $\frac{7}{10}$.
- 'What is the probability of picking out a colour that starts with 'b'? Students should say that there are three colours that start with 'b' (brown, blue and black) so the probability is $\frac{3}{10}$.
- 'What is the probability that you will pull out a gold ball.' The students might be confused initially but will probably say that you can't pull out a gold ball because there isn't one in the bag. One or two students might suggest that the probability is 0 because there isn't any chance of that happening (as this sample space does not include a gold ball). However if the students just use the correct language instead of 0 (no chance, impossible, no likelihood, not probable) then the teacher-researcher can explain that 0 is another way of saying that.

- Some additional events can be given to the students to make sure that they understand how to find probability (of straightforward events like these).

Altered Preparation for Lesson 2:

Small group (student 1, student 2, student 3, student 4, student 5, student 8 and student 11) sits at the front of the classroom in a semi-circle around the board with the teacher-researcher, while the rest of the class are busy with activities given to them. These activities are mathematical games that include Sudoku, Ken-ken/Inky puzzles (3×3 and 4×4 blocks), Jigoku, Galaxy and Jigsaw Sudoku (see Appendices 12-16) and were found and printed from Krazydad (n.d.).

Key things to remember when working with the front group:

- Make sure to ask each of them questions, especially the students who are 'hiding'
- Look out carefully for hands up for questions (or confused looks)
- Write in blue/green pen on the board (one of the observers mentioned in the discussion forum that red writing on the board is not an easy colour for students to see)

After covering the following content, switch to the second group (student 6, student 9, student 10, student 12, student 13, student 14 and student 15) and repeat the process.

- Recap the definitions of **outcome**, **event** and **probability** and what the difference is between **experimental** and **theoretical probability**. Discuss different synonyms for likeliness (probable, highly likely, good chance, high chance) and for unlikely (not very likely, improbable, low chance, not very probable).
- 'We looked at different possibilities but look how many different probabilities we came up with. We need a way to measure the probability of something that everyone can find the same likelihood. We don't know what number we're going to pull out but we need to start thinking about what possibilities are likely to happen.'
- 'We used English to describe how probable something was of happening for experimental and theoretical but now we're going to use Mathematics and numbers to talk about likelihood.'
- 'There is a field of mathematics that investigates whether an event is likely to happen or not. This field is called…?' (probability)
- 'The probability of an event ranges from zero to one. An event that is sure to happen has a probability of one, and an event that cannot happen has a probability of zero. The probability that I could reach in this bag and draw out an elephant is zero. The probability that tomorrow will come is 1' (The teacher-researcher decided to include this information so that it would make sense to the

students why probability 'answers' are always fractions between 0 and 1, and can be written in decimal or percentage form).

- 'The result of an experiment like this (reaching in the bag and pulling out a ball) is called an outcome. We mentioned before that each colour had an equal chance of being drawn and this is because there is one of each colour in the bag. Since the different colours have an equal chance of being selected, the outcomes are said to be equally likely'
- 'There is a device that can help us find the probability of an event which is called a sample space. A sample space lists all the possible outcomes, making it easier to find the outcomes that are possible in an event'.
- 'We can show the sample space by writing down all the possible outcomes (writing down all the different coloured balls that are in the bag)'
- How to use sample space to calculate a probability:
  - write out all the possible outcomes so that you can see the whole sample space
  - count the total number of outcomes in the sample space
  - how many of those satisfy the constraint (or condition) of the event given to you
- 'One or more outcomes of an experiment make up what we call an event. When the outcomes are each equally likely, the probability of an event is the number of each of the successful outcomes for the event divided by the total number of outcomes'
- Write on the board the following equation (taken from Hume, 1966, p. 130):

$$Probability\ of\ event\ A = \frac{number\ of\ outcomes\ in\ event\ A}{total\ number\ of\ outcomes}$$

- Once the different fractions (probability answers) have been discussed, show students that they can also be written as percentages or decimal
- Once finding probability has been discussed and is understood, clarify the concept of average and why we knew three of each outcome was the theoretical probability (for the experiment of picking out a number from 1-10 in a bag, in the first lesson).
- 'If there are ten possible outcomes in total, the probability of picking a seven is one in ten (if you draw out ten numbers you should get each number, including 7, just once). But if you draw twenty numbers then theoretically you should get each number (including 7) twice. We did thirty experiments in the previous lesson and so theoretically we should have got each number (including 7) three times.'

- 'The only reason the concept 'average' came up is because each group who did thirty experiments theoretically should have got the number seven three times. But of course every group got seven a different number of times.'
- 'When we put everyone's results together in the table we saw how, if it was still out of thirty experiments, then each number was drawn close to three times (either two, three or four).'
- 'This shows us that the more often the same experiment is carried out, the higher the likelihood is of getting close to the theoretical probability. When we added a second class's results to ours the increase in experiments made the average of getting each outcome even closer to three.'
- **The larger the number of experiments…the higher the likelihood of getting close to the 'mathematical answer' which is theoretical probability.**

Original preparation for Lesson 3 (too complicated and needed to be changed):

- 'We are going to investigate flipping a coin three times in a row which will count as one experiment. Can you work out how many different outcomes there are? One way we can do that is to use the fundamental counting principle.'
- 'What is the fundamental counting principle? (If an experiment or a problem has two steps and the first step has a number of possibilities and the second step also has a number of possibilities then the experiment will have the number of outcomes resulting from multiplying the first step's number of possibilities with those of the second step).'
- 'When I flip a coin the result can either be heads or tails so if I flip a coin three times in a row, there are two outcomes for the first flip, two outcomes for the second flip and two outcomes for the third flip. If we follow the fundamental counting principle you can work out the total number of outcomes by saying $2 \times 2 \times 2 = 8$.'
- 'The specific eight outcomes can be identified by using a tree diagram'
- Draw a tree diagram on the board and show the students how to find the eight possible outcomes by following each branch from left to right listing the two possible outcomes that occur for each throw, as illustrated in Figure 3.3 above.

The following questions can then be ask, with the tree diagram clear for all students to see.

- 'What is the probability that all three flips will result in heads?' Students will probably say that there is only one result which is all heads (HHH) so the probability is $\frac{1}{8}$.
- 'What is the probability that there are exactly 2 tails and 1 head in the outcomes of three flips?' Students will probably say that there are three outcomes with exactly 2 heads and one tail, so the probability is $\frac{3}{8}$.

Keep asking questions like the above to ensure the students understand how tree diagrams are used to help find the exact possible outcomes and also how sample spaces serve as reminders for how many total possible outcomes there are (which can be worked out for multiple flips/draws using the fundamental counting principle).

Altered preparation for lesson 3:

Divide the class into four groups. Have the four different experiments (with multiples of each experiment) ready to give one to each group (a tin to pick out from with eight different coloured balls, a pack of cards to draw out from, including the two jokers, a die to roll three times in a row, a coin to flip twice). The tables for recording the results from the experiment must be printed and ready for the students to use (see Appendices 17-20).

- Each group can be divided into pairs or groups of threes and given repeats of the same experiment (Group 1 is divided into a further three small groups and each small group is given their own pack of cards to use in experimentation. This is the same for all four groups).
- Each group is given a printed table to use to record their results as they carry out the experiments.
- Once they are finished thirty experiments, the teacher-researcher will give them an experiment that they haven't completed yet (so that each student has a chance to work with all the experiments).
- The teacher-researcher will sit mostly with the groups as they do the flipping of the coin experiment and discuss the different outcomes that occur with them, also asking for expectations of possible outcomes.
- Discuss with each group as they carry out the different experiments what their expectations of the possible outcomes are, how many different results they think they could get and how likely they think it is that they will get the same outcome more than once. Then ask them after they have done the experiments if they got the results they expected and if they got any repeated outcomes and what kinds of outcomes they are missing.

"…qualitative research methods benefit both the researched and researcher communities in diverse ways" (Ebbs, 1996, p. 219). This study was designed using an antipositivistic approach which uses qualitative research techniques and relies on the collection of qualitative data. A qualitative research paradigm is interpretative, naturalistic and subjective however this study recognises that all research is interpretative as only an interpretation, and not a perfect copy, of the world can be presented (Opie, 2004). The particular tool used for data analysis was *Interpretive analysis* (outlined by Hatch, 2002). While interpretation is a component of all qualitative research, the tool is structured as a specific set of steps that, when followed, cause data to be thoroughly reviewed (but having potential for modifications to suit the data). Action research was a necessary methodology as the research was conducted in the environment of the teacher-researcher who was, partially, seeking solutions for an aspect of teaching practice – effective lesson preparation. Action research “is also seen as a way to encourage the professional development of teachers either by providing them with skills that will allow them to be reflective and inquiring practitioners or through the knowledge that they will acquire from the completion of action research projects in their classrooms” (Feldman & Minstrell, 2000, p. 431). The teacher-researcher used metacognitive practices in order to develop the teaching preparation framework. Feldman and Minstrell (2000) explain that action research is used primarily by teachers doing research in their classrooms to determine what does or doesn’t work in the classroom, and whether any modifications need to be made. Teachers make their own plans and use the action research method for research unique to them.

The following chapter includes the data collected which were summarised and categorised into tables. Key points needed for developing the framework have been noted and gradually its development is seen being formed through the steps of the *Interpretive analysis* tool used for data analysis, synthesis and the structure of Chapter 4. A comprehensive description of how the findings were appraised and then amalgamated to form the metacognitive framework concludes the chapter.

IV

Findings, data analysis and data synthesis

This chapter presents the findings of data collected in the study together with their analyses and synthesis. Details of these findings include a summary of all critical incidents that drew the attention of the teacher-researcher, details of which are in Appendix 11. Following this are the comments made/suggestions given by the observers in the discussion fora (Table 4.1) which are classified appropriately under five subheadings (positive comments about the students, critical comments about the students, positive comments about the teaching practice/occurrences in the lesson, critiques of the teaching practice and suggestions for preparation and teaching practice). Notes taken directly from the teacher-researcher's journal comprising of all metacognitive preparation and reflection experiences, are highlighted in Tables 4.2, 4.3 and 4.4 with a discussion on associations (if any) between the two. Following this are specific critiques made and suggestions given by the observers on how the teacher-researcher could improve teaching practice. These have been tabulated with quotes directly from the transcripts and notes made for each one on how they were addressed in follow-up lessons. Finally, the critical incidents which were selected for deeper analysis are numbered and thereafter discussed, through the answering of a series of questions designed for their analyses. Systems 1 and 2 of thinking and how they proved relevant to the critical incidents and teaching practice, is briefly detailed. These findings were key to developing the framework which sought to be metacognitive (therefore automatically reflective) in nature, with the hopes of having value in assisting teachers for the in-depth preparation of lesson content and delivery. Figure 3.2, in the preceding chapter, presents steps of the *Interpretive Analysis* tool which are used in the structuring of this chapter so as to give a detailed picture of how the findings were evaluated and then integrated to lead to the developed framework.

4.1. Read the data for a sense of the whole

A number of transcriptions and journal entries were read so as to get a sense of the whole. It was necessary for the transcriptions of the three lessons together with the follow up discussion fora for each lesson, to be read. Reading the journal entries made before and after each lesson and discussion forum, was also essential so that a sense of the whole collection of data could be acquired. Multiple readings of these data were necessary to fully engage with the gathered data. Notes were made as the data were read so as to record second impressions of the data (first impressions being incomplete as they were made as lessons were taught, however it was not possible to write all impressions down immediately). During the reading of the data, some information was searched for and then categorised accordingly.

4.2. Review impressions previously recorded in research journals and/or bracketed in protocols and record these in memos

While metacognitive preparation and reflection experiences were taken directly from the journal notes, the associations that could be found between some of them were only discovered after memoranda were examined to find significant meanings. Therefore the journal notes for each lesson on preparation, reflection (Tables 4.2, 4.3 and 4.4) are presented under the subsection discussing salient interpretations further on in this chapter. The tables include commentary as to what the associations (if any) between preparation and reflection are. This was for the purpose of the research, to look for connections between the type of preparation and reflection conducted and incidents of importance in the lesson, so the framework for the guidance of other teachers in their preparation for and reflection of lessons could be developed accurately. The only data that could be found exactly as they were recorded in the research journals (without comments or deeper analysis) were a list of critical incidents from the lessons that were noted by the teacher-researcher after each lesson. They were noted specifically during reflection, for the purpose of deciding what needed to be changed in the preparation process for the following lesson, therefore the whole list of incidents were made critical. The critical incidents highlighted parts of the teacher-researcher's practice that might have needed to be altered for the benefit of the students.

Tripp (2003) points out that many teachers, when attempting to decide which incidents to analyse further, find it difficult to select those key events. Some techniques aid in what to look out for. Analysis of events is thought of by most people as the best way to commence, and assigning adjectives to describe events helps to generate critical incidents. If a teacher has decided, before going into the classroom, to make a note of anything unusual or incidental, they are essentially more vigilant in being receptive to a full array of incidents (as it is necessary to see the whole picture in order to notice specific ones). Therefore in the build up to the lessons, the teacher-researcher had a number of questions in mind and was looking for the unusual or unexpected incident, particularly the ones that seemed to be directly linked to a part of the preparation process. In addition to that, in reviewing the audio-recordings, the incidents that seemed to be significant were categorised according to their 'adjective' and/or relevance to the lesson and tabulated for easy access in this dissertation (see Appendix 11). The kinds of questions kept in mind, which led to detecting significant incidents during lessons and developing categories ('adjectives') to tabulate them, included the following.

1. 'Are there any explanations I give or clarifications I make that were not planned for? Are there unexpected questions that I'm not sure how to address?'
2. 'How are the explanations that were prepared received by the students?'

3. ‘Are there any moments where the students (individually or as a group) make a discovery or explore the mathematics in an enthusiastic way?’
4. ‘Are there any revelations by students where they understand something, big or small, all of a sudden?’
5. ‘Are there any moments where the students find something too complicated, or there is a lack of understanding by a large portion of the class on a specific concept prepared for the lesson?’
6. ‘Are there any moments where the students helped each other to understand something that was confusing or something that the teacher-researcher could not seem to make clear?’

Critical incidents that were highlighted included episodes when the teacher-researcher explained concepts either clearly or not very well. Moments when the students collaborated to develop knowledge came up quite often and there were a number of incidents where one student noticed something that the teacher-researcher was not aware of (a mistake in a table caused by automatic rounding) or had not prepared to explain to the students (fractions in probability). Another moment that seemed pertinent was when the teacher-researcher tried to get the students to use a different term from the one that they used intuitively and turned out to be incorrect in saying that the term wasn’t an acceptable one. Other incidents that stood out included when the teacher-researcher attempted to change the direction of the lesson when students asked questions or gave answers in a way that covered material deemed too difficult by the teacher-researcher. The incidents that were noted quite frequently were those where a particular student made a discovery, understood a new concept or went far ahead of the concepts planned for the lesson. A particular note was also made about the incidents where the teacher-researcher realised that part of her teaching practice should be different/improved on (speaking too quickly, not giving students enough time to reply, answering their own questions without giving the students any time to attempt an answer). Other incidents that stood out were when the teacher-researcher acted on information gleaned from discussions with the observers or personal reflections (on the previous lesson). There were multiple incidents where developing the understanding of one or more students took place with the teacher-researcher’s guidance. Other moments noticed included a student finding understanding themselves and then explaining how a concept worked to other students, which often resulted in the other students’ understanding.

All incidents noticed by the teacher-researcher were considered when preparation took place for follow-up lessons (therefore making all the incidents identified in this study ‘critical’) however some of them were analysed in more detail therefore effecting greater influence on the directions of the following lesson. The detailed table of critical incidents can be found in Appendix 11 and the intensity of their analysis can be seen in each description given. The critical incidents (in Appendix 11) which were attended to with more in-depth critical analysis have been underlined and their analyses are found in the next subsection where impressions of data are identified.

4.3. Read the data, identify impressions, and record impressions in memos

After reading the data, certain impressions could be identified and drawn out of the transcriptions of the three discussion fora which were comments made by the observers on the lessons. Table 4.1 summarises suggestions given by the observers which are grouped according to their nature: positive comments about the students; critical comments about the students/situation; positive comments about the teaching practice/occurrences in the lesson; critiques of the teaching practice; and suggestions for preparation and teaching practice. The suggestions given for preparation and teaching practice of the teacher-researcher can be found further on under salient interpretations found in studied memos together with how these suggestions were addressed by the teacher-researcher in follow-up lessons.

Table 4.1: Comments made/suggestions given by the observers in the discussion fora

Positive comments about the students	
1. Student 1 was very astute in calculating the input numbers on Microsoft Excel and noticing that they didn't add up to what they should have (this was due to Microsoft Excel's automatic rounding function). Student 1 was extremely involved in the whole lesson and picked up on the new concepts quickly, asking good questions throughout the lesson.	*O2: S1, he was really good.* *O1: I know with S1, I noticed he had to be told that the computer worked out that, um the score that all the stats are in, and I just think that if a little guy can think like that and um. It doesn't matter and even if he was wrong and the way you handled him, I thought it was just, um I just thought it was a very, very good lesson.*
2. The whole class responded well to the lesson and got quite involved.	*O3: The kids were quite involved in the lesson, they were quite responsive.*
3. There was good lateral thinking displayed especially by student 1 and student 2 and some others especially as they were given things to make them think.	*O1: And then of course I was particular impressed with umm the lateral thinking, um you know I mentioned you know S1 and S2 but they weren't the only two but I thought that was a particular bonus to the lesson*
4. Many of the students demonstrated a great ability to express themselves mathematically.	*O1: And maybe also you interacting with them every day but the ability to express themselves for what… 11 year old in maths.*
5. Student 1 asked a question that impressed one of the observers, regarding the Jokers in the pack and whether the black and red joker counted as the same outcome or not.	*O2: And I have to say, I was very impressed with S1's question, um about the 53 verse 54, I… 'cause it's exa… it's so funny, it was exactly what I was thinking in my head, and he said it and I was like…* *T-R: Wow.* *O2: …amazing!*
Critical comments about the students/situation	
6. Being at the end of the term and having some activities happen during the day meant that some students were not dressed in school uniform and many of the students were a bit restless and looking forward to the holiday coming up. It also made the lesson seem slightly more casual and some of the students were distracted.	*O1: Um, but also,I..I just think, and it's not your fault at all... Half the class are dressed in civvies…* *O3: ja, which is never good.* *O1: and… you can just see it, even like S1, the way that s/he was just sitting in the chair. And you just realise if s/he was wearing a school uniform s/he wouldn't be sitting like that.*
Positive comments about the teaching practice/occurrences in the lessons	
7. Once concepts were worked out there was no talking down by the teacher-researcher. Once the class understood what a concept was, the correct terminology was used throughout the lesson.	*O1: Also that you used terminology.... there was no talking down. Once we understood what a concept was, you used it throughout the lesson.*

8.	The teacher-researcher guided rather than instructed and encouraged students to build their own understanding.	*O3: You are teaching them thought guidance and not through instruction and that is the way you need to teach this age because if they haven't built their own understanding then they will never understand it*
9.	The students weren't afraid to ask questions (mostly the confident students) and the environment created by the teacher-researcher seemed to be a calming one that they were confident in.	*O1: But the other little subtleties, because it is not only just the subject, it's also your rapport with the kids, I somehow sensed the kids liked you, I am sure they do, but you could feel the warmth in the classroom, if you got them at a point where they were not scared to ask questions.* *O2: Yes and they not scared to give wrong answers. So straight away there is a love and a warmth – those subtleties that also come out or don't come out. It's fantastic the way you handle the questions.*
10.	The lesson flowed well and the teacher-researcher also addressed questions well.	*O2: I actually think you handled that very well I was very impressed and the questions you asked they answered perfectly well and they got it.*
11.	The teacher-researcher has a very nice rapport with the class and managed them very well. The discipline was also good and students were firmly but respectfully dealt with if they misbehaved.	*O1: One other little thing I just noticed was your discipline. There were one or two little moments when kids stepped out of line and just the way you got them back on line. Very quiet, very direct, to an individual. Lovely.* *O2: Ja I said here actually, you've got a really nice rapport with your class. Very Nice.* *O1: Ja, and fun. It's important. About the R 100 out of dad's wallet… I think they…they related to that.*
12.	A good summary of the main concepts was given at the end of the lesson.	*O1: What I also thought was very good in the lesson was your lovely summation at the end. We went through what you taught so that, um and we all know what happens, is that when you have your next lesson (have your handkerchief for this moment) half of them will have forgotten. Some of them… that's going to happen, you'll say what did I teach? You know? But it was lovely the way you… at the end you summarised the main concepts*
13.	The use of language by the teacher-researcher was very good and it was clear that they were very aware of how language was used.	*O2: But I was quite impressed with your use of language and you were clearly very aware of the way you used your language which was nice.*
14.	It was an excellent lesson where the students were kept involved and the teacher-researcher taught concepts by allowing the students to work out the concept definitions for themselves through experiments. The students were encouraged to think logically.	*O1: I thought it was an excellent lesson.* *O1: I just thought that the… the kids were so involved and it was wonderful that you taught concepts but they actually worked out the concepts.*
15.	A number of the students have been nurtured in being able to express themselves mathematically.	*O1: Um also just a maybe 'cause you're interacting with them every day but their ability to express themselves for what? 11 year olds in maths… um also that you used terminology.... there was no talking down.*
16.	Most of the students enjoyed the experiments very much initially and some got tired of them towards the end of the lesson.	*T-R: I thought it went well, um it was a little bit of a busy, noisy lesson. But I was ok with that, I think by the end maybe, but there were only one or two groups that looked like eugh, a bit tired of getting another experiment to do, so maybe it was too much in one lesson. Um and maybe I can split that over two lessons rather. So to do some experiments but then to have more discussion about it.* *O2: some of them, some of them were enjoying it and that's why, what you got was some groups that had completed and some groups that hadn't, but actually if that happens, I mean you, I think in any one class you always gonna get kids who do stuff faster than others.* *O1: But they were enthusiastic, they enjoyed the lesson, you know…*
17.	Very good questioning and waiting for student 16 to work something out by himself and he tried very hard to do it by himself and get to the right answer.	*O1: ja, ja, um.. but you asked him/her questions and you just realised s/he was digging very deep into his/her brain to work out your… the answer to your question.* *T-R: The questions were too hard?* *O1: No! I thought it was wonderful! S/he got there and eventually s/he got to fifty-four, s/he eventually got to fifty-four. But it was just lovely the way you asked a question and you… and you could just see this little guy/girl thinking, you know? You know… I'm not giving, and s/he almost knew, I'm not giving the T-R the right answer, which was*

	lovely. His/her logic was telling him/her that my first answer was not right.
18. The teacher-researcher did very well to go from group to group and ask questions that weren't too easy, pushing the students to think and figure things out for themselves.	*O1: Ja, ja. And I thought that was magic, the way you went from group to group and you asked questions that weren't uh, just, you know like…* *T-R: Weren't too leading?* *O1: uh no, weren't too easy.* *T-R: oh ok.* *O1: And I say like it was great to see that kind of thing, ja.*
19. The teacher-researcher gave them lessons to be enthusiastic about and they enjoyed the lessons, growing over time. 20. The teacher-researcher varied the activities keeping it interesting.	*O1: But they were enthusiastic, they enjoyed the lesson, you know…* *O3: Mmm, it's a nice group.* *O1: They grew, they grew. And you varied the activities which was lovely as well, you know. For interesting activities.* *T-R: Ok, awesome.*
Critiques of the teaching practice 21. There were a few students that were not engaged and not involved at all. The teacher-researcher needs to ensure that they are aware of these students to make sure they don't lose eye contact and fall below the radar.	*O3: You have your S2 and your S1 and you had the odd one around who were completely loving the lesson, and completely involved.... you have just go to be aware of the ones that are not....you got to find a way to engage them all. S3 was not involved at all.*
22. Questions were often asked only of those who had their hands up. One needs to be careful to pick the students to answer questions for the purpose of illustrating whether or not they don't understand (not just to get a correct answer). 23. The lesson flowed well but somehow needs to be balanced out with students who are not involved and not wanting to ask or answer questions in any way	*O2: It was ask the question and it was hands up, which is the way everyone teaches and is the way everyone was taught, but there has been research that shows that if you allow that to happen there are a lot of kids in your class you don't get engaged because it is the kids who put their heads down and just hope that they miss eye contact completely. There were about seven or eight of them who answered the questions, ok, and you used them it was great because you used them and they became an active part of the lesson. It is nice when that happens because then the lesson has momentum, but you also need to think about balancing that out with the kids who are not involved.*
24. Don't need to go into so much conceptual depth at a grade 6 level and the concepts that were introduced were too abstract for the students.	*O3: What I've said here is… should this depth happen in Grade Six? Now I think that it was too abstract. I don't think you have to go into that amount of detail about experimental and theoretical probability. They've got to be introduced to the concept of* ***p****robability and I think that at Junior School level,* ***P****robability should be more concrete.*
25. The definitions given in this lesson were the teacher-researchers definitions instead of being collaborative like in the previous lesson. They were difficult (and didn't really explain the concepts as easily as they could have). Only some students understood the new concepts.	*O2: Um, also I thought, the first three definitions from yesterday were nice because they were collaborative but then the rest of the definitions you gave today were yours. And they were actually quite… the language that you used was quite difficult.*
26. When one of the students said 'evenly', the teacher-researcher tried to dissuade them from using it saying it wasn't a good representation of saying the same likelihood (50/50 chance) however one of the observers said that it was a perfectly acceptable term for them to have used. After researching this further the teacher-researcher discovered that the term 'evenly' is regularly used to describe a 50% probability and therefore would relay this to the student who kept using the term quite innately to describe a 50% probability.	*O3: Even is a good word by the way.* *T-R: Even?* *O3: Even is a good word. Funnily enough when you said that it's like you said it's not a good word… it's… it's a dead good word.* *O2: Why…?* *T-R: What did he say?* *O3: He said he's got an even chance if it's in the middle he's got an even chance.* *O2: Even chance. So why didn't you like even?* *O3: But it's actually an accepted term.* *O2: No it is, but I want to know why she said she didn't like it.* *T-R: I don't know. He said there's an even…* *O2: So the fifty fifty…* *T-R: Yes.* *O2: He said it's an even chance.* *T-R: Oh, when it's halfway between zero and one.* *O2: Ja.* *T-R: I don't know. Because I thought of it in the space of fractions. Because I thought of it completely differently. I always turn away from using even because they [the students] say, four can go into twenty evenly.* *O3: In probability, it is a very accepted term.* *O2: Ja. It's acceptable.* *T-R: Ja. Ok, I must speak to S1 about that. Evenly can be used.*

27. The teacher-researcher was completely unaware of not engaging at all with one student who was sitting in the front group	*O3: …and S8 you lost. Did you know that you taught like this the whole lesson (showing teaching with back to S8)… you lost S8, k… be careful of that.*
28. The teacher-researcher needs to be more aware of the language used when covering probability concepts. Using the words ‘and’ and ‘or’ are important for later stages of probability and can cause confusion later if used incorrectly now. The students don’t need to know the difference but the teacher-researcher needs to understand them and be very careful how they are used.	*O2: And just be very aware of your language in probability. Because language in probability is actually the key to probability.* *T-R: Ok.* *O2: So when you use words like ‘and’ and ‘or’…* *T-R: Ja…* *O2: You in your head need to be very clear as to what ‘and’ and ‘or’ mean.* *T-R: mean…* *O2: because in the higher grades, ‘and’ and ‘or’ are very, very specific when it comes to probability. So even though you’re doing very simply examples now, the use of the words ‘and’ and ‘or’… you need to make sure that it doesn’t, that it’s not going to interfere with their understanding of what ‘and’ and ‘or’ actually mean in probability later on. So while they don’t need to understand ‘and’ and ‘or’ now…* *T-R: I do.* *O2: you need to understand it, so that you understand that you are not giving them a definition that might impede their understanding a bit…* *T-R: future understanding…* *O2: Ja.*
29. Could have still done a larger number of throws of die and mark result on tally tables in order to see the possible outcomes very clearly and then see that so many experimental outcomes will tend towards the theoretical (getting the same number of outcomes for each number on the die). This is important because there still doesn’t seem to be a definite understanding for every student that the more the experiments are done the closer the result will resemble the theoretical probability.	*O2: um…it may have been more interesting for them had you, at the beginning of the lesson said to them ‘ok, what are we expecting from the dice, what are we expecting from the cards, what are we expecting from the etcetera, etcetera’ and then given them a tally table on…as well as the, the table with their results, given them a tally table, say count how many ones you would have, how may twos, how many threes, and then create that as a percentage or as a fraction or whatever and see if it matches what we expected.* *T-R: Mmmm* *O1: Ja* *O3: Ja, I just, uh, I can see where you were going with doing three at a time but I’m just scared that, that its, you, you’ve missed… a step.* *T-R: they gained a big jump.* *O3: Ja, I would have liked a hundred and fifty throws of the dice ‘cause that’s what they did… and, and then do your tally table and just get back so they can see that it’s a six that they’re going towards in a dice and it’s a whatever in the cards, just to get that level sorted first. Because over the three lessons, I don’t know whether that’s been cemented in yet.* *T-R: What’s been cemented? The understanding of…* *O3: the understanding of how, the theoretical and the experimental should be… should.. should be the same, should…* *T-R: should tend towards each other.* *O3: should tend towards each other.* *T-R: Ok.*
30. The direction of the lesson wasn’t very clear as experiments were started but not all completed. 31. The lesson needed tighter time frames in order for the students to have more focus and for the content to be covered completely. It would have been better to leave time and space for a longer summary at the end. There was so much covered and there were some students who had picked up some finer points that could have been discussed in greater depth.	*O3: I understand, I exactly said that… what OR’s saying, where was the lesson going? ‘cause for me, we walked in and we started doing experiments, and the kids sort of like you know…* *T-R: Ok* *O2: some of them, some of them were enjoying it and that’s why, what you got was some groups that had completed and some groups that hadn’t, but actually if that happens, I mean you, I think in any one class you always gonna get kids who do stuff faster than others. So you need to see if you can then take that and turn it into a teaching moment. So then for example, if um then you said ok fifteen minutes is all you’re getting to do as many experiments as you can, some kids would have finished and some kids wouldn’t and um, what you could have done maybe was take… so say for example they had all managed to complete the cards, then you could look at the probability for the cards. And said ok well…* *Someone interrupted the meeting momentarily* *O2: um…so you could’ve said like this is, ok so this is… we’ve done a lot of experiments now on this, let’s see the probability, and then say for example only two groups had completed the dice and you can say ok, we haven’t got so many experiments for this one… um* *O3: let’s see how it affects the results.*

32. There could have even been more 'plenary' or summary sessions throughout the lesson even in the middle. This is especially important to have time to discuss, as a whole class, fun or interesting discoveries that are made through doing the experiments.	*O1: Ja, very, you know, and some of the comments, so, uh, that was good, but I, I'm making a suggestion that, that the plenary could have been, uh, expanded a bit more.* *T-R: Ok* *O1: There were seven minutes versus a forty-five minute lesson, and I just think you had covered such fantastic sort of ideas, and they learnt so much, that um either, and I like the idea that maybe you break up the lesson with a little bit more plenary sessions, a few mo… you know even if there was one in the middle…* *T-R: ok.* *O1: Because, it also helps get you back to centre stage, things that you're… that they're picking up, they're learning as the lesson goes on.* *T-R: ok.* *O3: Ja, [I] like that.* *O2: Ja I have to say I thought the discussion at the end was amazing, and I think that they really would have benefited from having a bit more time to think about and discuss it, ja.*
33. Fairness in probability kept being mentioned by the teacher-researcher but it was never spoken about specifically and the students' attention was never purposefully drawn to it. The card packs weren't shuffled and it should have been spoken about how it was necessary to shuffle them (and then shuffled) to make it a fair experiment when drawing a card.	*O2: 'cause another concept that you need to think about also is fairness and unbiased things, so, like for example, you, you keep mentioning it and it keeps coming up in discussions, um like you said, um, some child said six different numbers on the faces of the die.* *T-R: Oh.* *O2: And you said I'm glad you said six different numbers because it could have been six faces with two… all twos on them and then it would have been different… um… so and also in terms of like shuffling the pack, you didn't actually specifically say shuffle the pack…* *O3: And they weren't shuffled properly.* *O2: And they weren't shuffled. And, and….I know that O3 made a note about holding the, the cards in your hand as opposed to putting them on a table. What difference does that make in, in fair choice?* *O3: Was it fair choice?* *O2: Was it fair choice? Ja, uh, so that's also something that they need to understand…* *T-R: That it comes down for every single thing that you do it needs to be fair* *O2: It has to be fair.* *O2: Like it was briefly touched on in the very first lesson, and it was… it, it sort of keeps coming up, it keeps… I mean you keep mentioning it, but I don't think you're consciously aware of the fact that you're mentioning it, because I think you know about it. So you, you just keep bringing it up, but they don't know about it.* *Ok, this is the… you mean the fairness?* *O2: Ja.* *Ja.* *O2: And some of them may be doing it intuitively but again, I think that it needs to be brought into their consciousness.* *T-R: I purposefully mentioned it in the first lesson, but not since then.* *O2: Yes, that was aware… you were very aware of it then, but since then it's sort of been more underlying.*
34. The teacher-researcher needs to be aware of being consistent in definitions that are given to the students which must be accepted throughout discussions. New definitions can be added or refined but it needs to be monitored carefully that those that have already been discussed are still accepted.	*O2: And then in terms of your language actually I just remembered, there was a point that I didn't make on, on… when I last saw you. Um, the… in the first lesson you defined event as a series of outcomes and in lesson two when someone defined it as such, you said, no no it's a series of experiments.* *T-R: Oh I changed my definition…?* *O2: So just be very careful that you are consistent.* *T-R: K.*
Suggestions for preparation and teaching practice There were a number of suggestions made for the teacher-researcher to consider for their preparation and teaching practice. To avoid redundancy, these suggestions (including how they were addressed by the teacher-researcher) are laid out further on in this chapter where memos were studied for salient interpretations.	

4.4. Study memos for salient interpretations

The memos were studied for salient interpretations in a number of ways. Notes taken in the teacher-researcher's journal during metacognitive preparation and reflection for lessons are included below, with the related experiences kept together and their association, if any, discussed. Main suggestions given by the observers in the discussion fora critiquing the teacher-researcher's practice are presented and how they were addressed in follow-up lessons is detailed. This includes their influence on decisions made by the teacher-researcher for whether or not to change the preparation or flow for the following lessons. Using Appendix 11 to categorise them together with important observations from journal notes, final incidents have been selected for deeper analysis. The critical analysis is presented, immediately following each critical incident and includes development of thought around their connection with metacognitive preparation and reflection.

Journal notes for preparation and reflection of lessons with commentary on their association[1]

Table 4.2: Journal notes for preparation and reflection of first lesson with commentary

Important notes in preparation for first lesson	Important notes in reflection of first lesson	Association between preparation and reflection/other comments
My abilities as a teacher (developing and maintaining a good relationship with students; having a strong presence in the classroom and draw students' attention; the better an understanding I have on a topic, the better I am able to teach it; having a good ability to be flexible in the classroom and change the lesson on command and go with the flow if the students need to discuss something further or take a different direction in a necessary area of content; I struggle to prepare very well for lessons and have to put a lot of effort and focus into making sure I am well prepared for lessons on paper; I am a young teacher and feel excited about teaching and being in the classroom and am very passionate about Mathematics education)		A number of the comments made by the observers highlight some of the abilities that the teacher-researcher had noted, after careful self-assessment, during metacognitive preparation before the lessons commenced. These have been presented in Table 4.1 where comments made or suggestions given by the observers in the discussion fora are categorised. Critiques of the lessons and/or teaching style are also found there.
Good pedagogical content knowledge (I need to be very well read on probability and what is required at a Grade 6 level so that I can present the knowledge in a way that the students will grasp and will feel comfortable to ask questions and so that I can address any questions that come up efficiently)	*I'm so glad that I over prepared for the lesson and knew content quite far ahead of where I was planning to teach in depth. One of the students who noticed a number of things during the lesson (like the automatic rounding off of numbers in the table in Microsoft Excel and realising they didn't add up) brought up the topic of theoretical probability without me drawing attention to it. The student realised that the*	In the metacognitive preparation for the lesson the teacher-researcher thought about the depth of content knowledge necessary to present the lesson on probability and made the decision to prepare ahead of the level given in the lesson preparation. In the reflection straight away a point was made about a student who did bring up a level of content that had not yet been reached in the lesson but was thinking ahead.

[1] The preparation and reflection notes presented in the first two columns (of Tables 4.2, 4.3, and 4.4) are quoted directly from the teacher-researcher's journal and have therefore been written in the first person narrative. In addition, all grammar and punctuation has been taken verbatim from journal notes.

	probability of picking a number out of the bag doesn't change every time you pick a number out the bag because you always have the same sample when you start (in this case). In being over prepared for the lesson I was able to address his question which rounded the lesson off well in introducing theoretical probability.	The teacher-researcher had considered this and was prepared with a way to explain the different content to satisfy the student but not giving too much detail so the other students were not pulled towards a level of content they were not ready for.
Language (my language needs to be very carefully planned and structured so that the definitions that the students create are their own, but have been well guided; I want to have the different concept definitions well understood by the end of the sessions but have definitions that the students themselves have come up with by following the language that I use)	*Pleased with a good lesson where good language was used and together we built the definitions using correct terminology. It was nice that observers were very aware that I was careful with my language and noticed the students building the definitions themselves.*	A very clear association exists here between the metacognitive preparation and reflection for how to use language in presenting the lesson. The way that the new concepts were described to the students enabled them to grasp the new vocabulary and their meaning while building the definitions themselves. The observers also pointed out the good use of language in developing definitions and giving the students the correct vocabulary from the beginning and using those throughout the lesson.
Questioning techniques (I need to make sure that I question the students in a way that doesn't lead them directly to the answer but makes them think for themselves)		No questioning techniques were mentioned in the metacognitive reflection of the lesson however in the discussion forum after the lesson and careful analysis of the lesson transcription it should be noted that the students were carefully guided and allowed to develop their own understanding through concrete experimentation and group collaboration in developing definitions using new concepts learnt.
Whole class teaching or groups? (Should I teach in small groups or as a whole class lesson? Small group teaching has been more beneficial in a number of situations because of the focused attention on each student. However there needs to be a clear progression of knowledge taught and in three lessons there will be content taught repetitively to the small groups so that not as much content could be covered)	*Disappointed in not including all the students and not noticing that the weaker students were not following at all and not even slightly engaged according to observers* *Wondering whether better to always use the smaller groups instead of whole class teaching. I was going to do a similar whole class lesson for the next lesson but am now wondering if I should do a smaller group lesson, especially because one of the observers requested it.*	There was a brief thought process of whether it would be better to approach the lesson from a whole class or small group setup. The exclusion of some students during the lesson proved a disappointment for the teacher-researcher who was not even aware of their exclusion until the observers pointed it out in the discussion forum and they listened to the recording of the lesson noting that no questions were asked of a number of students. This made the teacher-researcher question whether a whole class lesson approach had been the best idea and at the encouragement from one of the observers (the teacher-researcher's Head of Department from school) decided to put a small group structure into play for the next lesson with the expectation that they could be better focused on the weaker students, if not being able to explain the concepts better at least becoming better aware of their level of understanding.
	Something that really surprised me was that I moved very slowly through the work I had planned to do with the class. I rushed towards the end, and some students hadn't finished with their tables. I also wasn't able to introduce theoretical probability as carefully as I had planned to.	There was no mention of pace in the metacognitive preparation for the lesson and the teacher-researcher was unexpected that the end of the lesson was somewhat rushed with some students not having completed the task. A suggestion from one of the observers was that the teacher-researcher prepare and print out any material needed for the lesson that was not the focus of the lesson as it was noticed that the students spent a large amount of time trying to draw their tables perfectly. This led to the incompletion of filled in tables with results from experiments (which was the actual focus of the lesson) and could have saved time for a

		more in depth introduction of theoretical probability.
Practical application (I need to research some clever applications of making probability concrete for the whole class of students so that they are all involved. They need a lot of concrete examples and experimenting before we move to abstract thinking – I will make sure to use this whole first lesson for concrete application)	*It took me a long time to get to explaining basic definitions and completing some experiments. I realise that I need to do many more concrete examples with the class before we move forward to multiple dice throws and coin flips and the abstract thinking in probability. I thought a whole lesson with multiple experiments from one event would be enough but the students need to keep experiencing more concrete examples.*	The preparation for the progression of lessons assumed that by the end of the third lesson, the concepts covered would be vaster that what was actually covered. The teacher-researcher thought that giving the whole first lesson for doing experimental probability was more than enough time for the students to grasp the concept in a concrete way. In reflection of the lesson they realised that this was not the case and many more concrete, experimental examples needed to be carried out. A suggestion from one of the observers supported this realisation.

Table 4.3: Journal notes for preparation and reflection of second lesson with commentary

Important notes in preparation for second lesson	Important notes in reflection of second lesson	Association between preparation and reflection/other comments
I will have a small group in the front of the classroom with me to teach the new content to (after revising the previous lesson's work) while the rest of the class, also in groups, is given different activities to do. The group with me in front will rotate with one or two other groups during lesson once everyone in the first group understands the new concepts introduced. I expect to get through two groups and possibly start with a third group.	*My first thought after this lesson is "What a disaster"! It took half the lesson just to recap the definitions from the first lesson and because I was trying so hard to include everyone and make sure the weaker student was contributing it took even longer and seemed to confuse the other students even more and the students who knew the answers seemed bored. I wasn't even able to move to the second group because it took so long to cover the material with the first group.*	The teacher-researcher didn't move to a second group because the planned work for the first group wasn't completed. It seems as though there was a bit of an impasse and perhaps the teacher-researcher could have changed the direction of the lesson slightly seeing as it went so off course.
It is important that the activities I give to the other groups to work on can be completed independently or in pairs or small groups. They need to be able to work on the activities so that I am not distracted from being completely focused on the group with me and be aware of who is understanding and who needs extra attention.	*The students working by themselves or in groups were given some activities that they hadn't done before and were therefore confused, got bored and started chatting to each other. I had planned to work with two groups (at least) and that meant there were two groups left to do something by themselves the whole lesson and I should have prepared better activities for them to do and definitely not given them new puzzles to work with but older games like 24 cards and simpler Sudoku. I should have supplemented these with revision work from the year's work covered in class whether or not I think they would have liked them (instead of just giving them fun mathematics games/activities to work on).* *Due to the students not being completely involved in the activities, I found it very difficult to keep focused on the small group I was working with when I had to keep telling the children at the back to work quietly. The front group working with me were definitely distracted by the students chatting at the back and I really struggled to focus as well on recapping the concepts from the previous lesson.*	The plan for the lesson did not work as well as expected and the activities given to the students did not seem appropriate given that it did not keep them occupied enough for the teacher-researcher to completely focus on the front group. The initial plan, to be focused on the front group and be aware of the students who did not understand the concepts, did not get implemented as intended and this caused further confusion with the front group whereby they took too long to revise the previously covered concepts, there was a rush to introduce the new concepts and the teacher-researcher was still not fully aware of the different levels of understanding for each child in the small front group. In addition to this, the front group was not switched out at all which meant few students were introduced to the new concepts.
I will start off with a summary of the previous lesson's concepts so that we can move forward positively to learning new concepts having a solid base knowledge. In realising that the students need to experience more concrete examples of probability, this	*It took so long for us to revise the previously covered concepts that by the time I started on the new concepts planned for teaching in this lesson, half the lesson was already finished and I couldn't switch to the second group as I hadn't even begun the new work with*	The teacher-researcher did not expect that the students would not have remembered the concepts very well from the previous lesson. What compounded the issue was that it wasn't a focused time for the front group due to the other groups not having

lesson will still focus on experimental probability with more practical application. I will move slowly towards introducing a sample space and how it is important to theoretical probability (but still using experimental probability as the base).	*the first group. By the end of the lesson I had only seen one group of students.*	activities that kept them busy and therefore became disruptive. This did not help the teacher-researchers efforts to revise the previous concepts or the students to remember them.
	The concepts from the previous lesson were not grasped as well as I had expected and few students in the front group with me remembered the definitions we had covered.	The teacher-researcher could have perhaps devised a better strategy to revising the previous concepts, maybe including some more practical experiments (which was how the students had collaboratively formed the definitions to begin with).
I need to make sure to include everyone in the small group and be especially aware of the weaker students and what their understanding is throughout the session. *I must remember not to pick only those with their hands up and to move well through the group keeping everyone involved and asking all the students questions to know if they're understanding.*	*My first thought after this lesson is "What a disaster"! It took half the lesson just to recap the definitions from the first lesson and because I was trying so hard to include everyone and make sure the weaker student was contributing it took even longer and seemed to confuse the other students even more and the students who knew the answers seemed bored. I wasn't even able to move to the second group because it took so long to cover the material with the first group.*	The teacher-researcher was not aware of the students' understanding from the previous lesson as it was expected that they would remember more than they did. The approach to including the weaker student was unsuccessful because instead of being quietly drawn into the conversation, too much pressure was put on them and they were still not able to remember the definitions of the previous concepts. In addition to this, so much focus was put on them that the other students who did remember the definitions were bored and just wanted to give the answers and move on. This did not happen and therefore the teacher-researcher only had one group throughout the lesson.
I will use a selection of coloured balls in a tin so that I don't confuse objects with the number of objects (getting a number of a number, i.e. four fours).		This strategy did work well and the students and teacher-researcher were not confusing the name of the objects with the number of objects as they did in the previous lesson.

Table 4.4: Journal notes for preparation and reflection of third lesson with commentary

Important notes in preparation for third lesson	Important notes in reflection of third lesson	Association between preparation and reflection/other comments
I need to be careful about changing the plan for/structure of the lesson at the last minute and not being prepared enough for the changed lesson. I need to make sure every aspect of the new lesson is planned very well and the activities are thought out very carefully.	*The lesson went very well and the students carried out the experiments efficiently (and with great enjoyment) and through the multiple experiments, there was a demonstration of a better understanding of the concepts through language use. Even though I had changed my lesson plan completely, the change was more carefully thought out than that from the first to the second lesson and was set at the right level for the class. Giving them a variety of experiments to do and using the probability language more seemed very helpful and by the end of the lesson I noticed most of the students using the language consistently to describe their activities.*	There is an especially important link here in that the teacher-researcher was very cautious as to how to go about changing the lesson plan structure so that the level of difficulty was reduced (as was discussed with the observers) and the students still benefited. The students in being given a variety of experiments to do, but which were all based on the same concepts, helped reinforce those concepts and good language use. This was the intention of the teacher-researcher in structuring the lesson with multiple experiments and it was carried out successfully.
	It only occurred to me during the lesson to check if the students knew the difference between the head and tail of a coin (which some of them didn't).	There were a few assumptions made before the lesson that were not even made consciously. Assuming the students knew the difference between the head and tail of a coin was one that needed to be corrected for the experiment to be carried out accurately (at least for the sake of agreement between all those involved if not accuracy of numbers as it is a 50% probability either way).
	I asked the students to write down the results of throwing the die with semi-colons in between so it was clear that	The teacher-researcher in realising that the students were being asked to do something that they themselves had not

	there were three separate digits however I did not write them in the table with semi-colons and only realised that I should have done so, in the moment that I was explaining the task.	done (writing a semi-colon in between the digits to make it clear what the separate numbers were) highlighted for the students that it was a mistake in the example given and explained why it was important to include the semi-colons.
The altered lesson is going to be very concrete with a lot of different experiments being rotated between the groups and the students needing to tally up their results. There will be a lot of discussion with each group as I move around the classroom. Ideally I would like the students to enjoy the lesson through the experimentation of activities of probability. I want them to see that they can experience mathematics in a way that is practical and fun and that doing it can help to understand it.	*It was so refreshing seeing how excited the students were at carrying out the activities. When I asked them if they thought the outcome could be repeated even though they were rolling the die three times to get an outcome (and so the outcome consisted of three rolls) and they said yes. They were then very excited to try roll a repeated outcome and would say "okay we either need to throw a one or a four to match the other outcome" and then "go, go, go, go, go…." followed by "aaaaaaah" or "yeeeeeeah" depending on the result of the roll.*	The teacher-researcher wanted to include many experiments that were different enough to each other that they could be enjoyed but similar enough that the same language with important concepts could be used. This was definitely the case and the students were excited to carry out more experiments to see if they could attain a repeated outcome (something that the teacher-researcher had asked the students whether it was possible to do, and if possible how likely). The enjoyment in carrying out the activities is clear to see in their spontaneous outbursts and in their conversations with each other.
Suggestions from one of the observers on simple, straightforward definitions for experimental and theoretical probability (experimental probability: this is what actually happened and theoretical probability: this is what should happen – how do these two match up?) By doing different additional experiments I want to gently bring this into the lesson by asking questions like, "What do you think could happen?" And then afterwards to ask, "What actually happened?"	*Using the simpler language to discuss the experiments (at the suggestion from one of the observers) was very helpful and the students definitely used them to better their probability language. I asked what kinds of outcomes they thought they would get and then what outcomes they did get and the question that brought about the most interesting thinking from a few of the students (one discussed in particular) was asking them what the total number of possible outcomes there are for two of the experiments.*	Getting input from teachers external to the situation was helpful in this case as a different outlook on the kinds of things the students needed to hear in order to click, was necessary. The simple language of expectation and then matching the reality of what happened to what they were expecting to happen, intuitively brought about good discussion on the probability of the experiments.
Tree diagrams are not part of the curriculum and even thought I thought they would be able to understand the concept of tree diagrams as related to theoretical probability, I can see how doing more experiments and letting them use the right language is more important. I will not introduce tree diagrams, therefore, at this stage and think that the multiple experiments will be good for their concept development.	*I had asked the students for how many possible outcomes they thought there were when drawing a card or the total possible number of outcomes for picking out a coloured ball. I had not asked them for what they thought the total possible number of outcomes for flipping a coin three times or rolling the die three times. However, towards the end of the lesson I noticed that student 7 had started drawing a tree diagram to try work out how many possible outcomes there are when rolling a dice three times. I had never mentioned using tree diagrams and she didn't seem to know that they are commonly used for probability challenges. This student had just realised that it would be more efficient in helping them 'see' and work out the total number of possible outcomes. It was very exciting to see this student searching and attempting different possibilities for finding a solution to a question that hadn't been asked by the teacher-researcher but a question asked themselves. A few other students also started exploring a few possibilities including tree diagrams to solve the same question but did not go as far as student 7. Student 7 also needed a little guidance to not miss outcomes that were 'branches of branches' in the tree diagram. I think that space on the whiteboard was a problem here – as the branches of branches could not be easily drawn and so were missed. However student 7 attempted a second tree diagram at my prompting which was much more*	This was an especially interesting situation in that after deciding not to introduce tree diagrams to the students, some of them, particularly student 7, realised that using them in attempting to answer one of the questions would be an efficient way to represent all possible outcomes. Student 7 was the most determined to use the tree diagrams successfully and even attempted a second tree diagram, once realising the flaws with the first one after a discussion with the teacher-researcher.

	accurate in calculating possible outcomes. After some guided questions and prompting they were able to calculate the correct number of possible outcomes for rolling a die three times (not taking into account repeated numbers).	
	A few of the students seemed a little tired of getting another event to do (after having done so many experiments over the course of the lesson). In future I could possibly break it up over two lessons and have a combination of doing experiments with having discussions about them.	It may have been too long for the students to sit doing so many different experiments. A different structure for the lesson for future could be designed. Allowing more time in the beginning for a review of what is being covered and more time at the end of the lesson for a summary of what has been realised in the lesson would also add to the value of the lesson's content.

Suggestions given in the discussion fora after each lesson

Once transcriptions of the discussion fora were completed, suggestions given by the observers were drawn from the data and categorised, as detailed above. One group of suggestions included comments that the observers made about how the teacher-researcher could improve teaching practice that might aid the flow of the lesson, understanding of the students or management of the lesson. These suggestions were noted at the time and effected changes for follow-up lessons (the degree of change, if any, was determined by the teacher-researcher). The suggestions made after each lesson and how they were addressed follows.

Table 4.5: Addressing suggestions post lesson 1

1. Think about teaching in small groups so as to have more one-on-one attention for students especially to pinpoint and help the weaker students. This suggestion was taken into consideration and used to change the plan for lesson 2 and also applied more carefully in lesson 3. The initial plan for lesson 2 was to have another whole class lesson however the teacher-researcher changed this completely by dividing the class into four groups and giving them activities to do while one group sat in the front with the teacher-researcher. The front group was to revise the definitions from the previous lesson and then be introduced to some new probability terms, such as sample space, through the use of a different event and set of experiments (drawing coloured balls out of a tin). The suggestion was helpful in that the small group provided a space for the teacher-researcher to revise terms with fewer students, introduce new terms clearly and then pinpoint which students did not understand well (or at all). The students who understood well would be sent to complete the same activities that the other small group had already started and those who did not understand would be kept with the teacher-researcher so that an attempt could be made to explain more carefully, possibly exploring the concepts concretely for longer.	*O3: I have to tell you though that I still strongly believe in the grouping, 'cause I looked at S3 and it was, she had cut right off.* *O2: The grouping thing requires a lot of planning* *T-R: Yes* *O2: And a lot of prep* *O3: And a lot of prep* *T-R: It is exhausting* *O3: But it is the best way to teach.* *O3: So and also I think… if S3, wasn't involved with the lesson at all, she is the one that you should be teaching to, because she is the one that needs the teaching.*
2. Think about how to stream if smaller groups are used (mixed ability groups or similar ability groups). This suggestion was taken into account in the second lesson and mixed ability groups were organised. A number of the weaker students the teacher-researcher had not included in the first lesson (noted by the observers) were included in the first group so that their understanding could be assessed and then developed.	*O2: So then the question is do you group them according to ability?* *T-R: I prefer ability… uh… range from sort of weakest to strongest and then divide in half and the bottom half to mix and the top half to mix.* *O3: I did it, I did it… but my classes are grouped. So within my classes I've got an ability range of about 30%. But I did it based on personality because mine are one just year older, one year more revolting.* *O1: Well it is November…* *O2: My question then is, the kids in the lower half of the class then never see an example of someone who's really, really good. And they never*

	get the chance to see that there may be more than what they are doing.
3. <u>Use letters or colours or other things for objects instead of numbers otherwise you have numbers as the object and then counting the objects gives a double number (i.e. four fours).</u> This suggestion was taken into account in both the second and third lesson and no more numbers were used as objects. In the second lesson the teacher-researcher brought small coloured balls to be picked out of a tin. In the third lesson the teacher-researcher gave four different events to the students to carry out experiments and none of them included numbers as their objects. The first was drawing a card out of a deck (which involved a suit with a number), picking a coloured ball out of a tin, throwing a die three times and flipping a coin twice. Even though rolling the die gave a number outcome, because the students were asked to roll it three times as one experiment, the outcome was a series of numbers (i.e. 4; 1; 6) and therefore number and object could not be confused as before. This was extremely helpful and in both lessons there was no confusion between the outcome and the number of outcomes.	*O2: Um…some of the other things I said here, like for example… in maths it's very easy to get stuck into using numbers as examples of objects to choose. But sometimes it can get quite confusing and you… you found that towards the end when saying one ones or like… four two's and it suddenly gets complicated between the number of objects and the object itself. So I just made a note here of saying you don't have to use numbers as objects as… as well as numbers. You can use cereal boxes… or colours or whatever. Ja.* *O2: I think especially… I don't even know if this was the beginning of this kind of lesson, like Probability, introducing Probability and things like that but, I think sometimes when you introduce a topic it's easier to steer clear of numbers as objects versus numbers as numbers um because it gives them a… a chance to separate out the concept of an object that you can choose and the number of times you choose that object.*
4. <u>To save time in the lesson, give the students blank tables already drawn (printed copies). This takes away unnecessary time spent on drawing the tables themselves (especially as the skill of drawing tables is not the focus of this lesson).</u> This suggestion was taken into consideration for lesson 2 and 3. In the second lesson the students were given activities to do and all activities were on cards that they could use to write on and then erase and reuse so that they did not have to draw out the figures. In the third lesson the students were given multiple data collection tables (one for each of the four events) and they did not have to draw them out first. This saved time in the lesson and the focus of the lesson was on carrying out the experiments and recording outcomes and not setting up tables.	*O2: Um… Maybe give them a table outline already, like just printed for them… because primary school, jeepers they take long.* *O2: Printing it out will save you… because what happened was they didn't have enough time to finish the 30 trials and then because they knew you wanted them to get to 30, they did it while you were explaining and if they're doing it while you're explaining they're not fully engaged in what you were saying.* *O2: If you're gonna give them that many to do then you need to take away the unnecessary things that are not part of the lesson focus*
5. <u>Keep reinforcing concepts throughout the lesson and using the new language throughout.</u> This suggestion was taken into consideration and attempted in lesson 2 but the focus lesson with the front group did not go as smoothly as the teacher-researcher had planned and therefore there was not much time to reinforce old or new concepts as simply remembering what the definitions learnt in the previous lesson were, was challenging enough and new work was not covered in full. The teacher-researcher did reinforce concepts throughout the lesson with each small group and did a summary with the whole class at the end of the lesson. One of the critiques after this lesson was that the teacher-researcher could have had more summaries with the whole class throughout the lesson (instead of only with the small groups). The students, however, did keep using the correct language and terminology throughout the time they did the experiments.	*O1: maybe just the thought here is that, in the lesson, um just to keep on (I know it sounds for S1 and some of the high-flyers a little bit boring) but that you go back to 'what does this word mean again'?* *T-R: Instead of once at the end?* *O1: Well you did it, but just… I thought maybe just a little bit more of… 'cause they're quite heavy concepts for little people, you know, and I think just… but I just thought your summation and your quick revision um… it was amazing, just everybody, the whole class got the message as it were.*
6. <u>Think about getting each student to have their own notebook where they write out mathematical definitions or important notes/formulae and this can be used over the years and kept as a valuable resource (not used – possibly a longer term option)</u> This suggestion was taken into consideration but not used in the second or third lesson as it was proposed as an idea for long term to include definitions, important notes or formulae made from the beginning to the end of the school year, over a period of a few years. This could be added to the teacher-researcher's teaching plan for future use (another point for the study's developing framework on how a third party can be helpful in growing the teacher through observation and discussion fora).	*O2: So one of the things we find in high school is that the kids are not - they are sort of okay with talking maths but they are horrendous at writing maths and everything in high school is written.* *T-R: Ok.* *O2: So um… one of the things to maybe think about is to start encouraging them to write mathematical notes, like but in English in full sentences, so those definitions that you gave that had been collaboratively developed in the class, get them to write them down in a notebook somewhere, so that they have a notebook. And even if it's like a mathematical dictionary that they have from grade 3 to grade 7, so that they don't burn it because they need it for the next year. Then at least they're encouraged to write maths in full sentences.* *T-R: Ok.* *O2: Obviously then you have to get buy in from your other teachers.* *O3: Good suggestion hey, 'cause I went to visit a school in Cape Town last year, and that's what they have. They have a maths dictionary. And I think it's great, 'cause you have an English dictionary. That's what they've started doing now that goes up the grades…* *O1: Ja that's a lovely idea, having a maths dictionary.*

Table 4.6: Addressing suggestions post lesson 2

1. <u>The teacher-researcher could have given the students the definitions to write down the previous lesson. At the beginning of this lesson the students in the front group could have been asked to write down what they remembered from the day before. This would have given a good indication of the different levels the students were at in terms of the new concepts.</u> This suggestion was forgotten about (due to another big change of teaching plan) and was therefore not implemented in the third lesson. The structure of the third lesson also did not allow for the small front group as the new structure for the lesson included all four groups being divided further into groups of three or four students carrying out different events around the classroom (as described above). The teacher-researcher could have still asked the students to write down what they remembered about the definitions discussed before and written those down individually and then gone around the classroom to get an idea of who had grasped the concepts and who hadn't (not necessarily needing to cite the definitions as discussed previously but just their understanding of each concept). The teacher-researcher had forgotten about this as a possibility in the planning of this lesson.	*O2: um… Also like, I think yesterday if you had made them write down what was event, what was etcetera, etcetera… and then maybe at the beginning of the lesson…I don't know, this is just throwing out ideas.* *T-R: mmmm* *O2: If you'd…at the beginning of the lesson, given them each a little sheet and they had to without looking at their notes, write down what they thought event was, what they thought probability was, what they tho…etcetera, etcetera… you can then also see how many of them have remembered from yesterday. And how much you still had to do.* *So it's like forma…formative assessment basically, feeding back to you as to what was clear and what was not clear, and then work from there again. Um, also I thought, the first three definitions from yesterday were nice because they were collaborative but then the rest of the definitions you gave today were yours. And they were actually quite… the language that you used was quite difficult.*
2. <u>The front group could be more fluid and the students who hadn't grasped the concepts from the previous day could have been given more time with the teacher-researcher. Those students who had could have been moved on with an activity to do themselves (maybe on the new concepts just covered).</u> This suggestion was taken into consideration and attempted somewhat in lesson three but as one small group did not sit in front with the teacher-researcher (as before) it could not quite be carried out in this lesson. The teacher-researcher however did attempt to do this by moving fluidly from group to group and having pertinent discussions with each group around their experiments and what kind of outcomes they expected and then asking them what kind of outcomes they actually got and how it related to what they were expecting.	*O3: So the ones that grasp it, say good you've got it. Go, there's a worksheet, go away and conquer the world. You know what I'm saying?* *T-R: mmm…* *O3: you lot, ok let's just go through the things again… um… k you seem to have got it now, you go away. So you've got… you've got fluidity in your classroom and like the S3 could sit there. But the other thing is when you asked her questions, there was S4 going and S2 going and S1 going (clicking fingers and waving hand to get attention) and S3 was going…(looking around in confusion). You know?* *O3: If you were working with groups and you find that you are getting stuck, it's actually a good idea to move them away because they're getting stuck, you're getting stuck, everybody's getting stuck. Go away and have a breath of fresh air. Let's try and… 'cause often it's like that group of kids or somebody's got up your nose or do you know what I'm saying? And let them go away and maybe do an experiment and refresh…* *T-R: bring the other group…* *O3: …bring the other group forward, 'cause it keeps the interest going, it keeps… you know they don't get bogged down in a… ja like they're sitting there under your beady eye for an hour. That's quite difficult for eight kids.* *T-R: Ja.* *O3: So perhaps, like if that ever happens again, just say look, you know what, we've got this far, here's a pack of cards, here are some questions.* *O2: go play.* *O3: Go and… go.* *T-R: go…* *O3: Work in groups.*
3. <u>The other students not in the front group could have conducted more experiments (concrete examples) at their tables and write their results on a large tally chart on the board (definitely used in lesson 3, multiple experiments done, maybe too many without enough discussion).</u> This suggestion was taken into consideration and definitely used in the third lesson whereby multiple experiments were done by every student in very small groups. There was possibly too much time given for carrying out the experiments and not enough discussion time as a whole class to explore their experiences and findings.	*O3: You could have also had your groups doing experiments.* *T-R: Oh instead of…* *O3: Which means they could have come to the board with results.* *T-R: Yes, yes…* *O3: Do you know what I'm saying? Instead of saying ok, pick one, pick one, pick one… you could have made your lesson more fluid.*
4. <u>More conceptual development using concrete experiments and discussions around the experiments is necessary to also let the students intuitively realise what the multiple experiments lead towards, instead of going into detail about how theoretical probability is calculated. Solidifying the language in probability is very important throughout this.</u>	*O3: What I've said here is… should this depth happen in Grade Six? Now I think that it was too abstract. I don't think you have to go into that amount of detail about experimental and theoretical probability. They've got to be introduced to the concept of probability and I think that at Junior School level, Probability should be more concrete.*

This suggestion was taken into consideration and definitely used in the third lesson as multiple experiments were done and the teacher-researcher guided the discussion with each small group asking them what their expectations were and then if the results matched those expectations. This unexpectedly and anticipated what would happen in the lesson as many of the students did intuitively realise what the multiple experiments would lead towards and their language around and for probability was solidified simply in their exchanging conversation on the experiments that they carried out.	*O2: And maybe a little bit more intuitive as well.* *O3: Ja. And I think that they didn't remember what had happened yesterday, except for the three bright ones. So in fact, the three challenged ones are still challenged.* *T-R: So wait, which part of that shouldn't I have done?* *O3: Well, for me, I would give them dice to throw and let them throw fifty dice…* *T-R: So do more experiments.* *O3: …do a tally table, do a frequency chart, do your probability of throwing a black one, do your probability of throwing a six… so that they understand… they understand the terms, they understand the… um you know…what was it… the event, the outcomes, the possible outcomes. That to me is enough knowledge for them at this level.* *O2: Ja. And solidify…so doing experiments and solidifying the language around probability.* *O3: Yes.*
5. <u>When working with small groups it is important to keep them fluid (switching them with the other groups). They could be given an opportunity to do more experiments so that they're not under such pressure for so long in the front group (and so that the teacher-researcher isn't under such pressure).</u> This suggestion was taken into consideration and the teacher-researcher attempted to move themselves fluidly between the small groups and having meaningful probability talk as they carried out experiments. It could be applied better to a future lesson where there is one group with the teacher explaining new concepts and other groups carrying out experiments themselves.	*O3: I would make the groups more fluid. So you're sitting there with kids that… you're sitting there with kids that hadn't grasped the concept and you're sitting there with kids putting pressure on them next to them. Now it's very nice to hear what the other kids have to say. And I mean… in life you're going to be sitting next to bright people. But, what I would have done is I would have moved those bright kids on. I… They… they grasped the concept, I would have given them a worksheet on… you know…* *T-R: On probability…* *O3: …given them a tin of your coloured balls and said…* *T-R: go…* *O3: go and… and I want to know the probability of red, the probability of …you could… and… so you could have moved them on.*
6. <u>It is important to know exactly what needs to be covered with the front group and if there is a struggle to keep the focus of the students or it is clear that they're not getting anything out of it, move them away and switch in another group. Once they do understand also move them away to carry on with practice examples or a fun mathematics activity. (suggestion to be used for future small-group lessons)</u> 7. <u>The key value-add to working with the small front group is to bring to the attention of the teacher-researcher the different levels that each of the students are at.</u> Both of the above suggestions were seriously taken into account for use in future lessons as a valuable teaching foci to have when working with a small group while other groups work on their own activities. They could not be applied to the third lesson but were added to the valuable considerations the teacher-researcher has for future teaching practice.	*O3: Know exactly what you're going to do at the board [with the front group] and if it's getting bogged down like I say, then either move them on, or move on.* *T-R: K.* *O2: Ja, make sure you, in your head, are very clear about what you want them to take away from the board, ten minutes with them.* *O3: 'cause I wasn't sure..* *O2: Ja, and if they, if they get, it let them go.* *O3: …move them on. Ja. Yes.* *T-R: ok.* *O3: For the group work to work, it's nice to be able to bring them to the front once you know… 'cause it's almost like a.. an ability to see, whose… who you need to extend, who you need to teach, who needs reteach.*

None of the suggestions given post lesson 3 (see Table 4.7 below) could be applied to lessons that were part of this study as only three lessons were planned and monitored for this research. The teacher-researcher however took them into consideration for professional development purposes, to keep in mind for future lessons and for her own development as a teacher. Receiving these suggestions from the observers also highlighted another point to be added to the study's developing framework – the importance of having a third party observe and critique a teacher's lesson.

Table 4.7: Suggestions post lesson 3

1. <u>The teacher-researcher could have started off the lesson with a whole class discussion asking the questions that were asked individually to the small groups. "What outcomes are we expecting from rolling the die or picking up the cards or flipping the coin…?"</u> 2. <u>The students could also have been given a tally chart to fill in how many of each outcome they got to create as a percentage/fraction how many of the expected outcomes they actually got.</u>	*O1: And maybe to start the lesson, you can see how old fashioned I am, but to start the lesson with a bit of revision…* *O2: Mmm.* *O1: …In other words, that you get them into the terminology very quickly…* *T-R: Ok.* *O1: […]but I just think, it… one it's revision and it also just helps with the focus.* *T-R: Ok* *O1: And um, ja it's just […] you come into a lesson and… and what were we chatting about last time… you know?* *T-R: Yes* *O1: You know, that kind of just… um, 'cause I think that maybe was the point, that… it took them a little bit of time I thought just to understand where are we going with this…* *O2: um…it may have been more interesting for them had you, at the beginning of the lesson said to them 'ok, what are we expecting from the dice, what are we expecting from the cards, what are we expecting from the etcetera, etcetera' and then given them a tally table on…as well as the, the table with their results, given them a tally table, say count how many ones you would have, how may twos, how many threes, and then create that as a percentage or as a fraction or whatever and see if it matches what we expected.*
3. <u>The teacher-researcher could give the students a sort of hypothesis at the start of the lesson to give them an idea of where they are headed towards. This is what we think happens and what we're trying to prove by doing all these experiments, but it might not happen. Let us prove/disprove the hypothesis together.</u>	*O1: and O2's point, you know about, that you, tell them almost like where they're going, in other words, almost give them like a bit of a hypothesis. This is, will this, is this what we're trying to prove, it might not happen.* *O3: Ja.* *O2: Actually that's really nice.* *O3: Think about it first.* *O1: Yes. So in other words, give them the, the hypothesis, this is what we think. Will it be true, will it not be true, you know, 'cause also remember the thing about a hypothesis, it can be proved… disproved and that doesn't make it less valid. You know?*
4. <u>It would be quite good for a future discussion around fairness to take place to let the students see how much it filters into everything. Whether drawing a card from the set held in someone's hands versus being drawn from lying on a desk, could even be brought up and discussed in terms of which is fairer and whether there is a difference at all.</u> 5. <u>The card experiment could be redone with fully shuffled packs and the outcomes compared to the first set of outcomes (done without being shuffled where most of the same suits were together). This could be followed with a discussion on whether it changed the outcome or not in having a shuffled (fairer) pack. The class could investigate whether the outcome would be different if all the cards were spread out on a table instead of being in a pile on the table, or if they were placed on a bigger table and spread out further.</u>	*O3: And maybe if you did your…'cause I'd be really interested to see the results of their cards… because they were sitting in spades, hearts, clubs, diamonds.* *T-R: Oh gosh, they weren't even shuffled.* *O3: No…* *T-R: I didn't even think of that.* *O3: No, so I would be really…and maybe it would be…what you could do is to, to indicate fairness is to redo that experiment in the completely shuffled packs….maybe line the, you know…on the table…you know…with that change, the outcome…* *T-R: And maybe, you know a bigger table, spreading it out…* *O3: Ja* *T-R: And completely going…* *O3: So one group, one group could do that. Because I would say that your results should be fairly skewed.* *T-R: Mmm…Ja, ja, no you're right because S1 kept going, I keep on getting the two of hearts, I keep on getting the two of hearts…(laughing)* *T-R: Some of them were shuffling, not very well, like you know the…forward back, forward back* *O3: If you do it all out on the table like that, ja…* *T-R: …and moving and changing… where? Anyway, ok. Ja that's a good point and I kept on pushing it but I didn't give them, talk about the experiment at all. Ok…*
6. <u>A combination of lesson two and three would have worked nicely where there was a session of whole class revision and helping the</u>	*O3: If we could… could have combined the two…*

students to focus on direction. Then the students could have been divided into groups and given the different experiments to do while one group was in the front with the teacher-researcher starting with some new concepts and if there was any stagnant moment that couldn't be moved past then they get switched out for one of the other groups.	*O2: It's hard as a teacher.* *O3: …combined lesson two and three…* *O2: Ja.* *O3: That would have been my desired result.* *O2: Ja.* *T-R: Oh, ok. With the one group in the front speaking detail, but experimenting in others…* *O2: And moving, experiments* *O1: And maybe to start the lesson, you can see how old fashioned I am, but to start the lesson with a bit of revision…*

Critical incidents selected for deeper analysis

Tripp (1993) suggests that diagnostic teaching could be summarised as developing one's professional judgement through identifying and analysing critical incidents. "A diagnostic teacher is one who can analyse their practice in a scholarly and academic fashion to produce expert interpretations upon which to base and justify their professional judgements" (p. 7). The use of critical incidents have two key functions: it is an exceptional system for developing and increasing awareness and handling of professional judgement, in so doing also improving practice; and it is also a way of finding a focal point for classroom action research, which is particularly pertinent to this study. Incidents are simply events that take place that might seem regular rather than 'critical' however they are made critical through analysis – once there is an attempt to find a more general meaning and classification/significance of the incident. "…critical incidents are not simply observed, they are literally created" (Tripp, 1993, p. 27). This is the kind of active analysis that was intended for this study – for incidents in the classroom to be selected and carefully critiqued to the point of making connections between lesson preparation and significance of the created critical incidents. Many of the incidents found to be significant were analysed in some way and they might have affected change in the follow-up lesson (see Appendix 11 for details). However the few critical incidents that follow were critiqued in greater depth and brought about significant change, either in the follow-up lessons or in the teacher-researcher's consciousness about improving her own teaching practice. These incidents were highlighted as they seemed in some way to be closely linked to the teacher-researcher's lesson preparation. A second and third selection step was made as they were looked at in more detail and the most relevant and 'critical' incidents (either effecting the biggest change in lesson preparation or teacher-researcher's practice) came to the fore.

"…analysis is a very personal affair" and "I find it is impossible to give an exhaustive account of what one does in analysing data" (Tripp, 1993, p. 66). Regardless of how true one finds this, the process of reading different researcher's views on how analysis of critical incidents should occur (Flanagan, 1954; Tripp, 1993; Butterfield, Borgen, Amundson, & Maglio, 2005; Brookfield, 2012), gives many ideas for structuring of analyses and possible questions to ask. The analysis procedure detailed below, therefore, was carefully developed to suit the study's purposes and an attempt was made to closely follow it. In the analyses of critical incidents any approach is considerably enriched by getting opinions from others and "…it is always useful, and often much easier, to include the ideas of others" (p. 66). In this study, the direct observers were directly involved in drawing the attention of the teacher-researcher to incidents that could be critical by making comments about the lesson and giving suggestions for teaching practice (which could have indirectly been based on incidents that they have, perhaps without knowing it, identified as critical).

In the analysis[2] of each critical incident there was an attempt to follow these stages. Questions posed by Tripp (1993) were also kept in mind for general consideration of the incidents, "Why do I see it like that? How else could I see it? What do I consider the right way(s) of seeing it?" (p. 66).

- Describe briefly important details of the incident.
- What made it significant/critical?
- Is there a link between the incident and metacognitive preparation for the lesson?
- What assumptions may have led to the incident?
- What was the immediate assumption after noticing it?
- After thinking for longer/researching/getting input from colleagues, was the assumption you made valid and accurate, or not? When was this verified?
- What are the other perspectives on the incident? Are there different ways the situation could have been viewed or your behaviour interpreted?
- In hindsight could you have responded differently to the incident? How?
- Do you want to repeat or avoid the incident? How can you change the lesson preparation to do this?

1. <u>I incorporated new terms and concepts (experiment, event, outcome, result) smoothly into the discussion so that the students could understand what I was saying even though I included the terms they hadn't heard before, particularly in this context. This was received extremely well by the students who naturally started using the terms themselves in describing what was happening in the lesson. (Incident 2, Appendix 11)</u>

This was significant, as in my metacognitive preparation session I was very careful to question my approach and think about the language that I was going to use to introduce new terms and definitions. I wanted to transmit them in a way that the correct formal language would be used but at the same

[2] The critical incidents and analysis were written in the first person as notes were taken straight from the research journal and questions were answered being directed personally to the teacher-researcher.

time, the students would be the ones formulating definitions themselves so that they could internalise the new concepts. This was recognised as successful afterwards by the observers in the discussion forum as the students used the formal terms, shared by the teacher indirectly in talk around the experiments, to collaboratively develop definitions. I knew that the language I used in a new section would be picked up by the students and so I was aware that I had to plan what I was going to say and speak very carefully. I also assumed that if I used the correct terminology repetitively, the students would start using it themselves. This assumption seemed to be correct, for the most part however in saying that I also must have assumed straight after the incident that the students were picking up the right language just because of my use of it. However it is possible that some students heard siblings use it or read it somewhere. Even though I considered this as a possibility, my assumption about using the language well was also validated by the observers who also pointed out that my use of language aided the students in defining the terms themselves but using the correct terminology. I also assumed in my preparation that all students would begin to use the language correctly however this assumption was incorrect as some students did not immediately pick up on the language being used and start using it themselves. From a different viewpoint, my behaviour could have been seen as unnatural as I used the language so repetitively and there might have been a more natural way to introduce the students to the language. My response to the incident was simply being excited that many of the students had picked up the new terms quite well and to carry on the lesson as I had prepared it. I could have perhaps paid more attention to the students who did not pick up the terminology quite as easily and question them around their understanding of the new terms and how they would describe or define them. This incident does affect the follow-up lesson as I would like to introduce other new terms (sample space, condition, constraint, equation for probability, event A) and need to think carefully about my language and how to talk about the new terms so that the students start incorporating it into their probability language as they did in this lesson.

2. <u>I explained the new term ‘sample space’ using a practical example of eight coloured balls in a tin. This was done with the aim of developing knowledge of how to calculate theoretical probability but my language use was not as effective as the previous day. The definitions I gave seemed exactly that, more like I had given them rather than letting the students develop them collaboratively as I had done in the previous lesson. (Incident 11, Appendix 11)</u>

This was a significant incident as it stood in direct contrast to an incident noted in the previous lesson on how well I had used language leading to the students collaboratively forming definitions with the correct terminology. It was in in fact one of the observers that pointed out that I had given my own definition rather than let the students develop it themselves (through listening to me use the language well as in the previous lesson) and I wasn’t aware of it until then. As referred to earlier, the analysis process is enriched by getting opinions from others on one's critical incidents. Assumptions that may have led to the incident are that the students picked up the language really well in the previous lesson and maybe I don't need to use the vocabulary as repetitively as I did before. In retrospect, this was a damaging assumption as I ended up not preparing the introductions to these concepts as well as I had done for the previous ones. To introduce these concepts, which were actually more abstract than the previous ones, would have required even better preparation so that they were incorporated naturally into the students' vocabulary. The immediate assumption I made after the incident was pointed out to me (turning it into a critical incident by analysing it further) and after listening to additional comments by the observer, was that I had not prepared definitions that explained the concepts well enough and they were too complicated. They simply included a different combination of the other new terms just learnt. This assumption was confirmed by the observers, one of them commenting that at this stage, Grade Six, any definitions trying to make a distinction between theoretical and experimental probability was unnecessary. In hindsight, I couldn't have made changes during the lesson as I did not notice the incident then however I should have kept the definitions more simple and prepared them as well as I had done for the previous lesson.

3. <u>Once all the data had been recorded in the table on Microsoft Excel, I proceeded to average the results and point out theoretically how many times each number should have been picked up. I had not planned to discuss this and it moved us as a class too quickly towards the relationship between experimental and theoretical probability. (Incident 4, Appendix 11)</u>

When I saw the results from the whole class and how they were tending towards theoretical probability, I mentioned to the students how close the average of the groups for each outcome was to picking three of each possible outcome (thirty experiments conducted with a $\frac{1}{10}$ probability for each number being picked). In mentioning this, the students wanted to know why this happens. This was significant as I had not clearly decided to make it a goal for the lesson for the students to understand this point. Due to my lack of preparation for it, I could not explain it simply to the students and they were also not ready for that content. I tried to explain it since it had been brought up but tried not to dwell too long on the explanation. Some of the students did understand it and there was a great moment of clarification for one student (that was unexpected) however most of the students were unnecessarily confused. There was no assumption made about teaching this concept before the lesson and that may have been the problem as I was not prepared for it to come up. Immediately after the incident occurred I was unsure about what to do and how much detail to go into and my uncertainty reflected in many of the students. In retrospect and after discussion with the direct observers, I realise that it was too complicated an issue to bring up and assuming that just because one student 'clicked' does not mean that it was the right level for the class. In hindsight this should have been addressed more carefully. I should have had a clearer goal and follow through action for this concept. If I had decided to teach it, then it should have been covered more thoroughly until all students understood (even if that meant pulling a few students aside to explain while those who understood carried on with other work). If I had decided for it not to be a goal for this lesson then I should not have pointed it out to the class and if a few students noticed it themselves and asked about it, then there could have been some attempt to guide them to understand how it works themselves, and separately from the rest of the class so as not to cause confusion.

4. A student used a term that the teacher-researcher hadn't – 'even chance' – which the teacher-researcher then said wasn't quite an appropriate term to use to describe likelihood. One of the observers mentioned that it was a perfectly acceptable term to use in probability and in researching this further the teacher-researcher came to find that it is a term used regularly to describe a fifty/fifty chance in probability. (Incident 26, Appendix 11)

I did not think that this was significant until one of the observers mentioned in the discussion forum that 'even chance' is an acceptable term. This became more significant when upon researching it, I discovered that it was in fact used regularly in probability and I had told a student that I'd prefer if they didn't use it. My assumption that the knowledge I had gained through research of probability content was all encompassing and sufficient, could have led to the incident. Another assumption I made was that the term was not technical enough to describe the probability of a 50% chance and so I asked the student to give me a better, more 'acceptable', term for me instead of letting them use what made sense and was intuitive to them. Immediately after realising the significance of the incident I assumed that perhaps the observers knew that it was an acceptable term but that it wasn't used very often. However on researching it and discovering that it is commonly used to describe fifty percent likelihood in probability, I was extremely surprised and knew I had to tell the student who used it (and the rest of the class) that it was an acceptable, and regularly used, term. I should not have been so quick to assume the term was incorrect and ask the student not to use it. My behaviour could have been viewed as hasty and arrogant in assuming that the terms I brought to the lesson were the only acceptable ones. In hindsight I could have said that I wasn't sure about it but would research it and let the class know the following lesson. My metacognitive knowledge was lacking in this instance. In preparing for the lesson, I should have investigated all the terms used in probability for different likelihoods. In preparation for the following lesson, I made sure to double-check all probability terms possibly related to the content to be covered.

5. Often I answered questions without giving students a chance to think or attempt to answer. I noticed this while I was transcribing and analysing lessons. One of these situations occurred when I asked the students why each outcome should tend towards being picked three times for theoretical probability and did not give them a chance to even think about it before answering the question myself. (Incident 7, Appendix 11)

This incident was significant as it highlighted a flaw in my teaching practice that I need to work on. I did not notice this incident immediately but only in careful reflection of the lesson and when listening to the audio-recording did I note the instances where I asked and answered questions too quickly without giving the students an opportunity to try. In addition to this, the speed at which the teacher-researcher spoke made it so difficult for transcription to take place that it makes sense that some (if not most) student would struggle to follow the pace set in the lesson and keep up with my talking as I introduce new concepts or revise older ones. This leads one to thinking that an essential part of the developing metacognitive framework should be to suggest to teachers to record their own lessons and reflect on them afterwards, taking special note of the speed of talking to students, not letting students have a chance to answer questions, not letting the students think long enough about a problem and answering too quickly, etc. I did not have any specific assumptions on this topic before the lesson however a possible assumption that could have led to me answering my own questions too quickly, is assuming that the students did not know any of the answers I was looking for. This is a flaw in my teaching practice especially if I am aspiring to teach with understanding. I cannot expect students to know all answers immediately and shouldn't want them to, but rather to let them think and talk out loud through their processing of a problem. In addition to this, in elaborating on Schoenfeld's (1987) comments about students' beliefs, I could add to the negative beliefs and views of students by behaving in a way that makes it look like I don't think they could come up with any logical thinking or good replies. In hindsight, I could have responded differently to the incident by giving the students a chance to answer questions that I pose and at least ask them what they are thinking, letting them know that the right answer does not matter as much as their processing of a problem to try understand it does. I will attempt to pause in future lessons when posing questions and talk slower and give students more time to think.

6. I tried to address what was discussed in the forum – the lack of understanding on the part of one or two students in the first lesson (who were selected specifically for the small group in the second lesson for that reason). I tried to draw them further into conversation and garner their understanding on concepts from the first lesson but I did not handle it very well and put too much pressure on the student. (Incident 43, Appendix 11)

I attempted to draw one of the weaker students, student 5, into recalling the definitions from the previous lesson and make sure they were not left out of the discussion. This was a significant incident as it did not have the desired result. I put too much pressure on this student who faltered and struggled to recall any of the correct information from the previous lesson. In trying to correct one of the teaching flaws from the previous lesson, I overcompensated and spent too much time trying to drag answers out of this student. This threw off the rest of the lesson as much time was wasted and other students who were also part of the small group and knew the correct answers grew bored. I made the assumptions that if I tried to include the weaker student more carefully that they would give all the correct answers needed to show that they did actually listen and understand in the previous lesson and we could move ahead with the lesson. This assumption was completely inaccurate and the student did not know what the concepts from the previous day were about and with the added pressure now focused completely on them, could not even seem to remember what the experiments were about. My immediate assumption was that the student could not remember anything because they are weak in mathematics, however further analysis of the situation led me to question all possible avenues for this result. They could have been that I was not patient enough as a teacher, I had not given enough focused time to this student the previous lesson when introducing the concepts or I was putting too much pressure and unnecessary focus on the student causing them to be flustered. I may have seemed like a bully, in front of the other students and to any other observers of the situation. The direct observers noticed this incident and pointed out a number of alternatives for how the situation could have been addressed. In hindsight, I could have asked one of the other students sitting there with their hands up to help out the student who was flustered. I could also have garnered the definitions from the students who were surer and then sent them off to complete a relevant activity and kept the students who were confused for a better explanation. For the follow-up lesson, in reflection of this incident and with some notes from the direct observers, I decided to go back to basics and back to experimenting and using the right vocabulary to solidify good language use for probability first.

7. <u>I had been attempting to explain to one of the groups how to work out the total number of different possible outcomes when pulling coloured balls out of a tin but they could not quite understand it. Student 10 came to understand it and started trying to explain it to the rest of the group. They described the scenario very simply to the other students and most of them understood. (Incident 38, Appendix 11)</u>

This incident was significant as it caused me to realise the value of a student's explanation to fellow classmates on concepts that we cover. Once student 10 was able to grasp understanding of the problem and try explain it to the other students at their level, they grasped that explanation much quicker than the one I had tried to give. The initial assumption that could have caused me to delay asking a student to try explain to the others, is that my explanation would work best. And it didn't. My assumption was invalid because I attempted to explain it a number of times and couldn't and student 10 was able to get through to the other students very quickly. My immediate assumption after the incident was that it just took longer for the students to understand and they needed it explained once more however this was not the case. Student 10 just had a better explanation that the other students could relate to. In hindsight I could have used the valuable explanation of student 10 in having a good whole class discussion, asking the rest of the class the same question and then letting student 10 explain it in their own words again. After walking round the class and trying to get an explanation from each group, a whole class consolidation with student 10's explanation would have made for a good summary. I could have also learnt from the student's language a better way to explain the concept to the rest of the class (as it seemed to make more sense to a few of the students not understanding in the small group than my explanation did). It gave me another valuable point to consider for my own teaching practice. Sometimes in not being able to explain something to the students, I need to step away and get a different perspective (perhaps a closer perspective from the mind of a classmate who might be seeing the problem in a similar way to the student who is not understanding).

8. <u>An amazing AHA moment where three of the students started using tree diagrams from their own initiative to try solve the total possible number of outcomes for rolling a die three times. It was very close to the end of the lesson and two of the students did not finish their investigations. However student 7 sketched an initial tree diagram and after some discussion with the teacher-researcher realised there were some inaccuracies. After seeing what the issue was student 7 then attempted a second tree diagram and was able to solve the problem for finding the total number of outcomes for rolling a die three times (going to the teacher-researcher at the end of the lesson to confirm the total number of different possible outcomes). (Incident 40, Appendix 11)</u>

This incident was significant as it highlighted the importance of me being open to changing my goals, and thereafter my actions. This was for the sake of a few students who push the limits and go further than the plan, and ask the questions that aren't expected or make the connections between concepts that I envisaged being too difficult. There were no assumptions made that could have led to this incident, rather the reverse where I assumed that the content was too difficult to cover and so it wasn't brought up. The original plan for this lesson was to introduce tree diagrams and this plan changed after serious consideration and discussion with the direct observers. Multiple throw of the die and flips of the coin were only kept as two of the experiments as it made for more interesting results however it was kept simple in that I only asked them to record results. When I asked how many total different possible outcomes they thought there were for the experiments, it was only asked about drawing a card from a deck and pulling a coloured ball out of a tin (that had eight different colours). It did not include asking for possible outcomes for the multiple coin flips or die throws (although it might have come up naturally in conversation following the other experiments). In attempting to go further to answer a question that

I posed about a different experiment, student 7 decided that an iconic representation would be the most effective way to go about it. I asked certain questions to guide the student to the correct use of the tree diagram for three rolls of a die and the student came to the correct answer themselves (see transcription of conversation in incident 40, Appendix 11 and the two tree diagrams created by student 7 in Appendices 21 and 22). Immediately after the incident I was excited at the student 7's progress and surprised at their persistence but I did not assume that the rest of the class would be ready for this content as student 7 was one of the stronger mathematics students in the class and did show initiative in other situations of learning. This was the correct assumption to make and the observers agreed that it was still too early to introduce that material to the students even though some students (in addition to student 7) had shown interest in solving the three die problem and also turned to tree-like representations to attempt it. In hindsight, I could have been better prepared for some of the students wanting to try answer those questions and pulled them aside to work with each other on it. It could have also presented a nice opportunity for collaboration on how the tree diagram worked for a problem like that instead of me giving leading questions. For future lessons, I can keep in mind the students who showed interest and initiative in solving this problem so as to remember to challenge them and give them extension problems.

9. <u>In the final summary, student 7 also pointed out that because there are so many more different possible outcomes when you roll the die three times (instead of only once), there is a much lower chance of getting a repeated outcome. (Incident 41, Appendix 11)</u>
10. <u>Student 11 had good thought processing that led to them understanding that changing the amounts of certain colours in a sample will affect the theoretical probability. This led to asking me that if there were more of a specific colour in a sample than another one, would there be a higher chance of pulling that colour out than the other colours. (Incident 33, Appendix 11)</u>

For both incidents I realised that a few of the students were definitely beginning to understand the concepts well and the connection between experimental and theoretical, but more students needed to be reached. They were critical because they identified moments in the lesson where understanding by specific individuals was reached but the goal for teaching in this environment it to reach more (if not all) of the students in the class. It is possible in finding these incidents that I was catering in my lesson preparation for the students who were more mathematically inclined rather than the whole class. This suggests to me that I need to reassess small group teaching in my practice and also be careful in how I prepare lesson content so that all students can be reached. I did not plan to teach either of the concepts displayed above and so it is encouraging to see that the students intuitively reached those understandings through the lesson's progression. My immediate assumption after students reached conclusions like the above on the concepts being discussed, was that they were the ones concentrating or listening carefully to my explanations or were just more mathematically inclined. However it is important to note that there are a number of possibilities that all have to do with me changing my teaching practice to try reach more students and be more attentive to those who seem like they are not listening but maybe are just lost because I didn’t prepare a definition carefully enough. I would have liked for more of the students to understand these points by the end of the lesson but perhaps I should have had a clearer goal set for the lesson so that the strategy could have been shaped to guide more of the students.

Considering the above critical incidents in terms of Systems 1 and 2, it is evident that those which reflected thorough metacognitive preparation, reflected the use of System 2 as well. There was focused attention to detail, and planning for a lesson was not carried out automatically and without thought. Where metacognitive preparation was involved, System 2 was involved and the language to be used together with the questions to be asked was structured carefully. This resulted in better explanations or a deeper understanding of a concept explained (see critical incidents 1, 9 and 10 above). The parts of the lesson where the teacher-researcher had not prepared carefully enough resulted in a confusing explanation, a lack of understanding by the students or the inability of the teacher-researcher to deal with a change in the direction of the lesson (see critical incidents 2, 3 and 6 above). It should be noted however that even in the critical incidents above where undesirable situations occurred, the recognition of what went wrong and an attempt to fix it could only take place through the use of System 2, which in the case of this study was through metacognitive reflection. In addition, for the critical incidents where there were positive outcomes, System 2 was used to assess them in an attempt to repeat the desired results.

4.5. Reread data, coding places where interpretations are supported or challenged

Data were reread for the purpose of confirming that interpretations were accurately gleaned. The final fully developed metacognitive framework (with a series of steps to follow) and details as to its efficacy for the teacher-researcher’s teaching practice are presented below. Dominowski (1998) discusses how research has revealed that verbalising in different ways affect problem solving differently and thinking aloud does not necessarily change task performance unless metacognitive processing is involved. This metacognitive processing can be accomplished through talking out loud or simply thinking. “ …requiring subjects to give reasons for their choices and actions often results in improved task performance” (p. 38). Dominowski (1998) refers to previous research of his and suggests that the latter may be due to focusing attention on processes that are related to the task (Berardi-Coletta, Buyer, Dominowski, & Rellinger, 1995). Questions that are focused on a problem leads the subject to attend to characteristics of that problem, however metacognitive questioning leads the subject to focus on their own processes. The importance of questioning in an attempt at being metacognitive has been substantiated by a number of researchers and therefore moments for questioning appear frequently in the framework (Berardi-Coletta, et. al., 1995; Carr & Biddlecomb, 1998; Davidson & Sternberg, 1998; Dominowski, 1998; Hacker, 1998; Pressley, Van Etten, Yokoi, Freebern, & Van Meter, 1998; Sitko, 1998). Much of the questioning described by these researchers is for the purpose of helping students to be more metacognitive, however this study used questioning in an attempt to aid the teacher-researcher in striving to be metacognitive for the purpose of developing the metacognitive framework.

4.6. Write a draft summary

A working framework began to form and was adjusted over time through careful analysis. This framework was developed based on the data collected and includes the four key metacognitive phenomena: metacognitive knowledge, metacognitive experiences, goals and actions. The main purpose of the framework is to guide a teacher who has no knowledge or practice in the metacognitive field, to use these phenomena to become more metacognitive (thinking deeply on their thought processes) in their preparation for and reflection of lessons. Another key aspect for the framework, noted during the reading and coding of data, will be to include points in time where the teacher can monitor their thinking through the practice of these phenomena. These points in time are PREP, BEFORE, DURING, AFTER AND LATER, which will be discussed in further detail. The final framework that this research would like to suggest to fill the above mentioned gap in the field of metacognition is presented in the following chapter.

4.7. Review interpretations with participants

The interpretations of the data collected from the observers were reviewed and validated by the observers themselves. A valid reason for reviewing interpretations with the direct observers was for the purpose of triangulation to validate the data. Having the interpretations reviewed, aided in correcting or confirming data interpretation leading to a more refined, revised summary. However it also supported that the teacher-researcher used valid data in concluding the study. The write-up of data regarding the comments made/suggestions given by the observers, together with how they were addressed by the teacher-researcher, was sent to the three observers for corroboration of validity. Their responses can be found in Appendices 23, 24 and 25. These responses corroborated conclusions drawn by the teacher-researcher and so the metacognitive framework could be finalised and presented. Chapter 5 outlines the series of steps that form the framework and clearly demonstrate how it can be used successfully. Detail of its value and effectiveness in the teacher-researcher's practice is then clarified.

V

Metacognitive framework and its efficacy

The final step of the *Interpretive Analysis* tool (Hatch, 2002, p. 180) is to **Write a revised summary and identify excerpts that support interpretations**. For this study the revised summary includes a) a workable, usable metacognitive framework developed through the process of this research, supported with applicable excerpts from the collected data and b) the efficacy of the framework for the practice of the teacher-researcher as tested in this study, also corroborated with evidence from the data. The cognitive goal of this metacognitive framework is to guide teachers in preparing for and reflecting on a chosen lesson metacognitively. If the main purpose of metacognition is to oversee whether a cognitive goal has been met, then this framework seeks to oversee the teacher in their preparation and reflection of the lesson/a series of lessons so as to improve practice.

5.1. Metacognitive framework (PBDAL[3])

Preparation notes for lessons (one subject for one grade) should be kept together in a journal where there can be a continuous flow of information and ideas. This journal can be used for the PREP, BEFORE and LATER stages. A separate notebook is suggested for use in the AFTER stage, some of the notes made here can then be transferred to the journal.

PREP (includes **metacognitive knowledge, goals and actions)**:
This stage encompasses preparation for the lesson which includes the development of the teacher's metacognitive knowledge. Each teacher possesses and develops their own metacognitive knowledge and this PREP section of the metacognitive framework seeks to help teachers consciously activate their metacognitive knowledge. In order to accomplish this effectively, the following questions should be carefully considered and answered (which can be detailed in the journal).

1. How do I teach…
    a. What are my strengths (that I know of/that others have told me)?
    b. What are my weaknesses (that I know of/that others have told me)?
2. What am I going to teach in this lesson?
3. What do I need to do or learn to teach this better?

[3] PREP, BEFORE, DURING, AFTER, LATER (5 main stages of the metacognitive framework)

4. What are the main concepts of the content? Do I know them very well?
5. Do I need to relearn some or all of the concepts?
6. Should I teach the class as a whole group or divide them into small groups?
7. Do I need to prepare worksheets or other resources?
8. How long will I need to prepare the content and resources for teaching?
9. Do I need to connect and use technology in the classroom in any way?
10. What is my goal for this lesson?
11. What will I do if this goal is not met…
    a. by everyone?
    b. by most students?
    c. by a few students?
12. What do I want each student to leave this lesson knowing?

<u>BEFORE, DURING, AFTER</u> (includes **metacognitive experiences**)

Each of these moments can be fleeting or lengthy thoughts about what I've prepared for the lesson, the goals I've set and the strategies I've put into place. It looks at future expectations of progress or completion of the lesson (BEFORE), internal feedback about current progress (DURING) and an assessment of how I've taught the concepts, connecting new information to old information by linking current and future lessons and deciding on the direction of lessons (AFTER).

<u>BEFORE</u>: This is the moment just before the lesson begins and is a moment's glance at what is about to be taught. This step is just in the mind of the teacher, on your own with no other input and a glance at preparation notes can take place here. This is an important step to focus the teacher in on the topic and goal for the lesson.

<u>DURING</u>: These are multiple moments during the lesson where the teacher is constantly evaluating oneself and checking the teaching process and responses from students, adapting the lesson where necessary. If a question comes up during the lesson that the teacher is not certain of how to answer, or a term is used by a student that may or may not be correct, the teacher can make a quick note of this during the lesson (in the notebook) in order to remember what to follow up on afterwards.

<u>AFTER</u>: This moment takes place straight after the lesson (if possible) and is again a brief glance at what was covered in the lesson, positive and negative responses from students, ideas for the following lesson and keeping in mind what needs to be addressed or readdressed in the next lesson. Also, which students

to follow up with on ideas grasped well (or not at all) and those who were extended or may have asked unexpected questions, need to be pinpointed. Also ask the following questions briefly AFTER the lesson. They can be answered in more detail during the LATER stage: "What stood out in the lesson that, as the teacher, I would want to repeat or avoid in future? How can I make sure to repeat or avoid it?"

Many of these thoughts could be as an extension of DURING (when the teacher was constantly evaluating the progress of the lesson and storing away ideas to be addressed at a later stage). It would be ideal if any of these thoughts could be briefly written down in the teacher's notebook (for accurate recording and simply to make for easier recall). This moment with any notes recorded make for a solid grounding of LATER, in preparation for the following lesson.

LATER (includes **metacognitive knowledge and metacognitive experiences**):
This is a longer period of time and is a mirror image of BEFORE except that there is more information to work with, which was gathered AFTER the lesson. The LATER also can include a repeat of the PREP process whereby the same questions are asked and changes made in preparation for the following lesson. Once again notes can be made in the journal showing a continuous flow from the previous lesson's PREP stage and pertinent notes from the notebook can be transferred to the journal. In this stage, make sure to include and answer the questions mentioned above.

1. What stood out in the lesson that, as the teacher, I would want to repeat or avoid in future?
2. How can I make sure to repeat or avoid it?

Additional questions for metacognitive preparation and reflection

Some of the following questions have been taken/adapted from Dominowski (1998).

1. What are you going to do? How are you going to do it?
2. How will I teach this? Why will I use this table or that example or…?
3. What order will I teach this in?
4. How much can I teach in this one hour lesson? Why?
5. How much could I get through in these three lessons?
6. What if they don't understand and I have to go back and repeat and it takes longer?
7. How can I find the balance in teaching between understanding the concepts and covering the whole syllabus?
8. If they don't understand a concept in this way, do I have a back-up plan – a different way to teach it?
9. Why did I do that?
10. How did I form this or put that together?

11. If I get stuck in the lesson preparation and am unsure of what resource to use or how to guide students to come up with a definition, what are my options? Can I ask someone else for guidance, possibly a colleague?
12. What am I struggling with here? What are my different options?
13. Explain the problem with deciding what to teach or how to teach it and write down different solutions that have been considered.

Additional elements of the framework

- It is necessary for colleagues or external teachers to observe your lessons and discuss them with you at certain intervals. One observation and discussion session per term is suggested as the minimum however the more often this can take place, the more feedback you can get on your teaching. This could help you to notice flaws in your teaching practice and learn how to improve.
- If this is not possible regularly (or at all depending on your situation) record your own lessons and listen to yourself. It is suggested that you record your lesson and listen to it afterwards, comparing its content to your PREP session, a minimum of twice a term. Listen carefully and note what you would critique if you were listening to a different teacher's lessons. Use the following questions as a guide:
    1. Do you talk too quickly? Or too slowly?
    2. Do you answer yourself without giving the students a chance to think of an answer?
    3. Do you give enough time for the students to think through your question before assuming they do not know something?
    4. Do you explain yourself as clearly as you thought you did or as carefully as you had planned in the PREP session?
    5. How much of what you wrote down for the PREP session did you actually cover in the lesson?
    6. Was your lesson content and delivery set at the right difficulty level for your students (weak students struggled, but were able to understand basic concepts, most student understood all concepts well, strong students showed deeper thinking and understanding, perhaps exploring the mathematics further, and you could extend them in the lesson)?

5.2. Efficacy of the metacognitive framework

The metacognitive framework developed over the time of this study proved to be useful in aiding the teacher-researcher in teaching preparation and practice for the probability lessons. This conclusion is corroborated by the action research process in the reflections of what was personally seen by myself as the participant and teacher-researcher. A number of critical incidents that occurred were assessed as constructive learning developments and attributed to the metacognitive preparation the teacher-researcher carried out for the lesson. This was verified by metacognitive reflections of the lessons and comments made by the direct observers in the discussion fora.

The metacognitive knowledge that the teacher-researcher had prior to the lessons brought about an awareness of how much research into probability was necessary. There was some knowledge on probability and how to teach it at a Grade Six level but the more in-depth knowledge needed to be enriched and investigated, especially to prepare for the students who questioned the concepts in greater depth. The teacher-researcher was also aware of the necessity to attempt to always have a better knowledge on any topic to be taught to students, needing to be the more knowledgeable authority in the classroom. This also gives students confidence in their teacher's ability and presence as a source of support. Metacognitive experiences referring to actual experiences in the classroom during the lessons were enriched by a diverse class, in terms of abilities, interests and learning methods. The goals set by the teacher-researcher were not always very clearly defined before each lesson and so the actions decided upon may not have always been knowledgeably chosen. Often when there was a moment that the teacher-researcher was flustered or unsure of how to approach a topic, it was due to not having prepared well enough for a specific concept or question.

This paper would like to suggest that the efficacy of the framework tried and tested by the teacher-researcher rests upon data which are indicative of thorough metacognitive preparation and reflection. The scenarios that seemed to rely on the teacher-researcher's preparation are responses from students during the lesson to the prepared content, namely accomplishments of the lesson. Positive scenarios include students asking probing questions, students pursuing deeper concepts, students attempting to engage deeply with new knowledge or students understanding new concepts well.

There were a number of these positive scenarios that came about in the duration of the three lessons. Some of these were when students explored the new theory learnt and attempted to apply it to a different scenario (see incidents 20 and 41 in Appendix 11). Some of these occurred when students pursued deeper concepts (see incidents 18 to 41 in Appendix 11). These also happened when students demonstrated a proficient understanding of new concepts, either when they collaborated in groups or when individual students made discoveries about the mathematics of Probability (see incidents 24, 28, 29, 32 and 33 in Appendix 11).

The scenarios that seemed to be based on the teacher-researcher's reflection are the kinds of changes that are made by the teacher-researcher afterwards, and the fact that changes are made at all. Positive scenarios include the teacher-researcher recognising that some concept or definition needs to be re-explained, that a section that was considered completed is revisited or restarted when necessary, that follow-up lessons are changed due to realisations made in reflection or that flaws in teaching practice are recognised and an attempt made to improve. The teacher-researcher saw a need to re-explain a number of things such as why the average of the class' results tending towards three showed that each outcome theoretically should have been picked three times, and why repeated experiments will result in finding the theoretical probability. It is important that these were not thought of as a waste of time, but preferably considered more valuable for students to gain the understanding in those lessons rather than it come up as an area of misunderstanding in later years of school.

Conducting experiments and recording results, the basic beginnings of probability, occurred in the first lesson and a little in the second lesson and the teacher-researcher thought that enough had been done and it was time to move ahead to more complex concepts. However after discussion with the observers and further metacognitive reflection, the teacher-researcher realised that it was too soon to move towards abstract concepts, therefore more experiments needed to be conducted. The entire third lesson was comprised of a number of different experiments that the students carried out in groups. This may prove cumbersome to some, however repeating parts of the curriculum only serves to benefit students as they will move ahead academically with the correct understanding of certain topics, hopefully without necessitating a future teacher to reteach concepts from earlier years that should have been consolidated.

The plans for both follow up lessons were changed in the course of the teacher-researcher's metacognitive processes. As has been seen in much detail above, two fully prepared lessons for lesson 2 and lesson 3 were completely withdrawn in favour of two lessons that had been planned using the progress from lessons 1 and 2 as feedback. The observers also assisted the teacher-researcher in her thought processes and metacognitive experience, in recognising what had been missing from lessons 1 and 2. This helped the teacher-researcher to comprehend what was content was indispensable in order to gradually scaffold the students. This needed to happen until they were ready for the content that had been prepared originally for lessons 2 and 3, even if that meant only covering that content much later, possibly after this study came to a conclusion.

The final constructive scenario that would seem to be based on the teacher-researcher's metacognitive preparation, is that flaws in teaching practice were recognised and an attempt made to correct them. Some of these flaws that the teacher-researcher became aware of through the metacognitive reflection processes central to this study, are talking too quickly and not giving students enough time to think about a problem. Another fault noticed was that the teacher-researcher answered some questions herself without giving the students any time to attempt to think or answer. These three flaws were not mentioned or seemingly noticed by the observers, but rather by the teacher-researcher during the transcription sessions while listening to the audio-recordings. It is for this precise reason that one of the further recommendations as part of the framework was that teachers should record themselves at least once a term and listen to their own lessons critically to identify faults that might be solved once they become known. These recording sessions could take place more often if the teacher is in the position to do so, and especially if there is no faculty to accommodate colleagues/observers to the lesson.

Another point that was picked up through careful metacognitive questioning in the teacher-researcher's reflection was that if parts of the lessons that involved specific questioning techniques for the students and these were not prepared well enough, they would not be asked well. This could be detrimental to the teacher's explanation and thus the students' understanding. This did happen in one of the lessons whereby language for a few definitions in the first lesson was prepared very carefully and the questions asked were managed very well, leading to valuable collaboration by the students. For different definitions planned to be covered in the second lesson, the preparation was not as thorough and the teacher-researcher did not present the language and questions as efficiently as had been done in the first lesson. The result was that the definitions were not as straightforward for the students to understand and the teacher-researcher had given the definitions, instead of guiding the students in forming the definitions themselves. This was picked up by the observers, who also pointed out that the teacher-researcher was planning content that was too abstract and difficult for the students. The teacher-researcher recognised this as another flaw and resolved for future lesson planning to be more meticulous as to the depth of content to prepare. The difficulty level needs to be carefully planned so that all students benefit fully from the lessons, as discussed above. In the short term, this led to both lessons 2 and 3 being changed to accommodate a more appropriate level of difficulty.

In examining the consequences that metacognitive preparation and reflection had on the teacher-researcher's teaching practice, one can confidently say that they have led to an awareness by the teacher-researcher of the students, the content and the teacher-researcher's own practice. Having an awareness of these led to positive learning experiences for the students, knowledge of the need for further investigation into the topic (Probability) by the teacher-researcher and knowledge of how to improve the teacher-researcher's practice further. It is thus deemed fair to infer that the metacognitive framework developed and applied in this study was efficacious for the teaching practice of the teacher-researcher. It is to be noted however, that the efficacy deemed has solely been shown for this teacher-researcher's practice and it would need further investigation to prove its efficacy in use by other teachers.

One of the motivations for this study was the teacher-researcher attempting to improve her teaching practice and Tripp (1993, Foreword) points out that “good teachers use good techniques and routines, but techniques and routines alone do not produce good teaching. The real art of teaching lies in teachers’ professional judgement because in teaching there is seldom one ‘right answer’”. It is the hope of this study that the metacognitive framework will be of use in aiding teacher’s professional judgement of their own teaching practice, advancing towards good teaching. The following, final, chapter concludes this dissertation by reviewing and summarising the research problem that this dissertation has addressed.

VI

Conclusion

This chapter seeks to review and summarise the dissertation, by drawing attention to the problem statement and how it was addressed. The methodology will also be briefly revisited and its implications in helping to shape the study, examined. A synthesis of findings that answered the research question is then addressed in two parts – a summary and then discussion of the results. The limitations, implications and generalisations of the study are detailed with suggested recommendations rounding off this chapter and the dissertation.

This study sought to address the gap in the literature by developing a practical metacognitive framework that teachers could use to be more metacognitive in preparation for or reflection of lessons. It also looked to include a method that would foster deeper understanding of concepts through carefully considered language use. Developing this framework and finding its correct placement in the literature was the focus. The study employed an action research methodology which was essential in order to substantiate the teacher-researcher being in the classroom, actively applying teaching methods and reflecting on her own teaching practice for the purpose of improving. The cyclical process was crucial for the teacher-researcher to be allowed to make plans, act on them, assess them afterwards and make changes based on the assessment, re-planning for follow up lessons with this process taking place again following the next lesson (Doerr & Tinto, 2000; Ferrance, 2000; McNiff & Whitehead, 2002; Opie, 2004; McNiff, 2005; Pine, 2009; Blair, 2010; Mertler, 2012). While the nature of the cyclical process was not drawn out, it was rigorous and carried out over the three lessons, their three discussion fora and any time outside of that used for preparation and reflection. The composition of the stages for action research, referred to by the aforementioned researchers, all detail a similar configuration and made an ideal foundation for the structuring of this study. The first stage of finding facts, planning, acting on that plan, assessing and then modifying the plan, before moving to the second stage, serves as a well-defined picture of the nature of this study in developing the framework and evaluating its efficacy for the teaching practice of the teacher-researcher.

6.1. Synthesis of findings

I restate my research question here for the purpose of presenting the synthesis of findings specific to its different aspects. *Does a developed framework for metacognitive lesson preparation and reflection have the potential to facilitate effective teaching practice?*

The framework was developed and thoroughly detailed in the previous chapter. This framework was proved effective for use by the teacher-researcher in this study. It outlines a series of steps and a straightforward process, which can be followed by teachers who are looking to be more metacognitive in their teaching preparation. While the use of the framework in this study was deemed successful in facilitating a more effective teaching practice for the teacher-researcher, it is the hope of this study that this tool will be applicable for use by any teacher. This would potentially occur only after further research is conducted, investigating the use of the framework by multiple teachers and its efficacy in their teaching practice. In addition, while this study was rooted in mathematics, it has the potential to be used by teachers of different subject disciplines and would need to be adapted accordingly.

The steps of the framework can be followed without difficulty and include a list of questions to be answered, suggestions for stages to complete certain reflection tasks and some specifics to reflect on. Ideas for how and where to take down notes have also been suggested such as actually writing in a journal or a notebook or simply considering certain aspects in thought. Some additional questions have also been offered for further metacognitive reflection which teachers can choose to use partially, completely or not at all. Additional elements of the framework have been outlined as recommendations, which teachers can use to further improve their teaching practice, taking into consideration what aided the teacher-researcher in the reflection process and practice. These elements include the teacher having observers of lessons regularly throughout the year to critique and offer guidance or suggestions, the teacher recording their own lesson and then listening to the recording critically to detect any flaws or teaching habits that should be changed. Some flaws that were discovered in the teacher-researcher's own teaching practice have been outlined so that teachers can be aware of the kinds of weaknesses to look out for when reflecting on a lesson. The Literature Review indicated that metacognition has not been applied in reflective action research, particularly on the part of the teacher. This action research study however, has been used to develop teacher agency whereby professional development was self-directed by the teacher-researcher. It is the hope of this study that the results presented can be used to aid reflective metacognitive practice for other teachers, so that their professional development can also be self-directed.

It is important to reiterate the results of this research: that the framework was successful in facilitating effective teaching practice in this study. This was due to a number of observations made by the teacher-researcher, acceptable in an action research setting, and the direct observers who were quite immersed in the action research process. The direct observers observed the lessons, contributed to the discussion fora, wrote observation notes on the lessons and reflected afterwards on the data collated by the teacher-researcher to confirm its validity.

The success of the framework in facilitating effective teaching practice for the teacher-researcher could mean its success for other teachers. While its efficacy for anyone apart from the teacher-researcher has not been established, the ability of the framework to prompt metacognitive reflection, by anyone who follows the reflection and questioning techniques, is evident. This is due to the metacognitive prompts being based on a number of techniques on how to be more metacognitive, detailed by several researchers deep in the field of metacognition, as discussed in the Literature review (Flavell, 1976; 1979; Brown, 1977; 1981; Schoenfeld, 1987; 1992; Hacker, 1998; Fox & Riconscente, 2008). The metacognitive framework was designed with an intense focus on asking questions and exploring one's own ideas deeply, an important strategy for being metacognitive as described by Berardi-Coletta, et. al. (1995), Carr and Biddlecomb (1998), Davidson and Sternberg (1998), Dominowski (1998), Hacker (1998), Pressley et. al. (1998) and Sitko (1998).

While the effectiveness of the framework for other teachers still needs to be researched in greater depth, teachers with an interest in being more metacognitive in their preparation for and reflection of lessons can engage with the framework, as detailed above. Additional points that were added to the framework may also be used by teachers in isolation from the framework. These include having observers critique lessons and recording and listening to one's own lessons to create an awareness of teaching flaws, which can be done more often if having observers isn't possible. This study reviewed, and put into practice, a different application of metacognition whereby it was applied to the teacher-researcher and assessment made of its effect on her teaching practice. This took place instead of investigating and exploring the outcomes of an attempt at teaching students to be metacognitive. This was key to the study as the primary purpose of developing the metacognitive framework was to acquire a tool that would aid the teacher-researcher, and other teachers, in striving to be more metacognitive in teaching preparation and reflection. Important areas that have been identified include the necessity of further research into the use of metacognition for improving teachers' practice.

6.2. Limitations

There were a number of limitations that impacted the process and/or results of this study due to the nature of the available sample or instruments. Each of these limitations along with the strategies used to minimise their impact is discussed here. The sample included a class of twenty-six children and three direct observers. While the absenteeism of one or more children was not a limitation as the study was conducted with a class of twenty-six students, a possible disadvantage to the study might have been the absenteeism of a student who was part of a critical incident on the first or second day, but might have benefited the study to have them as part of the remaining sessions. This however was not the case and all students who were part of critical incidents were present for the three lessons. The absence of a direct observer would have been more problematic, however it was thought that by having 3 direct observers if one was absent there were still two people to enter into a discussion with the teacher-researcher, to examine incidents from the lessons in the discussion fora. This did occur when Observer 1 was unable to make the second lesson's time slot. The discussion forum did commence with Observers 2 and 3, and multiple suggestions and comments regarding lesson 2 were given that benefited the teacher-researcher's reflection process.

The audio-recording of the lessons and the discussion fora were transcribed and there was the possibility that the teacher-researcher may not have recognised the voices, possibly pairing some of the comments with the incorrect pseudonyms. A method to minimise this limitation, that was thought of before the study commenced, was for the teacher-researcher to make notes in her journal during the discussion fora and reflect individually after the lesson. This did take place, and occurrences that stood out or comments made by the students that were prominent were covered, in order to be a reminder for who said them during the transcription process. These were referred to during the transcription process which aided accuracy. There were some moments in the transcription process when there was too much background noise to either pick up who was speaking or what was being said. In these cases, the students were not referred to specifically, by pseudonyms, or it was clearly written as 'inaudible' in the transcriptions.

The presence of the 3 direct observers in the classroom would always be unnatural and the children may not have acted as they typically would. They might have behaved in a silly manner to draw attention to themselves or not have participated as fully in the lesson as they normally would, which could have impacted the results of the data gathered. A way thought of to restrict this limitation was to ask the direct observers to attempt to be as unobtrusive as possible, sitting quietly in one place for as long as possible attempting not to miss out on important conversations between the students, with the hope that the students would forget their presence and act in a typical manner. This was the case, and apart from some excitement initially at the presence of unknown visitors in the classroom, being recorded and being part of a study, the students became quite relaxed and acted as they normally would have, as far as the teacher-researcher could see.

The lessons chosen to be used for data collection depended on a number of factors, mainly with what suited the normal timetable for that class and the teacher-researcher. This resulted in having the lessons very close to each other which left the teacher-researcher very little time in between to reflect and then prepare for the following lesson. In order to deal with this, some of the lesson preparation was done well in advance. After the lessons took place and the discussions and reflective metacognitive sessions occurred, there were at least between one and four days for each of the reflections to occur so that necessary changes could be made to the following lessons. This kept in line with the cyclical nature of the action research process. Another limitation was that there are always a number of unforeseen circumstances in any school environment that may change the days, times or length of the lessons planned for data collection. This could have happened with very short notice which may have impacted the collection of data negatively. It was decided that this would have had to be dealt with in the moment, with every attempt to minimise any changes made with timetabling that was planned well in advance. To the benefit of the study, these unforeseen circumstances did not occur and the lessons and discussion fora did take place smoothly and without interruption.

6.3. Implications for conducting the study

There are several implications that resulted from this study being carried out. In an attempt to seek a method for improving the personal teaching practice of the teacher-researcher, routines and a manner of teaching was developed that would be beneficial to the teacher-researcher and their future students. This study also led to a consideration for the teacher-researcher to make metacognitive strategies permanently part of her teaching practice and recommend it for other teachers. Another important implication of this study was the involvement, as a direct observer, of the primary school Mathematics Head of Department[4] which is positive for peer contribution and collaboration – a vital part of the action research methodology. This led to further implications of possible changes for improved teaching practice in Mathematics across the phases, and even school, as the Mathematics Head of Department is involved in staff development and assisting in standardising teaching methods across the grades. This implication could not be confirmed at the time of the conclusion of this study.

6.4. Generalising the study

As Hallberg (2008) points out in his reflections on qualitative research, "Applicability, transferability and fittingness are terms used to address what in quantitative studies are called generalizability of findings and representativeness of subjects" (p. 66). Therefore these three terms will be discussed in lieu of generalizability for this study. Lincoln & Lynham (2011) pair usefulness with applicability stating the importance of the study being useful and applicable to ordinary people to aid meaningful change in the specific occurrences being studied. It is important to search for models or new knowledge that can be

[4] The students are familiar with the Mathematics Head of Department, but simply as another educator at the school, and his/her presence should not have any worse effect than the other direct observers.

applied practically to improve performance in that area. This study was conducted with the intention to better serve teachers by defining and investigating a useful framework to improve teaching practice. In finding the role that metacognition plays in improving teaching practice, this study could be useful to ordinary people with practical implications for applying it to their teaching practices. Although this study has been written from a South African context, it may well have an appeal to a wider, international audience due to the struggles experienced in the mathematics education field.

6.5. Transferability of the study

Qazi (2011) discusses how transferability in qualitative research can be enhanced by providing the position of the researcher with a clear description of context, participants, selection and methods (detailed previously). Lincoln & Lynham (2011) suggest that for the study to be transferable, it needs to be as complete as possible, giving the premeditated scope so that readers may see the extent to which the theory can be useful in their own setting. It is also key that knowledge can be transferred by individuals from one environment to benefit another. The research design for this study was carefully constructed so that the span of the research is clearly visible to readers. The results have been portrayed in a way that makes the knowledge easy to access and use in a different environment, by giving clear examples of how metacognition aided the teacher-researcher's practice and how it can be put to use by any teacher. Although the efficacy of the framework has not been tested on multiple teachers, the set of steps to follow and questions to ask are easily accessible by any teacher looking to be more metacognitive.

The key to a study having fittingness is that it must be embedded in local context, with native perspectives, significances and descriptions, and acknowledge that there are multiple creative responses or solutions to any given problem and not just one (Lincoln & Lynham, 2011). This study was local by being in the teacher-researcher's school and involving South African pupils and 3 direct observers who are educators based in South Africa. Numerous responses were given in the classroom, and different descriptions and inputs of knowledge were put forward in the discussion fora. There are multiple places for metacognition to be used for developing effective thinkers and this study focused on just one of them, in its attempt to develop a metacognitive framework for teachers.

6.6. Recommendations

After reviewing the study and the developed metacognitive framework, a number of recommendations come to mind. A recommendation in light of this research is for teachers interested in being more metacognitive in their teaching practice to use the framework for preparation and reflection processes. Another recommendation is for teachers to attempt to be more metacognitive themselves before trying to teach students how to be metacognitive. It was also noticed that collegial teacher peer evaluation benefitted the teacher-researcher and their practice. This study would therefore like to suggest that more regular peer evaluation is utilised for improving teaching practice at schools.

One of the recommendations for the academic community, is that further research on metacognitive strategies and their application for improving teaching practice is much needed. The necessity to find more practical solutions that will help teachers to learn to be more metacognitive amidst their demanding timetable, is considerable. In terms of future research it should be noted that this metacognitive framework is still in its infancy and much testing is required to fully determine its efficacy, especially on a wider field. As the efficacy of this framework was only tested on the teacher-researcher in this study, doing further research where a large number of teachers use it to be more metacognitive in their teaching preparation and reflection would better test its effectiveness. A wider test field could also include a larger group of mathematics teachers for Grade Six, mathematics teachers across the grades and even teachers across different subject disciplines. The study has also provoked a number of future possible research questions to come to mind. What is the link between the teacher's metacognitive preparation for lessons and development of understanding in students? How does this metacognitive framework specifically impact the teacher's preparation process and practice? What link, if any, is there between the teacher's metacognitive preparation for lessons and students' results in formative assessments?

A practical application of this teaching preparation framework was used in its development and guidance for use. This study culminated in suggesting a metacognitive framework and outlining where it can be placed in the literature by showing the link between metacognitive preparation and reflection, and critical incidents identified in the classroom. This framework includes some practices that most teachers are familiar with but the use of some unfamiliar metacognitive strategies have been engaged with and suggested. This study also made a careful examination and analysis of the interaction between the teacher-researcher and the participants. Analysis of the teacher-researcher's preparations and reflections using a metacognitive approach, along with their link to what happens in the classroom (critical incidents), was essential for the steps to be established and improved thereafter in the development of the framework. It is the hope of this study that the developed metacognitive framework will be widely used and will successfully impact teaching practice, providing teachers with accessible knowledge to being metacognitive, proving its efficacy and contributing positively to the field of metacognition in education.

References

Adler, I. (1963). *Probability and statistics for everyman: How to understand and use the laws of chance*. New York: John Day Co.

Baumann, J. F., Seifert-Kessell, N., & Jones, L. A. (1992). Effect of think-aloud instruction on elementary students' comprehension monitoring abilities. *Journal of literacy research*, *24*(2), 143-172.

Berardi-Coletta, B., Buyer, L. S., Dominowski, R. L., & Rellinger, E. R. (1995). Metacognition and problem solving: A process-oriented approach. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *21*(1), 205-223.

Blair, E. (2010). How does telling the truth help educational action research? *Educational Action Research, 18*(3). doi:10.1080/09650792.2010.499810

Boekaerts, M., & Corno, L. (2005). Self-regulation in the classroom: A perspective on assessment and intervention. *Applied Psychology*, *54*(2), 199-231.

Brown, A. L. (1977). Knowing when, where, and how to remember: A problem of metacognition. National Institute of Education (DHEW), Washington, D.C.

Brown, A. L. (1981). Metacognition: The development of selective attention strategies for learning from texts. *Directions in reading: Research and instruction*, 21-43.

Brown, A. L. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F. E. Weinert, & R. H. Kluwe (Eds.), *Metacognition, motivation, and understanding*. Hillsdale, NJ: Lawrence Erlbaum.

Brown, A. L., & Campione, J. C. (1996). *Psychological theory and the design of innovative learning environments: On procedures, principles, and systems*. Lawrence Erlbaum Associates, Inc.

Brydon-Miller, M., Greenwood, D., & Maguire, P. (2003). Why action research?. *Action research*, *1*(1), 9-28.

Butler, D. L., & Winne, P. H. (1995). Feedback and self-regulated learning: A theoretical synthesis. *Review of educational research*, *65*(3), 245-281.

Call, P. E. (1991). SQ3R+ what I know sheet= one strong strategy. *Journal of Reading*, 50-52.

Carr, M., & Biddlecomb, B. (1998). Metacognition in mathematics from a constructivist perspective. In D. J. Hacker, J. Dunlosky, & A. C. Graesser (Eds.), *Metacognition in educational theory and practice*, 69-91. Lawrence Erlbaum Associates.

Cavanaugh, J. C., & Perlmutter, M. (1982). Metamemory: A critical examination. *Child development*, 11-28.

de Laplace, P. S. (1951). *A Philosophical Essay on Probabilities*. (F. W. Truscott & F. L. Emory, Trans.). New York: Dover Publications edition. (Original work published 1814).

Davidson, J. E., & Sternberg, R. J. (1998). Smart problem solving: How metacognition helps. In D. J. Hacker, J. Dunlosky, & A. C. Graesser (Eds.), *Metacognition in educational theory and practice*, 69-91. Lawrence Erlbaum Associates.

Department of Basic Education. (2011). *Curriculum and assessment policy statement (CAPS). Mathematics. Grade 4-6.* Retrieved from http://www.education.gov.za/LinkClick.aspx?fileticket=dr7zg3CFCr8%3d&tabid=572&mid=1568

Department of Basic Education. (2014). *Report on the 2013 National Senior Certificate Examination - National Diagnostic Report on Learner Performance*. Retrieved from http://www.education.gov.za/LinkClick.aspx?fileticket=hZL6MoEQvtQ%3D&tabid=358&mid=1325

Doerr, H. M., & Tinto, P. P. (2000). Paradigms for Teacher-Centred, Classroom based Research. In Kelly, A. E., Lesh, R. A. (Eds.) *Handbook of Research in Mathematics and Science Education.*

Dominowski, R. L. (1998). Verbalization and problem solving. In J. H. Douglas, J. Dunlosky, & A. C. Graesser (Eds.), *Metacognition in educational theory and practice*. Lawrence Erlbaum Associates.

Ebbs, C. A. (1996). Qualitative research inquiry: Issues of power and ethics. *Education, 117*(2), 217.

Feldman, A., & Minstrell, J. (2000). Action research as a research methodology for the study of the teaching and learning of science. In Kelly, A.E., Lesh, R.A. (Eds.) *Handbook of Research Design in Mathematics and Science Education*. Lawrence Erlbaum Associates.

Ferrance, E. (2000). *Action research*. LAB, Northeast and Island Regional Education Laboratory at Brown University.

Flanagan, J.C. (1954). The critical incident technique. *Psychological Bulletin,* 51(4): 327–358.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence, 12, 231-235.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist*, 34 (10), 906–911.

Fogarty, R. (1994). *The mindful school: How to teach for metacognitive reflection*. IRI/Skylight Publishing, Inc., 200 East Wood Street, Suite 274, Palatine, IL 60067.

Fox, E. & Riconscente, M. (2008). Metacognition and Self-Regulation in James, Piaget, and Vygotsky. Educational Psychology Review. 20:373–389. DOI 10.1007/s10648-008-9079-2

Gray, J., & Campbell-Evans, G. (2002). Beginning Teachers as Teacher-Researchers. *Australian Journal of Teacher Education, 27*(1). doi:10.14221/ajte.2002v27n1.4

Hacker, D. J. (1998). Definitions and Empirical Foundations. In J. H. Douglas, J. Dunlosky, & A. C. Graesser (Eds.), *Metacognition in Educational Theory and Practice*. Lawrence Erlbaum Associates.

Hallberg, L. (2008). Some reflections on qualitative research. *International Journal of Qualitative Studies on Health & Well-Being, 3*(2), 66-67. doi:10.1080/17482620802140984

Hatch, J. A. (2002). *Doing qualitative research in education settings.* Albany: SUNY Press.

Heller, M. F. (1986). How do you know what you know? Metacognitive modeling in the content areas. *Journal of Reading, 29*(5) 415-422.

Houghton Mifflin Company. (n.d.). Finding probability: Introducing the Concept. Math Background, Grade 6: Statistics and Probability. Retrieved from https://www.eduplace.com/math/mw/background/6/11/te_6_11_stats_ideas2.html

Huber, J. A. (2004). A closer look at SQ3R. *Reading Improvement*, *41*(2), 108-112.

Hume, B. (1966). *An introduction to probability and statistics*. Nedlands, Western Australia: University of Western Australia Press.

Israel, S. E. (2007). *Using Metacognitive Assessments to Create Individualized Reading Instruction*. Newark, USA: International Reading Association.

Jacobowitz, T. (1990). AIM: A metacognitive strategy for constructing the main idea of text. *Journal of Reading*, *33*(8), 620-624.

Jacobs, J. E. & Paris, S. G. (1987). Children's metacognition about reading: Issues in definition, measurement, and instruction. *Educational psychologist*,*22*(3-4), 255-278.

Jacobson, R. (1998). Teachers improving learning using metacognition with self-monitoring learning strategies. *Education journal, 118*(4), 579-563.

Johns, J. L., & McNamara, L. P. (1980). The SQ3R study technique: A forgotten research target. *Journal of Reading, 23*(8), 705-708.

Johnson, D. A. (1963). Probability and chance: Exploring mathematics on your own. London, United Kingdom: John Murray Publishers.

Kahneman, D. (2011). *Thinking, fast and slow*. New York: Farrar, Straus and Giroux.

Kemmis, S. (2010). What is to be done? The place of action research. *Educational action research*, *18*(4), 417-427.

Kluwe, R. H. (1982). Cognitive knowledge and executive control: Metacognition. In D. R. Griffin (Ed), *Animal mind—human mind* (pp. 201-224). Springer Berlin Heidelberg.

Krazydad. (n.d.). Printable puzzles, mazes and more. Retrieved from http://krazydad.com

Lewin, K. (1946). Action research and minority problems. *Journal of social issues, 2*(4), 34-46.

Lincoln, Y. S., & Lynham, S. A. (2011). Criteria for assessing theory in human resource development from an interpretive perspective. *Human Resource Development International, 14*(1), 3-22.

Mathison, S. (1988). Why triangulate?. *Educational Researcher*, *17*(2), 13-17.

Maxwell, J. A. (1996). *Qualitative research design: An interactive approach*. London: Sage Publications.

McNiff, J. (1995). *Action research for professional development: Concise advice for new action researchers.* Bournemouth: Hyde Publications.

McNiff, J. (2005). SAARMSTE Winter Research School – course notes. Langebaan.

McNiff, J., & Whitehead, A. J. (2002). *Action research: Principles and practice* (2nd ed.). London: Routledge.

Meadows, S. (2004). Models of cognition in childhood: Metaphors, achievements and problems. In Daniels, H. & Edwards. A. (Eds.). *The Routledge Falmer Reader in Psychology of Education*. Psychology Press.

Mertler, C. A. (2012). Action research: Improving schools and empowering educators (3rd ed.). Thousand Oaks, CA: Sage Publications.

Mevarech, Z. R., & Kramarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal*, 34(2), 365-395.

Nist, S. L., & Simpson, M. L. (1989). PLAE, a validated study strategy. *Journal of Reading, 33*(3), 182-186.

Nolan, T. E. (1991). Self-questioning and prediction: Combining metacognitive strategies. *Journal of Reading, 35*(2), 132-138.

Ogle, D. M. (1986). KWL: A teaching model that develops active reading of expository text. *The Reading Teacher, 39*(6), 564-570.

Opie, C. (2004). *Doing educational research – a guide to first time researchers*. London: Sage Publications.

Ormell, C. P. (1968). *An introduction to probability and statistics, including teacher's guide*. London: Oliver and Boyd.

Othman, Y. (2010). Application of metacognition strategies and awareness when reading texts. *The International Journal of Learning*, *17*(3), 457-471.

Oxford Dictionaries. (2015). Definition of *cognition* in English. Retrieved from the Oxford Dictionaries: Language Matters website: http://www.oxforddictionaries.com/definition/english/cognition?searchDictCode=all

Palinscar, A. S., & Brown, A. L. (1984). Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and instruction*, *1*(2), 117-175.

Paris, S. G., & Winograd, P. (1990). How metacognition can promote academic learning and instruction. In B. F. Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction*, *1*, 15-51. Hillsdale, NJ: Lawrence Erlbaum.

Piaget, J. (1997). Readings on the Development of children (2nd ed.). Gauvain, M., & Cole, M. (Eds.). New York: W. H. Freeman and Company. (Reprinted from *Piaget rediscovered* (pp. 7-20), by R. E. Ripple & V. N. Rockcastle (Eds.)., 1964, Ithaca, NY: Cornell University Press)

Pine, G. J. (2009). *Teacher action research: Building knowledge democracies*. Thousand Oaks, CA: Sage Publications.

Pressley, M., Van Etten, S., Yokoi, L., Freebern, G., & Van Meter, P. (1998). The metacognition of college studentship: A grounded theory approach. In J. H. Douglas, J. Dunlosky, & A. C. Graesser (Eds.), *Metacognition in Educational Theory and Practice*. Lawrence Erlbaum Associates.

Qazi, H. A. (2011). Evaluating goodness in qualitative researcher. *Bangladesh Journal of Medical Science*, *10*(1), 11-20.

Sagor, R. (2000). Guiding school improvement with action research. Alexandria, VA: Association for Supervision and Curriculum Development.

Schneider, W. (1985). Developmental trends in the metamemory-memory behavior relationship: An integrative review. In D. L. Forrest-Pressley, G. E. MacKinnon, & T. G. Waller (Eds.), *Metacognition, cognition, and human performance*, *1,* 57–109. New York: Academic.

Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. *ZDM Mathematics Education*, *42*(2), 149-161.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In Schoenfeld, A. H. (Ed.), *Cognitive science and mathematics education*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York: MacMillan.

Schraw, G. (1998). Promoting general metacognitive awareness. *Instructional Science, 26*(1-2), 113-125. Netherlands: Kluwer Academic Publishers.

Sitko, B. M. (1998). Knowing how to write: Metacognition and writing instruction. In D. J. Hacker, J. Dunlosky, & A. C. Graesser (Eds.), *Metacognition in educational theory and practice*, 69-91. Lawrence Erlbaum Associates.

Stanovich, K. E., & West, R. F. (2000). Individual differences in reasoning: Implications for the rationality debate?. *Behavioural and Brain Sciences, 23*(5), 645-726. USA: Cambridge University Press.

Steffe, L. P., & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education, 25*(6), 711-733.

Tripp, D. (2003). *Critical incidents in teaching: Developing professional judgement*. London: Routledge.

Van Someren, M. W., Barnard, Y. F., & Sandberg, J. A. C. (1994). *The think aloud method: A practical guide to modelling cognitive processes* (Vol. 2). London: Academic Press.

Von Glasersfeld, E. (1990). Chapter 2: An exposition of constructivism: Why some like it radical. *Journal for Research in Mathematics Education, 4*, 19-29; 195-210.

Von Glasersfeld, E. (1995). Book Review of Paul Cobb (ed.), Learning Mathematics: Constructivist and interactionist theories of mathematical development. *Zentralblatt für Didaktik der Mathematik 27*(4), 120–123.

Von Glasersfeld, E. (1996). Footnotes to" The Many Faces of Constructivism". *Educational researcher, 25*(6), 19. American Educational Research Association.

Von Glasersfeld, E. (1997). Homage to Jean Piaget (1896–1980). *Irish Journal of Psychology, 18*(3), 293-306.

Vygotsky, L. (1978). *Mind in Society*. Cambridge, MA: Harvard University Press.

Wilson, N. S., & Bai, H. (2010). The relationships and impact of teachers' metacognitive knowledge and pedagogical understandings of metacognition. *Metacognition & Learning*, 5(3), 269-288. doi:10.1007/s11409-010-9062-4

Wisner, R. J. (1973). *Elements of probability*. Illinois, USA: Scott, Foresman and Company.

Yeasmin, S. & Rahman, K. F. (2012). 'Triangulation' research method as the tool of social science research. *Bup Journal*, *1*(1), 154-163.

Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview.*Theory into practice*, *41*(2), 64-70.

Zohar, A. (1999). Teachers' metacognitive knowledge and the instruction of higher order thinking. *Teaching and Teacher Education, 15*, 413-429.

Zoghby, C. & van Jaarsveld, P. P. (2013). A metacognitive investigation of my personal mathematics cognition – a comparative analysis. In Kwofie, S. K. et al. (Eds.) *Proceedings of the 21st Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education*. (pp. 66-67) University of the Western Cape, Bellville. SAARMSTE.

Appendices

Demographics of Direct Observer 1

Gender: M / F

Age: 68

Education/ Qualification(s): BA, BEd, DEd, TTD, MEd

Number of years of teaching experience (Grade and Subject): 38 yrs Grades 4-7 Class teaching: Gr 4+5 English & History: Gr 6-7. Principal of 7 schools

What are qualities of a 'good teacher'? Classroom management that reflects a mix of firmness with natural friendliness. The teacher is sensitive to those children who need extra support and assist them. The teacher has a clear speaking voice and has the ability to explain simply.

What is a 'good lesson'? A lesson should be one where the learners are active participants – not passive listeners. There should be a number of different activities to keep the interest level high throughout the lesson. Children are thinking and asking questions beyond the lesson content (example: S2)

What is required to teach a 'good lesson'? Sound preparation! Teacher and learners working together in acquiring new knowledge. Very good revision at the end of a lesson which was done by the teacher to reinforce material taught. Teacher to have a sense of humour & fun – amply displayed in the lesson!

What is your vision about how Mathematics should be taught? Letting learners discover themselves Mathematical principles.

Demographics of Direct Observer 2

Gender: M/F

Age: 29

Education/ Qualification(s): BSc (Genetics & Biochemistry)
PGCE (Natural Sciences & Maths)

Number of years of teaching experience (Grade and Subject): 6
Grade 8 - 12. Maths

What are qualities of a 'good teacher'?
A lifetime learner, passionate about their subject, (good subject content knowledge,)

What is a 'good lesson'? A lesson in which each child takes away new knowledge, and has been engaged in the production of that knowledge.

What is required to teach a 'good lesson'?
You need to be able to be flexible, know your subject content, be able to read your class, have a good rapport with your class.

What is your vision about how Mathematics should be taught?
Maths should be taught in an holistic manner rather than compartmentalised. As a teacher, one needs to be aware of where one stands in the curriculum and where one is heading. (i.e. when teaching fractions, what has happened in fractions before & where do fractions go? Also, which other topics utilize fractions, and how?)

Demographics of Direct Observer 3

Gender: M / F

Age: 50

Education/ Qualification(s): *BSc (Computer Science); PGCE – Mathematics FET*

Number of years of teaching experience (Grade and Subject): *Grade 7 – 6 years; Grade 8 & 9: 2 years; Grade 10: first year; Grade 11 Maths Literacy: 2 years*

What are qualities of a 'good teacher'? *A life-long learner always striving to improve knowledge and teaching strategies. Looking to learn from those around you – open to criticism.*

What is a 'good lesson'*? Is well prepped and results in transfer of knowledge. When your pupils look forward to your Maths lesson! Where discussion leads to a deeper understanding for all.*

What is required to teach a 'good lesson'? *A deep understanding of the subject matter. Imagination to think of the best way to impart the knowledge to your pupils. A sense of humour! Flexibility to allow the lesson to depart from the prepared path based on the classroom experience.*

What is your vision about how Mathematics should be taught? *Maths should be the subject that everyone looks forward to. There should be no angst – pupils should love and understand the flexibility of the subject – that there are many methods available to achieve the desired outcome. It should be relevant and hands on.*

APPENDIX 4

Ethics clearance letter

WITS SCHOOL OF EDUCATION

UNIVERSITY OF THE WITWATERSRAND JOHANNESBURG

27 St Andrews Road, Parktown, Johannesburg, 2193•Private Bag 3, Wits 2050, Johannesburg, South Africa
Telephone: +27 11 717 3007 • Fax: +27 11 717 3009 • Website: www.wits.ac.za

Student number: 0708758W

Protocol Number: 2014ECE033M

28 May 2014

Dear Catherine Sylvester

Application for Ethics Clearance: Masters of Science by Dissertation

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

AN ACTION RESEARCH STUDY ON USING REFLECTION, DISCUSSION FOR A AND METACOGNITION TO IMPROVE MY PERSONAL TEACHING PRACTICE

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely

M Mabeta

Matsie Mabeta
Wits School of Education

011 717 3416

Cc: Supervisor – Dr. Pete Van Jaarsveld

APPENDIX 5

INFORMATION SHEET LEARNERS

Dear Grade 6 Learner

In addition to being your Mathematics teacher, I am also a part-time Masters student in the School of Education at the University of the Witwatersrand.

I am doing an action research study on using reflection, discussion fora and metacognition to improve my personal teaching practice.

My investigation involves three people observing three of our Mathematics lessons so that they can discuss better ways for me to teach you. I would also like to audiotape the three lessons so that I do not forget what happened in the lesson. I can use this information with the people that observe the lesson to see what I teach well and how I can teach you better. The people that will be observing our lessons will actually be looking at me and how I teach and not at you or what your answers are. They are there to give me advice on how I can be a better teacher. You would not need to change anything about how to participate in the lesson and it would be great if you just be yourself.

This information and consent form is to check if you wouldn't mind being observed in three lessons by these observers, which will also be audiotaped. If you would not like to be audiotaped then I will delete that part of the recording when I listen to it. I need your help with carrying out this study. If you wouldn't mind some people watching our lessons and me audiotaping our lessons then you would be choosing to be a participant in my study.

Remember that this is not a test, it is not for marks and it is voluntary, which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. The end results of this study will be written up in a Masters Dissertation which may be published by the University of Witwatersrand.

All collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but your permission is also required for you to be a participant in the study.

I look forward to working with you! Please feel free to contact me if you have any questions.

Thank you,

____

C M Sylvester

School contact details for the teacher-researcher, covered for anonymity.

APPENDIX 6

INFORMATION SHEET PARENTS

Dear Grade 6 Parent

In addition to being your child's Mathematics teacher, I am also a part-time Masters student in the School of Education at the University of the Witwatersrand.

I am doing an action research study on using reflection, discussion fora and metacognition to improve my personal teaching practice.

My research involves the observation (and audiotaping) of mathematics lessons by two educational researchers and one other teacher. The purpose of the observation and audiotaping is to analyse my teaching strategies and techniques and discuss ways to improve them so that I can better teach mathematics concepts. There is no focus on your child's knowledge or responses except to reflect on my teaching ability.

The reason why I have chosen your child's class is because I know the importance of Mathematics as an academic subject and would like to improve my teaching practice. Out of the three Grade 6 classes I will conduct my research on only one of them and this class will be selected randomly from the classes that I receive all the permission slips back from. Even though the observers will only be in one of the three Grade 6 classes, the knowledge and experience gained from data collection throughout the process will be applied to all three classes and the lessons prepared will be taught to all. I have chosen to do this in the school I teach at and with the children I have been teaching, as the classroom relationship is already developed. In addition, I want to improve my teaching instruction and this can be appropriately measured in the school that I am currently teaching at. This information and consent form serves to ask for your permission for your child to participate in my research study. This would require the observation and audiotaping of three mathematics lessons.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

The end results of this study will be written up in a Masters Dissertation which may be published by the University of Witwatersrand. All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.
Thank you very much for your help.

Yours sincerely,

____

C M Sylvester

School contact details for the teacher-researcher, covered for anonymity.

APPENDIX 7

Learner Consent Form

Please fill in the reply slip below if you agree to participate in my Masters Dissertation called:
An action research study on using reflection, discussion fora and metacognition to improve my personal teaching practice[5]

My name is: ____ and I give:

Circle one

Permission to be observed

I agree to be observed in class. YES/NO

Permission to be audiotaped

I agree to be audiotaped during the observation lesson. YES/NO

I know that the audiotapes will be used for this project only. YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of this Masters Dissertation.

Signature: _________________________________ Date: _______________________________

[5] Working title for the dissertation at the early stages of the research

APPENDIX 8

Parent's Consent Form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in my Masters Dissertation called:
An action research study on using reflection, discussion fora and metacognition to improve my personal teaching practice[6]

I, ___________________________________ the parent of ___________________________________ give:

	Circle one
Permission for my child to be observed	
I agree that my child may be observed in class.	YES/NO
Permission to be audiotaped	
I agree that my child may be audiotaped during class lessons.	YES/NO
I know that the audiotapes will be used for this project only.	YES/NO

Informed Consent
I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- my child does not have to answer every question and can withdraw from the study at any time.
- my child can ask not to be audiotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of this Masters Dissertation.

Signature: ___________________________________ Date: ________________________________

[6] Working title for the dissertation at the early stages of the research

APPENDIX 9

INFORMATION SHEET OBSERVERS

Dear

My name is Catherine Sylvester and I am a Masters student in the School of Education at the University of the Witwatersrand.

I am doing an action research study on using reflection, discussion fora and metacognition to improve my personal teaching practice.

My research involves the observation (and audiotaping) of three mathematics lessons by two educational researchers and one other teacher and I would like you to be one of these observers. The purpose of these research instruments is to analyse my teaching strategies and techniques and discuss ways to improve them so that I can better teach mathematics concepts. After each of the three mathematics lessons, I would like to conduct discussion fora which will look at how I taught the mathematics lesson (just observed) and what I could have done better. This will also help me to prepare thoroughly for how to approach and teach the following lesson.

I have chosen to do this in the school I teach at and with the children I have been teaching, as the classroom relationship is already developed. In addition, I want to improve my teaching instruction using an action research methodology and this can be appropriately measured and carried out in the school that I am currently teaching at. This information and consent forms serve to ask if you would be interested in being a participant for this study by observing three mathematics lessons and being part in three discussion fora. If you choose to accept this invitation, it would aid me in reflecting on a taught lesson and also preparing well for the following lesson.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study. I understand that there are transport costs involved in getting to the school for the study and I will reimburse you for this.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

The end results of this study will be written up in a Masters Dissertation which may be published by the University of Witwatersrand. All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. Thank you very much for your help.

Yours sincerely,

____

C M Sylvester

School contact details for the teacher-researcher, covered for anonymity.

APPENDIX 10

Observer's Consent Form

Please fill in and return the reply slip below indicating your willingness to be a participant in my Masters Dissertation called:
An action research study on using reflection, discussion fora and metacognition to improve my personal teaching practice[7]

I, ____, give my consent for the following:

	Circle one
Permission to be an observer of the researcher	
I agree to observe lessons for the abovementioned study.	YES/NO
Permission for my observation notes to be collected and analysed	
I agree that my observation notes can be used for this study only.	YES/NO
Permission to be part of a discussion forum	
I agree to be part of a discussion forum based on the lessons observed.	YES/NO
I know that I can stop taking part in the discussion forum at any time and don't have to answer all the questions asked.	YES/NO
Permission to be audiotaped	
I agree to be audiotaped during the abovementioned discussion fora.	YES/NO
I know that the audiotapes will be used for this project only.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of this Masters Dissertation.
- I will be compensated for my transport costs to get to the research venue.

Signature: ________________________________ Date: ______________________________

[7] Working title for the dissertation at the early stages of the research

<u>Teacher-researcher's explanations</u>	
Lesson 1 1. I reminded the students how to conduct experiments fairly before they started the activities.	*T-R: Ok. And then, if you have all your numbers ordered like this and let's say you said to someone 'ok, I'll give you a hundred rand if you can pick a three the first time' and you're able to somehow pick a three. And you pick your three up…find my three. And you're like, 'ha! Got my three, you owe me a hundred bucks'. Ok now you put it back in, and then, before you put it down the person says to you 'ok if you can pick a three again, I'll give you a thousand rand' k? But instead of putting your three back on the pile, you put it to the side of the pile and close your eyes. Now they go 'uh-uh that's not fair', you said 'but you didn't tell me where to put it, you just said I must pick it again. And so they close their eyes and they feel, and they feel and they…they know they put the three separately on the right. And they pick it up and say, aah there's a three, where's my thousand rand. 'K?* *Ss: some giggles* *T-R: Is that fair?* *Ss: No* *S: 'cause you like…cheating*
2. <u>I incorporated new terms and concepts (experiment, event, outcome, result) smoothly into the discussion so that the students could understand what I was saying even though I included the terms they hadn't heard before, particularly in this context. This was well received by the students who naturally started using the terms themselves in describing what was happening in the lesson.</u>	*T-R: And so, in order to conduct a fair experiment, when the person's mixing it up for you, you need to make sure you're mixing it around really well. You need to make sure they're all sort of in the same area, you haven't put it to the side. So if someone picks up this eight and then puts it back down, it's not in the corner, you mix it back into the group. And you need to make sure that they're not like stacked on top of each other, otherwise it's unfair for the ones on the bottom 'cause they won't get picked out first.* *T-R: K, who got a one? Two? Who got the result of a three? Three, noone got a three? Who got the result of a four? Five? Six? Seven? Eight? Nine? Noone got a nine? And a ten? A ten, good. Ok, cool.* (hands were put up to answer the teacher-researcher's questions) *T-R: Three had the lowest number of outcomes? In your event, how many had the highest number of outcomes?* *S: Seven.* *T-R: And the lowest number?* *S: Ten…four, five and ten.* *T-R: In your event what had the highest outcome?* *S: Eight.*
3. I clarified a topic discussed previously, on recurring numbers and the necessity to round off.	*T-R: Ok, let's pause there. Pens down. Experiments pause. I'll give you a few moments after to finish your thirty experiments. So you should have your tally charts in front of you. You should be able to look carefully and see how many on each… uh columns. 'k…so... Jason's group…what was the highest number of out…what had the highest number of outcomes?* *T-R: What had the highest number of outcomes?* *S: uh, We had a tie. Three and eight.* *T-R: Three and eight? Ok. In your event…* *S: Two.* *T-R: What was your lowest outcome…Lowest number of outcomes?* *Ss: Three, three.* *T-R: Three had the lowest number of outcomes? In your event, how many had the highest number of outcomes?* *S: Seven.*
4. <u>Once all the data had been recorded in the table on Microsoft Excel, I proceeded to average the results and point out theoretically how many times each number should have been picked up. I had not planned to discuss this and it moved us as a class too quickly towards the relationship between experimental and theoretical probability.</u>	*S: Uh Mrs T-R, I was just wondering, when it said three comma three three three, is that three comma three recurring?* *T-R: Yes. Good! Remember 'cause they'll round it off 'cause otherwise how… where does it go? To the end of the page?* *S: But Mrs T-R, isn't three um… three comma three recurring three comma three, three, three, three… carry on like forever?* *T-R: Yes, but can they write three comma three, three, three, three forever on the computer screen?* *S: Then they wouldn't know when to stop.* *T-R: They could if I dragged the box and made the box bigger, but then it would have to stop somewhere… so they've rounded off… automatically for me… very kindly.* *T-R: K, let's check out our probability – what we should have got. Three ones, four twos, three threes, three, three, three, four, four, two and two. So apart from three, one less, two, or one more, four, we don't actually have… um the average is not more than that for any of the numbers… or less than that. So on average, we either had two, three or four of each number. And so the more we add… so if I take 6C, 6J and this class and put all our events together, theoretically it will tend even more towards… three for every single number… picking it out.* *Ss: Woah; yoh; sjoe*
Lesson 2 5. At the start of the session with the front group, I tried to remind them of the concepts and definitions that were covered in the previous	*T-R: Ok, so an outcome is the result of an experiment.* *Ss: an experiment.* *T-R: Alright, S5 do you remember what an event is?* *S5: Um… an event is when… what… what number you… what… um… I don't know how to explain it.*

lesson, however it did not go smoothly.	*T-R: Ok, so let me remind you… when I asked you about event yesterday I said… who was in your group?* *S5: S2 and S7.* *T-R: You and… ok so I said… S2, S7 and S5, in your event… which outcome um… occurred the highest… number of times?* *S5: Um…* *T-R: So if I said in your event…what was I referring to?* *S5: What the highest one… what the highest number was…* *T-R: Ja I asked for the highest outcome… exactly! But I could have asked for the lowest outcome, in your event. So what do you think event refers to?* *S5: The whole thing.* *T-R: The whole thing? And what was the whole thing made up of? A whole lot of?* *S5: Numbers.* *T-R: What you… what did you do?* *S5: We chose numbers.* *T-R: Mmmhmmm. And every single time you did it? What was it called?* *S5: Um..uh…* *T-R: I said, ok, do another?* *S5: Event? No… um…* *T-R: Close… uh… S8 help her out?* *S8: An experiment.* *T-R: An experiment? Remember I said? 'k, now you're gonna do your next experiment, and your next experiment… so if event talks about all of those, what could we say? An event is?* *S5: Uhm.. a num.. another… another…Another try.* *T-R: Another try, or another experiment… and what if I'm talking about all the ones that you did.* *S5:* Inaudible *response* *T-R: The total, the total number of experiments? How else could we refer to it? Good!* *S2: A group of outcomes?* *T-R: A group of…?* *S2: Outcomes?* *T-R: Okay, but remember we're talking about all of them in general, no matter what the outcomes are. So we're actually talking about the… experiment that we did.*
6. I tried to give clarification to the front group on the difference between experimental and theoretical probability (as there was some confusion the day before) but again this was not approached well.	*T-R: So experimental probability… is finding… or not finding… is the probability based on the experiments… based on experiments that you've done. 'k? And then we have theoretical probability… and this is? Probability… based on… and I'm going to put here theory… um, and we're going to learn about what that theory of probability is.* *T-R: Do you remember how experimental probability and theoretical probat… probability were linked? What did we say about the two of them? 'k, so they both talked about the likelihood of something happening. The one's based on experiments that you've already done. The other one you don't have to do any experiments… but we talk about it… the whole world has the same way of measuring theoretical probability.* *S1: Based on experimental; numbers one to ten and you take the number one to ten, well one to nine and well there's still a… one… a…* inaudible. *Exactly! Good. Ok, so… what did we say about the more experiment… the more experiments we do? The more experiments we do, we have a certain experimental probability that we can see. And the more and more and more and more and more and more and more experiments that we do, what happens to our results? To our outcomes? What does it look more like?* *S4: Our results changed because we have a… a chance… you know… a chance to pick up the same number so it changed. So one time, we pick up a two, the next one we might pick up a two because they have an equal chance of being picked.*
7. <u>Often I answered questions without giving students a chance to think or attempt to answer. I noticed this while I was transcribing and analysing lessons. One of these situations occurred when I asked the students why each outcome should tend towards being picked three times for theoretical probability and did not give them a chance to even think about it before answering the question myself.</u>	*T-R: What… what started happening? In our average… look at your average line. What started happening?* *S: They started… uh…* *T-R: They all started getting closer to three. We're going to talk about why three was the key number now, but can you see how they all started getting closer to three?* *Ss: yes; yes; oh*
8. I attempted to clarify experimental probability and demonstrate how varied the outcomes can be, referring to different students' results.	*T-R: And so, the experiment that you did, and the experiment that S11 did and the experiment that S2, S5, S1… the experiments that each of you did, got all these numbers all over the place. That's experimental probability. They were all over the place! And you could have picked anything, and sometimes you picked the same number seven times in a row. But then… the more of those experiments we do and put together, and we put down thousands and thousands of experiments together, and we look at what the probability is… it gets closer to becoming what theoretical probability is. Does that make sense?* *Ss: Mmmm*

	T-R: The more experiments we do, the be… the more the probability tends towards becoming theoretical probability. And so if we were to almost imagine a situation where we have an infinite number of experiments going on all the time, all the time, they just keep on putting all this information together, you'd literally just have… wh… theoretical probability.
9. I asked the students for alternatives to giving a mathematical answer (theoretical) to a probability question and received very good input from the whole group	T-R: K, so what I want to do now, is I want to… use language… English language. We're not talking maths, we're not talking numbers… So like, very likely, unlikely… probable… not so… not so likely, not gonna happen really… what would you say? Probability of picking a one on the next try? S: Unlikely. T-R: Why do you say it's unlikely? S: 'cause there isn't…you haven't picked out one. T-R: It's not going to happen. 'k. Good – and if we say we can either have an answer of zero… which means… not at all, or we can have an answer of one which means it's definitely possible or we can have an answer somewhere in between. The closer it is to zero… is it going to be more likely or less likely to happen? Ss: Less; less; Less likely; unlikely T-R: Less likely. Ok. And S3, the closer it is to one… is it going to be more likely or less likely? S3: More likely. T-R: More likely, because one is definite so it's going to be more likely as it gets closer to one.
10. I explained alternatives to asking for the probability of something happening and the front group gave different alternatives.	T-R: K? What can we talk about… if I say what is the probability of this happening… what is the probability of that happening… what would you say, in there's another way to say that? What's another way to say probability? What is the… what is the probability of picking a red… ball? What is the probability of picking a yellow ball? What is the probability of picking a blue ball? How else can you say… ask that question… how else could I have asked you? What is the…? Ss: Likely…uh; likelihood. T-R: Likelihood. What is the…? Ss: Chance?; chance of something happening… T-R: What is the…? Ss: Probability, chance T-R: What is the chance of that happening? What is the likelihood of that happening? So, different words for probability. So is it fair to say then probability is… Ss: The chance of something happening; likelihood; chance S2: The chance of something happening.
11. I explained the new term 'sample space' with practical examples of coloured balls in a tin to develop new knowledge of how to calculate theoretical probability but the language use was not as effective as the previous day and the definitions seemed more like they were given by me rather than letting the students develop them together as I had done in the previous lesson.	T-R: Ok? So we're going to talk about converting it between all three of them. Alright so now let's look and say… So. When we're calculating probability and we're trying to work it out… something that helps us very, very much is called a sample space. T-R: We'll talk about numbers now… don't… let's not throw out random numbers. You… are on the right track but let's look at this. So if I talk about a sp… sample space… you gonna write this down as well as a definition… you can say a sample space is a device… or a tool… a tool, let's call it a tool. A tool that helps us find probability. 'k? find probability… T-R: Ok, they're all different colours. And my colours are orange, red, blue, yellow, green, purple, light pink and dark pink. Ok, so those are the colours I have in here. Now what a sample space is, is it says… in this space, in this area… what is the sample? What exactly is in this box? In total? Ss: Eight; all the… S: Seven. T-R: And you're going to write down every single one that's in here, in your sample space. So we will look here… so you're going to take it and pass it and each of you are gonna tell me one thing to write down… there a problem?
Lesson 3 12. I gave the class details at the beginning of the lesson on the four different activities that they were going to carry out experiments with. 13. I realised that I should clarify the difference between heads and tails of a coin which I did not expect to do and only thought of it doing it while I was explaining the coin experiment.	T-R: I've set up exper.. ah event sheet for you with the different experiments down the side and um what I've done, is for the coin test you going to flip the coin three times and that's going to be one experiment. So one person in a group, so you going to split into about two groups inside your group and you can each do your own event and, so if you look over here I've put there H for heads T for tails so depending on what you get, experim..I've given you experiment example it says experiment example outcome 1H which means the first times you flipped it you got a heads, second time you flipped it another H, heads, and the third time you flipped it says T for tails; so there is an example at the top. And then at the very end it says final outcome and so what I want you to tell me is, if you flip it three times... T-R: And so the first one says HHT, heads, heads, tails. That's experiment one then you'll do it again three times and that'll be experiment two which is three outcomes. Ok. Um, you're gonna do thirty experiments. So if this group starts, then ah, these two can do one event on their own and these three can do one event on their own. Um, and you can each put in your own table. Ok? So just to confirm, S5, S6! 'k, sorry I know it's difficult for you to look at me but just try.. until you can to see me. Who ah doesn't know or who does know rather what heads or tails of a coin are? S10? S: Um, heads is the coat of arms and tails is the picture of the... [Inaudible] S: It's the wrong way.. S: The other side... S: Other way around.. T-R: Other way around. So picture in the front. And, what else is in the front that's quite important for money? S: Ah…percent...inaudible; how much it's worth; inaudible... T-R: How much it's worth. 'k so we have the amount, how much its worth and the um picture whether its of well on the coins its ah Protea and other things and on the back our coat o arms ok. and so when you flip it make sure its on

14. I gave the class a reminder about using semi-colons to separate digits so that they did not look like the same number (I had forgotten to do this in the example given on the experiments recording sheets and only realised while I was explaining the activities that it should be pointed out that I should have used the semi-colons and they must not forget to do so).	*T-R: …flipping a coin …Same as flipping a coin. So you'll also do three outcomes. So you'll roll and see what number write it down. Roll again. See what number write it down, roll again, write it down, and then you'll write the three numbers next to each other, and final outcome, for the three different numbers you got. And make sure you leave a space because if you got 4, 1, 6, I don't want you writing you got 416. You got a 4, a 1 and a 6. Actually semicolons in between the numbers to list them. Ok, and the very last one… is… oh my [tins???] is back. It's going to be [tins??] with different colours and that's going to be thirty experiments where you pick one out, see what the colour is and record what the colour is. Ok. 30, cool. So the ones I've given you are going to start with and I'm just going to bring 'round your recording sheets so one person needs to ah two people in each group need to have a pen or pencil to record with.*
15. We had a discussion around repeated sequences from rolling the die three times. As a class we confirmed what some of the different possible outcomes could be and that there can be multiple repeats of the same outcome.	*T-R: How many final outcomes do you think you'll guess? How many different ones do you think there are?* *S: I'm two hundred and fifty seven years old!* *T-R: So you've got um… rolling your dice three times. So you've got 4, 4, 1… 5… 1, 5, 4… 6, 3, 4. They're all different so far. How many of the same ones do you think you'll get?* *S: A lot.* *T-R: A lot?* *S: Or, it goes only up to six* *T-R: Im, ja…* *S: [*Inaudible*] It's so far all the same… [*Inaudible*] What's the different between… [*Inaudible*] 1, 1, 1… 3…* *T-R: 1, 1, 6, or 1, 1, 1?* *S: 1, 1, 1…. 1, 1, 1.* *T-R: 1, 1, 1 and 1, 1, 6?* *S: So that's the first… [*Inaudible*]* *T-R: [*Inaudible*] But you don't have nines in between, or eights, or sevens?* *S: 3… [*Inaudible*] aaah… [*Inaudible*] 3…* *T-R: Ja, so you gotta take all of those out. Ok cool interesting. Do you think if you roll the dice three times every time for enough experiments eventually you'll get every single kind of outcome you could possibly get.* *S: Ja…* *T-R: If you just keep doing it forever and ever… ok, cool, well keep going.*
16. I gave a summary at the end of the lesson on the total number of different possible outcomes for picking a card out of a pack, picking a coloured ball out of a tin, throwing a die once (and then discussing the multiple outcomes when thrown more than once), tossing a coin once (and then discussing the multiple outcomes when tossing a coin twice or even three times).	*Um, so just to quickly summarise going through each one. You had a pack of cards, ok. There wer… how many, uh… different possibilities in a pack?* *Ss: fifty-four; fifty-two* *T-R: fifty-four?* *S2: fifty-two plus the two jokers* *S: plus the two jokers…* *T-R: ok, plus the two jokers, so you need to know when you're taking cards out of a pack if it includes the jokers, if it doesn't include the joke… the jokers, if it's only one pack, ok? If there fifty-four possibilities in total, then no matter how many times you pick a card out a pack, what is the maximum number of different possibilities, of different outcomes, that you can get?* *Ss: fifty-four?* *S1: fifty-four (more sure)* *Unsure answers of fifty-four from some students* *T-R: hands up, hands up… S20?* *S20: um…* *T-R: so how many different cards did we say are in this pack?* *S20: fifty-four* *T-R: so if I just keep on drawing, and eventually I get to draw at least one of every kind of that card, how many different possibilities are there? For me to draw?* *S20: uh…uh… fifty-four.* *T-R: fifty-four. Exactly. Good. Ok, so we know that there are only fifty-four possible different, different possible outcomes. Then in terms of the colours… so you were picking colours outside… out the tin, 'k? There were lots of different balls scrunched up, 'k? But there were only a certain number of colours, there was red, light pink, dark pink, blue, purple, orange, green and yellow. 'k? So if I said to you, if you kept on drawing one, picking one out from the tin, over and over and over and over again. And eventually you were able to draw out one of every single… ball that was in here. S21, what is the total possible number of outcomes could you get given… that you could get that's different to the other ones?* *S21:* inaudible *T-R: how many?* *S21: six.* *T-R: six? Why six? How many different colours were in here?* *S17: eight.* *S2: light pink, dark pink…* *S10: eight.* *S2: …blue, green, orange, red, yellow* *Ss: seven.* *Ss: eight.* *T-R: and purple*

	Ss: oh eight. T-R: so there were eight possible different colours. So when you wrote your answers down the outcomes si.. side, how many different possible colours could you have written down? Ss: eight. T-R: only eight? Because there were only eight colours. So there were only those eight colours to write down as a possible outcome. Ok? So the cards there were only fifty-four different kinds of cards, because you drew one, there was only one possibility so we had fifty-four different possible outcomes. So the colours, there were only eight different colours, so there was… there were only eight different possible outcomes that you could have written down. Ok?
17. I clarified that the two different coloured jokers were two different possible outcomes (after one of the students asked).	S1: For the cards, isn't it only fifty-three? Because there's two jokers but they're the same. T-R: but someone specified something to me, I think it was you or S4. S1: oh sorry, the colours. T-R: ok, so you're asking me that question but you've already answered S1: oh sorry. T-R: no it's fine, it's good. Ok, so S1 originally said, um… do we have fifty-three outcomes for cards or fifty-four? And I said well why… how is there a difference? And S1 said well, because there fifty-two normal cards and there two jokers, but they're both jokers, so it only counts as one outcome? And if they were the same colour then it would only count as one possible outcome. S2: oh like they're both jokers… T-R: they're both black jokers… but then S1 said ah, but one's red and one's black, must I specify? And because he did, there were fifty-four possible outcomes. Good. 'k, then the last two… one was the dice, 'k? The die…throwing the die. And the second one was flipping a coin. If I'd only asked you to roll the dice, the die once, how many possible outcomes are there? S4: six. T-R: sorry? S7: six possible outcomes. T-R: six. Why are there only six possible outcomes? S7: because there are only six sides on the cube, six different numbers that you can roll… T-R: six different numbers? Good. I'm glad you said six different numbers because if S7 had just said six different sides, what if I had a two on every single side? There's still only one outcome. A two every time. But because there's six different numbers, there's six different possible outcomes. Ok? And the same for the coin, if I'd only asked you to flip it once, how many outcomes? Ss: two; two… T-R: why? S2: well that's if it's a proper coin and it's got heads… T-R: k, two? S2: that's if it's a proper coin. T-R: if it's a proper coin, and it's got heads and tails. Ok? But I didn't ask you to throw or flip once, I asked you to throw or flip three time each. How did that change the number of possible outcomes? S16: would there be thirty-six possible outcomes then? T-R: I'm not going to tell you how many there are. But does everyone agree that there's a lot more than just six and two possible outcomes because you flipped… Ss: Ja… T-R: did you see how many different outcomes you got in your column of total final outcome, where you had the three numbers next to each other and the heads or tails, the three next to each other. Did you see how many different ones you got? Ss: Ja… S1: we only got one the same. T-R: you only got one the same? So very few people got the same one again, 'k? But even if you got the same one over and over, there's a certain amount of possible outcomes, and so we'll get to that later about how to do it.
<u>Exploration/development of the students (group or individual)</u>	
Lesson 1 18. There was very good development of concepts, casual chatting, in a friendly way the students arguing mathematically with each other, most students trying to contribute, clear enjoyment of carrying out the experiments.	T-R: We have done tally-charts before and you know how to tally, and you know how to draw a tally table. I'll put an example up of how to chart. But for now you're going to open your envelopes, make sure your desk's a bit flat..so..uh..rather prop something under it to make it flat so that the numbers don't fall off. Ok, and check that you all have numbers 1 to 10, one of each. RS: giggling, chatting, checking their numbers T-R: So what's going to happen is you're each going to have a turn. Um, now one of you is going to draw a number. So it's going to be upside down and one person's going to pick a number without looking. The person who isn't picking the number up is going to mix the numbers so that you, uh, so that they're nicely… evenly spaced and it's not numbers on top of each other so that it's fair for the person to pick. RS: Quiet chatting and sorting
19. The students developed a definition for an outcome using everyone's input, well.	S: We got a nine. S: No we got a six. S: That's a nine. T-R: Line at the bottom. It's a nine? So ok. Ss: Giggling at the 6 compared to a 9 T-R: If I had to ask you, what you thought… an outcome was. What would you say? S1? S1: Well, if you like your end result? What you get… T-R: Your end result?

	RS: Yes. *T-R: Good. End result in what?* *S2: I'd say the number you pick out in this…as in the number you pick out 'cause that would be kinda like your answer to the equation.* *T-R: Ok, the number you picked out.* *S: Your final number.* *T-R: Ok, your final number… um… I like that S1 said it's your result because that was also one of the words I used, remember? Remember I said 'What was your result? What was your result?', ok? What was it a result of? What had you been doing?* *S16: It was a result of the experiment?* *T-R: Awesome. 'k? So, does it make sense to say 'an outcome is the result of your experiment?'* *Ss: Yes.*
20. There was good exploration of new concepts and understanding was developed symbolically through the development of language.	*S2: Could it be many like different experiments?* *T-R: Say again?* *S2: Could it be like different ones, so like picking out numbers like here or picking a like a teddy-bear?* *T-R: Exactly. It could be, or picking socks out a drawer or choosing a… or randomly picking coins out of a bag. Ok? So as long as you're talking about an outcome, you're talking about something, a result of an experiment that you've carried out.*
21. There was a moment when student 1 was alert and looking closely and carefully at the progress of the lesson. This student noticed that the numbers on the board didn't quite add up and this was due to the automatic rounding off of the programme. It was a good reminder for me as a teacher to be careful not to make assumptions but to look ahead and try have a straightforward explanation for things that may be more difficult to explain.	*S1: There's one missing… 'cause you've got a…it's one… because you've got an extra four…for what's there… it's twenty-nine.* *T-R: Which row?* *S1: The bottom row.* *T-R: Ok.* *S1: For our probability because…* *T-R: No they're all thirty.* *S: The computer adds it up.* *S1: But…* *T-R: Oh! I know what you mean.* *S1: It's twenty-four.* *T-R: Ok, it's…'cause it's averaged… 'cause it's rounded, 'cause it's rounded up because I squished it from three comma three three three something…* *S1: Oh, ok…* *T-R: Ja, so it might have cut off one or two. Ok, cool*
22. I had introduced zero probability very subtly during the lesson and gave them the chance to show understanding discussing what possibilities. The students really enjoyed knowing that any number they shouted out for no probability was correct as long as it is not between one and ten.	*T-R: K. um, one other thing… if I said to you… looking at your numbers in front of you… zero, uh one to ten… and I said to you, which number has zero probability of being pulled out that bag… or being chosen.* *Ss: I dunno; no; mmm* *S: Eleven.* *S: We don't have…* *T-R: Eleven! What else?* *Ss: Zero; a hundred; twelve* *T-R: Zero! A hundred! Twelve! Why do they have probability of being chosen?* *S: Because they…; they aren't…* *T-R: One at a time. Hands.* *S2: Because in our… in the bag… we've only… well in like on the desk we've only got numbers from one to ten.*
23. Student 2 attempted to give a fraction as the answer to a probability question but I wanted to avoid using fractions until the concepts for probability were well developed for the whole class.	*T-R: And if I said to you what was the probability of pulling a seven out?* *Ss: Uhm; likely; three* *S2: One out of ten* *T-R: We can measure it and we'll talk about how to measure it. It is one out of ten. Alright. Um…k, so we're going to wrap up.*
24. There was fantastic development of thought through symbolic representations when student 10 had a moment of insight and understood why for the experiments that we were carrying out, theoretically each number should be pulled out three times (a total of ten outcomes, each outcome has the same probability of being picked and thirty experiments were carried out) by stating that if there	*S2: But how… how does that happen?* *T-R: Because when you start with your bag with your numbers in front of you, you can either pick a one… but you got the same chance of picking a one as you do picking out a two.* *S2: But what happens if you pick out like… out of the thirty… you pick out like twenty-five eights and…* *T-R: Exactly! And so that's what the experimental showed us. But you can pick out something that's not exactly theoretical. But the more times we do it, the more chance that it's going to be… a certain amount… for each number. And if you have a fair number… one to ten… it'll be the equal number… an equal chance for each number to be pulled out. In the same way as at the beginning when I said what if you had three hun… what if you have a hundred threes? You were like woah! Oh! Ok! Most… you know… you very likely to pull out a three rather than anything else. So the more there are of something, the higher the likelihood goes of pulling it out the bag. S10?* *S10: Um, you know you said you were getting closer to three?* *T-R: Yes.* *S10: Like is… because three is mentioned the most that we're getting closer to three? Or…*

were forty experiments carried out then each outcome theoretically should be picked out four times.	*T-R: Ok. So… when I show you in our next lesson how we're going to actually do the fractions… to actually work out what the answer is… what the measurable… what the way is to measure what the probability is… um… if you were to do the theoretical probability of what you drew out, you should have got three ones, three twos, three threes, three fours, three fives…* *S10: Oh…ok…* *T-R: And so now we look at our results and that's how close they were… each of them to three.* *S: That's really close!* *T-R: Why is it a chance? Because out of ten possibilities in your bag, how many… how many ones were there?* *S10: One.* *T-R: One. But because there were ten possibilities, and you did thirty experiments, there were three groups of… in your experiments? So you looked at the bag thirty times?* *S10: Oh ok.* *T-R: So you should have got three ones, three twos, three threes, three fours… so they should have all tailored towards…* *S10: But if it was forty, then it would be four. Cool, ok!*
25. In discussing how to find the theoretical probability, student 7 showed some understanding in how theoretical probability is more accurate for number and student 2 had an outburst when they made sense of a term, why theory is the base of theoretical probability.	*T-R: Ok, cool. And can you… I'm glad you said that… can you see that I've got zero ones? We've already discussed this. And I've got nine twos. 'k? A huge difference! And some of you got numbers in between but also got really high and low outcomes. But I want to show you something very interesting. Instead of just counting my event, I'm going to put all of our classes' events together…* *S19: Oh and then that's the average…* *T-R: And we're going to look what happens to that.* *S2; S10: … that's the probability; it's going to be the average;* *T-R: Well, we're going to have a look. And instead of calling it an average, what I'd like to think of it as… is our theoretical probability.* *S2: Theoretical!* *T-R: And it won't be exactly the theoretical probability, but the more events you do and the more events you put together…* *S7: The more accurate…* *T-R: The closer it gets…the more accurate it gets…* *S2: Oh it's like a theory!* *T-R: Exactly.*
26. <u>A student used a term that the teacher-researcher hadn't – 'even chance' – which the teacher-researcher then said wasn't quite an appropriate term to use to describe likelihood. One of the observers mentioned that it was a perfectly acceptable term to use in probability and in researching this further the teacher-researcher came to find that it is a term used regularly to describe a fifty/fifty chance in probability.</u>	*S1: And in the middle is it an even chance?* *T-R: Good! If it's exactly in the middle of zero and one it's an… it's an even chance, it's a… I don't know… I don't like the word even, how else can you say that?* *S2: But how do you get halfway?* *Ss: The same?; same; same?* *T-R: Ja, the same probability both ways, maybe… the same probability of either getting it or not getting it… and what is halfway between zero and one?* *Ss: zero comma five* *Ss: Point five.* *T-R: Point five. And how else can we write point five?* *S: Oh… A half?* *T-R: Half. Good. So you can say it's half a chance of getting this or half a chance of the other one.*
Lesson 2 27. There was a good group development of the term 'probability'.	*T-R: So then what is probability? In general, probability?* *S: A problem.* *T-R: S18.* *S18: Your chances of pulling out something?* *T-R: Your chances of pulling out one of the numbers? Describe it more generally… probability of anything in the world?* *S18: Your chances.* *T-R: The chances? Ok. How else can we say it? Instead of just saying the chances? How could we say?* *S13: Like the probability of an event?* *T-R: The probability of an event. What else? How do you describe what probability is though?* *Ss: Um…* *S: Probability….* *S4: Um… how often it's gonna happen?* *T-R: Ok, does probability tell you how often it happens? Or does probability tell you how often it could…* *S4: Could happen…* *T-R: Ok, the chances?* *S7: Could it be a hypothesis or a prediction?* *T-R: It could sort of be called a hypothesis or a prediction, but remember when you talk about a prediction, you saying once I've done the experiment I'm going to prove my prediction, but in this case…well I suppose, when you're tending towards your event… ja actually. Ok, so if I write it down, uh… we said… probability. How about likelihood?* *Ss: Oh ja; oh yes; ja*

	T-R: And since we're talking generally we're not going to say the likelihood of picking out a number between one and ten… 'cause that's what it might be for this one… *S: the chance…* *T-R: the likelihood of?* *Ss: Chances; the chances* *S1: Of one?* *S: Of something happening…* *S2: Oh ja 'cause you… uh…* *T-R: General. The likelihood of something happening. You got a drawer, you stick your hand in and you pull out a sock… the likelihood of it being green.* *Ss: Giggles*
28. There was very good development of knowledge about what theoretical probability means.	*T-R: What about theoretical? Oh that's a good point 'cause that's also about… what we said about theoretical and… and experimental probability.* *S: theory…* *S2: Oh you mean like, if like… if we do nothing experimentally…*inaudible *T-R: Say again?* *S2: Like, isn't the theo… theoretical one like… like our group got seven of those, then the theory of it is like we would get seven of all of them up to ten.* *T-R: Yes. Good.*
29. It was not very clearly stated but all of a sudden, student 2 realised what theoretical probability meant for a specific example, making the link between the total number of experiments and how many of each outcome should result.	*S2: I said no 'cause I was like I said oh 'cause then they'd all be three, but…* *T-R: Yes!* *S2: …depends what it is.* *T-R: So if the theoretical probability for this scenario was that you should get three outcomes for each one, then the more experiments that you do, pro… a large number of people doing lots of experiments together… you put that… all that information together, then it should become and one day it will be… three, three, three, three across the board.* *S2: Ja, but it depends 'cause like if it's out of.. like this one because we did 30 experiments we should get three, three, three… but if we did like…* *T-R: Forty experiments…* *S2: …forty experiments then you would get four.* *T-R: Exactly and we'll disc… you'll get four… is that what you said?* *S2: Yes.*
30. After a point raised by student 4 about numbers being spread on the desk, I clarified how to carry out the experiment fairly.	*S4: Um, when picking up those numbers, wouldn't it be unfair that some of those numbers were closer and some of the numbers were further away? Some of them were…far…* *S2: I went like that.* *T-R: Ok, good point. So you said when you're picking up the numbers, wasn't it unfair because some of them were in the same place or…or further away or closer to you and you might have been…so remember what we said at the beginning of the experiments.* *S2: You can't put them on top of each other.* *T-R: Do you remember what we said? What did we say at the beginning of the experiments?* *S1: Mix everything up.* *T-R: Why did I say mix everything up?* *S5: Because then…so that you don't get the same number over and over again.* *T-R: You might get the same number o…over and over again and that's ok if you do, but why is it still important to mix it up? To make the experiment something… to make it a…?* *Ss: Um…* *S5: To make it fair?* *T-R: To make it fair. Very good. To make it a fair experiment. 'k? Remember we spoke about that? And so that's why I didn't want to use the envelopes because th…the numbers kept on getting stuck in the corners…* *S2: Oh ja!* *T-R: And so otherwise you would have probably not pulled the numbers out the corners. So to make it a fair experiment, we wanted to mix them all around, to make sure it's* inaudible*…you shouldn't have pu…pushed the ones too far away from the person, you should have mixed them in a circle so that the person who kept on picking the close one, had a chance of getting all of them. The person picking, to make it a fair experiment shouldn't have gone for the same spot all the time, maybe like moved their hand all over. So, ja! It would have been unfair if it was in the same spot and wasn't mixed around. 'k, but…* *S2: But we had our eyes closed, so how would we have like…* *T-R: Yes, but… but.. like Ethan said, is what if you kept on putting a number back there and you kept on picking from that area.* *S2: But how would you know? 'cause like…* *T-R: Ah well, if you feel where the numbers are, and where the, you know, paper is on the table.* *S2: Oh ja, I see.*
31. There was an understanding of how multiple experiments tend towards theoretical probability which I clarified.	*S1: And so the more experiments you do, the more accurate you will be…* inaudible *T-R: Good. Exactly!* S1: inaudible *T-R: Yes. And when you say accurate, that's diff… you know when you say accurate you mean, it's not quite accurate.. the more experiments you do, the closer it gets to having a theoretical answer.* *S1: Ja.*
32. There was group collaboration to get to a wide range words that could be used for no chance/likelihood. I confirmed the vocabulary	*T-R: There are… there's a special way that you measure probability, 'k? And when you use a number to measure it, there are certain answers you can get. And the answer to measure probability… not the answer... the way that we measure probability ranges from zero to one. Say for example, if I took this and I took it in front of all of you and I said, what is the likelihood, what is the chance, what is the probability of pulling out a black paper?* *S: Is there any black?*

as the students built understanding	T-R: Is there any black. Good question! S1: Impossible! S2: It's a chance of zero out of… how ever many papers there are… S1: It's impossible… T-R: I like the answer impossible… 'k? S1: …because it's not like it's ever going to happen. T-R: It's not going to happen. 'k. Good – and if we say we can either have an answer of zero… which means… not at all, or we can have an answer of one which means it's definitely possible or we can have an answer somewhere in between. The closer it is to zero… is it going to be more likely or less likely to happen? Ss: Less; less; Less likely; unlikely T-R: Less likely. Ok. And S3, the closer it is to one… is it going to be more likely or less likely? S3: More likely. T-R: More likely, because one is definite so it's going to be more likely as it gets closer to one.
33. Student 11 had good thought processing that led to them understanding that changing the amounts of certain colours in a sample will affect the theoretical probability. This led to asking me that if there were more of a specific colour in a sample than another one, would there be a higher chance of pulling that colour out than the other colours.	S11: Mrs T-R, if there were more um… colours… uh… if there were more of a different colour than there were of other… let's say there was two black and then the rest were just white. Wouldn't there be a… a… T-R: Higher chance… S11: Higher chance… wouldn't there be… T-R: …of getting the colour that was more in the box? Yes of course! And we'll talk about the exact number of that higher chance now.
Lesson 3 34. There was much excitement shown in all groups at getting repeat outcomes and the students spurred each other on as they were rolling the dice or flipping the coin to 'try' get a repeated outcome.	S: ah that was so close… S1: king of diamonds, come on… king of diamonds S17: You ready? Go. You ready S15? Six… if anyone wants…I need six one, two… six, two, four... uh… one… where's it? six, one, two… two. Come on, two… two… S17: go, six, six… S: you have to get three… S17: six, six… ok wait, wait, wait… S: three… ok, three… S17: another six, another six, another six… S: roll again, roll again S: ok now you have to get a five Ss: six, six, six…. Ah. Ok, go again… S: go… four S17: four… roll a second number… S23: another four, another four S17: four, ok… S: one… S17: wait… five, four, one… Ss: chanting five, four, one… five, four, one… five, four, one… five, four, one… silence while they're looking at the die… Yeeees; yeah; boom S: are we supposed to connect them (the matching outcomes) S17: no I'm just doing it.
35. The students were confident in making suggestions/theorising to answer questions that I asked them about total number of different possible outcomes and whether or not they would get repeated outcomes. These suggestions could have been far off and some may have been complete guesses but they were happy to explore different options and then check later if they were correct.	T-R: I'm saying if you do it infinitely, if you keep on rolling that dice three times… What are those different outcomes that you can have without repeating once? S: ones, two, three, four, five, six… two, one, three, four, five, six… S: two…. T-R: There's a way to figure it out I'm gonna look at, but keep thinking about it S: You can have like a million T-R: a million… S: 'cause you start with all the number, every number and then you can change every number as well. You can have like… T-R: Good. S1: uh… as in like your… your final outcome, how many possibilities. S4: six times six… 'cause you've got… um a number of possibilities… over six. S1: no ways. It's over… it's like a hundred, a thousand, almost S4: and then one has a possibility of getting to… T-R: over a thousand? 'k, S7's gonna do another diagram trying to work out how many… S1: it's over a thousand… can we write this? T-R: How many final outcomes do you think you'll guess? How many different ones do you think there are? So you've got um… rolling your dice three times. So you've got four, four, one… five… one, five, four… six, three, four. They're all different so far. How many of the same ones do you think you'll get?

	S: A lot. *T-R: A lot?* *S17: Or… it goes only up to six…* *T-R: Mm, ja…* *S8: It's so far all the same but I'm not sure…*
36. Student 7 realised that to know how many total possible outcomes you could get when drawing one card out of a standard deck (including the two jokers), you would need to know the total number of different cards that there are to be drawn. Student 7 understood that since there are 54 different cards altogether (including the two jokers) there were 54 total possible outcomes.	*T-R: How many different outcomes do you think you can get?* *Ss: um…* *S11: thirty. No wait…* *T-R: thirty different outcomes?* *S7: fifty-four?* *T-R: why fifty-four?* *S7: aren't there fifty-four cards in a deck?* *T-R: including?* *S7: the jokers.* *T-R: including the jokers? There only fifty-four possibilities… so those are the only outcomes you can get.* *S11: oh jaa. I thought that… ok.* *T-R: good, well done. ok, right so I'm going to bring you a different one now.*
37. Student 6 knew that to work out how many total possible outcomes there were, they would need to first work out how many different cards there were in the pack. This student did not know this information immediately and gradually with the teacher-researcher's guidance worked out how many different cards there were (not fully recorded to end of working out).	*T-R: how many different possibilities do you think you can get, drawing a card from the pack?* *S: uh…* *S6: how many…* *S: one…* *S: how many cards there are?* *S6: how many cards are there?* inaudible *T-R: how many cards in total?* *S6: no, no, no I'm saying like how many, like… what does it go up to?* *S: each one…* *T-R: well, let's count… there's ace, two…* *S6: three, four, five, six, seven, eight, nine, ten…* *T-R: and then?* *S6: oh… jack, queen, king* *T-R: king… jack, queen, king* *S6: so it's eight… so one out of eighteen* *T-R: mmm? thirteen?* *S6: I mean sorry, uh one out of thirteen…* *T-R: So thirteen cards but what are the different suites?* *S6: there's three there's… no because there's jokers* *T-R: Jokers as well. How many jokers?* *S6: Two* *T-R: Two jokers. And then you said there's ace, two, king, right but how many different kinds of ace, two, kings do you have? How many ace, two, kings… what are the different suites you had? Clubs? So there's ace to thirteen clubs* *S6: There's diamonds, clubs, spades* *T-R: Ja* *S6: Um, hearts* *T-R: Hearts, so there's four different suites* *S6: Plus the joker* *T-R: Plus the two jokers. But if there's four different suites and each of them have ace to king, ace to king, ace to king, ace to king, then you can work out the total amount*
38. I had been attempting to explain to one of the groups how to work out the total number of different possible outcomes when pulling coloured balls out of a tin but they could not quite understand it. Student 10 came to understand it and started trying to explain it to the rest of the group. They described the scenario very simply to the other students and most of them understood.	*T-R: Yes, tell them why... uh right guys, listen to S10… for one second* *S10: there's eight different outcomes because there's eight different colours in here* *T-R: Why, why not thirty outcomes?* *S10: Um because if 'cause if you have thirty outcomes they could all be green but that would still be one out… um thirty experiments I mean but that would only be one outcome because there's only three, like there're eight different types of colours in here to get different outcomes.* *S17: Ok…* *S10: And that makes sense.* *S10: ok is it you?* *S17: ja, it's your turn.* *T-R: k did that make sense?* *S17: yip*
39. It was very nicely calculated as a whole group that there are 54 cards in a pack (including the jokers) and therefore there are 54 possible	*T-R: How many possible different outcomes could you get? If you were working with a normal pack of cards?* *S: over like…* *S2: there's the joker, there's the nine of hearts, there's…* *T-R: nine of each thing? Let's go through them…* *S2: Well there's the joker, the king and…* *S: there's four of each king… there's four of each one.* *S2: there's the… king and the jack…* *T-R: hu… why do you say four of each?* *S2: there's nine plus one…*

different outcomes that could result when drawing one card.	*S: four of each different suit…* *S: uh a jok… a jack, there's four…* *S16: ja, there's four of each king, four…* *T-R: ok, so four of each jack, four of each queen, four of each king…* *S16: four of each number.* *T-R: four of each number? How many numbers are there in total?* *S16: there's…* *S: there's twelve.* *S16: no there's ten, there's ten. It goes up to ten and then there's jack, queen, king* *Ss: oh ja; ja; king; king* *S16: So it's thirteen and then you got two jokers.* *S: how…? How…?* *S16: so thirteen times four plus two…* *S2: two goes forty…* *S: forty…* *S16: forty… fifty…* *S2: two…* *S16: four 'cause plus two jokers, so there fifty-four cards in the deck.* *T-R: so if you're just working with a normal deck, and you're drawing a card… how many possible options are there in total?* *S13: um…* *T-R: how many different cards did they just say there were?* *S16: fifty… fifty-four* *S13: fifty-four… and that's…* *T-R: fifty-four different cards? So how many possible outcomes would you have?* *S: fifty-four?*
40. An amazing AHA moment where three of the students started using tree diagrams from their own initiative to try solve the total possible number of outcomes for rolling a die three times. It was very close to the end of the lesson and two of the students did not finish their investigations. However student 7 sketched an initial tree diagram and after some discussion with the teacher-researcher realised there were some inaccuracies. After seeing what the issue was student 7 then attempted a second tree diagram and was able to solve the problem for finding the total number of outcomes for rolling a die three times (going to the teacher-researcher at the end of the lesson to confirm the total number of different possible outcomes).	*T-R: don't rub this out, ok, tell me. Explain it. (see Appendix 21)* *S7: uh so…* *T-R: Uh, right guys, take it down a notch. Only the person drawing the numbers talking and telling the other person what to write down. Chiara, only one person in your group.* *S7: so, each… you got six pos..* *T-R: K guys, listen to this.* *S7: six possibilities of a first number…* *T-R: yes* *S7: and then…* *T-R: what are your six possibilities?* *S7: one, two, three, four, five and six* *T-R: ok.* *S7: and then for each first number, so let's say it was one… it could have um… six polities… six possibilities for a second number…* *T-R: ok…* *S7: and six possibilities for a third number.* *T-R: ok…* *S7: so each first number has twelve possibilities for the following numbers…* *T-R: ok…* *S7: and so then if you times twelve by six, you get seventy-two…* *T-R: ok…* *S7: so then, so each of these has twelve possibilities of the following numbers.* *T-R: ok, so if you look at your first number, you saying… if your first number is one it could either go to one and one or it could go to one and two or it can go to… what about one and three?* *S7: oh ja… so…* *T-R: and one and four… so, so far you've said one goes to one and one, and one goes to two and two, and one goes to three and three, and one goes to four and four… but you've forgotten about one, one, two… and one, one, three… one, one, four… one, one, five..* *S7: so then would you times? Sjoe. Ok so wait… so…* *S1: how many possibilities there are…* *S7: so then this also has six possibilities as of the… so then you times…?* *T-R: do you want to do another diagram? So keep this diagram. Get another board and do another diagram for me. Do you know what I mean?* *S7: ok.* At the end of the lesson: *T-R: um S7, where's S7. Did you figure it out?* *S7: I got a hundred and eight, but I don't think it's right… (see Appendix 22)* *T-R: you got?* *S7: a hundred and eight* *T-R: sorry, sorry, sorry* *S7: but I don't know…* *T-R: six possible outcomes, six times six…* *S7: a hundred and eight? No, no, no that's times three. Laughing um times six, so uh… two hundred and sixteen?* *T-R: no times three, times six… ja it is. ja* *S1: how?* *S7: two hundred and eight? Ok.* *T-R: um… sixteen I think.* *S7: oh two hundred and sixteen. Ja*

	*S1: one, one, one… one, one, two… one, one, three… one, one, four… one, one, five… (*S1 quickly trying to work out how S7 got so many different possible outcomes) *S7: Thank you Mrs S.* *T-R: because ja, we'll talk about it, 'cause your diagram's good.*
41. <u>In the final summary, student 7 also pointed out that because there are so many more different possible outcomes when you roll the dice more three times (instead of only once), there is a much lower chance of getting a repeated outcome.</u>	*S7: Ja and… and because there's so many different possible outcomes if you're rolling it more often, then it's less likely to get um… the exact same, um… again.* *T-R: very nice.*
Lack of understanding/too complicated	
Lesson 1 42. Some students really understood how many of each possible outcome should have been expected theoretically (for a specific experiment), however there were a number of students who had no idea what I was talking about. It was very difficult in the whole class to help every student to understand without boring the rest who had it (which is where the small groups come in). This was discussed in the forum afterwards and an effort made to find a solution to aid the understanding of those who seemed a bit lost in this lesson.	*T-R: Eight. 'k. Alright I'm gonna take these out, or just double them. 'k, let's check out our probability – what we should have got. Three ones, four twos, three threes, three, three, three, four, four, two and two. So apart from three, one less, two, or one more, four, we don't actually have… um the average is not more than that for any of the numbers… or less than that. So on average, we either had two, three or four of each number. And so the more we add… so if I take Class X, Class Y and this class and put all our events together, theoretically it will tend even more towards… three for every single number… picking it out.* *MS: Woah; yoh; sjoe; quite cool* *S: So is it um..* *T-R: And if we averaged it, look still. On either side of three. Two, four, two, four, three, three, two, four, four.* *Ss: Yoh!* *T-R: And so…* *S2: But how… how does that happen?* *T-R: Because when you start with your bag with your numbers in front of you, you can either pick a one… but you got the same chance of picking a one as you do picking out a two.* *S2: But what happens if you pick out like… out of the thirty… you pick out like twenty-five eights and…* *T-R: Exactly! And so that's what the experimental showed us. But you can pick out something that's not exactly theoretical. But the more times we do it, the more chance that it's going to be… a certain amount… for each number. And if you have a fair number… one to ten… it'll be the equal number… an equal chance for each number to be pulled out. In the same way as at the beginning when I said what if you had three hun… what if you have a hundred threes? You were like woah! Oh! Ok! Most… you know… you very likely to pull out a three rather than anything else. So the more there are of something, the higher the likelihood goes of pulling it out the bag. S10?* *S10: Um, you know you said you were getting closer to three?* *T-R: Yes.* *S10: Like is… because three is mentioned the most that we're getting closer to three? Or…* *T-R: Ok. So… when I show you in our next lesson how we're going to actually do the fractions… to actually work out what the answer is… what the measurable… what the way is to measure what the probability is… um… if you were to do the theoretical probability of what you drew out, you should have got three ones, three twos, three threes, three fours, three fives…* *S10: Oh…ok…* *T-R: And so now we look at our results and that's how close they were… each of them to three.* *S2: That's really close!*
Lesson 2 43. <u>I tried to address what was discussed in the forum – the lack of understanding on the part of one or two students in the first lesson (who were selected specifically for the small group in the second lesson for that reason). I tried to draw them further into conversation and garner their understanding on concepts from the first lesson but I did not handle it very well and put too much pressure on the student.</u>	*T-R: Um, so… what I'm saying is… we spoke about you guys doing lots and lots of experiments, right? When we spoke about everyone's… when we spoke about everyone's event… we said in your event… so in your group of… a whole bunch of experiments that you did… which one was your highest outcome. And then I said in your event… who was in your group?* *S: S20.* *T-R: S20. So in your event with S20, the event that you two did together, what was your lowest number of outcomes? And I was talking about in your event, in your group of? What were you doing? What were you doing every single time?* *S: Picking?* *T-R: Ja, you were picking numbers. And every time you did that I said in this?* *S2: Scenario.* Ss: Giggle at the new word to attempt to describe the concept that the teacher-researcher was trying to get them to recall. *T-R: Experiment. Ok, so we spoke about…* *S2: It works! (*talking about the word 'scenario' to describe the concept) *T-R: Ok. So we spoke about doing an experiment. So I said ok guys, in this experiment you're gonna pick a number… and you're going to take that number and you're going to tally mark it to tell me how many of those outcomes you got. So, your experiment was to pick a number… and after doing this experiment, you got… what? What was your result called?* *Ss: An outcome; outcome* *T-R: An outcome, ok good. So you did an experiment, you got an outcome, you tally-marked what your outcome was. 'k? Then… you did another experiment. A second experiment. And it was the same experiment where you pulled out a number… again… and you also got?* *Ss: An outcome.*

	T-R: *An outcome. 'k? And you tally-marked that outcome. Ok? And you did thirty of these. Now when I said 'in your event'… I'm talking about each and every single experiment that you did.* *Ss:* *Oh!; so it would be group of experime… it was something…* *T-R:* *A group of experiments…* *S2:* *… it was something about a…* *T-R:* *… a collection of experiments…* *Ss:* *Collection! Oh ja!; yes!* *T-R:* *K, you don't have to use the word collections.* *S1:* *Outcomes!* *T-R:* *Ok, coll… collection is one way to say it, but if you say it's a group of experiments or a number of experiments or the total number of experiments that you do… that's also fine, ok?* *S2:* *So it could be just a group on its own?* *T-R:* *Exactly!* *S1:* *Isn't it outcomes? Outcomes.* *T-R:* *So after you've done the experiments you get outcomes. But when you referring back to the event that you did, you're talking about the certain number of experiments.* *Ss:* *Oh; Oh, what you did, not the outcome! Ok.*
44. In a discussion with one of the small groups, the students were confused. I attempted to clarify the difference between the total number of outcomes (depending on how many experiments are carried out – one outcome per experiment) and the total number of different possible outcomes (limited amount of outcomes that are different to each other – limited number of different colours or number of different cards).	*T-R:* *These outcomes, how many are there? How many different outcomes can you get?* *S:* *thirty…* *S17:* *one, two, three, four…* *T-R:* *Thirty? Thirty different outcomes?* *S17:* *No… one, two, three, four, five, six… you can get six* *T-R:* *Only six different… ?* *S17:* *so six colours.* *T-R:* *Only six different colours?* *S17:* *If each person picks a different… but isn't there thirteen because there's thirteen… or is that experiments?* *T-R:* *That's the number of experiments you do but what if you get a blue, a blue, a blue and a pink? You don't have four different outcomes that's only two outcomes because blue's the same outcome every time.* *S17:* *Well um, infinite 'cause…* *T-R:* *It can't be infinite because at some point you gonna have covered all your outcomes and you won't get more. S15, how many different outcomes are there? How many different possibilities?* *S15:* *Lots* *T-R:* *No, but you've got so many repeated look you've got a green and a green that's the one outcome there's another one that's green, all the greens are one outcome, one kind of outcome* *S:* *well there's* *T-R:* *'cause it's the same colour* *S17:* *thirteen, uh… can't you go green as thirteen then pink as um…* *T-R:* *I'm not saying how many outcomes are there in total that you get. I'm saying how many different kinds of outcomes are there?*

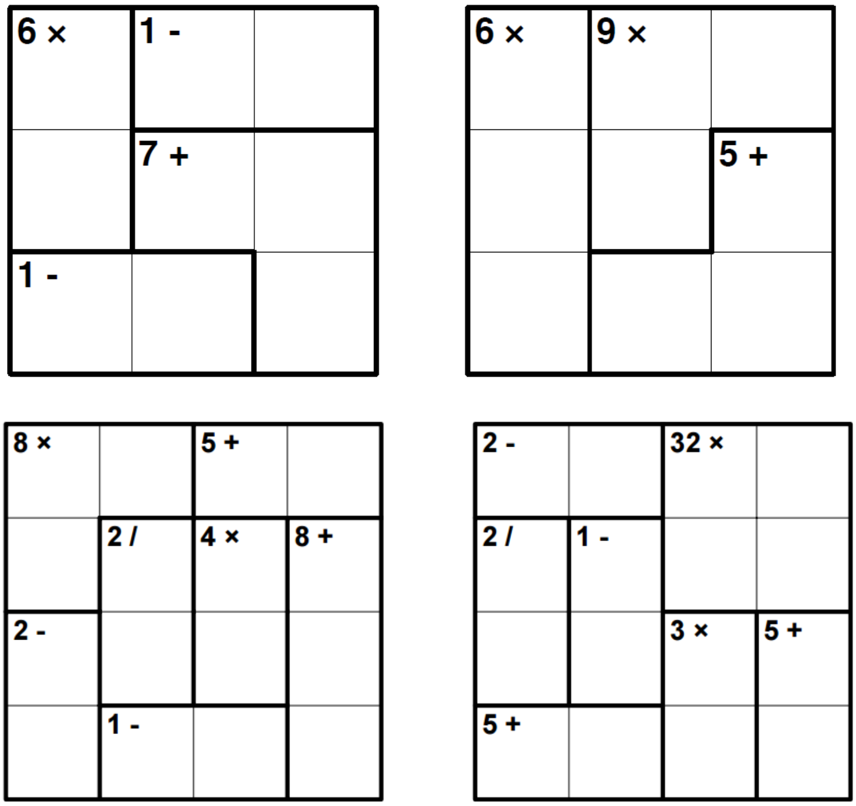
Mathematical game: Sudoku (taken from Krazydad, n.d.)

2		5			7			6
4			9	6			2	
				8			4	5
9	8			7	4			
5	7		8		2		6	9
			6	3			5	7
7	5			2				
	6			5	1			2
3			4			5		8

Fill in the blank squares so that each row, each column and each 3-by-3 block contain all of the digits 1 through 0. If you use logic you can solve the puzzle without guesswork.

APPENDIX 13

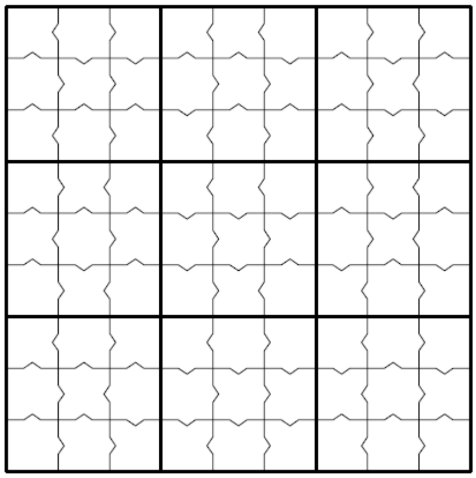
Mathematical game: Ken-ken/Inky puzzles: 3×3 and 4×4 blocks (taken from Krazydad, n.d.)



Fill in the blank squares so that each row and each column contain all of the digits 1 through 3. The heavy lines indicate areas (called cages) that contain groups of numbers that can be combined (in any order) to produce the result shown in the cage, with the indicated math operation. For example, 12X means you can multiply the values together to produce 12. Numbers in cages may repeat, as long as they are not in the same row or column.

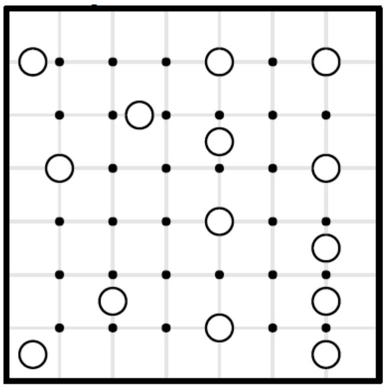
APPENDIX 14

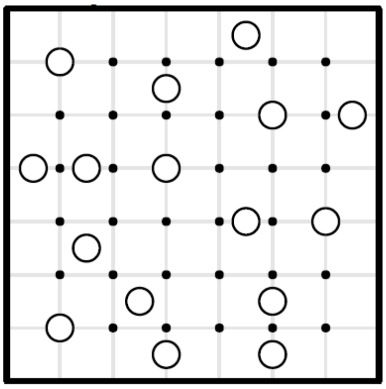
Mathematical game: Jigoku (taken from Krazydad, n.d.)



Fill in the blank squares so that each row, each column and each 3-by-3 block contain all of the digits 1 through 9. > and < connections between squares indicate that one number is greater than or less than another. If you use logic you can solve the puzzle without making guesses.

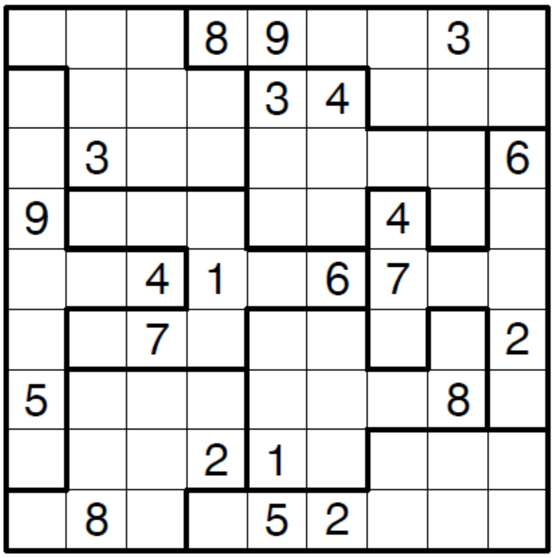
Mathematical game: Galaxy (taken from Krazydad, n.d.)





Connect the dots to make edges so that each circle is surrounded by a symmetrical galaxy shape, and the puzzle is completely tiled with galaxies. Each galaxy shape must be rotationally symmetric, having an identical appearance when rotated 180 degrees.

Mathematical game: Jigsaw Sudoku (taken from Krazydad, n.d.)



Fill in the blank squares so that each row, each column, and each jigsaw shape contain all the digits 1 through 9.

H for heads and T for tails

	Outcome 1	Outcome 2	Outcome 3	FINAL OUTCOME
Experiment eg.	H	H	T	HHT

Flipping a coin (3 times)

	Outcome 1	Outcome 2	Outcome 3	FINAL OUTCOME
Experiment 1				
Experiment 2				
Experiment 3				
Experiment 4				
Experiment 5				
Experiment 6				
Experiment 7				
Experiment 8				
Experiment 9				
Experiment 10				
Experiment 11				
Experiment 12				
Experiment 13				
Experiment 14				
Experiment 15				
Experiment 16				
Experiment 17				
Experiment 18				
Experiment 19				
Experiment 20				
Experiment 21				
Experiment 22				
Experiment 23				
Experiment 24				
Experiment 25				
Experiment 26				
Experiment 27				
Experiment 28				
Experiment 29				
Experiment 30				

H for heads and T for tails

	Outcome 1	Outcome 2	Outcome 3	FINAL OUTCOME
Experiment eg.	H	H	T	HHT

Flipping a coin (3 times)

	Outcome 1	Outcome 2	Outcome 3	FINAL OUTCOME
Experiment 1	H	T	T	HTT
Experiment 2	H	T	H	HTH
Experiment 3	T	T	T	TTT
Experiment 4	T	T	H	TTH
Experiment 5	H	H	T	HHT
Experiment 6	H	T	T	HTT
Experiment 7	T	H	H	THH
Experiment 8	T	H	H	THH
Experiment 9	T	H	H	THH
Experiment 10	H	H	H	HHH
Experiment 11	T	H	H	THH
Experiment 12	T	T	H	TTH
Experiment 13	H	H	H	HHH
Experiment 14	T	H	T	THT
Experiment 15	H	H	H	HHH
Experiment 16	T	T	T	TTT
Experiment 17	H	H	T	HHT
Experiment 18	H	H	T	HHT
Experiment 19	H	T	H	HTH
Experiment 20	T	H	H	THH
Experiment 21	H	T	T	HTT
Experiment 22	H	H	T	HHT
Experiment 23	H	T	T	HTT
Experiment 24	H	H	H	HHH
Experiment 25	H	T	H	HTH
Experiment 26	H	T	H	HTH
Experiment 27	H	T	T	HTT
Experiment 28	H	T	H	HTH
Experiment 29	H	T	H	HTH
Experiment 30	H	H	T	HHT

	Outcome (colour)
Experiment eg.	blue
Experiment eg.	dark pink
Experiment eg.	green
Picking coloured paper out a tin	
	Outcome (colour)
Experiment 1	
Experiment 2	
Experiment 3	
Experiment 4	
Experiment 5	
Experiment 6	
Experiment 7	
Experiment 8	
Experiment 9	
Experiment 10	
Experiment 11	
Experiment 12	
Experiment 13	
Experiment 14	
Experiment 15	
Experiment 16	
Experiment 17	
Experiment 18	
Experiment 19	
Experiment 20	
Experiment 21	
Experiment 22	
Experiment 23	
Experiment 24	
Experiment 25	
Experiment 26	
Experiment 27	
Experiment 28	
Experiment 29	
Experiment 30	

	Outcome (colour)
Experiment eg.	blue
Experiment eg.	dark pink
Experiment eg.	green
Picking coloured paper out a tin	
	Outcome (colour)
Experiment 1	red
Experiment 2	green
Experiment 3	purple
Experiment 4	red
Experiment 5	yellow
Experiment 6	pink
Experiment 7	orange
Experiment 8	yellow
Experiment 9	pink
Experiment 10	orange
Experiment 11	pink
Experiment 12	pink
Experiment 13	purple
Experiment 14	red
Experiment 15	pink
Experiment 16	purple
Experiment 17	pink
Experiment 18	yellow
Experiment 19	red
Experiment 20	blue
Experiment 21	green
Experiment 22	green
Experiment 23	pink
Experiment 24	orange
Experiment 25	purple
Experiment 26	pink
Experiment 27	pink
Experiment 28	yellow
Experiment 29	blue
Experiment 30	blue

	Number	Suit
Experiment eg.	Jack	spades
Experiment eg.	Four	diamonds
Experiment eg.	King	clubs

Drawing a card from a normal deck		
	Number	**Suit**
Experiment 1		
Experiment 2		
Experiment 3		
Experiment 4		
Experiment 5		
Experiment 6		
Experiment 7		
Experiment 8		
Experiment 9		
Experiment 10		
Experiment 11		
Experiment 12		
Experiment 13		
Experiment 14		
Experiment 15		
Experiment 16		
Experiment 17		
Experiment 18		
Experiment 19		
Experiment 20		
Experiment 21		
Experiment 22		
Experiment 23		
Experiment 24		
Experiment 25		
Experiment 26		
Experiment 27		
Experiment 28		
Experiment 29		
Experiment 30		

	Number	Suit
Experiment eg.	Jack	spades
Experiment eg.	Four	diamonds
Experiment eg.	King	clubs

Drawing a card from a normal deck		
	Number	**Suit**
Experiment 1	Ace	clubs
Experiment 2	Six	diamonds
Experiment 3	Ace	diamonds
Experiment 4	two	hearts
Experiment 5	nine	hearts
Experiment 6	eight	clubs
Experiment 7	Joker	
Experiment 8	seven	hearts
Experiment 9	King	clubs
Experiment 10	two	diamonds
Experiment 11	King	~~spades~~ clubs
Experiment 12	Four	hearts
Experiment 13	eight	spades
Experiment 14	three	hearts
Experiment 15	Joker	
Experiment 16	Four	spades
Experiment 17	two	clubs
Experiment 18	five	diamonds
Experiment 19	eight	clubs
Experiment 20	nine	diamond
Experiment 21	nine	diamonds
Experiment 22	three	diamonds
Experiment 23	seven	hearts
Experiment 24	queen	spades
Experiment 25	eight	diamond
Experiment 26	Joker	
Experiment 27	two	diamonds
Experiment 28	two	clubs
Experiment 29	eight	spades
Experiment 30	five	clubs

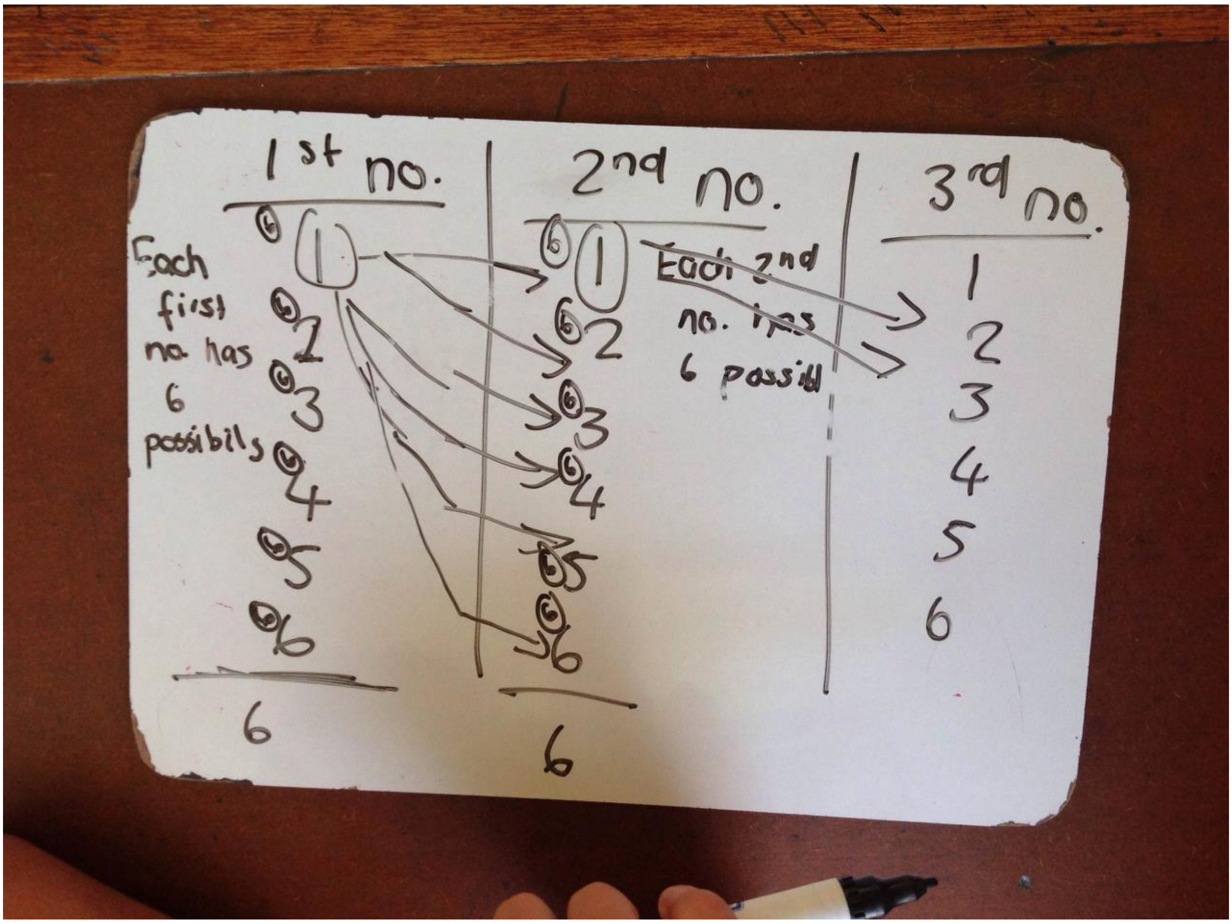
APPENDIX 20

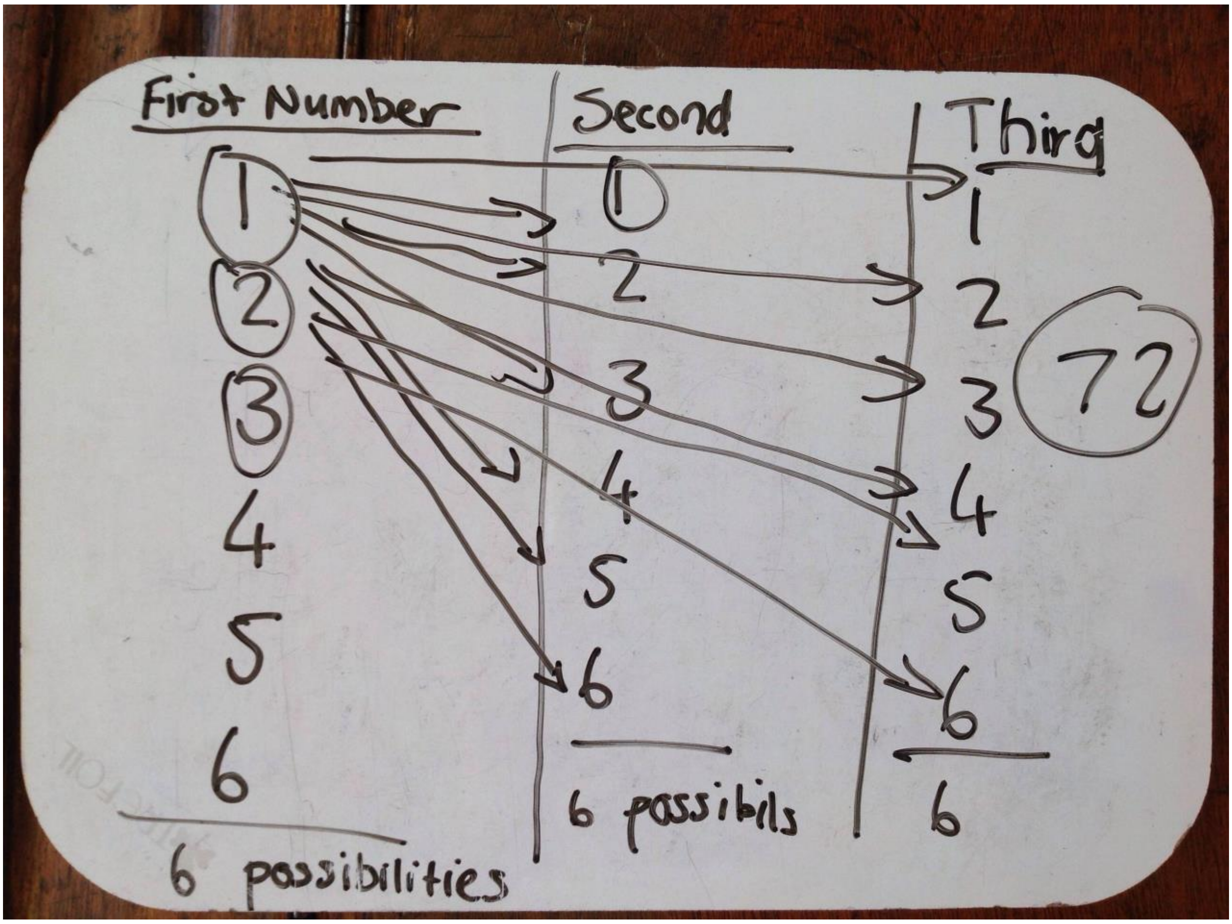
	Outcome 1	Outcome 2	Outcome 3	FINAL OUTCOME
Experiment eg.	4	1	6	416

Rolling a die (3 times)				
	Outcome 1	**Outcome 2**	**Outcome 3**	**FINAL OUTCOME**
Experiment 1				
Experiment 2				
Experiment 3				
Experiment 4				
Experiment 5				
Experiment 6				
Experiment 7				
Experiment 8				
Experiment 9				
Experiment 10				
Experiment 11				
Experiment 12				
Experiment 13				
Experiment 14				
Experiment 15				
Experiment 16				
Experiment 17				
Experiment 18				
Experiment 19				
Experiment 20				
Experiment 21				
Experiment 22				
Experiment 23				
Experiment 24				
Experiment 25				
Experiment 26				
Experiment 27				
Experiment 28				
Experiment 29				
Experiment 30				

	Outcome 1	Outcome 2	Outcome 3	FINAL OUTCOME
Experiment eg.	4	1	6	4;1;6

Rolling a die (3 times)				
	Outcome 1	**Outcome 2**	**Outcome 3**	**FINAL OUTCOME**
Experiment 1	1	1	5	1;1;5
Experiment 2	2	3	3	2;3;3
Experiment 3	1	3	1	1;3;1
Experiment 4	1	2	1	1;2;
Experiment 5	6	6	5	6;6;5
Experiment 6	1	3	1	1;3;1
Experiment 7	3	1	1	3;1;1
Experiment 8	6	6	6	6;6;6
Experiment 9	5	6	3	5;6;3
Experiment 10	3	2	5	3;2;5
Experiment 11	4	4	1	4;4;1
Experiment 12	6	3	4	6;3;4
Experiment 13	1	3	5	1;3;5
Experiment 14	2	1	6	2;1;6
Experiment 15	6	5	6	6;5;6
Experiment 16	5	4	6	5;4;6
Experiment 17	6	1	3	6;1;3
Experiment 18	5	6	1	5;6;1
Experiment 19	4	1	2	4;1;2
Experiment 20	4	5	1	4;5;1
Experiment 21	3	1	3	3;1;3
Experiment 22	3	5	3	3;5;3
Experiment 23	5	6	3	5;6;3
Experiment 24	6	2	1	6;2;1
Experiment 25	3	6	2	3;6;2
Experiment 26	4	1	6	4;1;6
Experiment 27	5	4	6	5;4;6
Experiment 28	6	1	2	6;1;2
Experiment 29	6	1	5	6;1;5
Experiment 30	6	6	4	6;6;4


1st no.
2nd no.
3rd no.
Each first no. has 6 possibils
Each 2nd no. has 6 possib
1
2
3
4
5
6
6
6


First Number
Second
Third
1
2
3
4
5
6
6 possibilities
1
2
3
4
5
6
6 possibils
1
2
3
4
5
6
6
72

APPENDIX 23

Review of data[8] by Observer 1

I concur with the attached report made of the lessons and make these further observations:

* When I observed lesson 3 I was very aware how much progress had been made since lesson 1. The children had a thorough grasp of concepts taught previously. They were using the correct terminology both confidently and effortlessly.

* Many hours of prior preparation by yourself of the lessons were evident. The group activities were absorbing for the children. it was most commendable how your questioning technique led the children towards understanding and then formulating their own definitions of the different concepts. there was minimal didactic teaching.

*There was one [student] in particular who asked probing questions that went considerably beyond the lesson content matter. It was good to see how you were able to answer his[9] questions but also able to set him further questions to force him to think even deeper. The lessons were significant conceptual growth experiences for him. (In an informal chat with him, he unaffectedly confirmed that he was a distinction Maths pupil in his Grade.)

* The report made a number of suggestions on how to further improve on the methodology. Although they might all be valid, I make the comment that there can be an 'over-analysis' of lessons. Good teaching is characterised by the lesson content being age- and ability- appropriate where the majority of the learners grasp what is being taught. Salient points of the lesson content should be clearly understood. The learners should find the lesson content absorbing, challenging, applicable to their young lives and hopefully ... enjoyable too! All of that was achieved in your lessons. Congratulations on your outstanding lessons!

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[8] The data given to the observers to review were the comments made by the observers in the discussion fora following each lesson and notes by the teacher-researcher on how each of these were addressed (which have been included in this study). The observers were asked to confirm the validity and authenticity of these data. They also had access to a developing stage of the framework presented in this study and made comments about it.

[9] Please note that for the sake of anonymity, all pronouns used are not specific to the actual participant and 'his' or 'he' has been used for the ease of reading.

Review of data[10] by Observer 2

I concur with the validity and authenticity of the observers' comments reported in this study and how the comments were addressed in follow-up lessons.

I think the framework is wonderful - well considered and described. My one comment would be more of a grammatical issue than a conceptual issue[11], and that is that you often use 'the teacher' and then talk about 'oneself'. This is an inconsistent use of 'person': the teacher representing 'he/she/it' and oneself representing 'I'. However, this is a truly minor issue, and has little bearing on the ideas behind your work.

[10] The data given to the observers to review were the comments made by the observers in the discussion fora following each lesson and notes by the teacher-researcher on how each of these were addressed (which have been included in this study). The observers were asked to confirm the validity and authenticity of these data. They also had access to a developing stage of the framework presented in this study and made comments about it.

[11] The issue raised here by Observer 2 was seriously considered in the stage of analysis 'review interpretations with participants'. The teacher-researcher decided to remove the one occasion where 'oneself' and rephrased the way the teacher who might use the framework is referred to.

APPENDIX 25

Review of data[12] by Observer 3

I concur with the validity and authenticity of the observers' comments reported in this study and how the comments were addressed in follow-up lessons.

Wow - what a paper! I really reflected on my own teaching strategies while reading it. What better teachers we would all be if we were to adopt this policy.

I think you have written as you teach. I'm not sure that you ever got feedback from your pupils as suggested[13] but I know that you did review what had been taught after brainstorming with us.

Thanks for that - its brilliant!

[12] The data given to the observers to review were the comments made by the observers in the discussion fora following each lesson and notes by the teacher-researcher on how each of these were addressed (which have been included in this study). The observers were asked to confirm the validity and authenticity of these data. They also had access to a developing stage of the framework presented in this study and made comments about it.

[13] Observer 3 commented on how the study progressed and their view on the formation of the framework (an earlier version of what has been presented in chapter 4 above). In the developing framework that the observer saw, it included the original plan for feedback to be gathered from the students on how the lesson had progressed. In the stage of analysis where data were reread and coding done for interpretations supported or challenged, the teacher-researcher realised that getting feedback from the students wouldn't work as originally thought and so this was removed from the framework. This is the part of the research process that the observer had noticed was not carried out and questioned its presence in the framework. Even though it had already been removed, the comment by the observer confirms the importance of triangulation and supported the teacher-researcher in the decision to remove it from the framework.