CULTURE, COGNITION AND UNCERTAINTY: METACOGNITION IN THE LEARNING AND TEACHING OF PROBABILITY THEORY

Irene Anne Broekmann

Degree awarded with Distinction 24 June 1993

A Research Report submitted to the *Faculty of Education*, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Master of Education by course-work and research report.

Johannesburg, 1992

CULTURE, COGNITION AND UNCERTAINTY: METACOGNITION IN THE LEARNING AND TEACHING OF PROBABILITY THEORY

Broekmann, Irene Anne, University of the Witwatersrand, Johannesburg, 1992

This research report investigates the psychological dimensions in the learning and teaching of probability theory. It begins by outlining some problems arising from the author's own experience in the learning and teaching of probability theory, and develops a theoretical position using the Theory of Activity. This theory places education within the broad social context and recognises the centrality of affective aspects of cognition.

The method of research is conceptual, using the theory to understand the problems outlined, and to develop further understanding of what constitutes good teaching in this discipline. Thus, elaborating on the theory using Vygotsky's notions of scientific and intuitive concepts, together with the differences between expert and novice, the report argues for teachers and students adopting a metacognitive attitude toward the learning and teaching of probability theory, where metacognition itself is seen as the reconstruction of not only cognitive skills themselves, but also of self-regulatory abilities. The report concludes that the meaning the students make of probability theory and their confidence in their own ability to learn it, need to be seen as units of analysis for concerns regarding the teaching and learning of probability theory.

DECLARATION

I declare that this research report is my own, unaided work. It is being submitted for the degree of Master of Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

Irene Anne Broekmann

This thirtieth day of September, 1992

PREFACE

This research was initiated while I was teaching mathematics and statistics courses to first year students in 'service-courses' at the University of the Witwatersrand. I was concerned that many students seemed to feel unable to even begin learning some of the sections required, that of probability theory in particular. I had already undertaken studies in psychology as I was becoming increasingly convinced that the psychological aspects of mathematics instruction were central. The purpose of the study was to develop knowledge and understanding of cognition and metacognition in learning and teaching in order to contribute positively to relationships in tertiary stochastics teaching. It is aimed at the reader who is interested in the applications of theories of cognitive development and instruction to the teaching and learning of mathematics-related disciplines.

ACKNOWLEDGEMENTS

Financial

The financial assistance of the Centre for Science Development of the Human Sciences Research Council towards this research is hereby acknowledged. Opinions expressed in this publication and conclusions arrived at are those of the author and are not necessarily to be attributed to the Centre for Science Development or the Human Sciences Research Council.

The University of the Witwatersrand and the Union of Jewish Women are thanked for their bursaries.

Thank you to my family for financial assistance while I was studying full-time.

Personal

I thank my family, many colleagues, friends and reading-group members who have helped me in various ways during this work. I wish to thank Professor Paul Fatti, Head of Statistics and Professor Paul Laridon of Mathematics for their early confirmation that the study would be worthwhile. In particular I wish to thank Dr Mike Kissack for assisting me to clarify my ideas; Ms Lynne Slonimsky and Dr Adele Gordon for ongoing support and encouragement, useful comment and direction; Professor Penny Enslin for the understanding; Mr Graham Walker for his guidance; my brother, Dr Reg Broekmann for his computing assistance; and my sister, parents, family and friends, especially Gail and Elaine, for the babysitting.

Finally, thank you to Dr Carol Macdonald who in supervising me has given so generously and supportively of her time and expertise.

To my husband Douglas Clerk, and our daughters, Sarah Jenelle and Caitlin Emma.

TABLE OF CONTENTS

1 CHAPTER ONE: INTRODUCTION

> "Probability is an obvious and simple subject. It is a baffling and complex subject. It is a subject we know a great deal about, and a subject we know nothing about. Kindergartners can study probability, and philosophers do ... . Such contradictions are the stuff of probability."
>
> Kerlinger, 1964, p.94

1.1 Conceptualising of the Problem

This research has been undertaken in an attempt to understand problems that became evident to me while teaching probability and statistics courses to 'service-course' students, students whose choice of subjects necessitated some knowledge of probability and statistics, but for whom this was not the major interest. These students were drawn from the fields of, inter alia, economics, psychology and medicine. Many of these students found the stochastic courses difficult, especially the probability theory component.

The existence of problems in the learning and teaching of Probability Theory that contribute to the difficulty experienced by students and teachers is well known. Dicks (1989, p.15) makes the claim that in South African universities only "a very small percentage of participants (rarely higher than 10%) really master the subject [and] [t]he majority (as high as 90%) are minimally competent." The problem in teaching and learning probability and statistics is both real and acknowledged.

1.2 Method of Procedure

> "To educate and teach human beings it is necessary to know their psychology. Conversely, a psychological knowledge of people ... is most successful in the process of education and

instruction. The unbroken unity between psychology and the educational process is reflected especially in this point."

Brushlinskii, 1987, p.78

This research attempts to provide a theoretical explanation for the difficulty service-course students experience with Probability Theory by explaining the processes involved in the learning of a scientific subject. In addition it attempts to orient teachers to their responsibilities arising from the theory. In order to do this I will consider the psychological subject (person) as primary.

While my express purpose is to address an existing problem for learners and teachers of probability in the service-type course, the theory has a far wider application. It has relevance for the learning and teaching of Probability Theory[1] in general and pertains broadly to the learning and teaching of mathematics-related disciplines.

The method of procedure will be conceptual, using empirical data drawn from my teaching and research experience at the University of the Witwatersrand as supportive or illustrative only. As this research is not intended to be quantitative, I will use terms like 'many' and 'often' as indicative of types of problems rather than as indicators of their pervasiveness. Structurally, topics introduced may be reintroduced and further developed.

1.3. Specific Problems Encountered with Students

Many types of problems exist for students in the area of Probability Theory, some more perplexing than others. Students often revealed that they felt that Probability Theory held no meaning and that they felt that they were incapable of solving problems requiring the quantification of uncertainty.

The problem that particularly concerned me was the inability of some students to search for that which was subjectively unknown-especially in problem solving. Many could not even begin to attempt to solve problems, even though they had seemingly been provided with the necessary knowledge. They felt that they could more easily cope with sections of the work where there were 'rules' one could apply to solve the problem.

> "There is a tendency for students to fall into "a 'number crunching' mode, plugging quantities into a computational formula or procedures without forming an internal representation of the problem ... They may be able to memorize the formulas and the steps to follow in familiar, well-defined problems but only seldom appear to get much sense of what the rationale is or how concepts can be applied in new situations."
>
> Garfield and Ahlgren, 1988, p.46

Struggling students often reported that they perceived Probability Theory as a collection of facts that must be used to solve the problems set in tutorials and examinations. I observed that this drive to use these 'facts' could override knowledge the students actually had and which they could have easily accessed before they were exposed to new information. Instead of gaining a new level of perspective on everyday probability understanding, the students were losing their intuitive notions and failing to employ principles, like counting, where they may have been appropriate. They would concentrate on the surface features of a problem and seemed unable to discard irrelevant information in categorizing a problem or in translating values into a model. Either episodic or algorithmic thinking seemed to dominate, not allowing the broad access to cognitive stores needed for solving some probability problems. Non-algorithmic approaches were beyond the reach of their confidence and feelings of helplessness were pervasive. Given that the conditions of a problem can never be exhaustively taught, and given that thought is necessarily independent in that there is always some new aspect (Brushlinskii, 1987, p.78), teachers are failing if they do not elicit a creative, productive process in the teaching and learning situation, and if they cannot assist students to be independent in the use of the structures that statistical theory provides.

I noticed that students sometimes failed to understand what they were doing from the vantage point of comparison with what others were doing. In one tutorial group, one student had tackled a problem from first principles, while the others had used Bayes Theorem. Although they were in effect 'doing the same thing', the group could not see tho similarity of the two approaches because the one approach had a name and the other did not. Somehow, the 'rule' and its name confounded the students' understanding. Their knowledge remained inert (Whitehead, 1929), knowledge that one can express but not use. Thus on the one hand the rules hindered students in their use of previously acquired knowledge and, on the other hand, they relied on rules as they lacked confidence and a way of approaching probability knowledge.

A serious problem was the failure of some students to detect fundamental inconsistencies in their work. Given that probability values range between 0 and 1, it is a matter of concern when tertiary students provide an answer greater than 1. Brown (1978, p.105) refers to these types of errors made by tertiary students as "symbol-shock symptoms", but they can also be seen as further evidence of "inert knowledge" (Whitehead, ibid). A student may well know that probability values range between 0 and 1, but will nevertheless return an answer greater than 1 if that is what they calculate. Thus there is a level at which students can comprehend, yet fail to be able to use, relevant knowledge. Many of these problems emerging from my own experience in teaching probability were not unique to my own, or an even broader South African, context. For example, Brown (1980), referring to universities in the United States of America, states: "College students are by no means free of checking failure as any teacher of elementary statistics will attest" (1980, p.475) and: "[N]egative probabilities or variances are readily accepted as solutions if the student believes the formula was followed correctly!" (Brown, 1978, p.105).

There is also evidence of a discrepancy between students' understanding of certain everyday notions and corresponding formal formulations in probabilistic terms. For example, a student will be able to explain that if two events are independent then the outcome of the one will not affect the probability of the outcome of the other. However, when it comes to formal definitions of independence, that if events A and B are independent, then $P(A \cap B)=P(A)P(B)$, any intuitive sense of meaning disappears.

In an attempt to understand why the students are experiencing problems of the types mentioned above, one can look to their general sense of the meaningfulness or of the meaninglessness of the subject matter. If the latter holds, then there would be little scope for reality testing or error detection, particularly if students do not see the problems within the context of a system, hence isolating concepts and being content to provide some numerical answer regardless of the relationships within the system that structure Probability Theory.

Questions arising from these types of problem that I needed to address were:

* why the students have no sense of the meaning of probability theory, and
* why they feel they have no control, or
* why in fact they have no control over their ability to cope with problem solving in probability.

1.4 The Difficult Nature of the Study of Probability

> "The experience of psychologists, educators, and statisticians alike is that a large proportion of students, even in college, do not understand many of the basic statistical concepts they have studied."
>
> Garfield and Ahlgren, 1988, p.44

Probability in particular seems to pose the greatest difficulties. These difficulties are partly problems with mathematics, namely difficulty with rational number concepts and proportional reasoning, difficulty in translating verbal problems, and in general, the involvement of complex skills and concepts. In addition to these, there is the problem created by the nature of probabilistic knowledge itself, especially in the seemingly contradictory concept of probability as "laws of chance".

Kitchener (1983) refers to three levels of cognitive processing, those of cognition, metacognition and epistemic cognition. The first level, that of cognition, involves reading, problem solving, computing, and so on. The second level, metacognition, involves the monitoring, comprehension or control of one's progress on the cognitive level. The third level, epistemic cognition, involves an understanding of the limits of knowledge, as well as knowledge of the significance, usefulness and limitations of the skills involved. In the domain of Probability Theory, this requires an understanding of the nature of the uncertainty of knowing, and how Probability Theory attempts to transcend those limits. Epistemic cognition is cognition about "ill-structured" problems (ibid., p.222), problems which have no possibility of a single defined answer, these latter being called puzzles. So, although the fundamental nature of probability problems are ill-structured, the way one has to deal with them are not. Probability Theory represents an imposition of puzzles upon an ill-structured situation in an attempt to "demarcate the amorphous state somewhere between the imagined extremes of total ignorance and perfect knowledge" (Konold, 1988, p.2).

Too frequently, cognition itself becomes the sole focus of the attention of the students, which undermines the very purpose of that focus, that of efficient cognitive functioning. I will argue that in order to create the conditions for efficient problem solving, the other two levels need to be included as part of a learning-teaching orientation at a late secondary or tertiary level.

1.5 Specific Problems Encountered with Teachers

The problems that I have observed the students experience have been delineated. My assumption (which will be developed below) is that learning and teaching are inextricably linked. My research would be incomplete if I did not also look at problems teachers experience.

The teaching of Probability Theory is problematic. Much of my work at the university involved teaching on a team, hence I was able to observe other teachers at work, either directly, or by helping a student after another member of staff had helped the student.

An aspect of teaching probability that came to my attention was that some tutors (and here I include student tutors, tutors and lecturers) were insecure in handling problems in probability. What I often observed was that when a student asked for assistance, the tutors would assist the student using methods that they themselves would have used, irrespective of what the students had already attempted, often ignoring these attempts (though not necessarily consciously). Thus the students' knowledge and how this related to reaching a solution were ignored in favour of merely closing the problem by providing a method of solving it and getting an answer. I noticed that in assisting students with problems, teachers often failed to go beyond the problem to other cognitive, metacognitive or epistemic levels. Students may be left with the correct answer to a problem but no sense that they would ever be able to do it on their own. They may be left with no new ways to look at problems and no way of sensing meaning of the problem or answer. Many teachers and students seemed to become more involved with solving the problem at hand than in increasing understanding.

Teachers frequently expect students to be able to identify or extract from problems the essential principles, without direct teaching on how to search for generalizations. Some of the teaching orientations I have observed classify as what Gal'perin (1969, p.251) refers to as the 'trial and error' type. Information may be presented systematically in lectures and textbooks but there is no input as to how examples presented vary the inessential aspects while maintaining the essence. The learner is left to formulate what is to be attained and under what conditions. The student may obtain the correct procedure, but it may be unstable in the face of altered conditions and in transfer to new tasks.

Myers (1982) has identified three dimensions by which to investigate texts of elementary probability: external connectedness, or the degree to which a text explicitly relates the concepts to the presumed real-world knowledge of the students; internal connectedness, or the degree to which relations among the concepts and formulae are explicitly developed; and explanation, the degree to which a rationale or intuitive justification is provided for a particular formula (p.379). Hansen, McCann and Myers (1985, p.372) state: "Although all our subjects were novices, those in the explanatory text condition differed from the others in their processing of story problems in ways that are similar to the ways in which experts differ from novices".

What I will argue is that teaching needs to become more self-conscious in itself, that we teach the rules and the interconnections as recommended by the 'high-explanatory' texts of Myers (1982), and that we also teach ways of approaching new knowledge, with consciousness of all these aspects. Clearly it is important to consider the interplay between formal and intuitive approaches to teaching.

While teachers give considerable thought to what to teach, I contest that they do not give equivalent thought to how to deal with the structures of probabilistic thinking in a way from which both the teachers and the students can benefit. It may be argued that by modelling the probabilistic thinking of the teachers, the students are given adequate exposure to means of approaching problems and to the management of learning probability. But many students do not learn from direct exposure in a 'trial and error' orientation, especially if they are from educationally different or underprepared backgrounds to the one in which they are expected to operate, or in subject areas involving unfamiliar thinking skills. Thus attention needs to be given to special forms of communication and the way that we help to structure our students' thinking. As teachers we have a responsibility to consider the orientation to a mental act that we elicit in our teaching. This report will argue for a metacognitive approach to teaching and learning Probability Theory. If teachers could understand the difficulties experienced by the students in learning probability, more specifically in relation to the teaching thereof, it could prevent summary dismissal of the problems or the pathologizing of the students.

2 CHAPTER TWO: SOCIO-HISTORICAL NATURE OF STOCHASTIC KNOWLEDGE - SCIENTIFIC AND INTUITIVE CONCEPTS

2.1 The Dynamic Nature of Probability Theory

Probability and statistical knowledge is knowledge that has been developed by communities of practitioners and academics over many years in order to quantify uncertainty. It is systematic in nature (albeit not necessarily systematic in its development). This system is historically constituted and socially transmitted and transformed. It is a sign and language system that, as part of a cultural system, has fundamentally altered people's perceptions of how to deal with uncertainty.

There are various interpretations of the very concept of probability: the classical or theoretical, the frequentist or empirical, and the subjectivist (Konold, 1988). Thus the very concept of probability itself is contested. According to Konold (1988, p.9), the term probability exists "in a web of discourse and related concepts". To teach probability as a product is to limit it to a phenomenon that is frozen in time, whereas the nature of the discipline is one of a developing body of knowledge, negotiated by intersubjective agreement by bodies of statisticians who have been given the authority to accept or reject theory. Statistical practice is undertaken as joint activity.

2.2 Probability Theory as Scientific Knowledge

> "In receiving instruction in a system of knowledge, the child learns of things that are not before his eyes, things that far exceed the limits of his actual and even potential immediate experience."
>
> Vygotsky, 1987, p.180

According to Vygotsky (1987) scientific concepts are those which are deliberately and systematically taught, beginning with verbal definitions as part of a system, while spontaneous concepts are acquired along life's way. Scientific concepts begin with a generalization, while spontaneous concepts begin with the empirical or experiential, an instantiation rather than a generalization. Scientific concepts are mediated by other concepts and people and are not in direct relation to objects in the everyday world.

Probability Theory is seen as a scientific discipline (see for example the South African Statistical Association Journal, March 1990, p.30), and can be classified as a scientific conceptual system according to Vygotsky's distinction between scientific and spontaneous or everyday concepts (or intuitive concepts using Piaget's (1975) nomenclature).

In essence, I will argue that as probability is a scientific subject, it must be taught self-consciously in its interrelationships. As a scientific system it is not 'discoverable' by the student but must be consciously learned, traditionally in an institutional situation. Thus by this very consciousness and deliberateness, it has a metacognitive aspect. While with everyday learning, unconscious acquisition of knowledge is followed by gradual increases in active conscious control, Probability Theory has to be transmitted as a system of knowledge to the novice as part of cultural transmission.

Konold (1988) argues that understanding how students think about probability before and during instruction is important. This research attempts to explain why.

2.3 The Relationship between Present and New Knowledge

In cognitive activity, the individual actively interprets information in the light of his or her present knowledge. Present knowledge thus both limits and makes possible new knowledge, while at the same time new information and processes can develop present knowledge. If present and new knowledge thus both constrain and enable development, an assumption underlying educational practice is that one's experiences in and out of an institution of learning affect the way one makes sense of new experiences and information. Noss (1988) reiterates Mellin-Olsen's claim that the "key task confronting mathematics education is to 'bridge the gap' between forms of knowledge which people possess, and those which are invariably offered at school ..." (Noss, 1988, p.404).

Students do not begin a study of probability with no intuitive sense of what probability means. Rather, the student has some a priori sense of probability which I have referred to as intuitive concepts, whether learned spontaneously or formally some time previously. These intuitive conceptions can be either correct or incorrect and serve as cognitive structures into which new information can be assimilated.

2.4 Misconceptions Regarding Probability Theory

Kahneman, Slovic and Tversky (1982) have investigated incorrect intuitive concepts, or misconceptions, in probability. These misconceptions are frequently coherent and stable and are meaningful to the thinkers within their current structures of knowledge. Konold (1988, drawing from Kahneman and Tversky, 1973) has developed a model of non-formal reasoning under uncertainty in an attempt to explain student thinking, even where that is in contradiction to the formal discipline. It is an attempt to understand the conceptual bases on which erroneous probabilistic interpretations are made.

A commonly held misconception is that probability values are meant to provide evidence for the outcome of a single or 'the next' trial. Other common erroneous intuitive concepts in Probability Theory include the 'representativeness heuristic' (degree of similarity between sample and parent population) and the 'availability heuristic' (relative ease of bringing to mind). For example, when considering the outcome of six spins of an unbiased coin, and if heads is designated H and tails T, the outcome HHTHHT looks more likely than HHHHHH, as it appears to be more representative, while formally they are equiprobable outcomes. It is not easy to convince the novice where the formal and informal are contradictory, if working primarily with the informal.

There are gaps between belief and knowledge, in knowing about probability yet not 'believing' in it, due to misunderstanding its systematic meaning. Knowledge and belief can be seen as inseparable until one finds students who can understand the theory of say the 'gambler's fallacy' and yet will still insist on betting on red in roulette if the last few spins landed on black. There is a belief that the outcomes are not independent.

According to Mevarech (1983) these misconceptions can be deeply ingrained in the student's knowledge base and the student may possess subjective feelings of understanding. Researchers (e.g. Shaughnessy, 1982; Mevarech, 1983) have acknowledged that simply more exposure to the correct rules or the laws of probability may not be sufficient to overcome misconceptions. Kahneman and Tversky (in Shaughnessy, 1982, p.788) corroborate this when they showed that students with substantial background in probability and statistics shared misconceptions with the more naive.

2.5 Correct Intuitive Conceptions

However, much intuitive probability knowledge acquired spontaneously is correct. Piaget and Inhelder (1975) divided the development of intuitive probabilistic knowledge into stages; from the inability to understand randomness, through to understanding the law of large numbers. Fong, Krantz and Nisbett (1986) speak of correct abstract intuitive concepts as "statistical heuristics" (Nisbett, 1983, p.225), an example of which is an intuitive version of the law of large numbers. Fong et al. (1986) try to explain intuitive heuristic approaches as being some universal generalized abstract rules that correspond with statistical rules. While these authors use the results of Kahneman and Tversky's (1972) research as an example of failure to use generalized cognitive structures, the research rather supports the notion that students are using these, and that they possess and use the necessary computational and procedural skills, but that their erroneous reasoning holds because they assimilate new knowledge into inappropriate structures.

Whether the intuitive versions are correct or incorrect, what is important is how these affect the learning of probability, the students' ability to solve probability problems, and their sense of the meaning of probability and its rules. In other words, if the intuitive approaches can converge with the formal they will make sense to the students and if the learning of the formal can lead to greater problem-solving ability and correctness in an everyday context, then the disjuncture between the scientific and intuitive closes at a higher level of generalization.

If we wish to sensitize our students to intuitive heuristics so as to provide them with the option of determining their usefulness or otherwise, we have to recognize the necessity of operating at a higher level of generality. Metacognitive awareness of the possibilities of rejecting preconceptions must be present if students are to gain control of their learning of probability.

2.6 Importance of Linking Intuitive and Scientific Concepts for Probability

Many teachers agonize over whether to start with abstract or empirical probability concepts (Steinbring and von Harten, 1982, p.278). Garfield and Ahlgren (1988) attribute the distaste students have toward Probability Theory to the students having been exposed to the study in a highly abstract and formal way. However, while some students do not even understand the goals of assigning probability values, a return to the empirical or the coaching of new knowledge in students' everyday knowledge (adequate or inadequate or erroneous) would be worthless. Thus in this report I contest the necessity of working from both the empirical and the formal, claiming that the abstract and formal ways are important for conscious, controlled understanding of the discipline, for enhancing and developing correct intuitive understanding, and for the development of the abstract concepts themselves, while the intuitive are important for a sense of meaning. Both formal and intuitive must be understood within a knowledge of the epistemic nature of probability knowledge. Kelly (1986) argues that when teaching concepts like 'independence' and 'mutually exclusive', it is important to provide not only the scientific but the intuitive as well, saying that "we seldom help students to bridge the gap between fuzzy distinctions apparent in nature and the very rigid distinctions made in mathematics" (p.3). My thesis claims and explains the necessity of the intuitive and scientific co-developing, so that the scientific helps to change any incorrect intuitive notions and the intuitive helps to provide meaning for the scientific. Regarding independence, this argument would claim that there is in fact some objective notion of independence within the practice of Probability Theory which gets embodiment from everyday notions and from theoretical formulations rather than remaining in separate discourses. So with the gambler's fallacy, ideally the scientific notion of independent events will protect the gambler from wishful thinking in the game of roulette, provided one assumes the randomness of spins. We need to link the empirical, the intuitive, and the formal in explanation, and in addition show the formal to be linked to itself in a consistent system. Some teachers of probability are certainly trying to do this. Walton and Walton (1987), for example, link student experimentation with computer simulation and mathematical methods. Comparison of the three sets of results (actual trials, computer simulation, and mathematical solution) "provides a surprising and motivating reinforcement for students ... The elegance of the mathematical solutions is contrasted with the awesome number-crunching ability of the computer and the "doing is believing" experience of the actual trials" (ibid., p.3). Provided all this is seen within the context of the goals of Probability Theory, the students may understand the conventions and in addition understand that they are more than just conventions.

2.7 A Psychological Explanation

I have mentioned the important role of the teacher, hinted at the importance of metacognition and introduced the notion of goals. Many teachers and researchers in probability and statistics or in even in mathematics in general recognise the need to take these areas into account; they attribute student failure to solve problems, given the apparently necessary knowledge, to non-cognitive and metacognitive factors inhibiting the use of that knowledge (Garofalo and Lester, 1987). However, these teachers and researchers fail to provide a comprehensive explanation of why this should be the case. Studies regarding affect in the literature have commonly involved the correlation of attitude and performance but without a theoretical reason for the importance of goals and motives and how they will link with components of problem solving, like confidence. As Garofalo and Lester (1987, p.3) claim, "it is safe to say that the overwhelming majority of problem-solving researchers have been content to restrict their investigations exclusively to cognitive aspects of performance." So that, "... despite enthusiastic development of new instructional materials, little seems to be known about how to teach probability and statistics effectively" (Garfield and Ahlgren, 1988, p.45). It is claimed that a "major reason for the limited success of remediation is that research ... has not attended to underlying psychological mechanisms" (ibid., p.55).

Narode (1987, p.15) asks: "What are the psychological processes that will enable students to use ideas, that they in fact have but cannot or do not use?" He claims that this is the question which motivates the study of metacognition in mathematics and science, but he does not answer the question in any way that elucidates the processes involved in needing to invoke the notion of metacognition. Again what is missing is a comprehensive psychological explanation for why the psychological aspects of learning must be considered as central to understanding the teaching and learning of Probability Theory.

Separating student and subject matter, process and product, learning and development seemed to contribute to the problems in learning or teaching thus delineated. I hypothesized that both students and teachers were less secure when searching for what was subjectively unknown for the student, and that the student as a psychological subject was being valued at less than the subject matter. Mellin-Olsen (1987) is critical of the 'liberal theories' of 'process' and 'child-centred' education. Yet one cannot deny the importance of placing the learner and the process of learning centrally when using the Theory of Activity, which Mellin-Olsen uses explicitly. What we are aiming for is a change in the students' thinking while still acknowledging that this cannot be without content, that "[t]he necessary content of thought is the "analysis, synthesis and generalization of something, ... the conditions and requirements of a problem to be solved, etc." (Brushlinskii, 1987, p.70). There is probabilistic knowledge to be acquired that must serve the students in their other subjects and in their future careers, but in order for this knowledge to be of use to them it should be meaningful in a subjective way as well as being formally correct.

I will provide an in-depth theoretical justification from a psychological perspective for teaching both the student <u>and</u> the subject matter. I too will use the Theory of Activity and the work of theorists who work within its framework. I will argue that educational activity needs to be treated in a unified way, with a focus on teachers and students as psychological subjects. I will argue that within educational activity, students and teachers both need to regain a sense of control and that this can be achieved via metacognition on two levels, that of the teachers' management of educational activity and that of students awareness of their own thinking processes.

My first step in the research was to understand the problems in theoretical terms; for this I returned to Vygotsky's notions of scientific and intuitive concepts which are embedded in the Theory of Activity. Vygotsky's theory provides a model which considers both the internal mechanisms of cognition, as well as the external mediation of an historically rooted discipline. Hence I first explicate some pertinent aspects of the Theory of Activity and then progress through a discussion of mediation, internalization and the 'Zone of Proximal Development', relating these to the learning and teaching of probability. These will be developed so as to incorporate the concept of metacognition into the framework of Activity Theory, taking due account of affect which will broadly incorporate motives and goals, confidence and control.

3 CHAPTER THREE: THE THEORY OF ACTIVITY

> A "study of culture and cognition must incorporate the study of both the systems of social relations and of internal (cognitive) activity".
>
> Cole, 1985, p.159

3.1 Introduction

I shall use Activity Theory both implicitly and explicitly in order to explain the problems arising in the teaching and learning of Probability Theory. I will draw extensively from the work of Vygotsky[2]. This chapter will deal explicitly with some of the fundamental tenets of the theory.

The Theory of Activity is itself fraught with controversy[3] and to enter into the debates surrounding central issues is clearly beyond the scope of this research report. However the controversies do not detract from the usefulness of the notion for my work, and there is sufficient common ground to proceed. This introduction will serve to outline those aspects of the Theory of Activity considered important to the study. Many of these will be developed further in this research report.

3.2 Activity Theory Explaining Consciousness

The title of this research report indicates that the relationship between culture and cognition is considered important. There are two major interpretations of the Theory of Activity, one broad and one narrow in nature.

The broad conception of Activity Theory will first be invoked. It provides an explanation of the relationship between culture and cognition by regarding mind and activity as a unity, and by theorizing how social relations contribute to the way in which mental activities are organized. Culture is used broadly to mean social relations. These social relations provide the external impetus for the construction of an internal plane of consciousness.

Vygotsky suggested the use of the concept 'activity' as a means of overcoming tautological explanations of consciousness by consciousness, and of overcoming explanations of behaviour by behaviour. Activity provides a different level of analysis, by providing a material means of 'accessing' consciousness. Mental acts are formed by and reflected in activity.

Thus activity can be seen as a unit of analysis which bridges the historically different approaches of sociology and psychology to the study of cognition. In essence, Activity Theory allows us to see the social and the individual as a unity and cognition as processes within this unity.

For Vygotsky, who operated within this tradition where mind is inseparably linked to culture, higher mental functions are related to particular social knowledge, while lower mental functions are biologically driven. It is into the category of higher mental functions that we place the kind of thinking involved in Probability Theory, that is, functions which are cultural in origin.

Consciousness is formed in successive activities but is not an aggregate of these. Rather, it is a system in itself, with structures which are inseparable from systems of human relations, and which contain within and between them the possibility for development. This is important for later assumptions about the development of concepts and the nature and development of cognitive structures.

3.3 Mediation

According to Vygotsky, consciousness is not determined directly by objects and actions but by the mediation of social beings in relation to these. The concept of mediation is central to a theory which posits the continuity of the social and the individual. It is primarily with respect to mediation that Vygotsky differs from Piaget. For Piaget, direct interaction with objects, material or mental, leads to development, while for Vygotsky, direct interaction needs to be given social meaning by mediation. In an extension of this, consciousness or higher mental functions can be mediated by mental reflection of the objective world using psychological tools or semiotic systems; for example probability itself, which uses the symbolic and linguistic aspects of the subject matter, which in turn are socially developed in order to impose structure on uncertainty.

There are various interpretations of the notion of mediation. There is the sense of mediation providing the necessary framework by which to interpret stimuli and act on the environment, that is the sense in which mediation is used by Feuerstein (1979, 1980). Or, in mediation, some sign or concept is given social meaning by another who is in command of mature functioning in that area, as would be the case with a teacher of probability creating for the learner a new sense of significance regarding subject matter, over which the learner must gain voluntary control. Consciousness of the importance of gaining this voluntary control and the means for doing so is to be part of the concept of metacognition which I will develop, arguing that metacognition is consciousness of control.

The subjects of activity are neither purely passive receivers nor creators. Knowledge is neither determined by the external world acting on innate or learned properties of the mind, nor is it acquired by conscious reflection only, but rather by a dialectic of the two in activity. Any external act is mediated by internal processes, which are reflected in external activity (Leont'ev, 1979). It is in the Theory of Activity that actions and schemata are the outer and inner perspectives of an activity (Cole, 1985, p.159).

3.4 Activity as the Fulcrum

Activity itself can be seen as the mediator, the middle link which mediates objective and subjective reality:

object<--->activity<--->subject

where both object and subject are seen historically and culturally. Objects are transformed into subjective form and activity is converted into objective results or products (Leont'ev, 1979, p.46). "The concept of activity thus was perceived as an actualization of culture in individual behaviour, embodied in the symbolic function of ... speech systems" (Kozulin, 1986, p.267). On the other hand "[t]he concretization of activity ... appears as a psychological mechanism that creates new symbols and word senses that may eventually be incorporated into the stock of culture (Kozulin, ibid., p.269). Probability Theory in the individual is an actualization of years of accumulated development that nevertheless contains within it the possibility of further development.

The potential of the Theory of Activity as an analytic tool comes about through the fundamental nature of action. People act, and in so doing alter the objective environment and the subjective self, while the acts themselves are constrained by internalization of the environment. In a sense, then, activities or tasks are the focal point or fulcrum of the dialectic between the person, in the full subjective sense and the environment, in the full social sense, in reciprocal transformation. Mental functioning develops and becomes manifest in the process of activity. This has a bearing on learning in that activity-based theories of learning include a study of "'the internal link' of the learning process, not as an aggregate of individual mental functions, but as the [learner's] active engagement as a subject and a personality ..."(Davydov and Markova, 1983, p.54).

3.5 Motives and Goals

In the narrower sense, the term 'activity' can be understood in terms of its structures, as outlined by Leont'ev (1979). Leont'ev, whose work on activity is regarded as seminal, claimed that there were different levels of analysis of activity: those of motives (will to action) in conjunction with activity, goals ("anticipatory reflection" (Lomov, 1982, p.72)) in conjunction with action, and operations in conjunction with constraints[4].

The active subject in interacting with the environment is not without motives and goals, at least as subjectively experienced in affect. Together these goals and motives play a determining role in development, organizing mental processes in activity, for example, the level of concentration, selectivity of perception, and so on. External conditions constrain the motives, goals, and operations of activity in that the objects, means and conditions are usually given.

3.6 Implications of the Theory of Activity

I will begin with a broad outline of some of the important implications of the theory.

Firstly, in that consciousness is developed in the process of human relationships and in the unity of subject and object, all aspects of intellectual development and learning are be seen in interrelationship.

Activity, by dealing with that which is specifically human, that is, self-consciousness and intention, presupposes purposiveness and the importance of the cultural meaningfulness of objects of social experience. In particular, there is continuity between the cognitive and the affective components of a mental act which are regarded as ontologically inseparable. "A cognized object, reflected in the mind, touches to some extent upon the needs, motives, and interests of the individual, evoking in him a specific emotional-volitional affective relation (aspiration, feeling). It is only to that extent that he knows himself." Brushlinskii (1987, pp.68-69) thus argues for the organic unity between the cognitive and affective aspects of mental phenomena and that it is this that must define "the new interpretation of the 'unit'... in psychology" (ibid., p.69) .

3.7 Affect and Meaning

Vygotsky made the relationship between cognition, affect and meaning explicit by referring to

> "...the existence of a dynamic system of meaning in which the affective and the intellectual unite ... that every idea

contains a transmuted affective attitude toward the bit of reality to which it refers."

Vygotsky, 1986, p.10

I will develop the idea of *affect* as self-knowledge through the notion of 'metacognitive experience'.

Some recognition of the importance of the interrelationship of affect, belief and culture has been addressed in the literature on mathematics education, for example Lester and Garofalo (1987). These authors however do not adequately deal with the essential dialectical relationship involved, as they assume a static view of socio-cultural conditions. According to their model (1987, p.11-12), affect and belief cannot influence the socio-cultural but can influence knowledge and control, whereas the Theory of Activity would place all these in dynamic interrelationship.

3.8 Communication and Meaning

For learning to take place it is necessary to "create conditions that will enable activity to acquire personal meaning" (Davydov and Markova, 1983, p.57). In order to generalize over uncertainty, by imposing structure on irregularity, probability concepts have been developed which can be communicated via language and signs.

The notion of joint activity is important, for it is within the entire system of social interrelatedness that activity takes its meaning. "Thus it is not just objects and tasks that guide an individual's acts. The individual 'fits' his act to the acts of others. This raises the dynamic level of activity" (Lomov, 1982, p.84). The learning of the concepts of probability needs to take place in co-operative communication between student and teacher in joint activity, whether this be direct communication with other people or indirect communication through books, tapes, and so on.

"[J]oint (as well as individual) activity presupposes planning, which is realized in the process of communication" (Lomov, 1982, p.84). Communication, as it were, pervades joint activity, playing an organizing role. Thus communication organizes and reflects organization in joint educational activity. It is in communication that meaning can be negotiated using the sign conventions and generalizations of the system of probabilistic knowledge as tools.

3.9 Educational Activity

In many activities the object of activity is that some product, be it material or ideal, is transformed by the execution of the activity. Assimilation during the course of activity is a by-product. In educational activity, however, the explicit goal is a change in the students themselves rather than a change in the objects of student activity. Students of probability in service-courses do need to know the theory and be able to solve problems independently, but the problems they are set are problems with pre-existing solutions, set and solved by many generations of teachers and students before them. Hence in the learning and teaching of Probability Theory, it is not the solution of the problem that is so important as what the process of solving the problem does for students' cognitive development or how it reflects their knowledge and abilities. The students thus become both the <u>subject</u> and the <u>object</u> of probability activity while the subject matter becomes both <u>object</u> and <u>means</u> for acquiring new knowledge and methods of action. The development of the subject is by means of the transformation of the object, which becomes a psychological tool. The transformation of the object then is a secondary goal in the educational setting. The product of probabilistic activity becomes important where it enables educators to understand the psychological processes involved in the students' reaching such a product. Clearly this perspective would be different for practising statisticians outside the educational context.

3.10 Aim of Educational Activity

Service-course students are taught Probability Theory as an adjunct to their other courses. Hence it is not intended that they operate as practising statisticians in the 'real world', although some may develop that interest. Goals vary from intentions that students develop basic statistical knowledge and skills in order to be at least partially 'literate' and 'numerate' where it concerns research in their area of study or practice in the future work-place, through the fostering of the acquisition of basic definitions and theorems, the development of skills and the ability to trace relationships, to including aims of facilitating the students' ability to be able to solve new problems independently. I contend that aims must include a consideration of the subjective sense of meaning of Probability Theory for students, as well as a consideration of their knowledge of their own ability to acquire probability knowledge[5]. Education cannot be reduced to the transferral of knowledge[6] or the acquisition of skills, but must include the development of the mental structures of the learner, cognitive, metacognitive and epistemic, coupled with an awareness of the importance of affect.

Where learning is to be deliberate and the content of the knowledge to be learned contains generalizations, mental development of probabilistic thinking is no longer a by-product of activity but its central goal. Educational activity can be seen as "a system of organization and methods for conveying to the individual socially formed experience ... " (Davydov and Markova, 1983, p.59).

> "The reversal in the relative priority of ... knowing how and knowing that, hinges on the ability to transmit culture, as a system of rules, independently of its material embodiment. It requires of the novice that he first learn not by experience, imitation and reinforcement from others who have done likewise ... but rather that he learn in advance the abstract principles by which he can generate artefacts of his own, in the light of a critical evaluation of the efforts of his predecessors. This is the difference between the passive absorption and active acquisition of knowledge .... It is also, and most fundamentally, the difference between learning by example and learning by teaching."
>
> Ingold, 1986, p.356

3.11 Usefulness of Activity Theory for Explaining Probability Problems

Thus if it is wished to understand the learning of probability it is useful to invoke the level of analysis of activity which addresses not only the practical actions of statistical behaviour but also the meaning of the activity for the subjects. In other words, use of the Theory of Activity will allow for a psychological study of the learning and teaching of Probability Theory. It denies the separability of activity, motive, action, goal, means and operation in the context of the inseparability of subject and object, society and the individual, and process and product.

We want individual change but that can come about only through social relations with those who are more competent in the discipline, and with the students' active involvement with the subject matter. Thus to ignore a psychological approach to teaching, concentrating only on the subject matter, is to distort educational activity. Misconceptions or a sense of meaninglessness and helplessness will abound if students do not know the how and why of theoretical probability approaches. They will then rely on algorithmic approaches and will not be able to cope with novel[7] problems, as evidenced in the classroom situation.

As educationists we need to understand how the development of concepts takes place within educational activity in order to facilitate it. In order to understand how to achieve this, we need to understand the transformation of activity into its subjective formulation in the psychological subject. What becomes clear is that it is necessary to look at the relationships that exist between learner and teacher and to decide how these can aid our understanding of the communication process. This in turn needs to be seen within the broader context of the relationship between the social and the individual, which is not oppositional. If we accept that Probability Theory is a symbolic tool that is culturally created and culturally transmitted, we need to understand how the learning of Probability Theory can fundamentally alter the nature of thinking about uncertainty. The teacher and student need to "ascertain and create conditions that will enable activity to acquire personal meaning, to become the source of the person's self-development" (Davydov and Markova, 1983, p.57).

To evaluate the effectiveness of educational activity in qualitative terms requires looking at not only the students' "actions with the to-be-learned material but also his methods of control, evaluation, and self-regulation when dealing with the materials" (Davydov and Markova, 1983, p.52, 63). In order to achieve this the student needs to become the subject of his or her own activity and assume "an active orientation to the real world around him, toward himself and other people" (ibid., p.57). Hence, the consideration of the notion of metacognition becomes important.

For human action to have the properties of activity, the subject must accept the goals toward which his or her actions are directed. Many of the goals of service-course students are prepared goals, not of their own making. It may require considerable self-control to accept them. This is again where the notion of metacognition demands investigation, for it is important that students are able to realize the limitations and constraints under which they are working as a prerequisite for dealing with them.

It is the task of a psychological analysis to elucidate how reality is subjectively reflected in the process of activity and what underlies mental regulation of that activity (Lomov, 1982, p.81).

4 CHAPTER FOUR LEARNING AND DEVELOPMENT

4.1 The Assumed Distinction Between Learning and Development

In order to address the problem of the development of probabilistic concepts, it is important to discuss what is meant by learning and development and to investigate the relationship between them. To address the problems associated with the concepts of learning and development is beyond the scope of this report and my dealing with the issue must therefore be superficial. Even within the work of a single constructivist theorist, learning and development can be used in different ways. My assumption is that learning is the assimilation of new information into existing cognitive structures or the acquisition of new actions during this process, while development is seen as more than learning, involving changes in cognitive structures characterized by "qualitative changes in the level and form of the abilities ..." (Davydov and Markova, 1983 p.59). If we make a distinction between the mastery of knowledge, skills and specific abilities, and the "mastery of methods and universal forms of mental activity" (Davydov and Markova, 1983, p.58), the latter will refer to development. In this way development is more general than learning, the former relating to the structures themselves and the latter to ways of organizing and utilizing those structures.

Development to Vygotsky was a dynamic process, uneven, with reversals and progressions. It was seen as the "progressive emergence of conscious awareness of concepts and thought operations" (Vygotsky, 1987, p.185), where consciousness itself is constructed by social awareness.

4.2 Internal or External Generators of Change

In consideration of what it is that generates cognitive development, Piaget looks at internal generators, that is, internal to the organism, while Vygotsky looks at external generators of cognitive change. Craig (1985) synthesizes these approaches by claiming that even if change is externally generated, it is the individual who needs to reconstruct the cognitive structures and processes internally. Thus cognitive development can be both externally and internally generated. While I acknowledge that maturation is a necessary prerequisite for coping with an increasing complexity of cognition, mathematical statistics is clearly social in origin and hence it is the socially transmitted aspects of development that concern us. As educationists we are looking at external generators that will elicit the appropriate internal changes in individual knowledge development.

We are thus seeking ways of eliciting internal changes by external means. Hence learning must lead development. The internalizing of social forms of knowledge and the mastery of specific knowledge skills and abilities must generate qualitative internal transformation allowing for a broader understanding of the cognitive, metacognitive and epistemic nature of probability. Although Vygotsky (1987, p.206) does not cite his research sources in this work, he makes the claim that "[r]esearch has shown that instruction always moves ahead of development" .

4.3 Scientific and Intuitive Concepts and their Relationship to Learning and Development

> "I shall call a culture unselfconscious if its form-making is learned informally, through imitation and correction. And I shall call a culture self-conscious if its form-making is taught academically, according to explicit rules."
>
> Alexander, in Ingold, [illegible] p.317

Vygotsky's theory can be used to advocate the developmental lead that systematic teaching can play. Vygotsky provides the analogy of learning a second language in order to explain how scientific and spontaneous concepts are connected. The concepts differ both in their developmental paths and in how they function, yet there is a mutual dependency. A second language by definition requires the existence of a first which was learned unselfconsciously.

Analogously, the learning of scientific concepts depends on the level of everyday concepts in that a "conceptual fabric" (1987, p.180) is presupposed. In the same way as the learning of a new language can enhance one's consciousness and knowledge of one's first language, so the learning of the scientific can raise consciousness about previously existing everyday knowledge, providing for greater control and voluntary use in more generalized ways. Thus teaching scientific concepts can prepare the way for the developmental path of the spontaneous by providing structures for their upward development, while the spontaneous can provide embodiment for the scientific. In a sense then consciousness moves downward from the scientific while meaning moves upward from the everyday.

> Having already travelled the long path of development from below to above, everyday concepts have blazed the trail for the continued downward growth of the scientific concepts; they have created the structures required for the emergence of the lower or more elementary characteristics of the scientific concept. In the same way, having covered a certain portion of the path from above to below, scientific concepts have blazed the trail for the development of everyday concepts. They have prepared the structural formations for the mastery of the higher characteristics of the everyday concept.
>
> Vygotsky, 1987, p.219

Many mathematics educators advocate moving from spontaneous concepts towards the scientific (e.g. Fenema, personal communication, 1989), starting with probability notions the students already have and moving toward the scientific. Reference to known classes of phenomena held by the student are said to provide the student with a sense of the meaningfulness of the new knowledge. This creates problems if students have misconceptions, for then the correct scientific concepts have to eradicate these spontaneously acquired concepts which are meaningful to the students. If the scientific and everyday concepts co-develop, motivation for restructuring can occur when cognitive conflicts appear between presently held concepts, and scientific concepts being introduced. For Vygotsky, spontaneous thought and non-spontaneous thought would have antagonistic <u>and</u> similar features. Vygotsky believes that deliberate instruction should introduce scientific concepts or generalizations that are not initially connected to personal meaning, but which rather by their abstract nature generate consciousness of their meaning within a system. From this co-development of both the scientific and everyday concepts, 'true' concepts are formed by the merging of consciousness and personal meaning:

Deliberate learning——scientific concepts——┐
　　　　　　　　　　　　　　　　　　　　　　├—true concepts
Spontaneous learning——everyday/intuitive concepts—┘

<u>Figure 1: The Development of True Concepts</u>

Believing Vygotsky's scientific and everyday concepts to be mutually developing does not devalue everyday concepts. It is not so much a displacement, nor a broadening or enriching, but a reconstruction. Everyday concepts may still have a function in everyday life but for an academic discipline the students need to develop 'true' concepts that will be both meaningful to the subject and over which the subject has conscious control.

This research argues that meaning and consciousness must both be primary considerations for teachers as they assist students in the construction and reconstruction of Probability Theory.

4.4 Development of Probability Knowledge

Development of probability knowledge then takes place through the assimilation of socially evolved or historically formed methods of dealing with the world of uncertainty. It is more than the simple accumulation of probabilistic 'facts' but rather the active engagement of the individual in reproducing and transforming or, in fact, reconstructing these in 'subjective' form.

4.5 The Nature of Understanding Probability Theory

It is necessary to consider what is meant by the term 'understanding' with respect to Probability Theory, as there is no precise definition of understanding nor any "specification of the processes such an understanding might affect" (Myers, 1982, p.379). The term 'understanding' is often used intuitively and hence I need to elaborate on its employment in Probability Theory.

Understanding in probability at the service-course level is commonly assessed by two broad categories of problem: 'formula' problems and 'story' problems. The former presents certain values and conditions and the students effectively substitute values. In examining knowledge of independent events, what follows is an exemplar. If $P(A)=0.6$, $P(B)=0.4$ and A and B are independent events then $P(A \cap B) = ?$ The story type, on the other, hand requires the student to categorize the problem appropriately first.

Clearly the ideal measure of understanding would be the ability of the students to come to correct problem solutions by the most elegant methods. I do not want to become involved in a discussion of assessment at this stage, but it does affect how students perceive their own understanding of probability and how teachers perceive the understanding of their students. If it is assumed that understanding may be partial, at what point do teachers consider students' understanding to be adequate, or at what point are students to be satisfied with their own understanding? Must the student be able to organize the subject matter so that accurate predictions or answers can be made, via accurate manipulations and transformations of the material? According to Myers (1982) understanding should aid categorization and translation of problems; but should the novice in a service-course be expected to be able to present complete problem solutions? To be able to do this as well as know that they are correct, places cognitive demands on the novice that may lead to the helplessness that I often witnessed in students.

As with reading a novel, understanding needs to be more than understanding the statements made by individual sentences, requiring an understanding of the meaning of the text produced by their interrelationship and significance. It is all very well to be able to do problems involving 'and' or 'or' in the text as key words, but if the students do not fully understand the notions of intersection and union they will not be able to do problems when these terms are implied by conjunction rather than explicit. The students need to understand more than just the definitions and theorems in isolation if they are to show understanding of the subject: an understanding that will manifest in the ability to transform, relate, extend and elaborate upon givens, or at least degrees of such. Thus understanding must include a consideration of a subjective sense of '<u>meaning</u>'. Bishop (1985) speaks of the personal nature of the meaning of a new mathematical idea, one that must make connections with present knowledge.

4.6 The Development of Concepts in General

The idea of the development of concepts is important for understanding difficulties experienced by the students and teachers of probability. A concept is not just a "collection of associative connections learned with the aid of memory" (Vygotsky, 1987, p.169); a concept is rather an act of generalization, a reconstruction in the individual. As generalizations, concepts are related within a system, with higher-order concepts mediated by lower-order concepts, which develop by means of interconnections and interdependencies. If a teacher attempts to teach a concept in its final form, the word but not the concept may be learned which then limits the ability to apply the concept. The group of students described in Chapter 1 who were all working with Bayes theorem failed to see that they were utilizing the same concept as the student working from first principles, as they were blinded by an empty label. They exemplify the failure of the development of the concept of conditional probability. Students who do not have a command of a concept feel helpless in even attempting the solution of problems that are new to them. On the contrary, those students who have mastered the functions by achieving consciousness of them in their interrelationships, have voluntary control as the "reverse side" of the "conscious awareness" (Vygotsky, 1987, p.187), or have at least conscious control to a certain extent. Development is then seen as the change of internal relationships among functions.

4.7 The Development of Scientific Concepts

If scientific concepts themselves develop, 'transmission' from teacher to student is clearly not the mechanism of learning. Markman (1981) cites research which indicates that the more facts one has on a topic the more they interfere with each other, making access to them difficult. Hence, paradoxically, the learner becomes less proficient unless the facts are integrated into coherent units, using organizational skills. This must be seen in a dialectical relationship with the freeing of cognitive capacity allowing for inferential processing, or additional control. It seems that the service-course students are not perceiving the system which constitutes probability, and I hypothesize that this is due to a failure by the teachers to organize and mediate the system and its relations to the students in a self-conscious manner. The students are thus unable to use the concepts deliberately (not being able to get going), unable to apprehend inconsistencies (by isolating concepts from their relations and thus becoming satisfied with a numerical answer albeit meaningless in the system), and are unable to make sense of the laws.

4.8 Strengths and Weaknesses of Everyday and Deliberate Routes to Learning

The weakness of everyday concepts is that they do not lend themselves easily to abstraction and consequently freedom in their use, while their strength lies in empirical understanding. On the other hand, the weakness of scientific concepts is that they can appear as empty verbalisms, as our students so frequently indicate to us. The strength of the scientific is that they can be used in general situations. But it appears that for the struggling service-course students the strengths are absent while the weaknesses are evident, that is, the meaninglessness of the general in terms of a paucity of interconnections with personal experience, facts which have not been incorporated into a system of conscious interrelationships, and knowledge which cannot easily be applied in concrete situations. Vygotsky (1987, p.191) says:

> "Scientific concepts have a unique relationship to the object. This relationship is mediated through other concepts or the generalization and mastery of concepts emerges for the first time. And once a new structure of generalization or abstraction has arisen in one sphere of thought, it can - like any structure - be transferred without training to all remaining domains of concepts and thought. Thus, conscious awareness enters through the gate opened up by the scientific concept".

4.9 The Necessity of the Co-development of Everyday and Scientific Concepts

Conscious instruction of concepts can provide the source and impetus for the development of concepts. By being introduced to formal relationships between concepts, the student "rethinks relationships between things ...[which] results in a radical reorganization of the thinking that provides for the reflection of reality and the very processes of human activity" (Luria, 1976, p.4, in Craig, 1985, p.80).

Students do have some intuitive sense of probability (see section 1.3 above) and they can deal with some of the scientific concepts at a 'rule' level, but the two do not merge to provide a sense of the meaningfulness of the body of knowledge. As a consequence the students lose confidence in their ability to cope with the expected standard of performance. The problems that I have listed indicate a continuing disjuncture between the lines of development of spontaneous and scientific concepts, contributing to the perpetuation of the weak aspects of the concepts, rather than the development of the two in a way that allows for the increase in their strengths. According to the Theory of Activity the two are necessarily united in a single system. If these concepts are taught with such a unity in mind, teaching can assist in raising student thinking to continually higher levels of generalization. Understanding and knowledge in probability can become belief. On the other hand, if the students are allowed to retain these two types of concepts in parallel, they will continue to feel helpless in the light of the perceived meaninglessness and uselessness of Probability Theory to them. I will argue that what is required is a level of self-consciousness that has to be part of deliberate instruction.

4.10 The Relationship between Teaching and Learning

This then requires an analysis of the relationship between teaching and learning.

> "The educational process has its own sequence, logic and complex organization ... It would be a tremendous error to assume that there is a complete correspondence between the external process and the internal structure of the developmental processes that it brings to life."
>
> Vygotsky, 1987, p.206

Conventionally, teaching is the activity of the teacher and learning the activity of the student. However, invoking the Theory of Activity inhibits the polarisation of the teacher and learner in educational activity. Both have to be actively engaged in the students' mastering of the socially evolved forms of dealing with uncertainty, that is, with self-consciousness leading to control and a subjective sense of meaningfulness.

I have often heard teachers of statistics say that the students just do not have what is needed to undertake the work. If teachers accept Vygotsky's research evidence as indicative that learning can lead development, attitudes to teaching must significantly alter and we as teachers must assume responsibility for the development of our students in order that they may undertake the work we expect of them. This is becoming an urgent issue to consider in South Africa, especially for service-course students where some statistical literacy is needed in order for them to contribute to the community in other ways.

4.11 Experts and Novices

The teacher is an expert and the student a novice as far as knowledge of the body of Probability Theory is concerned.

> "Experts, by definition, know more than novices. They have accumulated more specific facts in a given domain than novices, who may have more vague, general, and perhaps erroneous beliefs. What also comes with expertise, ... is a greater systematization of knowledge. Experts detect more patterns, see more relations between discrete events, and know more principles that unite diverse facts than do novices."
>
> Markman, 1981, p.76

The novice has to concentrate on aspects that the expert has routinized, thus has greater processing constraints, and cannot generate as many expectations as the expert. Novices, regardless of age "exhibit similar patterns of behaviour ... demonstrat[ing] the crucial role of experience and expertise in cognitive monitoring" (Brown and DeLoache, 1978, in Baker and Brown, 1984, p.358). As such there is what Flavell (1981a, p.57) refers to as a "phenomenological chasm" between the states of the minds of those who know and those who are to become knowledgeable.

4.12 Situation Definition and the Establishment of Intersubjectivity

According to Wertsch (1984) a situation definition is an active creation or representation of the context by participants. The reality of many a learning-teaching situation in Probability Theory in this country involves a mismatch of the way teachers and students perceive the learning context and the knowledge deriving therefrom. An extreme example of this comes from one of the students in the Academic Support Programme for Quantitative Techniques (formerly, Mathematics and Statistics) who refers to the probability course as "brutal" (see Appendix 1). The teacher has a sense of the system of Probability Theory while the students may only see a series of disintegrated statements. If development cannot be conceptualized in quantitative terms, but involves a shift in understanding regarding the objects and the actions by which one operates on those objects (Wertsch, 1984), then communication between teacher and student is required that will enable an understanding of the way the novices are constructing the activity in order to establish some grounds for negotiation toward intersubjectivity. Wertsch defines intersubjectivity as the state of sharing the same situation definition and knowing that you share the same situation definition (ibid., p.12), that is an 'interpsychological situation definition'. In order for teaching and learning to take place, intersubjectivity is needed. Wertsch refers to the mechanism which makes negotiated intersubjectivity possible as semiotic mediation, which relies on the use of signs and language.

Given that different situation definitions exist for students and teachers, how then is one to accomplish such a situation redefinition in order that the intersubjective condition of shared situation definitions can prevail? This again becomes the responsibility of the teachers who may have to temporarily suspend their own intrapsychological situation definition, and their more sophisticated views of probability, in order to enter into a communicative framework with the students, that is understanding the way the students are conceptualizing the work at hand. A temporary change on the part of the teacher might result in a permanent change of situation definition for the student, resulting in a qualitatively different ability to understand the activity. This does not necessarily mean entering at the students' actual level of development as a Piagetian model would suggest. Rather it refers to an understanding of how the students perceive the situation than to teaching at the level of their present state of knowledge. Vygotsky argues against teaching from the present state of knowledge alone and speaks of raising the students' level of understanding by teaching beyond their present knowledge, in fact by inducing consciousness through scientific concepts for true conceptual development. Vygotsky's sense of the downward path from the scientific with the embodiment by everyday concepts implies a leading of students from above, rather than an hierarchical ascent from below in a taxonomic sense.

From the communicative action of negotiating common ground, a dialectic between task and participants comes into being which leads to individual cognitive and social change, and to students' knowledge of Probability Theory, as well as teachers' understanding of the meaning of Probability Theory for the students, and the adjustment of their teaching to this. It becomes important to consider the process of how to transmit probabilistic knowledge from the expert teacher to the novice student that will enable the student to develop adequate control of the knowledge so as to be able to access it for problem solving.

5 CHAPTER FIVE: INTERNALIZATION

5.1 Introduction

We need to consider how culture affects cognition in order to determine how social knowledge is learned, and correspondingly how this can be best taught.

In this chapter I shall discuss the concept of internalization, the process which epitomizes the inseparability of culture and cognition. Internalization will be addressed in close relation to the notion of the 'Zone of Proximal Development'.

5.2 Social Origins of Consciousness

Vygotsky's 'law of cultural development' states:

> "Any function in the child's cultural development appears twice, or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category. This is equally true with regard to voluntary attention, logical memory, the formation of concepts, and the development of volition. We may consider this position as a law in the full sense of the word, but it goes without saying that internalization transforms the process itself and changes its structures and functions. Social relations or relations among people genetically underlie all higher functions and their relationships."
>
> Vygotsky, 1981, p.163

Vygotsky (in Wertsch, 1985, p.166) argued that "[a]ny higher function <u>necessarily</u> goes through an external stage in its development <u>because</u> it is initially a social function." (emphasis added).

The development of consciousness itself then starts with the external (social) which is constructed through self-reflection, by way of other-reflection. However, consciousness must not be seen as an internal reprint of relations in society, but is rather formed in the transformation of these relations in internalization.

For Vygotsky (in Wertsch, 1979, p.2), then, social reality in the broad sense plays a primary role in the development and nature of internal higher mental functions. The means for influencing or being aware of ourselves were originally means of influencing or being aware of others, or others' means of influencing or being aware of us. If we substitute the 'teaching of Probability Theory' for 'influence' and 'metacognition' for 'being aware of ourselves', we need to understand the impact of ~~the~~ theoretical position of the social origins of higher mental functions on educational practice, and in particular, how others' means of influence become internalized to become our own in an educational setting. We need to recognise that self-reflection itself has its origins in the way one reflects on others and how one internalizes others' reflections on oneself. We must thus make the claim that metacognition has its origin in social relationships, and that metacognition in the academic sense has its origins in the teacher-learner relationship.

5.3 Relationships as Central

Whereas Western psychology tended to focus on the individual, Vygotsky contended that the study of relationships is central, that an understanding of the social relations as the context of individual development is essential to understanding the individual. Hence this research report will consider social relationships in the learning and teaching situation.

5.4 Internalization as the Process which Transforms the Social to the Psychological.

Internalization is the transition from interpsychological knowledge to intrapsychological, 'truly mental' functioning, where patterns of activity that had been conducted externally come to be executed internally. This is the reason for the assertion in the title of this report of the importance of considering culture, and its mediation in communication, for cognition.

The choice of Vygotsky's theory for understanding the development of probability concepts becomes clearer when the social or cultural nature of the concepts themselves is noted. Part of the social processes to which Vygotsky refers are representational systems of a symbolic nature, and language. Thus in the construction and reconstruction of probability activity by the subject, probabilistic consciousness is itself produced by means of cultural mediation.

5.5 The Origins of Self-Regulation in the Deliberate Learning Situation

The origins of self-regulation then lie in the expert-novice interaction, where there is differential expertise and consequently differential control of the learning process. Initially the expert provides the 'other-regulation' necessary for the novice student to perform the required task. By taking over the communicative and regulative functions of the expert, that is, all the procedures in the 'language-game' of Probability Theory (cf. Wittgenstein, 1972), the novice develops self-regulatory abilities. There is a gradual shift of control from expert to novice. "That is, she/he has not simply mastered the ability to carry out one side of the communicative interaction by responding to the directives of others. She/he has taken over the rules and responsibilities of both participants ... The definitions of situation and the patterns of activity which formerly allowed the [novice] to participate in the problem-solving effort on the interpsychological plane now allow him/her to carry out the task on the intrapsychological plane" (Wertsch, 1979, p.18). In order to highlight the difficulties for the teacher in establishing common ground between expert and novice, I shall elaborate on the process of internalization drawing from Wertsch (ibid.) who theorises that there are four stages in the process.

5.6 The Process of Internalization

Mellin-Olsen (1987) asserts that the basic position for mathematics educators "is to introduce pupils to mathematical knowledge by connecting it with their existing culturally embedded knowledge and activities" (cited in Noss, 1988, p.404). This can be interpreted in different ways. It can mean that teachers must find mathematics in the knowledge students already possess, or it can mean the introduction of conventional mathematics without losing sight of the current knowledge, skills and interests of the students. While Mellin-Olsen refers to culturally embedded knowledge as practical knowledge the students already have, I will reinterpret the statement using the broader concept of culturally embedded knowledge as internalized mental acts.

The first level of transition from other- to self-regulation requires that the teacher provide the means by which the learner can develop a task definition that will allow for student participation in the learning situation. According to Wertsch (1979, p.12) "[a]n understanding of the communicative context provides the necessary foundation for any transfer of strategic responsibility from adult to child." Service-course students, as novices, have limited perspectives on the task, and their frequent need for step-wise direction and explicit help is an indication to teachers that an assessment of their situation definition is required. It is a task of the teacher to investigate the social and psychological realities of the students where these pertain to the learning of probability, especially for students who have adapted to an inadequately mediated mathematics, for example, those whose school teachers were themselves were trying to learn as they taught.

An important aspect of communication at this stage is related to the learner's interpretation of the teacher's directives. It may be that in familiar tasks, the teacher's intention and the student's interpretation correspond, but it is more likely that there will be incongruities between these in novel or complex tasks involving Probability Theory.

The second level in the transition from other-regulation to self-regulation is characterized by a degree of successful participation by the student who accepts some of the procedural responsibilities. However, the student's understanding of the task situation would not be in complete agreement with that of the teacher.[8] Although the students' understanding of the task situation will allow for interpsychological functioning, they do not see all the implications in the statements made by the teachers.

The third level is that in which the definition of the task is sufficiently mutual for participation by the student, and is characterized by the fact that the student can function adequately in what Wertsch (ibid.) refers to as the 'other-regulation' game. Applied to students studying probability, this is the level where the students can work independently provided they have 'recipes', or algorithms, that is, the student generates the solution but the steps are still provided by the teacher. They are apparently independent but are still 'other-regulated'. However they are now also taking a share of the strategic responsibility.

This is the point at which many students seem to cease developing, not moving on to the fourth stage, where the student "take[s] over complete responsibility for the problem-solving effort" (Wertsch, 1979, p.17), where the process shifts from the interpsychological to the intrapsychological, and where the shift from other-regulation to self-regulation is completed. As a consequence of the last stage of the formation of a mental act, different aspects of activity merge, that of the operation itself, that of thought concerning the operative, and metacognition, the last incorporating consciousness and control.

It is my contention that one reason that the fourth stage is not always reached by students with respect to Probability Theory is that students are not sufficiently exposed to an articulation by the teachers of the way that they, the teachers, approach problems in the conscious metacognitive sense. Students may believe that experts home in on the correct solution immediately, and while in fact this may be true in many instances for experts, it was not always the case when the teachers were novices.

Internalization then is the gaining of control over the external sign forms. It incorporates the transition of regulations, 'rules of the game' and responsibility from the interpsychological to intrapsychological which eventually become the internal property of the student.

5.7 Abbreviation while "Keeping in Mind"

Gal'perin (1969) speaks of 'abbreviation' as being the automatizing of cognitive processes. It is an important concept for this research because it contributes to the difference in situation definition between teacher and student. The more familiar one is with an operation, the more one knows the results in advance and can omit conscious processes, while obtaining and using the results. Difficulties arise for the teacher and student if abbreviation occurs in an uncontrolled way. If teachers have learned probability in an abbreviated form themselves or by experience have abbreviated, they may be unable or reluctant to acknowledge and explain the procedural steps involved in learning and problem solving. They may have difficulty in recognizing what aspect of the problem solving process or principle is inhibiting the students' development.

Egocentric thinking in particular can be a problem for teachers of mathematics and related disciplines because, in general, they have developed far beyond the level at which they are teaching. "The better you know something, the more risk there is of behaving egocentrically in relation to your knowledge. Thus the greater the gap between teacher and learner the harder teaching becomes, in this respect at least" (Donaldson, 1978, p.19).

If abbreviation occurs spontaneously without understanding of the underlying principle, then in each new situation the student has to guess a method or solution, or tasks have to be executed mechanically. Given that the student will tend to rely on the more stable cues, the danger arises that the verbal formula becomes fixated prematurely, and verbal form dominates content with the concomitant risk of empty verbalisms. One often hears such utterances from students, as evidenced in the problem cited regarding Bayes Theorem, where without the name attached to the method, the students could not recognize what they were doing. On the other hand, conscious control of the process of internalization allows for the control of the formation of the abbreviated reality. According to Gal'perin (1969) "only conscious mastery of the abbreviation process can guarantee extensive transfer and the development of new action ... only conscious development of abbreviation guarantees understanding of the connections between operations ... This connection not only permits the pupil to recover the full content of a mental act when he needs to, but also enables the student to keep the abbreviated contents "in mind" during the execution of the action" (p.256). Hence there is a difference between actions that operate only according to a formula and actions that operate with a formula while 'keeping in mind' the origins and connections of the formula, making the operation meaningful. As Gal'perin (<u>ibid</u>.) says, even though it is not certain what it means to keep something in mind, there is no doubt that it exists. Thus even though the students do not need to actively use any previous explanation, the fact that it co-exists 'in the mind' and may again be accessed, can enable them to perceive meaning.

Thus, in the development of a mental act, from external speech of others, or other-control, through external speech to oneself and thereafter internal speech, most of the intermediate operations are lost from consciousness, but can become evident when the automatic flow of thought stops due to processing difficulties. Inner speech is characterized by the automatic flow of abbreviated forms and becomes the "psychological interface between culturally fixated symbolic systems ... and the individual 'language' (Kozulin, 1986, p.269). Inner speech incorporates both the internalization of external or social forms and the linguistic form by which thoughts are communicated outwards.

Hence in speaking of different situation definitions for teacher and student we see how this theoretical position can elucidate the difficulties inherent in the teaching-learning situation, irrespective of different lived practices of students and teachers. The only means we have to access the way we learn to think probabilistically is from studying the formation of the mental function, by studying the process genetically, not by introspection of its final form.

5.8 The Zone of Proximal Development

Internalization is a process embedded within a context of joint activity between teacher and learner. I will use Vygotsky's concept of the 'Zone of Proximal Development' as the context in which internalization occurs, a context in which the present levels and potential future levels of student thinking interact in the development of knowledge.

Vygotsky defines the 'Zone of Proximal Development' as "... the difference between the child's actual level of development and the level of performance that he achieves in collaboration with the adult". The actual level is thus determined by independent problem solving, while the level of potential development is determined in extension with the help of the more capable (1987, p.209).

Wertsch (1984) argues that this definition of the Zone of Proximal Development is problematic without clarification of what is meant when we speak of the help of the more capable. Clearly being told what to do is at a different level of adult guidance than being given hints as to how to proceed.

Vygotsky's Zone of Proximal Development includes the changes that take place in cognition in an individual which are induced by an outside source who is in a particular relationship to that individual, namely that of someone more competent in that sphere of social knowledge. The teacher becomes the mediator and link between the sociocultural knowledge of Probability Theory and individual cognitive development. By mediating this knowledge at an appropriate level in terms of what the learner needs to know, rather than in terms purely of what the learner already knows, the Zone of Proximal Development is created. That is, instruction creates the Zone of Proximal Development by the creation of an interpsychological process by means of which intrapsychological development will be generated. While the level of actual development may be measured, the level of potential development will depend on the interpsychological situation entered into; in other words, it depends on the nature of teacher assistance.

Vygotsky (1987, p.211) also refers to the Zone of Proximal Development as that "which determines the domains of transition accessible to the [learner]...[It] is a defining feature of the relationship between instruction and development". It is the Zone of Proximal Development that looks at the process of development in which the teacher's orientation is to future levels of development rather than to past or present levels. It is thus clearly in the relationship between teacher and learner that one can conceptualise the zone of next development[9], that toward which the learner will move in terms of competence, from observation, to participation and independence by the internalization of the interactive activities, and of the interrogative and regulatory role of the teacher as model. The teacher imposes a structure on the world of mathematical stimuli for the learner through this social relationship and provides a focus for the development of meaning.

In conclusion, then, the Zone of Proximal Development is determined by both the student's present state of development and the form of instruction, that is, it entails both the student and the social relationships within in which the student learns.

5.9 Implications of the Notion of Zone of Proximal Development for the Teaching of Probability Theory

The first implication of the notion of the Zone of Proximal Development for teaching is that one needs to understand what the learner is presently capable of achieving, both independently and with assistance. The former can be assessed using traditional methods, but the latter is largely unaddressed in evaluation mechanisms found in the present educational systems, especially as it concerns the relationship between teacher and learner.

The second implication concerns development. Vygotsky was particularly concerned with how development can proceed. This is in sharp contrast to the psychology or education that seeks to be able to measure what already is. It places responsibility on the teachers to orient their work to the students' development rather than their present state of knowledge. It speaks strongly against academic arrogance. If one is taking responsibility for anticipating the next level of development of the students and organizing ways of achieving this, then as teachers we cannot take the attitude that students must 'sink or swim'. In fact, if one teaches to present levels of abilities, then one is not facilitating mental development as one could by organizing learning to create the pathway for new development. Just as the scientific concepts must be introduced in order to raise the level of generality of the intuitive and to allow for conscious control of the everyday concepts, and the intuitive concepts need recognition for their role in the provision of meaning for the scientific, so must teaching be arranged to develop new levels of mental ability while not losing sight of what abilities the students have currently. Vygotsky clearly asserts that learning precedes development, and good instruction must create the Zone of Proximal Development, albeit constrained by present knowledge.

5.10 Social Interaction as the Motive Force for the Development of Conceptual Thinking

Since the development of concepts involves the co-development of everyday notions and those socially developed and culturally transmitted concepts, the scientific, mediation within social interaction is the motive force for the development of conceptual thinking in probability. Social history and individual development are linked through instruction. Teachers then have to consider how to structure interpsychological functioning in the context of joint educational activity in order to facilitate intrapsychological functioning.

The Zone of Proximal Development can be seen as the interface between actual development of spontaneous concepts and the development of scientific concepts. Scientific concepts form "a zone of proximal possibilities for the development of everyday concepts" (Vygotsky, 1987, p.169) toward true concepts, involving consciousness, control and meaning. Some mathematics educators have addressed the significance of this relationship, for example, Garfield and Ahlgren (1988, p.54) who say: "It is likely that the real issue will be how to optimize the interplay of experience and rules".

While instruction must take account of the present capabilities of the learner, this does not limit the teacher to teaching from 'the known to the unknown'. It permits the teacher to introduce concepts and processes beyond the level at which students are independently operating and by the process of showing them how to respond to the new material, instruction can prepare the way for the development of new concepts and processes. In this way students are constructing knowledge and developing competence in the relationship with the more competent. Effective instruction must anticipate development.

It becomes apparent that scientific concepts lead to the development of control by consciousness, at a higher level of mental functioning than the concepts which submit to that control. It is to this higher level of functioning that I apply the term 'metacognition'.

Some mathematics education researchers have used a Vygotskian approach. Manning (1984), for example, speaks of the importance of inner speech, drawing on the process of development of higher mental functions via internalization of dialogue. Manning believes that teaching children to speak to themselves will enhance cognitive growth. Implicitly she says that children will come to internalize outer controls. Explicitly, however, she focuses on teaching the students to articulate 'inner speech'. I argue that we should be looking at an articulation of the outer controls, in order to enable students and teachers to develop intersubjectivity, for it is communication that is the point of contact between the teacher and the student, and between actual and potential levels of development.

Acknowledging the prevalence of misconceptions leads us to conclude how articulations of processes in communication and joint activity become important. The teacher and learner move to a greater understanding of probability and the process of teaching and learning probability respectively, by means of evaluating their current beliefs against their own other beliefs, other peoples' beliefs (including the experts), and empirical observations.

6 CHAPTER SIX: METACOGNITION

"... [T]he ability to monitor one's comprehension is necessary for academic excellence ... Without such knowledge about comprehension, comprehension itself will suffer"."

Markman, 1981, p.81 (emphasis mine)

"Perhaps the most telling feature of clinical interviews analyzed from the perspective of metacognitive processes is the absence of those processes in failed problem solutions."

Narode, 1987, p.21[10]

"Anyone who has ever taught a group of college students must know that their metacognitive skills in a variety of domains could stand considerable enhancing!"

Baker and Brown, 1984, p.380

6.1 Introduction

Metacognition is the rubric under which many questions related to thinking are discussed, in particular questions concerning whether one can help others to improve their thinking skills by encouraging reflection upon those skills in relation to cognitive goals. As educationists, our frequently expressed goal is to teach so that our students become independent thinkers and capable problem-solvers. How then does metacognition assist in the realization of this goal?

6.2 The Notion of Central Processors

Before embarking on any discussion of metacognition I must address the notion of generalized cognitive functions. Walkerdine (1988), drawing from Foucault, speaks in terms of separate discourses in learning. She argues:

> "that mathematics (and cognition more generally) is produced in practices ... [which] argues against a model of a single pathway of cognitive development, a central information processor, and for the possibility of specific 'skills' being learnt, produced and accomplished within the practices themselves".
>
> Walkerdine, 1990, p.248

This position challenges the notion of central executives or processors, although Walkerdine does address the question of transfer in terms of overlapping relations of signification (1988), allowing for the generalizations of some skills. Working with the development of probabilistic skills within the activity of learning Probability Theory, I assume that there is some central knowledge base that consists of skills and processes that are transferable across 'discourses', and I place metacognition into this category. Thus, while I accept that Probability Theory is historically developed as a 'discourse' different from, say, mathematics, I nevertheless assume that there are trans-historical structures within the broader academic domain. I am not assuming that these structures pre-exist learning within the field but rather that aspects of thought learned in one field may be usefully used in other fields and as such can become 'central'.

Given the difficulties that arise for students in making connections between alternative formulations of concepts within Probability Theory, for example the concept of independence to which I have been referring, one could argue that transfer of learning does not take place and that the formal and the intuitive remain separate discourses. However, I have chosen to read these rather in terms of the disjuncture between scientific and everyday knowledge (Vygotsky, 1987), and will argue that metacognitive awareness of this disjuncture can assist in closing the 'discourse gap' by development to qualitatively higher cognitive levels, toward 'true concepts' .

In addition, I assume some intrinsic and constructive level of mental operation, since if learning is purely experientially-based, novel (even in the sense of 'new to the student') solutions to problems would not be possible.

6.3 Bi-level Psychological Organization

Pascual-Leone, by his Theory of Constructive Operators (1979, and in Jukes, 1987), hypothesizes a bi-level psychological organisation, with learned and intrinsic operators. Pascual-Leone argues for intrinsic operators at the meta-level acting on learned content. Activity Theory argues for the necessity of the interrelatedness of constructs and operations, hence I assume a dialectical relationship between the learned and intrinsic at both the cognitive and metacognitive levels, in keeping with constructivist notions. Thus there are aspects of metaconstructs that are learned which operate to organize cognitive structures and to enable the subject to move beyond existing social knowledge. Placing these notions within the Theory of Activity would imply a dialectical relationship between the mechanisms of learning by internalization and intrinsic generative mechanisms, thus combining the learning of social (extrinsic) forms of knowledge with creativity. We thus have levels of mental operation which are not situation-specific and which are versatile with creative possibilities.

In summary, I am assuming that there are different levels of cognitive structures with both cognitive skills learned within activities and centralized skills, these latter being learned and intrinsic. These centralized skills are potentially transferable across discourses and operate on other levels of skills. Such a process-structural model is necessary to explain the choices made by the subject between the various possibilities that would accord with the subject's intentions and goal orientation. It enables us to move closer towards explaining the unique ways in which connections between mental structures must be made for cognitive development to occur.

6.4 Affect

According to Pascual-Leone (1979) the 'activation weight' of schemes can be boosted, energizing the meta-level schemes that are contextually free. Affect and belief can be seen as part of the superordinate structures which form the control executives. It could be argued that affect and belief may exist at the cognitive level, and that the levels of thought are not distinct. However, one cannot ignore the importance of affect and belief when looking at cognitive development, and it is this aspect of metacognition upon which I shall focus, in particular, those of consciousness and control as pertaining to confidence in the learning and teaching of probability. Hence I turn to a discussion of metacognition and draw from the discussion aspects of metacognition that are essential for independent problem solving in probability, in particular, a belief on the part of the students that they can manage probability at the level expected in a service course, and a commitment from the teachers that they can help the students.

6.5 Historical View of Metacognition

Metacognition is a term often used without adequate delimitation (Flavell, 1981a) and "has been used to refer to a variety of epistemic processes" (Flavell, 1981a, p.37). Flavell (<u>ibid</u>., p.37) defines metacognition broadly as "knowledge or cognition that takes as its object or regulates any aspect of any cognitive endeavour", in other words "cognition about cognition".
It encompasses

> "knowledge concerning one's own cognitive processes and products or anything related to them ... Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective."
>
> Flavell, 1976, p.232

Brown's (1980) understanding, too, is that "[m]etacognition refers to the deliberate conscious control of one's own cognitive actions ... " (p.453). According to these theorists, metacognition includes the three aspects of knowledge about cognition, monitoring of cognitive processes and regulation of these processes, by which

I understand them to mean self-regulation. However, within the theoretical position I have assumed this self-regulation must initially have aspects of other-regulation. I will argue that within the Zone of Proximal Development other-regulation of cognition is internalized to become self-regulation via self-reflection. Even though there may be interaction with intrinsic levels of control, what concerns us as teachers is that which can be socially developed.

The concept 'metacognition' thus can unite aspects of the historically different notions of 'consciousness' (in the sense of 'thinking about thinking')[11] and 'executive control', deriving theoretically from developmental psychology and information processing, respectively. It is embedded within a framework that is oriented to processual, goal oriented and task related activity.

6.6 Metacognitive Experience

Flavell (1981b, p.273) defines metacognitive experiences as conscious "cognitive and/or affective experiences or states of awareness that accompany or relate to a ... cognitive enterprise". Metacognitive experience then is the 'consciousness' rather than the 'executive control' aspect of cognition. This conscious awareness is usually triggered by feelings. Affect thus can facilitate the development of metacognitive knowledge and contribute to the monitoring and guiding of cognitive actions by boosting or activating the metacognitive superscheme. Metacognitive experiences can be feelings or thoughts of understanding or not understanding, like surprise when expectations are not fulfilled, bringing to consciousness the failure of the cognitive operation. Automatized processes, too, can involve metacognitive experience by the 'feeling' of the process moving in the right direction (Gal'perin, 1969, p.264), thus providing a control. Metacognitive experience unites the affective and intellectual, which is fundamental to a theory of Activity with meaning as a unit of analysis (cf. Vygotsky, 1986, p.10). If students' perceived sense of control and actual control over their learning of probability is in question, the relationship between affect and cognition needs to be investigated further. While metacognition contains the notion of control, control can be facilitated by metacognition, in a dialectical way.

6.7 Elaboration of the Relationship between Affect and Cognition and its Link with Metacognition

The work of Skemp (1976) is useful for developing the relationship between effect and intellect in metacognitive experience. Skemp speaks of a director system which operates on the environment using sensors to gain information from the environment. A comparator compares the present state with the goal state. He speaks of pleasure being experienced as one moves towards the goal state and unpleasure as one moves away from the goal state; of fear as one moves toward what he refers to as an anti-goal state and relief as one moves away from this anti-goal state. Second-level director systems operate on this first-level director system, at a metacognitive level. He speaks of confidence, frustration, security and anxiety in terms of one's ability to perform the goal-directed actions (p.13). Confidence is experienced in the ability to move towards the goal state, frustration in the inability to move toward the goal state, security in the ability to move away from the anti-goal state and anxiety in the inability to move away from the anti-goal state. Thus there are the levels of the change of state and the knowledge of the ability to change state, with the emotions as signposts. In the teaching and learning situation there are at least four director systems, two for the student and two for the teacher.

Skemp (*ibid.*) claims that 'consciousness' is the central issue of psychology with increased consciousness contributing to the increased adaptability of a director system (p.15). The deliberate heightening of consciousness will bring about improved functioning in new situations, as learning is seen as a change of state of the director system toward more accurate, reliable, efficient, extensive and complete functioning. Too much anxiety or frustration can lead to withdrawal of consciousness from the source of the emotions, and repeated failures will destroy confidence. My argument thus stresses the importance of students' feelings of confidence regarding their performance.

Metacognition can lead to four types of situations regarding knowledge: knowing that you know, knowing that you do not know, not knowing that you know, and not knowing that you do not know. Clearly, the first is the most desirable state for academic competence; the second points to the necessity of cognitive action; the third may lead to lack of lack of confidence or inefficient working; while the fourth is a breakdown on both the cognitive and metacognitive levels. Subjective feelings of understanding referred to as "misconceptions" are failure on the cognitive and metacognitive level. This research investigates the metacognitive experience of failure to comprehend or use knowledge.

Relating back to the types of problems I delineated in the first chapter, it is apparent that failing to be critical of a probability value greater than 1 is failure at a metacognitive level as well as at a cognitive level. However, in my experience few students of probability fall into the category of not knowing <u>that</u> they do not know. Rather they struggle to understand <u>why</u> they do not know, and consequently feel <u>helpless</u> in remedying the situation. Some of my students have reported a pervasive feeling of helplessness regarding probability problems, reporting feeling that they have no control over the resolution of problems. These students have no means of delimiting the problem areas. Instead of having clearly delineated and articulated problems, they have a sense of intellectual blur. They do not see themselves as capable of analysing what knowledge they have and where they are ignorant. It is probable that only the students who are already capable are able to ask questions as they monitor their progress and engage in strategic action.

6.8 Confidence and Metacognition

Clearly the above analysis speaks for the importance of providing some way for the students to sense and have some control over their learning, especially as affect, and I argue that confidence in particular, can activate certain cognitive functions (cf. Pacual-Leone, 1979). For example, Cavanaugh and Perlmutter (1982, p.19) cite high correlations between students' verbally expressed feelings of knowing' and short response latencies on memory tasks, the latter said to be indicative of confidence. The danger for students of a lack of confidence is that they may not only give up trying to do probability problems but may give up trying to learn how to do them, too.

Insecurity in the learning situation can be compensated for by external sources of confidence, including training of skills and control of those skills by heightening consciousness of the nature of the task and of the support; hence metacognition with mediation in the Zone of Proximal Development. It is in activity and through relations with others, in particular with mediators, that the subject comes to know himself or herself and thus becomes the object of self-knowledge (Brushlinskii, 1987, p.76). We have seen that internal control develops from the internalization of external control within the Zone of Proximal Development. Consciousness and control are different aspects of a unified Vygotskian concept of consciousness which is broader than metacognition as defined by Flavell. However, both contain the notion that awareness and control are in the service of goals. With acute consciousness (on a theoretical level) of the role of affect, motives and goals in the development of consciousness and control, I state that confidence can be seen as the metacognitive experience of internal control.

6.9 The Necessity of Metacognition in Deliberate Learning Situations

Brown (1978) justifies the use of the term 'metacognition' in the face of criticism of a proliferation of 'metas', by claiming that the 'meta'-level reflects an important change of emphasis, providing an important focus for a discussion of effective thinking. She acknowledges that conscious control of thinking is not necessary for all forms of knowing, but that it is necessary in deliberate learning and problem-solving situations. Thus for everyday interaction with meaningful environments, Brown would concede incidental learning, but in the scientific context in which probability has been classified, she would support the claim that consciousness is essential, "the underlying force that the observed routines reflect, are symptomatic of, and are epiphenomenal to" (Brown, 1978, p.79). Thus she claims that skills training alone is inadequate. One needs an orientation toward self-awareness and self-regulation, including awareness of the epistemic nature of the task in order to gradually increase conscious control over that knowledge. "Students who receive only instruction in the skills [cognitive] often fail to use them intelligently and on their own volition because they do not appreciate the reasons why such activities are useful, nor do they grasp where and when to use them" (Baker and Brown, 1984, p.381).

Metacognitive experience and goals or motives interrelate to the extent that "intention in action ... corresponds to our experience of doing" (Ingold, 1986, pp.318-319). Thus, according to Ingold, the 'subjective meaning' of action lies in intentionality.

This strengthens the argument for the importance of metacognition, for what we are aiming for is the embodiment of scientific concepts from everyday concepts and the structural readiness for the learning of scientific concepts based on the development of everyday concepts. In effect, a shift away from a purely cognitive orientation, to one which includes a metacognitive and epistemic orientation, allows for both 'lateral' and 'vertical' transfer, for the ability to integrate subskills with self-regulation. This results in an extension of the possibilities for broad synthesis and application of knowledge.

Baker and Brown (1984) cite research supporting positive effects on studying from this change of emphasis, claiming that it is not surprising that instructional programmes that specifically train for vertical and lateral transfer by attending to the students' awareness of the skill's utility and their ability to regulate their own activities are more successful than programmes that leave the problem of transfer entirely up to the learner. Hence the notion of the mediation of transfer of general skills at the cognitive, metacognitive and epistemic level moves into the foreground in the Zone of Proximal Development.

6.10 Metacognition and the Learning and Teaching of Probability Theory in Particular

Schoenfeld (1984) states that metacognition is an essential component of competent mathematical performance. He claims that students do not develop many metacognitive skills because mathematics instruction concentrates on the computational and procedural.

It may be argued that in probability classes students are getting feedback on the problems they attempt and that there is tutorial help, thus there would surely be metacognitive experience leading to the development of metacognitive knowledge. But conscious responses consequent upon cognitive failure, according to Flavell (1981a), are not necessary, and the capacity to notice and interpret difficulties with comprehension varies with age and, analogously, expertise as "[m]etacognitive deficiencies are the problem of the novice, regardless of age" (Brown, 1980, p.475). Thus it cannot be assumed that metacognition will develop spontaneously. One cannot present probabilistic information to the students expecting them to cope with it in the same way that they would everyday learning.

There is evidence that metacognitive knowledge does develop indirectly with school experience as a "by-product of active attempts to understand" (Markman, 1981, p.75). However, Brown (1978) points out that students' incompetence at assessing their own abilities indicates that this method of developing metacognitive knowledge is not efficient, a situation which would be exacerbated by the schooling many of our students have had. Flavell (1981b) claims that metacognitive experiences are most likely to occur in situations where there is intentional and conscious thinking or when expectations are not confirmed. Teachers need to concentrate on facilitating its development.

6.11 Self Regulation in Institutional Settings

In situations where learning takes place in large, often impersonal, institutions, as is traditionally the case in service course probability teaching, the dyadic notion underlying the Zone of Proximal Development becomes more of an ideal than a possibility. On the whole, the student is alone in his or her learning and hence self-regulation becomes vital. Thus it is imperative that the student hasten to assume the role of tutor as well as tutee - and hence the importance of teaching metacognitive skills, allowing for the internalization of the role of the teacher by the student. The students, although novices in probability, nevertheless have some sense that the responsibility for learning is largely their own but that they need the support of the more knowledgeable. This implies that the teacher needs to be sensitive to the students' needs and level of participation and hence communication between teacher and student needs to be open and clear, with teachers' responses being contingent upon the students' level of capability. Again this implies mutual responsibility.

We may conclude that there is a double need for a metacognitive orientation in the teaching and learning of probability, for not only is it the case that metacognition may not have developed adequately spontaneously, but it also has to become a conscious consideration for the teaching and learning of probability in particular, a scientific subject in an institutional setting.

6.12 Levels of Metacognition for Teacher and Learner

Referring to Figure 2, there are metacognitive aspects for both the teacher and the learner. The teacher metacognises at two levels, at the level of thinking about his or her own thinking and at the level of thinking about the students' thinking. In addition, the teacher has to assist the students in thinking about their own thinking during the process of coming to terms with knowledge of probability. This is the indirectness delineated by Skemp (1976), where the teacher has only indirect access to the students' cognitive processes, and that is via metacognitive processes.

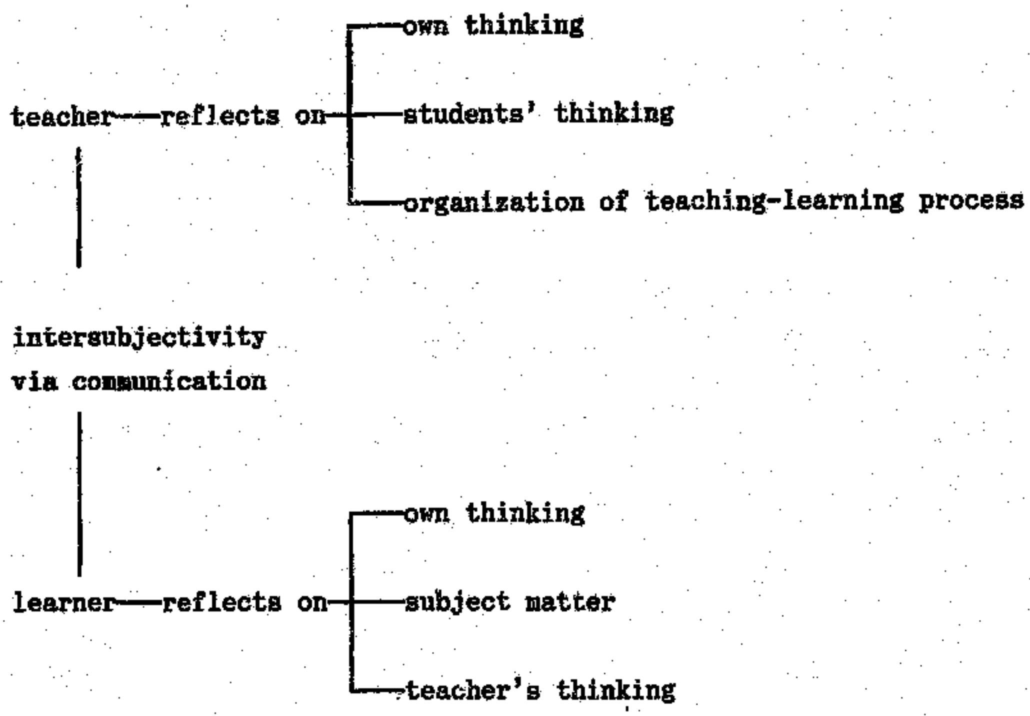

Figure 2: Metacognition in Learning and Teaching

It is the regulatory function of teaching, the teachers' responsibility for and awareness of an organizing role, which is realized in communication, that constitutes part of their metacognitive role. What the student needs to reconstruct is the interrogative mode of the teacher so that the consciousness and control of the teaching process becomes the consciousness and control of the learning process.

This is not an easy task, for the connections between external social interaction and intrapsychological processes involved in self-regulation become more distant the more expert one becomes in a particular task (Wertsch, 1979). Teachers' abbreviation of the language and skills of probability make access to the knowledge of the development of their own intrapsychological functioning difficult. A teacher may be glib in an explanation because he or she so clearly understands it. Mathematicians are famous for comments like "It is intuitively obvious that ...". Flavell (1981a, p.44) states that [t]he egocentrism-based feeling that, if it feels this clear to me, it must be clear to others probably cannot be extinguished, but it should not be allowed to engender egocentric judgements and actions". This is important, especially at present in South Africa. Responsible teachers simply do not have the option of being arrogant, claiming that as they managed to get where they did, others must sink or swim. Good teaching requires an understanding of learning and of what leads to competence for both the teachers and their students.

Wertsch (1979) argues that the mechanisms by which one moves from other-regulation to self-regulation lie in the mechanisms of communication. The novice first has to perform the task with other-regulation and from there attempt to establish consonance between their own action in the task and the task definition and demands. For the teacher this requires being aware that their own definition of the situation cannot be assumed to correspond with that of the learners, but that in the joint performance of the task, understanding on the part of the teacher and learner of the requirements of the situation will start to converge, and that the student will internalise the task requirements.

Thus, for the learning and teaching of Probability Theory, two levels of metacognitive analysis are required: self-awareness in both teacher and student; and communication and awareness as playing an organising role for teachers. There need to be metacognitive educators, where the planning is realized in the communication of the tasks. Metacognition for teachers includes organizing educational activity to facilitate the controlled learning of concepts. "By planning for instruction ... f fundamental concepts we can insure that the student's understanding is built up systematically. This provides a firmer foundation upon which the student can acquire a grasp of Probability Theory" (Kelly, 1986, p.9).

Finally, metacognitive awareness of the relationships between the empirical, theoretical, subjective and intersubjective concepts of probability can assist students to challenge their sense of the meaninglessness of the tasks. Thus by increasingly being made, or becoming, aware of their thinking in Probability Theory, students will develop their understanding of the essential scientific intentions of Probability Theory as a tool for dealing with uncertainty, and thus develop an understanding of the epistemic nature of probability.

6.13 Confidence, Consciousness and Control

Confidence is knowledge of the ability to be aware of and have control over cognition. By means of metacognition, and its unifying effect on epistemic, metacognitive and cognitive experience, knowledge and strategy, we can begin to close the disjuncture between the scientific and everyday notions of probability, and in so doing begin to push out that sense of meaningless so often spoken about by the students. Communication between learner and teacher, which results in contingent control of the learning process geared to students' needs and definition of the situation, can allow for the development of a sense of confidence in the students which is dialectically related to the metacognitive experience of being able to 'do probability'.

6.14 Teaching Metacognition

How then does one 'teach' metacognition? As with critiques and consciousness-raising in general, developing awareness of introspection and attention to metacognitive experiences is the starting point. These can then be interpreted in terms of meaning, both in the subjective and epistemic sense, and implication.

Various researchers have published articles on the teaching of metacognition relevant to Probability Theory, for example, Callahan (1987). However it is not my intention to go into detail. Rather my aim has been to explain the importance of a metacognitive orientation in the learning and teaching of Probability Theory, which would aim at assisting those students who have not spontaneously developed metacognitive strategies.

Direct questions regarding understanding need not be asked in order for metacognitive experiences to occur as (to use Skemp's model) the thinking process entails affective states which provide information regarding cognitive activity. Metacognitive experience that is attended to may assist the individual to make decisions regarding either the making or monitoring of cognitive progress and can generate further metacognitive experiences. But "[w]hat the learner makes of a feeling of confusion and what he or she decides to do about it are guided to varying degrees by the metacognitive knowledge base" (Garner, 1987, p.19). Here the teacher has the role of extending the metacognitive knowledge base.

Schoenfeld (1985, in Kameenui and Griffin, 1989) suggests that "models of control decisions" (p.580) with a blow-by-blow account of metacognitive processes, explicit and articulated, will help students. This is in keeping with the theoretical model of internalization. Students can be encouraged to be self-questioning, adopting "the self-interrogation mode of the expert" (Brown and DeLoache, in Markman, 1981, p.75). Following Vygotsky, we posit that learning is first social and it is the teacher who first asks the questions; but the students have to learn to be both the questioner and the questioned, questioning assumptions, locating important laws and generating questions about them, probing weak areas, finding applicable areas and counter-examples and so on. Students need to understand that it is not from a one-sided responding to directives (as so many of our students desire) that they will succeed, but rather from the <u>internalisation of both roles</u> in the learning and teaching of probability, that is, the role of the student and that of the other-regulation of teacher, in terms of <u>both rules and responsibilities</u>. Initially, the teachers must take control; but this control needs to be shifted to the student as and when the teachers perceive the students to be able to assume such control. Wood (1986) argues that the mark of a good teacher is that of contingent control - of knowing when to assume and when to relinquish control. Wertsch and Stone (1985) argue that in a Vygotskian approach structural properties of interpsychological functioning, such as its dialogical, question-answer organization are part of the resulting internal, intrapsychological plane of functioning via the mastery of external sign forms. Both students and teachers need to become aware of the social origins of internalized mathematical knowledge.

Lester and Garofalo (1987) outline the metacognitive role of the teacher as external monitor, facilitator and model. I believe all these are well explained by the theory. "[A]ll forms of external negotiation... all external prods to reflection have the effect of stimulating internal negotiation, reflection, metacognition" (Bruner, 1965, in Chipman et al, p.605).

6.15 Metacognition in the Classroom

Even though I have limited my research to a theoretical understanding of a teaching and learning problem, some practical applications entailed in the theory can be suggested. There are methods that university students can be encouraged to adopt to help them prevent themselves making some of the mistakes I have outlined. For example, consciously predicting a probability may be a worthwhile start so that the student can place the problem into a 'common sense' or everyday framework, which may allow for greater accessibility of knowledge stores. In any event, to do so will position the problem epistemically. Predictions that are wrong can be used to provide information about one's comprehension. When expectations are repeatedly confirmed, one can be more confident of one's understanding. Checking for internal consistency of answers will eliminate the type of problem where negative or probability values greater than 1 occur.

This may be all very well for the student who reaches some answer, but what can be done for the student who after realizing an incorrect probability cannot arrive at a correct value, or for the student who cannot get going at all? How can we hypothesize that metacognition will help? Students can be encouraged to make judgements about appropriate methods and to tentatively hold inferences while assessing their appropriateness for the problems at hand. Markman (1981) claims that the more deeply one attempts to understand, the more comprehension problems will be revealed and cognitive strategies can be developed.

Although it could be expected that years of schooling would result in abilities especially suited to academic learning, abilities that are broadly applicable, we are nevertheless faced again with the notion of disjuncture. From my experience in trying to understand the methods of education with which many students are familiar, I concluded that facts are taught, but are not adequately taught in their connectedness with others, nor in some sort of pivotal relationship around an action or goal that will allow the students to organise them. The more abstract the concept, the less it will have everyday meaning and thus will derive meaning only in its interrelationship with other concepts in a system and in relation to goals. The students need to understand what principles organise the material. It may be argued that Probability Theory is presented systematically and this I cannot contest, but within this ordered representation, its fundamental coherence and its epistemic foundation somehow fails to be conveyed. It is here that the necessity of mediation, that of the teacher providing selective mechanisms for the students to use in making sense of the theory, of providing frameworks from which they can structure their learning and develop cognitive structures appropriate to the theory of probability, is invoked. If the students had 'meta' knowledge of their knowledge stores and some idea of which were potentially utilisable in the solution of a problem, it might save them from fixating on a knowledge area, should that area fail to provide 'the solution'.

In the example I gave in the first chapter regarding Bayes theorem, those students who were in fact 'doing' the 'same thing' but were arguing who was right, could have been helped by the 'meta' knowledge that seemingly different knowledge areas may say the 'same thing'. "[I]t is the richness and structure of these connections that would seem to ... spell the difference between inert and usable knowledge." (Bereiter and Scardamalia, 1985, p. 77).

Algorithmic learning can be useful to a point, but given a problem solving area like Probability Theory, what is needed is a "concurrent understanding of the reason why the skill must be used" (Brown, 1978, pp. 136-137). This 'keeping in mind' referred to by Gal'perin (1969) allows not only for cognitive control but in particular allows for the subjective sense of meaning the students need.

Returning to the earlier problem of the students' inability to relate to the formalization of independence via $P(A \cap B)=P(A)P(B)$, the students may be assisted in their understanding of independence by linking the teaching thereof with conditional probability (see Kelly and Zwiers, 1986) or by making explicit the distinction between independent and mutually exclusive events, even though this may seem trivial to the teacher. In this way more connections between conceptions are made explicit, and students are more able to sense the meaning of independence subjectively but accurately.

6.16 A Metacognitive Orientation and Relationships

A metacognitive orientation will present a challenge to existing power relations between some teachers and students, for the teacher who is focusing on how he or she is to facilitate the student in acquiring expertise may threaten his or her own position. There is a definite sense of satisfaction amongst some teachers to be able to declare a student 'weak' and absolve themselves of further responsibility. However, a completely different power relation exists if one looks at the student at a tertiary level historically, to attempt to understand the nature of the students' cognitive abilities and how these may be developed. My contention is that for students and teachers, metacognition will shift the power relations to the advantage of the students' learning, and if this is our goal, it is surely desirable. If metacognitive experiences lead one to feel uncomfortable in this regard perhaps a re-examination of one's goals as a teacher are necessary.

Mathematics and related disciplines have been used as a 'sieve' to sort out the cognitively weak from the strong, or so it is believed. Many students have developed a learned helplessness by repeated failure (Miller and Seligman, 1974), or have developed 'inert knowledge'. If it is the case that students are giving up because the difficulty of the task is outside their zone of next development then this aspect clearly needs to be watched and recommendations made regarding curricula.

6.17 The Limits of the Notion and Use of Metacognition

Studies concerning metacognition reflect an important shift toward viewing both student and teacher as responsible active agents in the learning process. However, the generality of the approach cannot be ignored. Such generality makes the importance of such an approach difficult to refute in the learning and teaching of Probability Theory (in keeping with a Popperian (1959) approach to theory). My contention is that a metacognitive <u>orientation</u> by teacher and student can assist in providing the learner with at least the confidence to know that both teacher and student share the same goals - that of the student becoming adequately competent in basic Probability Theory and the solving of probability problems - and to sense that he or she and the teacher are not in opposition to one another.

Metacognitive knowledge does not simply become awakened during cognitive functioning, nor does the development of awareness of the metacognitive simply lead to improved cognitive functioning. Metacognitive knowledge cannot be taught without some underlying cognitive skills that will be managed and developed by these metacognitive skills. A dialectic relationship between the cognitive and metacognitive must necessarily exist in the light of the theoretical position that has been developed, and this is in keeping with the findings of research (see Butterfield and Belmont, 1974; Butterfield, Wambold and Belmont, 1973, in Brown, 1978). Thus teachers and learners need to concentrate on the development of knowledge, skills and strategies at both the cognitive and metacognitive levels. However, I have concentrated on the development of the metacognitive, as there has been less direct attention paid to this aspect in the literature on the learning and teaching of Probability Theory and because metacognition serves a unifying function for the three levels of thinking.

A significant difficulty with a metacognitive position relates to assumptions regarding the nature of understanding. If understanding is not necessarily 'all or nothing', and degrees of understanding exist (Markman, 1981), monitoring this comprehension becomes difficult and of course open to error. There is sometimes a lack of correspondence between what students say they could or should do and what they really do. Baker and Brown (1984) claim that the direction of the lack of correspondence is usually in the direction of knowing that some strategy is important but not using it. They make the traditional distinction between procedural and strategic knowledge, knowing that as opposed to knowing how, the latter being considerably more difficult to examine and describe.

6.18 The Necessity of Metacognition

The question arises as to whether metacognition is really necessary. This question can be interpreted in different ways. In the first place, it can be seen as questioning whether metacognition is necessary for effective thinking. Secondly, the question can be seen as asking whether metacognitive strategies need to be taught. Thirdly, it can be interpreted in relation to the importance of a metacognitive teaching orientation. Lastly, it can be seen to ask whether metacognition is essential for cognitive development.

The first has already been answered in the affirmative. Comprehension monitoring is essential but not necessarily conscious at all times. The second interpretation too has been answered in the affirmative by emphasising the dialectical nature of scientific and spontaneous development, of deliberate and incidental learning, and the internalizing of originally social forms of knowledge by individual reconstruction. In response to the third interpretation, it may well be that there are other useful and effective approaches to the teaching and learning of probability, but those are not the direct concern of a report on metacognition in the learning and teaching of probability. The last of the interpretations of the question, that of whether metacognitive approaches can lead to structural or qualitative changes, is more difficult to answer for it raises the question of the role of metacognition in the broad transfer of cognitive abilities, the possibility of which has been assumed. An attempt to answer this empirically would require, at least, thorough knowledge of what is required in both teaching metacognition and tests of transfer. Teachers would need to undertake task analyses in order to assess what the learning of probability requires in total, a daunting and difficult task. A full engagement with this issue is beyond the scope of this research report.

6.19 Metacognition and Control

The metacognitive approaches outlined above, those of giving attention to metacognitive experiences and eliciting self-awareness are important not only for Probability Theory but also for increasing students' sense of control over their learning in general. It may be argued that this is merely a deception and that the use of metacognition is confounding the real lack of control the students have. It may be that we are merely facilitating the illusion of control, and that in fact students' and teachers' thinking processes are controlled by broader social factors, and that it is twee to think that it can be otherwise while 'real' power is maintained elsewhere.

However, if our assumptions regarding Activity Theory are valid and that society constrains but also enables, then by facilitating the development of strategies in the students and teachers that will allow them to have, at least, some control over their learning processes, both groups may enabled to alter circumstances that are unacceptable. Clearly the roles of teacher and student in this regard are mutually dependent. Facilitating metacognitive knowledge in the students must be accompanied by metacognitive development in the teacher. If the teacher attempts to transcend the barriers created by not only the disjunctures evident in the teaching and learning of probability itself but in the broader societal context, that is the disjunctures created by insensitivity to the position of others in the society, then in fact metacognition may assist in enabling both students and teachers and could result in important changes in teaching and learning.

Clearly in terms of Probability Theory it would mean an entire reappraisal of the role of Probability Theory for service-course students, and how that reappraisal would affect what the students learn and teachers teach, and how and where this takes place. In that probability notions form the basis of inferential statistics, and that statistics is a widely used and often abused tool in society, I believe that good critical teaching of probability is very important.

It is doubtful whether the curriculum (used in the broad sense) can remain unaltered if a metacognitive orientation is assumed that puts teachers and students 'on the same side' of the mastery of knowledge, particularly for service-courses. As Bishop (1985) states "[r]ecognition of the social construction of phenomena leads me to propose a new orientation for mathematics education. This orientation views mathematics classroom teaching as controlling the organisation and dynamics of the classroom for the purposes of sharing and developing mathematical meaning" (p.26). He claims that this has desirable features for practice in the classroom including: emphasising the dynamic interactive nature of teaching; stressing the learner as well as learning; recognizing the notion of shared knowledge, content and context; taking into account pupils existing knowledge, abilities and feelings; taking account of the cognitive and affective aspects of the developmental process and emphasizing the development of mathematical meaning.

7 CHAPTER SEVEN: CONCLUSION

I have argued that Probability Theory is a unique discipline attempting to impose certainty upon uncertainty, changing ill-structured problems into puzzles. It is essential that the epistemic nature of probability be 'kept in mind' during cognitive action with the content and processes involved in probability. It is metacognition that can serve to unify the aspects of cognition, metacognition, and epistemic cognition.

Beginning with problems encountered by students and teachers, deriving from my own experience in the learning and teaching of Probability Theory, I hypothesized that students were often overwhelmed by feelings of helplessness and meaninglessness in tackling novel (in the sense of being new to them) probability problems and had become heavily reliant on algorithmic approaches. Some students were unable to synthesize across knowledge bases by abstractions to a higher level of generalization, and unable to synthesize across different types of knowledge or across different components of the system of probability knowledge. Thus they were unable to detect fundamental inconsistencies, for example, the returning of probability values greater than 1 when they could easily say that probability values range between 0 and 1, and could not identify two seemingly different approaches as being fundamentally the same (see chapter 1). In addition, it became clear that giving students more knowledge when they were struggling with systematizing their knowledge served only to make them less efficient in problem-solving than before.

The most perplexing problem was that students gave up trying to learn probability as they were unsure of their ability to do so. Teachers also were insecure about their ability to help the students to become proficient probability problem solvers, while others seemed not to care. In general there was a sense of lack of control over the learning process in this field and a consequent lack of confidence on the part of both students and teachers in their ability to change this. An outcome of this was that students and teachers merely tried to get problems 'done', rather than focus on their goals.

Placing the learning and teaching of Probability Theory within the theoretical assumptions underlying the Theory of Activity assisted in explaining why these problems occur. Firstly, Activity Theory focuses our understanding on intentionality in activity. Analyzing the goals of learning and teaching in Probability Theory raises the generalization that teachers and students were both aiming at the student gaining some probabilistic skills, and in addition, fostering independent problem-solving abilities in the students. Hence the student becomes the subject of educational activity, and the teacher becomes the link between the social knowledge of Probability Theory, existing in the broader culture, and the student's internalization of that knowledge. The teacher is at the interface of external knowledge and its internalization by the student. The relationship between the teacher and student thus becomes primary in educational activity, and the students and teachers need to perceive that they share the same goals.

In the use of Activity Theory, cognitive activity is ontologically inseparable from affect which gives activation weight to superschemes that control cognitive process-structures. Hence it is necessary that we give consideration to the affective aspects of the learning and teaching process within this educational relationship. In particular, this study has looked at the importance of confidence, the knowledge of the ability to learn probability. Metacognitive experience was seen as the consciousness of this capability of learning probability.

The notion of shared situation definition was invoked, that the expert and novice, by communication, must come to an intersubjective position in order to overcome the phenomenological chasm hypothesized to exist between those who have knowledge of a system and its structures and processes and those who are in the early stages of internalization, that of active engagement with the subject. Mediation becomes the motive force for development.

Internalization must occur not only in relation to actual content matter but also in relation to the interrogatory functions of the teacher. Internalization includes the internalization of responsibility, where the responsibility for student learning must be transferred to the student as part of student development. The learner has to internalize the language-game of probability, including self-reflection and self-regulation. Both these aspects can be seen as part of metacognition, which unites consciousness and control, consciousness and control being complementary sides of the same phenomenon.

The present and future interact in the learning and teaching relationship, creating the Zone of Proximal Development within which learning triggers development. With regard to Probability Theory, we have seen that an approach that begins with students' present knowledge may be inappropriate because misconceptions are common. Wh is required is rather that scientific concepts are taught which do not necessarily link with present knowledge. As these everyday and scientific concepts themselves develop, they allow for the reconstruction of knowledge, facilitating the development of true concepts, those that hold personal meaning for the students and are in addition objectively part of statistical practices. The development required is that of the mastery of methods of acquiring knowledge; metacognition regulates this process.

Metacognition, if deliberately taught, can be seen as the 'scientific' aspect of the teaching and learning of probability. Intuitive learning will take place though imitation and correction; but, more powerfully, learning with control and consciousness can take place with a metacognitive orientation where the knowledge, monitoring and regulation of cognitive activities takes place. Metacognition in this study has become the meta-level wherein the teachers have to become conscious not only of their own thinking processes, and those of the students', but also how to organize teaching in order to facilitate student development by means of reflection on their own thinking processes as well as those of the teachers.

For the teacher, this will involve showing students how to generalize, to extract essence, to interconnect, to make relationships explicit, and so on. For example teachers can make explicit the difference between mutually exclusive events and independent events, even though this may seem trivial for them. They have to suspend egocentric judgements and actions. It requires that the teacher respond to the abilities of the students in terms of the amount of responsibility they are able to assume at their existing level of development, and in general, by mediation, to provide the motive force for student development, encouraging students to reflect upon their own development as both the subject and object of educational activity.

Metacognitive experience can be used as a tool for the development of this awareness. Where repeated failure, and its consequent anxiety or withdrawal of consciousness, has been seen to lead to a lack of confidence, a positive role for failure has been found in this study, for these affective experiences themselves can be used to monitor future progress.

A metacognitive orientation, then, reflects an important change of emphasis in teaching, unifying motives and goals with actions. Teachers accepting the theoretical positions outlined in this report must assume a developmental approach to teaching and learning rather than an approach which passively accepts students' present abilities. It places the responsibility for student learning on both teacher and learner, but that these responsibilities change emphasis during internalization, with the teachers initially providing more support to compensate for insecurities, by providing external sources of confidence, on the assumption that the students will all acquire what is expected of them.

It requires that the goals of educational activity include the students' sense of the meaning of probability, as well as a consideration of the students' knowledge of their own ability to reflect on and acquire probability knowledge - both content and processes.

Practical suggestions regarding methods include teachers extending students' metacognitive knowledge bases, for example by articulating that seemingly different statements may be saying the same thing. Teachers may articulate their own thinking as objects of thought to enable students to see the processes involved in reaching a suitable method or model for a problem. Teachers can highlight aspects that are important, mediating a framework by which students can apprehend probability knowledge. If teachers can, by task analyses, deconstruct the processes involved in abbreviation, they will allow students greater control over the learning processes and facilitate transfer. These are major tasks, and a comprehensive description of them is clearly beyond the scope of a report such as this. But if teachers and students can get closer to knowing their strengths and limitations, knowing their repertoire of strategies and where they are appropriate, knowing how to plan, implement, monitor and regulate cognitive activities, within a context of knowledge of the complexities of probabilistic knowledge, they will be closer to effective teaching and learning of Probability Theory, which itself can become automatized. In a sense they will be invoking an epistemic level in assessing the possibility that their goals can be reached and devising methods of materializing these.

The strength of my argument lies finally in teachers' and learners' commitment to their goals, and their metacognitive knowledge that they, the teachers, can help students learn, and that they, the students, can help themselves to use teachers as a resource in an institutional setting so that they eventually become their own teachers. The injunction emerges that one may not dismiss learning difficulties in the students as limitations of their capacity until more is understood about learning and teaching and what the student brings to the learning situation in terms of knowledge bases of a specific discipline. Using the concepts of consciousness and control is important in order to take action towards improving teaching and learning in a situation where present methods are not adequate.

To conclude I quote Narode (1987, p.26), who writes of a metacognitive orientation:

> "Generally both the student and the teacher learn from the interaction, and while content knowledge may elude the student's long term memory, hopefully the metacognitive orientation will remain.

Meaningfulness and confidence must become units of analysis of good teaching.

Endnotes

1 This is especially pertinent for the introduction of Probability Theory into the high school syllabi.

2 Some of Vygotsky's central works on activity are not yet available in English.

3 For example, Lomov (1982, p.58) contests the validity of applying the theory of Activity to individual psychological processes rather than giving primacy to the fabric of social relations.

4 Rubenstein (cf. Kozulin, 1986, Leont'ev, 1979) contested that the levels could be split off from each other, claiming that both goals and motives operate at the level of both action and activity. However, Rubenstein concedes that their theories can be seen to correspond. Leont'ev on the other hand, accedes to the notion of different levels of goals.

5 'Meaningful' in the way used by Craig (1985, p.91), in the sense of conceptions of what probability ought to entail.

6 See section on the development of concepts.

7 'Novel' is used here in the sense of 'new for the student'.

8 Wertsch (1979) worked with mothers and children, but I have argued that an analogous situation obtains between teacher and learner or expert and novice.

9 Craig (1985) corroborates this interpretation.

10 Citing Schoenfeld and Lesh.

11 Vygotsky's view of consciousness is of course considerably more complex than this.

REFERENCES

Baker, L. and Brown, A.L. (1984) Metacognitive Skills and Reading, in P.D.Pearson (ed.) Handbook of Reading Research, pp.353-394, New York: Longman.

Bereiter, C. and Scardamalia, M. (1985) Cognitive Coping Strategies and the Problem of "Inert Knowledge", in S.F.Chipman, J.W.Segal and R.Glaser (eds.) Thinking and Learning Skills, vol.2: Research and Open Questions, Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Bishop, A. (1985) The Social Construction of Meaning-a Significant Development for Mathematics Education? For the Learning of Mathematics, vol.5, no.1, pp.24-28.

Brown, A.L. and DeLoache, J.S. (1978) Skills, Plans, and Self-Regulation, in R.S.Siegler (ed.) Children's Thinking: What Develops? pp.3-35, Hillsdale, New Jersey: Erlbaum.

Brown, A.L. (1978) Knowing When, Where, and How to Remember: A Problem of Metacognition, in R.Glaser (ed.), Advances in Instructional Psychology, vol.1, Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Brown, A.L. (1980) Metacognitive Development and Reading, in R.J Spiro, B.Bruce, and W.F.Brewer (eds.) Theoretical Issues in Reading Comprehension, Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Bruner, J. (1985) On Teaching Thinking: An Afterthought, in S.F.Chipman, J.W.Segal and R.Glaser (eds.) Thinking and Learning Skills, vol.2: Research and Open Questions, Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Brushlinskii, A.V. (1987) Activity, Action and Mind as Process, Soviet Psychology, vol.25, no.4, pp.59-81.

Callahan, L.G. (1987) Research Report: Metacognition and School Mathematics, Arithmetic Teacher, vol.34, no.9, pp.22-23.

Cavanaugh, J.C. and Perlmutter, M. (1982) Metamemory: A Critical Examination, Child Development, vol.53, pp.11-28.

Cole, M. (1985) The Zone of Proximal Development: Where Culture and Cognition Create Each Other, in J.V.Wertsch (ed.) Culture, Communication, and Cognition: Vygotskian Perspectives, Cambridge: Cambridge University Press.

Craig, A. (1985) Mothers and Children: An Analysis of Change, Doctoral Thesis, University of Natal, Durban.

Davydov, V.V. and Markova, A.K. (1983) A Concept of Educational Activity for Schoolchildren, Soviet Psychology, vol 21. no.2, pp.60 - 76.

Dicks, H. (1989) Service Course Teaching in Agriculture, South African Statistical Association Newsletter, June, p.15.

Donaldson, M. (1978) Children's Minds, Glasgow: Fontana.

Feuerstein, R. (1979) The Dynamic Assessment of Retarded Performers: The Learning Potential Assessment Device, Theory, Instruments, and Techniques, Baltimore: University Park Press.

Feuerstein, R. (1980) Instrumental Enrichment: An Intervention Program for Cognitive Modifiability, Baltimore: University Park Press.

Flavell, J.H. (1976) Metacognitive Aspects of Problem Solving, in L.B. Resnick (ed.), The Nature of Intelligence, Hillsdale: Lawrence Erlbaum Associates.

Flavell, J.H. (1981a) Cognitive Monitoring, in W.P.Dickson (ed.), Children's Oral Communication Skills, New York: Academic Press.

Flavell, J.H. (1981b) Monitoring Social Cognitive Enterprises: Something Else that may Develop in the Area of Social Cognition, in J.H.Flavell and I.Ross (eds.), Social Cognitive Development: Frontiers and Possible Futures, New York: Cambridge University Press.

Fong, G.T., Krantz, D.H., and Nisbett, R.E. (1986) The Effects of Statistical Training on Thinking about Everyday Problems, Cognitive Psychology, vol.18, pp.253-292.

Gal'perin, P.Ya. (1969) Stages in the Development of Mental Acts, in M.Cole and I.Maltzman (eds.), A Handbook of Contemporary Soviet Psychology, New York: Basic Books.

Garfield, J. and Ahlgren, A. (1988) Difficulties in Learning Basic Concepts in Probability and Statistics: Implications for Research, Journal for Research in Mathematics Education, vol.19, no.1, pp.44-63.

Lester, F.K. and Garofalo, J. (1987) The Influence of Affect, Beliefs, and Metacognition on Problem Solving Behaviour: Some Tentative Speculations, Paper presented at the Annual Meeting of the American Educational Research Association, ED 281 758.

Garner, R. (1987) Metacognition and Executive Control, in Metacognition and Reading Comprehension, New Jersey: Ablex Publishing Corporation.

Hansen, R.S., McCann, J., and Myers, J.L. (1985) Rote versus Conceptual Emphases in Teaching Elementary Probability, Journal for Reasearch in Mathematics Education, vol.16, no.5, pp.364-374.

Ingold, T. (1986) Evolution and Social Life, USA: Cambridge University Press.

Jukes, T.J. (1987) Mental Capacity and Executive Strategies Among Zulu Speaking Children, M.A.Dissertation, University of Natal, Durban.

Kahnemann, D., Slovic, P.,and Tversky, A. (1982) Judgement under Uncertainty: Heuristics and Biases, Cambridge: Cambridge University Press.

Kahnemann, D. and Tversky, A. (1972) Subjective Probability: A Judgement of Representativeness, Cognitive Psychology, vol.3, pp.430-454.

Kameenui, E.J. and Griffin, C.C. The National Crisis in Verbal Problem Solving in Mathematics: A Proposal for Examining the Role of Basal Mathematics Programs, The Elementary School Journal, vol. 89. no.5, pp.575-593.

Kelly, I.W. and Zwiers, F.W. (1986) Introducing Disjoint and Independent Events in Probability, Revised version of paper presented at Second International Conference on Teaching Statistics, Victoria, Canada.

Kerlinger, F.N. (1964), Foundations of Behavioral Research (2nd ed.), London: Holt, Rinehart and Winston.

Kitchener, K.S. (1983) Cognition, Metacognition and Epistemic Cognition, Human Development, vol.26, pp.222-232.

Konold, C. (1988) Understanding Students' Beliefs about Probability, Massachusetts, ED303358, Scientific Reasoning Research Institute.

Kozulin, A. (1986) The Concept of Activity in Soviet Psychology, American Psychologist, vol.41, no.3, pp.264-274.

Leont'ev, A.N. (1979). The Problem of Activity in Psychology, in J. V. Wertsch (ed.) The Concept of Activity in Soviet Psychology, New York: ME Sharpe.

Lomov, B.F. (1982) The Problem of Activity in Psychology, Soviet Psychology, vol.21, pp.55-91.

Manning, B.H. (1984) A Self-Communication Structure for Learning Mathematics, School Science and Mathematics, vol. 84(1), pp.43-51.

Markman, E.M. (1981) Comprehension Monitoring, in W.P.Dixon (ed.) Children's Oral Communication Skills, New York: Academic Press.

Mellin-Olsen, S. (1987) The Politics of Mathematics Education, Dordrecht: D. Reidel Publishing Company.

Miller, W.R. and Seligman, M.E.P. (1974) Depression and Learned Helplessness in Man, in J.M.Neale, G.C.Davison and K.P.Price (eds.) Contemporary Readings in Psychopathology, New York: Wiley and Sons.

Mevarech, Z.R. (1983) A Deep Structure Model of Students' Statistical Misconceptions, Educational Studies in Mathematics, vol.14, pp.415-429.

Myers, J.L. (1982) The Role of Explanation in Teaching Elementary Probability, pp.378-405.

Myers, J.L., Hansen, R.S., Robson, R.C. and McCann, J. (1983) The Role of Explanation in Learning Elementary Probability Theory, Journal of Educational Psychology, vol.75, no.3, pp.374-381.

Narode, R. (1987) Metacognition in Math and Science Education, Massachusetts, ED 291 558.

Noss, R. (1988) Book Reviews Educational Studies in Mathematics, A.J.Bishop (ed.) vol.19. no.3, pp.403-411.

Pascual-Leone, J. and Goodman D. (1979) Intelligence and Experience: a Neo-Piagetian Approach, Instructional Science, vol.8, pp.301-367.

Piaget, J. and Inhelder, B. (1975) The Origin of the Idea of Chance in Children, London: Routledge and Kegan Paul.

Popper, K.R. (1959) The Logic of Scientific Discovery, Great Britain: Anchor Press.

South African Statistical Association Journal, March 1990.

Schoenfeld, A.H. (1984) Beyond the Purely Cognitive: Belief Systems, Social Cognitions and Metacognitions as Driving Forces in Intellectual Performance, Cognitive Science, vol.7, pp.329-363.

Shaughnessy, J.M. (1982) Misconceptions of Probability, Systematic and Otherwise; Teaching Probability and Statistics so as to Overcome Some Misconceptions, in D.R.Grey (ed.), Proceedings of the First International Conference on Teaching Statistics, vol.2, pp.784-801.

Skemp, R.R. (1979) Intelligence, Learning and Action: A Foundation for Theory and Practice in Education, New York: Wiley.

Steinbring, H. and von Harten, G. (1982) Learning from Experience - Bayes Theorem: a Model for Stochastic Learning Situations, in D.R. Grey (ed.) Proceedings of the First International Conference on Teaching Statistics, vol.2, pp.278-290.

Vygotsky, L.S. (1981), in J.V.Wertsch (ed.) The Concept of Activity in Soviet Psychology, Armonk, New York: M.E. Sharpe.

Vygotsky, L. (1986) Thought and Language, (tr. A.Kozulin) Cambridge, Massachusetts: MIT Press.

Vygotsky, L.S. (1987) The Collected Works of L.S. Vygotsky, R.W. Weiber (ed.), vol.1, New York: Plenum Press.

Walkerdine, V. (1988) The Mastery of Reason: Cognitive Development and the Production of Rationality, London: Routledge.

Walkerdine, V. (1990) Subjectivity, Discourse and Practice in Mathematics Education, in Proceedings of the First International Conference on Political Dimensions of Mathematics Education, London: Department of Mathematics, Statistics, & Computing Institute of Education, University of London, pp.248-255.

Walton, K.D. and Walton, J.D. (1987) Probability: Actual Trials, Computer Simulations, and Mathematical Solutions, Reston, VA: Mathematics Education Trust.

Wertsch, J.V. (1979) From Social Interaction to Higher Psychological Processes. A Clarification and Application of Vygotsky's Theory, Human Development, vol.22, pp.1-22.

Wertsch, J.V. (1984) The Zone of Proximal Development: Some Conceptual Issues, in B.Rogoff and J.V.Wertsch (eds.) Children's Learning in the Zone of Proximal Development. No 23, New Directions for Child Development. San Francisco: Jossey-Bass.

Wertsch, J.V. and Stone, C.A. (1985) The Concept of Internalization in Vygotsky's Account of the Genesis of Higher Mental Functions, in J.V. Wertsch (ed.) Culture, Communication and Cognition, Vygotskian Perspectives, Cambridge: Cambridge University Press.

Whitehead, A.N. (1929) The Aims of Education, New York: Macmillan.

Wittgenstein, L. (1972) Philosophical Investigations, G.E.M.Anscombe (tr.), Oxford: Basil Blackwell.

Wood, D. (1986) Aspects of Teaching and Learning, in M.Richards and P.Light (eds.) Children of Social Worlds, Cambridge: Polity Press.

SELECT BIBLIOGRAPHY

Brown, A.L., Campione, J. and Day, J.D. (1981) Learning to Learn: On Training Students to Learn from Texts, Educational Researcher, vol.10, no.2, pp.14-21.

Brown, C., Brown, S., Cooney, T. and Smith, D. (1982) The Pursuit of Mathematics Teachers' Beliefs. In Wagner, S. (ed.) Proceedings of the Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, pp.203-215, Athens: University of Georgia, Department of Mathematics Education.

Bruner, J.S. (1972) The P[illegible]nce of Education New York: Allen & Unwin.

Bruner, J.S. (1974) Beyond [illegible] Information Given New York: Allen & Unwin.

Bruni, J.V. and Silverman, H.J. (1986) Developing Concepts in Probability and Statisitcs - and Much More, Arithmetic Teacher, vol.33, no.6, pp.34-37.

Cloete, N. (1984) Perspectives on Student Learning: Has the Long Awaited Paradigm-shift Occurred? Perspectives in Education, vol.8, no.2 (Dec.).

Cobb, P. (1986) Contexts, Goals, Beliefs, and Learning Mathematics, For the Learning of Mathematics, vol.6, no.2 (June), pp.2-9.

Cole, M. and Scribner, S. (1974) Culture and Thought: a Psychological Introduction, New York: Wiley.

DeGroot, M.H. (1986) A Conversation with Persi Diaconis, Statistical Science, vol.1, no.3, pp.319-334.

Dunlap, L.L. (1987) 'First Grade Children's Understanding of Probability', Paper presented at the Annual Meeting of the Eastern Psychological Assocation, Arlington, VA, April.

Fischbein, E. (1975) The Intuitive Sources of Probabilistic Thinking in Children, Dordrecht: D. Reidel Publishing Company.

Ginsburg, H.P., Posner, J.K. and Russell, R.L. (1981) The Development of Mental Addition as a Function of Schooling and Culture, Journal of Cross-Cultural Psychology, vol.12, pp.163-178.

Gray, M.J.(1987) Comprehension Monitoring: What the Teacher Should Know, The Clearing House vol. 61 (September), no.1, pp. 38-41.

Green, D.R., A Survey of Probability Concepts in 3000 Pupils Aged 11-16 Years, in D.R. Grey (ed.) Proceedings of the First International Conference on Teaching Statistics, vol. 2, pp. 343-360.

Luria, A.R. (1976) Cognitive Development: Its Cultural and Social Foundations, Cambridge: Harvard University Press.

Meichenbaum, D. (1976) Toward a Cognitive Theory of Self Control. In G.E. Schwartz and D. Shapiro (eds.) Consciousness and Self-Regulation, vol. 1, pp. 223-255. New York: Plenum.

Miller, R. (1984)' Reflections of Mind and Culture'. Inaugural Lecture, Department of Psychology, University of Natal.

Myers, J.L., Hansen, R.S., Robson, R.C. and McCann, J. (1983) The Role of Explanation in Learning Elementary Probability Theory, Journal of Educational Psychology, vol. 75, no. 3, pp. 374-381.

Saxe, G.B. (1982) Culture and the Development of Numerical Cognition: Studies among the Oksapmin in Papua, New Guinea. In C.J. Brainerd (ed.), Children's Logical and Mathematical Cognition,. New York: Springer-Verlag.

Schwartz, G.E. and Shapiro, D. (eds) (1978) Consciousness and Self-Regulation, Great Britain: Wiley. 2 vols.

Silver, E.A. (1982) Knowledge Organization and Mathematical Problem Solving. In F.K. Lester and J. Garofalo (eds.) Mathematical Problem Solving: Issues in Research, pp.15-25, Philadelphia: Franklin Institute Press.

Skemp, R.R. (1976) Relational and Instrumental Understanding, Mathematics Teaching, vol.77, pp.20-26.

Steiner, H-G. (1987) A Systems Approach to Mathematics Education, Journal for Research in Mathematics Education, vol.18, no.1, pp.46-52.

Wertsch, J.V. (1985a) Vygotsky and the Social Formation of Mind, Library of Congress Cataloging in Publication Data, USA.

Wertsch, J.V. (ed.). (1985b) Culture, Communication and Cognition. Vygotskian Perspectives. Cambridge: Cambridge University Press.

APPENDIX 1

Quotes from students' writing on the course Quantitative Techniques I, University of the Witwatersrand, 1989.

Student 1: "Reading notes is like an escape from the practicality and brutal character of the course".

Student 2: "... there is a lack of initiative on my part to reinforce the theory that I have acquired from the notes. ...The simplicity of the notes therefore lure (sic) me into indolency and ultimately condemn me (sic) misjudgement. To date it is unfortunate that I have not yet found a diagnosis treatment to this predicament - enslavement. In conclusion, my approach to the course is a duplex one. I therefore spend more time not doing my work but trying to rationalise, iron-out and subdue this (gross discrepency (sic) between the notes and the practicalities of the course - examples and applications) pernicious inconsistencies."