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Application of a constrained non-linear hydraulic gradient design tool to water reticulation network upgrade

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Southern Africa has embarked on substantial expansion of its water supply network in order to ensure safe, reliable, convenient and sufficient water for everyone. To achieve this, new systems are being built and many existing systems are being upgraded. The upgrade of many existing systems is required for two reasons: some currently functional systems may run dry if subjected to additional demands as these systems were not initially designed to cater for such demand, and some systems are currently non- or sub-functional as they were ill-designed and/or ill-implemented from inception. Many of the systems that require upgrade are underdesigned due to a lack of skill, tools and/or knowledge of designers, or from other extraneous factors (e.g., illegal connections and sabotage). It is hardly surprising therefore that the failures of water projects in developing countries are recorded to be as high as 80%. Ill-designed systems increase operation and maintenance costs significantly. In especially Southern Africa, designers require simple, yet rigorously tested tools to facilitate sustainable, yet cost-effective network designs. Presented in this paper is a simple, yet robust constrained non-linear hydraulic gradient network reticulation design tool. The design tool is calibrated using the New York City water supply problem that has served as a benchmark problem for other models and then applied to the Selebi–Phikwe (SP) water reticulation network (WRN) in Botswana, which was designed based on engineering judgement. The optimisation algorithm employed in the design tool is based on the concept that a hypothetical hydraulic gradient for a hydraulically balanced WRN exists that, when achieved iteratively, produces optimal pipe sizes and an optimal flow relation between each pipe. The unique problems and challenges of the SP WRN (pressure deficiencies in sections of the existing network and the proposed addition of three new residential developments) required determining the most appropriate peak and night flow operating scenarios, and optimal pipe sizes for the proposed expansion of the network. Optimisation by trial and error had been previously employed in the design of the SP WRN—a common practice amongst water system designers, and the results are compared with those generated with the design tool. The design tool achieved a 62% reduction in total pipe cost from that obtained by trial and error for the SP WRN problem. At the same time, the design tool gives comparable pipe costs to those published in literature for the New York City water supply tunnels problem.

Keywords: Water networks; Expansion; Cost savings

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1. Introduction

1.1 *Ideal water distribution system design*

Ideally, water distribution systems should be designed to cater for both present as well as future demands and staged development of the water system provides an effective way to achieve this. In practice, the basic steps employed in staged development should include the following:

i. Determine the water system's design period, typically 20–30 years.
ii. Calculate the projected demand of existing consumers in, e.g., five-year steps up to the final design year.
iii. On the model of the water system, add to each 'demand step' all anticipated new developments requiring supply. Do the same until the model for the final design year has been determined (this may be refined later).
iv. Determine design criteria, such as maximum and minimum allowable residual pressures, maximum flow velocities, preferred pipe diameters, storage requirements, etc.
v. Design the model based on predicted demands for the final design including all the anticipated nodes and links required to supply present as well as future developments.
vi. Identify and investigate various network configurations and how well they achieve future supply objectives in terms of cost minimisation and reliability of supply.
vii. Select the optimal design and ensure the system variables compare favourably with the design criteria.
viii. Having determined the final design, the demand in the model is progressively reduced, and at each time step, as many components as possible are removed whilst maintaining the design criteria. This is repeated back to the present to achieve a system effectively designed to cater for present as well as anticipated future developments.

For several reasons including budgetary constraints, many water systems in Southern Africa were not designed in the manner described above. It therefore becomes imperative for many of such existing systems to be redesigned and/or upgraded in order for communities for which they were designed to benefit fully from the services.

1.2 *Challenges in water distribution system upgrade*

The task of upgrading existing water systems presents several challenges, some of which include the following: firstly, the upgrade of water systems may require the use of pipe extensions (in parallel, or series) in WRNs which would result in the alteration of flow variables and residual node pressures. This alteration in network variables results from the complex inter-relationships that exist between WRN components and are exacerbated in loop or combined loop-branch network configurations where there are a large number of inter-connected links and nodes. While upgrading water systems, it is therefore imperative to ensure that the altered network variables satisfy design criteria. Secondly, to arrive at an optimal solution, several network configurations and designs need to be identified and investigated in relation to the design criteria in order to arrive at an optimal solution. This requires significant investment in time and computational resources especially when the system houses a large number of diverse components. It is these challenges that, in a resource-constrained environment (such as Southern Africa), make futile trial and error design underpinned by engineering judgement. Trial and error network design underpinned by engineering judgement occurs when a water designer adjusts the sizes (upwards or downwards) of one or more pipes in order to satisfy the design criteria and minimise costs. Decision support tools, on the other hand, facilitate network design using well-defined procedures and/or empirically proven algorithms. They have been found to be more efficient in saving design time and associated costs especially in large networks, and in generating significantly more feasible solutions, thus making the task of determining an optimal solution simpler. It is against this backdrop that the constrained non-linear pipe design tool described in this paper was developed.

2. Conceptualisation of the design tool's optimisation and previous work

The design tool's optimisation procedure presented herein is primarily adapted, with some modifications, from Featherstone and El-Jumaily's (1983) model, which is based on the concept that a hypothetical hydraulic gradient, S_o for a hydraulically balanced WRN exists by which an initial network design can be iteratively corrected to produce optimal pipe sizes and an optimal flow relation between each pipe. Deb and Sarker (1971), Wu (1975) and Alperovits and Shamir (1977) present design optimisation models that utilise a similar concept vis-à-vis the use of hydraulic gradients/surfaces to determine optimal system designs. The models presented by these authors provide theoretical anchorage for the concept proposed by Featherstone and El-Jumaily (1983).

Deb and Sarker's (1971) model is called the equivalent pipe diameter method for network optimisation. This method determines optimal equivalent pipe diameters for a network once the hydraulic surface (i.e., node pressures) and the head at inlet are known. By imposing the hydraulic surface over the network, pipe sizes are replaced by equivalent pipes of 100 m length and equivalent diameter, D_e^m using the Hazen Williams pipe formula. m is a constant derived from the pipe cost function employed by Deb and Sarker (1971). Prior to the output of results, these equivalent pipe diameters are converted into actual pipe diameters with actual pipe lengths. The major drawbacks that this method presents are that the cost functions used are related to equivalent and not actual pipes; that the hydraulic surface within the network is artificially created and not computed from the analysis; and that A^1 is obtained from hypothetical flows (Watanatada 1973, Featherstone and El-Jumaily 1983). A^1 is a constant for each network loop whose optimal value determines whether an appropriate network solution may be obtained ($A^1 = \Sigma\{D_e^m/Q\}$. Q represents pipe discharge in litres per minute).

Wu's (1975) model showed that for a single pipe main composed of lengths of different diameters delivering water to the sub-mains in an irrigation system, the optimal shape of the energy gradient producing minimum cost of the pipeline is a curve with a sag of 15% of the total head drop, below a straight (linear) line drawn between the inlet and outlet head elevations at the middle section of the main (figure 1). A number of energy gradient patterns including concave and convex curves and a straight line were imposed on the pipeline. The cost difference however, between the results of using the straight energy and the optimal (parabolic) energy gradient lines was found to be of the order of only 2% (figure 1) (Featherstone and El-Jumaily 1983).

The Linear Programming Gradient method proposed by Alperovits and Shamir (1977) uses the solution of a linear program as an intermediate step in a hydraulic gradient search. This technique requires that pipe flows be set to particular values before the linear program can be formulated. Once the linear program is solved, information available from this solution is then used to calculate a hydraulic gradient for the network which is then used to change pipe flows. Solving a new linear program using the improved pipe flows, results in a reduction in network cost. This process is iterative, and converges to a local optimum solution. The method by Alperovits and Shamir (1977) has the advantage of not requiring any substitution for continuously variable pipe diameters, as the solution can easily be limited to commercially available pipe diameters (Quindry *et al.* 1981). The LPG model is also capable of sizing major water system components, and determining optimal operating settings for pumps and valves under multiple loading conditions. Some weaknesses include the considerable skill required to set out and optimise a water system since several heuristics are employed, and the need to optimise from several starting points to avoid local optima.

The optimisation procedure proposed by Featherstone and El-Jumaily (1983) and adapted in the design tool presented herein overcomes certain limitations of previous methods in that the hydraulic gradient employed in the optimisation is not assumed, as done in Deb and Sarker's (1973) model, but calculated during the design optimisation (see equations (16)–(20)). Also, assumed pipe diameters are utilised and finally transformed into actual commercial sizes during optimisation as opposed to the concept of equivalent diameter. Since several runs are recommended while using the design tool to determine an optimal solution, concave and convex hydraulic gradients (in relation to network costs) that terminate at local optima are generated (see Figure 10)—a similar feature of Wu's (1975) study.

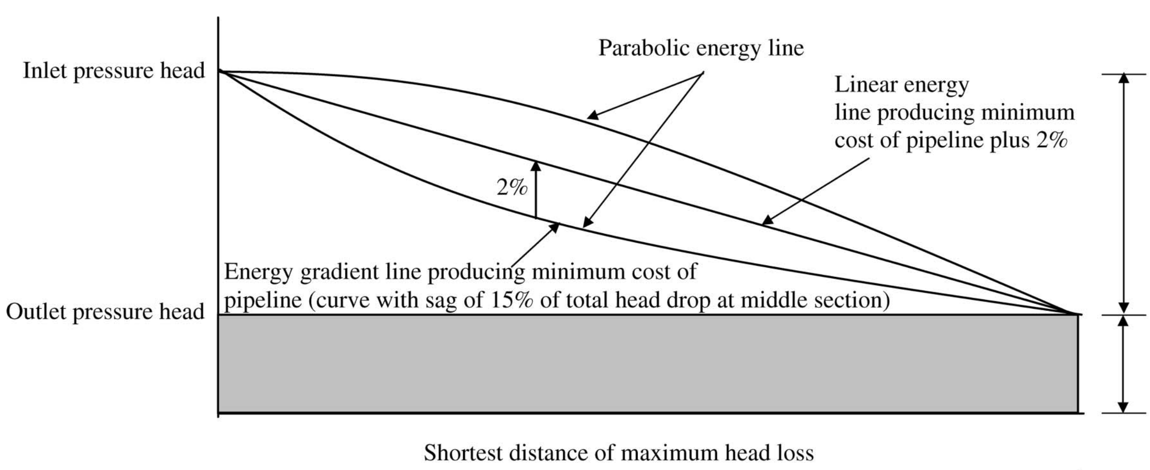

Figure 1. Rotation of linear and parabolic energy lines about network inlet and outlet (after Featherstone and El-Jumaily 1983).

Featherstone and El-Jumaily's (1983) optimisation, as well as the design tool, incorporates the cost functions (capital and operating) of the major components of the water distribution system (equations (1) and (7), respectively). The distinguishing features between the two is that Featherstone and El-Jumaily's (1983) model (i) is customised to UK conditions through the use of UK pipe, pump and tank cost functions; (ii) utilises the capital (pump and tank) and pump operating cost functions (and variables) directly in the objective function (equation (2)); and (iii) determines S_0, the dummy hydraulic gradient, by equating the first derivative of the objective function shown in equation (2) ($dC_{water\ distribution\ system}/dS_0$) to zero. The latter feature is based on the fact that while varying $C_{water\ distribution\ system}$ with respect to S_0, and the relationship $dC_{water\ distribution\ system}/dS_0$ becomes equal to 0, a minimum (optimum) solution has been reached.

$$C_{water\_distribution\_system} = \sum_{i=1}^{N} C_{pipe_i} + C_{pump} + C_{tank} + C_{operating} \tag{1}$$

$$C_{water\_distribution\_system} = K_1 \sum_{i=1}^{N} \frac{L_i^{a_2} Q_i^{0.4a3}}{S_0^{0.2a3}} + c_1 Q^{c2} + d_1 V^{d2} + \frac{\rho g Q(S_0 d + RP + ZG + h_{fp})}{1000\mu} TFY \tag{2}$$

K_1 is a variable dependent on lamda, λ the Darcy–Weisbach pipe friction coefficient; N is the number of pipes within the network; c_1 and c_2, and d_1 and d_2 represent pump installation and storage costing coefficients; V represents storage volume; RP and ZG are minimum residual pressure head above ground level, and depth of water in borehole below ground level respectively; T is number of hours of pump operation per annum; F is energy cost per KW.hr; and Y is design life of the pumps.

By equating the first derivative of the objective function to zero, S_0 in Featherstone and El-Jumaily's (1983) model becomes:

$$S_0 = \frac{0.2a_1 K_2 \sum_{i=1}^{N} \frac{(L_i^{a_2} Q_i^{0.4a_3})}{S_0^{0.2a_3}}}{K_3} \frac{1}{(0.2a_3 + 1)} \tag{3}$$

In contrast to the features of Featherstone and El-Jumaily's (1983) model highlighted above, the design tool presented herein has the following the features: (i) is customised to the Southern Africa condition by using Southern Africa cost functions; (ii) capital (pump and tank) and pump operating costs, although included in the objective function (equation (21)), are calculated in a separate program and inserted into the objective function after calculation. The hydraulic gradient, S_0 calculated does not therefore presume the presence of only one pumping and distribution mains from the source via the tank to the reticulation network, as is presumed in Featherstone and El-Jumaily's (1983) model. Capital (pump and tank) costs are calculated from the optimal relationship (achieved using successive approximation techniques) between the pump flow rate and tank storage volume required to supply consumer demands at the minimum cost. Pump operating costs, on the other hand, are calculated based on the average hours of operation anticipated each day to supply consumer's demands over the pump's design life, discounted to the present; (iii) determines S_0 by simply re-arranging the objective function with respect to S_0 (see equation (21)).

Several authors (e.g., Schaake and Lai 1969, Quindry *et al*. 1981, Gessler 1982, Bhave 1985, Morgan and Goulter 1985, Dandy *et al*. 1996, Savic and Walters 1997, Lippai *et al*. 1999, Wu and Simpson 2002, Eusuff and Lansey 2003) have attempted the design and upgrade of WRNs. The most recent works have employed Evolutionary Algorithms to facilitate the optimisation task(s). Evolutionary Algorithms (especially Genetic Algorithms) have become extremely effective in generating a host of feasible solutions for small and large systems which employ multiple variables, are stochastic in nature and operate under varying loading conditions such as is found in WRNs. They could however become computationally cumbersome in that they require a significantly large number of runs to ascertain an optimal solution since they employ several parameters that may be varied individually or collectively. Evolutionary Algorithms, because of the multiple variables involved, also require considerable skill to set out and optimise a water system.

Some additional advantages of the design tool presented herein is its simplicity of use for the Southern Africa situation (since, at each run, it requires the varying of only one variable—the maximum and minimum pipe size), the significant savings in design time, and its ability to generate comparable results with other tools. The simplicity of this design tool also presents promising opportunities for especially Southern Africa's designers who are constantly faced with a lack of resources to be trained in using more complicated network design software.

3. Formulation of the design tool

The basic equations of continuity (4), conservation of energy (5) and hydraulic head loss relation (6) are utilised in modelling WRNs:

$$\Sigma(Q_{in} - Q_{out}) = 0 \tag{4}$$

$$\Sigma h_f = \Delta E_{FGN} \tag{5}$$

$$h_f = g(Q_i) \tag{6}$$

Q_{in}, Q_{out} represent flows into and away from any node respectively; Σh_f is total energy loss around a loop; ΔE_{FGN} is difference in total hydraulic grade between fixed grade nodes (FGNs); and $g(Q_i)$ is pipe head loss equation (equations (17) or (18)) as a function of flow, Q_i.

To arrive at an optimal WRN design, an iterative hydraulic simulation-optimisation algorithm is employed. Efficient hydraulic simulation is based on modelling the WRN using equations (4)–(6) and determining the unknown variables (Q_i) or node residual pressure heads, H_j) using the Newton–Raphson iterative procedure on simultaneous equations generated using the nodal formulation method. Pipe sizes and other pipe parameters, consumer demands, network layout configuration, pump characteristics and FGN elevations are known or assumed prior to simulation. The Choleski Decomposition technique (Stoer and Bulirsch 1993) is employed to solve the matrix which calculates node pressure heads. Head loss is calculated based on the Darcy–Weisbach or Hazen–Williams pipe friction equations (17) and (18). At the end of each simulation, continuity is checked at each network node and if a violation exists, node pressures are corrected and the network simulated to determine new variable values. Output from the simulation includes pipe flows and orientation, pipe headlosses, friction factors, node pressure heads, draw-off at each source node, pumping head(s) and valve head losses.

3.1 *Objective function and optimisation*

The design tool presented is a module in a suite of software programs called ***Wadessy*** (an acronym for ***Wa***ter ***De***cision ***S***upport ***Sy***stem). The overall objective of ***Wadessy***'s suite of programs is to minimise the capital and recurrent costs of the major components of a water distribution system, and the system is modelled as follows (Ilemobade and Stephenson 2003):

$$\begin{aligned} &Minimise\ C_{water\ distribution\ system} \\ &\quad = (C_{WRN} + C_{pump\ and\ tank\ sub-system}) \end{aligned} \tag{7}$$

Where C represents cost, and

$$\begin{aligned} C_{pump\ and\ tank\ sub-system} = (&C_{pump\ installation} + C_{pump\ operation} \\ &+ C_{pumping\ mains} + C_{tank\ storage}) \end{aligned} \tag{8}$$

The minimisation of each major component is primarily a function of certain decision variables:

$$Minimise\ C_{WRN} = f(S_0, d_i) \tag{9}$$

$$Minimise\ C_{pump\ and\ tank\ sub-system} = f(Q_k, d_i) \tag{10}$$

Q_k represents pumping mains flow capacity; d_i, pipe diameter and S_0, hydraulic gradient.

The water system objective function is constrained by pipe sizes, nodal pressure heads and pump flow capacity as follows:

i.

$$d_{minimum} \leq d_i \leq d_{maximum} \tag{11}$$

$$d_i \in \{D\} \tag{12}$$

where $\{D\} = (d_1,\ d_2, \ldots, d_n)$ commercially available pipe diameters. The specified maximum and minimum diameter sizes from the list of commercially available pipes serves to narrow the range of sizes during the optimisation process thereby enhancing quicker runs and better quality solutions.

ii. Constraint on node pressure head requires that

$$H_{minimum} \leq H_j \leq H_{maximum} \tag{13}$$

where $H_{minimum}$ and $H_{maximum}$ represent the minimum and maximum allowable residual pressure heads at any node j. $H_j = f(Q_j + t)$ represents calculated pressure head at node j. Q_j represents demand at node j and t represents the demand tolerance prescribed for each node in the network. A demand tolerance is introduced to enhance network resilience to a given degree of variability in peak and night flows.

iii. The optimisation process attempts to achieve a local cost solution nearest to its starting point. That is,

$$C_{water\ distribution\ system\ minimum} \leq C_{water\ distribution\ system\ previous} \tag{14}$$

When a local optimum is reached, the optimisation procedure terminates. Several runs are recommended before selecting the optimal solution.

iv. Furthermore, the non-negativity constraint requires that

$$d_i, l_i, t_i \geq 0 \tag{15}$$

l_i represents pipe length; and t_i, pipe wall thickness.

v. A constraint on the pumping mains flow capacity requires that

$$Q_{Average\ Hourly\ Demand} \leq Q_k \leq Q_{Maximum\ Hourly\ Demand} \tag{16}$$

The Darcy–Weisbach (17) and Hazen–Williams (18) headloss equations are presented below,

$$d_i = \left(\frac{8\lambda Q_i^2}{g\pi^2 S_0}\right)^{0,20} \tag{17}$$

$$d_i = \left(\frac{10.7 Q_i^{1.85}}{C_{HW}^{1.85} S_0}\right)^{0,21} \tag{18}$$

S_0 represents hydraulic gradient (h_f/l_i), λ represents the Darcy–Weisbach pipe friction factor and C_{HW}, the Hazen–Williams pipe friction coefficient. While the Darcy–Weisbach equation is a much better equation (since it caters for the entire range of pipe flow in the turbulent flow zone), the Hazen–Williams equation provides an easy-to-use equation for determining headloss in pipe flow within the transitional turbulent zone only (where most pipe flow operates in practice). Many engineers often argue that the inherent uncertainties in water distribution systems (i.e., demands, pipe roughness, etc.) are much greater than the error made by using the simpler Hazen–Williams equation and hence their preference to use it during design. Equations (17) and (18) are therefore provided in ***Wadessy***'s design tool to give designers choice of headloss equation depending on their preference and pipe information available during design.

WRN pipe costs in South Africa are represented by the equation below (Barta and Rowse 1998):

$$C_{WRN} = \sum_{i=1}^{NP} \left[b_1 l_i^{b2} d_i^{b3} t_i^{b4}\right] \tag{19}$$

where b_1, b_2, b_3 and b_4 are pipe cost variables. By substituting d_i in equation (17) or (18) into equation (19), and equation (19) into equation (7), the objective function for the optimisation process becomes;

$$C_{water\_distribution\_system} = \sum_{i=1}^{NP} \left[b_1 l_i^{b_2} \left(\frac{R_3 Q_i^{R_1}}{S_0^{R_2}}\right)^{b_3} t_i^{b_4}\right] + C_{pump\_and\_tank\_sub-system} \tag{20}$$

for the Darcy–Weisbach equation, $R_1 = 0.40$; $R_2 = 0.20$; $R_3 = 0.61\lambda_2^R$;
for the Hazen–Williams equation, $R_1 = 0.38$; $R_2 = 0.21$; $R_3 = C_{HW}^{-R_1}$.

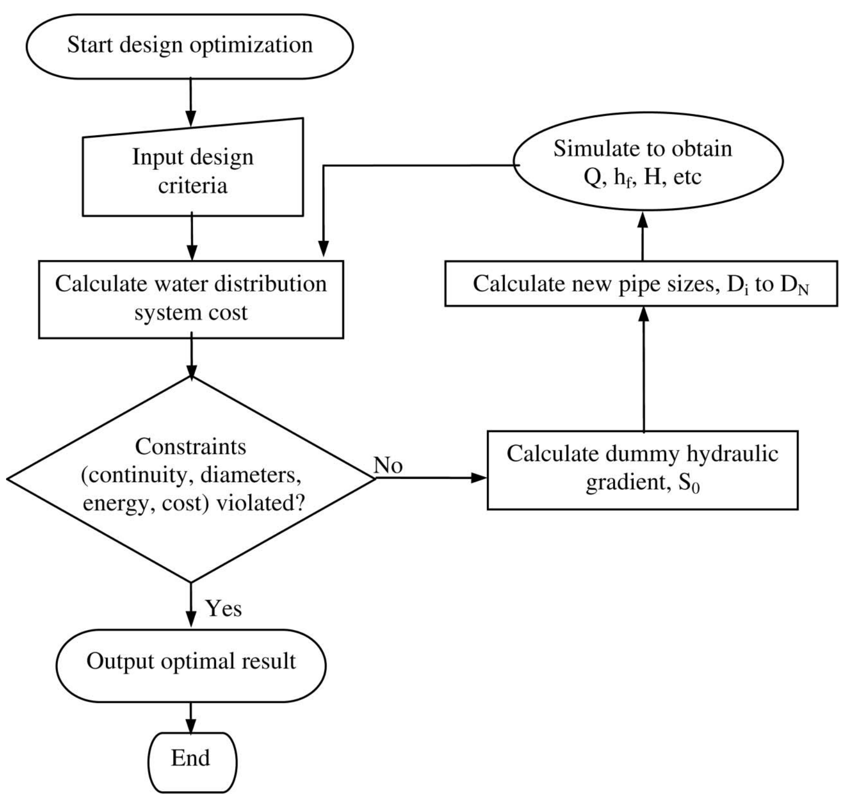

Figure 2. Design optimisation process.

Equation (21) results from re-arranging equation (20) with respect to S_0:

$$S_0 = \left(\frac{\sum_{i=1}^{NP} \left[b_1 l_i^{b_2} \left(R_3 Q_i^{R_1} \right)^{b_3} t_i^{b_4} \right]}{C_{water\_distribution\_system} - C_{pump\_and\_tank\_sub-system}} \right)^{\frac{1}{R_2 b_3}} \tag{21}$$

The dummy hydraulic gradient, S_0 thus becomes the variable that iteratively corrects WRN pipe diameters, d_i until an optimal solution is reached. S_0 is initially calculated based on a hydraulically balanced WRN of initial pipe sizes. This value is substituted into the designer pre-selected head loss equation (17) or (18) and new pipe sizes are calculated. The WRN comprising the corrected pipe sizes is then simulated to determine pipe flows, headlosses, and node residual pressures. If design criteria (equations (11)–(15)) are violated, the optimisation terminates. Otherwise, S_0 is recalculated and the iterative procedure is repeated (see figure 2). A separate methodology is used to calculate $C_{pump\ and\ tank\ sub\text{-}system}$ the optimal value is simply inserted into equation (21) if available).

Wadessy's design tool has been employed in the two problems presented in the sections below, i.e., the New York City water supply tunnels problem and the SP WRN upgrade. The New York City water supply tunnels problem was primarily employed to calibrate ***Wadessy***'s design tool.

4. Calibration of *Wadessy*'s design tool using the New York City water supply tunnels problem

A number of studies in pipe network optimisation have examined the expansion of the New York City water supply

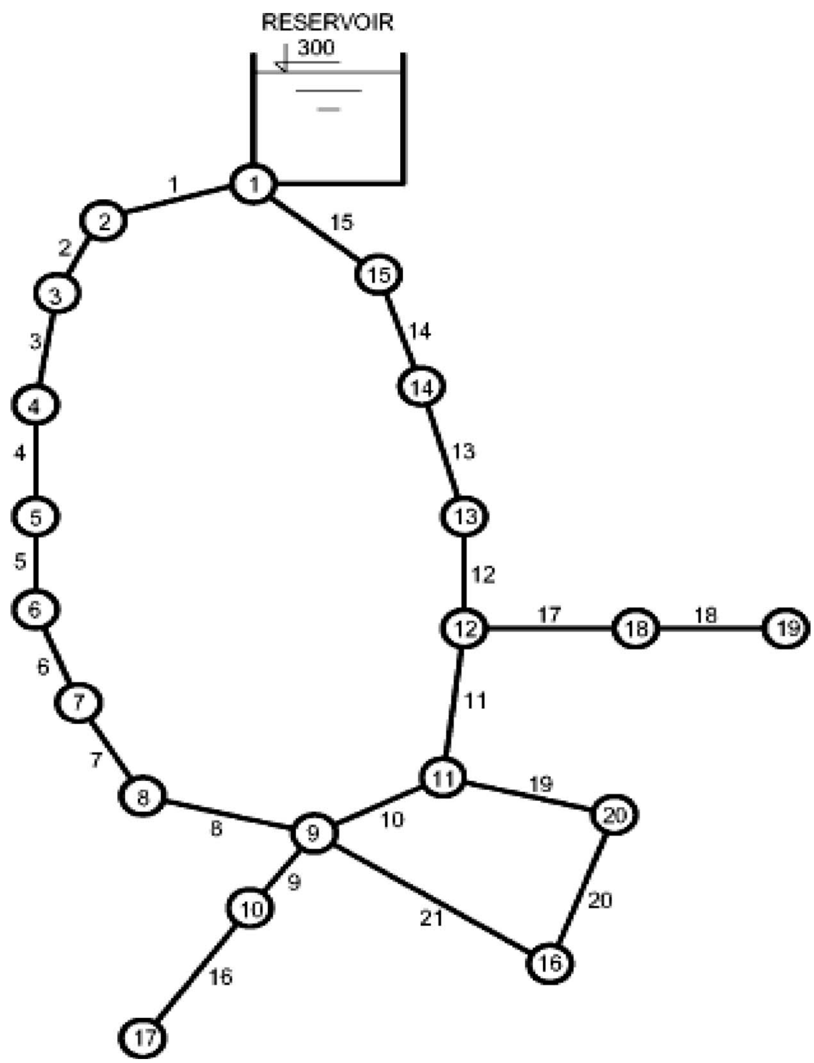

Figure 3. Schematic of the New York City water supply tunnels.

tunnels (Schaake and Lai 1969, Quindry *et al.* 1981, Gessler 1982, Bhave 1985, Morgan and Goulter 1985, Dandy *et al.* 1996, Savic and Walters 1997, Lippai *et al.* 1999, Wu and Simpson 2002, Eusuff and Lansey 2003). It now serves as a benchmark case study for calibrating ***Wadessy***'s design tool. The common objective of the studies was to determine the most economically effective design for additions to the existing system of tunnels that constituted the primary water distribution system of the city of New York (figure 3). Because of age and increased demands, the existing gravity flow tunnels were found to be inadequate to meet the pressure requirements at nodes 16, 17, 18, 19, and 20 for the projected consumption level. The proposed method of upgrade is the same as in previous studies, i.e., to reinforce the system by constructing tunnels parallel to the existing tunnels. The existing WRN of pipes and nodes (table 1) and pipe costs (table 2) are the same input data used in other studies. Optimisation runs in ***Wadessy***'s design tool were done using the Metric system of units. These were later converted to the Imperial system of units for easy comparison with previous studies.

The optimisation process was started from several starting designs in order to generate several optimised solutions and ensure that the optimal was of good quality. In ***Wadessy***'s design tool, the nodes of the proposed expansion to the existing network are pre-determined by the designer. The optimal sizes of the pipe links are however determined using the design tool. In this study, the designer pre-determined nodes 16, 17, 18, 19, and 20 as nodes connecting parallel pipes to the existing network. While attempting to determine an optimal upgraded network, ***Wadessy***'s design tool freezes existing pipe sizes. The optimal results found in previous studies and the present study are summarised in tables 3 and 4. It is apparent from table 3 that the solutions obtained from the literature differ in the number of pipes to be duplicated. The solutions by all authors (except Wu and Simpson (2002) and ***Wadessy***) identify 6 pipes, while Wu and Simpson (2002) identify 7 pipes and ***Wadessy***, 5 pipes. Even the publications that found the same number of pipes to be laid in parallel differ as to which pipes are identified, e.g., Dandy *et al.* (1996) identify pipes 15, 16, 17, 18, 19, and 21, while solution a of Savic and Walters (1997) identifies pipes 7, 16, 17, 18, 19, and 21. Utilisation of the maximum and minimum diameter bounds (equation (11) and (12) in ***Wadessy***'s design tool reduced the search space of the optimisation process and facilitated the determination of an optimum solution each time. From Table 3, ***Wadessy***'s design tool can be seen to perform at least as well as other techniques in determining optimal pipe sizes and total cost. In terms of design time, ***Wadessy***'s design tool, at each run, generates a local optimum solution in an average time of 4 seconds on a 1.51 MB (Centrino), 400 MHz processor computer. The optimal results

Table 1. Pipe and node data for the New York City water supply tunnels.

	Length		Existing diameters			Demand		Minimum allowable pressure head	
Pipe	ft	(m)	inch	(mm)	Node	ft^3/s	(m^3/s)	ft	(m)
1	11600	(3536)	180	(4570)	1		Tank	300	(91.4)
2	19800	(6035)	180	(4570)	2	92.4	(2.62)	255	(77.7)
3	7300	(2225)	180	(4570)	3	92.4	(2.62)	255	(77.7)
4	8300	(2530)	180	(4570)	4	88.2	(2.5)	255	(77.7)
5	8600	(2621)	180	(4570)	5	88.2	(2.5)	255	(77.7)
6	19100	(5822)	180	(4570)	6	88.2	(2.5)	255	(77.7)
7	9600	(2926)	132	(3350)	7	88.2	(2.5)	255	(77.7)
8	12500	(3810)	132	(3350)	8	88.2	(2.5)	255	(77.7)
9	9600	(2926)	180	(4570)	9	170.0	(4.81)	255	(77.7)
10	11200	(3414)	204	(5180)	10	1.0	(0.03)	255	(77.7)
11	14500	(4420)	204	(5180)	11	170.0	(4.81)	255	(77.7)
12	12200	(3719)	204	(5180)	12	117.1	(3.32)	255	(77.7)
13	24100	(7346)	204	(5180)	13	117.1	(3.32)	255	(77.7)
14	21100	(6431)	204	(5180)	14	92.4	(2.62)	255	(77.7)
15	15500	(4724)	204	(5180)	15	92.4	(2.62)	255	(77.7)
16	26400	(8047)	72	(1830)	16	170.0	(4.81)	260	(79.2)
17	31200	(9510)	72	(1830)	17	57.5	(1.63)	273	(83.2)
18	24000	(7315)	60	(1520)	18	117.1	(3.32)	255	(77.7)
19	14400	(4389)	60	(1520)	19	117.1	(3.32)	255	(77.7)
20	38400	(11704)	60	(1520)	20	170.0	(4.81)	255	(77.7)
21	26400	(8047)	72	(1830)					

All pipes including existing and new pipes are assumed to have a Hazen–Williams $C_{HW} = 100$.

presented by Lippai *et al*. (1999) and Eusuff and Lansey (2003) in table 4 violate the minimum pressures specified at nodes 16, 17 and 19.

5. Case study: The Selebi – Phikwe, Botswana water reticulation network upgrade

The second study employed using ***Wadessy***'s design tool is the Selebi – Phikwe (SP) WRN upgrade. SP has been chosen by the government of Botswana as a priority centre for regional industrial development. With a population of about 50,000 people (and a 80,000 projected population when the three new developments are implemented), it is the third largest town in Botswana and is its principal location for large-scale light manufacturing industries. The mining and processing of copper – nickel by BCL Ltd., Botswana's largest single employer, is the town's major industry. In fact, the SP community evolved around the mining activities of BCL Ltd. Other industries that are established in SP include garment manufacturing, furniture, mining accessories, sanitary ware, automotive accessories, structural engineering and jewellery. A reliable water supply therefore is essential for the development of the town.

Two design problems were addressed during the SP WRN upgrade process:

i. Very low pressures were experienced at certain nodes within the existing WRN: since inception, the current WRN has been upgraded several times to cater for increased demands and new developments. The majority of this upgrade has been haphazard resulting in certain areas of the WRN (especially at the Botswana Defence Force, BDF) experiencing no flow at especially peak demand periods (figure 4). The BDF plays a strategic role in the training of Botswana's security forces, and hence should not lack water supply at any time.
ii. Three new residential developments are planned for the east and south-eastern sections of SP, south of Botshabelo. These new developments are Mekoro

Table 2. Available pipe sizes and costs for the New York City water supply tunnels.

Diameter		Pipe Cost	
inch	(mm)	$/ft	($/m)*
36	(910)	93.5	(306.8)
48	(1220)	134.0	(439.6)
60	(1520)	176.0	(577.4)
72	(1830)	221.0	(725.1)
84	(2130)	267.0	(876.0)
96	(2440)	316.0	(1,036.8)
108	(2740)	365.0	(1,197.5)
120	(3050)	417.0	(1,368.1)
132	(3350)	469.0	(1,538.7)
144	(3660)	522.0	(1,712.6)
156	(3960)	577.0	(1,893.0)
168	(4270)	632.0	(2,073.5)
180	(4570)	689.0	(2,260.5)
192	(4880)	746.0	(2,447.5)
204	(5180)	804.0	(2,637.8)

Table 3. Comparison of the parallel pipes required for the New York City water supply tunnels.

				Wu and Simpson (2002)					
	Dandy *et al.* (1996)	Savic and Walters (1997)		fmGA* with SABS	fmGA* without sabs	fmGA* without sabs	Lippai *et al.* (1999) and Eusuff and Lansey (2003)	***Wadessy***'s design tool	
		a	b	a	b	c		Diam.	
Pipe	Diam. in	Diam. in	Diam. in	Diam. in	Diam. in	Diam. in	Diam. in	in	(mm)
1	–	–	–	–	–	72	–		–
7	–	108	–	–	108	108	132		–
8	–	–	–	–	–	–	–		–
15	120	–	144	120	–	–	–		–
16	84	96	84	84	108	96	96	84	(2130)
17	96	96	96	96	108	96	96	108	(2740)
18	84	84	84	84	72	84	84	96	(2440)
19	72	72	72	72	60	72	72	108	(2740)
21	72	72	72	72	84	72	72	96	(2440)
Cost $ m	38.8	37.13	40.42	38.8	39.42	39.69	38.13	39.62	
Function evaluation (iterations)	125,000	N/A	N/A	30,000	21,200	18,800	46,016 and 31,267	(49)**	

*fast messy Genetic Algorithm with or without self-adaptive boundary search.
**average of 4 seconds for each run.

(426 hectares), Block A (170 hectares) and Block B (188 hectares). Each of these developments will require the provision of potable water through the existing SP WRN. Mekoro is scheduled to be developed first (before Blocks A and B) and as such, the upgraded pipe sections in this paper were restricted to Mekoro only, while only the cumulative demands of Blocks A and B were taken into consideration during design. Figure 4 shows the existing SP WRN (skeletal), the proposed Mekoro addition to the SP WRN, and the proposed sites for both Blocks A and B residential developments. Figure 5 depicts detailed Mekoro node references and elevations above mean sea level (MSL) utilised in ***Wadessy***'s design tool.

5.1 *Analysis, results and discussion*

Table 5 presents pipe and node data for violating nodes in the existing SP WRN while table 6 presents pipe and node data for the proposed Mekoro WRN. Pipe costs are given in table 7. With the implementation of the proposed residential developments, it is estimated that peak and night

Table 4. Node pressure heads for critical nodes on the New York City water supply tunnels.

			Savic and Walters (1997)		Wu and Simpson (2002) fmGA with sabs	Lippai *et al.* (1999) and Eusuff and Lansey (2003)	***Wadessy***'s design tool	
	Minimum Pressure head	Dandy *et al.* (1996) Pressure head	a Pressure head	b Pressure head	a Pressure head	Pressure head	Pressure head	
Node	ft	ft	ft	ft	ft	ft	ft	(m)
16	260.0	N/A	260.2	261.5	260.5	259.8*	269.3	(82.1)
17	272.8	N/A	272.9	273.8	272.8	272.6*	275.1	(83.8)
18	255.0	N/A	NA	NA	261.8	261.1	265.4	(80.9)
19	255.0	N/A	255.2	256.8	255.7	254.8*	260.1	(79.3)
20	255.0	N/A	NA	NA	261.2	260.7	273.8	(83.4)

*Implies violation of the minimum pressure head constraints (Eusuff and Lansey 2003).
NA represents Not Available.

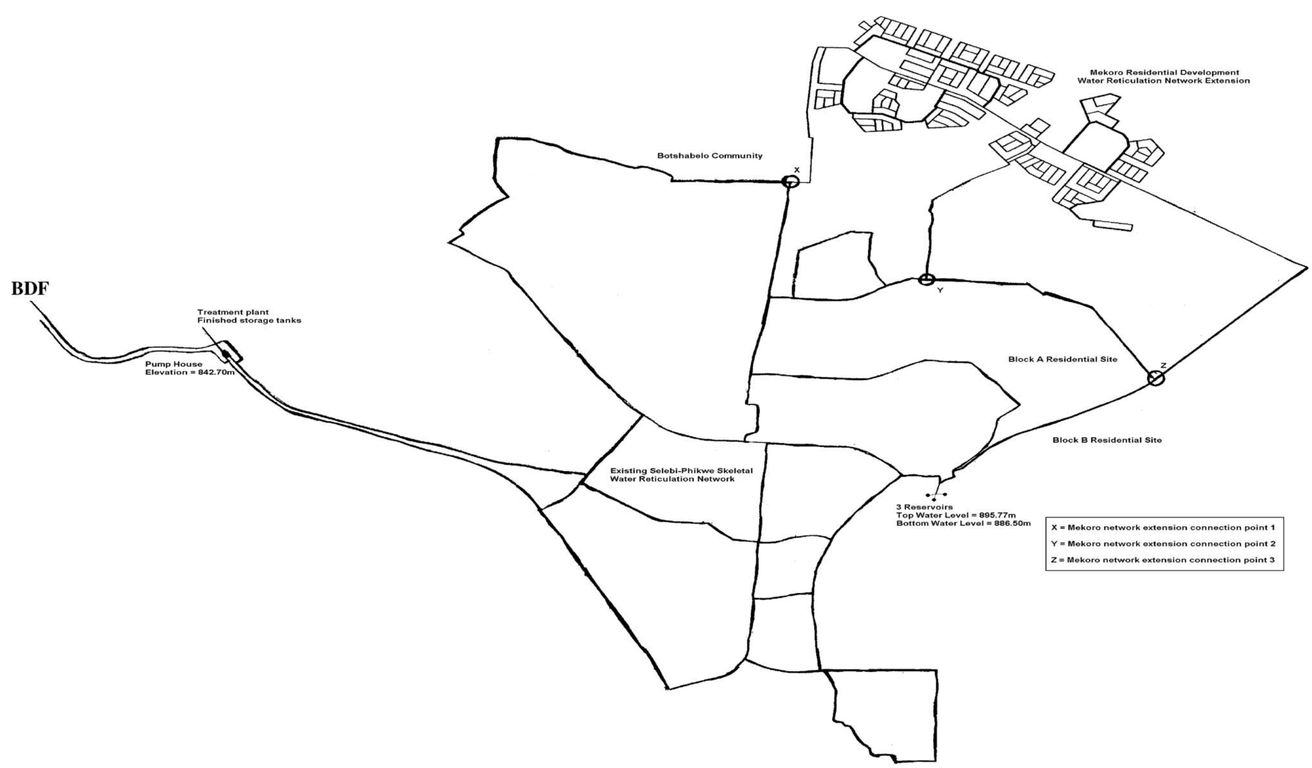

Figure 4. Skeletal layout of the existing SP WRN and proposed Mekoro, Block A and Blocks B developments.

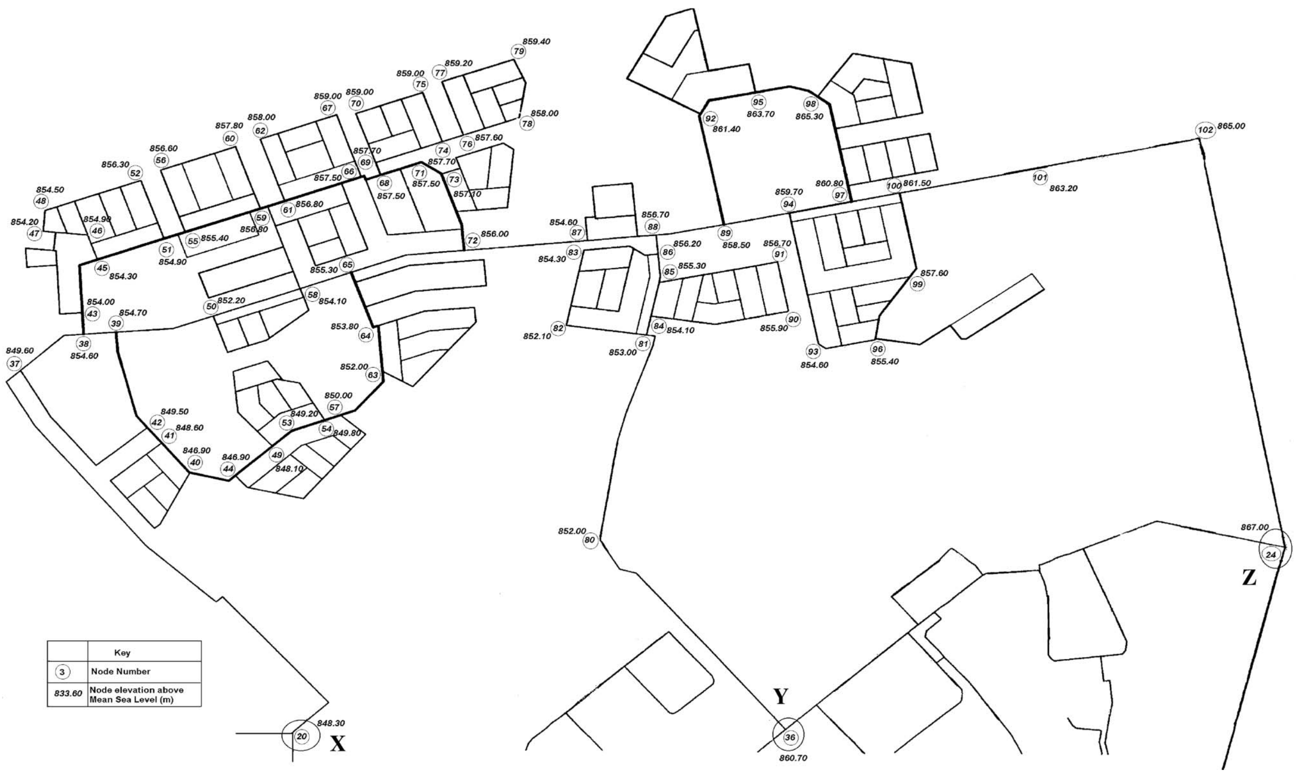

Figure 5. Detailed node characteristics for the proposed Mekoro WRN.

Table 5. Pipe and node data (nodes that violate the minimum and maximum permissible pressure heads) in the existing SP WRN.

					Peak flow		Night flow	
Pipe	Start node	Length (m)	Existing Diameter (mm)	End node	Demand (m^3/s)	Pressure head (m)	Demand (m^3/s)	Pressure head (m)
2	3	1800.00	110	4	0.025	10.66^{+}	0	61.22
41	2	2420.00	75	27	0.005	48.30	0	106.58^{+}
42	2	2180.00	75	28	0.008	6.04^{+}	0	106.58^{+}

$^{+}$Violating node pressure heads.
All pipes are of uPVC material.
Minimum pressure head at each network node (except source node) is 15 metres.

flow demands for the combined upgraded SP WRN will sum to 0.4162 m^3/s and 0.1555 m^3/s, respectively. Night flow simulation was performed assuming the extreme condition of no draw-off at all demand nodes and the total volume of water supplied from the source going to the storage. 20 Ml of storage is provided in 3 tanks situated at the south-eastern section of SP (Node 22). Minimum and maximum allowable node residual pressure heads in the system are 15 m and 90 m, respectively, and minimum pipe sizes connecting users to the reticulated mains and on which fire hydrants are expected are 63 mm and 75 mm, respectively (Botswana Water Utilities Corporation 1995).

Prior to employing ***Wadessy***'s design tool, the optimal upgrade of the SP WRN had been attempted by a consulting firm using optimisation by trial and error. As a result, it was difficult to verify the optimal solution obtained, and a large amount of computational time and effort was expended. While attempting to determine the optimal solution using trial and error, it was also necessary to determine the most appropriate peak and night flow scenarios at which the upgraded SP WRN should be operated by. To this end, the following scenarios were considered in the simulations:

i. **Peak flow scenarios.**
   a. Two source nodes (water supply from the existing storage tanks located at node 1 through two pumps operating in parallel and from the existing

Table 6. Pipe and node data for the proposed Mekoro WRN.

Pipe	Length (m)	Initial Diameters (mm)	Node	Peak flow demand (m^3/s)
48	1710.40	200	36	0.0140
49	238.40	200	37	0.0026
50	98.90	200	38	0.0000
51	347.30	160	39	0.0002
52	68.00	160	40	0.0004
53	132.90	160	41	0.0012
54	148.40	160	42	0.0003
55	151.20	160	43	0.0004
56	78.90	160	44	0.0011
57	108.70	160	45	0.0003
58	51.90	160	46	0.0004
59	213.60	160	47	0.0006
60	156.10	160	48	0.0005
61	216.30	160	49	0.0007
62	303.70	200	50	0.0010
63	280.80	200	51	0.0003
64	166.60	200	52	0.0005
65	73.60	160	53	0.0002
66	221.00	160	54	0.0007
67	89.50	63	55	0.0010
68	98.20	63	56	0.0005
69	146.90	90	57	0.0010
70	273.10	90	58	0.0014
71	215.50	160	59	0.0012
72	212.70	90	60	0.0004
73	68.00	160	61	0.0007
74	225.90	90	62	0.0005
75	241.70	90	63	0.0009
76	241.70	160	64	0.0026
77	225.90	90	65	0.0013
78	78.80	160	66	0.0028
79	225.90	90	67	0.0007
80	248.90	90	68	0.0013
81	248.90	160	69	0.0007
82	225.90	90	70	0.0005
83	71.10	160	71	0.0003
84	45.00	110	72	0.0013
85	178.80	90	73	0.0007
86	213.50	90	74	0.0003
87	213.50	90	75	0.0002
88	178.80	90	76	0.0005
89	68.00	90	77	0.0006
90	194.50	90	78	0.0004
91	231.00	90	79	0.0006
92	180.10	90	80	0.0133
93	209.90	90	81	0.0006
94	130.40	160	82	0.0009
95	73.20	160	83	0.0006
96	273.50	160	84	0.0004
97	352.50	200	85	0.0006
98	378.10	200	86	0.0004
99	209.00	200	87	0.0010
100	862.60	250	88	0.0017
101	749.70	250	89	0.0000
102	246.90	90	90	0.0007
103	263.40	90	91	0.0007
104	235.30	90	92	0.0038
105	65.60	250	93	0.0007
106	117.80	250	94	0.0030
107	97.80	250	95	0.0031
108	423.70	110	96	0.0011
109	172.50	110	97	0.0034
110	366.50	110	98	0.0044
111	66.00	250	99	0.0007
112	207.80	200	100	0.0012
113	387.90	160	101	0.0015
114	196.80	160	102	0.0000
115	200.60	160		
116	386.80	160		
117	204.00	200		
118	193.40	200		
119	457.80	90		
120	183.20	90		
121	241.50	90		
122	299.20	90		
123	148.10	200		
124	1575.00	250		
125	640.00	250		
126	456.70	250		

Existing SP WRN: pipes 1–47 and nodes 1–35; Proposed Mekoro WRN: pipes 48–126 and nodes 36–102.
All pipes are to be of uPVC material.
Minimum pressure head at each network node (except source node) is 15 metres.
Night flow at each draw-off node (except the storage tank) is 0 m^3/s.

tanks located at node 22) supply the upgraded SP WRN during peak flow.

b. One source node (water supply from the existing storage tanks located at node 1 through two pumps operating in parallel) supplies the upgraded SP WRN during peak flow.

c. One source node (water supply from the existing tanks located at node 22) supplies the upgraded SP WRN during peak flow.

d. One source node (water supply from the finished storage tanks located at node 1 through two pumps operating in parallel) supplies the existing SP WRN only and the other source node (water supply from the existing tanks located at node 22) supplies only Mekoro, Block A and Block B developments during peak flow. A dedicated feeder mains is proposed in this case, to run from the existing tank located at node 22 to the proposed developments.

ii. **Night flow scenarios.**

a. One source node (water supply from the finished storage tanks located at node 1 through two pumps operating in parallel) supplies the upgraded SP WRN during night flow.

b. One source node (water supply from the finished storage tanks located at node 1 through one operational pump) supplies the upgraded SP WRN during night flow.

From the results obtained, the most appropriate peak and night flow scenarios for the proposed upgraded SP WRN determined using optimisation by trial and error were scenarios (1a) and (2b), respectively. The other scenarios were eliminated as potential candidates primarily due to the fact that the minimum node pressure head constraints were violated in one or more nodes during either or both loading conditions. Figures 6 and 7 present results for each of the scenarios listed above when operated on the optimal upgraded SP WRN determined using optimisation by trial and error.

Wadessy's design tool was subsequently employed to determine the optimal design for the proposed upgraded SP WRN based on the peak and night flow scenarios determined above (i.e., scenarios 1a and 2b). Figure 8 presents pipe sizes generated for the optimal upgraded SP WRN using optimisation by trial and error and the ***Wadessy*** design tool. Node residual pressure heads generated by ***Wadessy***'s design tool during peak and night flow conditions on the optimal upgraded SP WRN is presented in figure 9. A demand tolerance of 20% was input during each run. As a result, the optimal network design is enabled to accommodate a maximum variance in the peak and night flows by 20%, hence increasing network resilience (see figure 9).

Nodes 4 and 28 in the existing SP WRN (see scenario ia, figure 6) generate, during peak flows, residual pressures that violate the minimum specified 15 metres (i.e., 11.21 and 6.31 metres, respectively). Nodes 4 and 28 supply the BDF which experiences low flows during peak flows. This problem was incorporated in the upgrade exercise. The optimal result generated using ***Wadessy***'s design tool also generated parallel pipe sizes of 50 mm, 75 mm and 150 mm to pipes 2, 41 and 42, respectively (figure 8).

To determine the optimal upgraded SP WRN, 14 optimisation runs were undertaken (figure 10). Each run converged towards a local optimum solution. However, the narrower the search space, through the manipulation of the maximum and minimum diameter constraints, the better the quality of the solution generated. Maximum and minimum diameter constraints of 450 mm and 50 mm, respectively, permitted a broad search space for the optimisation and hence, generated solutions that were within that space. The optimal solution was however generated from a narrower search space with maximum and

Table 7. October 2001 pipe costs for the Selebi – Phikwe pipes in Pula[++].

	Diam (mm)	50	63	75	90	100	110	150	160	200	250	300	350	400	450	500
AC	**Class**	–	–	12[+]	–	12	12[+]	12[+]	–	12	–	12	12[+]	12[+]	12[+]	12[+]
	Cost/m	–	–	11.4*	–	15.8	25.7*	27.4	–	29.4	–	161.0	183.0	200.0*	220.0*	230.0*
uPVC	**Class**	9	9	9	9	9	9	–	9	9	6	9[+]	–	–	–	–
	Cost/m	5.3	8.0	11.4	2.4	7.2	25.7	–	45.5	43.1	169.0	161.0*	–	–	–	–

[++]1 Pula is equivalent to about 0.1 British Pound.
[+]class of pipe assumed.
*pipe cost per meter assumed as actual cost not available.

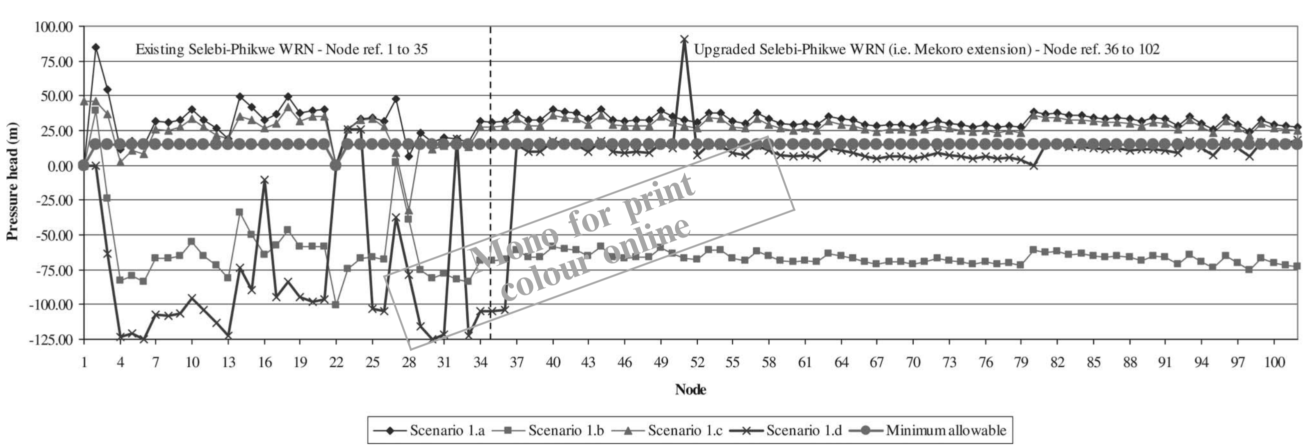

Figure 6. Peak flow scenario (1.a to 1.d) on the optimal upgraded SP WRN determined using optimisation by trial and error.

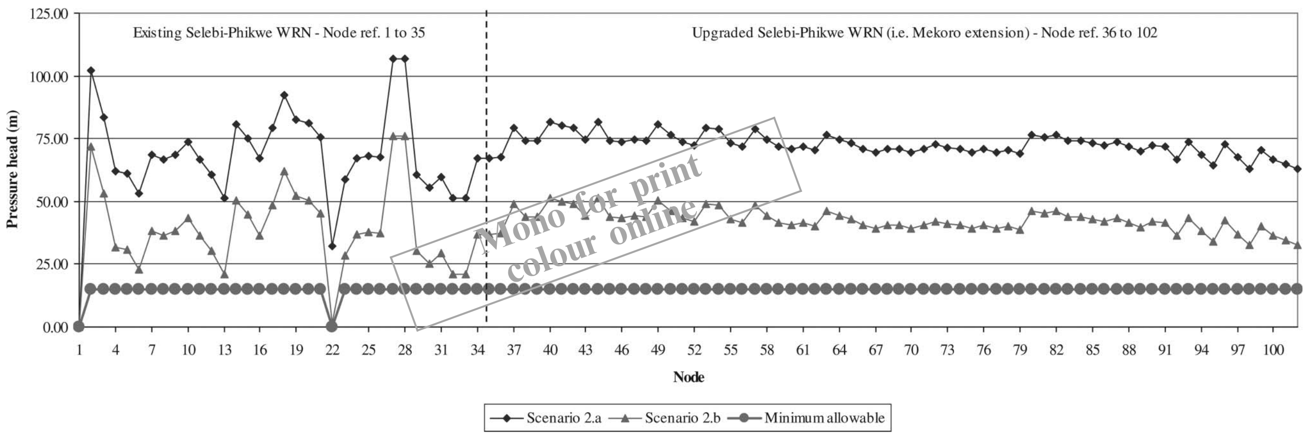

Figure 7. Night flow scenarios (2.a to 2.b) on the optimal upgraded SP WRN determined using optimisation by trial and error.

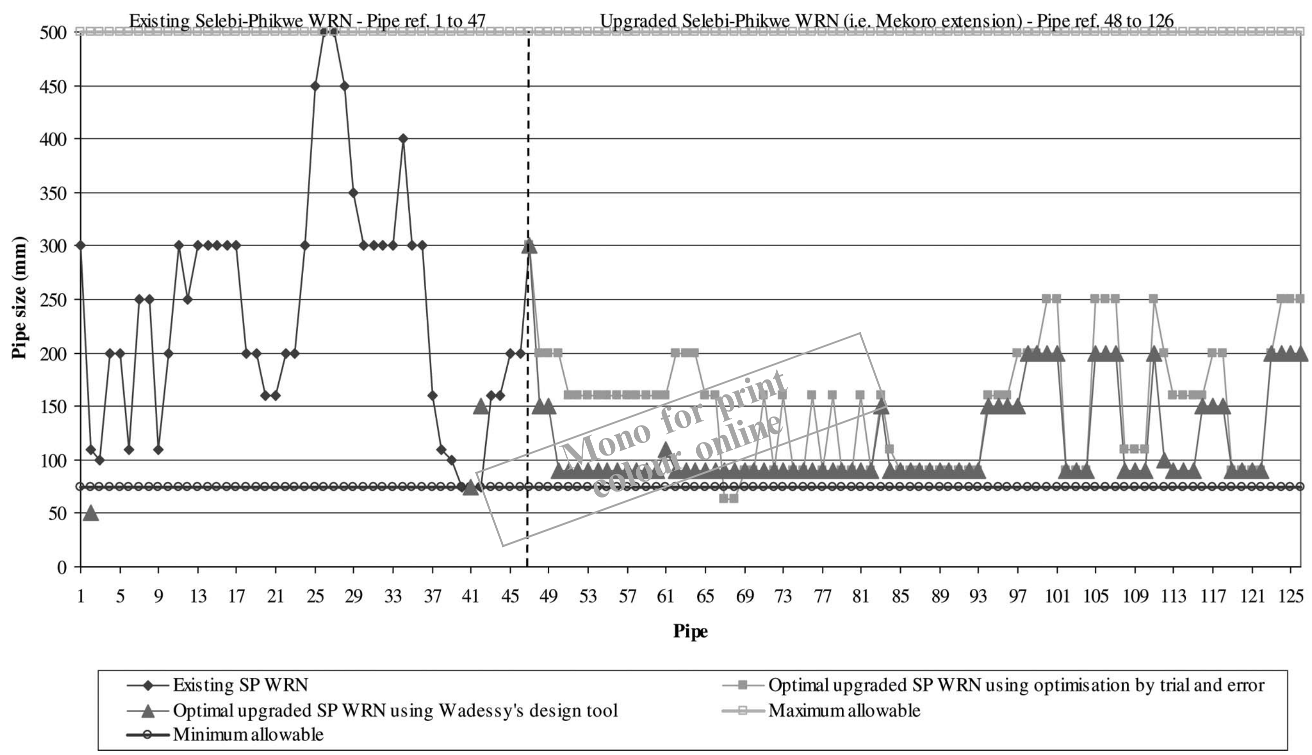

Figure 8. The optimal upgraded SP WRN pipes determined using optimisation by trial and error and ***Wadessy***'s design tool.

minimum diameter constraints of 200 mm and 100 mm, respectively.

During peak flows, minimum allowable node pressure heads of 15 metres are satisfied in the optimal network. During night flows however, maximum allowable node pressure heads are violated in nodes 18, 2, 27 and 28 (the latter three being downstream nodes of upgraded pipes). The installation of pressure reducing appurtenances at these nodes would facilitate the reduction of above maximum pressures during night flow conditions. The requirement that the minimum pipe size (63 mm and 75 mm) connecting users to the reticulated mains and on which fire hydrants are expected, was satisfied in the optimal solution generated using ***Wadessy***'s design tool.

Unplasticised Polyvinyl Chloride pipes (uPVC) and Asbestos Cement (AC) pipes were used in the existing SP WRN while uPVC pipes are preferred for use in the pipe extensions due to ease of availability and lower cost.

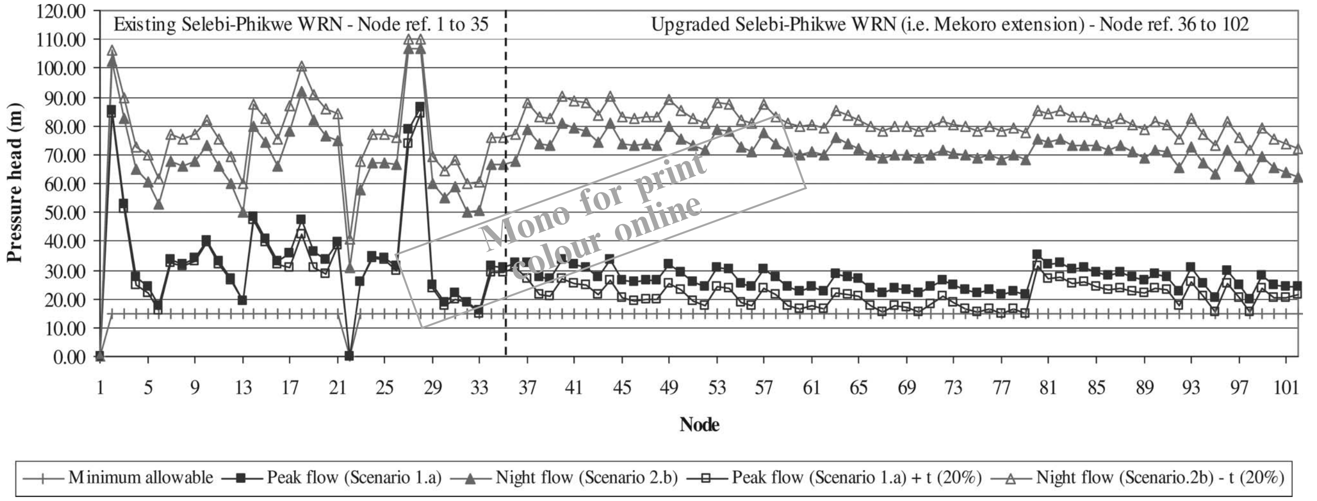

Figure 9. Node residual pressure heads for the optimal upgraded SP WRN determined using ***Wadessy***'s design tool for peak flow and night flow. t represents tolerance.

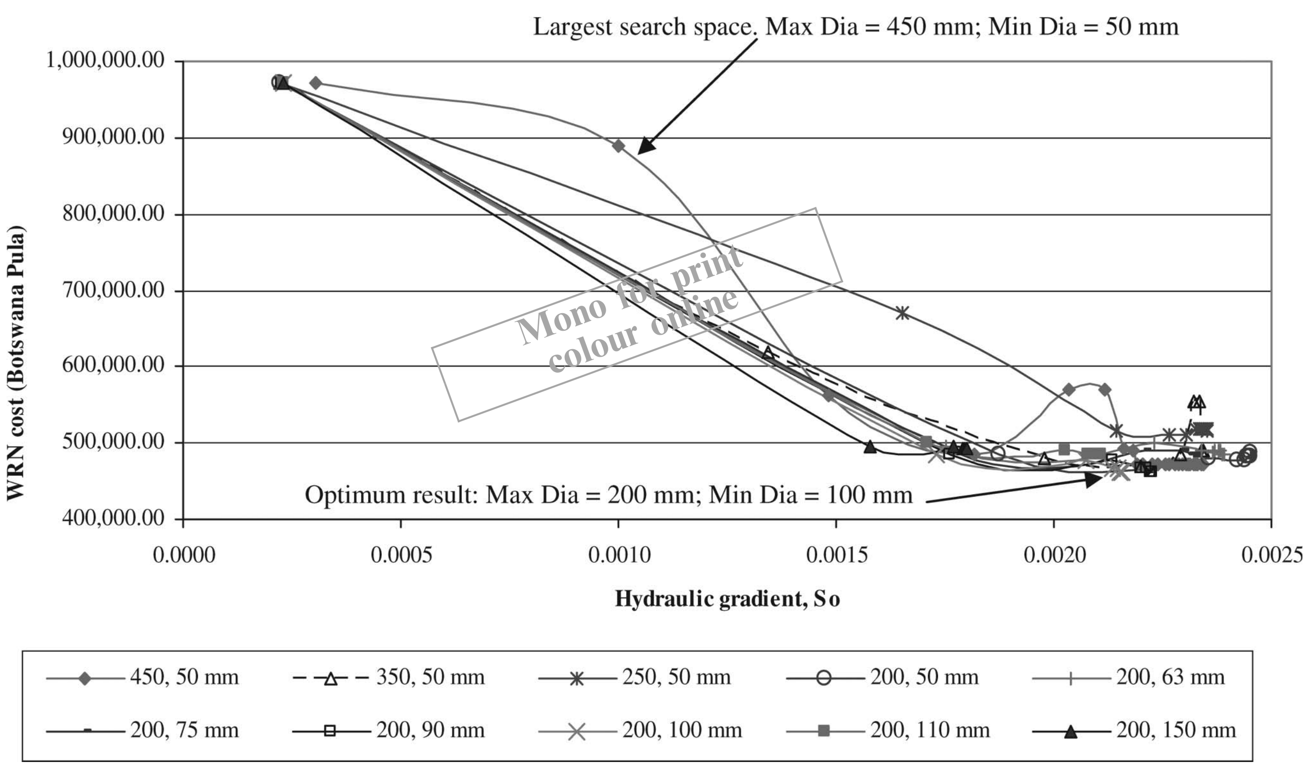

Figure 10. Non-linear variation of total WRN cost with hydraulic gradient, S_0 using maximum and minimum diameter constraints.

Optimisation by trial and error generated an upgraded SP WRN design costing 1 223 406 Botswana Pula while the optimisation process employing ***Wadessy***'s design tool generated an upgraded SP WRN design costing 461 039 Pula (including the parallel pipes for pipes 4, 41, and 42 costing a total of 96 812 Pula). A cost saving of 62%, i.e., 762 367 Pula was therefore achieved using ***Wadessy***'s design tool. The repeated use of both 90 and 100 mm uPVC pipes in the Mekoro WRN facilitated the large reduction in the network's cost, as 90 and 100 mm pipes, mostly utilised in ***Wadessy***'s optimal network, cost much less than other pipe sizes. In terms of efficiency, ***Wadessy***'s design tool achieved an optimum solution at each run in approximately 50 iterations (approximately 9 seconds on a 1.51 MB (Centrino), 400 MHz processor computer).

6. Conclusion

Presented here is a simple tool to facilitate the design (including expansion) of water networks. The tool employs an efficient, non-linear algorithm that determines a network's hypothetical hydraulic gradient which in turn optimises the network's pipe sizes within prescribed constraints. The tool is calibrated using the New York City water supply problem that has served as a benchmark problem for other models. It is then applied to the Selebi–Phikwe (SP) water reticulation network (WRN) in Botswana, which was designed based on engineering judgement. A 62% reduction in total pipe cost from that obtained by engineering judgement for the SP WRN problem was achieved. At the same time, comparable pipe costs to those published in literature for the New York City water supply tunnels problem was achieved.

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