

# Using Coloured Rods

## TO TEACH ARITHMETIC

### SOME FUNDAMENTAL PRINCIPLES

By J. GEDDES

*Mr. J. Geddes is one of the Province's more progressive primary school principals, keen, as this article shows, to experiment with new techniques in education. Formerly vice-principal of the H. A. Jack School and past principal of the Sir Edmund Hillary School, he is now principal of the young and rapidly growing Blairgowrie Primary School, Johannesburg.*

**I**N this article, which does not claim to be authoritative, an attempt will be made to discuss the teaching of arithmetic by the use of Coloured Rods. The use of Rods at Blairgowrie School is still in the experimental stage and what follows is merely a suggested approach. Coloured Rods were introduced at this school into a Grade II class at the beginning of 1961 and have since been provided for both grades and for Standard I.

#### Sources

The following reference books and documents have been used:

"*Course in Pure Number*"<sup>1</sup> based on Cuisenaire/Cattegno Methods. This document, which was reproduced for the Federal Ministry of Education, Rhodesia and Nyasaland by courtesy of the Education Department of Victoria, Australia, has proved to be of great value, and quotations from it appear from time to time in this article.

"*Arithmetic with Numbers in Colour*"<sup>2</sup> by C. Cattegno.

"*The Young Mathematician*"<sup>3</sup> series by C. E. D. Birrell. It is suggested that if this series of books is going to be used, it be introduced only after all operations, using colour only, have been dealt with, i.e. introduce the first book when the pupils are attaching number values to the rods. The reason for this is that Birrell gives illustrations showing a picture of the rod, the relevant numeral and the written word. It is felt that the number symbols are associated with the rods too soon. The illustrations in these books which show that one can get "two" balls, "two" beans, "two" cars, etc., and not just the rod which has the "value of two", is an excellent idea. The other numbers are dealt with in a similar manner.

"*Colour Factor Mathematics*"<sup>4</sup> by H. A. Thompson. This system introduces a completely different set of colours. Here, too, the children first become familiar with the coloured rods and then they sub-

stitute the initial letter for each colour, e.g. instead of writing yellow + scarlet = light-blue + violet, they write  $y + x = l + v$ . The rods or "blocks" in this series range from 1 to 12.

There is not the slightest doubt that other coloured rod systems will soon appear, but it is not the purpose of this article to advocate the exclusive use of any one system.

#### Apparatus

Teachers have always used coloured blocks, beads, and a variety of concrete objects in their efforts to teach number work. Irrespective of whether teachers introduce coloured rods as a system or not, they still have to teach Grade I children the sequence of number, i.e. 1, 2, 3 up to 10. Children must still learn number games, the counting of different objects, number rhymes, etc. It is essential that pupils become completely familiar with number sequence from 1 to 10, before using the rods. Similarly when dealing with the "limiting" and "grouping" stages indicated later in this article, a variety of objects (other than coloured rods) should be used.

#### Using Rods

The learning of sequence of number, limiting and grouping, will probably take up the whole of the first term, and *coloured rods will come into operation during the second term.*

When coloured rods are introduced, the following steps are made possible:—

- (a) Coloured rods enable pupils to "discover" number, and number combinations, for themselves—a very important development!
- (b) Pupils see number as "wholes" or "groups" (cardinal number).
- (c) They are able to learn the significance of "equality".

(d) They develop an understanding of the "nature" of the four basic operations, i.e. addition, multiplication, subtraction and division.

It must be emphasised that the coloured rods enable the children to develop the concept of "wholes", i.e. the "fourness" or "sixness" of number, and unless this is fully grasped, none of the operations can be understood, because every operation involves *wholes*.

The coloured rods are merely a means to an end. The pupils must "discover" the significance of the four operations. When rods are used without the complication of number, pupils are able to "discover" the significance of addition, multiplication, subtraction and division.

Only when the significance of the four operations has been clearly grasped by the pupils, and only then, should they substitute symbols (numbers) for the colours. They will learn that what was true of colours remains true of numbers when these are used instead.

### Sequence

The following questions might well be asked: "What sequence should be followed in the teaching of the four operations? Does the sequence fit in with our present Arithmetic Syllabus?"

The Grade I syllabus deals with addition and then subtraction—it is felt that the sequence should be addition, *multiplication*, subtraction and then division! There is a natural development from addition to multiplication. Subtraction, too, is an extremely difficult concept. Grade I children should actually deal with all *four* operations orally, using the methods suggested later in this article.

As the *nature* of one operation becomes clear to a pupil, so he will progress to the next. This will be *in terms of colour only*, as number values are not associated with the colour *until all four operations have been taught in terms of colour only*. The child at this stage is concerned with the "nature" of the operation, *not number*.

Children will naturally proceed at different rates, and so grouping is essential. As the various groups proceed from one stage to another, those children who have grasped the nature of the four operations may now be shown that symbols (numbers) can be attached to each colour.

*The children will then attach number value to the rods and will go through the same stages as they did when they attached colour values.* The teacher will have to use her own discretion as to how much time will be spent at the "colour only" level, but must ensure that the "nature of the operations" is fully grasped before proceeding.

It has been suggested that multiplication be dealt with after addition, and that sequence will be followed in this article. It is possible that the T.E.D. Syllabuses as they now stand may well have to be

modified if the teaching of Arithmetic, using coloured rods, is to be introduced as a standard approach in all schools. Only time will tell what modifications have to be made.

It is probable that most children in Grade I will do *written work* involving addition and multiplication only. Certain children in Grade I, who progress more quickly, will do written work in subtraction as well.

It must be emphasised at this point that, "the apparatus and system is modified and adjusted to fit the child—the child is not forced to fit the apparatus and system." This makes it imperative to group in Arithmetic: the individual approach is obviously the best, the big classes rarely make this possible.

It is suggested that the following sequence be used:—

It is logical to work on the basis that ordinal number, i.e. numbers in sequence 1, 2, 3, say up to 10, should be studied before dealing with number in groups or "wholes" (cardinal number). The children must also have a full understanding of the meaning of *equality*, and so it is essential that there be a full understanding of *ordinal number* (one to one sequence), *cardinal number* (wholes) and *equality*, before there can be an understanding of mathematical operations.

Once the children have learnt number sequence up to 10, and have experienced number in as many ways as possible, they should use concrete objects (not coloured rods) to learn:—

- (a) "*Limiting*" i.e. selecting a specified number of objects from a larger group, e.g. 4 objects from a group of 6.
- (b) "*Grouping*" i.e. constructing groups of a specified number of objects, e.g. two groups of three objects.

The object behind grouping is that it leads up to an understanding of number seen as a "whole" or "group" (cardinal number).

When the teacher feels that the limiting and grouping concepts have been fully understood, coloured rods can be introduced for the *first time*. This stage, where the rods are introduced, would probably be during the second term. The rods could be used on two days of the week and on those days should be used to the exclusion of all other number work.

### Further Stages

This first stage is known as the "Free-Play Stage". No written work in connection with the rods is done at this stage, but written work involving ordinal number should be continued in the normal way. During the "Free-Play" stage, the children learn the *colour of the rods*, and become aware of *their relative lengths*. The children should have a great deal of practice in becoming familiar with the relative sizes of the rods. They should identify

rods with the eyes closed, hold the fingers wide enough apart to fit exactly a specified rod, and other similar exercises. A pupil should reach the stage where, holding a handful of rods behind his back, he is able to remove rods, one at a time, and without looking at them, indicate their respective colours. This would indicate a thorough familiarity with the rods.

Teachers should ask:—

- (i) "Show me the smallest rod—what colour is it?"
- (ii) "Show me the biggest rod—what colour is it?"
- (iii) "Show me a rod a little bigger than the white, much bigger than the white," and so on.

The next step should be "Directed Building", and this can be done by introducing "Staircases". Staircases serve two purposes:—

- (i) To ensure a thorough knowledge of colour names.
- (ii) To demonstrate very clearly the relative size of the rods.

"This stage is organised, so that the child progresses from an immature awareness of the relative lengths of the rods, to a stage where his concept of the 'whole' is so clear, that he can work abstractly, with visual images of the absolute lengths of the rods."<sup>1</sup>

The next step is a true understanding of "equality". Children often use the "equals sign" quite indiscriminately, and without real understanding. The use of coloured rods enables children to perceive for themselves the significance of equality. They will "discover" that an orange rod equals a yellow and a yellow, and so on. Children should be allowed to "discover" as many combinations as possible, and it is suggested that the "mats" made at this stage, should not consist of more than two rows of rods.

Once the significance of equality has been discovered, a natural development after this will be an understanding of the nature of *addition*.

The teacher should select a rod, e.g., orange, and allow pupils to build a "mat" with as many combinations as possible—yellow and yellow equal orange; green and red and green equal orange, and so on. Pupils should be encouraged to explore and discover all the ways in which rods can be combined, to equal each of the other colours. They should be taught to read their "mats" in as many ways as possible.

When "equality" was being taught, attention was *directed* at the fact that rods were "equal". Here, however, the use of "mats" emphasises the fact that the rods are "combined"—the combination of "wholes". To emphasise this, introduce the term "plus".

The next stage which develops naturally is "*multiplication*", because multiplication is "the addition of equal numbers". The teacher may again

select the orange rod, and the child should read as follows: Yellow plus yellow equals orange; two yellows equal orange; five reds equal orange; ten whites equal orange, and so on.

Repeat this process using other rods, and thus stress the link between addition and multiplication. This can be done by reading the mats, in terms of addition and multiplication, in as many ways as possible. The approach should be: "Read your mat in another way." It is preferable not to introduce the term "multiplied by" at this stage.

When the teacher feels that there is a true understanding of the nature of addition and multiplication, and their relationship to each other, proceed to teach the concept of subtraction. The approach should be:—

"What do we need to add to black, to equal orange?"

"What do we need to add to light-green, to equal orange?"

"Can I add anything to orange, to equal black?" (to reinforce complementary addition).

"What do I need to add to one red and one crimson, to equal orange?"

One should now substitute the term "minus" for "What do I need to add to..." (The term "take away" should be avoided). "By using the complementary addition approach, it is far easier to explain the technique of subtraction."<sup>1</sup>

Using the term "minus", instead of "what do I need to add to...", all the above steps should be repeated.

The process should be repeated using other rods, until the concept of the nature of subtraction has been clearly grasped.

Before proceeding, the child should be led to read addition, multiplication and subtraction situations with rods, i.e. they should build "mats" and read them in a variety of ways—read them, in terms of addition, multiplication and subtraction. This allows the pupils to see the relationship of one operation to another.

"The next step is to develop an understanding of the nature of *Division*, and its relation to the other processes."

The approach could be as follows:—

- (i) Ask the questions:
  - "How many yellow rods are equal to one orange rod?"
  - "How many red rods are equal to one orange rod?"
  - "How many light greens are equal to one blue rod?"
  - (No remainders involved here).
- (ii) Then introduce remainders, e.g.,
  - "How many light-greens equal the orange rod?"Repeat this step using other rods.

(iii) "What does orange minus one light-green equal?"

"Orange minus two light-greens equal: Orange minus three light-greens, equal?"  
Repeat using other rods.

(iv) "Can you find two rods of the same length equal to an orange rod?"

"What colours are the two rods?"

"Can you find three rods of the same length, equal to a blue rod?"

A pupil should reach the stage where he can read as follows:—"Orange, minus two reds, equals three reds."

"When this stage is reached, there is a true understanding of division."<sup>1</sup>

### Consolidation

At this stage, children should build "mats" and read them in as many ways as possible. They should read them in terms of addition, multiplication, subtraction and division.

At this juncture, THE FOUR BASIC OPERATIONS have been dealt with in terms of colour.

### Introduction to Fractions

At this stage teachers could well introduce pupils to simple fractions. Here too, they should be handled on a *colour basis only*, e.g., we can see "Two white rods equal one red rod". We can also say, "One white rod is 'half' a red rod." All other rods up to 10 should be handled in a similar manner.

"*Arithmetic with Numbers in Colour*" Introductory Stage, Book I, page 24, and "A Teacher's Introduction to the Cuisenaire—Cattegno Method of Teaching Arithmetic" page 35, both by C. Cattegno, show very clearly how to handle the introduction of fractions. In this way, fractions take their proper place in number work. Symbols, e.g.  $\frac{1}{2}$ ,  $\frac{1}{4}$ , etc., should be used only after the teacher has covered the four operations in terms of number.

### The Four Basic Operations in Terms of Number

Here again the object is *not* to study number directly. Numbers here, as colours previously, are merely "a means to an end". Work must *not be organised so as to cover basic number facts*.

Teachers are still concerned with synthesis and the *nature of the operations*. The next step is to "attach" number values to the coloured rods. "Number facts are not deduced from the rods, but ideas gained in the previous stage are imposed upon them. The ideas must be imposed by the children—hence all teaching (i.e., telling) defeats the purpose of the work."<sup>1</sup> Here the aim should be to develop the ability to recognise rods in terms as follows:—"If white is one, what is this rod?" Repeat this process with all rods, to ensure that the relationships

are grasped. The children now return to building staircases and all the steps connected with that stage, *but instead of using colour names, use number values*. "It is very important that the visual images of the rods that are attached to colour-names are carried across to number-names."<sup>1</sup>

When this has been done, repeat all steps which were used to study equality, but now use number-names instead of the colour-names previously used. Read "mats" in terms of addition (as was done with colour-names) but now use number-names instead. Deal similarly with multiplication, subtraction and division and then finally read "mats" in terms of the four operations. *Up to this stage all the work has been done orally.*

### The Introduction of Written Work

The teacher now writes an addition sum *from the rods*, in the form of *7 plus 3 equals 10*.

Now introduce the form:  $7+3=10$ . Do a variety of exercises, e.g.,  $2+3+2=7$ ;  $2+2+4=8$ , until the written form of addition has been consolidated. This can be done by allowing the children to build and write their own sums.

### Multiplication

The teacher writes a multiplication sum *from the rods*, in the form: *2 times 2 equals 4*. Now introduce the form:  $2 \times 2 = 4$ . (Verbalised as "Two twos equal four".) Consolidate written form by doing a variety of exercises.

### Subtraction

The teacher writes a subtraction sum *from the rods*, in the form of *8 minus 2 equals 6*. Now introduce the form:  $8-2=6$ . Consolidate once again by doing many written examples.

The children soon realise that the signs  $+$ ,  $-$ ,  $\times$  have the same meaning as "plus", "minus" and "times".

### Interpretation of Written Equations with Rods

"The teacher writes a simple equation such as:  $7+3=10$  (in this form) on the blackboard.

"The children make this equation with their rods and then write it in their exercise books."<sup>1</sup>

Ensure that they get enough practice in reading and writing addition sums and then deal similarly with multiplication and subtraction. Here, once again use mats, and allow pupils to read and write sums which include all three operations.

The next stage is where pupils write answers to written equations *without using rods*. (N.B. this stage is introduced only when the pupils themselves have indicated that they no longer require rods.) Exercises from Part III of Cattegno's Introductory Stage (Book I) may be used and brackets are introduced.

## Fractions in Terms of Number

Instead of colour, now introduce symbols  $\frac{1}{2}$ ,  $\frac{1}{4}$ , etc., and show that what was true when using colour, is also true when using number. (Refer to Cattegno's "Arithmetic with Numbers in Colour" Introductory Stage, Book I, page 24 and "Teacher's Introduction to the Cuisenaire—Cattegno Method of Teaching Arithmetic" page 35.)

When teachers wish to teach the basic mathematical ideas from 10 to 20 and beyond, the teacher will create situations showing that concepts which have been formed and expressed with numbers below 10 also apply to numbers above 10. In other words, use the same approach for numbers from 11—20 as was used for numbers below 10.

## Progress of Children

Throughout the year children will be working at different stages, and by the end of the year many might well have outstripped the rest of the class. It appears, therefore, that teachers would have to keep an individual record of each child's progress in Arithmetic. This record should pass up with the

child, so that the new teacher is immediately aware of the child's ability in number work.

From time to time fantastic claims have been made for the colour rod system. Any person who has not used the rods would be quite entitled to look upon many of the claims as being impossible, but once they have used the rods, they too, will see what possibilities exist when this system is used. This does not mean that children should be submitted to various forms of "mental gymnastics". Understanding comes with maturity, and this must always be borne in mind.

## REFERENCES

- (1) "Course in Pure Number". Based on Cuisenaire/Cattegno Methods. (This was reproduced for the Federal Ministry of Education, Rhodesia and Nyasaland. By the courtesy of the Education Department of Victoria, Australia.)
- (2) "Arithmetic with Number in Colour" Books 1, 2 and 3.
- (3) "The Young Mathematician". Introductory Book and Books I and II by C. E. D. Birrell.
- (4) "Colour-Factor Mathematics". The First Year Part I and II and "The First Year". Teacher's Handbook by H. A. Thompson.

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## BOOK REVIEWS

### *World Perspectives in Education*

Edmund J. King (Methuen).

University Departments of Education and Teacher Training Colleges should examine Dr. King's latest book, which as a study in comparative education and a most perceptive analysis of the less frequently recognised problems that beset us, represents quite one of the most lucid and well-informed surveys we have seen. His range of interest is wide; he devotes, for instance, many pages to the question *Who is Educated?* He examines the impact of technological change on our traditional ways of teaching. He considers the world-wide problem of teacher recruitment and training. He finds it strange that "most parents and politicians do not effectively demand a better or different education for most of the population than it now receives . . . for industrialists are convinced that before 1980 there must be a rapid shift towards a more highly skilled labour force;

but the nation's leaders have not translated this conviction into terms of popular education." Britain's teacher shortage in 1961 was 90,000, and experts calculate that—as advanced thinkers in South Africa have always proposed should be done—classes should be reduced to a maximum of 30 children. But to effect this, 110,000 more teachers would be required in Britain by 1965. Dr. King, who is a lecturer in Education at the University of London, comments that the teaching profession must look forward to a decreasing popularity and a general worsening of quality in its new recruits. "That seems bound to happen," he maintains, "unless prospects and esteem are greatly enhanced. The sort of bright young man or woman who in France or Italy would have become a secondary school teacher now tends in Britain and the U.S.A. to dash off and join the managerial or scientific 'power' elite. It is undoubtedly true that many grammar school heads in Britain implicitly assess the teaching profession very