

Effect of the non-thermal Sunyaev–Zel’dovich effect on the temperature determination of galaxy clusters

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ABSTRACT

A recent stacking analysis of Planck HFI data of galaxy clusters led to the derivation of the cluster temperatures using the relativistic corrections to the Sunyaev–Zel’dovich effect (SZE). However, the temperatures of high-temperature clusters, as derived from this analysis, were basically higher than the temperatures derived from X-ray measurements, at a moderate statistical significance of 1.5σ . This discrepancy has been attributed by Hurier to calibration issues. In this paper, we discuss an alternative explanation for this discrepancy in terms of a non-thermal SZE astrophysical component. We find that this explanation can work if non-thermal electrons in galaxy clusters have a low minimum momentum ($p_1 \sim 0.5-1$), and if their pressure is of the order of 20–30 per cent of the thermal gas pressure. Both these conditions are hard to obtain if the non-thermal electrons are mixed with the hot gas in the intracluster medium, but can be possibly obtained if the non-thermal electrons are mainly confined in bubbles with a high amount of non-thermal plasma and a low amount of thermal plasma, or are in giant radio lobes/relics in the outskirts of the clusters. To derive more precise results on the properties of the non-thermal electrons in clusters, and in view of more solid detections of a discrepancy between X-ray- and SZE-derived cluster temperatures that cannot be explained in other ways, it would be necessary to reproduce the full analysis done by Hurier by systematically adding the non-thermal component of the SZE.

Key words: galaxies: clusters: general – galaxies: clusters: intracluster medium – cosmic background radiation.

1 INTRODUCTION

The Sunyaev–Zel’dovich effect (SZE) is the distortion of the spectrum of cosmic microwave background photons produced by inverse Compton scattering off the electrons in the hot plasma in galaxy clusters (Zel’dovich & Sunyaev 1969). It is well known that it can be used as a powerful probe of the properties of the thermal plasma in galaxy clusters (see e.g. Pointecouteau, Giard & Barret 1998; Birkinshaw 1999), and also of a non-thermal plasma, like relativistic electron populations (e.g. Colafrancesco, Marchegiani & Palladino 2003; Colafrancesco, Marchegiani & Buonanno 2011).

Recently, Hurier (2016; hereafter H16) analysed the Planck HFI data of galaxy clusters (Planck Collaboration I 2011) to derive the temperature of a large sample of galaxy clusters using the relativistic corrections of the thermal SZE, which, being dependent on the cluster temperature, can break the degeneracy between the cluster temperature and the optical depth that is present in the non-relativistic expression of the SZE (see Colafrancesco et al. 2003 for

a complete discussion). By stacking the Planck data of thousands of clusters, H16 found a signature of the relativistic corrections to the thermal SZE with the highest statistical significance obtained to date. H16 was also able to determine the average temperature of temperature-binned clusters. H16 found that for high-temperature clusters, the 545-GHz Planck channel shows a higher emission compared to low-temperature clusters, and that the temperature that can be estimated from the SZE is basically higher than the temperature estimated from X-ray observations, though with a moderate statistical significance ($\lesssim 1.5\sigma$). Since the effects of other contamination (like, e.g. kinematic SZE, background and foreground contamination and discrete radio sources) have been considered and eliminated from the analysis, H16 attributed this discrepancy to the effect of calibration uncertainties.

However, another physical possibility, which we will discuss in this paper, is that this discrepancy is due to the presence of a non-thermal component of the SZE, as produced by non-thermal relativistic electrons, like the electrons that produce the radio haloes observed in many galaxy clusters, especially those with high temperatures (e.g. Feretti et al. 2012). Colafrancesco et al. (2003, 2011) found that the deviations from the spectral shape of a pure thermal SZE due to a non-thermal electron contribution are more evident

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at high frequencies. Since the discrepancy between the SZE- and the X-ray-determined temperatures in H16 is observed mainly in the 545-GHz channel and for high-temperature clusters, where the non-thermal phenomena are usually more relevant (e.g. Feretti et al. 2012), it is natural, therefore, to suppose that the non-thermal SZE is the origin of this discrepancy. For example, Colafrancesco et al. (2011) showed that in the Bullet cluster, a single gas temperature fit to the SZE data provides the best-fitting value of $k_B T_e \sim 22$ keV, whereas by considering the additional contribution of a non-thermal SZE produced by non-thermal electrons with properties like those producing the radio halo, it is possible to obtain a better fit with a thermal electron population with temperature of the order of the X-ray-derived one, i.e. $k_B T_e \sim 14$ keV, and with the addition of a non-thermal electron population providing the main contribution to the SZE at high frequencies.

In this paper, we explore, therefore, this possibility. For this aim, as preparatory work for future and more significant detections of this discrepancy, we attribute the discrepancy suggested by the H16 results to the contribution of a non-thermal SZE, and we derive the properties of the electrons that are required to produce this effect. Finally, we discuss if these physical requirements are reasonable and we discuss their possible impact on the physics of galaxy clusters, as well as other possible explanations of this discrepancy.

2 METHODS

Following the general approach of Colafrancesco et al. (2003), the SZE can be written in the general form

$$\Delta I(x) = I_0 y g(x) \quad (1)$$

where $I_0 = 2(k_B T_0)^3 / (hc)^2$, $x = (h\nu) / (k_B T_0)$, and the Compton parameter y , in its general form, is given by

$$y = \frac{\sigma_T}{m_e c^2} \int P_e d\ell, \quad (2)$$

where P_e is the electron pressure, which in the general case can be due to thermal or non-thermal electrons, and the integral is performed along the line of sight ℓ . The function $g(x)$ contains the spectral dependence of the SZE, and depends on the electron temperature for the thermal SZE, and on the properties (spectral shape and minimum electron momentum) of non-thermal electrons for the non-thermal SZE.

Notice that H16 used an approach where the thermal SZE is given by the sum of the non-relativistic part, where the function $g(x)$ is independent of the cluster temperature, and relativistic corrections, which instead depend on the cluster temperature (see e.g. Nozawa et al. 2000). In the following, we use a different approach (Wright 1979; Enßlin & Kaiser 2000; Colafrancesco et al. 2003), where the SZE is derived following a full relativistic calculation that can be applied to both the thermal and the non-thermal SZE. The two approaches have been shown to be equivalent within the ranges of validity of the fitting formulae used for the relativistic corrections (see e.g. Boehm & Lavalle 2009). Therefore, in our approach the total SZE can be written as

$$\Delta I_{\text{tot}}(x) = \Delta I_{\text{th}}(x) + \Delta I_{\text{nt}}(x), \quad (3)$$

where the quantity ΔI_{th} includes both the non-relativistic part and the relativistic corrections to the thermal SZE, and ΔI_{nt} is the non-thermal SZE.

With these assumptions, it is possible to calculate the spectral shape of the function $g(x)$, which, in the general form and working at the first order in the optical depth τ , can be written as

$$g(x) = \frac{m_e c^2}{(k_B T_e)} [j_1(x) - j_0(x)]. \quad (4)$$

In this expression, the quantity $\langle k_B T_e \rangle$ is analogous to the electron temperature for a general electron population (Colafrancesco et al. 2003). For thermal electrons, the equivalence $\langle k_B T_e \rangle = k_B T_e$ holds, while for non-thermal electrons we find that

$$\langle k_B T_e \rangle = \frac{1}{3} \int_0^\infty dp f_e(p) p \beta m_e c^2, \quad (5)$$

where $f_e(p)$ is the normalized spectrum of the electrons written as a function of their normalized momentum $p = \beta\gamma$.

In equation (4), the quantity $j_0(x)$ is the spectrum of the cosmic microwave background normalized to I_0 ,

$$j_0(x) = \frac{x^3}{e^x - 1}, \quad (6)$$

and the quantity $j_1(x)$ is given by

$$j_1(x) = \int_{-\infty}^{+\infty} j_0(x e^{-s}) P_1(s) ds, \quad (7)$$

where the function $P_1(s)$ is given by

$$P_1(s) = \int_0^\infty f_e(p) P_s(s, p) dp, \quad (8)$$

and where the function $P_s(s, p)$ includes the full relativistic physics of the inverse Compton scattering process (see e.g. Enßlin & Kaiser 2000; Colafrancesco et al. 2003). In the following, we use for the non-thermal electron spectrum a single power-law shape with a minimum momentum p_1 :

$$f_e(p) \propto p^{-s_e}, \quad p \geq p_1. \quad (9)$$

Colafrancesco et al. (2003) found that in this case the function $g(x)$ depends on the values of both s_e and p_1 .

Following these expressions, we can write the total SZE as:

$$\frac{\Delta I_{\text{tot}}(x)}{I_0} = g_{\text{th}}(x; T_e) y_{\text{th}} + g_{\text{nt}}(x; s_e, p_1) y_{\text{nt}}, \quad (10)$$

where we define g_{th} and g_{nt} as the function $g(x)$ for thermal and non-thermal electrons, respectively, and we have highlighted the parametric dependence on the electron properties. We note that, if we neglect the non-thermal component of the SZE, the SZE can be written as

$$\frac{\Delta I_{\text{tot}}(x)}{I_0} = g_{\text{th}}(x; T_e^*) y_{\text{th}}^* \quad (11)$$

and, as a result, determining the electron temperature gives a value of T_e^* different from the true one T_e .

In principle, a different value of the temperature T_e will impact also the Compton parameter y_{th}^* , but we note that this parameter is given by the product of the temperature and the optical depth (Colafrancesco et al. 2003), and since we are dealing with the results obtained from the stacking of a large number of clusters, therefore, the average optical depth, and as a consequence y_{th} , is just a free parameter in this analysis.

For this reason, we estimate the contribution of the relativistic corrections to the thermal SZE and of the non-thermal SZE from the ratio of the signal at two frequencies, x_1 and x_2 . The ratio of the total signal at the two frequencies is given by:

$$\frac{\Delta I(x_1)}{\Delta I(x_2)} = \frac{g_{\text{th}}(x_1; T_e) y_{\text{th}} + g_{\text{nt}}(x_1; s_e, p_1) y_{\text{nt}}}{g_{\text{th}}(x_2; T_e) y_{\text{th}} + g_{\text{nt}}(x_2; s_e, p_1) y_{\text{nt}}}, \quad (12)$$

while the same ratio obtained by neglecting the non-thermal contribution is

$$\frac{\Delta I(x_1)}{\Delta I(x_2)} = \frac{g_{\text{th}}(x_1; T_e^*) y_{\text{th}}^*}{g_{\text{th}}(x_2; T_e^*) y_{\text{th}}^*} = \frac{g_{\text{th}}(x_1; T_e^*)}{g_{\text{th}}(x_2; T_e^*)}. \quad (13)$$

By defining

$$R \equiv \frac{g_{\text{th}}(x_1; T_e^*)}{g_{\text{th}}(x_2; T_e^*)} \quad (14)$$

and

$$X = \frac{y_{\text{nt}}}{y_{\text{th}}}, \quad (15)$$

and equating equations (12) and (13), we obtain

$$\frac{y_{\text{th}}(g_{\text{th}}(x_1; T_e) + X g_{\text{nt}}(x_1; s_e, p_1))}{y_{\text{th}}(g_{\text{th}}(x_2; T_e) + X g_{\text{nt}}(x_2; s_e, p_1))} = R, \quad (16)$$

from which we obtain

$$X = \frac{R g_{\text{th}}(x_2; T_e) - g_{\text{th}}(x_1; T_e)}{g_{\text{nt}}(x_1; s_e, p_1) - R g_{\text{nt}}(x_2; s_e, p_1)}. \quad (17)$$

3 RESULTS

We apply equation (17) to the results presented in H16; we specifically refer to the results presented in fig. 4 of H16, where the temperature obtained from the SZE is compared to that obtained from X-ray spectroscopic measurements. We consider that this case is more solid than the other case presented in fig. 3 of H16, where the temperature obtained from the scaling relation between the temperature and X-ray luminosity is used. We also use frequencies of 545 and 353 GHz (i.e. $x_1 \sim 9.59$ and $x_2 \sim 6.21$), where the non-thermal contribution for high-temperature clusters is expected to be more evident. Note also that H16 observed that the ratio of the SZE signal at these two frequencies changes with the temperature because of the relativistic corrections. Therefore, we consider that this ratio is a good indicator of the amount of relativistic correction.

We consider the temperature bin where the discrepancy between the X-ray- and SZE-obtained temperatures is more evident, i.e. $k_B T_X = 11$ keV in fig. 4 of H16, and assume that the real cluster temperature is $k_B T_e = k_B T_X$, while we fix $k_B T_e^* = k_B T_{\text{SZ}}$, i.e. 15 keV.

For the non-thermal electron population, we chose – as a reference case – a population like the best case found from the SZE data analysis of the Bullet cluster, i.e. with spectral index $s_e = 3.7$ (Marchegiani & Colafrancesco 2015), and calculate the value of X by changing p_1 . We will discuss later the effect of changing the value of the spectral index.

We found that the condition $X < 1$ (i.e. the non-thermal pressure is lower than the thermal one) is satisfied only in a narrow range of values of p_1 , i.e. $0.2 < p_1 < 3$ (see Fig. 1). In this range, the minimum possible value of X is obtained for $p_1 = 0.7$, where $X = 0.228$. Including the error bars on T_{SZ} shown in the plot of H16, we obtain the value $X = 0.228 \pm 0.012$ for $p_1 = 0.7$.

We must stress that the uncertainties on the properties of the non-thermal electron population produce an uncertainty on the strength of these estimates. By changing the value of s_e , the value of X that can be derived is changed, as well as the corresponding value of p_1 , even if this last quantity changes by a small factor. For example, for $s_e = 2.7$, we find that the minimum possible value of X is obtained for $p_1 = 0.5$, for which $X = 0.341 \pm 0.018$. Anyway, we note that this value is of the same order of magnitude as when $s_e = 3.7$, suggesting that, if the non-thermal SZE interpretation of these data is correct, a pressure ratio of the order of 20–30 per cent is quite a strong requirement.

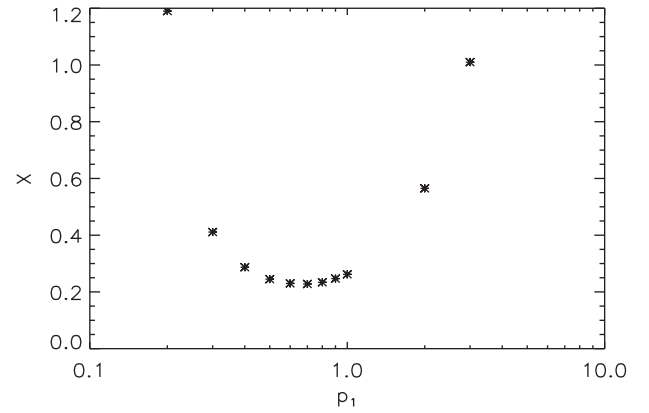


Figure 1. Pressure ratio $X = y_{\text{nt}}/y_{\text{th}}$ required to explain the discrepancy between the SZE- and X-ray-derived cluster temperatures in terms of a non-thermal SZE component, calculated according to equation (17) for a non-thermal population with $s_e = 3.7$ and minimum momentum p_1 , and using the ratio of the SZE signals at the frequencies 545 and 353 GHz.

We stress also that these results are obtained by working with the ratio of the signal at only two frequencies (i.e. 353 and 545 GHz), and using only the final results of the analysis of H16, where the discrepancy between the X-ray- and SZE-determined temperatures was suggested at only the $\sim 1.5\sigma$ level. Once this discrepancy is detected at a higher statistical significance, then to obtain more precise results for the possibility that the data indicate a contribution from the non-thermal SZE and to determine the parameters of the non-thermal electrons in galaxy clusters (i.e. X as well as s_e and p_1 separately), it would be necessary to reanalyse the whole sample of clusters by considering the contribution of the non-thermal SZE at all the frequencies considered. This work is outside the scope of this paper and will be considered in future work.

An even better possibility would be to measure the SZE signal at a high number of frequencies along the whole range of frequencies until ~ 1 THz, because in this way it would be possible to derive much more precise constraints on both the thermal and non-thermal SZE (Colafrancesco & Marchegiani 2010). An instrument like Millimetron (Kardashev et al. 2014) will allow this analysis.

4 DISCUSSION AND CONCLUSIONS

We have explored the possibility that the discrepancy between the SZE- and X-ray-estimated values of the cluster temperature suggested by H16 is due to the presence of a physical non-thermal SZE component instead of calibration issues. The main result of our analysis is that the non-thermal SZE can explain this discrepancy under the requirement that the non-thermal electrons have a very low minimum momentum, of the order of $p_1 = 0.5$ –1. This result is in agreement with the results obtained from the SZE analysis of the Bullet cluster (Colafrancesco et al. 2011; Marchegiani & Colafrancesco 2015); however, it is difficult to maintain this condition in a large number of clusters, considering that electrons with this low energy are expected to have a very small lifetime because of energy losses due to non-thermal bremsstrahlung and Coulomb interactions with the thermal gas (see Sarazin 1999), and also considering that the cluster sample here considered should contain mainly clusters without a radio halo (or with low-power radio haloes), where the number of non-thermal electrons is expected to be lower.

In addition, the derived ratio between the non-thermal and thermal pressures, of the order of 20–30 per cent, is an issue for our

knowledge of the physics of non-thermal processes in galaxy clusters. In fact, from the upper limits derived from a stacked analysis of Fermi-LAT data in galaxy clusters, Huber et al. (2013) derived an upper limit of the order of 5 per cent for the ratio between the energy content of non-thermal and thermal protons. Since processes accelerating cosmic rays in galaxy clusters are believed to provide an energy ratio of the order of 0.01 or smaller between electrons and protons, there is a problem in finding a source of acceleration of the electrons that is selectively not effective for protons, which is an analogous problem to that in the comparison between the properties of radio haloes and the Fermi-LAT upper limits in galaxy clusters (Vazza et al. 2016). However, we note that from a statistical study of SZE and X-ray properties of a sample of 23 radio halo clusters, an average value of $X \sim 0.55$ was found (Colafrancesco et al. 2014), and that in the Bullet cluster, the SZE data seem to indicate values of $p_1 = 1$ and $X = 91$ per cent (Marchegiani & Colafrancesco 2015). Therefore, these problems seem to be present in all the cases where the SZE is used as a probe of the non-thermal electrons in galaxy clusters, suggesting that something might be still missing in our understanding of the properties of non-thermal phenomena in galaxy clusters. Alternatively, this use of the SZE measured at a very limited number of frequencies, instead of using many frequencies (Colafrancesco & Marchegiani 2010), may not be sufficiently good for obtaining reliable information on the non-thermal content in galaxy clusters.

Note that the solution recently suggested for radio haloes, i.e. electrons produced by dark matter annihilation (Marchegiani & Colafrancesco 2016), does not work effectively for the SZE signal, because the spectrum at low energies of dark matter-produced electrons does not follow a simple power law (their spectrum is quite flatter), and hence they do not produce a significant SZE signal (Marchegiani & Colafrancesco 2015).

A possible solution is that the electrons producing the non-thermal SZE are not completely diffused into the intracluster medium, but are mainly confined in bubbles of a relativistic plasma where the density of the thermal gas is low, as observed in several clusters (see e.g. Fabian et al. 2006). In this case, the low density of the target thermal nuclei would allow the low-energy non-thermal electrons to survive for longer and they would have a power-law spectral shape down to low values of p_1 . The injection of electrons from radio galaxy jets with the formation of lobes seems to be the most likely mechanism to produce this kind of structure. If the jets are mainly leptonic, or if some mechanism can confine the protons in these bubbles, then we would not expect strong gamma-ray emission to be produced in the bubbles because of the lack of target nuclei. However, if these bubbles have an internal pressure similar to the pressure of the surrounding intracluster medium, to obtain a value of $X \sim 20$ – 30 per cent it would be necessary for the bubbles to cover a volume of the order of a quarter of the cluster volume, a condition that might be difficult to obtain. This problem can be partially alleviated if a strong contribution comes from bubbles of a relativistic plasma in the outskirts of the cluster, like giant radio galaxy lobes or radio relics.

We conclude that, once a detection with a higher significance of the discrepancy between X-ray- and SZE-estimated temperatures is obtained, there are two ways to derive stronger conclusions from this analysis. The first is to reproduce the full analysis done in H16 by considering the additional contribution of the non-thermal SZE over a wide range of frequencies and considering a number of possible models for the non-thermal electrons, with the aim of determining not only the pressure but also the spectral properties of the non-thermal electrons. Another possibility is to perform multi-frequency and spatially resolved observations of the SZE in galaxy clusters,

and to compare the results with X-ray and radio observations, to check if the non-thermal SZE is mainly produced in bubbles with a high non-thermal and low thermal plasma content. Instruments like Millimetron (see e.g. Kardashev et al. 2014) are suitable for achieving this goal.

Another issue that will need to be considered is the effect of temperature non-uniformities within the clusters, which can create a bias in the estimation of the mean temperature of a cluster considered as isothermal. For example, it has been shown that the X-ray temperature determinations based on a single temperature fit are biased towards high-density and low-temperature regions (e.g. Mazzotta et al. 2004), because the emissivity of the thermal bremsstrahlung is proportional to $n_e^2 \sqrt{T_e}$. The thermal SZE is, in the non-relativistic formulation, linearly proportional to both n_e and T_e , and therefore, one would expect that the SZE-derived temperatures would be less biased than the X-ray-derived ones. However, the relativistic formulation makes the dependence on T_e non-linear, and this translates into a bias towards high values of the temperature. As in the X-ray analysis, a spatially resolved measure of the temperature along the cluster through the SZE can allow the recovery of the true distribution of the temperature through a de-projection technique (Colafrancesco & Marchegiani 2010). However, when working with the spatially integrated SZE values in clusters that are assumed to be isothermal, the bias introduced by a single temperature fit to the SZE can be high, as well the variance of the temperature, which, especially in disturbed clusters with a big range of temperatures, can have values of the order of the average temperature (Prokhorov & Colafrancesco 2012).

As an example of a simple and idealized case, Vikhlinin (2006) found that, assuming a cluster with two thermal populations with the same bremsstrahlung emission measures and temperatures of 4 and 16 keV, the resulting X-ray emission can be fitted by a single thermal population with a temperature of ~ 7 keV. To compare the SZE bias with this result, we obtained an analogous estimate of the temperature derived from a single temperature fit to the SZE produced by two populations with the same optical depth and temperatures of 4 and 16 keV. We found that the SZE-derived temperature is ~ 13 keV. Therefore, in this case, both these estimates deviate from the average temperature of 10 keV by a factor of the order of ~ 30 per cent in opposite directions, which would be sufficient to account for all the discrepancy claimed as a result of H16. However, when working with the stacked analysis of a large sample of clusters, which may have different temperature structures (including e.g. temperature gradients, cooling flows, shock waves and cold fronts), estimating the bias of the discrepancy between X-ray- and SZE-derived temperatures is a complex issue that will be addressed in a future paper.

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