THE APPLICATION OF MACRO CO-KRIGING AND COMPOUND LOGNORMAL THEORY TO LONG RANGE GRADE FORECASTS FOR THE CARBON LEADER REEF

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DECLARATION

I declare that this project report is my own, unaided work. It is being submitted for the degree Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

V A Chamberlain

SEVENTH day of OCTOBER 1997

ABSTRACT

Due to the extreme costs of establishing new shaft systems in Witwatersrand gold mines it is essential that the resource estimation is optimised. The result of poor or sub-optimal estimation could be catastrophic even to the largest of mining companies.

This project examines the application of Compound Lognormal Distribution theory and shows the advantages of this distribution model over more traditional models, for the Carbon Leader Reef. The incorporation of information from mined out areas of a deposit in resource estimation is demonstrated. The critical role played by accurate geological modelling is highlighted.

The process of Macro Co-Kriging in conjunction with Compound Lognormal Theory is discussed in detail and is shown to be a more accurate estimation technique than traditional techniques using Lognormal theory.

Finally the use of the Macro Co-kriged limits are shown to be useful in the classification of Mineral Resources.

ACKNOWLEDGEMENTS

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My wife, Moira, and our children for their patience and encouragement.

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1 INTRODUCTION

The grade estimates are of the utmost importance in planning new shafts on a Witwatersrand Gold Mine. The cost of a new shaft system may be in excess of R 3 000 000 000. If the resource estimation is not optimised the resultant loss could be catastrophic even to the largest of mining companies. Long range grade forecasts for the Carbon Leader Reef have in the past proved to be unreliable and this project will attempt to address this situation by considering the following:

1 The amount of information available for long range grade prediction in Carbon Leader Reef mines is extremely restricted due to the depth of the ore body and the cost of drilling. A large volume of data is however available from mined out areas in the form of routine channel sampling. A technique which allows for the integration of the limited borehole information ahead of the face with the masses of data in the mined out areas is therefore needed. This approach was first suggested by Sichel (1949) and later formalised by Krige et al (1990).

2 A distribution model (not necessarily lognormal) that accurately represents the gold value distribution of Carbon Leader Reef needs to be applied in the estimation process. Possible alternative models, to the lognormal distribution, were first suggested by Sichel (1990) and documented by Sichel et al (1992) and Dohm (1995).

1.1 Proposed Solution

Using all available data from both Carbon Leader boreholes and routine underground channel sampling an attempt will be made to determine the following:

1 The most applicable geological model to be applied to the Carbon Leader

Reef for long range grade forecasts.

2 The statistical probability density model (distribution) which best fits the geological subdivision of the Carbon Leader Reef.

3 The spatial variability of the distribution parameters for the geological subdivision and their integration in the estimation process.

4 The extent of spatial correlation of variables to be used in the estimation process.

A macro co-kriging process will then be used to project grades into unmined areas and the results will then be tested as follows :

1 Theoretical grade tonnage curves will be compared with those determined from kriging results.

2 Comparison of actual block values with estimated block values in mined out areas to determine if a regression effect is present.

3 Comparison of estimated block values with actual block values in adjacent areas - where data was excluded from the co-kriging process.

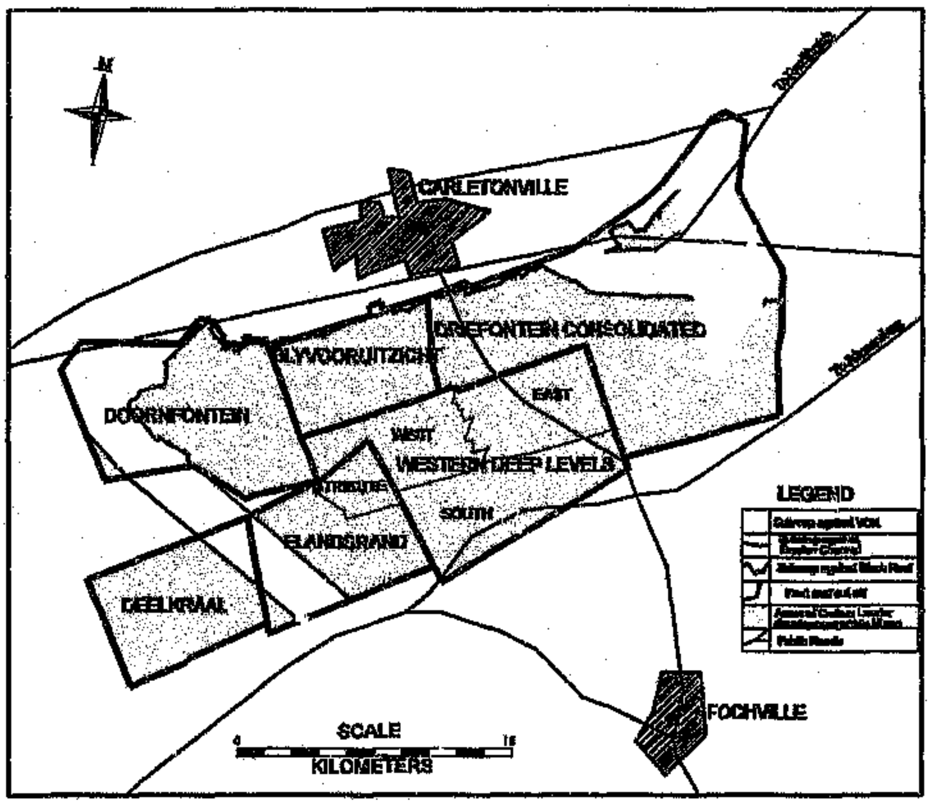
4 Comparison of the above technique to more traditional techniques of long range forecasting to determine if the this process is in fact more reliable for long range forecasting of Carbon Leader grades.

An attempt will also be made to determine confidence limits to be applied to this technique and to relate these limits to Mineral Resource Classification categories.

1.2 Location of Project Area

The project area is located within the lease area of Western Deep Levels East Mine. Western Deep Levels lies some 90 km west of Johannesburg and 8 km south of Carletonville (Figure 1.1).

Figure 1.1 Locality plan - Western Deep Levels.



1.3 Data Locality

The reef under consideration is the Carbon Leader Reef. The following data was used in this project :

1 Digitised routine sampling results (224 148 samples) from Western Deep

Levels East, Western Deep Levels West and Blyvooruitzight. A scale transformation was applied to the data to protect the confidentiality thereof. A regularised plot of the data is shown in Figure 1.2

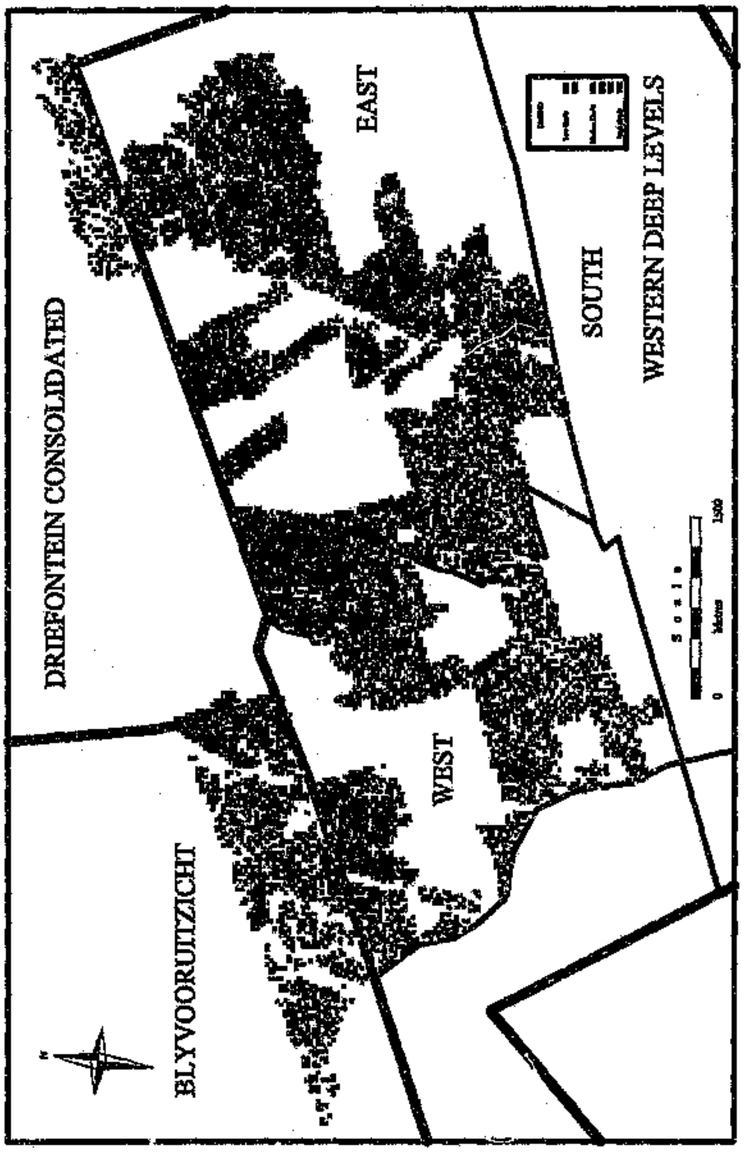


Figure 1.2 Plot showing 30 x 30 regularised block gold grades for Carbon Leader Reef

2 All available surface borehole drilling results for intersections of the Carbon Leader Reef. For Western Deep Levels the data was accessed from mine data bases and for surrounding mines published records were used. The same scale transformation was applied to the borehole data.

2 REGIONAL STRATIGRAPHY

The Carbon Leader Reef is situated at the base of the Langlaagte Formation (SACS 1993), which in turn forms the base of the Johannesburg Subgroup of the Central Rand Group. It rests unconformably on the sediments of the Blyvooruitzicht Formation, the base of which is marked by the North Leader Reef (Figure 2.1).

The lithic quartzites which form the bulk of the Blyvooruitzicht Formation are coarse grained and yellow brown in colour. They frequently contain, locally discontinuous, pebble bands which are high in detrital pyrite but contain trace gold values. These coarse grained quartzites of the footwall frequently contain phyllonitic zones in the vicinity of the Carbon Leader Reef.

The Carbon Leader Reef is overlain by approximately 1.8 m of proto to ortho quartzites, informally known as Hangingwall of Carbon Leader. The Hangingwall of Carbon Leader quartzites display low angle trough cross bedding and contain grits and/or small pebbles. This unit is overlain by 0.2 to 0.5 m of angular grits, referred to as the Rice Pebble Marker. These grits are abruptly overlain by the Green Bar, an approximately 2.0 m thick chloritoid rich siltstone. Above the Green Bar lies a very clean ortho quartzite which varies in thickness from 0.5 to 2.0 m.

The Carbon Leader Reef has a strike of approximately 075° and dips to the south at approximately 22°. The Carbon Leader Reef basal unconformity eliminates

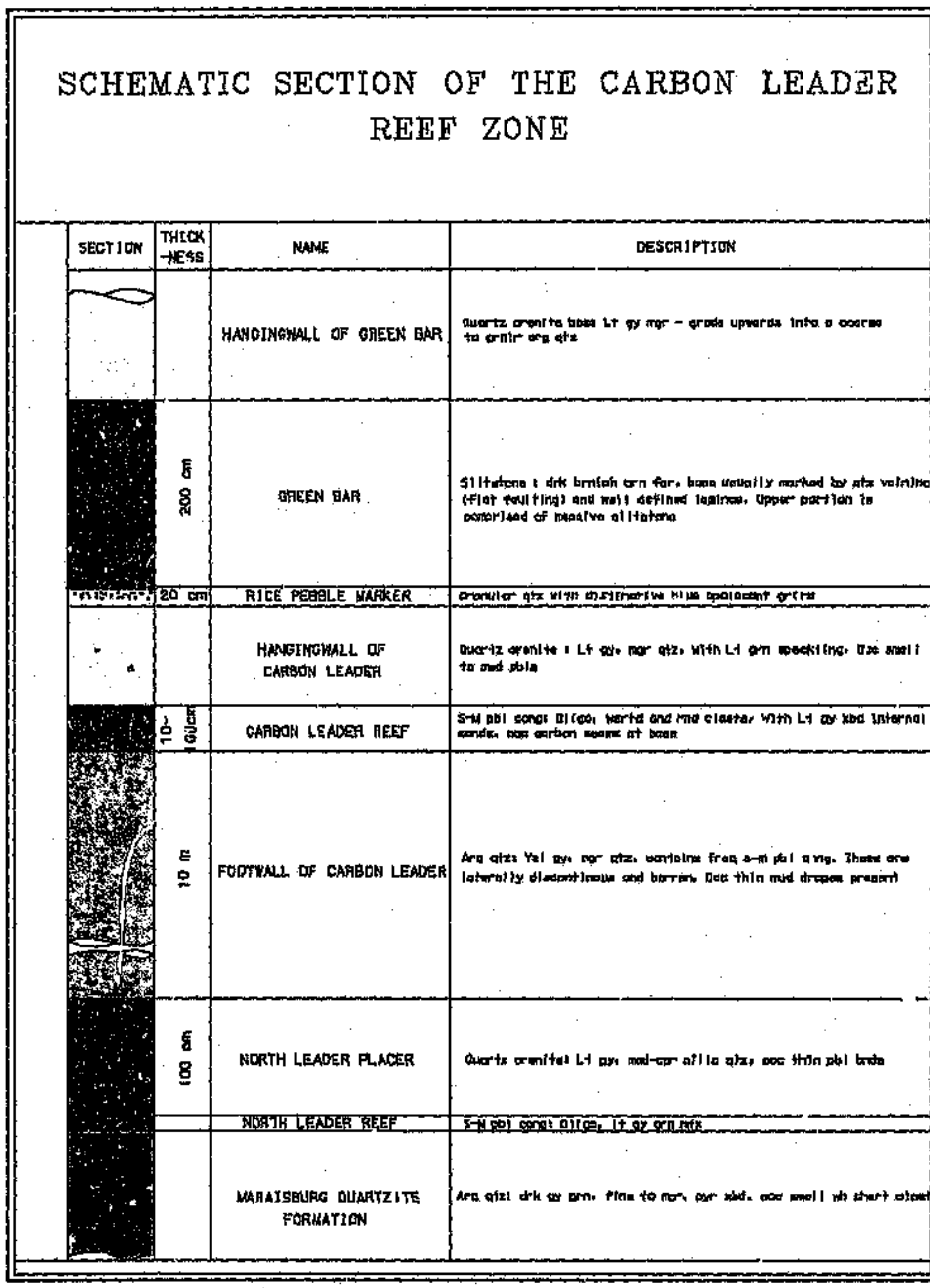


Figure 2.1 Stratigraphic column of Carbon Leader Reef zone

the Blyvooruitzicht Formation to the northeast of Western Deep Levels East.

3 SEDIMENTOLOGY OF THE CARBON LEADER

The existing method of facies delineation on Western Deep Levels was developed by members of the mine geology department and first documented by T Rowland (1990). It involves subdividing the reef into one of three categories based on channel width (Plateau Facies < 20 cm, Transitional Facies 20 - 40 cm and Channel Facies > 40 cm) and does not take into account the sedimentological characteristics.

Rowland and previous authors however concluded that the Carbon Leader Reef displayed characteristics of a high energy braided stream environment (Channel Facies) with low energy interchannel areas (Plateau Facies) consistent with an anastomosing system (Adam 1991), acting upon an alluvial fan.

This subdivision of the Carbon Leader Reef resulted in the generation of a facies plan, showing a series of NE-SW trending channels (up to 150 m wide, with a maximum depth of 1.2 m) separated by large areas of thin plateau reef (Figure 3.1). Rowland, Adam and others (Buck and Minter, 1985), concentrated upon the areal distribution of grades and lithofacies and did not investigate the detailed stratigraphic subdivision of the Carbon Leader Reef.

The only work on a detailed stratigraphic subdivision of the Carbon Leader Reef was published by Nami (1981,1982) who undertook a detailed study of the Carbon Leader Reef on Blyvooruitzicht Gold Mine. In this investigation two subfacies were identified. The uppermost of these (subfacies A) consisted of a matrix supported, laterally continuous conglomerate with uniformly high gold grades, an essentially non erosive basal contact and frequent basal carbon (when developed upon the footwall quartzites). The lowermost unit (subfacies B) comprised a series of lenticular conglomerate bodies which were poorly sorted and matrix to pebble supported - overlain by trough cross bedded quartzites. This unit was seen to possess highly erratic gold grades.

Nami interpreted subfacies B as representing channel deposits whilst subfacies A represented deposits formed by overbank flooding from channels.

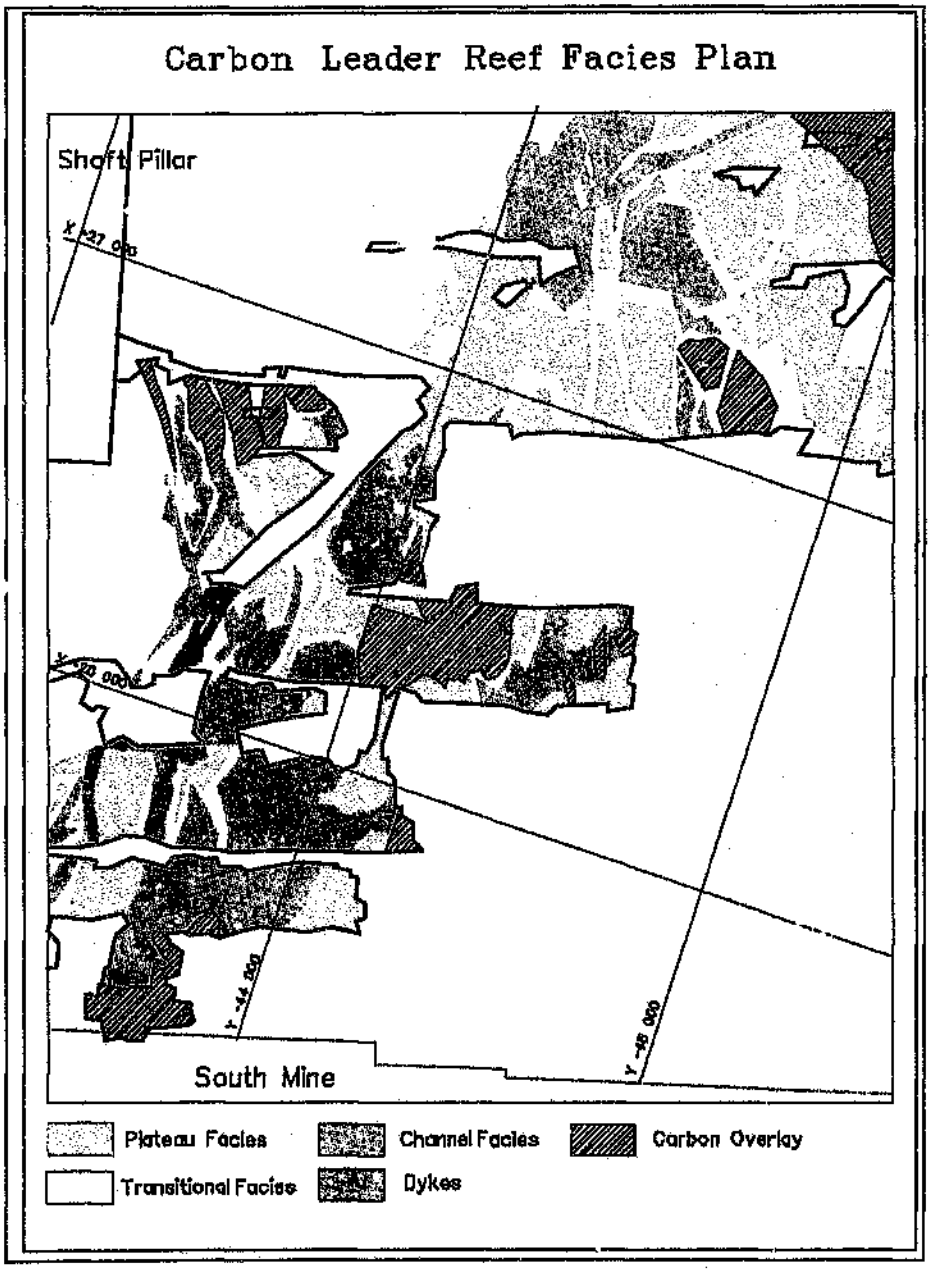


Figure 3.1 Carbon Leader Reef facies plan

A multi disciplinary study of the vertical distribution of facies within the Carbon Leader Reef was recently completed at Western Deep Levels East. The first phase of the study proposed a model for the Carbon Leader Reef which subdivided the reef into four stratigraphic subfacies units (Kingsley 1992). These were numbered 1 to 4 from the base upwards.

It was observed that "channel" reef generally contained all four subfacies units, whilst "plateau" reef comprised only the upper two subfacies units (Figure 3.2).

The second phase of the study examined the lateral variations of the four subfacies units and concluded (Chamberlain and Young, 1996) that the lower two units are erosive channel features, probably deposited on alluvial fans. The third unit is more laterally extensive but still shows evidence of basal scouring and is interpreted as a flood deposit. The uppermost unit shows a non erosive base, is laterally extensive and is conformably overlain by mature "marine" sands. This unit is interpreted as the base of a marine transgressive event which culminated in the deposition of the Green Bar silts. A similar conclusion was reached by Braun (1988).

The NE/SW channel trends as delineated by the pre-existing facies model (Rowland 1990 and Adam 1991) are seen to be the composite of multiple erosional and depositional events, with the individual facies subunits actually displaying N/S and E/W channel trends, in which normal channel features can be recognised.

As is apparent in Figure 3.1 the NE/SW channel trends are commonly blind in nature and terminate/start in an unpredictable manner, this being due to the composite nature of the channels. It is therefore not possible to project any of the channel/plateau areas into unmined areas. For the purposes of long range forecast it is therefore necessary to use a more global approach to geological modelling .

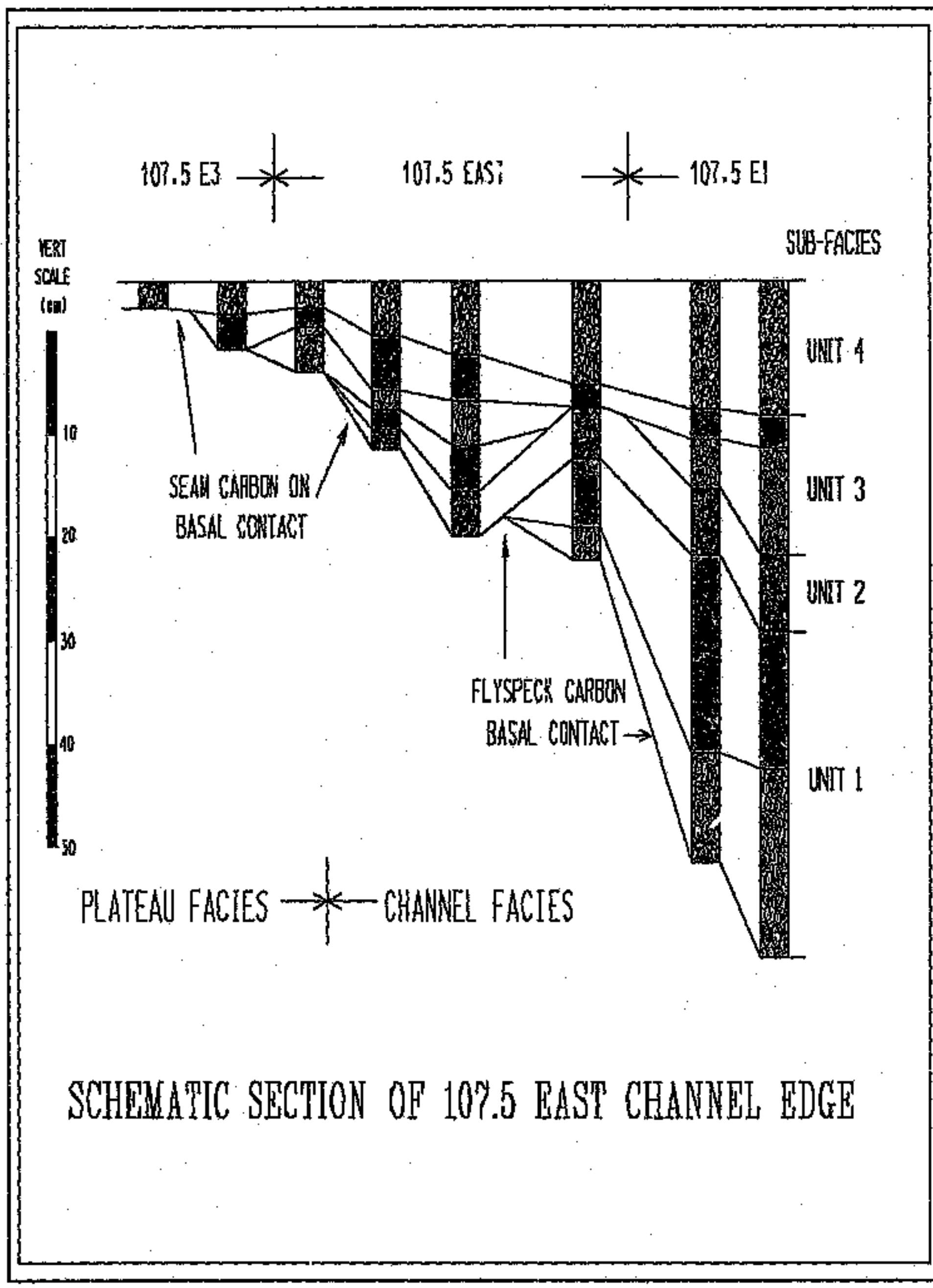


Figure 3.2 Schematic section of Carbon Leader Channel edge showing vertical distribution of subfacies units

Currently the Western Deep Levels lease area is split into two subdivisions, an easterly high grade geozone and a westerly moderate grade geozone. The area under consideration for this project lies entirely within the easterly high grade zone (Figure 3.3).

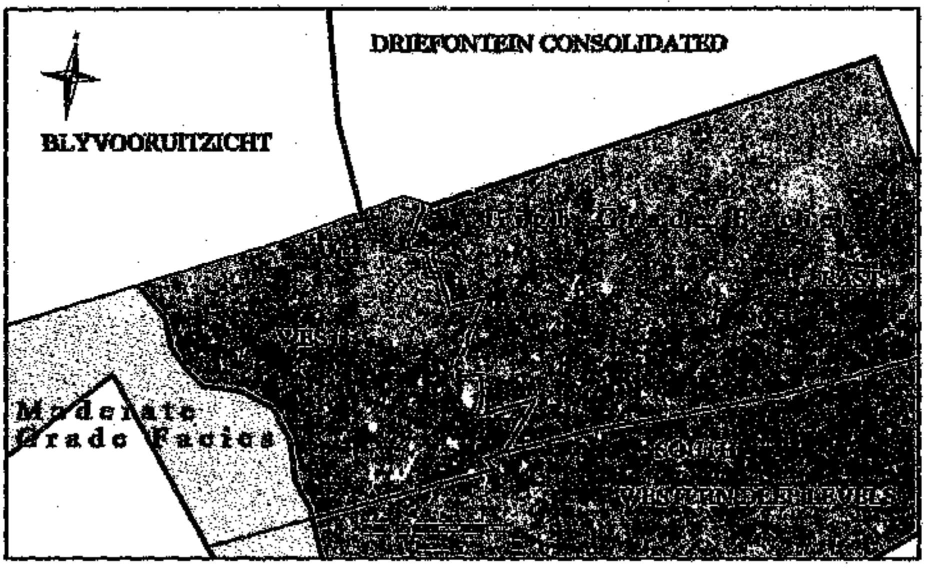


Figure 3.3 Distribution of global Carbon Leader Reef Geozones

4 STATISTICAL ANALYSIS OF UNDERGROUND CHANNEL SAMPLING DATA

4.1 Descriptive Statistics

The channel sampling data for the Carbon Leader Reef was collected as part of routine mine sampling in the manner summarised by Storrar (1977). The data was then digitised from stope sheets. A total of 224 148 samples were available for the high grade geozone of the Carbon Leader Reef. The frequency diagram (Figure 4.1) and summary statistics (Table 4.1) are shown below :

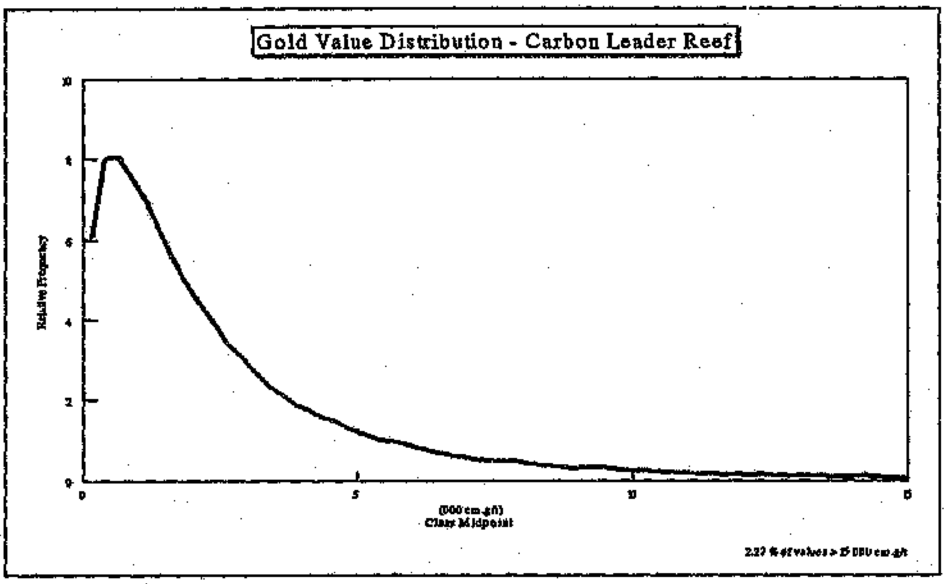


Figure 4.1 Distribution of gold values (cm.g/t)

Statistical Parameter	Value
Mean	3 131 cm.g/t
Median	1 826 cm.g/t
Mode	624 cm.g/t
Variance	21 120 242 cm.g/t ^{2}
Standard Deviation	4596 cm.g/t
Coefficient of Variation	146.77 %
Skewness	8.615
Kurtosis	247.388
Minimum	1 cm.g/t
Maximum	32 347 cm.g/t

Table 4.1 Summary statistics of gold values (cm.g/t)

From the above it is apparent that the distribution of cm.g/t values is extremely positively skewed with a skewness of 8.6 and a coefficient of variation greater than 100 %. The upper tail is not well developed above 10 000 cm.g/t. The median of the data is 1826 cm.g/t whereas the mean is 3131 cm.g/t, again a reflection of the skew nature of the data.

Considering the log transformed data set, such that x = ln z. The distribution would be expected to be closer to normality (Krige, 1960). The frequency diagram (Figure 4.2) shows this to be true, as do the summary statistics (Table 4.2).

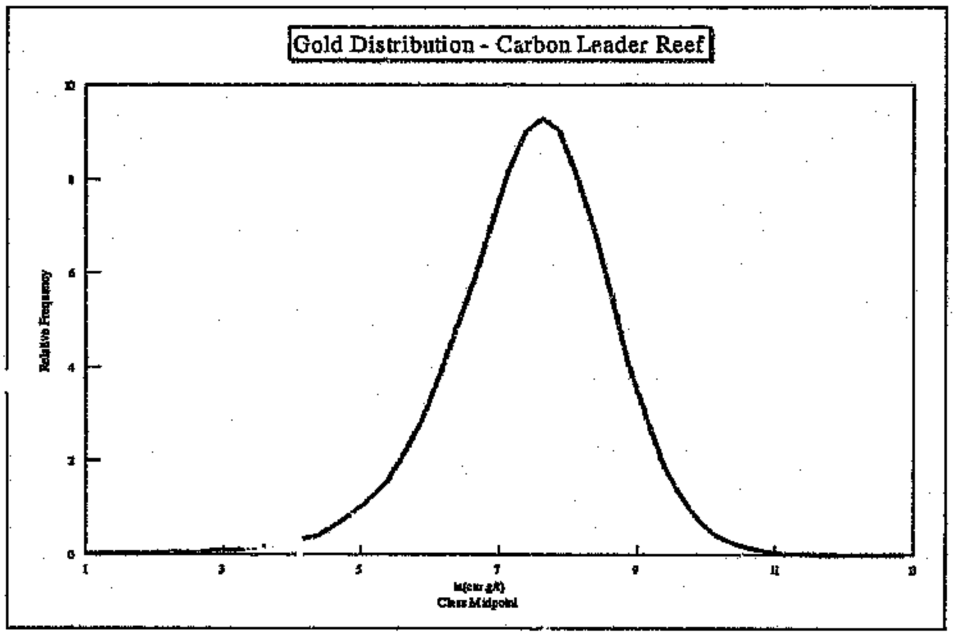


Figure 4.2 Distribution of ln(cm.g/t) gold values

Statistical Parameter	Value
Mean ξ	7.427
Median	7.676
Mode	7.626
Variance σ^2	1.428

Standard Deviation σ	1.195
Coefficient of Variation	16.08 %
Skewness $\sqrt{\beta_1}$	-0.568
Kurtosis β_2	4.285
Range	12.687
Minimum	0
Maximum	12.687

Table 4.2 Summary statistics of ln (cm.g/t) gold values

However, though this distribution appears to approach normality the conditions of true normality are not met. The skewness is not equal to zero and the kurtosis is not equal to three. The distribution of log values is negatively skewed and is too peaked to be considered normal.

The deviation of this distribution from normality can also be illustrated by a lo; probability plot, where a straight line would be expected for a normal distribution. It is apparent from Figure 4.3 that the log values do not plot as a straight line.

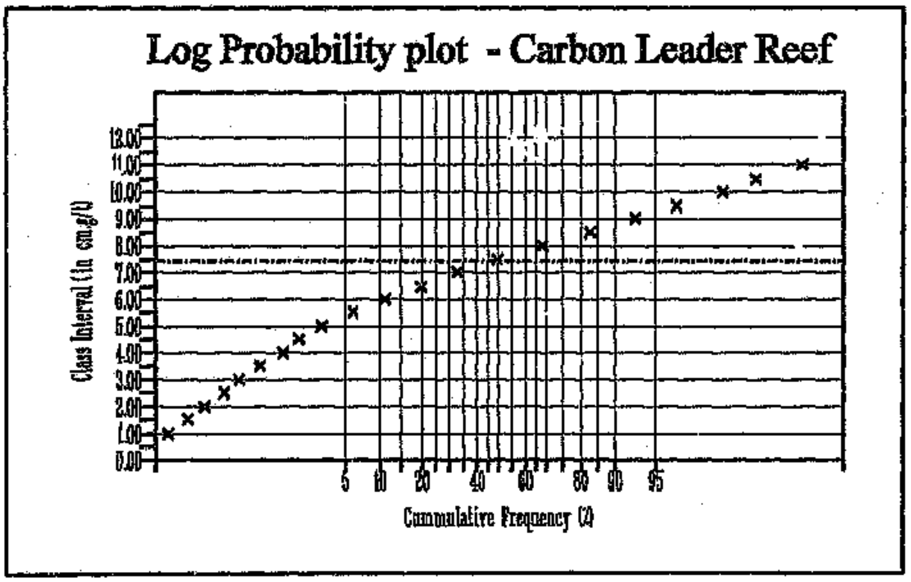


Figure 4.3 Log probability plot of gold values (cm.g/t)

4.2 Distributional Models

There are four possible distribution model tha' have been applied to Witwatersrand gold data. These are the Two Parameter Lognormal Distribution (Krige, 1960) , Three Parameter Lognormal Distribution (Sichel, 1966), the Log-Generalised Inverse Gaussian Distribution (LNGIG) (Sichel et al, 1992) and the Compound Lognormal Distribution (CLN) (Sichel et al, 1992).

The probability density function of gold values is graphically illustrated by a histogram and described by the following statistical parameters :

1 The mean - a measure of the position of the data

2 The variance - a measure of how the data varies about the mean

3 The skewness - a measure of the tails of the distribution

4 The kurtosis - a measure of the peakedness of the distribution as well as the thickness of the tails.

A distributional model is essentially a mathematical equation which defines the shape of the histogram of the observed data values. The essential elements of the four models discussed above are as follows :

4.2.1 Lognormal Distribution

The two and three parameter distribution function has the following form :

$$f(z) = \frac{1}{\sqrt{2\pi}\,\sigma(z+\beta)}\, e^{\frac{-1}{2}\left[\frac{\ln(z+\beta)-\xi}{\sigma}\right]^2}$$

Where	z	is the observed value in cm.g/t
	β	is the additive constant or third parameter
	ξ	is the mean of the logtransformed data
	σ^2	is the variance of the logtransformed data

The mean of the two parameter lognormal distribution (μ) is :

$$\mu = e^{\xi + \frac{\sigma^2}{2}}$$

and the mean of the three parameter lognormal distribution is :

$$\mu = e^{\xi + \frac{\sigma^2}{2}} - \beta$$

The following assumptions apply to lognormal theory :

1 The skewness of the logtransform is equal to zero for a two parameter lognormal distribution or is made to equal zero by the addition of an additive constant in the three parameter case.

2 The kurtosis of the logtransform is equal to three for the two parameter case or is made to equal to three in the three parameter case.

3 The above is reflected by a straight line log probability plot.

4 The log mean and the log variance are independent.

4.2.2 Compound Lognormal Distribution

The distribution has the following form :

$$\Omega(x) = \frac{s(1-c)^{\nu+\frac{1}{2}}}{2^{\nu}\sqrt{\pi}\,\Gamma(\nu+\frac{1}{2})} \; e[\pm\sqrt{c}\; s(x-a)] \;\; (s|x-a|)^{\nu}\; K_{\nu}(s|x-a|)$$

Where $\quad x \quad$ is the logtransformed grade in cm.g/t

And $\quad K_{\nu}(.) \quad$ is the modified Bessel function of second kind of order ν

The four parameters of the distribution are:

a = location parameter ($-\infty < x < \infty$)

s = spread parameter ($s > 0$)

c = skewness parameter ($0 \leq c < 1$)

ν = kurtosis parameter ($\nu > -½$)

Estimation of the parameters of the CLN distribution are based on the pearson moments ie. log mean (ξ), log variance (σ^2), skewness ($\sqrt{\beta_1}$) and kurtosis (β_2) of the log transformed gold (cm.g/t) values.

From the skewness ($\sqrt{\beta_1}$) and kurtosis (β_2) calculate q from

$$q = \frac{3\beta_1}{2\beta_2 - 3\beta_1 - 6}$$

where $\beta_1 = (\sqrt{\beta_1})^2$

and by iteration determine an estimate for c

$$q = c\left(\frac{3+c}{1-c}\right)^2$$

Then calculate

$$\nu = \frac{2c\,(3+c)^2}{\beta_1\,(1+c)^3} - \frac{1}{2}$$

and

$$s = \sqrt{\frac{(2\nu+1)\,(1+c)}{\sigma^2\,(1-c)^2}}$$

finally

$$a = \xi - \frac{(2\nu+1)\sqrt{c}}{s\,(1-c)}$$

The expected value is calculated as follows :

$$\text{Expected Value} = \left[\frac{(1-c)}{1-\left(\frac{1}{s}+\sqrt{c}\right)^2}\right]^{\left(\nu+\frac{1}{2}\right)} e^{a}$$

4.2.3 Lognormal Generalised Inverse Gaussian Distribution

The density distribution of the log transformed values is given by :

$$\varphi(x) = \frac{1}{2sK_\gamma(b)} e^{\gamma\left(\frac{x-\xi}{s}\right) - b\cosh\left(\frac{x-\xi}{s}\right)}$$

Where ξ is the location parameter ($-\infty < \xi < \infty$)

s is the scale parameter ($s > 0$)

b is the kurtosis parameter ($b > 0$)

γ is the skewness parameter ($-\infty < \gamma < \infty$)

The definition of these parameters is discussed in detail by Dohm (1995).

4.3 Distribution Model Selection

Of major importance in the project is the decision as to which distributional model is most applicable to the Carbon Leader Reef. There are many techniques to decide on which model to use. A few of these are summarised below :

1 A graphical test is conducted by plotting the cumulative gold distribution on log-probability paper. If the distribution conforms to a lognormal distribution this plot should be a straight line. If the graph clearly deviates from a straight line it indicates that lognormal assumptions do not hold. If the line represents an elongated reversed s-shape the underlying distribution may be Compound Lognormal (CLN) and if the line shows an elongate s-shape the underlying distribution may be Lognormal Generalised Inverse Gaussian (LNGIG).

2 Statistical parameter comparison can indicate if the distribution of

logtransformed values is normal or not. If both the conditions of normality (Skewne·ˢ = 0 and Kurtosis = 3) are met then the distribution of gold values can be assumed to be lognormal. The addition of an additive constant can be attempted but as will be shown is unlikely to satisfy the above conditions in the case of the Carbon Leader. The distribution is therefore probably CLN or LNGIG.

The statistical significance of the deviation of the shape parameters can be tested by using the assumption of a parent normal distribution and the asymptotic standard errors for the parameters (Kendall and Stuart, 1958).

3 If the conditions of lognormality are not met a theoretical test to differentiate between CLN and LNGIG distributions was detailed by Dohm (1995). The theoretical test value is given by

$$\text{TEST VALUE} = \frac{1}{2}\left(3\beta_1 + 6\right)$$

Where x = ln z values

$\beta_1 = (\text{skewness of } x)^2$

β_2 = kurtosis of x

If the TEST VALUE $\leq \beta_2$ then the distribution may be CLN.

If $\sqrt{\beta_1} = 0$, it means that the distribution of the log transformed data is symmetrical. If also $\beta_2 = 3$, the TEST VALUE will equal 3, and indicating that the distribution is lognormal.

If the TEST VALUE $> \beta_2$ it indicates that the underlying distribution may be LNGIG.

This test can be graphically represented (Figure 4.4) where it is apparent that there is an area of overlap between the models. The limiting line for the CLN distribution is at $\frac{1}{2}(3\beta_1 + 6)$ and for the LNGIG distribution is the Log-Gamma distribution obtained when the b-parameter tends to zero (Dohm, 1995).

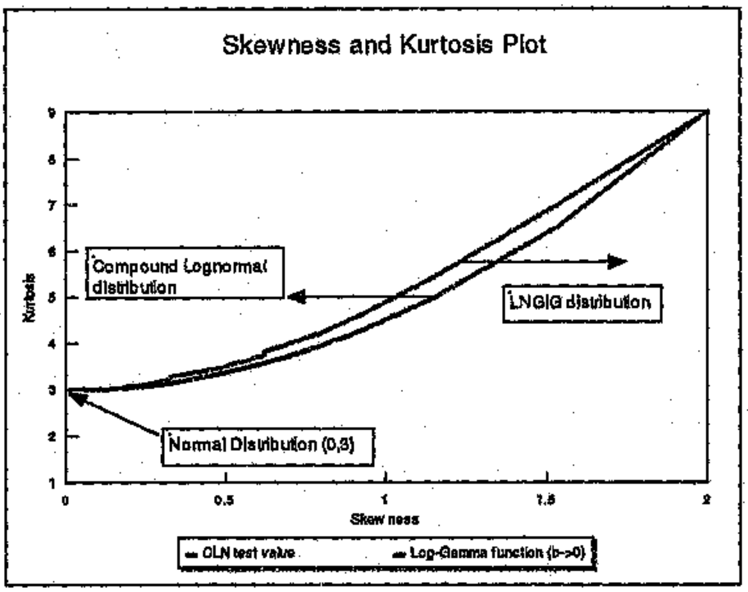


Figure 4.4 Graphical test for CLN / LNGIG distribution

In deciding which distributional model is more applicable to the Carbon Leader the following procedures were applied :

1 A log probability plot (Figure 4.3) shows the distribution does not plot as a straight line. This together with the fact that the base statistics do not show a skewness of 0 and a kurtosis of 3 preclude the use of the two parameter log normal distribution. A further test is to calculate the two parameter lognormal estimate :

$$\text{Estimate} = e^{7.427 + \frac{1}{2}(1.428)}$$
$$= 3\ 432 \text{ cm.g/t}$$

This estimate is 9.6 % higher than the arithmetic mean of 3 131 cm.g/t.

2 In an effort to apply the three parameter lognormal distribution a series of additive coefficients were applied . For each additive constant the statistical parameters of the log distribution were calculated (Table 4.3) These values are shown graphically in Figure 4.5.

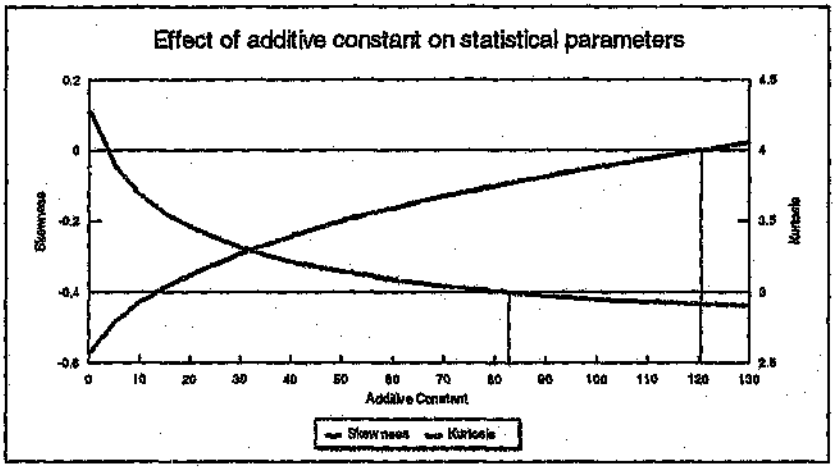


Figure 4.5 Change in skewness and kurtosis with increased additive constant

Beta	Mean	Var	Skewness	Kurtosis	Estimate
0	7.4268	1.4278	-0.5680	4.2846	3431
5	7.4346	1.3914	-0.4815	3.8813	3391
10	7.4417	1.3634	-0.4274	3.6875	3363
15	7.4484	1.3393	-0.3851	3.5565	3339
20	7.4547	1.3177	-0.3494	3.4583	3319
30	7.4668	1.2795	-0.2905	3.3169	3286
40	7.4783	1.2460	-0.2403	3.2180	3259
50	7.4892	1.2160	-0.2000	3.1443	3235
60	7.4997	1.1887	-0.1635	3.0874	3215
70	7.5099	1.1635	-0.1304	3.0424	3197
80	7.5246	1.1289	-0.0859	2.9908[2]	3174
85	7.5198	1.1401	-0.1001	3.0063[2]	3181
90	7.5294	1.1182	-0.0723	2.9769	3167
100	7.5387	1.0975	-0.0463	2.9530	3153
110	7.5478	1.0781	-0.0221	2.9334	3141
120	7.5567	1.0596	0.0007[1]	2.9174	3130
130	7.5654	1.0421	0.0223	2.9044	3120

Table 4.3 Statistical parameters for increasing additive constants

1 - Not significantly different from 0 2 - Not significantly different from 3

From Table 4.3 it is apparent that there is no single additive constant for which the carbon leader distribution assumes a normal shape. At an additive constant of 85 the kurtosis approximates three and the estimate is 3 174 cm.g/t - an over estimate of 1.4 %. While at an additive constant of 120 the skewness approximates zero and the estimate is 3 130 - a negligible error.

The log probability plot of ln z + β (120) (Figure 4.6) still shows a skewed line and indicates that the conditions of normality of x still do not apply, the same is indicated by Table 4.3. Although with an additive constant of 120 cm.g/t a good estimate of the population mean is obtained, there remains a problem that after the back transformation from log to normal space negative values will be created. Using the formula as discussed by Dohm (1995) the percentage of negative values can be calculated.

$$\%\ \text{Negative Values} = \Phi\left[\frac{(\ln\beta - \xi_\beta)}{\sigma_\beta}\right] \times 100$$

Where Φ is the standard normal probability

For an additive constant of 120 the % of negative values = 0.48 %. The above factors all negate the use of any of the traditional lognormal models for modelling the Carbon Leader Reef gold values.

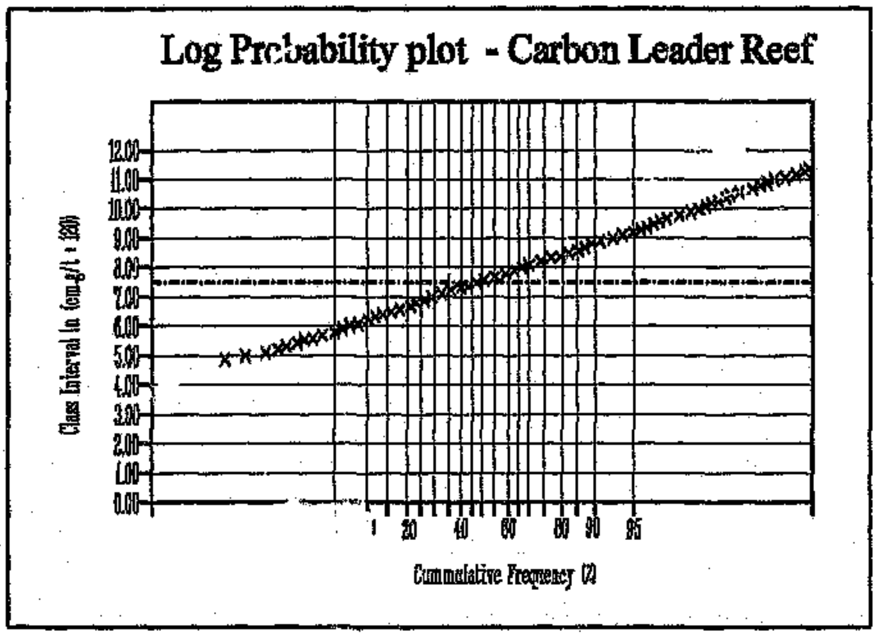


Figure 4.6 Log probability plot for ln(cm.g/t + 120)

3 The next step was to test which of the CLN or the LN-GIG distribution models will be more appropriate to fitting the Carbon Leader Reef gold distribution. The theoretical CLN test value for the distribution is discussed above and the calculated test value for this distribution is :

$$\text{Test value} = \tfrac{1}{2}(3(0.322624)+6) = 3.4839$$

The kurtosis of the distribution = 4.285

The most appropriate distribution model, of those currently available, for the high grade Carbon Leader Reef is therefore the Compound Lognormal Model. The three paran.. .r lognormal model may have given a reasonable estimate of the population mean but the negative values resulting from the back transformation will lead to inaccurate grade tonnage curve prediction.

4.4 Distribution Fitting

As the CLN distribution model has been shown to be the most appropriate model, the following step is to fit the distributio⌐. The process of distribution fitting was discussed in section 4.2 and the results of each step for the Carbou Leader Reef are summarised below.

Using the CLN parameters as shown in Table 4.4, the expected CLN estimate was calculated : 3 133 cm.g/t, which is a 0.06% over estimate. By calculating the expected frequency for the C‵ ˙ distribution a further test of the fit is possible. Figure 4.7 shows a comparative plot of the observed frequencies against the expected frequencies for the log transformed gold values.

q	0.7781
c	0.0712
Root c	- 0.8493
ν	2.8143
s	2.4742
a	8.2311

Table 4.4 CLN parameters for Carbon Leader Reef

This fit was statistically tested using a χ^2 - test. The test showed the fit not to be statistically significant. The...ore the three parameter lognormal (B=120) model was compared wit· the CLN model. The χ^2 test value for the CLN model is 1181 whereas for the 3 Pa.ameter Lognormal model the value is 68 048. The above shows the CLN model to be a better fit and for practical purposes the fit was accepted on this basis and that of the graphical comparison shown in Figure 4.7.

4.4 Distribution Fitting

As the CLN distribution model has been shown to be the most appropriate model, the following step is to fit the distribution. The process of distribution fitting was discussed in section 4.2 and the results of each step for the Carbon Leader Reef are summarised below.

Using the CLN parameters as shown in Table 4.4, the expected CLN estimate was calculated : 3 133 cm.g/t, which is a 0.06% over estimate. By calculating the expected frequency for the CLN distribution a further test of the fit is possible. Figure 4.7 shows a comparative plot of the observed frequencies against the expected frequencies for the log transformed gold values.

q	0.7781
c	0.0712
Root c	- 0.[illegible]493
v	2.8143
s	2.4742
a	8.2311

Table 4.4 CLN parameters for Carbon Leader Reef

This fit was statistically tested using a χ^2 - test. The test showed the fit not to be statistically significant. Therefore the three parameter lognormal (B=120) model was compared with the CLN model. The χ^2 test value for the CLN model is 1181 whereas for the 3 Parameter Lognormal model the value is 68 048. The above shows the CLN model to be a better fit and for practical purposes the fit was accepted on this basis and that of the graphical comparison shown in Figure 4.7.

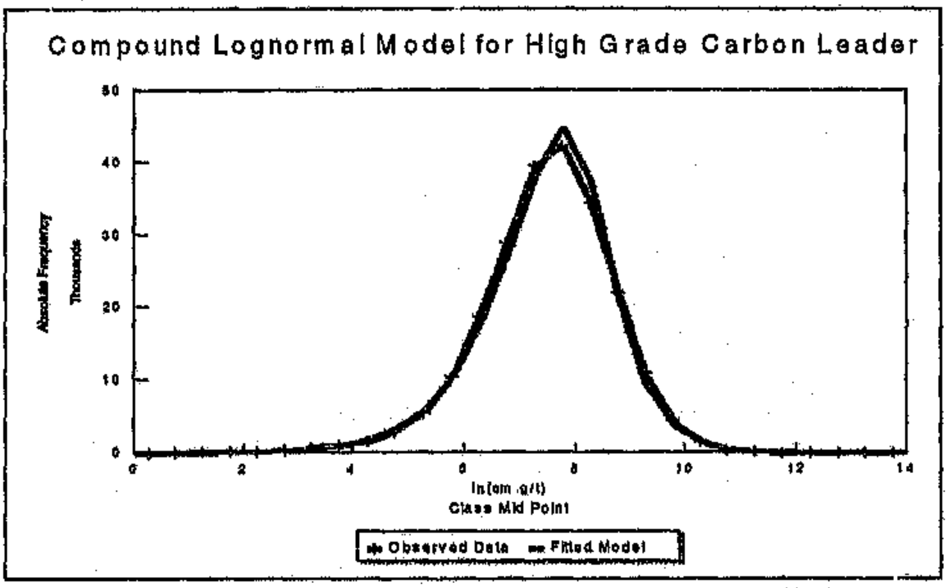


Figure 4.7 Comparison between observed frequencies and CLN Model - ln(cm.g/t)

The CLN model was then back transformed to real (cm.g/t) space and compared to the observed frequency distribution. Figure 4.8 shows a close correspondence between the expected and the observed frequencies. The CLN model as calculated was then accepted and use for all further estimation.

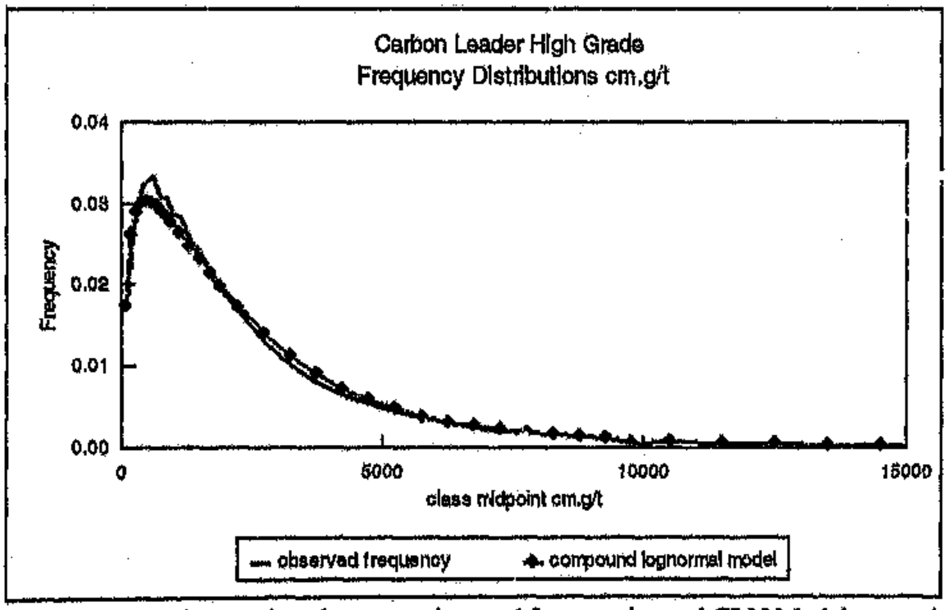


Figure 4.8 Comparison between observed frequencies and CLN Model - cm.g/t

4.5 Additional characteristics of the underground channel sampling

Two other important characteristics of the underground sampling need to be considered. These are the Variance Size of Area relationship (Krige 1951) and the log mean (ξ) versus log variance (σ^2) relationship (Dohm, 1995). Both these relationships need to be examined before any estimation can take place.

4.5.1 Variance Size of Area Relationship

The variance size of area was traditionally thought to represent a straight line relationship with no upper bound. However as the size of available data sets has increased it has become apparent that this relationship has as its vertical upper bound - the population variance (Dohm, 1995). As the variance is an important parameter in the CLN estimation process and the variance is related to the area over which it is calculated, this relationship must be accounted for. For the Carbon Leader Reef the Variance size of area relationship is tabulated in Table 4.5 and shown graphically in Figure 4.9

Block Side(m)	Area (m^2)	Avg Log Variance	No. Blocks
25	625	0.861232	5360
27.5	756.25	0.943318	1873
30	900	0.798485	1445
50	2500	0.971922	1651
75	5625	1.058218	1334
100	10000	1.177228	1101
150	22500	1.237687	725
200	40000	1.250095	519
250	62500	1.276179	403
300	90000	1.302953	315
400	160000	1.310541	214
500	250000	1.328273	150
750	562500	1.353592	79
1000	1000000	1.383995	50
1500	2250000	1.430183	25
2000	4000000	1.415778	17
5000	25000000	1.559939	5

Table 4.5 Tabulated Variance size of area relationship

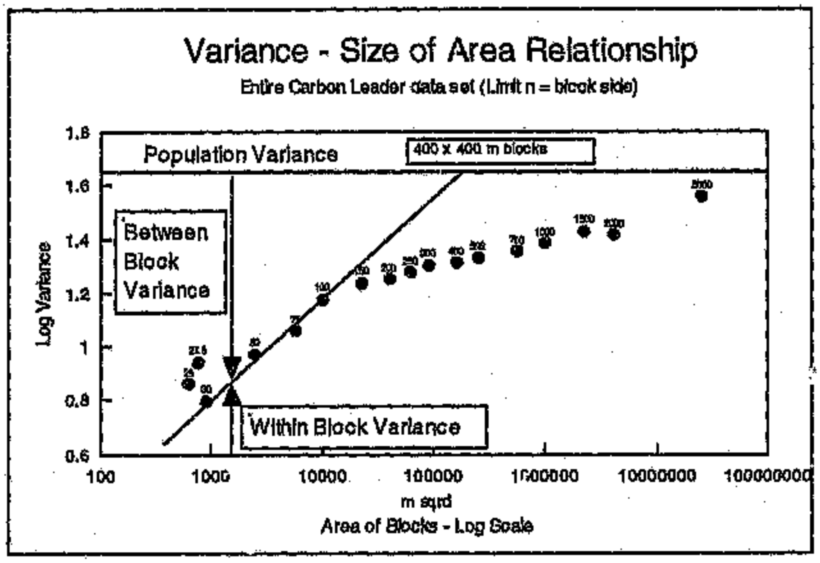


Figure 4.9 Variance size of area relationship - Entire Carbon Leader data set

For the Carbon Leader Reef the slope of the relationship appears to break at a block size of 150 x 150 m and if this slope is projected to the population variance the intercept is at a block size of 400 x 400 m. The influence of size over the estimation of variance, at this block size, was then assumed to have been minimised and this block size was used for all further analysis.

4.5.2 Log Mean - Log Variance Relationship

It was noted by Dohm (1995) that for the CLN distribution a linear relationship exists between the log Mean (ξ) and the log Variance (σ^2). This relationship is best defined for block sizes greater than or equal to the minimum block size as determined by the variance size of area relationship. For the Carbon Leader a block size of 400 x 400 m was used. The relationship for the High grade Carbon Leader is illustrated in Figure 4.10 and the regression summary is shown in Table 4.6.

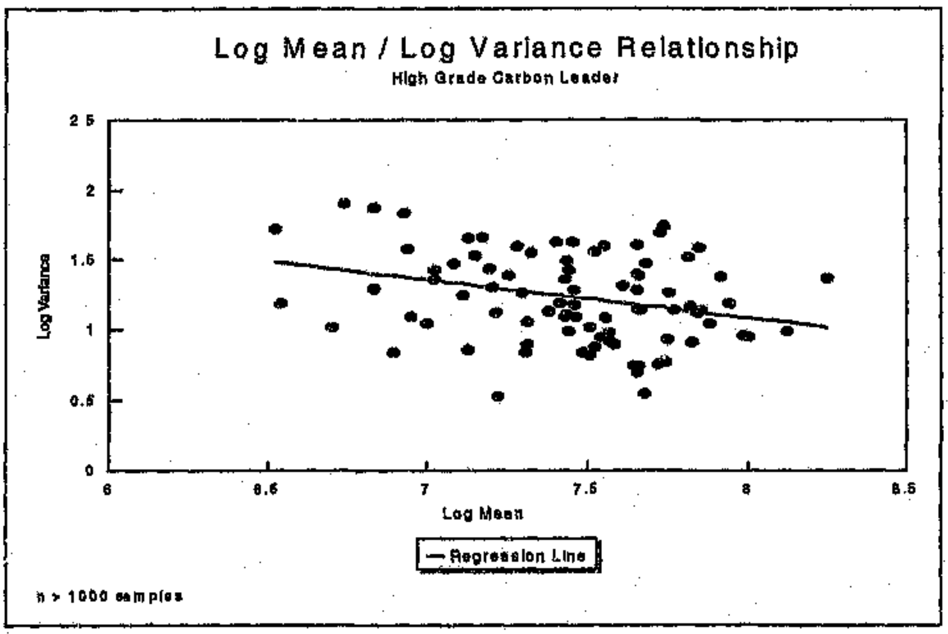


Figure 4.10 Log mean versus log variance relationship

Regression Summary	
Constant	3.2553
Std Err of Y Est	0.3393
R Squared	0.0743
No. of Observations	87
Degrees of Freedom	85
Correlation Coefficient	0.2726
X Coefficient(s)	-0.2711
Std Err of Coef.	0.1038

Table 4.6 Summarised relationship between log mean (ξ) and log variance (σ^2)

It is apparent from the above that there is a negative relationship between the log mean and the log variance. However the relationship is not clearly defined and there is only a 27 % correlation. The importance of the above is that if lognormal theory was utilised for estimation, a constant variance would be assumed, resulting in the under-estimation of low grade and over estimation of high grades (Table 4.7). This is further support for the application of CLN theory in estimation of the Carbon Leader Reef.

Parameters	Low Grade	High Grade
Log Mean	6.50	8.25
Log population varianc	1.43	1.43
Log variance ex regression	1.45	0.97
Estimates (cm.g/t)		
Lognormal estimate	1358	7817
Regression estimate	1376	6214
Lognormal vs regression estimates	**1 % Under**	**26 % Over**

Table 4.7 Over/under estimation resulting from lognormal assumptions

5 ANALYSIS OF BOREHOLE SAMPLING DATA

Borehole analysis consists of two phase:

1 Borehole cluster analysis

2 Borehole Intersection analysis.

These processes are discussed in detail below:

5.1 Borehole Intersection Analysis

Borehole intersection clusters from long deflections are compared to those cluster values obtained from the original holes using analysis of variance (ANOVA) on the natural log of the intersection values (O'Brien, 1996). The comparison enables a decision as to wether the gold values within the original cluster are sufficiently different from the goid values in the long deflection (of the same borehole) for the clusters to be considered as independent samples.

This process is required as long deflections seldom achieve a separation from the original borehole greater than the range of the point support semivariogram and hence cannot be considered as independent samples. The theory of distributional/ classical statistics rests on the independence of samples and hence clusters need to be tested for independence before these techniques can be used in estimation.

A standard ANOVA table of the form shown in Table 5.1 is used and a "F-test" determines at what level of probability the distributions are similar (Till, 1974). All the considered boreholes are not detailed but an example is shown for borehole UD33.

	d.f	Sum of Squares	Mean Square	Experimental F	Critical F (95 %)
Between Clusters	1	4.0187	4.0187	12.1843	5.3177
Within Clusters	8	2.6386	0.3298		
Total	10	6.6573	0.6657		

Table 5.1 ANOVA table for UD33

The intersection data for UD33 is tabulated in Table 5.2.

Borehole	Intersection	Cluster	Acceptability	Mean Grade
UD33	1	1	1	1692
UD33	2	1	0	1463
UD33	3	1	0	941
UD33	4	1	1	5023
UD33	5	2	1	369
UD33	6	2	1	893
UD33	7	2	1	1077
UD33	8	2	1	2518
UD33	9	2	1	6707
UD33	10	2	1	9117
UD33	11	2	1	1932

Table 5.2 Intersection data for UD33

The calculated ANOVA is shown in Table 5.1.

The critical F value at 95 % confidence limit for 1 and 8 degrees of freedom is 5.3177. The calculated value of F is 12.1843 which is greater than 5.3177 and the null hypothesis (H_0) is rejected. The original cluster and the long deflection cluster are therefore regarded as independent boreholes.

It is to be noted that the separation distance between the original cluster and the long deflection cluster can be used to modify the results of the ANOVA.

5.2 Intersection Analysis

Traditionally, any cored reef cut is classified as to its acceptability. The acceptability is determined by:

1 The mechanical acceptability. This is a measure of the completeness of the cut ie. are chips missing?, is the core ground within the reef zone?, has the core been fractured by drilling?

2 The geological acceptability. This is a measure of the geological completeness of the cut ie. is the intersection faulted or has the reef been sheared?

If a sample is classified as either mechanically or geologically unacceptable, then the entire intersection is classified as unacceptable.

The problem is how to decide if a particular intersection should be used in the calculation of a cluster mean or not. The Anglo American standard method of calculating a borehole average was as follows :

1 Determine the arithmetic average value (accumulation) of the acceptable

intersections.

2 Determine which of the unacceptable intersections has a value higher than the mean of the acceptable values.

3 Recalculation of the mean value, based on all the acceptable values and the unacceptable intersections with gold values greater the average of the acceptable values.

This technique is unscientific and is considered to be inappropriate for the following reasons :

1 The definition of an acceptable intersection is subjective and can vary with time and personnel.

2 The visual inspection of borehole core cannot determine whether gold has been lost or not. Especially, when considering that one is attempting to quantify a possible loss in something that is actually missing.

3 The inclusion of unacceptable values higher than the average of the acceptables can lead to a positively biased borehole average value and consequently to an over estimation of a resource.

The technique currently in use is summarised below. Using a technique adapted from Heyns (1958), within-cluster variances are calculated using the following formula :

$$\hat{\sigma}^2 = \frac{1}{2(r-1)} \sum_{i=1}^{r} \left[\ln R_{01i} - \left(\frac{1}{r} \sum_{i=1}^{r} \ln R_{01i} \right) \right]^2$$

R_{01i} - ratio of a pair of acceptable and non acceptable intersections

r - total number of pairs

As the distribution of gold values (accumulations) within Witwatersrand Reefs tend toward log normality, the ln Au values are used in this procedure as their distribution will approximate a normal distribution. The 90 % confidence limits are then calculated using normal probability tables and the calculated variance. These limits, the individual borehole log means and the intersection log values are then plotted so as to identify outliers. This process is carried out iteratively to continually improve the confidence limits by removing extreme non-acceptable outliers (Chamberlain and O'Brien, 1995).

No acceptable intersections are removed from the data set. The iterative process is stopped as soon as five percent of the acceptable values fall outside the confidence limits.

For the Carbon Leader Reef the following procedure was adopted:

1 The borehole intersection analysis, as described above was applied and extreme unacceptable and undefined outliers were excluded.

2 The distributions of natural logs of intersection values for each classification ie. acceptable, unacceptable and unknown were compared This second stage was carried out to see if there was a negative bias toward the unacceptable or undefined intersections.

It would be expected that the unacceptable intersections would be of lower grade than the acceptable intersections. This is because gold would have been expected to have been removed by the faulting or mechanical damage, which rendered the intersection unacceptable. It is important to note that is not the case for the

Carbon Leader Reef (Figure 5.1) where the acceptable and unacceptable intersection show a similar distribution with no indication of a negative bias. The unclassified intersections show a bimodal distribution but this may be attributed to the smaller data set of unclassified intersections.

The results of the intersection analysis are shown in Figure 5.2. Only five intersection values were excluded from cluster average calculations. These where intersections that had been classified as unacceptable and that lie outside the 90 % confidence limits, intersections classified as acceptable but outside the 90 %

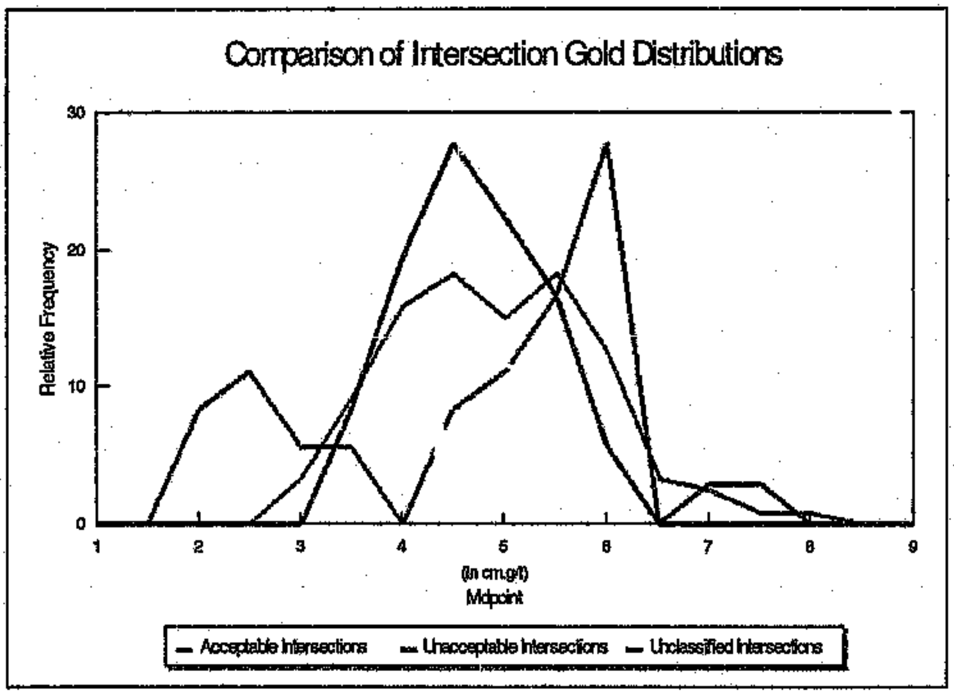


Figure 5.1 Comparison of ln(gold) distributions for acceptability classes

confidence limits were kept in the data set. If the traditional method had been used a total of 72 intersections would have been excluded.

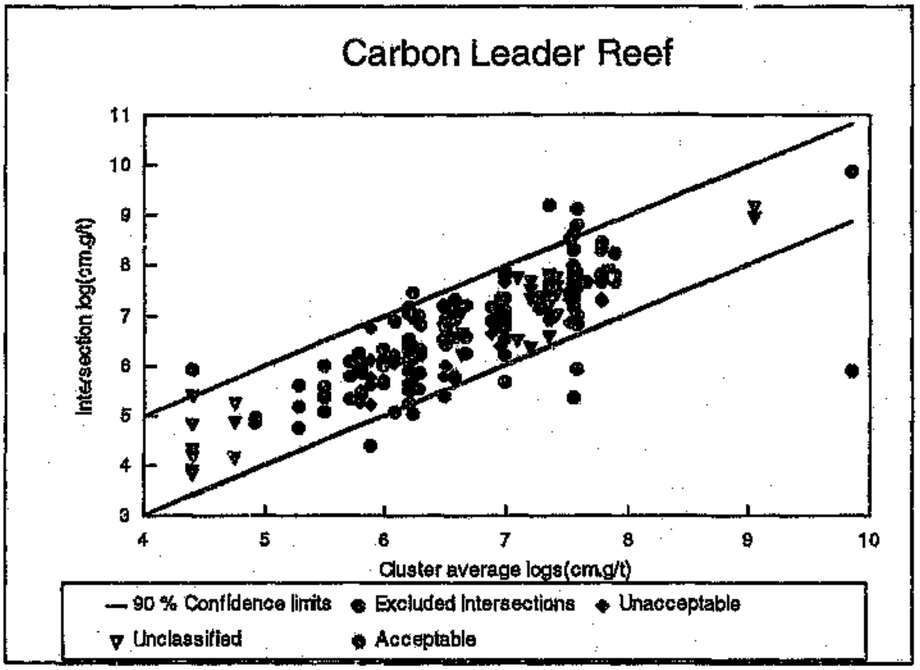


Figure 5.2 Intersection analysis

5.3 Calculation of cluster mean grades

The analysis of the borehole information having been completed, cluster averages could be calculated. These averages were calculated for statistically independent borehole clusters and only on statistically acceptable samples. For the cm.g/t average the mean was based on the arithmetic average of the intersection values. However, the problem of calculating the log mean remains. To examine this problem a borehole data set was simulated from the high grade Carbon Leader data set. A series of 'boreholes' consisting of five intersections were selected from the available underground channel sampling for each 400 x 400 m block. The selected samples were all taken, from a radius of 10 m, so as to mimic a true surface borehole. The change in support effect between underground channel samples and borehole cuts is negligible and can hence be ignored.

There are two possibilities for calculation of the log mean for a cluster :

1 Ln (arithmetic average of cm.g/t values of intersections)

2 Average of the log cm.g/t values for each intersection.

The relationship between the two methods was examined by means of a regression analysis (Figure 5.3) and the distributions were also compared (Figure 5.4). It is apparent from these plots that in all cases the average log values is less than the log of the cm.g/t average and that major discrepancies could exist depending on which calculation technique is used. The decision in the case of the Carbon Leader reef was based firstly on the estimates calculated from the borehole sets and secondly on the theoretical explanation. Lognormal estimates were calculated for both of the data sets and the results are summarised in Table 5.3

Data Set	Log (cm.g/t average)	Avg (log)
Log mean	7.7429	7.3739
Log var	0.7363	0.7709
lognormal estimate	3331 cm.g/t	2343 cm.g/t
Sample Mean	3127 cm.g/t	3127 cm.g/t
Bias	+ 6.5 %	- 25.1 %

Table 5.3 Lognormal estimates

From the above table it is apparent that the log(cm.g/t average) gives the best estimate of the average grade.

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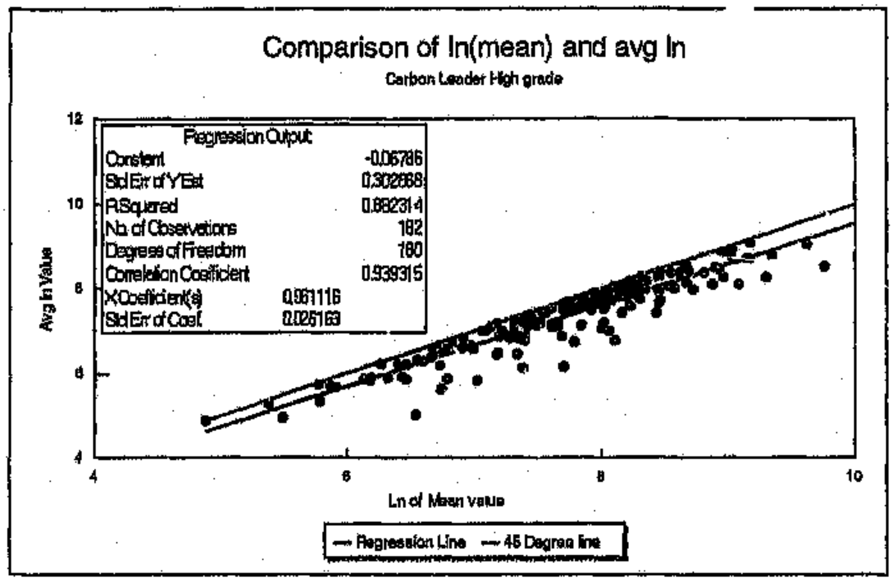

Figure 5.3 Regression analysis between ln(avg cm.g/t) and avg (ln cm.g/t)

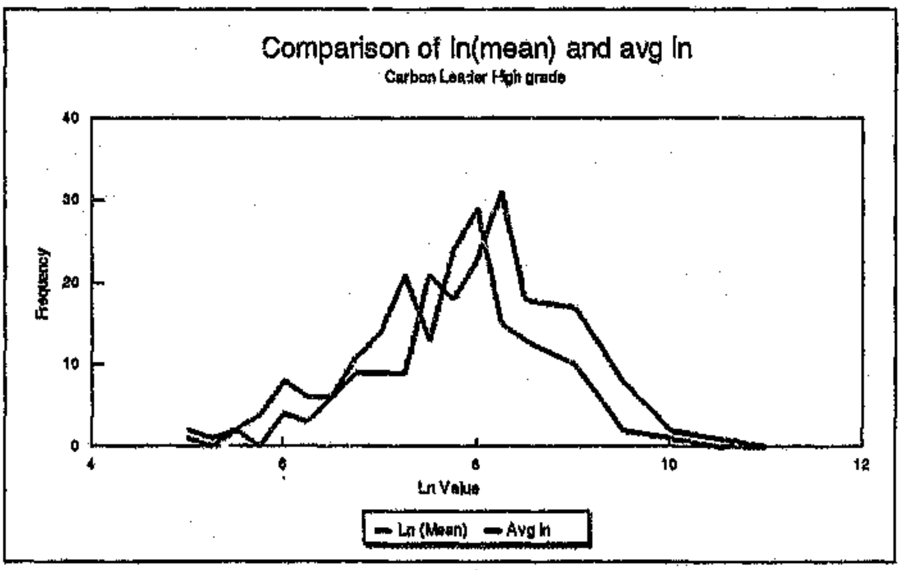

Figure 5.4 Comparative distributions for ln (avg cm.g/t) and avg (ln cm.g/t)

6 MACRO CO-KRIGING

Macro co-kriging is a kriging process where both block data and borehole data are included as input data. The block data is based on regularised underground channel samples and each block may represent many hundreds of samples, where as the borehole data represents a small number, rarely more than 10, clustered samples. These two data types can obviously not be given the same weighting in the kriging process. Hence the use of a mixed support type kriging implemented as a form of co-kriging. The change in support is accounted for by applying a differential nugget effect to blocks and the boreholes, thereby reducing the weight allocated to borehole samples.

In this project the block size used for estimation is 400 x 400 m, as determined by the Variance Size of Area relationship. To utilise the CLN model as fitted to the underground data, four parameters are necessary. Usually only the log mean is kriged, the log variance is determined by the log mean - log variance relationship and the skewness and kurtosis are held constant. This is an application of the Bayesian approach to estimation as proposed by Krige and Assibey-Bonsu, 1992. The underlying assumption is that the log mean and log variance of a deposit vary from locality to locality but the shape of the distribution, as defined by the log skewness and the log kurtosis, remains constant. As the log mean - log variance relationship is not clearly defined in the case of the Carbon Leader Reef the log variance was also estimated by kriging.

6.1 Determination of Kriging Parameters

6.1.1 Calculation of Block variograms

The block variograms were calculated for 400 x 400 m blocks. The size of the block was determined by the Variance Size of Area relationship. The log data was regularised into the determined block size and the resultant block data, both log mean and log variance, were used to calculate the block variograms. The variogram parameters are summarised in Table 6.1.

	Nugget	Sill 1	Range 1	Sill 2	Range 2
Log Mean ξ	0	0.05	775 m	0.06	2000 m
Log Variance σ^2	0	0.05	630 m	0.05	2000 m

Table 6.1 400 x 400 block variogram model

As expected the nugget effect was shown to be zero. The ranges and sill values for the log mean and the log variance are very similar, further indication that the two variables are related.

6.1.2 Calculation of Borehole variograms

Due to the sparsity of borehole data there is rarely sufficient data to calculate a borehole variogram. It is therefore necessary to simulate a borehole data set from which a variogram can be calculated. It was proposed by Krige and Assibey-Bonsu (1992) that five random samples be taken from each block to represent a borehole with five deflections. I propose however that the random sampling does not accurately represent a borehole and therefore selected five adjacent samples from the centre of each block. These samples were all taken with in a radius of 10 m so as to realistically mimic a typical borehole configuration. The change in support between a channel sample and a borehole intersection is not significant and can be ignored.

A mean accumulation (cm.g/t) was calculated for each borehole and these values were then log transformed, a log variance was also calculated for each borehole. The log transformed data was then used for the calculation of the 'borehole' variograms. The parameters of the 'borehole' variograms are summarised in Table 6.2.

	Nugget	Sill 1	Range 1
Log Mean	0.54	0.21	1394
Log Variance	0.18	0.11	572

Table 6.2 Borehole variogram model

The only parameter of importance in table 6.2 is the nugget effect which is used in the kriging process to reduce the weight allocated to the borehole samples. This is achieved by allocating weight to blocks according to a zero nugget effect and reducing the weight allocated to boreholes according to the measured nugget effect.

6.2 Kriging

The log mean and log variance were kriged using the macro kriging approach. A neighbourhood of 2 000 m was used with an 8 x 8 discretization. All borehole and block information was used (Figure 6.1) Block log mean and log variance estimates were calculated.

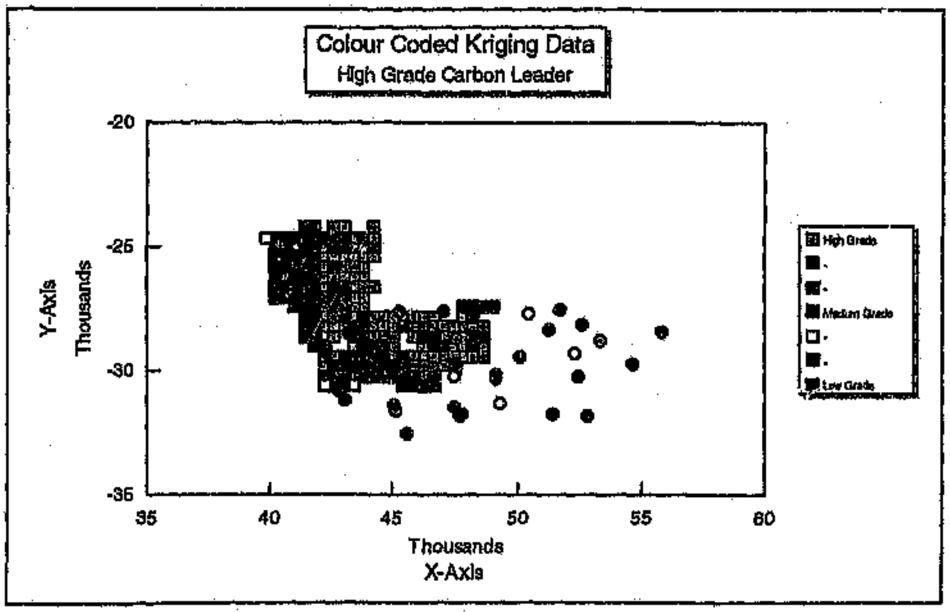

Figure 6.1 Location of block and borehole data used in Macro Kriging

6.3 Compound Lognormal Estimation

Compound Lognormal block estimates were calculated using the following information :

1 Kriged 400 x 400 m block log means

2 Kriged 400 x 400 m block variances

3 Skewness parameter (c) as determined by model fitting

4 Kurtosis parameter (v) as determined by model fitting

5 Kriging variances for both log mean and log variance kriging

The following were then calculated :

1 CLN block estimates based on the log mean, log variance, the skewness and kurtosis (Figure 6.2). The following formula was used :

$$\textit{Expected Value} = \left[\frac{(1-c)}{1-\left(\frac{1}{s}+\sqrt{c}\right)^2}\right]^{\left(\nu+\frac{1}{2}\right)} e^{a}$$

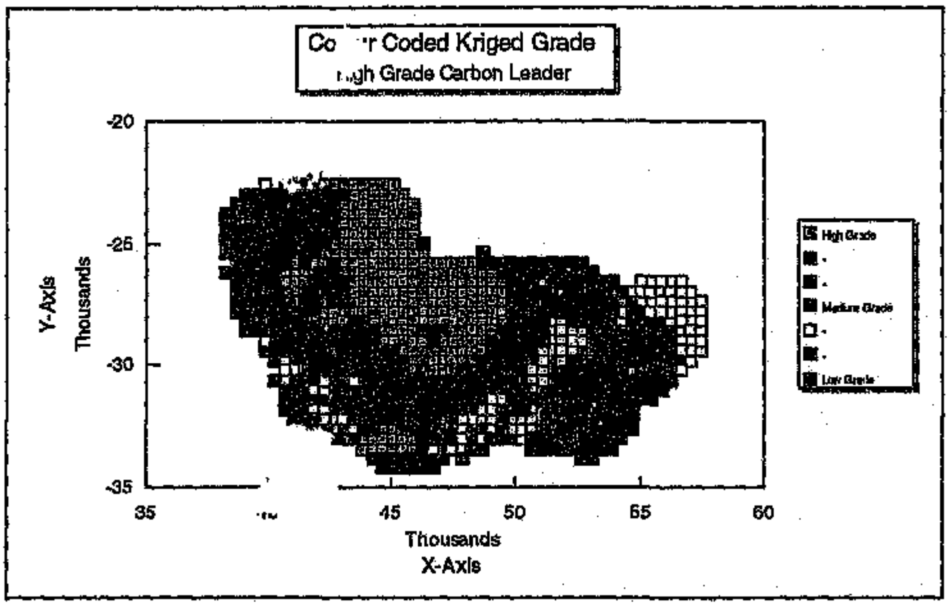

Figure 6.2 Colour coded kriging results

2 95 % confidence limits. The limits were calculated as follows :

$$\xi_{LL} = \xi_{est} - 1.96\,\sigma_k \quad \text{and} \quad \xi_{UL} = \xi_{est} + 1.96\,\sigma_k$$
$$\sigma^2_{LL} = \sigma^2_{est} - 1.96\,\sigma_k \quad \text{and} \quad \sigma^2_{UL} = \sigma^2_{est} + 1.96\,\sigma_k$$

For the purposes of this process the distribution of variances within 400 x 400 m blocks is assumed to approach normality and hence the calculation methodology used.

These values were then input into the CLN model to calculate the limits (Figure 6.3). The limits are expressed as a percentage of the mean estimate and show a general increase in uncertainty as the distance from the block data increases. The block data is shown in Figure 6.3 as blue blocks. The small islands of increased confidence are caused by the presence of borehole information.

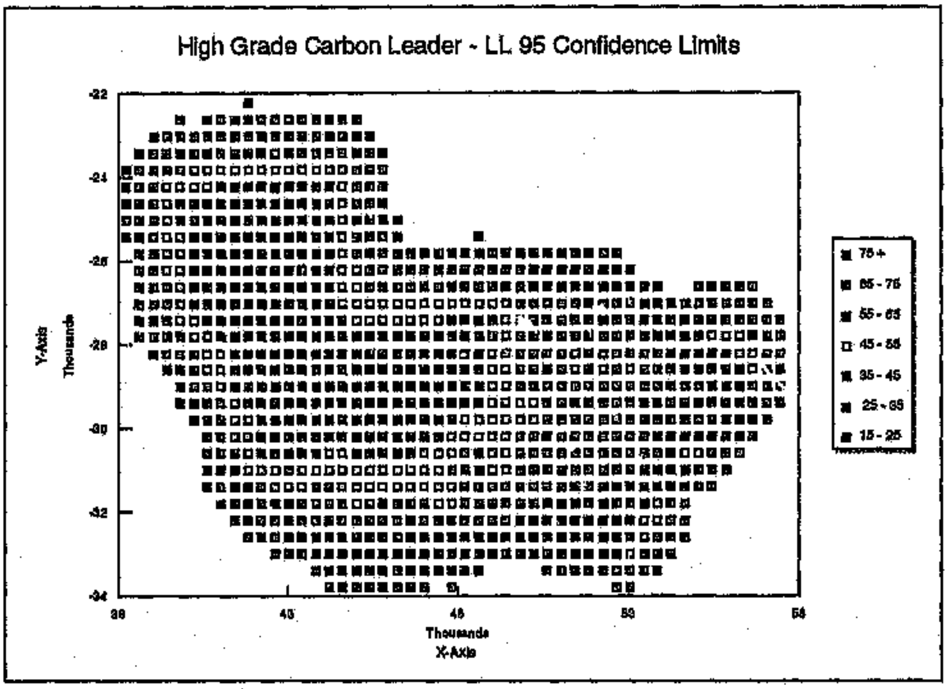

Figure 6.3 Colour coded lower 95% confidence limits expressed as percentage of kriged estimates

Concerns as to the use of the kriging variance in calculation of limits are commonly found in the literature. However, the methodology used above will be shown to provide reliable limit estimates for the Carbon Leader Reef, in Section 7.

7 VALIDATION OF COMPOUND LOGNORMAL MACRO KRIGED ESTIMATES

Numerous methods of validation were used to test the effectiveness of the Macro Co-kriging and these are all discussed below.

7.1 Block on block reconciliation

For each of the mined out blocks a CLN kriged estimate was calculated and these values were compared with the regularised block values (Figure 7.1).

It is apparent that a small regression effect is still present and this was corrected for by using the following relationship :

Corrected Estimate = Kriged Estimate * 0.819969 + 437.3437

The possibility of dip or strike based bias was also examined by considering dip and strike sections. An example is shown in figure 7.2 where a dip section shows no bias towards up/down dip sides of the deposit. Similar results were seen for all dip and strike sections examined.

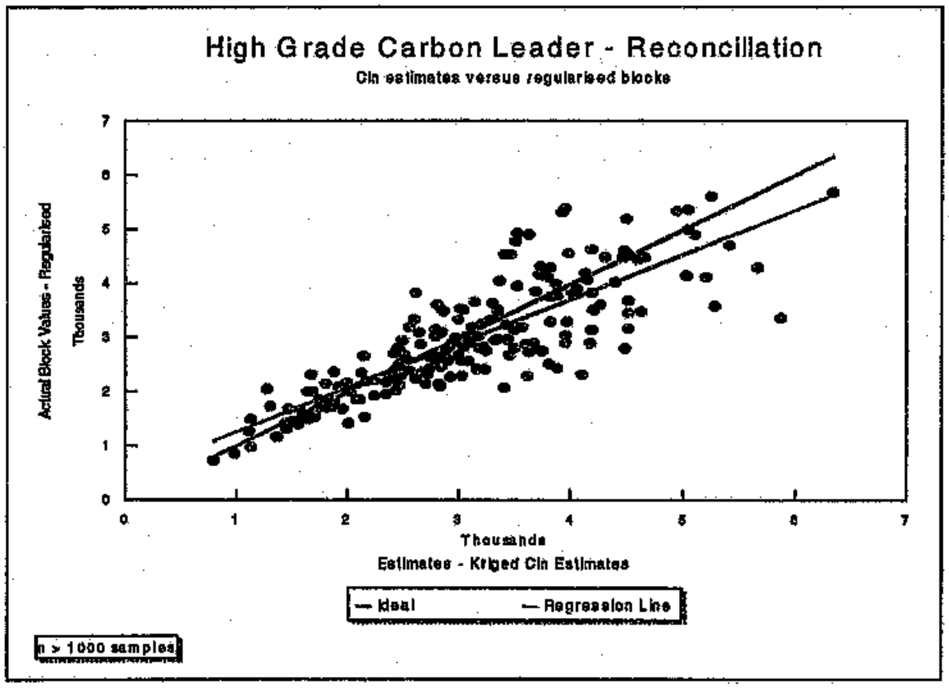

Figure 7.1 Block on block reconciliation

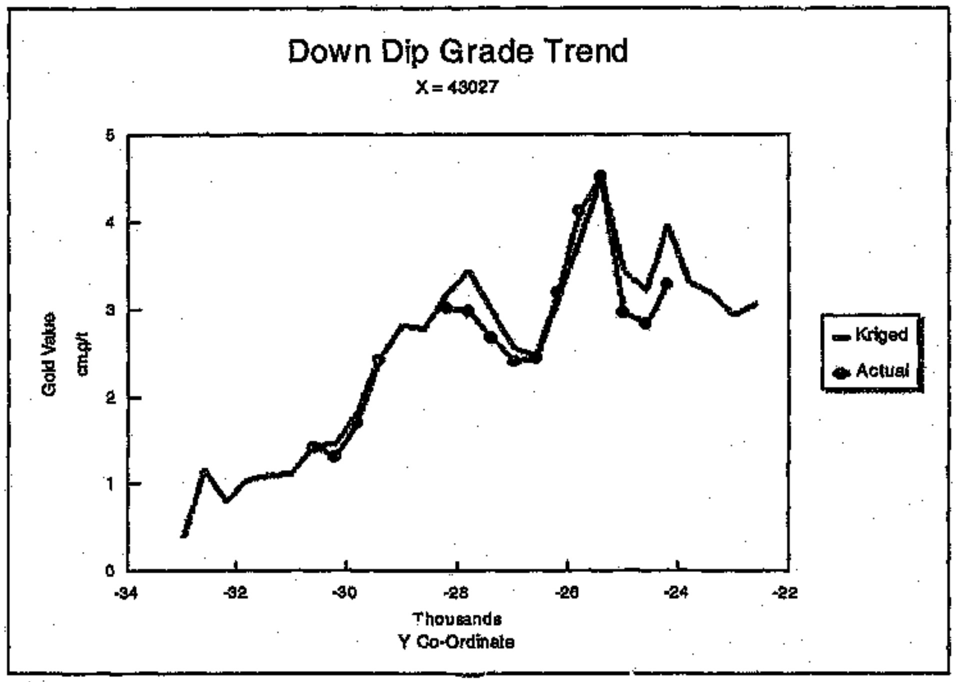

Figure 7.2 Dip grade section showing comparison between kriged estimates and actual block values

7.2 Grade Tonnage Reconciliation

A further test was to compare the grade tonnage curve as predicted by kriging with a theoretical grade tonnage curve based on the mean grade of the block estimates and the CLN model. The following procedure was followed in comparison.

1 The regression corrected block estimates were sorted and a grade tonnage curve was created for the kriged blocks.

2 The mean of the kriged blocks was used to create a CLN distribution of point values with the same mean grade.

3 The histogram was converted to a normal distribution via a Gaussian anamorphosis.

4 The grade tonnage relationship was calculated for the normalised distribution. The distribution was adjusted to take into account the change in support between points and blocks, by using a block variance of 1 084 935. This variance was determined from the point semivariogram and verified by examining the variance of the regularised underground block data.

The theoretical and kriged grade tonnage curves were compared (Table 7.1) and a plot of the relationship (Figure 7.3) shows that up to cut-off grades of 15 g/t the kriging provides accurate estimates. Above 15 g/t the regressed kriged results appear to be slight underestimates.

	Kriged Results		Theoretical Results	
Cut-off	% Tonnage	Increase in grade relative to mean grade (g/t)	% Tonnage	Increase in grade relative to mean grade (g/t)
0	100.00	0.00	100.00	0.00
2	100.00	0.00	100.00	0.00
4	100.00	0.00	99.86	0.03
6	98.68	0.23	99.12	0.15
8	97.15	0.46	97.04	0.48
10	93.18	1.04	93.08	1.06
12	83.60	2.42	87.10	1.90
14	75.25	3.71	79.37	2.98
16	64.97	5.47	70.46	4.27
18	57.84	6.81	61.07	5.72
20	52.44	7.84	51.81	7.32
22	46.13	9.07	43.16	9.04
24	41.24	10.06	35.43	10.85
26	34.93	11.38	28.74	12.73
28	27.29	13.26	23.10	14.68
30	23.32	14.38	18.46	16.67
32	19.45	15.47	14.69	18.71
34	15.27	16.79	11.66	20.78
36	10.59	18.56	9.25	22.88
38	6.82	20.64	7.35	25.00
40	4.79	22.35	5.84	27.15

Table 7.1 Tabulated comparison between kriged and theoretical grade tonnage curves

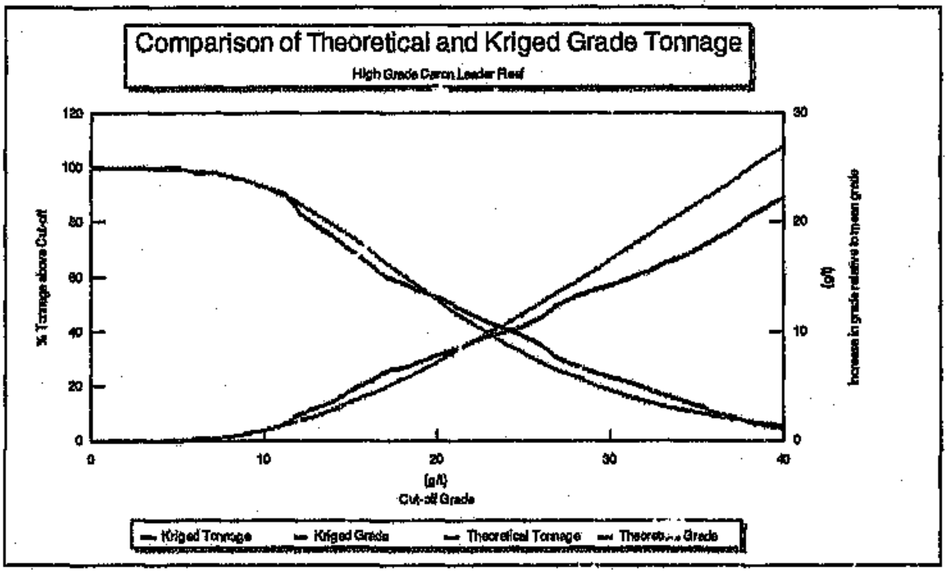

Figure 7.3 Comparative theoretical and kriged grade tonnage curves

7.3 Reconciliation of Kriged Estimates and block data excluded from Macro co-kriging

On completion of the project a unique opportunity presented itself. A large additional data set was received from an adjacent mine into which estimation had been done. All borehole information from the mine had been included in the estimation. This afforded the opportunity to verify the estimation technique as well the validity of the limit estimates.

When comparing the block estimates with the new regularised block data it is important to remember that not all blocks have the same confidence in the estimate, as was shown in Figure 6.3. The new data was therefore split, and colour coded, into zones according to their distance from the nearest data block as used in the Macro Kriging (Figure 7.4). A study of the adjacent mine has shown that a deep Carbon Leader Channel is developed and this area is also shown in Figure 7.4. It was expected that the estimation for this zone would be unsatisfactory due to the limited data available and the probable change in distributional model associated with a facios change.

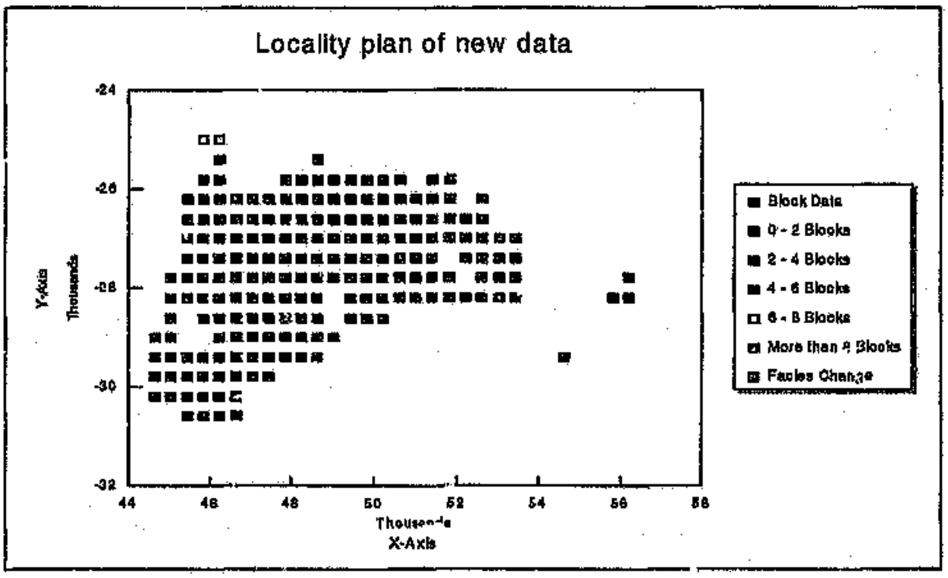

Figure 7.4 Locality plan of new data - colour coded according to distance from block data

The relationship between the macro co-kriged estimates and the regularised block values was then examined (Figure 7.5). The same colour coding as used in Figure 7.4 was applied.

The following can be observed in Figure 7.5 :

1 The estimation as a whole was successful.

2 The further away the estimated block lies from the block data used in the estimation, the less reliable the estimation process. However, this was predicted to occur via the limit calculations. The effectiveness of the estimation extends, at least, up to 5 blocks (2 000 m) from the mined out area.

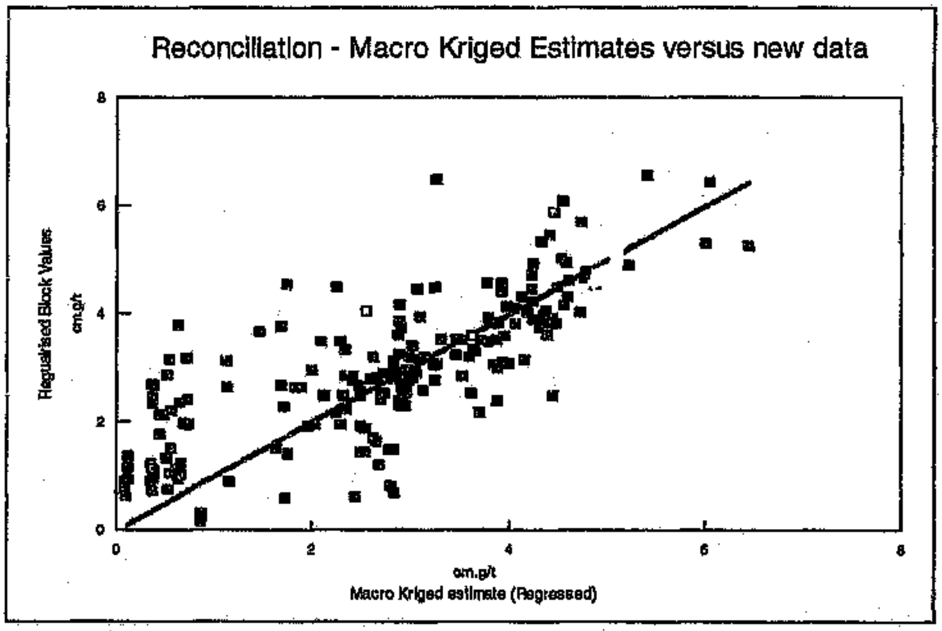

Figure 7.5 Reconciliation of macro kriged estimates with new block data

3 The Carbon Leader Channel was poorly estimated. This is an indication of the importance of accurate geological modelling . All facies changes must be taken account of in any estimation process.

4 Although the estimation becomes less acceptable beyond a 2 000 m range, no bias appears to be present. Therefore composited values for larger areas should still provide good estimates for large blocks of ground.

As far as the limit calculations are concerned, a plot of the blocks sorted on the basis of the Lower 95 % Limit shows the estimate to be acceptable (Figure 7.6). The regularised new block data points are colour coded using the same schema as Figure 7.4. It is evident from the plot that the vast majority of points that fall outside the limits are from the Carbon Leader Channel. If these points are ignored it is clear that the estimated limits are a good approximation of the real limits. A total of 9 blocks fall outside the calculated limits, being a total of 5.6 % of the blocks (excluding the Carbon Leader Channel data). This is as would be expected for 95 % confidence limits.

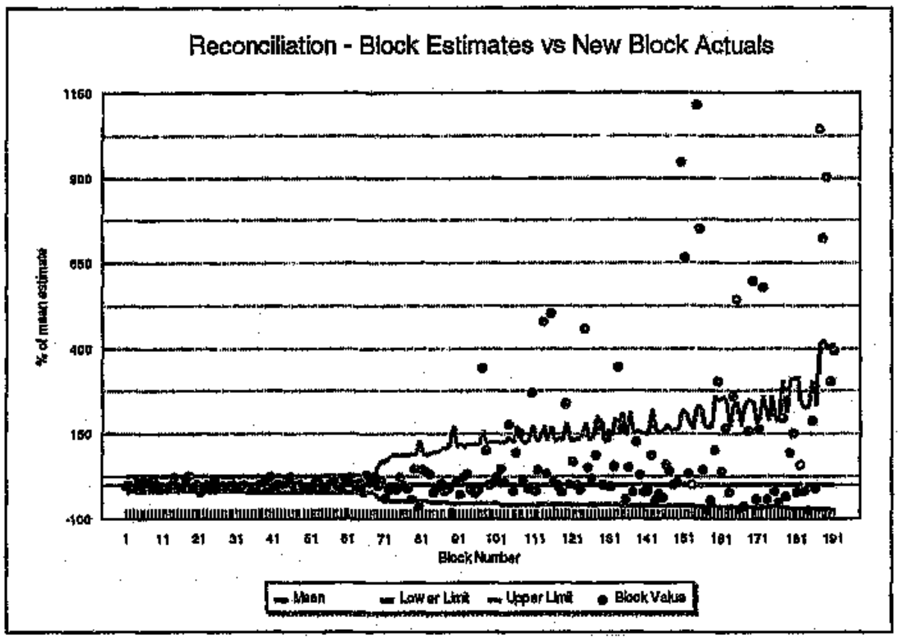

Figure 7.6 Reconciliation of kriged limit estimates with new block data

In conclusion it can be stated that methodology of limit estimation as applied to the Carbon Leader Reef Macro Kriging estimates can be regarded as acceptable. This conclusion only applies while in a geologically homogenous area.

7.4 Comparison with other techniques

For a subset of the area consisting of 96 data blocks a comparison of the CL based Macro Kriging was compared with Lognormal Macro Kriging. Three approaches were considered in the Lognormal case :

1 Macro Kriged log mean (ξ) and Macro Kriged log variance (σ^2)

2 Macro Kriged log mean (ξ) with an a Priori log variance based on the population variance

3 Macro Kriged log mean (ξ) with the log variance derived from the log

mean / log variance relationship.

A regression analysis was then conducted for each technique, between the estimates and the regularised block values. The results are tabulated in Table 7.2 and plotted in Figure 7.7.

Model	Technique	Reconciliation		
		Correlation Coef.	Regression Slope	Type
CLN	Fixed skewness and kurtosis	0.94	0.89	1
Lognormal	Kriged Variance	0.90	0.68	2
	A Priori Variance	0.83	0.42	3
	Variance ex relationship	0.83	1.04	4

Table 7.2 Summarised comparison between estimation techniques

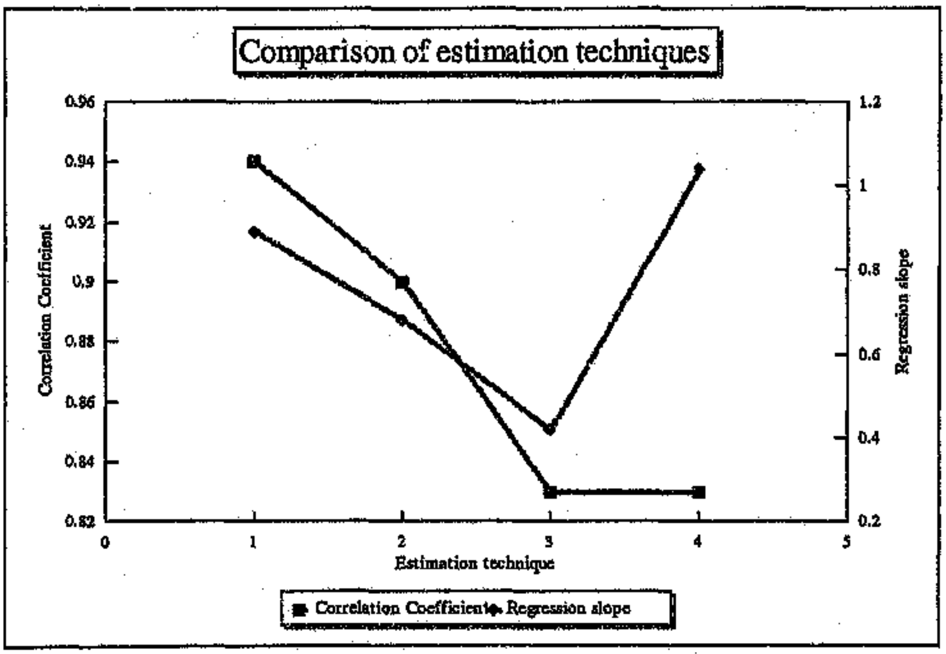

Figure 7.7 Comparison of regression parameters for estimation techniques - types numbered as per Table 7.2

From the above it is clear that the CLN based Macro Kriging provides the best estimates for the mined out areas. The only technique that approaches the efficiency of the CLN estimation is the Lognormal estimation based on the Log mean / Log variance relationship. This occurs due to the correction of the variance via the regression equation and is a hybrid approach as there should be no relationship if the distribution was lognormal.

8 MINERAL RESOURCE CATEGORISATION

The categorisation of the Mineral Resource for the Carbon Leader is based on the Anglo American standard which in turn is based on draft six of the proposed South African Mineral Resource Classification (Hammerbeck, 1996). A diagram of the classification scheme is shown in Figure 8.1 The Anglo American schema is modified relative to Figure 8.1 in that the Indicated Category has been subdivided into three categories. These are in increasing order of confidence : Indicated III, Indicated II and Indicated I.

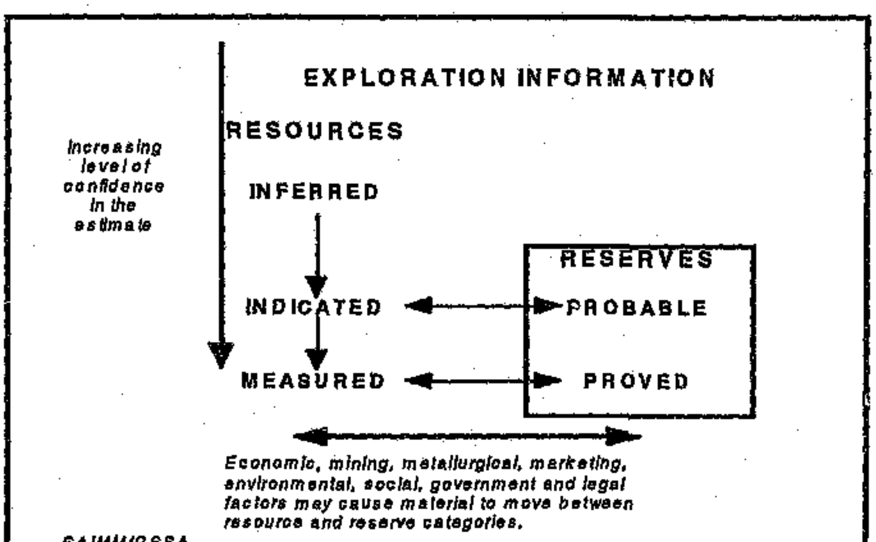

Figure 8.1 Proposed Mineral Resource classification schema

As has been shown in Section 7 the Macro Kriged limits appear to be accurate estimates of the real confidence limits and they can therefore be used to classify the resource. The benefit of applying such a schema is that planning and exploration can be driven by a quantifiable parameter and hence real comparison between mines is possible.

In the case of the Carbon Leader Reef the lower 95% limit is used to categorise the resource and the schema is shown in Table 8.1, after McWha and Chamberlain (1996).

RESOURCE CATEGORY	LOWER 95 % LIMIT AS PERCENTAGE OF MEAN ESTIMATE	TOP RATED PLANNING PROCESS
INFERRED	> 80 %	Exploration and Feasibility
INDICATED III	60 - 80 %	Shaft sinking
INDICATED II	40 - 60 %	Major capital, re-establishment
INDICATED I	20 - 40 %	Medium term development

Table 8.1 Mineral Resource classification criteria

The actual limit boundaries to be used are still being refined and the system is currently being tested on the Carbon Leader Reef.

9 CONCLUSIONS

By using a Bayesian estimation process and an appropriate distribution density model in long range grade forecasts for the Carbon Leader Reef, it is shown in this project report that reliable estimates can be derived. The reliability of the method as applied in this project is shown to be dependant on the following factors :

- Accurate geological modelling.
- The selection of the correct distribution density function in modelling sampling populations.
- A full understanding of the statistical and spatial parameters of underground sampling data.
- A statistically valid technique of treating borehole information.
- Application of macro co-kriging

The danger of estimating across geological boundaries is illustrated and the applicability of log normal theory to the Carbon Leader Reef gold grades is shown to be questionable.

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