The University of the Witwatersrand, Johannesburg

Doctoral Thesis

Coalition formation in a virtual buying cooperative: A case for formal grammars

Author:
Mpho Ivy Raborife

Supervisor:
Prof Sigrid Ewert and
Dr. Suna Bensch

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Declaration of Authorship

I, Mpho Ivy Raborife, declare that this thesis titled, "Coalition formation in a virtual buying cooperative: A case for formal grammars" and the work presented in it are my own. I confirm that:

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- Where I have consulted the published work of others, this is always clearly attributed.
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Abstract

Faculty of Science
School of Computer Science and Applied Mathematics

Doctor of Philosophy

Coalition formation in a virtual buying cooperative: A case for formal grammars

by Mpho Ivy Raborife

We report on a study that investigates the applicability of formal grammars in modelling coalition formation. This particular coalition formation is amongst a group of physically distributed enterprises intending to purchase items from a supplier as a single entity, termed a virtual buying cooperative (VBC). We investigate several grammars with regard to their appropriateness in modelling the interaction strategy amongst the enterprises during the formation of a VBC. A regular grammar, context-free grammars, a random permitting context grammar, random forbidding context grammars, and random context grammars are used to model the formation of a VBC in this study. The adequacy and limitations in modelling the formation of a VBC by these grammars is explored. The results demonstrate that random context grammars are adequate in modelling a VBC environment. In addition to generating the specified languages representing a formed coalition, the production rules of all the three random context grammars investigated in this study, at every derivation step, adhere to the interaction strategy of a VBC during its formation. The strategy excludes enterprises that have not been invited to join the coalition from participating in the coalition. Furthermore, if an enterprise has been invited to join the coalition by multiple enterprises, it can only accept one invitation. This study aims to bridge the gap between formal grammars and technological applications.

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Contents

Declaration of Authorship **i**

Abstract **ii**

Acknowledgements **iii**

Contents **iv**

List of Figures **vi**

Abbreviations **viii**

Preface **x**

1 Introduction **1**
- 1.1 Introduction 1
- 1.2 Research Motivation and Rationale 2
- 1.3 Research Aims 2
- 1.4 Research Methodology 4
- 1.5 Contribution of the Research Study 6
- 1.6 Structure of the Document 7

2 Literature Review **8**
- 2.1 Virtual Buying Cooperative 8
  - 2.1.1 Introduction 8
  - 2.1.2 Background 9
  - 2.1.3 Virtual Buying Cooperative 12
- 2.2 Grammars 14
  - 2.2.1 Introduction 14
  - 2.2.2 Application to Coalition Formation 16

3 Definitions and Preliminaries **18**

4 Research Basis **21**
- 4.1 Regular Grammar 21

4.2 Context-Free Grammars . . . 26
4.2.1 Basic Language . . . 26
4.2.2 Informative Language . . . 33
4.3 Conclusion . . . 39

5 Modelling a Basic VBC **41**
5.1 Introduction . . . 41
5.2 Random Forbidding Context Grammar . . . 43
5.3 Random Permitting Context Grammar . . . 53
5.4 Random Context Grammar . . . 62
5.5 Discussion . . . 73

6 Modelling a Structured VBC **76**
6.1 Introduction . . . 76
6.2 Random Forbidding Context Grammar . . . 78
6.3 Random Context Grammar . . . 90
6.4 Discussion . . . 104

7 Modelling a Detailed VBC **106**
7.1 Introduction . . . 106
7.2 Random Forbidding Context Grammar . . . 108
7.3 Random Context Grammar . . . 121
7.4 Discussion . . . 140

8 Conclusion **142**

References **148**

List of Figures

4.1 Rule to initiate coalition formation . . . 23
4.2 Rules for claiming an item, inviting another enterprise, or opting out . . . 24
4.3 Rules for claiming an item, inviting another enterprise, or opting out . . . 30
4.4 Rules for claiming an item, inviting another enterprise, or opting out . . . 36

5.1 rFcg generating L_{basic} . . . 44
5.2 Initiating coalition formation . . . 46
5.3 Enterprises performing their operations . . . 47
5.4 Enterprises claiming items . . . 47
5.5 Balancing the items claimed by the enterprises and the items made available to them . . . 48
5.6 Introducing the r's to the sentential form . . . 48
5.7 Introducing items to the sentential form . . . 48
5.8 Dissolving the coalition . . . 49
5.9 rPcg generating L_{basic} . . . 54
5.10 Production rules for enterprise m . . . 56
5.11 Initiating coalition formation . . . 57
5.12 Enterprises performing their operations . . . 57
5.13 Enterprises claiming items . . . 58
5.14 Balancing the items claimed by the enterprises and the items made available to them . . . 59
5.15 Introducing the r's to the sentential form . . . 59
5.16 Dissolving the coalition . . . 60
5.17 Rcg generating L_{basic} . . . 63
5.18 Initiating coalition formation . . . 66
5.22 Dissolution stage of a coalition . . . 66
5.19 Initiation stage of a coalition . . . 67
5.20 Operational stage of a coalition . . . 68
5.21 Operational stage of a coalition . . . 69

6.1 rFcg generating $L_{\text{structure}}$ . . . 80
6.2 Initiate coalition formation . . . 82
6.3 Enterprises performing their operations . . . 82
6.4 Enterprises claiming items . . . 83
6.5 Balancing the items claimed by the enterprises to the items made available to them . . . 84
6.6 Introducing the r's to the sentential form . . . 85
6.7 Introducing items to the sentential form . . . 85

6.8 Dissolving a coalition . . . 86
6.9 Rcg generating $L_{\text{structure}}$ . . . 92
6.10 Initiating a coalition . . . 94
6.11 Initiation stage of a coalition . . . 95
6.12 Operational stage of a coalition . . . 96
6.13 Operational stage of a coalition . . . 97
6.14 Dissolution stage of a coalition . . . 97
6.15 Restructuring the items claimed by the coalition . . . 98
6.16 Replacing the central marker . . . 98

7.1 rFcg generating L_{detailed} . . . 109
7.2 Cont: rFcg generating L_{detailed} . . . 110
7.3 Initiating a coalition . . . 113
7.4 Enterprises preparing to claim items, or opting out of the coalition . . . 114
7.5 Enterprises claiming items . . . 115
7.6 Dealing with items . . . 116
7.7 Introducing the r's to the sentential form . . . 116
7.8 Dissolving the coalition . . . 117
7.9 Rcg generating L_{detailed} . . . 123
7.10 Cont: Rcg generating L_{detailed} . . . 124
7.11 Initiating a coalition . . . 127
7.12 Initiation stage of a coalition . . . 128
7.13 Operational stage of a coalition . . . 129
7.14 Operational stage of a coalition . . . 130
7.15 Operational stage of a coalition . . . 130
7.16 Restructuring the items claimed by the coalition . . . 131
7.17 Rules for reordering the information about enterprises that invited other enterprises without claiming items . . . 132
7.18 Rules for reordering the information about enterprises that claimed items without inviting other enterprises . . . 133
7.19 Rules for reordering the information about enterprises that opted out of the coalition . . . 134
7.20 Rules for reordering the information about enterprises that could not claim items . . . 135
7.21 Rules for reordering the information about enterprises that were invited more than once . . . 136
7.22 Rule to introduce central marker . . . 136

Abbreviations

CFG	**C**ontext-**F**ree **G**rammar
CFL	**C**ontext-**F**ree **L**anguage
RG	**R**egular **G**rammar
RL	**R**egular **L**anguage
RCG	**R**andom **C**ontext **G**rammar
RCL	**R**andom **C**ontext **L**anguage
RPCG	**R**andom **P**ermitting **C**ontext **G**rammar
RPCL	**R**andom **P**ermitting **C**ontext **L**anguage
RFCG	**R**andom **F**orbidding **C**ontext **G**rammar
RFCL	**R**andom **F**orbidding **C**ontext **L**anguage
VBC	**V**irtual **B**uying **C**ooperative

I dedicate this work to my family and many friends. A special feeling of gratitude to my loving parents, Tshidiso and Madira Raborife for their support and encouragement throughout this journey.

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Preface

A paper based on this study has been presented at a conference.

- E. Ngassam and M. Raborife, “Virtual Buying Cooperative: a procurement model for improving the sustainability of very small retailers in emerging economies,” in Proceedings of *IST-Africa 2013 Conference*. Nairobi, Kenya, 2013.

Chapter 1

Introduction

1.1 Introduction

A coalition is an alliance amongst a group of entities joining forces in order to execute a task as a single larger entity in order to increase efficiency, with each individual entity pursuing its own interests. In our study we focus on a virtual buying cooperative (VBC). This is a single-level alliance amongst a group of physically distributed enterprises with a common interest in purchasing the same goods at negotiated pricing as agreed upon with the supplier [Ngassam and Raborife 2013]. A VBC is a single-level alliance since once the enterprises have made the purchase, the coalition disbands and another one can be formed.

A VBC is especially beneficial to very small enterprises (VSEs) where the owners usually work in isolation and are disconnected to economically strong regions, and markets. These enterprises are usually run by one owner with approximately 10 to 20 employees depending on the type of industry [Africa 1996]. Due to the size of the businesses, they do not buy large amounts of goods [Ngassam and Raborife 2013]. This hinders their ability to fully aggregate demand and negotiate discounted prices from their suppliers. A study by Hewitt [2009] further reveals that VSE owners prefer not to work in partnerships. This also hampers the process of a buying coalition amongst such enterprises.

In a VBC model, VSEs meet in a virtual marketplace, form a coalition as and when needed, and once a purchase has been made, the coalition disbands and another one can be formed [Ngassam and Raborife 2013]. This model lowers transactional costs since enterprises do not have to travel to place their orders. Since the group buys a larger amount than an individual enterprise, the group can negotiate favourable pricing. This enables buyers to leverage on group purchasing power. Ngassam and Raborife

[2013] highlight the socio-economic importance of this coalition, particularly for small business owners who are typically located far away from their supplier, and purchase small quantities of items frequently from their suppliers, increasing transactional costs. Tsvetovat *et al.* [2000] implemented a test bed that can be used for such a model. In addition, this study also presents a number of discount pricing models. In the next section, the motivation behind our research study is discussed.

1.2 Research Motivation and Rationale

In emerging economies such as the Republic of South Africa (RSA), VSEs are essential in driving economic growth, and creating employment [Ngassam and Raborife 2013]. Although these businesses are crucial to the economy of RSA, they are usually operated in informal environments. This environment is typically characterised by poor infrastructure, poor inventory management, bad working habits, and lack of direct access to markets. This leads to the exploitation of such enterprises by their suppliers [Merz *et al.* 2007; Merz 2010].

Current group purchasing tools such as Groupon are based on daily deals that are initiated by suppliers [Dholakia 2011; Edelman *et al.* 2014]. Each marketplace contains daily deals; goods/services on discount. Potential buyers are contacted typically via email; advertising the deals based on the buyers' preference. The daily deal discount is only available if a certain number of individuals sign up for the offer. The sale goes through only if a predetermined number of individuals sign up for the deal.

There are limitations on the quantity of goods each buyer can purchase. The discount is predetermined and remains static regardless of the quantity of goods to be sold. These models are based solely on group purchasing power, that is, the power lies in the number of participants. Such strategies cannot be relied upon to increase profit margins for the sellers. In addition, they cannot be used to purchase large amounts of products due to the limit on the number of goods that can be purchased. This excludes enterprises who would ideally purchase their stock through these group purchasing platforms.

1.3 Research Aims

A VBC consists of geographically distributed buyers and a seller interacting in a virtual marketplace in order to achieve a common goal. This goal is to facilitate the buying and selling processes. During the operation of a VBC, buyers appear to be a single entity, but in fact they are several autonomous entities. The formed single entity takes full responsibility for the entire value chain of its product, even though the task is carried out by many participants, and for that reason their cooperation must be harmonic.

In a VBC, forming a coalition involves an enterprise (termed the *initiator enterprise*) approaching the supplier with an intent to purchase items [Ngassam and Raborife 2013]. The supplier in turn replies with the overall available quantity of the requested items. The initiator enterprise then purchases items, and invites selected associates, who in turn invite their associates, etc., to join the coalition in order to purchase the items from the supplier. The total number of items purchased by members of the coalition cannot be more than the quantity made available to them by the supplier. In a coalition, only invited enterprises may purchase items and/or invite other enterprises. An enterprise has the following four options if it is invited to join a coalition:

1. Purchase a number of items, and invite other enterprises.
2. Purchase a number of items without inviting other enterprises.
3. Invite other enterprises without purchasing any items.
4. Neither purchase items, nor invite other enterprises.

This study aims to investigate suitable formal grammars that generate a language representing a formed coalition, and whose production rules (generating that language) model the interaction strategy amongst enterprises during the formation of a VBC as described in Ngassam and Raborife [2013]. The following are the conditions enterprises need to adhere to during the formation of a VBC.

- For each coalition, an enterprise may only participate once.
- Only invited enterprises can participate in a coalition.
- An enterprise may invite an unlimited number of its known associates.
- An enterprise may claim as many items as it requires provided that there are still items available.
- Collectively, members of a coalition cannot claim more items than were allocated to them by the supplier.

1.4 Research Methodology

A VBC is defined as a temporal group of enterprises with a common interest in purchasing the goods from a supplier [Ngassam and Raborife 2013]. The aim of this study is to model the interaction strategy employed by enterprises during the formation of a VBC, and adhere to the conditions of a VBC as specified in Section 1.3. This interaction strategy is modelled by production rules of a grammar that generates a language representing a formed coalition. At every derivation step, an applicable rule needs to ensure that the interaction strategies amongst enterprises during the formation of a VBC as specified in the previous section are adhered to. For instance, if an enterprise has already performed its operation and is invited to join the coalition again, the production rules that allow the enterprise to either opt out of the coalition, claim items, and / or invite other enterprises should not be able to apply. The only rule that should apply in this instance is the one that signals that this is a repeated invitation. In this research study, the production rules model the interaction strategy employed by enterprises during the formation of a VBC.

We modelled the coalition formation process of this model using formal grammars. We used five different types of grammars, namely, a regular grammar, context-free grammars, a random permitting context grammar, random forbidding context grammars, and random context grammars. The regular grammar (rg) presented in this study could only generate a language that has information about enterprises that have claimed items. In this language, the number of items that can be claimed by members of a coalition is not bounded as it is meant to be in a VBC. Once a bound is placed on the quantity, then an rg cannot generate the language. In such a case, we found that a context-free grammar (cfg) could. The cfgs explored in this study could only generate languages that provide the following information:

- Enterprises that have claimed items.
- Whether there is an enterprise that has claimed items, but did not invite other enterprises.
- Whether there is an enterprise that has invited other enterprises without claiming any items.
- Whether there is an enterprise that has opted out of the coalition.
- Total number of items that have been claimed by the enterprises in a coalition.
- Total number of items made available to the coalition.

The initiation strategy employed by the rg, and the cfgs used in our study only allowed an enterprise to invite one other enterprise. This is also the case with the random permitting context grammar (rPcg) used in this study. In the cfgs, and the rPcg an item is made available as it is claimed by the enterprise. This implies that at the beginning of the coalition formation process, the total number of items that can be claimed by members of coalition is not known. This interaction strategy is not in accordance to the specified interaction strategy employed by enterprises in a VBC environment.

We found that random forbidding context grammars (rFcgs), and random context grammars (rcgs) could generate the languages that present all the following information about a formed coalition. We continued to demonstrate that an rg, and a cfg could not generate such languages.

- Enterprises that have claimed items.
- Number of enterprises that have claimed items, but did not invite other enterprises.
- Number of enterprises that have invited other enterprises without claiming any items.
- Number of enterprises that have opted out of the coalition.
- Number of times enterprises have been invited to join the coalition more than once.
- Number of enterprises that could not join the coalition as there were no items left to claim.
- Number of items each enterprise in a coalition has claimed.
- Total number of items that have been claimed by the enterprises in a coalition.
- Total number of items made available to the coalition.

The rPcg, and the rFcgs generate the languages presenting the above-mentioned information about a formed coalition. However, their production rules do not model the interaction strategy amongst enterprise during the formation of a VBC as described by Ngassam and Raborife [2013]. The production rules of the rFcgs presented in this study modelled a coalition in which all enterprises are invited to join the coalition at the same time. These enterprises cannot invite their associates. In addition, an available item is generated as it is claimed by an enterprise, that is, the overall quantity of items that may be claimed by enterprise is not known at the start of the coalition. This is not a strategy employed by enterprises in a VBC during its formation as expressed in Ngassam and Raborife [2013]. The rcgs could generate the languages, and the production rules of these grammars modelled the interaction strategy amongst enterprises during the formation of a VBC as specified by Ngassam and Raborife [2013].

The production rules of the rcgs used in our study enabled the initiator enterprise to start the coalition formation process. Each invited enterprise could invite as many of its associates, but enterprises were only allowed to participate once per formed coalition, that is, the enterprise could only accept one invitation. The total number of items that could be claimed by members of a coalition was known before the coalition formation can begin, and enterprises could not claim more items than were made available to them. At any derivation step, until the rules that apply once all the enterprises have performed their operations, the rcgs reflected the following:

- Enterprises that have been invited.
- Enterprises that have not performed their operations yet.
- Enterprises that claimed items.
- Enterprises that have claimed items, but did not invite other enterprises.
- Enterprises that have opted out of a coalition.
- Enterprises that have invited other enterprises without claiming items.
- Enterprises that could not perform any operations due to lack of available items.
- The number of repeated invitations. In the language presented in Section 7, enterprises that have been invited to join the coalition more than once are explicitly represented in a word of the language.
- The number of items that may still be claimed by members of a coalition.

This study has demonstrated that random context grammars are adequate in modelling the coalition formation process in a VBC. In addition, the production rules in the random context grammars adhere to the conditions that govern the interaction amongst enterprises during the formation of a VBC, and model all four options available to an enterprise upon invitation.

1.5 Contribution of the Research Study

According to the author's knowledge this study constitutes the first attempt in using grammars, specifically random context grammars to model coalition formation for a specific technology in the group purchasing domain. Ngassam and Raborife [2013] provide the socio-economic need for a VBC model, whilst Tsvetovat *et al.* [2000] describes various economic models for coalitions in the group purchasing domain. Csuhaj-Varjú and Salomaa [1997] proposed a formal model for agents in a multi-agent system that collaborate with each other via a network and for the behaviour of agents and agent communities using a network for cooperation. This study also describes tools that enable the development of languages that support text processing via these networks, facilitating communication. However, it does not offer an application of how the proposed model works for a clearly defined system such as a VBC. Our study is a first step towards bridging the gap between random context grammars and real world applications. The implementation of the proposed random context grammars is suggested for future work.

1.6 Structure of the Document

The rest of the document is structured as follows:

- Chapter 2 provides the background literature.
- Chapter 3 presents the formal definitions of the concepts used in this study.
- Chapter 4 provides the basis of our research study. A regular grammar and two context-free grammars that model a language representing a formed coalition are presented in this chapter.
- Chapter 5 presents a simple formal language that describes a formed coalition. In the information about the coalition represented in this language, each enterprise that has been invited to join the coalition, has the information about its actions grouped together.
- Chapter 6 presents a structured formal language that describes a formed coalition. The information about all the enterprises that have claimed items is grouped together in the language presented in this chapter.
- Chapter 7 presents an informative formal language that describes a formed coalition. All the information about each enterprise that has been invited to join the coalition is explicitly represented in this language.
- Chapter 8 presents a summary of the major points raised in our study.

Chapter 2

Literature Review

2.1 Virtual Buying Cooperative

2.1.1 Introduction

Coalitions are temporary alliances among individuals or groups with a shared purpose [Gamson 1961]. They are task-oriented and are formed with a purpose in mind. Once that purpose no longer exists, the coalition dissolves [Horling and Lesser 2005]. They are most useful in situations where a single entity cannot perform a particular task, or the efficiency of the task is increased if more than one entity performs it. This is typically the case in multi-agent systems (MAS) where an agent would need the help of other agents in order to perform a task efficiently. These types of coalitions have been thoroughly investigated using game theory [Shenoy 1979; Peleg 1984; Rosenschein and Zlotkin 1994; Chalkiadakis *et al.* 2010].

We view a VBC as a distributed, multi-agent system. The agents represent the buyers and sellers and work in the best interest of the entities they represent. In a multi-agent distributed system, there are three major goals that need to be achieved:

1. Efficiency - effective communication protocols and task allocations amongst agents.
2. Consistency - the predictability of the system's behaviour and its ability to handle failure.
3. Robustness - fault tolerance.

In a VBC, we focus on the first two goals that are elaborated on as follows:

- Enterprises place their order sequentially. If an enterprise is in the process of placing its order, no other enterprise may place any order. That is, efficiency excludes two or more enterprises from placing orders at the same time. In addition, an enterprise can only participate if it is invited to join the coalition.

- In a VBC, the initiator enterprise always initiates the formation of a coalition. The supplier cannot initiate the formation of a coalition. In addition, the formation of a coalition can only occur if there are items to be purchased. If there are no items available, the coalition dissolves. If an enterprise is invited more than once to join a coalition, it can only accept one invitation. Consistency ensures that for every formed coalition, an enterprise can only participate once, and members of a coalition cannot claim more items than were initially made available to them. Furthermore, in our model, all enterprises have the same rule templates. If invited, an enterprise can only select an option from a limited set of rules. It cannot respond in a manner that is not consistent with other enterprises.

A coalition may be a single-level, or a multi-level alliance. In a multi-level coalition, agents form a coalition, and then, coalitions form coalitions, such as in Muller *et al.* [2006]; Haque *et al.* [2010 2013]; Lau and Zhang [2004] to name a few. In a single-level alliance, agents form a coalition, perform a task and disband after the task is completed. A case in point is in Beer and Appelrath [2013] where agents form dynamic coalitions for the supply and demand of power products in electricity markets. In our study, the coalition is a single-level alliance as explained in the next paragraph.

In a VBC, the coalition formation process involves temporarily grouping independent enterprises whose sole mandate is to purchase items from a supplier as a single entity. These enterprises meet at a virtual marketplace, and form coalitions as and when needed, based on the items they are interested in. The enterprises pool their buying power, and negotiate a favourable pricing based on the number of items that they will purchase. Once they have made the purchase, the coalition is disbanded and another coalition can be formed.

2.1.2 Background

The VBC model is proposed with the aim of assisting small enterprises in emerging economies to access markets and trading partners. In addition, it aims to reduce supply risks which are the result of small enterprises not having access to a wide range of trusted suppliers leading them to purchase products from suppliers that may supply products at high prices. These enterprises are typically not located in close proximity to their suppliers. Furthermore, due to the size of small enterprises, products are usually purchased frequently giving rise to high transactional costs. These are some of the factors that hamper the success, and potential growth of these enterprises which are essential to the economic growth of developing countries.

Virtual buying cooperatives are a form of collaborative networked enterprises (CNE's). A CNE is a network of enterprises collaborating to achieve a common goal such as sharing specific tools which might be relevant to their individual organisations [Saetta *et al.* 2012]. In these networks, enterprises are typically geographically distributed and their interactions are usually supported by computerized means [Saetta *et al.* 2012]. Virtual enterprises are also a form of CNE's whose variant are VBC's. Virtual enterprises are a temporary network of independent enterprises linked through computer networks with a goal of exploiting an apparent market opportunity [Kasper-Fuehrer and Ashkanasy 2001]. Advantages of such a network include the following:

- Access to competitive markets.
- Collaboration amongst independent enterprises.
- Shared costs and resources.
- Reduction in transactional costs.

In a virtual enterprise, the relationship amongst the connected individual enterprises is determined by a common need [Migliarese and Corvello 2006]. In a VBC, this common need is solely to purchase items as a single entity. According to Migliarese and Corvello [2006], an important aspect of a virtual enterprise is how the individual enterprises link up with one another. This is typically how the connections are formed [Mintzberg 1983];

- Business opportunity arises in a market. In a VBC, this opportunity arises when the initiator enterprise approaches the supplier with the intent to purchase items.
- Competencies needed to exploit the opportunity are identified. In a VBC, the competency is the overall quantity of the requested items available from the supplier that the initiator enterprise can purchase.
- Potential partners to form the connection are identified, and then integrate to form the virtual enterprise. In a VBC, this phase involves the initiator enterprise purchasing items and inviting selected associates, who in turn invite their associates, etc., to join the coalition in order to purchase the items from a supplier.

- Once the opportunity has been seized and exploited, the connection seizes to exist. This is involves the dissolution of a VBC, that is, when all invited enterprises have performed their operations. In a VE, if the opportunity is long-term, the enterprise then transforms into a stable form of organisation. In a VBC, the opportunity is short-term since the target market intended for this model (VSEs) is not comfortable working in partnerships [Hewitt 2009]. Once a sale is made, the coalition disbands.

In order for collaboration in a VBC to be harmonic, and to successfully achieve its goals, there must be optimal cooperation, coordination and communication amongst its member enterprises. In a VE, this is usually hampered by complex communication channels, issues with trust, business opportunity identification, procedures to set up the virtual enterprise and partner selection.

In a VE, interaction is by computerized means such as audio/video conferencing, email and file sharing. Such computerized means are viable for established businesses with access to computing devices such as desktops as well as the skills to use such devices. However, for a VSE in a remote area (such as the rural areas in South Africa), where business owners typically have no access to a desktop/laptop and have limited computing skills (if any), this is not a viable solution.

At a glance, our proposed model possesses similar characteristics as other group purchasing platforms (GPPs) such as Groupon; on which the technological model for group purchasing platforms is based on. Group purchasing platforms are electronic commerce websites offering group deals to consumers. A VBC model is an e-commerce virtual marketplace comprising of virtual agents representing buyers and sellers facilitating temporal coalition purchasing [Ngassam and Raborife 2013].

In a GPP, suppliers offer discounted coupons on their products, and individual buyers are approached depending on their preferences [Edelman *et al.* 2014]. The coupon can only be purchased if a minimum number of buyers have bid on the product at the end of the bidding period. Interested subscribers express intention via the website; once a certain number of people sign up for the offer, the deal becomes available to all. If a predetermined minimum is not met, no one gets the deal that day. However, there is a predetermined number on the quantity of goods that can be purchased on each marketplace [Edelman *et al.* 2014].

The model for Groupon is not appropriate for VSEs since it is supplier-driven: the supplier initiates the formation, and is in charge of the entire value chain. If a VSE is to use this GPP to purchase items, it may purchase unnecessary items in order to compensate when the supplier does not put them on discounted pricing. In turn, this will lead to profit loss associated with overstocking. In addition, since this model is supplier-driven, a supplier might put discounts on products that are not relevant to the VSE's business. This would not be of any use to the VSEs.

Another disadvantage of the Groupon model for VSEs is that a predetermined number of participants need to sign up in order for the goods to be available for sale, if not; the goods are not available for sale. Since VSEs require stock for their business operations, this model might lead to a situation in which VSEs do not have goods when coalitions are not formed in time and with the predetermined minimum number of participants. In a VBC, each marketplace represents a supplier and multiple buyers (VSEs). Our model is buyer-driven to protect VSEs from exploitation and purchasing stock that they do not need. A buyer expresses his/her intent to buy and the seller responds with discount given based on quantity of goods bought. Potential partners are approached and a coalition is formed. Goods can be purchased, irrespective of the number of interested buyers.

VSEs are a classification of Small, Micro and Medium-sized Enterprises (SMMEs) in South Africa. SMMEs include a variety of businesses, ranging from established traditional family businesses employing over a hundred people, down to the survivalist informal enterprises with a single owner and no employees. Although VSEs are viable formal small businesses, their success and sustainability is usually hampered by their inventory control mechanisms [Ngassam and Raborife 2013]. According to Ngassam and Raborife [2013], it has been revealed that VSEs use about three hours per week on purchasing goods, and lose a large amount of money on supply risks. Furthermore, these enterprises may overstock in order to compensate for supply risks that might happen. This also results in a loss of money caused by VSEs purchasing unnecessarily excessively large amounts of stock for their business.

The aim of a VBC is to provide VSEs with direct access to the market and trading partners, reducing supply risks, and risks associated with inventory control. In this study, we provide a reference framework that details the interaction strategy amongst enterprises during the formation of a VBC. Future applications of a VBC may be informed, and developed based on this framework.

2.1.3 Virtual Buying Cooperative

Recall that a virtual buying cooperative is a temporary, single-level alliance amongst a group of independent enterprises with a common need to purchase items from a supplier as a single entity; thus improving their ability to negotiate favourable pricing [Ngassam and Raborife 2013]. Tsvetovat *et al.* [2000] presents an economic model that can be generalised to a VBC coalition, which includes a pricing model, a discount model, and an implementation test-bed for such a model. In Tsvetovat *et al.* [2000], a marketplace consist of multiple buyers and multiple sellers. During the bidding period, the multiple buyers act as a single entity, whilst the sellers act for their own interest with the aim of winning the bid. In contrast, in a VBC, a marketplace consists of multiple buyers and a single seller. Tsvetovat *et al.* [2000] focus on the economic viability and incentive of forming a coalition such as a VBC. Our study explores the communication protocols that may be employed by enterprises in a VBC using formal grammars.

In a VBC, the supplier predetermines the discount to be allocated to the members of a coalition if they can purchase the total quantity of the products allocated to them [Ngassam and Raborife 2013]. If members of the coalition purchase less than the pre-allocated quantity from the supplier, the negotiation process is initiated in order to allocate the discount accordingly. In our study, we do not investigate this weighted discount model.

Generally, all coalition models have the following phases [Tsvetovat *et al.* 2000; Ngassam and Raborife 2013].

- Negotiation - In a coalition, the leader arranges with one or more suppliers to provide the goods or service. In a virtual buying cooperative, the initiator enterprise approaches a supplier to provide goods and starts the formation process.
- Coalition Formation - The coalition leader approaches its associates to join the coalition. In a virtual buying cooperative, not only does the initiator enterprise have the power to invite its associates, its associates may also invite their associates, etc.
- Coalition Stability - In this phase, designers of a coalition model need to specify if members of a coalition are allowed to leave during operation, and the consequences of leaving a coalition during formation. In a VBC, an enterprise can decide to opt out upon invitation to join the coalition. An enterprise, however, cannot decide to withdraw from the coalition once it has started to participate in it.
- Distribution of Gain - In this phase, one specifies, how, if there is any, difference between retail and wholesale prices of a good distributed to the members of the coalition. In a VBC, maximum gain is achieved if enterprises take as many items, within the quantity provided by the supplier, as possible.
- Distribution of Costs and Utility - This concerns the bearer of distribution and logistics costs. There is no viable economic study of a VBC model, therefore one can assume that the cost would be borne by the supplier since the target market

for this model is VSEs. That is, the supplier would be in charge of dropping off the goods, either at a central location where all enterprises can pick them up, or at the location of individual enterprises.

- Distribution of Risk - This concerns the bearer of the financial risks as the transaction is executed. There are no viable economic studies that have been conducted on this model, however, with the rise of mobile banking in emerging markets, to reduce risks, once the coalition is formed, each enterprise can use mobile banking to pay the supplier for their required goods.
- Trust and Certification - This concerns trust in three stages: negotiation stage, payment collection, and in the distribution stage.

Based on the above-mentioned stages of coalition formation, we have reduced the phases of a VBC formation to three phases.

- Creation - involves the initiator enterprise approaching the supplier with the intent to purchase items. In turn the supplier replies with the overall available quantity of the requested items.
- Operation - involves invited enterprises purchasing items with/without inviting other enterprises, inviting other enterprises with/without purchasing items, or opting out of the coalition.
- Dissolution - marks the end of a VBC. This can be brought about by a successful transaction being made.
- Post-dissolution - involves the grouping of the information about the formed coalition according to the specifications of the supplier.

In the next section, the applicability of formal grammars to modelling coalition formation in a VBC is presented.

2.2 Grammars

2.2.1 Introduction

Chomsky [1959] proposed a hierarchy that categorizes formal languages into classes with increasing expressive power[1], i.e. each successive class can generate a larger set

[1]Expressive power refers to the capacity of a language to represent concepts.

of formal languages than the one before. The categories of languages with increasing complexity are regular languages, context-free languages, context-sensitive languages, and recursively-enumerable languages. These languages are generated by grammars of their respective type. There are also grammars with regulated rewriting, in which the rules are context-free but the application of the rules is not [Dassow and Păun 1989]. In this study we use, where appropriate, regular grammars, context-free grammars, and random context grammars to model the formation of a VBC.

Random context grammars (rcgs) [van der Walt 1972] belong to the class of context-free grammars with regulated rewriting [Dassow and Păun 1989], i.e., the productions of a grammar are context-free, but are applied in a non-context-free manner.

In the case of rcgs, the application of a production at any step in a derivation depends on the set of symbols that appear in the sentential form of the derivation at that step. As opposed to context-sensitive grammars, the context may be distributed in a random manner in the sentential form. Context is classified as either permitting or forbidding: permitting context enables the application of a production, while forbidding context inhibits it. When a grammar uses permitting context only or forbidding context only, it is called a random permitting context grammar (rPcg) or random forbidding context grammar (rFcg), respectively. The corresponding languages are called random permitting context languages (rPcls) and random forbidding context languages (rFcls).

Dassow and Păun [1989] showed that rcgs without erasing productions lie strictly between the context-free and context-sensitive grammars. When erasing productions are allowed, rcgs are as powerful as the recursively-enumerable grammars. It is not known if rFcgs without erasing productions rules have an erasing equivalent. This implies that we do not know if they generate the same language class, or if they are equivalent to other grammars such as rcgs. In our study, we use rFcgs with erasing rules. However, every rPcg with erasing production rules has a non-erasing equivalent [Zetzsche 2010].

A context-free grammar (cfg) is an rcg where no context is used. A regular grammar (rg) is a cfg with either left- or right-linear production rules. Left-linear refers to an instance where the non-terminal symbol on the right-hand side of a production rule is at the left end. Right-linear refers to an instance where the non-terminal symbol on the right-hand side of a production rule is at the right end. Context-free grammars generate context-free languages (cfls), and regular grammars generate regular languages (rls). Regular languages are a strict subclass of context-free languages. This implies that for every regular language, there exists a context-free grammar that can generate it.

2.2.2 Application to Coalition Formation

We investigate the extent to which each of the classes of formal grammars described in the previous section can model coalition formation in a VBC. In our study, this investigation begins with a formal language that represents a formed coalition. This language presents information about a formed coalition, such as, the number of enterprises that have purchased items, the number of items purchased by member of the coalition, etc. Then we follow the following process:

1. Build a grammar that generates the language.
2. Examine the production rules of the grammar to determine if they model the interaction strategy amongst the enterprises during the formation of a VBC as specified by Ngassam and Raborife [2013].
3. If the production rules do not model the interaction strategy as specified by Ngassam and Raborife [2013], build another grammar of a different class and look into its interaction strategy, and so forth. This is an incremental process.

In this study, formal grammars are used to model the various stages of a VBC life cycle. The initiator enterprise approaches the supplier with the intent to purchase items. In turn the supplier replies with the total available quantity of the requested goods. Then the initiator is enabled to purchase the goods within the total quantity. The production rules involved in this process ensure that only one enterprise may be the initiator enterprise, and that the total quantity of items to be purchased by members of the coalition is known before the formation process can begin. This is the initiation phase.

Once the initiator has been enabled to claim items, it may also invite other enterprises that may also purchase items and/or invite other enterprises. These processes may only happen if there are still items available for purchase. In our study, the members of the coalition purchase items of the same type. The production rules in this phase ensure the following:

- Once an enterprise has opted out of the coalition, it cannot invite other enterprises nor claim any items.
- An enterprise can only participate once per formed coalition. Participation includes opting out, inviting other enterprises with/without claiming items, and claiming items. If an enterprise has been invited more than once, it can only accept one invitation, the other invitations are void.

- Enterprises do not claim more items than were made available to them.

This is referred to as the operation phase.

The dissolution phase involves production rules that apply after all invited enterprise have performed their operations. This includes “removing” all the items that were not claimed. These items are rewritten to terminals.

The post-dissolution phase involves production rules that group the information about the formed coalition according to the specifications of the supplier. These specification from a supplier may entail grouping the information of all enterprises that opted out of the coalition. The production rules used in this phase are referred to as restructuring rules.

In the next section, we present the necessary formal definitions and concepts used in our study.

Chapter 3

Definitions and Preliminaries

This chapter presents theorems and formal definitions of the concepts used in this study.

Definition 3.1. Let $\mathbb{N}$ denote the integers, and $\mathbb{N}_+ = \{1, 2, \ldots\}$. Moreover, for $m \in \mathbb{N}_+$, let $[m] = \{1, 2, \ldots, m\}$.

Definition 3.2. An alphabet is a finite set of symbols. A word over an alphabet Σ is a finite ordered list of symbols chosen from the set Σ. A language L is a set of words over some alphabet [Xavier 2005].

Definition 3.3. For a word w and a symbol a, let $\mathrm{n}_a(w)$ indicate the total number of occurrences of the symbol a in w.

Definition 3.4. For a word w and a symbol a, $\mathrm{p}_a(w)$ indicates if there is at least a single occurrence of a in w. This is defined as follows:

$$\mathrm{p}_a(w) = \begin{cases} 0 & \text{if} \quad \mathrm{n}_a(w) = 0 \\ 1 & \text{if} \quad \mathrm{n}_a(w) > 0 \end{cases}$$

Definition 3.5. For a word w and a set $S = \{a_1, a_2, a_3, \ldots, a_m\}$ where $m \geq 1$, $\mathrm{np}_S(w)$ is defined as follows:

$$\mathrm{np}_S(w) = \sum_{i=1}^{m} \mathrm{p}_{a_i}(w)$$

Definition 3.6. For a word $w = b_{j_1} b_{j_2} \ldots b_{j_m}$, the term linear order of w is defined as follows.

$$\mathrm{LinOrder}(w) \iff 1 \leq j_1 < j_2 \ldots < j_q \leq m$$

Definition 3.7. For a word $w = b_{j_1}b_{j_2}\dots b_{j_m}$, and a word $v = z_{t_1}z_{t_2}\dots z_{t_m}$, w and v are disjoint if the following holds:

$$\text{DisJoint}\,(w, v) \iff \{j_1, j_2, \dots, j_m\} \cap \{t_1, t_2, \dots, t_m\} = \emptyset$$

Definition 3.8. The length of a word w, denoted by $|w|$, is the number of symbols in the word. The length of the null string λ is zero.

Definition 3.9. A random context grammar (rcg) is a quadruple [Ewert and van der Walt 2002] $G = (V_N, V_T, P, S)$, where

1. V_N is a finite set of non-terminals,
2. V_T is a finite set of terminals,
3. P is a finite set of productions of the form $A \to x\ (\mathcal{P}; \mathcal{F})$, where $A \in V_N$, $x \in (V_N \cup V_T)^*$ and $\mathcal{P}, \mathcal{F} \subseteq V_N$, and
4. $S \in V_N$ is the start symbol.

Let V denote $V_N \cup V_T$. For two strings $y_1, y_2 \in V^*$ and a production $A \to x\ (\mathcal{P}; \mathcal{F})$ in P, we may write $y_1Ay_2 \implies y_1xy_2$ if every $B \in \mathcal{P}$ is in the string y_1y_2 and no $B \in \mathcal{F}$ is in the string y_1y_2.

The reflexive and transitive closure of $\implies$ is denoted by $\stackrel{*}{\implies}$.

Definition 3.10. A random permitting context grammar (rPcg) is a random context grammar $G = (V_N, V_T, P, S)$, where for each production $A \to x\ (\mathcal{P}; \mathcal{F}) \in P$, $\mathcal{F} = \emptyset$ [Ewert and van der Walt 2002].

Definition 3.11. A random forbidding context grammar (rFcg) is a random context grammar $G = (V_N, V_T, P, S)$, where for each production $A \to x\ (\mathcal{P}; \mathcal{F}) \in P$, $\mathcal{P} = \emptyset$ [Ewert and van der Walt 2002].

Definition 3.12. A context-free grammar (cfg) is a random context grammar $G = (V_N, V_T, P, S)$, where $\mathcal{P} = \mathcal{F} = \emptyset$ for each production $A \to x\ (\mathcal{P}; \mathcal{F}) \in P$ [Ewert and van der Walt 2002].

Definition 3.13. A context-free grammar $G = (V_N, V_T, P, S)$ is regular if every production is of the form $A \to xB$ or $A \to x$, where $A, B \in V_N$ and $x \in V_T$ [Martin 1997].

A language is regular, context-free, random permitting context, random forbidding context, or random context if it is generated by the regular, context-free, random permitting context, random forbidding context, or random context grammar of the respective type.

Theorem 3.14. *(from Martin [1997], page 145) Pumping Lemma for Regular Languages - Let L be a regular language. Then there is an integer h so that for any $u \in L$ with $|u| \geq h$, there are strings p, q and r so that;*

1. $u = pqr$,
2. $|pq| \leq h$,
3. $|q| > 0$, *and*
4. *for any* $m \geq 0$, $pq^m r$ *is in* L.

Theorem 3.15. *(from Martin [1997], page 239) Pumping Lemma for Context-Free Languages - Let L be a context-free language. Then there is an integer h so that for any $u \in L$ with $|u| \geq h$, there are strings p, q, r, s, and t so that;*

1. $u = pqrst$,
2. $|qs| > 0$,
3. $|qrs| \leq h$, *and*
4. *for any* $m \geq 0$, $pq^m rs^m t$ *is in* L.

Chapter 4

Research Basis

In this chapter, a regular language, and two context-free languages are presented which form a baseline of our study. These language do not provide enough information about a formed coalition, nor do the grammars that generate them adhere to the formation process of a VBC as set out by Ngassam and Raborife [2013].

Section 4.1 presents a language (representing a formed coalition) in which the number of items that can be claimed by members of a coalition is not bounded. Enterprises can claim as many items as they require. In the languages presented in Section 4.2, the items that can be claimed by members of a coalition is bounded.

4.1 Regular Grammar

In this section we present a regular grammar, and the language it generates which represents a formed coalition. Let $m \in \mathbb{N}_+$ represent the number of enterprises in a coalition, and let

$$L_1 = \{v \mid v = a_{j_1}^{n_{j_1}} a_{j_2}^{n_{j_2}} \ldots a_{j_m}^{n_{j_m}} \,;\, n_{j_i} \geq 1 \text{ for } i \in [m] \,;\, \text{LinOrder}(v)\}.$$

Assume $w \in L_1$. The following information about a formed coalition is represented in a word $w \in L_1$.

1. The allocation of n_{j_i} items to enterprise a_{j_i} is represented by $a_{j_i}^{n_{j_i}}$.
2. The condition $n_{j_i} \geq 1$ for $i \in [m]$ implies that only enterprises that have claimed at least one item are part of the formed coalition.

Example 4.1 illustrates one of the words in L_1.

Example 4.1. $w = a_1^2 a_2 a_4^3$

In this word, there are three enterprises (a_1, a_2, and a_4) that have claimed items. The enterprise represented by a_1 has claimed two items (a_1^2), the enterprise represented by a_2 has claimed one item (a_2), and the enterprise represented by a_4 has claimed three items (a_4^3). The sum of all a_i's (where $i \in [4]$) is six, which is the total number of items claimed by the enterprises in the coalition.

L_1 is generated by a regular grammar that has the following rule templates. The enterprises are represented by non-terminal symbols. In particular, enterprise i is represented by non-terminal S_i.

The following regular grammar generates L_1.

Let $G = (V_\mathrm{N}, V_\mathrm{T}, P, S)$:

1. $V_\mathrm{N} = \{S, S_1, S_2, \ldots, S_m\}$.
2. $V_\mathrm{T} = \{a_1, a_2, \ldots, a_m\}$.
3. P is the set of productions defined as follows:

   For every S_i, where $i \in [m-1]$,

$$\begin{aligned} S &\rightarrow S_i \\ S_i &\rightarrow a_i S_i \mid \\ &\rightarrow S_{i+1} \mid \\ &\rightarrow \lambda \end{aligned}$$

In our grammar, all enterprises have the same template for rules with the exception of enterprise m. Each enterprise i ($i \in [m]$) has the rule template $S_i \rightarrow a_i S_i$. This denotes claiming a single item. To invite another enterprise, each enterprise i ($i \in [m-1]$) has the rule template $S_i \rightarrow S_{i+1}$. Based on this rule, we can deduce that the word $a_1^2 a_2 a_4^3$ in Example 4.1 must have involved at least four enterprises. If an enterprise decides to opt out of the coalition, the rule template $S_i \rightarrow \lambda$ applies. Opting out of the coalition implies that the enterprise has finished claiming its items, and does not want to invite another enterprise. In addition, this is applicable if an enterprise does not want to be a part of the coalition, that is, neither claim items, nor invite other enterprises.

The rule templates for enterprise m are as follows. This is the last enterprise to join the coalition, therefore, it cannot invite another enterprise. If we allow this enterprise to invite another enterprise, the coalition formation process may end in an endless loop.

$$\begin{aligned} S_m &\rightarrow a_m S_m \mid \\ &\rightarrow \lambda \end{aligned}$$

We demonstrate these concepts and the formation of a coalition in the following example.

Example 4.2. *In the following regular grammar we have five enterprises ($m = 5$). Each enterprise can claim items, invite another enterprise with/without claiming items, or opt out of the coalition.*

Let $G = (V_{\mathrm{N}}, V_{\mathrm{T}}, P, S)$:

1. $V_{\mathrm{N}} = \{S, S_1, S_2, S_3, S_4, S_5\}$.
2. $V_{\mathrm{T}} = \{a_1, a_2, a_3, a_4, a_5\}$.
3. *P is the set of productions defined in Figures 4.1-4.2.*

Figure 4.1 refers to the rule initiating the formation of a coalition. The supplier gives control to the initiator enterprise as shown by rule 4.1 enabling the initiator to begin claiming items. In this example, the enterprise represented by S_1 is the initiator enterprise.

$$S \rightarrow S_1 \tag{4.1}$$

FIGURE 4.1: Rule to initiate coalition formation

In Figure 4.2, an enterprise can claim an item as exemplified by rule 4.2, invite another enterprise as illustrated by rule 4.3, or opt out of the coalition as exemplified by rule 4.4.

Consider the following situation: S_1 claims three items and then invites S_2. S_2 claims one item and invites S_3. S_3 invites S_4 without claiming items. S_4 claims two items and invites S_5. S_5 opts out of the coalition.

The following derivation illustrates S_1 being introduced into the sentential form. This signals that the enterprise represented by S_1 can claim items. In this instance, the

$$
\begin{aligned}
S_1 &\rightarrow a_1S_1 \mid && (4.2)\\
&\rightarrow S_2 \mid && (4.3)\\
&\rightarrow \lambda && (4.4)\\
S_2 &\rightarrow a_2S_2 \mid && (4.5)\\
&\rightarrow S_3 \mid && (4.6)\\
&\rightarrow \lambda && (4.7)\\
S_3 &\rightarrow a_3S_3 \mid && (4.8)\\
&\rightarrow S_4 \mid && (4.9)\\
&\rightarrow \lambda && (4.10)\\
S_4 &\rightarrow a_4S_4 \mid && (4.11)\\
&\rightarrow S_5 \mid && (4.12)\\
&\rightarrow \lambda && (4.13)\\
S_5 &\rightarrow a_5S_5 \mid && (4.14)\\
&\rightarrow \lambda && (4.15)
\end{aligned}
$$

FIGURE 4.2: Rules for claiming an item, inviting another enterprise, or opting out

enterprise represented by S_1 *is the initiator enterprise; it is the first enterprise to be put in a position to claim items.*

$$S \implies S_1$$

The initiator then claims three items as follows:

$$
\begin{aligned}
S_1 &\implies a_1S_1 && (\textit{rule } (4.2))\\
&\implies a_1a_1S_1 && (\textit{rule } (4.2))\\
&\implies a_1a_1a_1S_1 && (\textit{rule } (4.2))
\end{aligned}
$$

S_1 *then invites* S_2.

$$a_1a_1a_1S_1 \implies a_1a_1a_1S_2 \qquad (\textit{rule } (4.3))$$

S_2 *claims an item.*

$$a_1a_1a_1S_2 \implies a_1a_1a_1a_2S_2 \qquad \text{(rule (4.5))}$$

S_2 then invites S_3.

$$a_1a_1a_1a_2S_2 \implies a_1a_1a_1a_2S_3 \qquad \text{(rule (4.6))}$$

S_3 then invites S_4 without claiming any items.

$$a_1a_1a_1a_2S_3 \implies a_1a_1a_1a_2S_4 \qquad \text{(rule (4.9))}$$

S_4 then claims two items as follows:

$$\begin{aligned} a_1a_1a_1a_2S_4 &\implies a_1a_1a_1a_2a_4S_4 && \text{(rule (4.11))} \\ &\implies a_1a_1a_1a_2a_4a_4S_4 && \text{(rule (4.11))} \end{aligned}$$

S_5 is then invited by S_4 as follows.

$$a_1a_1a_1a_2a_4a_4S_4 \implies a_1a_1a_1a_2a_4a_4S_5 \qquad \text{(rule (4.12))}$$

S_5 opts out of the coalition.

$$a_1a_1a_1a_2a_4a_4S_5 \implies a_1a_1a_1a_2a_4a_4 \qquad \text{(rule (4.15))}$$

The word generated by this particular example is

$$\implies a_1^3a_2a_4^2$$

This represents a coalition formed by three enterprises, represented by a_1, a_2, and a_4. The enterprise represented by a_1 has claimed three items, the enterprise represented by a_2 has claimed one item, and the enterprise represented by a_4 has claimed two items.

The strategy employed in this grammar enables an enterprise to invite only one other enterprise with a label higher than it. In addition, enterprises can claim as many items as they want. This demonstrates that regular grammars are appropriate in modelling a coalition in which the number of items that can be claimed by members of a coalition is not bounded. This is not applicable for a VBC since the supplier limits the total quantity of items that members of the coalition can claim. In the next section, we present context-free languages in which the number of items that can be claimed by members of the coalition is bounded.

4.2 Context-Free Grammars

In this section we present two context-free grammars that generate different languages. In the first language,we have information regarding the number of items made available to the coalition as well as the enterprises who have claimed items. Furthermore, there is condition that limits the number of items claimed by members of a coalition to the quantity made available to them. In the second language, we have additional information about a formed coalition, such as the number of enterprises who neither claimed items nor invited another enterprise.

4.2.1 Basic Language

Let $m \in \mathbb{N}_+$ represent the number of enterprises in a coalition, and

$$\begin{aligned} L_2 \;=\; & \{vx^k \mid v = a_{j_1}^{n_{j_1}} a_{j_2}^{n_{j_2}} \dots a_{j_m}^{n_{j_m}} \; ; \; n_{j_i} \geq 1 \text{ for } i \in [m] \; ; \; \text{LinOrder}\,(v) \; ; \; k \in \mathbb{N}_+ \; ; \\ & \sum_{i=1}^{m} n_{j_i} \leq k\}. \end{aligned}$$

Assume $w \in L_2$. The following information about a formed coalition is represented in a word $w \in L_2$.

1. The allocation of n_{j_i} items to enterprise a_{j_i} is represented by $a_{j_i}^{n_{j_i}}$.

2. k represents the total number of items that can be claimed by members of a coalition.

3. The condition $n_{j_i} \geq 1$ for $i \in [m]$ implies that only enterprises that have claimed at least one item are part of the formed coalition.

4. The condition $\sum_{i=1}^{m} n_{j_i} \leq k$ implies that the total number of items claimed by members of a coalition cannot be more than the pre-allocated quantity from the supplier.

Example 4.3 illustrates a word in L_2.

Example 4.3. $w = a_1 a_3 x^3$

This word represents a coalition comprising of two enterprises (a_1 and a_3) that have claimed items. Each of these enterprises has claimed one item. The total number of items that was made available to the coalition is three (x^3).

L_2 can be generated by the following context-free grammar. The enterprises are represented by non-terminal symbols.

Let $G = (V_N, V_T, P, S)$:

1. $V_N = \{S, S_1, S_2, \ldots, S_m\} \cup \{S'_1, S'_2, \ldots, S'_m\} \cup \{S''_1, S''_2, \ldots, S''_m\} \cup \{X, X'\}$.

2. $V_T = \{a_1, a_2, \ldots, a_m\} \cup \{x\}$.

3. P is the set of productions defined as follows:

   For every S_i, where $i \in [m-1]$,

$$
\begin{aligned}
S &\rightarrow S_i \\
S_i &\rightarrow a_i S'_i x \mid \\
&\rightarrow S''_i \mid \\
&\rightarrow X' \\
S'_i &\rightarrow S_i \mid \\
&\rightarrow S''_i \mid \\
&\rightarrow X \\
S''_i &\rightarrow S_{i+1} \\
X' &\rightarrow xX \\
X &\rightarrow xX \mid \\
&\rightarrow \lambda
\end{aligned}
$$

In this grammar, all enterprises, except for the last enterprise to be invited to join the coalition (that is enterprise m), have the same rule templates. This is because the last enterprise to join the coalition cannot invite any other enterprise. If we allow enterprise m to invite another enterprise, then we cannot know how many enterprises will form part of the coalition, and thus the formation of this coalition may never end.

1. $S \to S_i$ introduces the initiator enterprise i to the sentential form.
2. $S_i \to a_i S_i' x$ applies when enterprise i claims an item. For every item claimed, an item is generated as represented by x.
3. $S_i \to S_i''$ applies if enterprise i wants to invite another enterprise without claiming items.
4. $S_i \to X'$ applies if enterprise i opts out of the coalition.
5. $S_i' \to S_i$ applies if enterprise i wants to claim another item.
6. $S_i' \to S_i''$ applies when enterprise i wants to invite another enterprise after claiming at least one item.
7. $S_i' \to X$ applies when enterprise i claims at least one item, and does not invite another enterprise to join the coalition.
8. $S_i'' \to S_{i+1}$ applies if enterprise i invites another enterprise. Based on this rule, we can conclude that the word $a_1 a_3 x^3$ in Example 4.3 involved at least three enterprises.
9. $X' \to xX$ produces an item after an enterprise has opted out of the coalition.
10. $X \to xX$ produces an item after an enterprise has completed claiming items. This enterprise did not invite another enterprise after claiming items.
11. $X \to \lambda$ signals the end of the coalition formation process.

Enterprise m has the following rule templates. This enterprise cannot invite another enterprise.

$$
\begin{aligned}
S_m &\rightarrow a_m S'_m x \mid \\
&\rightarrow X' \\
S'_m &\rightarrow S_m \mid \\
&\rightarrow X \\
X' &\rightarrow xX \\
X &\rightarrow xX \mid \\
&\rightarrow \lambda
\end{aligned}
$$

We demonstrate these notions using the following example.

Example 4.4. *In the following context-free grammar, we have five enterprises, $m = 5$. Each enterprise can claim items, and/or invite another enterprise, or not join the coalition.*

Let $G = (V_{\mathrm{N}}, V_{\mathrm{T}}, P, S)$:

1. $V_{\mathrm{N}} = \{S, S_1, S_2, S_3, S_4, S_5\} \cup \{S'_1, S'_2, S'_3, S'_4, S'_5\} \cup \{S''_1, S''_2, S''_3, S''_4, S''_5\} \cup \{X, X'\}$.
2. $V_{\mathrm{T}} = \{a_1, a_2, a_3, a_4, a_5\} \cup \{x\}$.
3. *P is the set of productions defined in Figure 4.3.*

Consider the following situation: There are four items available. S_1 claims one item and invites S_2. S_2 invites S_3 without claiming any items. S_3 claims two items and then invites S_4. S_4 invites S_5 without claiming any items. S_5 opts out of the coalition.

The following derivation shows S_1 being introduced to the sentential form as follows.

$$S \implies S_1$$

S_1 then claims an item in the following way.

$$S_1 \implies a_1 S'_1 x \qquad \text{(rule (4.17))}$$

S_1 invites S_2 as shown below.

$$\begin{aligned}
S &\rightarrow S_1 && (4.16)\\
S_1 &\rightarrow a_1S_1'x \mid && (4.17)\\
&\rightarrow S_1'' \mid && (4.18)\\
&\rightarrow X' && (4.19)\\
S_1' &\rightarrow S_1 \mid && (4.20)\\
&\rightarrow S_1'' \mid && (4.21)\\
&\rightarrow X && (4.22)\\
S_1'' &\rightarrow S_2 && (4.23)\\
S_2 &\rightarrow a_2S_2'x \mid && (4.24)\\
&\rightarrow S_2'' \mid && (4.25)\\
&\rightarrow X' && (4.26)\\
S_2' &\rightarrow S_2 \mid && (4.27)\\
&\rightarrow S_2'' \mid && (4.28)\\
&\rightarrow X && (4.29)\\
S_2'' &\rightarrow S_3 && (4.30)\\
S_3 &\rightarrow a_3S_3'x \mid && (4.31)\\
&\rightarrow S_3'' \mid && (4.32)\\
&\rightarrow X' && (4.33)
\end{aligned}$$

$$\begin{aligned}
S_3' &\rightarrow S_3 \mid && (4.34)\\
&\rightarrow S_3'' \mid && (4.35)\\
&\rightarrow X && (4.36)\\
S_3'' &\rightarrow S_4 && (4.37)\\
S_4 &\rightarrow a_4S_4'x \mid && (4.38)\\
&\rightarrow S_4'' \mid && (4.39)\\
&\rightarrow X' && (4.40)\\
S_4' &\rightarrow S_4 \mid && (4.41)\\
&\rightarrow S_4'' \mid && (4.42)\\
&\rightarrow X && (4.43)\\
S_4'' &\rightarrow S_5 && (4.44)\\
S_5 &\rightarrow a_5S_5'x \mid && (4.45)\\
&\rightarrow X' && (4.46)\\
S_5' &\rightarrow S_5 \mid && (4.47)\\
&\rightarrow X && (4.48)\\
X' &\rightarrow xX && (4.49)\\
X &\rightarrow xX \mid && (4.50)\\
&\rightarrow \lambda && (4.51)
\end{aligned}$$

FIGURE 4.3: Rules for claiming an item, inviting another enterprise, or opting out

$$\begin{aligned}
a_1S_1'x &\Longrightarrow a_1S_1''x && \textit{(rule (4.21))}\\
&\Longrightarrow a_1S_2x && \textit{(rule (4.23))}
\end{aligned}$$

S_2 invites S_3 as follows.

$$\begin{aligned}
a_1S_2x &\Longrightarrow a_1S_2''x && \textit{(rule (4.25))}\\
&\Longrightarrow a_1S_3x && \textit{(rule (4.30))}
\end{aligned}$$

S_3 then claims two items as follows:

$$\begin{aligned} a_1 S_3 x &\Longrightarrow a_1 a_3 S_3' x x && (\textit{rule } (4.31)) \\ &\Longrightarrow a_1 a_3 S_3 x x && (\textit{rule } (4.34)) \\ &\Longrightarrow a_1 a_3 a_3 S_3' x x x && (\textit{rule } (4.31)) \end{aligned}$$

S_3 then invites S_4 as follows.

$$\begin{aligned} a_1 a_3 a_3 S_3' x x x &\Longrightarrow a_1 a_3 a_3 S_3'' x x x && (\textit{rule } (4.35)) \\ &\Longrightarrow a_1 a_3 a_3 S_4 x x x && (\textit{rule } (4.37)) \end{aligned}$$

S_4 then invites S_5 without claiming any items.

$$\begin{aligned} a_1 a_3 a_3 S_4 x x x &\Longrightarrow a_1 a_3 a_3 S_4'' x x x && (\textit{rule } (4.39)) \\ &\Longrightarrow a_1 a_3 a_3 S_5 x x x && (\textit{rule } (4.44)) \end{aligned}$$

Finally, S_5 opts out of the coalition as follows.

$$a_1 a_3 a_3 S_5 x x x \Longrightarrow a_1 a_3 a_3 X' x x x \qquad (\textit{rule } (4.46))$$

Our coalition has four available items, three of the claimed items have been generated. We generate the fourth as follows.

$$a_1 a_3 a_3 X' x x x \Longrightarrow a_1 a_3 a_3 x X x x x \qquad (\textit{rule } (4.49))$$

Finally, rule (4.51) is applied as follows.

$$a_1a_3a_3xXxxx \implies a_1a_3a_3xxxx$$

The word generated by this example is

$$a_1a_3^2x^4$$

This represents a coalition that has two enterprises, represented by a_1 and a_3. The enterprise represented by a_1 has claimed one item, and the enterprise represented by a_3 has claimed two items. There are four items that were available to be claimed by members of the coalition.

The strategy employed by this cfg allows enterprises to invite one other enterprise with a label higher than it. In addition, the number of items that can be claimed by members of a coalition is bounded as per language definition (L_2); the grammar generating the language produces an item as it is claimed by an enterprise. This is exemplified by rule 4.17 in which for an item claimed (a generated a_1) by the enterprise represented by S_1, an x is also generated. This implies that at the begin of the coalition, the items are not bounded. The cfg presented in this section, does not model coalition formation for a VBC as described by Ngassam and Raborife [2013].

We have shown that L_2 is a context-free language. We now continue to show that this language cannot be generated by a regular grammar. This indicates regular grammars cannot model a coalition in which the number of items that can be claimed is bounded by the supplier. The pumping lemma for regular languages is defined in Theorem 3.14.

Theorem 4.1. *L_2 is not a regular language.*

Proof. Assume L_2 is a regular language.

Let h be the integer of Theorem 3.14.

Let $u = a_1^h x^h$, then $u \in L_2$.

According to the definition of the pumping lemma for regular languages, there is a decomposition of u into pqr, such that $|pq| \leq h$.

In this word, an enterprise represented by a_1 has claimed h items, which is equal to the number of items that were made available to the coalition by the supplier, x^h.

According to condition 2 of Theorem 3.14, $pq = a_1^h$, maximally.

The conditions $|pq| \leq h$, and $|q| > 0$ of the pumping lemma for regular languages imply that $q = a_1^i$, $1 \leq i \leq h$.

Then for $m > 1$, the resulting word u' will have more a_1's than x's. Thus $u' \notin L_2$. We can then conclude that L_2 is not a regular language. □

In the following section, we present a language that extends L_2. In addition to the information presented in L_2, this language provides information about the number of enterprises that have opted out of the coalition. Furthermore, it provides information about the number of enterprises that have claimed items without inviting another enterprise.

4.2.2 Informative Language

Let the number of enterprises in a coalition be represented by $m \in \mathbb{N}_+$, and let $S = \{a_1, a_2, a_3, \ldots, a_m\}$. Let

$$
\begin{aligned}
L_3 \quad = \quad & \{v\beta x^k \mid v = a_{j_1}^{n_{j_1}} a_{j_2}^{n_{j_2}} \ldots a_{j_m}^{n_{jm}} \; ; \; n_{j_i} \geq 1 \text{ for } i \in [m] \; ; \; \mathrm{LinOrder}\,(v) \; ; \; \beta \in \{z, e\}^+ \; ; \\
& \mathrm{n}_z\,(\beta) \leq \mathrm{np}_S\,(v) \; ; \; k \in \mathbb{N}_+ \, ; \; \sum_{i=1}^{m} n_{j_i} \leq k\} \; .
\end{aligned}
$$

Assume $w \in L_3$. The following information about a formed coalition is represented in a word $w \in L_3$.

1. $a_{j_i}^{n_{j_i}}$ represents the allocation of n_{j_i} items to enterprise a_{j_i}.
2. z represents an enterprise that has claimed items without inviting another enterprise.
3. e represents an enterprise that was invited to join the coalition, but neither claimed items nor invited another enterprise.
4. k represents the total number of items that can be claimed by the members of a coalition.
5. The condition $n_{j_i} \geq 1$ for $i \in [m]$ implies that only enterprises that have claimed at least one item are part of the formed coalition.
6. The condition $\sum_{i=1}^{m} n_{j_i} \leq k$ implies that the total number of items claimed by members of a coalition cannot be more than the pre-allocated quantity from the supplier.

Example 4.5 illustrates one of the words in L_3.

Example 4.5. $w = a_1^2 a_2 a_3^3 z x^6$

This word reflects that three enterprises (a_1, a_2, and a_3) claimed items. The enterprise represented by a_1 has claimed two items, the enterprise represented by a_2 has claimed one item, and the enterprise represented by a_3 has claimed three items.

The single occurrence of z implies that there is one enterprise that did not invite another enterprise. An enterprise a_i has to claim at least one item before a z is introduced, signalling that a_i does not invite another enterprise. The placement of z in a word w is such that z occurs to the right of the sequence of a_i's (where a_i is representing the enterprise that does not invite another enterprise). The total number of items that was made available to the coalition is six (x^6).

The following context-free grammar generates L_3, and an enterprise i is represented by a non-terminal S_i.

Let $G = (V_{\mathrm{N}}, V_{\mathrm{T}}, P, S)$:

1. $V_{\mathrm{N}} = \{S, S_1, S_2, \ldots, S_m\} \cup \{S'_1, S'_2, \ldots, S'_m\} \cup \{S''_1, S''_2, \ldots, S''_m\} \cup \{X, X'\}$.
2. $V_{\mathrm{T}} = \{a_1, a_2, \ldots, a_m\} \cup \{x, e, z\}$.
3. P is the set of productions defined as follows:

   For every S_i, where $i \in [m-1]$,

$$
\begin{aligned}
S &\rightarrow S_i \\
S_i &\rightarrow a_i S'_i x \mid \\
&\rightarrow S''_i \mid \\
&\rightarrow eX' \\
S'_i &\rightarrow S_i \mid \\
&\rightarrow S''_i \mid \\
&\rightarrow zX \\
S''_i &\rightarrow S_{i+1} \\
X' &\rightarrow xX \\
X &\rightarrow xX \mid \\
&\rightarrow \lambda
\end{aligned}
$$

As in the previous section, in this grammar all enterprises, except for enterprise m, have the following rule templates.

1. $S \rightarrow S_i$ introduces the initiator enterprise i to the sentential form.
2. $S_i \rightarrow a_i S_i' x$ applies when enterprise i claims an item. For every item claimed, an item is generated as represented by x.
3. $S_i \rightarrow S_i''$ applies if enterprise i wants to invite another enterprise without claiming items.
4. $S_i \rightarrow eX'$ applies when enterprise i opts out of the coalition.
5. $S_i' \rightarrow S_i$ applies if enterprise i wants to claim another item.
6. $S_i' \rightarrow S_i''$ applies when enterprise i wants to invite another enterprise after claiming at least one item.
7. $S_i' \rightarrow zX$ applies when enterprise i claims at least one item, and does not invite another enterprise to join the coalition. Since in our grammar, an enterprise can only invite one other enterprise, the application of this rule signals the end of the coalition formation process. Based on this rule, we can deduce that the occurrence of z in the word $a_1^2 a_2 a_3^3 z x^6$ shown in example 4.5 signals that the enterprise represented by a_3 did not invite another enterprise after claiming items.
8. $S_i'' \rightarrow S_{i+1}$ applies if enterprise i invites another enterprise.
9. $X' \rightarrow xX$ produces an item after an enterprise has opted out of the coalition.
10. $X \rightarrow xX$ produces more items.
11. $X \rightarrow \lambda$ signals the end of the formation process.

Enterprise m has the following rule templates. This enterprise cannot invite another enterprise.

$$
\begin{aligned}
S_m &\rightarrow a_m S_m' x \mid \\
&\rightarrow eX' \\
S_m' &\rightarrow S_m \mid \\
&\rightarrow zX \\
X' &\rightarrow xX \\
X &\rightarrow xX \mid \\
&\rightarrow \lambda
\end{aligned}
$$

We exemplify the formation of the coalition using five enterprises ($m = 5$) as shown in Example 4.6.

Example 4.6. *Let* $G = (V_N, V_T, P, S)$*:*

1. $V_N = \{S, S_1, S_2, S_3, S_4, S_5\} \cup \{S'_1, S'_2, S'_3, S'_4, S'_5\} \cup \{S''_1, S''_2, S''_3, S''_4, S''_5\} \cup \{X, X'\}$.
2. $V_T = \{a_1, a_2, a_3, a_4, a_5\} \cup \{z, e, x\}$.
3. *P is the set of productions defined in Figure 4.4.*

S	$\rightarrow$	S_1	(4.52)	S'_3	$\rightarrow$	$S_3 \mid$	(4.70)
S_1	$\rightarrow$	$a_1 S'_1 x \mid$	(4.53)		$\rightarrow$	$S''_3 \mid$	(4.71)
	$\rightarrow$	$S''_1 \mid$	(4.54)		$\rightarrow$	zX	(4.72)
	$\rightarrow$	eX'	(4.55)	S''_3	$\rightarrow$	S_4	(4.73)
S'_1	$\rightarrow$	$S_1 \mid$	(4.56)	S_4	$\rightarrow$	$a_4 S'_4 x \mid$	(4.74)
	$\rightarrow$	$S''_1 \mid$	(4.57)		$\rightarrow$	$S''_4 \mid$	(4.75)
	$\rightarrow$	zX	(4.58)		$\rightarrow$	eX'	(4.76)
S''_1	$\rightarrow$	S_2	(4.59)	S'_4	$\rightarrow$	$S_4 \mid$	(4.77)
S_2	$\rightarrow$	$a_2 S'_2 x \mid$	(4.60)		$\rightarrow$	$S''_4 \mid$	(4.78)
	$\rightarrow$	$S''_2 \mid$	(4.61)		$\rightarrow$	zX	(4.79)
	$\rightarrow$	eX'	(4.62)	S''_4	$\rightarrow$	S_5	(4.80)
S'_2	$\rightarrow$	$S_2 \mid$	(4.63)	S_5	$\rightarrow$	$a_5 S'_5 x \mid$	(4.81)
	$\rightarrow$	$S''_2 \mid$	(4.64)		$\rightarrow$	eX'	(4.82)
	$\rightarrow$	zX	(4.65)	S'_5	$\rightarrow$	$S_5 \mid$	(4.83)
S''_2	$\rightarrow$	S_3	(4.66)		$\rightarrow$	zX	(4.84)
S_3	$\rightarrow$	$a_3 S'_3 x \mid$	(4.67)	X'	$\rightarrow$	xX	(4.85)
	$\rightarrow$	$S''_3 \mid$	(4.68)	X	$\rightarrow$	$xX \mid$	(4.86)
	$\rightarrow$	eX'	(4.69)		$\rightarrow$	λ	(4.87)

FIGURE 4.4: Rules for claiming an item, inviting another enterprise, or opting out

Consider the following situation: There are seven items available. S_1 *claims two items and then invites* S_2*.* S_2 *claims one item and then invites* S_3*.* S_3 *claims two items and invites* S_4*.* S_4 *claims one item and then invites* S_5*.* S_5 *opts out of the coalition.*

The following derivation step illustrates S_1 *being introduced into the sentential form. This signals that the enterprise represented by* S_1 *can claim items. In this example, the enterprise represented by* S_1 *is the initiator enterprise; it is the first enterprise to be put in a position to claim items.*

$$S \implies S_1$$

The initiator then claims two items as follows:

$$\begin{aligned} S_1 &\implies a_1S_1'x && \text{(rule (4.53))} \\ &\implies a_1S_1x && \text{(rule (4.56))} \\ &\implies a_1a_1S_1'xx && \text{(rule (4.53))} \end{aligned}$$

S_1 then invites S_2 to join the coalition by applying rules (4.57) and (4.59) as follows.

$$a_1a_1S_1'xx \implies a_1a_1S_1''xx \implies a_1a_1S_2xx$$

S_2 then claims an item as follows.

$$\implies a_1a_1a_2S_2'xxx \qquad \text{(rule (4.60))}$$

S_2 then invites S_3 to join the coalition by applying rules (4.64) and (4.66).

$$a_1a_1a_2S_2'xxx \implies a_1a_1a_2S_2''xxx \implies a_1a_1a_2S_3xxx$$

Rules (4.67)–(4.80) are applied as follows: S_3 claims two items and then invites S_4 to join the coalition. S_4 then claims an item and invites S_5.

$$\begin{aligned}
&\Longrightarrow a_1a_1a_2a_3S_3'xxxx && (\textit{rule } (4.67))\\
&\Longrightarrow a_1a_1a_2a_3S_3xxxx && (\textit{rule } (4.70))\\
&\Longrightarrow a_1a_1a_2a_3a_3S_3'xxxxx && (\textit{rule } (4.67))\\
&\Longrightarrow a_1a_1a_2a_3a_3S_3''xxxxx && (\textit{rule } (4.71))\\
&\Longrightarrow a_1a_1a_2a_3a_3S_4xxxxx && (\textit{rule } (4.73))\\
&\Longrightarrow a_1a_1a_2a_3a_3a_4S_4'xxxxxx && (\textit{rule } (4.74))\\
&\Longrightarrow a_1a_1a_2a_3a_3a_4S_4''xxxxxx && (\textit{rule } (4.78))\\
&\Longrightarrow a_1a_1a_2a_3a_3a_4S_5xxxxxx && (\textit{rule } (4.80))
\end{aligned}$$

S_5 opts out of the coalition by applying rule (4.82).

$$a_1a_1a_2a_3a_3a_4S_5xxxxxx \Longrightarrow a_1a_1a_2a_3a_3a_4eX'xxxxxx$$

Rule (4.85) then applies as follows.

$$a_1a_1a_2a_3a_3a_4eX'xxxxxx \Longrightarrow a_1a_1a_2a_3a_3a_4exXxxxxxx$$

Rule (4.87) then applies as follows.

$$a_1a_1a_2a_3a_3a_4exXxxxxxx \Longrightarrow a_1a_1a_2a_3a_3a_4exxxxxxxx$$

The resulting word is:

$$a_1a_1a_2a_3a_3a_4exxxxxxx$$

This represents a coalition made up of four enterprises, represented by a_1, a_2, a_3, and a_4.

We now employ the pumping lemma for regular languages to show that L_3 cannot be generated by a regular grammar.

Theorem 4.2. *L_3 is not a regular language.*

Proof. Assume L_3 is a regular language.

Let h be the integer of Theorem 3.14.

Consider $u \in L_3$.

According to the definition of the pumping lemma for regular languages, there is a decomposition of u into pqr, such that $|pq| \leq h$. Let $u = pqr = a_1^h z x^h$, $u \in L_3$.

In this word, an enterprise represented by a_1 has claimed h items, which is equal to the number of items that were made available to the coalition by the supplier, x^h.

The conditions $|pq| \leq h$, and $|q| > 0$ of the pumping lemma for regular languages imply that implies that $q = a_1^i$, $1 \leq i \leq h$.

Then for $m > 1$, the resulting word u' will have more a_1's than x's. Thus $u' \notin L_3$. We can then conclude that L_3 is not a regular language. □

The relationship that exists between the number of items that can be claimed by the members of a coalition (that is, the sum of all a_i's), and the number of items made available to them (k in x^k), is the reason that L_2 and L_3 are not regular languages. This case can be likened to one of the most widely known cfls, $L = \{a^n b^m \mid n \leq m\}$ where the number of a's is less than or equal to the number of b's. This demonstrates that a regular grammar cannot model a coalition in which the number of items that can be claimed is limited by the supplier.

4.3 Conclusion

In the grammars presented in this chapter, an enterprise i only invites one other enterprise with the next higher label than it, enterprise $i + 1$. In the description of a virtual buying coalition as defined by Ngassam and Raborife [2013], an enterprise can invite as many enterprises as it wants, provided there are still items to be claimed. However, an enterprise can only be part of a formed coalition once, that is, if an enterprise has been invited by more than one enterprise to join the coalition, it can only accept one invitation.

In order to enable enterprises to invite multiple other enterprises without violating the conditions of a VBC, the inviting enterprise would have to check that the enterprises that it is inviting have not already been invited to join the coalition before inviting them. Alternatively, the invited enterprises would have to signal that they have already been invited and cannot participate again. This cannot be modelled by regular grammars and a context-free grammars. To counter this, one would have to include some form of context in the grammars enabling enterprises to check that they are eligible to join the coalition when invited before performing any actions.

In this chapter, we showed that a context-free grammar can help in modelling a coalition in which there is a limit on the number of goods that the members of the coalition can claim. In contrast, a regular grammar is appropriate in a coalition in which enterprises can claim as many items as they need. The next chapter shows a grammar that can model a virtual buying cooperative when enterprises are allowed to invite multiple other enterprises without violating the conditions of the description of a VBC as stated by Ngassam and Raborife [2013]. In addition, we present grammars that can generate the language in the next chapter, but their production rules do not model the interaction strategy employed by enterprises during the formation of a VBC.

Chapter 5

Modelling a Basic VBC

5.1 Introduction

In this chapter we present an rPcg, an rFcg, and an rcg that generate the same language representing a formed coalition. Although the first two grammars presented in this chapter generate our language, they do not model a VBC environment. As with the grammars presented in the previous chapter, an available item is generated as it is claimed by an enterprise. The production rules of the rPcg allow enterprises to invite one other enterprise to join the coalition. In the rFcg, the invitation strategy does not allow enterprises to invite each other, all enterprises are invited at the same time. The rcg presented in this chapter models a VBC environment. In this grammar, an invited enterprise can invite as many of its associates, and claim as many items as it requires as long as there are still items available. In addition, invited enterprises participate once per formed coalition. However, the rcg only models first three phases of the formation VBC, that is, the initiation, operation, and dissolution phases. It does not model the post-dissolution has of a VBC since the structure of the language presented in this section does not require it.

Let $m \in \mathbb{N}_+$ be the number of enterprises. Then,

$$\begin{aligned} L_{\text{basic}} \;=\; & \{vx^k \mid v = a_{j_1}^{n_{j_1}} z^{t_1} \beta_1 a_{j_2}^{n_{j_2}} z^{t_2} \beta_2 a_{j_3}^{n_{j_3}} z^{t_3} \beta_3 \ldots a_{j_q}^{n_{j_q}} z^{t_q} \beta_q \,;\, k \in \mathbb{N}_+ \,;\, q \in [m]; n_{j_i} \geq 0 \\ & \text{and } t_i \in \{0,1\} \,;\, i \in [q] \,;\, \text{LinOrder}\,(v) \,;\, \sum_{i=1}^{q} n_{j_i} \leq k \,;\, \text{if } n_{j_i} = 0, \text{then } t_i = 0 \,; \\ & \beta_i \in \{e,f,d\} \cup \{r\}^* \,;\, \text{if } n_{j_i} \neq 0, \text{then } \mathrm{n}_e\,(\beta_i) = \mathrm{n}_f\,(\beta_i) = 0 \,;\, \text{if } \mathrm{n}_e\,(\beta_i) \neq 0, \text{then} \\ & \mathrm{n}_f\,(\beta_i) = 0 \,;\, \text{if } \mathrm{n}_d\,(\beta_i) > 0, \text{then } \sum_{i=1}^{q} n_{j_i} = k \,;\, 0 \leq \mathrm{n}_r\,(v) \leq m(m-1)\} \;. \end{aligned}$$

In L_{basic}, a string z^{t_i} always follows a string of a_{j_i}, where i represents an enterprise. In L_{basic}, t_i may be a zero or one, that is, there may be a z following a string of a_i's. If there is a z ($t_i = 1$) following a string of a_{j_i}'s, this implies that the enterprise i has claimed n_{j_i} items, but did not invite other enterprises to join the coalition.

If enterprise i has invited at least one other enterprise after claiming items, then $t_i = 0$. In a case where enterprise i has invited at least one other enterprise without claiming any items, $n_{j_i} = t_i = 0$. In this case β_i will contain an f, signaling that the invitation was forwarded to at least one other enterprise during the formation of a coalition.

If enterprise i has opted out of the coalition – neither claimed items, nor invited other enterprises – then β_i cannot contain an f, and $n_{j_i} = t_i = 0$. To show that enterprise i has opted out of the coalition, β_i will contain an e.

A situation may arise in which enterprise i did not claim the quantity of items it required (due to lack of available items), this is represented by a d in β_i. If a d occurs in β_i, then the total number of items claimed by all members of the coalition must be equal to the number of items made available to the coalition.

The invitation strategy in a VBC allows enterprises to invite their known associates. It is possible that these enterprises may share associates, worst case scenario being that all enterprises are associates of each other. In this case, all these enterprises may invite each other during the formation process. In such a scenario, if we have m enterprises, then there will be in a total of $m(m-1)$ repeated invitations. A repeated invitation is denoted by r.

Consider a word w in L_{basic}. It represents the following properties about the formed coalition:

1. k represents the number of items that may be claimed by the enterprises in a cooperative.

2. Each occurrence of a_i denotes that the enterprise represented by i has claimed an item.

3. An occurrence of z^{t_i} denotes that the enterprise represented by i has claimed at least one item, but did not invite other enterprises.

4. $\mathrm{n}_e(w)$ represents the number of enterprises that were invited, but neither claimed items nor invited other enterprises (e stands for exit).

5. $\mathrm{n}_f(w)$ represents the number of enterprises that did not claim items, but invited other enterprises (f stands for forward).

6. $n_d(w)$ represents the total number of enterprises that could not perform any actions, because there were no items available (d stands for depleted).

7. $n_r(w)$ represents the total number of times that enterprises were invited to join the cooperative more than once (r stands for repeat).

Example 5.1 shows a word in L_{basic}.

Example 5.1. $w = a_1 edra_5^2 dfx^3$

This word reflects two enterprises (a_1 and a_5) that have claimed items. The enterprise represented by a_1 has claimed one item, and the enterprise represented by a_5 has claimed two items. This is represented by the number of their occurrences in w.

There is an enterprise that was invited to claim items, but opted out of the coalition. This is represented by the e in w. The occurrence of r in w implies that an enterprise was invited to join the coalition twice. The f in w implies that there is an enterprise that invited at least one other enterprise without claiming any items. There are enterprises that were invited to join the coalition, but could not join as there were no items available. This is represented by the two occurrences of d in w.

The sum of all occurrences of a_i in w is three, which is the total number of items claimed by the enterprises in the coalition. The total number of x's in w is also three. This is the total number of items that were made available to the coalition.

In the subsequent sections, we present grammars that generate this language. In addition, we also discuss the extent to which the production rules these grammars use to generate L_{basic} can model the formation of a VBC.

5.2 Random Forbidding Context Grammar

Let $G_{\text{rFcg}} = (V_{\text{N}}, V_{\text{T}}, P, S)$:

For ease of notation, let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$, $\mathcal{S}' = \{S'_1, S'_2, \ldots, S'_m\}$, $\mathcal{S}^o = \{S^o_1, S^o_2, \ldots, S^o_m\}$, $\mathcal{D} = \{D_1, D_2, \ldots, D_m\}$, $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$, $\mathcal{F} = \{F_1, F_2, \ldots, F_m\}$, $\mathcal{X} = \{X_1, X_2, \ldots, X_m\}$ and $\delta = \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}^o$.

1. $V_{\text{N}} = \{S\} \cup \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}^o \cup \mathcal{D} \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{X} \cup \{X, X', X'', R\}$.

2. $V_{\text{T}} = \{a_1, a_2, \ldots, a_m\} \cup \{z, f, e, x, d, r\}$.

3. P is the set of productions defined in Figure 5.1. Please note:
   For $i \in [m]$, in the production rule $S \to S^o_{j_{n_1}} R S^o_{j_{n_2}} R \ldots S^o_{j_{n_i}} RX$,
   - all $j_{n_1}, j_{n_2}, \ldots, j_{n_i}$ are distinct, and
   - $1 \leq n_i \leq m$.

$$
\begin{aligned}
S &\to S^o_{j_{n_1}} R S^o_{j_{n_2}} R \ldots S^o_{j_{n_i}} RX && (5.1)\\
S^o_i &\to S_i && (5.2)\\
&\to D_i \ (; \{X, X'\}) && (5.3)\\
&\to E_i && (5.4)\\
&\to F_i && (5.5)\\
S_i &\to a_i S'_i \ (; \mathcal{S}' \cup \{X', X''\}) && (5.6)\\
&\to D_i \ (; \{X, X'\}) && (5.7)\\
S'_i &\to z \ (; \{X\}) && (5.8)\\
&\to \lambda \ (; \{X\}) && (5.9)\\
&\to S_i \ (; \{X\}) && (5.10)\\
X &\to X_i X' \ (; \{D_i, E_i, F_i, S_i, S^o_i\}) && (5.11)\\
X' &\to X \ (; \mathcal{S}') && (5.12)\\
X' &\to X'' \ (; \mathcal{S}') && (5.13)\\
R &\to \lambda && (5.14)\\
R &\to r && (5.15)\\
R &\to r^2 && (5.16)\\
&\vdots\\
R &\to r^{m-1} && (5.17)\\
X &\to xX \ (; \delta) && (5.18)\\
X &\to x \ (; \delta) && (5.19)\\
D_i &\to d \ (; \delta) && (5.20)\\
E_i &\to e \ (; \delta) && (5.21)\\
F_i &\to f \ (; \delta) && (5.22)\\
X_i &\to x \ (; \delta) && (5.23)\\
X'' &\to \lambda \ (; \delta) && (5.24)
\end{aligned}
$$

FIGURE 5.1: rFcg generating L_{basic}

In G_{rFcg}, all enterprises have the same rule templates. Rule 5.1 introduces the enterprises to the coalition. Rule 5.2 allows enterprise i to claim items.

Rule 5.3 applies if there are no items available to be claimed. Forbidding context ensures that this rule cannot apply if there is at least one item available as represented by the non-terminal X, or if there is an enterprise in the process of claiming an item, as represented by the non-terminal X'. Production rule 5.4 enables enterprise i to opt out of the coalition. In a VBC, rule 5.5 is relevant when enterprise i invites other enterprises without claiming items. However, in this grammar it introduces f since all enterprises are invited at the same time.

Production rule 5.6 applies if enterprise i claims an item. Forbidding context makes certain that this rule cannot apply if there is another enterprise claiming an item. This avoids the situation in which two or more enterprises claim the same item. Rule 5.7 is applicable if enterprise i does not find any items available to be claimed. Forbidding context is used as in rule 5.3.

Production rule 5.8 introduces a z after a string of a_i's. In a VBC model, this would indicate that enterprise i has claimed at least one item without inviting another enterprise. Rule 5.9 relates to the aspect of the language definition that there may also be no z after a string of a_i's. Rule 5.10 enables enterprise i to claim another item. In these three rules, forbidding context ensures that once an item has been claimed, an X is immediately marked as claimed before any of these rules can apply.

Rule 5.11 matches a claimed item to an available item. Forbidding context ensures that this rule applies immediately after enterprise i has claimed an item. Rule 5.12 introduces an item to the sentential form that can be claimed by a member of the coalition. Forbidding context ensures that this rule applies before an enterprise can claim any item. Rule 5.13 removes an available item from the sentential form. Forbidding context is used as in rule 5.12.

Rules 5.14–5.17 introduce the r's to the sentential form. In a VBC, this would imply that enterprise i has been invited to join the coalition more than once. In this grammar, this would not be possible since all enterprises are invited at the same time.

Rules 5.18–5.19 generate more x's. Forbidding context is used to ensure that these rules only apply once all invited enterprises have performed their actions.

Rules 5.20–5.23 remove all non-terminals associated with enterprise i. Forbidding context is used to ensure that these rules only apply once all invited enterprises have performed their actions.

Rule 5.24 removes the unavailable item from the sentential form.

We exemplify these concepts and the formation of a coalition as follows.

Example 5.2. *In the following random forbidding context grammar we have five enterprises ($m = 5$). Each enterprise can claim items, or opt out of the coalition.*

Let $G = (V_\mathrm{N}, V_\mathrm{T}, P, S)$*:*

For ease of notation, let $\mathcal{S} = \{S_1, S_2, S_3, S_4, S_5\}$, $\mathcal{S}' = \{S_1', S_2', S_3', S_4', S_5'\}$, $\mathcal{S}^o = \{S_1^o, S_2^o, S_3^o, S_4^o, S_5^o\}$, $\mathcal{D} = \{D_1, D_2, D_3, D_4, D_5\}$, $\mathcal{E} = \{E_1, E_2, E_3, E_4, E_5\}$, $\mathcal{F} = \{F_1, F_2, F_3, F_4, F_5\}$, $\mathcal{X} = \{X_1, X_2, X_3, X_4, X_5\}$ *and* $\delta = \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}^o$.

1. $V_{\mathrm{N}} = \{S\} \cup \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}^o \cup \mathcal{D} \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{X} \cup \{X, X', X'', R\}$.
2. $V_{\mathrm{T}} = \{a_1, a_2, a_3, a_4, a_5\} \cup \{z, f, e, x, d, r\}$.
3. *P is the set of productions defined in Figures 5.2-5.8.*

Figure 5.2 refers to the rule template in 5.1. In this example, all five enterprises are invited to join the coalition.

$$S \quad \rightarrow \quad S_1^o R S_2^o R S_3^o R S_4^o R S_5^o R X \tag{5.25}$$

FIGURE 5.2: Initiating coalition formation

Figure 5.3 refers to the rule templates 5.2– 5.5.

Figure 5.4 refers to the rule templates 5.6– 5.7.

Figure 5.5 refers to the rule templates 5.8– 5.13.

Figure 5.6 refers to the rule templates 5.14– 5.17.

Figure 5.7 refers to the rule templates 5.18– 5.19.

Figure 5.8 refers to the rule templates 5.20– 5.24.

Consider the following situation: There are six items available, of which S_1 *wants to claim two items,* S_2 *wants to claim one item,* S_3 *wants to claim two items,* S_4 *opts out and* S_5 *wants to claim one item.*

According to our grammar, the coalition formation process commences as follows.

$$S \implies S_1^o B S_2^o B S_3^o B S_4^o B S_5^o B X$$

S_1^o *claims two items as follows:*

$$
\begin{aligned}
S_1^o &\rightarrow S_1 && (5.26)\\
&\rightarrow D_1\ \big(;\{X, X'\}\big) && (5.27)\\
&\rightarrow E_1 && (5.28)\\
&\rightarrow F_1 && (5.29)\\
S_2^o &\rightarrow S_2 && (5.30)\\
&\rightarrow D_2\ \big(;\{X, X'\}\big) && (5.31)\\
&\rightarrow E_2 && (5.32)\\
&\rightarrow F_2 && (5.33)\\
S_3^o &\rightarrow S_3 && (5.34)\\
&\rightarrow D_3\ \big(;\{X, X'\}\big) && (5.35)\\
&\rightarrow E_3 && (5.36)\\
&\rightarrow F_3 && (5.37)\\
S_4^o &\rightarrow S_4 && (5.38)\\
&\rightarrow D_4\ \big(;\{X, X'\}\big) && (5.39)\\
&\rightarrow E_4 && (5.40)\\
&\rightarrow F_4 && (5.41)\\
S_5^o &\rightarrow S_5 && (5.42)\\
&\rightarrow D_5\ \big(;\{X, X'\}\big) && (5.43)\\
&\rightarrow E_5 && (5.44)\\
&\rightarrow F_5 && (5.45)
\end{aligned}
$$

FIGURE 5.3: Enterprises performing their operations

$$
\begin{aligned}
S_1 &\rightarrow a_1 S_1'\ (;\mathcal{S}' \cup \{X', X''\}) && (5.46)\\
&\rightarrow D_1\ \big(;\{X, X'\}\big) && (5.47)\\
S_2 &\rightarrow a_2 S_2'\ (;\mathcal{S}' \cup \{X', X''\}) && (5.48)\\
&\rightarrow D_2\ \big(;\{X, X'\}\big) && (5.49)\\
S_3 &\rightarrow a_3 S_3'\ (;\mathcal{S}' \cup \{X', X''\}) && (5.50)\\
&\rightarrow D_3\ \big(;\{X, X'\}\big) && (5.51)\\
S_4 &\rightarrow a_4 S_4'\ (;\mathcal{S}' \cup \{X', X''\}) && (5.52)\\
&\rightarrow D_4\ \big(;\{X, X'\}\big) && (5.53)\\
S_5 &\rightarrow a_5 S_5'\ (;\mathcal{S}' \cup \{X', X''\}) && (5.54)\\
&\rightarrow D_5\ \big(;\{X, X'\}\big) && (5.55)
\end{aligned}
$$

FIGURE 5.4: Enterprises claiming items

$$\begin{aligned}
S_1' &\rightarrow z\ (;\{X\}) && (5.56)\\
&\rightarrow \lambda\ (;\{X\}) && (5.57)\\
&\rightarrow S_1\ (;\{X\}) && (5.58)\\
X &\rightarrow X_1X'\{D_1,E_1,F_1,S_1,S_1^o\} && (5.59)\\
S_2' &\rightarrow z\ (;\{X\}) && (5.60)\\
&\rightarrow \lambda\ (;\{X\}) && (5.61)\\
&\rightarrow S_2\ (;\{X\}) && (5.62)\\
X &\rightarrow X_2X'\{D_2,E_2,F_2,S_2,S_2^o\} && (5.63)\\
S_3' &\rightarrow z\ (;\{X\}) && (5.64)\\
&\rightarrow \lambda\ (;\{X\}) && (5.65)\\
&\rightarrow S_3\ (;\{X\}) && (5.66)\\
X &\rightarrow X_3X'\{D_3,E_3,F_3,S_3,S_3^o\} && (5.67)\\
S_4' &\rightarrow z\ (;\{X\}) && (5.68)\\
&\rightarrow \lambda\ (;\{X\}) && (5.69)\\
&\rightarrow S_4\ (;\{X\}) && (5.70)\\
X &\rightarrow X_4X'\{D_4,E_4,F_4,S_4,S_4^o\} && (5.71)\\
S_5' &\rightarrow z\ (;\{X\}) && (5.72)\\
&\rightarrow \lambda\ (;\{X\}) && (5.73)\\
&\rightarrow S_5\ (;\{X\}) && (5.74)\\
X &\rightarrow X_5X'\{D_5,E_5,F_5,S_5,S_5^o\} && (5.75)\\
X' &\rightarrow X\ (;\mathcal{S}') && (5.76)\\
X' &\rightarrow X''\ (;\mathcal{S}') && (5.77)
\end{aligned}$$

FIGURE 5.5: Balancing the items claimed by the enterprises and the items made available to them

$$\begin{aligned}
R &\rightarrow \lambda && (5.78)\\
R &\rightarrow r && (5.79)\\
R &\rightarrow r^2 && (5.80)\\
R &\rightarrow r^3 && (5.81)\\
R &\rightarrow r^4 && (5.82)
\end{aligned}$$

FIGURE 5.6: Introducing the r's to the sentential form

$$\begin{aligned}
X &\rightarrow xX\ (;\delta) && (5.83)\\
X &\rightarrow x\ (;\delta) && (5.84)
\end{aligned}$$

FIGURE 5.7: Introducing items to the sentential form

$$D_1 \rightarrow d\,(;\delta) \tag{5.85}$$
$$E_1 \rightarrow e\,(;\delta) \tag{5.86}$$
$$F_1 \rightarrow f\,(;\delta) \tag{5.87}$$
$$X_1 \rightarrow x\,(;\delta) \tag{5.88}$$
$$D_2 \rightarrow d\,(;\delta) \tag{5.89}$$
$$E_2 \rightarrow e\,(;\delta) \tag{5.90}$$
$$F_2 \rightarrow f\,(;\delta) \tag{5.91}$$
$$X_2 \rightarrow x\,(;\delta) \tag{5.92}$$
$$D_3 \rightarrow d\,(;\delta) \tag{5.93}$$
$$E_3 \rightarrow e\,(;\delta) \tag{5.94}$$
$$F_3 \rightarrow f\,(;\delta) \tag{5.95}$$
$$X_3 \rightarrow x\,(;\delta) \tag{5.96}$$
$$D_4 \rightarrow d\,(;\delta) \tag{5.97}$$
$$E_4 \rightarrow e\,(;\delta) \tag{5.98}$$
$$F_4 \rightarrow f\,(;\delta) \tag{5.99}$$
$$X_4 \rightarrow x\,(;\delta) \tag{5.100}$$
$$D_5 \rightarrow d\,(;\delta) \tag{5.101}$$
$$E_5 \rightarrow e\,(;\delta) \tag{5.102}$$
$$F_5 \rightarrow f\,(;\delta) \tag{5.103}$$
$$X_5 \rightarrow x\,(;\delta) \tag{5.104}$$
$$X'' \rightarrow \lambda\,(;\delta) \tag{5.105}$$

FIGURE 5.8: Dissolving the coalition

$$\Longrightarrow S_1RS_2^oRS_3^oRS_4^oRS_5^oRX \qquad (rule\ (5.26))$$
$$\Longrightarrow a_1S_1'RS_2^orS_3^oRS_4^oRS_5^oRX \qquad (rule\ (5.46))$$
$$\Longrightarrow a_1S_1'RS_2^oRS_3^oRS_4^oRS_5^oRX_1X' \qquad (rule\ (5.59))$$
$$\Longrightarrow a_1S_1RS_2^oRS_3^oRS_4^oRS_5^oRX_1X' \qquad (rule\ (5.58))$$
$$\Longrightarrow a_1S_1RS_2^oRS_3^oRS_4^oRS_5^oRX_1X \qquad (rule\ (5.76))$$
$$\Longrightarrow a_1a_1S_1'RS_2^oRS_3^oRS_4^oRS_5^oRX_1X \qquad (rule\ (5.46))$$
$$\Longrightarrow a_1a_1S_1'RS_2^oRS_3^oRS_4^oRS_5^oRX_1X_1X' \qquad (rule\ (5.59))$$

S_1' is then replaced by a z by applying rule (5.56).

$$a_1a_1S_1'RS_2^oRS_3^oRS_4^oRS_5^oRX_1X_1X' \Longrightarrow a_1a_1zRS_2^oRS_3^oRS_4^oRS_5^oRX_1X_1X'$$

An item is made available by applying rule (5.76).

$$a_1a_1zRS_2^oRS_3^oRS_4^oRS_5^oRX_1X_1X' \implies a_1a_1zRS_2^oRS_3^oRS_4^oRS_5^oRX_1X_1X$$

S_2 *claims an items as follows:*

$$\begin{aligned}
&\implies a_1a_1zRS_2RS_3^oRS_4^oRS_5^oRX_1X_1X && \text{(rule (5.30))}\\
&\implies a_1a_1zRa_2S_2'RS_3^oRS_4^oRS_5^oRX_1X_1X && \text{(rule (5.48))}\\
&\implies a_1a_1zRa_2S_2'RS_3^oRS_4^oRS_5^oRX_1X_1X_2X' && \text{(rule (5.63))}
\end{aligned}$$

S_2' *is removed by applying rule (5.61).*

$$a_1a_1zRa_2S_2'RS_3^oRS_4^oRS_5^oRX_1X_1X_2X' \implies a_1a_1zRa_2RS_3^oRS_4^oRS_5^oRX_1X_1X_2X'$$

An item is made available by applying rule (5.76).

$$a_1a_1zRa_2RS_3^oRS_4^oRS_5^oRX_1X_1X_2X' \implies a_1a_1zRa_2RS_3^oRS_4^oRS_5^oRX_1X_1X_2X$$

S_3 *then claims two items as follows:*

$$\begin{aligned}
&\implies a_1a_1zRa_2RS_3RS_4^oRS_5^oRX_1X_1X_2X && \text{(rule (5.34))}\\
&\implies a_1a_1zRa_2Ra_3S_3'RS_4^oRS_5^oRX_1X_1X_2X && \text{(rule (5.50))}\\
&\implies a_1a_1zRa_2Ra_3S_3'RS_4^oRS_5^oRX_1X_1X_2X_3X' && \text{(rule (5.67))}\\
&\implies a_1a_1zRa_2Ra_3S_3RS_4^oRS_5^oRX_1X_1X_2X_3X' && \text{(rule (5.66))}\\
&\implies a_1a_1zRa_2Ra_3S_3RS_4^oRS_5^oRX_1X_1X_2X_3X && \text{(rule (5.76))}\\
&\implies a_1a_1zRa_2Ra_3a_3S_3'RS_4^oRS_5^oRX_1X_1X_2X_3X && \text{(rule (5.50))}\\
&\implies a_1a_1zRa_2Ra_3a_3S_3'RS_4^oRS_5^oRX_1X_1X_2X_3X_3X' && \text{(rule (5.67))}
\end{aligned}$$

S_3' *is then replaced by a* z *by applying rule (5.64).*

$$\Longrightarrow a_1a_1zRa_2Ra_3a_3zRS_4^oRS_5^oRX_1X_1X_2X_3X_3X'$$

An item available by applying rule (5.76).

$$\Longrightarrow a_1a_1zRa_2Ra_3a_3zRS_4^oRS_5^oRX_1X_1X_2X_3X_3X$$

S_4 opts out of the coalition by applying rule (5.40).

$$\Longrightarrow a_1a_1zRa_2Ra_3a_3zRE_4RS_5^oRX_1X_1X_2X_3X_3X$$

S_5 then claims an item as follows.

$$\begin{aligned}
&\Longrightarrow a_1a_1zRa_2Ra_3a_3zRE_4RS_5RX_1X_1X_2X_3X_3X && (\text{rule } (5.42))\\
&\Longrightarrow a_1a_1zRa_2Ra_3a_3zRE_4Ra_5S_5'RX_1X_1X_2X_3X_3X && (\text{rule } (5.54))\\
&\Longrightarrow a_1a_1zRa_2Ra_3a_3zRE_4Ra_5S_5'RX_1X_1X_2X_3X_3X_5X' && (\text{rule } (5.75))
\end{aligned}$$

We remove S_5' by applying rule (5.73).

$$\Longrightarrow a_1a_1zRa_2Ra_3a_3zRE_4Ra_5RX_1X_1X_2X_3X_3X_5X'$$

We get rid of X' as follows.

$$\begin{aligned}
&\Longrightarrow a_1a_1zRa_2Ra_3a_3zRE_4Ra_5RX_1X_1X_2X_3X_3X_5X'' && (\text{rule } (5.77))\\
&\Longrightarrow a_1a_1zRa_2Ra_3a_3zRE_4Ra_5RX_1X_1X_2X_3X_3X_5 && (\text{rule } (5.105))
\end{aligned}$$

Since all the enterprises have performed their actions, the following rules apply.

$$\Longrightarrow a_1a_1zra_2Ra_3a_3zRE_4Ra_5RX_1X_1X_2X_3X_3X_5 \qquad (rule\ (5.79))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zRE_4Ra_5RX_1X_1X_2X_3X_3X_5 \qquad (rule\ (5.78))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrE_4Ra_5RX_1X_1X_2X_3X_3X_5 \qquad (rule\ (5.80))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrE_4a_5RX_1X_1X_2X_3X_3X_5 \qquad (rule\ (5.78))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrE_4a_5rX_1X_1X_2X_3X_3X_5 \qquad (rule\ (5.79))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrea_5rX_1X_1X_2X_3X_3X_5 \qquad (rule\ (5.98))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrea_5rxX_1X_2X_3X_3X_5 \qquad (rule\ (5.88))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrea_5rxxX_2X_3X_3X_5 \qquad (rule\ (5.88))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrea_5rxxxX_3X_3X_5 \qquad (rule\ (5.92))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrea_5rxxxxX_3X_5 \qquad (rule\ (5.96))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrea_5rxxxxxX_5 \qquad (rule\ (5.96))$$
$$\Longrightarrow a_1a_1zra_2a_3a_3zrrea_5rxxxxxx \qquad (rule\ (5.104))$$

The generated word is $a_1^2zra_2a_3^2zr^2ea_5rx^6$

This word represents a coalition comprising of four enterprises that have claimed items (a_1, a_2, a_3, *and* a_5)*, and one enterprise that has opted out of the coalition.*

Although the rFcg presented in this section generates L_{basic}, it does not model coalition formation in a VBC. In this rFcg, enterprises are invited to join the coalition at the same time, and the invited enterprises cannot invite other enterprises. It is, however, worth noting that using an rFcg, one could model a coalition in which each enterprise invites one other enterprise, and still generates L_{basic}. In a VBC, an enterprise initiates the first interaction with the supplier, and it is up to the initiator enterprise to invite other enterprises, who in turn may invite other enterprises, etc. However, we could not build an rFcg in which enterprises are allowed to invite multiple other enterprises. This may be due to the structure of L_{basic}, where all enterprises that have claimed items are in a linear order, and all items that are claimed by an individual enterprise are grouped together.

In the rFcg provided in this section, items claimed by members of a coalition are generated as they are claimed. In a VBC, the supplier places a bound on the number of items that can be claimed. This bound is known to the initiator enterprise before a coalition can be formed.

In the next section, we provide a rPcg that generates L_{basic}. However, this rPcg does not model coalition in a VBC.

5.3 Random Permitting Context Grammar

Let $G_{\text{rPcg}} = (V_{\text{N}}, V_{\text{T}}, P, S)$:

1. $V_{\text{N}} = \{S, S_1, S_2, \ldots, S_m\} \cup \{S_1^o, S_2^o, \ldots, S_m^o\} \cup \{S_1', S_2', \ldots, S_m'\} \cup \{S_1'', S_2'', \ldots, S_m''\} \cup \{S_1''', S_2''', \ldots, S_m'''\} \cup \{X, X', X'', X''', R\}$.
2. $V_{\text{T}} = \{a_1, a_2, \ldots, a_m\} \cup \{z, f, e, x, d, r\}$.
3. P is the set of productions defined in Figure 5.9. In the production rules, $i \in [m-1]$.

In G_{rPcg}, all enterprises, except the last enterprise to be invited to join the coalition, have the same rule templates. The last enterprise to be invited to join the coalition cannot invite another enterprise, we present the production rules for enterprise m in Figure 5.10.

Rule 5.106 introduces the initiator enterprise to the coalition. An item that can be claimed by this enterprise is also introduced, as represented by the non-terminal X. Rule 5.107 allows enterprise i to claim items. Permitting context ensures that this rule only applies if there is an item to be claimed.

Rule 5.108 applies if there are no items to be claimed by enterprise i. Permitting context ensures that this rule only applies if there are no items to be claimed. The non-terminal X'' represents the lack of items. Rule 5.109 applies if enterprise i does not want to be a part of the coalition, that is, neither claim items nor invite other enterprises. Rule 5.110 applies if enterprise i invites another enterprise without claiming any items. In a VBC, an enterprise can invite as many enterprises as it wants. However, in this model, an enterprise only invites one other enterprise. The non-terminal R is used to introduced the r's to the sentential form. In a VBC, this would imply that enterprise i has been invited to join the coalition more than once.

In rule 5.111, enterprise i claims an item. Permitting context is used to ensure that this rule can only apply if there is an item available, as represented by X. Rule 5.112 applies if enterprise i does not find any items available to be claimed. Permitting context is used as in rule 5.108. Rule 5.113 generates an item x for every item claimed by enterprise i. Permitting context is used to ensure that this rule only applies if enterprise i has claimed an item, as represented by S_i'. Rule 5.114 prepares enterprise i to perform its

$$
\begin{aligned}
S &\rightarrow S_i^o X && (5.106)\\
S_i^o &\rightarrow S_i\ (\{X\}\,;) && (5.107)\\
&\rightarrow dR\ \left(\{X''\}\,;\right) && (5.108)\\
&\rightarrow eR && (5.109)\\
&\rightarrow fRS_{i+1}^o && (5.110)\\
S_i &\rightarrow a_i S_i'\ (\{X\}\,;) && (5.111)\\
&\rightarrow dR\ \left(\{X''\}\,;\right) && (5.112)\\
X &\rightarrow xX'\ \left(\{S_i'\}\,;\right) && (5.113)\\
S_i' &\rightarrow S_i''\ \left(\{X'\}\,;\right) && (5.114)\\
&\rightarrow RS_{i+1}^o\ \left(\{X'\}\,;\right) && (5.115)\\
&\rightarrow S_i'''\ \left(\{X'\}\,;\right) && (5.116)\\
S_i'' &\rightarrow S_i\ (\{X\}\,;) && (5.117)\\
X' &\rightarrow X\ \left(\{S_i''\}\,;\right) && (5.118)\\
&\rightarrow X\ \left(\{S_{i+1}^o\}\,;\right) && (5.119)\\
X' &\rightarrow X''\ \left(\{S_i''\}\,;\right) && (5.120)\\
&\rightarrow X''\ \left(\{S_{i+1}^o\}\,;\right) && (5.121)\\
&\rightarrow X'''\ \left(\{S_i'''\}\,;\right) && (5.122)\\
S_i'' &\rightarrow dR\ \left(\{X''\}\,;\right) && (5.123)\\
S_i'' &\rightarrow RS_{i+1}^o\ \left(\{X'''\}\,;\right) && (5.124)\\
S_i''' &\rightarrow zR\ \left(\{X'''\}\,;\right) && (5.125)\\
R &\rightarrow \lambda && (5.126)\\
R &\rightarrow r && (5.127)\\
R &\rightarrow r^2 && (5.128)\\
&\vdots\\
R &\rightarrow r^{m-1} && (5.129)\\
X''' &\rightarrow xX''' && (5.130)\\
X''' &\rightarrow x && (5.131)\\
X'' &\rightarrow \lambda && (5.132)
\end{aligned}
$$

FIGURE 5.9: rPcg generating L_{basic}

operations again. Permitting context ensures that this rule only applies if an item has been generated for enterprise i.

Rule 5.115 allows enterprise i to invite another enterprise after claiming at least one item. Permitting context is used as in rule 5.114. Rule 5.116 applies if enterprise i does not want to invite another enterprise after claiming at least one item. Permitting context is used as in rule 5.114.

Rule 5.117 applies if enterprise i wants to claim another item. Permitting context ensures that this rule only applies if there is an item available, as represented by the non-terminal X. Rule 5.118 introduces an unclaimed item to the sentential form. Permitting context ensures that this rule only applies if enterprise i wants to claim another item. Rule 5.119 also introduces an unclaimed item to the sentential form. Permitting context is used to ensure that this rule only applies if there is an invited enterprise $i+1$ ready to perform its operations.

Rule 5.120 introduces an unavailable to the sentential form, as represented by the non-terminal X''. Permitting context is used as in rule 5.118. Rule 5.121 also introduces an unavailable to the sentential form, as represented by the non-terminal X''. Permitting context is used as in rule 5.119. Rule 5.122 allows for the provision of introducing more items to the sentential form. Permitting context is used to make sure that this rule applies if enterprise i has finished claiming items, and did not invite another enterprise. This is represented by S_i'''.

Rule 5.123 applies if enterprise i find that there are no available items, when it wants to claim another item. Permitting context is used as in rule 5.108. Rule 5.124 allows enterprise i to invite another enterprise. Permitting context ensures that this rule only applies if there are unclaimed items, as represented by X'''. Rule 5.125 applies if enterprise i does not invite another enterprise after claiming at least one item.

Rules 5.126–5.129 introduce the r's to the sentential form. In a VBC, this would imply that enterprise i has been invited to join the coalition more than once.

Rule 5.130 and rule 5.131 generate more x's. Rule 5.132 removes the unavailable item from the sentential form.

We present the production rule templates for the last enterprise to be invited to join the cooperative, that is, enterprise m in Figure 5.10.

The following example demonstrates these concepts, and the formation of a coalition.

Example 5.3. *In this example, we have five enterprises, $m = 5$. Each enterprise can claim items with/without inviting one other enterprise, or opt of the coalition.*

$$
\begin{aligned}
S_m^o &\rightarrow S_m\ (\{X\};) && (5.133)\\
&\rightarrow dR\ \left(\{X''\};\right) && (5.134)\\
&\rightarrow eR && (5.135)\\
S_m &\rightarrow a_m S_m'\ (\{X\};) && (5.136)\\
&\rightarrow dR\ \left(\{X''\};\right) && (5.137)\\
X &\rightarrow xX'\ \left(\{S_m'\};\right) && (5.138)\\
S_m' &\rightarrow S_m''\ \left(\{X'\};\right) && (5.139)\\
&\rightarrow S_m'''\ \left(\{X'\};\right) && (5.140)\\
S_m'' &\rightarrow S_m\ (\{X\};) && (5.141)\\
X' &\rightarrow X\ \left(\{S_m''\};\right) && (5.142)\\
&\rightarrow X''\ \left(\{S_m''\};\right) && (5.143)\\
&\rightarrow X'''\ \left(\{S_m'''\};\right) && (5.144)\\
S_m'' &\rightarrow dR\ \left(\{X''\};\right) && (5.145)\\
S_m''' &\rightarrow zR\ \left(\{X'''\};\right) && (5.146)\\
R &\rightarrow \lambda && (5.147)\\
R &\rightarrow r && (5.148)\\
R &\rightarrow r^2 && (5.149)\\
R &\rightarrow r^3 && (5.150)\\
&\vdots\\
R &\rightarrow r^{m-1} && (5.151)\\
X''' &\rightarrow xX''' && (5.152)\\
X''' &\rightarrow x && (5.153)\\
X'' &\rightarrow \lambda && (5.154)
\end{aligned}
$$

FIGURE 5.10: Production rules for enterprise m

Let $G = (V_{\mathrm{N}}, V_{\mathrm{T}}, P, S)$*:*

1. $V_{\mathrm{N}} = \{S, S_1, S_2, S_3, S_4, S_5\} \cup \{S_1^o, S_2^o, S_3^o, S_4^o, S_5^o\} \cup \{S_1', S_2', S_3', S_4', S_5'\} \cup \{S_1'', S_2'', S_3'', S_4'', S_5''\} \cup \{S_1''', S_2''', S_3''', S_4''', S_5'''\} \cup \{X, X', X'', X''', R\}$.
2. $V_{\mathrm{T}} = \{a_1, a_2, a_3, a_5, a_5\} \cup \{z, f, e, x, d, r\}$.
3. *P is the set of productions defined in Figure 5.11 – 5.16. In the production rules,* $i \in [5]$.

Figure 5.11 shows the initiator enterprise being introduced to the sentential form. In our example, the initiator enterprise is represented by S_1.

$$S \rightarrow S_1^o X \tag{5.155}$$

FIGURE 5.11: Initiating coalition formation

In Figure 5.12, the production rules 5.156 – 5.171 refer to rule templates 5.107 – 5.110 for enterpises one, two, three, and four. The production rules 5.172 – 5.174 refer to the production rule templates 5.133 – 5.135 for enterprise five.

$$S_1^o \rightarrow S_1 (\{X\};) \tag{5.156}$$
$$\rightarrow dR \left(\{X''\};\right) \tag{5.157}$$
$$\rightarrow eR \tag{5.158}$$
$$\rightarrow fRS_2^o \tag{5.159}$$
$$S_2^o \rightarrow S_2 (\{X\};) \tag{5.160}$$
$$\rightarrow dR \left(\{X''\};\right) \tag{5.161}$$
$$\rightarrow eR \tag{5.162}$$
$$\rightarrow fRS_3^o \tag{5.163}$$
$$S_3^o \rightarrow S_3 (\{X\};) \tag{5.164}$$
$$\rightarrow dR \left(\{X''\};\right) \tag{5.165}$$
$$S_3^o \rightarrow eR \tag{5.166}$$
$$\rightarrow fRS_4^o \tag{5.167}$$
$$S_4^o \rightarrow S_4 (\{X\};) \tag{5.168}$$
$$\rightarrow dR \left(\{X''\};\right) \tag{5.169}$$
$$\rightarrow eR \tag{5.170}$$
$$\rightarrow fRS_4^o \tag{5.171}$$
$$S_5^o \rightarrow S_5 (\{X\};) \tag{5.172}$$
$$\rightarrow dR \left(\{X''\};\right) \tag{5.173}$$
$$\rightarrow eR \tag{5.174}$$

FIGURE 5.12: Enterprises performing their operations

Figure 5.13 presents an instance of the production rule templates 5.111 – 5.117, specifically the production rules 5.175 – 5.202 for enterprises S_1, S_2, S_3, and S_4. The production rules 5.203 – 5.208 refer to the templates 5.136 – 5.141 for the enterprise represented *by* S_5.

$$S_1 \to a_1 S_1' \left(\{X\}\,;\right) \quad (5.175)$$
$$\to dR \left(\{X''\}\,;\right) \quad (5.176)$$
$$X \to xX' \left(\{S_1'\}\,;\right) \quad (5.177)$$
$$S_1' \to S_1'' \left(\{X'\}\,;\right) \quad (5.178)$$
$$\to RS_2^o \left(\{X'\}\,;\right) \quad (5.179)$$
$$\to S_1''' \left(\{X'\}\,;\right) \quad (5.180)$$
$$S_1'' \to S_1 \left(\{X\}\,;\right) \quad (5.181)$$
$$S_2 \to a_2 S_2' \left(\{X\}\,;\right) \quad (5.182)$$
$$\to dR \left(\{X''\}\,;\right) \quad (5.183)$$
$$X \to xX' \left(\{S_2'\}\,;\right) \quad (5.184)$$
$$S_2' \to S_2'' \left(\{X'\}\,;\right) \quad (5.185)$$
$$\to RS_3^o \left(\{X'\}\,;\right) \quad (5.186)$$
$$\to S_3''' \left(\{X'\}\,;\right) \quad (5.187)$$
$$S_2'' \to S_2 \left(\{X\}\,;\right) \quad (5.188)$$
$$S_3 \to a_3 S_3' \left(\{X\}\,;\right) \quad (5.189)$$
$$\to dR \left(\{X''\}\,;\right) \quad (5.190)$$
$$X \to xX' \left(\{S_3'\}\,;\right) \quad (5.191)$$
$$S_3' \to S_3'' \left(\{X'\}\,;\right) \quad (5.192)$$
$$\to RS_4^o \left(\{X'\}\,;\right) \quad (5.193)$$
$$\to S_4''' \left(\{X'\}\,;\right) \quad (5.194)$$
$$S_3'' \to S_3 \left(\{X\}\,;\right) \quad (5.195)$$
$$S_4 \to a_4 S_4' \left(\{X\}\,;\right) \quad (5.196)$$
$$\to dR \left(\{X''\}\,;\right) \quad (5.197)$$
$$X \to xX' \left(\{S_4'\}\,;\right) \quad (5.198)$$
$$S_4' \to S_4'' \left(\{X'\}\,;\right) \quad (5.199)$$
$$\to RS_5^o \left(\{X'\}\,;\right) \quad (5.200)$$
$$\to S_4''' \left(\{X'\}\,;\right) \quad (5.201)$$
$$S_4'' \to S_4 \left(\{X\}\,;\right) \quad (5.202)$$
$$S_m \to a_m S_m' \left(\{X\}\,;\right) \quad (5.203)$$
$$\to dR \left(\{X''\}\,;\right) \quad (5.204)$$
$$X \to xX' \left(\{S_m'\}\,;\right) \quad (5.205)$$
$$S_5' \to S_5'' \left(\{X'\}\,;\right) \quad (5.206)$$
$$\to S_5''' \left(\{X'\}\,;\right) \quad (5.207)$$
$$S_5'' \to S_5 \left(\{X\}\,;\right) \quad (5.208)$$

FIGURE 5.13: Enterprises claiming items

Figure 5.14 presents an instance of production rule templates 5.118 – 5.125 for enterprises S_1, S_2, S_3, *and* S_4. *The production rules for enterprise* S_5 *are an instance of the rule templates 5.142 – 5.146.*

$$X' \rightarrow X\left(\{S_1''\};\right) \quad (5.209)$$
$$\rightarrow X\left(\{S_2^o\};\right) \quad (5.210)$$
$$X' \rightarrow X''\left(\{S_1''\};\right) \quad (5.211)$$
$$\rightarrow X''\left(\{S_2^o\};\right) \quad (5.212)$$
$$X' \rightarrow X'''\left(\{S_i'''\};\right) \quad (5.213)$$
$$S_1'' \rightarrow dR\left(\{X''\};\right) \quad (5.214)$$
$$S_1'' \rightarrow RS_2^o\left(\{X'''\};\right) \quad (5.215)$$
$$S_1''' \rightarrow zR\left(\{X'''\};\right) \quad (5.216)$$
$$X' \rightarrow X\left(\{S_2''\};\right) \quad (5.217)$$
$$\rightarrow X\left(\{S_3^o\};\right) \quad (5.218)$$
$$X' \rightarrow X''\left(\{S_2''\};\right) \quad (5.219)$$
$$\rightarrow X''\left(\{S_3^o\};\right) \quad (5.220)$$
$$X' \rightarrow X'''\left(\{S_i'''\};\right) \quad (5.221)$$
$$S_2'' \rightarrow dR\left(\{X''\};\right) \quad (5.222)$$
$$S_2'' \rightarrow RS_3^o\left(\{X'''\};\right) \quad (5.223)$$
$$S_2''' \rightarrow zR\left(\{X'''\};\right) \quad (5.224)$$
$$X' \rightarrow X\left(\{S_3''\};\right) \quad (5.225)$$
$$\rightarrow X\left(\{S_4^o\};\right) \quad (5.226)$$
$$\rightarrow X''\left(\{S_3''\};\right) \quad (5.227)$$
$$X' \rightarrow X''\left(\{S_4^o\};\right) \quad (5.228)$$
$$\rightarrow X'''\left(\{S_3'''\};\right) \quad (5.229)$$
$$S_3'' \rightarrow dR\left(\{X''\};\right) \quad (5.230)$$
$$S_3'' \rightarrow RS_4^o\left(\{X'''\};\right) \quad (5.231)$$
$$S_3''' \rightarrow zR\left(\{X'''\};\right) \quad (5.232)$$
$$X' \rightarrow X\left(\{S_4''\};\right) \quad (5.233)$$
$$\rightarrow X\left(\{S_5^o\};\right) \quad (5.234)$$
$$X' \rightarrow X''\left(\{S_1''\};\right) \quad (5.235)$$
$$\rightarrow X''\left(\{S_5^o\};\right) \quad (5.236)$$
$$X' \rightarrow X'''\left(\{S_3'''\};\right) \quad (5.237)$$
$$S_4'' \rightarrow dR\left(\{X''\};\right) \quad (5.238)$$
$$S_4'' \rightarrow RS_5^o\left(\{X'''\};\right) \quad (5.239)$$
$$S_4''' \rightarrow zR\left(\{X'''\};\right) \quad (5.240)$$
$$X' \rightarrow X\left(\{S_5''\};\right) \quad (5.241)$$
$$\rightarrow X''\left(\{S_5''\};\right) \quad (5.242)$$
$$\rightarrow X'''\left(\{S_5'''\};\right) \quad (5.243)$$
$$S_5'' \rightarrow dR\left(\{X''\};\right) \quad (5.244)$$
$$S_5''' \rightarrow zR\left(\{X'''\};\right) \quad (5.245)$$

FIGURE 5.14: Balancing the items claimed by the enterprises and the items made available to them

Figure 5.15 refers to the production rule templates 5.126 – 5.129.

$$R \rightarrow \lambda \quad (5.246)$$
$$R \rightarrow r \quad (5.247)$$
$$R \rightarrow r^2 \quad (5.248)$$
$$R \rightarrow r^3 \quad (5.249)$$
$$R \rightarrow r^4 \quad (5.250)$$

FIGURE 5.15: Introducing the r's to the sentential form

Figure 5.16 refers to the production rule templates 5.130 – 5.132.

$$X''' \rightarrow xX''' \quad (5.251)$$
$$X''' \rightarrow x \quad (5.252)$$
$$X'' \rightarrow \lambda \quad (5.253)$$

FIGURE 5.16: Dissolving the coalition

Consider the following situation: There are three items available, of which S_1 wants to claim two, and then invite S_2. S_2 invite S_3 without claiming any items. S_3 wants to claim one item, and then invite S_4. S_4 invites S_5 without claiming any items. S_5 opts out of the coalition.

According to our grammar, our coalition formation process commences by introducing the initiator enterprise to the sentential form. In this example, our initiator enterprise is represented by S_1.

$$S \Longrightarrow S_1^o X$$

S_1 claims two items, and then invites S_2 as follows.

$$\Longrightarrow S_1X \quad (rule\ (5.156))$$
$$\Longrightarrow a_1S_1'X \quad (rule\ (5.175))$$
$$\Longrightarrow a_1S_1'xX' \quad (rule\ (5.177))$$
$$\Longrightarrow a_1S_1''xX' \quad (rule\ (5.178))$$
$$\Longrightarrow a_1S_1''xX \quad (rule\ (5.209))$$
$$\Longrightarrow a_1S_1xX \quad (rule\ (5.181))$$
$$\Longrightarrow a_1a_1S_1'xX \quad (rule\ (5.175))$$
$$\Longrightarrow a_1a_1S_1'xxX' \quad (rule\ (5.177))$$
$$\Longrightarrow a_1a_1RS_2^oxxX' \quad (rule\ (5.179))$$
$$\Longrightarrow a_1a_1RS_2^oxxX \quad (rule\ (5.210))$$

S_2 invites S_3 without claiming any items by applying rule (5.163).

$$a_1a_1RS_2^o xxX \implies a_1a_1RfRS_3^o xxX$$

S_3 claims an item, and then invites S_4 as follows.

$$\begin{aligned}
&\implies a_1a_1RfRS_3xxX && \text{(rule (5.164))}\\
&\implies a_1a_1RfRa_3S_3'xxX && \text{(rule (5.189))}\\
&\implies a_1a_1RfRa_3S_3'xxxX' && \text{(rule (5.191))}\\
&\implies a_1a_1RfRa_3S_4^o xxxX' && \text{(rule (5.193))}\\
&\implies a_1a_1RfRa_3S_4^o xxxX'' && \text{(rule (5.236))}
\end{aligned}$$

S_4 invites S_5 without claiming any items by applying rule (5.171).

$$a_1a_1RfRa_3S_4^o xxxX'' \implies a_1a_1RfRa_3fRS_5^o xxxX''$$

S_5 opts out of the coalition by applying rule (5.174).

$$a_1a_1RfRa_3fRS_5^o xxxX'' \implies a_1a_1RfRa_3fReRxxxX''$$

The r's are introduced as follows.

$$\begin{aligned}
&\implies a_1a_1fRa_3fReRxxxX'' && \text{(rule (5.246))}\\
&\implies a_1a_1fa_3fReRxxxX'' && \text{(rule (5.246))}\\
&\implies a_1a_1fa_3feRxxxX'' && \text{(rule (5.246))}\\
&\implies a_1a_1fa_3ferrxxxX'' && \text{(rule (5.248))}
\end{aligned}$$

The non-terminal X'' is removed as follows.

$$\Longrightarrow a_1 a_1 f a_3 f e r r x x x \qquad \textit{(rule (5.253))}$$

The generated word is $a_1^2 r f a_3 f r^2 e x^3$

This coalition comprises of two enterprises that have claimed items. It also has one enterprise that has opted out of the coalition.

In this random permitting context grammar, an enterprise may invite one other enterprise; the enterprise with a higher label than it. In addition, items generated as they are claimed by members of a coalition. In a VBC, an enterprise can invite multiple associates. However, an enterprise can only participate once per formed cooperative. In the next section, we present a random context grammar that models coalition formation as described by Ngassam and Raborife [2013]

5.4 Random Context Grammar

The main aim of our study is to model a VBC as described by Ngassam and Raborife [2013]. In that description, enterprises invite each other, and each enterprise can claim as many items as it wants, provided there are still items to be purchased. An enterprise may be invited several times, but it may only accept the invitation once. The production rules of the rcg presented in this section that generates L_{basic} model these restrictions.

In this section we present a random context grammar that models the formation of a VBC as described by Ngassam and Raborife [2013], and generates L_{basic}.

Let $G_{\text{rcg}} = (V_{\text{N}}, V_{\text{T}}, P, S)$:

For ease of notation, let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$, $\mathcal{S}^o = \{S_1^o, S_2^o, \ldots, S_m^o\}$, $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$, $\mathcal{A}' = \{A_1', A_2', \ldots, A_m'\}$ $\mathcal{D} = \{D_1, D_2, \ldots, D_m\}$, $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$, $\mathcal{F} = \{F_1, F_2, \ldots, F_m\}$, $\mathcal{X} = \{X_1, X_2, \ldots, X_m\}$, and $\delta = \mathcal{S} \cup \mathcal{X} \cup \mathcal{S}^o \cup \mathcal{A} \cup \{T\}$.

1. $V_{\text{N}} = \{S\} \cup \mathcal{S} \cup \mathcal{S}^o \cup \mathcal{A} \cup \mathcal{A}' \cup \mathcal{D} \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{X} \cup \{X, T\}$.
2. $V_{\text{T}} = \{a_1, a_2, \ldots, a_m\} \cup \{z, f, e, x, d, r\}$.
3. P is the set of productions defined in Figure 5.17. Please note:
   For $i \in [m]$, in the productions $S_i \to S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o(\{A_i'\}; \mathcal{A} \cup \mathcal{X})$ and $S_i \to S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o F_i(; \{A_i, A_i'\})$,

- all $j_{n_1}, j_{n_2}, \ldots, j_{n_i}$ are distinct,
- $i \notin \{j_{n_1}, j_{n_2}, \ldots, j_{n_i}\}$, and
- $1 \leq n_i \leq m$.

$$\begin{aligned}
S &\rightarrow TX && (5.254)\\
T &\rightarrow TX| && (5.255)\\
&\rightarrow S_i^o && (5.256)\\
S_i^o &\rightarrow S_i\ \left(;\left\{A_i', D_i, E_i, F_i, S_i^o\right\}\right) && (5.257)\\
&\rightarrow r\ \left(\left\{A_i'\right\};\right) && (5.258)\\
&\rightarrow r\ (\{D_i\};) && (5.259)\\
&\rightarrow r\ (\{E_i\};) && (5.260)\\
&\rightarrow r\ (\{F_i\};) && (5.261)\\
&\rightarrow r\ (\{S_i^o\};) && (5.262)\\
S_i &\rightarrow A_iS_i\ (\{X\}; \mathcal{A} \cup \mathcal{X}) && (5.263)\\
&\rightarrow D_i\ (;\{X\}) && (5.264)\\
&\rightarrow E_i\ \left(;\left\{A_i, A_i'\right\}\right) && (5.265)\\
&\rightarrow z\ (\{A_i'\}; \mathcal{A} \cup \mathcal{X}) && (5.266)\\
&\rightarrow S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o\ (\{A_i'\}; \mathcal{A} \cup \mathcal{X}) && (5.267)\\
&\rightarrow S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o F_i\ \left(;\left\{A_i, A_i'\right\}\right) && (5.268)\\
X &\rightarrow X_i\ (\{A_i\}; \{X_i\}) && (5.269)\\
A_i &\rightarrow A_i'\ (\{X_i\};) && (5.270)\\
X_i &\rightarrow x\ (;\{A_i\}) && (5.271)\\
A_i' &\rightarrow a_i\ (;\delta) && (5.272)\\
D_i &\rightarrow d\ (;\delta) && (5.273)\\
E_i &\rightarrow e\ (;\delta) && (5.274)\\
F_i &\rightarrow f\ (;\delta) && (5.275)\\
X &\rightarrow x\ (;\delta) && (5.276)
\end{aligned}$$

FIGURE 5.17: Rcg generating L_{basic}

In the grammar G_{rcg}, all enterprises have the same rule templates. The number of items that can be claimed by members of a coalition is provided before the coalition can be formed. In the production rules 5.254–5.256, the initiator enterprise is invited to join the coalition after the overall quantity of the required items has been placed in the sentential form.

According to production rules 5.257–5.262, an invited enterprise, say enterprise i, has the following options:

1. In rule 5.257, enterprise i is put into the position to perform its actions, that is, claim items with / without inviting other enterprises, invite other enterprises without claiming items, or opt out of the coalition. Forbidding context is used to ensure that this rule does not apply if enterprise i has already been invited before the current invitation. This is in accordance to the VBC description that an enterprise may only participate once per formed coalition.

2. Production rules 5.258–5.262 are applicable if enterprise i is already a part of the coalition. This scenario arises when an enterprise is invited to join the coalition by more than one enterprise. Permitting context is used to check whether or not enterprise i has been a part of the coalition before this invitation. This is achieved by examining the sentential form for non-terminal symbols associated with the enterprise i.

Production rules 5.263–5.268 are applicable if enterprise i has not been part of the coalition before the current invitation. These rules illustrate the actions that may be performed by enterprise i.

1. Rule 5.263 applies when enterprise i claims an item. Permitting context is used to ensure that this rule only applies if there is at least one item available. Forbidding context is used to ensure that if enterprise i is claiming an item, no other enterprises are in the process of claiming items. This is to avoid a situation in which two enterprises claim the same item.

2. Production rule 5.264 is applicable if enterprise i does not find any items available to be claimed. We use forbidding context to ensure that this rule does not apply if there are items available.

3. Rule 5.265 applies if enterprise i decides to opt out of the coalition. Forbidding context is used to ensure that this rule does not apply if enterprise i has claimed at least one item.

4. Rule 5.266 applies if enterprise i claims at least one item without ever inviting other enterprises. Permitting context is used to ensure that this rule only applies if enterprise i has claimed at least one item, represented by the existence of an A_i' in the sentential form. Forbidding context is used to ensure that this rule does not apply if enterprise i or any other enterprise is claiming items at that point in time.

5. Production rule 5.267 is applicable if enterprise i invites other enterprises after claiming at least one item. Permitting and forbidding contexts are used as in rule 5.266.

6. Rule 5.268 is employed when enterprise i invites other enterprises without claiming items. Forbidding context is used to make sure that this rule does not apply if enterprise i has claimed at least one item.

Production rules 5.269–5.271 apply as follows:

1. Rule 5.269 ensures that once an item has been claimed, the X corresponding to it is immediately marked as claimed. Permitting context ensures that this rule only applies if an item has been claimed as represented by A_i. Forbidding context is used to ensure that no more than one X for each claimed item is marked.

2. Rule 5.270 ensures that an A_i does not produce more than one X_i.

3. Rule 5.271 replaces each marked claimed item with a terminal symbol x.

Rules 5.272–5.275 remove all non-terminals associated with enterprise i. Forbidding context is used to ensure that these rules only apply once all invited enterprises have performed their actions.

Rule 5.276 rewrites all the items left unclaimed once all the invited enterprises have performed their actions.

We demonstrate these concepts and the formation of a coalition in the following example.

Example 5.4. *In the following random context grammar we have five enterprises. Each enterprise can claim items and invite other enterprises.*

Let $G = (V_{\mathrm{N}}, V_{\mathrm{T}}, P, S)$*:*

For ease of notation, let $\mathcal{S} = \{S_1, S_2, S_3, S_4, S_5\}$, $\mathcal{S}^o = \{S_1^o, S_2^o, S_3^o, S_4^o, S_5^o\}$, $\mathcal{A} = \{A_1, A_2, A_3, A_4, A_5\}$, $\mathcal{A}' = \{A_1', A_2', A_3', A_4', A_5'\}$ $\mathcal{D} = \{D_1, D_2, D_3, D_4, D_5\}$, $\mathcal{E} = \{E_1, E_2, E_3, E_4, E_5\}$, $\mathcal{F} = \{F_1, F_2, F_3, F_4, F_5\}$, $\mathcal{X} = \{X_1, X_2, X_3, X_4, X_5\}$, *and* $\delta = \mathcal{S} \cup \mathcal{X} \cup \mathcal{S}^o \cup \mathcal{A} \cup \{T\}$.

1. $V_{\mathrm{N}} = \{S\} \cup \mathcal{S} \cup \mathcal{S}^o \cup \mathcal{A} \cup \mathcal{A}' \cup \mathcal{D} \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{X} \cup \{X, T\}$.

2. $V_{\mathrm{T}} = \{a_1, a_2, a_3, a_4, a_5\} \cup \{z, f, e, x, d, r\}$.

3. P is the set of productions defined in Figures 5.18-5.22.

Figure 5.18 refers to the rule templates 5.254 – 5.256. In this example, enterprise one is the initiator enterprise represented by S_1^o.

$$S \rightarrow TX \quad (5.277)$$
$$T \rightarrow TX \mid \quad (5.278)$$
$$S_1^o \quad (5.279)$$

FIGURE 5.18: Initiating coalition formation

The rule templates 5.257 – 5.262 are exemplified in Figure 5.19.

The rule templates 5.263 – 5.268 are exemplified in Figure 5.20.

The rule templates 5.269 – 5.271 are exemplified in Figure 5.21.

The rule templates 5.272 – 5.276 are exemplified in Figure 5.22.

$$E_1 \rightarrow e(;\delta) \quad (5.355)$$
$$A'_1 \rightarrow a_1(;\delta) \quad (5.356)$$
$$F_1 \rightarrow f(;\delta) \quad (5.357)$$
$$E_2 \rightarrow e(;\delta) \quad (5.358)$$
$$A'_2 \rightarrow a_2(;\delta) \quad (5.359)$$
$$F_2 \rightarrow f(;\delta) \quad (5.360)$$
$$E_3 \rightarrow e(;\delta) \quad (5.361)$$
$$A'_3 \rightarrow a_3(;\delta) \quad (5.362)$$
$$F_3 \rightarrow f(;\delta) \quad (5.363)$$
$$E_4 \rightarrow e(;\delta) \quad (5.364)$$
$$A'_4 \rightarrow a_4(;\delta) \quad (5.365)$$
$$F_4 \rightarrow f(;\delta) \quad (5.366)$$
$$E_5 \rightarrow e(;\delta) \quad (5.367)$$
$$A'_5 \rightarrow a_5(;\delta) \quad (5.368)$$
$$F_5 \rightarrow f(;\delta) \quad (5.369)$$
$$X \rightarrow x(;\delta) \quad (5.370)$$

FIGURE 5.22: Dissolution stage of a coalition

$$
\begin{aligned}
S_1^o &\rightarrow S_1(;\{E_1, F_1, A_1', S_1^o, D_1\}) \mid && (5.280)\\
&\rightarrow r\ (\{D_1\}\,;) \mid && (5.281)\\
&\rightarrow r\ (\{F_1\}\,;) \mid && (5.282)\\
&\rightarrow r\ (\{E_1\}\,;) \mid && (5.283)\\
&\rightarrow r\ \left(\{A_1'\}\,;\right) \mid && (5.284)\\
&\rightarrow r\ (\{S_1^o\}\,;) && (5.285)\\
S_2^o &\rightarrow S_2(;\{E_2, F_2, A_2', S_2^o, D_2\}) \mid && (5.286)\\
&\rightarrow r\ (\{D_2\}\,;) \mid && (5.287)\\
&\rightarrow r\ (\{F_2\}\,;) \mid && (5.288)\\
&\rightarrow r\ (\{E_2\}\,;) \mid && (5.289)\\
&\rightarrow r\ \left(\{A_2'\}\,;\right) \mid && (5.290)\\
&\rightarrow r\ (\{S_2^o\}\,;) && (5.291)\\
S_3^o &\rightarrow S_3(;\{E_3, F_3, A_3', S_3^o, D_3\}) \mid && (5.292)\\
&\rightarrow r\ (\{D_3\}\,;) \mid && (5.293)\\
&\rightarrow r\ (\{F_3\}\,;) \mid && (5.294)\\
&\rightarrow r\ (\{E_3\}\,;) \mid && (5.295)\\
&\rightarrow r\ \left(\{A_3'\}\,;\right) \mid && (5.296)\\
&\rightarrow r\ (\{S_3^o\}\,;) && (5.297)\\
S_4^o &\rightarrow S_4(;\{E_4, F_4, A_4', S_4^o, D_4\}) \mid && (5.298)\\
&\rightarrow r\ (\{D_4\}\,;) \mid && (5.299)\\
&\rightarrow r\ (\{F_4\}\,;) \mid && (5.300)\\
&\rightarrow r\ (\{E_4\}\,;) \mid && (5.301)\\
&\rightarrow r\ \left(\{A_4'\}\,;\right) \mid && (5.302)\\
&\rightarrow r\ (\{S_4^o\}\,;) && (5.303)\\
S_5^o &\rightarrow S_5(;\{E_5, F_5, A_5', S_5^o, D_5\}) \mid && (5.304)\\
&\rightarrow r\ (\{D_5\}\,;) \mid && (5.305)\\
&\rightarrow r\ (\{F_5\}\,;) \mid && (5.306)\\
&\rightarrow r\ (\{E_5\}\,;) \mid && (5.307)\\
&\rightarrow r\ \left(\{A_5'\}\,;\right) \mid && (5.308)\\
&\rightarrow r\ (\{S_5^o\}\,;) && (5.309)
\end{aligned}
$$

Figure 5.19: Initiation stage of a coalition

$$\begin{aligned}
S_1 &\rightarrow A_1 S_1(\{X\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.310)\\
&\rightarrow D_1\ (; \{X\})\ \mid && (5.311)\\
&\rightarrow E_1\ \left(; \left\{A_1, A_1'\right\}\right)\ \mid && (5.312)\\
&\rightarrow z(\{A_1'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.313)\\
&\rightarrow S_3^o S_2^o(\{A_1'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.314)\\
&\rightarrow S_3^o S_2^o F_1(; \{A_1, A_1'\} && (5.315)\\
S_2 &\rightarrow A_2 S_2(\{X\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.316)\\
&\rightarrow D_2\ (; \{X\})\ \mid && (5.317)\\
&\rightarrow E_2\ \left(; \left\{A_2, A_2'\right\}\right)\ \mid && (5.318)\\
&\rightarrow z(\{A_2'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.319)\\
&\rightarrow S_3^o S_5^o(\{A_2'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.320)\\
&\rightarrow S_3^o S_5^o F_2(; \{A_2, A_2'\}) && (5.321)\\
S_3 &\rightarrow A_3 S_3(\{X\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.322)\\
&\rightarrow D_3\ (; \{X\})\ \mid && (5.323)\\
&\rightarrow E_3\ \left(; \left\{A_3, A_3'\right\}\right)\ \mid && (5.324)\\
S_3 &\rightarrow z(\{A_3'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.325)\\
&\rightarrow S_1^o S_4^o(\{A_3'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.326)\\
&\rightarrow S_1^o S_4^o F_3(; \{A_3, A_3'\}) && (5.327)\\
S_4 &\rightarrow A_4 S_4(\{X\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.328)\\
&\rightarrow D_4\ (; \{X\})\ \mid && (5.329)\\
&\rightarrow E_4\ \left(; \left\{A_4, A_4'\right\}\right)\ \mid && (5.330)\\
&\rightarrow z(\{A_4'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.331)\\
&\rightarrow S_2^o(\{A_4'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.332)\\
&\rightarrow S_2^o F_4(; \{A_4, A_4'\}) && (5.333)\\
S_5 &\rightarrow A_5 S_5(\{X\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.334)\\
&\rightarrow D_5\ (; \{X\})\ \mid && (5.335)\\
&\rightarrow E_5\ \left(; \left\{A_5, A_5'\right\}\right)\ \mid && (5.336)\\
&\rightarrow z(\{A_5'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.337)\\
&\rightarrow S_1^o(\{A_4'\}; \mathcal{A} \cup \mathcal{X}) \mid && (5.338)\\
&\rightarrow S_1^o F_4(; \{A_5, A_5'\}) && (5.339)
\end{aligned}$$

FIGURE 5.20: Operational stage of a coalition

$$X \rightarrow X_1(\{A_1\};\{X_1\}) \quad (5.340)$$
$$A_1 \rightarrow A_1' \; (\{X_1\};) \quad (5.341)$$
$$X_1 \rightarrow x \; (;\{A_1\}) \quad (5.342)$$
$$X \rightarrow X_2(\{A_2\};\{X_2\}) \quad (5.343)$$
$$A_2 \rightarrow A_2' \; (\{X_2\};) \quad (5.344)$$
$$X_2 \rightarrow x \; (;\{A_2\}) \quad (5.345)$$
$$X \rightarrow X_3(\{A_3\};\{X_3\}) \quad (5.346)$$
$$A_3 \rightarrow A_3' \; (\{X_3\};) \quad (5.347)$$
$$X_3 \rightarrow x \; (;\{A_3\}) \quad (5.348)$$
$$X \rightarrow X_4(\{A_4\};\{X_4\}) \quad (5.349)$$
$$A_4 \rightarrow A_4' \; (\{X_4\};) \quad (5.350)$$
$$X_4 \rightarrow x \; (;\{A_4\}) \quad (5.351)$$
$$X \rightarrow X_5(\{A_5\};\{X_5\}) \quad (5.352)$$
$$A_5 \rightarrow A_5' \; (\{X_5\};) \quad (5.353)$$
$$X_5 \rightarrow x \; (;\{A_5\}) \quad (5.354)$$

FIGURE 5.21: Operational stage of a coalition

Consider the following situation: There are six items available, of which S_1 wants two, S_2 wants one, S_3 wants two, S_4 opts out and S_5 wants two.

According to our grammar, S starts rewriting the start symbol into six copies of the non-terminal X. These non-terminals indicate the total number available to the coalition.

$$S \implies TX \implies^5 TXXXXXX$$

The initiator enterprise (S_1^o) is introduced to the sentential form, giving it the highest priority to claim items.

$$\implies S_1^o XXXXXX$$

S_1^o claims two items as follows:

$$\Longrightarrow S_1XXXXXX \qquad (rule\ (5.280))$$
$$\Longrightarrow A_1S_1XXXXXX \qquad (rule\ (5.310))$$
$$\Longrightarrow A_1S_1XXXXX_1X \qquad (rule\ (5.340))$$
$$\Longrightarrow A_1'S_1XXXXX_1X \qquad (rule\ (5.341))$$
$$\Longrightarrow A_1'S_1XXXXxX \qquad (rule\ (5.342))$$
$$\Longrightarrow A_1'A_1S_1XXXXxX \qquad (rule\ (5.310))$$
$$\Longrightarrow A_1'A_1S_1X_1XXXxX \qquad (rule\ (5.340))$$
$$\Longrightarrow A_1'A_1'S_1X_1XXXxX \qquad (rule\ (5.341))$$
$$\Longrightarrow A_1'A_1'S_1xXXXxX \qquad (rule\ (5.342))$$

S_1 then invites S_2 and S_3 by applying a rule in rule (5.314) .

$$A_1'A_1'S_1xXXXxX \Longrightarrow A_1'A_1'S_3^oS_2^oxXXXxX$$

S_2 claims an items as follows:

$$\Longrightarrow A_1'A_1'S_3^oS_2xXXXxX \qquad (rule\ (5.286))$$
$$\Longrightarrow A_1'A_1'S_3^oA_2S_2xXXXxX \qquad (rule\ (5.316))$$
$$\Longrightarrow A_1'A_1'S_3^oA_2S_2xX_2XXxX \qquad (rule\ (5.343))$$
$$\Longrightarrow A_1'A_1'S_3^oA_2'S_2xX_2XXxX \qquad (rule\ (5.344))$$
$$\Longrightarrow A_1'A_1'S_3^oA_2'S_2xxXXxX \qquad (rule\ (5.345))$$

S_2 then invites S_3 and S_5 by applying rule (5.320).

$$A_1'A_1'S_3^oA_2'S_2xxXXxX \Longrightarrow A_1'A_1'S_3^oA_2'S_3^oS_5^oxxXXxX$$

The enterprise represented by S_3^o now appears twice on the sentential form. Rule (5.297) applies as follows:

$$A_1'A_1'S_3^oA_2'S_3^oS_5^oxxXXxX \Longrightarrow A_1'A_1'rA_2'S_3^oS_5^oxxXXxX$$

S_3 then claims two items as follows:

$$\Longrightarrow A_1'A_1'rA_2'S_3S_5^oxxXXxX \quad (\text{rule } (5.292))$$
$$\Longrightarrow A_1'A_1'rA_2'A_3S_3S_5^oxxXXxX \quad (\text{rule } (5.322))$$
$$\Longrightarrow A_1'A_1'rA_2'A_3S_3S_5^oxxX_3XxX \quad (\text{rule } (5.346))$$
$$\Longrightarrow A_1'A_1'rA_2'A_3'S_3S_5^oxxX_3XxX \quad (\text{rule } (5.347))$$
$$\Longrightarrow A_1'A_1'rA_2'A_3'S_3S_5^oxxxXxX \quad (\text{rule } (5.348))$$
$$\Longrightarrow A_1'A_1'rA_2'A_3'A_3S_3S_5^oxxxXxX \quad (\text{rule } (5.322))$$
$$\Longrightarrow A_1'A_1'rA_2'A_3'A_3S_3S_5^oxxxX_3xX \quad (\text{rule } (5.346))$$
$$\Longrightarrow A_1'A_1'rA_2'A_3'A_3'S_3S_5^oxxxX_3xX \quad (\text{rule } (5.347))$$
$$\Longrightarrow A_1'A_1'rA_2'A_3'A_3'S_3S_5^oxxxxxX \quad (\text{rule } (5.348))$$

S_3 invites S_1 and S_4 by applying rule (5.326) .

$$A_1'A_1'rA_2'A_3'A_3'S_3S_5^oxxxxxX \Longrightarrow A_1'A_1'rA_2'A_3'A_3'S_1^oS_4^oS_5^oxxxxxX$$

S_4 opts out of the coalition by applying rule (5.298) and rule (5.330).

$$\Longrightarrow A_1'A_1'rA_2'A_3'A_3'S_1^oS_4S_5^oxxxxxX \Longrightarrow A_1'A_1'rA_2'A_3'A_3'S_1^oE_4S_5^oxxxxxX$$

Since the enterprise represented by S_1 has already performed its actions (we have two A_1's in the sentential form), rule (5.285) applies as follows.

$$A_1'A_1'rA_2'A_3'A_3'S_1^oE_4S_5^oxxxxxX \Longrightarrow A_1'A_1'rA_2'A_3'A_3'rE_4S_5^oxxxxxX$$

S_5 claims an item as follows:

$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4S_5xxxxxX \qquad \text{(rule (5.304))}$$
$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A_5S_5xxxxxX \qquad \text{(rule (5.334))}$$
$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A_5S_5xxxxxX_5 \qquad \text{(rule (5.352))}$$
$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A'_5S_5xxxxxX_5 \qquad \text{(rule (5.353))}$$
$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A'_5S_5xxxxxx \qquad \text{(rule (5.354))}$$

Since there are no items left available, S_5 has to apply rule (5.337).

$$A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A'_5S_5xxxxxx \Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A'_5zxxxxxx$$

Since all the enterprises have performed their actions, the following rules apply.

$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3reA'_5zxxxxxx \qquad \text{(rule (5.364))}$$
$$\Longrightarrow A'_1a_1rA'_2A'_3A'_3reA'_5zxxxxxx \qquad \text{(rule (5.356))}$$
$$\Longrightarrow A'_1a_1ra_2A'_3A'_3reA'_5zxxxxxx \qquad \text{(rule (5.359))}$$
$$\Longrightarrow a_1a_1ra_2A'_3A'_3reA'_5zxxxxxx \qquad \text{(rule (5.356))}$$
$$\Longrightarrow a_1a_1ra_2a_3A'_3reA'_5zxxxxxx \qquad \text{(rule (5.362))}$$
$$\Longrightarrow a_1a_1ra_2a_3a_3reA'_5zxxxxxx \qquad \text{(rule (5.362))}$$
$$\Longrightarrow a_1a_1ra_2a_3a_3rea_5zxxxxxx \qquad \text{(rule (5.368))}$$

The generated word is $a_1^2ra_2a_3^2rea_5zx^6$

Based solely on this word, we can deduce the following about the formed VBC. The VBC represented by this word has four enterprises that have claimed items (a_1, a_2, a_3, and a_5). The enterprise represented by a_1 was invited to join the coalition twice. This enterprise also invited at least one other enterprise after claiming two items. The enterprise represented by a_2 claimed an item, and then invited at least one other enterprise to join the coalition. The enterprise represented by a_3 was invited to join the coalition twice. This enterprise invited at least one other enterprise after claiming three items. There is an enterprise that was invited to join the coalition, but opted out. This enterprise is represented by the e in our word. The enterprise represented by a_5 claimed an item *without inviting other enterprises to join the coalition. There were six items made available to this coalition, this is also the overall quantity of items claimed by members of the coalition.*

5.5 Discussion

We have shown that L_{basic} can be generated by an rFcg, an rPcg, and an rcg. We have demonstrated that a random context grammar models coalition formation in a virtual buying cooperative more closely to the description by Ngassam and Raborife [2013] than the two other grammars presented in this chapter. It is also worth mentioning that L_{basic} can be generated by a cfg as shown by G_{cfg}. However, the production rules of this grammar, as with the rPcg and the rFcg presented in this chapter, do not model the interaction strategy amongst enterprises during the formation of a VBC. In this grammar, each enterprise can invite one other enterprise, and each available item is generated as it is claimed by an enterprise.

Let $G_{\text{cfg}} = (V_{\text{N}}, V_{\text{T}}, P, S)$:

1. $V_{\text{N}} = \{S, S_1, S_2, \ldots, S_m\} \cup \{S'_1, S'_2, \ldots, S'_m\} \cup \{X, X', R\}$.
2. $V_{\text{T}} = \{a_1, a_2, \ldots, a_m\} \cup \{x, e, z, f, r, d\}$.
3. P is the set of productions defined as follows:

   For every S_i, where $i \in [m-1]$,

$$
\begin{aligned}
S &\rightarrow S_i \\
S_i &\rightarrow a_i S_i' x \mid \\
&\rightarrow eRX' \mid \\
&\rightarrow fRS_{i+1} \\
S_i' &\rightarrow a_i S_i' x \mid \\
&\rightarrow zRX \mid \\
&\rightarrow dR \mid \\
&\rightarrow RS_{i+1} \\
R &\rightarrow \lambda \\
R &\rightarrow r \\
R &\rightarrow r^2 \\
&\vdots \\
R &\rightarrow r^{m-1} \\
X' &\rightarrow xX \\
X &\rightarrow xX \mid \\
&\rightarrow \lambda
\end{aligned}
$$

In this grammar all enterprises, except for enterprise m, have the following rule templates.

1. $S \rightarrow S_i$ introduces the initiator enterprise i to the sentential form.
2. $S_i \rightarrow a_i S_i' x$ applies when enterprise i claims an item. For every item claimed, an item is generated as represented by x.
3. $S_i \rightarrow eRX'$ applies when enterprise i opts out of the coalition.
4. $S_i \rightarrow fRS_{i+1}$ applies if enterprise i wants to invite another enterprise without claiming items.
5. $S_i' \rightarrow a_i S_i' x$ applies when enterprise i claims another item.
6. $S_i' \rightarrow zRX$ applies when enterprise i claims at least one item, and does not invite another enterprise to join the coalition. Since in our grammar, an enterprise can only invite one other enterprise, the application of this rule signals the end of the coalition formation process.
7. $S_i' \rightarrow dR$ introduces the terminal symbol d to the sentential form.

8. $S'_i \rightarrow RS_{i+1}$ applies when enterprise i invites enterprise $i+1$ after claiming at least one item.

9. The rule templates $R \rightarrow \lambda$, $R \rightarrow r$, $R \rightarrow r^2$, $\ldots$, $R \rightarrow r^{m-1}$ introduce terminal symbols r to the sentential form.

10. $X' \rightarrow xX$ produces an item after an enterprise has opted out of the coalition.

11. $X \rightarrow xX$ produces more items.

12. $X \rightarrow \lambda$ signals the end of the formation process.

Enterprise m has the following rule templates. This enterprise cannot invite another enterprise.

$$
\begin{array}{rcl}
S_m & \rightarrow & a_i S'_m x \mid \\
 & \rightarrow & eRX' \mid \\
S'_m & \rightarrow & a_i S'_m x \mid \\
 & \rightarrow & zRX \mid \\
 & \rightarrow & dR \\
R & \rightarrow & \lambda \\
R & \rightarrow & r \\
R & \rightarrow & r^3 \\
\vdots & & \\
R & \rightarrow & r^{m-1} \\
X' & \rightarrow & xX \\
X & \rightarrow & xX \mid \\
 & \rightarrow & \lambda
\end{array}
$$

In the next section, we present two grammars that generate a language, different from L_{basic}, representing a formed coalition.

Chapter 6

Modelling a Structured VBC

6.1 Introduction

An rFcg, and an rcg that generate the same language representing a formed coalition are presented in this chapter. The rFcg generates this language, but it does not model a VBC environment. However, the rcg does. The production rules of the rFcg generate an available as it is claimed by an enterprise. In addition, all enterprises are invited to join the coalition at the same time. Once invited, these enterprises cannot invite other enterprises to join the coalition. It remains an open question as to whether this language can be generated by an rPcg. The rcg presented in this chapter models all four phases relating to the formation of a VBC. That is, the initiation, operational, dissolution, and post dissolution phases. In the post dissolution phase, the information about the enterprises that have claimed items is grouped together.

Let $\delta = \lambda + z$, and $\rho = e + f + d$ be regular expressions. Let $m \in \mathbb{N}_+$ be the number of enterprises. Then,

$$\begin{aligned} L_{\text{structure}} \quad = \quad & \{vc\alpha x^k \mid v = a_{j_1}^{n_{j_1}} a_{j_2}^{n_{j_2}} \ldots a_{j_q}^{n_{j_q}} \,;\, q \in [m] \,;\, i \in [q] \,;\, \text{LinOrder}\,(v) \,;\, k \in \mathbb{N}_+ \,;\, \alpha \\ & = \alpha_1 \alpha_2 \ldots \alpha_q \,;\, \alpha_i = y^{n_{j_i}} \beta_i \,;\, \beta_i = \delta r^* \rho r^* \,;\, 0 \leq \text{n}_r\,(v) \leq m(m-1) \,;\, n_{j_i} \geq 0 \,; \\ & \sum_{i=1}^{q} n_{j_i} = \text{n}_y\,(\alpha) \leq k \,;\, \text{if } n_{j_i} = 0, \text{then } \text{n}_z\,(\beta_i) = 0 \,;\, \text{if } n_{j_i} \neq 0, \text{then } \text{n}_e\,(\beta_i) = \\ & \text{n}_f\,(\beta_i) = 0 \,;\, \text{if } \text{n}_e\,(\beta_i) \neq 0, \text{then } \text{n}_f\,(\beta_i) = 0 \,;\, \text{if } \text{n}_d\,(\beta_i) > 0, \text{then } \sum_{i=1}^{q} n_{j_i} = k\} \ . \end{aligned}$$

In $L_{\text{structure}}$, we present a situation in which a supplier is mostly interested in the information about enterprises that have claimed items. In the language presented in the previous chapter, information about whether an enterprise has invited other enterprises (with or without claiming items), or opted out of the coalition appears as an entity. In $L_{\text{structure}}$, information regarding the enterprises that have claimed items is on the right-hand side of the central marker c. This makes it easier for a supplier to gather information about these enterprises. The total number of y's in $L_{\text{structure}}$ is always equal to the total number of items claimed by the enterprises. During the restructuring process, the a_{j_i}'s ($i \in [m]$) are replaced by y's on the left-hand side of c. After the grouping the information regarding the enterprises that have claimed items, all the a_{j_i}'s ($i \in [m]$) appear on the right-hand side of c.

Assume $w \in L_{\text{structure}}$. The following information about a formed coalition is represented in w.

1. $a_{j_i}^{n_{j_i}}$ represents the allocation of n_{j_i} items to enterprise a_{j_i}.
2. c is a marker that separates information on enterprises that have claimed items from the information relating to if an enterprise has invited other enterprises (with or without claiming items), opted out of the coalition, the number of repeated invitations, number of enterprises that could not perform their operation due to insufficient number of available items, and the total number of items made available to the coalition (c stands for central marker).
3. $\mathrm{n}_z(w)$ represents the number of enterprises that claimed items, but did not invite other enterprises.
4. $\mathrm{n}_e(w)$ represents the number of enterprises that were invited, but neither claimed items nor invited other enterprises (e stands for exit).
5. $\mathrm{n}_f(w)$ represents the number of enterprises that did not claim items, but invited other enterprises (f stands for forward).
6. $\mathrm{n}_d(w)$ represents the total number of enterprises that were invited to join the coalition but could not join as there were no items available (d stands for depleted).
7. $\mathrm{n}_r(w)$ represents the total number of enterprises that were invited to join the coalition more than once (r stands for repeat).
8. $\mathrm{n}_y(w)$ represents the total number of items claimed by the coalition.
9. $\mathrm{n}_x(w)$ represents the total number of items that were made available to the coalition.

10. k represents the number of items that may be claimed by the enterprises in a coalition.

Example 6.1 illustrates a word in $L_{\text{structure}}$.

Example 6.1. $w = a_1a_2a_5cyzedyzfyrx^3$

In this word, there are three enterprises (a_1, a_2 and a_5) that have claimed items. Each of these enterprises has claimed one item. Two of these enterprises – we do not know which ones – did not invite other enterprises to join the coalition. This is represented by the two occurrences of z in w.

There is an enterprise that was invited to claim items, but opted out of the coalition, which is represented by the e in w. The r in w implies that an enterprise was invited to join the coalition more than once. The f in w implies that there is an enterprise that invited at least one other enterprise without claiming any items. There is an enterprise that was invited to join the coalition, but could not join as there were no items available. This enterprise is represented by the d in w.

The sum of all occurrences of a_i's in w is three, which is the total number of items claimed by the enterprises in the coalition. This is equal to the total number of items that were made available to the coalition, the number of x's. The number of y's in w is three, this is equal to the number of items claimed by members of a coalition.

6.2 Random Forbidding Context Grammar

We present an rFcg that generates $L_{\text{structure}}$:

Let $G_{\text{rFcg}} = (V_{\text{N}}, V_{\text{T}}, P, S)$:

For ease of notation, let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$, $\mathcal{S}^o = \{S_1^o, S_2^o, \ldots, S_m^o\}$, $\mathcal{S}' = \{S_1', S_2', \ldots, S_m'\}$, $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$, $\mathcal{D} = \{D_1, D_2, \ldots, D_m\}$, $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$, $\mathcal{F} = \{F_1, F_2, \ldots, F_m\}$, $\mathcal{Z} = \{Z_1, Z_2, \ldots, Z_m\}$, $\mathcal{X} = \{X_1, X_2, \ldots, X_m\}$, $\mathcal{T} = \{T_1, T_2, \ldots, T_m\}$, $\mathcal{B} = \{B_1, B_2, \ldots, B_m\}$, $\mathcal{B}' = \{B_1', B_2', \ldots, B_m'\}$, $\mathcal{B}'' = \{B_1'', B_2'', ..., B_m''\}$, and $\delta = \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}^o$.

1. $V_{\text{N}} = \{S\} \cup \mathcal{S} \cup \mathcal{S}^o \cup \mathcal{S}' \cup \mathcal{A} \cup \mathcal{D} \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{X} \cup \mathcal{T} \cup \mathcal{B} \cup \mathcal{B}' \cup \mathcal{B}'' \cup \{X, X', X'', B\}$.

2. $V_{\text{T}} = \{a_1, a_2, \ldots, a_m\} \cup \{z, f, e, x, d, c, r, y\}$, and

3. P is the set of productions defined in Figure 6.1, where $i \in [m]$. For $i \in [m]$, in the production rule $S \rightarrow S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o cB_{j_{n_1}} B_{j_{n_2}} \ldots B_{j_{n_i}} X$,

- all $j_{n_1}, j_{n_2}, \ldots, j_{n_i}$ are distinct, and
- $1 \leq n_i \leq m$.

In G_{rFcg}, all enterprises have the same rule templates. Rule 6.1 models the invitation to all enterprises to join the coalition. Rule 6.2 allows enterprise i to claim items for the first time. Forbidding context is used to ensure that this rule does not apply if enterprise i could not perform its operations due to lack of available items as represented by D_i, enterprise i has opted out of the coalition as represented by E_i, there is an F_i[1], if there is an enterprise claiming an item as represented by X', or there are no available items as represented by X''. Rule 6.3 applies if enterprise i opts out of the coalition. Rule 6.4 applies if enterprise i cannot claim items due to the lack of available items. Forbidding context is used to ensure that this rule cannot apply if there are items available, or if this is not the first time enterprise i has been invited to join the coalition. Rule 6.5 applies if enterprise i has opted out of the coalition. Forbidding context ensures that this rule does not apply if enterprise i has claimed at least one item, or has already participated in this coalition. In a VBC, rule 6.6 is relevant when enterprise i invites other enterprises without claiming items. However, in this grammar it prepares for the introduction of an f since all enterprises are invited at the same time.

In rule 6.7, enterprise i claims an item. Forbidding context makes certain that this rule cannot apply if there is another enterprise claiming an item. This avoids the situation in which two or more enterprises claim the same item. In addition, it ensures that this rule does not apply if the enterprise has already opted out of the coalition. Rule 6.8 is applicable if enterprise i does not find any items available to be claimed. Forbidding context is used to ensure that this rule does not apply if there are items available.

Rule 6.9 applies if enterprise i wants to claim an item, after already claiming, but finds that there are no items available. Forbidding context is used to ensure that this rule does not apply if there are items available, or if there is an enterprise claiming an item. Rule 6.10 enables enterprise i to claim another item. Forbidding context is used to ensure that this rule does not apply if there are no items available.

Rule 6.11 generates an item for every claimed item. Forbidding context ensures that this rule applies immediately after enterprise i has claimed an item. Rule 6.12 generates a y for every item claimed. Forbidding context enforces the application of this rule immediately after an item has been claimed.

[1]In a grammar in which its production rules model a VBC environment, this would signal that enterprise i has invited at least one other enterprise without claiming items.

$$
\begin{aligned}
S &\rightarrow S^o_{j_{n_1}} S^o_{j_{n_2}} \dots S^o_{j_{n_i}} c B_{j_{n_1}} B_{j_{n_2}} \dots B_{j_{n_i}} X && (6.1)\\
S^o_i &\rightarrow S_i \ \big(; \big\{D_i, E_i, F_i, X', X''\big\}\big) && (6.2)\\
&\rightarrow T_i && (6.3)\\
B_i &\rightarrow D_i B''_i \ \big(; \big\{D_i, E_i, F_i, S^o_i, X, X'\big\}\big) && (6.4)\\
&\rightarrow E_i B''_i \ \big(; \big\{A_i, D_i, E_i, F_i, S_i, S^o_i, S'_i, Z_i\big\}\big) && (6.5)\\
&\rightarrow F_i B''_i \ \big(; \big\{A_i, D_i, E_i, F_i, S_i, S^o_i, S'_i, Z_i\big\}\big) && (6.6)\\
S_i &\rightarrow A_i S'_i \ (; \mathcal{S}' \cup \{D_i, E_i, F_i, X', X'', Z_i, B'_i, B''_i\}) && (6.7)\\
&\rightarrow T_i \ (; \{X', X\}) && (6.8)\\
S'_i &\rightarrow T_i \ (; \{X, B_i\}) && (6.9)\\
&\rightarrow S_i \ (; \{X, B_i\}) && (6.10)\\
X &\rightarrow X_i X' \ (; \{D_i, E_i, F_i, S_i, S^o_i, T_i\}) && (6.11)\\
B_i &\rightarrow y B'_i \ \big(; \big\{D_i, E_i, F_i, S_i, S^o_i, Z_i, X'', T_i\big\}\big) && (6.12)\\
B'_i &\rightarrow Z_i B''_i \ \big(; \big\{\mathcal{S}'\big\}\big) && (6.13)\\
&\rightarrow B_i \ \big(; \big\{\mathcal{S}'\big\}\big) && (6.14)\\
&\rightarrow B''_i \ \big(; \big\{\mathcal{S}'\big\}\big) && (6.15)\\
X' &\rightarrow X \ (; \mathcal{S}' \cup \{B'_i\}) && (6.16)\\
&\rightarrow X'' \ (; \mathcal{S}' \cup \{B'_i\}) && (6.17)\\
B''_i &\rightarrow \lambda \ (; \delta) && (6.18)\\
&\rightarrow r \ (; \delta) && (6.19)\\
&\rightarrow r^2 \ (; \delta) && (6.20)\\
&\rightarrow r^3 \ (; \delta) && (6.21)\\
&\ \vdots \\
&\rightarrow r^{m-1} \ (; \delta) && (6.22)\\
X &\rightarrow xX \ (; \delta) && (6.23)\\
&\rightarrow x \ (; \delta) && (6.24)\\
A_i &\rightarrow a_i \ (; \delta) && (6.25)\\
Z_i &\rightarrow z \ (; \delta) && (6.26)\\
D_i &\rightarrow d \ (; \delta) && (6.27)\\
E_i &\rightarrow e \ (; \delta) && (6.28)\\
F_i &\rightarrow f \ (; \delta) && (6.29)\\
X_i &\rightarrow x \ (; \delta) && (6.30)\\
T_i &\rightarrow \lambda \ (; \delta) && (6.31)\\
X'' &\rightarrow \lambda \ (; \delta) && (6.32)
\end{aligned}
$$

FIGURE 6.1: rFcg generating $L_{\text{structure}}$

Production rule 6.13 introduces a Z after a string of a_i's. In a VBC model, this would indicate that enterprise i has claimed at least one item without inviting another enterprise. Rule 6.14 allows for the generation of another y if an enterprise claims another item. Rule 6.15 applies if enterprise i has finished claiming items. This rule relates to the aspect of the language definition that there may be no z after a string of y's. In these three rules, forbidding context ensures that these rules do not apply until an item X is generated for the claimed item, and a y corresponding claimed item is also generated.

Rule 6.16 introduces an item to the sentential form that can be claimed by a member of the coalition. Forbidding context ensures that this rule applies before an enterprise can claim any item. Rule 6.17 "marks" an available item in the sentential once all enterprises have performed their operations. This item will be removed later from the sentential form. Forbidding context is used as in rule 6.16.

Rules 6.18–6.22 introduce the r's to the sentential form. In a VBC, this would imply that enterprise i has been invited to join the coalition more than once. In this grammar, this would not be possible since all enterprises are invited at the same time.

Rules 6.23–6.24 generate more x's. Forbidding context is used to ensure that these rules only apply once after all invited enterprises have performed their actions.

Rules 6.25–6.31 remove all non-terminals associated with enterprise i. Forbidding context is used to ensure that these rules only apply once all invited enterprises have performed their actions.

Rule 6.32 removes the unavailable item from the sentential form.

The following example demonstrates these concepts.

Example 6.2. *In this example we have five enterprises ($m = 5$). Each enterprise can either claim items, or opt out of the coalition.*

Let $G = (V_\mathrm{N}, V_\mathrm{T}, P, S)$, where

For ease of notation, let $\mathcal{S} = \{S_1, S_2, S_3, S_4, S_5\}$, $\mathcal{S}^o = \{S_1^o, S_2^o, S_3^o, S_4^o, S_5^o\}$, $\mathcal{S}' = \{S_1', S_2', S_3', S_4', S_5'\}$, $\mathcal{A} = \{A_1, A_2, A_3, A_4, A_5\}$, $\mathcal{D} = \{D_1, D_2, D_3, D_4, D_5\}$, $\mathcal{E} = \{E_1, E_2, E_3, E_4, E_5\}$, $\mathcal{F} = \{F_1, F_2, F_3, F_4, F_5\}$, $\mathcal{Z} = \{Z_1, Z_2, Z_3, Z_4, Z_5\}$, $\mathcal{X} = \{X_1, X_2, X_3, X_4, X_5\}$, $\mathcal{T} = \{T_1, T_2, T_3, T_4, T_5\}$, $\mathcal{B} = \{B_1, B_2, B_3, B_4, B_5\}$, $\mathcal{B}' = \{B_1', B_2', B_3', B_4', B_5'\}$, $\mathcal{B}'' = \{B_1'', B_2'', B_3'', B_4'', B_5''\}$, and $\delta = \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}^o$.

1. *$V_\mathrm{N} = \{S\} \cup \mathcal{S} \cup \mathcal{S}^o \cup \mathcal{S}' \cup \mathcal{A} \cup \mathcal{D} \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{X} \cup \mathcal{T} \cup \mathcal{B} \cup \mathcal{B}' \cup \mathcal{B}'' \cup \{X, X', X'', B\}$.*
2. *$V_\mathrm{T} = \{a_1, a_2, a_3, a_4, a_5\} \cup \{z, f, e, x, d, r, c, y\}$, and*

3. P is the set of productions defined in Figure 6.2 – 6.8, where $i \in [5]$.

Figure 6.2 refers to the rule template 6.1. All five enterprises are invited to join the coalition at the same time.

$$S \rightarrow S_1^o S_2^o S_3^o S_4^o S_5^o c B_1 B_2 B_3 B_4 B_5 X \tag{6.33}$$

FIGURE 6.2: Initiate coalition formation

Figure 6.3 refers to the rule templates 6.2 – 6.6.

$$
\begin{aligned}
S_1^o &\rightarrow S_1\ \left(;\left\{D_1, E_1, F_1, X', X''\right\}\right) && (6.34)\\
&\rightarrow T_1 && (6.35)\\
B_1 &\rightarrow D_1 B_1''\ \left(;\left\{D_1, E_1, F_1, S_1^o, X, X'\right\}\right) && (6.36)\\
&\rightarrow E_1 B_1''\ \left(;\left\{A_1, D_1, E_1, F_1, S_1, S_1^o, S_1', Z_1\right\}\right) && (6.37)\\
&\rightarrow F_1 B_1''\ \left(;\left\{A_1, D_1, E_1, F_1, S_1, S_1^o, S_1', Z_1\right\}\right) && (6.38)\\
S_2^o &\rightarrow S_2\ \left(;\left\{D_2, E_2, F_2, X', X''\right\}\right) && (6.39)\\
&\rightarrow T_2 && (6.40)\\
B_2 &\rightarrow D_2 B_2''\ \left(;\left\{D_2, E_2, F_2, S_2^o, X, X'\right\}\right) && (6.41)\\
&\rightarrow E_2 B_2''\ \left(;\left\{A_2, D_2, E_2, F_2, S_2, S_2^o, S_2', Z_2\right\}\right) && (6.42)\\
&\rightarrow F_2 B_2''\ \left(;\left\{A_2, D_2, E_2, F_2, S_2, S_2^o, S_2', Z_2\right\}\right) && (6.43)\\
S_3^o &\rightarrow S_3\ \left(;\left\{D_3, E_3, F_3, X', X''\right\}\right) && (6.44)\\
&\rightarrow T_3 && (6.45)\\
B_3 &\rightarrow D_3 B_3''\ \left(;\left\{D_3, E_3, F_3, S_3^o, X, X'\right\}\right) && (6.46)\\
&\rightarrow E_3 B_3''\ \left(;\left\{A_3, D_3, E_3, F_3, S_3, S_3^o, S_3', Z_3\right\}\right) && (6.47)\\
&\rightarrow F_3 B_3''\ \left(;\left\{A_3, D_3, E_3, F_3, S_3, S_3^o, S_3', Z_3\right\}\right) && (6.48)\\
S_4^o &\rightarrow S_4\ \left(;\left\{D_4, E_4, F_4, X', X''\right\}\right) && (6.49)\\
&\rightarrow T_4 && (6.50)\\
B_4 &\rightarrow D_4 B_4''\ \left(;\left\{D_4, E_4, F_4, S_4^o, X, X'\right\}\right) && (6.51)\\
&\rightarrow E_4 B_4''\ \left(;\left\{A_4, D_4, E_4, F_4, S_4, S_4^o, S_4', Z_4\right\}\right) && (6.52)\\
&\rightarrow F_4 B_4''\ \left(;\left\{A_4, D_4, E_4, F_4, S_4, S_4^o, S_4', Z_4\right\}\right) && (6.53)\\
S_5^o &\rightarrow S_5\ \left(;\left\{D_5, E_5, F_5, X', X''\right\}\right) && (6.54)\\
&\rightarrow T_5 && (6.55)\\
B_5 &\rightarrow D_5 B_5''\ \left(;\left\{D_5, E_5, F_5, S_5^o, X, X'\right\}\right) && (6.56)\\
&\rightarrow E_5 B_5''\ \left(;\left\{A_5, D_5, E_5, F_5, S_5, S_5^o, S_5', Z_5\right\}\right) && (6.57)\\
&\rightarrow F_5 B_5''\ \left(;\left\{A_5, D_5, E_5, F_5, S_5, S_5^o, S_5', Z_5\right\}\right) && (6.58)
\end{aligned}
$$

FIGURE 6.3: Enterprises performing their operations

Figure 6.4 refers to the rule templates 6.7 – 6.12.

$$
\begin{aligned}
S_1 &\rightarrow A_1 S_1' \; (; \mathcal{S}' \cup \{D_1, E_1, F_1, X', X'', Z_1, B_1', B_1''\}) && (6.59)\\
&\rightarrow T_1 \; (; \{X', X\}) && (6.60)\\
S_1' &\rightarrow T_1 \; (; \{X, B_1\}) && (6.61)\\
&\rightarrow S_1 \; (; \{X, B_1\}) && (6.62)\\
X &\rightarrow X_1 X' \; (; \{D_1, E_1, F_1, S_1, S_1^o, T_1\}) && (6.63)\\
B_1 &\rightarrow y B_1' \; \big(; \big\{D_1, E_1, F_1, S_1, S_1^o, Z_1, X'', T_1\big\}\big) && (6.64)\\
S_2 &\rightarrow A_2 S_2' \; (; \mathcal{S}' \cup \{D_2, E_2, F_2, X', X'', Z_2, B_2', B_2''\}) && (6.65)\\
&\rightarrow T_2 \; (; \{X', X\}) && (6.66)\\
S_2' &\rightarrow T_2 \; (; \{X, B_2\}) && (6.67)\\
&\rightarrow S_2 \; (; \{X, B_2\}) && (6.68)\\
X &\rightarrow X_2 X' \; (; \{D_2, E_2, F_2, S_2, S_2^o, T_2\}) && (6.69)\\
B_2 &\rightarrow y B_2' \; \big(; \big\{D_2, E_2, F_2, S_2, S_2^o, Z_2, X'', T_2\big\}\big) && (6.70)\\
S_3 &\rightarrow A_3 S_3' \; (; \mathcal{S}' \cup \{D_3, E_3, F_3, X', X'', Z_3, B_3', B_3''\}) && (6.71)\\
&\rightarrow T_3 \; (; \{X', X\}) && (6.72)\\
S_3' &\rightarrow T_3 \; (; \{X, B_3\}) && (6.73)\\
&\rightarrow S_3 \; (; \{X, B_3\}) && (6.74)\\
X &\rightarrow X_3 X' \; (; \{D_3, E_3, F_3, S_3, S_3^o, T_3\}) && (6.75)\\
B_3 &\rightarrow y B_3' \; \big(; \big\{D_3, E_3, F_3, S_3, S_3^o, Z_3, X'', T_3\big\}\big) && (6.76)\\
S_4 &\rightarrow A_4 S_4' \; (; \mathcal{S}' \cup \{D_4, E_4, F_4, X', X'', Z_4, B_4', B_4''\}) && (6.77)\\
&\rightarrow T_4 \; (; \{X', X\}) && (6.78)\\
S_4' &\rightarrow T_4 \; (; \{X, B_4\}) && (6.79)\\
&\rightarrow S_4 \; (; \{X, B_4\}) && (6.80)\\
X &\rightarrow X_4 X' \; (; \{D_4, E_4, F_4, S_4, S_4^o, T_4\}) && (6.81)\\
B_4 &\rightarrow y B_4' \; \big(; \big\{D_4, E_4, F_4, S_4, S_4^o, Z_4, X'', T_4\big\}\big) && (6.82)\\
S_5 &\rightarrow A_5 S_5' \; (; \mathcal{S}' \cup \{D_5, E_5, F_5, X', X'', Z_5, B_5', B_5''\}) && (6.83)\\
&\rightarrow T_5 \; (; \{X', X\}) && (6.84)\\
S_5' &\rightarrow T_5 \; (; \{X, B_5\}) && (6.85)\\
&\rightarrow S_5 \; (; \{X, B_5\}) && (6.86)\\
X &\rightarrow X_5 X' \; (; \{D_5, E_5, F_5, S_5, S_5^o, T_5\}) && (6.87)\\
B_5 &\rightarrow y B_5' \; \big(; \big\{D_5, E_5, F_5, S_5, S_5^o, Z_5, X'', T_5\big\}\big) && (6.88)
\end{aligned}
$$

Figure 6.4: Enterprises claiming items

Figure 6.5 refers to the rule templates 6.13 – 6.17.

$$
\begin{aligned}
B_1' &\rightarrow Z_1 B_1'' \ (;\{\mathcal{S}'\}) && (6.89)\\
&\rightarrow B_1 \ (;\{\mathcal{S}'\}) && (6.90)\\
&\rightarrow B_1'' \ (;\{\mathcal{S}'\}) && (6.91)\\
X' &\rightarrow X \ (;\mathcal{S}' \cup \{B_1'\}) && (6.92)\\
&\rightarrow X'' \ (;\mathcal{S}' \cup \{B_1'\}) && (6.93)\\
B_2' &\rightarrow Z_2 B_2'' \ (;\{\mathcal{S}'\}) && (6.94)\\
&\rightarrow B_2 \ (;\{\mathcal{S}'\}) && (6.95)\\
&\rightarrow B_2'' \ (;\{\mathcal{S}'\}) && (6.96)\\
X' &\rightarrow X \ (;\mathcal{S}' \cup \{B_2'\}) && (6.97)\\
&\rightarrow X'' \ (;\mathcal{S}' \cup \{B_2'\}) && (6.98)\\
B_3' &\rightarrow Z_3 B_3'' \ (;\{\mathcal{S}'\}) && (6.99)\\
&\rightarrow B_3 \ (;\{\mathcal{S}'\}) && (6.100)\\
B_3' &\rightarrow B_3'' \ (;\{\mathcal{S}'\}) && (6.101)\\
X' &\rightarrow X \ (;\mathcal{S}' \cup \{B_3'\}) && (6.102)\\
&\rightarrow X'' \ (;\mathcal{S}' \cup \{B_3'\}) && (6.103)\\
B_4' &\rightarrow Z_4 B_4'' \ (;\{\mathcal{S}'\}) && (6.104)\\
&\rightarrow B_4 \ (;\{\mathcal{S}'\}) && (6.105)\\
&\rightarrow B_4'' \ (;\{\mathcal{S}'\}) && (6.106)\\
X' &\rightarrow X \ (;\mathcal{S}' \cup \{B_4'\}) && (6.107)\\
&\rightarrow X'' \ (;\mathcal{S}' \cup \{B_4'\}) && (6.108)\\
B_5' &\rightarrow Z_5 B_5'' \ (;\{\mathcal{S}'\}) && (6.109)\\
&\rightarrow B_5 \ (;\{\mathcal{S}'\}) && (6.110)\\
&\rightarrow B_5'' \ (;\{\mathcal{S}'\}) && (6.111)\\
X' &\rightarrow X \ (;\mathcal{S}' \cup \{B_5'\}) && (6.112)\\
&\rightarrow X'' \ (;\mathcal{S}' \cup \{B_5'\}) && (6.113)
\end{aligned}
$$

FIGURE 6.5: Balancing the items claimed by the enterprises to the items made available to them

Figure 6.6 refers to the rule templates 6.18 – 6.22.

Figure 6.7 refers to the rule templates 6.23 – 6.24.

Figure 6.8 refers to the rule templates 6.25 – 6.32.

Consider the following situation: There are five items available, of which S_1 wants to claim two items, S_2 wants to claim one item, S_3 wants to claim two items, S_4 opts out and S_5 wants to claim one item.

According to our grammar, the coalition formation process commences as follows.

$$
\begin{aligned}
B_i'' &\rightarrow \lambda \, (; \delta) && (6.114)\\
&\rightarrow r \, (; \delta) && (6.115)\\
&\rightarrow r^2 \, (; \delta) && (6.116)\\
&\rightarrow r^3 \, (; \delta) && (6.117)\\
&\rightarrow r^4 \, (; \delta) && (6.118)
\end{aligned}
$$

FIGURE 6.6: Introducing the r's to the sentential form

$$
\begin{aligned}
X &\rightarrow xX \, (; \delta) && (6.119)\\
&\rightarrow x \, (; \delta) && (6.120)
\end{aligned}
$$

FIGURE 6.7: Introducing items to the sentential form

$$S \Longrightarrow S_1^o S_2^o S_3^o S_4^o S_5^o c B_1 B_2 B_3 B_4 B_5 X$$

The enterprise represented by S_1 claims two items as follows.

$$
\begin{aligned}
&\Longrightarrow S_1 S_2^o S_3^o S_4^o S_5^o c B_1 B_2 B_3 B_4 B_5 X && (\textit{rule } (6.34))\\
&\Longrightarrow A_1 S_1' S_2^o S_3^o S_4^o S_5^o c B_1 B_2 B_3 B_4 B_5 X && (\textit{rule } (6.59))\\
&\Longrightarrow A_1 S_1' S_2^o S_3^o S_4^o S_5^o c B_1 B_2 B_3 B_4 B_5 X_1 X' && (\textit{rule } (6.63))\\
&\Longrightarrow A_1 S_1' S_2^o S_3^o S_4^o S_5^o c y B_1' B_2 B_3 B_4 B_5 X_1 X' && (\textit{rule } (6.64))\\
&\Longrightarrow A_1 S_1 S_2^o S_3^o S_4^o S_5^o c y B_1' B_2 B_3 B_4 B_5 X_1 X' && (\textit{rule } (6.62))\\
&\Longrightarrow A_1 S_1 S_2^o S_3^o S_4^o S_5^o c y B_1 B_2 B_3 B_4 B_5 X_1 X' && (\textit{rule } (6.90))\\
&\Longrightarrow A_1 S_1 S_2^o S_3^o S_4^o S_5^o c y B_1 B_2 B_3 B_4 B_5 X_1 X && (\textit{rule } (6.92))\\
&\Longrightarrow A_1 A_1 S_1' S_2^o S_3^o S_4^o S_5^o c y B_1 B_2 B_3 B_4 B_5 X_1 X && (\textit{rule } (6.59))\\
&\Longrightarrow A_1 A_1 S_1' S_2^o S_3^o S_4^o S_5^o c y B_1 B_2 B_3 B_4 B_5 X_1 X_1 X' && (\textit{rule } (6.63))\\
&\Longrightarrow A_1 A_1 S_1' S_2^o S_3^o S_4^o S_5^o c y y B_1' B_2 B_3 B_4 B_5 X_1 X_1 X' && (\textit{rule } (6.64))
\end{aligned}
$$

The enterprise represented by S_1 signals that it has finished claiming items as follows

$$A_1 \rightarrow a_1\ (;\delta) \tag{6.121}$$
$$Z_1 \rightarrow z\ (;\delta) \tag{6.122}$$
$$D_1 \rightarrow d\ (;\delta) \tag{6.123}$$
$$E_1 \rightarrow e\ (;\delta) \tag{6.124}$$
$$F_1 \rightarrow f\ (;\delta) \tag{6.125}$$
$$X_1 \rightarrow x\ (;\delta) \tag{6.126}$$
$$T_1 \rightarrow \lambda\ (;\delta) \tag{6.127}$$
$$A_2 \rightarrow a_2\ (;\delta) \tag{6.128}$$
$$Z_2 \rightarrow z\ (;\delta) \tag{6.129}$$
$$D_2 \rightarrow d\ (;\delta) \tag{6.130}$$
$$E_2 \rightarrow e\ (;\delta) \tag{6.131}$$
$$F_2 \rightarrow f\ (;\delta) \tag{6.132}$$
$$X_2 \rightarrow x\ (;\delta) \tag{6.133}$$
$$T_2 \rightarrow \lambda\ (;\delta) \tag{6.134}$$
$$A_3 \rightarrow a_3\ (;\delta) \tag{6.135}$$
$$Z_3 \rightarrow z\ (;\delta) \tag{6.136}$$
$$D_3 \rightarrow d\ (;\delta) \tag{6.137}$$
$$E_3 \rightarrow e\ (;\delta) \tag{6.138}$$
$$F_3 \rightarrow f\ (;\delta) \tag{6.139}$$
$$X_3 \rightarrow x\ (;\delta) \tag{6.140}$$
$$T_3 \rightarrow \lambda\ (;\delta) \tag{6.141}$$
$$A_4 \rightarrow a_4\ (;\delta) \tag{6.142}$$
$$Z_4 \rightarrow z\ (;\delta) \tag{6.143}$$
$$D_4 \rightarrow d\ (;\delta) \tag{6.144}$$
$$E_4 \rightarrow e\ (;\delta) \tag{6.145}$$
$$F_4 \rightarrow f\ (;\delta) \tag{6.146}$$
$$X_4 \rightarrow x\ (;\delta) \tag{6.147}$$
$$T_4 \rightarrow \lambda\ (;\delta) \tag{6.148}$$
$$A_5 \rightarrow a_5\ (;\delta) \tag{6.149}$$
$$Z_5 \rightarrow z\ (;\delta) \tag{6.150}$$
$$D_5 \rightarrow d\ (;\delta) \tag{6.151}$$
$$E_5 \rightarrow e\ (;\delta) \tag{6.152}$$
$$F_5 \rightarrow f\ (;\delta) \tag{6.153}$$
$$X_5 \rightarrow x\ (;\delta) \tag{6.154}$$
$$T_5 \rightarrow \lambda\ (;\delta) \tag{6.155}$$
$$X'' \rightarrow \lambda\ (;\delta) \tag{6.156}$$

FIGURE 6.8: Dissolving a coalition

$$\Longrightarrow A_1A_1T_1S_2^oS_3^oS_4^oS_5^ocyyB_1'B_2B_3B_4B_5X_1X_1X' \qquad (rule\ (6.61))$$

Before the enterprise represented by S_2 can claim an item, we have to "free" an item as follows.

$$\Longrightarrow A_1A_1T_1S_2^oS_3^oS_4^oS_5^ocyyB_1''B_2B_3B_4B_5X_1X_1X' \qquad (rule\ (6.91))$$
$$\Longrightarrow A_1A_1T_1S_2^oS_3^oS_4^oS_5^ocyyB_1''B_2B_3B_4B_5X_1X_1X \qquad (rule\ (6.92))$$

The enterprise represented by S_2 claims one item as follows.

$$\Longrightarrow A_1A_1T_1S_2S_3^oS_4^oS_5^ocyyB_1''B_2B_3B_4B_5X_iX_iX \qquad (rule\ (6.39))$$
$$\Longrightarrow A_1A_1T_1A_2S_2'S_3^oS_4^oS_5^ocyyB_1''B_2B_3B_4B_5X_1X_1X \qquad (rule\ (6.65))$$
$$\Longrightarrow A_1A_1T_1A_2S_2'S_3^oS_4^oS_5^ocyyB_1''B_2B_3B_4B_5X_1X_1X_2X' \qquad (rule\ (6.69))$$
$$\Longrightarrow A_1A_1T_1A_2S_2'S_3^oS_4^oS_5^ocyyB_1''yB_2'B_3B_4B_5X_1X_1X_2X' \qquad (rule\ (6.70))$$

Before the enterprise represented by S_3 can claim its items, we need to "free" an item as follows.

$$\Longrightarrow A_1A_1T_1A_2T_2S_3^oS_4^oS_5^ocyyB_1''yB_2'B_3B_4B_5X_1X_1X_2X' \qquad (rule\ (6.67))$$
$$\Longrightarrow A_1A_1T_1A_2T_2S_3^oS_4^oS_5^ocyyB_1''yZ_2B_2''B_3B_4B_5X_1X_1X_2X' \qquad (rule\ (6.94))$$
$$\Longrightarrow A_1A_1T_1A_2T_2S_3^oS_4^oS_5^ocyyB_1''yZ_2B_2''B_3B_4B_5X_1X_1X_2X \qquad (rule\ (6.97))$$

The enterprise represented by S_3 claims two items as follows.

$$\Longrightarrow A_1A_1T_1A_2T_2S_3S_4^oS_5^ocyyB_1''yZ_2B_2''B_3B_4B_5X_1X_1X_2X \quad (rule\ (6.44))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3S_3'S_4^oS_5^ocyyB_1''yZ_2B_2''B_3B_4B_5X_1X_1X_2X \quad (rule\ (6.71))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3S_3'S_4^oS_5^ocyyB_1''yZ_2B_2''B_3B_4B_5X_1X_1X_2X_3X' \quad (rule\ (6.75))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3S_3'S_4^oS_5^ocyyB_1''yZ_2B_2''yB_3'B_4B_5X_1X_1X_2X_3X' \quad (rule\ (6.76))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3S_3S_4^oS_5^ocyyB_1''yZ_2B_2''yB_3'B_4B_5X_1X_1X_2X_3X' \quad (rule\ (6.74))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3S_3S_4^oS_5^ocyyB_1''yZ_2B_2''yB_3B_4B_5X_1X_1X_2X_3X' \quad (rule\ (6.100))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3S_3S_4^oS_5^ocyyB_1''yZ_2B_2''yB_3B_4B_5X_1X_1X_2X_3X \quad (rule\ (6.102))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3S_3'S_4^oS_5^ocyyB_1''yZ_2B_2''yB_3B_4B_5X_1X_1X_2X_3X \quad (rule\ (6.71))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3S_3'S_4^oS_5^ocyyB_1''yZ_2B_2''yB_3B_4B_5X_1X_1X_2X_3X_3X' \quad (rule\ (6.75))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3S_3'S_4^oS_5^ocyyB_1''yZ_2B_2''yyB_3'B_4B_5X_1X_1X_2X_3X_3X' \quad (rule\ (6.76))$$

The enterprise represented by S_4 opts out of the coalition.

$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3S_3'T_4S_5^ocyyB_1''yZ_2B_2''yyB_3'B_4B_5X_1X_1X_2X_3X_3X' \quad (rule\ (6.50))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3S_3'T_4S_5^ocyyB_1''yZ_2B_2''yyB_3'E_4B_4''B_5X_1X_1X_2X_3X_3X' \quad (rule\ (6.52))$$

In this example, we have five items available, and all the available items have been claimed. The following process removes an item as follows.

$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4S_5^ocyyB_1''yZ_2B_2''yyB_3'E_4B_4''B_5X_1X_1X_2X_3X_3X' \quad (rule\ (6.73))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4S_5^ocyyB_1''yZ_2B_2''yyB_3''E_4B_4''B_5X_1X_1X_2X_3X_3X' \quad (rule\ (6.101))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4S_5^ocyyB_1''yZ_2B_2''yyB_3''E_4B_4''B_5X_1X_1X_2X_3X_3X'' \quad (rule\ (6.103))$$

The enterprise represented by S_5 cannot claim any items as there are no items available.

$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4T_5cyyB_1''yZ_2B_2''yyB_3''E_4B_4''B_5X_1X_1X_2X_3X_3X'' \quad (rule\ (6.55))$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4T_5cyyB_1''yZ_2B_2''yyB_3''E_4B_4''D_5B_5''X_1X_1X_2X_3X_3X'' \quad (rule\ (6.56))$$

The r's are introduced to the sentential form as follows.

$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4T_5cyyB_1''yZ_2yyB_3''E_4B_4''D_5B_5''X_1X_1X_2X_3X_3X'' \quad \text{(rule (6.114))}$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyB_3''E_4B_4''D_5B_5''X_1X_1X_2X_3X_3X'' \quad \text{(rule (6.115))}$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4B_4''D_5B_5''X_1X_1X_2X_3X_3X'' \quad \text{(rule (6.116))}$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5B_5''X_1X_1X_2X_3X_3X'' \quad \text{(rule (6.114))}$$
$$\Longrightarrow A_1A_1T_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5X_1X_1X_2X_3X_3X'' \quad \text{(rule (6.114))}$$

Since all the enterprises have performed their operations, the following rules apply.

$$\Longrightarrow a_1A_1T_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5X_1X_1X_2X_3X_3X'' \quad \text{(rule (6.121))}$$
$$\Longrightarrow a_1a_1T_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5X_1X_1X_2X_3X_3X'' \quad \text{(rule (6.121))}$$
$$\Longrightarrow a_1a_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5X_1X_1X_2X_3X_3X'' \quad \text{(rule (6.127))}$$
$$\Longrightarrow a_1a_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5xX_1X_2X_3X_3X'' \quad \text{(rule (6.126))}$$
$$\Longrightarrow a_1a_1A_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5xxX_2X_3X_3X'' \quad \text{(rule (6.126))}$$
$$\Longrightarrow a_1a_1a_2T_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5xxX_2X_3X_3X'' \quad \text{(rule (6.128))}$$
$$\Longrightarrow a_1a_1a_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5xxX_2X_3X_3X'' \quad \text{(rule (6.134))}$$
$$\Longrightarrow a_1a_1a_2A_3A_3T_3T_4T_5cyyryZ_2yyrrE_4D_5xxxX_3X_3X'' \quad \text{(rule (6.133))}$$
$$\Longrightarrow a_1a_1a_2A_3A_3T_3T_4T_5cyyryzyyrrE_4D_5xxxX_3X_3X'' \quad \text{(rule (6.129))}$$
$$\Longrightarrow a_1a_1a_2a_3A_3T_3T_4T_5cyyryzyyrrE_4D_5xxxX_3X_3X'' \quad \text{(rule (6.135))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3T_3T_4T_5cyyryzyyrrE_4D_5xxxX_3X_3X'' \quad \text{(rule (6.135))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3T_4T_5cyyryzyyrrE_4D_5xxxX_3X_3X'' \quad \text{(rule (6.141))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3T_4T_5cyyryzyyrrE_4D_5xxxxX_3X'' \quad \text{(rule (6.140))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3T_4T_5cyyryzyyrrE_4D_5xxxxxX'' \quad \text{(rule (6.140))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3T_5cyyryzyyrrE_4D_5xxxxxX'' \quad \text{(rule (6.147))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3T_5cyyryzyyrreD_5xxxxxX'' \quad \text{(rule (6.145))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3cyyryzyyrreD_5xxxxxX'' \quad \text{(rule (6.155))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3cyyryzyyrredxxxxxX'' \quad \text{(rule (6.151))}$$
$$\Longrightarrow a_1a_1a_2a_3a_3cyyryzyyrredxxxxx \quad \text{(rule (6.156))}$$

The generated word is $a_1^2a_3^2cy^2ryzy^2r^2edx^5$

This word represents a coalition in which two enterprises have claimed items, with each enterprise having claimed two items which is represented by $a_1^2a_3^2$*. There is an enterprise that was invited to join the coalition, but it opted out of the coalition. This is represented by an occurrence of* e*.*

Even though the rFcg presented in this section generates $L_{\text{structure}}$, its production rules do not model a VBC environment. For instance, all enterprises are invited to join the coalition at the same time, and an available item is generated as it is claimed by an enterprise. In the next section, we present an rcg that generates $L_{\text{structure}}$, and models coalition formation in a VBC.

6.3 Random Context Grammar

The main aim of this study is to model coalition formation in a VBC as specified by Ngassam and Raborife [2013]. In that description, forming a coalition involves an enterprise approaching a supplier with an intent to purchase items. The supplier in turn replies with the overall available quantity of the requested items. The initiator enterprise then purchases items, and invites selected associates, who in turn invite their associates, etc., to join the coalition in order to purchase the items from a supplier. The total number of items purchased by members of the coalition cannot be more than the quantity made available to them by the supplier. In a coalition, only invited enterprises may purchase items and/or invite other enterprises.

In the rFcg presented in this chapter, the enterprises are invited at the same same. In addition, items are generated as they are claimed by members of the coalition. In the rcg presented in this section, enterprises invite as many enterprises they want without violating the condition that an enterprise can only participate in a coalition once. In addition, the items that can be claimed by members of the coalition are generated before the coalition formation process begins. This coalition formation process aligns to the description by Ngassam and Raborife [2013].

The following rcg generates $L_{\text{structure}}$:

Let $G_{\text{rcg}} = (V_{\text{N}}, V_{\text{T}}, P, S)$, where

For ease of notation, let $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$, $\mathcal{A}' = \{A'_1, A'_2, \ldots, A'_m\}$, $\mathcal{A}'_i = \{A'_1, A'_2, \ldots, A'_i\}$, $\mathcal{S} = \{S_1, S_2, \ldots S_m\}$, $\mathcal{S}^o = \{S^o_1, S^o_2, \ldots, S^o_m\}$, $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$, $\mathcal{F} = \{F_1, F_2, \ldots, F_m\}$, $\mathcal{X} = \{X_1, X_2, \ldots, X_m\}$, $\mathcal{B} = \{B_1, B_2, \ldots, B_m\}$, $\mathcal{B}' = \{B'_1, B'_2, \ldots, B'_m\}$, $\mathcal{A}'' = \{A''_1, A''_2, \ldots, A''_m\}$, and $\delta = \mathcal{A} \cup \mathcal{S} \cup \mathcal{S}^o \cup T$.

1. $V_\mathrm{N} = \{S\} \cup \mathcal{S} \cup \mathcal{A} \cup \mathcal{A}' \cup \mathcal{S}^o \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{X} \cup \mathcal{B} \cup \mathcal{B}' \cup \mathcal{A}'' \cup \{X, T\}$.

2. $V_\mathrm{T} = \{a_1, a_2, \dots, a_m\} \cup \{c, z, f, e, x, d, r, y\}$.

3. P is the set of productions defined in Figure 6.9: Please note that for any non-terminal symbol P, $P_0 = \emptyset$.
   For $i \in [m]$, in the productions $S_i \to S^o_{j_{n_1}} S^o_{j_{n_2}} \dots S^o_{j_{n_i}}(\{A'_i\}; \mathcal{A} \cup \mathcal{X})$ and $S_i \to S^o_{j_{n_1}} S^o_{j_{n_2}} \dots S^o_{j_{n_i}} F_i(; \{A_i, A'_i\})$,
   - all $j_{n_1}, j_{n_2}, \dots, j_{n_i}$ are distinct,
   - $i \notin \{j_{n_1}, j_{n_2}, \dots, j_{n_i}\}$, and
   - $1 \le n_i \le m$.

4. S is the start symbol

In G_rcg, all enterprises have the same set of rules. Rules 6.157–6.159 ensure that the initiator enterprise S^o_i is invited to join the coalition after the overall quantity of the required items has been placed in the sentential form.

An invited enterprise, say enterprise i, has the following options:

1. In rule 6.160, enterprise i prepares to perform its operations. Forbidding context is used to ensure that this rule cannot be applied if enterprise i has already been invited before the current invitation.

2. Production rules 6.161–6.165 are applicable if invited enterprise i is already a part of coalition. Permitting context is used to check that the invited enterprise i has been a part of the coalition before this invitation. This is achieved by examining the sentential form for non-terminal symbols associated with the enterprise i.

Productions rules 6.166–6.171 are applicable if enterprise i has not been part of the coalition before the current invitation. These rules illustrate the actions that may be performed by an invited enterprise i.

1. Rule 6.166 applies when enterprise i claims an item. Permitting context is used to ensure that this rule only applies if there is at least one item available. Forbidding context is used to ensure that if enterprise i is claiming an item, no other enterprises are in the process of claiming items. This is to avoid a situation in which two enterprises claim the same item.

$$
\begin{aligned}
S &\rightarrow S_0TX && (6.157)\\
T &\rightarrow TX && (6.158)\\
&\rightarrow S_i^o && (6.159)\\
S_i^o &\rightarrow S_i\ \left(;\left\{A_i', D_i, E_i, F_i, S_i^o\right\}\right) && (6.160)\\
&\rightarrow r\ \left(\left\{A_i'\right\};\right) && (6.161)\\
&\rightarrow r\ (\{D_i\};) && (6.162)\\
&\rightarrow r\ (\{E_i\};) && (6.163)\\
&\rightarrow r\ (\{F_i\};) && (6.164)\\
&\rightarrow r\ (\{S_i^o\};) && (6.165)\\
S_i &\rightarrow A_iS_i\ (\{X\}; \mathcal{A} \cup \mathcal{X}) && (6.166)\\
&\rightarrow D_i\ (;\{X\}) && (6.167)\\
&\rightarrow E_i\ \left(;\left\{A_i, A_i'\right\}\right) && (6.168)\\
&\rightarrow z\ (\{A_i'\}; \mathcal{A} \cup \mathcal{X}) && (6.169)\\
&\rightarrow S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o\ (\{A_i'\}; \mathcal{A} \cup \mathcal{X}) && (6.170)\\
&\rightarrow S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o F_i\ \left(;\left\{A_i, A_i'\right\}\right) && (6.171)\\
X &\rightarrow X_i\ (\{A_i\}; \{X_i\}) && (6.172)\\
A_i &\rightarrow A_i'\ (\{X_i\};) && (6.173)\\
X_i &\rightarrow x\ (;\{A_i\}) && (6.174)\\
E_i &\rightarrow e\ (;\delta) && (6.175)\\
F_i &\rightarrow f\ (;\delta) && (6.176)\\
X &\rightarrow x\ (;\delta) && (6.177)\\
D_i &\rightarrow d\ (;\delta) && (6.178)\\
S_0 &\rightarrow B_iS_0\ (\{A_i'\}; \delta \cup \mathcal{A}'_{i-1} \cup \{A_i'', B_i\}) && (6.179)\\
A_i' &\rightarrow A_i''\ (\{B_i\}; \{A_i''\}) && (6.180)\\
B_i &\rightarrow a_i\ \left(\left\{A_i''\right\};\right) && (6.181)\\
A_i'' &\rightarrow y\ (;\{B_i\}) && (6.182)\\
S_0 &\rightarrow c\ (;\delta \cup \mathcal{A}') && (6.183)
\end{aligned}
$$

FIGURE 6.9: Rcg generating $L_{\text{structure}}$

2. Production rule 6.167 is applicable if enterprise i does not find any items available to be claimed. We use forbidding context to ensure that this rule does not apply if there are items available.

3. Rule 6.168 applies if enterprise i decides to opt out of the coalition. Forbidding context is used to ensure that this rule does not apply if enterprise i has claimed at least one item, represented by either A_i, or A_i' in the sentential form.

4. Rule 6.169 applies if enterprise i claims at least one item without ever inviting other enterprises. Permitting context is used to ensure that this rule only applies

if enterprise i has claimed at least one item, represented by the existence of an A'_i in the sentential form. Forbidding context is used to ensure that this rule does not apply if enterprise i or any other enterprise is claiming items at that point in time.

5. Production rule 6.170 is applicable if enterprise i invites other enterprises after claiming at least one item. Permitting and forbidding contexts are used as in rule 6.169 above.

6. Rule 6.171 is employed when enterprise i invites other enterprises without claiming items. Forbidding context is used to make sure that this rule does not apply if enterprise i has claimed at least one item.

Rules 6.172 – 6.174 ensure that for every item claimed, exactly one X is marked as claimed. This guarantees that enterprises do not claim more items than were made available by the supplier. These rules apply as follows:

1. Rule 6.172 ensures that once an item has been claimed, exactly one X is marked as claimed (permitting context). Forbidding context is used to ensure that no more than one X for each claimed item is marked.

2. Rule 6.173 ensures that an A_i does not produce more than one X_i.

3. Rule 6.174 replaces each claimed item with a terminal symbol x.

Rules 6.175–6.178 apply once all invited enterprises have performed their actions. This is achieved by checking if there are any non-terminals associated with these enterprises in the sentential form, using forbidding context.

In rules 6.179–6.182, S_0 is used to restructure all the items claimed by the enterprises. This operation only commences once all invited enterprises have performed their operations. For each enterprise i, each item claimed by the enterprise is replicated on the left-hand side of S_0 with the non-terminal B_i. This is a result of the application of rule 6.179. In addition, items claimed by enterprise i can only be restructured once the items claimed by enterprises $[i-1]$ have been restructured.

The production rules 6.180 and 6.181 ensure that for each A_i, exactly one B_i is produced. Both permitting and forbidding context are used to ensure that the matching is correct. Once the matching has been completed for enterprise i, A''_i is replaced by an y. This is accomplished by rule 6.182.

Once the restructuring has been concluded, we replace the S_0 with a c as in rule 6.183.

We demonstrate these concepts and the formation of a coalition in the following example.

Example 6.3. *In the following rcg , we have five enterprises. Each enterprise can claim items, or opt out of the coalition.*

Let $G = (V_\mathrm{N}, V_\mathrm{T}, P, S)$*:*

For ease of notation, let $\mathcal{A} = \{A_1, A_2, A_3, A_4, A_5\}$*,* $\mathcal{A}' = \{A'_1, A'_2, A'_3, A'_4, A'_5\}$*,* $\mathcal{A}'_i = \{A'_1, A'_2, \ldots, A'_i\}$*,* $\mathcal{S} = \{S_1, S_2, S_3, S_4, S_5\}$*,* $\mathcal{S}^o = \{S^o_1, S^o_2, S^o_3, S^o_4, S^o_5\}$*,* $\mathcal{E} = \{E_1, E_2, E_3, E_4, E_5\}$*,* $\mathcal{F} = \{F_1, F_2, F_3, F_4, F_5\}$*,* $\mathcal{X} = \{X_1, X_2, X_3, X_4, X_5\}$*,* $\mathcal{B} = \{B_1, B_2, B_3, B_4, B_5\}$*,* $\mathcal{B}' = \{B'_1, B'_2, B'_3, B'_4, B'_5\}$ $\mathcal{A}'' = \{A''_1, A''_2, A''_3, A''_4, A''_5\}$*, and* $\delta = \mathcal{A} \cup \mathcal{S} \cup \mathcal{S}^o \cup T$*.*

1. $V_\mathrm{N} = \{S\} \cup \mathcal{S} \cup \mathcal{A} \cup \mathcal{A}' \cup \mathcal{S}^o \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{X} \cup \mathcal{B} \cup \mathcal{B}' \cup \mathcal{A}'' \cup \{X, T\}$.
2. $V_\mathrm{T} = \{a_1, a_2, a_3, a_4, a_5\} \cup \{z, f, e, x, y, d, r\}$.
3. *P is the set of productions defined in Figures 6.10 – 6.16.*

The rule templates 6.157 – 6.159 are exemplified in Figure 6.10. In this example, the enterprise represented by S_1 *is the initiator enterprise.*

$$S \rightarrow S_0 T X \tag{6.184}$$

$$T \rightarrow T X \tag{6.185}$$

$$\rightarrow S^o_1 \tag{6.186}$$

FIGURE 6.10: Initiating a coalition

The production rule templates 6.160 – 6.165 are exemplified in Figure 6.11.

The production rule templates 6.166 – 6.171 are exemplified in Figure 6.12.

The production rule templates 6.172 – 6.174 are exemplified in Figure 6.13.

The rule templates 6.175 – 6.178 are exemplified in Figure 6.14.

Figure 6.15 exemplifies the rule templates 6.179 – 6.182.

Once the restructuring has been concluded, we replace the S_0 *with a c as in Figure 6.16.*

Consider the following situation: There are six items available, of which S_1 *wants two,* S_2 *wants one,* S_3 *wants two,* S_4 *opts out and* S_5 *wants two.*

According to our grammar, S starts rewriting the start symbol into six copies of the non-terminal X. These non-terminals indicate the total number available to the coalition.

$$
\begin{aligned}
S_1^o &\rightarrow S_1\left(;\{E_1, F_1, A_1', S_1^o, D_1\}\right) \mid && (6.187)\\
&\rightarrow r\left(\{D_1\};\right) \mid && (6.188)\\
&\rightarrow r\left(\{F_1\};\right) \mid && (6.189)\\
&\rightarrow r\left(\{E_1\};\right) \mid && (6.190)\\
&\rightarrow r\left(\{A_1'\};\right) \mid && (6.191)\\
&\rightarrow r\left(\{S_1^o\};\right) && (6.192)\\
S_2^o &\rightarrow S_2\left(;\{E_2, F_2, A_2', S_2^o, D_2\}\right) \mid && (6.193)\\
&\rightarrow r\left(\{D_2\};\right) \mid && (6.194)\\
&\rightarrow r\left(\{F_2\};\right) \mid && (6.195)\\
&\rightarrow r\left(\{E_2\};\right) \mid && (6.196)\\
&\rightarrow r\left(\{A_2'\};\right) \mid && (6.197)\\
&\rightarrow r\left(\{S_2^o\};\right) && (6.198)\\
S_3^o &\rightarrow S_3\left(;\{E_3, F_3, A_3', S_3^o, D_3\}\right) \mid && (6.199)\\
&\rightarrow r\left(\{D_3\};\right) \mid && (6.200)\\
&\rightarrow r\left(\{F_3\};\right) \mid && (6.201)\\
&\rightarrow r\left(\{E_3\};\right) \mid && (6.202)\\
&\rightarrow r\left(\{A_3'\};\right) \mid && (6.203)\\
&\rightarrow r\left(\{S_3^o\};\right) && (6.204)\\
S_4^o &\rightarrow S_4\left(;\{E_4, F_4, A_4', S_4^o, D_4\}\right) \mid && (6.205)\\
&\rightarrow r\left(\{D_4\};\right) \mid && (6.206)\\
&\rightarrow r\left(\{F_4\};\right) \mid && (6.207)\\
&\rightarrow r\left(\{E_4\};\right) \mid && (6.208)\\
&\rightarrow r\left(\{A_4'\};\right) \mid && (6.209)\\
&\rightarrow r\left(\{S_4^o\};\right) && (6.210)\\
S_5^o &\rightarrow S_5\left(;\{E_5, F_5, A_5', S_5^o, D_5\}\right) \mid && (6.211)\\
&\rightarrow r\left(\{D_5\};\right) \mid && (6.212)\\
&\rightarrow r\left(\{F_5\};\right) \mid && (6.213)\\
&\rightarrow r\left(\{E_5\};\right) \mid && (6.214)\\
&\rightarrow r\left(\{A_5'\};\right) \mid && (6.215)\\
&\rightarrow r\left(\{S_5^o\};\right) && (6.216)
\end{aligned}
$$

FIGURE 6.11: Initiation stage of a coalition

$$
\begin{aligned}
S_1 &\rightarrow A_1 S_1 \left(\{X\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.217)\\
&\rightarrow D_1 \left(; \{X\}\right) \mid && (6.218)\\
&\rightarrow E_1 \left(; \left\{A_1, A_1'\right\}\right) \mid && (6.219)\\
&\rightarrow z \left(\{A_1'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.220)\\
&\rightarrow S_3^o S_2^o \left(\{A_1'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.221)\\
&\rightarrow S_3^o S_2^o F_1 \left(; \{A_1, A_1'\}\right. && (6.222)\\
S_2 &\rightarrow A_2 S_2 \left(\{X\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.223)\\
&\rightarrow D_2 \left(; \{X\}\right) \mid && (6.224)\\
&\rightarrow E_2 \left(; \left\{A_2, A_2'\right\}\right) \mid && (6.225)\\
&\rightarrow z \left(\{A_2'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.226)\\
&\rightarrow S_3^o S_5^o \left(\{A_2'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.227)\\
&\rightarrow S_3^o S_5^o F_2 \left(; \{A_2, A_2'\}\right) && (6.228)\\
S_3 &\rightarrow A_3 S_3 \left(\{X\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.229)\\
&\rightarrow D_3 \left(; \{X\}\right) \mid && (6.230)\\
&\rightarrow E_3 \left(; \left\{A_3, A_3'\right\}\right) \mid && (6.231)\\
S_3 &\rightarrow z \left(\{A_3'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.232)\\
&\rightarrow S_1^o S_4^o \left(\{A_3'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.233)\\
&\rightarrow S_1^o S_4^o F_3 \left(; \{A_3, A_3'\}\right) && (6.234)\\
S_4 &\rightarrow A_4 S_4 \left(\{X\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.235)\\
&\rightarrow D_4 \left(; \{X\}\right) \mid && (6.236)\\
&\rightarrow E_4 \left(; \left\{A_4, A_4'\right\}\right) \mid && (6.237)\\
&\rightarrow z \left(\{A_4'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.238)\\
&\rightarrow S_2^o \left(\{A_4'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.239)\\
&\rightarrow S_2^o F_4 \left(; \{A_4, A_4'\}\right) && (6.240)\\
S_5 &\rightarrow A_5 S_5 \left(\{X\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.241)\\
&\rightarrow D_5 \left(; \{X\}\right) \mid && (6.242)\\
&\rightarrow E_5 \left(; \left\{A_5, A_5'\right\}\right) \mid && (6.243)\\
&\rightarrow z \left(\{A_5'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.244)\\
&\rightarrow S_1^o \left(\{A_4'\}; \mathcal{A} \cup \mathcal{X}\right) \mid && (6.245)\\
&\rightarrow S_1^o F_4 \left(; \{A_5, A_5'\}\right) && (6.246)
\end{aligned}
$$

FIGURE 6.12: Operational stage of a coalition

$$\begin{aligned}
X &\rightarrow X_1\ (\{A_1\};\{X_1\}) && (6.247)\\
A_1 &\rightarrow A_1'\ (\{X_1\};) && (6.248)\\
X_1 &\rightarrow x\ (;\{A_1\}) && (6.249)\\
X &\rightarrow X_2\ (\{A_2\};\{X_2\}) && (6.250)\\
A_2 &\rightarrow A_2'\ (\{X_2\};) && (6.251)\\
X_2 &\rightarrow x\ (;\{A_2\}) && (6.252)\\
X &\rightarrow X_3\ (\{A_3\};\{X_3\}) && (6.253)\\
A_3 &\rightarrow A_3'\ (\{X_3\};) && (6.254)\\
X_3 &\rightarrow x\ (;\{A_3\}) && (6.255)\\
X &\rightarrow X_4\ (\{A_4\};\{X_4\}) && (6.256)\\
A_4 &\rightarrow A_4'\ (\{X_4\};) && (6.257)\\
X_4 &\rightarrow x\ (;\{A_4\}) && (6.258)\\
X &\rightarrow X_5\ (\{A_5\};\{X_5\}) && (6.259)\\
A_5 &\rightarrow A_5'\ (\{X_5\};) && (6.260)\\
X_5 &\rightarrow x\ (;\{A_5\}) && (6.261)
\end{aligned}$$

FIGURE 6.13: Operational stage of a coalition

$$\begin{aligned}
E_1 &\rightarrow e\ (;\delta) && (6.262)\\
F_1 &\rightarrow f\ (;\delta) && (6.263)\\
E_2 &\rightarrow e\ (;\delta) && (6.264)\\
F_2 &\rightarrow f\ (;\delta) && (6.265)\\
E_3 &\rightarrow e\ (;\delta) && (6.266)\\
F_3 &\rightarrow f\ (;\delta) && (6.267)\\
E_4 &\rightarrow e\ (;\delta) && (6.268)\\
F_4 &\rightarrow f\ (;\delta) && (6.269)\\
E_5 &\rightarrow e\ (;\delta) && (6.270)\\
F_5 &\rightarrow f\ (;\delta) && (6.271)\\
X &\rightarrow x\ (;\delta) && (6.272)
\end{aligned}$$

FIGURE 6.14: Dissolution stage of a coalition

$$
\begin{aligned}
S_0 &\rightarrow B_1' S_0 \left(\{A_1'\}; \delta \cup \{A_1'', B_1'\}\right) && (6.273)\\
A_1' &\rightarrow A_1'' \left(\{B_1'\}; \{A_1''\}\right) && (6.274)\\
B_1' &\rightarrow B_1 \left(\{A_1''\}\,;\right) && (6.275)\\
A_1'' &\rightarrow y \left(;\{B_1'\}\right) && (6.276)\\
B_1 &\rightarrow a_1 \left(;\{A_1''\}\right) && (6.277)\\
S_0 &\rightarrow B_2' S_0 \left(\{A_2'\}; \delta \cup \mathcal{A}'_1 \cup \{A_2'', B_2'\}\right) && (6.278)\\
A_2' &\rightarrow A_2'' \left(\{B_2'\}; \{A_2''\}\right) && (6.279)\\
B_2' &\rightarrow B_2 \left(\{A_2''\}\,;\right) && (6.280)\\
A_2'' &\rightarrow y \left(;\{B_2'\}\right) && (6.281)\\
B_2 &\rightarrow a_2 \left(;\{A_2''\}\right) && (6.282)\\
S_0 &\rightarrow B_3' S_0 \left(\{A_3'\}; \delta \cup \mathcal{A}'_2 \cup \{A_3'', B_3'\}\right) && (6.283)\\
A_3' &\rightarrow A_3'' \left(\{B_3'\}; \{A_3''\}\right) && (6.284)\\
B_3' &\rightarrow B_3 \left(\{A_3''\}\,;\right) && (6.285)\\
A_3'' &\rightarrow y \left(;\{B_3'\}\right) && (6.286)\\
B_3 &\rightarrow a_3 \left(;\{A_3''\}\right) && (6.287)\\
S_0 &\rightarrow B_4' S_0 \left(\{A_4'\}; \delta \cup \mathcal{A}'_3 \cup \{A_4'', B_4'\}\right) && (6.288)\\
A_4' &\rightarrow A_4'' \left(\{B_4'\}; \{A_4''\}\right) && (6.289)\\
B_4' &\rightarrow B_4 \left(\{A_4''\}\,;\right) && (6.290)\\
A_4'' &\rightarrow y \left(;\{B_4'\}\right) && (6.291)\\
B_4 &\rightarrow a_4 \left(;\{A_4''\}\right) && (6.292)\\
S_0 &\rightarrow B_5' S_0 \left(\{A_5'\}; \delta \cup \mathcal{A}'_4 \cup \{A_5'', B_5'\}\right) && (6.293)\\
A_5' &\rightarrow A_5''(\{B_5'\}; \{A_5''\}) && (6.294)\\
B_5' &\rightarrow B_5 \left(\{A_5''\}\,;\right) && (6.295)\\
A_5'' &\rightarrow y \left(;\{B_5'\}\right) && (6.296)\\
B_5 &\rightarrow a_5 \left(;\{A_5''\}\right) && (6.297)\\
& && (6.298)
\end{aligned}
$$

FIGURE 6.15: Restructuring the items claimed by the coalition

$$S_0 \rightarrow c \left(;\{\delta \cup \mathcal{A}'\}\right) \qquad (6.299)$$

FIGURE 6.16: Replacing the central marker

$$S \Longrightarrow S_0TX \Longrightarrow^5 S_0TXXXXXX$$

The initiator enterprise (S_1^o) is introduced to the sentential form, giving it the highest priority to claim items.

$$\Longrightarrow S_0S_1^oXXXXXX$$

S_1^o claims two items as follows:

$$\begin{aligned}
&\Longrightarrow S_0S_1XXXXXX && \text{(rule (6.187))}\\
&\Longrightarrow S_0A_1S_1XXXXXX && \text{(rule (6.217))}\\
&\Longrightarrow S_0A_1S_1XXXXX_1X && \text{(rule (6.247))}\\
&\Longrightarrow S_0A_1'S_1XXXXX_1X && \text{(rule (6.248))}\\
&\Longrightarrow S_0A_1'S_1XXXXxX && \text{(rule (6.249))}\\
&\Longrightarrow S_0A_1'A_1S_1XXXXxX && \text{(rule (6.217))}\\
&\Longrightarrow S_0A_1'A_1S_1X_1XXXxX && \text{(rule (6.247))}\\
&\Longrightarrow S_0A_1'A_1'S_1X_1XXXxX && \text{(rule (6.248))}\\
&\Longrightarrow S_0A_1'A_1'S_1xXXXxX && \text{(rule (6.249))}
\end{aligned}$$

S_1 then invites S_2 and S_3 by applying rule (6.221).

$$S_0A_1'A_1'S_1xXXXxX \Longrightarrow S_0A_1'A_1'S_3^oS_2^oxXXXxX$$

S_2 claims an items as follows:

$$\begin{aligned}
&\Longrightarrow S_0A_1'A_1'S_3^oS_2xXXXxX && \text{(rule (6.193))}\\
&\Longrightarrow S_0A_1'A_1'S_3^oA_2S_2xXXXxX && \text{(rule (6.223))}\\
&\Longrightarrow S_0A_1'A_1'S_3^oA_2S_2xX_2XXxX && \text{(rule (6.250))}\\
&\Longrightarrow S_0A_1'A_1'S_3^oA_2'S_2xX_2XXxX && \text{(rule (6.251))}\\
&\Longrightarrow S_0A_1'A_1'S_3^oA_2'S_2xxXXxX && \text{(rule (6.252))}
\end{aligned}$$

S_2 then invites S_3 and S_5 by applying rule (6.227).

$$S_0A_1'A_1'S_3^oA_2'S_2xxXXxX \Longrightarrow S_0A_1'A_1'S_3^oA_2'S_3^oS_5^oxxXXxX$$

The enterprise represented by S_3^o now appears twice on the sentential form. Rule (6.204) applies as follows:

$$S_0A_1'A_1'S_3^oA_2'S_3^oS_5^oxxXXxX \Longrightarrow S_0A_1'A_1'rA_2'S_3^oS_5^oxxXXxX$$

S_3 then claims two items as follows:

$$\begin{aligned}
&\Longrightarrow S_0A_1'A_1'rA_2'S_3S_5^oxxXXxX && \text{(rule (6.199))}\\
&\Longrightarrow S_0A_1'A_1'rA_2'A_3S_3S_5^oxxXXxX && \text{(rule (6.229))}\\
&\Longrightarrow S_0A_1'A_1'rA_2'A_3S_3S_5^oxxX_3XxX && \text{(rule (6.253))}\\
&\Longrightarrow S_0A_1'A_1'rA_2'A_3'S_3S_5^oxxX_3XxX && \text{(rule (6.254))}\\
&\Longrightarrow S_0A_1'A_1'rA_2'A_3'S_3S_5^oxxxXxX && \text{(rule (6.255))}\\
&\Longrightarrow S_0A_1'A_1'rA_2'A_3'A_3S_3S_5^oxxxXxX && \text{(rule (6.229))}\\
&\Longrightarrow S_0A_1'A_1'rA_2'A_3'A_3S_3S_5^oxxxX_3xX && \text{(rule (6.253))}\\
&\Longrightarrow S_0A_1'A_1'rA_2'A_3'A_3'S_3S_5^oxxxX_3xX && \text{(rule (6.254))}\\
&\Longrightarrow S_0A_1'A_1'rA_2'A_3'A_3'S_3S_5^oxxxxxX && \text{(rule (6.255))}
\end{aligned}$$

S_3 invites S_1 and S_4 by applying rule (6.233).

$$S_0A_1'A_1'rA_2'A_3'A_3'S_3S_5^oxxxxxX \Longrightarrow S_0A_1'A_1'rA_2'A_3'A_3'S_1^oS_4^oS_5^oxxxxxX$$

S_4 opts out of the coalition by applying rule (6.205) and rule (6.237).

$$\Longrightarrow S_0A_1'A_1'rA_2'A_3'A_3'S_1^oS_4S_5^oxxxxxX \Longrightarrow S_0A_1'A_1'rA_2'A_3'A_3'S_1^oE_4S_5^oxxxxxX$$

Since the enterprise represented by S_1 has already performed its actions (we have two A_1's in the sentential form), rule (6.191) applies as follows.

$$S_0A_1'A_1'rA_2'A_3'A_3'S_1^oE_4S_5^oxxxxxX \Longrightarrow S_0A_1'A_1'rA_2'A_3'A_3'rE_4S_5^oxxxxxX$$

S_5 claims an item as follows:

$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4S_5xxxxxX \qquad (rule\ (6.211))$$
$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A_5S_5xxxxxX \qquad (rule\ (6.241))$$
$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A_5S_5xxxxxX_5 \qquad (rule\ (6.259))$$
$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A'_5S_5xxxxxX_5 \qquad (rule\ (6.260))$$
$$\Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A'_5S_5xxxxxx \qquad (rule\ (6.261))$$

S_5 does not want to invite other enterprises. Therefore it applies rule (6.244).

$$A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A'_5S_5xxxxxx \Longrightarrow A'_1A'_1rA'_2A'_3A'_3S^o_1E_4A'_5zxxxxxx$$

Since all the enterprises have performed their actions, rule (6.268) applies.

$$S_0A'_1A'_1rA'_2A'_3A'_3rE_4A'_5zxxxxxx \Longrightarrow S_0A'_1A'_1rA'_2A'_3A'_3reA'_5zxxxxxx$$

The restructure process writes an B_i on the left-hand side of c that corresponds to an A'_i on the right-hand side of c. The B_i is later rewritten to a terminal symbol a_i and the A'_i is later rewritten to a y.

To restructure the A'_1's first, rules (6.273) – (6.293) apply as follows:

$$\Longrightarrow B_1'S_0A_1'A_1'rA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.273))$$
$$\Longrightarrow B_1'S_0A_1'A_1''rA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.274))$$
$$\Longrightarrow B_1S_0A_1'A_1''rA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.275))$$
$$\Longrightarrow B_1S_0A_1'yrA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.276))$$
$$\Longrightarrow a_1S_0A_1'yrA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.277))$$
$$\Longrightarrow a_1B_1'S_0A_1'yrA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.273))$$
$$\Longrightarrow a_1B_1'S_0A_1''yrA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.274))$$
$$\Longrightarrow a_1B_1S_0A_1''yrA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.275))$$
$$\Longrightarrow a_1B_1S_0yyrA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.276))$$
$$\Longrightarrow a_1a_1S_0yyrA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.277))$$

A_2' is restructured as follows:

$$\Longrightarrow a_1a_1B_2'S_0yyrA_2'A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.278))$$
$$\Longrightarrow a_1a_1B_2'S_0yyrA_2''A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.279))$$
$$\Longrightarrow a_1a_1B_2S_0yyrA_2''A_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.280))$$
$$\Longrightarrow a_1a_1B_2S_0yyryA_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.281))$$
$$\Longrightarrow a_1a_1a_2S_0yyryA_3'A_3'reA_5'zxxxxxx \qquad (rule\ (6.282))$$

The A'_3's are restructured as follows:

$$\Longrightarrow a_1a_1a_2B'_3S_0yyryA'_3A'_3reA'_5zxxxxxx \qquad (rule\ (6.283))$$

$$\Longrightarrow a_1a_1a_2B'_3S_0yyryA''_3A'_3reA'_5zxxxxxx \qquad (rule\ (6.284))$$

$$\Longrightarrow a_1a_1a_2B_3S_0yyryA''_3A'_3reA'_5zxxxxxx \qquad (rule\ (6.285))$$

$$\Longrightarrow a_1a_1a_2B_3S_0yyryyA'_3reA'_5zxxxxxx \qquad (rule\ (6.286))$$

$$\Longrightarrow a_1a_1a_2a_3S_0yyryyA'_3reA'_5zxxxxxx \qquad (rule\ (6.287))$$

$$\Longrightarrow a_1a_1a_2a_3B'_3S_0yyryyA'_3reA'_5zxxxxxx \qquad (rule\ (6.283))$$

$$\Longrightarrow a_1a_1a_2a_3B'_3S_0yyryyA''_3reA'_5zxxxxxx \qquad (rule\ (6.284))$$

$$\Longrightarrow a_1a_1a_2a_3B_3S_0yyryyA''_3reA'_5zxxxxxx \qquad (rule\ (6.285))$$

$$\Longrightarrow a_1a_1a_2a_3B_3S_0yyryyyreA'_5zxxxxxx \qquad (rule\ (6.286))$$

$$\Longrightarrow a_1a_1a_2a_3a_3S_0yyryyyreA'_5zxxxxxx \qquad (rule\ (6.287))$$

Finally, A'_5 is restructured as follows:

$$\Longrightarrow a_1a_1a_2a_3a_3B'_5S_0yyryyyreA'_5zxxxxxx \qquad (rule\ (6.293))$$

$$\Longrightarrow a_1a_1a_2a_3a_3B'_5S_0yyryyyreA''_5zxxxxxx \qquad (rule\ (6.294))$$

$$\Longrightarrow a_1a_1a_2a_3a_3B_5S_0yyryyyreA''_5zxxxxxx \qquad (rule\ (6.295))$$

$$\Longrightarrow a_1a_1a_2a_3a_3B_5S_0yyryyyreyzxxxxxx \qquad (rule\ (6.296))$$

$$\Longrightarrow a_1a_1a_2a_3a_3a_5S_0yyryyyreyzxxxxxx \qquad (rule\ (6.297))$$

Now that all items claimed by members of the coalition have been structured, S_0 is replaced by a central marker by applying rule (6.299).

$$a_1a_1a_2a_3a_3a_5S_0yyryyyreyzxxxxxx \Longrightarrow a_1a_1a_2a_3a_3a_5cyyryyyreyzxxxxxx$$

Based solely on this word, we can deduce the following about the formed VBC.

- *There are four enterprises that have claimed items (a_1, a_2, a_3, and a_5).*
- *There is an enterprise that was invited to join the coalition, but opted out. This enterprise is represented by the e in our word.*

- *There is an enterprise that claimed items without inviting other enterprises to join the coalition. This is represented by the z in our word.*
- *There were six items made available to this coalition, this is also the overall quantity of items claimed by members of the coalition.*
- *There are two repeated invitations. This is represented by the two occurrences of r in our word.*

6.4 Discussion

We have demonstrated that rcgs are the appropriate grammars in modelling coalition formation in a VBC. We now continue to show that $L_{\text{structure}}$ cannot be generated by a cfg using the pumping lemma for context-free languages as defined in Theorem 3.15. This implies that cfgs cannot model coalition formation in a VBC, or any coalition that is represented by $L_{\text{structure}}$.

Theorem 6.1. *$L_{\text{structure}}$ is not a context-free language (cfl).*

Proof. Assume that $L_{\text{structure}}$ is a cfl.

Let h be the integer of Theorem 3.15.

Let $u = a_1^h c y^h z x^h$, then $u \in L_{\text{structure}}$.

According to the definition of the pumping lemma for context-free languages, there is a decomposition of u into qrs, such that $|qrs| \leq h$.

Consider $qrs : |qrs| \leq h$.

i) Let qrs contain a_1's only. Then for $m = 2$, the resulting word u' will have more a_1's than x's. This implies that the enterprises have claimed more items that were available, thus $u' \notin L_{\text{structure}}$.

ii) Let qrs contain the c. Then for any value $m \neq 1$, the resulting word u' is not in $L_{\text{structure}}$.

iii) Let qrs contain y's only. Then for $m = 2$, the resulting word u' will have more y's than x's. Thus $u' \notin L_{\text{structure}}$.

iv) Let qrs contain the z. Then for $m = 2$, the resulting word u' will have two z's, while the number of enterprises remain at a constant one. This resulting word $u' \notin L_{\text{structure}}$.

v) Let qrs contain x's only. Then for any $m = 0$, the resulting word u' will have fewer x's than a_i's. Thus, $u' \notin L_{\text{structure}}$.

Therefore, $L_{\text{structure}}$ is not a context-free language. □

Chapter 7

Modelling a Detailed VBC

7.1 Introduction

An rFcg, and an rcg that generate the same language representing a formed coalition are presented in this chapter. The rFcg generates this language, but, it does not model a VBC environment. However, the rcg does. The rFcg generates an available item as it is claimed by an enterprise. Furthermore, in the rFcg, all enterprises are invited to join the coalition at the same time. It remains an open question as to whether this language can be generated by an rPcg. The rcg presented in this chapter models all four phases relating to the formation of a VBC. That is, the initiation, operational, dissolution, and post dissolution phase.

$$\begin{aligned} L_{\text{detailed}} \;=\; & \{\mathcal{AFZEDR}cy^g x^k \mid g, k \in \mathbb{N}_+ \,;\, \mathcal{A} = a_{j_1}^{n_{j_1}} a_{j_2}^{n_{j_2}} \ldots a_{j_q}^{n_{j_q}} \,;\, n_{j_i} \geq 1 \text{ for } i \in q \,;\, q \in \\ & [m] \,;\, \text{LinOrder}\,(A) \,;\, \sum_{i=1}^{q} n_{j_i} \leq k \,;\, \mathcal{F} = f_{l_1} f_{l_2} \ldots f_{l_s} \,;\, s \in [m] \,;\, \text{LinOrder}\,(F) \,; \\ & \text{DisJoint}\,(F, A) \,;\, \mathcal{Z} = z_{o_1} z_{o_2} \ldots z_{o_r} \,;\, r \in [m] \,;\, \text{LinOrder}\,(Z) \,;\, \{o_1, o_2, \ldots, o_r\} \\ & \subseteq \{j_1, j_2, \ldots, j_q\} \,;\, \mathcal{E} = e_{u_1} e_{u_2} \ldots e_{u_t} \,;\, t \in [m] \,;\, \text{DisJoint}\,(E, A) \,; \\ & \text{LinOrder}\,(E) \,;\, \text{DisJoint}\,(E, F) \,;\, \mathcal{D} = d_{t_1} d_{t_2} \ldots d_{t_v} \,;\, v \in [m] \,;\, \text{LinOrder}\,(D) \,; \\ & \text{DisJoint}\,(D, E) \,;\, \text{DisJoint}\,(D, F) \,;\, \text{DisJoint}\,(D, Z) \,;\, \mathcal{R} = r_{p_1}^{n_{p_1}} r_{p_2}^{n_{p_2}} \ldots r_{p_b}^{n_{p_b}} \,; \\ & n_{p_i} \geq 1 \text{ for } i \in b \,;\, b \in [m] \,;\, |\mathcal{R}| \leq m(m-1) \,;\, \text{LinOrder}\,(R) \,;\, \{p_1, p_2, \ldots, p_b\} \\ & \subseteq \{j_1, j_2, \ldots, j_q\} \cup \{l_1, l_2, \ldots, l_s\} \cup \{o_1, o_2, \ldots, o_r\} \cup \{u_1, u_2, \ldots, u_t\} \,;\, |\mathcal{A}| + \\ & |\mathcal{F}| + |\mathcal{Z}| + |\mathcal{E}| + |\mathcal{D}| + |\mathcal{R}| = g\} \;. \end{aligned}$$

In L_{detailed}, the items claimed by each enterprise are ordered and grouped as an entity. In addition, the information about the behaviour of enterprises during the formation of the coalition is also grouped. This implies that the supplier can inquire about a specific enterprise's behaviour. For instance, the supplier can check if the enterprise represented by a_{j_i} has invited other enterprises by checking if there is a z_{o_i} in a word representing a formed coalition. If there is a z_{o_i}, then there should be at least one a_{j_i}.

If enterprise i has invited at least one other enterprise after claiming items, then there will be no z_{o_i}. If enterprise i has invited at least one other enterprise without claiming any items, then $n_{j_i} = 0$, and there will be no z_{o_i}. In this case, there will be an f_{li}, signaling that the invitation was passed over to at least one other enterprise during the formation of a coalition.

If enterprise i has opted out of the coalition, then $n_{j_i} = 0$, and there will be no z_{o_i}, or f_{li}. To show that enterprise i has opted out of the coalition, the word in a language will contain e_{u_i}.

In a situation in which enterprise i cannot claim the quantity of items it requires (due to lack of available items), this is represented by d_{t_i}. If d_{t_i} occurs in a word $\in L_{\text{detailed}}$, then the total number of items claimed by all members of the coalition must be equal to the number of items made available to the coalition.

The invitation strategy in a VBC allows enterprises to invite their known associates. It is possible that enterprise i may be invited to join the coalition more than once. $r_{p_i}^{n_{p_i}}$ denotes that enterprise i has been invited n_{p_i} times after the first invitation.

For any word w of L_{detailed}, the following holds:

1. k represents the number of items that may be claimed by the enterprises in a coalition.
2. $a_{j_i}^{n_{j_i}}$ represents the allocation of n_{j_i} items to the enterprise represented by a_{j_i}.
3. f_{l_i} denotes that enterprise i invited other enterprises but did not claim items.
4. z_{o_i} denotes that enterprise i claimed items but did not invite other enterprises.
5. e_{u_i} denotes that enterprise i neither claimed items nor invited other enterprises.
6. d_{t_i} denotes that enterprise i could not perform any operations as there were no items available.
7. $r_{p_i}^{n_{p_i}}$ represents the number of times n_{p_i}, enterprise i was invited to join the coalition after it was first invited.

8. c is a marker that separates information on enterprises that have claimed items, the behaviour of enterprises during the formation of the coalition, and the total number of items made available to the coalition.

In Example 7.1, we give a word in L_{detailed} that involves six enterprises.

Example 7.1. $w = a_1^2 a_2 a_5 f_3 z_2 z_5 e_6 r_1 c y^9 x^8$

In this word, there are three enterprises (a_1, a_2, and a_5) that have claimed items. The enterprise represented by a_1 has claimed two items, and the enterprises represented by a_2, and a_5 have each claimed one item respectively. The enterprises a_2 and a_5 did not invite other enterprises to join the coalition, which is represented by z_2 and z_5, respectively.

The symbol f_3 in w represents that enterprise a_3 invited at least one other enterprise without claiming any items. There is an enterprise that was invited to claim items, but opted out of the coalition, which is represented by the symbol e_5 in w. The symbol r_1 in w implies that enterprise a_1 was invited to join the coalition more than once.

The sum of all occurrences of a_i's in w is four, which is the total number of items claimed by the enterprises in the coalition. The total number of items made available to the coalition is eight, which is represented by the substring x^8.

7.2 Random Forbidding Context Grammar

The random forbidding context grammar that generates L_{detailed} is defined as follows.

Let $G_{\text{rFcg}} = (V_{\text{N}}, V_{\text{T}}, P, S)$:

For the sake of brevity, let $\mathcal{S}^o = \{S_1^o, S_2^o, \ldots, S_m^o\}$, $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$, $\mathcal{S}' = \{S_1', S_2', \ldots, S_m'\}$, $\mathcal{Q}^o = \{Q_1^o, Q_2^o, \ldots, Q_m^o\}$, $\mathcal{Q} = \{Q_1, Q_2, \ldots, Q_m\}$, $\mathcal{Q}' = \{Q_1', Q_2', \ldots, Q_m'\}$, $\mathcal{Q}'' = \{Q_1'', Q_2'', \ldots, Q_m''\}$, $\mathcal{Q}''' = \{Q_1''', Q_2''', \ldots, Q_m'''\}$, $\mathcal{Q}'''' = \{Q_1'''', Q_2'''', \ldots, Q_m''''\}$, $\mathcal{D} = \{D_1, D_2, \ldots, D_m\}$, $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$, $\mathcal{F} = \{F_1, F_2, \ldots, F_m\}$, $\mathcal{D}' = \{D_1', D_2', \ldots, D_m'\}$, $\mathcal{R} = \{R_1, R_2, \ldots, R_m\}$, $\mathcal{R}^o = \{R_1^o, R_2^o, \ldots, R_m^o\}$, $\mathcal{Z}' = \{Z_1', Z_2', \ldots, Z_m'\}$, $\mathcal{Z} = \{Z_1, Z_2, \ldots, Z_m\}$, and $\delta = \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}'' \cup \mathcal{S}^o \cup T$.

1. $V_N = \{S\} \cup \mathcal{S} \cup \mathcal{S}^o \cup \mathcal{S}' \cup \mathcal{Q}^o \cup \mathcal{Q} \cup \mathcal{Q}' \cup \mathcal{Q}'' \cup \mathcal{Q}''' \cup \mathcal{Q}'''' \cup \mathcal{D} \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{D}' \cup \mathcal{R} \cup \mathcal{R}^o \cup \mathcal{Z}' \cup \mathcal{Z} \cup \{X, X'\}$.

2. $V_T = \{a_1, a_2, \ldots, a_m\} \cup \{f_1, f_2, \ldots, f_m\} \cup \{z_1, z_2, \ldots, z_m\} \cup \{e_1, e_2, \ldots, e_m\} \cup \{d_1, d_2, \ldots, d_m\} \cup \{r_1, r_2, \ldots, r_m\} \cup \{c, x, y\}$.

3. P is the set of productions defined as in Figure 7.1 – Figure 7.2. Please note: For $i \in [m]$, in the production rule $S \rightarrow S^o_{j_{n_1}} \ldots S^o_{j_{n_1}} F_{j_{n_1}} \ldots F_{j_{n_i}} Z_{j_{n_1}} \ldots Z_{j_{n_i}} E_{j_{n_i}} \ldots E_{j_{n_i}} D_{j_{n_1}} \ldots D_{j_{n_i}} R_{j_{n_1}} \ldots R_{j_{n_i}} c Q_{j_{n_1}} Q'_{j_{n_1}} Q''_{j_{n_1}} Q'''_{j_{n_1}} Q''''_{j_{n_1}} \ldots Q_{j_{n_i}} Q'_{j_{n_i}} Q''_{j_{n_i}} Q'''_{j_{n_i}} Q''''_{j_{n_i}} X$,
   - all $j_{n_1}, j_{n_2}, \ldots, j_{n_i}$ are distinct, and
   - $1 \leq n_i \leq m$.

$$
\begin{aligned}
S &\rightarrow S^o_{j_{n_1}} \ldots S^o_{j_{n_1}} F_{j_{n_1}} \ldots F_{j_{n_i}} Z_{j_{n_1}} \ldots Z_{j_{n_i}} \\
&\quad E_{j_{n_i}} \ldots E_{j_{n_i}} D_{j_{n_1}} \ldots D_{j_{n_i}} R_{j_{n_1}} \ldots R_{j_{n_i}} c \\
&\quad Q_{j_{n_1}} Q'_{j_{n_1}} Q''_{j_{n_1}} Q'''_{j_{n_1}} Q''''_{j_{n_1}} \ldots Q_{j_{n_i}} Q'_{j_{n_i}} Q''_{j_{n_i}} Q'''_{j_{n_i}} Q''''_{j_{n_i}} X && (7.1)\\
S^o_i &\rightarrow S_i \ (; \{D'_i, E'_i, F'_i, X'', X'\}) && (7.2)\\
&\rightarrow \lambda && (7.3)\\
E_i &\rightarrow E'_i \ (; \{A_i, D'_i, F'_i, S_i, S^o_i, S'_i, Z'_i\}) && (7.4)\\
Q''_i &\rightarrow y \ (; \{E_i, E''_i\}) && (7.5)\\
E_i &\rightarrow E''_i && (7.6)\\
D_i &\rightarrow D'_i \ (; \{X, X'\}) && (7.7)\\
Q'''_i &\rightarrow y \ (; \{D_i\}) && (7.8)\\
F_i &\rightarrow F'_i \ (; \{A_i, D'_i, E'_i, S_i, S^o_i, S'_i, Z'_i\}) && (7.9)\\
Q'_i &\rightarrow y \ (; \{F_i, F''_i\}) && (7.10)\\
F_i &\rightarrow F''_i && (7.11)\\
S_i &\rightarrow A_i S'_i \ (; \mathcal{S}' \cup \{D'_i, E'_i, F'_i, X', X'', Z'_i, Q'_i\}) && (7.12)\\
&\rightarrow \lambda \ (; \{X', X\}) && (7.13)\\
S'_i &\rightarrow \lambda \ (; \{X, Q_i\}) && (7.14)\\
&\rightarrow S_i \ (; \{X, Q_i\}) && (7.15)\\
X &\rightarrow X_i X' \ (; \{D_i, E_i, F_i, S_i, S^o_i, D'_i, E'_i, F'_i\}) && (7.16)\\
Q_i &\rightarrow y Q'_i \ (; \{D_i, E_i, F_i, S_i, S^o_i, X'', D'_i, E'_i, F'_i\}) && (7.17)\\
Q'_i &\rightarrow Q_i \ (; \{\mathcal{S}'\}) && (7.18)\\
Z_i &\rightarrow Z'_i \ (; \{D'_i, E'_i, F'_i, S_i, S^o_i, S'_i, Q'_i\}) && (7.19)\\
Z_i &\rightarrow Z''_i \ (; \{D'_i, E'_i, F'_i, S_i, S^o_i, S'_i, Q'_i\}) && (7.20)\\
Q_i &\rightarrow y \ (; \{Z''_i, Z_i\}) && (7.21)\\
X' &\rightarrow X \ (; \mathcal{S}' \cup \{Q'_i\}) && (7.22)\\
X' &\rightarrow X'' \ (; \mathcal{S}' \cup \{Q'_i\}) && (7.23)
\end{aligned}
$$

FIGURE 7.1: rFcg generating L_{detailed}

In G_{rFcg}, all enterprises have the same rule templates. In this grammar, an invited enterprise has two options, claim items or opt out of the coalition. Rule 7.1 denotes the invitation to all enterprises to join the coalition. Rule 7.2 enables enterprise i to claim items. Forbidding context is used to ensure that this rule does not apply if enterprise

$$
\begin{aligned}
R_i &\rightarrow R_i^o && (7.24)\\
Q_i'''' &\rightarrow \lambda\,(;\{R_i, R_i^2, R_i^3, \ldots, R_i^{m-1}\}) && (7.25)\\
R_i &\rightarrow R_i && (7.26)\\
Q_i'''' &\rightarrow y\,(;\{R_i^o, R_i^2, R_i^3, \ldots, R_i^{m-1}\}) && (7.27)\\
R_i &\rightarrow R_i^2 && (7.28)\\
Q_i'''' &\rightarrow yy\,(;\{R_i^o, R_i, R_i^3, \ldots, R_i^{m-1}\}) && (7.29)\\
R_i &\rightarrow R_i^3 && (7.30)\\
Q_i'''' &\rightarrow yyy\,(;\{R_i^o, R_i, R_i^2, R_i^4, \ldots, R_i^{m-1}\}) && (7.31)\\
&\vdots\\
R_i &\rightarrow R_i^{m-1} && (7.32)\\
Q_i'''' &\rightarrow y^{m-1}\,(;\{R_i^o, R_i, R_i^2, R_i^3, \ldots, R_i^{m-2}\}) && (7.33)\\
R_i &\rightarrow R_i^o\,(;\delta) && (7.34)\\
R_i &\rightarrow r_i\,(;\delta) && (7.35)\\
R_i^2 &\rightarrow r_i^2\,(;\delta) && (7.36)\\
R_i^3 &\rightarrow r_i^3\,(;\delta) && (7.37)\\
&\vdots\\
R_i^{m-1} &\rightarrow r_i^{m-1}\,(;\delta) && (7.38)\\
X &\rightarrow xX\,(;\delta) && (7.39)\\
X &\rightarrow x\,(;\delta) && (7.40)\\
A_i &\rightarrow a_i\,(;\delta) && (7.41)\\
Z_i' &\rightarrow z_i\,(;\delta) && (7.42)\\
Z_i'' &\rightarrow \lambda\,(;\delta) && (7.43)\\
Z_i &\rightarrow \lambda\,(;\delta) && (7.44)\\
D_i &\rightarrow \lambda\,(;\delta) && (7.45)\\
D_i' &\rightarrow d_i\,(;\delta) && (7.46)\\
E_i' &\rightarrow e_i\,(;\delta) && (7.47)\\
F_i' &\rightarrow f_i\,(;\delta) && (7.48)\\
D_i &\rightarrow \lambda\,(;\delta) && (7.49)\\
E_i'' &\rightarrow \lambda\,(;\delta) && (7.50)\\
F_i'' &\rightarrow \lambda\,(;\delta) && (7.51)\\
Q_i &\; to \; \lambda\,(;\delta) && (7.52)\\
Q_i' &\rightarrow \lambda\,(;\delta) && (7.53)\\
Q_i'' &\rightarrow \lambda\,(;\delta) && (7.54)\\
Q_i''' &\rightarrow \lambda\,(;\delta) && (7.55)\\
Q_i'''' &\rightarrow \lambda\,(;\delta) && (7.56)\\
X_i &\rightarrow x(;\delta) && (7.57)\\
X'' &\rightarrow \lambda\,(;\delta) && (7.58)
\end{aligned}
$$

FIGURE 7.2: Cont: rFcg generating L_{detailed}

i has opted out of the coalition (E_i), forwarded the invitation (F_i), there are no items available (X''), or if there is an enterprise already claiming items (X').

Rule 7.3 applies if enterprise i does not want to claim any items. Rule 7.4 applies if enterprise i opts out of the coalition. Forbidding context ensures that this rule does not apply if enterprise i has claimed items, passed on the invitation or is yet to claim items. Rule 7.5 generates a y if an enterprise has opted out of the coalition. Forbidding context makes certain that this rule does not apply if enterprise i has not opted out of the coalition. Rule 7.6 applies if enterprise i does not want to opt out of the coalition.

Rule 7.7 applies if enterprise i cannot claim items due to lack their lack of availability. Forbidding context ensures that this rule does not apply if there is an item available (X), or there is an enterprise in the process of claiming an item (X'). Rule 7.8 generates a y for a D_i'. Forbidding context ensures that this rule does not apply if rule 7.7 has not applied.

In a VBC, rule 7.9 would apply if enterprise i invites other enterprises without claiming items. In this grammar, it introduces an F_i. Forbidding context ensures that this rule does not apply if enterprise i has already claimed an item, could not claim items (due to lack of available items), or has opted out of the coalition. Rule 7.10 generates a y for an F_i'. Forbidding context ensures that this rule does not apply before rule 7.9. Rule 7.11 applies if we do not introduce an F_i'.

In rule 7.12, enterprise i claims an item. Forbidding context ensures that this rule does not apply if enterprise i has already opted out of the coalition, "forwarded" the invitation, there is an enterprise in the process of claiming an item, or there is no item available to be claimed. Rule 7.13 applies if there are no items to be claimed. Forbidding context ensures that this rule does not apply if there is an item to be claimed (X), or there is an enterprise in the process of claiming an item (X').

Rule 7.14 applies if enterprise i claims an item, and then leaves the coalition. Forbidding context ensures that this rule does not apply if an available item has not been assigned to the previously claimed item, and a y has not been generated for the item. Rule 7.15 allows enterprise i to claim another item. Forbidding context is used as in rule 7.14.

Rule 7.16 generates an item for every claimed item. Forbidding context ensures that this rule applies immediately after enterprise i has claimed an item. Rule 7.17 generates a y for every item claimed. Forbidding context enforces the application of this rule immediately after an item has been claimed. Rule 7.18 enables enterprise i to resume its operations regarding claiming items. Forbidding context ensures that this rule does not apply if there is an enterprise claiming an item.

Rule 7.19 introduces a Z_i that will eventually become a z_i. In a VBC model, this would indicate that enterprise i has claimed at least one item without inviting another enterprise. This rule prevents enterprise i from claiming any more items. Forbidding context ensures that this rule does not apply if enterprise i has not claimed any items. Rule 7.20 relates to the aspect of the language definition that there may be no z in a word of a language after a string of y's. Forbidding context is used as in rule 7.19. Rule 7.21 generates a y for a Z_i'. Forbidding context ensures that this rule does not apply if there is no Z_i'.

Rule 7.22 introduces an item to the sentential form that can be claimed by a member of the coalition. Forbidding context ensures that this rule does not apply if there is an enterprise in the process of claiming an item. Rule 7.23 removes an available item from the sentential form. Forbidding context is used as in rule 7.22.

Rules 7.24–7.33 introduce the r's to the sentential form. In a VBC, this would imply that enterprise i has been invited to join the coalition more than once. In this grammar, this would not be possible since all enterprises are invited at the same time. Rules 7.35 – 7.38 rewrite the generated R's to their non-terminals. Forbidding context ensures that these rules do not apply until all enterprises have performed their operations.

Rules 7.39–7.40 generate more x's. Forbidding context is used to ensure that these rules only apply once all invited enterprises have performed their actions.

Rules 7.41–7.57 remove all non-terminals associated with enterprise i. Forbidding context is used to ensure that these rules only apply once all invited enterprises have performed their actions.

Rule 7.58 removes the unavailable item from the sentential form.

We exemplify these concepts in the following example.

Example 7.2. *In this example, there are three enterprises. Each enterprise has the option to either claim items, or opt out of the coalition.*

Let $G_{\text{rFcg}} = (V_{\text{N}}, V_{\text{T}}, P, S)$*:*

For the sake of brevity, let $\mathcal{S}^o = \{S_1^o, S_2^o, S_3^o\}$, $\mathcal{S} = \{S_1, S_2, S_3\}$, $\mathcal{S}' = \{S_1', S_2', S_3'\}$, $\mathcal{Q}^o = \{Q_1^o, Q_2^o, Q_3^o\}$, $\mathcal{Q} = \{Q_1, Q_2, Q_3\}$, $\mathcal{Q}' = \{Q_1', Q_2', Q_3'\}$, $\mathcal{Q}'' = \{Q_1'', Q_2'', Q_3''\}$, $\mathcal{Q}''' = \{Q_1''', Q_2''', Q_3'''\}$, $\mathcal{Q}'''' = \{Q_1'''', Q_2'''', Q_3''''\}$, $\mathcal{D} = \{D_1, D_2, D_3\}$, $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$, $\mathcal{F} = \{F_1, F_2, \ldots, F_m\}$, $\mathcal{D}' = \{D_1', D_2', D_3'\}$, $\mathcal{R} = \{R_1, R_2, R_3\}$, $\mathcal{R}^o = \{R_1^o, R_2^o, R_3^o\}$, $\mathcal{Z}' = \{Z_1', Z_2', Z_3'\}$, $\mathcal{Z} = \{Z_1, Z_2, Z_3\}$, *and* $\delta = \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}'' \cup \mathcal{S}^o \cup T$.

1. $\Sigma = \{S\} \cup \mathcal{S} \cup \mathcal{S}^o \cup \mathcal{S}' \cup \mathcal{Q}^o \cup \mathcal{Q} \cup \mathcal{Q}' \cup \mathcal{Q}'' \cup \mathcal{Q}''' \cup \mathcal{Q}'''' \cup \mathcal{D} \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{D}' \cup \mathcal{R} \cup \mathcal{R}^o \cup \mathcal{Z}' \cup \mathcal{Z} \cup \{X, X'\}$.

2. $V_T = \{a_1, a_2, a_3\} \cup \{f_1, f_2, f_3\} \cup \{z_1, z_2, z_3\} \cup \{e_1, e_2, e_3\} \cup \{d_1, d_2, d_3\} \cup \{r_1, r_2, r_3\} \cup \{c, x, y\}$.

3. *P is the set of productions defined as in Figure 7.3 – 7.8.*

Figure 7.3 refers to the rule template 7.1.

$$S \rightarrow S_1^o S_2^o S_3^o F_1 F_2 F_3 Z_1 Z_2 Z_3 E_1 E_2 E_3 D_1 D_2 D_3 R_1 R_2 R_3 c Q_1 Q_1' Q_1'' Q_1''' Q_1'''' Q_2 Q_2' Q_2'' Q_2''' Q_2'''' Q_3 Q_3' Q_3'' Q_3''' Q_3'''' X \tag{7.59}$$

FIGURE 7.3: Initiating a coalition

Figure 7.4 refers to the rule template 7.2 – 7.11.

Figure 7.5 refers to rule templates 7.12 – 7.21.

Figure 7.6 refers to rule templates 7.22 – 7.23.

Figure 7.7 refers to the rule templates 7.24 – 7.38.

Figure 7.8 refers to the rule templates 7.41 – 7.58.

Consider the following situation: There are four items available, of which S_1 wants to claim two items, S_2 wants to claim one item, and S_3 opts out.

According to our grammar, the coalition formation begins as follows.

$$S \Longrightarrow S_1^o S_2^o S_3^o F_1 F_2 F_3 Z_1 Z_2 Z_3 E_1 E_2 E_3 D_1 D_2 D_3 R_1 R_2 R_3 c Q_1 Q_1' Q_1'' Q_1''' Q_1'''' Q_2 Q_2' Q_2'' Q_2''' Q_2'''' Q_3 Q_3' Q_3'' Q_3''' Q_3'''' X$$

The enterprise represented by S_1 claims two items as follows.

$\Longrightarrow S_1 S_2^o S_3^o F_1 F_2 F_3 Z_1 Z_2 Z_3 E_1 E_2 E_3 D_1 D_2 D_3 R_1 R_2 R_3 c Q_1 Q_1' Q_1'' Q_1''' Q_1'''' Q_2 Q_2' Q_2'' Q_2''' Q_2'''' Q_3 Q_3' Q_3'' Q_3''' Q_3'''' X$ (*rule* (7.60))
$\Longrightarrow A_1 S_1' S_2^o S_3^o F_1 F_2 F_3 Z_1 Z_2 Z_3 E_1 E_2 E_3 D_1 D_2 D_3 R_1 R_2 R_3 c Q_1 Q_1' Q_1'' Q_1''' Q_1'''' Q_2 Q_2' Q_2'' Q_2''' Q_2'''' Q_3 Q_3' Q_3'' Q_3''' Q_3'''' X$ (*rule* (7.90))
$\Longrightarrow A_1 S_1' S_2^o S_3^o F_1 F_2 F_3 Z_1 Z_2 Z_3 E_1 E_2 E_3 D_1 D_2 D_3 R_1 R_2 R_3 c Q_1 Q_1' Q_1'' Q_1''' Q_1'''' Q_2 Q_2' Q_2'' Q_2''' Q_2'''' Q_3 Q_3' Q_3'' Q_3''' Q_3'''' X_1 X'$ (*rule* (7.94))
$\Longrightarrow A_1 S_1' S_2^o S_3^o F_1 F_2 F_3 Z_1 Z_2 Z_3 E_1 E_2 E_3 D_1 D_2 D_3 R_1 R_2 R_3 c y Q_1^o Q_1' Q_1'' Q_1''' Q_1'''' Q_2 Q_2' Q_2'' Q_2''' Q_2'''' Q_3 Q_3' Q_3'' Q_3''' Q_3'''' X_1 X'$ (*rule* (7.95))

$$S_1^o \rightarrow S_1\ (;\{D_1', E_1', F_1', X'', X'\} \tag{7.60}$$
$$\rightarrow \lambda \tag{7.61}$$
$$E_1 \rightarrow E_1'\ (;\{A_1, D_1', F_1', S_1, S_1^o, S_1', Z_1'\}) \tag{7.62}$$
$$Q_1'' \rightarrow y\ (;\{E_1, E_1''\}) \tag{7.63}$$
$$E_1 \rightarrow E_1'' \tag{7.64}$$
$$D_1 \rightarrow D_1'\ (;\{X, X'\}) \tag{7.65}$$
$$Q_1''' \rightarrow y\ (;\{D_1\}) \tag{7.66}$$
$$F_1 \rightarrow F_1'\ (;\{A_1, D_1', E_1', S_1, S_1^o, S_1', Z_1'\}) \tag{7.67}$$
$$Q_1' \rightarrow y\ (;\{F_1, F_1''\}) \tag{7.68}$$
$$F_1 \rightarrow F_1'' \tag{7.69}$$
$$S_2^o \rightarrow S_2\ (;\{D_2', E_2', F_2', X'', X'\}) \tag{7.70}$$
$$\rightarrow \lambda \tag{7.71}$$
$$E_2 \rightarrow E_2'\ (;\{A_2, D_2', F_2', S_2, S_2^o, S_2', Z_2'\}) \tag{7.72}$$
$$Q_2'' \rightarrow y\ (;\{E_2, E_2''\}) \tag{7.73}$$
$$E_2 \rightarrow E_2'' \tag{7.74}$$
$$D_2 \rightarrow D_2'\ (;\{X, X'\}) \tag{7.75}$$
$$Q_2''' \rightarrow y\ (;\{D_2\}) \tag{7.76}$$
$$F_2 \rightarrow F_2'\ (;\{A_2, D_2', E_2', S_2, S_2^o, S_2', Z_2'\}) \tag{7.77}$$
$$Q_2' \rightarrow y\ (;\{F_2, F_2''\}) \tag{7.78}$$
$$F_2 \rightarrow F_2'' \tag{7.79}$$
$$S_3^o \rightarrow S_3\ (;\{D_3', E_3', F_3', X'', X'\}) \tag{7.80}$$
$$\rightarrow \lambda \tag{7.81}$$
$$E_3 \rightarrow E_3'\ (;\{A_3, D_3', F_3', S_3, S_3^o, S_3', Z_3'\}) \tag{7.82}$$
$$Q_3'' \rightarrow y\ (;\{E_3, E_3''\}) \tag{7.83}$$
$$E_3 \rightarrow E_3'' \tag{7.84}$$
$$D_3 \rightarrow D_3'\ (;\{X, X'\}) \tag{7.85}$$
$$Q_3''' \rightarrow y\ (;\{D_3\}) \tag{7.86}$$
$$F_3 \rightarrow F_3'\ (;\{A_3, D_3', E_3', S_3, S_3^o, S_3', Z_3'\}) \tag{7.87}$$
$$Q_3' \rightarrow y\ (;\{F_3, F_3''\}) \tag{7.88}$$
$$F_3 \rightarrow F_3'' \tag{7.89}$$

FIGURE 7.4: Enterprises preparing to claim items, or opting out of the coalition

$$
\begin{aligned}
S_1 &\rightarrow A_1S'_1\ (;\mathcal{S}' \cup \{D'_1, E'_1, F'_1, X', X'', Z'_1, Q'_1\}) && (7.90)\\
&\rightarrow \lambda\ (;\{X', X\}) && (7.91)\\
S'_1 &\rightarrow \lambda\ (;\{X, Q_1\}) && (7.92)\\
&\rightarrow S_1\ (;\{X, Q_1\}) && (7.93)\\
X &\rightarrow X_1X'\ (;\{D_1, E_1, F_1, S_1, S^o_1, D'_1, E'_1, F'_1\}) && (7.94)\\
Q_1 &\rightarrow yQ'_1\ (;\{D_1, E_1, F_1, S_1, S^o_1, X'', D'_1, E'_1, F'_1\}) && (7.95)\\
Q'_1 &\rightarrow Q_1\ \big(;\{\mathcal{S}'\}\big) && (7.96)\\
Z_1 &\rightarrow Z'_1\ (;\{D'_1, E'_1, F'_1, S_1, S^o_1, S'_1, Q'_1\}) && (7.97)\\
Z_1 &\rightarrow Z''_1\ (;\{D'_1, E'_1, F'_1, S_1, S^o_1, S'_1, Q'_1\}) && (7.98)\\
Q_1 &\rightarrow y\ (;\{Z''_1, Z_1\}) && (7.99)\\
S_2 &\rightarrow A_2S'_2\ (;\mathcal{S}' \cup \{D'_2, E'_2, F'_2, X', X'', Z'_2, Q'_2\}) && (7.100)\\
&\rightarrow \lambda(;\{X', X\}) && (7.101)\\
S'_2 &\rightarrow \lambda\ (;\{X, Q_2\}) && (7.102)\\
&\rightarrow S_2\ (;\{X, Q_2\}) && (7.103)\\
X &\rightarrow X_2X'\ (;\{D_2, E_2, F_2, S_2, S^o_2, D'_2, E'_2, F'_2\}) && (7.104)\\
Q_2 &\rightarrow yQ'_2\ (;\{D_2, E_2, F_2, S_2, S^o_2, X'', D'_2, E'_2, F'_2\}) && (7.105)\\
Q'_2 &\rightarrow Q_2\ \big(;\{\mathcal{S}'\}\big) && (7.106)\\
Z_2 &\rightarrow Z'_2\ (;\{D'_2, E'_2, F'_2, S_2, S^o_2, S'_2, Q'_2\}) && (7.107)\\
Z_2 &\rightarrow Z''_2\ (;\{D'_2, E'_2, F'_2, S_2, S^o_2, S'_2, Q'_2\}) && (7.108)\\
Q_2 &\rightarrow y\ (;\{Z''_2, Z_2\}) && (7.109)\\
S_3 &\rightarrow A_3S'_3\ (;\mathcal{S}' \cup \{D'_3, E'_3, F'_3, X', X'', Z'_3, Q'_3\}) && (7.110)\\
&\rightarrow \lambda\ (;\{X', X\}) && (7.111)\\
S'_3 &\rightarrow \lambda\ (;\{X, Q_3\}) && (7.112)\\
&\rightarrow S_3\ (;\{X, Q_3\}) && (7.113)\\
X &\rightarrow X_3X'\ (;\{D_3, E_3, F_3, S_3, S^o_3, D'_3, E'_3, F'_3\}) && (7.114)\\
Q_3 &\rightarrow yQ'_3\ (;\{D_3, E_3, F_3, S_3, S^o_3, X'', D'_3, E'_3, F'_3\}) && (7.115)\\
Q'_3 &\rightarrow Q_3\ \big(;\{\mathcal{S}'\}\big) && (7.116)\\
Z_3 &\rightarrow Z'_3\ (;\{D'_3, E'_3, F'_3, S_3, S^o_3, S'_3, Q'_3\}) && (7.117)\\
Z_3 &\rightarrow Z''_3\ (;\{D'_3, E'_3, F'_3, S_3, S^o_3, S'_3, Q'_3\}) && (7.118)\\
Q_3 &\rightarrow y\ (;\{Z''_3, Z_3\}) && (7.119)
\end{aligned}
$$

FIGURE 7.5: Enterprises claiming items

$$
\begin{aligned}
X' &\rightarrow X\ (;\mathcal{S}' \cup \{Q_1'\}) && (7.120)\\
X' &\rightarrow X''\ (;\mathcal{S}' \cup \{Q_1'\}) && (7.121)\\
X' &\rightarrow X\ (;\mathcal{S}' \cup \{Q_2'\}) && (7.122)\\
X' &\rightarrow X''\ (;\mathcal{S}' \cup \{Q_2'\}) && (7.123)\\
X' &\rightarrow X\ (;\mathcal{S}' \cup \{Q_3'\}) && (7.124)\\
X' &\rightarrow X''\ (;\mathcal{S}' \cup \{Q_3'\}) && (7.125)
\end{aligned}
$$

FIGURE 7.6: Dealing with items

$$
\begin{aligned}
R_1 &\rightarrow R_1^o && (7.126)\\
Q_1'''' &\rightarrow \lambda\ (;\{R_1, R_1^2\}) && (7.127)\\
R_1 &\rightarrow R_1 && (7.128)\\
Q_1'''' &\rightarrow y\ (;\{R_1^o, R_1^2,\}) && (7.129)\\
R_1 &\rightarrow R_1^2 && (7.130)\\
Q_1'''' &\rightarrow yy\ (;\{R_1^o, R_1\}) && (7.131)\\
R_1^o &\rightarrow \lambda\ \ (;\delta) && (7.132)\\
R_1 &\rightarrow r_1\ \ (;\delta) && (7.133)\\
R_1^2 &\rightarrow r_1^2\ \ (;\delta) && (7.134)\\
R_2 &\rightarrow R_2^o && (7.135)\\
Q_2'''' &\rightarrow \lambda\ (;\{R_2, R_2^2\}) && (7.136)\\
R_2 &\rightarrow R_2 && (7.137)\\
Q_2'''' &\rightarrow y\ (;\{R_2^o, R_2^2\}) && (7.138)\\
R_2 &\rightarrow R_2^2 && (7.139)\\
Q_2'''' &\rightarrow yy\ (;\{R_2^o, R_2\}) && (7.140)\\
R_2^o &\rightarrow \lambda\ \ (;\delta) && (7.141)\\
R_2 &\rightarrow r_2\ \ (;\delta) && (7.142)\\
R_2^2 &\rightarrow r_2^2\ \ (;\delta) && (7.143)\\
R_3 &\rightarrow R_3^o && (7.144)\\
Q_3'''' &\rightarrow \lambda\ (;\{R_3, R_3^2\}) && (7.145)\\
R_3 &\rightarrow R_3 && (7.146)\\
Q_3'''' &\rightarrow y\ (;\{R_3^o, R_3^2\}) && (7.147)\\
R_3 &\rightarrow R_3^2 && (7.148)\\
Q_3'''' &\rightarrow yy\ (;\{R_3^o, R_3\}) && (7.149)\\
R_3^o &\rightarrow \lambda\ (;\delta) && (7.150)\\
R_3 &\rightarrow r_3\ (;\delta) && (7.151)\\
R_3^2 &\rightarrow r_3^2\ (;\delta) && (7.152)
\end{aligned}
$$

FIGURE 7.7: Introducing the r's to the sentential form

$$
\begin{aligned}
X &\rightarrow xX\ (;\delta) && (7.153)\\
X &\rightarrow x\ (;\delta) && (7.154)\\
A_1 &\rightarrow a_1\ (;\delta) && (7.155)\\
Z_1' &\rightarrow z_1\ (;\delta) && (7.156)\\
Z_1'' &\rightarrow \lambda\ (;\delta) && (7.157)\\
Z_1 &\rightarrow \lambda\ (;\delta) && (7.158)\\
D_1' &\rightarrow d_1\ (;\delta) && (7.159)\\
E_1' &\rightarrow e_1\ (;\delta) && (7.160)\\
F_1' &\rightarrow f_1\ (;\delta) && (7.161)\\
D_1 &\rightarrow \lambda\ (;\delta) && (7.162)\\
E_1'' &\rightarrow \lambda\ (;\delta) && (7.163)\\
F_1'' &\rightarrow \lambda\ (;\delta) && (7.164)\\
X_1 &\rightarrow x\ (;\delta) && (7.165)\\
A_2 &\rightarrow a_2\ (;\delta) && (7.166)\\
Z_2' &\rightarrow z_2\ (;\delta) && (7.167)\\
Z_2'' &\rightarrow \lambda\ (;\delta) && (7.168)\\
Z_2 &\rightarrow \lambda\ (;\delta) && (7.169)\\
D_2' &\rightarrow d_2\ (;\delta) && (7.170)\\
E_2' &\rightarrow e_2\ (;\delta) && (7.171)\\
F_2' &\rightarrow f_2\ (;\delta) && (7.172)\\
D_2 &\rightarrow \lambda\ (;\delta) && (7.173)\\
E_2'' &\rightarrow \lambda\ (;\delta) && (7.174)\\
F_2'' &\rightarrow \lambda\ (;\delta) && (7.175)\\
X_2 &\rightarrow x\ (;\delta) && (7.176)\\
A_3 &\rightarrow a_3\ (;\delta) && (7.177)
\end{aligned}
$$

$$
\begin{aligned}
Z_3' &\rightarrow z_3\ (;\delta) && (7.178)\\
Z_3'' &\rightarrow \lambda\ (;\delta) && (7.179)\\
Z_3 &\rightarrow \lambda\ (;\delta) && (7.180)\\
D_3' &\rightarrow d_3\ (;\delta) && (7.181)\\
E_3' &\rightarrow e_3\ (;\delta) && (7.182)\\
F_3' &\rightarrow f_3\ (;\delta) && (7.183)\\
D_3 &\rightarrow \lambda\ (;\delta) && (7.184)\\
E_3'' &\rightarrow \lambda\ (;\delta) && (7.185)\\
F_3'' &\rightarrow \lambda\ (;\delta) && (7.186)\\
X_3 &\rightarrow x\ (;\delta) && (7.187)\\
Q_1 &\rightarrow \lambda\ (;\delta) && (7.188)\\
Q_1' &\rightarrow \lambda\ (;\delta) && (7.189)\\
Q_1'' &\rightarrow \lambda\ (;\delta) && (7.190)\\
Q_1''' &\rightarrow \lambda\ (;\delta) && (7.191)\\
Q_1'''' &\rightarrow \lambda\ (;\delta) && (7.192)\\
Q_2 &\rightarrow \lambda\ (;\delta) && (7.193)\\
Q_2' &\rightarrow \lambda\ (;\delta) && (7.194)\\
Q_2'' &\rightarrow \lambda\ (;\delta) && (7.195)\\
Q_2''' &\rightarrow \lambda\ (;\delta) && (7.196)\\
Q_2'''' &\rightarrow \lambda\ (;\delta) && (7.197)\\
Q_3 &\rightarrow \lambda\ (;\delta) && (7.198)\\
Q_3' &\rightarrow \lambda\ (;\delta) && (7.199)\\
Q_3'' &\rightarrow \lambda\ (;\delta) && (7.200)\\
Q_3''' &\rightarrow \lambda\ (;\delta) && (7.201)\\
Q_3'''' &\rightarrow \lambda\ (;\delta) && (7.202)\\
X'' &\rightarrow \lambda\ (;\delta) && (7.203)
\end{aligned}
$$

FIGURE 7.8: Dissolving the coalition

$\Longrightarrow A_1S_1S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyQ_1^oQ_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X'$ (*rule (7.93)*)

$\Longrightarrow A_1S_1S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X'$ (*rule (7.96)*)

$\Longrightarrow A_1S_1S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X$ (*rule (7.120)*)

$\Longrightarrow A_1A_1S_1'S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X$ (*rule (7.90)*)

$\Longrightarrow A_1A_1S_1'S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X'$ (*rule (7.94)*)

$\Longrightarrow A_1A_1S_1'S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1^oQ_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X'$ (*rule* (7.95))
$\Longrightarrow A_1A_1S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1^oQ_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X'$ (*rule* (7.92))
$\Longrightarrow A_1A_1S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X'$ (*rule* (7.96))
$\Longrightarrow A_1A_1S_2^oS_3^oF_1F_2F_3Z_1Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X$ (*rule* (7.120))

The enterprise represented by S_1 has claimed items. The following rule applies to deal with Z_1.

$\Longrightarrow A_1A_1S_2^oS_3^oF_1F_2F_3Z_1''Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X$ (*rule* (7.98))

The enterprise represented by S_2 claims an item as follows.

$\Longrightarrow A_1A_1S_2S_3^oF_1F_2F_3Z_1''Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X$ (*rule* (7.70))
$\Longrightarrow A_1A_1A_2S_2'S_3^oF_1F_2F_3Z_1''Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X$ (*rule* (7.100))
$\Longrightarrow A_1A_1A_2S_2'S_3^oF_1F_2F_3Z_1''Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''Q_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X_2X'$ (*rule* (7.104))
$\Longrightarrow A_1A_1A_2S_2'S_3^oF_1F_2F_3Z_1''Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''yQ_2^oQ_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X_2X'$ (*rule* (7.105))
$\Longrightarrow A_1A_1A_2S_3^oF_1F_2F_3Z_1''Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''yQ_2^oQ_2'Q_2''$
$Q_2'''Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X_2X'$ (*rule* (7.102))
$\Longrightarrow A_1A_1A_2S_3^oF_1F_2F_3Z_1''Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''yQ_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X_2X'$ (*rule* (7.106))
$\Longrightarrow A_1A_1A_2S_3^oF_1F_2F_3Z_1''Z_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''yQ_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X_2X$ (*rule* (7.122))

The enterprise represented by S_2 has claimed items. The following rule applies to deal with Z_2.

$\Longrightarrow A_1A_1A_2S_3^oF_1F_2F_3Z_1''Z_2'Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''yQ_2Q_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X_2X$ (*rule* (7.107))
$\Longrightarrow A_1A_1A_2S_3^oF_1F_2F_3Z_1''Z_2'Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q_1'Q_1''Q_1'''Q_1''''yyQ_2'Q_2''Q_2'''$
$Q_2''''Q_3Q_3'Q_3''Q_3'''Q_3''''X_1X_1X_2X$ (*rule* (7.109))

The enterprise represented by S_3 opts out as follows.

$\Longrightarrow A_1A_1A_2F_1F_2F_3Z''_1Z'_2Z_3E_1E_2E_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3Q''_3Q'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.81))
$\Longrightarrow A_1A_1A_2F_1F_2F_3Z''_1Z'_2Z_3E_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2Q_3$
$Q'_3Q''_3Q'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.82))
$\Longrightarrow A_1A_1A_2F_1F_2F_3Z''_1Z'_2Z_3E_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2Q_3$
$Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.82))
$\Longrightarrow A_1A_1A_2F_1F_2F''_3Z''_1Z'_2Z_3E_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2Q_3$
$Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.89))
$\Longrightarrow A_1A_1A_2F_1F_2F''_3Z''_1Z'_2Z_3E_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.85))

Since the enterprises represented by S_1, and S_2 have claimed items, the following rules apply.

$\Longrightarrow A_1A_1A_2F_1F_2F''_3Z''_1Z'_2Z_3E_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.65))
$\Longrightarrow A_1A_1A_2F_1F_2F''_3Z''_1Z'_2Z_3E_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.75))
$\Longrightarrow A_1A_1A_2F''_1F_2F''_3Z''_1Z'_2Z_3E_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.69))
$\Longrightarrow A_1A_1A_2F''_1F''_2F''_3Z''_1Z'_2Z_3E_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.79))
$\Longrightarrow A_1A_1A_2F''_1F''_2F''_3Z''_1Z'_2Z_3E''_1E_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.64))
$\Longrightarrow A_1A_1A_2F''_1F''_2F''_3Z''_1Z'_2Z_3E''_1E''_2E'_3D_1D_2D_3R_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.74))

The R_i's are dealt with as follows.

$\Longrightarrow A_1A_1A_2F''_1F''_2F''_3Z''_1Z'_2Z_3E''_1E''_2E'_3D_1D_2D_3R^o_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1Q''''_1yyQ'_2Q''_2Q'''_2Q''''_2$
$Q_3Q'_3yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.126))
$\Longrightarrow A_1A_1A_2F''_1F''_2F''_3Z''_1Z'_2Z_3E''_1E''_2E'_3D_1D_2D_3R^o_1R_2R_3cyyQ_1Q'_1Q''_1Q'''_1yyQ'_2Q''_2Q'''_2Q''''_2Q_3Q'_3$
$yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.127))
$\Longrightarrow A_1A_1A_2F''_1F''_2F''_3Z''_1Z'_2Z_3E''_1E''_2E'_3D_1D_2D_3R^o_1R^o_2R_3cyyQ_1Q'_1Q''_1Q'''_1yyQ'_2Q''_2Q'''_2Q''''_2Q_3Q'_3$
$yQ'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.137))
$\Longrightarrow A_1A_1A_2F''_1F''_2F''_3Z''_1Z'_2Z_3E''_1E''_2E'_3D_1D_2D_3R^o_1R^o_2R_3cyyQ_1Q'_1Q''_1Q'''_1yyQ'_2Q''_2Q'''_2Q_3Q'_3y$
$Q'''_3Q''''_3X_1X_1X_2X$ (*rule* (7.138))
$\Longrightarrow A_1A_1A_2F''_1F''_2F''_3Z''_1Z'_2Z_3E''_1E''_2E'_3D_1D_2D_3R^o_1R^o_2R^o_3cyyQ_1Q'_1Q''_1Q'''_1yyQ'_2Q''_2Q'''_2Q_3Q'_3y$

$Q_3''' Q_3'''' X_1 X_1 X_2 X$ (*rule* (7.148))
$\Longrightarrow A_1A_1A_2F_1''F_2''F_3''Z_1''Z_2'Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'y$
$Q_3''' X_1X_1X_2X$ (*rule* (7.149))

Finally, all enterprises have performed their operations, then we can rewrite all the non-terminals from the sentential form.

$\Longrightarrow a_1A_1A_2F_1''F_2''F_3''Z_1''Z_2'Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'$
$yQ_3'''X_1X_1X_2X$ (*rule* (7.155))
$\Longrightarrow a_1a_1A_2F_1''F_2''F_3''Z_1''Z_2'Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'$
$yQ_3'''X_1X_1X_2X$ (*rule* (7.155))
$\Longrightarrow a_1a_1a_2F_1''F_2''F_3''Z_1''Z_2'Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'y$
$Q_3'''X_1X_1X_2X$ (*rule* (7.166))
$\Longrightarrow a_1a_1a_2F_2''F_3''Z_1''Z_2'Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''$
$X_1X_1X_2X$ (*rule* (7.164))
$\Longrightarrow a_1a_1a_2F_3''Z_1''Z_2'Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1$
X_1X_2X (*rule* (7.175))
$\Longrightarrow a_1a_1a_2Z_1''Z_2'Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1$
X_2X (*rule* (7.186))
$\Longrightarrow a_1a_1a_2Z_2'Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2$
X (*rule* (7.157))
$\Longrightarrow a_1a_1a_2z_2Z_3E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2$
X (*rule* (7.167))
$\Longrightarrow a_1a_1a_2z_2E_1''E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2$
X (*rule* (7.180))
$\Longrightarrow a_1a_1a_2z_2E_2''E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2$
X (*rule* (7.163))
$\Longrightarrow a_1a_1a_2z_2E_3'D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2$
X (*rule* (7.174))
$\Longrightarrow a_1a_1a_2z_2e_3D_1D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2$
X (*rule* (7.182))
$\Longrightarrow a_1a_1a_2z_2e_3D_2D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2$
X (*rule* (7.162))
$\Longrightarrow a_1a_1a_2z_2e_3D_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2$
X (*rule* (7.173))
$\Longrightarrow a_1a_1a_2z_2e_3R_1^oR_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.184))
$\Longrightarrow a_1a_1a_2z_2e_3R_2^oR_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.132))
$\Longrightarrow a_1a_1a_2z_2e_3R_3^ocyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.141))
$\Longrightarrow a_1a_1a_2z_2e_3cyyQ_1Q_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.150))
$\Longrightarrow a_1a_1a_2z_2e_3cyyQ_1'Q_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.188))

$\Longrightarrow a_1a_1a_2z_2e_3cyyQ_1''Q_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.189))
$\Longrightarrow a_1a_1a_2z_2e_3cyyQ_1'''yyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.190))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyQ_2'Q_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.191))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyQ_2''Q_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.194))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyQ_2'''Q_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.195))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyQ_3Q_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.196))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyQ_3'yQ_3'''X_1X_1X_2X$ (*rule* (7.198))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyyQ_3'''X_1X_1X_2X$ (*rule* (7.199))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyyX_1X_1X_2X$ (*rule* (7.201))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyyxX_1X_2X$ (*rule* (7.165))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyyxxX_2X$ (*rule* (7.165))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyyxxxX$ (*rule* (7.176))
$\Longrightarrow a_1a_1a_2z_2e_3cyyyyyxxxx$ (*rule* (7.154))

Based solely on the generated word, one can deduce the following.

- *There are two enterprises that have claimed items, of which enterprise one has claimed two items and enterprise two has claimed one item.*
- *In a VBC, z_2 would imply that enterprise two did not invite another enterprise to join the coalition after claiming an item.*
- *The enterprise represented by three opted out of the coalition (e_3).*
- *There were four items made available to the coalition, of which, only three were claimed.*

The rFcg presented in this section does not model coalition formation in a VBC. This is because it offers invited enterprises two of the four options that are available in VBC, that is, claim items, or opt out of the coalition. In addition, each available item is generated as it is claimed by an enterprise. In the next section, we present an rcg that models coalition formation in a VBC as described by Ngassam and Raborife [2013].

7.3 Random Context Grammar

In the following random context grammar, the enterprises are represented by non-terminal symbols. In particular, enterprise i is represented by non-terminal S_i. Each enterprise has the following options when invited to join the coalition.

- Claim items.
- Opt out of the coalition.
- Invite other enterprises with/without claiming items.

In this grammar, an enterprise can invite its known associates, who may also invite their known associates, etc. Each invited enterprise can perform only once per formed coalition. The number of items available to members of the coalition is known before the coalition formation process, and the enterprises do not claim more than was made available to them.

The total number of enterprises in a coalition is m.

Let $G = (V_{\mathrm{N}}, V_{\mathrm{T}}, P, S)$, where

For the sake of brevity, let $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$, $\mathcal{A}' = \{A'_1, A'_2, \ldots, A'_m\}$, $\mathcal{A}'' = \{A''_1, A''_2, \ldots, A''_m\}$, $\mathcal{A}'_i = \{A'_1, A'_2, \ldots, A'_i\}$, $A'_0 = \lambda$, $\mathcal{S} = \{S_1, S_2, \ldots S_m\}$, $\mathcal{S}' = \{S'_1, S'_2, \ldots, S'_m\}$, $\mathcal{S}'' = \{S''_1, S''_2, \ldots, S''_m\}$, $\mathcal{X} = \{X_1, X_2, \ldots, X_m\}$, $\mathcal{S}^o = \{S^o_1, S^o_2, \ldots, S^o_m\}$, $\mathcal{F} = \{F_1, F_2, \ldots, F_m\}$, $\mathcal{F}' = \{F'_1, F'_2, \ldots, F'_m\}$, $\mathcal{F}_i = \{F_1, F_2, \ldots, F_i\}$, $\mathcal{Z} = \{Z_1, Z_2, \ldots, Z_m\}$, $\mathcal{Z}' = \{Z'_1, Z'_2, \ldots, Z'_m\}$, $\mathcal{Z}_i = \{Z_1, Z_2, \ldots, Z_i\}$, $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$, $\mathcal{E}' = \{E'_1, E'_2, \ldots, E'_m\}$, $\mathcal{E}_i = \{E_1, E_2, \ldots, E_i\}$, $\mathcal{B} = \{B_1, B_2, \ldots, B_m\}$, $\mathcal{B}' = \{B'_1, B'_2, \ldots, B'_m\}$, $\mathcal{D} = \{D_1, D_2, \ldots, D_m\}$, $\mathcal{D}' = \{D'_1, D'_2, \ldots, D'_m\}$, $\mathcal{D}_i = \{D_1, D_2, \ldots, D_i\}$, $\mathcal{R} = \{R_1, R_2, \ldots, R_m\}$, $\mathcal{R}' = \{R'_1, R'_2, \ldots, R'_m\}$, $\mathcal{R}_i = \{R_1, R_2, ..., R_i\}$, and $\delta = \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}'' \cup \mathcal{S}^o \cup \mathcal{A} \cup T$.

1. $V_{\mathrm{N}} = \{S\} \cup \mathcal{S} \cup \mathcal{A} \cup \mathcal{A}' \cup \mathcal{A}'' \cup \mathcal{S}^o \cup \mathcal{S}' \cup \mathcal{S}'' \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{X} \cup \mathcal{F}' \cup \mathcal{E}' \cup \mathcal{Z} \cup \mathcal{Z}' \cup \mathcal{B} \cup \mathcal{B}' \cup \mathcal{R} \cup \mathcal{R}' \cup \mathcal{D} \cup \mathcal{D}' \cup \{X, T\}$.
2. $V_{\mathrm{T}} = \{a_1, a_2, \ldots, a_m\} \cup \{f_1, f_2, \ldots, f_m\} \cup \{z_1, z_2, \ldots, z_m\} \cup \{e_1, e_2, \ldots, e_m\} \cup \{d_1, d_2, \ldots, d_m\} \cup \{r_1, r_2, \ldots, r_m\} \cup \{c, x, y\}$, and
3. P is the set of productions defined as in Figure 7.9. Please note that for any non-terminal symbol P, $P_0 = \emptyset$. In the productions $S_i \to S^o_{j_{n_1}} S^o_{j_{n_2}} \ldots S^o_{j_{n_i}} (\{A'_i\}; \mathcal{A} \cup \mathcal{X})$ and $S_i \to S^o_{j_{n_1}} S^o_{j_{n_2}} \ldots S^o_{j_{n_i}} F_i(; \{A_i, A'_i\})$.
   - All $j_{n_1}, j_{n_2}, \ldots, j_{n_i}$ are distinct,
   - $i \notin \{j_{n_1}, j_{n_2}, \ldots, j_{n_i}\}$, and
   - $1 \leq n_i \leq m$.

In the grammar presented above, all enterprises have the same rule templates. The production rules 7.204–7.206 introduce the initiator enterprise i to the sentential form after the items have been placed on the sentential form as represented by X. The number

$$
\begin{aligned}
S &\rightarrow S_0TX && (7.204)\\
T &\rightarrow TX && (7.205)\\
&\rightarrow S_i^o && (7.206)\\
S_i^o &\rightarrow S_i(;\{A_i', D_i, E_i, F_i, S_i^o\}) && (7.207)\\
&\rightarrow R_i\ \left(\{A_i'\}\,;\right) && (7.208)\\
&\rightarrow R_i\ (\{D_i\}\,;) && (7.209)\\
&\rightarrow R_i\ (\{E_i\}\,;) && (7.210)\\
&\rightarrow R_i\ (\{F_i\}\,;) && (7.211)\\
&\rightarrow R_i\ (\{S_i^o\}\,;) && (7.212)\\
S_i &\rightarrow A_iS_i\ (\{X\}; \mathcal{A}\cup\mathcal{X}) && (7.213)\\
&\rightarrow D_i\ (;\{X\}) && (7.214)\\
&\rightarrow E_i\ \left(;\{A_i, A_i'\}\right) && (7.215)\\
&\rightarrow Z_i\ (\{A_i'\}; \mathcal{A}\cup\mathcal{X}) && (7.216)\\
&\rightarrow S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o\ (\{A_i'\}; \mathcal{A}\cup\mathcal{X}) && (7.217)\\
&\rightarrow S_{j_{n_1}}^o S_{j_{n_2}}^o \ldots S_{j_{n_i}}^o F_i(;\{A_i, A_i'\}) && (7.218)\\
X &\rightarrow X_i\ (\{A_i\}; \{X_i\}) && (7.219)\\
A_i &\rightarrow A_i'\ (\{X_i\}\,;) && (7.220)\\
X_i &\rightarrow x\ (;\{A_i\}) && (7.221)\\
X &\rightarrow x\ (;\delta) && (7.222)\\
S_0 &\rightarrow B_iS_0\ (\{A_i'\}; \delta\cup\mathcal{A}'_{i-1}\cup\{A_i'', B_i\}) && (7.223)\\
A_i' &\rightarrow A_i''\ (\{B_i\}; \{A_i''\}) && (7.224)\\
B_i &\rightarrow a_i\ \left(\{A_i''\}\,;\right) && (7.225)\\
A_i'' &\rightarrow y\ (;\{B_i\}) && (7.226)\\
S_0 &\rightarrow F_i'S_0\ (\{F_i\}; \delta\cup\mathcal{A}'\cup\mathcal{F}_{i-1}) && (7.227)\\
F_i &\rightarrow y\ \left(\{F_i'\}\,;\right) && (7.228)\\
F_i' &\rightarrow f_i\ (;\{F_i\}) && (7.229)\\
S_0 &\rightarrow Z_i'S_0\ (\{Z_i\}; \delta\cup\mathcal{A}'\cup\mathcal{F}\cup\mathcal{Z}_{i-1}) && (7.230)\\
Z_i &\rightarrow y\ \left(\{Z_i'\}\,;\right) && (7.231)\\
Z_i' &\rightarrow z_i\ (;\{Z_i\}) && (7.232)\\
S_0 &\rightarrow E_i'S_0\ (\{E_i\}; \delta\cup\mathcal{A}'\cup\mathcal{F}\cup\mathcal{Z}\cup\mathcal{E}_{i-1}) && (7.233)\\
E_i &\rightarrow y\ \left(\{E_i'\}\,;\right) && (7.234)\\
E_i' &\rightarrow e_i\ (;\{E_i\}) && (7.235)
\end{aligned}
$$

FIGURE 7.9: Rcg generating L_{detailed}

$$S_0 \rightarrow D_i' S_0 \left(\{D_i\}; \delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D}_{i-1}\right) \quad (7.236)$$

$$D_i \rightarrow y \left(\{D_i'\};\right) \quad (7.237)$$

$$D_i' \rightarrow d_i \left(; \{D_i\}\right) \quad (7.238)$$

$$S_0 \rightarrow R_i' S_0 \left(\{R_i\}; \delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D} \cup \mathcal{R}_{i-1}\right) \quad (7.239)$$

$$R_i \rightarrow R_i'' \left(\{R_i'\}; \{R_i''\}\right) \quad (7.240)$$

$$R_i' \rightarrow r_i \left(\{R_i''\};\right) \quad (7.241)$$

$$R_i'' \rightarrow y \left(; \{R_i'\}\right) \quad (7.242)$$

$$S_0 \rightarrow c \left(; \delta \cup \mathcal{S} \cup \mathcal{A} \cup \mathcal{F} \cup \mathcal{E} \cup \mathcal{R} \cup \mathcal{D}\right) \quad (7.243)$$

FIGURE 7.10: Cont: Rcg generating L_{detailed}

of items available to the coalition is bounded. S_0 will be used later to restructure the items.

Rule 7.207 allows enterprise i to perform its operations. Permitting context is used to ensure that this rule applies if there is at least one item available. Forbidding context is used to ensure that this rule does not apply if enterprise i has had an opportunity to perform in this coalition before.

Production rules 7.208–7.212 are applicable if enterprise i is already a part of coalition. This scenario arises when an enterprise is invited to join the coalition by more than one enterprise. Permitting context is used to check that enterprise i has been a part of the coalition before this invitation.

In rule 7.213, enterprise i claims an item. Permitting context is used to ensure that this rule only applies if there is at least one item available. Forbidding context is used to ensure that if an enterprise is claiming an item, no other enterprises are allowed to claim any items. This is to avoid a situation in which two enterprises claim the same item. In addition, it ensures that for every item claimed, there is only one X corresponding to it. This is to ensure that the quantity generated at the start is never exceeded.

Rule 7.214 is applicable if enterprise i does not find any items available to be claimed. We use forbidding context to ensure that this rule does not apply if there are items available.

In rule 7.215, enterprise i opts out of the coalition. Forbidding context is used to ensure that this rule does not apply if an enterprise has claimed at least one item.

Rule 7.216 applies if enterprise i claims at least one item without inviting other enterprises. Permitting context is used to ensure that this rule only applies if an enterprise i has claimed at least one item, represented by the existence of A_i' in the sentential form.

Forbidding context is used to ensure that this rule is not applied if enterprise i or any other enterprise is claiming items at that point in time.

In rule 7.217, enterprise i invites other enterprises after claiming at least one item. Permitting and forbidding contexts are used as in the previously defined rule.

Rule 7.218 is employed when an enterprise i invites other enterprises without claiming items. Forbidding context is used to make sure that this rule does not apply if an enterprise has claimed at least one item by checking for any A_i's or A_i''s in the sentential form.

Rule 7.219 matches a claimed item to an available item. Permitting context is used to ensure that once an item has been claimed, the X corresponding to it is immediately marked as claimed. Forbidding context is used to ensure that no more than one X for each claimed item is marked.

Rule 7.220 ensures that no more than one X_i is produced for an A_i.

Rule 7.221 replaces each marked claimed item with a terminal symbol x.

Rule 7.222 rewrites all the unclaimed items to the terminal symbol. Forbidding context is used to ensure that this rule does not apply if enterprises are still performing their operations.

In the rule 7.223, S_0 is used to restructure all the items claimed by the enterprises. This operation only commences once all invited enterprises have performed their operations. For each enterprise i, each item claimed by the enterprise is replicated on the left-hand side of S_0 with the variable B_i'. In addition, items claimed by enterprise i can only be restructured once the items claimed by enterprise $i-1$ have been restructured.

In the rules 7.224–7.225 for each A_i, exactly one B_i is produced, that is, the B_i's are correctly matched to the A_i's. Both permitting and forbidding context are used to ensure that the matching is correct. Once the matching has been completed for an enterprise i, rule 7.226 applies.

Once all the items claimed by the enterprises have been restructured (the a_i's) as shown in the previous paragraph, the symbols pertaining to the information about enterprises that invited other enterprises without claiming any items (the f_i's) are ordered. To check if an enterprise has invited another enterprise without claiming items, rule 7.227 applies. In this case, permitting context is used to check if an F_i exists and forbidding context is used to ensure that the restructuring of the F_i's can only proceed once all the A_i''s have been restructured. In addition, it is also used to ensure that a specific enterprise F_i can only begin to restructure after its predecessors[1] $\mathcal{F}_{i-1}$ have done so. Rule 7.228 ensures that once an F_i' has been added on the left-hand side of S_0, it is then "removed" from the right-hand side of S_0. Rule 7.229 rewrites F_i' to a terminal symbol.

Rule 7.230 applies if an enterprise i has claimed items without inviting other enterprises by checking if there is a Z_i in the sentential form using permitting context. Forbidding context is used to ensure that the restructuring of Z_i's only commences once all the A_i''s and the F_i's have been restructured, and that an enterprise represented by Z_i is restructured after its predecessors $\mathcal{Z}_i$ have been restructured. Rule 7.231 ensures that once an Z_i has been added on the left-hand side of S_0, it is then "removed" from the right-hand side of S_0. Rule 7.232 rewrites Z_i to a terminal.

Rule 7.233 applies if an enterprise i has opted out of the coalition using permitting context. Forbidding context is used to ensure that the restructuring of E_i's commences once all the A_i''s, F_i's and the Z_i's have been restructured, and that a particular E_i is only restructured after its predecessors $\mathcal{E}_{i-1}$ have been restructured. Rule 7.234 ensures that once an E_i has been added on the left-hand side of S_0, it is then "removed" from the right-hand side of S_0. Rule 7.235 rewrites E_i to a terminal.

Rule 7.236 checks if an enterprise i could not perform any actions because there were no items available. Forbidding context is used to ensure that the restructuring of D_i's only commences once all the A_i''s, F_i's, Z_i's and the E_i's have been restructured, and that a specific D_i is only restructured after its predecessors $\mathcal{D}_{i-1}$ have been restructured. Rule 7.237 ensures that once an D_i has been added on the left-hand side of S_0, it is then "removed" from the right-hand side of S_0. Rule 7.238 rewrites D_i to a terminal.

Each time an enterprise i has been invited after its first invitation, it is replicated on the left-hand side of S_0 with the variable R_i'. This is a result of the application of rule 7.239. This rule can only apply once all the A_i''s, F_i's, Z_i's and the E_i's have been restructured. The production rules 7.240 and 7.241 ensure that for each R_i, exactly one R_i' is produced. Once the matching has been completed for an enterprise i, rule 7.242 applies.

Rule 7.243 introduces the central marker c that delineates between the enterprises who have claimed items and the behaviour of the enterprises during the formation of the coalition.

We exemplify the random context grammar described above with a virtual buying co-operative involving five enterprises.

[1]Predecessors: enterprises that have a label with a lower number

Let $G = (V_\mathrm{N}, V_\mathrm{T}, P, S)$, where For the sake of brevity, let $\mathcal{A} = \{A_1, A_2, A_3, A_4, A_5\}$, $\mathcal{A}' = \{A'_1, A'_2, A'_3, A'_4, A'_5\}$, $\mathcal{A}'' = \{A''_1, A''_2, A''_3, A''_4, A''_5\}$, $\mathcal{A}'_i = \{A'_1, A'_2, \ldots, A'_i\}$, $A'_0 = \lambda$, $\mathcal{S} = \{S_1, S_2, S_3, S_4, S_5\}$, $\mathcal{S}' = \{S'_1, S'_2, S'_3, S'_4, S'_5\}$, $\mathcal{S}'' = \{S''_1, S''_2, S''_3, S''_4, S''_5\}$, $\mathcal{X} = \{X_1, X_2, X_3, X_4, X_5\}$, $\mathcal{S}^o = \{S^o_1, S^o_2, S^o_3, S^o_4, S^o_5\}$, $\mathcal{F} = \{F_1, F_2, F_3, F_4, F_5\}$, $\mathcal{F}' = \{F'_1, F'_2, F'_3, F'_4, F'_5\}$, $\mathcal{F}_i = \{F_1, F_2, \ldots, F_i\}$, $\mathcal{Z} = \{Z_1, Z_2, Z_3, Z_4, Z_5\}$, $\mathcal{Z}' = \{Z'_1, Z'_2, Z'_3, Z'_4, Z'_5\}$, $\mathcal{Z}_i = \{Z_1, Z_2, \ldots, Z_i\}$, $\mathcal{E} = \{E_1, E_2, E_3, E_4, E_5\}$, $\mathcal{E}' = \{E'_1, E'_2, E'_3, E'_4, E'_5\}$, $\mathcal{E}_i = \{E_1, E_2, \ldots, E_i\}$, $\mathcal{B} = \{B_1, B_2, B_3, B_4, B_5\}$, $\mathcal{B}' = \{B'_1, B'_2, B'_3, B'_4, B'_5\}$, $\mathcal{D} = \{D_1, D_2, D_3, D_4, D_5\}$, $\mathcal{D}' = \{D'_1, D'_2, D'_3, D'_4, D'_5\}$, $\mathcal{D}_i = \{D_1, D_2, \ldots, D_i\}$, $\mathcal{R} = \{R_1, R_2, R_3, R_4, R_5\}$, $\mathcal{R}' = \{R'_1, R'_2, R'_3, R'_4, R'_5\}$, $\mathcal{R}_i = \{R_1, R_2, ..., R_i\}$, and $\delta = \mathcal{S} \cup \mathcal{S}' \cup \mathcal{S}'' \cup \mathcal{S}^o \cup \mathcal{A} \cup T$.

1. $V_\mathrm{N} = \{S\} \cup \mathcal{S} \cup \mathcal{A} \cup \mathcal{A}' \cup \mathcal{A}'' \cup \mathcal{S}^o \cup \mathcal{S}' \cup \mathcal{S}'' \cup \mathcal{E} \cup \mathcal{F} \cup \mathcal{X} \cup \mathcal{F}' \cup \mathcal{E}' \cup \mathcal{Z} \cup \mathcal{Z}' \cup \mathcal{B} \cup \mathcal{B}' \cup \mathcal{R} \cup \mathcal{R}' \cup \mathcal{D} \cup \mathcal{D}' \cup \{X, T\}$.
2. $V_\mathrm{T} = \{a_1, a_2, a_3, a_4, a_5\} \cup \{f_1, f_2, f_3, f_4, f_5\} \cup \{z_1, z_2, z_3, z_4, z_5\} \cup \{e_1, e_2, e_3, e_4, e_5\} \cup \{d_1, d_2, d_3, d_4, d_5\} \cup \{r_1, r_2, r_3, r_4, r_5\} \cup \{c, x, y\}$, and
3. P is the set of productions defined in Figures 7.11–7.22.

Rules 7.204–7.206 are shown in Figure 7.11. In this example, the enterprise represented by S_1 is the initiator enterprise.

$$
\begin{aligned}
S &\rightarrow S_0 T X && (7.244)\\
T &\rightarrow T X && (7.245)\\
&\rightarrow S^o_1 && (7.246)
\end{aligned}
$$

FIGURE 7.11: Initiating a coalition

Rules 7.207–7.212 are exemplified in Figure 7.12.

Rules 7.213–7.218 are exemplified in Figure 7.13.

The rules 7.219–7.221 are demonstrated in Figure 7.14.

Rule 7.222 is shown in Figure 7.15

The rules 7.223–7.226 are exemplified in Figure 7.16.

Figure 7.17 presents an instance of rules 7.227–7.229.

In Figure 7.18, the rule 7.230–7.232 are illustrated for the five enterprises.

An instance of rules 7.233–7.235 is presented in Figure 7.19.

$$S_1^o \rightarrow S_1\left(;\left\{E_1, F_1, A_1', S_1^o, D_1\right\}\right) \mid \tag{7.247}$$
$$\rightarrow R_1\left(\{D_1\};\right) \mid \tag{7.248}$$
$$\rightarrow R_1\left(\{F_1\};\right) \mid \tag{7.249}$$
$$\rightarrow R_1\left(\{E_1\};\right) \mid \tag{7.250}$$
$$\rightarrow R_1\left(\left\{A_1'\right\};\right) \mid \tag{7.251}$$
$$\rightarrow R_1\left(\{S_1^o\};\right) \tag{7.252}$$
$$S_2^o \rightarrow S_2\left(;\left\{E_2, F_2, A_2', S_2^o, D_2\right\}\right) \mid \tag{7.253}$$
$$\rightarrow R_2\left(\{D_2\};\right) \mid \tag{7.254}$$
$$\rightarrow R_2\left(\{F_2\};\right) \mid \tag{7.255}$$
$$\rightarrow R_2\left(\{E_2\};\right) \mid \tag{7.256}$$
$$\rightarrow R_2\left(\left\{A_2'\right\};\right) \mid \tag{7.257}$$
$$\rightarrow R_2\left(\{S_2^o\};\right) \tag{7.258}$$
$$S_3^o \rightarrow S_3\left(;\left\{E_3, F_3, A_3', S_3^o, D_3\right\}\right) \mid \tag{7.259}$$
$$\rightarrow R_3\left(\{D_3\};\right) \mid \tag{7.260}$$
$$\rightarrow R_3\left(\{F_3\};\right) \mid \tag{7.261}$$
$$\rightarrow R_3\left(\{E_3\};\right) \mid \tag{7.262}$$
$$\rightarrow R_3\left(\left\{A_3'\right\};\right) \mid \tag{7.263}$$
$$\rightarrow R_3\left(\{S_3^o\};\right) \tag{7.264}$$
$$S_4^o \rightarrow S_4\left(;\left\{E_4, F_4, A_4', S_4^o, D_4\right\}\right) \mid \tag{7.265}$$
$$\rightarrow R_4\left(\{D_4\};\right) \mid \tag{7.266}$$
$$\rightarrow R_4\left(\{F_4\};\right) \mid \tag{7.267}$$
$$\rightarrow R_4\left(\{E_4\};\right) \mid \tag{7.268}$$
$$\rightarrow R_4\left(\left\{A_4'\right\};\right) \mid \tag{7.269}$$
$$\rightarrow R_4\left(\{S_4^o\};\right) \tag{7.270}$$
$$S_5^o \rightarrow S_5\left(;\left\{E_5, F_5, A_5', S_5^o, D_5\right\}\right) \mid \tag{7.271}$$
$$\rightarrow R_5\left(\{D_5\};\right) \mid \tag{7.272}$$
$$\rightarrow R_5\left(\{F_5\};\right) \mid \tag{7.273}$$
$$\rightarrow R_5\left(\{E_5\};\right) \mid \tag{7.274}$$
$$\rightarrow R_5\left(\left\{A_5'\right\};\right) \mid \tag{7.275}$$
$$\rightarrow R_5\left(\{S_5^o\};\right) \tag{7.276}$$

FIGURE 7.12: Initiation stage of a coalition

An example of rules 7.236–7.238 is presented in Figure 7.20.

An example of the rules 7.239–7.242 is presented in Figure 7.21 for five enterprises.

Figure 7.22 shows the rule template 7.243.

Consider the following situation: There are six items available, of which S_1 wants two, S_2 wants one, S_3 wants two, S_4 opts out and S_5 wants two.

$$
\begin{aligned}
S_1 &\rightarrow A_1S_1\ (\{X\};\mathcal{A}\cup\mathcal{X})\ | && (7.277)\\
&\rightarrow D_1\ (;\{X\})\ | && (7.278)\\
&\rightarrow E_1\ \big(;\{A_1,A_1'\}\big)\ | && (7.279)\\
&\rightarrow Z_1\ (\{A_1'\};\mathcal{A}\cup\mathcal{X})\ | && (7.280)\\
&\rightarrow S_3^oS_2^o\ (\{A_1'\};\mathcal{A}\cup\mathcal{X})\ | && (7.281)\\
&\rightarrow S_3^oS_2^oF_1\ \big(;\{A_1,A_1'\}\big) && (7.282)\\
S_2 &\rightarrow A_2S_2\ (\{X\};\mathcal{A}\cup\mathcal{X})\ | && (7.283)\\
&\rightarrow D_2\ (;\{X\})\ | && (7.284)\\
&\rightarrow E_2\ \big(;\{A_2,A_2'\}\big)\ | && (7.285)\\
&\rightarrow Z_2\ (\{A_2'\};\mathcal{A}\cup\mathcal{X})\ | && (7.286)\\
&\rightarrow S_3^oS_5^o\ (\{A_2'\};\mathcal{A}\cup\mathcal{X})\ | && (7.287)\\
&\rightarrow S_3^oS_5^oF_2\ \big(;\{A_2,A_2'\}\big) && (7.288)\\
S_3 &\rightarrow A_3S_3\ (\{X\};\mathcal{A}\cup\mathcal{X})\ | && (7.289)\\
&\rightarrow D_3\ (;\{X\})\ | && (7.290)\\
&\rightarrow E_3\ \big(;\{A_3,A_3'\}\big)\ | && (7.291)\\
&\rightarrow Z_3\ (\{A_3'\};\mathcal{A}\cup\mathcal{X})\ | && (7.292)\\
&\rightarrow S_1^oS_4^o\ (\{A_3'\};\mathcal{A}\cup\mathcal{X})\ | && (7.293)\\
&\rightarrow S_1^oS_4^oF_3\ \big(;\{A_3,A_3'\}\big) && (7.294)\\
S_4 &\rightarrow A_4S_4\ (\{X\};\mathcal{A}\cup\mathcal{X})\ | && (7.295)\\
&\rightarrow D_4\ (;\{X\})\ | && (7.296)\\
&\rightarrow E_4\ \big(;\{A_4,A_4'\}\big)\ | && (7.297)\\
&\rightarrow Z_4\ (\{A_4'\};\mathcal{A}\cup\mathcal{X})\ | && (7.298)\\
&\rightarrow S_2^o\ (\{A_4'\};\mathcal{A}\cup\mathcal{X})\ | && (7.299)\\
&\rightarrow S_2^oF_4\ \big(;\{A_4,A_4'\}\big) && (7.300)\\
S_5 &\rightarrow A_5S_5\ (\{X\};\mathcal{A}\cup\mathcal{X})\ | && (7.301)\\
&\rightarrow D_5\ (;\{X\})\ | && (7.302)\\
&\rightarrow E_5\ \big(;\{A_5,A_5'\}\big)\ | && (7.303)\\
&\rightarrow Z_5\ (\{A_5'\};\mathcal{A}\cup\mathcal{X})\ | && (7.304)\\
&\rightarrow S_1^o\ (\{A_4'\};\mathcal{A}\cup\mathcal{X})\ | && (7.305)\\
&\rightarrow S_1^oF_4\ \big(;\{A_4,A_4'\}\big) && (7.306)
\end{aligned}
$$

FIGURE 7.13: Operational stage of a coalition

According to our grammar, S starts rewriting the start symbol into six copies of the non-terminal X. These non-terminals indicate the total number available to the coalition.

$$S \Longrightarrow S_0TX \Longrightarrow^5 S_0TXXXXXX$$

$$X \rightarrow X_1 \, (\{A_1\}; \{X_1\}) \tag{7.307}$$
$$A_1 \rightarrow A'_1 \, (\{X_1\};) \tag{7.308}$$
$$X_1 \rightarrow x \, (; \{A_1\}) \tag{7.309}$$
$$X \rightarrow X_2 \, (\{A_2\}; \{X_2\}) \tag{7.310}$$
$$A_2 \rightarrow A'_2 \, (\{X_2\};) \tag{7.311}$$
$$X_2 \rightarrow x \, (; \{A_2\}) \tag{7.312}$$
$$X \rightarrow X_3 \, (\{A_3\}; \{X_3\}) \tag{7.313}$$
$$A_3 \rightarrow A'_3 \, (\{X_3\};) \tag{7.314}$$
$$X_3 \rightarrow x \, (; \{A_3\}) \tag{7.315}$$
$$X \rightarrow X_4 \, (\{A_4\}; \{X_4\}) \tag{7.316}$$
$$A_4 \rightarrow A'_4 \, (\{X_4\};) \tag{7.317}$$
$$X_4 \rightarrow x \, (; \{A_4\}) \tag{7.318}$$
$$X \rightarrow X_5 \, (\{A_5\}; \{X_5\}) \tag{7.319}$$
$$A_5 \rightarrow A'_5 \, (\{X_5\};) \tag{7.320}$$
$$X_5 \rightarrow x \, (; \{A_5\}) \tag{7.321}$$

FIGURE 7.14: Operational stage of a coalition

$$X \rightarrow x \, (; \delta) \tag{7.322}$$

FIGURE 7.15: Operational stage of a coalition

The initiator enterprise (S^o_1) is introduced to the sentential form, giving it the highest priority to claim items.

$$\Longrightarrow S^o_1 XXXXXX$$

S^o_1 claims two items as follows:

$$
\begin{aligned}
S_0 &\rightarrow B_1 S_0\ (\{A_1'\}; \delta \cup \mathcal{A}'_0 \cup \{A_1'', B_1\}) && (7.323)\\
A_1' &\rightarrow A_1''\ (\{B_1\}; \{A_1''\}) && (7.324)\\
B_1 &\rightarrow a_1\ \left(\{A_1''\}\,;\right) && (7.325)\\
A_1'' &\rightarrow y\ (;\{B_1\}) && (7.326)\\
S_0 &\rightarrow B_2 S_0\ (\{A_2'\}; \delta \cup \mathcal{A}'_1 \cup \{A_2'', B_2\}) && (7.327)\\
A_2' &\rightarrow A_2''\ (\{B_2\}; \{A_2''\}) && (7.328)\\
B_2 &\rightarrow a_2\ \left(\{A_2''\}\,;\right) && (7.329)\\
A_2'' &\rightarrow y\ (;\{B_2\}) && (7.330)\\
S_0 &\rightarrow B_3 S_0\ (\{A_3'\}; \delta \cup \mathcal{A}'_2 \cup \{A_3'', B_3\}) && (7.331)\\
A_3' &\rightarrow A_3''\ (\{B_3\}; \{A_3''\}) && (7.332)\\
B_3 &\rightarrow a_3\ \left(\{A_3''\}\,;\right) && (7.333)\\
A_3'' &\rightarrow y\ (;\{B_3\}) && (7.334)\\
S_0 &\rightarrow B_4 S_0\ (\{A_4'\}; \delta \cup \mathcal{A}'_3 \cup \{A_4'', B_4\}) && (7.335)\\
A_4' &\rightarrow A_4''(\{B_4\}; \{A_4''\}) && (7.336)\\
B_4 &\rightarrow a_4\ \left(\{A_4''\}\,;\right) && (7.337)\\
A_4'' &\rightarrow y\ (;\{B_4\}) && (7.338)\\
S_0 &\rightarrow B_5 S_0\ (\{A_5'\}; \delta \cup \mathcal{A}'_4 \cup \{A_5'', B_5\}) && (7.339)\\
A_5' &\rightarrow A_5''(\{B_5\}; \{A_5''\}) && (7.340)\\
B_5 &\rightarrow a_5\ \left(\{A_5''\}\,;\right) && (7.341)\\
A_5'' &\rightarrow y\ (;\{B_5\}) && (7.342)\\
& && (7.343)
\end{aligned}
$$

FIGURE 7.16: Restructuring the items claimed by the coalition

$$
\begin{aligned}
&\Longrightarrow S_0 S_1 XXXXXX\\
&\Longrightarrow S_0 A_1 S_1 XXXXXX\\
&\Longrightarrow S_0 A_1 S_1 XXXXX_1 X\\
&\Longrightarrow S_0 A_1' S_1 XXXXX_1 X\\
&\Longrightarrow S_0 A_1' S_1 XXXXxX\\
&\Longrightarrow S_0 A_1' A_1 S_1 XXXXxX\\
&\Longrightarrow S_0 A_1' A_1 S_1 X_1 XXXxX\\
&\Longrightarrow A_1' A_1' S_1 X_1 XXXxX\\
&\Longrightarrow S_0 A_1' A_1' S_1 xXXXxX
\end{aligned}
$$

$$S_0 \rightarrow F_1' S_0 \ (\{F_1\}; \{\mathcal{A}_5\}) \quad (7.344)$$
$$F_1 \rightarrow y \ \left(\{F_1'\};\right) \quad (7.345)$$
$$F_1' \rightarrow f_1 \ (; \{F_1\}) \quad (7.346)$$
$$S_0 \rightarrow F_2' S_0 \ (\{F_2\}; \{\mathcal{F}_1\}) \quad (7.347)$$
$$F_2 \rightarrow y \ \left(\{F_2'\};\right) \quad (7.348)$$
$$F_2' \rightarrow f_2 \ (; \{F_2\}) \quad (7.349)$$
$$S_0 \rightarrow F_3' S_0 \ (\{F_3\}; \{\mathcal{F}_2\}) \quad (7.350)$$
$$F_3 \rightarrow y \ \left(\{F_3'\};\right) \quad (7.351)$$
$$F_3' \rightarrow f_3 \ (; \{F_3\}) \quad (7.352)$$
$$S_0 \rightarrow F_4' S_0 \ (\{F_4\}; \{\mathcal{F}_3\}) \quad (7.353)$$
$$F_4 \rightarrow y \ \left(\{F_4'\};\right) \quad (7.354)$$
$$F_4' \rightarrow f_4 \ (; \{F_4\}) \quad (7.355)$$
$$S_0 \rightarrow F_5' S_0 \ (\{F_5\}; \{\mathcal{F}_4\}) \quad (7.356)$$
$$F_5 \rightarrow y \ \left(\{F_5'\};\right) \quad (7.357)$$
$$F_5' \rightarrow f_5 \ (; \{F_5\}) \quad (7.358)$$

FIGURE 7.17: Rules for reordering the information about enterprises that invited other enterprises without claiming items

S_1 then invites S_2 and S_3 by applying rule (7.281).

$$S_0 A_1' A_1' S_1 xXXXxX \Longrightarrow S_0 A_1' A_1' S_3^o S_2^o xXXXxX$$

S_2 claims an items as follows:

$$\Longrightarrow S_0 A_1' A_1' S_3^o S_2 xXXXxX$$
$$\Longrightarrow S_0 A_1' A_1' S_3^o A_2 S_2 xXXXxX$$
$$\Longrightarrow S_0 A_1' A_1' S_3^o A_2 S_2 xX_2XXxX$$
$$\Longrightarrow S_0 A_1' A_1' S_3^o A_2' S_2 xX_2XXxX$$
$$\Longrightarrow S_0 A_1' A_1' S_3^o A_2' S_2 xxXXxX$$

S_2 then invites S_3 and S_5 by applying rule (7.287).

$$\Longrightarrow S_0 A_1' A_1' S_3^o A_2' S_3^o S_5^o xxXXxX$$

$$S_0 \rightarrow Z_1' S_0 \; (\{Z_1\}; \{\mathcal{F}_5\}) \tag{7.359}$$
$$Z_1 \rightarrow y \; \left(\{Z_1'\}\,;\right) \tag{7.360}$$
$$Z_1' \rightarrow z_1 \; (; \{Z_1\}) \tag{7.361}$$
$$S_0 \rightarrow Z_2' S_0 \; (\{Z_2\}; \{\mathcal{F}_5, \mathcal{Z}_1\}) \tag{7.362}$$
$$Z_2 \rightarrow y \; \left(; \{Z_2'\}\right) \tag{7.363}$$
$$Z_2' \rightarrow z_2 \; (\{Z_2\}\,;) \tag{7.364}$$
$$S_0 \rightarrow Z_3' S_0 \; (\{Z_3\}; \{\mathcal{F}_5, \mathcal{Z}_2\}) \tag{7.365}$$
$$Z_3 \rightarrow y \; \left(; \{Z_3'\}\right) \tag{7.366}$$
$$Z_3' \rightarrow z_3 \; (\{Z_3\}\,;) \tag{7.367}$$
$$S_0 \rightarrow Z_4' S_0 \; (\{Z_4\}; \{\mathcal{F}_5, \mathcal{Z}_3\}) \tag{7.368}$$
$$Z_4 \rightarrow y \; \left(; \{Z_4'\}\right) \tag{7.369}$$
$$Z_4' \rightarrow z_4 \; (\{Z_4\}\,;) \tag{7.370}$$
$$S_0 \rightarrow Z_5' S_0 \; (\{Z_5\}; \{\mathcal{F}_5, \mathcal{Z}_4\}) \tag{7.371}$$
$$Z_5 \rightarrow y \; \left(; \{Z_5'\}\right) \tag{7.372}$$
$$Z_5' \rightarrow z_5 \; (\{Z_5\}\,;) \tag{7.373}$$

FIGURE 7.18: Rules for reordering the information about enterprises that claimed items without inviting other enterprises

The enterprise represented by S_3^o now appears twice on the sentential form. Rule (7.264) applies as follows:

$$\Longrightarrow S_0 A_1' A_1' R_3 A_2' S_3^o S_5^o xxXXxX$$

S_3 then claims two items as follows:

$$S_0 \rightarrow E_1'S_0\ (\{E_1\};\{\mathcal{F}_5) \quad (7.374)$$
$$E_1 \rightarrow y\ \left(\{E_1'\};\right) \quad (7.375)$$
$$E_1' \rightarrow e_1\ (;\{E_1\}) \quad (7.376)$$
$$S_0 \rightarrow E_2'S_0\ (\{E_2\};\{\mathcal{F}_5,\mathcal{E}_1\}) \quad (7.377)$$
$$E_2 \rightarrow y\ \left(\{E_2'\};\right) \quad (7.378)$$
$$E_2' \rightarrow e_2\ (\{E_2\};) \quad (7.379)$$
$$S_0 \rightarrow E_3'S_0\ (\{E_3\};\{\mathcal{F}_5,\mathcal{E}_2\}) \quad (7.380)$$
$$E_3 \rightarrow y\ \left(;\{E_3'\}\right) \quad (7.381)$$
$$E_3' \rightarrow e_3\ (\{E_3\};) \quad (7.382)$$
$$S_0 \rightarrow E_4'S_0\ (\{E_4\};\{\mathcal{F}_5,\mathcal{E}_3\}) \quad (7.383)$$
$$E_4 \rightarrow y\ \left(;\{E_4'\}\right) \quad (7.384)$$
$$E_4' \rightarrow e_4\ (\{E_4\};) \quad (7.385)$$
$$S_0 \rightarrow E_5'S_0\ (\{E_5\};\{\mathcal{F}_5,\mathcal{E}_4\}) \quad (7.386)$$
$$E_5 \rightarrow y\ \left(;\{E_5'\}\right) \quad (7.387)$$
$$E_5' \rightarrow e_5\ (\{E_5\};) \quad (7.388)$$

FIGURE 7.19: Rules for reordering the information about enterprises that opted out of the coalition

$$\Longrightarrow S_0A_1'A_1'R_3A_2'S_3S_5^oxxXXxX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3S_3S_5^oxxXXxX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3S_3S_5^oxxX_3XxX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'S_3S_5^oxxX_3XxX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'S_3S_5^oxxxXxX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3S_3S_5^oxxxXxX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3S_3S_5^oxxxX_3xX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3'S_3S_5^oxxxX_3xX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3'S_3S_5^oxxxxxX$$

S_3 invites S_1 and S_4 by applying rule (7.293).

$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3'S_1^oS_4^oS_5^oxxxxxX$$

S_4 opts out of the coalition by applying rule (7.265) and rule (7.297).

$$S_0 \rightarrow D_1' S_0 \ (\{D_1\}; \delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D}_0) \quad (7.389)$$
$$D_1 \rightarrow y \ (\{D_1'\};) \quad (7.390)$$
$$D_1' \rightarrow d_1 \ (; \{D_1\}) \quad (7.391)$$
$$S_0 \rightarrow D_2' S_0 \ (\{D_2\}; \delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D}_1) \quad (7.392)$$
$$D_2 \rightarrow y \ (\{D_2'\};) \quad (7.393)$$
$$D_2' \rightarrow d_2 \ (; \{D_2\}) \quad (7.394)$$
$$S_0 \rightarrow D_3' S_0 \ (\{D_3\}; \delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D}_2) \quad (7.395)$$
$$D_3 \rightarrow y \ (\{D_3'\};) \quad (7.396)$$
$$D_3' \rightarrow d_3 \ (; \{D_3\}) \quad (7.397)$$
$$S_0 \rightarrow D_4' S_0 \ (\{D_4\}; \delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D}_3) \quad (7.398)$$
$$D_4 \rightarrow y \ (\{D_4'\};) \quad (7.399)$$
$$D_4' \rightarrow d_4 \ (; \{D_4\}) \quad (7.400)$$
$$S_0 \rightarrow D_5' S_0 \ (\{D_5\}; \delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D}_4) \quad (7.401)$$
$$D_5 \rightarrow y \ (\{D_5'\};) \quad (7.402)$$
$$D_5' \rightarrow d_5 \ (; \{D_5\}) \quad (7.403)$$

FIGURE 7.20: Rules for reordering the information about enterprises that could not claim items

$$\Longrightarrow S_0 A_1' A_1' R_3 A_2' A_3' A_3' S_1^o S_4 S_5^o xxxxxX$$

$$\Longrightarrow S_0 A_1' A_1' R_3 A_2' A_3' A_3' S_1^o E_4 S_5^o xxxxxX$$

Since the enterprise represented by S_1 has already performed its actions (we have two A_1's in the sentential form), rule (7.252) applies as follows.

$$\Longrightarrow S_0 A_1' A_1' R_3 A_2' A_3' A_3' R_1 E_4 S_5^o xxxxxX$$

S_5 claims an item as follows:

$$S_0 \rightarrow R_1'S_0\,(\{R_1\};\delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D} \cup \mathcal{R}_0) \tag{7.404}$$
$$R_1 \rightarrow R_1''\,(\{R_1'\};\{R_1''\}) \tag{7.405}$$
$$R_1' \rightarrow r_1\,\left(\{R_1''\};\right) \tag{7.406}$$
$$R_1'' \rightarrow y\,\left(;\{R_1'\}\right) \tag{7.407}$$
$$S_0 \rightarrow R_2'S_0\,(\{R_2\};\delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D} \cup \mathcal{R}_1) \tag{7.408}$$
$$R_2 \rightarrow R_2''\,(\{R_2'\};\{R_2''\}) \tag{7.409}$$
$$R_2' \rightarrow r_2\,\left(\{R_2''\};\right) \tag{7.410}$$
$$R_2'' \rightarrow y\,\left(;\{R_2'\}\right) \tag{7.411}$$
$$S_0 \rightarrow R_3'S_0\,(\{R_3\};\delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D} \cup \mathcal{R}_2) \tag{7.412}$$
$$R_3 \rightarrow R_3''\,(\{R_3'\};\{R_3''\}) \tag{7.413}$$
$$R_3' \rightarrow r_3\,\left(\{R_3''\};\right) \tag{7.414}$$
$$R_3'' \rightarrow y\,\left(;\{R_3'\}\right) \tag{7.415}$$
$$S_0 \rightarrow R_4'S_0\,(\{R_4\};\delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D} \cup \mathcal{R}_3) \tag{7.416}$$
$$R_4 \rightarrow R_4''\,(\{R_4'\};\{R_4''\}) \tag{7.417}$$
$$R_4' \rightarrow r_4\,\left(\{R_4''\};\right) \tag{7.418}$$
$$R_4'' \rightarrow y\,\left(;\{R_4'\}\right) \tag{7.419}$$
$$S_0 \rightarrow R_5'S_0\,(\{R_5\};\delta \cup \mathcal{A}' \cup \mathcal{F} \cup \mathcal{Z} \cup \mathcal{E} \cup \mathcal{D} \cup \mathcal{R}_4) \tag{7.420}$$
$$R_5 \rightarrow R_5''\,(\{R_5'\};\{R_4''\}) \tag{7.421}$$
$$R_5' \rightarrow r_5\,\left(\{R_5''\};\right) \tag{7.422}$$
$$R_5'' \rightarrow y\,\left(;\{R_5'\}\right) \tag{7.423}$$

FIGURE 7.21: Rules for reordering the information about enterprises that were invited more than once

$$S_0 \rightarrow c\,(;\delta \cup \mathcal{S} \cup \mathcal{A} \cup \mathcal{F} \cup \mathcal{E} \cup \mathcal{Z} \cup \mathcal{R} \cup \mathcal{D}) \tag{7.424}$$

FIGURE 7.22: Rule to introduce central marker

$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3'R_1E_4S_5xxxxxX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3'R_1E_4A_5S_5xxxxxX$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3'R_1E_4A_5S_5xxxxxX_5$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3'R_1E_4A_5'S_5xxxxxX_5$$
$$\Longrightarrow S_0A_1'A_1'R_3A_2'A_3'A_3'R_1E_4A_5'S_5xxxxxx$$

Since there are no items left available, S_5 has to apply rule (7.304).

$$\Longrightarrow S_0 A_1' A_1' R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$

To restructure the A_1''s first, rules (7.323) – (7.326) apply as follows:

$$\Longrightarrow B_1 S_0 A_1' A_1' R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow B_1 S_0 A_1' A_1'' R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 S_0 A_1' A_1'' R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 S_0 A_1' y R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 B_1 S_0 A_1' y R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 B_1 S_0 A_1'' y R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 a_1 S_0 A_1'' y R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 a_1 S_0 yy R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$

A_2' is restructured as follows:

$$\Longrightarrow a_1 a_1 B_2 S_0 yy R_3 A_2' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 a_1 B_2 S_0 yy R_3 A_2'' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 a_1 a_2 S_0 yy R_3 A_2'' A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$
$$\Longrightarrow a_1 a_1 a_2 S_0 yy R_3 y A_3' A_3' R_1 E_4 A_5' Z_5 xxxxxx$$

The A'_3's are restructured as follows:

$$\Longrightarrow a_1a_1a_2B_3S_0yyR_3yA'_3A'_3R_1E_4A'_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2B_3S_0yyR_3yA''_3A'_3R_1E_4A'_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3S_0yyR_3yA''_3A'_3R_1E_4A'_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3S_0yyR_3yyA'_3R_1E_4A'_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3B_3S_0yyR_3yyA'_3R_1E_4A'_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3B_3S_0yyR_3yyA''_3R_1E_4A'_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3S_0yyR_3yyA''_3R_1E_4A'_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3S_0yyR_3yyyR_1E_4A'_5Z_5xxxxxx$$

Finally, A'_5 is restructured as follows:

$$\Longrightarrow a_1a_1a_2a_3a_3B_5S_0yyR_3yyyR_1E_4A'_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3B_5S_0yyR_3yyyR_1E_4A''_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5S_0yyR_3yyyR_1E_4A''_5Z_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5S_0yyR_3yyyR_1E_4yZ_5xxxxxx$$

The Z_5 is restructured as follows:

$$\Longrightarrow a_1a_1a_2a_3a_3a_5Z'_5S_0yyR_3yyyR_1E_4yZ_5xxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5Z'_5S_0yyR_3yyyR_1E_4yyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5S_0yyR_3yyyR_1E_4yyxxxxxx$$

The E_4 is restructured as follows:

$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5E'_4S_0yyR_3yyyR_1E_4yyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5E'_4S_0yyR_3yyyR_1yyyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4S_0yyR_3yyyR_1yyyxxxxxx$$

The R_1 is restructured as follows:

$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4R'_1S_0yyR_3yyyR_1yyyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4R'_1S_0yyR_3yyyR''_1yyyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4r_1S_0yyR_3yyyR''_1yyyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4r_1S_0yyR_3yyyyyyyxxxxxx$$

Finally, the R_3 is restructured as follows:

$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4r_1R'_3S_0yyR_3yyyyyyyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4r_1R'_3S_0yyR''_3yyyyyyyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4r_1r_3S_0yyR''_3yyyyyyyxxxxxx$$
$$\Longrightarrow a_1a_1a_2a_3a_3a_5z_5e_4r_1r_3S_0yyyyyyyyyyxxxxxx$$

Now that all items claimed by members of the coalition have been structured, S_0 is replaced by a central marker by applying rule (7.424).

$$\Longrightarrow a_1a_1a_1a_1a_2a_3a_3a_5z_5e_4r_1r_3cyyyyyyyyyyxxxxxx$$

The generated word in this example is

$$\Longrightarrow a^2{}_1a_2a_3^2a_5z_5e_4r_1r_3cy^{10}x^6$$

Based solely on this word, we can deduce the following about the formed VBC.

- There are four enterprises that have claimed items (a_1, a_2, a_3, and a_5).
- Enterprise five claimed an item without inviting other enterprises to join the coalition.
- There is an enterprise (enterprise four) that was invited to join the coalition, but opted out.
- Enterprise one was invited to join the coalition twice.
- Enterprise three was invited to join the coalition twice.
- There were six items made available to this coalition, members of a coalition claimed all six of these items.

7.4 Discussion

We have demonstrated that rcgs are an appropriate grammar class for modelling coalition formation in a VBC. When using rcgs, we have shown an invited enterprise can either claim items, opt out of the coalition, or invite other enterprises with/without claiming items. In addition, in this grammar, in comparison to the rFcg, an enterprise can invite as many of its known associates as it requires, and it is ensured that each invited enterprise participates once per formed coalition. The number of items available to members of the coalition is known before the coalition formation process begins, and the enterprises do not claim more than was made available to them.

We now demonstrate that L_{detailed} cannot be generated by a cfg using the pumping lemma for context-free languages as defined in Theorem 3.15. This implies that cfgs cannot model coalition formation in a VBC, or any coalition that is represented by L_{detailed}.

Theorem 7.1. *L_{detailed} is not a context-free language (cfl).*

Proof. Assume that L_{detailed} is a cfl.

Let h be the integer of Theorem 3.15.

Let $u = pqrst = a_1^h z_1 c y^{h+1} x^h$.

Consider $qrs : |qrs| \leq h$.

i) Let qrs contain a_1's only. Then for $m = 2$, the resulting word u' will have more a_1's than x's. This implies that the enterprises have claimed more items than were available, thus $u' \notin L_{\text{detailed}}$.

ii) Let qrs contain the z_1. Then for $m = 2$, the resulting word u' will have two z_1's. This resulting word $u' \notin L_{\text{detailed}}$.

iii) Let qrs contain the c. Then for any value $m \neq 1$, the resulting word u' is not in L_{detailed}.

iv) Let qrs contain y's only. Then for $m = 2$, the resulting word u' will have more y's than x's. Thus $u' \notin L$.

v) Let qrs contain x's only. Then for any $m = 0$, the resulting word u' will have fewer x's than a_i's. Thus, $u' \notin L$.

vi) Let qrs contain y's and x's. Then for any $m = 0$, the resulting word u' will have fewer x's, and fewer y's than a_i's. Thus, $u' \notin L$.

Therefore, L_{detailed} is not a context-free language. □

Chapter 8

Conclusion

A virtual buying cooperative (VBC) is a temporal, single-level alliance amongst a group of physically distributed enterprises intending to purchase goods from a single supplier as a single larger entity [Ngassam and Raborife 2013]. In a VBC, an initiator enterprise initiates the coalition formation process by approaching the supplier with the intent to purchase items. The supplier then in turn replies with the total available number of requested items. The initiator enterprise then in turn invites its known associates, who may also invite their known associates, etc. to join the coalition. Once an enterprise has been invited to join the coalition, it has the following four options:

- Claim a number of items, and invite other enterprises.
- Claim a number of items without inviting other enterprises.
- Invite other enterprises without claiming any items.
- Neither claim items, nor invite other enterprises.

The aim of our study was to build a grammar whose production rules model these four options. In addition, a VBC environment as described by Ngassam and Raborife [2013] has interaction strategy rules amongst enterprises. These rules are as follows:

- Only invited enterprises can participate in a coalition.
- An invited enterprise can only participate once per formed coalition. In a case in which an enterprise is invited to join the coalition by more than one associate, this enterprise can only accept one invitation.
- An enterprise may invite an unlimited number of its known associates.

- An invited enterprise can claim as many items as it requires, provided that there are still items to be claimed.
- The total number of items claimed by all members of the coalition cannot exceed the total number of items made available to them by the supplier.

The production rules of the grammars used in our study also needed to ensure that these rules are adhered to during the formation of a coalition. A VBC may be viewed as a multi-agent system in which an agent (initiator enterprise) enlists the help of other agents (its associates, their associates, etc.) in order to perform a task efficiently. In the case of a VBC, this task involves purchasing items from a supplier at negotiated pricing. Such coalitions have been thoroughly investigated using game theory [Shenoy 1979; Peleg 1984; Rosenschein and Zlotkin 1994; Chalkiadakis *et al.* 2010]. Csuhaj-Varjú and Salomaa [1997] proposed a formal model for agents in a multi-agent system that collaborate with each other using a network for collaboration. In this study, tools that allow for the development of languages that support text processing via these networks, facilitating communication are described. However, it does not offer an application of how the proposed model works for a clearly defined system such a VBC.

This study presents five formal grammars being built in an attempt to model coalition formation in a VBC. The choice of grammars used depended on the given language which represents a formed coalition. For a given language, we built a grammar which generated that language. Based on the grammar, we deduced, based on its production rules, if the grammar models the interaction strategy employed by enterprises during the formation of the coalition as specified by Ngassam and Raborife [2013]. If this grammar did not model a VBC environment, we then built another grammar that generates languages of a higher expressive power than the previous grammar, examined its interaction strategy, and so on and so forth. This is an incremental process.

The first language presented in our study was a regular language. The coalition represented in this language comprises of enterprises that have claimed items, the number of items that can be claimed by members of a coalition is not reflected in the language. This language was used as a basis for the use of formal grammars in modelling coalition formation processes. We also used this language to demonstrate that although a regular grammar can model coalition formations, they cannot model a VBC coalition formation environment. This grammar can only model a coalition in which enterprises can claim as many items as they require. In addition, due to the linearity property of the production rules in a regular grammar, an enterprise can only invite one other enterprise. The linearity property refers to the fact that there can only be one non-terminal symbol on the right-hand side of a production rule in a regular grammar. In our study, each enterprise is represented by a non-terminal. Therefore, an enterprise that is part of the coalition can only invite one other enterprise.

In a VBC, the number of items enterprises can claim cannot exceed the number of items made available to them. In our next step, we placed a bound on the number of items that can be claimed by members of the coalition. This information gave rise to another language that represented a formed coalition comprising of enterprises that have claimed items. In addition, this language reflected the total number that was made available to members of the coalition as well as the condition that this number cannot be exceeded by the total number of items claimed by members of a coalition. We demonstrated, using the pumping lemma for regular languages, that once this restriction is imposed on the language, a regular grammar cannot generate it. This language can be likened to the context-free language, $L = \{a^m b^n \mid m \leq n\}$, where the number of a's is less than or equal to the number of b's. This implies that in a coalition where there is a bound on the number of items that can be claimed by members of the coalition, one needs a grammar with a higher expressive power than a regular grammar.

We then demonstrated that context-free grammars are sufficient in modelling such a coalition. However, the context-free grammars also had the same limitations in terms of modelling the coalition formation process in a VBC as the regular grammar. In the context-free grammars, an invited enterprise could invite only one other enterprise. In a VBC, enterprises are allowed to multiple other enterprises with each invited enterprise participating only once per formed coalition. If we allow enterprises to invite multiple other enterprises when modelling the coalition formation process using a regular grammar, or a context-free grammar, there may be a case in which an enterprise is invited to join the coalition by more than one enterprise. In this case, this enterprise may participate more than once in this coalition. This is because, we cannot, when using either a regular or context-free grammar, check if this enterprise has already been part of the coalition before that current invitation, allowing us to void the current invitation if that is the case.

In this study, there is a fixed number of enterprises (m) that may form part of the coalition. We noticed that when using a grammar in which an enterprise can only invite one other enterprise[1], enterprise m has a smaller set of rules than the other enterprises. This is due to the fact that this enterprise cannot invite another enterprise to join the coalition. If this enterprise were to invite another enterprise ($m + 1$), and enterprise $m + 1$ were to invite another enterprise, etc., we might end up in a situation in which

[1]As it is the case with the regular grammar, context-free grammars, and the random permitting context grammar presented in this study

the coalition formation process may never be concluded. That is, we have an infinite number of enterprises inviting each other, resulting in an endless loop.

In the context-free grammars investigated in this study, the items claimed by the enterprises are generated as they are claimed, that is, there is no bound on the number of items that can be claimed. In a VBC, before the coalition formation process can commence, the number of items that can be claimed is generated, and enterprises cannot claim more than was made available to them. We could build a context-free grammar in which the number of items that may be claimed by members of a coalition is generated before the coalition process can begin. However, we cannot ensure that enterprises cannot claim more than was made available to them during the coalition formation process. If we allow the items that may be claimed by members of a coalition to be generated before the coalition formation process begins, and also disallow for any generation of items once the process has begun, using a context-free grammar, an enterprise cannot check if there is an item available before it claims. This requires some form of context, and as per the definition of context-free grammars, these grammars do not have any context.

A random permitting context grammar allows the application of certain production rules if there are certain symbols in the sentential form. In the random permitting context grammar used to model coalition formation in our study, each enterprise can only invite one other enterprise. If we allow enterprises to invite multiple other enterprises, we cannot stop enterprises from participating more than once per formed coalition using an rPcg, as it is the case with the regular grammar, and the context-free grammars. This is because of the fact that although random permitting context grammars have context conditions, this condition only permits the application of a production, and cannot prohibit an application of a production rule. If we were to allow multiple invitations when using random permitting context grammars, we cannot prevent an enterprise from accepting all the invitations, thus participating in a single coalition more than once. Additionally, when using a random permitting context grammar, we cannot invite all the enterprises to join the coalition at the same time. This is due to the fact that we cannot ensure that these enterprises do not claim the same items, since we cannot prohibit an enterprise from claiming an item if another enterprise is already claiming that item. This may result in two or more enterprises claiming the same item, and at the end of the coalition formation process, enterprises having claimed more items than were made available to them.

In order to enable enterprises to invite multiple other enterprises without violating the interaction strategy rules during the formation of a VBC, the inviting enterprise would have to check that the enterprises it is inviting have not already been invited to join the coalition before the current invitation. Alternatively, the invited enterprises would have to signal that they have already been invited and cannot participate the second time around. To enable this, one would have to include some form of context in the grammar that would enable enterprises to check if they are eligible to join the coalition when invited before performing any actions. This would ensure that for each formed coalition, an enterprise participates once. Consider a situation whereby an enterprise is invited more than once, and for instance, on more than one occasion claims items without inviting other enterprises (that is, replaces its associated non-terminal with a z). This would imply that this enterprise will be associated with more than one z, which is not in accordance with any of the languages presented in this study. Thus, these three grammars, regular grammar, context-free grammar, and a random permitting context grammar, do not model coalition formation in a VBC.

In our study, when random forbidding context grammars are used, all enterprises are invited at the same time, and enterprises work sequentially to claim items. This implies that enterprises have only two options when invited to join the coalition, either opt out, or claim items. However, in a VBC, enterprises have two more options, that is; claim items and invite other enterprises, and invite other enterprises without claiming any items. It is possible to model all these options using random forbidding context grammars. One can easily prohibit an enterprise from participating more than once per formed coalition, when this enterprise has been invited by more than one associate using random forbidding context grammars.

In the languages representing a formed coalition presented in this study, the total number of items claimed by a specific individual enterprise appears as a single block of symbols. For instance, a coalition comprising of two enterprises, a_1 and a_2, with each enterprise having claimed two items is represented by $w = a_1^2 a_2^2$. In the languages presented in this study, a coalition comprising of two enterprises, a_1 and a_2, with each enterprise having claimed two items cannot be represented by $w = a_1 a_2 a_1 a_2$, or $w = a_2^2 a_1^2$, or $w = a_2 a_1^2 a_2$, or $w = a_1 a_2^2 a_1$, or $w = a_2 a_1 a_2 a_1$.

If we enable multiple invitations amongst enterprises when using random forbidding context grammars, the information regarding a formed coalition would still need to be restructured according to a specific language's definition. For instance, consider a situation in which our coalition comprises of two enterprises, a_1 and a_2, with each enterprise having claimed two items, and both enterprises having been invited by more than one enterprise resulting in $w = a_2 a_1 a_2 a_1$. At the end of the coalition formation process, we still need to restructure this information such that it is of the form $w = a_1^2 a_2^2$, in order to adhere to the structure of all the languages presented in this study. If we were to restructure using a random forbidding context grammar, we cannot guarantee at the end of the restructuring process, the correct number of a_1's and a_2's will be represented in the final word. In the random forbidding context grammars that generate our languages, each item that can be claimed by an enterprise is generated as it is claimed. As it is with rPcgs, this mitigates the risk of having two or more enterprises claiming the same, resulting in members of a coalition claiming more items than were available to them.

The main aim of our study is to model coalition formation in a VBC as specified by Ngassam and Raborife [2013] using formal grammars. This study has demonstrated that rcgs are appropriate in modelling the coalition formation process in a VBC adhering to the conditions and interaction strategies amongst the enterprises during its formation as described by Ngassam and Raborife [2013]. Formal grammars have been developed over the years with real-world applications in mind, such as Csuhaj-Varjú *et al.* [1994]; Csuhaj-Varjú and Salomaa [1997] to name a few. However, we are not aware of a study that investigates the applicability of random context grammars in modelling real-world technological applications whose functionality is fully specified. According to our knowledge, this study constitutes the first attempt at modelling coalition formation using random context grammars.

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