Survival Analysis of Bank Loans and Credit Risk Prognosis

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Dedication

I would like to dedicate this project to God Almighty for having this project as part of his grand plan for my life, and to my parents, Mr. Josephat and Mrs. Cecilia Dzikiti who always believed in me since my childhood. My daughter Kudzai Munemo, thank you for being patient with me throughout my studies. For this reason, I dedicate this project to you.

Acknowledgements

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Abstract

Standard survival analysis methods model lifetime data where cohorts are tracked from the point of origin, until the occurrence of an event. If more than one event occurs, a special model is chosen to handle competing risks. Moreover, if the events are defined such that most subjects are not susceptible to the event(s) of interest, standard survival methods may not be appropriate. This project is an application of survival analysis in a consumer credit context. The data used in this study was obtained from a major South African financial institution covering a five year observation period from April 2009 to March 2014. The aim of the project was to follow up on cohorts from the point where vehicle finance loans originated to either default or early settlement events and compare survival and logistic modeling methodologies. As evidenced by the empirical Kaplain Meier survival curve, the data typically had long term survivors with heavy censoring as at March 2014. Cause specific Cox regression models were fitted and an adjustment was made for each model, to accommodate a proportion p of long term survivors. The corresponding Cumulative Incidence Curves were calculated per model, to determine probabilities at a fixed horizon of 48 months. Given the complexity of the consumer credit lifetime data at hand, we investigated how logistic regression methods would compare. Logistic regression models were fitted per event type. The models were assessed for goodness of fit. Their ability to differentiate risk were determined using the model Gini Statistics. Model assessment results were satisfactory. Methodologies were compared for each event type using Receiver Operating Characteristic curves and area under the curves. The Results show that survival methods perform better than logistic regression methods when modelling lifetime data in the presence of competing risks and long term survivors.

List of Abbreviations

Term	Description
AFT	Accelerated Failure Time
AIC	Akaike Information Criteria
BCBS	Basel Committee on Banking Supervision
CIC	Cumulative Incidence Curve
DLR	Dichotomous Logistic Regression
EAD	Exposure at Default
EDA	Exploratory Data Analysis
ECOA	Equal Credit Opportunity Act
ES	Early Settlement
ESF	Empirical Survival Function
GS	Gini Statistic
HR	Hazard Ratio
IV	Information Value
KM	Kaplan Meier
LGD	Loss Given Default
LMH	Log-rank Mantel-Haenszel
LR	Logistic Regression
OLS	Ordinary Least Squares
PD	Probability of Default
PH	Proportional Hazards
ROC	Receiver Operating Characteristic
SBC	Schwarz Bayesian Criterion
VIF	Variance Inflation Factor
WoE	Weight of Evidence

Chapter 1

Introduction

Survival analysis is a statistical data analysis technique, designed to analyse the amount of **time** it takes for an **event** to occur, over an observation period. The technique models lifetime data. The units of time in survival analysis range from days, weeks, months, years and even decades from the beginning of follow up till an event occurs or until observation ceases (censorship). Survival analysis originated in the biomedical research discipline where the event of interest was death of biological organisms, (Smith and Smith, 2000). Nowadays, the event refers to various other aspects depending on the domain of study and its context. In the social sciences, the event may refer to a change in social status, for example marital status. In engineering, the event may be decommissioning of machines. Natural disasters may be taken as the event in geosciences and the onset of a disease is an event in Epidemiology. Where the event is a negative experience, for example relapse or death in cancer patients, the event is usually referred to as **failure**.

In this study, survival analysis is applied to consumer credit data, a case for a leading South African banking institution. Consumer credit data is analogous to lifetime data as it concerns the credit status of a cohort of customers with different loan repayment behaviours over a given observation period. A single money lending product offering instalment loans is considered in this case, whereby a customer repays the loan in instalments, on a monthly basis over a predetermined repayment period.

We apply survival methodologies to the prediction of two mutually exclusive events, default and Early Settlement (ES). The occurrence of these two events over the observation period impacts negatively on profitability. Lenders prefer a longer time to default as the acquired interest will compensate for, or even exceed losses due to default, (Stepanova and Thomas, 2002). In the consumer credit context, survival methods results can be used as input into the computation of credit risk parameters. These play a crucial role in risk management, and include the Probability of Default (PD), Loss Given Default (LGD) and Exposure at Default (EAD) models.

1.1 Why Survival Analysis?

Survival analysis is time to event analysis of lifetime data. It is applicable in any scientific domain of study where researchers are interested in measuring the likelihood of an event and when it is likely to occur. Conventional statistical modelling techniques are not compatible with the nature of survival data, since survival data is not strictly *normally* distributed and its format includes details of *censoring*. Censoring occurs when the survival time of some respondents is partly known, that is, the survival time is incomplete for some individual cases (Hubber and Patetta, 2013). Thus, survival data cannot be analyzed using the traditional statistical procedures such as linear regression where the normality assumption is key and censoring is not accounted for. According to Tableman (2008), prior to the survival analysis method, subjects with incomplete data were deleted before the analysis. Deletion of censored observations results in loss of valuable information and consequently underestimation or overestimation of parameters of interest. Survival methods are believed to give more accurate estimates as they accommodate the censored data obtained throughout the observation period.

Survival methodology is appealing for its flexibility in the usage of parametric and semi parametric models depending on the choice of the researcher and the underlying nature of the data. Where semi parametric models are used, as often is the case, a minimum of assumptions is required to obtain the key features desired from survival data, these are the survival and hazard functions. Semi parametric methodologies of survival analysis make no distributional assumption as to the appropriateness of the response variable, lifetime of bank loans in this case. This feature saves a lot on data assessment, preparation and computational time. It is less computationally intensive in the case of huge data sets where computer and software performance is key (Hubber and Patetta, 2013).

1.2 Background of Study

Vehicle Finance, which will be referred to as the "product" in this study, is mainly done through the mainstream banking system in South Africa. The product extends loans for the purchase of motor vehicles. This study focuses on the retail end of the market which finances the purchase of light-delivery vehicles, taxis, agricultural vehicles, motorcycles, watercraft, caravans, and trailers. It finances among other retail specifications, self-employed persons, taxi finance, and small businesses. The Instalment Sale Agreement product is considered in this study. Financing terms range between 12 and 72 months with/without a deposit depending on credit worthiness of individual customers at the time of application.

Figure 1.1 is a biplot constructed by the author based on a survey recently conducted on the South African automobile market concerning a sample of budget brands. A biplot is a graphical display of multivariate data where rows and columns are depicted as points. It is analogous to a scatter plot in a bivariate scenario (Le Roux and Gardner, 2005). It represents relationships and association among multiple variables. The biplot in Figure 1.1 depicts some factors which drive particular vehicle brands. The squares in the plot represent vehicle brands offered on the South African market. The bulk of the bank loans in terms of volumes are associated with popular brands represented in the biplot. The circles represent attributes. Each vehicle brand is mainly associated with attributes in close proximity to the respective square points.

According to the survey, Volkswagen offers powerful, high quality vehicles and customers readily recommend the brand. Nissan and Ford are environmentally friendly vehicles and motorists feel confident with the brands. Toyota and Mazda are fuel efficient and they offer good after sales service plans. Honda and Renault offer modern vehicles with attractive styling, prestigious and exciting. Hyundai, Volkswagen and Kia manufacture high quality and safe vehicles. Customers feel proud to be seen driving vehicles from the Suzuki range. Opel, Nissan and Ford vehicles are environmentally friendly budget cars and customers feel confident when driving them.

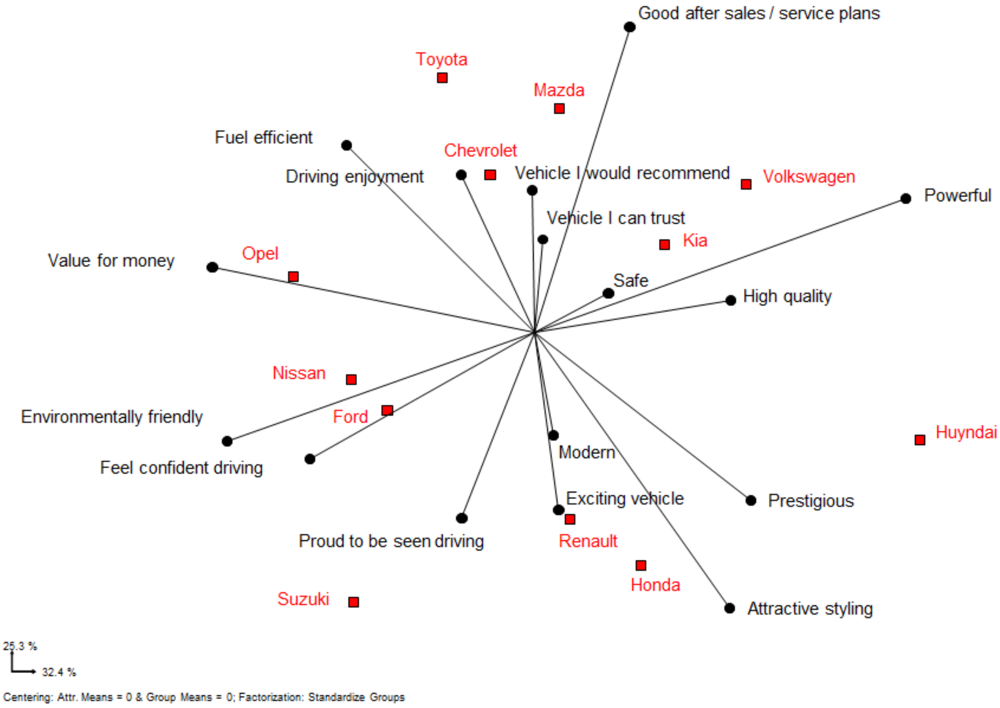


Figure 1.1: Biplot of Vehicle Brands

In practise, budget vehicles are cheap, fuel efficient and they offer good after sales/service plans. These are in demand mainly by the lower end of the market and the vehicles sales move in high volumes. For most consumers, budget vehicles are mainly used for day to day chores owing to reasonably low maintenance costs. Manufacturers well known to produce budget vehicles include Toyota, Mazda, Opel and Chevrolet. Luxurious vehicles are often expensive and sales volumes account for a smaller percentage.

These are in demand by the upper end of the market. Customers in this category are usually driven by exciting, high quality, attractive styling, powerful and prestigious vehicles.

Banks in South Africa play a very important role in financing vehicle purchases. The bulk of recipients of vehicle loans are “good” customers as they repay the loans over the agreed repayment period without impairments. However, it is inevitable to find “bad” customers within the system. This study focuses on the volume of bad customers and quantifies losses anticipated on such customers in a specified time interval. For compliance purposes and consistency with international standards, the two events analysed in this study, default and ES are defined in line with the Basel Accord (defined below) and bank definitions respectively.

1.2.1 Overview of the Basel Accord

Following the messy banking system collapse in the 1970s, the G10 major western economies agreed on bank supervison rules which regulates finance and banking internationally. The rules are set by a committee that meets in Basel, a city in the northwestern Switzerland, at the Bank for International Settlements. The Basel Committee on Banking Supervision (BCBS) (2006) issues and maintains Basel Accords.

Basel Accords are a series of recommendations on banking laws and regulations set out to ensure that international banks maintain adequate capital to sustain themselves during the periods of economic strain. According to Cai and Wheale (2009), the first accord, ‘Basel I’ was reached around 1988. The improved ‘Basel II’ was issued in 2004 and is currently in use. The Basel standards are not enforced by the Committee; however, countries that adopt the standards are expected to create and enforce regulations created from their specifications.

The South African banking community adopted the standards and this governs capital markets and model building standards. Accordingly, this study follows descriptions by the BCBS (2006) to define default. The definition of default is restated as follows:

1.2.2 Definition of *Default*

A default is considered to have occurred with regard to a particular borrower when either or both of the two following events have taken place.

1. The bank considers that the obligor is unlikely to repay his/her credit obligations to the bank in full.
2. The obligor is more than 90 days past the due date on any credit obligation to the banking group.

1.2.3 Definition of *Early Settlement*

Early settlement refers to early closure of loan accounts. The customer “settles” the outstanding amount ahead of the original repayment period. The reasons for early closure of accounts differ from customer to customer. Some customers close accounts by switching to another lender. In South Africa, some customers upgrade to newly released car models causing early closure of existing accounts and the opening of new accounts. A completely new set of customer specific and vehicle specific application variables are captured. As such, new accounts are treated separately from the old account for the same individual.

Default and ES events impact negatively on the lender because they both cut out a proportion of anticipated interest.

1.3 Motivation

Conventional models of credit risk were often built on static variables obtained from application data. Logistic Regression (LR) has been the cornerstone of credit models. It plays a very important role in building scorecards, which determine whether an applicant should be granted a loan or not. The scorecards, coupled with capital models, PD, EAD and LGD serve as input into the pricing models to calculate the amount the applicant qualifies for and the relevant interest rate at application.

Even though LR methods have been in use for model building, Belloti and Crook (2007) showed that survival analysis methods are more competitive and often superior to the LR approach. Survival methods use more information in model building than LR. More information includes details of censoring as well as survival time of every subject under study. Survival analysis techniques are applied in this study because of the existence of lifetime history of bank loans and censored observations in consumer credit data. Survival lifetime per account is the response variable.

Two events of interest are defined and considered simultaneously. These are default and ES. Analysis of more than one event in the same study is a variation of survival analysis known as ***competing risks*** analysis. A single customer can only experience one of the events and not both or gets censored. In this case, censorship occurs when a customer neither defaults nor pays off early such that the event of interest is never observed (Stepanova and Thomas, 2002). Censored subjects in this case are "good" customers. Thus, while logistic regression indicates if a customer will experience an event, leading to credit scoring, survival methods determine not only if, but also predict when, a customer will experience an event of interest (Banasik, Crook and Thomas, 1999). Survival analysis is applicable to both credit and profit scoring.

The likelihood of default and ES is highly influenced by general economic conditions that are measured by time-varying macroeconomic variables such as earnings, all-share price index, unemployment index, interest rates and so on. Time-varying variables cannot readily be included in logistic regression models. Belloti and Crook (2007) conducted experiments to prove that the inclusion of macroeconomic variables using survival methods gives statistically significant models and uplifts model predictive performance compared to models which exclude the macroeconomic variables. Survival analysis accommodates dynamic survival models by allowing for time-dependent variables such as macroeconomic and behavioral variables. Inclusion of time-dependent variables in survival methods is beyond the scope of this study due to data complexity in the consumer credit context.

1.4 Aims and Objectives

The purpose of this study is to analyse competing risks in consumer credit data with two events of interest, default and ES. The specific objectives are as follows:

1. Perform a univariate analysis using the Gini Statistic (GS) for each candidate covariate in order to identify and select variables capable of differentiating risk.
2. Conduct a multivariate analysis of the predictor variables, including correlation analysis and the Variance Inflation Factor (VIF) to determine the statistical relationships among them.
3. Use stepwise regression to select a combination of variables which strive to give optimum statistical power when used together in a model.
4. Fit the Cox regression model to the development data set for the default event assuming early repayment as the censored event.
5. Fit the Cox regression model to the development data set for the early repayment event assuming default as the censored event.
6. For each model, assess the validity of the proportional hazards assumption.
7. Use the cause specific hazards as an intermediate step into the calculation of the Cumulative Incidence Curve (CIC) for each model.
8. Fit a logistic regression model for each of the events.
9. Compare Cox regression and logistic regression based on estimating which loans are likely to default/pay off early in a fixed outcome period.

1.5 Research Data

This study explores a data set obtained in the consumer credit context. The analysis looks at facility level information, rather than at customer level. That means in the event that a customers holds more than one account, this study treats each account separately. The facility level rule is in line with the Basel Accord, “For retail exposures, the definition of default can be applied at the level of a particular facility, rather than at the level of the obligor. As such, default by a borrower on one obligation does not require a bank to treat all other obligations to the banking group as defaulted”, BCBS (2006, page 101).

The data set will consist of all active accounts between 01 April 2009 and 31 March 2014. Application and behavioural variables are provided per account in the data set. The repayment status is given per account per month under observation. A fixed workout/outcome period will be determined and used in the calculation of forward looking probabilities. A workout period is the amount of time it takes for the bulk of accounts to be absorbed into the events of interest.

1.6 Data Source

This study utilises consumer credit data obtained from a leading South African financial institution. The institution adopted standards outlined in the Basel Accord. This implies that the data complies with the international standards and that the data is credible for study purposes.

1.7 Limitations of Study

International legislation prevents the use of certain covariates such as gender and population group in credit granting decisions (Hand and Henley, 1997). This is meant to curb irrational prejudices. Classification is based solely on merit (past behaviour and credit record of applicants). Such variables prohibited by the law will not be used as covariates in this study.

The study considers customers whose loans were approved. No information is available for rejected applicants and for customers who rejected the offer (in attrition).

1.8 Summary

Chapter one introduced the reader to the methodologies proposed for use in this dissertation, the background of study, motivation, aims and objectives as well as limitations of study. Survival analysis originated in biomedical research but in this case it is being applied to consumer credit data which is analogous to lifetime data as it concerns a follow up on the behaviour of cohorts over time. Survival analysis is believed to be a more appealing approach in studies involving lifetime data as compared to logistic regression. This study is aimed at investigating the notion and verify whether survival analysis outperforms logistic regression in the presence of competing risks which are default and ES.

Chapter 2

Literature Review

2.1 Introduction

This chapter features the history, development and redevelopment of credit scoring systems spanning from the pre computer era to the current methodologies employed by financial institutions. Emphasis is placed on statistically sound approaches to modelling credit risk, possibilities, pitfalls and limitations of various techniques. Credit scoring systems have been redeveloped continuously to conform to the ever changing world.

Of major importance motivating this report is the critical statistical analysis of bank loans, not only if, but when, customers will default. This section zooms into the origins and progression of survival analysis techniques up to its application in models of credit. A theoretical review of survival analysis methods highlights the terminology and notation often used and describes relevant mathematical models applied. An overview of the features and theoretical properties of survivor curves is outlined as well as statistical approaches used to compare survivor curves and to test significance. Variations of the modeling technique arise due to the purpose of study, the nature of the outcome variable, the event(s) of interest and also the type of input variables available for the analysis.

2.2 Background to Credit Scoring

The term *credit* refers to an amount of money loaned to a customer by a granter, which must be repaid, with interest, under agreed terms and conditions. Loans may be fixed term, rolling or revolving, where loan amounts can be increased flexibly. *Credit scoring* is the name used for customer's classification into risk classes according to their ability to repay debt. Credit scores are usually estimated using the creditor's historic data under the presumption that the future is like the past. Prediction and modelling of the future is thus determined by the past historical information. The ability to offer credit by financial institutions has important profit implications while the ability to obtain credit by the customers has important quality of life implications (Jilek, 2008).

Prior to the computer age, credit granting decisions were based on subjective human assessment in a process called judgemental methods. There was no legislation in place to govern and control decisions made. According to Capon (1982), before the launch of the Equal Credit Opportunity Act (ECOA), passed in 1974, credit systems discriminated granting of loans based on gender and marital status. ECOA enforced equal opportunity in accessing loans by customers irrespective of gender and marital status. Judgemental methods of granting credit which involved individual judgement by a credit officer on a case to case basis were replaced by an automated way of making credit decisions, referred to as credit scoring. Not only banks adopted credit scoring, retailers, oil companies, travel and entertainment entities also utilised the credit scoring system.

Running concurrently to judgemental methods were numerical scoring systems, first developed in the mail order industry around 1930 (Capon, 1982). Certain characteristics were chosen for their ability to differentiate risk. Points were awarded to different levels of the characteristics. Decisions were based on the summated scores of individual applicants across characteristics and a predetermined set of fail to reject/reject cut-off values. Characteristics used in early systems included, inter alia, income, rent, marital status, life insurance ownership, collateral, occupation and length of employment. Numerical methods represented an important advance compared to judgemental methods but diffusion of quantitative methods only occurred after the development of computer technology in the 1960s.

Since the development of computer based systems, the use of credit scoring systems has expanded enormously. While judgemental methods were subject to credit evaluators, decisions made with credit scoring systems are objective and free from arbitration. The use of credit scoring systems by credit granters reduces bad debt losses as more customers are granted credit, improves consistency in decision making and costs of granting credit are reduced as automation of systems cuts down human effort (Capon, 1982).

Credit scoring is often used for *application scoring* (whether to fail to reject or reject a new applicant) and *behavioural scoring* (prediction of likelihood of default by already accepted customers). It can also be applied in other fields such as fraud risk and facility risk. This study is inclined towards behavioural scoring where the prediction of likelihood to default and early repayment is of prime interest. With the emergence of computer usage, statistical methods were developed to help granters identify and model good and bad risks (Hand and Henley, 1997).

In practice, classical statistical approaches were used to build credit models. Discriminant analysis and linear regression techniques were popular for being conceptually straightforward and widely available in most statistical computer packages. Regression coefficients and numerical attributes of variables/characteristics are combined and added to give an overall score (Capon,1982).

Other statistical approaches explored in credit scoring include logistic regression, probit analysis, nonparametric smoothing, mathematical programming, Markov Chain models, recursive partitioning, expert systems, genetic algorithms, neural networks and survival analysis (Hand and Henley, 1997). In the statistical fraternity, the methodologists search possibilities and pitfalls in different approaches, hence seek development and redevelopment of models. There is no overall "best" method. The choice of model depends on nature of the problem: data structure, costs involved, time, research question to be addressed, nature and the number of predictor variables.

In banks, the most widely used statistical methods of modeling credit risk include: linear regression, logistic regression and survival analysis. Expert systems are not statistically based models. They are often intuitive, based on expert knowledge, not only one expert but several experts with different experiences and views are involved. The expert system method is often time consuming and involves a lot of logistics properly carried out prior to discussions and brainstorming. Ordinary linear regression has been used in credit modelling for its simplistic nature, ease of computation and interpretation.

Consider a linear regression model with a dependent variable Y_i, k independent variables (X_1, X_2, X_3, ... , X_k) and n observations:

$$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + ... + \beta_k X_{ki} + \epsilon_i, \quad i = 1, 2, 3...., n \tag{2.1}$$

where β_i's are the parameter estimates and ϵ_i is the error term. The dependent variable is assumed to be continuous and responses can be in the range (-∞ , +∞). The use of linear regression however requires some principal assumptions:

- *Linearity* of the relationship between the dependent and the independent variables
- *Independence* of errors (no serial correlation)
- *Homoscedasticity* (constant variance of errors)
- *Normality* of the error distribution

To avoid inefficient or biased regression models, the above assumptions should not be violated. Often heteroscedasticity occurs in the consumer credit context where the dependent variable, default is binomial (Jilek, 2008). Hence, most financial institutions adopted logistic regression which gives more accurate results for a binary response variable. The scoring function in logistic regression is the probability of default or ES, whereas in linear regression, the researcher has to convert the results to a probability.

2.3 Logistic Regression

The Logistic Regression technique is deemed a superior statistical technique to Ordinary Least Squares (OLS) regression method when the response variable is categorical (Jilek, 2008). OLS regression suffers due to its strict statistical assumptions given in the previous section. The most commonly used LR is the dichotomous (binary response) logistic regression, the methodology can be generalised to polytomous outcomes (more than two categories in the response variable). In this study, we focus on Dichotomous Logistic Regression (DLR).

Various articles on probability models, including Peng, Lee and Ingersoll (2002), Lottes, DeMaris and Adler (1996) outline theoretical processes to show the strength of LR over OLS regression. In the consumer credit context, Jilek (2008) supports the superiority of LR over OLS regression given the nature of the data and the research questions inclined to seek and investigate probabilities. However, it is believed that some banks and other credit scoring companies are still using the inferior OLS regression, it is acceptable even though better statistical approaches are in place (Jilek, 2008).

In a dichotomous outcome variable, a plot against a continuous independent variable may result in two parallel lines due to the responses. A line of best fit in such cases may not be correctly estimated by OLS regression especially when the line is curved on its extreme ends. Such a shape, usually a sigmoidal or S-shaped does not follow a linear trend on the extreme ends, the errors are neither normally distributed nor of constant variance across the entire range of data. Logistic regression handles the problems of non-normality and non linearity using its logit transformation on the dependent variable expressed as follows (for simple logistic model):

$$logit(Y) = log_e(odds) = ln\left(\frac{p}{1-p}\right) = \alpha + \beta X \tag{2.2}$$

where the response variable Y is coded 1 for event and 0 for non event, X is the independent variable. The odds (event) = $\frac{p(events)}{p(non\ events)}$, p represents the probability of event = $\frac{number\ of\ events}{total(events,non\ events)}$, α is the intercept, β is the regression coefficient of X and $e = 2.71828$ is the base of the system of natural logarithms.

The odds (non event) = $\frac{p(non\ events)}{p(events)}$. The p(non event), $1 - p = \frac{number\ of\ non\ events}{total(events, non\ events)}$. Hence the $p(event) + p(nonevent) = 1$. The odds of events, odds(event) is the reciprocal of the odds(non event), thus, the odds(event) multiplied by odds(non event) = 1. An odds ratio, which is a measure of effect in LR, is a quotient of two odds and is used to compare the two odds. An odds ratio greater than 1, indicates an increased likelihood of an event while an odds ratio of less than 1, indicates a decreased likelihood of an event, (Lottes, *et al*, 1996).

Taking the antilog on both sides of equation 2.2, the logistic regression equation to predict the probability of the outcome of interest given x, a specific value of X, becomes a nonlinear relationship between the probability of Y and X:

$$p = Probability(Y = event\ of\ interest|X = x) = \frac{e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}} \tag{2.3}$$

Extending the above logic to multiple predictors, the expression for logistic regression given a vector of predictors X_1 to X_k is thus,

$$p = Probability(Y = event\ of\ interest|\mathbf{X}) = \frac{e^{\alpha+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3+\ldots+\beta_k x_k}}{1 + e^{\alpha+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3+\ldots\beta_k x_k}} \tag{2.4}$$

In probability models, often used in risk models, the dependent variable values must lie between 0 and 1. The right hand side of OLS equation 2.1 has no such limitation and as such, it is possible to generate probabilities outside the [0,1] limits. Moreover model 2.1 implies constant slopes for the predictors. The output effectively loses logical sense at extreme values of the response variable. On the contrary, logistic regression model 2.4 constrains the predicted probabilities to lie within the range [0,1] and it allows the predictors to have a diminishing effect at extreme values of the dependent variable (Lottes, *et al*, 1996).

Considering the right side of model 2.4, the exponential function is always non negative and always falls between 0 and 1. Moreover, the slopes of the predictor variables in the LR model are estimated using the maximum likelihood function, relating observed responses to predicted probabilities, compared to the weighted least squares approach in OLS, which minimises the sum of squared deviations between observed and predicted values.

The goal of LR is to correctly predict the category of the outcome using a parsimonious model. The logistic regression procedure has the ability to perform stepwise regression where model fit is assessed with addition or deletion of a possible candidate variable. The process results in parameter estimates that have optimum properties such as lack of bias and minimum variances. Nonetheless, while logistic regression tells us if the customer will default/settle early, survival methods suggest not only if but when customers will experience an event (Stepanova and Thomas, 2002). Statisticians developed further probability models to handle lifetime data.

Survival analysis methods recently gained popularity in the credit modeling context. In the literature, many authors, for example Bellotti and Crook (2007), have conducted analyses to show that survival methods are often superior to the conventional statistical methods of modeling credit risk. The more advanced survival methodology uses more information than conventional models as it allows details of censoring and time which cannot be easily incorporated in either linear or logistic regression models. Survival methods use the fewest assumptions to obtain the required key analysis. No distributional assumptions of the response variable are required. This study compares LR and survival techniques in modelling competing risks.

Survival methods are believed to be superior to LR. Researchers such as Stepanova and Thomas (2002) and Belloti and Crook (2007) conducted studies to show the limitations of LR in handling survival data and how it is inferior to survival analysis. This study reports on the notion using consumer credit data. The following sections focus on survival analysis methodology.

2.4 Survival Analysis

Survival analysis comprises a pool of specialised methods used to analyse lifetime data. The response variable is time until an event occurs and/or time to censorship. Censorship is the unique feature of survival analysis where survival experience is partly known. The response variable can be continuous or discrete. Events can be positive, where subjects recover from an event or negative, where subjects relapse, die or contract diseases.

Other terms referring to survival analysis include event history analysis, durability analysis, reliability analysis, lifetime analysis, time to event analysis and so forth. It is possible to use survival methods in cases where the outcome is different from time. For example, the case where a researcher wishes to analyse the amount of mileage until a tyre bursts, the number of cycles until an engine requires repair (Hubber and Patetta, 2013).

Survival Analysis dates back to life and mortality tables mainly used in actuarial science and demography from around the 17^{th} century. It led to the true meaning of "survival" through mortality rates. According to Odd, Andersen, Borgan, Gill, and Keiding (2009), the original life tables method was based on wide time intervals and large data sets. Around the 1950's Kaplan and Meier proposed an estimator of survival curves (Odd, *et al*, 2009). They developed a method of short time intervals and smaller sample sizes compared to those used in the actuarial and demographic studies. The 20^{th} century saw further developments in handling survival data.

Cox (1972) introduced a method of incorporating covariates in the analysis of survival data in what is known as the Cox Proportional Hazards (PH) model. This model uses time independent covariates or static variables and assumes constant proportional hazards. However, in reality we are bound to have time dependent variables in the modelling of survival data. Time dependent covariates violate the constant proportional hazard assumption, thus the Cox PH model was further developed into the **extended** Cox and the **stratified** Cox models which measure the interaction of exposure with time. This study uses the Cox PH model to fit survival models and to analyse competing risks in consumer credit data.

2.5 Theoretical Review of Survival Analysis

The primary information desired from survival data includes survival and hazard functions. This is obtained using survival functions and hazard functions respectively. Survival curves are monotone decreasing over time and their values range from 1 at the beginning of study and approximate 0 as time goes to infinity. These describe the probability of survival of subjects over time. On the other hand, hazard functions focus on failure and they describe a *rate* of failure, and not a *probability* (Kleinbaum and Klein, 2005). Further details onto the computation of survival and hazard curves are given in the sections below.

Survival Time

Considering a follow up of customers to see how long they take before they default on bank loans, the survival time describes the amount of time a customer takes before they default. Survival time is denoted by a random variable **T**. Time is non negative therefore **T** takes values greater than or equal to zero ($\mathbf{T} \geq 0$). Subsequently, small t represents a specific value of **T**. The cumulative distribution function (c.d.f) of **T** according to Tableman (2008) is given by

$$F(t) = P(T \leq t) = \int_0^t f(x)dx \tag{2.5}$$

where $f(x)$ represents probability density function of time.

Survivor Function

The survivor function, also referred to as the *reliability function*, denoted by $S(t)$ is the probability that a respondent survives beyond a specified time t. Survival probabilities at different time lags help in summarizing survival data (Kleinbaum and Klein, 2005). The expression for the survivor function is given by:

$$S(t) = P(T > t) = 1 - F(t) = \int_t^\infty f(x)dx \tag{2.6}$$

Theoretically, $S(t)$ is a monotone decreasing probability function, that is: at $t = 0$, $S(t) = S(0) = 1$ and at $t = \infty$, $S(t) = S(\infty) = 0$. Thus, $S(t) \in$ [0,1]. Therefore $S(t)$ is essentially a probability of surviving beyond time t.

Hazard Function

The hazard function, also called the *mortality rate* or *conditional failure rate*, is the measure of potential failure at time t given that the respondent has survived up to some time t. The hazard function $h(t)$ is a rate expressed as the ratio of $f(t)$ to $S(t)$ and it is not a probability. Hence $h(t)$ takes nonnegative infinite values $[0, \infty)$. The hazard function is mathematically expressed as follows:

$$h(t) = lim_{\Delta t \to 0^+} \frac{P(t \leq T < t + \Delta t \mid T \geq t)}{\Delta t} = \frac{f(t)}{S(t)} \tag{2.7}$$

where Δt represents a small time interval. The hazard rate gives the key/primary information in survival analysis as it determines the occurrence and timing of events.

Survivor Function versus Hazard Function

The survivor function is a probability, its values range from 0 to 1. The hazard function is a rate and it takes any value from 0 to ∞. A hazard function expresses the instantaneous potential for an event/failure to occur given that a respondent has survived up to a specified t, while the survivor function simply gives a survival probability at time t. The two functions give opposite information about the survival data under study (Kleinbaum and Klein, 2005). The relationship between the survivor function and the hazard function is defined as:

$$S(t) = exp\left[-\int_0^t h(u)du\right] \tag{2.8}$$

$$h(t) = -\left[\frac{dS(t)/dt}{S(t)}\right] \tag{2.9}$$

Thus, the survivor function can be derived from the hazard function, and vice versa. The hazard function is the widely used tool instead of the survivor function because:

- It gives the instantaneous potential at a specific time t and it is more informative with regards to the underlying failure mechanism.

- It can be used to determine the underlying statistical model in a data set. The models include exponential, lognormal or increasing/decreasing Weibull among other survival models.

- Survival models are mostly expressed in terms of the hazard function and this becomes handy in summarizing survival data.

The Hazard Ratio

In survival analysis, the Hazard Ratio (HR) is the measure of the effect of explanatory variables. The corresponding measure of the effect in a logistic regression is the odds ratio and β measures the effect of independent variables in ordinary linear regression. In survival analysis, when two samples are exposed to different treatments, say 0 and 1, the HR is given by:

$$HR(t|1,0) = \frac{h(t|1)}{h(t|0)} \tag{2.10}$$

HR = 1, means there is no treatment effect. For values of HR less than 1, it implies the sample receiving treatment 0 has more risk of failing than the sample receiving treatment 1, and vice versa for HR values greater than 1. The HR plays a crucial role in survival analysis (Kleinbaum and Klein, 2005).

2.6 Censoring

In survival analysis, **censoring** is a term used to express incomplete data. This occurs when we do not have the entire information about some respondents over a specified follow up period. Only a part of information about the individual is known. In survival analysis, censoring is denoted by a dichotomous variable δ which takes values 0 or 1, to indicate whether an object was censored or not respectively. Censoring can either be fixed or random.

Fixed censoring is defined in two scenarios. **Type I** censoring is a fixed form of censoring whereby subjects under investigation are subjected to the same test at the same time and the experiment is terminated at predetermined time t. Failure times are either observed or censored. In Type I censoring, *t* is fixed. **Type II** censoring involves running an experiment until a predetermined number of failures, say *r* of the total objects *n* have been achieved. The remaining observations are all censored and *r* is fixed.

Random censoring occurs when objects enter the study at different times. Censoring occurs when:

- A respondent relocates without traceable contacts or may fail to appear due to some other cause. This is called loss to follow up.
- A respondent discontinues participation abruptly and drops out.
- An event hasn't occurred when the study is terminated.

When a respondent's information is censored using any of the above scenarios, that is, censored at the right side of observed time, we call this **right censoring**. **Left censored** data occurs when the true survival time is less than or equal to the total follow up period. For example, the event occurred to the respondent before the start of the study and the actual event time is unknown. **Interval censored** data is obtained when the event is known to have occurred between two time points but the exact time is unknown. Random censoring with right censored information is common in the consumer credit context.

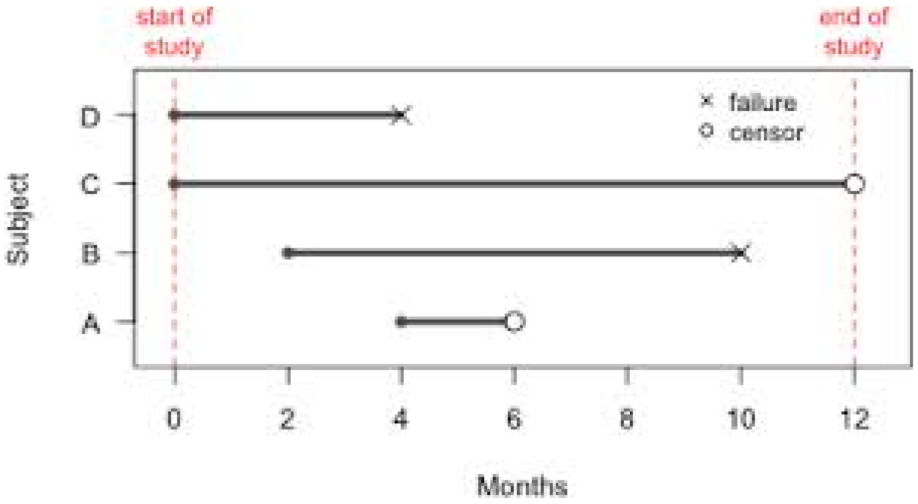


Figure 2.1: Censoring

Figure 2.1 is an illustration of censorship in survival data. Four participants entered the study at different calender times. Subjects B and D experienced the event before the termination of study. Subject A withdrew from the study before the event. The study was terminated before subject C experienced an event. Both subjects A and C are right censored.

2.7 The Empirical Survivor Function

Survival data is composed of complete data (where an event took place) and incomplete (censored) data. The Empirical Survival Function (ESF) calculation ignores the censoring aspect. All observations are regarded as complete and they are ordered with respect to the length of stay in the study, from the minimum to maximum.

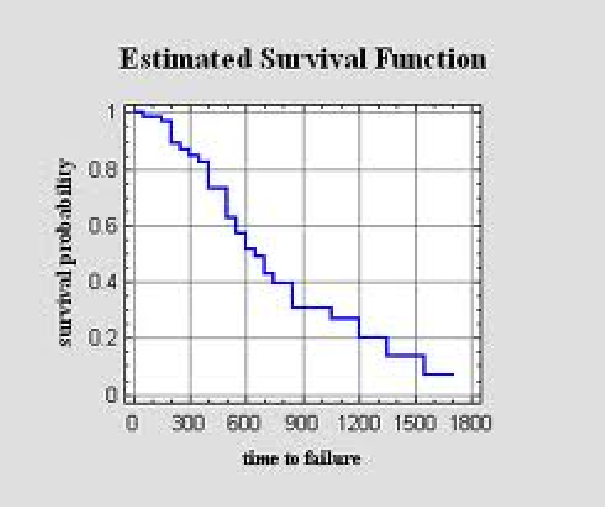


Figure 2.2: Empirical Survival Curve

The ESF is denoted by $S_n(t)$ and is calculated as follows:

$$S_n(t) = \frac{Number\ of\ observations\ at\ \mathbf{T} > t}{n} \tag{2.11}$$

where n is the total number of objects under study. Plotting $S_n(t)$ against time spent in the study gives a step function, stepping down at each time t as shown in Figure 2.2. The mean of the area under the curve is calculated as follows:

$$\widehat{mean} = \int_0^{\infty} S_n(t)dt \tag{2.12}$$

2.8 The Kaplan Meier Survival Estimator

The Kaplan Meier (KM) survival estimator is an extension of the ESF which generalises the ESF and adjusts for censored observations. The KM formula for survival probability at any specified time t_i is limited to product terms of survival probabilities prior to and including the survival probability of subjects surviving beyond t_i. Thus, the KM survival curve is also referred to as the product-limit estimator of survival.

For KM computation purposes, the survival data is sorted in ascending order of failure times. The first entry begins at survival time equals zero. At time zero, the probability of surviving beyond t_0 is 1. Subjects y_i, are eliminated at each failure point in time due to either failure or censorship. The **risk set**, denoted by $R(t_i)$ is the total number of individuals who survived up to at least a specified failure time point t_j.

The KM survival estimator considers: n_i = number of observations in the risk set and d_i = number of subjects failing at failure time t_j. The KM survival estimator is defined as:

$$\begin{aligned}
\widehat{S}(t_j) \quad &= \prod_{i=1}^{j} \widehat{Pr}\left[T > t_i | T \geq t_i\right] \\
&= \widehat{S}(t_{j-1}) \times \widehat{Pr}\left[T > t_i | T \geq t_i\right] \\
&= \prod_{i=1}^{j} \left(\frac{n_i \; - \; d_i}{n_i} \right) \qquad (2.13)
\end{aligned}$$

As obtained in the ESF, the KM estimator of survival produces a step function which steps down only at observed failure times. The risk set at time t_i excludes both failed and censored observations prior to the specified time point. Thus, the KM curve is greater than or equal to the ESF curve. If the data set is complete and there are no censored observations, the KM reduces to the ESF. The mean and median estimates are higher when using the KM function compared to the ESF in the presence of censored observations. A KM type estimate of hazard at time point t_i is given by:

$$\widehat{h}(t_i) = \frac{d_i}{n_i} \tag{2.14}$$

Comparison of Survivor Curves

Given two-sample (two treatment) survival curves, the graphs may differ judging by the eye. An example is shown in Figure 2.3. In order to investigate if the difference between the two curves is statistically significant, the most commonly used tests include the Log-rank Mantel-Haenszel (LMH) test and the Gehan test that adjusts for censored data (Kleinbaum and Klein, 2005). The LMH test is the generalized log-rank test adjusting for covariates. It gives the overall comparison of survivor curves. The null hypothesis states that there is no difference between the two survival curves, that is: $H_0 : F_1 = F_2$. The values of the two samples are combined and listed in ascending order.

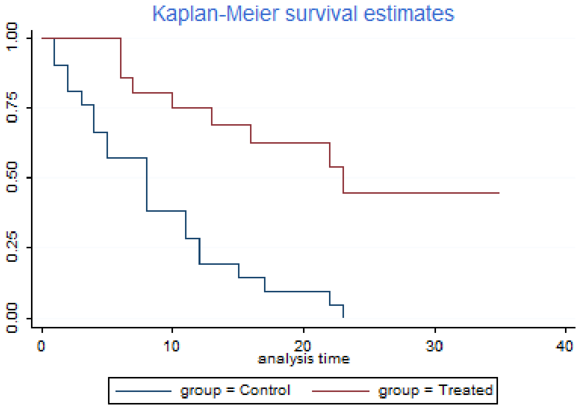


Figure 2.3: Comparison of Survivor Curves

The LMH statistic computation is presented in Tableman (2008). Let z denote the combined ordered values of the two samples; n is the total number of objects at risk for the combined data; m_1 is the total number who failed at point z; n_1 the number at risk for treatment 1 at point z; a = 1 if event A occurred on a treatment 1 value or 0 if event occurred in treatment two value. The expectation and variance are calculated as follows:

$$E_0(A) = \frac{m_1 n_1}{n} \tag{2.15}$$

and

$$Var_0(A) = \frac{m_1(n - m_1)}{n - 1} \times \frac{n_1}{n}\left(1 - \frac{n_1}{n}\right) \tag{2.16}$$

The LMH statistic is thus obtained as,

$$MH = \frac{\sum_{i=1}^{k}(a_i - E_0(A_i))}{\sqrt{\sum_{i=1}^{k} Var_0(A_i)}} \tag{2.17}$$

The LMH and the log-rank tests employ the concept of a Chi-square test and make use of observed minus expected counts as seen in equation 2.17. A *p-value* is defined as the probability of obtaining a value as extreme as the test statistic. A p-value more than 0.05 implies that there is sufficient evidence to fail to reject the null hypothesis, otherwise the two treatments give different effects to objects under study. These tests can be applied to more than two groups. The null hypothesis still states that there are no differences among all curves in question. Other tests include the Wilcoxon, the Tarone-Ware, the Peto and the Flemington-Harrington test. All of which are variations to the log-rank test and may be used according to the purpose. They differ mostly in weighting. For example, the Peto test weights more on earlier failures while the log-rank put more weight on later failures (Kleinbaum and Klein, 2005). Thus, the choice of a test depends on the purpose and context of the study.

2.9 The Cox Proportional Hazards Regression Model

The previous section detailed KM survival curves. Recall that the computation of KM survival curves do not adjust for covariates and there is no regression model fit to obtain survival estimates. In a real life scenario, covariates are inevitable and they influence survival estimates. Exclusion of covariates in the calculation of survival curves may result in less accurate results. Cox (1972) developed a regression model which outputs *adjusted survival curves* by including covariates in the computation of survival estimates. The Cox PH regression methodology has gained popularity for it is flexibility and use of a small number of assumptions to obtain the basic information required from survival analysis, namely the HR which is obtained from the Cox hazard function and survival information obtained from the Cox model survival function.

The Cox model hazard function calculates the hazard at time *t* of a subject, adjusted for possible explanatory variables. The formula is expressed as the product of the baseline hazard function of time and an exponential function of covariates. The baseline hazard is an unspecified form of the Cox model and the distribution of the outcome (survival time) is unknown. This makes the Cox PH regression a **semiparametric** model. The semiparametric property of the Cox PH model makes it a robust model which can closely approximate parametric models. It is therefore regarded the "safe" model, when in doubt of the best fitting model. The baseline hazard function is expressed in terms of time and the exponent part ensures that only non-negative estimates are obtained.

The Cox PH model hazard function is:

$$h(t, \mathbf{X}) = h_0(t) \times \exp\left[\sum_{i=1}^{p} \beta_i X_i\right] \tag{2.18}$$

where **X** is a vector of predictor variables $X_1, X_2, X_3, ...X_p$. $h_0(t)$ is the **baseline hazard** which involves t only and no covariates, $\exp\left[\sum_{i=1}^{p} \beta_i X_i\right]$ is an exponential component of the model that involves time independent covariates **X**. Time independent variables do not change over time, for example population group and nationality. In the absence of explanatory variables, the Cox PH model reduces to the baseline hazard $h_0(t)$. The Cox PH model survival function is given by:

$$S(t|\mathbf{X}) = [S_0(t)]^{\exp\left[\sum_{i=1}^{p} \beta_i X_i\right]} \tag{2.19}$$

2.9.1 Maximum Likelihood Estimation of the Cox Regression Model

Typical parametric regression models depend on some specified distribution of the response variable which forms the basis for the likelihood function. A full likelihood function is derived for parametric regression models whereas Cox regression uses the partial likelihood function. Cox regression makes no distributional assumption of the dependent variable (time to event). The regression coefficients, $\widehat{\beta}'s$ in a Cox regression model are the maximum likelihood estimates. The coefficients are derived by maximising a likelihood function $L = L(\beta)$ which is equal to the joint probability of observed data. L_j is a “partial” likelihood function as its computation considers probabilities for subjects who fail together with elements in the risk set. It does not consider the probabilities of censored subjects. Partial likelihood is determined at each failure time and is expressed as the product of likelihoods per each failure time. Thus, for k failure times,

$$L = L_1 \times L_2 \times L_3 \times ... \times L_k = \prod_{j=1}^{k} L_j \tag{2.20}$$

Once the likelihood function is obtained, the next step maximises this function by maximising the natural log of L through a series of iterations. The maximization process is done through the mathematical differentiation process and setting the differential equal to zero.

$$\frac{dln(L)}{d\beta_i} = 0 \qquad i = 1, 2, 3, ..., p. \tag{2.21}$$

2.9.2 Tied Event Times in Cox PH Regression

The concept of partial likelihood estimation of regression parameters assumes no tied event times. However, tied event times are inevitable in consumer credit data and there are various approaches to treat tied events in survival analysis. Hubber and Patetta (2013) briefly described the different approaches used to modify the partial likelihood estimation to accommodate ties.

The *exact method* considers all possible orderings of the tied event times to compute the partial likelihood. It assumes that ties are due to lack of precision in measuring survival time. The exact method is computer intensive with large data sets. The *discrete method* assumes events occurred at exactly the same time. It replaces the proportional hazards model with the logistic model and computes the probabilities that the events occurred to a set of subjects with tied event times. It is also very computer intensive for large data sets with many ties.

The *Breslow method* approximates the exact method (Hubber and Patetta, 2013). It yields coefficients biased towards zero when the number of ties is large. It is less computer intensive compared to exact and discrete methods. The *Efron method* yields coefficients that are closer to the exact method and give a close approximation to the exact method. It uses less computer time compared with the Breslow method. If there are no ties in the data set, all methods described above result in the same likelihood and yield identical parameter estimates. Where the data set is big and there is a large number of ties, the Efron method is preferred.

According to Hertz-Picciotto and Rockhill (1997), the Breslow and Efron estimates are computed as follows: Let x_l be the vector of explanatory variables for the l^{th} individual. Let the ordered failure times be $t_1 < t_2 < t_3 < ... < t_k$. Let D_i be the set of individuals who failed at time t_i, d_i be the size of set D_i and R_i be the risk set at time t_i. Denote $\upsilon_i = exp\left[\left(\sum_{l\epsilon D_i} x_l\right)' \beta\right]$.

The likelihood for the Breslow approximation is:

$$L(\beta) = \prod_{i=1}^{k} \left\{ \frac{\upsilon_i}{\left[\sum_{l\epsilon R_i} exp(x_l'\beta)\right]^{d_i}} \right\}. \tag{2.22}$$

The likelihood for the Efron approximation is:

$$L(\beta) = \prod_{i=1}^{k} \left\{ \frac{\upsilon_i}{\prod\limits_{j=i}^{d_i} \left[\sum_{l\epsilon R_i} exp(x_l'\beta) - \frac{j-1}{d_i} \sum_{l\epsilon R_i} exp(x_l'\beta)\right]} \right\}. \tag{2.23}$$

2.9.3 Computation of the Hazard Ratio

Recall that, the HR is the measure of effect in survival analysis. HR in a Cox PH model is determined using the $\beta' s$ of the exponential part of the formula. It shows us the effect of explanatory variables even without estimating the baseline hazard function. We can thus, obtain the hazard and subsequently the survival function using a minimum of assumptions (Kleinbaum and Klein, 2005). This makes the Cox model the most appealing approach to analysing survival data. However, the Cox PH model assumes a constant HR for any two subjects over time.

HR is defined as the hazard for one subject divided by the hazard for another subject in the same study. The two subjects are distinguished by their values for the explanatory variables. Suppose two subjects' predictor values are denoted by $\mathbf{X}^{'}$ and $\mathbf{X}$ respectively, where

$$\mathbf{X}^{'} = (X_1^{'}, X_2^{'}, X_3^{'}, ..., X_p^{'}) \qquad \text{and} \qquad \mathbf{X} = (X_1, X_2, X_3, ..., X_p)$$

then the HR comparing the above subjects is computed in terms of regression coefficients obtained using the Cox hazard function as illustrated below:

$$\widehat{HR} = \frac{\widehat{h}(t, \mathbf{X}^{'})}{\widehat{h}(t, \mathbf{X})} = \frac{\widehat{h}_0(t) \times \exp\left[\sum_{i=1}^{p} \beta_i X_i^{'}\right]}{\widehat{h}_0(t) \times \exp\left[\sum_{i=1}^{p} \beta_i X_i\right]} \tag{2.24}$$

The baseline hazard function is the same for both subjects in the same study thefore it cancels out and $\widehat{HR}$ is computed using the exponential part of the Cox hazard formula. Using the mathematical rules of algebra on the exponent part, the $\widehat{HR}$ is further reduced to:

$$\widehat{HR} = \exp\left[\sum_{i=1}^{p} \beta_i (X_i^{'} \; - \; X_i)\right] \tag{2.25}$$

Thus, from the above computation, the $\widehat{HR}$ is independent of the baseline hazard function. It is also independent of time if and only if the predictor variables are not time varying.

2.9.4 The Cox Proportional Hazards Assumption

The Cox PH assumption states that the HR for any two individuals in the same study is constant over time. In other words, the hazard for a subject is proportional to the hazard for another subject in the same study where the proportionality constant, say $\widehat{\theta}$ is independent of time (Kleinbaum and Klein, 2005).

$$\widehat{\theta} = \exp\left[\sum_{i=1}^{p} \beta_i(\mathbf{X}_i^{'} - \mathbf{X}_i)\right] \tag{2.26}$$

This implies that

$$\widehat{h}(t, \mathbf{X}^{'}) = \widehat{\theta}\widehat{h}(t, \mathbf{X}) \tag{2.27}$$

The Cox PH model is appropriate for use when the PH assumption is met. When the HR vary with time, for example where hazards cross or when time varying confounding variables are present the PH assumption maybe violated, making it inappropriate to use the Cox PH model. Where the Cox PH assumption is not met, variations of the Cox model can be used, for example the **extended Cox regression** or the **stratified Cox regression** depending on the context.

2.9.5 Assessment of the Cox Proportional Hazards Assumption

The Cox PH model assumes a constant HR comparing any two subjects in the same study over time. There are various approaches used to evaluate the reasonableness of Cox PH assumption. These include, inter alia the graphical approach, goodness of fit tests as well as the time dependant variables assessment.

The graphical approach is the most widely used technique to evaluate the Cox PH assumption. Given a set of categorised or coarse classified covariates as the predictors in a Cox PH model, the estimated $-ln(-ln)$ survivor curves over different categories of covariates are compared. The PH assumption is satisfied when *parallel curves* for $-ln(-ln)$ survivor curves of different categories of the same covariate are obtained. The $-ln(-ln)$ survivor curves are popularly known as the *log-log* plots. A *log-log* survival curve is a transformation that results from taking the natural logarithm of an estimated probability curve twice. That is:

$$-ln(-ln(\widehat{S})) = -ln\left(exp\left[-\int_0^t h(u)du\right]\right) \tag{2.28}$$

where $\int_0^t h(u)du$ is the cumulative hazard function resulting from the formula for the relationship between survival curves and hazard function that is given by:

$$S(t) = exp\left[-\int_0^t h(u)du\right]$$

The first log of a survival curve is always negative because mathematically the log of a fraction is negative. Therefore the first log is negated and the value becomes positive to allow for the second log. Recall from calculus that the log of a negative number is undefined. However, after taking the log twice the final result can be positive or negative. While the scale of the y axis of a survival curve ranges between 0 and 1 for the survival curve being a probability, the corresponding scale for a $-ln(-ln)$ range between $-\infty$ to $+\infty$. This allows more flexibility and inferences regarding the PH assessment (Kleinbaum and Klein, 2005).

The algebraic formulation of the log-log curves stems from the Cox PH survival curve given by:

$$\begin{aligned}
S(t|\mathbf{X}) &= [S_0(t)]^{\exp\left[\sum_{i=1}^p \beta_i X_i\right]} \\
Taking\ the\ first\ log: & \\
ln(S(t|\mathbf{X})) &= \exp\left[\sum_{i=1}^p \beta_i X_i\right] \times ln\left[S_0(t)\right] \\
Taking\ the\ second\ log: & \\
ln\left[-ln(S(t|\mathbf{X}))\right] &= -\sum_{i=1}^p \beta_i X_i - ln\left[-lnS_0(t)\right] \\
-ln\left[-ln(S(t|\mathbf{X}))\right] &= +\sum_{i=1}^p \beta_i X_i + ln\left[-lnS_0(t)\right] \qquad (2.29)
\end{aligned}$$

Hence, after taking the log twice on the survival probability, the $log - log$ curve can be defined as the summation of two terms, which are the linear sum of $\beta_i X_i$ and the $log(-log)$ of the baseline hazard function. The $log - log$ curve comparing two subjects with different specifications of predictors $\mathbf{X}_1$ and $\mathbf{X}_2$ where $\mathbf{X}_1 = (X_{11}, X_{12}, X_{13}, ..., X_{1p})$

$and\ \mathbf{X}_2 = (X_{21}, X_{22}, X_{23}, ..., X_{2p})$ is thus computationally reduced to an expression that drops out the baseline hazard function and therefore does not involve time.

$$-ln[-ln(S(t, \mathbf{X}_1))] = -ln[-ln(S(t, X_2))] + \sum_{i=1}^{p} \beta_i (X_{2i} - X_{1i}) \tag{2.30}$$

$$-ln[-ln(S(t, \mathbf{X}_2))] = -ln[-ln(S(t, X_1))] + \sum_{i=1}^{p} \beta_i (X_{1i} - X_{2i}) \tag{2.31}$$

The algebraic reduction of terms yields a linear sum of the differences in corresponding predictor values for the two subjects. Hence if the Cox PH model is used and the log-log survival curves of the above two subjects are plotted on the same graph, the two plots would be approximately parallel. The distance between the two curves is the linear expression involving the differences in predictor values which does not involve time. If the vertical distance between the curves is constant over time, then the curves are parallel. (Kleinbaum and Klein, 2005). Cox PH model is appropriate if the empirical plots of log-log survival curves are parallel.

2.10 Model Building Process

The first step in model building is Exploratory Data Analysis (EDA). This involves examining the empirical Kaplan Meier survival curves, assessing the results of the log-rank test, distribution of survival time, univariate and multivariate analysis of covariates. EDA is crucial in identifying numerical issues and potential errors embedded in the data set. The distribution of individual covariates is considered over time and consistency is assessed overtime. Covariates are examined one on one with the dependent variable using Weight of Evidence (WoE), which is discussed in section 2.10.2. WoE is the most widely used variable transformation tool in credit scoring (Hubber and Patetta, 2013). The ability of each variable to differentiate risk is determined using the GS.

2.10.1 Exploratory Data Analysis

Selection of variables to include in modelling involves exploration of individual variables in a process called univariate analysis. The one on one relationship between the dependant variable and each of the candidate covariate is analysed using *WoE* and *GS*. Binning of individual variables is performed at the univariate analysis stage. Multivariate analysis selects the final and supposedly the "best" variables which go together in the final model. *Stepwise regression* works out partial associations and may deal with cases of multicollinearity (Hubber and Patetta, 2013). *Correlation analysis* analyses the one on one relationship between variables and the result usually complements stepwise regression analysis. The decision to select which variables to use is up to the researcher.

2.10.2 Univariate Data Analysis

In a typical data set with credit information, ordinal, continuous and nominal variables exist. For modelling purposes, often only the interval independent variables are required. Therefore it might be necessary to transform and create dummy variables. The most commonly used transformation method is called WoE. The WoE process converts any variable into a numeric interval variable. Groups/classes/bins are assigned according to the risk of the group expressed by the logarithm of likelihood ratios, that is logarithm of a portion of, say defaulted versus non defaulted subjects (Jilek, 2008). WoE refers to the set of "goods", customers who do not default and the "bads", customers who default or settle accounts early.

$$w_{ij} = ln\left(\frac{p_{ij}}{q_{ij}}\right) \tag{2.32}$$

where w_{ij} is the WoE, p_{ij} is the number of good risks in level/attribute j of variable/characteristic i divided by the total number of good risks who responded to i and q_{ij} is the number of bad risks in attribute j of characteristic i divided by the number of bad risks in attribute j who responded to characteristic i. The WoE curve should be a monotonic function across the categories of a covariate.

Variables can be selected using the Information Value (IV). It measures the difference between distributions of the good and the bad risk on a specific covariate. Characteristics with an information value of over 0.1 will be considered for model development

(Hand and Henley, 1997). Using WoE definitions described above,

$$IV = \sum_{ij}(p_{ij} - q_{ij}) \times w_{ij} \tag{2.33}$$

Gini (1936) was an Italian demographer, sociologist and statistician who developed a measure of statistical dispersion, known as the Gini coefficient. It determines the level of inequality of a distribution. A value of 0 represents total equality and a value of 1 represent maximal inequality. The values are usually expressed as a percentage. In the area of economics, the coefficient was developed mainly to measure inequality of income or wealth of many different nations worldwide. The Gini Coeffient gained popularity in other domains including engineering, sociology, chemistry, ecology and so on.

Similarly, in the field of Statistics, the GS, also known as the Somer's D, measures a variable's ability to differentiate risk when fitting a univariate (single input variable) logistic model. It measures uniformity of a distribution. The lower the GS, the more uniformly distributed the variable (Jilek, 2008). In the consumer credit context, the GS statistic is used to measure how equal the event rates are across the attributes of a variable. The higher the GS, the higher the ability of the characteristic to differentiate risk. All variables with a GS of less than five percent maybe excluded (Migut, Jakubowski and Stout (2013)), however, due to the small number of covariates considered, this research will lower down the cut GS cut off to four percent.

In the calculation of the GS, the attributes $i = 1, 2, 3, ..., m$ are sorted in ascending order of their event rates. For each of the attributes, the number of events is given by n_i^{event}, the non-event by $n_i^{non-event}$. The total number of events = N^{event} and the total number of non-events is given by $N^{non-event}$. Then the GS is calculated as follows:

$$GS = \left(1 - \frac{2 \times \sum_{i=2}^{m}\left(n_i^{event} \times \sum_{j=1}^{i-1} n_j^{non-event}\right) + \sum_{k=1}^{m}\left(n_k^{event} \times n_k^{non-event}\right)}{N^{event} \times N^{non-event}}\right) \times 100 \tag{2.34}$$

2.10.3 Multivariate Data Analysis

Variables that are not significant in univariate analysis may become significant in a multivariate model because of partial associations. Partial association refers to the effect of one variable changes in the presence of other variables. Partial association of candidate covariates is determined using, inter alia, stepwise regression, backward or forward elimination and best subsets selection.

Best subsets selection is computer intensive when we have big data sets. Each covariate should have at most two categories. This study may not use best subset selection because of the existence of more than two classes in some variables. Among other methodologies, this study uses stepwise regression to identify subsets of covariates befitting plausible models.

Covariates are assessed for multicollinearity using the Variance Inflation Factor (VIF) as well as correlation analysis. It is imperative to assess multicollinearity among covariates before analysts conduct a multivariate regression analysis (Mansfield and Helms, 1982). If covariates within the model are highly correlated with each other, this would be reflected through a high VIF value. It is recommended that VIF values should lie below five (Migut et al (2013)), if covariates are to be considered for model development. However the author revised the VIF cut off to three in order to eliminate any form of multicollinearity. Any candidate covariate with a VIF of at least 3 should be excluded from further analysis.

2.10.4 Goodness of Fit Measures

Type 3 tests are used to test the hypothesis that all regression coefficients in the model are zero. Type 3 tests are Wald, Score and the Likelihood ratio tests. The **Wald statistic** also known as the Z statistic is used with the maximum likelihood estimates to compute the p-value (the probability of obtaining a value as extreme as the test statistic). The p-value helps the researcher to decide whether to fail to reject or reject the underlying hypothesis about the variables being assessed usually if the p-value is less than $\alpha = 0.001$. The **Score** test is similar to the Wald test. The Likelihood Ratio is an alternative statistic, which makes use of the log likelihood statistic, $-2 \times lnL$.

The p-values for Wald static and Likelihood Ratio may not yield the same p-value but generally the conclusion is the same unless there is multicollinearity present. From expert opinion, Kleinbaum and Klein (2005) suggest that when in doubt, the researcher should use Likelihood Ratio test.

The Likelihood Ratio is also used to compare model fit. For example, the Likelihood Ratio comparing model one and model two is computed as follows: Likelihood Ratio = -2ln(L_{model1}) - (-2ln(L_{model2})). Likelihood Ratio follows a χ^2 distribution and the corresponding p-value is obtained, leading to conclusions on the hypothesis H_0: Model 1 = Model 2 and H_1: Models differ. P-values less than $\alpha = 0.001$ suggest that two models are significantly different. The Akaike Information Criterion (AIC) and the Schwarz Bayesian Criterion (SBC), also known as Bayesian Information Criteria are also goodness of fit measures used to compare one model to another. The difference between AIC and SBC lie in the penalties used for extra variables. SBC penalty is more severe and therefore uses more parsimonious models (Hubber and Patetta, 2013). The lower the goodness of fit measures the better the model.

Other model diagnostics measures include the assessment of residuals. In survival analysis, the mostly commonly used residuals are the Cox-Snell (Cox and Snell, 1968) residuals and the Schoenfeld residuals (Schoenfeld, 1982).

Cox-Snell residuals are computed as follows:

$$r_{c_i} = exp(\widehat{\beta} x_i)\widehat{H}_0(t_i) = \widehat{H}_i(t_i) = -log(\widehat{S}_i(t_i)) \tag{2.35}$$

where $\widehat{H}_0(t_i)$ is the cumulative baseline hazard and $\widehat{H}_i(t_i)$ is the estimated cumulative hazard for the i^{th} individual at time t_i and $\widehat{S}_i(t_i)$ is the estimated survivor function of the i^{th} individual at time t_i. $\widehat{S}_i(t_i)$ has an exponential distribution with unit mean (Stepanova and Thomas, 2002). If the model is adequate, a plot of $log(-log(\widehat{S}_i(t_i)))$ against r_{c_i} should give a straight line with a unit slope and zero intercept. Schoenfeld residuals are interpreted in a similar way except that their output includes residuals and values for every covariate conditional on the risk set.

2.10.5 Assessment of Model Performance

Model assessment is based on the validation set. The basic methods of assessment include separability measures and the counting methods. Separability measures such as the divergence statistic (the value of the sample t-statistic between the two design set classes) are not reviewed in this study. Rather, counting measures are used for model performance assessment.

Counting measures are based on a two by two contingency table of predicted by actual classes. A Lorentz curve shows the cumulative proportion of true goods plotted against the proportion of true bads as the threshold varies. A similar computation is done for the Receiver Operating Characteristic (ROC) curve wherein the true positive rate is plotted against the false positive rate (Hand and Henley, 1997). The area between the curve and the axes is used as the discriminatory power of the model. A transformation, called GS is a measure of twice the area between the curve and the diagonal (ideal classifier). A better fitting model scores the higher value.

2.11 Competing Risks Survival Analysis

Standard survival analysis methodology considers a single event in a study. Survival curves in such cases can be computed using the Kaplan Meier approach if no covariate adjustment is desired. However, when two or more events are determined in a single study, the statistical problem is defined as a *competing risk* problem as each event “competes” to be the cause of *failure*.

The KM approach may not be used in the presence of competing risks as it becomes very sensitive and may produce biased results. The modeling methodologies ideal for competing risks include the Cox PH model, parametric survival models and the CIC. The Cox PH regression cause specific methodology is widely used to model competing risks. Where each event type is modeled separately and other event types are treated as censored categories, the approach is called the **“cause specific”** method.

2.11.1 The Cause Specific Approach

The most widely used approach to competing risks is the cause specific approach. Stepanova and Thomas (2002) illustrated the "cause-specific" methodology to competing risks. The analysis was based on the U.K consumer credit data, obtained from a major U.K financial institution.

Consider default as event type 1 and ES as event type two. The cause specific approach fits two separate Cox regression models, one for each failure type. Stepanova and Thomas (2002) estimated the time until default $\mathbf{T}_1$ and assumed the rest of observed lifetimes to be censored, including the subjects who entered into the early repayment bucket. A second model analogous to the first one was fit and estimated time until repayment $\mathbf{T}_2$, assuming other observed lifetimes to be censored, including the subjects who defaulted. A Cox PH model was fit in each case, on $\mathbf{T}_1$ and $\mathbf{T}_2$. From literature, the predicted lifetime of a loan is thus $\mathbf{T}$ = min($\mathbf{T}_1, \mathbf{T}_2$, term of the loan). In this analysis, the cause specific hazard functions of the two events are:

$$h_1(t) = lim_{\Delta t \to 0} P(t \leq T_1 < t + \Delta t | T_1 \geq t)/\Delta t \tag{2.36}$$

$$h_2(t) = lim_{\Delta t \to 0} P(t \leq T_2 < t + \Delta t | T_2 \geq t)/\Delta t \tag{2.37}$$

where the random variable T_1 = time to failure from default, and T_2 = time to failure due to early repayment. $h_1(t)$ and $h_2(t)$ give the instantaneous failure rates at t for default and early repayment respectively.

In general, given c events in an analysis, the Cox PH cause specific model is given by:

$$h_c(t, \mathbf{X}) = h_{0c}(t) \exp \left[\sum_{i=1}^{p} \beta_{ic} X_i \right] \tag{2.38}$$

In this analysis, when c = 1 a default event is modelled and the rest of the observed lifetimes are treated as censored. If c = 2, a model for early repayment is obtained, holding other observations as censored. $\mathbf{X} = (X_1, X_2, X_3, ..., X_p)$ is a vector of explanatory variables included in the study. The β_{ic}'s are event specific regressions parameters.

2.11.2 The Cumulative Incidence Curve

The CIC is an alternative summary curve to the KM survival curve. CIC estimates the "marginal probability" of an event. CIC has more meaningful interpretation in terms of treatment utility inspite of whether competing risks are independent or not. The marginal probability for each event type c at failure time t_i is computed as follows:

$$CIC_c(t_i) = \sum_{i=1}^{i} \widehat{S}(t_{i-1})\widehat{h}_c(t_i) \tag{2.39}$$

where

$\widehat{S}(t_{i-1})$ is the overall survival probability of surviving previous time t_{i-1}. This computes subjects surviving all competing risks. The hazard estimate $\widehat{h}_c(t_i)$ for event type c is the proportion of subjects failing from event c at time t_i.

$$\widehat{h}_c(t_i) = \frac{m_{ci}}{n_i} \tag{2.40}$$

where m_{ci} is the number of subjects failing from event type c and n_i is the risk set.

When computing CIC of a single event type, the CIC reduces to a $(1 - (KM))$ curve. CIC does not rely on the independence assumption of competing risks unless the "cause specific" hazards model is used as an intermediate step to obtain HR estimates for individual competing risks. Fine and Gray (1999) modelled the CIC and regressed directly on the CIC and not on the cause specific hazards and their methodology is regarded as an improvement on cause specific hazards as it corrects the mismatch effect of covariates on cause specific hazards and CIC when hazard functions are used as an intermediate step. The resultant curve was referred to as the subdistribution, the marginal probability function, the crude incidence and sometimes the absolute cause specific risk.

Thus, the CIC is a generalisation of the (1-(KM)) curve. It accommodates multiple event types in its computation. In the absence of competing risks, it reduces to (1-

(KM)). In this analysis, the CIC curve for default is given by:

$$CIC(t_i) = \sum_{i=1}^{i} \widehat{S}(t_{i-1})\widehat{h}_1(t_i) \tag{2.41}$$

where

$\widehat{S}(t_{i-1})$ is the overall survival probability of surviving up to a previous time t_{i-1}. This computes subjects surviving both default and ES. The hazard estimate $\widehat{h}_1(t_i)$ for default event is the proportion of subjects failing from event default at time t_i.

$$\widehat{h}_1(t_i) = \frac{m_{1i}}{n_i} \tag{2.42}$$

where
m_{1i} is the number of subjects failing from default event and n_i is the risk set.

A separate CIC curve of ES (event type = 2) is fit with steps analogous to the default event one described above. Fine and Gray (1999) used adjusted hazard estimates of each event type as input into the CIC curves and noted that the effect of a covariate on the cause specific hazard of a particular failure type maybe very different from the effect of the covariate on the corresponding CIC. Moreover, when cause specific formulation is used as the intermediate step in CIC construction, testing for covariate effects on the subdistribution CIC is not possible. To this effect, the authors introduced a proportional hazard model for the subdistribution grounded in the log(-log) transformation of the univariate survival data.

2.12 Parametric Models of Survival

Parametric survival models are based on specified distribution of the outcome (survival time) expressed in terms of unknown parameters. Most commonly used distributions in parametric survival models include exponential, Weibull, log-logistic, lognormal and generalised gamma. Parametric models are mainly Accelerated Failure Time (AFT) models whereby the outcome is expressed in terms of explanatory variables. The AFT model shows the effect of explanatory variables on survival time. AFT models are designed to accommodate left, right and interval censored data while the PH models can only handle right censored data (Hubber and Patetta, 2013). AFT model assumes the effect of covariates is proportional over time. Recall that the PH model assumes a constant HR over time and it gives the effect of predictor variables on the hazard. Thus, AFT models compare survival times while the PH models compare hazards using the numeric *acceleration factor* and the *HR* respectively.

In mathematical transformation theory, the acceleration factor in an AFT model is reflected as the "stretch" factor between two survivor functions. It either stretches or contracts survival functions being compared. If sample one receives treatment one and sample two receives treatment two, $S_2(t)$ can be expressed in terms of $S_1(t)$ using a constant λ which is the acceleration factor in an AFT model. That is $S_2(t) = \lambda S_1(t)$. An acceleration factor greater than one implies treatment two promotes survival λ times, the reverse is true for λ less than one.

2.12.1 The Weibull Distribution

The survival $S(t)$ and hazard $h(t)$ functions of Weibull distribution are:

$$S(t) = e^{-\lambda t^p} \quad and \quad h(t) = \lambda p t^{p-1}$$

p is the shape parameter and it governs the the shape of the hazard function for example the "bathtub" and the "cup" shapes. A special and unique property of the Weibull model is, "if the AFT assumption holds then the PH assumption also holds", (Kleinbaum and Klein, 2005). It holds for a fixed shape parameter p.

2.12.2 The Exponential Distribution

The survival $S(t)$ and hazard $h(t)$ functions of exponential distribution are:

$$S(t) = e^{-\lambda t} \quad and \quad h(t) = \lambda$$

The exponential distribution is regarded as the simplest because its hazard rate is constant over time. It is a special case of a weibull model where $p = 1$ (see Weibull) the hazard function becomes a constant. Exponential model can allow for both the PH and AFT assumptions. The constant hazard function λ can be reparameterized and expressed as a PH model.

2.12.3 The Log-logistic Distribution

The survival $S(t)$ and hazard $h(t)$ functions of the Log-logistic distribution are:

$$S(t) = \frac{1}{1+\lambda t^p} \quad and \quad h(t) = \frac{\lambda p t^{p-1}}{1+\lambda t^p}$$

2.13 Mixture Models of Survival

Before the use of survival regression models, it is imperative to explore the non-parametric KM survival and hazard curves. Survival methods discussed in the previous section assume the empirical survival curve levels off at zero as time goes to $+\infty$. If the survival curve levels off to non zero proportions, then the standard survival methodologies may be inappropriate.

Empirical survival curves may level off to non zero proportions in cases where some subjects in study are not susceptible to the event(s) of interest. That is, given an extended observation period, the bulk of accounts may never default nor pay-off early. These are called long-term survivors. As most customers are “good” customers, we are likely to have long-term survivors in this study. Approaches to modelling lifetime data in the presence of long-term survivors are called **cure models** or **mixture models**.

Literature on mixture models is found in the works of Farewell (1982) as well as Sy and Taylor (2000). Mixture or cure models are designed to cater for a sizeable proportion of subjects who do not experience the event of interest at the end of the observation period. “A KM survival curve that shows a long and stable plateau with heavy censoring at the tail maybe taken as empirical evidence of a cured fraction”, Sy and Taylor (2000, page 228).

2.13.1 The Proportional Hazards Mixture Model

A mixture of two populations is considered, the susceptible, will be denoted population **A** and the non-susceptible (long-term survivors) population **B**. A binary indicator is added to distinguish between subjects falling in the two populations. Let Y = 1 if the account defaults/ pay-off early eventually and Y = 0 otherwise. Define p = Pr(Y =1), T = time to an event of subjects only in population **A**. The proportion of **B** = 1-p. The survivor function of the entire population (**A** + **B**) is given by:

$$S(t) = (1 - p) + pS_A(t) \tag{2.43}$$

where $S_A(t)$ is the survivor function of population **A**.

2.14 Conclusion

In this section, methods of modeling credit risk were discussed, from the pre computer era to present day methodologies. Judgemental methods were based on expert opinion of experienced officers. Credit granting decisions were gender and marital status sensitive, before the ECOA was introduced. The current methodologies are guided by legislature and all individuals are granted equal opportunities. Statistical models of credit highlights the possible statistical methods of credit scoring including multiple linear regression, logistic regression and survival analysis. The progression of one method used to another was detailed in this section indicating the strength and weaknesses of techniques when employed in credit modelling. Survival analysis methods are believed to be superior to most statistical methods in building credit risk models.

The usage of parametric survival methods require an accurate specification of the model before usage. This is oftenly difficult to determine, hence the use of a semi-parametric survival method, the Cox PH regression methodology. The Cox PH regression technique is known to be a robust method which can closely approximate parametric regression estimates. In the presence of competing risks, the Cox PH regression method is employed for each cause of failure in a cause-specific regression. Cause specific models are used as intermediate step into the calculation of CIC which estimates the marginal probabilities of each event in the presence of the other events.

The CIC is ideal in credit risk for the calculation of PD, LGD and EAD models. In the presence of long term survivors, mixture models of survival are the most appropriate.

Chapter 3

Methodology

3.1 Introduction

The scope of this study is statistical analysis of loans extended for the purpose of vehicle purchases in a retail framework. The purpose is to analyse competing risks in a consumer credit context. Data set is obtained from a major South African financial institution. As in a normal statistical analysis scenario, EDA is necessary for us to engage and familiarise ourselves with the data, to detect any possible anomalies, to obtain the trends and structure of the data. EDA involves univariate and multivariate data analysis as well as trend analysis over time. Models are built to compare performance of methods currently applied by financial institutions for behavioural scoring. Logistic regression and Cox regression methods are compared to determine the superior technique in the presence of competing risks, default and early repayment. A detailed analysis focuses on the possibilities and pitfalls in different modelling approaches.

3.2 Data

Vehicle finance data is obtained from a leading South African financial institution. All the information required is extracted from the institution data warehouse, for the period 01 April 2009 to 31 March 2014. The information includes details from the applicants and details from vehicle manufacturers. The raw data set comprise all active accounts. Accounts entering the study after the observation start date, 01 April 2009 are treated as left truncated entries. Granular detail is given per month per account.

The data set comprise the standard, application and behavioural variables. Standard variables are customer key identifiers. These include the customer account number, ID and address, inter alia similar identifiers. Some standard variables shall be masked in this study for the purposes of commercial confidentiality.

Application variables are further divided into customer specific and vehicle specific variables. Behavioural variables are mainly customer specific, detailing repayment behaviour after obtaining the vehicle finance. For the purposes of this study more variables are derived for example age of asset, survival time and account status indicator. Some of the derived fields shall be added as independent or standard variables used to enhance model development and ease of computation respectively. The standard variables are given in Table 3.1.

Table 3.1: Standard Variables

Variable Name	Description/Usage
Account Number	Masked customer key and unique identifier
Month	Cohort indentifier
Product Code	Instalment Sale product Identifier
Balancing Segment	Retail Accounts Identifier
Start Month	Start to follow up date
Default Month	Event month
ES Month	Event month
Account Balance	Calculates exposure
Accumulated Interest Amount	Calculates exposure
Survival Time	Number of months between the entry and exit points

Table 3.2 is a list of customer and vehicle specific application and behavioural variables which make up part of possible candidate covariates. Additional covariates will be derived.

Table 3.2: Customer and Vehicle Specific Covariates

Variable Short Name	Variable Name	Description
Vage	Age of Vehicle	Age of vehicle in years
Amt Fin	Amount financed	Loan amount extended
ATC	Article Type Code	Vehicle Type
DTC	Dwelling Type Code	Describes the residential environment
Inst Freq	Instalment Frequency	Frequency of instalments
Maint Code	Maintenance Code	Indicates maintenance of the vehicle
Manf Code	Manufacturer Code	Vehicle brand
Marital Status	Marital Status Code	Applicant's Marital status
M&M Code	M&M Code	TransUnion's Mead and McGrouther Code
New Old Used	New/Old/Used	Vehicle Status
Num Dep	Number of Dependants	Total number of dependants
Num Ins	Number of Insurances	Count of Insurances held by applicant
Orig Term	Original Term	Original Term
SA Cit	SA Citizen Ind	SA Citizen or not
Spse Emp Ind	Spouse Employed Indicator	Indicates if spouse is employed or not
Time Curr Add	Time at current address	Time at current address in years
Time Curr Job	Time in current Job	Time at current job in years
Yr Mod	Year Model	Year model of vehicle
LTV	Loan to Value	Outstanding balance as a percentage of current value of vehicle
Pay Mthd Cd	Payment Method Code	Indicates payment method
Out Term	Outstanding Term	Remaining Term
PrevD	Number of Previous Defaults	Count of previous defaults for each account

3.3 Exploratory Data Analysis

Data extraction and addition of derived fields is processed in SAS®. Additional fields include, inter alia, retail indicator, survival time, status variable and other input variables deemed necessary. Survival data structure is assessed through a SAS® procedure, proc lifetest, to determine the shapes of hazard and survivor curves and to make inferences around the underlying model. The full data set is randomly split into 80 percent development set and 20 percent validation set. The split is consistent with the institution's internal model building standards. Model development uses the development set and models are evaluated based on the validation set.

The development set is imported into SAS® Enterprise Miner. Credit scoring option is selected and the interactive binning node is used for WoE calculation, binning of variables and calculation of the GS for every candidate variable. Variables passing the univariate analysis are assessed using stepwise regression and correlation analysis in Enterprise Guide. After selection and deletion, the final set of variables selected are then used for model development.

3.4 Model Building

Let *D* be the generalised cause of failure, Event 1 = Default, and Event 2 = ES. If an account neither defaults nor settles early, then the account is right censored (c) due to termination of study. There is no possibility of withdrawals and loss to follow up censoring in the current data as all the accounts are kept in check to final classification. In competing risks analysis, the random variable *T* = survival time is thus determined as $T = min(T_1, T_2, T_c)$. The time period of interest is the time to the first event that occurred to each account.

3.4.1 Cox Regression Models

The fundamental *cause-specific hazard function* say the hazard of failing from default in the presence of ES is a joint distribution of *T* and *D* is given by:

$$h_{default}(t) = lim_{\Delta t \to 0^+} \frac{P(t \leq T < t + \Delta t, D = default \mid T \geq t)}{\Delta t} \tag{3.1}$$

Final variables are tested for proportional hazards assumption using the graphical log-log plots. The Cox proportional hazards regression is performed on the cause specific hazards, given covariates described in the table of covariates and $\widehat{h}_{0,1}(t)$ and $\widehat{h}_{0,2}(t)$ being the baseline cause specific hazards of default and ES respectively. Note that the baseline population is generated using a combination of bins/classes of covariates with the highest population in each model. Recall that the bins are created at the univariate stage.

The cause specific Cox PH regression model for default is given by:

$$\widehat{h_1}(t|\mathbf{X}, \mathbf{Y}) = \widehat{h}_{0,1}(t) \times exp(\beta_1 Vage + \beta_2 Amt\ Fin + \beta_3 ATC + ... + \beta_{45} PrevD + \beta_{46} Arr) \tag{3.2}$$

The cause specific Cox PH regression model for ES is given by:

$$\widehat{h_2}(t|\mathbf{X}, \mathbf{Y}) = \widehat{h}_{0,2}(t) \times exp(\beta_1 Vage + \beta_2 Amt\ Fin + \beta_3 ATC + ... + \beta_{45} PrevD + \beta_{46} Arr) \tag{3.3}$$

Recall that the cause specific hazard functions together with the overall survival curve are used as inputs into the calculation of the CIC for each event. The overall survival curve is the probability of not having failed from any cause at time t_i. Thus the unconditional probability of failing from default in the presence of competing risks at time t_i is estimated as:

$$\mathbf{CIC}_1(t_i) = \sum_{i=1}^{i} \widehat{S}(t_{i-1})\widehat{h}_1(t_i) \tag{3.4}$$

where $\widehat{S}(t_{i-1})$ is the overall survival probability of surviving previous time t_{i-1}. The cause specific hazard estimate $\widehat{h}_1(t_i)$ for the default event is the proportion of subjects failing from event default at time t_i with reference equation 3.2.

The unconditional probability of failing from ES in the presence of default event at time t_i is estimated as:

$$\mathbf{CIC}_2(t_i) = \sum_{i=1}^{i} \widehat{S}(t_{i-1})\widehat{h}_2(t_i) \tag{3.5}$$

3.4.2 Logistic Regression Models

Logistic Regression models are perfomed per event over the workout period (48 months period). The probability of failing from default event is estimated as follows:

$$p(D = 1|\mathbf{X,Y}) = \frac{e^{\alpha+(\beta_1 Vage+\beta_2 Amt\ Fin+\beta_3 ATC+...+\beta_{44} AIP+\beta_{45} PrevD+\beta_{46} Arr)}}{1 + e^{\alpha+(\beta_1 Vage+\beta_2 Amt\ Fin+\beta_3 ATC+...+\beta_{44} AIP+\beta_{45} PrevD+\beta_{46} Arr)}} \quad (3.6)$$

The probability of failing from ES event is estimated as follows:

$$p(D = 2|\mathbf{X,Y}) = \frac{e^{\alpha+(\beta_1 Vage+\beta_2 Amt\ Fin+\beta_3 ATC+...+\beta_{44} AIP+\beta_{45} PrevD+\beta_{46} Arr)}}{1 + e^{\alpha+(\beta_1 Vage+\beta_2 Amt\ Fin+\beta_3 ATC+...+\beta_{44} AIP+\beta_{45} PrevD+\beta_{46} Arr)}} \quad (3.7)$$

3.4.3 Model Assessment

Logistic regression and Cox regression methods are compared to determine the superior technique in the presence of competing risks, default and early repayment. We will compare Cox regression and logistic regression in estimating which loans are likely to default/pay off early within a fixed outcome period. The ROC curves are used to compare model performance. However the area under each ROC curve can be used to summarise model performance. This is equivalent to the overall model GS. Thus we will compare the models ability to differentiate risk using model GS. Plots of actual versus expected events will be used to asses the ability of the models to rank order risk groups. For LR models, the Hosmer-Lemeshow test will be used to assess goodness of fit of the models under the hypothesis:

$$H_0 : No\ model\ misfit$$

$$H_1 : Model\ misfit$$

Chapter 4

Model Development

4.1 Introduction

This chapter takes the reader through the step by step model building process used in this study. There exists a complex data structure containing a wealth of information covering every calender month over a five year period of observation. Model development and analysis will be implemented in the SAS® environment. In financial institutions, individual accounts are observed and tracked every month from the point of entry (open date) to the point of exit into either default or ES or to the termination of study at the end of March 2014. This information illustrates a longitudinal study where individual customers from the same cohort go though different kinds of behaviours leading to default or ES or censorship at different time periods.

Survival analysis methodologies are well designed to model time to an event. In the presence of competing risks and long term survivors, we will also investigate how Logistic regression methods compare. It is interesting however, to first look at the composition of the vehicle finance book in terms of brands on the market. We investigate whether the composition complements the information obtained from an independent survey whose results are summarised in a biplot in Figure 1.1 of Chapter 1. Recall that the survey was conducted on a sample of popular brands of the South African automobile market. Do the "popular" brands in Section 1.2 command higher volumes in the vehicle finance portfolio under investigation?

4.2 A Look at the Vehicle Brands on Book

Manufacturer Code indicates the make or the brand of a vehicle. It is derived from the Mead and McGrouther (M&M) code. Every vehicle tradeable on South African automobile market is associated with an M&M code. The codes are developed and handled by an international organisation called TransUnion, a registered credit bureau and a repository of credit information on consumers and businesses. TransUnion releases a monthly publication called the Auto Dealer's Guide containing, among other information, the M&M code and the approximated trade and retail prices, sometimes new prices of each vehicle as well.

The M&M Code contains eight digits. The first three represent the Manufacturer Code/ Brand name, e.g Volkswagen (VW), the next three digits represent the model, e.g Golf and the last two digits represents the derivative of a particular vehicle, e.g 2.0 DSG. In addition to the eight digits, users may add two or four digits to represent the year model. In this case, four trailing digits were added to make a total of twelve digits for the M&M code. An example of an M&M Code is given below:

$$\underbrace{6\ |\ 4\ |\ 0}_{\textbf{VW}}\ \ \underbrace{4\ |\ 5\ |\ 9}_{\textbf{GTI VI}}\ \ \underbrace{2\ |\ 8}_{\textbf{2.0 DSG}}\ \ \underbrace{2\ |\ 0\ |\ 1\ |\ 2}_{\textbf{2012}}$$

The M&M Code was not considered as a covariate as it contains 23710 levels, too many to be transformed and grouped. However, two covariates were derived from the M&M Code. These are the Manufacturer Code and Year Model. The Year Model was then used to calculate Age of Vehicle at the point of application. Manufacture Code is a list of brands on the book which gives a reflection of the vehicle brands offered on the South African automobile market. About 150 different brands exist. The top 20 brands in terms of volumes were drawn to check if the brands match the popular brands in the biplot given in Figure 1.1 in Section 1.2. Table 4.1 shows a list of the top 20 brands and the contribution of the rest of the brands. All the brands in the biplot are also in the top 20 in Table 4.1 indicating that they are indeed the most popular brands trading on the South African automobile market.

Table 4.1: Brands on Book

MANUFACTURER CODE	**BRAND**	**VOLUME**	**FREQUENCY**(%)
600	TOYOTA	48899	16.64
640	VOLKSWAGEN	42802	14.57
470	NISSAN	21634	7.36
100	CHEVROLET	17038	5.80
265	HYUNDAI	16989	5.78
050	BMW	16661	5.67
440	MBENZ	16068	5.47
040	AUDI	11028	3.75
220	FORD	10868	3.70
480	OPEL	9733	3.31
280	ISUZU	8462	2.88
321	KIA	7872	2.68
540	RENAULT	6578	2.24
250	HONDA	5072	1.73
450	COLT	4832	1.64
300	JEEP	4350	1.48
350	LAND ROVER	4279	1.46
430	MAZDA	4240	1.44
598	TATA	3104	1.06
590	SUZUKI	2772	0.94
The Rest	Other	30526	10.39
Total	All	293807	100

A graphical view is given in the pie chart in Figure 4.1. The top five brands account for roughly 50% of the financial institution's portfolio. These are Toyota, Volkswagen, Nissan, Chevrolet and Hyundai. Other popular brands occupy about 40% while the unpopular brands account for just about 10% of the portfolio. The unpopular brands category include the luxury, niche and other small brands. Brands associated with luxurious vehicles include Lamborghini, Maserati, Maybach, Rolls Royce and Ferrari among others. Niche refers to a specific, specialised population of the market, example agricultural and trucking businesses are specialised areas. Niche brands in this case include Massey Ferguson, John Deere, Leyland, DAF, Ducati and Kawasaki.

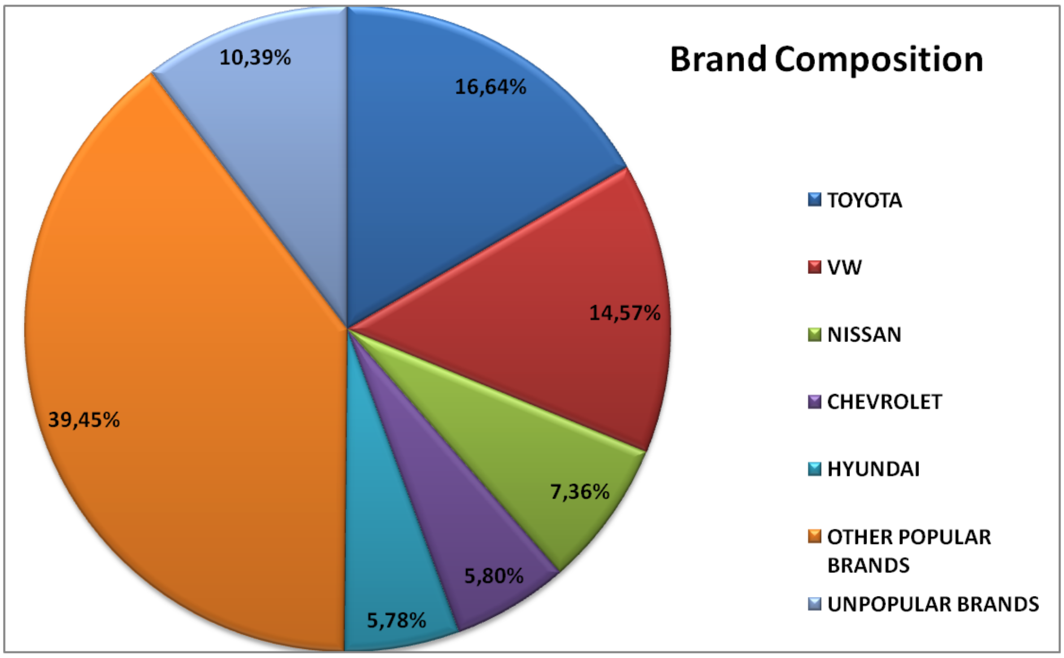


Figure 4.1: Brands on Book

4.3 Model Building Process Flow

Table 4.2 shows the steps taken in the model building process and the location of the SAS® code at each step. Data extraction and preparation processes were covered in steps one to three. This entails merging different application and performance data sets and putting together all available records/observations and fields/variables related to vehicle finance. Some variables were derived from the crop of raw variables in the data sets. These include such critical fields as the Default and ES flags and additional covariates such as Manufacturer Code and Age of Vehicle.

EDA was performed in steps four and five. Further detail is provided in Section 4.5. Multivariate analysis was performed on covariates passing the EDA satisfactorily. Final variables to include in the final models were established in step six. Step seven covered cause specific Cox PH regression models. Competing risks analysis was addressed by the CIC in step eight. Logistic regression was performed at step nine and finally, step ten focussed on model assessment and comparisons.

Table 4.2: Model Building Process Flow

Step	Process	SAS® Code Location
1	Data extraction	Appendix B.1
2	Data preparation	Appendix B.2
3	Derivation of critical fields	Appendix B.3
4	Covariate selection	Appendix B.4
5	Univariate analysis	Appendix B.5
6	Multivariate analysis	Appendix B.6
7	Cox PH regression	Appendix B.7
8	CIC	Appendix B.8
9	Logistic regression	Appendix B.9
10	Model Assessment and Comparisons	Appendix B.10

4.4 Data Extraction and Preparation

All the data were extracted from the financial institution's data warehouse for the period April 2009 to March 2014. The following fields were either derived or extracted:

- Entry and exit point. The entry point defines the month when the account entered the study starting from April 2009 onwards, while the exit point depicts the event or censorship month. For each account, these points were determined.

- Survival time. Total number of months between entry and exit points for each account.

- Account status. Status of the account at its point of exit, whether a loan is in default or ES at the end of study.

- Censorship flag. Accounts not absorbed into default or ES were right censored at the end of March 2014 due to termination of study and flagged with a censorship indicator (whether the loan was censored or experienced an event).

4.4.1 Definition of Events

An account is deemed to have exited the observation period on the first occurrence of any of the following events:

- Default event: Any account where the default definition is met is deemed defaulted.

- Early Settlement event: Any account triggering early settlement definition was identified accordingly.

4.4.2 Outcome Period Analysis

The outcome period, also known as the workout period was determined on a cohort basis. Using the full historical book, cohorts were tracked from the point of entry to absorption into different events. The outcome period depicts the amount of time it takes for the accounts to be absorbed into default or ES. Figure 4.2 shows the cumulative frequencies of each event in the study period. The majority of accounts were absorbed within the first 48 months from the entry point. Three percent of the accounts susceptible to the events of interest exceeded 48 months in study. It was therefore decided to use 48 months as the outcome period. The outcome period will be used as the fixed time horizon to determine our probabilities.

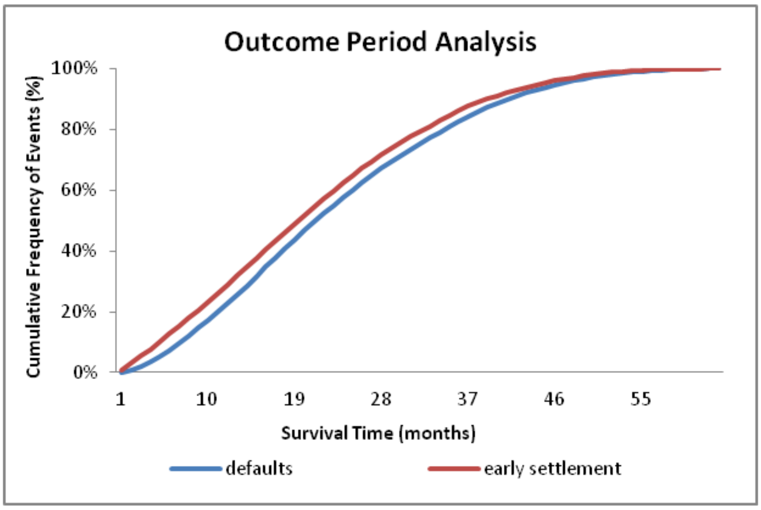


Figure 4.2: Outcome Period Analysis

Besides giving an indication of the outcome period, Figure 4.2 also shows that in the portfolio under investigation, ES event occurs more frequently than default. Early settlement curve is consistently higher than the default curve over survival time. This implies that there are more early settlement events in the portfolio than there are defaults, both at an overall level and at each survival time point t as the lines do not cross at any point. The gap between the two curves diminishes as t exceeds 48 months implying that the difference between the number of defaults and ES events eventually reduces as $t \to \infty$.

It is important to note that only the default and the ES events defined above were modelled. For incomplete accounts (accounts 48 months or more in observation), the probabilities at 48 were applied.

4.4.3 Sampling Approach

In order to check the performance of the models, an independent holdout validation data set was set aside from the development data set. The models were developed on the development data set. The models were then applied to the validation set. This was done to assess the performance of the models and to ensure that no over/under-fitting occurred. The available data set was randomly split using simple random sampling into a development and a validation data set in the ratio 80:20 respectively (Migut et al (2013)).

Given the outcome period detailed in Section 4.4.2, it was decided to use 48 months horizon period to determine accounts qualifying in the LR models. Thus, accounts should have stayed for at least 48 months in observation before being considered for LR. In other words, LR methodology requires a "waiting" period to maturity, which in this case is 48 months. The binary response variable gives the status of the account at month 48. A one indicates an event and a zero indicates a non event. On the contrary, Cox PH regression does not require the waiting period as in LR. The baseline function and the CIC helps to calculate forward looking probabilities at month 48. Thus, Cox PH method makes use of all available data including the most recently entered accounts. Table 4.3 shows the development and validation sets and the total number of accounts used in each case.

Table 4.3: Sample Selection

Set	Data Set Name	Number of Accounts
1	Cox Development	293807
2	Cox Validation	73452
3	Logistic Development	40000
4	Logistic Validation	10000

4.5 Covariate Selection

A complete list of variables available on the data warehouse was established. All variables were considered potential covariates until proven otherwise by logical and statistical analysis. Part of the covariate list is in Table 3.2. Variables containing missing values of 15 percent and greater were excluded from the analysis. Variables which were not populated consistently back in time were excluded as well. Variables populated with the same value throughout all the records were dropped. Categorical variables with number of levels exceeding 150 were excluded.

Logical assessment on covariates was performed separately for numerical and categorical variables. The summary statistics are provided in Tables 4.4 and 4.5 respectively. To illustrate this process, a small number of covariates were selected in each case. For numerical data, variables with minimum value = maximum vale and a zero standard deviation were excluded as this implies that the variable was populated with the same value throughout the records. Such variables do not qualify to be used as covariates.

Table 4.4: Selection of Numerical Covariates

NUMERICAL VARIABLE	MIN	MAX	STANDARD DEVIATION	DECISION MADE
DEBIT INTEREST RATE	0	29.50	2.71	KEEP
NUMBER OF DEPENDANTS	0	9	0.48	KEEP
NUMBER OF ENQUIRIES	0	0	0	DROP
NUMBER OF INSURANCES	0	4	0.81	KEEP
SPOUSE INCOME AMT	0	1401195	5968.29	KEEP
TIME AT CURRENT ADD	0	0	0	DROP
TIME IN CURRENT JOB	0	99	8.71	KEEP

A similar logical analysis was performed on categorical variables. Covariates with one level were excluded. As discussed in the first paragraph of Section 4.5, categorical variables with more than 150 levels were excluded, a special exception was made for Manufacturer Code and it was considered for further analysis. Variables satisfying the logical assessment process were kept for further univariate analyses.

Dates are absolute-valued variables which cannot be used in their raw form, relative measures were determined from them. Dates are provided in actual date, month, year and time (where applicable) formats. A date part component can be extracted such as the year only, the month only or the month and year or the date, month and year depending on the researcher's choice. Variables derived from dates include age of asset,

Table 4.5: Selection of Categorical Covariates

CATEGORICAL VARIABLE	LEVELS	MISSING LEVELS	NON MISSING LEVELS	DECISION MADE
ARTICLE TYPE CODE	11	1	10	KEEP
CUSTOMER KEY	325924	1	325923	DROP
DWELLING TYPE CODE	5	1	4	KEEP
M&M CODE	23710	0	23710	DROP
MAINTENANCE CODE	3	1	2	KEEP
SA CITIZEN IND	3	1	2	KEEP
SUB PRODUCT CODE	1	0	1	DROP

where the year model part was extracted from the M&M Code and the year part at any date of observation was determined. The difference between the two is the age of asset at any t. The age of customer was calculated in years from the birthdate to the date at t.

Rand-value variables cannot be used in their raw form owing to future inflation. Once again more covariates were derived from rand-value variables. Ratios were calculated from the rand-value variables. The ratio of the loan amount to its latest valuation at any t was calculated to obtain LTV. Similarly, other ratios were derived. These include Deposit to loan (deposit:loan) and Balloon to loan (residual:loan).

4.5.1 Covariate Binning Process

Variables were grouped using the interactive grouping node in SAS® Enterprise Miner. Continuous variables were grouped into decile groups according to volumes. Groups with similar event rates were combined manually to improve the discriminatory power of the covariates. Categorical variables were grouped manually. Each level represented a category. Frequencies were determined for each category.The first set of categories were checked for logic and ability to rank order. Where the conditions were not met, variables were then re-grouped manually, according to meaning, volumes and event rate. All covariates were assessed using GS to determine their ability to differentiate risk. For each model, all variables were assessed for PH assumption resulting in further exclusions, regrouping and retention of satisfactorily grouped covariates. An example of the binning process is given in Table 4.6 and Figure 4.3. For the New Old Used variable, existing levels were taken as the initial categories. However the event rates in the Old and Used categories are very close. This is also seen in overlapping proportional hazards of the same variable in Figure 4.4. Thus the Old and the Used categories were combined to create only two categories for the variable. A similar approach was employed for the rest of the covariates in Appendix B.5.

Table 4.6: Binning Process

New Old Used	Population (%)	Event Rate (%)
New	41.70	8.46
Old	10.61	9.50
Used	47.60	9.58

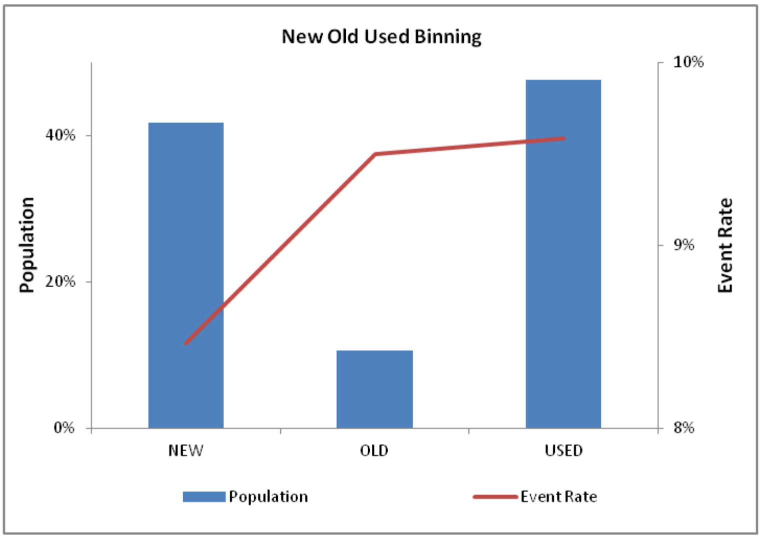


Figure 4.3: Binning Process

4.5.2 Default Model Univariate Analysis

Table 4.7 lists each variable considered, the decision made and the reason for exclusion where applicable.

Table 4.7: Default Model - Univariate Gini Statistics

Variable	Gini Statistic	Decision Made
Debit Interest Rate	38.050	Included
Dwelling Type Code	10.942	Included
Equipment Category Code	10.764	Included
Manufacturer Code	7.852	Included
Original Term	7.116	Included
Rate Type Code	6.521	Included
Age of Vehicle	6.196	Included
Marital Status Code	6.159	Included
New Old Used	5.560	Included
Article Type Code	5.142	Included
Deposit to Loan	4.152	Included
Instalment Method Code	2.612	Excluded due to failure to differentiate risk
Baloon to Loan	2.594	Excluded due to failure to differentiate risk
Number of Insurances	2.060	Excluded due to failure to differentiate risk
Discounting Code	0.000	Excluded due to failure to differentiate risk
Instalment Frequency Code	0.000	Excluded due to failure to differentiate risk
Maintenance Code	0.000	Excluded due to failure to differentiate risk
Product Code	0.000	Excluded due to failure to differentiate risk
SA Citizen Indicator	0.000	Excluded due to failure to differentiate risk
Site Type	0.000	Excluded due to failure to differentiate risk
Spouse Employed Indicator	0.000	Excluded due to failure to differentiate risk

Covariates for which the GS was greater than four were assessed for PH assumption. Article Type Code was removed from the list as evidenced in Figure 4.4 by overlapping hazards across the two categories. New Old Used was retained but the variable was regrouped by combining categories with similar event rates. As seen in the New Used and Old plot in Figure 4.4, the hazards for Used and Old categories overlap and cross in the early stages therefore it made sense to combine the Old and Used categories thereby reducing the number of categories for this variable from three to two. Plots in this Section are for illustrative purposes. The rest of the plots are provided in Appendix A.

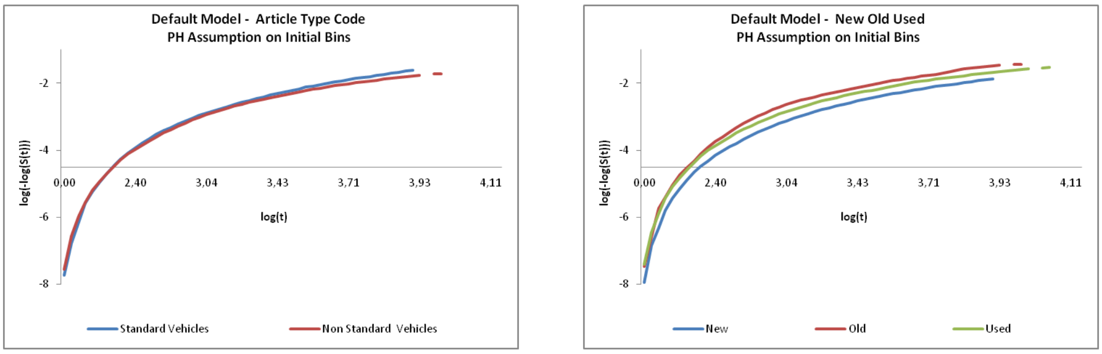


Figure 4.4: PH Assessment - Default Model

Variables satisfying the PH assumption were further assessed for population stability over time, WoE and event rate across categories. Population stability in this study is being used as a univariate assessment tool which shows us how much the volume in each category has shifted over a specified observation period. A fairly stable change in volumes is expected. This is assessed through intuitive graphical approach in this study. Further to this, we expect the WoE and event rate of each variable to be monotonic across categories. This shows the ability of the covariate to rank order risk. The population in each group should be at least five percent. No discrepancies were observed in all covariates assessed.For illustration purposes, the charts in Figure 4.5 show the univariate assessment plots for Debit Interest Rate. The rest of the univariate assessment plots are in Appendix A.

In Figure 4.5, Debit Interest Rate was assessed for all the univariate requirements discussed above. There is no evidence of crossing nor overlapping hazards in the PH assumption plot. The lines are almost parallel, indicating that Debit Interest Rate satisfies the PH assumption. The population and event rate plot is satisfactory as each category has a population greater than five percent. The monotonic event rate and the WoE curves show that the variable has the ability to rank order. Population stability plot gives an intuitive assessment to show that there are no unreasonable trends in categories across the observation period. With all the conditions satisfied, Debit Interest Rate qualifies in the multivariate analysis stage for the default model.

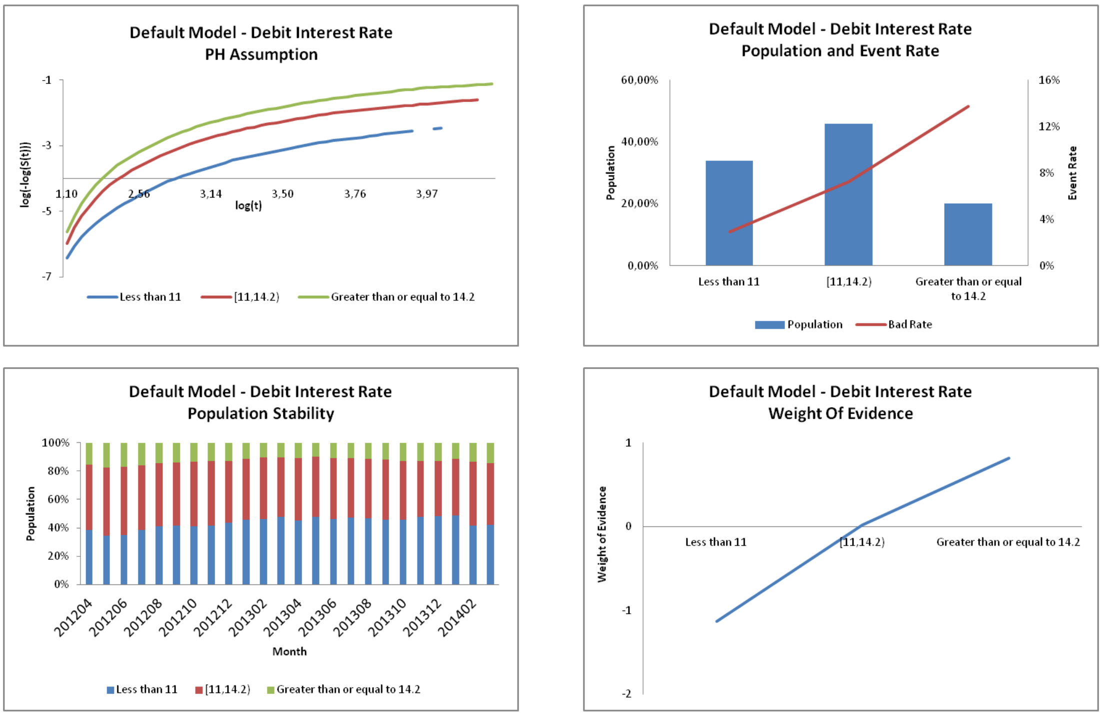


Figure 4.5: Univarite Assessment Plots - Default Model

Note that for each model the visual population stability was assessed over the full observation period. However the bars became inconspicuous with too many bars as seen in Figure 4.6, where we illustrate this using the Dwelling Type Code variable. The variable maintained consistent volumes in each category from April 2009 to March 2014. For a clearer view the plots trend will shown over the latest two years, from April 2012 to March 2014. The rest of the population stability assessment plots are in the Appendix A. No discrepancies were observed.

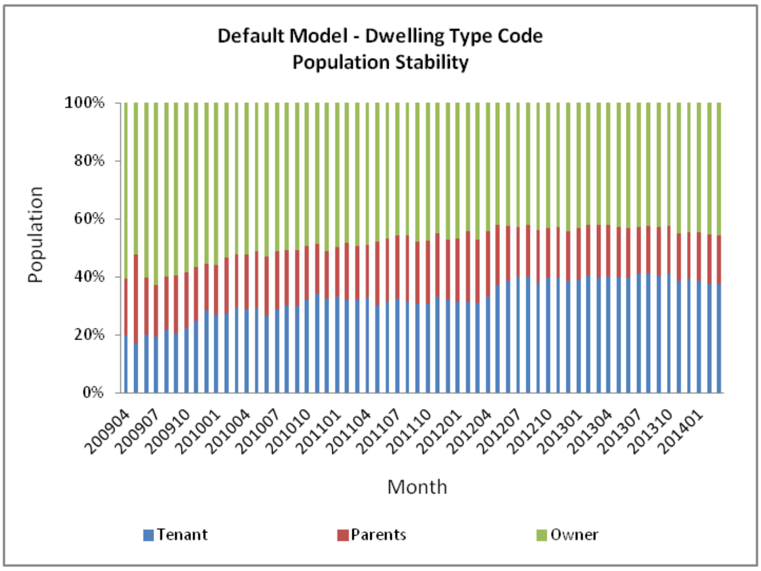


Figure 4.6: Population Stability

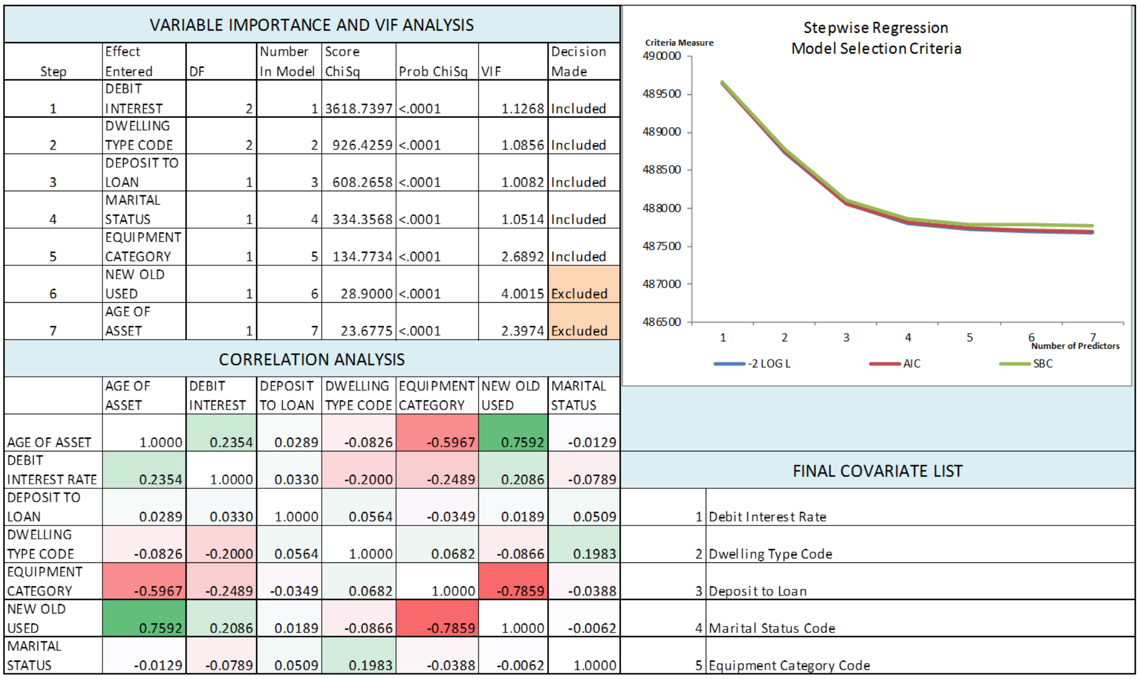
4.5.3 Default Model Multivariate Analysis - Cox Regression

Varibles satisfying all the univariate assessment tests were considered for multivariate analysis. Figure 4.7 summarises results from the multivariate assessment on the basis of the following criteria.

- Variable Importance and VIF Analysis table: Covariates are listed in order of their importance in determining default, and the order in which they were entered into the model at stepwise regression. Debit Interest Rate is the most significant variable in the Default model, followed by Dwelling Type Code and the least important is Age of Vehicle. Variable New Old Used was excluded from the analysis due to a high VIF value which is greater than three, suggesting that it is correlated with one or more of the other predictors, which leads to parameter instability.

- Model Selection Criteria chart: At each step through stepwise regression, the model selection criteria are determined. These are the AIC, SBC and -2 Log-likelihood. The most parsimonious model is reflected in the lower values of these criteria. However the point at which the graph levels off, gives an indication of where adding more covariates beyond that point will not improve model performance. As the plot levels off at point six, it makes sense to exclude variables added at any point later than the sixth step. This criteria suggested that the variable Age of Vehicle should be excluded from the analysis at this stage.

- Correlation Matrix: The values in this table complement the analyses in the above steps. New Old Used is highly correlated with Age of Vehicle (0.7592) and Equipment Category Code (-0.7859). Age of Vehicle is also highly correlated with Equipment Category Code (-0.5967). This means dropping Age of Vehicle does not lead to information loss as this information is already contained in Equipment Category Code and New Old Used.



VARIABLE IMPORTANCE AND VIF ANALYSIS

Step	Effect Entered	DF	Number In Model	Score ChiSq	Prob ChiSq	VIF	Decision Made
1	DEBIT INTEREST	2	1	3618.7397	<.0001	1.1268	Included
2	DWELLING TYPE CODE	2	2	926.4259	<.0001	1.0856	Included
3	DEPOSIT TO LOAN	1	3	608.2658	<.0001	1.0082	Included
4	MARITAL STATUS	1	4	334.3568	<.0001	1.0514	Included
5	EQUIPMENT CATEGORY	1	5	134.7734	<.0001	2.6892	Included
6	NEW OLD USED	1	6	28.9000	<.0001	4.0015	Excluded
7	AGE OF ASSET	1	7	23.6775	<.0001	2.3974	Excluded

CORRELATION ANALYSIS

	AGE OF ASSET	DEBIT INTEREST	DEPOSIT TO LOAN	DWELLING TYPE CODE	EQUIPMENT CATEGORY	NEW OLD USED	MARITAL STATUS
AGE OF ASSET	1.0000	0.2354	0.0289	-0.0826	-0.5967	0.7592	-0.0129
DEBIT INTEREST RATE	0.2354	1.0000	0.0330	-0.2000	-0.2489	0.2086	-0.0789
DEPOSIT TO LOAN	0.0289	0.0330	1.0000	0.0564	-0.0349	0.0189	0.0509
DWELLING TYPE CODE	-0.0826	-0.2000	0.0564	1.0000	0.0682	-0.0866	0.1983
EQUIPMENT CATEGORY	-0.5967	-0.2489	-0.0349	0.0682	1.0000	-0.7859	-0.0388
NEW OLD USED	0.7592	0.2086	0.0189	-0.0866	-0.7859	1.0000	-0.0062
MARITAL STATUS	-0.0129	-0.0789	0.0509	0.1983	-0.0388	-0.0062	1.0000

FINAL COVARIATE LIST

1	Debit Interest Rate
2	Dwelling Type Code
3	Deposit to Loan
4	Marital Status Code
5	Equipment Category Code

Figure 4.7: Multivariate Analysis - Default Model

Taking all these criteria suggests that the most significant predictors for the default model are the ones listed in Table 4.8. Recall that for each covariate, the final categories were reached through a thorough univariate assessment as discussed extensively in Section 4.5.2. As a result each variable making into the final model is provided in Table 4.8 with a description of each of the categories.

Table 4.8: Final Covariates - Default Model

Variable	Category	Value	Description
Debit Interest Rate	1	Less than 11	Less than 11
	2	[11,14.2)	Greater than or equal to 11 but less than 14.2
	3	14.2+	Greater than 14.2
Deposit to Loan	1	No deposit paid	No deposit paid
	2	Deposit paid	Deposit paid
Dwelling Type Code	1	Tenant	Customer is a tenant
	2	Parents	Customer living with parents
	3	Owner	Customer owns a residential property
Equipment Category Code	1	Used Vehicles	Demo and Preowned vehicles
	2	New Vehicles	Brand new Vehicles
Marital Status Code	1	Married	The customer is married
	2	Other	Includes Single, Divorced, Widowed customers

4.5.4 Early Settlement Model Univariate Analysis

Univariate assessment conducted in the default model was adopted for the ES model. Variables were grouped using the interactive grouping node in SAS® Enterprise Miner. The first set of categories were checked for logic and ability to rank order, where the conditions were not met, variables were then re-grouped manually, according to meaning, volumes and event rate. All covariates were assessed using GS to determine their ability to differentiate risk. For ES, all variables were assessed for PH assumption resulting in further exclusions, regrouping and retention of satisfactorily grouped covariates. Table 4.9 lists each variable considered and the decision made. Exclusions were made due to failure of the covariate to differentiate risk.

Table 4.9: Early Settlement Model - Univariate Gini Statistics

Variable	Gini Statistic	Decision Made
Original Term	20.982	Included
Ballon to Loan	18.117	Included
Deposit to Loan	16.247	Included
Equipment Category Code	14.406	Included
Age of Vehicle	12.769	Included
New Old Used	11.858	Included
Dwelling Type Code	9.952	Included
Marital Status Code	8.411	Included
Number of Insurances	8.053	Included
Rate Type Code	7.585	Included
Manufacturer Code	6.550	Included
Article Type Code	4.605	Included
Debit Interest Rate	4.321	Included
Instalment Method Code	0.000	Excluded
Discounting Code	0.000	Excluded
Instalment Frequency Code	0.000	Excluded
Maintenance Code	0.000	Excluded
Product Code	0.000	Excluded
SA Citizen Indicator	0.000	Excluded
Site Type	0.000	Excluded
Spouse Employed Indicator	0.000	Excluded

Covariates passing the GS were assessed for PH assumption. Variables were regrouped where necessary. The PH plots used to select predictors are provided in Appendix A.3. Covariates which were excluded due to failure to satisfy PH assumption in the ES model are: Article Type Code, Manufacturer Code, Rate Type Code, Number of Insurances, Marital Status Code and Balloon to Loan. Variables passing the PH assumption were further assessed for population stability over time. The WoE and event rate should be monotonic across groups. The population in each group should be at least 5 percent. No discrepancies were observed. The rest of the univariate assessment plots are in the Appendix A.3.

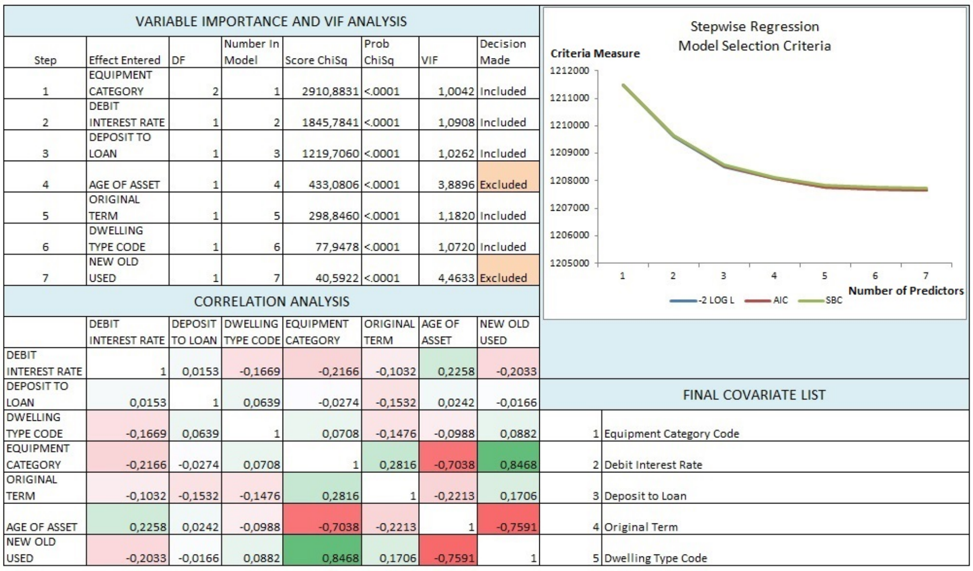
4.5.5 Early Settlement Model Multivariate Analysis - Cox Regression

Varibles satisfying all the univariate assessment tests were considered for multivariate analysis. As in the Default model, the bucket combination with the largest population was manually selected and used as the baseline. Figure 4.8 summarises results from the multivariate assessment.

- Variable Importance and VIF Analysis table: Equipment Category Code is the strongest variable in the ES model, followed by Debit Interest Rate and the least important is New Old Used. The variables Age of Vehicle and New Old Used were excluded from the analysis due to their high VIF value which is greater than 3 in each case. This was done to avoid parameter instability.

- Model Selection Criteria chart: The model selection criteria were determined. The AIC, SBC and -2 Loglikelihood plots level off at point 6, it makes sense to exclude variables added at any point later than the 6^{th} step. This criteria suggests that the variable New Old Used should be excluded from the analysis at this stage. This complements information in the VIF analysis and correlation matrix as well.

- Correlation Matrix: Variables with high correlation were already excluded due to high VIF. Age of Vehicle and Equipment Category Code (-0.7038). New Old Used is highly correlated with Equipment Category Code (0.8468). Dropping Age of Vehicle does not lead to information loss as this information is already contained in Equipment Category Code.



VARIABLE IMPORTANCE AND VIF ANALYSIS

Step	Effect Entered	DF	Number In Model	Score ChiSq	Prob ChiSq	VIF	Decision Made
1	EQUIPMENT CATEGORY	2	1	2910,8831	<.0001	1,0042	Included
2	DEBIT INTEREST RATE	1	2	1845,7841	<.0001	1,0908	Included
3	DEPOSIT TO LOAN	1	3	1219,7060	<.0001	1,0262	Included
4	AGE OF ASSET	1	4	433,0806	<.0001	3,8896	Excluded
5	ORIGINAL TERM	1	5	298,8460	<.0001	1,1820	Included
6	DWELLING TYPE CODE	1	6	77,9478	<.0001	1,0720	Included
7	NEW OLD USED	1	7	40,5922	<.0001	4,4633	Excluded

CORRELATION ANALYSIS

	DEBIT INTEREST RATE	DEPOSIT TO LOAN	DWELLING TYPE CODE	EQUIPMENT CATEGORY	ORIGINAL TERM	AGE OF ASSET	NEW OLD USED
DEBIT INTEREST RATE	1	0,0153	-0,1669	-0,2166	-0,1032	0,2258	-0,2033
DEPOSIT TO LOAN	0,0153	1	0,0639	-0,0274	-0,1532	0,0242	-0,0166
DWELLING TYPE CODE	-0,1669	0,0639	1	0,0708	-0,1476	-0,0988	0,0882
EQUIPMENT CATEGORY	-0,2166	-0,0274	0,0708	1	0,2816	-0,7038	0,8468
ORIGINAL TERM	-0,1032	-0,1532	-0,1476	0,2816	1	-0,2213	0,1706
AGE OF ASSET	0,2258	0,0242	-0,0988	-0,7038	-0,2213	1	-0,7591
NEW OLD USED	-0,2033	-0,0166	0,0882	0,8468	0,1706	-0,7591	1

FINAL COVARIATE LIST

1	Equipment Category Code
2	Debit Interest Rate
3	Deposit to Loan
4	Original Term
5	Dwelling Type Code

Figure 4.8: Multivariate Analysis - Early Settlement Model

Taking all this criteria suggests that the most significant predictors for the ES model are those listed in Table 4.10.

Table 4.10: Final Covariates - Early Settlement Model

Variable	Class	Value	Description
Debit Interest Rate	1 2	Less than 12.55 12.55+	Less than 12.55 Greater than or equal to 12.55
Deposit to Loan Ratio	1 2	Less than 0.45 0.45+	less than 45 Greater than or equal to 45
Dwelling Type Code	1 2	Tenant Owner,Parents	Customer is a tenant Owner or lives with parents
Equipment Category Code	1 2 3	Old Used New and LD	Vehicle older than 5 years Demo vehicles or less than 5 years New and Light Utility Vehicles
Original Term	1 2	Less than or equal to 60 Greater than 60	Account tenure less than or equal 60 months Original term more than 60 months

4.6 Summary

The composition of the vehicle finance book is given in Table 4.1 and in Figure 4.1. These complement the information obtained from an independent survey whose results are summarised in a biplot in Figure 1.1 of Chapter 1 and the brands appearing in the biplot are indeed the "popular" brands of the South African automobile market. Over a 5 year observation period, individual accounts were tracked every month from the point of entry to the point of exit into either default, ES or to the termination of study at the end of March 2014. Covariate selection process was thoroughly done and all the covariates selected satisfied the conditions required in a univariate analysis. Variables satisfying multivariate approaches were then used in fitting the models. Further detail is provided in Chapter 5.

Chapter 5

Results and Analysis

5.1 Introduction

Credit risk refers to potential loss associated with loans issued to consumers due to borrower's failure to meet contractual obligations. Among other risks (e.g liquidity buffer, operational and market risks), credit risk is the most significant source of regulatory capital demand. In order to hold adequate capital against credit risk, financial institutions should adopt the use of advanced statistical capital models to calculate capital demand. To comply with the requirements of the Basel Accord, the organisation set up internal model building standards detailed in Section 5.7. These were adopted in this study, to assess adequacy and accuracy of each model. Statistical assumptions of the models built were verified. Model comparisons were made and the results presented graphically and numerically. Analyses were conducted to determine the methodology striving to perform better for a consumer credit cohort data in the presence of competing risks and long term survivors.

5.2 Model Fitting

In fitting all the models, it was opted to manually select the categories to be considered as the baseline rather than opting for an automatic selection. This was done to optimise the volumes in the baseline to ensure maximum statistical significance for as many variables as possible. Therefore the category combination with the largest population was manually selected and used as the baseline for each event. For each model, the category combination giving the highest population is given in Table 5.1.

Table 5.1: Baseline Population

Default Model		*ES Model*	
Variable	**Category**	**Variable**	**Category**
Debit Interest Rate	2	Debit Interest Rate	1
Deposit to Loan Ratio	1	Deposit to Loan Ratio	1
Dwelling Type Code	1	Dwelling Type Code	2
Equipment Category Code	1	Equipment Category Code	3
Marital Status Code	1	Original Term	2

Following the model building process, each model variant was fitted. Logistic regression was fitted for each event type. Cox PH regression was fitted for each event type as well. No discrepancies were observed in all models. For each covariate, category specific p-values indicated that all variables were significant at granular level.

5.3 Logistic Regression

Models were fitted separately for each event type. In each case, accounts used for model development were allowed at least 48 months to perform. This is the “waiting” performance period to maturity, required for the purposes of LR. The account-level probabilities were calculated based on the event observed at month 48. Results and analyses are provided below.

5.3.1 Default Model Multivariate Analysis - Logistic Regression

A stepwise multivariate analysis was conducted to select the final covariates and categories for use in the default Logistic regression model. All covariates met the 0.05 significance level for entry into the model and the order of importance is given in Table 5.2.

Table 5.2: Variable Importance - Default Model

Step	Variable	ScoreChiSquare
1	Debit Interest Rate	844.96
2	Deposit to Loan	108.46
3	Dwelling Type Code	61.28
4	Marital Status Code	21.55
5	Equipment Category Code	18.02

The AIC plot is given in Figure 5.1. The graph does not level off hence no covariates were removed based on the AIC selection criterion.

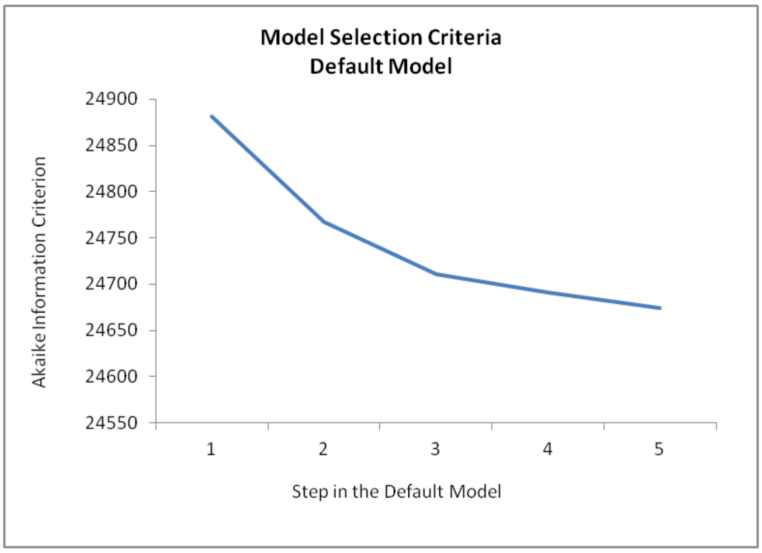


Figure 5.1: Default Model Selection Criteria

The Hosmer and Lemeshow test failed to reject H_0 with a Chi-square statistic of 14.8138 and a p-value of 0.0628 indicating a good model fit. However, Dwelling Type Code category two was not significantly different from the baseline category with a p-value greater than 0.05 as shown in Table 5.3, it was decided to collapse the category and combine it with the baseline category one. Thus the number of categories in Dwelling Type code were reduced from three to two for the default Logistic regression model.

Table 5.3: Logistic Stepwise Regression - Default Model

Variable	Class	Estimate	Odds Ratio	P Value
Intercept		-2.0826		
Debit Interest Rate	1	-0.5969	0.5504	<.0001
	2	0.0000	1.0000	
	3	0.7293	2.0737	<.0001
Deposit to Loan	1	0.0000	1.0000	
	2	-0.4752	0.6217	<.0001
Dwelling Type Code	1	0.0000	1.0000	
	2	-0.0285	0.9718	0.5650
	3	-0.2514	0.7776	<.0001
Equipment Category Code	1	0.0000	1.0000	
	2	-0.1906	0.8264	<.0001
Marital Status Code	1	0.0000	1.0000	
	2	-0.1731	0.8409	<.0001

5.3.2 Default Model - Logistic Regression

The maximum likelihood estimates are provided in Table 5.4. The parameter estimates for the baseline population are all valued at 0 as the baseline is used as the empirical reference population, and should not be influenced by the effect of covariates. The odds ratio is obtained by exponentiating the corresponding parameter estimate for each category. Subsequently, the odds ratios of the baseline population take up the value of 1. Thus, the movement in odds ratio from the baseline is relative to the movement in parameter estimates of the respective category.

Odds are calculated based on probabilities and they range between 0 and $+\infty$. They give us a ratio of the probability of default versus the probability of non default in each category. Reading from Table 5.4, the ratio of odds of defaulting for accounts with a Debit Interest Rate of 14.2 versus those with [11,14.2) is 2.0792 to 1. This shows the extent to which accounts with Debit Interest Rate of 14.2 are more prone to default than those in the baseline population. In this case its 2.0792 times more. Parameter estimates in Table 5.4 show that there is sufficient evidence to conclude that all the categories are statistically significant as the p-value is less than 0.001 across all categories.

Table 5.4: Logistic Regression - Default Model Estimates

Variable	Category	Estimate	Odds Ratio	P Value
Intercept		-2.0946	0.1231	<.0001
Debit Interest Rate	Less than 11	-0.5971	0.5504	<.0001
	[11, 14.2)	0.0000	1.0000	
	14.2+	0.7320	2.0792	<.0001
Deposit to Loan	No deposit paid	0.0000	1.0000	
	Deposit paid	-0.4753	0.6217	<.0001
Dwelling Type Code	Tenant	0.0000	1.0000	
	Owner	-0.2388	0.7876	<.0001
Equipment Category Code	Used vehicles	0.0000	1.0000	
	New vehicles	-0.1895	0.8274	<.0001
Marital Status Code	Married	0.0000	1.0000	
	Other	-0.1766	0.8381	<.0001

The logical trend in each parameter was checked for intuitiveness by analysing the direction of each parameter estimate and its corresponding odds ratios relative to the baseline population. The final parameter estimates complements the trends observed during the univariate analysis process. The detailed explanation is given in Table 5.5.

5.3.3 Goodness of fit Statistics - Default Logistic Model

To assess goodness of fit of the Logistic default model, the Hosmer and Lemeshow test was conducted. It measures how well predicted events align with the observed events. The population is subdivided into decile groups according to the probabilities. Each decile group is compared on the expected versus observed events. Low values of the Hosmer and Lemeshow statistic and high p-values (greater than 0.05) indicate a

Table 5.5: Intuitiveness of Signs of the Default Model - Logistic Regression

Variable	Relation to Baseline	Explanation
Debit Interest Rate	Positive	From table 5.4, the odds ratios show an increasing trend over the rank ordered categories. The odds of 14.2+ category are 1.08 higher than the odds of category [11,14.2). This implies that the rate of default increases with increasing debit interest rate. Thus, more expensive loans due to high interest rate are more prone to default than those with lower interest rate.
Dwelling Type Code	Negative	The odds of owners are 22% lower than the odds of tenants. Home owners have a lower probability of default than tenants. The greater the equity associated with customers, the lower the probability of default.
Deposit to Loan Ratio	Negative	The odds of deposit payers are 38% lower than the odds of customers who did not pay a deposit. The rate of default decreases with increasing deposit paid. If a deposit is paid, the capital loan amount reduces and the cost of borrowing becomes cheaper and thus the rate of default is minimal where the deposit has been paid.
Equipment Category Code	Negative	The odds of new vehicles are 18% lower than the odds of new vehicles. Old vehicles are associated with higher default rate compared to new vehicles as the demand for ageing vehicles diminishes.
Marital Status Code	Negative	The odds of married customers are 17% higher than the odds of the unmarried. Marriage comes with greater responsibilities, customers become more prone to default on loans.

good fit of the observed versus predicted event rates. For the default model, the Chi-square statistic of 15,2447 was obtained, with a corresponding p-value of 0,0546. This indicates that this model does fit the data. The Hosmer and Lemeshow test partitions data into decile groups according to risk levels. Results in Table 5.6 indicate that the observed and expected rates of default are similar by population deciles.

Table 5.6: The Hosmer and Lemeshow Partition - Default Model

Group	Total Population	Events Observed	Events Expected	Nonevents Observed	Nonevents Expected
1	3643	110	124.73	3533	3518.27
2	4298	193	198.42	4105	4099.58
3	3900	217	231.39	3683	3668.61
4	5717	429	422.23	5288	5294.77
5	3634	309	316.09	3325	3317.91
6	4247	424	398.82	3823	3848.18
7	3548	414	401.45	3134	3146.55
8	4216	626	602.00	3590	3614.00
9	4499	799	772.51	3700	3726.49
10	2298	415	468.36	1883	1829.64

For each risk group, the actual versus observed event rates were calculated based on the total population. This was done to determine the ability of the model to rank order risk and to establish how accurate the model is, in predicting risk. Thus the values of actual and expected event rates were plotted across range of the risk. For the models to be accurate, the actual versus expected plots should not deviate significantly from the 45 degree diagonal. To check the ability of the model to rank order risk, the points should lie in increasing order of the risk group. Accuracy and rank ordering metrics were determined for both development and validation data sets. The accuracy plots are provided in Figure 5.2. No discrepancies were observed as all the points in both data sets are satisfactorily rank ordered and they all lie close to the 45 degree diagonal.

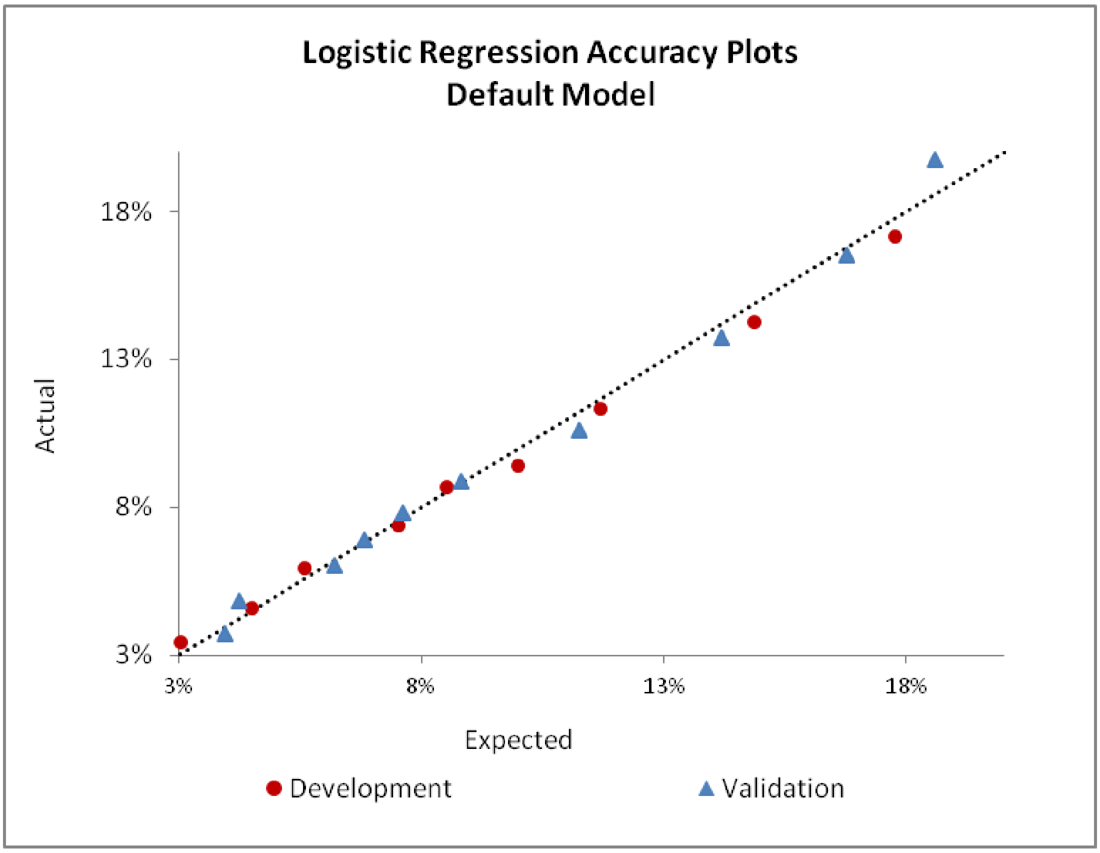


Figure 5.2: Accuracy Plots Default Model

5.3.4 Early Settlement Model Multivariate Analysis - Logistic Regression

As in the default model, a stepwise multivariate analysis was conducted to select the final covariates and categories for the ES Logistic regression model. Original Term failed to meet the 0.05 significance level for entry into the model at stepwise regression stage, hence it was dropped. The remaining covariates are listed in Table 5.7.

Table 5.7: Variable Importance - ES Model

Step	EffectEntered	ScoreChiSq
1	Equipment Category Code	647.69
2	Debit Interest Rate	501.65
3	Deposit to Loan	271.44
4	Dwelling Type Code	10.26

The selection criteria plot of the ES model is provided in Figure 5.3. As seen in the plot, the graph does not level off, thus no further covariates were removed.

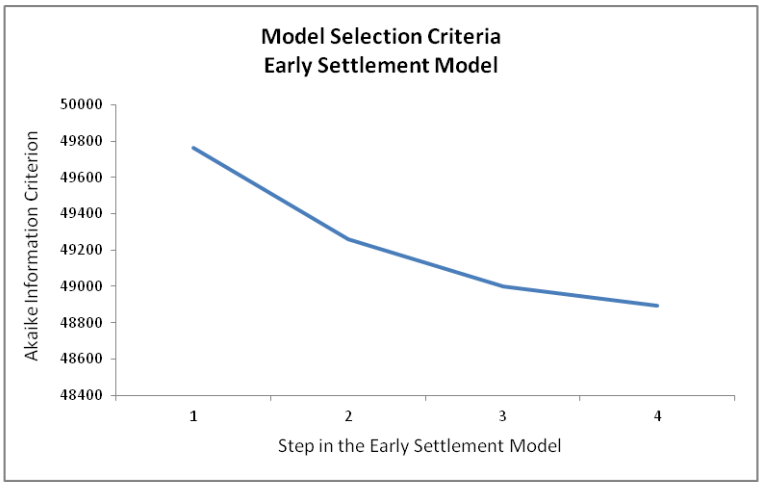


Figure 5.3: Early Settlement Model Selection Criteria

The Hosmer and Lemeshow test was conducted and we obtained a Chi-square statistic of 66.9723 and a p-value less than 0.05 leading to the rejection of the null hypothesis. To correct model misfit, it was decided to re-run the stepwise regression using all possible two way interaction terms. Insignificant categories were removed until a final model with some interaction terms, was achieved. The descriptions of the Early Settlement covariates and their corresponding categories are detailed in Table 4.10, with the exception of Original Term. The final categories and interaction terms are provided in Chapter 5, Table 5.8.

5.3.5 Early Settlement Logistic Regression Model

Table 5.8 shows the maximum likelihood estimates of the Logistic regression ES model. Category1 represents the category of the first covariate. Category2 represents the category of the interaction term. Some variable names were shortened in order to optimise space in Table 5.8. Debit Interest Rate was shortened to Debit Interest and Equipment Category Code to Equip Cat. In the case of Equipment Category Code, LDV refers to Light Delivery Vehicles. All covariates and interaction terms in the final model were statistically significant at granular level as reflected in p-values less than 0.05 across all categories . The intuitiveness of signs is explained in Table 5.9.

Table 5.8: Logistic Regression - ES Model Estimates

Variable	Category1	Category2	Estimate	Odds Ratio	P Value
Intercept			-0.5386	0.5835	<.0001
Equipment Category Code	Old		0.4025	1.4955	<.0001
	Used		0.3001	1.3500	<.0001
	New, LDV		0.0000	1.0000	
Debit Interest Rate	< 12.55		0.0000	1.0000	
	12.55+		-0.4254	0.6535	<.0001
Deposit to Loan	< 0.45		0.0000	1.0000	
	0.45+		0.3680	1.4449	<.0001
Dwelling Type Code	Tenant		-0.0502	0.9511	0.001
	Owner		0.0000	1.0000	
Debit Interest × Equip Cat	< 12.55	New, LDV	0.0000	1.0000	
	12.55+	Old	0.2030	1.2250	<.0001
	12.55+	Used	0.1562	1.1691	<.0001
Deposit to Loan × Equip Cat	< 0.45	New, LDV	0.0000	1.0000	
	0.45+	Old	-0.2770	0.7581	<.0001
	0.45+	Used	-0.1604	0.8518	<.0001
Debit Interest × Deposit to Loan	< 12.55	< 0.45	0.0000	1.0000	
	12.55+	0.45+	0.1008	1.1061	0.008

Table 5.9: Intuitiveness of Signs of the Early Settlement Model - Logistic Regression

Variable	Relation to baseline	Explanation
Debit Interest Rate	Negative	The odds of customers whose interest rate is 12.55+ settling early is 35% lower than the odds of customers whose interest rate is lower than 12.55. It is difficult to settle early on expensive loans hence the ES odds are lower in the higher category of debit interest rate.
Deposit to Loan Ratio	Positive	The ES odds of customers who paid deposit to loan ratio of 0.45+ are 44% higher than the odds of customers who paid less than 0.45. If a deposit amount is paid, it reduces the capital amount of the loan and it becomes easier to settle the balance early on lower capital base.
Dwelling Type Code	Negative	The odds of settling early are 5% lower for home owners than tenants. Home owners have a higher tendency to settle early than tenants. The greater the equity associated with customers, the higher the probability of ES as they perhaps shift focus to longer term home loans.
Equipment Category Code	Positive	The odds to settle early are 35% higher in used vehicles compared to new and 49% higher in old vehicles than new. Old vehicles have a highest probability of ES compared to Used and New. As customers upgrade to new vehicle models, settlement occurs most oftenly on old vehicles.
Debit Int × Equipment Cat	Positive	Old vehicles coupled with high interest rate have the highest odds of ES as there is higher urge to avoid high interest and dispose old asset.

5.3.6 Goodness of fit Statistics - ES Logistic Model

The Hosmer and Lemeshow test was conducted for the final ES Logistic regression model. A low Chi-square statistic of 3,0120 was obtained, associated with a high p-value of 0,9336. This indicates that the observed and expected ES rates are similar by population deciles and the model fits very well.

Table 5.10: The Hosmer and Lemeshow Partition - ES Logistic Model

Group	Total	Events Observed	Events Expected	Nonevents Observed	Nonevents Expected
1	1890	295	293.44	1595	1596.56
2	4401	729	737.29	3672	3663.71
3	3955	1109	1131.09	2846	2823.91
4	5771	1738	1702.86	4033	4068.14
5	4937	1502	1509.92	3435	3427.08
6	2385	816	793.90	1569	1591.10
7	4095	1469	1474.08	2626	2620.92
8	5569	2236	2241.27	3333	3327.73
9	3736	1597	1593.59	2139	2142.41
10	3261	1487	1500.20	1774	1760.80

As in the default Logistic model, the actual versus observed event rates were calculated based on the total population. Accuracy and rank ordering metrics were determined for both development and validation data sets. The accuracy plots are provided in Figure 5.4. No discrepancies were observed as all the points in both development and validation sets are satisfactorily rank ordered and they all lie close to the 45 degree diagonal. Risk groups one and two had very low event rate as seen on the points lying close to the origin. The points lie closer to the 45 degree diagonal compared to those in the default model. This shows that the ES Logistic model has higher accuracy compared to the default Logistic model.

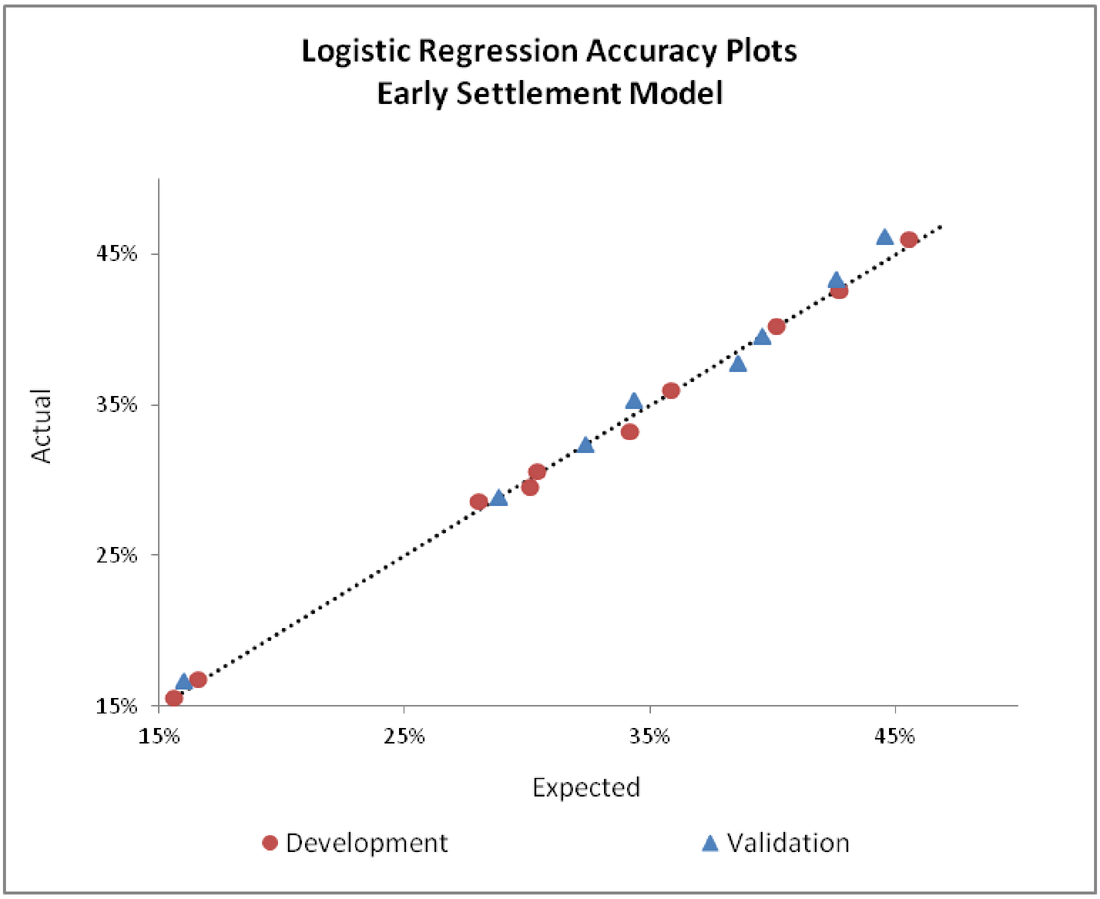


Figure 5.4: Accuracy Plots Early Settlement Model

Both LR models were fitted satisfactorily. For each model the ROCs curves were calculated and the corresponding area under the curves were determined. The model GSs were also calculated to determine the ability of each model to differentiate risk. These were then compared to the those obtained in the corresponding Cox PH models to determine the methodology striving to perform better given lifetime data in a consumer credit setting in the presence of competing risks and long term survivors. More detail is provided in Section 5.6.

5.4 Cox PH Regression

Unlike the Logistic development data selection process, the Cox regression model does not consider a “waiting” period. All accounts are eligible for inclusion in the development. Accounts recently entered into the study also play a very important role. If not absorbed into any of the event then they can be classified as censored and form part of the risk set according to time spent in study. Two cause specific PH models were fitted separately for each event. A CIC was calculated in each case to determine the marginal probabilities of an event happening at a fixed workout period of 48 months in the presence of competing risks and long term survivors.

5.4.1 Long Term Survivors

As evidenced by the empirical KM curve in Figure 5.5, the overall survival plot levels off at non zero values. This indicates that the bulk of accounts are not susceptible to the events of interest. It makes business sense as most of the customers on the vehicle finance book are good customers and statistically, that prompts heavy censoring at the end of the study. A proportion (p) of good customers is chosen such that the overall survival curve levels of to p. In this case the minimum value of the survival curve was selected as p. Thus $p = 0.49179045$. Using the Cure model approach, the survival functions will be adjusted to allow for long term survivors. The proportion of susceptible population **A** $= p$ and that of non-susceptible population **B** $= 1 - p$. The survivor function of the entire population (**A** + **B**) is given by:

$$S(t) = (1 - p) + pS_A(t) \tag{5.1}$$

where $S_A(t)$ is the survivor function of population **A**. For each model, the hazard function was derived from the survivor function and the corresponding CIC’s were calculated. The final models were tested on the validation set.

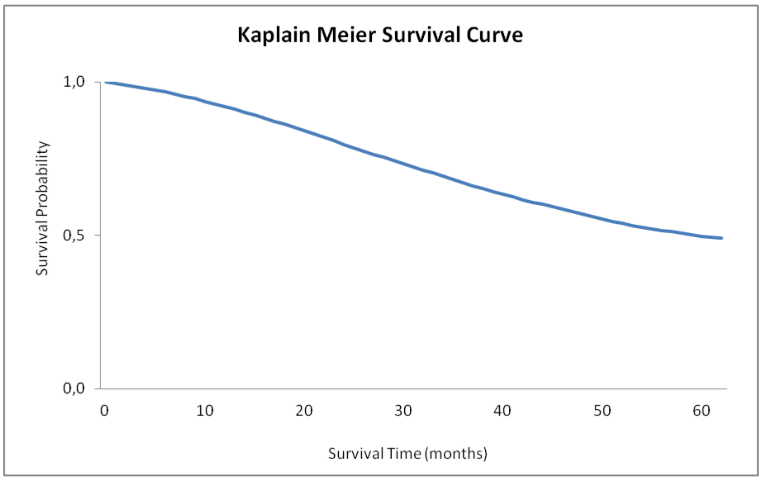


Figure 5.5: Kaplain Meier Survival Curve

5.4.2 Analysis of The Hazard Functions

In survival analysis, we assume that neither of the events can happen at the point of entry. Thus the survival probability at time 0 is equal to 1 and conversely, the hazard function at time 0 is equal to 0. As the survival time in this study is discrete, we expect the events to start occurring at month 1 onwards. Figures 5.7 and 5.6 show the empirical probability mass and hazard functions for ES and default events respectively. For both events, the functions start at zero as there are no events recorded at the entry points. The probability mass function of ES is the number of accounts settling early at any t, from 0 to 48, relative to the total number of early settlements in the book. It is a function of the total number of ES events which shows the probability distribution of the early settlement event over time. The probability distribution functions were determined for both events. The hazard function is the instantaneous rate of occurrence of an event. This is a function of the accounts at risk at any t. The hazard functions were calculated and plotted for each event as well.

5.4.2.1 Default Event

With reference to Figure 5.6, the probability mass function of the default event increases sharply in the first 18 months of the loans. The same trend is reflected in the corresponding hazard function which increases sharply up to 18 months and generally stabilises beyond the 18-month point. This is due to the fact that at the point of application, the selected customers have low risk of default but, as time goes on, customers experience various social and economic events leading to default and the risk of default increases. This trend is experienced in the first 1 and half years of loans for the vehicle finance product. As the accounts grow older than 18 months, the rate of default decreases, accounts passing this point have a lower chance of default. This is attributable to the fact that most customers improve their financial status with time and the original repayment amount becomes insignificant with time and hence the chance of default diminishes as time approaches 48 months.

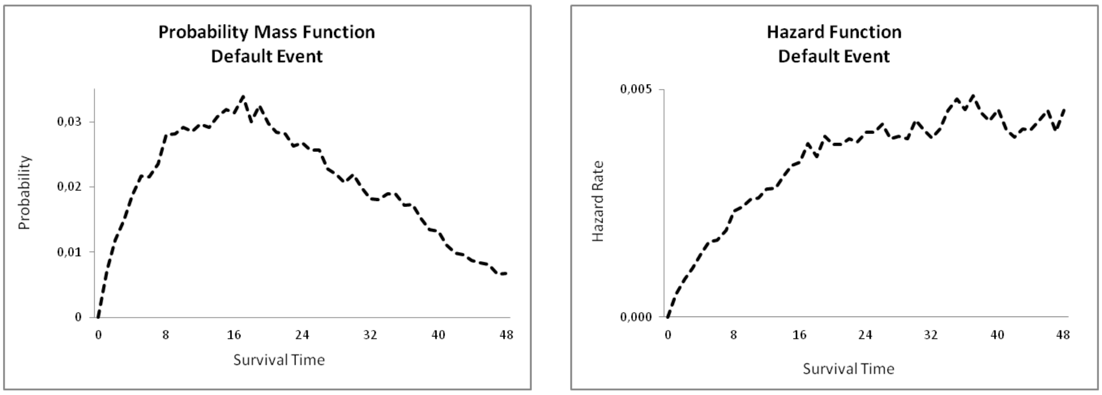


Figure 5.6: Default Event - Hazard Function

5.4.2.2 Early Settlement Event

The probability mass function in Figure 5.7 increases steadily in the first 18 months, decreases at a steady rate beyond 18 months and diminishes towards 48 months. This is also reflected in the hazard function that is on an increasing trend for the same 18 months and rather stabilizes later. For the first 1 and half years from the entry point, the risk of early settlement increases with increasing time. As the vehicle ages, the chances of early settlement increases as customers upgrade to new vehicle models. However, as the accounts approach maturity, the risk of early settlement lessens as it becomes easier to complete the originally agreed term of repayment and customers rather complete the repayment normally instead of settling early, to avoid penalty charges associated with the event.

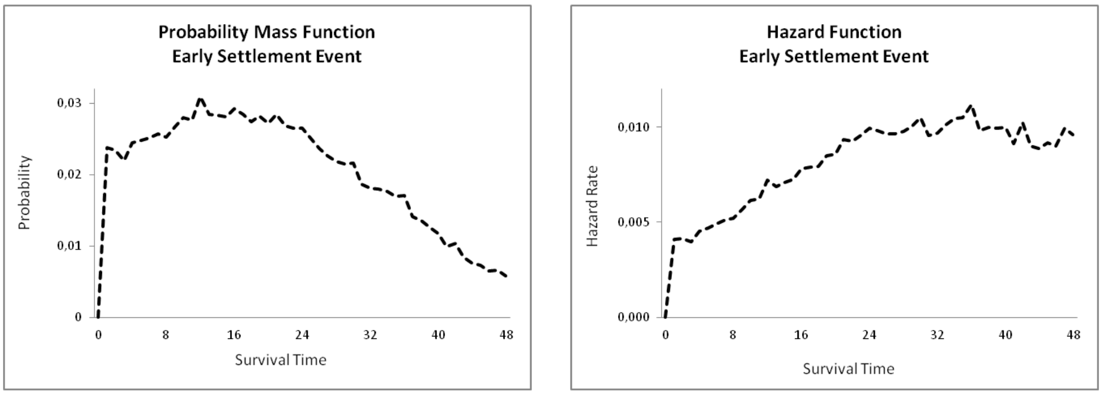


Figure 5.7: Early Settlement Event - Hazard Function

5.4.3 Default Model

Cox PH regression was run for the default model using PROC PHREG in SAS$^{\circledR}$. All covariates selected for the default event in the previous chapter were entered into the model. Below is the model output for the default model.

The global null hypothesis shows each statistic with a Chi-Square distribution and degrees of freedom. Based on the p value of less than 5% for each test, we conclude that at least one variable in each model is different from zero as we are testing the global null hypothesis that all coefficients are equal to zero.

Table 5.11: Global Null Hypothesis

Test	ChiSquare	Degrees of Freedom	P Value
Likelihood Ratio	45701.21	7	<.001
Score	43765.52	7	<.001
Wald	41400.33	7	<.001

The Type 3 tests for classed and categorical variables indicate that each variable is justifiably included into each model as the p value of less than 5% for each variable is statistically significant. However, Type 3 tests are an “overall” test indicating significant differences in event rates across any levels of a covariate. The test does not inform as to which level is different, thus a test for the significance of individual levels

Table 5.12: Type 3 Tests

Variable	Degrees of Freedom	WaldChiSq	P Value
Debit Interest Rate	2	18530.94	<.001
Deposit to Loan Ratio	1	5728.34	<.001
Dwelling Type Code	2	4658.23	<.001
Equipment Category Code	1	1143.74	<.001
Marital Status Code	1	2294.85	<.001

is required. This is addressed by the maximum likelihood estimates of each category. As evidenced from the Analysis of Maximum Likelihood Estimates in Table 5.13.

Table 5.13: Cox PH - Default Model Estimates

Variable	Category	Estimate	P Value	Hazard Ratio
Debit Interest Rate	<11	-0.721	<.0001	0.486
	[11,14.2)	0.000		1.000
	14.2+	0.416	<.0001	1.515
Deposit to Loan	No deposit paid	0.000		1.000
	Deposit paid	-0.538	<.0001	0.584
Dwelling Type Code	Tenant	0.000		1.000
	Parents	-0.150	<.0001	0.861
	Owner	-0.394	<.0001	0.674
Equipment Category Code	Used Vehicles	0.000		1.000
	New Vehicles	-0.199	<.0001	0.819
Marital Status Code	Married	0.000		1.000
	Other	-0.272	<.0001	0.762

The analysis of maximum likelihood estimates show the coefficient values and their associated p-values. The p-values are based on the Wald Chi Square tests for the null hypothesis that each coefficient is equal to zero. The test statistic is calculated by squaring the ratio of each coefficient (beta) to its standard error. All the categories are statistically significantly different from the baseline groups in the final model.

The hazard ratio gives the measure of effect of explanatory variables. The corresponding measure of the effect in a Logistic regression environment is the odds ratio. Hazard ratios are calculated based on instantaneous rate of default happening in predefined subgroups, in this case the reference group and each of the categories at any t, given that the subject survived up to t.

Given parameter estimates, hazard ratios can be obtained by exponentiating parameter estimates in each category. This is analogous to the calculation of odds ratios in Logistic regression. Their values range between 0 and $+\infty$ as well. The baseline population have 0 parameter estimates with hazard ratios of 1. For values of HR less than 1, it implies the category has less risk of default than the baseline and vice versa for HR values greater than 1.

Reading from Table 5.13, the ratio of hazards of defaulting for accounts with a Debit Interest Rate of 14.2 versus those with [11,14.2) is 1.515 to 1. This shows the extent to which accounts with Debit Interest Rate of 14.2 are more prone to default than those in the baseline population. In this case its 1.515 times more. We look at the direction of estimates and the corresponding hazard ratios to determine intuitiveness and relate to business sense in Section 5.4.4.

5.4.4 Intuitiveness of Signs - Default Cox PH Model

The logical trend in each parameter is checked for intuitiveness by analysing the direction of each parameter estimate relative to the baseline. The final parameter estimates complements the trends observed during the univariate analysis process. The detailed explanation is given in Table 5.14.

Table 5.14: Intuitiveness of Signs of the Default Model

Variable	Relation to Baseline	Explanation
Debit Interest Rate	Positive	The hazards of customers with Debit Interest of 14+ are 50% higher than those in [11,14.2). Default rate increases with increasing debit interest rate. Higher interest rate implies more expensive loans and the accounts in that category have higher probability of default.
Dwelling Type Code	Negative	The hazards of default of home owners and customers living with parents are 33% and 14% lower than tenants respectively. Tenants have a higher probability of default than home owners and customers living with parents. The greater the equity associated with customers, the lower the probability of default.
Deposit to Loan Ratio	Negative	The default hazards of deposit payers are 42% less than deposit non payers' hazards. The rate of default decreases with increasing deposit paid. If a deposit is paid, the capital loan amount reduces and the cost of borrowing becomes cheaper and thus the rate of default is minimal where the deposit has been paid.
Equipment Category Code	Negative	Old vehicles have 18% higher hazards of default than new vehicles. This implies that old vehicles are associated with higher default rate as the demand for ageing vehicles diminishes.
Marital Status Code	Negative	The hazards of default for married customers are 24% higher than the unmarried because marriage comes with greater responsibilities, customers become more prone to default on loans.

5.4.5 Goodness of Fit Statistics - Cox PH Default Model

The accuracy measures were calculated for the Cox default model. This was performed on both the development and validation data sets. Accounts were ranked separately for each data set into deciles based on their default cumulative probabilities at month 48. For each decile group the actual and expected observations were determined. The default rate in each group was determined based on the total volumes. For the models to be accurate, the actual versus expected plots should not deviate significantly from the 45 degree diagonal. The accuracy plots for the default model are provided in Figure 5.8. No discrepancies were observed.

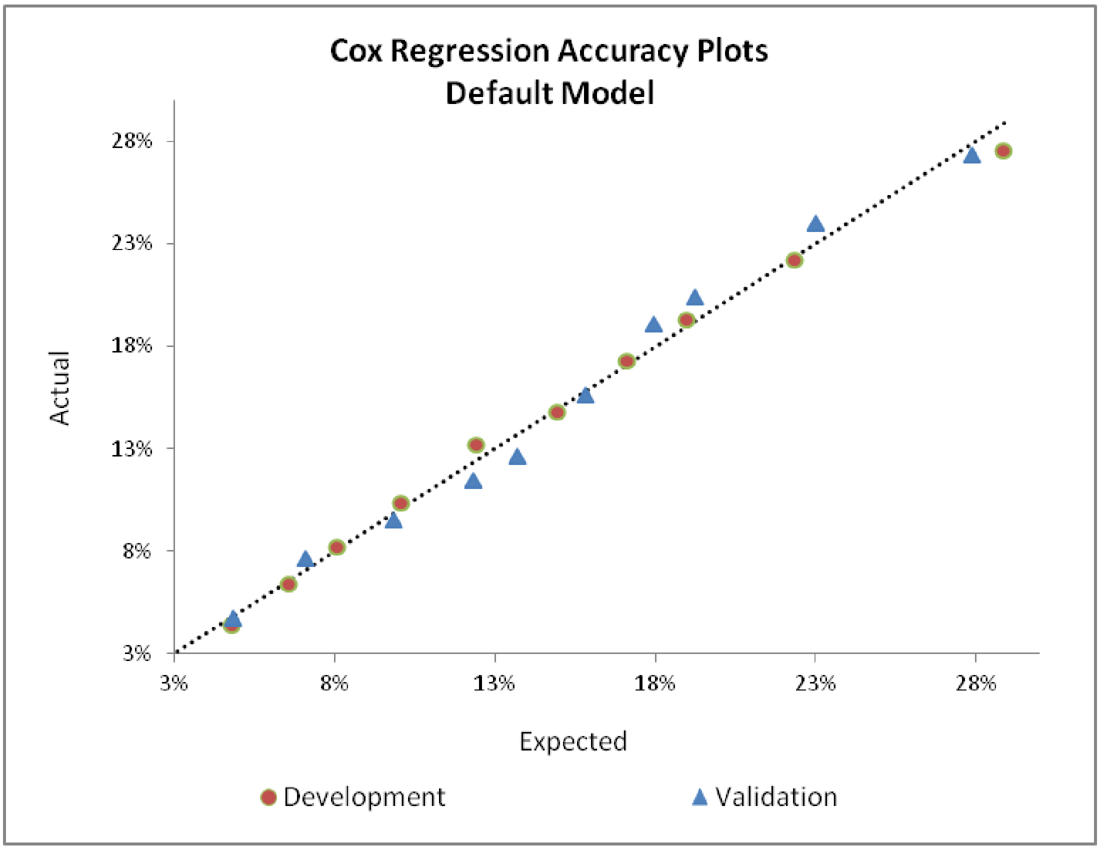


Figure 5.8: Accuracy Plots Default Cox Model

5.4.6 Early Settlement Model

Cox PH regression was run for the ES model using PROC PHREG in SAS®. All covariates selected for the ES event in the previous chapter were entered into the model. With p values of less than 0.001 for each level, there is sufficient evidence to conclude that at granular level, all the covariates are significant. Following is the ES model output.

The global null hypothesis shows each statistic with a Chi-Square distribution and degrees of freedom. Based on the p value of less than 5% for each test, we conclude that at least one variable in each model is different from zero as we are testing the global null hypothesis that all coefficients are equal to zero.

Table 5.15: Globall Null Hypothesis

Test	ChiSquare	Degrees of Freedom	P Value
Likelihood Ratio	6335.92	6	<.001
Score	6388.02	6	<.001
Wald	6293.77	6	<.001

The Type 3 tests for classed and categorical variables indicate that each variable is justifiably included into the ES model as the p value of less than 5% for each variable is statistically significant.

Table 5.16: Type 3 Tests

Variable	Degrees of Freedom	WaldChiSq	P Value
Debit Interest Rate	1	1671.29	<.001
Deposit to Loan Ratio	1	989.42	<.001
Dwelling Type Code	1	60.88	<.001
Equipment Category Code	2	2950.09	<.001
Original Term	1	333.79	<.001

The analysis of maximum likelihood estimates, shows the coefficient vales and their associated p values. As evidenced from the Analysis of Maximum Likelihood Estimates

Table 5.17: Cox PH - Early Settlement Model Estimates

Variable	Category	Estimate	P Value	HazardRatio
Debit Interest Rate	<12.55	0.000		1.000
	12.5+	-0.384	<.0001	0.681
Deposit to Loan	<0.45	0.000		1.000
	0.45+	0.378	<.0001	1.460
Dwelling Type Code	Tenant	-0,078	<.0001	0.925
	Owner, Parents	0.000		1.000
Equipment Category Code	Old	0.617	<.0001	1.854
	Used	0.463	<.0001	1.589
	New, LDV	0.000		1.000
Original Term	≤ 60	0.180	<.0001	1.197
	60+	0.000		1.000

in Table 5.17, all the levels are statistically significantly different from the baseline groups in the final with p-values less than 0.05.

5.4.7 Intuitiveness of Signs - Early Settlement Model

As in all the models discussed above, the ES Cox regression models was also analysed for intuitiveness of signs. More detail is provided in Table 5.18. The logical trend in each covariate was determined. Again in the ES Cox model, the final parameter estimates complements the trends observed during the univariate analysis process.

Table 5.18: Intuitiveness of Signs of the Early Settlement Model

Variable	Relation to Baseline	Explanation
Debit Interest Rate	Negative	The hazards of ES of customers with 12.5+ interest rate are 32% lower than those with debit interest rate less than 12.55. Thus, ES rate decreases as debit interest rate increases. The greater the interest rate the lesser the probability of ES. It is easier to settle early on cheaper loans with lower interest rate.
Deposit to Loan Ratio	Positive	The hazards of customers with deposit to loan ratio of 0.45+ are 46% higher than those with deposit to loan ratio less than 0.45. The ES rate increases with increasing amount of deposit paid. If a deposit amount is paid, it reduces the capital amount of the loan and it becomes easier to settle the balance early.
Dwelling Type Code	Negative	Tenants have 7% lower hazards of ES than home owners. The greater the equity associated with customers, the higher the ES probability.
Equipment Category Code	Positive	Old and Used vehicles have 85% and 58% higher hazards than New, LDV's respectively. As customers upgrade to new vehicle models, settlement occurs oftenly on old and used vehicles.
Original Term	Negative	With a negative trend in hazard ratios, that is the hazards of 60+ are 20% lower than those in less than 60% category. The longer the tenure of the loan, the more interest amount charged and it becomes more difficult to settle early.

5.5 Goodness of Fit Statistics - ES Cox PH Model

Accuracy measures were also determined for the Cox early settlement model. This was again performed on both the development and validation data sets. Accounts were ranked separately for each data set into deciles based on their ES cumulative probabilities at month 48. For each decile group the actual and expected observations were determined. The ES rate in each group was determined based on the total volumes. For the models to be accurate, the actual versus expected plots should not deviate significantly from the 45 degree diagonal. The accuracy plots for the early settlement model are provided in Figure 5.9. No discrepancies were observed.

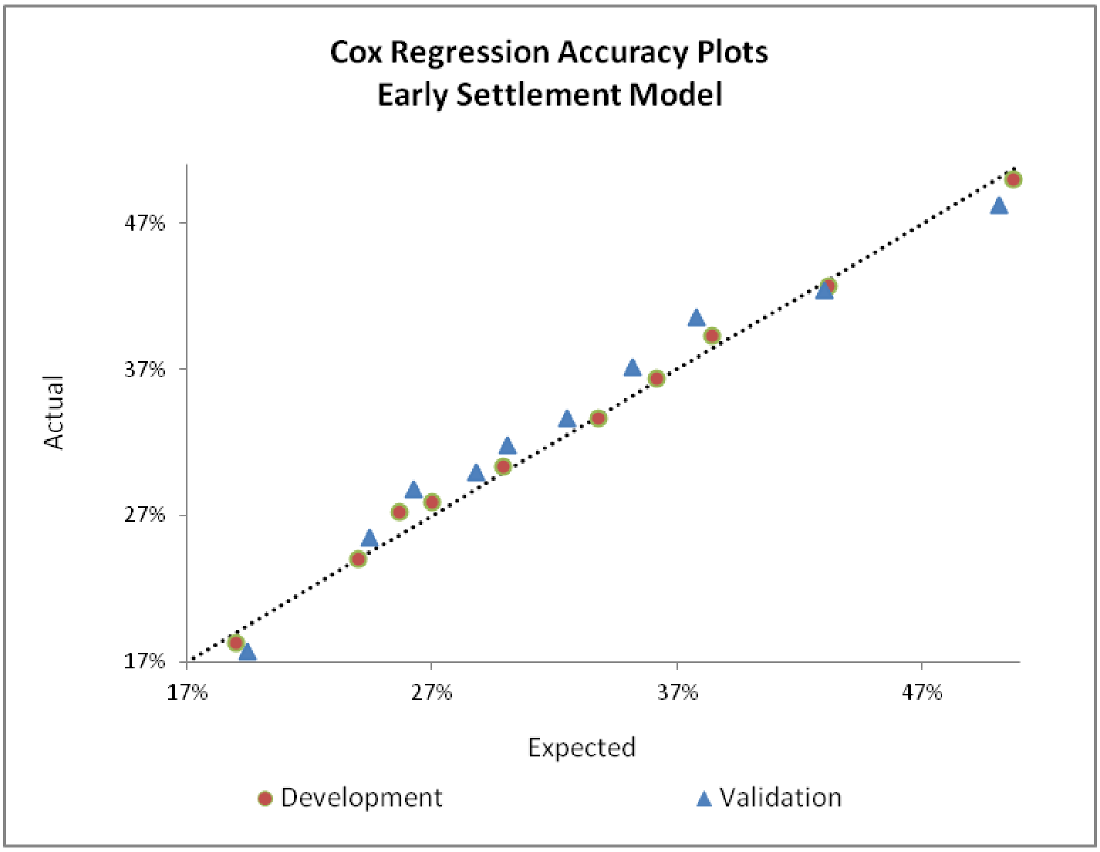


Figure 5.9: Accuracy Plots Early Settlement Cox Model

5.6 Model Comparison and Validation Metrics

As discussed in Section 5.7, models were assessed for accuracy and ability to differentiate, and rank order risk using accuracy plots and overall model GS. The ROC curves and area under the ROC curve metrics were used to compare performance of Logistic versus Cox PH regression models in a consumer credit setting.

5.6.1 Rank Ordering Metrics

Model specific accuracy plots were provided under each model variant discussion. With particular reference to Figures 5.2, 5.4, 5.8 and 5.9, reflecting accuracy of the default Logistic, ES Logistic, default Cox and ES Cox models respectively, it is evident that all models were built satisfactorily and accurately with each having the ability to rank order risk.

5.6.2 Model Gini Statistics

The overall model GS's were calculated for each model as a generalised metric to measure the ability of the models to differentiate risk. Results are given in Table 5.19 for each model. The lower GS's for LR models are attributable to the use of older vintages for development as the accounts should be allowed sufficient performance period (48 months in this case) before they can be considered for modelling. The fact that the overall GS for the Cox PH models are higher, suggest that Cox PH performs better than LR. The Cox PH model strength is enhanced by the inclusion of censored observations and the use of the most recent data in Cox PH regression.

Table 5.19: Model Gini Statistics

Reference Data Set	Default Model (%)	Early Settlement Model (%)
Cox Development	44.78	51.27
Cox Validation	44.50	51.17
Logistic Development	31.10	22.40
Logistic Validation	30.90	20.40

5.6.3 The ROC Curves

The ROC test plots the sensitivity against 1-specificity of the models at various cut-off values of risk. For the default event, sensitivity refers to a fraction of accounts in default that the model correctly identifies as defaulted. The same goes for the ES event. Specificity refers to a fraction of accounts not in default that the model correctly identifies as not in default. The ROC curves for the Logistic and Cox regression models are provided in Figure 5.10.

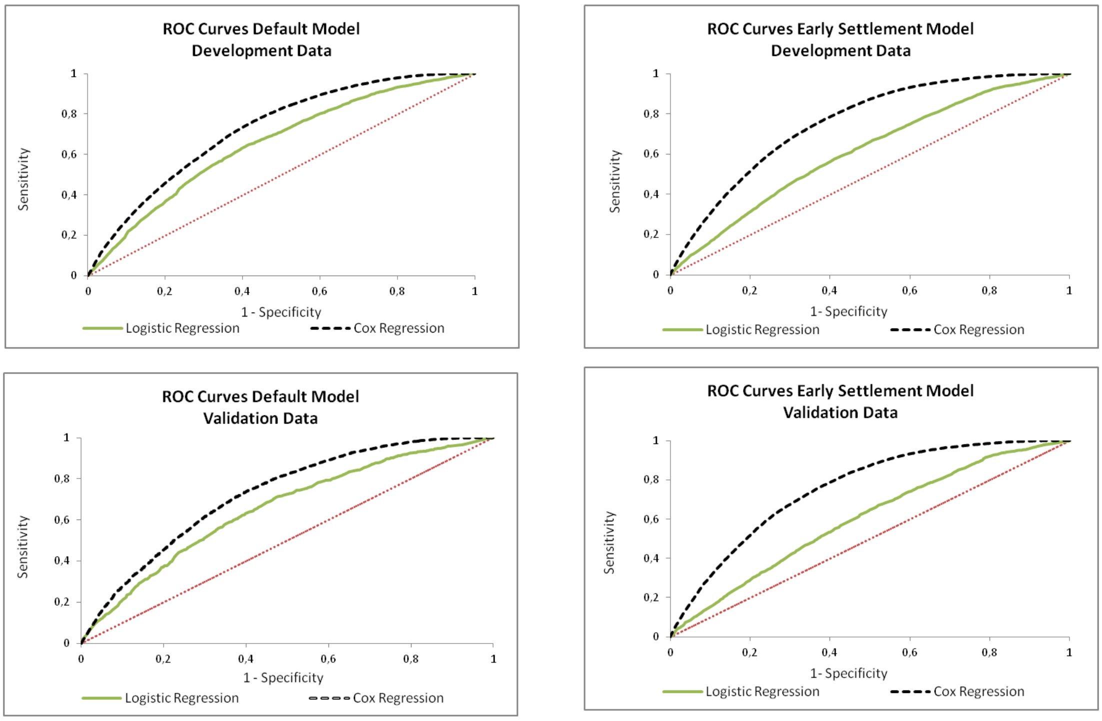


Figure 5.10: ROC Curves

The vertical axis represents sensitivity and the horizontal axis represents 1- specificity values at each cut-off point. Both axes range from 0 to 1. The diagonal divides the ROC cartesian plane. Curves above the diagonal line represent good classification model whereas points below the line represent poor results. Points along the diagonal represent a random model. In this case all the curves lie above the diagonal indicating that good classification models were developed in this study.

When comparing models, a better model is the one whose ROC curve lie closer to the upper end of the ROC space. In both models, it is clearly seen that Cox PH models perform better than the LR models. We therefore conclude that it is better to use Cox regression than LR in a lifetime data analysis.

Another metric that can be used to compare the performance of different models at this instance is the area under a ROC curve (AUC). It quantifies the overall ability of the model to discriminate between those accounts in default and those not in default. A completely useless model (one no better at identifying true positives than flipping a coin) has an area of 0.5. A perfect model (one that has zero false positives and zero false negatives) has an area of 1.00. In this study, the area under the curves were determined for Logistic regression models on both development and validation sets as follows: The closer the area is to the perfect model of area = 1, the better the model. The AUC can be represented by the overall model GS values and it has been stated that the Cox models have higher GS and subsequently higher AUC compared to LR models. Comparing the logistic model AUCs in Table 5.20, the default model is estimated better using Logistic regression than the ES model. This is also reflected in the ROC plots in Figure 5.10. The Early Settlement ROC plots lie closer to the diagonal line as compared to the default model curves.

Table 5.20: Area under ROC Curves

Reference Data Set	Default Model (%)	Early Settlement Model (%)
Cox Development	72.40	75.60
Cox Validation	72.30	75.20
Logistic Development	65.60	61.20
Logistic Validation	65.50	60.20

5.7 Credit Risk Model Standards

The model standards include:

- Minimum data history should be generally at least five years. Models in this study were built based on the observation period covering 5 years (April 2009 to March 2014). Every account was tracked on a monthly basis from the point of entry to exit.

- The data used in the model is representative of the population to be graded. All available data were used.

- A consistent definition of default. The definition of default was consistently applied across the observation period, the same was applied to the definition of early settlement.

- Use of the relevant information and data sources. The data used in this study was obtained from a credible data warehouse which is fully controlled and governed by international standards. All the data extracted relates to the vehicle finance product.

- Model drivers that are intuitive and plausible. Model specific covariates were selected satisfactorily. Evidence is presented, both graphically and numerically.

- Models should meaningfully differentiate risk. Overall model GS for each model was determined. The higher the model GS the more capable it is to differentiate risk.

- Models should be predictive and accurate. An assessment of the actual and predicted observations was conducted. No discrepancies were observed.

- Model outputs that are intuitive and plausible. As in the model drivers, model specific output is satisfactory statistically and it makes sound business sense. Results are provided both graphically and numerically.

5.8 Summary

The model building standards were satisfactorily addressed in each model. Models and drivers were intuitive and plausible as detailed under each model discussion wherein intuitiveness complements sound business logic. All models have high GS implying that they are capable of differentiating risk. Models predictive and accurate metrics were satisfactory. An assessment of the actual and predicted observations was conducted. Model GS and ROC curves were calculated to determine the methodology striving to perform better for a consumer credit cohort data in the presence of competing risks and long term survivors. It was concluded that Cox regression performs better than Logistic regression.

Chapter 6

Summary, Conclusion and Recommendations

6.1 Introduction

The aims and objectives of this study were to analyse competing risks in a consumer credit context with two events of interest, default and early settlement. These events were to be modelled using statistically sound techniques. The bulk of accounts under investigation were not susceptible to the events of interest. The data typically had long term survivors with heavy censoring at the end of the observation period. Two methodologies were compared in order to establish which works better, given a complex longitudinal cohort data set in the presence of competing risks and long term survivors. The two methodologies are Logistic regression and Cox regression. Model building standards in the credit risk environment were followed to ensure that the models developed were plausible and accurate. Models were developed and methodologies were compared using the model Gini statistics, receiver operating characteristic curves and area under the curves. Model output complements theoretical aspects detailed in Chapter 2. The data structure is analogous to lifetime data in other domains of study such as engineering and biomedical research, making the statistical methodologies versatile.

6.2 Summary

All available vehicle finance data were extracted from the financial institution's data warehouse. Data credibility is guaranteed as the data is governed and monitored according to the internal, national and international standards. Data preparation included selection of retail accounts opened and recorded in the period April 2009 to March 2014, outcome period analysis and derivation of critical variables such as the default and early settlement flags. In addition, all available variables were extracted and considered potential covariates unless proven otherwise by statistical and logical arguments, all of which were detailed in Chapter 4.

The simple random sampling approach was used to determine the development and hold out independent validation sets in the ratio of 80:20 respectively for each methodology. For Logistic regression, accounts were allowed 48 months to perform and the binary target variable was fully observable whereas in Cox regression, the period to maturity was not considered. Accounts with partially known survival times were included in the modelling as censored observations. This allowed the use of more information in Cox regression and less information for Logistic modelling. The volumes used in each method and data set are provided in Table 4.3.

A complete list of covariates was established. Variables with missing values greater than 15% were excluded as well as variables which were not consistently populated back in time. Variables populated with the same value across all observations were excluded as well as categorical covariates which had the number of categories exceeding 150. Univariate analysis thoroughly checked the remaining set of potential covariates, each for proportional hazards assumption, population stability, weight of evidence and ability to rank order and differentiate risk.

The univariate analysis results are available in Appendix A. Each covariate was advanced to the multivariate analysis process once it had satisfied the univariate analysis conditions. A multivariate analysis was conducted per model per method. The covariates which satisfied the variance inflation factor analysis, correlation analysis and statistical significance through stepwise regression were used in the modelling process.

All these were event and method specific.

Logistic regression models were built for each event type. Models were each tested for accuracy and ability to rank order and differentiate risk. The results were satisfactory. Logistic default and early settlement models were then compared in terms of performance. Table 5.20 shows a comparison of Logistic models in terms of area under the curves. It was noted that the Logistic regression methodology was better in predicting default than early settlement as the areas under the default model curves are higher than early settlement area under the curves. This is also reflected in the receiver operating characteristic curves where early settlement plots lie closer to the diagonal compared to default curves for the Logistic regression method.

Cause specific Cox regression models were built for each event type and the corresponding marginal cumulative probabilities were determined using the CIC. The code to calculate the CIC was created as it is not present in any of the commercially available statistical applications such as SAS, SPSS and STATA. Models were adjusted to accommodate long term survivors. The performance of the models was compared using overall model Gini statistics and receiver operating characteristic curves. Clearly Cox regression outperforms Logistic regression as evidenced by higher Gini statistics and better receiver operating characteristic curves in both default and early settlement models. This project was typeset in LaTeX and the analysis was conducted in SAS®.

6.3 Conclusion

Both LR and Cox PH models were developed based on statistically sound techniques, supported by literature. In all models, there is strong empirical support for the results as evidenced by the actual versus predicted analyses. All models satisfied conditions laid out as the Models Building Standards by the internal processes of the financial institution involved. The models managed to predict and correctly classify events in the validation set. The models can be used to determine and compare survival prognosis of different risk groups in a consumer credit context. However, LR uses older vintages in model building, therefore it becomes more difficult to capture the most recent activities. The dependant variable in LR is binary and does not consider time.

The use of survival methods to model credit risk data is motivated by the existence of lifetime loans which can be observed from the point of origin to an event of interest. Survival methods thus, estimate not only if, as in Logistic regression, but also when borrowers will default. This enhances flexibility as the model generates probabilities of each event happening at various points in time. For any given observation period, some customers default and some pay-off earlier than the originally agreed term. Where the event occurs before the end of the observation period, the lifetime of such credits are observable. For customers who do not default or pay-off early, before the end of the observation period, it is not possible to observe the time instant when the event occurs. This causes a lack of information due to right censoring.

Censoring allows the response variable to be incompletely determined for some accounts. Unlike in the conventional statistical methodologies, censored accounts are not discarded in survival analysis but contributes information to the study. Censoring is the defining feature of survival analysis, making it distinct from other kinds of analysis. Logistic regression particularly tends to miss censoring information. The response variable is binary and it should be fully observable. Although in terms of predictive performances the models are substantially similar, survival analysis gives more valuable information such as a whole predicted survival function rather than a single predicted survival probability. Survival analysis is superior to Logistic regression in that, a better credit granting decision is made if supported by the estimated survival times.

6.4 Recommendations

At a global level, conclusions drawn from the two methods used in this study are essentially the same. For the default model, Tables 5.5 and 5.14 provide conclusions drawn from Logistic and Cox regression respectively. The same logical trend was reached implying that the methodologies complement each other at a global level. This is further supported by the early settlement models. Tables 5.9 and 5.18 detail Logistic and

Cox regression summaries of the early settlement models respectively showing that the overall conclusion is the same, regardless of the methodology. It is therefore recommend that, if an assessment at a high level is desired, either of the two methodologies can be utilised, depending on the resources available and simplicity of the methodology selected.

The outcome or workout period is product and event specific. In this project, the details of the outcome period are provided in Section 4.4.2 where the product is vehicle finance and events are default and early settlement. It takes 48 months for the bulk of the accounts to be absorbed into the events of interest. This outcome period is then used as the fixed time horizon for which the probabilities are determined. It is interesting to see the shift in model performance if the product changes or events are defined such that the outcome period is shortened to 24, 12 or 6 months. It is recommend to conduct further analysis with varying products, event types and outcome periods.

The occurrence of early settlement and default events impact negatively on profitability as part of the anticipated interest (income) will not be realised. Even though both events are not good for business, early settlement is a better event than default. The lender is likely to suffer more in the case of default than in early settlement as there is a possibility of losing a fraction of capital in addition to the interest amount whereas in early settlement, the lender cannot lose more than the original capital amount issued. There are also penalties charged against the borrower for settling an account early leaving the lender in a better position as compared to further losses which may be incurred by the lender in case of default due to follow up and legal costs. It is recommended that the financial institution be lenient and waiver some charges against the customers settling early so that they can retain them in their customer base for future deals.

There are various reasons for customers to settle early. These include switching to another lender, upgrade to newly released models or settlement due to legal and insurance claims. With reference to the Type 3 tests in Table 5.16, the main driver of early settlement in this model is Equipment Category Code. This classifies new, old and used vehicles. The old vehicles category entails vehicles older than 5 years at the point of application. This category has the highest rate of early settlement. Customers purchasing old vehicles have a higher chance of settling accounts early as they upgrade to new models. Our recommendation to the financial institution involved in this study is to cap the age of pre-owned vehicles to at most 5 years at the point of purchase.

An analysis was conducted to identify the highest combination of categories with early settlement hazard ratio greater than the baseline. These are the customers where the hazards of early settlement happening is greater than those accounts in the baseline and they conversely have a lesser survival prognosis than the customers in the reference group. These customers are living with parents, purchased old vehicles with lower debit interest rate and shorter tenure. This conforms to the results explained in Table 5.18. If a customer is still living with parents it implies that they are younger, energetic, with minimal responsibilities and would still want to start new deals to purchase new vehicles in future. It is recommended that the financial institution to be more lenient in dealing with this group of customers in order to retain and improve market share in the future.

As discussed earlier, the default event is worse than the early settlement event. Table 5.12 shows Type 3 tests of the default model. The main driver of the default event is Debit Interest Rate. Debit Interest rate is usually calculated based on the risk profile of the customer at point of application. If the risk profile tends to high risk, the customer is penalised and allocated a higher interest rate. In practice, debit interest rate is also associated with type of vehicles. The lenders consider the make and model of cars. For high risk vehicles, the debit interest rate is higher. To reach a trade-off between profitability and minimising losses due to default, high risk customers may be advised not to purchase vehicles classified as high risk. By so doing the lender avoids penalising the customer twice and therefore prevents default and optimises on debit interest.

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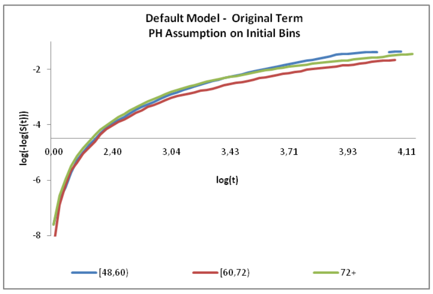
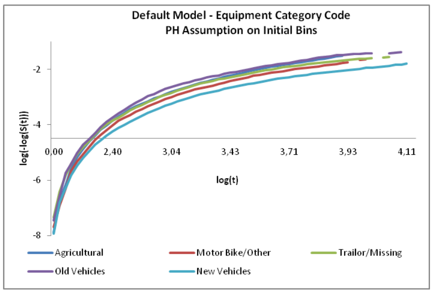
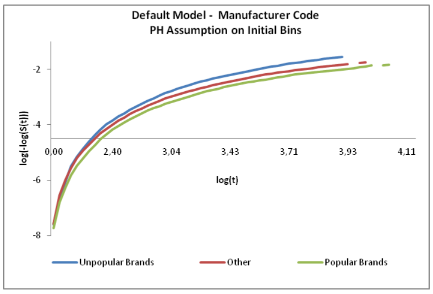
Appendix A

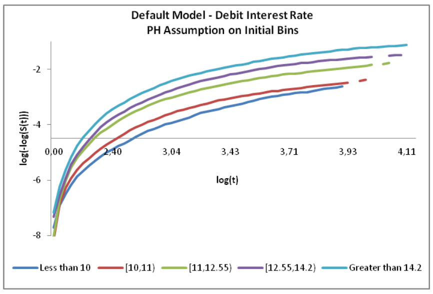
Univariate Assessment

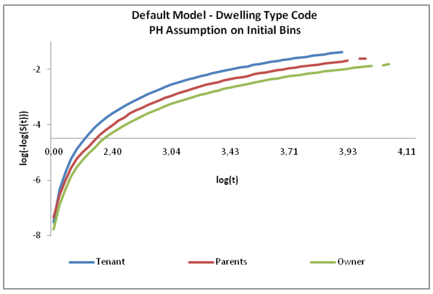
A.1 Default Model - Initial PH Assessment

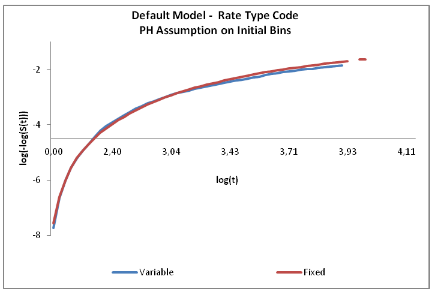












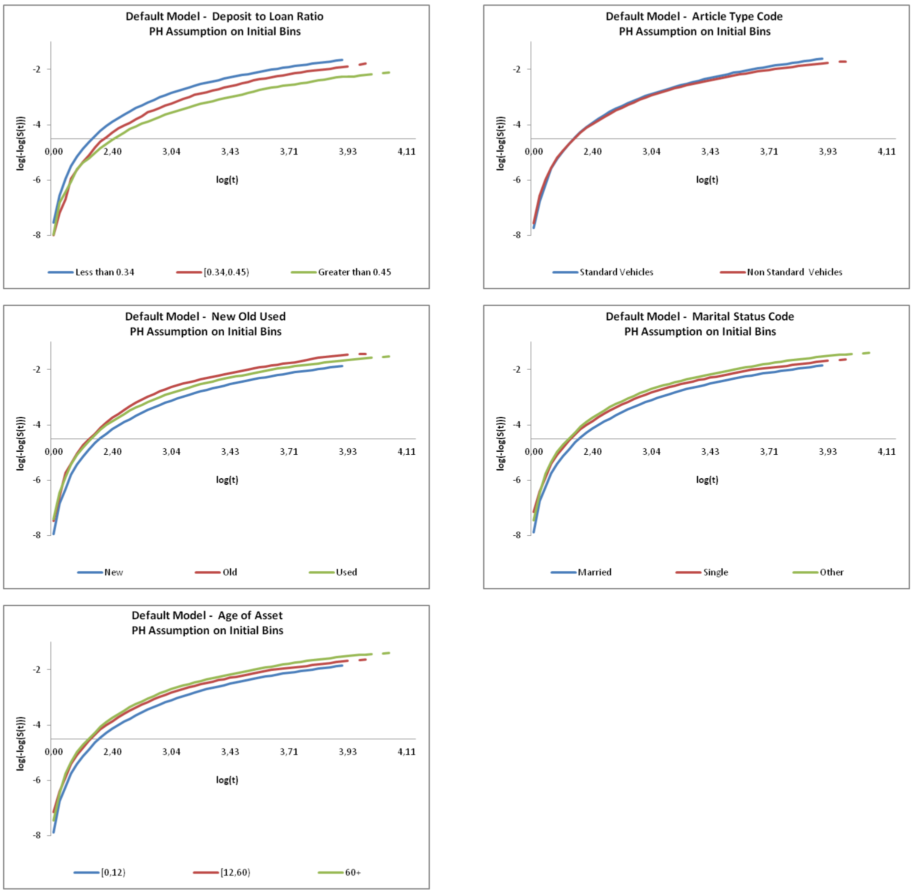
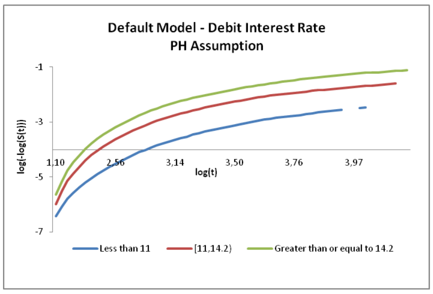
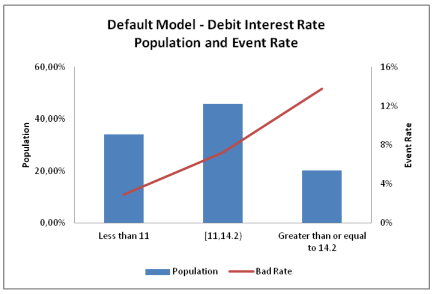
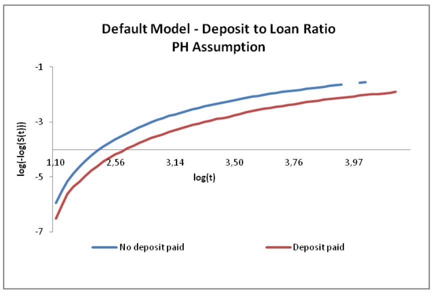


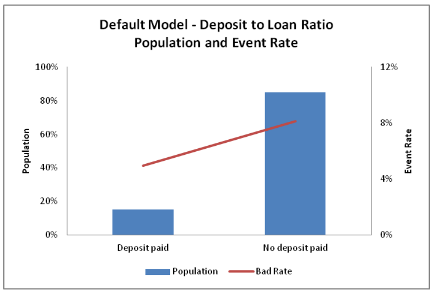
Figure A.1: Initial PH Assessment - Default Model

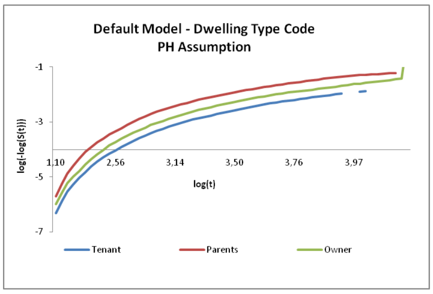
A.2 Default Model - Final PH, Event Rate, Population Stability and WoE Assessment

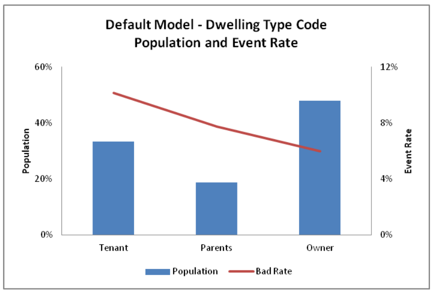

Default Model - Debit Interest Rate PH Assumption


Default Model - Debit Interest Rate Population and Event Rate


Default Model - Deposit to Loan Ratio PH Assumption


Default Model - Deposit to Loan Ratio Population and Event Rate


Default Model - Dwelling Type Code PH Assumption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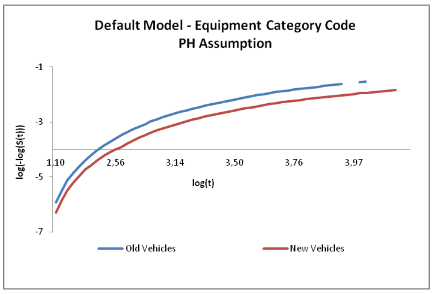
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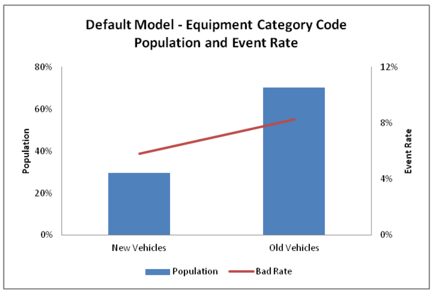
PH Assumption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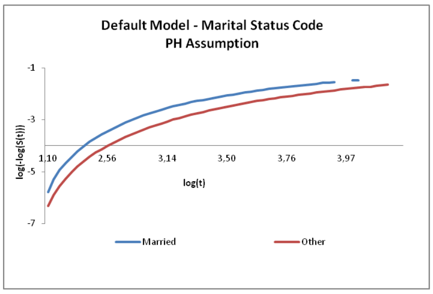
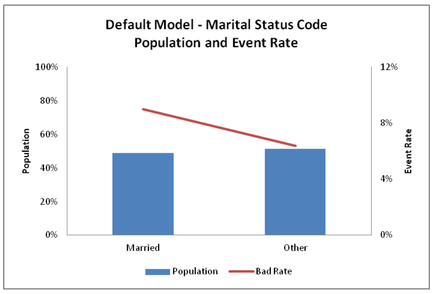
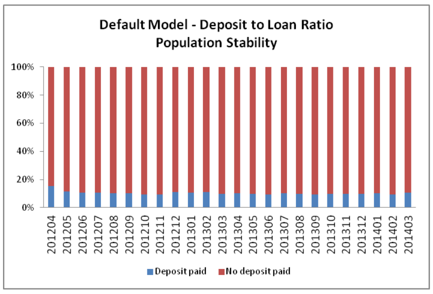
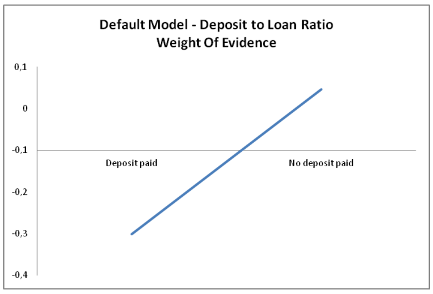
Population and Event Rate


Default Model - Marital Status Code
PH Assumption


Default Model - Marital Status Code
Population and Event Rate


Default Model - Deposit to Loan Ratio
Population Stability


Default Model - Deposit to Loan Ratio
Weight Of Evidence

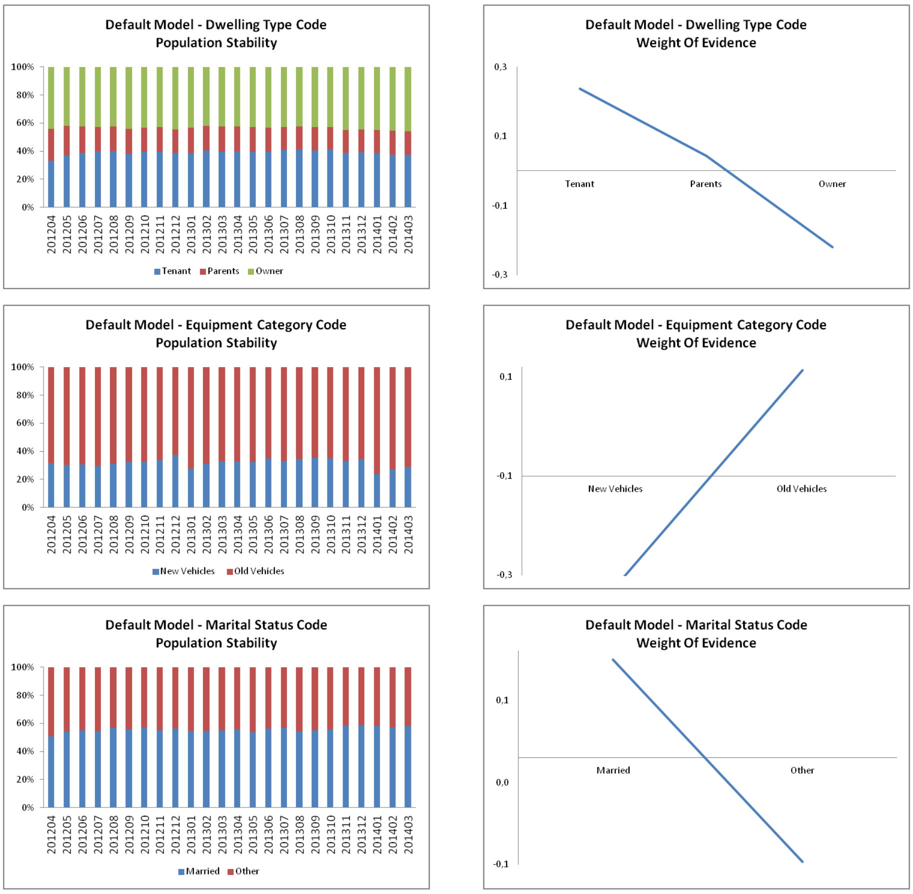
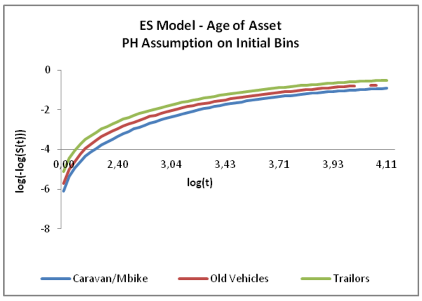
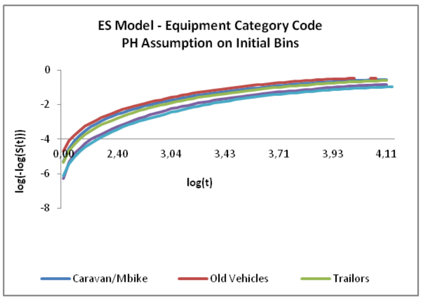
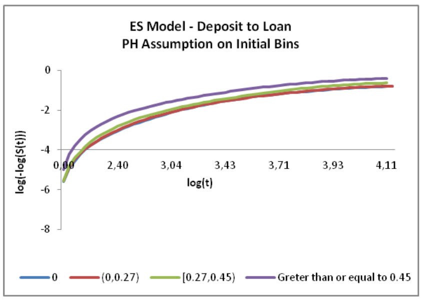


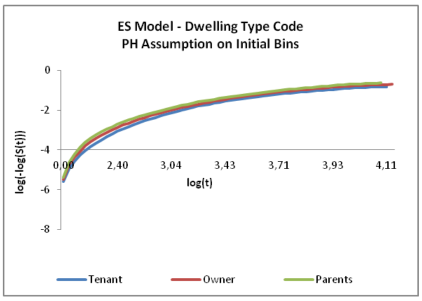
Figure A.2: Univariate Assessment - Default Model

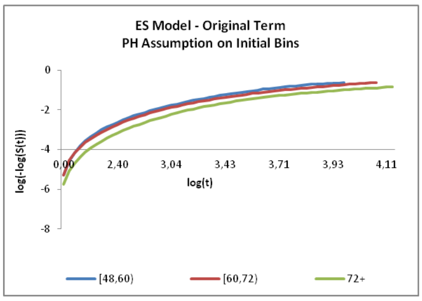
A.3 Early Settlement Model - Initial PH Assessment

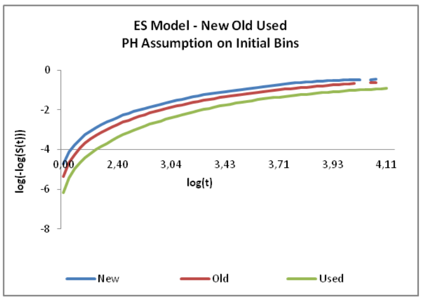












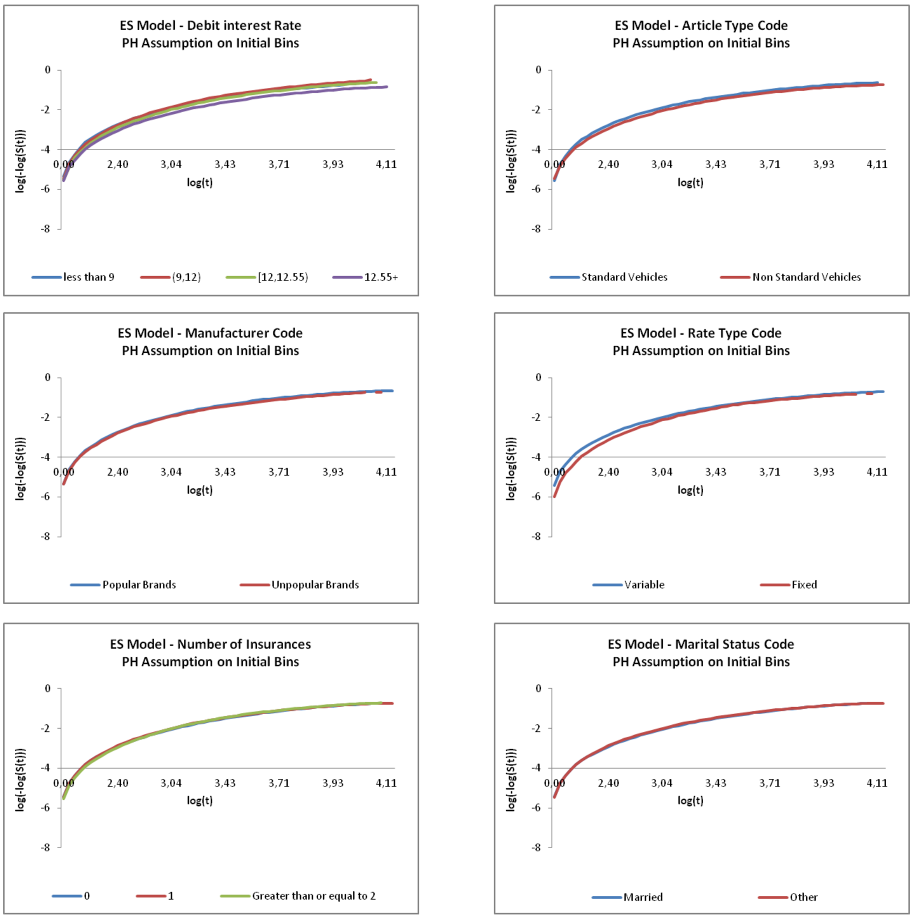
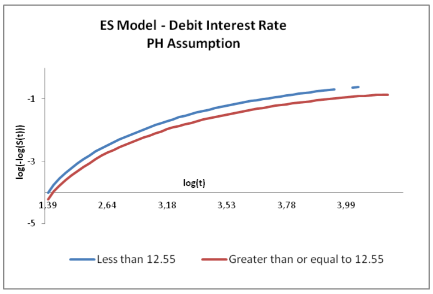
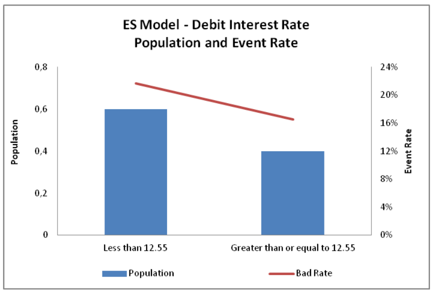
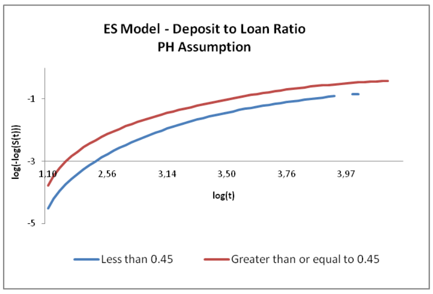


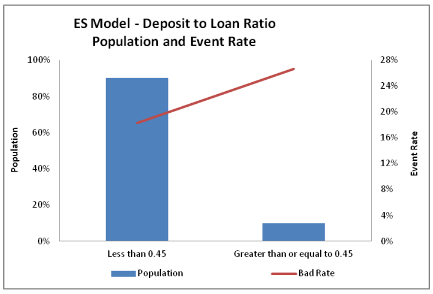
Figure A.3: Initial PH Assessment - ES Model

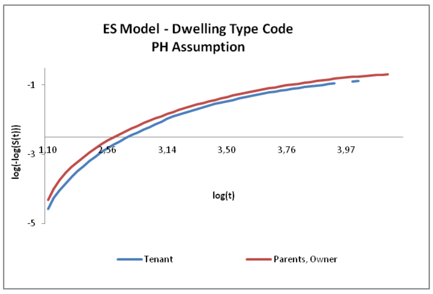
A.4 ES Model - Final PH, Event Rate, Population Stability and WoE Assessment

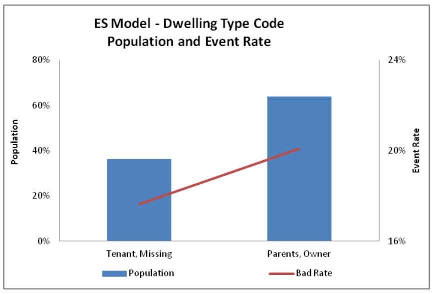


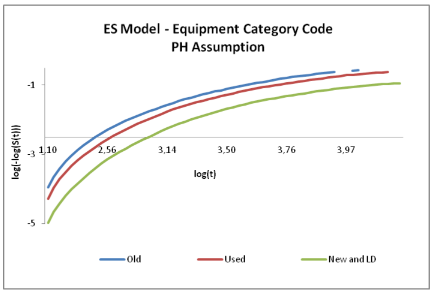


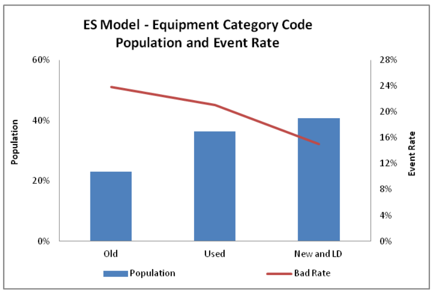


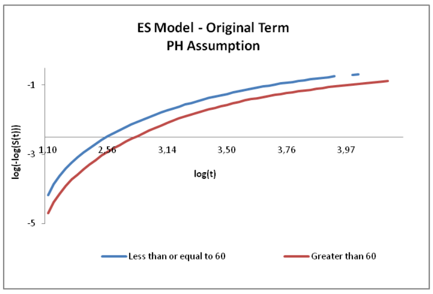


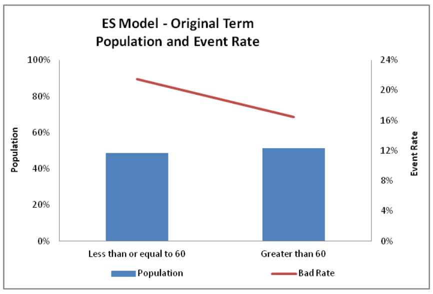


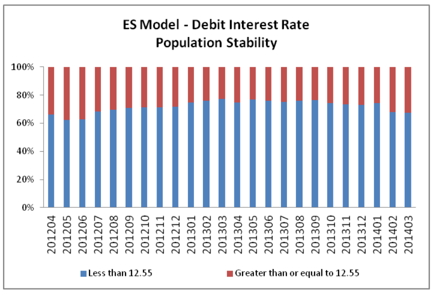


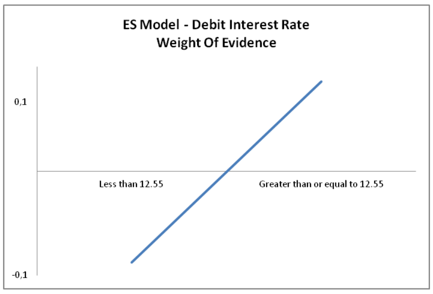

ES Model - Equipment Category Code
PH Assumption
log[-log(S(t))]
-1
-3
-5
1,10
2,56
3,14
3,50
3,76
3,97
log(t)
Old
Used
New and LD


ES Model - Equipment Category Code
Population and Event Rate
Population
60%
40%
20%
0%
Event Rate
28%
24%
20%
16%
12%
8%
4%
0%
Old
Used
New and LD
Population
Bad Rate


ES Model - Original Term
PH Assumption
log[-log(S(t))]
-1
-3
-5
1,10
2,56
3,14
3,50
3,76
3,97
log(t)
Less than or equal to 60
Greater than 60


ES Model - Original Term
Population and Event Rate
Population
100%
80%
60%
40%
20%
0%
Event Rate
24%
20%
16%
12%
8%
4%
0%
Less than or equal to 60
Greater than 60
Population
Bad Rate


ES Model - Debit Interest Rate
Population Stability
100%
80%
60%
40%
20%
0%
201204
201205
201206
201207
201208
201209
201210
201211
201212
201301
201302
201303
201304
201305
201306
201307
201308
201309
201310
201311
201312
201401
201402
201403
Less than 12.55
Greater than or equal to 12.55


ES Model - Debit Interest Rate
Weight Of Evidence
0.1
-0.1
Less than 12.55
Greater than or equal to 12.55

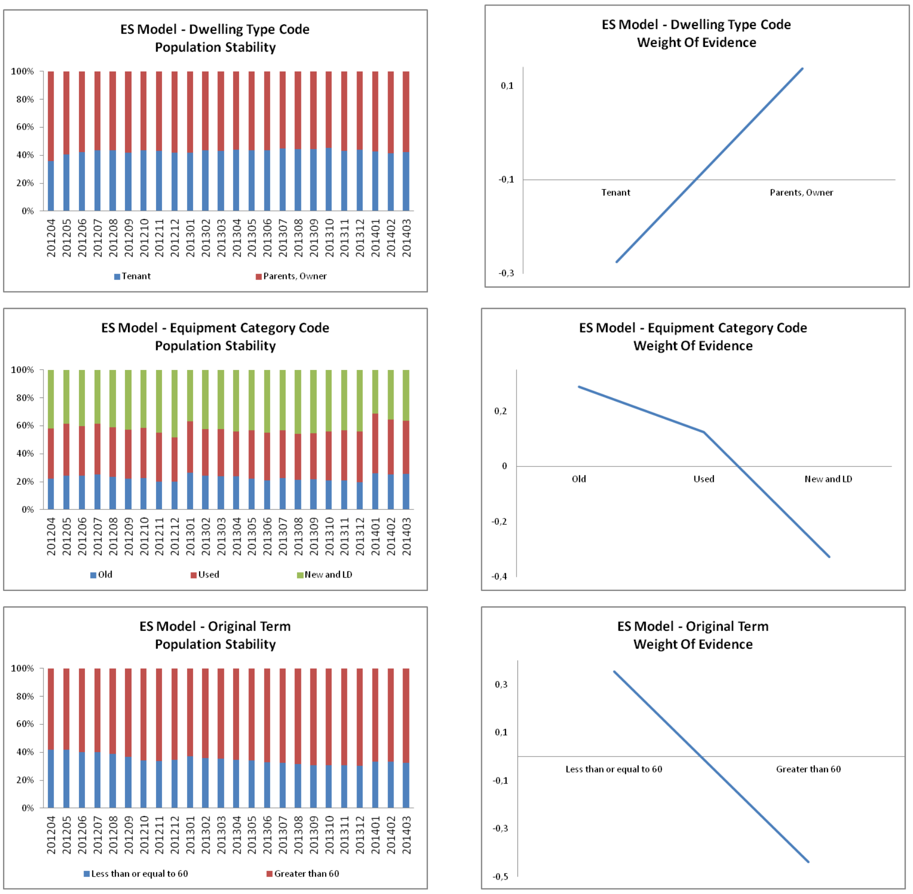


Figure A.4: Univariate Assessment - ES Model

Appendix B

The SAS® Code

B.1 Data Extraction

```
PROC SQL;
CREATE TABLE LGD.DATA_ACCOUNT_MONTHLY AS
SELECT * FROM CONNECTION TO ORACLE
( SELECT
SITE_CODE ,
CLOSED_DATE as  CLOSED ,
to_number(Account_number) as Account_number,
DATA_COMPANY_CODE as COMPANY_CODE  ,
OPEN_DATE as OPEN ,
ACCOUNT_BALANCE ,
ACCUMULATED_INTEREST_AMT ,
OUTSTANDING_CAPITAL_AMT  ,
OUTSTANDING_COMMISSION_AMT ,
SUB_PRODUCT_CODE ,
RESIDUAL_VALUE_AMT  as RESIDUAL_VALUE ,
YEAR_MODEL ,
CUSTOMER_KEY ,
OUTSTANDING_TERM  ,
to_number(to_char(Information_Date,'yyyymm')) as Month,
gross_rate as DEBIT_INTEREST_RATE,
Source_system_id,
DATA_status_code,
deposit_paid,
status_7x_date,
original_capital_amt,
article_type_code,
SUB_COMPANY_CODE,
INSTALMENT_DUE_AMT,
INTEREST_PAYABLE_AMT,
Information_Date,
SUB_REGION_CODE,
SITE.FC_AREA_NAME,
acc.site_code,
         M_M_Code,
CMS_WRITE_OFF_AMT,
RISK_HOLD_CODE,
            SITE_TYPE,
Retail_Price ,
Trade_Price,
New_price,
DORMANT_IND
FROM
DATA.DATA_ACCOUNT_MONTHLY@READONLY.DWH.PROD.IM
order by account_number;
QUIT;
***CHECK FOR DUPLICATE ENTRIES*
proc sort data=LGD.DATA_ACCOUNT_MONTHLY nodupkey dupout=dup1;
by account_number month;
run;
***NOTE: 0 observations with duplicate key values were deleted.
***CHECK FOR MISSING MONTHS IN BETWEEN
PROC FREQ DATA= LGD.DATA_ACCOUNT_MONTHLY;
```

```
TABLE MONTH;
RUN;
***no missing months
```

B.2 Data Preparation

```
/*Retail Indicator*/
data lgd.Retail_Accounts;
length division_code $100. DIVISION $100.;
set data.data_account_monthly(drop=OPEN  CLOSED);
if closed_date ^= . then do;
if account_balance > 0 then
account_balance= 0;
if outstanding_term  ^=0 then
outstanding_term = 0;end;
accumulated_interest_amt = accumulated_interest_amt*-1;
if (information_date ge '31JAN2008:00:00:00'dt
and information_date le '30NOV2008:00:00:00'dt)
or (information_date ge '31JAN2009:00:00:00'dt
and information_date le '31AUG2009:00:00:00'dt)
then do; if SOURCE_SYSTEM_ID='80' then
Exposur_BALANCE=-sum(OUTSTANDING_CAPITAL_AMT,0)
+sum(INSTALMENT_DUE_AMT,0)+sum(INTEREST_PAYABLE_AMT,0);
if SOURCE_SYSTEM_ID='99' then
Exposur_BALANCE=-sum(OUTSTANDING_CAPITAL_AMT,0)
-sum(INSTALMENT_DUE_AMT,0);end;
else do; if SOURCE_SYSTEM_ID='99' then
Exposur_BALANCE=-sum(OUTSTANDING_CAPITAL_AMT,0)
-sum(INSTALMENT_DUE_AMT,0);
else Exposur_BALANCE = account_balance; end;
Exposure=-min(0,Exposur_BALANCE);
balancing_segment= put(COMPANY_CODE,cmpy.);
if COMPANY_CODE=5 then do;
```

```
DIVISION_CODE='5';
if      SUB_COMPANY_CODE=4 then
SUB_DIVISION_CODE='FCR';
else if SUB_COMPANY_CODE=5  then
SUB_DIVISION_CODE='MAF';
else if SUB_COMPANY_CODE=19 then
SUB_DIVISION_CODE='VCF';
else if SUB_COMPANY_CODE=23 then
SUB_DIVISION_CODE='LRF';
else if SUB_COMPANY_CODE=35 then
SUB_DIVISION_CODE='JAG';
else if SUB_COMPANY_CODE=42 then
SUB_DIVISION_CODE='42'; end;
else if COMPANY_CODE=39 then do;
DIVISION_CODE='39';
if      SITE_CODE=5488 then
SUB_DIVISION_CODE='CRD';
else if SITE_CODE=5552 then
SUB_DIVISION_CODE='EPL';
else    SUB_DIVISION_CODE='NWB';
end;
else if COMPANY_CODE>100 then do;
DIVISION_CODE='OTH';
if COMPANY_CODE=144 then
SUB_DIVISION_CODE='PVB';end;
else if
SITE_CODE in ('5352','5351','5596','5498','5307','5571')
then do;
DIVISION_CODE='ENT';
SUB_DIVISION_CODE='ENT';end;
else if
COMPANY_CODE
in (22,28,31,36,37,38,40,41,45,51,52,54,55,70) then do;
```

```
if
REGION in
('AF LARGE','AF MEDIUM','AF OTHER','AF S&M','FLEET')
or substr(REGION,1,3)='CS '
or SUB_REGION_CODE=3 then do;
DIVISION_CODE='ALI';
SUB_DIVISION_CODE=put(COMPANY_CODE,2.);
end; else do;
DIVISION_CODE='ALR';
SUB_DIVISION_CODE
=trim('R')||trim(put(COMPANY_CODE,2.));
end; end;
else if REGION in
('AF LARGE','AF MEDIUM','AF OTHER','AF S&M','FLEET')
or substr(REGION,1,3)='CS '
or SUB_REGION_CODE=3 then do;
DIVISION_CODE='COM';
if      REGION='AF LARGE' then
SUB_DIVISION_CODE='LAR';
else if REGION='AF MEDIUM' then
SUB_DIVISION_CODE='MED';
else if REGION='AF OTHER' then
SUB_DIVISION_CODE='AOT';
else if REGION='AF S&M' then
SUB_DIVISION_CODE='ASM';
else if REGION='FLEET' then
SUB_DIVISION_CODE='FLT';
else if
substr(REGION,1,3)='CS ' or SUB_REGION_CODE=3 then
SUB_DIVISION_CODE='AOT'; end;
else if REGION in
('AF SMALL','BPC','CENTRALISED SALES'
,'DATA FINANCIAL SERVICE','HEAD OFFICE','OTHER',
'Personal Sales
Dealers','Personal Sales Internal','Personal Sales Small',
'Risk Management','', 'SMALL','SMALL-Dormant' ,'Private Bank')
then do;
DIVISION_CODE='RET';
if REGION in ('AF SMALL','SMALL') then
SUB_DIVISION_CODE='SML';
else if REGION ='BPC' then
SUB_DIVISION_CODE='BPC';
else if
 REGION in ('CENTRALISED SALES')
  or substr(REGION,1,3)='CS ' then
SUB_DIVISION_CODE='CEN';
else if REGION ='DATA FINANCIAL SERVICE' then
SUB_DIVISION_CODE='DATA';
else if REGION ='HEAD OFFICE' then
SUB_DIVISION_CODE='HOF';
else if REGION ='OTHER' then
SUB_DIVISION_CODE='OTH';
else if REGION ='Personal Sales Dealers' then
SUB_DIVISION_CODE='PSD';
else if REGION ='Personal Sales Internal' then
SUB_DIVISION_CODE='PSI';
else
if REGION in
('Personal Sales Small','SMALL-Dormant') then
SUB_DIVISION_CODE='PSS';
else if REGION ='Risk Management' then
SUB_DIVISION_CODE='RSM';
else if REGION ='Private Bank' then
SUB_DIVISION_CODE='AFL';
else if REGION ='' then
SUB_DIVISION_CODE='UNK'; end; else
```

```
do; put '*** New value found for Region ***';
put SITE_CODE= REGION=;
abort; end;
select (DIVISION_CODE);
 when('5') do; DIVISION='Ford Credit'; ORDERING=5; end;
when('15')do; DIVISION='Unitrans'; ORDERING=8; end;
when('21')do; DIVISION='MEEG'; ORDERING=7; end;
when('39')do; DIVISION='MAN'; ORDERING=4; end;
when('SEC')
do;DIVISION='Securitisation';ORDERING=6;end;
when('OTH')do;DIVISION='Other';ORDERING=14;end;
when('ALI')do;DIVISION='Alliances';ORDERING=15; end;
when('ALR')do;
DIVISION='Alliances Retail';ORDERING=13;end;
when('COM')do;DIVISION='Commercial';
ORDERING=2;end;
when('COR')do;DIVISION='Corporate';ORDERING=1;end;
when('RET') do;DIVISION='Retail';ORDERING=3;end;
when('ENT')do;DIVISION='Enterprise';ORDERING=9; end;
otherwise;end;
if balancing_segment ^= 'SEC'
then gl_division=balancing_segment;
else if COMPANY_CODE = 58 thenR
gl_division='CAR2';
else if COMPANY_CODE = 95 then gl_division='CAR1';
else gl_division='SEC'; if (balancing_segment in ('DATA')
and compress(division) in ('','Retail')
or  balancing_segment in ('DATA')
and compress(DIVISION_CODE) in ('','ALR')
Or  balancing_segment in ('SEC')
and compress(division) in ('','Retail','Securitisation')
Or  balancing_segment in ('BOT','DATA','VODA'))
and commercial_ind ^= "YES" then
```

```
Retail_Indicator = 1;
```

B.3 Derivation of Critical Fields

```
/*Default flag*/
Data LGD.DEFAULT_ACCOUNTS
(rename =
(DATA_status_code =
DATA_sc_org DATA_status_code1 = DATA_status_code));
set LGD.RETAIL_ACCOUNTS;
if source_system_id = '80' then do;
if status_7x_date ne . then
DATA_status_code1 = 70;
else DATA_status_code1 = DATA_status_code;
if arrear_payments_due_count >= 3 then
DATA_status_code1 = 40; end;
if source_system_id = '99' then do;
if risk_hold_code = 'FWO' then
DATA_status_code1 = 70;
else if site_type in ('071','087') then
DATA_status_code1 = 69;
else if arrear_payments_due_count >= 7 then
DATA_status_code1 = 40;
else DATA_status_code1 = 10; end;
run;

data LGD.BASEDATA;
set LGD.DEFAULT_ACCOUNTS;
if (DATA_status_code >= 40 and DATA_status_code <= 79)
then default = 1; else default = 0; run;
data workingfile;
```

```
set LGD.BASEDATA;
by account_number;
if first.account_number then do;
if close ne . and exposure = 0
and interest_payable_amt = 0
and instalment_due_amt = 0
and residual_value_amt = 0
and close = month
and outstanding_term = 0
then do; closed_ind = 1;
             closed_month = close;end;
else do; closed_ind = 0; closed_month = .; end; end;
retain closed_ind closed_month;
lifetime = -1*((-1*intck('month',input(put(open,best6.)
 ,yymmn6.) ,input(put(closed_month,best6.),yymmn6.)))); run;
proc sort
data=workingfile; by account_number descending month ; run;
data sa.accountage;
set workingfile;
age_of_account =
 -1*((-1*intck('month',input(put(open,best6.),yymmn6.)
,input(put(month,best6.),yymmn6.))));
by account_number;
if first.account_number
 do;
if closed_ind = 1 and
default ne 1 and lifetime lt original_term then do;
es = 1; es_month = closed_month;
end; else do; es = 0; es_month = .; end; end;
retain es es_month;

***Censoring
proc sort data=LGD.BASEDATA; by account_number descending month;run;
```

```
data sa.censored;
     set LGD.BASEDATA;
     by account_number;
     if first.account_number then do;
                if default ne 1 and es ne 1 then do; censor = 1;
                           censor_month = month; end;else do;
                           censor = 0;
                           censor_month = .; end; end;
     retain censor censor_month;
    if def_month ne . then
           event_month = def_month;
     else if es_month ne . then
           event_month = es_month;
     else event_month = censor_month;
survtime=-1*((-1*intck('month',input(put(open,best6.),yymmn6.)
 ,input(put(event_month,best6.),yymmn6.))));
run;
```

B.4 Covariate Selection

```
PROC SQL;
CREATE TABLE SA.STATS_ACCOUNT_MONTHLY AS
SELECT * FROM CONNECTION TO ORACLE
(SELECT
to_number(Account_number) as Account_number ,
to_number(to_char(Information_Date,'yyyymm')) as Month,
RESIDUAL_VALUE_AMT,
ADDITIONAL_CAPITAL_AMT,
OPEN_DATE,
DATA_COMPANY_CODE,
DEPOSIT_PAID,
EXTRAS_FINANCED_AMT,
INSTALMENT_FREQUENCY_CODE,
ORIGINAL_BALANCE,
ORIGINAL_CAPITAL_AMT,
ORIGINAL_INSTALMENT_AMT,
ORIGINAL_INTEREST_AMT,
ORIGINAL_PRIME_RATE,
ORIGINAL_TERM,
YEAR_MODEL,
ACCOUNT_TYPE_CODE,
ARTICLE_TYPE_CODE,
ASSET_LIABILITY_IND,
DATA_STATUS_CODE,
DATA_INSTALMENT_METHOD_CODE,
DATA_RATE_TYPE_CODE,
CREDIT_AGREEMENT_LAW_IND,
DISCOUNTING_CODE,
EQUIPMENT_CATEGORY_CODE,
NEW_OLD_USED,
RE_CREATION_IND,
ACCOUNT_SECURITY,
            ARTICLE_TYPE_CODE,
DATE_OF_BIRTH,
DEPOSIT_PAID,
DWELLING_TYPE_CODE,
EXTRAS_FINANCED_AMT,
INSTALMENT_FREQUENCY_CODE,
MAINTENANCE_CODE,
MANUFACTURER_CODE,
MARITAL_STATUS_CODE,
NET_INCOME,
NUMBER_OF_DEPENDANTS,
NUMBER_OF_INQUIRIES,
NUMBER_OF_INSURANCES,
OCCUPATION_CODE,
PRODUCT_CODE,
RESIDUAL_VALUE,
SA_CITIZEN_IND,
SPOUSE_EMPLOYED_IND,
SPOUSE_INCOME_AMT,
TIME_AT_CURRENT_ADDRESS,
TIME_IN_CURRENT_JOB,
TOTAL_DEFAULTS_EVER_10,
TOTAL_LIVING_EXPENSES
FROM DATA.DATA_ACCOUNT_MONTHLY@READONLY.DWH.PROD.IM )
where
account_number in
(select account_number from SA.BASEDATA4)
order by account_number;
DISCONNECT FROM ORACLE; QUIT;
proc sql;
create table sa.basedata5 as
select a. *,
   b. *,
   c. *
from (select * from sa.base5 as a
left join STATS_ACCOUNT_MONTHLY as b
on a. account_number = b. account_number
left join STATIC_MANUFACTURER_MONTHLY as c
on a. M_M_Code = c. M_M_Code and a. month = c. month;
quit;
*** Derived Covariates
DATA sa.basedata5;
SET sa.basedata5;
     model_month = year_model * 100 + 1;
     Age_of_asset
      = (-1*((-1*intck('month',input(put(model_month,$6.),yymmn6.)
      ,input(put(month,$6.),yymmn6.)))))/12;
     Application_IIR = ORIGINAL_INSTALMENT_AMT/CUSTOMER_INCOME_AMT;
     Current_IIR = INSTALMENT_DUE_AMT/CUSTOMER_INCOME_AMT;
     Deposit_to_Loan = Deposit_Paid/ORIGINAL_CAPITAL_AMT;
     Ballon_to_Loan = RESIDUAL_VALUE_AMT/ORIGINAL_CAPITAL_AMT;
     Age_of_Applicant = (-1*((-1*intck('month',input(put(DOB,$6.),
     yymmn6.)
      ,input(put(month,$6.),yymmn6.)))))/12;
     LTV = Original_Balance/Trade_Price; run;
***Numerical Variable Selection
proc summary data=sa.base3 nway missing; ar _numeric_;
output out=descriptivestats;quit;
data base3;
set sa.base3;drop
residual_value
information_date
status_7x_date
status_70_loss
number_instalment_in_arrear
cms_write_off_amt
ordering
age_of_account
NUMBER_OF_INQUIRIES
NUMBER_OF_DEPENDANTS
TIME_AT_CURRENT_ADDRESS
Application_IIR
Current_IIR
TOTAL_DEFAULTS_EVER_10
Customer_Income_amt; run;
/*1) count number of missing numeric values for each variable*/
data missing_nums;
set base3; format _numeric_ best8.;
      array nums _numeric_;
      keep _numeric_;
      do i = 1 to dim(nums);
           if nums[i] = . then
                nums[i] = 1;
           else nums[i] = 0; end;  drop i; run;
proc summary data= missing_nums nway missing;
     var _numeric_;  output sum=  out=missing_nums; quit;
proc transpose data=missing_nums (drop=_type_ _freq_)
  out=missing_nums name=variable; run;
***create a
variable for the number of missing values greater than threshold*/
proc sql noprint;
     select count(*) into:numvar
           from missing_nums;
     select count(*) into:devnobs
           from lgd.account_monthly_covariates; quit;
data missing_nums (drop=_label_);
     set missing_nums;
     format concat $10240.;
     retain concat '';
     pmissing = col1/&devnobs.;
     if pmissing > 0.15
     and variable not in ("account_number", "month" ,"closed_ind",
"closed_month","default","def_month",
"es", "es_month", "censor", "censor_month",
 "event_month", "survtime") then
           concat = trim(concat)||' '||compress(variable);
     if _n_ = &numvar. then
           call symput("numvar",put(trim(concat),$10240.));run;
***Calculate the number of missing levels for each variable
ods output nlevels=levels;
proc freq data=sa.base3  nlevels;
     tables _char_/ noprint; run; ods output close;
***Create a variable for the number of variables with number
```

```
  of levels greater than threshold and where a variable
   has a single level*/
proc sql noprint;
     select count(*) into:charvar from levels; quit;
data levels; set levels (rename=(tablevar=variable));
     format concat $10240.; retain concat '';
     plevels = nlevels/&devnobs.;
     if nlevels > 150 or nlevels = 1  then
           concat = trim(concat)||' '||compress(variable);
     if _n_ = &charvar. then
           call symput("charvar",put(trim(concat),$10240.)); run;
data base4;
set base3;
drop &numvar. &charvar. run;
```

B.5 Univariate Analysis

```
****Sampling
/*Validation set (20%) = 73452 accounts*/
proc surveyselect
data=base4 out=sa.validation method=srs n=73452 seed=20140922; run;
proc sql;
create table sa.development
as select * from base4 where account_number
 not in (select account_number from sa.validation); quit;
data manufacturer;
set sa.base3;
manufacturer_code = substr(M_M_Code,1,3); run;
proc sql;
create table development as
select a.*,
b. manufacturer_code
from sa.development as a
```

```
left join manufacturer as b
on a. account_number = b. account_number; quit;
data sa.default;
set development;
length _UFormat $200;
drop _UFormat;
_UFormat=’’;
*------------------------------------------------------------*;
* Variable: ARTICLE_TYPE_CODE;
*------------------------------------------------------------*;
LABEL GRP_ARTICLE_TYPE_CODE = ’Grouped: ARTICLE_TYPE_CODE’;
LABEL
WOE_ARTICLE_TYPE_CODE = ’Weight of Evidence: ARTICLE_TYPE_CODE’;
_UFormat = put(ARTICLE_TYPE_CODE,$3.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_ARTICLE_TYPE_CODE = 1;
WOE_ARTICLE_TYPE_CODE = -0.190237734;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq ’CRV’ OR _UFORMAT
eq ’HOR’ OR _UFORMAT eq ’LDV’ OR _UFORMAT eq ’MBK’
 OR _UFORMAT eq ’MIN’ OR _UFORMAT
 eq ’TRK’ OR _UFORMAT eq ’TRL’ OR _UFORMAT eq ’TRS’
) then do;
GRP_ARTICLE_TYPE_CODE = 1;
WOE_ARTICLE_TYPE_CODE = -0.190237734;
end;
else
if (_UFORMAT eq ’BOT’ OR _UFORMAT eq ’MVH’
) then do;
GRP_ARTICLE_TYPE_CODE = 2;
WOE_ARTICLE_TYPE_CODE =  0.070567735;
```

```
end;
else do;
GRP_ARTICLE_TYPE_CODE = 1;
WOE_ARTICLE_TYPE_CODE = -0.190237734;
end;
end;
*------------------------------------------------------------*;
* Variable: Age_of_asset;
*------------------------------------------------------------*;
LABEL GRP_Age_of_asset = 'Grouped: Age_of_asset';
LABEL WOE_Age_of_asset = 'Weight of Evidence: Age_of_asset';
if MISSING(Age_of_asset) then do;
GRP_Age_of_asset = 3;
WOE_Age_of_asset = -0.138195006;
end;
else if NOT MISSING(Age_of_asset) then do;
if Age_of_asset < 2 then do;
GRP_Age_of_asset = 1;
WOE_Age_of_asset = 0.1158359209;
end;
else
if 2 <= Age_of_asset AND Age_of_asset < 3.67 then do;
GRP_Age_of_asset = 2;
WOE_Age_of_asset = -0.095049539;
end;
else
if 3.67 <= Age_of_asset then do;
GRP_Age_of_asset = 3;
WOE_Age_of_asset = -0.138195006;
end;
end;
*------------------------------------------------------------*;
* Variable: DEBIT_INTEREST_RATE;
```

```
*------------------------------------------------------------*;
LABEL GRP_DEBIT_INTEREST_RATE = 'Grouped: DEBIT_INTEREST_RATE';
LABEL WOE_DEBIT_INTEREST_RATE
 = 'Weight of Evidence: DEBIT_INTEREST_RATE';
if MISSING(DEBIT_INTEREST_RATE) then do;
GRP_DEBIT_INTEREST_RATE = 3;
WOE_DEBIT_INTEREST_RATE = 0.2476004225;
end;
else if NOT MISSING(DEBIT_INTEREST_RATE) then do;
if DEBIT_INTEREST_RATE < 10 then do;
GRP_DEBIT_INTEREST_RATE = 1;
WOE_DEBIT_INTEREST_RATE =  1.316778244;
end;
else
if 10 <= DEBIT_INTEREST_RATE AND DEBIT_INTEREST_RATE < 11 then do;
GRP_DEBIT_INTEREST_RATE = 2;
WOE_DEBIT_INTEREST_RATE = 0.8624454296;
end;
else
if 11 <= DEBIT_INTEREST_RATE AND
DEBIT_INTEREST_RATE < 12.55 then do;
GRP_DEBIT_INTEREST_RATE = 3;
WOE_DEBIT_INTEREST_RATE = 0.2476004225;
end;
else
if 12.55 <= DEBIT_INTEREST_RATE
AND DEBIT_INTEREST_RATE < 14.2 then do;
GRP_DEBIT_INTEREST_RATE = 4;
WOE_DEBIT_INTEREST_RATE = -0.298184443;
end;
else
if 14.2 <= DEBIT_INTEREST_RATE then do;
GRP_DEBIT_INTEREST_RATE = 5;
```

```
WOE_DEBIT_INTEREST_RATE = -0.814975572;
end;
end;
*------------------------------------------------------------*;
* Variable: DEPOSIT_PAID;
*------------------------------------------------------------*;
LABEL GRP_DEPOSIT_PAID = ’Grouped: DEPOSIT_PAID’;
LABEL WOE_DEPOSIT_PAID = ’Weight of Evidence: DEPOSIT_PAID’;
if MISSING(DEPOSIT_PAID) then do;
GRP_DEPOSIT_PAID = 1;
WOE_DEPOSIT_PAID =  0.064037842;
end;
else if NOT MISSING(DEPOSIT_PAID) then do;
if 0 <= DEPOSIT_PAID AND DEPOSIT_PAID < 9000 then do;
GRP_DEPOSIT_PAID = 1;
WOE_DEPOSIT_PAID =  0.064037842;
end;
else
if 9000 <= DEPOSIT_PAID AND DEPOSIT_PAID < 20000 then do;
GRP_DEPOSIT_PAID = 2;
WOE_DEPOSIT_PAID = -0.337951621;
end;
else
if 20000 <= DEPOSIT_PAID AND DEPOSIT_PAID < 30000 then do;
GRP_DEPOSIT_PAID = 3;
WOE_DEPOSIT_PAID = -0.189797344;
end;
else
if 30000 <= DEPOSIT_PAID AND DEPOSIT_PAID < 68000 then do;
GRP_DEPOSIT_PAID = 4;
WOE_DEPOSIT_PAID = -0.015375292;
end;
else
```

```
if 68000 <= DEPOSIT_PAID then do;
GRP_DEPOSIT_PAID = 5;
WOE_DEPOSIT_PAID = 0.5235175903;
end;
end;
*------------------------------------------------------------*;
* Variable: Deposit_to_Loan;
*------------------------------------------------------------*;
LABEL GRP_Deposit_to_Loan = 'Grouped: Deposit_to_Loan';
LABEL WOE_Deposit_to_Loan = 'Weight of Evidence: Deposit_to_Loan';
if MISSING(Deposit_to_Loan) then do;
GRP_Deposit_to_Loan = 3;
WOE_Deposit_to_Loan = 0.4668041512;
end;
else if NOT MISSING(Deposit_to_Loan) then do;
if Deposit_to_Loan < 0.34 then do;
GRP_Deposit_to_Loan = 1;
WOE_Deposit_to_Loan = -0.045864396;
end;
else
if 0.34 <= Deposit_to_Loan AND Deposit_to_Loan < 0.45 then do;
GRP_Deposit_to_Loan = 2;
WOE_Deposit_to_Loan = 0.0348316203;
end;
else
if 0.45 <= Deposit_to_Loan then do;
GRP_Deposit_to_Loan = 3;
WOE_Deposit_to_Loan = 0.4668041512;
end;
end;
*------------------------------------------------------------*;
* Variable: Exposure;
*------------------------------------------------------------*;
```

```
LABEL GRP_Exposure = 'Grouped: Exposure';
LABEL WOE_Exposure = 'Weight of Evidence: Exposure';
if MISSING(Exposure) then do;
GRP_Exposure = 4;
WOE_Exposure = 0.1346572287;
end;
else if NOT MISSING(Exposure) then do;
if Exposure < 98140.01 then do;
GRP_Exposure = 1;
WOE_Exposure = -0.229444959;
end;
else
if 98140.01 <= Exposure AND Exposure < 133558.66 then do;
GRP_Exposure = 2;
WOE_Exposure = -0.079484352;
end;
else
if 133558.66 <= Exposure AND Exposure < 152482.08 then do;
GRP_Exposure = 3;
WOE_Exposure = 0.0358432893;
end;
else
if 152482.08 <= Exposure then do;
GRP_Exposure = 4;
WOE_Exposure = 0.1346572287;
end;
end;
*------------------------------------------------------------*;
* Variable: MARITAL_STATUS_CODE;
*------------------------------------------------------------*;
LABEL GRP_MARITAL_STATUS_CODE = 'Grouped: MARITAL_STATUS_CODE';
LABEL WOE_MARITAL_STATUS_CODE =
'Weight of Evidence: MARITAL_STATUS_CODE';
```

```
if MISSING(MARITAL_STATUS_CODE) then do;
GRP_MARITAL_STATUS_CODE = 1;
WOE_MARITAL_STATUS_CODE = -0.119971765;
end;
else if NOT MISSING(MARITAL_STATUS_CODE) then do;
if 1 <= MARITAL_STATUS_CODE AND MARITAL_STATUS_CODE < 3 then do;
GRP_MARITAL_STATUS_CODE = 1;
WOE_MARITAL_STATUS_CODE = -0.119971765;
end;
else
if 3 <= MARITAL_STATUS_CODE then do;
GRP_MARITAL_STATUS_CODE = 2;
WOE_MARITAL_STATUS_CODE = 0.1269054955;
end;
end;
*------------------------------------------------------------*;
* Variable: NUMBER_OF_INSURANCES;
*------------------------------------------------------------*;
LABEL GRP_NUMBER_OF_INSURANCES = 'Grouped: NUMBER_OF_INSURANCES';
LABEL WOE_NUMBER_OF_INSURANCES =
'Weight of Evidence: NUMBER_OF_INSURANCES';
if MISSING(NUMBER_OF_INSURANCES) then do;
GRP_NUMBER_OF_INSURANCES = 2;
WOE_NUMBER_OF_INSURANCES =  -0.27230291;
end;
else if NOT MISSING(NUMBER_OF_INSURANCES) then do;
if 0 <= NUMBER_OF_INSURANCES AND NUMBER_OF_INSURANCES < 3 then do;
GRP_NUMBER_OF_INSURANCES = 1;
WOE_NUMBER_OF_INSURANCES = 0.0222955252;
end;
else
if 3 <= NUMBER_OF_INSURANCES then do;
GRP_NUMBER_OF_INSURANCES = 2;
```

```
WOE_NUMBER_OF_INSURANCES =  -0.27230291;
end;
end;
*------------------------------------------------------------*;
* Variable: ORIGINAL_BALANCE;
*------------------------------------------------------------*;
LABEL GRP_ORIGINAL_BALANCE = 'Grouped: ORIGINAL_BALANCE';
LABEL WOE_ORIGINAL_BALANCE = 'Weight of Evidence: ORIGINAL_BALANCE';
if MISSING(ORIGINAL_BALANCE) then do;
GRP_ORIGINAL_BALANCE = 2;
WOE_ORIGINAL_BALANCE = -0.052907499;
end;
else if NOT MISSING(ORIGINAL_BALANCE) then do;
if ORIGINAL_BALANCE < 122358.98 then do;
GRP_ORIGINAL_BALANCE = 1;
WOE_ORIGINAL_BALANCE = -0.171879447;
end;
else
if 122358.98
<= ORIGINAL_BALANCE AND ORIGINAL_BALANCE < 247063.33 then do;
GRP_ORIGINAL_BALANCE = 2;
WOE_ORIGINAL_BALANCE = -0.052907499;
end;
else
if 247063.33 <=
ORIGINAL_BALANCE AND ORIGINAL_BALANCE < 293882.74 then do;
GRP_ORIGINAL_BALANCE = 3;
WOE_ORIGINAL_BALANCE = 0.0708255812;
end;
else
if 293882.74 <=
ORIGINAL_BALANCE AND ORIGINAL_BALANCE < 575945.07 then do;
GRP_ORIGINAL_BALANCE = 4;
```

```
WOE_ORIGINAL_BALANCE = 0.2070221793;
end;
else
if 575945.07 <= ORIGINAL_BALANCE then do;
GRP_ORIGINAL_BALANCE = 5;
WOE_ORIGINAL_BALANCE = -0.041999079;
end;
end;
*------------------------------------------------------------*;
* Variable: ORIGINAL_TERM;
*------------------------------------------------------------*;
LABEL GRP_ORIGINAL_TERM = ’Grouped: ORIGINAL_TERM’;
LABEL WOE_ORIGINAL_TERM = ’Weight of Evidence: ORIGINAL_TERM’;
if MISSING(ORIGINAL_TERM) then do;
GRP_ORIGINAL_TERM = 4;
WOE_ORIGINAL_TERM =  0.113416106;
end;
else if NOT MISSING(ORIGINAL_TERM) then do;
if 48 <= ORIGINAL_TERM AND ORIGINAL_TERM < 54 then do;
GRP_ORIGINAL_TERM = 1;
WOE_ORIGINAL_TERM = -0.477796284;
end;
else
if 54 <= ORIGINAL_TERM AND ORIGINAL_TERM < 60 then do;
GRP_ORIGINAL_TERM = 2;
WOE_ORIGINAL_TERM = -0.104678453;
end;
else
if 60 <= ORIGINAL_TERM AND ORIGINAL_TERM < 72 then do;
GRP_ORIGINAL_TERM = 3;
WOE_ORIGINAL_TERM = -0.033932584;
end;
else
```

```
if 72 <= ORIGINAL_TERM then do;
GRP_ORIGINAL_TERM = 4;
WOE_ORIGINAL_TERM =  0.113416106;
end;
end;
*------------------------------------------------------------*;
* Variable: DATA_RATE_TYPE_CODE;
*------------------------------------------------------------*;
LABEL GRP_DATA_RATE_TYPE_CODE = 'Grouped: DATA_RATE_TYPE_CODE';
LABEL WOE_DATA_RATE_TYPE_CODE =
'Weight of Evidence: DATA_RATE_TYPE_CODE';
_UFormat = put(DATA_RATE_TYPE_CODE,$1.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_DATA_RATE_TYPE_CODE = 2;
WOE_DATA_RATE_TYPE_CODE = -0.078813438;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq 'B' OR _UFORMAT eq 'G'
) then do;
GRP_DATA_RATE_TYPE_CODE = 1;
WOE_DATA_RATE_TYPE_CODE = 0.3833158125;
end;
else
if (_UFORMAT eq 'O'
) then do;
GRP_DATA_RATE_TYPE_CODE = 2;
WOE_DATA_RATE_TYPE_CODE = -0.078813438;
end;
else do;
GRP_DATA_RATE_TYPE_CODE = 2;
WOE_DATA_RATE_TYPE_CODE = -0.078813438;
end;
```

```
end;
*------------------------------------------------------------*;
* Variable: DWELLING_TYPE_CODE;
*------------------------------------------------------------*;
LABEL GRP_DWELLING_TYPE_CODE = 'Grouped: DWELLING_TYPE_CODE';
LABEL WOE_DWELLING_TYPE_CODE =
'Weight of Evidence: DWELLING_TYPE_CODE';
_UFormat = put(DWELLING_TYPE_CODE,$1.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_DWELLING_TYPE_CODE = 3;
WOE_DWELLING_TYPE_CODE = 0.2198800801;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq 'T'
) then do;
GRP_DWELLING_TYPE_CODE = 1;
WOE_DWELLING_TYPE_CODE = -0.236749091;
end;
else
if (_UFORMAT eq 'B' OR _UFORMAT eq 'P'
) then do;
GRP_DWELLING_TYPE_CODE = 2;
WOE_DWELLING_TYPE_CODE =  -0.04388855;
end;
else
if (_UFORMAT eq 'O'
) then do;
GRP_DWELLING_TYPE_CODE = 3;
WOE_DWELLING_TYPE_CODE = 0.2198800801;
end;
else do;
GRP_DWELLING_TYPE_CODE = 3;
```

```
WOE_DWELLING_TYPE_CODE = 0.2198800801;
end;
end;
*------------------------------------------------------------*;
* Variable: EQUIPMENT_CATEGORY_CODE;
*------------------------------------------------------------*;
LABEL GRP_EQUIPMENT_CATEGORY_CODE =
'Grouped: EQUIPMENT_CATEGORY_CODE';
LABEL WOE_EQUIPMENT_CATEGORY_CODE =
 'Weight of Evidence: EQUIPMENT_CATEGORY_CODE';
_UFormat = put(EQUIPMENT_CATEGORY_CODE,$6.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 3;
WOE_EQUIPMENT_CATEGORY_CODE = -0.071746202;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq 'AGRICU'
OR _UFORMAT eq 'CARAVN' OR _UFORMAT eq 'LDV<5Y'
 OR _UFORMAT eq 'OTHER') then do;
GRP_EQUIPMENT_CATEGORY_CODE = 1;
WOE_EQUIPMENT_CATEGORY_CODE =  -0.29965709;
end;
else
if (_UFORMAT eq 'LDVNEW' OR _UFORMAT eq 'MTBIKE'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 2;
WOE_EQUIPMENT_CATEGORY_CODE = -0.099150242;
end;
else
if (_UFORMAT eq 'MVH<5Y' OR _UFORMAT eq 'TRAILR'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 3;
```

```
WOE_EQUIPMENT_CATEGORY_CODE = -0.071746202;
end;
else
if (_UFORMAT eq 'LDV>5Y' OR _UFORMAT eq 'MVH>5Y'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 4;
WOE_EQUIPMENT_CATEGORY_CODE = -0.025044586;
end;
else
if (_UFORMAT eq 'BOATS' OR _UFORMAT eq 'MVHNEW'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 5;
WOE_EQUIPMENT_CATEGORY_CODE = 0.3239856317;
end;
else do;
GRP_EQUIPMENT_CATEGORY_CODE = 3;
WOE_EQUIPMENT_CATEGORY_CODE = -0.071746202;
end;
end;
*------------------------------------------------------------*;
* Variable: NEW_OLD_USED;
*------------------------------------------------------------*;
LABEL GRP_NEW_OLD_USED = 'Grouped: NEW_OLD_USED';
LABEL WOE_NEW_OLD_USED = 'Weight of Evidence: NEW_OLD_USED';
_UFormat = put(NEW_OLD_USED,$4.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_NEW_OLD_USED = 3;
WOE_NEW_OLD_USED = -0.097270071;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq 'NEW'
) then do;
```

```
GRP_NEW_OLD_USED = 1;
WOE_NEW_OLD_USED = 0.1339093009;
end;
else
if (_UFORMAT eq ’OLD’
) then do;
GRP_NEW_OLD_USED = 2;
WOE_NEW_OLD_USED = -0.021696506;
end;
else
if (_UFORMAT eq ’DEMO’ OR _UFORMAT eq ’USED’
) then do;
GRP_NEW_OLD_USED = 3;
WOE_NEW_OLD_USED = -0.097270071;
end;
else do;
GRP_NEW_OLD_USED = 3;
WOE_NEW_OLD_USED = -0.097270071;
end;
end;
*------------------------------------------------------------*;
* Special Code Values
*------------------------------------------------------------*;
data sa.es;
set development;
length _UFormat $200;
drop _UFormat;
_UFormat=’’;
*------------------------------------------------------------*;
* Variable: ARTICLE_TYPE_CODE;
*------------------------------------------------------------*;
LABEL GRP_ARTICLE_TYPE_CODE = ’Grouped: ARTICLE_TYPE_CODE’;
LABEL WOE_ARTICLE_TYPE_CODE =
'Weight of Evidence: ARTICLE_TYPE_CODE';
_UFormat = put(ARTICLE_TYPE_CODE,$3.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_ARTICLE_TYPE_CODE = 2;
WOE_ARTICLE_TYPE_CODE = 0.0600793848;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq 'BOT' OR _UFORMAT
eq 'CRV' OR _UFORMAT eq 'HOR' OR _UFORMAT eq 'LDV'
OR _UFORMAT eq 'MBK'
) then do;
GRP_ARTICLE_TYPE_CODE = 1;
WOE_ARTICLE_TYPE_CODE =  -0.19799383;
end;
else
if (_UFORMAT eq 'MIN'
OR _UFORMAT eq 'MVH' OR _UFORMAT
eq 'TRK' OR _UFORMAT eq 'TRL' OR _UFORMAT eq 'TRS'
) then do;
GRP_ARTICLE_TYPE_CODE = 2;
WOE_ARTICLE_TYPE_CODE = 0.0600793848;
end;
else do;
GRP_ARTICLE_TYPE_CODE = 2;
WOE_ARTICLE_TYPE_CODE = 0.0600793848;
end;
end;
*------------------------------------------------------------*;
* Variable: Age_of_asset;
*------------------------------------------------------------*;
LABEL GRP_Age_of_asset = 'Grouped: Age_of_asset';
LABEL WOE_Age_of_asset = 'Weight of Evidence: Age_of_asset';
```

```
if MISSING(Age_of_asset) then do;
GRP_Age_of_asset = 3;
WOE_Age_of_asset = -0.240770078;
end;
else if NOT MISSING(Age_of_asset) then do;
if Age_of_asset < 0.92 then do;
GRP_Age_of_asset = 1;
WOE_Age_of_asset = 0.2809112341;
end;
else
if 1 <= Age_of_asset AND Age_of_asset < 2 then do;
GRP_Age_of_asset = 2;
WOE_Age_of_asset =  0.066325612;
end;
else
if 2 <= Age_of_asset then do;
GRP_Age_of_asset = 3;
WOE_Age_of_asset = -0.240770078;
end;
end;
*------------------------------------------------------------*;
* Variable: Ballon_to_Loan;
*------------------------------------------------------------*;
LABEL GRP_Ballon_to_Loan = 'Grouped: Ballon_to_Loan';
LABEL WOE_Ballon_to_Loan = 'Weight of Evidence: Ballon_to_Loan';
if MISSING(Ballon_to_Loan) then do;
GRP_Ballon_to_Loan = 2;
WOE_Ballon_to_Loan = 8.0384835102;
end;
else if NOT MISSING(Ballon_to_Loan) then do;
if Ballon_to_Loan < 0.19 then do;
GRP_Ballon_to_Loan = 1;
WOE_Ballon_to_Loan = -0.199895453;
```

```
end;
else
if 0.19 <= Ballon_to_Loan then do;
GRP_Ballon_to_Loan = 2;
WOE_Ballon_to_Loan = 8.0384835102;
end;
end;
*------------------------------------------------------------*;
* Variable: DEBIT_INTEREST_RATE;
*------------------------------------------------------------*;
LABEL GRP_DEBIT_INTEREST_RATE = 'Grouped: DEBIT_INTEREST_RATE';
LABEL WOE_DEBIT_INTEREST_RATE =
'Weight of Evidence: DEBIT_INTEREST_RATE';
if MISSING(DEBIT_INTEREST_RATE) then do;
GRP_DEBIT_INTEREST_RATE = 4;
WOE_DEBIT_INTEREST_RATE = -0.061690313;
end;
else if NOT MISSING(DEBIT_INTEREST_RATE) then do;
if DEBIT_INTEREST_RATE < 9 then do;
GRP_DEBIT_INTEREST_RATE = 1;
WOE_DEBIT_INTEREST_RATE = 0.2977797976;
end;
else
if 9 <= DEBIT_INTEREST_RATE AND DEBIT_INTEREST_RATE < 11.2 then do;
GRP_DEBIT_INTEREST_RATE = 2;
WOE_DEBIT_INTEREST_RATE = -0.009312748;
end;
else
if 11.2 <=
DEBIT_INTEREST_RATE AND DEBIT_INTEREST_RATE < 12.55 then do;
GRP_DEBIT_INTEREST_RATE = 3;
WOE_DEBIT_INTEREST_RATE = -0.028272011;
end;
```

```
else
if 12.55 <= DEBIT_INTEREST_RATE then do;
GRP_DEBIT_INTEREST_RATE = 4;
WOE_DEBIT_INTEREST_RATE = -0.061690313;
end;
end;
*------------------------------------------------------------*;
* Variable: DEPOSIT_PAID;
*------------------------------------------------------------*;
LABEL GRP_DEPOSIT_PAID = 'Grouped: DEPOSIT_PAID';
LABEL WOE_DEPOSIT_PAID = 'Weight of Evidence: DEPOSIT_PAID';
if MISSING(DEPOSIT_PAID) then do;
GRP_DEPOSIT_PAID = 1;
WOE_DEPOSIT_PAID = 0.2339184311;
end;
else if NOT MISSING(DEPOSIT_PAID) then do;
if 0 <= DEPOSIT_PAID AND DEPOSIT_PAID < 9000 then do;
GRP_DEPOSIT_PAID = 1;
WOE_DEPOSIT_PAID = 0.2339184311;
end;
else
if 9000 <= DEPOSIT_PAID AND DEPOSIT_PAID < 10237.41 then do;
GRP_DEPOSIT_PAID = 2;
WOE_DEPOSIT_PAID = 0.0552648552;
end;
else
if 10237.41 <= DEPOSIT_PAID AND DEPOSIT_PAID < 25000 then do;
GRP_DEPOSIT_PAID = 3;
WOE_DEPOSIT_PAID = -0.126963253;
end;
else
if 25000 <= DEPOSIT_PAID AND DEPOSIT_PAID < 68000 then do;
GRP_DEPOSIT_PAID = 4;
```

```
WOE_DEPOSIT_PAID = -0.218446239;
end;
else
if 68000 <= DEPOSIT_PAID then do;
GRP_DEPOSIT_PAID = 5;
WOE_DEPOSIT_PAID = -0.383823001;
end;
end;
*------------------------------------------------------------*;
* Variable: Deposit_to_Loan;
*------------------------------------------------------------*;
LABEL GRP_Deposit_to_Loan = 'Grouped: Deposit_to_Loan';
LABEL WOE_Deposit_to_Loan = 'Weight of Evidence: Deposit_to_Loan';
if MISSING(Deposit_to_Loan) then do;
GRP_Deposit_to_Loan = 4;
WOE_Deposit_to_Loan = -0.662801391;
end;
else if NOT MISSING(Deposit_to_Loan) then do;
if Deposit_to_Loan < 0.1 then do;
GRP_Deposit_to_Loan = 1;
WOE_Deposit_to_Loan = 0.2645913378;
end;
else
if 0.1 <= Deposit_to_Loan AND Deposit_to_Loan < 0.27 then do;
GRP_Deposit_to_Loan = 2;
WOE_Deposit_to_Loan = -0.088808411;
end;
else
if 0.27 <= Deposit_to_Loan AND Deposit_to_Loan < 0.45 then do;
GRP_Deposit_to_Loan = 3;
WOE_Deposit_to_Loan = -0.291998735;
end;
else
```

```
if 0.45 <= Deposit_to_Loan then do;
GRP_Deposit_to_Loan = 4;
WOE_Deposit_to_Loan = -0.662801391;
end;
end;
*------------------------------------------------------------*;
* Variable: Exposure;
*------------------------------------------------------------*;
LABEL GRP_Exposure = 'Grouped: Exposure';
LABEL WOE_Exposure = 'Weight of Evidence: Exposure';
if MISSING(Exposure) then do;
GRP_Exposure = 4;
WOE_Exposure = 0.1257168092;
end;
else if NOT MISSING(Exposure) then do;
if Exposure < 59896.02 then do;
GRP_Exposure = 1;
WOE_Exposure = -0.771138811;
end;
else
if 59896.02 <= Exposure AND Exposure < 87430 then do;
GRP_Exposure = 2;
WOE_Exposure =  -0.41018573;
end;
else
if 87430 <= Exposure AND Exposure < 107239 then do;
GRP_Exposure = 3;
WOE_Exposure = -0.144308354;
end;
else
if 107239 <= Exposure AND Exposure < 302979.99 then do;
GRP_Exposure = 4;
WOE_Exposure = 0.1257168092;
```

```
end;
else
if 302979.99 <= Exposure then do;
GRP_Exposure = 5;
WOE_Exposure = 0.2749438614;
end;
end;
*------------------------------------------------------------*;
* Variable: MARITAL_STATUS_CODE;
*------------------------------------------------------------*;
LABEL GRP_MARITAL_STATUS_CODE = 'Grouped: MARITAL_STATUS_CODE';
LABEL WOE_MARITAL_STATUS_CODE =
'Weight of Evidence: MARITAL_STATUS_CODE';
if MISSING(MARITAL_STATUS_CODE) then do;
GRP_MARITAL_STATUS_CODE = 2;
WOE_MARITAL_STATUS_CODE = -0.194096763;
end;
else if NOT MISSING(MARITAL_STATUS_CODE) then do;
if 1 <= MARITAL_STATUS_CODE AND MARITAL_STATUS_CODE < 6 then do;
GRP_MARITAL_STATUS_CODE = 1;
WOE_MARITAL_STATUS_CODE = 0.1490584888;
end;
else
if 6 <= MARITAL_STATUS_CODE then do;
GRP_MARITAL_STATUS_CODE = 2;
WOE_MARITAL_STATUS_CODE = -0.194096763;
end;
end;
*------------------------------------------------------------*;
* Variable: NUMBER_OF_INSURANCES;
*------------------------------------------------------------*;
LABEL GRP_NUMBER_OF_INSURANCES = 'Grouped: NUMBER_OF_INSURANCES';
LABEL WOE_NUMBER_OF_INSURANCES =
'Weight of Evidence: NUMBER_OF_INSURANCES';
if MISSING(NUMBER_OF_INSURANCES) then do;
GRP_NUMBER_OF_INSURANCES = 3;
WOE_NUMBER_OF_INSURANCES = 0.1596414305;
end;
else if NOT MISSING(NUMBER_OF_INSURANCES) then do;
if 0 <= NUMBER_OF_INSURANCES AND NUMBER_OF_INSURANCES < 1 then do;
GRP_NUMBER_OF_INSURANCES = 1;
WOE_NUMBER_OF_INSURANCES = -0.304323348;
end;
else
if 1 <= NUMBER_OF_INSURANCES AND NUMBER_OF_INSURANCES < 2 then do;
GRP_NUMBER_OF_INSURANCES = 2;
WOE_NUMBER_OF_INSURANCES = 0.0131218294;
end;
else
if 2 <= NUMBER_OF_INSURANCES then do;
GRP_NUMBER_OF_INSURANCES = 3;
WOE_NUMBER_OF_INSURANCES = 0.1596414305;
end;
end;
*------------------------------------------------------------*;
* Variable: ORIGINAL_BALANCE;
*------------------------------------------------------------*;
LABEL GRP_ORIGINAL_BALANCE = 'Grouped: ORIGINAL_BALANCE';
LABEL WOE_ORIGINAL_BALANCE = 'Weight of Evidence: ORIGINAL_BALANCE';
if MISSING(ORIGINAL_BALANCE) then do;
GRP_ORIGINAL_BALANCE = 4;
WOE_ORIGINAL_BALANCE = 0.1520340757;
end;
else if NOT MISSING(ORIGINAL_BALANCE) then do;
if ORIGINAL_BALANCE < 84411.26 then do;
GRP_ORIGINAL_BALANCE = 1;
```

```
WOE_ORIGINAL_BALANCE = -0.790670253;
end;
else
if 84411.26 <= ORIGINAL_BALANCE
AND ORIGINAL_BALANCE < 122358.98 then do;
GRP_ORIGINAL_BALANCE = 2;
WOE_ORIGINAL_BALANCE = -0.435465698;
end;
else
if 122358.98 <= ORIGINAL_BALANCE
AND ORIGINAL_BALANCE < 162013.66 then do;
GRP_ORIGINAL_BALANCE = 3;
WOE_ORIGINAL_BALANCE =  -0.12987777;
end;
else
if 162013.66 <= ORIGINAL_BALANCE
AND ORIGINAL_BALANCE < 474562.69 then do;
GRP_ORIGINAL_BALANCE = 4;
WOE_ORIGINAL_BALANCE = 0.1520340757;
end;
else
if 474562.69 <= ORIGINAL_BALANCE then do;
GRP_ORIGINAL_BALANCE = 5;
WOE_ORIGINAL_BALANCE = 0.3759302492;
end;
end;
*------------------------------------------------------------*;
* Variable: ORIGINAL_TERM;
*------------------------------------------------------------*;
LABEL GRP_ORIGINAL_TERM = ’Grouped: ORIGINAL_TERM’;
LABEL WOE_ORIGINAL_TERM = ’Weight of Evidence: ORIGINAL_TERM’;
if MISSING(ORIGINAL_TERM) then do;
GRP_ORIGINAL_TERM = 3;
```

```
WOE_ORIGINAL_TERM = 0.4382467786;
end;
else if NOT MISSING(ORIGINAL_TERM) then do;
if 48 <= ORIGINAL_TERM AND ORIGINAL_TERM < 60 then do;
GRP_ORIGINAL_TERM = 1;
WOE_ORIGINAL_TERM = -0.562700224;
end;
else
if 60 <= ORIGINAL_TERM AND ORIGINAL_TERM < 72 then do;
GRP_ORIGINAL_TERM = 2;
WOE_ORIGINAL_TERM = -0.279623884;
end;
else
if 72 <= ORIGINAL_TERM then do;
GRP_ORIGINAL_TERM = 3;
WOE_ORIGINAL_TERM = 0.4382467786;
end;
end;
*------------------------------------------------------------*;
* Variable: DATA_RATE_TYPE_CODE;
*------------------------------------------------------------*;
LABEL GRP_DATA_RATE_TYPE_CODE = 'Grouped: DATA_RATE_TYPE_CODE';
LABEL WOE_DATA_RATE_TYPE_CODE =
'Weight of Evidence: DATA_RATE_TYPE_CODE';
_UFormat = put(DATA_RATE_TYPE_CODE,$1.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_DATA_RATE_TYPE_CODE = 2;
WOE_DATA_RATE_TYPE_CODE = 0.5912971246;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq 'B' OR _UFORMAT eq 'O'
) then do;
```

```
GRP_DATA_RATE_TYPE_CODE = 1;
WOE_DATA_RATE_TYPE_CODE = -0.087441637;
end;
else
if (_UFORMAT eq 'G'
) then do;
GRP_DATA_RATE_TYPE_CODE = 2;
WOE_DATA_RATE_TYPE_CODE = 0.5912971246;
end;
else do;
GRP_DATA_RATE_TYPE_CODE = 2;
WOE_DATA_RATE_TYPE_CODE = 0.5912971246;
end;
end;
*------------------------------------------------------------*;
* Variable: DWELLING_TYPE_CODE;
*------------------------------------------------------------*;
LABEL GRP_DWELLING_TYPE_CODE = 'Grouped: DWELLING_TYPE_CODE';
LABEL WOE_DWELLING_TYPE_CODE =
'Weight of Evidence: DWELLING_TYPE_CODE';
_UFormat = put(DWELLING_TYPE_CODE,$1.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_DWELLING_TYPE_CODE = 3;
WOE_DWELLING_TYPE_CODE = -0.212109212;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq 'B' OR _UFORMAT eq 'T'
) then do;
GRP_DWELLING_TYPE_CODE = 1;
WOE_DWELLING_TYPE_CODE = 0.2754640218;
end;
else
```

```
if (_UFORMAT eq 'O'
) then do;
GRP_DWELLING_TYPE_CODE = 2;
WOE_DWELLING_TYPE_CODE = -0.110103555;
end;
else
if (_UFORMAT eq 'P'
) then do;
GRP_DWELLING_TYPE_CODE = 3;
WOE_DWELLING_TYPE_CODE = -0.212109212;
end;
else do;
GRP_DWELLING_TYPE_CODE = 3;
WOE_DWELLING_TYPE_CODE = -0.212109212;
end;
end;
*------------------------------------------------------------*;
* Variable: EQUIPMENT_CATEGORY_CODE;
*------------------------------------------------------------*;
LABEL GRP_EQUIPMENT_CATEGORY_CODE =
'Grouped: EQUIPMENT_CATEGORY_CODE';
LABEL WOE_EQUIPMENT_CATEGORY_CODE =
'Weight of Evidence: EQUIPMENT_CATEGORY_CODE';
_UFormat = put(EQUIPMENT_CATEGORY_CODE,$6.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 3;
WOE_EQUIPMENT_CATEGORY_CODE = -0.123577521;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq 'BOATS' OR _UFORMAT
eq 'CARAVN' OR _UFORMAT eq 'LDV<5Y' OR _UFORMAT
eq 'LDV>5Y' OR _UFORMAT eq  'MTBIKE'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 1;
WOE_EQUIPMENT_CATEGORY_CODE = -0.335662295;
end;
else
if (_UFORMAT eq 'MVH>5Y'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 2;
WOE_EQUIPMENT_CATEGORY_CODE = -0.216798722;
end;
else
if (_UFORMAT eq 'MVH<5Y' OR _UFORMAT eq 'TRAILR'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 3;
WOE_EQUIPMENT_CATEGORY_CODE = -0.123577521;
end;
else
if (_UFORMAT eq 'LDVNEW'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 4;
WOE_EQUIPMENT_CATEGORY_CODE = 0.1004088237;
end;
else
if (_UFORMAT eq 'AGRICU' OR _UFORMAT eq 'MVHNEW' OR _UFORMAT eq 'OTHER'
) then do;
GRP_EQUIPMENT_CATEGORY_CODE = 5;
WOE_EQUIPMENT_CATEGORY_CODE = 0.3964204139;
end;
else do;
GRP_EQUIPMENT_CATEGORY_CODE = 3;
WOE_EQUIPMENT_CATEGORY_CODE = -0.123577521;
end;
end;
```

```
*------------------------------------------------------------*;
* Variable: NEW_OLD_USED;
*------------------------------------------------------------*;
LABEL GRP_NEW_OLD_USED = ’Grouped: NEW_OLD_USED’;
LABEL WOE_NEW_OLD_USED = ’Weight of Evidence: NEW_OLD_USED’;
_UFormat = put(NEW_OLD_USED,$4.0);
%dmnormip(_UFormat);
if MISSING(_UFORMAT) then do;
GRP_NEW_OLD_USED = 3;
WOE_NEW_OLD_USED =  0.312998795;
end;
else if NOT MISSING(_UFORMAT) then do;
if (_UFORMAT eq ’OLD’
) then do;
GRP_NEW_OLD_USED = 1;
WOE_NEW_OLD_USED =  -0.23738566;
end;
else
if (_UFORMAT eq ’DEMO’ OR _UFORMAT eq ’USED’
) then do;
GRP_NEW_OLD_USED = 2;
WOE_NEW_OLD_USED = -0.167441695;
end;
else
if (_UFORMAT eq ’NEW’
) then do;
GRP_NEW_OLD_USED = 3;
WOE_NEW_OLD_USED =  0.312998795;
end;
else do;
GRP_NEW_OLD_USED = 3;
WOE_NEW_OLD_USED =  0.312998795;
end;
```

```
end;
*------------------------------------------------------------*;
* Special Code Values
*------------------------------------------------------------*;
run;
proc sql;
   create table sum as
         select MANUFACTURER_CODE
              ,sum(ES) as Event_Count
              ,sum(1 - ES) as Non_Event_Count
              ,count(*) as Frequency
              ,mean(es) as Event_Rate
         from SA.ES
              group by MANUFACTURER_CODE; quit;
data sa.es;
set sa.es;
   if manufacturer_code = ’480’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’050’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’220’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’250’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’440’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’280’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’470’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’040’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’600’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’265’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’640’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’540’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’321’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’100’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’450’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’300’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’350’  then grp_manufacturer_code = 1;
else if manufacturer_code = '430'  then grp_manufacturer_code = 1;
else if manufacturer_code = '598'  then grp_manufacturer_code = 1;
else if manufacturer_code = '235'  then grp_manufacturer_code = 1;
else if manufacturer_code = '590'  then grp_manufacturer_code = 1;
else if manufacturer_code = '500'  then grp_manufacturer_code = 1;
else if manufacturer_code = '650'  then grp_manufacturer_code = 1;
else if manufacturer_code = '200'  then grp_manufacturer_code = 1;
else if manufacturer_code = '000'  then grp_manufacturer_code = 1;
else if manufacturer_code = '415'  then grp_manufacturer_code = 1;
else if manufacturer_code = '130'  then grp_manufacturer_code = 1;
else if manufacturer_code = '160'  then grp_manufacturer_code = 1;
else if manufacturer_code = '457'  then grp_manufacturer_code = 1;
else if manufacturer_code = '120'  then grp_manufacturer_code = 1;
else if manufacturer_code = '103'  then grp_manufacturer_code = 1;
else if manufacturer_code = '115'  then grp_manufacturer_code = 1;
else if manufacturer_code = '585'  then grp_manufacturer_code = 1;
else if manufacturer_code = '110'  then grp_manufacturer_code = 1;
else if manufacturer_code = '381'  then grp_manufacturer_code = 1;
else if manufacturer_code = '075'  then grp_manufacturer_code = 1;
else if manufacturer_code = '670'  then grp_manufacturer_code = 1;
else if manufacturer_code = '234'  then grp_manufacturer_code = 1;
else if manufacturer_code = '505'  then grp_manufacturer_code = 1;
else if manufacturer_code = '020'  then grp_manufacturer_code = 1;
else if manufacturer_code = '290'  then grp_manufacturer_code = 1;
else if manufacturer_code = '095'  then grp_manufacturer_code = 1;
else if manufacturer_code = '304'  then grp_manufacturer_code = 1;
else if manufacturer_code = '320'  then grp_manufacturer_code = 1;
else if manufacturer_code = '583'  then grp_manufacturer_code = 1;
else if manufacturer_code = '506'  then grp_manufacturer_code = 1;
else if manufacturer_code = '228'  then grp_manufacturer_code = 1;
else if manufacturer_code = '232'  then grp_manufacturer_code = 1;
else if manufacturer_code = '331'  then grp_manufacturer_code = 1;
else if manufacturer_code = '215'  then grp_manufacturer_code = 1;
else if manufacturer_code = ’578’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’240’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’266’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’315’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’285’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’581’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’242’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’445’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’222’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’378’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’073’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’610’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’502’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’155’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’125’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’046’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’270’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’057’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’390’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’875’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’186’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’153’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’855’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’584’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’033’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’442’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’241’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’360’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’577’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’170’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’550’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’580’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’225’  then grp_manufacturer_code = 1;
else if manufacturer_code = '275'  then grp_manufacturer_code = 1;
else if manufacturer_code = '190'  then grp_manufacturer_code = 1;
else if manufacturer_code = '129'  then grp_manufacturer_code = 1;
else if manufacturer_code = '218'  then grp_manufacturer_code = 1;
else if manufacturer_code = '310'  then grp_manufacturer_code = 1;
else if manufacturer_code = '562'  then grp_manufacturer_code = 1;
else if manufacturer_code = '022'  then grp_manufacturer_code = 1;
else if manufacturer_code = '048'  then grp_manufacturer_code = 1;
else if manufacturer_code = '466'  then grp_manufacturer_code = 1;
else if manufacturer_code = '263'  then grp_manufacturer_code = 1;
else if manufacturer_code = '128'  then grp_manufacturer_code = 1;
else if manufacturer_code = '206'  then grp_manufacturer_code = 1;
else if manufacturer_code = '260'  then grp_manufacturer_code = 1;
else if manufacturer_code = '652'  then grp_manufacturer_code = 1;
else if manufacturer_code = '035'  then grp_manufacturer_code = 1;
else if manufacturer_code = '385'  then grp_manufacturer_code = 1;
else if manufacturer_code = '420'  then grp_manufacturer_code = 1;
else if manufacturer_code = '090'  then grp_manufacturer_code = 1;
else if manufacturer_code = '165'  then grp_manufacturer_code = 1;
else if manufacturer_code = '410'  then grp_manufacturer_code = 1;
else if manufacturer_code = '245'  then grp_manufacturer_code = 1;
else if manufacturer_code = '462'  then grp_manufacturer_code = 1;
else if manufacturer_code = '467'  then grp_manufacturer_code = 1;
else if manufacturer_code = '620'  then grp_manufacturer_code = 1;
else if manufacturer_code = '037'  then grp_manufacturer_code = 1;
else if manufacturer_code = '244'  then grp_manufacturer_code = 1;
else if manufacturer_code = '508'  then grp_manufacturer_code = 1;
else if manufacturer_code = '008'  then grp_manufacturer_code = 1;
else if manufacturer_code = '055'  then grp_manufacturer_code = 1;
else if manufacturer_code = '313'  then grp_manufacturer_code = 1;
else if manufacturer_code = '403'  then grp_manufacturer_code = 1;
else if manufacturer_code = '435'  then grp_manufacturer_code = 1;
else if manufacturer_code = '465'  then grp_manufacturer_code = 1;
```

```
 else if manufacturer_code = ’503’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’535’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’560’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’612’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’025’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’038’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’121’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’224’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’323’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’333’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’343’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’460’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’611’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’032’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’044’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’049’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’132’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’158’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’159’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’188’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’230’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’236’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’238’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’239’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’308’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’318’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’330’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’464’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’515’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’582’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’597’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’631’  then grp_manufacturer_code = 1;
else grp_manufacturer_code = 1;
```

```
 run;
proc sql;
   create table sum as
         select MANUFACTURER_CODE
              ,sum(default) as Event_Count
              ,sum(1 - default) as Non_Event_Count
              ,count(*) as Frequency
              ,mean(default) as Event_Rate
         from SA.DEFAULT
              group by MANUFACTURER_CODE;
quit;
data sa.default;
set sa.default;
 if manufacturer_code = ’280’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’480’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’050’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’540’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’220’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’100’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’640’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’470’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’440’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’600’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’040’  then grp_manufacturer_code = 2;
 else if manufacturer_code = ’265’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’321’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’250’  then grp_manufacturer_code = 3;
 else if manufacturer_code = ’450’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’300’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’350’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’430’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’598’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’235’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’590’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’500’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’650’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’200’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’000’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’415’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’130’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’160’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’457’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’120’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’103’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’115’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’585’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’110’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’381’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’075’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’670’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’234’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’505’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’020’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’290’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’095’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’304’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’320’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’583’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’506’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’228’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’232’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’331’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’215’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’578’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’240’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’266’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’315’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’285’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’581’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’242’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’445’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’222’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’378’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’073’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’610’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’502’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’155’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’125’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’046’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’270’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’057’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’390’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’875’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’186’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’153’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’855’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’584’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’033’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’442’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’241’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’360’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’577’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’170’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’550’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’580’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’225’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’275’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’190’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’129’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’218’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’310’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’562’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’022’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’048’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’466’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’263’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’128’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’206’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’260’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’652’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’035’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’385’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’420’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’090’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’165’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’410’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’245’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’462’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’467’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’620’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’037’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’244’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’508’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’008’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’055’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’313’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’403’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’435’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’465’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’503’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’535’  then grp_manufacturer_code = 1;
else if manufacturer_code = ’560’  then grp_manufacturer_code = 1;
```

```
 else if manufacturer_code = ’612’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’025’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’038’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’121’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’224’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’323’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’333’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’343’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’460’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’611’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’032’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’044’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’049’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’132’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’158’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’159’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’188’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’230’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’236’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’238’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’239’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’308’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’318’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’330’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’464’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’515’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’582’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’597’  then grp_manufacturer_code = 1;
 else if manufacturer_code = ’631’  then grp_manufacturer_code = 1;
 else grp_manufacturer_code = 1;
 run;
***PH Assumption
%macro ph_assumption(infile=,target=,outfile=);
```

```
proc contents data=
&infile.(keep=grp_: &target. survtime) out=_variables_ noprint;
run;
proc sql noprint;
      create table _variables_ as
           select * from _variables_
           where lowcase(name) not in ("&target.","survtime");
      select count(*) into:nvar from _variables_;
quit;
data &outfile.;
      format variable $32.;
      format level best8.;
run;
%do i = 1 %to &nvar.;

      data _variables_;
           set _variables_;
           if _n_ = &i. then
                call symput("var",put(compress(name),$32.));
      run;
      proc sql noprint;
           select count(*)
            into:nlvl from
            (select distinct &var. from &infile.); quit;
      proc sql noprint;
           create table level as
                select distinct &var.
                      from &infile.; quit;
      %do j = 1 %to &nlvl.;
           data level;
                set level;
                if _n_ = &j. then
                      call symput("level",put(&var.,best8.));
                run;
                proc phreg data=&infile.
                (where=(&var. eq &level.)) noprint;
                     model survtime * &target. (0) =
                     ;
                     baseline out=h
                   survival=survival loglogs=loglog cumhaz=cumhaz;
                run;
                data h;
                     format variable $32. level best8.;
                     set h;
                     variable = compress("&var.");
                     level = &level.;
                run;
                data &outfile.;
                     set &outfile. h;
                run;
           %end;
     %end;
%mend;

%ph_assumption(infile=SA.es,target=es,outfile=PH);
***Final Grouping
data sa.default;
set sa.default;
if grp_age_of_asset in (2,3) then
dm_age_of_asset = 2;
else dm_age_of_asset = 1;
if grp_debit_interest_rate in (1,2) then
dm_debit_interest_rate = 1;
else if grp_debit_interest_rate in (3,4) then
dm_debit_interest_rate = 2;
else dm_debit_interest_rate = 3;
```

```
if grp_deposit_to_loan in (2,3) then
dm_deposit_to_loan = 2;
else dm_deposit_to_loan = 1;
dm_dwelling_type_code = grp_dwelling_type_code;
if grp_equipment_category_code in (1,2,3,4) then
dm_equipment_category_code = 1;
else dm_equipment_category_code = 2;
if grp_manufacturer_code in (2,3) then
dm_manufacturer_code = 2;
else dm_manufacturer_code = 1;
if grp_new_old_used in (2,3) then
dm_new_old_used = 2;
else dm_new_old_used = 1;run;
data sa.es;
set sa.es (drop=em_age_of_asset em_new_old_used);
 if grp_age_of_asset in (1,2)
 then em_age_of_asset = 1; else em_age_of_asset = 2;
if grp_debit_interest_rate in (1,2,3) then
em_debit_interest_rate = 1;
else em_debit_interest_rate = 2;
if grp_deposit_to_loan in (1,2,3) then
em_deposit_to_loan = 1;
else em_deposit_to_loan = 2;
if grp_new_old_used in (1,2) then em_new_old_used = 1;
else em_new_old_used = 2;
if grp_dwelling_type_code in (2,3) then
em_dwelling_type_code = 2;
else em_dwelling_type_code = 1;
if grp_equipment_category_code in (1,2) then
em_equipment_category_code = 1;
else if grp_equipment_category_code in (4,5) then
em_equipment_category_code = 3;
else em_equipment_category_code = 2;
```

```
if grp_original_term in (1,2) then
em_original_term = 1;
else em_original_term = 2; run;

***Population Stability
%macro fields(infile=, target=, outfile=);
proc contents data=&infile.
(keep=dm_:) out=fields (keep=name) noprint; run;
proc sql;
     select count(*) into:nvar from fields; quit;
data &outfile.;
     set _null_; run;
%do i = 1 %to &nvar.;
     data _null_;
           set fields;
           if _n_  eq &i. then
                call symput("variable",name); run;
     proc sql;
           create table sum as
                select "&variable." as variable
                     ,&variable. as level
                     ,sum(&target.) as Event_Count
                     ,count(*) as Frequency
                     ,mean(&target.) as Event_Rate
                from &infile.
                     group by &variable.; quit;
     data &outfile.;
           set &outfile. sum;
     run; %end;
%mend;
%fields(infile=SA.default,target=default,outfile=Buckets);

%macro fields(infile=, target=, outfile=);
```

```
proc contents data=&infile.
(keep=em_:) out=fields (keep=name) noprint; run;
proc sql;
     select count(*) into:nvar from fields; quit;
data &outfile.;
     set _null_; run;
%do i = 1 %to &nvar.;
     data _null_;
           set fields;
           if _n_  eq &i. then
                call symput("variable",name); run;
     proc sql;
           create table sum as
                select "&variable." as variable
                     ,&variable. as level
                     ,sum(&target.) as Event_Count
                     ,count(*) as Frequency
                     ,mean(&target.) as Event_Rate
                from &infile.
                     group by &variable.; quit;
     data &outfile.;
           set &outfile. sum; run;
%end;
%mend;
%fields(infile=SA.es,target=es,outfile=Buckets);

%macro stability(infile=);
     proc contents
     data=&infile.(keep=open em_:) out=_variables_ noprint;
     run;
     proc sql noprint;
           create table _variables_ as
                select * from _variables_ where
           lowcase(name) not in ("open");
      select count(*) into:nvar from _variables_;
quit;
data monthly_plots;
      format variable $32.;
run;
%do i = 1 %to &nvar.;
      data _variables_;
           set _variables_;
           if _n_ = &i.
           then
                do; call symput
                ("var",put(compress(name),$32.));
                      call symput("lbl",put(label,$32.));
                end;
      run;
      %put &var.;
      proc sql;
           create table h as
                select     "&var." as variable
                      ,&var. as level
                      ,open
                      ,count(*) as observation
                from &infile.
                      group by &var. ,open;
      quit;
      data h(drop=lvl);
           set h(rename=(level=lvl));
           level = compress(lvl);
      run;

      data monthly_plots;
           set monthly_plots h;
```

```
            run;
      %end;
      proc sort data=monthly_plots (where = (variable ne ''));
            by variable;
      run;
%mend;
%stability(infile=sa.es);
```

B.6 Multivariate Analysis

```
***Overall Kaplain Meier Curve
data development;
set sa.development;
if censor = 0 then status = 1; else status = 0;
run;
proc phreg data=development;
model survtime*status(0) = ;
baseline out=sa.km survival=Overall_KM ; run;
***Default Model Multivariate Analysis
/*select the baseline*/
proc contents data=sa.default noprint out=vars(keep=name); run;
data default_model;
set sa.default;
dm_MARITAL_STATUS_CODE = grp_MARITAL_STATUS_CODE;
keep ACCOUNT_NUMBER
MONTH
default
censor
open
event_month
dm_age_of_asset
dm_debit_interest_rate
```

```
dm_deposit_to_loan
dm_dwelling_type_code
dm_equipment_category_code
dm_MARITAL_STATUS_CODE
dm_new_old_used
survtime; run;
proc sql;
create table basepop as
select
dm_age_of_asset,
dm_debit_interest_rate,
dm_deposit_to_loan,
dm_dwelling_type_code,
dm_equipment_category_code,
dm_MARITAL_STATUS_CODE,
dm_new_old_used,
count(*) as volume
from default_model
group by
dm_age_of_asset,
dm_debit_interest_rate,
dm_deposit_to_loan,
dm_dwelling_type_code,
dm_equipment_category_code,
dm_MARITAL_STATUS_CODE,
dm_new_old_used
order by volume desc;
quit;
/*Stepwise regression Step 1*/
ods output fitstatistics = fitness;
ods output modelbuildingsummary  = build;
ods output classlevelinfo = levels;
ods output ParameterEstimates = estimates;
```

```
proc phreg data=default_model;
class
dm_age_of_asset ( ref = '2' )
dm_debit_interest_rate ( ref = '2' )
dm_deposit_to_loan ( ref = '1' )
dm_dwelling_type_code ( ref = '1' )
dm_equipment_category_code ( ref = '1' )
dm_MARITAL_STATUS_CODE ( ref = '1' )
dm_new_old_used ( ref = '2' ) ;
model survtime*default(0) =
dm_age_of_asset
dm_debit_interest_rate
dm_deposit_to_loan
dm_dwelling_type_code
dm_equipment_category_code
dm_MARITAL_STATUS_CODE
dm_new_old_used
/selection=stepwise;
baseline out=out survival=s2 loglogs=ls2;
run;
/*Correlation Coefficient*/
proc corr data=default_model
(keep=dm_age_of_asset
dm_debit_interest_rate
dm_deposit_to_loan
dm_dwelling_type_code
dm_equipment_category_code
dm_MARITAL_STATUS_CODE
dm_new_old_used)
pearson out=correlations;
run;
proc sql;
create table basepop as
```

```
select
dm_debit_interest_rate,
dm_dwelling_type_code,
dm_deposit_to_loan,
dm_MARITAL_STATUS_CODE,
dm_equipment_category_code,
count(*) as volume
from default_model
group by
dm_debit_interest_rate,
dm_dwelling_type_code,
dm_deposit_to_loan,
dm_MARITAL_STATUS_CODE,
dm_equipment_category_code
order by volume desc;
quit;
***ES Model
proc contents data=sa.es noprint out=vars(keep=name); run;
data es_model;
set sa.es;
keep
ACCOUNT_NUMBER
MONTH
em_age_of_asset
em_debit_interest_rate
em_deposit_to_loan
em_dwelling_type_code
em_equipment_category_code
em_new_old_used
em_original_term
es
survtime;
run;
```

```
/*select the baseline*/
proc sql;
create table basepop as
select
em_age_of_asset,
em_debit_interest_rate,
em_deposit_to_loan,
em_dwelling_type_code,
em_equipment_category_code,
em_new_old_used,
em_original_term,
count(*) as volume
from es_model
group by
em_age_of_asset,
em_debit_interest_rate,
em_deposit_to_loan,
em_dwelling_type_code,
em_equipment_category_code,
em_new_old_used,
em_original_term
order by volume desc;
quit;
/*Stepwise regression Step 1*/
ods output fitstatistics = fitness;
ods output modelbuildingsummary  = build;
ods output classlevelinfo = levels;
ods output ParameterEstimates = estimates;
proc phreg data=es_model;
class
em_age_of_asset ( ref = '1' )
em_debit_interest_rate ( ref = '1' )
em_deposit_to_loan ( ref = '1' )
```

```
em_dwelling_type_code ( ref = '2' )
em_equipment_category_code ( ref = '3' )
em_new_old_used ( ref = '2' )
em_original_term ( ref = '2' ) ;
model survtime*es(0) =
em_age_of_asset
em_debit_interest_rate
em_deposit_to_loan
em_dwelling_type_code
em_equipment_category_code
em_new_old_used
em_original_term
/selection=stepwise;
baseline out=out survival=s2 loglogs=ls2;
run;
/*Correlation Coefficient*/
proc corr data=es_model
(keep= em_age_of_asset
em_debit_interest_rate
em_deposit_to_loan
em_dwelling_type_code
em_equipment_category_code
em_new_old_used
em_original_term)
pearson out=correlations;
run;
proc sql;
create table basepop as
select
em_debit_interest_rate,
em_deposit_to_loan,
em_dwelling_type_code,
em_equipment_category_code,
em_original_term,
count(*) as volume
from es_model
group by
em_debit_interest_rate,
em_deposit_to_loan,
em_dwelling_type_code,
em_equipment_category_code,
em_original_term
order by volume desc; quit;
***Default Logistic
ods output fitstatistics = sa.logistic_fitness;
ods output modelbuildingsummary = sa.logistic_build;
ods output classlevelinfo = sa.logistic_levels;
ods output parameterestimates = sa.logistic_parameters;
ods output type3 = sa.logistic_importance;
ods output globaltests = sa.logistic_globaltest;
ods output lackfitchisq = sa.logistic_chisq;
ods output lackfitpartition = sa.logistic_hosmer;
proc logistic data = sa.logistic_combined ;
     class  dm_debit_interest_rate ( ref = '2' )
dm_dwelling_type_code ( ref = '1' )
dm_deposit_to_loan ( ref = '1' )
dm_MARITAL_STATUS_CODE ( ref = '1' )
dm_equipment_category_code ( ref = '1' ) /param=ref ;
     model default(event='1') =
dm_debit_interest_rate
dm_deposit_to_loan dm_dwelling_type_code
dm_equipment_category_code dm_MARITAL_STATUS_CODE
           / selection=stepwise link=logit
           scale=deviance
           lackfit
           rsq
```

```
   outroc=rocs;
     score data = sa.logistic_combined out = development_default;
/*     score data = validation_grp out = validation_default;*/ run;
***ES Logistic
ods output fitstatistics = sa.logistic_es_fitness;
ods output modelbuildingsummary = sa.logistic__es_build;
ods output classlevelinfo = sa.logistic_es_levels;
ods output parameterestimates = sa.logistic_es_parameters;
ods output type3 = sa.logistic_es_importance;
ods output globaltests = sa.logistic_es_globaltest;
ods output lackfitchisq = sa.logistic_es_chisq;
ods output lackfitpartition = sa.logistic_es_hosmer;
proc logistic data = sa.logistic_combined
 /*(where =(em_equipment_category_code ne 2
 or em_original_term ne 2 or em_deposit_to_loan ne 2))*/;
     class em_debit_interest_rate ( ref = ’1’ )
   em_deposit_to_loan ( ref = ’1’ )
   em_dwelling_type_code ( ref = ’2’ )
   em_equipment_category_code ( ref = ’3’ )
   em_original_term ( ref = ’2’ ) /param=ref ;
     model es(event=’1’) =
em_debit_interest_rate
em_deposit_to_loan
em_debit_interest_rate*em_deposit_to_loan
em_dwelling_type_code
em_equipment_category_code
em_debit_interest_rate*em_equipment_category_code
em_deposit_to_loan*em_equipment_category_code
           / selection=STEPWISE link=logit
           scale=deviance
           lackfit rsq
    outroc=rocs2;
     score data = sa.logistic_combined out = development_es;
```

```
/*     score data = validation_grp out = validation_es;*/
run;
```

B.7 Cox PH regression

```
/*Final Default Model*/
ods output fitstatistics = fitness;
ods output modelbuildingsummary  = build;
ods output classlevelinfo = levels;
ods output ParameterEstimates = estimates;
proc phreg data=default_model;
class
dm_debit_interest_rate ( ref = ’2’ )
dm_dwelling_type_code ( ref = ’1’ )
dm_deposit_to_loan ( ref = ’1’ )
dm_MARITAL_STATUS_CODE ( ref = ’1’ )
dm_equipment_category_code ( ref = ’1’ );
model survtime*default(0) =
dm_debit_interest_rate
dm_deposit_to_loan
dm_dwelling_type_code
dm_equipment_category_code
dm_MARITAL_STATUS_CODE;
baseline out=sa.def_baseline survival=s2 loglogs=ls2;
run;
proc reg data=default_model;
model SURVTIME =
        dm_age_of_asset
dm_debit_interest_rate
dm_deposit_to_loan
dm_dwelling_type_code
dm_equipment_category_code
dm_MARITAL_STATUS_CODE
```

```
dm_new_old_used
/vif collin;
run;
/*Final Model*/
ods output fitstatistics = fitness;
ods output modelbuildingsummary  = build;
ods output classlevelinfo = levels;
ods output ParameterEstimates = estimates;
proc phreg data=es_model;
class
em_debit_interest_rate ( ref = '1' )
em_deposit_to_loan ( ref = '1' )
em_dwelling_type_code ( ref = '2' )
em_equipment_category_code ( ref = '3' )
em_original_term ( ref = '2' );
model survtime*es(0) =
em_debit_interest_rate
em_deposit_to_loan
em_dwelling_type_code
em_equipment_category_code
em_original_term;
baseline out=sa.es_baseline survival=s2 loglogs=ls2; run;
proc reg data=es_model;
model SURVTIME =
em_age_of_asset
em_debit_interest_rate
em_deposit_to_loan
em_dwelling_type_code
em_equipment_category_code
em_new_old_used
em_original_term
/vif collin;
run;
```

B.8 CIC

```
***Adjustment Factors
proc format;
/*Default Model Estimates*/
value dm_debit_interest_rate_est
1 = -0.72123
2  = 0
3 = 0.41568;
value dm_deposit_to_loan_est
1  = 0
2  =  -0.53807;
value dm_dwelling_type_code_est
1  =  0
2  = -0.14953
3  =  -0.39423;
value dm_equipment_category_code_est
1  =  0
2 = -0.19984;
value dm_marital_status_code_est
1  = 0
2 =  -0.2722;

/*ES Model Estimates*/

value em_debit_interest_rate_est
1  = 0
2  =   -0.38364;
value em_deposit_to_loan_est
1  =  0
2  = 0.37831;
value em_dwelling_type_code_est
1  =  -0.07833
2  = 0;
```

```
value em_equipment_category_code_est
1   =  0.61721
2  =  0.46329
3  =  0;
value em_original_term_est
1 = 0.18008
2 = 0;

DATA IN;
SET sa.combined;
time_in_study = 0;
DM_HR = EXP(PUT(dm_debit_interest_rate,
dm_debit_interest_rate_est.)
+ PUT(dm_deposit_to_loan,dm_deposit_to_loan_est.)
+ PUT(dm_dwelling_type_code,dm_dwelling_type_code_est.)
+ PUT(dm_equipment_category_code,
dm_equipment_category_code_est.)
+ PUT(dm_marital_status_code,dm_marital_status_code_est.));
EM_HR = EXP(PUT(em_debit_interest_rate,
em_debit_interest_rate_est.)
+ PUT(em_deposit_to_loan,em_deposit_to_loan_est.)
+ PUT(em_dwelling_type_code,em_dwelling_type_code_est.)
+ PUT(em_equipment_category_code,
em_equipment_category_code_est.)
+ PUT(em_original_term,em_original_term_est.)); RUN;
data in ;
set in;
if survtime gt 48 then
do;
survtime = 48;
default = 0;
es = 0;
event_date = intnx('month',
input(put(open,$6.),yymmn6.),48);
event_month = year(event_date)
* 100 + month(event_date);
censor_month = event_month;
default_month = .;
es_month = .;
end;run;
PROC SQL;
CREATE TABLE baseline_survival AS
SELECT A. SURVTIME,
A. s2 as survdm,
B. s2 as survem
FROM sa.def_baseline AS A
LEFT JOIN sa.es_baseline AS B ON
A. SURVTIME = B. SURVTIME; QUIT;
/*S(t) = (1 - p) + pS(t I Y = 1)*/
data baseline_survival;
set baseline_survival;
 p = 0.4917904496;
if survdm eq . then
survdm = 1;
if survem eq . then
survem = 1;
dm_survival  = 1 - p +  p * survdm;
em_survival  = 1 - p +  p *survem;run;
data out;
length survtime dm_survival em_survival 8;
if _n_ = 1 then
do; declare hash baseline(dataset:'baseline_survival');
baseline.defineKey('survtime');
baseline.defineData('dm_survival', 'em_survival');
baseline.defineDone(); end;
set in(rename=(survtime=survival_time));
```

```
actual_time_in_study = time_in_study;
time_in_study = min(time_in_study,47);
survival = 1;
em_probability = 0;
dm_probability = 0;
o_dm_survival = 1;
o_em_survival = 1;
cn = min(intck('month',input(put(month,$6.),yymmn6.),
'31Mar2014'd),48);
/*cn = 48;*/
do survtime = 0 to cn;
rc = baseline.find();
dm_hazard = 1 -
(dm_survival ** dm_hr)/(o_dm_survival ** dm_hr);
em_hazard = 1 -
(em_survival ** em_hr)/(o_em_survival ** em_hr);
emincident = em_hazard * survival;
dmincident = dm_hazard * survival;
survival = survival - emincident - dmincident;
if survtime = time_in_study then
obs_survival = survival;
if survtime gt time_in_study then do;
em_probability = em_probability + emincident;
dm_probability = dm_probability + dmincident;
end;
o_dm_survival = dm_survival;
o_em_survival = em_survival; end;
time_in_study = actual_time_in_study;
em_probability = em_probability / obs_survival;
dm_probability = dm_probability / obs_survival;
incomplete = 1 - (em_probability + dm_probability);
em_probability_adj = em_probability / (1 - incomplete);
dm_probability_adj = dm_probability / (1 - incomplete); run;
```

```
ods output measures=gini (where=(statistic="Somers’ D R|C"));
proc freq data=out;
table dm_probability * default
em_probability * es
/measures; run;
proc sql;
create table default_actual_vs_expected as
select survtime
,mean(default) as actual_defaults
,mean(dm_probability_adj) as expected_defaults
from out (where=(censor ne 1 and open = month))
group by survtime;
create table es_actual_vs_expected as
select survtime
,mean(es) as actual_es
,mean(em_probability_adj) as expected_es
from out (where=(censor ne 1 and open = month))
group by survtime; quit;
```

B.9 Logistic Regression

```
***Default Logistic
ods output fitstatistics = sa.logistic_fitness;
ods output modelbuildingsummary = sa.logistic_build;
ods output classlevelinfo = sa.logistic_levels;
ods output parameterestimates = sa.logistic_parameters;
ods output type3 = sa.logistic_importance;
ods output globaltests = sa.logistic_globaltest;
ods output lackfitchisq = sa.logistic_chisq;
ods output lackfitpartition = sa.logistic_hosmer;
proc logistic data = sa.logistic_combined ;
     class  dm_debit_interest_rate ( ref = ’2’ )
dm_dwelling_type_code ( ref = ’1’ )
```

```
dm_deposit_to_loan ( ref = '1' )
dm_MARITAL_STATUS_CODE ( ref = '1' )
dm_equipment_category_code ( ref = '1' ) /param=ref ;
     model default(event='1') =
dm_debit_interest_rate dm_deposit_to_loan
dm_dwelling_type_code
dm_equipment_category_code dm_MARITAL_STATUS_CODE
           / selection=none link=logit
           scale=deviance
           lackfit
           rsq
   outroc=rocs;
     score data = sa.logistic_combined out = development_default;
/*     score data = validation_grp out = validation_default;*/
run;
***ES Logistic
ods output fitstatistics = sa.logistic_es_fitness;
ods output modelbuildingsummary = sa.logistic__es_build;
ods output classlevelinfo = sa.logistic_es_levels;
ods output parameterestimates = sa.logistic_es_parameters;
ods output type3 = sa.logistic_es_importance;
ods output globaltests = sa.logistic_es_globaltest;
ods output lackfitchisq = sa.logistic_es_chisq;
ods output lackfitpartition = sa.logistic_es_hosmer;
proc logistic data = sa.logistic_combined
 /*(where (em_equipment_category_code  ne 2
 or em_original_term ne 2 or em_deposit_to_loan ne 2))*/;
     class em_debit_interest_rate ( ref = '1' )
   em_deposit_to_loan ( ref = '1' )
   em_dwelling_type_code ( ref = '2' )
   em_equipment_category_code ( ref = '3' )
   em_original_term ( ref = '2' ) /param=ref ;
     model es(event='1') =
em_debit_interest_rate
em_deposit_to_loan
em_debit_interest_rate*em_deposit_to_loan
em_dwelling_type_code
em_equipment_category_code
em_debit_interest_rate*em_equipment_category_code
em_deposit_to_loan*em_equipment_category_code
           / selection=none link=logit
           scale=deviance
           lackfit rsq
    outroc=rocs2;
     score data = sa.logistic_combined out = development_es;
/*     score data = validation_grp out = validation_es;*/run;
```

B.10 Model Assessment and Comparisons

```
/*Default ROC Curve*/
proc sql;
create table dm_summary as
select dm_probability
,count(*) as trials
,sum(default) as events
from out
group by dm_probability
order by dm_probability desc; quit;
proc sql;
create table dm_summary as
select *
,sum(trials) as tot_trials
,sum(events) as tot_events
from dm_summary;
quit;
data _dmsummary_
```

```
(rename=(Yes_Yes = _POS_ No_No = _NEG_ Yes_No =
_FALSEPOS_ No_Yes = _FALSENEG_));
set dm_summary;
Yes_Yes = sum(Yes_Yes,events);
Yes_No = sum(Yes_No,trials - events);
No_Yes = tot_events - Yes_Yes;
No_No = tot_trials - tot_events - Yes_No;
_1MSPEC_ = 1 - No_No/(No_No + Yes_No);
_SENSIT_ = Yes_Yes / (Yes_Yes + No_Yes);
retain Yes_Yes Yes_No;
keep _1MSPEC_ _SENSIT_; run;
/*ES ROC Curve*/
proc sql;
create table em_summary as
select em_probability
,count(*) as trials
,sum(es) as events
from out
group by em_probability
order by em_probability desc; quit;
proc sql;
create table em_summary as
select *
,sum(trials) as tot_trials
,sum(events) as tot_events
from em_summary; quit;
data _emsummary_
 (rename=(Yes_Yes = _POS_ No_No = _NEG_ Yes_No
 = _FALSEPOS_ No_Yes = _FALSENEG_));
set em_summary;
Yes_Yes = sum(Yes_Yes,events);
Yes_No = sum(Yes_No,trials - events);
No_Yes = tot_events - Yes_Yes;
```

```
No_No = tot_trials - tot_events - Yes_No;
_1MSPEC_ = 1 - No_No/(No_No + Yes_No);
_SENSIT_ = Yes_Yes / (Yes_Yes + No_Yes);
retain Yes_Yes Yes_No;
keep _1MSPEC_ _SENSIT_; run;
```