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Negative differential resistance associated with hot phonons

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We predict the existence of a hot-phonon negative differential resistance (NDR) in GaN. We show that this is a consequence of a wave-vector dependence of lifetime caused by the effect of coupled plasmon-phonons. Anti-screened long-wavelength modes have shorter lifetimes, screened shorter-wavelength modes have longer lifetimes, the boundary between them being determined by the temperature-dependent Landau damping. The higher density of screened modes means that the average lifetime is of order of the lifetime of the bare phonon. Its increase with electron temperature (field) is responsible for the NDR. We also find that the momentum relaxation rate (MRR) associated with the absorption of phonons can be negative in some circumstances, which can be seen to be a consequence of the non-uniform distribution of hot phonons in wave-vector space. We also point out that the ultra-short lifetimes sometimes deduced from experiment should more properly be regarded as electron energy- relaxation times. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4754012]

I. INTRODUCTION

It is well-known that the performance of high-power semiconductor devices can be significantly degraded by the production of hot phonons. In III-V compounds at high electric fields, hot electrons interact strongly with polar optical phonons, with the result that the phonon population is driven far from thermodynamic equilibrium and can be described as being hot. The increase in phonon number results naturally in an increase of electrical resistance, which affects device performance. How strong this effect is depends on the lifetime of these hot phonons. In recent years, the measurement of the lifetime of optical phonons in the nitrides has been the focus of a number of investigations. There have been three main techniques that are used: Raman scattering,[1–4] electrical noise,[5] and photoluminescence/mobility.[6,7] It has become clear that the lifetime depends upon the lattice temperature and the density of electrons, with clear evidence of the presence of coupled plasmon-phonon modes.

What is actually measured (or deduced) is a lifetime averaged over those phonon modes most strongly coupled to the electrons. Different techniques probe different regions of the phonon spectrum and can be expected to give different results. This is particularly the case for Raman techniques, which probe only those modes with wave vectors near to zero $q \leq 10^8\,\mathrm{m}^{-1}$. Time-resolved measurements give the phonon lifetime of long-wavelength modes directly.[1] The same is true of measurements of bandwidth, given the assumptions associated with the uncertainty principle.[2–4] Other techniques yield lifetimes that are averages over the range of wave vectors most strongly coupled to the hot electrons, typically $0 \leq q \leq 10^9\,\mathrm{m}^{-1}$ in GaN. Extrapolation of Raman results to this regime would be valid only if the lifetime was not a function of wave vector, but, as we will show, coupled-mode effects make such an extrapolation invalid. Other methods rely essentially on the assumption that the hot-electron distribution can be described by an electron temperature which, given the high densities typically involved, is usually taken to be a good approximation. It assumes that the rate of energy exchange between electrons is greater than the rate of energy-relaxation rates associated with the emission of phonons.[8] Measurement of electron temperature and the electric power dissipated allows an energy-relaxation time to be deduced in the hydrodynamic model,

$$P = \frac{3k_B}{2}\frac{(T_e - T_0)}{\tau_E}, \tag{1}$$

P is the power dissipated per electron, k_B is Boltzmann's constant, T_e is the electron temperature, T_0 is the lattice temperature, and τ_E is the energy-relaxation time. Ensemble Monte Carlo analysis in the absence of hot phonons shows that for mean energies equal to or greater than the optical phonon energy, τ_E increases rapidly with energy from 200 fs to around 1 ps.[9] The phonon lifetime comes into the picture only when the hot-phonon temperature equals the electron temperature, for in that case the rate of dissipation of power is limited by the rate at which optical phonons decay into lattice modes that can transfer energy rapidly to the thermal bath. The power dissipated per mode is

$$P_p = \frac{\hbar\omega_L}{\tau_p}[n(\omega_L, T_e) - n(\omega_L, T_0)] \tag{2}$$

Here, $\hbar\omega_L$ is the phonon energy, $n(\omega_L, T)$ is the Bose-Einstein phonon number, and τ_p is the lifetime of the mode. If N_p is the effective number of hot-phonon modes involved, then $P = P_p N_p$, and the lifetime can be deduced. Unfortunately, N_p is not directly measurable.

The formation of coupled modes makes things more complicated; its effect is to introduce a new spectrum of

polar modes. Instead of modes all of which can be taken to have the same frequency and to exhibit negligible dispersion (i.e., no dependence on wave vector), coupled modes have a frequency range and exhibit dispersion. This implies that their lifetime, as determined by the anharmonicity of the lattice, varies with wave vector.[10] Previous analyses by us have taken into account the frequency-dependent coupling strengths, but have assumed a constant lifetime.[11] Even with this simplification, intensive numerical analysis is involved. Our purpose here is to use a single-pole model to take into account the wave-vector dependence of coupled-mode lifetime and to present a transparent account of hot-phonon effects.

According to the fluctuation-dissipation theorem, the electron-coupled-mode interaction strength is proportional to the imaginary part of the negative reciprocal of the frequency and wave-vector dependent dielectric function

$$\mathrm{Im}\left(-\frac{1}{\varepsilon(\omega,q)}\right)=\frac{\mathrm{Im}\,\varepsilon(\omega,q)}{[\mathrm{Re}\,\varepsilon(\omega,q)]^2+[\mathrm{Im}\,\varepsilon(\omega,q)]^2}. \quad (3)$$

The imaginary part of the dielectric function quantifies the Landau damping of the mode; when it is large the mode ceases to be well-defined, and when it is small the interaction strength maximises when the real part vanishes. The first part of the single-pole model is to confine the interaction to modes with $Re\,\varepsilon(\omega,q)=0$. These are of two types, phonon-like with frequencies near ω_L and plasmon-like with frequencies lower than the TO frequency. The second part is to ignore the effect of the plasmon-like mode entirely; it is, in any case, severely restricted to wave-vectors near zero by Landau damping. In contrast, the phonon-like mode is unrestricted in wave vector. The third part of the model is to assume that phonon-like modes decay via the same anharmonic process that defines the lifetime of the bare mode. This assumption has yielded predictions of the dependence on temperature of lifetime that agrees with experiment for AlN, GaN, and InN and also for the dependence on density in GaN and GaAs.[10] In a further simplification, we treat GaN as a cubic crystal with a parabolic conduction band, and ignore the relatively small asymmetry of the hexagonal lattice.

In what follows, we treat the dispersion of the single mode as a function of electron temperature and electron density and then obtain the lifetime as a function of wave vector. These results are incorporated into an account of power dissipation in which we first obtain the rate of spontaneous emission of a mode with a particular wave vector, and then compute the phonon occupation number of that mode. From this, we obtain the power dissipated per electron for a particular temperature and electron density, and hence the energy-relaxation time. A count of the number of modes involved allows us to obtain an effective mode lifetime, which turns out to be of the order of the lifetime of the bare phonon. Finally, we turn to standard transport theory and obtain the momentum-relaxation time using the Callen scattering-out model[12] and the mobility. From the power density, we obtain the applied field and then the drift velocity. We show that hot coupled modes produce a negative differential resistance (NDR), and that, under certain circumstances, the momentum-relaxation rate (MRR) associated with phonon absorption is negative.

II. DISPERSION

The coupling between plasma and polar-optical modes is described in terms of the dielectric function. We limit the description to the case of the zinc-blende-type lattice, and a non-degenerate electron distribution. The dielectric function is the sum of electron and lattice components

$$\varepsilon=\varepsilon_\infty+\varepsilon_{elec}+\varepsilon_{lat}.$$

The lattice component is

$$\varepsilon_{lat}=-\varepsilon_\infty\frac{\omega_L^2-\omega_T^2}{(\omega+i\gamma)^2-\omega_T^2}, \quad (4)$$

where γ is the inverse lifetime of the lattice mode and ω_L, ω_T are the longitudinal and transverse optical phonon frequencies, respectively. The electronic contribution in RPA for a non-degenerate gas is given by

$$\frac{\varepsilon(q,\omega)}{\varepsilon_\infty}=\left(\frac{q_D}{q}\right)^2\frac{Z(S_-)-Z(S_+)}{2a}, \quad (5)$$

q is the wave-vector and Z(s) is the plasma dispersion function[13]

$$Z(s)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}dt\frac{e^{-t^2}}{t-s}, \quad (6)$$

where s is complex.

The real arguments of the plasma dispersion function are

$$S_\pm=y\pm a/2,\ \ y=\left(\frac{m^*}{2k_BT}\right)^{1/2}\frac{\omega}{q},\ \ a=\left(\frac{\hbar^2q^2}{2m^*k_BT}\right)^{1/2}, \quad (7)$$

The reciprocal screening length is obtained from $q_D^2=\frac{e^2N}{\varepsilon_\infty k_BT}$ and the plasma frequency is $\omega_p=(e^2N/\varepsilon_\infty m^*)^{1/2}$. Use of the Padé approximant method gives the following expressions for the real and imaginary parts:[14]

$$Re\left(\frac{\varepsilon(\omega,q)}{\varepsilon_\infty}\right)=\left(\frac{q_D}{q}\right)^2\frac{1}{2a}\left[\frac{S_+\{2+(\pi-2)^2S_+^2\}}{d_+}-\frac{S_-\{2+(\pi-2)^2S_-^2\}}{d_-}\right]+\frac{\omega^2-\omega_L^2}{\omega^2-\omega_T^2},$$
$$Im\left(\frac{\varepsilon(\omega,q)}{\varepsilon_\infty}\right)=\pi^{1/2}\left(\frac{q_D}{q}\right)^2\frac{1}{2a}\left[\frac{1}{d_-}-\frac{1}{d_+}\right], \quad (8)$$

where

$$d_\pm=1+(4-\pi)S_\pm^2+(\pi-2)^2S_\pm^4. \quad (9)$$

Fig. 1 shows the Re = 0 coupled-mode dispersion for electron temperatures 300 K and 3000 K and for electron

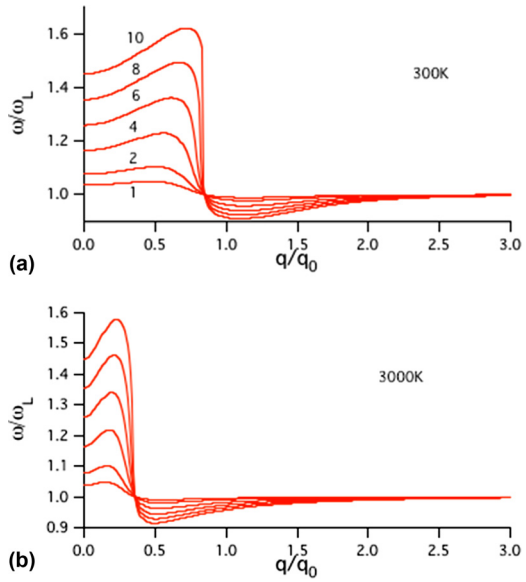

FIG. 1. Coupled-mode dispersion; frequency (normalised to the LO phonon frequency) as a function of wave-vector (normalised to q_0, where $q_0 = (2m * \omega_L/\hbar)^{1/2}$).

densities $1–10 \times 10^{18}\,\mathrm{cm}^{-3}$. Landau damping divides the dispersion into two regimes, the boundary being at the density-independent cross-over point, which we can label q*. For $q < q^*$, the frequencies are all greater than the bare LO frequency ω_L, and vice versa for $q > q^*$, and at $q = q^*$, $\omega = \omega_L$. With rising temperature, Landau damping becomes increasingly effective towards small q. For $q < q^*$, the interaction is anti-screened; for $q > q^*$, it is screened. Note that for $q > 3q_0$ $(q_0 = (2m^*\omega_L/\hbar)^{1/2})$, $\omega(q) \approx \omega_L$.

III. LIFETIME

Assuming that the modes decay via the lattice anharmonicity by the Ridley channel[15] i.e., $LO \rightarrow TO + (LA\ or\ TA)$, the rate of decay can be written

$$\frac{1}{\tau_p} = \frac{1}{\tau_0}\left(\frac{\omega - \omega_T}{\omega_L - \omega_T}\right)^3 \frac{\omega}{\omega_L}\frac{\bar{n}(\omega - \omega_T)\bar{n}(\omega_L)}{\bar{n}(\omega_L - \omega_T)\bar{n}(\omega)}, \tag{10}$$

where the bar over the phonon number denotes the value at thermodynamic equilibrium at the lattice temperature. We are assuming here that the daughter modes of the decay remain at equilibrium, an assumption that requires further analysis in future. τ_0 is the lifetime of the bare phonon, which we take as 2.5 ps.[1]

Because of the dispersion, the lifetime becomes a function of wave vector. Fig. 2 illustrates this for 300 K and 3000 K and for the same range of densities. Landau damping once more divides the behaviour into two regimes: in the anti-screening regime ($q < q^*$) the lifetime shortens with increasing density, and lengthens in the screening regime ($q > q^*$). For $q > 3q_0$, $\tau(q) \approx \tau_0$. It is clear that attempts to describe hot-phonon effects using a simple lifetime are not likely to be useful.

IV. HOT PHONONS

The rate of production of hot phonons, distinguishing its wave-vector dependence, is given by

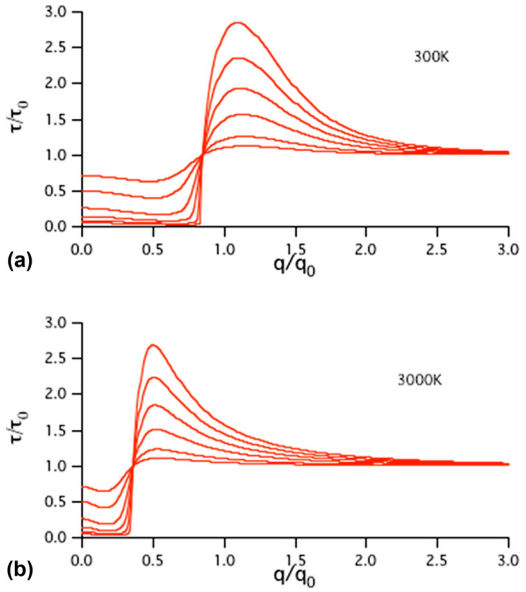

FIG. 2. Mode lifetime.

$$\frac{dn(q,\omega)}{dt} = G(q,\omega) - \frac{n(q,\omega) - n(q,\omega,T_L)}{\tau_p(q)}. \tag{11}$$

Here, $n(q,\omega)$ is the occupation number, $G(q,\omega)$ is the net rate of emission with wave-vector q and frequency ω, $n(q,\omega,T_L)$ is the thermodynamic-equilibrium number at the lattice temperature, and $\tau_p(q)$ is the coupled-mode lifetime. The net emission rate is

$$G(q,\omega) = W_{em}(q,\omega)[n(q,\omega)+1] - W_{abs}(q,\omega)n(q,\omega). \tag{12}$$

We can relate the rates in Eq. (12) using the principle of detailed balance

$$W_{abs}(q,\omega) = W_{em}(q,\omega)\frac{n(q,\omega,T_e)+1}{n(q,\omega,T_e)}, \tag{13}$$

where $n(q,\omega,T_e)$ is the number at the temperature of the electron gas. The occupation number at the steady state is then

$$n(q,\omega) = n(q,\omega,T_e)\frac{n(q,\omega,T_L)+\lambda(q,\omega)}{n(q,\omega,T_e)+\lambda(q,\omega)}, \tag{14}$$

$$\lambda(q,\omega) = W_{em}(q,\omega)\tau_p(q,\omega). \tag{15}$$

Note that W_{em} (q) is the total spontaneous emission rate for the mode of wave vector q. For a non-degenerate electron gas, it is given by[16]

$$W_{em}(q) = \frac{1}{2}W_0(q)\left(\frac{\hbar\omega(q)}{E_q^3}\right)^{1/2}\frac{N_e}{N_c}k_B T_e e^{-E_1/k_B T_e},$$
$$E_q = \frac{\hbar^2 q^2}{2m^*},\ E_1 = \frac{[\hbar\omega(q)+E_q]^2}{4E_q}, \tag{16}$$

where N_c is the effective density of states (including spin degeneracy) and N_e is the electron density. The rate $W_0(q)$ is related to the bare-phonon rate, W_0, as follows:

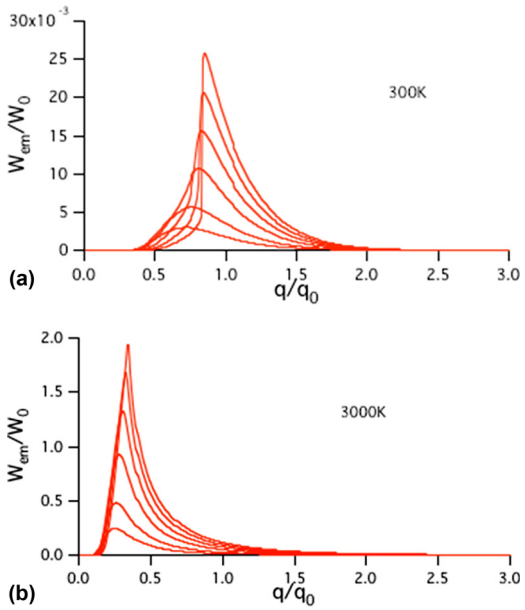

FIG. 3. Spontaneous emission rate.

$$W_0(q) = W_0\left(\frac{\omega_L}{\omega}\right)^{3/2}\frac{\omega^2-\omega_T^2}{\omega_L^2-\omega_T^2} \tag{17}$$

($W_0 = 1.36\times10^{14}\,s^{-1}$, $m^* = 0.21m_0$, $\hbar\omega_L = 0.091\,eV$).

Fig. 3 shows $W_{em}(q)$ for 300 K and 3000 K and for the same range of densities. $W_{em}(q)$ defines the wave-vector range of excited modes, roughly between 0 and $3q_0$. There is the expected increase in rate with density, and there is the now-familiar shift towards low q due to Landau damping. The increasing spikiness with density reflects the rapid transition from anti-screening to screening.

Hot-phonon effects occur only at electron temperatures above the lattice temperature, in our case 300 K. Fig. 4 shows phonon occupation number versus wave vector for 600 K and 3000 K and the usual densities. The shorter lifetimes for $q < q^*$ more than compensate for the effect of anti-screening, resulting in the production of phonons in this regime being curtailed. On the other hand, the longer lifetimes for $q > q^*$ more than compensate the effect of screening, and phonon number rises with density. The dashed line denotes the number of bare phonons at the temperature of the electrons. The actual number exceeds this as a consequence of the smaller frequency of the screened mode.

V. POWER

The energy transferred per electron to a particular mode, wave vector q, dissipates at a rate given by

$$P(q,T_e) = \frac{\hbar\omega(q)}{N_eV}\frac{[n(q)-\bar{n}(\omega_L)]}{\tau_p(q)} = \frac{\hbar\omega(q)}{N_eV}\frac{[n(q,T_e)-\bar{n}(\omega_L)]W_{em}(q)}{n(q,T_e)+\lambda(q)}, \tag{18}$$

where V is the volume. Summing over q gives the total rate

$$P(T_e) = \frac{V}{2\pi^2}\int P(q,T_e)q^2dq. \tag{19}$$

In order to illustrate the effect of hot phonons with wave-vector dependent lifetime on electron transport, we focus on two densities: $N_e = 1\times10^{18}\,cm^{-3}$ and $N_e = 6\times10^{18}\,cm^{-3}$. In the integration for power, we need to avoid cut-off errors, so we have calculated $W_{em}(q)$ in each case for $0 < q < 5q_0$. The corresponding powers are depicted in Fig. 5.

Hydrodynamic transport theory suggests that we can define an energy-relaxation time

$$\frac{3k_B}{2}\frac{(T_e-T_0)}{\tau_E} = P(T_e). \tag{20}$$

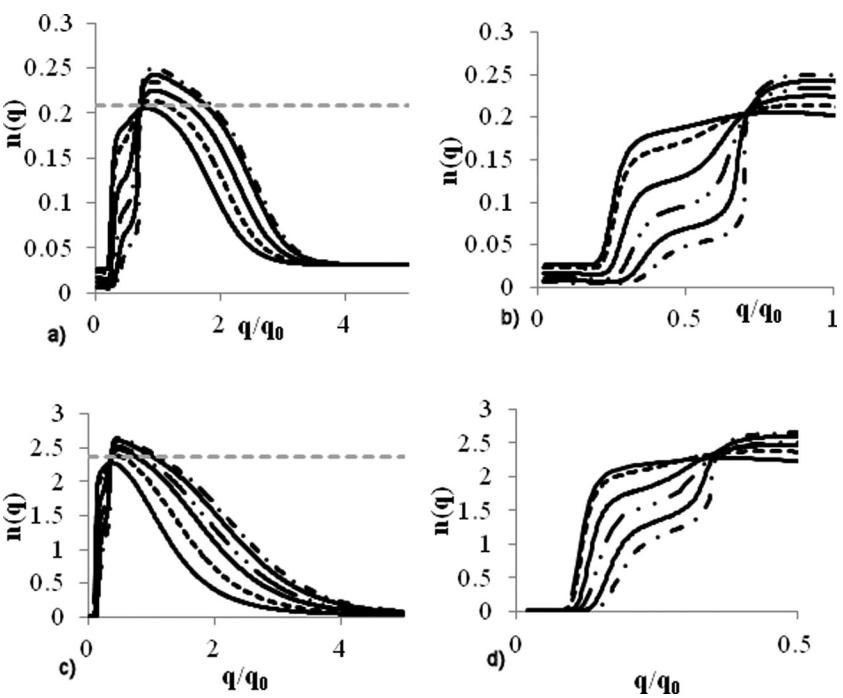

FIG. 4. Phonon occupation number. At 600 K (a) and (b), and 3000 K (c) and (d) for $N_e = 1$, 2, 4, 6, 8, $10\times10^{18}\,cm^{-3}$. The dashed line denotes the number of bare phonons at the temperature of the electrons. (a) and (c) full q range, while (b) and (d) show detail for low q.

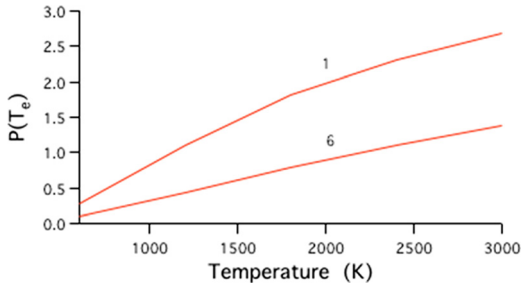

FIG. 5. Power dissipated per electron, eV/ps; numbers relate to electron densities in $\times 10^{18}\,\mathrm{cm}^{-3}$.

From the standpoint of phonon decay, a time-constant is often defined as follows:[17]

$$\frac{\hbar\omega_L[n(\omega_L,T_e)-\bar{n}(\omega_L)]}{\tau^*}=P(T_e). \tag{21}$$

Fig. 6 shows τ_E as a function of temperature for electron densities of 1 and $6\times10^{18}\,\mathrm{cm}^{-3}$. The energy relaxation time increases with density and shows a weak temperature dependence as the density increases. The left-hand side of Eq. (21) represents the rate of loss of energy of a single mode, whereas the right-hand side represents the rate of loss of all active modes. These are compatible if τ^* quantifies the total loss rate, i.e.,

$$\frac{1}{\tau^*}=\sum_i\frac{1}{\tau_{pi}}, \tag{22}$$

where τ_{pi} is the lifetime of the mode i.

VI. TRANSPORT

We need to obtain the drift velocity as a function of field. A simple estimate assuming the constancy of frequency and lifetime, (that is, for the case of bare phonons) leads to a drift velocity that saturates with field. Thus, if we assume that the mobility is inversely proportional to the phonon number and the power is proportional to the phonon number, then from $P=e\mu F^2$ we get F proportional to n and the drift velocity, v, is a constant which is the well-known solution for the hot-phonon effect. However, in the case of coupled modes, $P\propto(\hbar\omega/\tau_p)n$ and so $F\propto(\hbar\omega/\tau_p)^{1/2}$, therefore v instead of being constant, is $v\propto(\hbar\omega/\tau_p)^{1/2}$. As the electron temperature increases and the single-carrier excitation boundary shifts to smaller q, more and more of the modes become screened, with the result that the frequency reduces and the lifetime increases. The drift velocity therefore decreases with temperature and, consequently, with field, exhibiting a NDR. Our numerical solution supports the conclusion of this rough model.

We assume that the net momentum-relaxation rate, W_m, is adequately given by the scattering-in and scattering-out rates, taking into account the frequency dependence on wave vector and on coupling strength. The integrals for absorption and emission are

$$\left(\frac{W_m}{W_0}\right)_{abs}=\frac{1}{4}\left(\frac{\hbar\omega_L}{E}\right)^{3/2}\left[\int_{q_1}^{q_2}\frac{\omega}{\omega_L}\frac{(\omega/\omega_L)^2-(\omega_T/\omega_L)^2}{(\omega/\omega_L)^2(1-(\omega_T/\omega_L)^2)}\frac{n(q)}{(q/q_0)}((q/q_0)^2-\omega/\omega_L)d(q/q_0)\right]$$

$$\left(\frac{W_m}{W_0}\right)_{em}=\frac{1}{4}\left(\frac{\hbar\omega_L}{E}\right)^{3/2}\left[\int_{q_3}^{q_4}\frac{\omega}{\omega_L}\frac{(\omega/\omega_L)^2-(\omega_T/\omega_L)^2}{(\omega/\omega_L)^2(1-(\omega_T/\omega_L)^2)}\frac{(n(q)+1)}{(q/q_0)}((q/q_0)^2+\omega/\omega_L)d(q/q_0)\right]$$

$$\frac{W_{mtot}}{W_0}=\left(\frac{W_m}{W_0}\right)_{abs}+\left(\frac{W_m}{W_0}\right)_{em}, \tag{23}$$

where W_0 is the scattering rate for the bare phonon. The limits on q arising from the conservation of energy and momentum must be obtained numerically

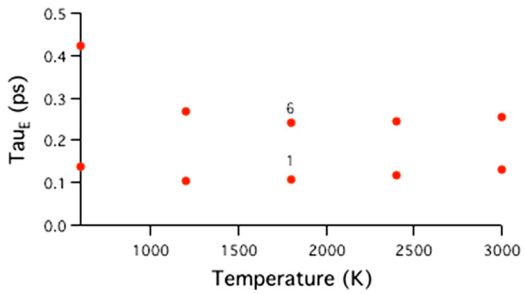

FIG. 6. Energy relaxation time, $N_e=1$ and $6\times10^{18}\,\mathrm{cm}^{-3}$.

$$q_1/q_0=\left(\frac{E}{\hbar\omega_L}\right)^{1/2}\left[\left(1+\frac{\hbar\omega_L}{E}\frac{\omega}{\omega_L}\right)^{1/2}-1\right],$$

$$q_2/q_0=\left(\frac{E}{\hbar\omega_L}\right)^{1/2}\left[\left(1+\frac{\hbar\omega_L}{E}\frac{\omega}{\omega_L}\right)^{1/2}+1\right],$$

$$q_3/q_0=\left(\frac{E}{\hbar\omega_L}\right)^{1/2}\left[1-\left(1-\frac{\hbar\omega_L}{E}\frac{\omega}{\omega_L}\right)^{1/2}\right],$$

$$q_4/q_0=\left(\frac{E}{\hbar\omega_L}\right)^{1/2}\left[1+\left(1-\frac{\hbar\omega_L}{E}\frac{\omega}{\omega_L}\right)^{1/2}\right], \tag{24}$$

Figs. 7(a) and 7(b) show examples of the energy dependence of the rates for 600 K and 3000 K for a density of $6\times10^{18}\,\mathrm{cm}^{-3}$. From W_m, we can calculate the mobility, μ,

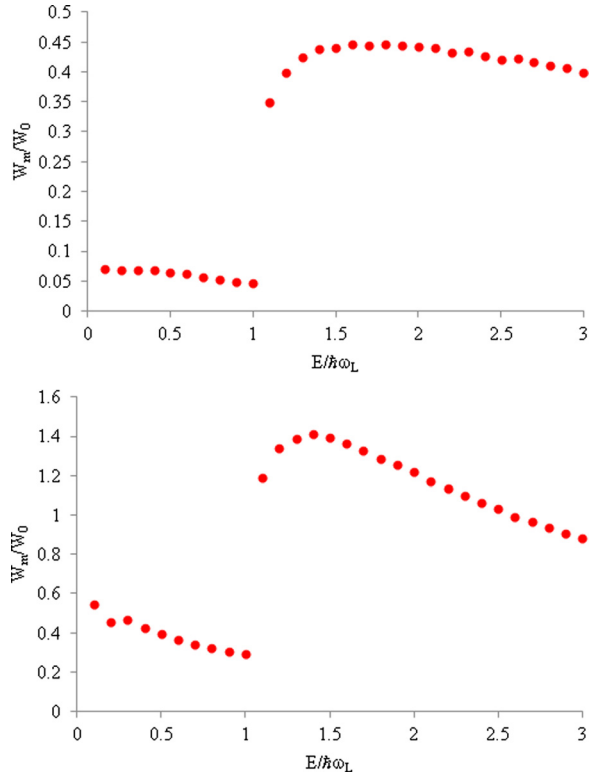

FIG. 7. Momentum relaxation rate $N_e = 6 \times 10^{18}\,cm^{-3}$ (a) 600 K, (b) 3000 K.

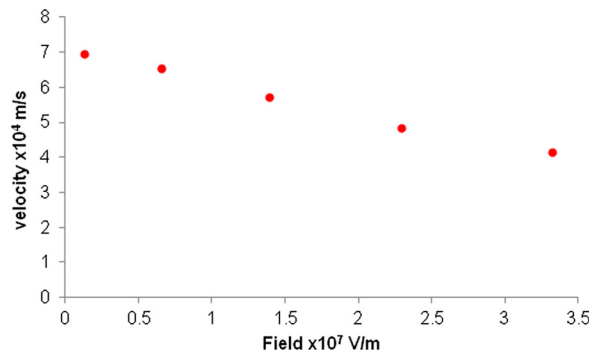

FIG. 8. Drift velocity as a function of field, $N_e = 6 \times 10^{18}\,cm^{-3}$.

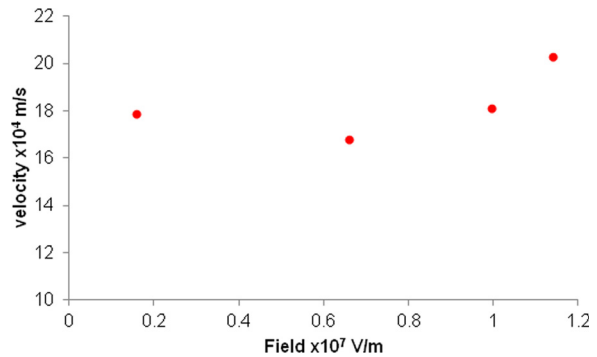

FIG. 9. Drift velocity as a function of field, $N_e = 1 \times 10^{18}\,cm^{-3}$.

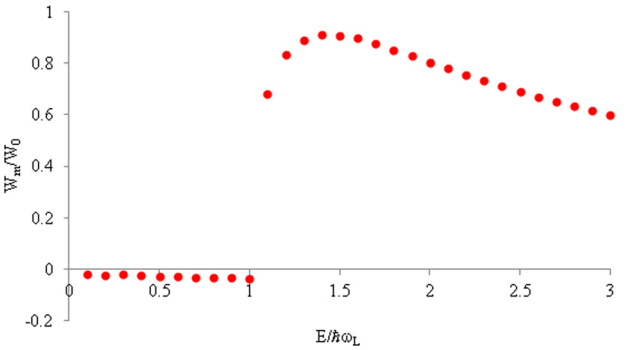

FIG. 10. Momentum relaxation rate, $N_e = 1 \times 10^{18}\,cm^{-3}$ and 3000 K.

as a function of temperature and hence obtain the field, F, corresponding to the power dissipated

$$P(T_e) = e\mu F^2 \tag{25}$$

Finally, we calculate the drift velocity as a function of the field. Fig. 8 shows the result for $N_e = 6 \times 10^{18}\,cm^{-3}$ and it exhibits a pronounced NDR. For $N = 1 \times 10^{18}\,cm^{-3}$, the NDR is much weaker (Fig. 9). Towards high fields the velocity increases rapidly. This increase is a consequence of the MRR associated with phonon absorption becoming negative (Fig. 10). The appearance of negative MRR for absorption can be attributed to the non-uniform distribution of phonons in q-space. Small-q phonons are always responsible for *increasing* the electron momentum via absorption. In the familiar case where the distribution of phonons in q-space is uniform, this negative component of MRR is more than compensated by the effect of larger-q phonons because of their higher density of states, leading to a positive MRR. (The MRR associated with emission is always positive.) In the case of hot phonons, the concentration at small q always exceeds the concentration at large q (Fig. 4). In certain cases, this non-uniformity can be large enough to counteract the effect of large-q phonons and produce a negative MRR for absorption. The resultant enhancement of mobility causes the drift velocity to increase (Fig. 9).

VII. SUMMARY

Defining coupled modes in a non-degenerate electron gas by the vanishing of the real part of the dielectric function has allowed us to associate a frequency and a lifetime with each wave-vector defined mode. Landau damping has the effect of producing two regimes, one for small q ($q < q^*$) characterised by anti-screening and a short lifetime, and another for large q ($q > q^*$) characterised by screening and lifetimes as long or longer than the lifetime of the bare mode. The rate of production of phonons marginally favours $q > q^*$ and the net effect is that this regime tends to determine hot-phonon effects. Because of this, data from Raman scattering have little bearing.

Hot-phonon effects are determined by the power dissipated by the electrons drifting in the applied electric field.

From the calculation of power dissipated per electron in GaN, regarded as a cubic crystal, we deduce an energy relaxation time of order hundreds of femtoseconds. Another time-constant can be deduced from the power that is associated with phonon decay. This is not the phonon lifetime but, rather, the reciprocal of the sum of the rates of decay of all the active modes.

The calculated phonon occupation numbers and the frequency dispersion determine the momentum-relaxation rate and subsequently the mobility of the non-degenerate electron gas. Given the mobility, the applied field associated with the power per electron can be calculated and, thereafter, the drift velocity. We find that the velocity-field curve exhibits a significant NDR. Previous works in this area have predicted velocities lower than expected for bare phonons, as we have here, and this is clearly a consequence of the increased scattering role of hot phonons. However, their velocities rise monotonically towards a saturation, a consequence of their use of a constant phonon lifetime. The NDR obtained by us is associated with a lifetime that increases with temperature and, therefore, field. A longer lifetime means more phonons and a lower mobility, hence NDR. In certain cases, the presence of a negative MRR ameliorates the effect of the longer lifetime resulting in a weak minimum in the velocity field characteristics. The longer lifetime confirms what we have said about the region of wave vector $q > q^*$ being dominant. We conclude that hot-electron effects are primarily associated with screened polar modes.

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