Quantifying the Role of Inactive Links in Social Networks

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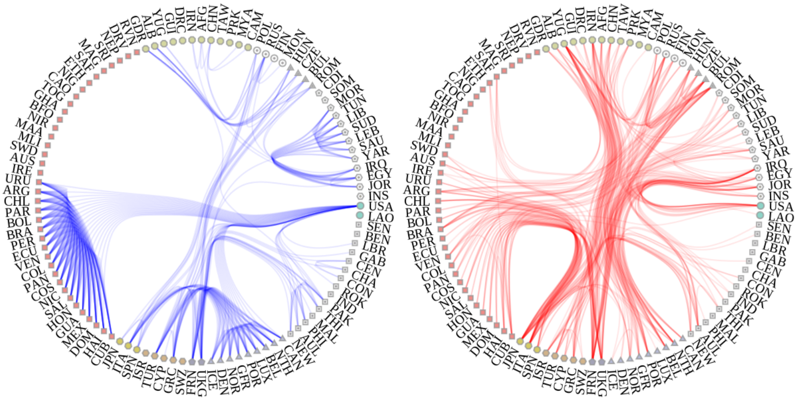
The 7th International Conference on Complex Networks and Their Applications

Cambridge, United Kingdom

2018, Dec 13

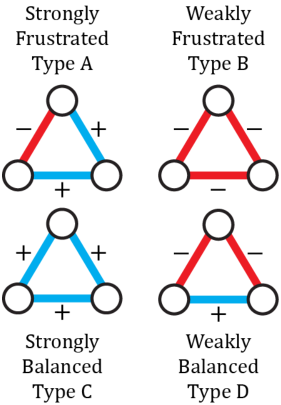
Overview

Political Networks



1960 Alliances and Disputes network of countries.

"Social Balance": emergent properties in the network of relationships



Balanced triads:

- *"A friend of a friend is also a friend."*
- *"An enemy of my enemy is my friend."*

Problem: sometimes the unbalanced (or frustrated) triangles are common.

Virtual worlds and the real world

Datasets for political networks:

Real World

- International relationships during the **Cold War (CW) era** (1949-1993).
- Extracted from the Correlates of War database.

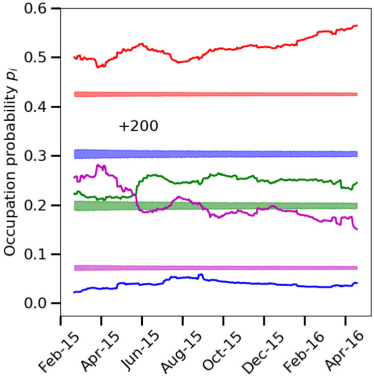
Virtual World

- From **EVE Online**: an MMORPG.
- Alliances raise taxes and control territory: player-created alliances play a role similar to that of a state.
- Diplomatic relationships between alliances of players from March 2015 to April 2016.
- Two networks: “Big alliances” (+200 members) and “Alliances with sovereignty” (SOV).

Balance: real network versus random network

In EVE Online

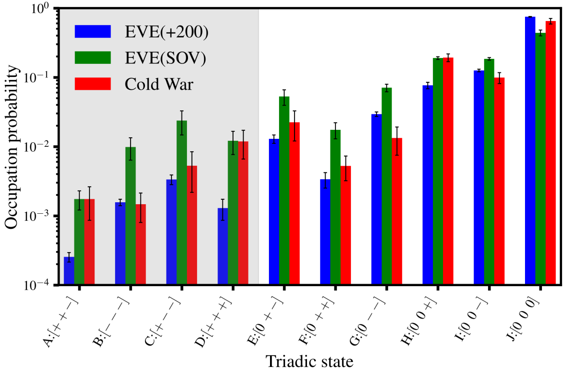
- Balanced triads [+ − −] (red) and [+ + +] (magenta) are more common than random.
- Strongly frustrated triads [+ + −] (blue) are much less common than random.
- Lowly unbalanced triads [− − −] (green) are slightly more common than random.
- Emergent features: hierarchy between triads is persistent over time and across networks.



Occupation probabilities for four types of triads for the relationships between the alliances in EVE Online

The occurrence of inactive links in political networks

- Three-node cycles with active edges typically represent a **few percent** of all possible triads in the political networks.
- The bulk of the triads involve **inactive** edges.
- Information contained in the inactive edges?



What is the effect of inactive (neutral or nonexistent) edges in political networks?

Hamiltonian approach to extended social balance

- Energy $\sim$ occupation probability (population of lower-energy states is higher)
- Adding inactive edges: edge attribute $s_{ij} = \{-1, 0, +1\}$
- Hamiltonian for the generative mechanisms in the network

$$\mathcal{H}\left(\{s_{ij}\}\right) = \frac{1}{6}\sum_{i\neq j\neq k=1}^{N}\Bigg[\underbrace{-\,\alpha\; s_{ij}s_{ik}s_{jk}}_{\text{three-edge interaction}}\;\underbrace{-\,\gamma\;\left(s_{ij}s_{ik}+s_{ij}s_{jk}+s_{jk}s_{ik}\right)}_{\text{two-edge interaction}}\Bigg]$$
$$+\frac{1}{2}\sum_{i\neq j=1}^{N}\Bigg[\underbrace{+\omega\; s_{ij}}_{\text{one-edge interaction}}\;\underbrace{+\,\mu\; s_{ij}^2}_{\text{chemical potential}}\Bigg],$$

Two applications (predictive power):

1. Average sign of a link (L) and the fraction of active links (A)
2. Transition probabilities between different triadic states

Mean-field approach to the proposed Hamiltonian

Using a mean-field approximation and the global network properties:

- Magnetization $L \equiv \langle s_{ij} \rangle \qquad (-1 \leq L \leq +1)$
- Activation $A \equiv \left\langle s_{ij}^2 \right\rangle \qquad (0 \leq A \leq +1)$
- Non-trivial relationship between network properties (data collapse?)

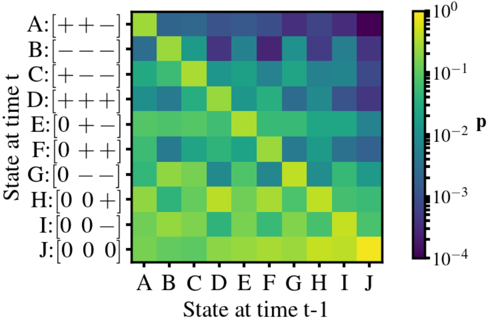
$$\mathcal{G}_{MF}(L, A) \equiv \frac{\operatorname{arctanh}\left(\frac{-L}{A}\right)}{\ln\left(\left[\frac{1}{A} - 1\right] 2\cosh\left[\operatorname{arctanh}\left(\frac{-L}{A}\right)\right]\right)} \approx \frac{\omega}{\mu} - \frac{2\gamma}{3\mu}(N-2)L \, .$$

Measuring transition probabilities between triadic states

The transition probabilities between the triadic states σ and σ':

$$P(\sigma \to \sigma') \propto \exp -\beta \left(E_{\sigma'} - E_{\sigma} \right)$$

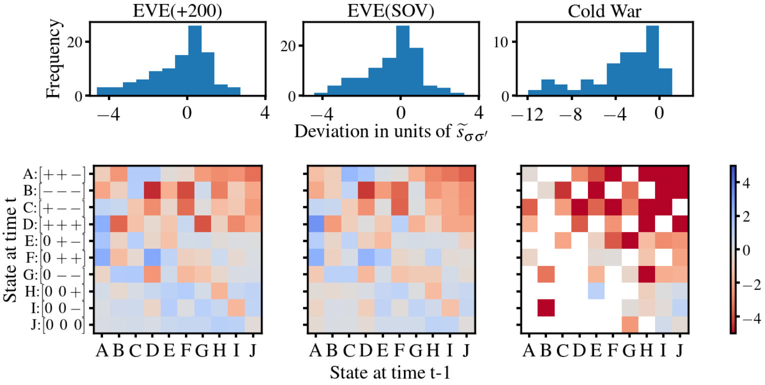
Energy decreasing transitions are more common than energy increasing ones.



Example of transition probabilities (consecutive days) between triadic states for alliances in EVE Online.

Predicting transition probabilities between triadic states

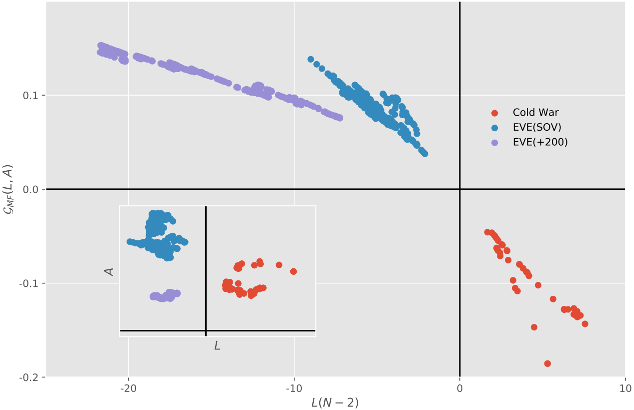
The transitions are consistent with the energy differences.



In units of the standard deviation, differences between the predicted and measured transition probability.

Relation between global network measures

$$\mathcal{G}_{MF}(L,A) \equiv \frac{\mathrm{arctanh}\left(\frac{-L}{A}\right)}{\ln\left(\left[\frac{1}{A}-1\right]2\cosh\left[\mathrm{arctanh}\left(\frac{-L}{A}\right)\right]\right)} \approx \frac{\omega}{\mu} - \frac{2\gamma}{3\mu}(N-2)L$$



- L: magnetization (average sign of a link)
- A: activation (fraction of active links)

Conclusions

- Inactive edges in political networks are a source of information.
- Selected properties of relationships in political networks are remarkably constant across time and networks.
- A Hamiltonian approach to social balance is proposed.
- The proposed model has predictive power and can uncover generative mechanisms
  1. The activation of links can be related to an "activation energy".
  2. The transition probabilities in our data are consistent with the differences in energies.
  3. The mean-field approximation allows one to define and calculate the systems magnetization (L) and average activation (A).

Thanks

- Belaza, Andres M. et. al. (2017)

  Statistical physics of balance theory
  PLOSOne 12(8): e0183696 .
- Belaza, Andres M. et. al. (2018)
  Social Stability and Extended Social Balance -
  Quantifying the Role of Inactive Links in Social Networks
  arXiv:1807.09042

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