VARIABLE AND CLASS-DEPENDENT SERVICE CAPACITY WITH A MULTI-CLASS ARRIVAL PROCESS

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KEYWORDS

Batch Service,Variable Service Capacity, N customer types

ABSTRACT

In manufacturing, a single batch server can often group a number of customers that require the same type of service. In this paper, a shared queue without customer reordering is used in order to reduce the variability of throughput time of material throughout the manufacturing process which guarantees a global First-Come-First-Served (FCFS) service discipline. This is a significant difference with the more common polling systems where each type of customer has a dedicated queue. The batch server in this paper has a variable service capacity that depends on the classes of the customers in the queue. This paper extends previous work by considering a general number of N customer classes. During the analysis, we focus on the system occupancy of this system at random slot boundaries.

INTRODUCTION

Customer differentiation has been studied mostly in the context of polling systems, for instance by Boxma et al. (2008) , Goswami et al. (2006), Dorsman et al. (2012) or Fowler et al. (2002). These types of models use a different queue for each class of customer. However, this is not always feasible because a more complicated structure is needed to filter the arrivals which increases the operational cost of the system. In this paper, we use a single shared queue for all customer classes and there is no reordering of customers allowed. This results in a global FCFS service discipline which also ensures a consistent throughput time flow of products throughout the manufacturing process. A consistent flow can be a system requirement in manufacturing for accurately predicting the delay until order completion. A global FCFS service discipline can also be used in telecommunications where strict fairness rules are required. This is described in more detail by, for instance, Avi-Itzhak and Levy (2004).

The previously mentioned papers on polling systems also incorporated batch service but they assumed the service capacity to be constant. We will look at a batch server with a stochastic capacity. Examples of these types of models can be found in Chaudhry and Chang (2004), Sikdar and Samanta (2016) or Pradhan et al. (2015). All these examples have in common that the service capacity is independent of the state of the system and the customers in the queue. In Germs and Foreest (2013), the authors looked at a system where many of the parameters such as the service capacity and service time are dependent on the number of waiting customers.

The batch server in this paper is capable of grouping all customers of the same class but only up to the first customer of another class due to the lack of customer reordering in the shared queue. This results in a stochastic service capacity that depends on the classes of the waiting customers. Since the server can group only same-class customers, the length of such a sequence will have a key impact on the performance of this system. For this reason, we incorporate a tendency for same-class clustering in the arrival process which has been studied before for the case of two classes and without batch service in, for instance, Maertens et al. (2012). In manufacturing, this tendency often occurs due to sorting the schedule over short intervals resulting in correlation between the classes of consecutive customers.

In Baetens et al. (2016), we looked at the system occupancy of a system that combines the previously described variable capacity batch server and 2-class arrival process with clustering. The delay of this model has been studied in Baetens et al. (2018). The main contribution of our paper is extending previous work that only looked at arrival processes with 2 classes to a more generic case with N different customer classes which significantly alters the analysis. An overview of the analysed system is shown in Figure 1. The focus of the paper is the probability generating function (pgf) of the system occupancy at random slot boundaries. Differentiating between N customer classes changes the behaviour of the system since the alternation of customer classes is no longer guaranteed which significantly increases the complexity. An important part of the analysis is the proof that the denominator of the pgf of the system occupancy has N zeroes inside the unit circle which allows us to find a unique solution for all unknowns we introduced during the analysis.

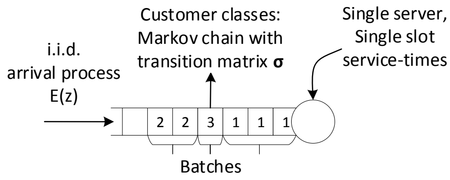

Figure 1: Overview of the system

We first give a more detailed description of the discrete-time queueing system. During the analysis of this system, we focus on the stability condition and the system occupancy at random slot boundaries. Afterwards, we present a numerical example in order to look at the impact of variance in the arrival process, and finish with drawing some conclusions.

MODEL DESCRIPTION

We start by looking at the arrival process of the discrete-time queueing system being analysed. The number of arrivals in consecutive slots is independent and identically distributed and the total number of arrivals in a single slot follows a generic distribution with the probability mass function (pmf) $e(n)$ and generating function $E(z)$ with a mean arrival rate of $\lambda = E'(1)$. We distinguish N different customer classes in the arrival stream of packets. In order to model the tendency for clustering in the arrival process, we introduce correlation between the classes of consecutive customers by using the transition matrix $\boldsymbol{\sigma}$ given by

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{1,1} & \cdots & \sigma_{N,1} \\ \vdots & \sigma_{i,j} & \cdots \\ \sigma_{1,N} & \cdots & \sigma_{N,N} \end{bmatrix} ,$$

where $\sigma_{i,j}$ is the probability that the class of a random customer is of class j given that its predecessor is of class i. When $\sigma_{i,i}$ is larger than the probability that a random customer belongs to class i, then the expected length of a sequence of class i customers is larger than if there would be no correlation between consecutive customers which results in a tendency for clustering of class i customers.

As mentioned earlier, we use a single shared queue for customers of all classes and do not allow reordering of customers in this queue. The result of these restrictions is a global FCFS-service discipline. An advantage of using a shared queue is that the complexity of the system is lower than with dedicated queues resulting in a reduced cost. A disadvantage of the decreased complexity is that no optimizing of the arrival stream is possible, generally resulting in a decreased performance.

The last part of the model is the service process. The system has a single batch server that can take all same-class customers that are waiting at the head of the queue. We place no limit on the maximum service capacity but in practice the service capacity is limited by the length of a sequence of same-class customers, which follows a geometric distribution with parameter $\sigma_{i,i}$. This means that the degree of clustering, determined by the probability $\sigma_{i,i}$ will play an important role in the performance of the system. The service time of a batch of class i customers does not depend on the size of the batch, nor on the class of the customers in it, and is always equal to a single slot.

ANALYSIS

During the analysis, we will first give the system equations that capture the behaviour of the system at random slot boundaries. Then we will look at the stability condition by looking at a saturated system. At the end of the analysis, we will obtain a closed-form expression for the steady-state probability generating function of the system occupancy at random slot boundaries.

System Equations

In order to find the equations that capture the behaviour of the system, we will first define three random variables u_k, t_k and c_k. The random variable u_k represents the system occupancy at the k-th slot boundary. Because the server only groups customers that belong to the same class, we can define the type or class of a batch as the class of the customers in it. The class of the most recently initiated batch is denoted by t_k. This corresponds with the class of the batch currently being processed if $u_k > 0$ or the previous batch if $u_k = 0$. Lastly, the variable c_k corresponds to the number of customers in service during slot k which follows a geometric distribution with parameter $\sigma_{i,i}$ for a batch of class i customers limited by the system occupancy.

We can distinguish three cases in the system equations. The first case occurs when either the server is idle in the k-th slot or all customers in the system are in service, $u_k = c_k$, which results in an empty queue. If there were also no arrivals during slot k, then the system will be idle in slot $k+1$ and $t_{k+1} = t_k$. Otherwise, if there was at least one arrival during slot k, then a new service will be initiated and the class of the batch will be determined by the class of the first arrival and the system occupancy will be equal to the total number of arrivals during the k-th slot. Since the single server can only group customers belonging to the same class, the service capacity c_{k+1} follows a geometric distribution limited by the number of customers in the system. Lastly, when not all customers in the system at the k-th slot boundary belonged to the same class or $c_k < u_k$ then there will be at least one customer left behind in the queue at the next slot boundary which means a new service can be initiated and the class of the customers in this batch cannot be the same as the previous batch. Otherwise the customer(s) at the head of the queue would also have been processed in the k-th slot.

If we now assume that $t_k = i$, we obtain the following system equations

$$(u_{k+1}, t_{k+1}, c_{k+1}) = \begin{cases} (0, i, 0) \text{ if } u_k = c_k \ \& \ e_k = 0 \ , \\ (e_k, T_{i,k}, \min(G(t_{k+1}), e_k) \\ \qquad \text{if } u_k = c_k \ \& \ e_k > 0 \ , \\ (u_k - c_k + e_k, T'_{i,k}, \min(G(t_{k+1}), u_{k+1})) \\ \qquad \text{if } 0 < c_k < u_k \ , \end{cases} \tag{1}$$

where $T_{i,k} \in \{1, \ldots, N\}$ is class j with probability $\sigma_{i,j}$, $T'_{i,k} \in \{1, \ldots, N\} \backslash \{i\}$ is of class j with probability $\sigma_{i,j}/(1 - \sigma_{i,i})$ and $G(i)$ is a geometrically distributed random variable with parameter $\sigma_{i,i}$.

Stability Condition

In order to obtain a condition under which the system is stable, we will look at a saturated system. That is a system where there are always more than enough waiting customers so that the system is never idle and the service capacity is not limited by the number of waiting customers. More details on this method can be found in Baccelli and Foss (1995). With this assumption, the system equations of Eq. are reduced to the last line since $0 < c_k < u_k$ always holds true. If we denote the steady-state probability that a random batch contains class i customers by $Pr[t = i]$, then by using the definition

$$[\boldsymbol{\sigma'}]_{i,j} := \begin{cases} 0 \text{ if } i = j, \\ \frac{\sigma_{i,j}}{1-\sigma_{i,i}} \text{ otherwise} \ , \end{cases}$$

where i and j are respectively the row and column indices, we obtain the following equations for these probabilities

$$\begin{bmatrix} Pr[t = 1] \\ Pr[t = 2] \\ \vdots \\ Pr[t = N] \end{bmatrix} = \boldsymbol{\sigma'} \begin{bmatrix} Pr[t = 1] \\ Pr[t = 2] \\ \vdots \\ Pr[t = N] \end{bmatrix} \ .$$

We obtain the probability that a random batch contains class i customers by solving this set of N equations with the additional equation for the sum of the probabilities $\sum_{i=1}^{N} Pr[t = i] = 1$.

The stability condition dictates that the mean number of customers arriving in a slot, defined in the model description as λ, must be smaller than the mean number of customer leaving. Since the size of a sequence of class i customers follows a geometric distribution with the parameter $\sigma_{i,i}$ and expected length of $\frac{1}{1-\sigma_{i,i}}$ customers, the system will only be stable if and only if the following condition holds

$$\lambda < \sum_{i=1}^{N} Pr[t = i] \frac{1}{1 - \sigma_{i,i}} \ . \tag{2}$$

System Occupancy

As mentioned earlier, we will focus on obtaining the steady-state pgf of the system occupancy at random slot boundaries. The first step is to find the probabilities that the system is idle in a random slot and the most recently initiated batch contained class i customers, denoted by $U_{I,i}$. Then we will calculate the partial pgfs $U_i(z)$, with the corresponding pmf $u_i(n) = \lim_{k\to\infty} Pr[u_k = n, t_k = i]$, of the system occupancy at random slot boundaries on which the server initiated a batch of class i customers. The probabilities $U_{I,i}$ can be calculated by using the first case in Eq. (1). We obtain that

$$\begin{aligned} U_{I,i} =& U_{I,i} E(0) + E(0) \sum_{m=1}^{\infty} u_i(m) \sigma_{i,i}^{m-1} \\ U_{I,i} =& \frac{E(0)}{1 - E(0)} \frac{U_i(\sigma_{i,i})}{\sigma_{i,i}} \ . \end{aligned} \tag{3}$$

The partial pgfs $U_i(z) = \sum_{m=1}^{\infty} u_i(m) z^m$ $(1 \le i \le N)$ of the system occupancy at random slot boundaries in which a batch of class i customers is initiated, are obtained by using the system equations that lead to a service initiation. These system equations correspond with the second and third case of Eq.

(1), resulting in

$$U_i(z) = \sum_{j=1}^{N} \lim_{k\to\infty} \sigma_{j,i} E[z^{e_k} | u_k = c_k, e_k > 0,$$
$$t_k = j, t_{k+1} = i] + \sum_{j\neq i} \lim_{k\to\infty} \frac{\sigma_{j,i}}{1-\sigma_{j,j}}$$
$$\cdot E[z^{u_{k+1}} | 0 < c_k < u_k, t_k = j, t_{k+1} = i] \ .$$

Since the service times of classes of all batches are single slots, this leads to

$$\begin{aligned} U_i(z) &= \sum_{j=1}^{N} U_{I,j}(E(z) - E(0))\sigma_{j,i} \\ &+ \sum_{j=1}^{N} \frac{U_j(\sigma_{j,j})}{\sigma_{j,j}}(E(z) - E(0))\sigma_{j,i} \\ &+ \sum_{j\neq i} \sum_{m=1}^{\infty} \sum_{n=1}^{m-1} u_j(m) \sigma_{j,j}^{n-1} z^{m-n} E(z) \sigma_{j,i} \\ &= \sum_{j=1}^{N} \frac{\sigma_{j,i} U_j(\sigma_{j,j})}{\sigma_{j,j}} \frac{E(z) - E(0)}{1 - E(0)} \\ &+ \sigma_{j,i} E(z) \sum_{j\neq i} \frac{z U_j(\sigma_{j,j}) - \sigma_{j,j} U_j(z)}{\sigma_{j,j}(\sigma_{j,j} - z)} \ . \qquad (4) \end{aligned}$$

Each class i has such an equation, we obtain the following matrix equation

$$[\boldsymbol{A}]_{i,j} := \begin{cases} 1 \text{ if } i = j, \\ \frac{\sigma_{j,i} E(z)}{\sigma_{j,j} - z} \text{ otherwise} \end{cases},$$
$$\boldsymbol{A} \cdot \begin{bmatrix} U_1(z) \\ \vdots \\ U_N(z) \end{bmatrix} = \begin{bmatrix} B_1(z) \\ \vdots \\ B_N(z) \end{bmatrix} \ ,$$

where i and j respectively correspond with the class of the new and previous service, and $B_i(z)$ is equal to

$$B_i(z) := \sum_{j=1}^{N} \frac{\sigma_{j,i} U_j(\sigma_{j,j})}{\sigma_{j,j}} \frac{E(z) - E(0)}{1 - E(0)} + \sum_{j\neq i} \frac{z\sigma_{j,i} E(z)}{\sigma_{j,j} - z} \frac{U_j(\sigma_{j,j})}{\sigma_{j,j}} \ .$$

At this point, we have a set of N equations for the N partial pgfs $U_i(z)$ which allows us to find a unique expression for each $U_i(z)$. By summing all partial pgfs $U_i(z)$ and the idle probabilities in Eq. (3), we obtain the steady-state pgf $U(z)$ of the system occupancy at random slot boundaries

$$U(z) = \sum_{i=1}^{N} U_{I,i} + \sum_{i=1}^{N} U_i(z) \ .$$

From Eq. (3) and (4), it is clear that there are still N remaining unknowns, namely $U_i(\sigma_{i,i})$ for $1 \leq i \leq N$, in this expression. The denominator $D_N(z)$ of the pgf $U(z)$ of the system occupancy at random slot boundaries is equal to the determinant of $\boldsymbol{A}$ multiplied with the term $\prod_{i=1}^{N}(z - \sigma_{i,i})$ in order to eliminate all denominators in $B_i(z)$. Due to the properties of the determinant, this is equal to multiplying each column j with $(z - \sigma_{j,j})$, which results in

$$[\boldsymbol{D}(z)]_{i,j} = \begin{cases} z - \sigma_{i,i} \text{ if } i = j, \\ -\sigma_{j,i} E(z) \text{ otherwise} \end{cases},$$
$$D_N(z) = |\boldsymbol{D}(z)| \ .$$

In the last part of the analysis, we will prove that this denominator has N zeroes inside the circle $|z| = 1 + \epsilon$. These N zeroes will be used to obtain a set of N equations that will allow us to find the N remaining unknowns $U_i(\sigma_{i,i})$, for $1 \leq i \leq N$. This proof is based on work done by Chaudhry et al. (2016) where a similar result was obtained by using induction on the size of the matrix. For the case that N is equal to 1, we obtain $(z - \sigma_{1,1})$ for the denominator which clearly has a single zero inside the unit circle. In the next part, we assume that $D_{N-1}(z)$ has $N-1$ zeroes inside or on the unit circle and try to use this assumption to prove that $D_N(z)$ has N zeroes in the same area. We first write $D_N(z)$ as

$$D_N(z) = (z - \sigma_{N,N}) D_{N-1}(z) - \sum_{i=1}^{N-1} \sigma_{N,i} E(z) C_{N,i}(z) \ ,$$

where $C_{N,i}(z)$ corresponds with the cofactor of the element at row i and column N. By substituting $|y_{N,i}(z)| = \frac{|C_{N,i}(z)|}{|D_{N-1}(z)|}$, we obtain

$$\left|\frac{D_N(z) - (z - \sigma_{N,N})D_{N-1}(z)}{(z - \sigma_{N,N})D_{N-1}(z)}\right| = \left|\frac{\sum_{i=1}^{N-1} \sigma_{N,i} E(z) C_{N,i}(z)}{(z - \sigma_{N,N})D_{N-1}(z)}\right| \leq \frac{\sum_{i=1}^{N-1} \sigma_{N,i} |E(z)||y_{N,i}(z)|}{|z - \sigma_{N,N}|} \quad , \tag{5}$$

We can also prove that the following inequality holds for $1 \leq i \leq N$

$$|z - \sigma_{i,i}| >| \sum_{j=1, j\neq i}^{N} \sigma_{i,j} E(z)| \quad . \tag{6}$$

By using the Taylor expansion, the left-hand and right-hand side of the inequality can be approximated respectively by

$$\begin{aligned} |z - \sigma_{i,i}| &\geq 1 - \sigma_{i,i} + \epsilon \\ | \sum_{j=1, j\neq i}^{N} \sigma_{i,j} E(z)| &\leq (1 + \lambda\epsilon + O(\epsilon^2))(1 - \sigma_{i,i}) \quad . \end{aligned}$$

By trying to prove the opposite of Eq. (6), we get a result that violates the stability condition of Eq. (2) which proves that the inequality holds on the circle $|z| = 1 + \epsilon$. By combining Eq. (6) with the observation that the modulus of each entry of $D_N(z)$ is less than 1, we see that the entries satisfy Hadamard's condition on the circle $|z| = 1 + \epsilon$. This implies that $|y_{N,i}(z)| < 1$ for $1 \leq i < N$ which, based on Eq. (5), results in the following inequality

$$\left|\frac{D_N(z) - (z - \sigma_{N,N})D_{N-1}(z)}{(z - \sigma_{N,N})D_{N-1}(z)}\right| < \frac{\sum_{i=1}^{N-1} \sigma_{N,i} |E(z)|}{|z - \sigma_{N,N}|} < 1 \quad ,$$

where we again used Eq. (6) that indicates that the modulus of the diagonal elements is larger than the sum of the modulus of the remaining elements on the row. This means that $|f(z)| > |g(z)|$, where $f(z) = (z - \sigma_{N,N})D_{N-1}(z)$ and $g(z) = D_N(z) - (z - \sigma_{N,N})D_{N-1}(z)$. Rouché's theorem, see Adan et al. (2006), then indicates that $f(z)$ and $f(z) + g(z)$ have the same number of zeroes inside the unit circle $|z| = 1$. Since we assumed at the start of the proof that $D_{N-1}(z)$ has $N - 1$ zeroes and $(z - \sigma_{N,N})$ clearly has a single zero in the same area, $D_N(z) = f(z) + g(z)$ has N zeroes. These N zeroes, assuming they are all distinct, allow us to form a set of N equations to find a unique solution for the remaining unknowns. However, if not all zeroes are distinct, then it is an indication of a high degree of symmetry between two or more classes in the arrival process. This symmetry can be used to complete the set of equations necessary for finding a unique solution.

NUMERICAL EXAMPLES

During our discussion of some numerical examples, we will focus on the impact of variance in the arrival process. For this reason, we use an arrival process that is the weighted sum of two geometrics with the following pgf

$$E(z) = \frac{a}{1 + \frac{\lambda}{2a}(1 - z)} + \frac{1 - a}{1 + \frac{\lambda}{2(1-a)}(1 - z)} \quad ,$$

where the parameter a is chosen so that the variance in the arrival process with parameter a is equal to $\sigma_a^2 = \nu\sigma_{0.5}^2$ where $\sigma_{0.5}^2$ is the variance of the geometric distribution with mean λ. In our examples we look at three customer classes, $N = 3$, and the matrix $\boldsymbol{\sigma}$ that contains the transition probabilities, is given by

$$\boldsymbol{\sigma} = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \quad .$$

In Figure 2 and 3, we look at the impact of variance in the arrival process on the mean system occupancy and the idle probability (which is the sum of all $U_{I,i}$ with $1 \leq i \leq N$) by changing the parameter ν. The parameter ν is used to compare the variance in the

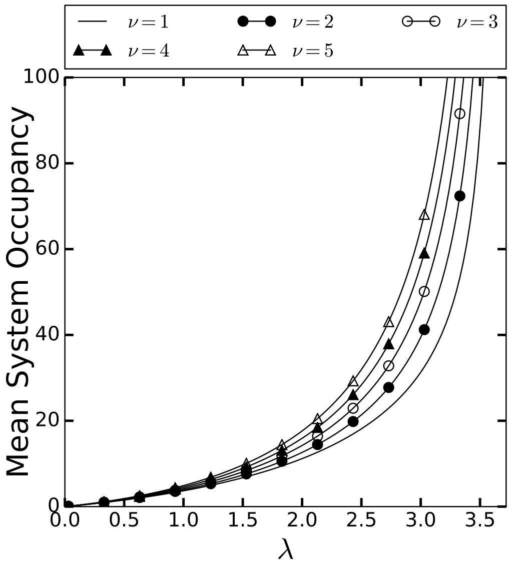

Figure 2: Impact of increased variance in the arrival process on the mean system occupancy

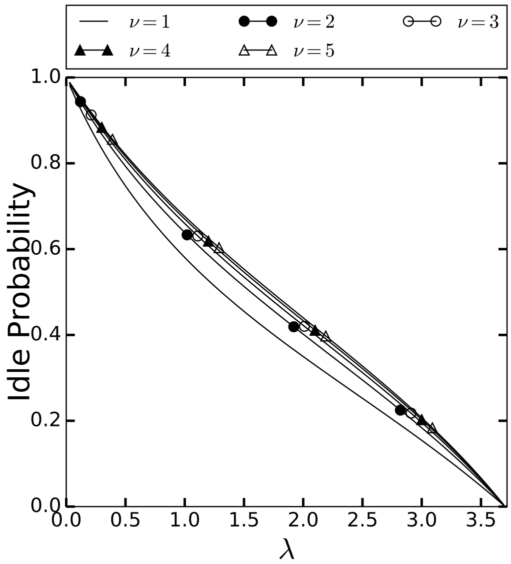

Figure 3: Impact of increased variance in the arrival process on the idle probability

arrival process to the variance of a normal geometric distribution. In Figure 2, we first notice that an increased variance in the arrival process significantly increases the mean system occupancy. One important note is that the impact of higher variance becomes stronger when the arrival rate is higher. This observation is however not true for the probability that the server is idle in a random slot. In Figure 3, we notice that although a higher variance always increases the probability that the server is idle, a point is rapidly reached at which increasing the variance further only leads to a negligible change in the idle probability. We also observe that the line for $\nu = 1$ (minimal variance) first decreases faster than the other lines but for higher arrival rates, the lines again move closer together. This means that there is an arrival rate at which the impact of the degree of variance in the arrival process reaches a maximum value and increasing the arrival rate does not further increase this impact which is different from what we observed for the mean system occupancy in Figure 2.

CONCLUSIONS

In this paper, we analysed a multi-class system with a batch server that has a service capacity that depends on the content of the queue. More specific, the batch server can group all waiting customers at the head of the queue as long as they belong to the same class. Due to the way customers are grouped together, we also incorporated a method to include clustering of same-class customers in the arrival stream. During the analysis, we looked at the condition under which the system is stable and we obtained the steady-state pgf of the system occupancy at random slot boundaries. In the numerical examples, we focused on the impact of variance in the arrival process on the idle probability and mean system occupancy. We noticed that an increase of the variance in the arrival process always leads to a significant increase of the mean system occupancy but that this is not the case when we look at the idle probability. For this performance value, a point is quickly reached at which further increasing the variance in the arrival process has a negligible impact. The main extension that we are working towards is to include a more complex service process with class-dependent service and switch-over times which would allow us to more accurately model systems where switching between types can take a long time, for instance in manufacturing.

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