INTEGRATED EPQ AND PERIODIC CONDITION-BASED MAINTENANCE

Alp Darendeliler
Dieter Claeys
El-Houssaine Aghezzaf
Department of Industrial Systems
Engineering and Product Design
Ghent University, Belgium
and
Industrial Systems Engineering
(ISyE), Flanders Make

Abdelhakim Khatab
Industrial Engineering and Production
Laboratory, Lorraine University
Metz, France

KEYWORDS
Condition-based maintenance, Economic Production Quantity.

ABSTRACT

In this paper, a stochastically deteriorating production system is studied under condition-based maintenance. Periodic monitoring is carried out to observe the degradation level of the system. If the degradation level exceeds failure threshold, nonconforming items are produced and a high corrective maintenance cost is incurred. Preventive maintenance actions are performed to reduce the possibility of failures. By considering inspection interval, preventive maintenance level and lot-size as decision variables, an integrated model is developed to minimize long-run average cost rate consisting of inspection costs, maintenance cost, cost of producing nonconforming items, inventory holding cost and setup costs. An illustrative example is presented to analyze the model.

INTRODUCTION

Condition-based maintenance (CBM) is an approach which recommends maintenance actions according to the current status of the production system through condition monitoring (Jardine et al., 2006). Due to the development of the sensor technologies, the current degradation level of the manufacturing systems can be monitored. The other maintenance approaches are classified as unplanned (breakdown) maintenance and time-based (planned) preventive maintenance (Martin, 1994). Unplanned maintenance takes place when a failure occurs. In this case, unexpected failures can interrupt the production plan and cause lost sales. Thus, it is an inefficient approach. Under time-based maintenance, periodic preventive maintenance actions are performed at certain points in time to reduce the possibility of failures. The current health status of the system is not taken into account. Therefore, unnecessary preventive maintenance actions could be done when there is remaining health of the system. CBM can eliminate these unnecessary maintenance actions by taking maintenance actions according to the state of the system through conditional monitoring.

The degradation of the components can either be monitored via continuously, periodically or non-periodically. Continuous monitoring gives real-time data about the degradation level of the system. However, in some production systems like pipelines buried underground in oil and gas industries, performing continuous monitoring is not applicable (Alaswad and Xiang 2017). These systems can be monitored at certain points in time to assess the degradation level of the components. For the cases where the inspection costs are high, an economic inspection policy needs to be found to reduce the overall cost.

Ben-Daya and Makhdoum (1998) consider an integrated production and quality model for different inspection policies. The deterioration process is modeled by a hazard rate function. Another integrated optimization of lot-sizing and preventive maintenance level, is developed by Ben-Daya (2002). The age-based maintenance policy is applied. In that model, optimal preventive maintenance level and inspection intervals are found to minimize the total cost of production and maintenance. Chen (2013) constructed a model to optimize EPQ and preventive maintenance level under Weibull shock model. Jafari and Makis (2015) studied optimal lot-sizing and preventive maintenance policy where the deterioration is modeled by the proportion hazards model.

In the literature, joint optimization EPQ and CBM has been studied. Peng and Van Houtum (2016) proposes a model to jointly optimize the total cost rate associated with the production lot-sizing and condition-based maintenance. In their model, they assume that the degradation process is continuously monitored and it is modeled as a continuous time and continuous state stochastic process. Khatab et al. (2017) develop a model to minimize long-run average cost rate of total production and maintenance costs by finding optimal values of preventive maintenance level and inspection intervals. Gamma process is used for modeling the degradation. However, the lot-size is not optimized. Cheng et. al (2017) develop a model for joint optimization of lot-sizing and CBM for multiple dependent items that are economically dependent. Inspections are carried out at the end of the production lots. They use simulation and a genetic algorithm to find the optimal values of lot-size and preventive maintenance level.

In the literature, periodic monitoring has not been considered for the optimization of lot-sizing and CBM. In this study, an integrated model is developed to optimize production and maintenance costs simultaneously under condition-based maintenance where the system is monitored periodically to observe the degradation level. The degradation of the component is modeled as a stationary Gamma Process which fits well for modeling temporal variability of deterioration (Van Noortwijk, 2009). A considerable cost is charged for each inspection so it is necessary to determine an optimal inspection interval in order to minimize the total cost rate. Nonconforming items are produced in case the system degradation level exceeds the failure threshold. Additional cost is incurred due to the production of nonconforming items. The setup cost, inventory holding cost, preventive and corrective maintenance cost, inspection cost and cost of producing nonconforming items are considered for the minimization of the total long-run average cost rate.

MODEL DESCRIPTION

A single component and single item production system is considered in this paper. The deterioration level of the component can only be observed upon completion of the production lot and it is modeled as a stationary Gamma process $X(t)$, which is a monotonically increasing function. The system is out of control when the degradation level exceeds failure threshold X_f. In this case, nonconforming items are produced and additional cost is incurred for the production of each nonconforming item. We assume that the manufacturing system produces at a constant production rate p and a constant demand rate d.

In the model, the time length of production for a lot t_0 ($Q = (p-d)t_0$), the preventive maintenance threshold X_p and the inspection interval τ are the decision variables that should be determined to minimize the long-run average cost rate $C(\tau, t_0, X_p)$. The frequency of inspection τ, is an integer multiple of the time length of production t_0. Inspections are carried out at times $\tau, 2\tau, \ldots$, when the production of a lot ends. Deterioration occurs during the production time so its level remains same during the idle time of the lots which starts after the inventory level $I(t)$ reaches $(p-d)t_0$. Initial inventory level is assumed to be zero. Preventive and corrective maintenance actions take a fixed amount of time l and carried out during the idle times in order not to interrupt the production. It is assumed that after preventive and corrective maintenance, the system is "as good as new".

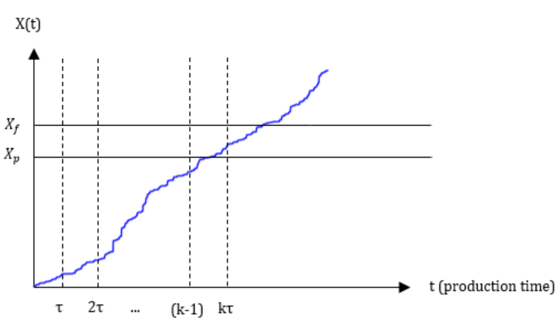

Figure 1. Sample degradation path crossing preventive maintenance level

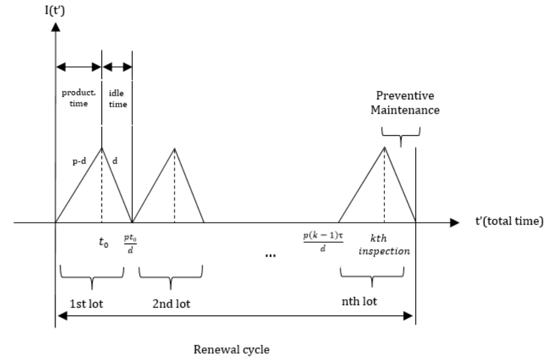

Figure 2. Inventory level in one cycle in case of preventive maintenance

A sample degradation path with respect to production time, is presented in Figure 1. X_p is the preventive maintenance threshold; if the degradation level is observed to be higher, then preventive maintenance is performed. In Figure 1, since $X(t)$ is between X_p and X_f at time $k\tau$, preventive maintenance is performed right after the k^{th} inspection to avoid failure and costly corrective maintenance. In this graph, only production time is considered because the degradation level remains unaltered during the idle times. The corresponding inventory level with respect to the total time t', is shown in Figure 2. The setup cost C_s, is incurred for each production lot. Time t' consists of production and idle times.

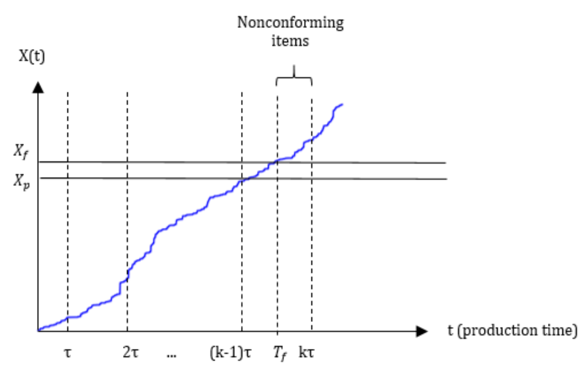

Figure 3. Sample degradation path crossing failure threshold level

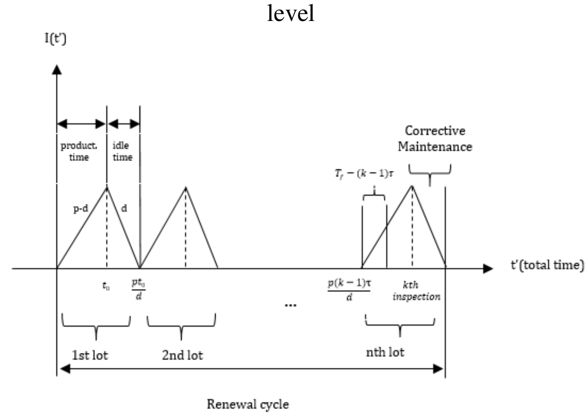

Figure 4. Inventory level in one cycle in case of corrective maintenance

The path in Figure 3 illustrates the case where corrective maintenance actions are to be performed at the end of the kth inspection. The deterioration does not reach the level X_p before the $(k-1)^{th}$ inspection. In the production of the lot corresponding to the k^{th} inspection, it exceeds the failure threshold. Thus, nonconforming items are produced which incur additional costs. Figure 4 shows the corresponding graph of inventory level over total time.

Notations

i	index of inspection intervals
τ	inspection interval
p	constant production rate
d	constant demand rate
t_0	production time for a lot
Q	lot-size
l	constant duration of predictive and corrective maintenance ($l \leq \frac{pt_0}{d} - t_0$)
$X(t)$	degradation level of the production system at time t
$F(x, t-s)$	cumulative distribution function of $X(t)$ – $X(s)$
$f(x, t-s)$	probability density function of $X(t)$ – $X(s)$
X_p	predetermined threshold level for preventive maintenance
X_f	failure threshold level
C_h	inventory holding cost per unit of time
C_I	cost for one inspection
C_s	setup cost per lot
C_{nc}	cost of producing one nonconforming unit
C_p	cost of the predictive maintenance action
C_c	cost of the corrective maintenance action
T_f	first passage time to the failure threshold
α	percentage of producing nonconforming items when the degradation level is above X_f
$E[S_c]$	expected setup cost per cycle
$E[H_c]$	expected inventory holding cost per cycle
$E[M_c]$	expected maintenance cost per cycle
$E[NC_c]$	expected cost of producing nonconforming items per cycle
$E[T]$	expected cycle length
$E[C]$	expected total cost in one cycle
$\mathrm{C}(\tau, t_0, X_p)$	long-run average total cost per unit of time

FORMULATION OF THE OPTIMIZATION MODEL

The degradation of the component is modeled as a stationary Gamma process with shape and scale parameters α and β respectively. It suits well with condition-based maintenance models where inspections are carried out in discrete time points (Van Noortwijk, 2009). The density function of the deterioration of $X(t_1) - X(t_2)$ between times t_1 and t_2 is as follows:

$$f(x, t_1 - t_2) = \frac{x^{((t_1-t_2)k-1)}\exp(-\frac{x}{\theta})}{\Gamma\big((t_1 - t_2)\,k\big)\theta^{(t_1-t_2)k)}} \qquad (1)$$

and the cumulative density function is computed by the equation,

$$F(x, t_1 - t_2) = \frac{\Gamma((t_1 - t_2)k,\ \frac{x}{\theta})}{\Gamma((t_1 - t_2)k)} \qquad (2)$$

where $\Gamma\big((t_1 - t_2)\,k\big)$ is the gamma function and $\Gamma((t_1 - t_2)\,k, \frac{x}{\theta})$ is the lower incomplete gamma function. The cumulative density function of the first passage time to failure threshold T_f, is

$$G(t) = P\{T_f \leq t\} = P\{X(t) > X_f\} = \overline{F}(X_f, t) \qquad (3)$$

The density function of the first passage time to the failure threshold is $g(t) = \frac{d}{dt}\overline{F}(X_f, t)$. Khatab et al. (2017) express this density function as

$$g(t) = \frac{k}{\Gamma(kt)} \int_{\frac{X_f}{\theta}}^{\infty} [\ln(u) - \Psi(kt)]u^{kt-1} \times \exp(-u)\,du \qquad (4)$$

where $\Psi(u) = dln(\Gamma(u))/du$ is the digamma function.

After either preventive or corrective maintenance, the degradation level becomes zero. Figure 2 shows an example of a renewal cycle. After the maintenance action is completed and the inventory level becomes zero, the renewal cycle restarts. The renewal reward theorem is used to compute the long-run average cost rate by dividing the average total accumulated cost in a renewal cycle by the average cycle length.

The expected cycle length is calculated by finding probability of the event that degradation level is lower than X_p before $(i-1)^{th}$ inspection and it exceeds X_p between $(i-1)^{th}$ and i^{th} inspections (Figure 1). It is given by

$$E[T] = \sum_{i=1}^{\infty} \frac{i\tau p}{d} \int_{0}^{X_p} f(x, (i-1)\tau)\, \overline{F}(X_p - x, \tau)dx \qquad (5)$$

where $(\tau p)/d$ is the total time length between two consecutive inspections.

The expected inventory holding cost per cycle is computed by

$$E[H_c] = C_h E[T]\frac{(p-d)t_0}{2} \qquad (6)$$

The expected sum of inspection and maintenance cost per cycle is,

$$E[M_c] = C_I \frac{E[T]}{\left(\frac{\tau p}{d}\right)} + \sum_{i=1}^{\infty} \int_0^{X_p} f(x,(i-1)\tau)(C_p \bar{F}(X_p - x, \tau) + \left(C_c - C_p\right)\bar{F}\left(X_f - x, \tau\right) dx \quad (7)$$

where the probability of performing maintenance right after the i^{th} inspection is $\int_0^{X_p} f(x,(i-1)\tau)\bar{F}(X_p - x, \tau)dx$. The probability of performing corrective maintenance in a cycle is as follows:

$$P\{E_i^c\} = P\{((X(i\tau) \geq X_f) \;\&\; X\left((i-1)\tau\right) < X_p)\}$$

$$= \int_0^{X_p} f(x,(i-1)\tau)\bar{F}\left(X_f - x, \tau\right)dx \quad (8)$$

The expected setup cost per cycle is given by

$$E[S_c] = C_s \frac{E[T]}{(pt_0/d)} \quad (9)$$

where (pt_0/d) is the time length of one production lot.

If the degradation level $X(i\tau)$ is observed to be x at any inspection, then probability density function of the remaining time to failure T_f given that $X(i\tau) = x$, is as follows:

$$g(x,t) = \frac{d}{dt} P\{T_f \leq t \middle| X(0) = x\}$$

$$= \frac{d}{dt} \bar{F}\left(X_f - x, t\right) \quad (10)$$

The conditional density function of T_f given that E_i^c can be expressed as

$$f_{T_f}\left((i-1)\tau + t \middle| E_i^c\right) = \frac{\int_0^{X_p} f(x,(i-1)\tau)g(x,t)dx}{P\{E_i^c\}} \quad (11)$$

By using the above probability, the expected cost of producing nonconforming items given the event E_i^c can be calculated as

$$E[NC_c|E_i^c] = C_{nc}\alpha p \int_0^{\tau} (\tau - t) \frac{\int_0^{X_p} f(x,(i-1)\tau)g(x,t)dx}{P\{E_i^c\}} dt \quad (12)$$

where α is the percentage of the produced nonconforming items and C_{nc} is the cost of producing a nonconforming item. After the degradation level reaches X_f, nonconforming items are produced up to the time of i^{th} inspection. An example path is shown in Figure 3. By multiplying the above equation by $P\{E_i^c\}$ for each inspection i and summing over all probabilities, the expected cost of producing nonconforming items is computed. It is expressed as

$$E[NC_c] = \sum_{i=1}^{\infty} C_{nc}\alpha p \int_0^{\tau} \int_0^{X_p} (\tau - t) \times f(x,(i-1)\tau)g(x,t)dxdt \quad (13)$$

For each value of the inspection interval ($\tau = nt_0$), the optimization problem is to minimize the long-run expected cost rate $C(\tau, t_0, X_p)$ by finding the optimal values of t_0, X_p subject to the constraint that idle time period $(p-d)t_0/d$, is longer than the time length of the predictive and corrective maintenance actions. Otherwise, shortages occur. The model is as follows:

$$\textit{minimize} \quad C\left(\tau, t_0, X_p\right)$$

$$\textit{subject to}$$

$$\frac{(p-d)t_0}{d} \geq l \quad (14)$$

$$X_p \geq 0 \quad (15)$$

$$X_f - X_p \geq 0 \quad (16)$$

where the long-run average cost rate is,

$$C\left(\tau, t_0, X_p\right) = \frac{E[C]}{E[T]}$$

$$= \frac{E[H_c] + E[M_c] + E[S_c] + E[NC_c]}{E[T]} \quad (17)$$

and the optimal objective function value of the above optimization problem is $C\left(\tau, t_0^*, X_p^*\right)$.

The optimal long-run average cost rate of $C\left(\tau, t_0, X_p\right)$, is found by solving the optimization problem for each value of $\tau = t_0, 2t_0, \ldots, Mt_0$ where M is a sufficiently big integer. Thus, the optimal value is,

$$C\left(\tau,^* t_0^*, X_p^*\right) = \min_{\tau = t_0, 2t_0, \ldots, Mt_0} C\left(\tau, t_0^*, X_p^*\right) \quad (18)$$

The objective function of the optimization problem is differentiable. Examples of different data sets show that the objective function is not convex so the local minimums might not be the global minimum. The Frank-Wolfe algorithm is used to solve this problem (Hillier and Lieberman 2001). The algorithm uses the linear approximation of the nonlinear objective function that are obtained by the first-order Taylor series expansion. Different initial points are chosen to find the local minimum with the smallest objective function value.

AN ILLUSTRATIVE EXAMPLE

A computational result of the model is presented in this section. The deterioration of the system $X(t)$ is modeled as a stationary Gamma process. The shape and scale parameters are $k = 1.2$ and $\theta = 0.8$ respectively. Inspection interval τ, is set as the integer multiple of t_0. Thus, inspections are carried out right after the completion of the production. The inspection, preventive maintenance and corrective maintenance costs are $C_i = 10$, $C_p = 202$ and $C_f = 550$. The cost of producing nonconforming items is $C_{nc} = 100$. Failure threshold level of the component is set as $X_f = 5.15$. The constant production and demand rates are $p = 2$ and $d = 1$ respectively. Inventory holding cost per item per unit time $C_h = 5$. The setup cost per lot is $C_s = 50$. Time length of the preventive maintenance and corrective maintenance actions is $l = 1.39$.

A solution, $X_p^* = 2.49, t_0^* = 2.7263,\ \tau^* = t_0^*$, is obtained by the Frank-Wolfe algorithm. The model is solved for each inspection interval which are $\tau = t_0, 2t_0, \dots Mt_0$. Optimal values of X_p and t_0 are found for each inspection interval ($\tau = nt_0$) and the one that minimizes the long-run average cost rate is chosen as an optimal solution. The long-run average cost rate with respect to X_p and t_0 is shown in Figure 5.

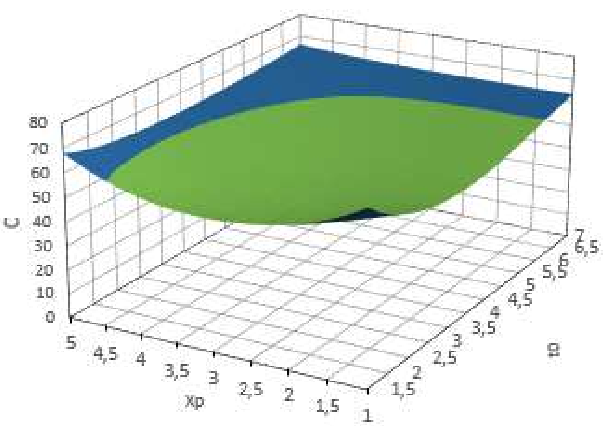

Figure 5. $C(\tau = t_0\ , t_0, X_p)$ with $X_f = 5.15,\ a = 1.2,\ b = 0.8$

CONCLUSION

In this study, a model is constructed for the joint optimization of lot-sizing and condition-based maintenance. Degradation process of the production system is modeled as a stationary Gamma process. Inspections are done periodically to observe the degradation level. Inspection cost is considerable so appropriate length of inspection period needs to be selected to minimize the overall cost rate. Maintenance actions are conducted in idle time periods in order not to interrupt the production plan. Renewal Reward Theory is used to compute the average long-run total cost rate. For a given τ, optimal values of X_p and t_0 are found by solving optimization problems with a nonlinear objective function and linear constraints. The Frank-Wolfe algorithm is used to solve this problem. Enumeration is done on τ to find the minimum value of the cost rate.

REFERENCES

Alaswad S., Xiang Y. (2017). A review on condition-based maintenance optimization models for stochastically deterioration system. Reliability Engineering and System Safety, 157, 54-63.

Ben-Daya M., Makhdoum M. (1998). Integrated production and quality model under various preventive maintenance policies. Journal of the Operational Research Society, 49, 840-853.

Ben-Daya M. (2002). The economic production lot-sizing problem with imperfect production process and imperfect maintenance. International Journal of Production Economics, 76, 257, 264.

Chen Y (2013). An optimal production and inspection modeling with minimal repair and rework considerations, Applied Mathematical Modeling, 37, 1618-1626.

Cheng Q. C., Zhou B. H., Li L. Joint optimization of lot sizing and condition-based maintenance for multi-component production systems, Computers & Industrial Engineering, 110, 538-549.

Hillier F. S. , Lieberman G. J. (2001). Introduction to Operations Research (Seventh Edition). McGraw-Hill.

Jafari L., Makis M. (2015). Joint optimal lot sizing and preventive maintenance policy for a production facility subject to condition monitoring, International Journal of Production Economics, 169, 156-168.

Jardine A.K.S, Lin D., Banjevic D (2006). A review on machinery diagnostics and prognostics implementing condition-based maintenance, Mechanical Systems and Signal Processing, 20, 1483-1510.

Khatab A., Diallo C., Aghezzaf E.H. and Venkatadri U. (2017). Optimizing the integrated economic production quantity for a stochastically deterioration production system under condition-based maintenance, 7^{th} IESM Conference.

Martin K.F (1994). A review by discussion of condition monitoring and fault-diagnosis in machine-tools, International Journal of Machine Tools and Manufacture 34 (1994) 527–551.

Peng, H. and Houtum, V. G. J. (2016). Joint optimization of condition-based maintenance and production lot-sizing. European Journal of Operational Research, 253, 94–107.

Van Noortwijk J. (2009). A survey of the application of gamma processes in maintenance. Reliability Engineering & System Safety, 94 , 2–21 .

BIOGRAPHIES

ALP DARENDELİLER is a doctoral student in the Industrial Engineering and Operations Research Program at the Department of Industrial Systems Engineering and Product Design, Faculty of Engineering and Architecture, Ghent University. He obtained his master degree in Industrial Engineering from Middle East Technical University, Turkey and bachelor degree in Industrial Engineering from Bilkent University, Turkey.

DIETER CLAEYS obtained the masters degree in Computer Science and the Ph.D. degree in Engineering in 2006 and 2011 respectively, both from Ghent University, Belgium. In 2007, he received the ORBEL award, which is accredited annually to the best Belgian master thesis in the field of Operations Research. From 2006-2017, he has been working at the SMACS Research Group within the Department of Telecommunications and Information Processing at Ghent University. From 2014-2017 he has been a postdoctoral research fellow from FWO. Since October 2015, he is an assistant professor at the department of Industrial Systems Engineering and Product Design at Ghent University. His main research interests include maintenance engineering, and the analysis of discrete-time queueing models and its applications to operations research and telecommunications systems.

ABDELHAKIM KHATAB is an Associate Professor at the University of Lorraine (France). He received a Ph.D. degree in Industrial Automation from the National Institute of Applied Science (INSA) of Lyon (France), in 2000. He held a researcher-lecturer position at INSA of Lyon from 2000-2004. After that (2005-2008), he held a post-doc position in Mechanical Engineering Department at Laval University, Quebec (Canada). His current research involves reliability design and analysis, and maintenance optimization of manufacturing and production systems.

EL-HOUSSAINE AGHEZZAF is full professor of industrial systems engineering and operations research at the Faculty of Engineering and Architecture of Ghent University. He holds a MS. and a Ph.D. in applied mathematics and operation research from CORE - Center for Operations Research and Econometrics at UCL-Université catholique de Louvain. He is currently heading the department of Industrial Systems Engineering and Product Design (ISyE-PD). His main research interests are in integrated systems optimization and simulation, stochastics and data analytics, supply and utility networks engineering.