Target safety levels for insulated steel beams exposed to fire, based on Lifetime Cost Optimisation

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ABSTRACT: The absence of clear target safety levels for structural fire safety severely hampers probabilistic structural fire design. In support of a generalized definition of target safety levels for structural fire safety engineering, optimum target safety levels for insulated steel beams are determined as a function of the fire characteristics by applying lifetime cost optimization (LCO) techniques. Where the fire development characteristics support the prospect of flashover, the Eurocode parametric fire curve is considered, otherwise fires are assumed to roam in search of fuel, leading to spatial variations in temperature, with thermal exposure to structural elements described via travelling fire methods. Fragility curves are derived as a function of, amongst others, the insulation thickness and fire load density, and applied in the LCO evaluations. The LCO results in an assessment of the optimum investment level as a function of the fire, damage and investment cost parameters characterizing the building. It is intended that the current contribution can be a stepping stone towards rational and validated reliability targets for PBD in structural fire safety engineering.

1 INTRODUCTION

Structural fire safety requirements implicitly balance up-front investments in materials (protection or element sizing) with improved performance (loss reductions) in the unlikely event of a fire. For traditional prescriptive fire safety recommendations, the underlying target safety levels are not clear to the designer, nor is the associated balancing of risk and investment costs. While easy to apply, the prescriptive guidance has the severe disadvantage that the level of safety investment is not tailored to the specifics of the case, resulting in large overinvestments in some cases, and possibly insufficient structural fire safety in others (Spinardi et al., 2017). This observation is a major driver for the use of performance-based-design (PBD) methodologies, where the fire safety design is tailored to the needs of the building. The lack of a clear definition of the target safety level, however, severely hampers probabilistic fire safety applications (Hopkin et al., 2017).

For normal design conditions, EN 1990 lists target reliability indices in function of the building's consequence class (CEN, 2002a). These target safety levels are compatible with lifetime cost-optimization calculations, where up-front investments in structural safety are balanced against reductions in uncertain future failure costs (Vrouwenvelder, 2002). Although the normal design condition target reliability index is not directly applicable to structural fire design (Van Coile et al., 2017), the concept of lifetime cost optimization can nevertheless be applied to inform target values for structural fire engineering (Fischer, 2014).

2 LIFETIME COST OPTIMISATION FOR STRUCTURAL FIRE SAFETY

2.1 *Concept of lifetime cost optimisation*

Lifetime cost optimization (LCO) for structural fire safety minimizes the lifetime costs associated with aspects of structural fire design and fire-induced failure. This total lifetime cost, Y, constitutes on the one hand upfront safety investments and maintenance costs, and on the other hand damage costs incurred in the uncertain event of a fire (Van Coile et al., 2014). In mathematical terms, Y is given by Equation (1), derived from (Rackwitz, 2000). The constituent terms are explained in Table 1.

As costs and benefits accrue over the lifetime of the structure, all terms are assessed through their net present value, taking into account a discount rate γ. Assuming a continued need for similar structures, the present value assessment considers systematic renewal after failure or obsolescence and an infinite time horizon (Rackwitz, 2000, Fischer et al., 2013). Considering the goal of code calibration, all terms are evaluated from a societal perspective (Van Coile and Pandey, 2017).

$$Y = C + A + D_M + D_L + D_R \tag{1}$$

Table 1. Constituent terms lifetime cost Y

Symbol	*Description*
C	Total building construction and maintenance cost
A	Obsolescence cost
D_M	Fire-induced material damages
D_L	Fire-induced loss to human life and limb
D_R	Reconstruction cost after fire-induced failure

2.2 *Fire protection insulation for steel beams*

Insulation is commonly provided to steel beams to achieve a predetermined fire rating advocated in prescriptive guidance documents. The fire rating is defined with respect to the ISO 834 standard fire curve, as specified in EN 1991-1-2 (CEN, 2002b), where the associated required insulation thickness corresponds with a maximum allowable steel beam temperature in function of the utilisation.

Considering the steel section and insulation properties of Table 2, and a design governed by steel yielding, insulation thicknesses are given in Figure 1 as a function of the fire rating and fire utilization u_{fi}. The fire utilization is defined by Equation (2), where u is the utilization in normal design conditions, γ_R is the global (i.e. aggregated) safety factor in normal design conditions (for steel beam bending design γ_R = 1), and η_{fi} as defined by equation (2.5) in EN 1993-1-2 (CEN, 2005). Steel temperatures have been calculated with the simplified iterative procedure of EN 1993-1-2. Further explanation is given in (Hopkin and Van Coile, 2018).

Table 2. Thermal analysis parameters

Symbol	*Description*	*Value*
d_p	Insulation thickness	variable [mm]
k_p	Insulation thermal conductivity	0.2 [W/mK]
c_p	Insulation specific heat	1700 [J/kgK]
ρ_p	Insulation density	800 [kg/m^3]
A/V*	Section factor	130 [m^{-1}]

* for example: a UB 533x210x101 profile heated from 3 sides

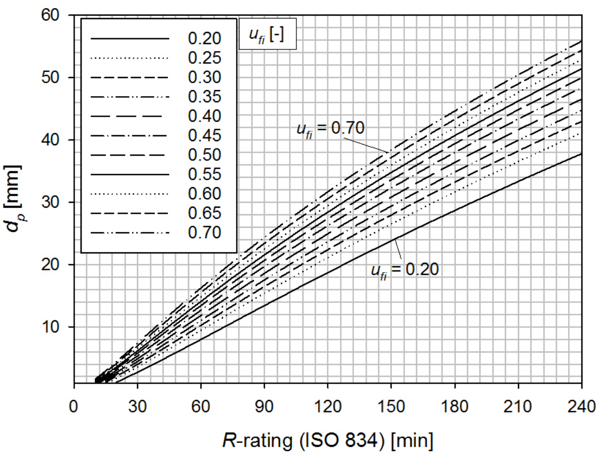

Figure 1. Required insulation thickness d_p in function of the ISO 834 duration and fire utilization u_{fi}, considering Table 2.

$$u_{fi} = \frac{E_{fi,d}}{R_{fi,d}\left(t=0\right)} = \frac{\eta_{fi} u}{\gamma_R} \tag{2}$$

The insulation applied for fire-rating steel beams, however, comes at a cost. Based on indicative reported costs for the UK, listed in Table 3, the tentative quadratic cost function of Equation (3) is derived, denoting the insulation cost per m^2 of protected beam surface area in function of the insulation thickness in mm. For generality, the cost coefficients a_0 and a_2 will be used in the derivations further.

Table 3. Indicative costs fire rating steel beams UK

Fire rating	*Cost* [GBP/m^2]	*Indicative thickness* d_p [mm]*	*Eq. (3)* [GBP/m^2]
30 min	5-8	5	6.0
60 min	8-12	12	10.8
90 min	18-20	19	19.4
120 min	30-35	25	30.0

* calculated

$$C_p \ [\frac{GBP}{m^2_{Ap}}] = 5 + 0.04 d_p^2 \left[\frac{GBP}{m^2_{Ap}}\right] = a_0 + a_2 d_p^2 \tag{3}$$

2.3 *Cost optimisation of fire protection thickness*

The concepts introduced in 2.1 are applied to assess the optimum level of fire protection for steel beams in an open plan office building floor. Specifics of the problem are presented further in Section 3. Here, the equations defining the optimum design solution are further elaborated.

The total single floor construction cost is specified to Equation (4), where C_0 is the base construction cost per m^2 floor area, A_f is the floor area, and A_p is the total surface area of protected steel beams.

The building obsolescence cost is considered relative to the total construction cost through an obsolescence rate ω, resulting in the present net value valuation of Equation (5).

The fire-induced material damages are associated with structural failure in case of fully developed fires. Damages incurred in case of fire ignition which does not grow into a fully developed fire are considered not affected by the level of structural fire protection and are thus omitted from the cost-optimization. Damages considered in case of a fully developed fire are only those that result from structural failure. This assumption will be further evaluated in follow up research.

While fully developed fires occur with a rate λ_{fi}, the conditional probability of an associated fire induced structural failure is denoted by P_f and is d_p-dependent. Incurred material damages are highly variable (Fischer, 2014), but only the average failure cost μ_M is needed for societal cost-optimization (Van Coile and Pandey, 2017), resulting in the present net value evaluation of Equation (6).

Losses to human life and limb are taken into account through the Life Quality Index (Pandey et al., 2006). More specifically risk to life is valued monetarily through the Societal Capacity to Commit Resources (SCCR, also known as Societal Willingness To Pay or SWTP), see (Pandey and Nathwani, 2004). This procedure for the valuation of risk to life acknowledges that a society's capacity to pay for safety is limited by its resources and by the efficiency of the safety investment it buys. This valuation of (changes of) risk to life has received increasing international support, as shown for example by its recent inclusion in ISO2394:2015 (ISO, 2015). Expressing the human consequence of fire-induced failure in terms of average number of fatalities μ_F resulting from structural failure, D_L is given by Equation (7) with μ_L the LQI-based valuation of risk to human lives. The SCCR is given by Equation (9) with g the gross domestic product per capita, q a work-leisure trade-off factor taken as 0.18, and C_x a demographic constant taken as 16.5 years. The SCCR has a 2016 value of about 2.6 million GBP per fatality averted.

Reconstruction costs are taken into account through Equation (8).

Equations (4)-(8) have been established on the level of a single floor. The failure costs will depend on the specifics of the situation, with for example μ_F small in case of low-rise buildings, but potentially very high for high-rise structures.

$$C = C_0 A_f + A_p\left(a_0 + a_2 d_p^2\right) \tag{4}$$

$$A = C\frac{\omega}{\gamma} \tag{5}$$

$$D_M = \mu_M \frac{\lambda_{fi} P_f\left(d_p\right)}{\gamma} \tag{6}$$

$$D_L = SCCR \cdot \mu_F \frac{\lambda_{fi} P_f\left(d_p\right)}{\gamma} = \mu_L \frac{\lambda_{fi} P_f\left(d_p\right)}{\gamma} \tag{7}$$

$$D_R = C\frac{\lambda_{fi} P_f\left(d_p\right)}{\gamma} \tag{8}$$

$$SCCR = \frac{g}{q} C_X \tag{9}$$

Combining the above, the total lifetime cost Y can be evaluated. To obtain more general results, however, combined parameters are introduced as listed in Table 4, and the total cost is normalized by the total base floor plate cost C_0A_f, resulting in Equation (10).

Furthermore, constant terms which are not dependent on thickness of the insulation do not influence the optimization. Omitting these terms results in Equation (11). Restructuring Equation (11) results in the equivalent formulation (12) which however has dimension of mm^2.

The optimum design criterion is then given by Equation (13). Note that Equations (11)-(13) are not dependent on the base insulation cost a_0. This implies that the obtained optimum value is not necessarily economically feasible, as the economic feasibility of applying fire protection should (in principle) compare the total cost of the protected steel beam to the total cost in case fire protection is omitted. Thus, the economic feasibility does depend on the base fire protection cost a_0. However, given the decision/assumption to apply any fire protection (for example, for reasons of jurisdictional conventions), the optimum level of fire protection is governed by Equation (13).

Table 4. Aggregate cost parameters

Symbol	*Description*	*Calculation*
ξ_M [-]	Relative total material failure cost	$\frac{\mu_M + C}{C_0 A_f}$
ξ_L [-]	Relative human losses	$\frac{\mu_L}{C_0 A_f}$
$a_{0,N}$ [-]	Relative base fire protection cost	$\frac{a_0}{C_0}\cdot\frac{A_p}{A_f}$
$a_{2,N}$ [mm^{-2}]	Relative rate of the marginal fire protection cost	$\frac{a_2}{C_0}\cdot\frac{A_p}{A_f}$
DII [mm^2]	Damage to Investment Indicator	$\frac{\lambda_{fi}\left(\xi_M + \xi_F\right)}{a_{2,N}\left(\gamma+\omega\right)}$

$$Y_N = \left(1 + a_{0,N} + a_{2,N} d_p^2\right)\left(1 + \frac{\omega}{\gamma}\right) + \lambda_{fi}\frac{\xi_M + \xi_L}{\gamma} P_f \tag{10}$$

$$Y_{N,0} = a_{2,N} d_p^2\left(1 + \frac{\omega}{\gamma}\right) + \frac{\lambda_{fi}}{\gamma}\left(\xi_M + \xi_L\right) P_f \tag{11}$$

$$Y_{N,0}^{*} = d_p^2 + DII \cdot P_f \tag{12}$$

$$\min_{d_p}\left\{Y_{N,0}^{*}\right\} \Rightarrow \frac{dY_{N,0}^{*}}{dd_p} = 0 \Rightarrow d_{p,opt} = -\frac{DII}{2}\cdot\frac{dP_f}{dd_p} \tag{13}$$

3 APPLICATION TO OPEN PLAN OFFICE

3.1 *Case description and evaluation of P_f*

A compartment with dimensions as listed in Table 5 is considered. Fire exposure is taken into account considering the methodology described in (Hopkin and Van Coile, 2018) taking into account the travelling fire (TF) methodology presented in (Hopkin, 2013). Uncertainties with respect to the glass breakage fraction, combustion efficiency, spread rate and TF near field temperature are considered, and the beam position within the compartment is varied. The fire load density q_F is described by a Gumbel distribution with coefficient of variation equal to 0.3 and mean value specified by a nominal fire load density $q_{F,nom}$, in accordance with (CEN, 2002b). Uncertainties in the steel yield stress at ambient temperatures, the yield stress retention factor at elevated temperature, structural load effects and model uncertainties are taken into account in accordance with (Khorasani, 2015, JCSS, 2015).

Considering the above, fragility curves have been calculated in (Hopkin and Van Coile, 2018) describing P_f considering Table 2 and Table 5 in function of the insulation thickness d_p and nominal fire load density $q_{F,nom}$. Examples are given in Figure 2 and Figure 3. These graphs make reference to the load ratio χ, which is defined as Q_k / (Q_k + G_k), with Q_k the load effect resulting from the characteristic imposed load and G_k the load effect resulting from the characteristic permanent load.

Table 5. Compartment characteristics

Symbol	*Description*	*Value*
w	Compartment width	22.4 [m]
l	Compartment depth	44.7 [m]
h	Compartment height	3.4 [m]
w_w	Total window width	130 [m]
w_h	Average window height	3.1 [m]

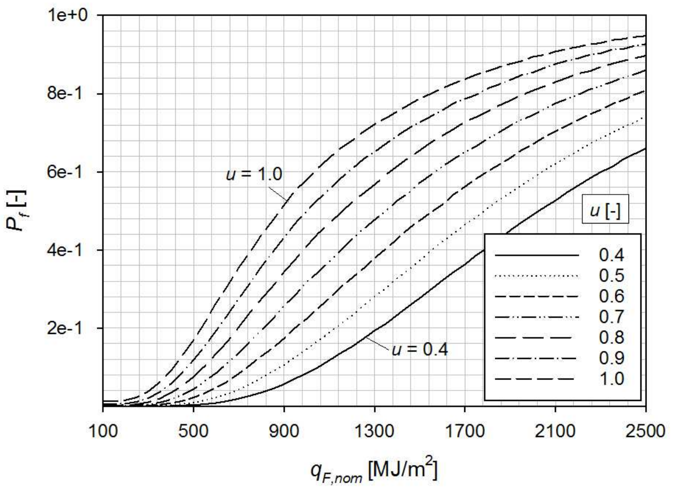

Figure 2. Fragility curve in function of $q_{F,nom}$, for d_p = 16 mm and χ = 0.50, with u the ambient design utilization ratio.

Table 6. Specific example application parameters

Description & reference	*Value*	*Units*
Number of occupied storeys	5	[-]
Building height	< 30	[m]
Ignition rate per floor (BSI, 2003)	$6 \cdot 10^{-3}$	[y^{-1}]
Probability of ignition resulting in a fully developed fire (EC, 2002)	0.9	[-]
Nominal fire load density (CEN, 2002b)	400	[MJ/m^2]
Building cost (Turner & Townsend, 2016)	2,700	[£/m^2]
Structural grid	7.5x7.5	[m x m]
Ambient utilisation (u)	0.55	[-]
Load ration (χ)	0.42	[-]
Fire utilisation (u_{fi})	0.31	[-]
Relative total material failure cost (ξ_M), (Kanda and Shah, 1997)	7.0	[-]

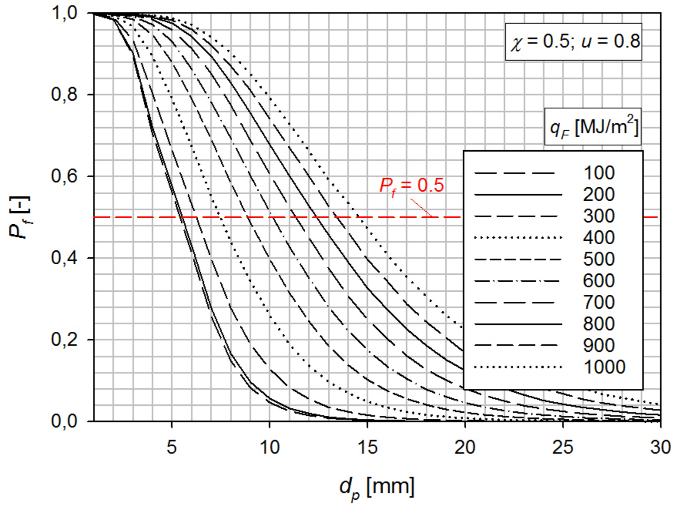

Figure 3. Fragility curve in function of d_p, for u = 0.8 and χ = 0.50, with $q_{F,nom}$ the nominal fire load density.

3.2 *Specific example application optimum d_p*

Equation (13) identifies the optimum insulation thickness in function of *DII* and the rate of P_f. The optimization and lifetime cost evaluation will be performed in general terms in the next section for a range of parameters. In the following, a specific evaluation is performed to illustrate the assessment as well as the order of magnitude for different input parameters.

The optimum insulation thickness is assessed for a non-sprinkler protected UK medium-rise office floor plate, with compartment characteristics as given in Table 5 and example application parameters as per Table 6. Steel section profiles are assumed to be UKB 457x152x82 throughout.

The relative total material failure cost considers a cost of 1.4 per floor affected and assumes all floors are affected by a structural failure (i.e. critical element). As many sources of uncertainty are associated with the assessment of ξ_M, further sensitivity analysis is recommended. This type of parameter study will be performed for *DII* in the next Section.

Considering a medium-rise building, a well-maintained fire alarm and no sleeping accommodation, the building can be assumed to be evacuated swiftly in case of a fire. Consequently, μ_F in case of fire-induced failure can be considered proportionally low. Its effect on the optimum insulation thickness is further evaluated by considering an average number of casualties in case of fire-induced failure, ranging from a low of 0 to a high of 10. Taking into account an SCCR of 2.6 million GBP per fatality, ξ_L is in the range 0-9.6.

A societal discount rate γ of 0.02 is considered, as well as an obsolescence rate ω of 0.02. The parameter $a_{2,N}$ is evaluated as $7.5 \cdot 10^{-6}$ mm^{-2} considering the cost assessment of Equation (6).

The above results in a *DII* of $1.4 \cdot 10^4$ mm^2 to $3.3 \cdot 10^4$ mm^2 in function of the average number of casualties.

The natural logarithm of the fragility curves (e.g. Figure 3) can be approximated by a linear curve for $P_f < 0.5$. This provides a practical way to evaluate Equation (13): P_f is evaluated in accordance with the procedure in (Hopkin and Van Coile, 2018) for two d_p-values, after which the approximate curve of Equation (14) is fitted. The optimum insulation thickness then corresponds with Equation (15), when substituting the approximation of Equation (14) in the general optimum design criterion of Equation (13). This approximation is applied here as a realistic and straightforward procedure for practical application of the presented methodology.

Specifically, P_f is evaluated numerically for $d_p = 6$ mm and $d_p = 12$ mm, with results listed in Table 7. These two evaluations are then used to evaluate b_0 and b_1 considering Equation (14) – results again listed in Table 7. The optimum insulation thickness is then directly defined by Equation (15), which can be readily solved iteratively. Considering the *DII*-range identified above, the obtained optimum insulation thickness ranges from 17 to 19 mm. Considering $u_{fi} = 0.31$, this corresponds with an optimum fire resistance rating ranging from approximately 90 to 100 min (see also Figure 1), slightly above the recommended value in UK prescriptive guidance (DCLG, 2006). Furthermore, for this type of medium-rise building with swift evacuation, the precise evaluation of the average number of casualties in case of fire-induced structural failure is found not to drastically affect the optimum level of fire protection.

$$\ln\left(P_f\right) = b_0 + b_1 d_p \tag{14}$$

$$d_{p,opt} = -\frac{DII}{2} b_1 \exp\left(b_0 + b_1 d_{p,opt}\right) \tag{15}$$

Table 7. Intermediate calculation results

Calculated values
$P_f\left(d_p = 6mm\right) = 4.5 \cdot 10^{-1}$; $P_f\left(d_p = 12mm\right) = 4.6 \cdot 10^{-2}$
$b_0 = 1.48$; $b_1 = -0.38$

3.3 *Optimum fire resistance in function of DII*

The example above illustrates the application of the optimization methodology to a specific case. More general results are presented in the following. These results have been obtained through a full numerical evaluation of P_f, i.e. the approximation of Equation (14) is not applied. All evaluations have been performed considering the parameters in Table 2 and Table 5 and stochastic distributions as described in (Hopkin and Van Coile, 2018).

Firstly, Figure 4 visualizes the normalized total lifetime cost $Y^*_{N,0}$, i.e. Equation (12), for a *DII* of 10^4 mm^2, considering an ambient design utilisation of 0.8 and load ratio of 0.5 ($u_{fi} = 0.42$). The optimum investment levels (i.e. optimum insulation thickness d_p) correspond with the minima (smallest $Y^*_{N,0}$) indicated in the graph. To improve the interpretability of the graph, the *R*-rating (ISO 834 standard fire exposure) corresponding with d_p is used instead as X-axis, considering the calculation procedure mentioned in 2.2 (Figure 1) for $u_{fi} = 0.42$.

The influence of *DII* on the optimum *R*-rating is evaluated in Figure 5 (all other parameters as in Figure 4). As discussed, the optimum *R*-rating relates to optimum insulation thicknesses. It is acknowledged that the large thicknesses corresponding with large *R*-ratings may not be realistic depending on the application. Nevertheless, the calculation results are considered here to provide insight in the general trade-off of increased fire-rating at an increased cost.

Note that the *DII* will differ not just between building types, but also between countries in function of attitudes to fire prevention and fire brigade intervention rates (both affecting occurrence rates), and socio-economic and construction cost parameters (e.g. through the SCCR of Equation (9)). Consequently, the optimum *R*-rating for a given building layout may differ between countries. Furthermore, *DII* is influenced by for example the height of the structure, e.g. for high-rise structures, both ξ_L and ξ_M can be reasonably considered significantly higher than in the specific medium-rise example of 3.2. On the other hand, improved management procedures and the implementation of sprinkler systems (as recommended for high-rise buildings in the UK) reduce λ_{fi} and thus lower *DII*. To improve the readability of Figure 5, a second X-axis has been included indicating $\lambda_{fi}\cdot(\xi_M+\xi_L)$, applicable for $a_{2,N}\cdot(\gamma+\omega) = 3\cdot 10^{-7}$ mm^{-2} as in the specific example of 3.2.

As can be expected, Figure 5 indicates a major influence of the nominal fire load on the optimum fire resistance rating. However, for a *DII* of 10^4 mm^2, the effect is about 10 min fire rating for every 100 MJ/m^2 nominal fire load. Taking into account the limited number of fire ratings used in practice (e.g. 15 min increments have only newly been introduced into UK practice via (BSI, 2008)), the optima obtained in Figure 5 can be considered relatively insensitive with respect to the precise evaluation of $q_{F,nom}$. Similarly, for $DII = 10^4$ mm^2 and $q_{F,nom} = 400$ MJ/m^2 as is representative for an office fire load (CEN, 2002b), R_{opt} decreases by circa 10 minutes when halving *DII* and increases by circa 10 minutes when *DII* doubles. Consequently, also with respect to the precise evaluation of *DII*, the results in Figure 5 are found to be relatively insensitive.

The secondary X-axis (as well as the definition of *DII* in Table 4) directly indicates the effect of increased or reduced fire occurrence frequencies, e.g. through the inclusion of sprinkler protection, or uncertainty with respect to the assumed sprinkler reliability, where such a system is provided, as well as the effect of material or human losses.

For completeness, the effect of the load ratio χ and ambient design utilization u are investigated in Figure 6 for $q_{F,nom}$ = 400 MJ/m^2. For lower DII (e.g. up to 10^4 mm^2) the influence is from a practical perspective small, i.e. considering the limited number of fire resistance ratings applied in practice. With increasing DII the differentiation increases.

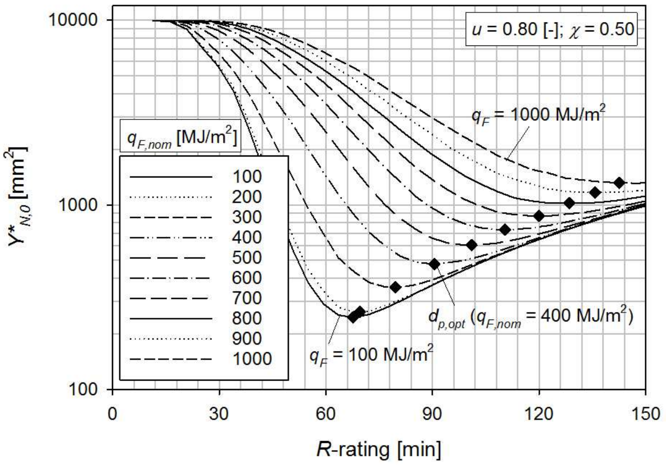

Figure 4. Normalized lifetime cost in function of R-rating, for u = 0.8, χ = 0.50, $DII = 10^4$, with $q_{F,nom}$ nominal fire load density.

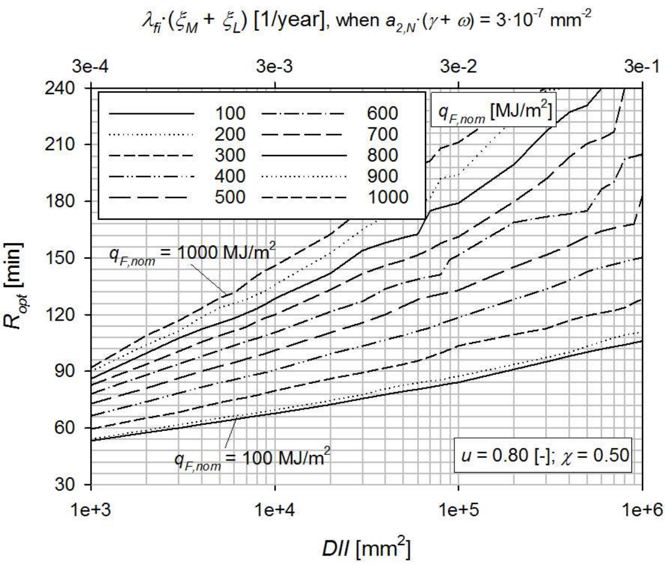

Figure 5. Optimum fire resistance rating R_{opt} in function of DII, u = 0.8 and χ = 0.50, with $q_{F,nom}$ the nominal fire load density.

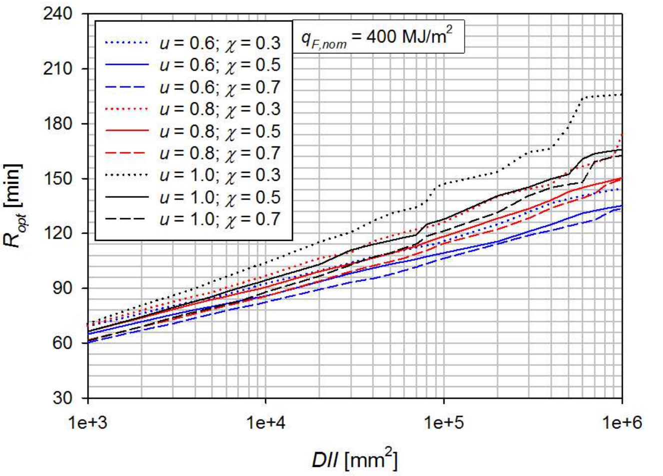

Figure 6. Optimum fire resistance rating R_{opt} in function of DII, $q_{F,nom}$ = 400 MJ/m^2, different χ and u.

3.4 *Tentative implications for current UK practice*

For $q_{F,nom}$ > 300 MJ/m^2, Figure 5 indicates that fire protection ratings of 60 minutes or higher are optimal, even for low DII. This result suggests that societal benefit may be obtained by increasing the lowest current fire resistance recommendations for office buildings in the UK (30 minutes for low rise office building), see (DCLG, 2006) or (BSI, 2008).

For high-rise office buildings, UK prescriptive guidance considers 120 minutes fire rating + sprinkler protection. Considering $q_{F,nom}$ = 400 MJ/m^2 as a representative office fire load (CEN, 2002b), Figure 5 indicates optimum fire resistance exceeding 120 minutes for DII in exceedance of 10^5 mm^2. Given all other parameters as in the example of 3.2, the secondary X-axis in Figure 5 applies, indicating a value of $\lambda_{fi}\cdot(\xi_M+\xi_L)$ = 0.03 corresponding with the noted threshold value for DII. Considering the fire ignition frequency and non-sprinkler fire suppression rate of 3.2 and taking into account a 99% probability of sprinklers successfully preventing a structurally significant fire from developing, the above implies exceedance of the 120 min threshold when $\xi_M + \xi_L$ exceeds $5\cdot10^3$. Assuming (for an order of magnitude assessment) 100 stories affected, the corresponding $\xi_{M,s} + \xi_{L,s}$ per floor equals 50. Neglecting the contribution of material losses ($\xi_{M,s}$ = 0 for illustrative purposes), the optimum fire resistance will exceed 120 minutes when the number of fatalities per floor exceeds 52. In case of a complete structural failure of a high-rise office building, it cannot a priori be ruled out that the expected number of fatalities per floor affected may exceed this number.

In conclusion, based upon the presented simplified assessment above there is a recommendation to further investigate whether increasing structural fire resistance requirements for high-rise office buildings in the UK might be appropriate. The preliminary results presented here suggest that the avoidance of fire-induced structural failure for high-rise office buildings in the UK may warrant increased safety investments.

Additional parameter studies and case studies must be performed to generalize the conclusions. Furthermore, as noted in (Hopkin et al., 2017) cost-optimization needs to be preceded by a tolerability assessment.

3.5 *Target reliability levels*

The optimum fire resistance (R) ratings indicated in Figure 5 can be translated into optimum (target) values for the reliability index in case of a fully developed fire, β_{fi}, considering Equation (16), with Φ the cumulative standard normal distribution function (CEN, 2002a).

$$P_f = \Phi(-\beta) \tag{16}$$

Establishing a commonly applicable target reliability index for structural fire design would greatly enhance the feasibility of actively demonstrating safety in PBD (Hopkin et al., 2017).

Figure 7 visualizes the optimum β_{fi} corresponding with Figure 5. As can be expected, the optimum safety level increases with *DII*, i.e. higher relative damage costs increase the optimum level of safety investment. The results, however, also indicate a decrease in $\beta_{fi,opt}$ with increasing fire load density. This is logical as obtaining a given β_{fi} for a higher nominal fire load density will correspond with a higher investment cost. For a given failure cost, there is thus a higher permissible failure rate if the investment costs are higher, see also (Rackwitz, 2000, Van Coile et al., 2017). Linking the above with the results presented in Figure 5, higher nominal fire load densities are, for a given *DII*, associated with on the one hand an increased optimum investment level (*R*-rating), and on the other hand a lower optimum safety level ($\beta_{fi,opt}$), illustrating the trade-off being made between safety investment and the acceptance of damage.

For interpretability, Figure 7 again includes a secondary X-axis applicable for the specific case $a_{2,N}\cdot(\gamma+\omega) = 3\cdot 10^{-7}$ mm^{-2} as in 3.2.

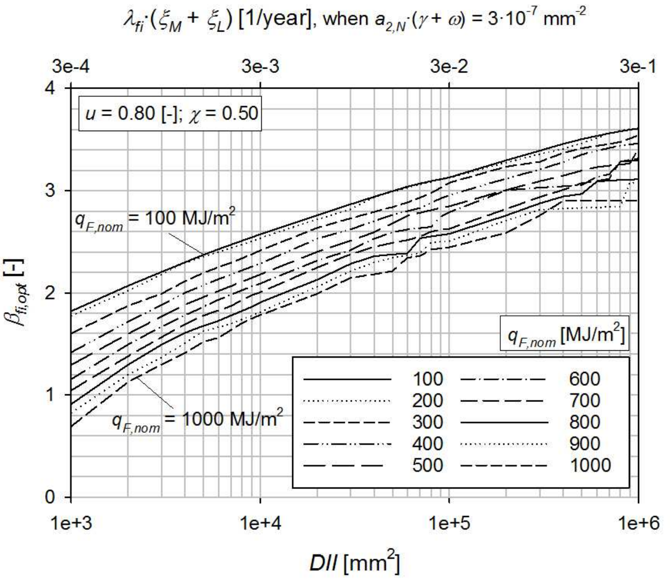

Figure 7. Optimum reliability index given a fully developed fire, $\beta_{fi,opt}$, in function of *DII*, u = 0.8 and χ = 0.50, with $q_{F,nom}$ the nominal fire load density.

Considering the standard nominal office fire load of 400 MJ/m^2 mentioned earlier, the effect of the load ratio χ and ambient design utilization u on $\beta_{fi,opt}$ are investigated in Figure 8. The load ratio χ slightly affects the optimum reliability index for a given utilization u. The ambient utilization u further influences the optimum reliability index in case of fire, β_{fi}, with a higher utilization resulting in a lower optimum reliability index. As with the reduction of $\beta_{fi,opt}$ with increasing $q_{F,nom}$, larger investments are required to obtain a given β_{fi} for a larger utilization u, and thus the optimum safety level is lower (for given failure costs).

Importantly, both in Figure 7 and Figure 8 observed values for $\beta_{fi,opt}$ are for a given *DII* in a bandwidth of $\Delta\beta \pm 0.5$. This tolerance has been accepted as underlying the simplified Level II method (König and Hosser, 1982), applied for deriving safety factors for the Eurocode design format (CEN, 2002a), and thus the observed tolerance may provide valuable insight for the definition of commonly accepted target safety levels for structural fire design.

Finally, Figure 9 represents the optimum failure probabilities $P_{f,opt}$ corresponding with the $\beta_{fi,opt}$ in Figure 7. The results in Figure 9 are of particular interest as they indicate a linear relationship between *DII* and $P_{f,opt}$ when considering logarithmic axes. Furthermore, Figure 9 confirms that the linear scaling of target failure probabilities with the fire occurrence rate, as postulated by the Natural Fire Safety Concept (EC, 2002), is inappropriate, as has been stated in (Van Coile et al., 2017) based on general cost assessments.

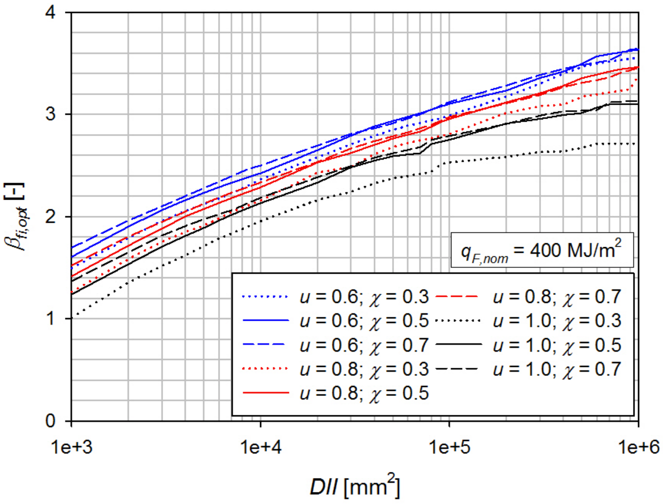

Figure 8. Optimum reliability index given a fully developed fire, $\beta_{fi,opt,}$ in function of *DII*, $q_{F,nom}$ = 400 MJ/m^2, different χ and u.

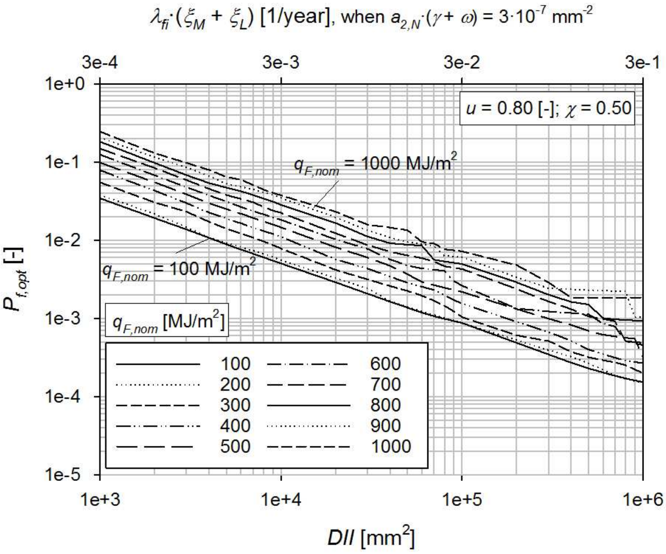

Figure 9. Optimum probability of failure given a fully developed fire, $\beta_{fi,opt,}$ in function of *DII*, u = 0.8 and χ = 0.50, with $q_{F,nom}$ the nominal fire load density.

4 DISCUSSION & CONCLUSIONS

Structural fire safety requirements balance up-front investments in materials (e.g. protective insulation) with improved performance in the unlikely event of a fire. In support of the development of risk- and reliability-based design approaches for structural fire safety, optimum insulation thicknesses have been determined for a protected steel beam, considering an open-plan office floorplate.

Expressing the results in an equivalent fire resistance rating *R*, current UK guidance recommended *R*-ratings were tentatively found below optimum for low rise office buildings. For medium-rise non-sprinklered office buildings, the 90 minutes prescribed rating was found to approximately correspond with optimum safety levels, while for sprinkler-protected high-rise office buildings preliminary results suggest that life safety considerations may result in an optimum fire resistance in exceedance of 120 minutes. It is noted that cost-optimization does not consider the issue of tolerability of low-probability high-consequence events.

A final proposal of target reliability levels for structural fire design should be made by a code-making committee, informed by detailed studies such as that presented herein. While further evaluations are necessary to generalize the results, the following conclusions are derived from the current study:

- The optimum level of structural fire safety is highly dependent on both the severity of fire-induced damage as on the cost of improving fire resistance;
- The logarithm of the optimum failure probability is linearly related to the logarithm of the damage-to-investment ratio;
- Optimum fire resistance ratings are not sensitive to precise evaluations of input parameters. It is expected that safety targets can be based on order-of-magnitude assessments for input parameters.

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