IEEE Transactions on Instrumentation and Measurement, Volume 69, Issue 1, January 2020, pp. 286-300
DOI: 10.1109/TIM.2019.2894048

Distributed Joint Probabilistic Data Association Filter with Hybrid Fusion Strategy

Shaoming He, Hyo-Sang Shin* and Antonios Tsourdos

Abstract

This paper investigates the problem of distributed multi-target tracking over a large-scale sensor network, consisting of low-cost sensors. Each local sensor runs a joint probabilistic data association filter to obtain local estimates and communicates with its neighbours for information fusion. The conventional fusion strategies, i.e., consensus on measurement and consensus on information, are extended to multi-target tracking scenarios. This means that data association uncertainty and sensor fusion problems are solved simultaneously. Motivated by the complementary characteristics of these two different fusion approaches, a novel distributed multi-target tracking algorithm using a hybrid fusion strategy, e.g., a mix between consensus on measurement and consensus on information, is proposed. A distributed counting algorithm is incorporated into the tracker to provide the knowledge of the total number of sensor nodes. The new algorithm developed shows advantages in preserving boundedness of local estimates, guaranteeing global convergence to the optimal centralised version and being implemented without requiring no global information, compared with other fusion approaches. Simulations clearly demonstrate the characteristics and tracking performance of the proposed algorithm.

Index Terms

Multi-target tracking, Multi-sensor fusion, Distributed fusion, Joint probabilistic data association, Hybrid fusion

I. Introduction

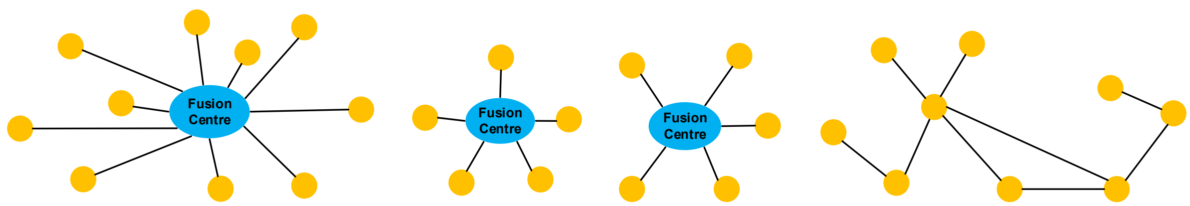
Wireless sensor networks have attracted great attention in recent decades thanks to their critical importance in a wide range of applications, including environmental monitoring [1], ground vehicle tracking [2]–[4], air traffic control [5], spacecraft navigation [6], vision-based pedestrian tracking [7], etc. The availability of low-cost sensors has enabled employment of multiple sensor nodes to large-scale sensing tasks [8]. Low-cost sensors, however, are generally subject to high clutter rate and low detection probability, leading to performance degradation, especially in multi-target tracking (MTT) scenarios [9]. Leveraging proper fusion algorithms over the sensor network could counteract the drawbacks of low-cost sensors and thus enhance the tracking performance. To this end, this paper aims to address the problem of distributed MTT in a sensor network.

The multi-sensor data integration or fusion can be categorised into three architectures in general: centralised, decentralised and distributed [10], [11], as shown in Fig. 1. The centralised fusion architecture simultaneously

Shaoming He, Hyo-Sang Shin and Antonios Tsourdos are with the School of Aerospace, Transport and Manufacturing, Cranfield University, Cranfield MK43 0AL, UK (email: {shaoming.he,h.shin,a.tsourdos}@cranfield.ac.uk)

*Corresponding Author.

processes the measurements provided by all sensors in a fusion centre, which directly connects with all sensor nodes. Although data fusion through a fusion centre is ideally Bayesian optimal, the fusion centre cannot effectively communicate with all sensors for large-scale sensor networks because of physical constraints, e.g., communication delay, limiting communication range. Unlike centralised fusion, the decentralised architecture utilises several fusion centres, capable of communicating with several local sensors, as backups in data integration, thus showing improved robustness against system failure. Each sensor node in the distributed architecture performs fusion using the information only obtained from locally connected neighbours in a peer-to-peer fashion. As the distributed fusion architecture requires no fusion centre, it could provide enhanced robustness to sensor failure and great flexibility, compared with the other two types of architectures. For this reason, this paper adapts distributed estimation framework as the fusion architecture.



(a) (b) (c)

Fig. 1. Different multi-sensor fusion architectures. The circle denotes the local sensor node and the black solid lines refer to the communication between one local sensor and the fusion centre or two sensors. (a) Centralised fusion architecture: all sensor nodes are connected the fusion centre. (b) Decentralised fusion architecture: sensor nodes are allocated to several fusion centres either statically or dynamically. (c) Distributed fusion architecture: sensor nodes only communicate with their neighbours in a peer-to-peer fashion.

Generally, a distributed MTT algorithm contains two main components: the local multi-target tracker for each sensor node and the distributed information fusion algorithm for locally connected sensors. There are several well-established algorithms available in the literature to address local MTT problems: nearest neighbour (NN) filter [12], probabilistic data association (PDA) filter [13], joint probabilistic data association (JPDA) filter [14]–[17] and multiple hypothesis tracking (MHT) [18]–[21]. JPDA filter is used as the baseline local tracker in this paper because of its balance between performance and computational cost. In the fusion stage, each sensor node communicates with its neighbours through a network topology to perform estimation fusion. Note that although distributed multi-sensor target tracking for single target is well-established, its direct extension to the MTT scenario is not straightforward due to the measurement origin uncertainty and hence requires careful adjustment. This is the main concern of this paper.

A. *Related Work*

In distributed fusion, the control-theoretic consensus algorithm is a powerful tool for performing network-wide computation tasks, such as averaging of quantities and functions [22]–[25]. The promising feature of consensus-based framework is its global convergence and flexibility: the consensus algorithm can be applied to any connected sensor network for information fusion with guaranteed global convergence. The Kalman consensus filter (KCF) is the first paradigm that exploits the benefits of average consensus algorithm in distributed state estimation [23], [26], [27]. KCF consists of two steps in fusion: the first step utilised an information Kalman filter to update the local estimates by summation of the information received from local neighbours; the second step applies the consensus on estimates (CE) strategy to compute the average mean of all local estimates. This simple averaging computation, however, cannot guarantee satisfactory performance for some scenarios, since the CE fusion strategy never exploits the useful covariance matrix [28]. This issue is partially resolved by a convex combination of local estimates in diffusion Kalman filter (DKF) via covariance intersection [29]. Improvements over KCF were also found in [30]–[32], where an algorithm, termed as information consensus filter (ICF), was proposed to address the issue of naive sensors (e.g., target is outside the sensor's field-of-view). Apart from CE, the well-known covariance intersection provides a different point of view in information fusion [33]. This conservative approach computes the geometric mean of local probability density functions via the minimisation of a weighted Kullback-Leibler divergence. Distributed implementation of covariance intersection applies average consensus algorithm on information-related terms [33], thus, termed as consensus on information (CI). However, this strategy has never been extended to multi-target tracking scenario, where data association uncertainties need to be carefully addressed.

A multi-sensor multi-target tracking algorithm was proposed in [34] by using a modified DBSCAN clustering for track-to-track association. However, this algorithm requires a fusion centre to collect all information from local sensors and therefore is not robust against the system failure. In [35], a distributed multi-target tracker was proposed based on the CE strategy. This method, therefore, requires every sensor node and its neighbours have joint observability or at least detectability about the target of interest. Improved results were reported in [36] by combining PDA filter with consensus on information weighted local estimates. However, this approach requires the global information on the total number of sensors N_s, e.g., partially distributed. In practice, an unexpected sensor failure will inevitably change the total number of nodes, leading to performance degradation if the original value of N_s is used. Distributed NN filter for multi-target tracking to address track ambiguity was proposed in [37]. By incorporating consensus algorithm with Random Finite Set (RFS) filters (e.g., Probability Hypothesis Density filter [38], [39], multi-Bernoulli filter [40]), distributed MTT algorithms were proposed without data association. These approaches, however, cannot preserve track continuity, e.g., no target identity information. Apart from consensus, another mainstream in distributed multi-target tracking is sequentially fuse the information between two connected sensors [9], [41], [42]. Although this strategy is scalable, it requires each sensor's field-of-view to cover the entire surveillance region, which is not practical [42].

B. Contribution and Organisation

This paper develops distributed multi-target tracking algorithms by exploiting different fusion strategies: distributed JPDA with consensus on measurement (DJPDA-CM), distributed JPDA with consensus on information (DJPDA-CI) and distributed JPDA with hybrid consensus (DJPDA-HC). The main contributions are:

(1) The conventional consensus on measurement (CM) and CI strategies are extended to the MTT scenario, addressing the inherent data association uncertainty issue. Specifically, except for local estimates, the data association uncertainty-related term of each sensor node is also shared with its neighbours for information fusion.

(2) A new distributed multi-target tracking algorithm with hybrid fusion strategy, termed as DJPDA-HC, is proposed by exploiting the benefits of both DJPDA-CM and DJPDA-CI. The proposed DJPDA-HC filter is a fully distributed tracker without utilising any global information; thus, this approach has strong robustness against sensor failures. Theoretical analysis and extensive simulations demonstrate that DJPDA-HC outperforms DJPDA-CM and DJPDA-CI under various conditions.

Note that this paper also derives the centralised JPDA as a reference for performance evaluation. Performance of all the distributed JPDA algorithms is compared to that of the centralised one through extensive numerical simulations. Results reveal that the proposed DJPDA-HC algorithm outperforms others under various conditions.

The rest of the paper is organised as follows. Section II presents some preliminaries and backgrounds. Section III derives the centralised JPDA filter as a performance benchmark. In Sec. IV, distributed JPDA algorithms are proposed by using CM and CI fusion strategies, followed by the proposed DJPDA-HC shown in Sec. V. Finally, numerical results from various simulations are demonstrated.

II. Backgrounds and Preliminaries

This section firstly presents a brief description of the system model that will be utilised in the following sections. Then, the problem formulation of this paper will be addressed.

A. System Modelling

The set of target states and measurements received at scan k are, respectively, defined as

$$X_k = \left\{ x_k^1, ..., x_k^{N_k} \right\}, \quad Z_k = \left\{ z_k^0, z_k^1, ..., z_k^{M_k} \right\} \tag{1}$$

where N_k denotes the number of targets at scan k, x_k^i the ith target at scan k, M_k the number of measurements received at scan k, $z_k^j\ (j \neq 0)$ the jth measurement received at scan k, z_k^0 the dummy measurement for convenient representation of miss detection.

Consider the following dynamical system

$$\begin{aligned} x_k^i &= f_{k-1}^i \left(x_{k-1}^i \right) + w_{k-1}^i \\ z_k^i &= h_k^i \left(x_k^i \right) + \eta_k^i \end{aligned} \tag{2}$$

where $x_k^i \in \mathbb{R}^n$ and $z_k^i \in \mathbb{R}^m$ denote the system state and the corresponding measurement at time step k, respectively. The nonlinear functions $f_k^i \left(x_k^i \right)$ and $h_k^i \left(x_k^i \right)$ correspond to the system state evolution and measurement equations, respectively. The signals w_k^i and η_k^i are process noise and measurement noise, which are assumed to be zero-mean Gaussian with covariances Q_k^i and R_k^i. For convenience, we make the following general assumptions, which are widely-accepted in MTT problems.

Assumption 1. Each target can generate at most one measurement and each measurement can originate from at most one target. Each target-generated measurement is independent of each other and is detected with probability P_D with measurement likelihood $p(z\,|x\,)$.

Assumption 2. The clutter distribution is assumed to be unknown *a priori* and is thus considered as Poisson distribution. Clutters or false alarms are modelled by a local Poisson point process (PPP) with intensity $\lambda_{FA} = N_{FA}/V_s$ with N_{FA} being the average number of clutters of one scan and V_s being the sensor detection volume.

B. Problem Formulation

Suppose that N_s sensors participate in a cooperative distributed estimation mission, e.g., actively broadcasting their local information to their neighbours. For this multi-sensor system, we use a partially connected undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ to represent the communication topology, where $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_{N_s}\}$ is a set of vertices that represent N_s sensors and $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V}\}$ is a set of edges that stand for the relationship between two neighbouring sensors in the topology. If two sensors (i and j) are adjacent, namely, they can communicate with each other, then $(\nu_i, \nu_j) \in \mathcal{E}$. Denote $\mathcal{N}_i$ as the set of the neighbours (locally connected) of sensor i including sensor i.

The aim of this paper is to design a distributed multi-target tracking algorithm using a partially connected sensor network. Each sensor node runs a local JPDA algorithm and the local estimations are then fused in a distributed way. Note that in a partially or not fully connected sensor network, each sensor can only communicate with its neighbours.

III. Centralised JPDA Filter: A Benchmark

An optimal fusion strategy and benchmark for performance evaluation of distributed state estimation algorithms is centralised estimation, which processes the measurements from all sensors simultaneously through a fusion centre. For centralised implementation of multi-sensor JPDA filter, we will utilise the information filter, which is proved to be useful in information fusion [43].

JPDA assumes that each measurement can originate from a number of candidate targets in data association. Therefore, the posterior probability distribution of each target obtained from JPDA filter is a Gaussian mixture distribution. Since propagation of the Gaussian mixture distribution over time is practically intractable due to the explosion of mixture components, JPDA utilises a single Gaussian distribution to approximate the Gaussian mixture at each time instant to reduce the computational burden. More specifically, the state estimation is updated by utilising a pseudo innovation term, e.g., a weighted sum of the original innovation terms,

$$\tilde{z}_k^i = \sum_{j=1}^{M_k} \beta_j^i \tilde{z}_{j,k}^i = \sum_{j=1}^{M_k} \beta_j^i \left(z_{j,k} - \hat{z}_k^i \right) \tag{3}$$

where $\hat{z}_k^i = h_k^i \left(x_{k|k-1}^i \right)$ stands for the predicted measurement of the ith target and β_j^i represents the marginal association probability that the jth measurement is associated with the ith target. Details of how to calculate the marginal association probability can be found in [14], [15].

Applying the pseudo innovation term $\tilde{z}_k^i$ to standard extended Kalman filter paradigm generalises the classical JPDA filter to accommodate nonlinear system (3), via linearising the state and measurement equations, as

(1) Prediction:

$$
\begin{aligned}
x^i_{k|k-1} &= f^i_{k-1}\left(x^i_{k-1|k-1}\right) \\
P^i_{k|k-1} &= F^i_{k-1}P^i_{k-1|k-1}\left(F^i_{k-1}\right)^T + Q^i_{k-1}
\end{aligned} \tag{4}
$$

(2) Measurement Update:

$$
\begin{aligned}
S^i_k &= H^i_k P^i_{k|k-1}\left(H^i_k\right)^T + R^i_k \\
K^i_k &= P^i_{k|k-1}\left(H^i_k\right)^T\left(S^i_k\right)^{-1} \\
x^i_{k|k} &= x^i_{k|k-1} + K^i_k \tilde{z}^i_k \\
P^i_{k|k} &= P^i_{k|k-1} - K^i_k\left(1-\beta^i_0\right)S^i_k\left(K^i_k\right)^T + K^i_k\bar{P}^i_k\left(K^i_k\right)^T
\end{aligned} \tag{5}
$$

where $F^i_k = \frac{\partial f^i_k}{\partial x_k}\left(x_{k|k}\right)$ is the linearised system matrix, $H^i_k = \frac{\partial h^i_k}{\partial x_k}\left(x_{k|k-1}\right)$ the linearised measurement matrix, and $\bar{P}_k$ a positive semi-definite matrix representing the measurement origin uncertainty and takes the form

$$
\bar{P}_k = \sum_{j=1}^{M_k}\beta^i_j\tilde{z}^i_{j,k}\left(\tilde{z}^i_{j,k}\right)^T - \tilde{z}^i_k\left(\tilde{z}^i_k\right)^T \tag{6}
$$

For the purpose of deriving information JPDA filter, define $Y^i_{k|k} = \left(P^i_{k|k}\right)^{-1}$ and $y^i_{k|k} = \left(P^i_{k|k}\right)^{-1}x^i_{k|k}$ as the information matrix and the information vector, respectively. As derived in Appendix A, the information form of JPDA filter is given by

$$
\begin{aligned}
Y^i_{k|k} &= Y^i_{k|k-1} + \bar{\mathrm{I}}^i_k \\
x^i_{k|k} &= \left(Y^i_{k|k-1} + \mathrm{I}^i_k\right)^{-1}\left(y^i_{k|k-1} + \mathrm{i}^i_k + \beta^i_0\mathrm{I}^i_k x^i_{k|k-1}\right)
\end{aligned} \tag{7}
$$

where the measurement-related terms are defined as

$$
\begin{aligned}
\mathrm{I}^i_k &= \left(H^i_k\right)^T\left(R^i_k\right)^{-1}H^i_k, \quad \mathrm{i}^i_k = \left(H^i_k\right)^T\left(R^i_k\right)^{-1}\sum_{j=1}^{M_k}\beta^i_j z^i_{j,k} \\
\bar{\mathrm{I}}^i_k &= Y^i_{k|k-1}K^i_k\left\{\left[\left(1-\beta^i_0\right)S^i_k - \bar{P}^i_k\right]^{-1} - \left(K^i_k\right)^T Y^i_{k|k-1}K^i_k\right\}^{-1}\left(K^i_k\right)^T Y^i_{k|k-1}
\end{aligned} \tag{8}
$$

Note that i^i_k in Eq. (7) is the measurement information, which clearly reveals that JPDA utilises a weighted sum of all candidate measurements. The term $\beta^i_0\mathrm{I}^i_k x^i_{k|k-1}$ quantifies target miss detection. If the ith target is miss detected, then, $\beta^i_0 = 1$ and $\beta^i_j = 0$, which means $\mathrm{i}^i_k = 0$. Therefore, only prior information can be utilised for update. The summation of i^i_k and $\beta^i_0\mathrm{I}^i_k x^i_{k|k-1}$ determines the total amount of information that JPDA can leverage to update the ith target. Due to data association uncertainty, there exist two information matrix contributions, e.g., I^i_k and $\bar{\mathrm{I}}^i_k$ in information JPDA filter. Notice that the inverse of $Y^i_{k|k-1} + \bar{\mathrm{I}}^i_k$ is the posterior covariance matrix, including data association uncertainty, of the ith target, provided by JPDA filter, whereas the inverse of $Y^i_{k|k-1} + \mathrm{I}^i_k$ can be interpreted as the ideal covariance matrix of the ith target without any association uncertainty. It is easy to verify that $\bar{\mathrm{I}}^i_{l,k} = \mathrm{I}^i_{l,k}$ when $\beta^i_{l,0} = 0$ and $\bar{P}^i_{l,k} = 0$, e.g., no data association uncertainty. This means that the nonlinear information JPDA filter reduces to classical extended information filter for single target tracking without any clutters.

In MTT over a sensor network, each sensor node orders its estimated tracks differently and therefore track-to-track association is required to associate the tracks from different sensors that represent the ith target [44]. There are a number of elegant choices for solving the track-to-track association problem in combinatorial optimisation by reformulating the problem as a network flow [45] or using approximate Lagrangian relaxation approach [46]–[48]. Considering the balance between accuracy and efficiency, the Lagrangian relaxation method is utilised in this paper. After finding the same source origin of local tracks, information fusion can be performed based on the property of information filter: incorporating additional information from other sensors could be achieved by summation of the corresponding information terms. This implies that the optimal centralised JPDA over N_s sensors can be implemented as

$$\begin{aligned} Y^{i,c}_{k|k} &= Y^{i}_{k|k-1} + \sum_{l=1}^{N_s} \bar{\mathrm{I}}^{i}_{l,k} \\ x^{i,c}_{k|k} &= \left(Y^{i}_{k|k-1} + \sum_{l=1}^{N_s} \mathrm{I}^{i}_{l,k} \right)^{-1} \left[y^{i}_{k|k-1} + \sum_{l=1}^{N_s} \left(\mathrm{i}^{i}_{l,k} + \beta^{i}_{l,0} \mathrm{I}^{i}_{l,k} x^{i}_{k|k-1} \right) \right] \end{aligned} \tag{9}$$

It is clear that that centralised estimation requires full information of all sensors. Considering the fact that each sensor usually can only communicate with its neighbours due to communication limit, this paper will develop distributed implementation algorithms based on average consensus algorithm to recover the performance of the centralised estimation (9). The centralised JPDA filter will be utilised as a benchmark for the performance evaluation of the algorithms developed.

IV. Distributed JPDA Filter Using Consensus on Measurement and Consensus on Information

This section extends conventional fusion strategies, e.g., consensus on measurement and consensus on information, to MTT scenarios on the basis of JPDA. Before providing the main results, the average consensus algorithm is briefly reviewed first.

A. Average Consensus

To perform estimation fusion in a distributed way, the concept of average consensus is adopted here. The average consensus algorithm is used to obtain the mean value of the information of all sensors in a distributed way without all-to-all communications; thus consensus-based distributed estimation can be applied to any generic connected sensor network. Denote a_l as the available information from the lth sensor and a_l is initialised as $a_l(0)$. Then, the distributed average consensus algorithm [22], [23] at the mth iteration is defined as

$$a_l\left(m\right) = \sum_{l' \in \mathcal{N}_l} \pi_{l,l'} a_{l'}\left(m-1\right) \tag{10}$$

where $\pi_{l,l'}$ is the consensus gain, which satisfies the condition $\sum\limits_{l' \in \mathcal{N}_l} \pi_{l,l'} = 1$ with $\pi_{l,l'} \geq 0$. The convergence rate of average consensus algorithm depends on the consensus gain and the algebraic connectivity of graph $\mathcal{G}$. Typical choices of $\pi_{l,l'}$ that guarantee the stability of the consensus phase are Metropolis weight and maximum-degree weight [49].

Lemma 1. *[22], [23] Under the assumption that the sensor network is strongly connected, by running the iterative algorithm as in Eq. (10), the information of all sensors asymptotically converges to the initial average value as*

$$\lim_{m \to \infty} a_l\left(m\right) = \frac{1}{N_s} \sum_{l'=1}^{N_s} a_{l'}\left(0\right) \tag{11}$$

B. Distributed JPDA Filter Using Consensus on Measurement

DJPDA-CM aims to compute the summations $\sum_{l=1}^{N_s} \bar{\mathrm{I}}_{l,k}^i$, $\sum_{l=1}^{N_s} \mathrm{I}_{l,k}^i$ and $\sum_{l=1}^{N_s} \left(\mathrm{i}_{l,k}^i + \beta_{l,0}^i \mathrm{I}_{l,k}^i x_{l,k|k-1}^i\right)$ in Eq. (9) through a distributed way to recover the performance of centralised JPDA filter. Since these three terms are measurement-related, this fusion strategy is therefore called consensus on measurement. Define consensus variables $V_{l,k}^i$, $G_{l,k}^i$, $v_{l,k}^i$, which are initialised as

$$V_{l,k}^i(0) = \bar{\mathrm{I}}_{l,k}^i, \quad G_{l,k}^i(0) = \mathrm{I}_{l,k}^i, \quad v_{l,k}^i(0) = \mathrm{i}_{l,k}^i + \beta_{l,0}^i \mathrm{I}_{l,k}^i x_{l,k|k-1}^i \tag{12}$$

By sharing $V_{l,k}^i$, $G_{l,k}^i$, $v_{l,k}^i$ with locally connected sensors using L communication iterations at every time instant, the measurement update of DJPDA-CM is given by

$$\begin{aligned} Y_{l,k|k}^i &= Y_{l,k|k-1}^i + N_s V_{l,k}(L) \\ x_{l,k|k}^i &= \left(Y_{l,k|k-1}^i + N_s G_{l,k}(L)\right)^{-1} \left(y_{l,k|k-1}^i + N_s v_{l,k}(L)\right) \end{aligned} \tag{13}$$

where $V_{l,k}(L)$, $G_{l,k}(L)$, $v_{l,k}(L)$ denote the values of $V_{l,k}^i$, $G_{l,k}^i$, $v_{l,k}^i$ after performing L steps of average consensus.

The detailed implementation of the proposed DJPDA-CM algorithm is summarised in Algorithm 1. The following lemma analyses the asymptotic performance of DJPDA-CM.

Lemma 2. *Under the assumption that the sensor network is strongly connected with previously converged local estimates, e.g.,* $x_{l,k-1|k-1}^i = x_{k-1|k-1}^{i,c}$, $Y_{l,k-1|k-1}^i = Y_{k-1|k-1}^{i,c}$, *DJPDA-CM (13) will asymptotically converge to the centralised JPDA (9) at current time instant.*

Proof. According to Lemma 1, applying average consensus algorithm to $V_{l,k}^i$, $G_{l,k}^i$, $v_{l,k}^i$ gives

$$\begin{aligned} \lim_{m\to\infty} V_{l,k}^i(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} \bar{\mathrm{I}}_{l',k}^i \\ \lim_{m\to\infty} G_{l,k}^i(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} \mathrm{I}_{l',k}^i \\ \lim_{m\to\infty} v_{l,k}^i(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} \left(\mathrm{i}_{l',k}^i + \beta_{l',0}^i \mathrm{I}_{l',k}^i x_{l',k|k-1}^i\right) \end{aligned} \tag{14}$$

Therefore, the measurement update of DJPDA-CM with infinite number of iterations can be formulated as

$$\begin{aligned} Y_{l,k|k}^i &= Y_{l,k|k-1}^i + \sum_{l'=1}^{N_s} \bar{\mathrm{I}}_{l',k}^i \\ x_{l,k|k}^i &= \left(Y_{l,k|k-1}^i + \sum_{l'=1}^{N_s} \mathrm{I}_{l',k}^i\right)^{-1} \left[y_{l,k|k-1}^i + \sum_{l'=1}^{N_s} \left(\mathrm{i}_{l',k}^i + \beta_{l',0}^i \mathrm{I}_{l',k}^i x_{l',k|k-1}^i\right)\right] \end{aligned} \tag{15}$$

Under the assumption that the previous local estimates are equal to the centralised fusion as $x_{l,k-1|k-1}^i = x_{k-1|k-1}^{i,c}$, $Y_{l,k-1|k-1}^i = Y_{k-1|k-1}^{i,c}$, it is straightforward to verify that $x_{l,k|k}^i = x_{k|k}^{i,c}$, $Y_{l,k|k}^i = Y_{k|k}^{i,c}$ from Eq (15). □

Remark 1. Lemma 2 reveals that the advantage of DJPDA-CM is that it is asymptotically optimal at each time instant provided that the priors are converged. However, since only finite number of consensus iterations is tractable in practice, convergence will not be fully achieved. The performance of DJPDA-CM with small number of consensus

Algorithm 1 DJPDA-CM for target i at sensor node l at time step k

Input: Previous target estimation $\left\{x^i_{l,k-1|k-1}, P^i_{l,k-1|k-1}\right\}$, received measurements Z_k

(1) Linearise system equation: $F^i_{l,k-1} = \frac{\partial f^i_{k-1}}{\partial x_{k-1}}\left(x^i_{l,k-1|k-1}\right)$

(2) Prediction:

$$\begin{aligned} x^i_{l,k|k-1} &= f^i_{k-1}\left(x^i_{l,k-1|k-1}\right) \\ P^i_{l,k|k-1} &= F^i_{l,k-1}P^i_{l,k-1|k-1}\left(F^i_{l,k-1}\right)^T + Q^i_{k-1} \\ Y^i_{l,k|k-1} &= \left(P^i_{l,k|k-1}\right)^{-1}, y^i_{l,k|k-1} = \left(P^i_{l,k|k-1}\right)^{-1} x_{l,k|k-1} \end{aligned}$$

(3) Linearise measurement equation: $H^i_{l,k} = \frac{\partial h^i_k}{\partial x_k}\left(x^i_{l,k|k-1}\right)$

(4) Compute the measurement-related terms: $\bar{\mathrm{I}}^i_{l,k}$, $\mathrm{I}^i_{l,k}$, $\mathrm{i}^i_{l,k}$

(5) Compute the initial values of consensus variables:

$$V^i_{l,k}(0) = \bar{\mathrm{I}}^i_{l,k}, \quad G^i_{l,k}(0) = \mathrm{I}^i_{l,k}, \quad v^i_{l,k}(0) = \mathrm{i}^i_{l,k} + \beta^i_{l,0}\mathrm{I}^i_{l,k}x^i_{l,k|k-1}$$

(6) **for** $m = 0, 1, \cdots, L$ **do**

Broadcast information $V^i_{l,k}$, $G^i_{l,k}$, $v^i_{l,k}$ to locally connected sensor nodes

Applying average consensus algorithm to $V^i_{l,k}$, $G^i_{l,k}$, $v^i_{l,k}$

(7) Measurement update:

$$\begin{aligned} Y^i_{l,k|k} &= Y^i_{l,k|k-1} + N_s V_{l,k}(L) \\ x^i_{l,k|k} &= \left(Y^i_{l,k|k-1} + N_s G_{l,k}(L)\right)^{-1}\left(y^i_{l,k|k-1} + N_s v_{l,k}(L)\right) \\ P^i_{l,k|k} &= \left(Y^i_{l,k|k}\right)^{-1} \end{aligned}$$

Output: Current estimation $\left\{x^i_{l,k|k}, P^i_{l,k|k}\right\}$

iterations will degrade significantly. To see this, define Π as the consensus matrix, whose elements are the consensus gains $\pi_{l,l'}$ and let $\pi^m_{l,l'} \in \Pi^m$ with Π^m being the mth power of matrix Π. Further, define $\mathcal{N}^m_l$ as the set of sensor nodes that the lth sensor can be connected within m hops. Then, only when $l' \in \mathcal{N}^m_l$, $\pi^m_{l,l'} \neq 0$. This means that only when the observability of sensor set $\mathcal{N}^m_l$ is satisfied, the strategy of consensus on measurement is meaningful for data fusion since no local prior knowledge is utilised for fusion in DJPDA-CM. Therefore, DJPDA-CM constrains the posterior estimates as the prior estimates if the sensor and its neighbours cannot detect the target due to limited field-of-view. Note that the observability condition of sensor set $\mathcal{N}^m_l$ can only be ensured with enough number of iterations for sparse networks.

Remark 2. As stated in Lemma 2, the convergence of DJPDA-CM to the centralised fusion requires the condition that the previous local estimates are converged. In real applications, however, only finite number of consensus iterations are acceptable, which means that $x^i_{l,k-1|k-1} \neq x^{i,c}_{k-1|k-1}$ and $Y^i_{l,k-1|k-1} \neq Y^{i,c}_{k-1|k-1}$. Therefore, all local estimates are auto-correlated during the fusion phase and thus DJPDA-CM suffers from the well-known auto-correlation problem.

C. Distributed JPDA Filter Using Consensus on Information

Apart from DJPDA-CM, the covariance intersection approach [8] suggests an alternative way to design a distributed JPDA filter, that is, consensus on information matrix and information vector. Let us rewrite the measurement update of information JPDA filter, given in Eq. (7), as

$$\begin{aligned} Y_{l,k|k}^{i} &= Y_{l,k|k-1}^{i} + \bar{\mathrm{I}}_{l,k}^{i} \\ \left(Y_{l,k|k-1}^{i} + \mathrm{I}_{l,k}^{i}\right) x_{l,k|k}^{i} &= y_{l,k|k-1}^{i} + \mathrm{i}_{l,k}^{i} + \beta_{l,0}^{i} \mathrm{I}_{l,k}^{i} x_{l,k|k-1}^{i} \end{aligned} \tag{16}$$

Since covariance intersection utilises a convex combination of local estimates for data fusion, the consensus variables in DJPDA-CI are related to information matrix and information vector. More specifically, the consensus variables $W_{l,k}^{i}$, $Q_{l,k}^{i}$, $q_{l,k}^{i}$ are initialised as

$$W_{l,k}^{i}(0) = Y_{l,k|k-1}^{i} + \bar{\mathrm{I}}_{l,k}^{i}, \quad Q_{l,k}^{i}(0) = Y_{l,k|k-1}^{i} + \mathrm{I}_{l,k}^{i}, \quad q_{l,k}^{i}(0) = y_{l,k|k-1}^{i} + \mathrm{i}_{l,k}^{i} + \beta_{l,0}^{i} \mathrm{I}_{l,k}^{i} x_{l,k|k-1}^{i} \tag{17}$$

After receiving $W_{l,k}^{i}$, $Q_{l,k}^{i}$, $q_{l,k}^{i}$ from neighbours and performing L steps of average consensus at every time instant, the measurement update of DJPDA-CI at time instant k is given by

$$\begin{aligned} Y_{l,k|k}^{i} &= W_{l,k}(L) \\ x_{l,k|k}^{i} &= \left[Q_{l,k}(L)\right]^{-1} q_{l,k}(L) \end{aligned} \tag{18}$$

Notice that DJPDA-CI with single consensus iteration, e.g., $L = 1$ reduces to the general covariance intersection applied to locally connected sensors. By utilising multiple consensus iteration steps, e.g., $L > 1$, the information of each sensor can be transmitted to more local sensors, thus improving the overall performance. Different from single target tracking, the distributed covariance intersection of JPDA requires performing consensus on two different information matrices, e.g., $W_{l,k}^{i}$ and $Q_{l,k}^{i}$. This fact can be attributed to the intrinsic data association uncertainty property of MTT.

The detailed implementation of the proposed DJPDA-CI algorithm is summarised in Algorithm 2. The following lemma analyses the asymptotical performance of DJPDA-CI.

Lemma 3. *Under the assumption that the sensor network is strongly connected, DJPDA-CI (18) cannot recover the performance of the centralised JPDA (9) even with infinite number of consensus iterations at current time instant.*

Proof. Using Lemma 1, the consensus variables $W_{l,k}^{i}$, $Q_{l,k}^{i}$, $q_{l,k}^{i}$ asymptotically converge to

$$\begin{aligned} \lim_{m\to\infty} W_{l,k}^{i}(m) &= \frac{1}{N_s} \sum_{l'=1}^{N_s} \left(Y_{l',k|k-1}^{i} + \bar{\mathrm{I}}_{l',k}^{i}\right) \\ \lim_{m\to\infty} Q_{l,k}^{i}(m) &= \frac{1}{N_s} \sum_{l'=1}^{N_s} \left(Y_{l',k|k-1}^{i} + \mathrm{I}_{l',k}^{i}\right) \\ \lim_{m\to\infty} q_{l,k}^{i}(m) &= \frac{1}{N_s} \sum_{l'=1}^{N_s} \left(y_{l',k|k-1}^{i} + \mathrm{i}_{l',k}^{i} + \beta_{l',0}^{i} \mathrm{I}_{l',k}^{i} x_{l',k|k-1}^{i}\right) \end{aligned} \tag{19}$$

Algorithm 2 DJPDA-CI for target i at sensor node l at time step k

Input: Previous target estimation $\left\{x^i_{l,k-1|k-1}, P^i_{l,k-1|k-1}\right\}$, received measurements Z_k

(1) Linearise system equation: $F^i_{l,k-1} = \frac{\partial f^i_{k-1}}{\partial x_{k-1}}\left(x^i_{l,k-1|k-1}\right)$

(2) Prediction:

$$\begin{aligned} x^i_{l,k|k-1} &= f^i_{k-1}\left(x^i_{l,k-1|k-1}\right) \\ P^i_{l,k|k-1} &= F^i_{l,k-1}P^i_{l,k-1|k-1}\left(F^i_{l,k-1}\right)^T + Q^i_{k-1} \\ Y^i_{l,k|k-1} &= \left(P^i_{l,k|k-1}\right)^{-1}, y^i_{l,k|k-1} = \left(P^i_{l,k|k-1}\right)^{-1} x_{l,k|k-1} \end{aligned}$$

(3) Linearise measurement equation: $H^i_{l,k} = \frac{\partial h^i_k}{\partial x_k}\left(x^i_{l,k|k-1}\right)$

(4) Compute the measurement-related terms: $\bar{\mathrm{I}}^i_{l,k}$, $\mathrm{I}^i_{l,k}$, $\mathrm{i}^i_{l,k}$

(5) Compute the initial values of consensus variables:

$$\begin{aligned} &W^i_{l,k}(0) = Y^i_{l,k|k-1} + \bar{\mathrm{I}}^i_{l,k}, \quad Q^i_{l,k}(0) = Y^i_{l,k|k-1} + \mathrm{I}^i_{l,k} \\ &q^i_{l,k}(0) = y^i_{l,k|k-1} + \mathrm{i}^i_{l,k} + \beta^i_{l,0}\mathrm{I}^i_{l,k}x^i_{l,k|k-1} \end{aligned}$$

(6) **for** $m = 0, 1, \cdots, L$ **do**

Broadcast information $W^i_{l,k}$, $Q^i_{l,k}$, $q^i_{l,k}$ to locally connected sensor nodes

Applying average consensus algorithm to $W^i_{l,k}$, $Q^i_{l,k}$, $q^i_{l,k}$

(7) Measurement update:

$$\begin{aligned} Y^i_{l,k|k} &= W_{l,k}(L) \\ x^i_{l,k|k} &= \left[Q_{l,k}(L)\right]^{-1} q_{l,k}(L) \\ P^i_{l,k|k} &= \left(Y^i_{l,k|k}\right)^{-1} \end{aligned}$$

Output: Current estimation $\left\{x^i_{l,k|k}, P^i_{l,k|k}\right\}$

Therefore, the measurement update of DJPDA-CI with infinite number of consensus iterations is determined as

$$\begin{aligned} Y^i_{l,k|k} &= \frac{1}{N_s}\sum_{l'=1}^{N_s}\left(Y^i_{l',k|k-1} + \bar{\mathrm{I}}^i_{l',k}\right) \\ x^i_{l,k|k} &= \left[\sum_{l'=1}^{N_s}\left(Y^i_{l',k|k-1} + \mathrm{I}^i_{l',k}\right)\right]^{-1}\left[\sum_{l'=1}^{N_s}\left(y^i_{l',k|k-1} + \mathrm{i}^i_{l',k} + \beta^i_{l',0}\mathrm{I}^i_{l',k}x^i_{l',k|k-1}\right)\right] \end{aligned} \tag{20}$$

Equation (20) can be reformulated as

$$\begin{aligned} Y^i_{l,k|k} &= \frac{1}{N_s}\sum_{l'=1}^{N_s}Y^i_{l',k|k-1} + \frac{1}{N_s}\sum_{l'=1}^{N_s}\bar{\mathrm{I}}^i_{l',k} \\ x^i_{l,k|k} &= \left(\frac{1}{N_s}\sum_{l'=1}^{N_s}Y^i_{l',k|k-1} + \frac{1}{N_s}\sum_{l'=1}^{N_s}\mathrm{I}^i_{l',k}\right)^{-1}\left[\frac{1}{N_s}\sum_{l'=1}^{N_s}y^i_{l',k|k-1} + \frac{1}{N_s}\sum_{l'=1}^{N_s}\left(\mathrm{i}^i_{l',k} + \beta^i_{l',0}\mathrm{I}^i_{l',k}x^i_{l',k|k-1}\right)\right] \end{aligned} \tag{21}$$

which differs from Eq. (9). Therefore, DJPDA-CI is not asymptotically optimal. □

Remark 3. From Eq. (16), we know that the term $y^i_{l,k|k-1} + \mathrm{i}^i_{l,k} + \beta^i_{l,0}\mathrm{I}^i_{l,k}x^i_{l,k|k-1}$ corresponds to local estimate weighted by its updated information matrix. Then, it is clear that, in every consensus iteration of DJPDA-CI, the lth sensor leverages local prior information as well as local measurements to compute a regional average, that is a convex combination of local estimates in $\mathcal{N}_l$ with suitable weights $\pi_{l,l'}$. This is advantageous in ensuring bounded estimation errors for any number (even with single one) of consensus steps due to the consistency property of covariance intersection. Note that the term 'consistency' here indicates that the actual local covariance is always bounded by the fused local covariance. Another benefit of DJPDA-CI is that it is robust against the auto-correlation among local estimates due to the property of covariance intersection [8]. Apart from its advantages, Lemma 3 demonstrates that DJPDA-CI is a conservative fusion algorithm as the information terms $\sum_{l=1}^{N_s} \bar{\mathrm{I}}^i_{l,k}$, $\sum_{l=1}^{N_s} \mathrm{I}^i_{l,k}$ and $\sum_{l=1}^{N_s} \left(\mathrm{i}^i_{l,k} + \beta^i_{l,0}\mathrm{I}^i_{l,k}x^i_{k|k-1}\right)$ are underweighted by a scalar $1/N_s$, compared to the centralised solution (9).

V. Distributed JPDA Filter Using Hybrid Fusion Strategy

A. Distributed JPDA Filter Using Hybrid Consensus

Comparing DJPDA-CM and DJPDA-CI, we can observe that these two algorithms have complementary properties: DJPDA-CM is asymptotically optimal but its performance degrades significantly when the number of consensus iterations is small; DJPDA-CI is beneficial for ensuring the consistency of fused estimates but cannot recover the performance of centralised JPDA. Furthermore, DJPDA-CM reveals that recovering the performance of centralised JPDA requires the knowledge of network size, e.g., the total number of sensor nodes. In practice, an unexpected sensor failure might happen and this changes the total number of nodes. Motivated by these observations, this paper proposes a fully distributed JPDA filter using a hybrid consensus strategy to fully exploits the benefits of both DJPDA-CM and DJPDA-CI without the knowledge of N_s.

Let us define the consensus variables $U^i_{l,k}$, $u^i_{l,k}$, $V^i_{l,k}$, $G^i_{l,k}$, $v^i_{l,k}$, $b_{l,k}$, $c_{l,k}$, which are initialised as

$$\begin{aligned}
&U^i_{l,k}(0) = Y^i_{l,k|k-1}, \quad u^i_{l,k}(0) = y^i_{l,k|k-1} \\
&V^i_{l,k}(0) = \bar{\mathrm{I}}^i_{l,k}, \quad G^i_{l,k}(0) = \mathrm{I}^i_{l,k}, \quad v^i_{l,k}(0) = \mathrm{i}^i_{l,k} + \beta^i_{l,0}\mathrm{I}^i_{l,k}x^i_{l,k|k-1} \\
&b_{l,k}(0) = 1, \quad c_{l',k}(0) = 1\,(\exists! l'), \quad c_{l,k}(0) = 0\,(l \neq l')
\end{aligned} \tag{22}$$

where $b_{l,k}$, $c_{l,k}$ are utilised to estimate the network size and $\exists! l'$ denotes that there exists only one sensor node l'.

The proposed hybrid fusion scheme consists of three different types of data fusion: (1) consensus on prior $U^i_{l,k}$, $u^i_{l,k}$; (2) consensus on measurement $V^i_{l,k}$, $G^i_{l,k}$, $v^i_{l,k}$; and (3) consensus on counting variables $b_{l,k}$, $c_{l,k}$. The measurement update of DJPDA-HC at time instant k with finite L consensus iterations is given by

$$\begin{aligned}
Y^i_{l,k|k} &= U_{l,k}(L) + \frac{b_{l,k}(L)}{c_{l,k}(L)}V_{l,k}(L) \\
x^i_{l,k|k} &= \left[U_{l,k}(L) + \frac{b_{l,k}(L)}{c_{l,k}(L)}G_{l,k}(L)\right]^{-1}\left(u_{l,k}(L) + \frac{b_{l,k}(L)}{c_{l,k}(L)}v_{l,k}(L)\right)
\end{aligned} \tag{23}$$

Note from Eq. (22) that implementing DJPDA-HC requires selecting a special sensor node l that is initialised as $b_{l,k}(0) = 1$, $c_{l,k}(0) = 1$. Obviously, different choices of this special sensor affect the overall estimation performance. To address this problem, the random max consensus algorithm [50] is utilised here as an alternative way to find the index of the special node in a probabilistic manner. Specifically, define consensus variables $b_{l,k}$, $c_{l,k}$, $\lambda_{l,k}$ which are initialised as $b_{l,k}(0)=1$, $c_{l,k}(0)=1$, $\lambda_{l,k}(0)\sim\mathcal{U}(0,1)$, where $\mathcal{U}(0,1)$ denotes the uniform distribution with lower and upper bounds being, respectively, 0 and 1. This random initialisation guarantees the diversity of the proposed algorithm: every sensor node has the possibility to become the special one. At each average consensus iteration, $\lambda_{l,k}$ is updated as the output of random max consensus algorithm, denoted as $\lambda_{l,\max}$, which is the maximum value of $\lambda_{l',k}$, $\forall l'\in\mathcal{N}_l$. Then, the first time that $\lambda_{l,k}(m+1)\neq\lambda_{l,k}(m)$, we subtract 1 from $c_{l,k}(m+1)$. Since the random max consensus algorithm will eventually find the sensor node that has the largest value of $\lambda_{l,k}$ with certain m' steps [50], we have $\sum\limits_{l=1}^{N_s} c_{l,k}(m)=1$ when $m>m'$. Therefore, the total number of sensors can be approximated by $N_s=b_{l,k}(L)/c_{l,k}(L)$, with $L>m'$ consensus steps.

The detailed implementation of the proposed DJPDA-CI algorithm is summarised in Algorithm 3. The following lemma analyses the asymptotical performance of DJPDA-HC.

Lemma 4. *Under the assumption that the sensor network is strongly connected, DJPDA-HC (23) will asymptotically converge to the centralised JPDA (9) at current time instant.*

Proof. Applying average consensus algorithm to $U^i_{l,k}$, $u^i_{l,k}$, $V^i_{l,k}$, $G^i_{l,k}$, $v^i_{l,k}$, we have

$$\begin{aligned}
\lim_{m\to\infty} U^i_{l,k}(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} Y^i_{l',k|k-1}\\
\lim_{m\to\infty} v^i_{l,k}(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} y^i_{l',k|k-1}\\
\lim_{m\to\infty} V^i_{l,k}(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} \bar{\mathrm{I}}^i_{l',k}\\
\lim_{m\to\infty} G^i_{l,k}(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} \mathrm{I}^i_{l',k}\\
\lim_{m\to\infty} v^i_{l,k}(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} \left(\mathrm{i}^i_{l',k}+\beta^i_{l',0}\mathrm{I}^i_{l',k}x^i_{l',k|k-1}\right)
\end{aligned}\tag{24}$$

Since the random max consensus algorithm will eventually find the special sensor node with largest value of $\lambda_{l,k}$, average consensus algorithm guarantees

$$\begin{aligned}
\lim_{m\to\infty} b_{l,k}(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} b_{l',k}(0)=1\\
\lim_{m\to\infty} c_{l,k}(m) &= \frac{1}{N_s}\sum_{l'=1}^{N_s} c_{l',k}(0)=\frac{1}{N_s}
\end{aligned}\tag{25}$$

Substituting Eqs. (24) and (25) into Eq. (23) gives the proposed DJPDA-HC with infinite consensus iterations as

$$\begin{aligned}
Y^i_{l,k|k} &= \frac{1}{N_s}\sum_{l'=1}^{N_s} Y^i_{l',k|k-1}+\sum_{l'=1}^{N_s}\bar{\mathrm{I}}^i_{l',k}\\
x^i_{l,k|k} &= \left(\frac{1}{N_s}\sum_{l'=1}^{N_s} Y^i_{l',k|k-1}+\sum_{l'=1}^{N_s}\mathrm{I}^i_{l',k}\right)^{-1}\left[\frac{1}{N_s}\sum_{l'=1}^{N_s} y^i_{l',k|k-1}+\sum_{l'=1}^{N_s}\left(\mathrm{i}^i_{l',k}+\beta^i_{l',0}\mathrm{I}^i_{l',k}x^i_{l',k|k-1}\right)\right]
\end{aligned}\tag{26}$$

Algorithm 3 DJPDA-HC for target i at sensor node l at time step k

Input: Previous target estimation $\left\{x^i_{l,k-1|k-1}, P^i_{l,k-1|k-1}\right\}$, received measurements Z_k

(1) Linearise system equation: $F^i_{l,k-1} = \frac{\partial f^i_{k-1}}{\partial x_{k-1}}\left(x^i_{l,k-1|k-1}\right)$

(2) Prediction:

$$x^i_{l,k|k-1} = f^i_{k-1}\left(x^i_{l,k-1|k-1}\right)$$

$$P^i_{l,k|k-1} = F^i_{l,k-1} P^i_{l,k-1|k-1}\left(F^i_{l,k-1}\right)^T + Q^i_{k-1}$$

$$Y^i_{l,k|k-1} = \left(P^i_{l,k|k-1}\right)^{-1}, y^i_{l,k|k-1} = \left(P^i_{l,k|k-1}\right)^{-1} x_{l,k|k-1}$$

(3) Linearise measurement equation: $H^i_{l,k} = \frac{\partial h^i_k}{\partial x_k}\left(x^i_{l,k|k-1}\right)$

(4) Compute the measurement-related terms: $\bar{\mathrm{I}}^i_{l,k}$, $\mathrm{I}^i_{l,k}$, $\mathrm{i}^i_{l,k}$

(5) Compute the initial values of consensus variables:

$$U^i_{l,k}(0) = Y^i_{l,k|k-1}, \quad u^i_{l,k}(0) = y^i_{l,k|k-1}$$

$$V^i_{l,k}(0) = \bar{\mathrm{I}}^i_{l,k}, \quad G^i_{l,k}(0) = \mathrm{I}^i_{l,k}, \quad v^i_{l,k}(0) = \mathrm{i}^i_{l,k} + \beta^i_{l,0}\mathrm{I}^i_{l,k} x^i_{l,k|k-1}$$

$$b_{l,k}(0) = 1, \quad c_{l,k}(0) = 1, \quad \lambda_{l,k}(0) \sim \mathcal{U}(0,1)$$

(6) Initialise the subtraction flag for sensor node counting algorithm: $\alpha_{l,k} = 1$

(7) **for** $m = 0, 1, \cdots, L$ **do**

  Broadcast information $U^i_{l,k}$, $u^i_{l,k}$, $V^i_{l,k}$, $G^i_{l,k}$, $v^i_{l,k}$, $b_{l,k}$, $c_{l,k}$, $\lambda_{l,k}$ to locally connected sensor nodes

  Apply average consensus algorithm to $U^i_{l,k}$, $u^i_{l,k}$, $V^i_{l,k}$, $G^i_{l,k}$, $v_{l,k}$, $b_{l,k}$, $c_{l,k}$

  Apply random max consensus algorithm to $\lambda_{l,k}$ and update $\lambda_{l,k}(m+1)$ as $\lambda_{l,\max}$

  if $\lambda_{l,k}(m+1) \neq \lambda_{l,k}(m)$ and $\alpha_{l,k} = 1$ **do**

    $c_{l,k}(m+1) = c_{l,k}(m+1) - 1$

    $\alpha_{l,k} = 0$

(8) Measurement update:

$$Y^i_{l,k|k} = U_{l,k}(L) + \frac{b_{l,k}(L)}{c_{l,k}(L)} V_{l,k}(L)$$

$$x^i_{l,k|k} = \left[U_{l,k}(L) + \frac{b_{l,k}(L)}{c_{l,k}(L)} G_{l,k}(L)\right]^{-1}\left(u_{l,k}(L) + \frac{b_{l,k}(L)}{c_{l,k}(L)} v_{l,k}(L)\right)$$

$$P^i_{l,k|k} = \left(Y^i_{l,k|k}\right)^{-1}$$

Output: Current estimation $\left\{x^i_{l,k|k}, P^i_{l,k|k}\right\}$

Since consensus on priors guarantees that all local sensors have the same priori estimates, e.g., $Y^i_{l,k|k-1} = \frac{1}{N_s}\sum_{l'=1}^{N_s} Y^i_{l',k|k-1}$ and $y^i_{l,k|k-1} = \frac{1}{N_s}\sum_{l'=1}^{N_s} y^i_{l',k|k-1}$, it is immediate to see that DJPDA-HC with infinite number of consensus iterations can recover the performance of centralised JPDA. That is, DJPDA-HC is asymptotically optimal. □

Remark 4. It is worth to point out that, with small number of iterations, $b_{l,k}(L)$ is very close to $c_{l,k}(L)$ due to the same initialisations. For example, setting $L = 1$ for a sparse sensor network results in $b_{l,k}(L) \approx c_{l,k}(L)$. Under this condition, DJPDA-HC shows similar performance as DJPDA-CI. With the increase of L, we have $b_{l,k}(L)/c_{l,k}(L) \approx N_s$ and DJPDA-HC gradually converges to optimal centralised JPDA, similar to DJPDA-CM. Therefore, DJPDA-HC fully exploits the benefits of both DJPDA-CM, e.g., global convergence to the optimal centralised solution with infinite consensus iterations, and DJPDA-CI, e.g., preserving local consistency when the consensus horizon is limited.

B. Algorithm Analysis

This subsection analyses the differences between DJPDA-CM, DJPDA-CI and DJPDA-HC theoretically. For simplicity and convenience of analysis, the measurement updates of DJPDA-CM, DJPDA-CI and DJPDA-HC with a single consensus iteration, e.g., $L = 1$, are derived from Eqs. (13), (18) and (23) as

- DJPDA-CM:

$$\begin{aligned} Y^i_{l,k|k} &= Y^i_{l,k|k-1} + N_s \sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \bar{\mathrm{I}}^i_{l',k} \\ x^i_{l,k|k} &= \left(Y^i_{l,k|k-1} + N_s \sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \mathrm{I}^i_{l',k} \right)^{-1} \left[y^i_{l,k|k-1} + N_s \sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \left(\mathrm{i}^i_{l',k} + \beta^i_{l',0} \mathrm{I}^i_{l',k} x^i_{l',k|k-1} \right) \right] \end{aligned} \tag{27}$$

- DJPDA-CI:

$$\begin{aligned} Y^i_{l,k|k} &= \sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \left(Y^i_{l',k|k-1} + \bar{\mathrm{I}}^i_{l',k} \right) \\ x^i_{l,k|k} &= \left[\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \left(Y^i_{l',k|k-1} + \mathrm{I}^i_{l',k} \right) \right]^{-1} \left[\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \left(y^i_{l',k|k-1} + \mathrm{i}^i_{l',k} + \beta^i_{l',0} \mathrm{I}^i_{l',k} x^i_{l',k|k-1} \right) \right] \end{aligned} \tag{28}$$

- DJPDA-HC:

$$\begin{aligned} Y^i_{l,k|k} &= \sum_{l'\in\mathcal{N}_l} \pi_{l,l'} Y^i_{l',k|k-1} + \frac{\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} b_{l',k}}{\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} c_{l',k}} \sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \bar{\mathrm{I}}^i_{l',k} \\ x^i_{l,k|k} &= \left(\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} Y^i_{l',k|k-1} + \frac{\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} b_{l',k}}{\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} c_{l',k}} \sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \mathrm{I}^i_{l',k} \right)^{-1} \\ &\quad \times \left[\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} y^i_{l',k|k-1} + \frac{\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} b_{l',k}}{\sum_{l'\in\mathcal{N}_l} \pi_{l,l'} c_{l',k}} \sum_{l'\in\mathcal{N}_l} \pi_{l,l'} \left(\mathrm{i}^i_{l',k} + \beta^i_{l',0} \mathrm{I}^i_{l',k} x^i_{l',k|k-1} \right) \right] \end{aligned} \tag{29}$$

From the measurement updates of DJPDA-CM, DJPDA-CI and DJPDA-HC, we have the following important observations:

(1) Comparing with DJPDA-CM (27), the prior information vectors between locally connected sensor nodes of DJPDA-CI and DJPDA-HC are weighted by their corresponding prior information matrix as $\sum_{l' \in \mathcal{N}_l} \pi_{l,l'} y^i_{l',k|k-1} = \sum_{l' \in \mathcal{N}_l} \pi_{l,l'} Y^i_{l',k|k-1} x^i_{l',k|k-1}$. This handles the issue of naive sensors, e.g., target is outside of the sensor's field-of-view, by placing less weight when receiving the information from a naive neighbour sensor. Therefore, the proposed DJPDA-HC is helpful in ensuring the consistency of local estimates. It is clear that the implementation of DJPDA-CM requires the global information, e.g., the total number of sensors, whereas DJPDA-HC dynamically estimate N_s in a distributed way.

(2) Comparing with DJPDA-CI (28), the proposed DJPDA-HC remedies the drawback of measurement under-estimation by multiplying a scaling factor $\sum_{l' \in \mathcal{N}_l} \pi_{l,l'} b_{l',k} / \sum_{l' \in \mathcal{N}_l} \pi_{l,l'} c_{l',k}$, providing the possibility of performance recovery of optimal centralised estimation. Note that the exact total information is the summation of measurements from all sensors, which is critical in recovering the performance of optimal centralised estimation.

(3) As the proposed DJPDA-HC algorithm requires more consensus variables for measurement update, the computational complexity of DJPDA-HC is higher than that of DJPDA-CM and DJPDA-CI. However, consensus on prior and consensus on measurement can be implemented in parallel to reduce the computational burden because prior estimates are independent of current measurements.

(4) For DJPDA-CM, DJPDA-CI and DJPDA-HC, it is worth to point out that both computational complexity and communication burden increase linearly with the increase in the number of consensus iteration steps L. For this reason, the parameter L should be selected as a suitable trade-off between cost and estimation performance in practical applications.

VI. Numerical Simulations

This section presents a performance evaluation of the proposed DJPDA-CM, DJPDA-CM and DJPDA-HC algorithms using Monte-Carlo simulations and experiments on real-life data. The optimal centralised JPDA filer is used as a performance benchmark for the proposed algorithms. The performance is evaluated in terms of mean position estimation error as well as root-mean-square position estimation error.

A. Simulation Setup

The considered scenario considers 5 targets randomly moving in a $500m \times 500m$ rectangular area. The surveillance area is monitored by a sensor network with $N_s = 30$ sensors and each sensor's field-of-view is a $200m \times 200m$ rectangle. Each sensor in the network has the same degree of 2, which means that each sensor is locally connected with other two sensors. Under this condition, the communication topology of the sensor network is randomly generated. All sensors are randomly placed inside the $500m \times 500m$ rectangle to cover the entire surveillance area. One sample of the considered scenario is given in Fig. 2. The number of consensus iterations is set as $L = 10$ and

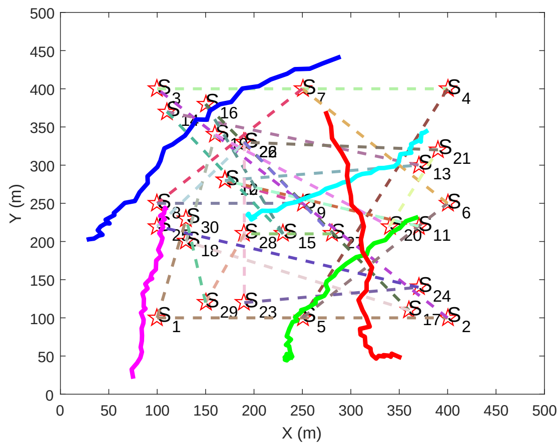


Fig. 2. Snapshots of the considered scenario with pentagrams as sensors, colour dashed lines as network connection, and solid colour lines as target trajectories.

the Metropolis weights [49] are leveraged for running average consensus algorithm as

$$\pi_{i,j} = \begin{cases} \dfrac{1}{\max\{|\mathcal{N}_i|, |\mathcal{N}_j|\}}, & \text{if } (i,j) \in \mathcal{E} \\ 1 - \sum\limits_{(i,j)\in\mathcal{E}} \pi_{i,j}, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \tag{30}$$

where $|\mathcal{N}_i|$ denotes the cardinality, e.g., number of elements, of set $\mathcal{N}_i$.

Each target's state is represented by a 4-D vector, with 2-D position and 2-D velocity components. In estimation update, the system equation is assumed to be the well-known constant velocity model $f_k^i\left(x_k^i\right) = F_k^i x_k^i$, where the system matrix F_k^i is given by

$$F_k^i = \begin{bmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{31}$$

with $T_s = 1s$ being the sampling time. The variance of process noise of the considered constant velocity model is determined as

$$Q_k^i = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{32}$$

For each sensor, if the target is located inside its field-of-view, the target-generated measurements are generated with a detection probability $P_D = 0.9$. Each sensor collects range as well as bearing measurements at regular time instants $t_k = kT_s,\ k \in \{1, 2, \cdots, 40\}$, as

$$h_k^i\left(x_k^i\right) = \begin{bmatrix} \sqrt{\left(x_k^i\left(1\right) - x_s^l\right)^2 + \left(x_k^i\left(2\right) - y_s^l\right)^2} \\ \arctan\left(\frac{x_k^i(2) - y_s^l}{x_k^i(1) - x_s^l}\right) \end{bmatrix} \tag{33}$$

where $\left(x_s^l, y_s^l\right)$ denotes the position of the lth sensor. The measurement noise is subject to a Gaussian white noise as $v_k \sim \mathcal{N}\left(\cdot; 0, R_k^i\right)$ with $R_k^i = diag\left(\sigma_r^2, \sigma_a^2\right)$, $\sigma_r = 3m$, $\sigma_a = 0.5\left(\pi/180\right)rad$. The clutter is assumed to be uniformly distributed in the surveillance region with its number being Poisson with 2 average returns per sensor at each scan. Gating is performed with a threshold such that the gating probability is $P_G = 0.999$.

For initialisation, the covariance matrix of the ith target at sensor node l is chosen as $P_{l,0|-1}^i = diag\left(100, 100, 10, 10\right)$. The initial state estimates are generated from a Gaussian distribution around the true target state with the covariance $P_{l,0|-1}^i$. Note that the starting point of each target is randomly generated inside the surveillance region at every Monte-Carlo run.

In order to evaluate the performance of the proposed algorithms with different conditions, one parameter is varied while others are set as their aforementioned default values. The simulations are obtained over 1000 Monte-Carlo runs for each parameter setting.

B. Performance Metric

Let $x_{l,k|k}^{i,j}$ denote the estimated state of the ith target at sensor node l at time instant k at the jth Monte Carlo run and $x_k^{i,j}$ be the true state of the ith target at time instant k at the jth Monte Carlo run. The mean error (ME) of position estimation and root-mean-square error (RMSE) of position estimation at time instant k, averaged over M Monte-Carlo runs, N_s sensors and N_k targets, are defined as

$$\begin{aligned} \text{ME}_k^{\text{pos}} &= \frac{1}{MN_sN_k} \sum_{i=1}^{N_k} \sum_{l=1}^{N_s} \sum_{j=1}^{M} \left\| p_{l,k|k}^{i,j} - p_k^{i,j} \right\| \\ \text{RMSE}_k^{\text{pos}} &= \left(\frac{1}{MN_sN_k} \sum_{i=1}^{N_k} \sum_{l=1}^{N_s} \sum_{j=1}^{M} \left\| p_{l,k|k}^{i,j} - p_k^{i,j} \right\|^2 \right)^{\frac{1}{2}} \end{aligned} \tag{34}$$

where $p_k^{i,j} = x_k^{i,j}\left(1:2\right)$ and $p_{l,k|k}^{i,j} = x_{l,k|k}^{i,j}\left(1:2\right)$ are true and estimated target positions.

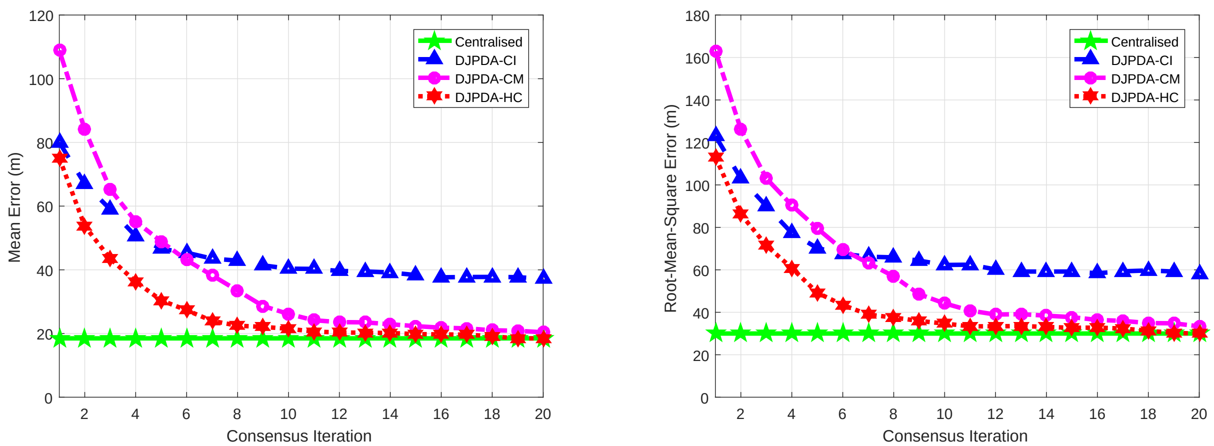
For performance evaluation of the proposed algorithms, the time averaged ME and RMSE are utilised. These two metrics are computed as

$$\begin{aligned} \text{ME}_{\text{avg}}^{\text{pos}} &= \frac{1}{T} \sum_{k=1}^{T} \text{ME}_k^{\text{pos}} \\ \text{RMSE}_{\text{avg}}^{\text{pos}} &= \frac{1}{T} \sum_{k=1}^{T} \text{RMSE}_k^{\text{pos}} \end{aligned} \tag{35}$$

where $T = 40$ is the total number of time instants during the tracking period.

C. Performance of Proposed Algorithms

In consensus-based distributed estimation, information transmission via multiple communications among locally connected sensors are required and the performance is highly-related to the number of iteration steps L. In order

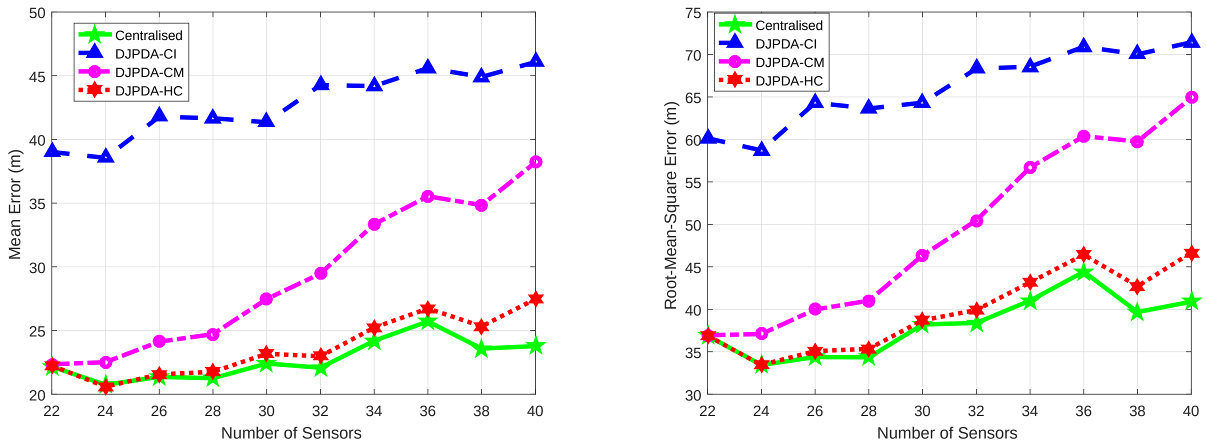


(a) Average ME (b) Average RMSE

Fig. 3. Average target position estimation ME and RMSE versus number of consensus iterations.

to investigate the effect of the parameter L on filtering performance, Monte-Carlo simulations are performed with respect to different iterations $L = 1, 2, \cdots, 20$. The simulation results of average target position estimation ME and RMSE are depicted in Fig. 3. From this figure, it can be noted that the performance of DJPDA-CM, DJPDA-CI and DJPDA-HC improves with the increase of iteration step L. The average MEs and RMSEs of DJPDA-CM, DJPDA-CI and DJPDA-HC all converge to some certain constants when the consensus iteration step becomes large enough. Note that DJPDA-CI generates more accurate estimation, compared to DJPDA-CM, with small number of consensus steps. This fact can be attributed to that DJPDA-CI utilises a convex combination of the prior estimates while DJPDA-CM only leverages the novel information for fusion. Fig. 3 also reveals that both DJPDA-CM and DJPDA-HC outperform DJPDA-CI with enough number of consensus iterations and guarantee a global convergence to recover the performance of the optimal centralised JPDA filter. This confirms the theoretical analysis shown in Sec. IV. With finite number of consensus iterations, the proposed DJPDA-HC provides better estimation performance, compared to both DJPDA-CM and DJPDA-CI, in terms of average ME and RMSE, demonstrating the advantages of DJPDA-HC algorithm. Notice that improvement in estimation can be obtained by increasing the number of consensus steps L. However, there is not much performance difference for the proposed DJPDA-HC with enough consensus steps, e.g., $L \geq 10$ in the considered setup. Typically, the communication rate is much faster than the sampling rate [51], meaning that certain consensus steps between two consecutive time instants can be ensured to guarantee the fusion performance.

Now, let us investigate the effect of the total number of sensors on tracking performance. Fig. 4 presents the Monte-Carlo simulation results of average target position estimation ME and RMSE with different number of sensors $N_s = 22, 24, \cdots, 40$. Note that all sensors are placed randomly inside the surveillance area. Therefore, if one sensor is close to another, their range and angle observations share similar qualities. Intuitively, the total amount of information increases with more sensors, which should generate improved performance. However, more sensors with the same graph degree inevitably requires larger number of consensus iterations for convergence. Due



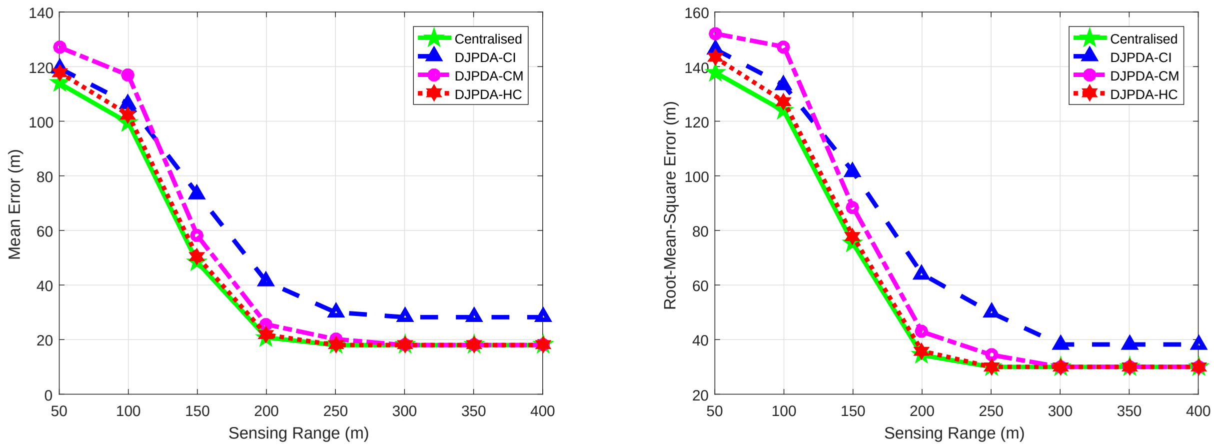
(a) Average ME (b) Average RMSE

Fig. 4. Average target position estimation ME and RMSE versus number of sensors.

to these contradictory facts, the performance of proposed algorithms do not show much difference for small number of sensors, e.g., $N_s \leq 30$. Compared to DJPDA-CI and DJPDA-HC, DJPDA-CM is more sensitive to the variation of total sensor number because this algorithm only utilises the available measurement information. It should be pointed out that the centralised JPDA filter also shows performance degradation with the increase of network size. This fact can be attributed to the increasing of the possibility of track-to-track association failure.

In network-based sensing and tracking, it is clear that the sensor's available field-of-view would affect the overall estimation performance: if the sensor's field-of-view is too narrow, then it cannot detect the target of interest. Fig. 5 compares the performance of different tracking algorithms with respect to different sensing range $50m, 100m, \cdots, 400m$. Here, the sensor's field-of-view is defined as a rectangle, depending on the sensing range. For example, if the sensing range is $200m$, the sensor's field-of-view is a $200m \times 200m$ rectangle. From Fig. 5, it can be observed that, with narrow field-of-view, e.g., sensing range $\leq 100m$, both DJPDA-CI and DJPDA-HC outperform DJPDA-CM, demonstrating that fusing the prior information is helpful to preserve the consistency of local estimates. Note that the centralised JPDA filter also shows apparent performance degradation with narrow sensor's field-of-view. The reason is that most sensor nodes miss detect the targets due to narrow field-of-view. Therefore, only small amount of information can be utilised in the fusion centre for data integration. With longer sensing range, the performance of DJPDA-CM and DJPDA-HC improves significantly and converges to the optimal centralised JPDA since more information is available for data integration. With enough sensing range, e.g., $\geq 250m$, the performance of all tested distributed JPDA filters converges to certain steady-state performance. However, it is clear that DJPDA-CI is conservative and cannot converge to optimal centralised fusion.

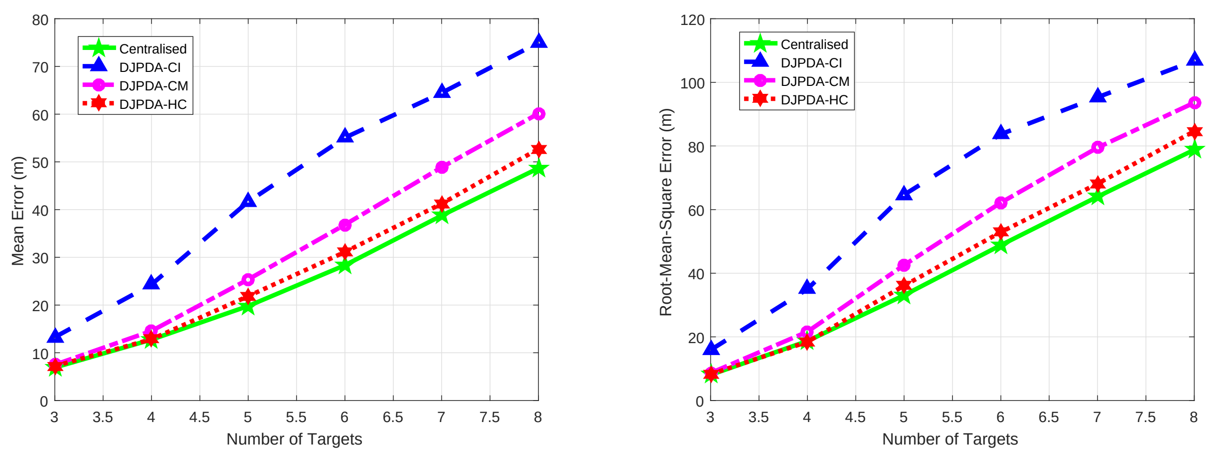
In MTT, the number of targets determines the complexity or size of the problem and greatly affects the tracking performance since data association becomes more challenging with the increase of problem complexity. Fig. 6 presents the Monte-Carlo simulation results of average target position estimation ME and RMSE with different number of targets $N_k = 3, 4, \ldots, 8$. Unsurprisingly, the performance of all tested tracking algorithms degrades with



(a) Average ME

(b) Average RMSE

Fig. 5. Average target position estimation ME and RMSE versus sensing range.



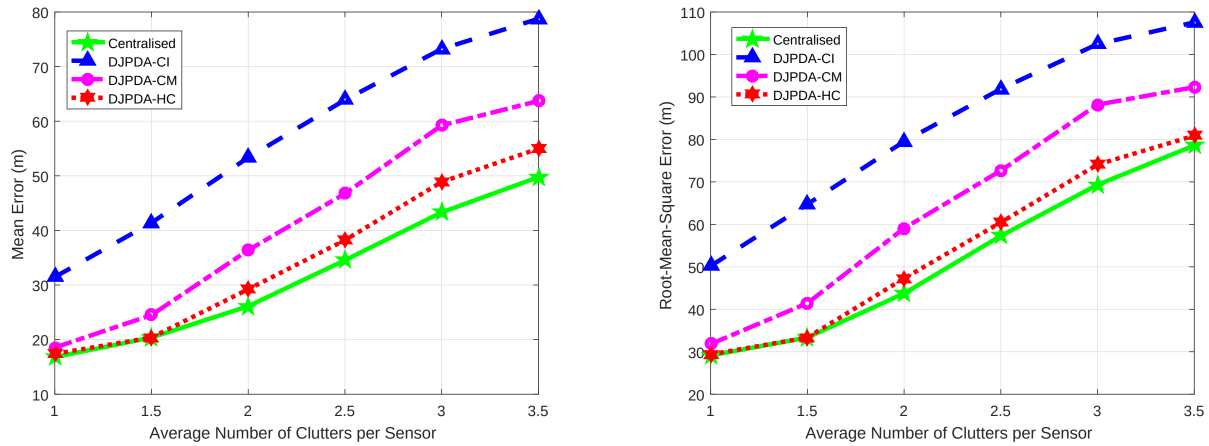
(a) Average ME

(b) Average RMSE

Fig. 6. Average target position estimation ME and RMSE versus number of targets.

more targets. The reason is that the chance of data association (measurement-to-track as well as track-to-track) failure rises with the increase in the number of targets, which has an adverse effect in JPDA. Interestingly, the proposed DJPDA-HC exhibits very close performance as centralised JPDA and outperforms both DJPDA-CM and DJPDA-CI even with large number of targets.

High clutter rate is a typical characteristics of low-cost sensors, which would have adverse effect on the tracking performance. To investigate the robustness of different algorithms against the variation of clutter rate, Fig. 7 presents the Monte-Carlo simulation results of average target position estimation ME and RMSE with different number of clutters per sensor per frame $N_{FA} = 1, 1.5, \cdots, 3.5$. Intuitively, the increase of clutter rate will give rise to high possibility of data association failure (measurement-to-track as well as track-to-track), thus the degradation in tracking performance. This can be clearly observed from Fig. 7 for all tested algorithms. This figure also reveals



(a) Average ME (b) Average RMSE

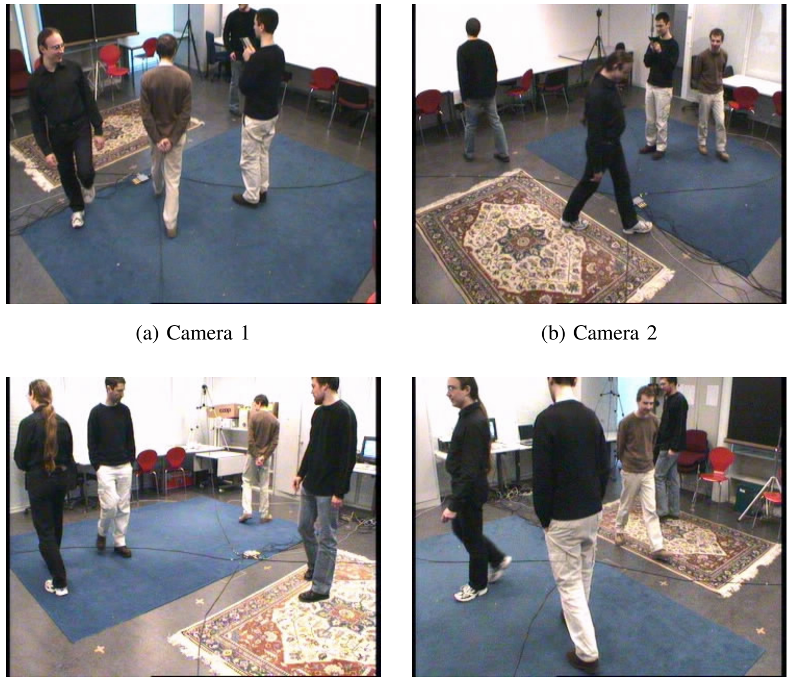
Fig. 7. Average target position estimation ME and RMSE versus number of clutters per sensor.

that DJPDA-CM is more sensitive to other algorithms. Furthermore, the proposed DJPDA-HC algorithm is strongly robust even to a high clutter rate and shows very close performance as the centralised JPDA filter.

D. Experiments on Real-Life Data

To further validate the effectiveness of the proposed method, we evaluate it with publicly available multiple target tracking datasets. We choose the popular EPFL laboratory video sequences [52] for pedestrian tracking. The EPFL laboratory dataset, filmed from four different cameras, offers four pedestrians walking around in a room. Figure 8 presents the snapshots from these four different cameras.

In all experiments, the pedestrian position measurements are obtained by running the state-of-the-art discriminatively trained part-based model (DTPM) detector [53]. The detections are obtained on a post-processing basis for all collected data to allow for fair comparisons with exactly the same inputs, i.e., the DTPM detector is applied to all videos in an offline fashion to get the measurements of all frames. Each detection is represented by a bounding box with its centroid, length and width as the measurement information of each detected target. For valid fusion, all DJPDA algorithms are performed using a ground-based inertial coordinate. Under this condition, the nonlinear measurement model $h_{l,k}^{i}\left(x_k^i\right)$ of the lth camera is a function of the camera calibration parameters [52]. The tracking performance are evaluated by comparing with the annotated ground truth, provided by [54]. The mean position error, averaged over four cameras and four persons, obtained from three different DJPDA algorithms are depicted in Fig. 9. It can be noted from this figure that both DJPDA-CM and DJPDA-HC asymptotically converge to the optimal centralised solution. Although DJPDA-CI has better tracking accuracy than DJPDA-CM with small number of consensus iterations, it cannot recover the performance of centralised fusion. Furthermore, the experimental study also reveals that the proposed DJPDA-HC outperforms both DJPDA-CI and DJPDA-CM in terms of tracking accuracy. These results clearly validate the theoretical study shown in previous sections.



(a) Camera 1 (b) Camera 2

(c) Camera 3 (d) Camera 4

Fig. 8. Snapshots of EPFL laboratory dataset from four different cameras.

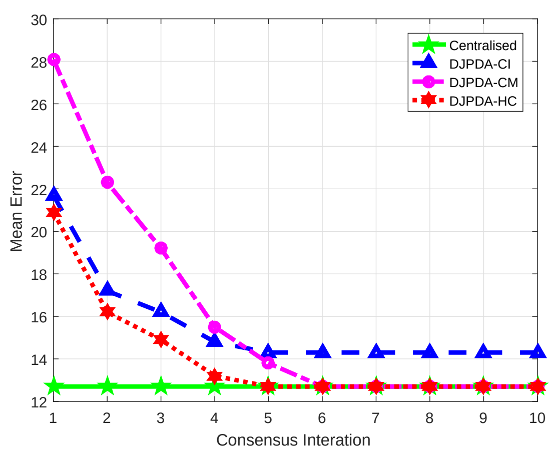


Fig. 9. Mean position error averaged over four cameras and four persons.

VII. CONCLUSIONS

The problem of distributed multiple targets tracking over a sensor network is investigated in this paper. The proposed algorithms utilise the baseline JPDA filter in conjunction with different fusion strategies, e.g., CM, CI and HC, via the average consensus algorithm. A distributed node counting algorithm is also proposed to support the implementation of DJPDA-HC. Theoretical analysis reveals that the proposed DJPDA-HC filter asymptotically converges to the optimal centralised JPDA and also provides the benefits of preserving the consistency of local estimates by a novel hybrid fusion strategy. Extensive simulations demonstrate that the proposed DJPDA-HC outperforms other algorithms under various different conditions. Future work will consider practical issues, such as time delay and packet loss, in the distributed JPDA.

APPENDIX A
INFORMATION FORM JOINT PROBABILISTIC DATA ASSOCIATION FILTER

Using matrix inversion lemma, the gain of JPDA filter K_k^i can be obtained as

$$\begin{aligned} K_k^i &= P_{k|k-1}^i \left(H_k^i\right)^T \left(H_k^i P_{k|k-1}^i \left(H_k^i\right)^T + R_k^i\right)^{-1} \\ &= P_{k|k-1}^i \left(H_k^i\right)^T \left(\left(R_k^i\right)^{-1} - \left(R_k^i\right)^{-1} H_k^i \left[\left(P_{k|k-1}^i\right)^{-1} + \left(H_k^i\right)^T \left(R_k^i\right)^{-1} H_k^i\right]^{-1} \left(H_k^i\right)^T \left(R_k^i\right)^{-1}\right) \\ &= \left[\left(P_{k|k-1}^i\right)^{-1} + \left(H_k^i\right)^T \left(R_k^i\right)^{-1} H_k^i\right]^{-1} \left(H_k^i\right)^T \left(R_k^i\right)^{-1} \end{aligned} \tag{36}$$

Substituting Eq. (36) in Eq. (5) gives the state estimation as

$$\begin{aligned} x_{k|k}^i &= x_{k|k-1}^i + \left[\left(P_{k|k-1}^i\right)^{-1} + \left(H_k^i\right)^T \left(R_k^i\right)^{-1} H_k^i\right]^{-1} \left(H_k^i\right)^T \left(R_k^i\right)^{-1} \tilde{z}_k^i \\ &= x_{k|k-1}^i + \left[\left(P_{k|k-1}^i\right)^{-1} + \left(H_k^i\right)^T \left(R_k^i\right)^{-1} H_k^i\right]^{-1} \\ &\qquad \times \left(H_k^i\right)^T \left(R_k^i\right)^{-1} \left(\sum_{j=1}^{M_k} \beta_j^i z_{j,k}^i - \left(1-\beta_0^i\right) H_k^i x_{k|k-1}^i\right) \\ &= x_{k|k-1}^i + \left(Y_{k|k-1}^i + \mathrm{I}_k^i\right)^{-1} \left[\mathrm{i}_k^i - \left(1-\beta_0^i\right) \mathrm{I}_k^i x_{k|k-1}^i\right] \\ &= \left(Y_{k|k-1}^i + \mathrm{I}_k^i\right)^{-1} \left(y_{k|k-1}^i + \mathrm{i}_k^i + \beta_0^i \mathrm{I}_k^i x_{k|k-1}^i\right) \end{aligned} \tag{37}$$

where the information-related terms are defined as

$$\begin{aligned} Y_{k|k-1}^i &= \left(P_{k|k-1}^i\right)^{-1}, y_{k|k-1}^i = \left(P_{k|k-1}^i\right)^{-1} x_{k|k-1}^i \\ \mathrm{I}_k^i &= \left(H_k^i\right)^T \left(R_k^i\right)^{-1} H_k^i, \quad \mathrm{i}_k^i = \left(H_k^i\right)^T \left(R_k^i\right)^{-1} \sum_{j=1}^{M_k} \beta_j^i z_{j,k}^i \end{aligned} \tag{38}$$

Based on the matrix inversion lemma, the update of the information matrix $Y_{k|k-1}^i$ is derived as

$$\begin{aligned} Y_{k|k}^i &= \left\{P_{k|k-1}^i - K_k^i \left(1-\beta_0^i\right) \left(K_k^i\right)^T + K_k^i \bar{P}_k^i \left(K_k^i\right)^T\right\}^{-1} \\ &= \left\{P_{k|k-1}^i - K_k^i \left[\left(1-\beta_0^i\right) S_k^i - \bar{P}_k^i\right] \left(K_k^i\right)^T\right\}^{-1} \\ &= Y_{k|k-1}^i + \bar{\mathrm{I}}_k^i \end{aligned} \tag{39}$$

where the information matrix contribution $\bar{\mathrm{I}}_k^i$ is given by

$$\bar{\mathrm{I}}_k^i = Y_{k|k-1}^i K_k^i \left\{\left[\left(1-\beta_0^i\right) S_k^i - \bar{P}_k^i\right]^{-1} - \left(K_k^i\right)^T Y_{k|k-1}^i K_k^i\right\} \left(K_k^i\right)^T Y_{k|k-1}^i \tag{40}$$

REFERENCES

[1] B. Silva, R. M. Fisher, A. Kumar, and G. P. Hancke, "Experimental link quality characterization of wireless sensor networks for underground monitoring," *IEEE Transactions on Industrial Informatics*, vol. 11, no. 5, pp. 1099–1110, 2015.

[2] S. He, D.-H. Shin, J. Zhang, J. Chen, and Y. Sun, "Full-view area coverage in camera sensor networks: Dimension reduction and near-optimal solutions," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 9, pp. 7448–7461, 2016.

[3] T. Sun and M. Xin, "Multiple uav target tracking using consensus-based distributed high degree cubature information filter," in *AIAA Guidance, Navigation, and Control Conference*, 2015.

[4] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.

[5] S. Oh, I. Hwang, and S. Sastry, "Distributed multitarget tracking and identity management," *Journal of Guidance, Control, and Dynamics*, vol. 31, no. 1, pp. 12–29, 2008.

[6] T. T. Vu and A. R. Rahmani, "Distributed consensus-based kalman filter estimation and control of formation flying spacecraft: Simulation and validation," in *AIAA Guidance, Navigation, and Control Conference*, 2015.

[7] Y. Chen, Q. Zhao, Z. An, P. Lv, and L. Zhao, "Distributed multi-target tracking based on the k-mtscf algorithm in camera networks," *IEEE Sensors Journal*, vol. 16, no. 13, pp. 5481–5490, 2016.

[8] J. Sequeira, A. Tsourdos, and S. B. Lazarus, "Robust covariance estimation for data fusion from multiple sensors," *IEEE Transactions on Instrumentation and Measurement*, vol. 60, no. 12, pp. 3833–3844, Dec 2011.

[9] T. Song, H. Kim, and D. Musicki, "Distributed (nonlinear) target tracking in clutter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 1, pp. 654–668, 2015.

[10] M. E. Liggins, C.-Y. Chong, I. Kadar, M. G. Alford, V. Vannicola, and S. Thomopoulos, "Distributed fusion architectures and algorithms for target tracking," *Proceedings of the IEEE*, vol. 85, no. 1, pp. 95–107, 1997.

[11] D.-J. Lee, "Unscented information filtering for distributed estimation and multiple sensor fusion," in *AIAA Guidance, Navigation and Control Conference and Exhibit*, 2008.

[12] R. Singer, R. Sea, and K. Housewright, "Derivation and evaluation of improved tracking filter for use in dense multitarget environments," *IEEE Transactions on Information Theory*, vol. 20, no. 4, pp. 423–432, 1974.

[13] D. Musicki, R. Evans, and S. Stankovic, "Integrated probabilistic data association," *IEEE Transactions on Automatic Control*, vol. 39, no. 6, pp. 1237–1241, 1994.

[14] T. E. Fortmann, Y. Bar-Shalom, and M. Scheffe, "Sonar tracking of multiple targets using joint probabilistic data association," *IEEE Engineering of Oceanic Engineering*, vol. 8, no. 3, pp. 173–184, 1983.

[15] J. Stauch, T. Bessell, M. Rutten, J. Baldwin, M. Jah, and K. Hill, "Joint probabilistic data association and smoothing applied to multiple space object tracking," *Journal of Guidance, Control, and Dynamics*, vol. 41, no. 1, pp. 19–33, 2018.

[16] S. He, H.-S. Shin, and A. Tsourdos, "Joint probabilistic data association filter with unknown detection probability and clutter rate," *Sensors*, vol. 18, no. 1, p. 269, 2018.

[17] ——, "Joint probabilistic data association filter with unknown detection probability and clutter rate," in *Multisensor Fusion and Integration for Intelligent Systems (MFI), 2017 IEEE International Conference on*. IEEE, 2017, pp. 559–564.

[18] D. Reid, "An algorithm for tracking multiple targets," *IEEE Transactions on Automatic Control*, vol. 24, no. 6, pp. 843–854, 1979.

[19] I. J. Cox and S. L. Hingorani, "An Efficient implementation of Reid's multiple hypothesis tracking algorithm and its evaluation for the purpose of visual tracking," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 18, no. 2, pp. 138–150, 1996.

[20] C. Kim, F. Li, A. Ciptadi, and J. M. Rehg, "Multiple hypothesis tracking revisited," *Proceedings of the IEEE International Conference on Computer Vision*, vol. 11-18-Dece, pp. 4696–4704, 2016.

[21] S. He, H.-S. Shin, and A. Tsourdos, "Track-oriented multiple hypothesis tracking based on tabu search and gibbs sampling," *IEEE Sensors Journal*, vol. 18, no. 1, pp. 328–339, 2017.

[22] R. Olfati-Saber and J. Shamma, "Consensus Filters for Sensor Networks and Distributed Sensor Fusion," *Proceedings of the 44th IEEE Conference on Decision and Control*, no. 0, pp. 6698–6703, 2005.

[23] R. Olfati-Saber, "Distributed Kalman filtering for sensor networks," *Proceedings of the IEEE Conference on Decision and Control*, pp. 5492–5498, 2007.

[24] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[25] G. Battistelli, L. Chisci, G. Mugnai, A. Farina, and A. Graziano, "Consensus-based linear and nonlinear filtering," *IEEE Transactions on Automatic Control*, vol. 60, no. 5, pp. 1410–1415, 2015.

[26] R. Carli, A. Chiuso, L. Schenato, and S. Zampieri, "Distributed Kalman filtering based on consensus strategies," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 4, pp. 622–633, 2008.

[27] A. T. Kamal, C. Ding, B. Song, J. A. Farrell, and A. K. Roy-Chowdhury, "A Generalized Kalman Consensus Filter for wide-area video networks," *Proceedings of the IEEE Conference on Decision and Control*, pp. 7863–7869, 2011.

[28] V. P. Bhuvana, M. Schranz, M. Huemer, and B. Rinner, "Distributed object tracking based on cubature kalman filter," in *Signals, Systems and Computers, 2013 Asilomar Conference on*. IEEE, 2013, pp. 423–427.

[29] J. Hu, L. Xie, and C. Zhang, "Diffusion kalman filtering based on covariance intersection," *IEEE Transactions on Signal Processing*, vol. 60, no. 2, pp. 891–902, 2012.

[30] A. T. Kamal, J. A. Farrell, and A. K. Roy-Chowdhury, "Information weighted consensus filters and their application in distributed camera networks," *IEEE Transactions on Automatic Control*, vol. 58, no. 12, pp. 3112–3125, 2013.

[31] P. Yao, G. Liu, and Y. Liu, "Average information-weighted consensus filter for target tracking in distributed sensor networks with naivety issues," *International Journal of Adaptive Control and Signal Processing*, vol. 32, no. 5, pp. 681–699, 2018.

[32] Y. Yu, "Consensus-based distributed linear filter for target tracking with uncertain noise statistics," *IEEE Sensors Journal*, vol. 17, no. 15, pp. 4875–4885, 2017.

[33] G. Battistelli and L. Chisci, "Kullback–leibler average, consensus on probability densities, and distributed state estimation with guaranteed stability," *Automatica*, vol. 50, no. 3, pp. 707–718, 2014.

[34] S. He, H.-S. Shin, and A. Tsourdos, "Multi-sensor multi-target tracking using domain knowledge and clustering," *IEEE Sensors Journal*, vol. 18, no. 19, pp. 8074–8084, 2018.

[35] N. F. Sandell and R. Olfati-Saber, "Distributed data association for Multi-target tracking in sensor networks," *IEEE Conference on Decision and Control (CDC)*, pp. 1085–1090, 2008.

[36] A. T. Kamal, J. A. Farrell, and A. K. Roy-Chowdhury, "Information consensus for distributed multi-target tracking," *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, pp. 2403–2410, 2013.

[37] Y. Yu, "Distributed target tracking in wireless sensor networks with data association uncertainty," *IEEE Communications Letters*, vol. 21, no. 6, pp. 1281–1284, 2017.

[38] G. Battistelli, L. Chisci, C. Fantacci, A. Farina, and A. Graziano, "Consensus cphd filter for distributed multitarget tracking," *IEEE Journal of Selected Topics in Signal Processing*, vol. 7, no. 3, pp. 508–520, 2013.

[39] L. Liu, H. Ji, and Z. Fan, "A cardinality modified product multi-sensor phd," *Information Fusion*, vol. 31, pp. 87–99, 2016.

[40] B. Wei and B. Nener, "Consensus labeled multi-bernoulli filtering for distributed space debris tracking," in *Control, Automation and Information Sciences (ICCAIS), 2017 International Conference on*. IEEE, 2017, pp. 203–208.

[41] L. Y. Pao and L. Trailovic, "The optimal order of processing sensor information in sequential multisensor fusion algorithms," *IEEE Transactions on Automatic Control*, vol. 45, no. 8, pp. 1532–1536, 2000.

[42] L. Trailovic and L. Y. Pao, "Computing budget allocation for efficient ranking and selection of variances with application to target tracking algorithms," *IEEE Transactions on Automatic Control*, vol. 49, no. 1, pp. 58–67, 2004.

[43] D. E. George and A. Unnikrishnan, "On the divergence of information filter for multi sensors fusion," *Information Fusion*, vol. 27, pp. 76–84, 2016.

[44] D. Mušicki, T. L. Song, H. H. Lee, X. Chen, and T. Kirubarajan, "Track-to-track fusion with target existence," *IET Radar, Sonar & Navigation*, vol. 9, no. 3, pp. 241–248, 2014.

[45] G. Battistelli, L. Chisci, F. Papi, A. Benavoli, A. Farina, and A. Graziano, "Optimal flow models for multiscan data association," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 4, pp. 2405–2422, 2011.

[46] S. Deb, K. R. Pattipati, and Y. Bar-Shalom, "A multisensor-multitarget data association algorithm for heterogeneous sensors," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 29, no. 2, pp. 560–568, 1993.

[47] S. Deb, M. Yeddanapudi, K. Pattipati, and Y. Bar-Shalom, "A generalized sd assignment algorithm for multisensor-multitarget state estimation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 33, no. 2, pp. 523–538, 1997.

[48] A. B. Poore and A. J. Robertson III, "A New Lagrangian Relaxation Based Algorithm for a Class of Multidimensional Assignment Problems," *Computational Optimization and Applications*, vol. 8, no. 2, pp. 129–150, 1997.

[49] S. Boyd, P. Diaconis, J. Sun, and L. Xiao, "Fastest mixing markov chain on a path," *American Mathematical Monthly*, vol. 113, no. 1, pp. 70–74, 2006.

[50] F. Iutzeler, P. Ciblat, and J. Jakubowicz, "Analysis of max-consensus algorithms in wireless channels," *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 6103–6107, 2012.

[51] W. Li and Y. Jia, "Distributed estimation for markov jump systems via diffusion strategies," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 53, no. 1, pp. 448–460, 2017.

[52] J. Berclaz, F. Fleuret, E. Turetken, and P. Fua, "Multiple object tracking using k-shortest paths optimization," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 33, no. 9, pp. 1806–1819, 2011.

[53] P. F. Felzenszwalb, R. B. Girshick, D. McAllester, and D. Ramanan, "Object detection with discriminatively trained part-based models," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 32, no. 9, pp. 1627–1645, 2010.

[54] Y. Xu, X. Liu, Y. Liu, and S.-C. Zhu, "Multi-view people tracking via hierarchical trajectory composition," in *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 2016, pp. 4256–4265.

Shaoming He was born in 1991. He received his MSc degree in Aerospace Engineering from Beijing Institute of Technology in 2016. He is currently a PhD student in the School of Aerospace, Transport and Manufacturing, Cranfield University. His research interests include multi-target tracking, guidance and control.

Hyo-Sang Shin received his BSc from Pusan National University in 2004 and gained an MSc on flight dynamics, guidance and control in Aerospace Engineering from KAIST and a PhD on cooperative missile guidance from Cranfield University in 2006 and 2010, respectively. He is currently Reader on Guidance, Control and Navigation Systems in Autonomous and Intelligent Systems Group at Cranfield University. His current research interests include multiple target tracking, probabilistic target detection and distributed control of multiple agent systems.

Antonios Tsourdos obtained a MEng in electronic, control and systems engineering from the University of Sheffield (1995), an MSc in systems engineering from Cardiff University (1996), and a PhD in nonlinear robust missile autopilot design and analysis from Cranfield University (1999). He is a Professor of Control Engineering with Cranfield University, and was appointed Head of the Centre for Cyber-Physical Systems in 2013. He was a member of the Team Stellar, the winning team for the UK MoD Grand Challenge (2008) and the IET Innovation Award (Category Team, 2009).