CRANFIELD UNIVERSITY

UNITED KINGDOM

OLIVER SCHWABE

A GEOMETRICAL FRAMEWORK FOR FORECASTING COST UNCERTAINTY IN INNOVATIVE HIGH VALUE MANUFACTURING

SCHOOL OF AEROSPACE, TRANSPORT AND MANUFACTURING

PhD
Academic Year: 2013-2018

Supervisors
Prof. Essam Shehab and Dr. John Erkoyuncu

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This thesis is submitted in partial fulfilment of the requirements for the Degree of PhD.

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ABSTRACT

Increasing competition and regulation are raising the pressure on manufacturing organisations to innovate their products. Innovation is fraught by significant uncertainty of whole product life cycle costs and this can lead to hesitance in investing which may result in a loss of competitive advantage. Innovative products exist when the minimum information for creating accurate cost models through contemporary forecasting methods does not exist. The scientific research challenge is that there are no forecasting methods available where cost data from only one time period suffices for their application.

The aim of this research study was to develop a framework for forecasting cost uncertainty using cost data from only one time period. The developed framework consists of components that prepare minimum information for conversion into a future uncertainty range, forecast a future uncertainty range, and propagate the uncertainty range over time. The uncertainty range is represented as a vector space representing the state space of actual cost variance for 3 to n reasons, the dimensionality of that space is reduced through vector addition and a series of basic operators is applied to the aggregated vector in order to create a future state space of probable cost variance. The framework was validated through three case studies drawn from the United States Department of Defense.

The novelty of the framework is found in the use of geometry to increase the amount of insights drawn from the cost data from only one time period and the propagation of cost uncertainty based on the geometric shape of uncertainty ranges. In order to demonstrate its benefits to industry, the framework was implemented at an aerospace manufacturing company for identifying potentially inaccurate cost estimates in early stages of the whole product life cycle.

Key words: Cost estimation; Cost uncertainty forecasting; Geometric forecasting; Scarce data

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ACKNOWLEDGEMENTS

I would like to thank the many people who have helped me through the course of my research studies. Foremost I would like to thank my academic supervisors Prof. Essam Shehab and Dr. John Erkoyuncu who have provided a home for my personal research passion at Cranfield and continue to challenge me to slow down and escape from the problem-solving mode of a challenging day job.

My thanks also go out to Prof. Bruce Jefferson, Dr. Peter Ball (previously Cranfield University, now University of York) and Dr. Rajkumar Roy as independent chairman, Dr. Yuchun Xu (previously Cranfield University now Aston University) and Professor Tetsuo Tomiyama as subject advisors, and Georg Prpich as pastoral adviser. Further appreciation goes to Gary Glennon-Alty (dstl), Andrew Nolan (Rolls-Royce), Nadia Saleh (Rolls-Royce), Richard Parker (BAE Systems), Fabian Eilingsfeld (PRICE Systems) and Robert Mills (previously Jaguar LandRover) for their industrial perspectives, as well as Paul Baguley (Cranfield University) for his insights into the mathematical realm of probability spaces.

Above all I also want to express my continued deepest appreciation for the willingness of my wife Christine, and my daughters Natascha and Annika, to support me on this journey and to respect the red light that hangs above my home office door and which broadcast my being “on air” and hence not to be disturbed on many evenings and weekends being dedicated to studying.

I also extend special thanks to the over 70 members of the LinkedIn Group “Cost Risk and Uncertainty” and all authors of materials referenced.

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LIST OF PUBLICATIONS

Journal Papers

- Schwabe, O., Shehab, E., Erkoyuncu, J.A. (2015) ´Uncertainty Quantification Metrics for Whole Product Life Cycle Cost Estimates in Aerospace Innovation´, *Journal Progress in Aerospace Sciences*, 77, pp. 1-24.

- Schwabe, O., Shehab, E., Erkoyuncu, J.A. (2016) ´A Framework for Early Life Cycle Visualisation, Quantification and Forecasting of Cost Uncertainty in the Aerospace Industry´, *Journal Progress in Aerospace Sciences*, 84, pp. 29-47.

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- Schwabe, O., Shehab, E., Erkoyuncu, J. (2016) ´Quantification and Forecasting of Whole Product Life Cycle Cost Uncertainty in Aerospace Industry Innovations´, Uncertainty Quantification: Research Opportunities Workshop, 10 February, 2016, Cranfield University, Cranfield, U.K.

- Schwabe, O., Shehab, E., Erkoyuncu, J. (2017) ´Cost Risk and Uncertainty Research´, Special Interest Group Uncertainty Quantification and Management in High Value Manufacturing Workshop, 19 January, 2017, Coventry, U.K.

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LIST OF ACRONYMS

HM	Her Majesty´s
DoD	Department of Defence
MoD	Ministry Of Defence
NAO	National Audit Office
NASA	National Aeronautics and Space Administration
NATO	North Atlantic Treaty Organisation
SAR	Selected Acquisition Reports
U.K.	United Kingdom
U.S.	United States

KEY TERMS AND DEFINITIONS

Term	Definition
Central Limit Theorem	The scientific principle based on the Law of Large Numbers which states that under certain conditions the arithmetic mean of a sufficiently large population will exhibit a normal distribution.
Cost Dimension	The cost variance type reported on, i.e. quantity, schedule, engineering, estimating, other, and support.
Cost Uncertainty	Unplanned future cost variance of an unknown quantity.
Cost Variance Vector	The magnitude and direction of cost variance for a cost variance dimension as related to the spatial centre.
Dimensions	The types of cost variance measured.
High Value Manufacturing Products	Products which are the result of "…the application of leading edge technical knowledge and expertise…" and result in "…the creation of products, production processes, and associated services which have strong potential to bring sustainable growth and high economic value…" (United Kingdom Technology Strategy Board, 2012).
Innovation Hesitance	The unwillingness to invest in products without a verified and accurate cost model.
Innovative	A condition of products or services where no (repeatable), robust verified cost model exist. This may (re-) occur at multiple times during the whole product life cycle.
Polar Force Field	A vector space with topological invariants related to the spatial centre, the dimensional sequence, the radial degree and dimensional scaling.
Robust forecasts	Forecasts that meet the needs of a cost estimating activity.
Small Cost Data	Exists if the estimation occurs with a data set from a single time period.
Uncertainty Quantification	The process of determining the single point actual prediction error of a technical baseline estimate.

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CHAPTER 1: INTRODUCTION

1.1 Introduction: Background and Motivation

Increasing competition and regulation are raising the pressure on high value manufacturing organisations of all sizes to innovate their products and portfolios. Innovation is, by default, fraught by uncertainty of whole product life cycle costs and this can lead to hesitance in investing in innovations at all technology readiness levels which may result in a significant loss of competitive advantage on a temporal and spatial scale. Temporal scales are related to different time horizons and spatial scales refer to differing sizes and natures of the organisations involved. High value manufacturing products are products that are the result of "…the application of leading edge technical knowledge and expertise…" and result in "…the creation of products, production processes, and associated services which have strong potential to bring sustainable growth and high economic value…" (United Kingdom Technology Strategy Board, 2012). "Innovative" products are considered to be such products where no robust and repeatable verified cost models exist at any specific point in time during their whole product life cycle, whereby such conditions may occur repetitively. Examples of such products investigated include advanced aircraft, military ships with novel mission paths and enhanced armoured fighting vehicles. This condition is one which leads to uncertainty regarding the accuracy of planned financial investments whereby uncertainty is understood to be unintended cost variance with an unknown impact at a future point in time.

The relevance and extent of the problem being addressed by this research study can best be summarised by the focus of major guides issued to industry by governmental purchasing institutions that are typically the most significant customers of innovative high value manufacturing products:

- "Our program assessments have too often revealed that not integrating cost estimation, system development oversight, and risk management—three key disciplines, interrelated and essential to effective acquisition management—has resulted in programs costing more than planned and delivering less than promised." (U.S. GAO, 2009)

- "UK National Audit Office, Parliamentary and internal UK Ministry of Defence (MoD) defence programme acquisition reports since 1946 clearly show that remarkably few programmes/ projects have entered service on time, on cost and with the required performance." (U.K. DoD, 2009)

- "Life cycle cost estimates of defence programmes are inherently uncertain and risky. Estimates are often made when information and data is sparse. Estimates, in turn, are based on historical samples of data that are almost always messy, of limited size, and difficult and costly to obtain. And no matter what estimation tool or method is used, historical observations never perfectly fit a smooth line or surface, but instead fall above and below an estimated value. To complicate matters, the weapon system under study is often of sketchy design." (NATO, 2007)

The financial uncertainty for investments in innovative high value manufacturing products is especially relevant in an age where the growing interdependence of such products and relevant industry infrastructure evolves rapidly and continuously. Reflecting on this context, while the expectation is that an engineering break-down cost estimation approach should achieve the most robust cost estimation it often does not. Over the past decades alternatives such as parametric estimation techniques as summarised by Foussier (2006a) have thus arisen in order to compensate for this situation although the robustness there remains heavily dependent on the existence of sufficient historical information for regression analysis and normalisation. Gathering sufficient data is an expensive effort that takes significant time investment by experts while in many cases even then being thwarted by the lack of data in the first place. This information, however, does not exist by definition for innovative high value manufacturing products.

Applied cost estimation approaches by default assume that cost estimation data is available, will follow the Law of Large Numbers and present standard probability density functions to which normalisation can reasonably fit the data available so that it represents future reality in a defensible manner (Foussier, P. M. M. (2006a). The Law of Large Numbers is a principle which proposes that if an experiment is conducted a sufficient number of times the average result of the experiment will normalise to a single value. Practice seems to indicate that this assumption only holds for manufacturing products of high technical and cost readiness, where the more units which are produced and brought in-service, the more data is available for cost evaluation of incremental changes. Considering that the costs of the first units are significant (especially since these will include non-recurrent research and development costs), finding a different approach to quantifying the potential uncertainty of early cost estimates is growing in importance.

The complexity of the challenge in the presented context is especially relevant since high value manufacturing products need to be understood as “systems-of-systems” which Haskins (2007) defines as an “interoperating collection of component systems that produce results unachievable by the individual systems alone”. The management of systems-of-systems is typically challenged by the interdependent operation of system elements and the differing whole product life cycles of these, whereby requirements mature significantly during the phases leading up to in-service. Management of systems-of-systems is also typically a highly distributed complex collaboration task with unclear boundaries and lacking halting rules especially in respect to requirements engineering (Haskins, 2007). Such a systems-of-systems view is helpful to understand that all requirements are essentially interdependent and the more innovative a product the more the development of new requirements over the product life cycle dominates the uncertainty calculation. The maturing concept of the engineering product service system provides insights into how these relational complexities might best be dealt with as suggested by Du et al. (2004) or Settanni et al. (2014). Why cost estimation for these types of products differs considerably from other products is seen in

Table 1-1 based on Haskins (2007) where the recommended activities for cost estimation between commodity and innovative products are compared.

Table 1-1: Commodity product versus aerospace innovation cost estimation activities

Activity (from Haskins, 2007)	Commodity Product (based on Haskins,2007)	Innovative Product
1. "Obtain a complete definition of the system, elements, and their subsystems."	Requirements largely defined and understood based on market maturity of earlier products.	Requirements partially known and high volume of changes expected far into life cycle due to lacking experience and historical data.
2. "Determine the total number of production units of each element to develop parametric cost data for operations."	Product order magnitude large since application, reliability etc. are clear with low uncertainty in respect to performance.	Product order magnitude low since actual performance is unclear.
3. "Obtain the life cycle program schedule."	Schedule is the "standard" schedule with experience in managing it.	Schedule is "standard" however there is significant uncertainty in respect to how long the various phases will last.
4. "Obtain manpower estimates for each phase of the entire program and, if possible, for each element and subsystem."	Estimates based on operational experience with very similar products.	Estimates difficult to provide due to novel requirements.
5. "Obtain approximate / actual overhead, general and administrative burden rates and fees that should be applied to hardware and manpower estimates."	Overheads generally known.	Overheads not necessarily impacted – no difference to traditional product.
6. "Develop cost estimates for each subsystem of each system element for each phase of the program."	Relatively reliable historical data with low uncertainty ranges is available.	Little or no historical data is available in the first place.

The bottom-up estimation approach traditionally used depends on "…a complete definition of the system, elements, and their subsystems." (Haskins, 2007) which is, by default, not possible for innovative products where the requirements are only partially known and a high volume of changes are expected far into life cycle due to the lack of experience and historical data.

The motivation for investigating the research problem arose out of semi-structured interviews in the first cycle of the the research method. It can be summarised as concerns of industry executives involved in the manufacture of innovative high value manufacturing products regarding the inability to predict the costs of manufacturing innovations in a defensible manner (EVP Supply Chain Finance, aerospace manufacturer, personal communication, 2013). Also mentioned is the resulting willingness to accept financial losses on initial production series due to this situation (CEO, defence manufacturer, personal communication, 2013). Although interviewees were active in different industries from civil aerospace through power generation to defence, the tenor remained similar in all cases.

When examining the challenge of forecasting cost and its uncertainty for innovative high value manufacturing products, it is helpful to revisit the basic principles involved. The principle is that the investigation examines a change over "time". The investigation is hence less focused on the start or end point, but interested in what happens between the two. The change over time which is of interest is cost "uncertainty" which is considered to be manifested and unplanned future cost variance. The change over time is called "propagation" and defined as the actual iterative change in uncertainty of the technical baseline estimate from the time of estimation to the time of verification. The aim is to quantify this in a robust manner, whereby quantification is focused on the process of determining the single point actual prediction error of a technical baseline estimate. The attribute of "unplanned" is of importance because "planned" variance is typically considered in explicit contingency setting of budgets. This investigation examines change over a single time period as a relevant starting point whereby the method permits extensions over multiple time periods. This extension does, however, result in building forecasts on forecasts. This results in the compounding of uncertainties over each time period and, therefore, increasing forecast uncertainty ranges significantly.

The first step is to understand that the technical cost estimate, which is baselined at some point in time, does not change until a new baseline is established. The cost uncertainty is hereby fixed at the outset of the process and considered to be static for purposes of the estimate. The next step is the evaluation of risks to cost forecasts which exhibits certain dynamic tendencies since risks may be triggered, and disappear at various points across the whole product life cycle. Based on the technical baseline estimate and the cost risk profile a budgetary decision is made. It is hereby not unusual for flat rate management reserves to be applied (for example 5% contingency as seen in one major aerospace company). In order to gain a better understanding of how this correlates with the research effort, the budgetary decision can be considered to resemble a point estimate which is constant over time, the combination of technical baseline estimate and cost risk can be considered to resemble a range estimate which fluctuates over time and the research focus to discuss a plane or space estimate which encompasses multiple future plausible scenarios (each fluctuating over time as well). A "plane" is hereby considered to represent two cost variance dimensions and a "space" any number of such above two. The difference is made due to the difference in underlying forecasting concepts and dynamics. This view is illustrated in Figure 1-1.

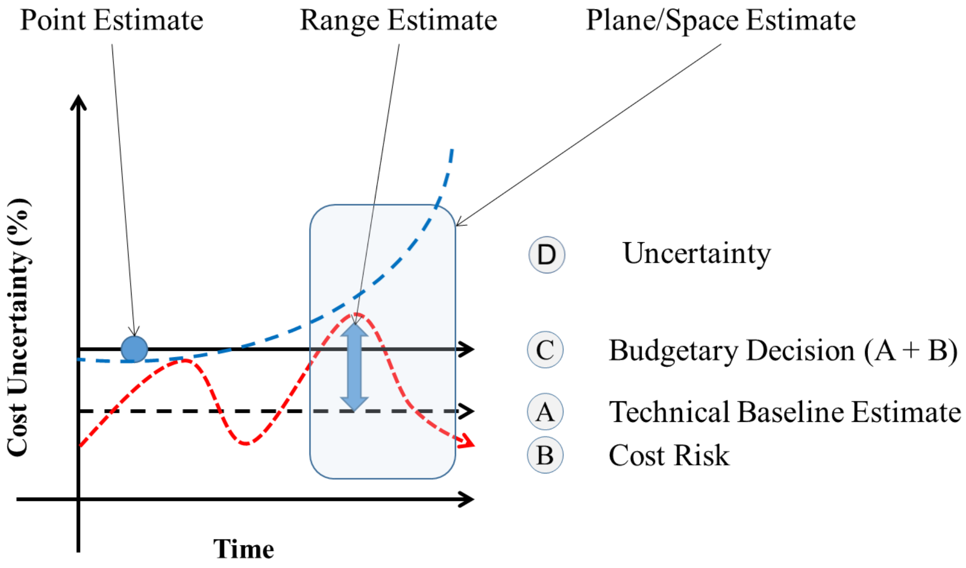

Figure 1-1: Cost uncertainty propagation and views

In Figure 1-1the dotted line A indicates the degree of cost uncertainty assigned to the technical baseline estimate. While often zero some considerations may be made to account for missing clarities. The dotted line B indicates the varying uncertainty due to identified cost risk and which is typically seen as a range. The solid line C indicates the budgetary decision made based on the input from A and B. The dotted line D indicates the uncertainty propagation behaviour identified during the case study data research related to multiple time window forecasts.

1.2 Quantifying Uncertainty in Cost Estimation

For the purposes of the research study, the key determinant of the investment hesitance is declared to be the inability of cost estimation techniques currently used in practice to robustly determine and forecast cost uncertainty (propagation) for innovative high value manufacturing products. Cost uncertainty is defined as unplanned cost variance with an unknown impact manifested at a future point in time. Its propagation is understood as the actual iterative change in uncertainty of the technical baseline estimate from the time of estimation to the time of verification. The challenges involved are declared to be due to conditions of small cost data and the absence of appropriate estimation techniques.

Small cost data exists if the estimation occurs with cost data from only one time period and having at least three cost variance dimensions. Under such conditions any uncertainty quantification techniques relying on the principles of the Law of Large Numbers are not reasonably applicable. The prevalence of such conditions in the context of innovative high value manufacturing products as identified by Schwabe et al. (2016b & 2016c) validates the importance of clearly identifying the uniqueness of the cost estimation context.

1.3 Research Questions and Hypothesis

The scientific research challenge gives rise to the research question. The research question is whether, for innovative high value manufacturing products, the geometry (shape) of small cost data is a viable data attribute for forecasting the propagation of cost estimate uncertainty over time and leads to robust results. “Viable” means that the technique is at least as repeatable, robust and fast as current practice while not relying on the applicability of the Central Limit Theorem. “Robust” forecasts are forecasts that meet the needs of a cost estimating activity which is typically the lowest possible difference between estimated and actual cost.

The research question is investigated through a research hypothesis. At its highest level the research study investigates the hypothesis that if cost variance of one time period is visualised as a shape, the geometric attributes of that shape can be used to forecast the cost variance of the next time period. The specific shape being investigated in the research study is a probability field represented as a vector space where all vectors originate at the same point and are radially arranged with a constant degree of separation. A vector space thus arranged is termed a polar force field. Cost variance presented in a table containing numbers is termed an arithmetic representation while presenting such as a histogram or spider chart would be a geometric representation. If cost variance data from a specific time period is used then that time period is termed a "state space" in contrast to a "dynamic space" which could be considered to describe the change over time between two state spaces. Vector spaces can be described both graphically in pictures / images and mathematically using vector algebra. The detailed phrasing of the research hypothesis is, therefore, that IF the arithmetic state space of actual cost variance is represented as a polar force field, THEN the state space of future cost variance can be derived through principles of vector algebra. The testing of the hypothesis is accomplished by experiments which convert cost variance data from one time period into a polar force field, use basic principles of vector algebra to forecast the future form of the polar force field and then convert the polar force field back into cost variance data for the following time period..

1.4 Scientific Research Challenge

The research problem is that in current practice, for innovative high value manufacturing products, the forecasting of cost estimate uncertainty over time is performed with techniques which are dependent on the Central Limit Theory being applicable. The Central Limit theory is a scientific principle based on the Law of Large Numbers which states that under certain conditions the arithmetic mean of a sufficiently large population will exhibit a normal distribution. The Central Limit Theory however does not reasonably apply since the forecasting effort is based on small cost data which represents the minimum sample population of one (small "n") with many parameters of unknown influence (large “p”). Following Spiegelhalter (2014) “…traditional statistical problems could be termed “large n, small p” ”. Investigations of small cost data on the other hand are considered as “small n, large p” problems where few observations are subject to many parameters and many hypotheses. Investigating “small n, large p” problems requires the capability to easily interact with data and rapidly explore large numbers of at times contradictory hypotheses (Spiegelhalter, 2014). The relevant scientific research challenge is that there are no forecasting methods available where the minimum information of cost data from only one time period suffices for reasonably robust cost uncertainty forecasting.

1.5 Research Aim and Objectives

The aim of the investigation was to develop a framework for forecasting cost uncertainty in innovative high value manufacturing products in order to help reduce innovation hesitance. The specific objectives were to:

- Capture and understand current methods and metrics for estimating cost uncertainty in the high value manufacturing industry.

- Classify the key metrics for visualising, quantifying and forecasting cost uncertainty and its propagation.

- Develop a framework for visualising, quantifying and forecasting cost uncertainty and its propagation in the form of a mathematical model.

- Validate and verify the framework and model using real life industrial case studies and experts’ opinion.

1.6 Summary

Chapter 1 discussed the background and motivation of the study including the context of quantifying uncertainty in cost estimation. The scientific research challenge, aims and objectives, research questions, hypothesis and thesis layout were introduced.

Chapter 2 presents the results of the literature review with an emphasis on the metrics for uncertainty quantification which were identified as part of uncertainty quantification methods in practice, an introduction to probability fields, force field analysis, and a discussion of the research gaps identified and their significance.

1.7 Thesis Layout

The remainder of this thesis is structured into seven chapters as illustrated in Figure 1-2. Chapter 2 covers the literature review for cost estimation methods and uncertainty quantification metrics used in practice and potentially available in the future. A typology of uncertainty quantification metrics related to the conditions under which they might be applied is presented and the concept of polar force fields which underlies the results of the investigation introduced. Chapter 3 presents the research methodology applied to test the research hypothesis based on the four primary modes of knowledge conversion. Chapter 4 examines current practice with its associated challenges and identifies the condition of "small cost data" as the object of the investigation. Chapter 5 contains the details of the integrated forecasting framework developed from various perspectives of growing depth to then explain the forecasting algorithms applied and the dependency model which are enabled to explain the forecasting results. Chapter 6 is focused on verification and validation through three primary case studies, interviews and surveys, the serious game used for qualitative validation and the thought experiment for qualitative validation of the explanatory dependency model approach. Chapter 7 discusses the research findings and examines their potential benefits for research and industry. Chapter 8 concludes the report by examining the degree that set objectives were fulfilled, sharing conclusions and recommending future work.

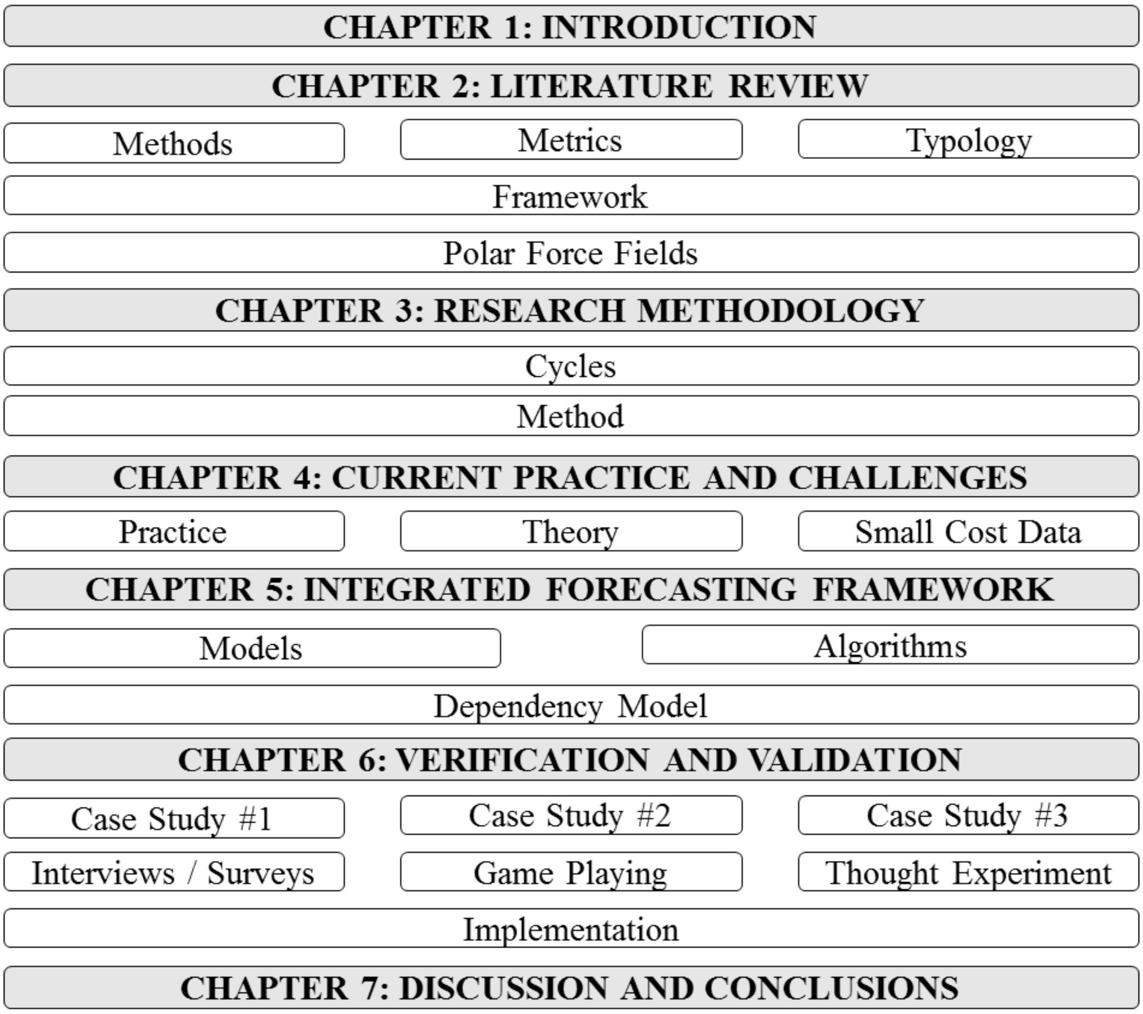

Figure 1-2: Thesis structure

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter presents the results of the literature review focusing on contemporary cost estimation methods and the concepts of uncertainty quantification and probability. This is followed by an in-depth review of metrics for uncertainty quantification over time leading to a summary of metrics typically applied and available for practice. A typology of uncertainty quantification metrics is presented and a framework for understanding their applicability in different contexts discussed. This is followed by an introduction to polar force field analysis, a discussion of the research gaps identified and a reflection on their significance. The content of this chapter is predominantly drawn from the literature review as published by Schwabe et al. (2015b).

During the course of the research study multiple iterative literature reviews were performed. The starting point was an exploration of the foundations of risk and uncertainty in an industrial context (Schwabe et al., 2014a) which set the stage for the following in-depth review of uncertainty quantification metrics for whole product life cycle cost estimates in aerospace innovation (Schwabe et al., 2015b).

Evolving out of the in-depth review of uncertainty quantification metrics a review of literature was performed with an emphasis on elements relevant to creating a framework for whole product life cycle visualisation, quantification and forecasting of cost uncertainty in the aerospace industry (Schwabe et al., 2016a) followed by an extension to the specific challenge of geometric quantification of cost uncertainty propagation (Schwabe et al., 2015a). The insights of the research study at that point in time led to a review of literature relevant to selecting cost estimation techniques for innovative high value manufacturing products (Schwabe et al., 2016b). Based upon that a review of literature in respect to boundary setting and short interval control for the cost estimate baseline of innovative high value manufacturing products was conducted (Schwabe et al., 2016c).

Further reviews were conducted in areas related to forecasting the uncertainty of cost estimates through symmetrisation and exploring the cost estimate contingency conundrum. Finally, reviews were conducted to explore the nature of polar force fields and the application of state and dynamic state principles and their pictures to the research findings. The different areas of the primary literature review (Schwabe et al., 2015b) are shown in Table 2-1:

Table 2-1: Different areas of the literature review

Areas of the reference	Number of references
Engineering	85
Mathematics	47
Risk	29
Policy	23
Finance	1

2.2 Cost Estimation Methods and Uncertainty Quantification

Any estimation attempts to forecast a future condition with the greatest possible degree of certainty in order to support relevant decision making. Since the future by default cannot be predicted with absolute certainty, an estimate will always contain a a degree of uncertainty the causes of which may be contained in explicit and tacit assumptions accompanying the estimate.

This assumption is not only relevant to the context of the estimate (i.e. that the inflation in the next accounting period may average 3%), but is also inherent in the cost estimation method applied in respect to technique and metrics used (i.e that a data set will correlate highly to a uniform probability density function applied by default in a Monte Carlo simulation). The review of uncertainty quantification metrics thus needs to be seen in the context of contemporary cost estimation methods.

For purposes of the investigation the primary cost estimation methods investigated and related to the research findings were analogy and expert opinion, parametrics and regression. The attributes discussed are summarised in Appendix F including a comparison to research findings.

2.2.1 Analogy and Expert Opinion Based Estimating

Estimating with analogies or expert opinion is based on identifying a historical product with cost information which is comparable to the one being estimated for. This method is primarily used when no relevant historical data sets are available for basing an estimate of the product upon or as a complementary method to other estimation techniques.

This method is used in practice especially in the early concept phase where future scenarios for the product and the corresponding business model are explored. This is largely within the context of previous experience whereby the specific future business model is typically considered an unknown and the emphasis lies on the identification of initial boundaries for the probability space of the future product. The method can be applied for the estimation of cost estimate uncertainty across a large range of whole product life cycle phases and is driven by human dynamics derived through qualitative identification of shared views regarding the context of the cost uncertainty estimate. The dynamics are given by the exchange of tacit knowledge between the stakeholders involved in an estimation effort. The method will generally focus on total cost variance (uncertainty) and may point to its major drivers. In relation to the other methods it presents the lowest level of granularity and has the highest level of review.

Stories known to the estimators are fundamental whereby these may be based on analogies drawn between the relevant product and those known-known experiences with others of sufficient similarity or on the expert experience of the estimator with individual factors related to these.

The method reveals lessons learned through past experiences with products which are deemed similar enough to warrant the use of analogies. The assumption is that the current estimating context is similar to the historical estimating context whereby a series of assumptions may be drawn explicitly and / or tacitly in order to identify these. The primary strength of this method is its speed since due to the lack of estimating history the estimator can only draw on the stories they and their peers are familiar with. The associated weakness however is the suitability of the relevant analogy.

While typically the most powerful qualitative influencer of stakeholder confidence in estimates it is at the same time the least rigorous though fastest method. Uncertainty quantification is typically based on the experience of different estimators regarding the difference between forecast and verified costs for comparable products under similar conditions.

2.2.2 Parametric Based Estimating

Parametric estimation methods are reasonably used under conditions where 4 to 41 relevant historical data sets for cost variance are available for basing an estimate upon. The boundaries are given based on the principles of computational complexity of short strings in respect to lossless (de-) compression as discussed by Soler-Toscano et al. (2014). The emphasis is placed on the relationship of variables in the cost estimating relationship model. This relationship is statistical in nature and build on the interaction of multiple mathematical models, as demonstrated in the investigation through the dependency model. The method is thus driven by the correlation of the mathematical models underlying each included variable from the perspective of cost estimating relationships.

In practice, the cost estimating relationship model is based on the work-breakdown structure of the technical estimate and exhibits large numbers of different variables so that the corresponding granularity is higher and the review altitude lower than analogy and expert opinion based approaches. The method refines the potential range of future conditions to those most plausible within the possible probability space. This is done primarily through detailed propagation analyses enabled through the dependency model. Since this method uses co-variate calculation to determine the dependencies between variables in the dependency model and then uses mathematical simulation to determine aggregated behaviour over time, it is able to identify statistically significant correlations for further evaluations.

By default this method primarily reveals unknown-knowns or unknown-unknowns related to dependencies in the cost estimating relationship model. The primary strength of this method is that its principles are widely understood in the cost estimation community and fundamental to the offerings of many supporting tools and techniques.

It is however significantly less trusted by decision makers than the work-breakdown approaches underlying regression methods and the data normalisation commonly applied suggests single modal data distributions while removing outliers that indicate potentially relevant multiple data centres (see also Schwabe et al., 2014b).

Uncertainty in parametric based estimation efforts is based on the different estimation results of various plausible scenarios examined, i.e. the difference in values between the scenario generating the highest and the lowest estimate.

2.2.3 Regression Based Estimating

Regression based methods are reasonably used when 42 or more relevant historical data sets are available so that the minimum prior data requirements of the relevant statistical techniques are met (see also Soler-Toscano et al., 2014). This method is based on having minimum prior data as described by the Central Limit Theorem when drawing upon the attributes of the Law of Large Numbers in order to create arithmetic forecasts.

For this method, the organisation of data is performed through probability density functions whereby the normalisation of the data to default single modal distributions is preferred and embedded in the relevant supporting tools and techniques such as the Monte Carlo simulation (Foussier, P.M.M., 2006a). Compared to analogy and expert opinion and parametrics, this method exhibits the highest degree of granularity due to typically being not only based on the variables included in the cost estimating relationship model, but also adding normalised attributes of data behaviour due to the use of statistical techniques based on the Central Limit Theorem. In principle less customised than parametric models, this method hence exhibits the greatest degree of granularity and correspondingly the lowest review altitude. The method is based on a narrow range of plausible future scenarios and applies statistical techniques to arrive at the most likely specific scenarios. Through the application of statistical techniques, which are predominantly based on the Central Limit Theorem, a single centre of data distribution is the focus whereby typically only a two-dimensional co-ordinate system is used for visualisation and quantification.

The knowledge revealed through this method is related to the cost forecast as given by statistical confidence levels which allow the estimator to identify the single point estimate to emphasise in further calculations. Not revealed, however, is which confidence level is most relevant for the decision at hand and how to manage the setting of related contingencies etc. The primary strength of this method is that it is based on significant and relevant experience which defines a stable context where statistical approaches provide reasonable and reliable planning orientation. The corresponding weakness of relying on the minimum prior data requirements of the Central Limit Theorem is derived from a generally lacking review of its applicability under such conditions. Uncertainty in regression based estimation efforts is typically based on the concept of statistical confidence values as generated by simulations such as the Monte Carlo technique.

2.3 The Uncertainty Quantification Challenge

The metrics that are available for describing and quantifying uncertainty under different conditions and in relation to various methods are discussed by Schwabe et al. (2015b). The main objective of uncertainty quantification is hereby assumed to be the forecast of the actual prediction error of the cost estimate as robustly as possible. The concept of “robustness” is used to describe the ability of forecasts to meet the needs of a cost estimating activity. While the actual prediction error can only be verified once the effort in question has been completed, the deviance forecast itself occurs in a dynamic and evolving context during the whole product life cycle. This results in the deviance forecast requiring continuous adjustment. As discussed by Michalski & Winston (1985), this adjustment is difficult to predict due to the wide range of potential influencers and the resulting rise in computational complexity.

While uncertainty quantification in practice is typically at best considered the result of a risk assessment process based around a technical baseline estimate, this uncertainty however typically represents a static snap-shot of current conditions without defensible explanation of future development. It is, however, the future development of this uncertainty that is critical to understand when it comes to manufacturing innovations, where the time between estimation and the point when verification can occur may often be measured in years and the financial investments involved require short-term decisions which are significant enough to threaten the future of the relevant company if judged wrongly.

When determining the uncertainty of a cost estimate, the estimator, similar to when creating the cost estimate itself, thus has several fundamental decisions to make (related to the research questions), whereby in general it is perceived that "…no one solution is theoretically better than the other ones…" (Foussier, 2006b).The estimation of uncertainty is the outcome of a computational process which is influenced by further choices related to various techniques such as propagation and prediction methods, random sampling methods, and experimental design. The computational complexity class is also relevant from objective and subjective perspectives, whereby the latter not only refers to capabilities, but also to cognitive biases of stakeholders. These decisions must be made by the estimator without defensible theoretical guidance regarding which choices are the most suited for the context of the estimate and the identification or matching of patterns potentially found in available data. As discussed by Golkarl & Crawley (2014) in respect to distributions for pattern matching, "The assumption of a distribution is arbitrary in this context, as there is no firm rationale on how to choose a distribution over another." Indeed established practices of estimators for uncertainty quantification are hence perhaps no more than culturally embedded inductive inferences that set the foundation for exploring plausible scenarios which describe an expected future as robustly as possible.

It is important to hereby remember that uncertainty typically increases the magnitude of a technical baseline estimate because the technical baseline estimate is the outcome of a dedicated technically focused estimating process which is then used as the input for a cost risk or cost threat assessment process (U.S. NASA, 2015). The treatment of opportunities which reduce uncertainty is considered to require separate assessment, i.e. a cost opportunity process which appears to find no explicit consideration in literature other than through the concept of the learning curve. As explained by Curran (1989) the next question is how to best describe this error. Error description is hereby dependent on the metric being applied and while a range of potentially suitable metrics exists, the literature review insights suggest that uncertainty at different technical and cost readiness levels is best described by different metrics. The less data suited for forward propagation is available for regression analysis, the lower the technical readiness level is by default. The less data is available the less sure the estimator can be that it can be normalised sufficiently to admit a probability density function that is based on the Central Limit Theorem and hence the more they must tend to metrics not dependent on these. The suitability of metrics therefore depends on the amount of data required for defensible pattern recognition, whereby it must be remembered that most statistical pattern recognition software algorithms in fact use Central Limit Theorem based regression techniques in their algorithms in the first place. On the other hand it might actually be argued that the more data is available the more difficult it becomes to find the right or most relevant pattern in the first place (Kostko, 1993; Taleb, 2010).

2.4 Describing Probability

Important for an effective literature review is an understanding of how probability ranges are described since this is a significant influencer of how probability is perceived and assessed by the cost estimation and general stakeholder community. While emphasis is often put on the use of probability density functions (Haskins, 2007), a combination of natural language terms with descriptive attributes is often recommended in the project management space in order to help stakeholders avoid the need for working with quantified probabilities they are typically not used to (Patt & Schrag, 2003). On the other hand, arguments are put forward that this approach is not valid across the complete range of probability scoring in that the higher the probability the more relevant the use of probability density functions becomes since they represent the intellectual rigor expected by stakeholders for the data to be used analytically to make decisions to be perceived as reliable in respect to interventions (Dieckmann & Slovic, 2010). Without data which is considered by stakeholders to be reliable, any probability estimate could be considered as unsuited for decision making (NATO, 2007). The concept of “reliable” is hereby usually understood at a minimum as an accuracy portrayed by a three point estimate based on real data with discussion of the risks and uncertainties related to this range so that a corresponding subjective and relevant statistical confidence level can be determined (Reeves et al., 2013). In parallel such an approach enhances the credibility of the risk assessment process as a whole (Hillson & Hulett, 2004). In essence, however, no standard measures for probability estimation appear to have emerged since different sources and contexts of probability information and ranges demand different encoding approaches to ensure predication quality (Haase et al., 2013).

2.5 Review of Uncertainty Quantification Metrics

The review of uncertainty quantification metrics is based on an initial definition of probability fields as the context of relevance, the Central Limit Theorem as foundational concept, a review of historical developments in the field. A detailed presentation of uncertainty quantification metrics identified by literature source is then followed by a review of metrics used in practice, metrics that are available for use in practice, and a comparison of these.

2.5.1 Probability Fields

The range of values a single point technical baseline estimate for different scenarios may have and the probability of the magnitude of these, values is described by uncertainty quantification metrics and considered to represent a multi-dimensional probability field. This probability field and its associated values may change over time as the variables influencing it change. The probability field is defined by lower and upper boundaries which are set by subjective threshold parameters, i.e. desired confidence levels. The desired probability field is the smallest range containing both estimate and verified value. From this perspective boundaries are not necessarily linear and may be defined by polynomial and scenario sensitive functions.

The probability field of the single point technical baseline estimate generated by a cost estimation process represents a zero dimensional point consisting of the expected cost at 100% probability for the point in time being estimated for. The cost risk process uses the single point technical baseline estimate as the lower bound (assuming only threats which increase cost are evaluated) and identifies an upper cost bound at a 100% confidence level. The progression to the 100% confidence level is described by the cumulative density function. The cost risk process adds a cost range to generate a one dimensional line on the probability / cost plane. The previous evaluation of cost and cost risk is then expanded to include a spectrum of probability based on the minimum confidence level demanded for decision making and generates a two dimensional space. Since the probability field changes over time this dimension needs to be added. Probability fields typically do not have straight line boundaries and the information distributed within it is not uniform.

Uncertainty quantification metrics hence not only need to be able to describe probability field boundaries (dynamic response surfaces) as they propagate over time, but also be suitable for predicting their development. The shape and the unfolding propagation behaviour of cost uncertainty is deterministic within the fidelity of the effort itself, yet due to the complexity of the probability field, a bottom-up predication represents a computational complexity class that is not solvable in polynomial time and parametric efforts also fail due to the lack of knowledge of the needed cost estimating relationships (which themselves may not be discoverable in polynomial time). The pattern which appears “hidden” in the probability field is hereby less related to the information distribution itself, and more to the manner in which this evolves / emerges or the rules which apply to this.

What remains is the question whether the techniques and metrics commonly used in this context are sufficient, or whether alternatives exist which are more suitable for uncovering and forecasting the propagating cost uncertainty patterns over time.

2.5.2 The Role of the Central Limit Theorem

A fundamental question raised is that of when the Central Limit Theorem can be used defensibly to determine the probability of a future event occurring. The Central Limit Theorem essentially states that given a sufficiently large number of observations the probability distribution of events will follow a single modal Gaussian pattern. Each observation must hereby be randomly and independently generated.

The Central Limit Theorem primarily describes the behaviour of the single centre of the data and is a special case of the Law of Large Numbers which proposes that if an experiment is conducted a sufficient number of times the average result of the experiment will normalise to a single value. The key reason for this question being fundamental is that in order to determine the (un-) certainty of an estimate most cost estimators will use Monte Carlo simulations applying Central Limit Theorem based probability density functions although the required (minimum) number of independent observations verifying this will not be available. Especially in respect to high value manufacturing innovations, very few if any actual observations will be available within the specific context and the analogous use of observations from other contexts, as offered through comparative databases of various software solutions, does not always reasonably meet these criteria either.

A further important reason for this question being fundamental is that the type of observation commonly used is financial cost for individual work-breakdown structure elements. This stands in contrast to the recommendations related to metrics that could be used in practice and which put forward the use of a risk management process which is based on effort level scoring schemes and custom probability / likelihood ranges (see also U.S. Naval Center for Cost Analysis (2014) and U.S. NASA (2015)). While the outcome may be a financial range on effort level, the unit of measurement is based on patterns of categories of impact and probability which is fundamentally different from technical baseline estimation efforts.

2.5.3 Uncertainty Quantification from Past to Present to Future

Uncertainty quantification metrics of the past are those implemented since the advent of the industrial age in the early 1900s. The fundamental schism of interest in this research overall is the phase change from mass manufacturing, where Central Limit Theorem principles can be applied with relative confidence, to an economy where rapidly growing global interdependence, information, knowledge and innovation are driving low volume production of innovative high value manufacturing products in increasingly short whole product life cycle (phases).

Uncertainty metrics used in practice are put forward in industry guides, although it must be questioned whether a dedicated cost risk process for uncertainty quantification as advocated among others by the U.S. NASA (2008) or U.K. JSP 507 (U.K. MoD, 2014) is being applied at all in industry, since in practice the boundary to the technical baseline estimate creation process is often unclear. The current approaches in respect to uncertainty quantification can hence be summarised as being the addition of a single figure (typically called “contingency”) to a technical baseline estimate, whereby the metric is a single point estimate in financial figures, a contingency in % and financial figures, and a final single point estimate in financial figures. The review of case studies mirrored this perspective in that virtually only single point estimates could be identified. The phase change from the uncertainty quantification metrics used in the past is apparently not yet in full swing especially since the education of the workforce is still heavily influenced by industrial paradigms.

When reviewing the uncertainty quantification metrics that are available for use in practice the focus lies on the same time window as those used in practice with an emphasis on journal and conference contributions. Two points of interest arise:

- The preferred metric for uncertainty quantification is the probability density function whereby the single point probability density function is considered separately and discussed less frequently.

- The clear separation of uncertainty quantification from technical baseline estimate.

These two points, as evidenced by the discussions around feasible uncertainty quantification metrics, point to the phase change in paradigms being well underway. Uncertainty quantification approaches in current use primarily reflect metrics developed in the past, and regression decisions of estimators to the fundamental questions raised in the introduction. In current practice, the estimator, out of tradition and without theoretical guidance, typically choses cost information based on work-breakdown structures and standard dispersion metrics based on subjectively chosen most fitting default single centre probability density functions whereby these are most likely to be of normal, triangular or log-normal nature. Commonly found metrics in contemporary use are confidence level, interquartile range, mean / median / mode, minimax, the co-efficient of dispersion, and standard deviation.

Uncertainty quantification metrics that may become available in the future can best be understood by examining PhD theses since 2000 and current research activities in various relevant research institutes. A general perspective can be taken in that across industries various uncertainty quantification metrics are being explored with the question of whether they may be more suitable to forecasting long term uncertainty. While continued investigation of the probability density function as seen from a Central Limit Theorem perspective remains an integral element, the general trend appears to be towards understanding at which point such paradigms are no longer defensible and beyond that point the suitability of approaches such as:

- Fuzzy theory (Zadeh & Kacprzyk, 1992; Kostko, 1993; Klir, & Yuan, 1995; Klir & Wierman, 1998),

- Bayesian belief networks (Kennedy & O´Hagan, 2001; Hamdan et al., 2009; Minunno et al., 2013; Khodakarami & Abdi, 2014),

- The concepts of entropy (Zurek, 1989; Grenn et al., 2014),

- Complexity (Hofmann, 2005; Snowden & Boone, 2007; Banazadeh & Jafari, 2012) and

- Tail-weight (Foss et al., 2011).

The use of probability spaces or geometrical approaches (such as force fields) is not evident. Future uncertainty quantification metrics of potential relevance point to a slowly arising paradigm shift in that the estimator, accepting the difference between cost and cost risk estimation, will chose information based on risk assessments (i.e. probability and impact) with metrics based on custom probability density functions which accept multiple data centres. Commonly found metrics potentially relevant for future use are the correlation co-efficient, kurtosis and skew.

The future therefore invites the estimator to progress in that while the information source remains cost risk focused, the concept of probability density functions is abandoned in favour of multi-dimensional response surfaces (which form the boundaries of probability spaces) that change over time. Commonly found metrics related to future application are related to homogeneity, density, compression, and complexity. It is these metrics that the presented research approach builds upon whereby the shape of the field the response surface is applied to is generated using polar force fields. In this case the response surface is the perimeter of the shape created by connecting the vertices of polar vectors and summarised by an aggregated vector.

2.5.4 The Evolution of Uncertainty Quantification

Drawing on work by Fienberg (1992), the time period from approximately the mid-16th century to the present day was considered whereby several boundaries were drawn based on:

- The assumption that the rise of probabilistic research can be seen as beginning with the work of Cardano on games of chance and then Laplace on the Law of Large Numbers.

- The work of Reverend Thomas Bayes with the Bayes Theorem marked a significant evolution from the original concept of the Law of Large Numbers, and

- that the growing understanding of (information) entropy as explored by Shannon (1948) marked a turning point into current paradigms of cost estimation.

A further turning point in the development of uncertainty quantification metrics might also be seen in the introduction of calculable uncertainty into economic theory in the 1930s as discussed by Boy (2009) and the growth of statistical approaches in industry (Pearson, 1935), followed by the “Theory of Games and Economic Behavior” by von Neumann and Morgenstern (1953). This reached a turning point with the Nobel prize for efforts in modern portfolio theory and the capital asset pricing model in 1995. The

Second World War accelerated the development of techniques, especially in the field of cryptology, followed by the growth in global trade and stock markets.

From the research perspective, these developments are historically fundamental although it is accepted that many different perspectives can indeed be taken. It is also important to note that developments in all sciences can seldom be identified as linear progressions with defensible key authors since publication overviews have not been maintained with rigor over the decades / centuries. There are no doubt many thinkers and authors who have achieved significant insights and influence but have fallen out of sight.

Table 2-2 displays a high level timeline of leading scholars and research in uncertainty quantification based upon authors and sources identified during the literature research.

Table 2-2: Map of the leading scholars and areas of research in uncertainty quantification

Foundations		Historical Roots		Future Perspectives	
- Pacioli, F.L. (1380) “Summa de arithmetica, Geometrica, Proportiono, et Proportionalita” - Cardano, G. (mid-16th century) “The Book on Games of Chance” - Pascal, B., de Fermat, P. (1654) on Fair Prices - Graunt, J. (1662) “Natural and political observations upon the bills of mortality” - Arbuthnot (1712) on devine providence - Bernouilli, J. (1713) “Ars Conjectandi” - Bernoulli, J. (1713) on subjective probability - De Moivre, A. (1718) “The Doctrine of Chances”	- Bernoulli, D. (1738) on utility theory - Bayes, T. (1764) on inverse probability method - Legendre (1805) on the method of least squares - Gauss, P. (1809) on normal distribution errors and least squares - Laplace, P.S. (1810) on the Central Limit Theorem - Quetelt, A. (1835) on the concept of the average man - Maxwell (1859) work on the kinetic theory of gases and law errors - Galton, F. (1869) “Hereditary Genius: An Inquiry into its Laws and Consequences”	- Galton, F. (1885) on regression towards the mean - Galton, F. (1888) on the concept of correlation - Pearson, W. (1900) on the chi-square test - Fisher, R.A. (nd) on significance testing - Gosset, W.S. (1908) on the student t-distribution - Knight, F.H. (1921) “Risk, Uncertainty and Profit” - Neyman, J. (1923) “On the application of probability theory to agricultural experiments. Essay on principles”	- Fisher, R.A. (1925) “Statistical methods for research workers” - Pearson, W. (1935) “The Application of Statistical Methods to Industrial Standardization and Quality Control” - Neyman, J. (1934) on the confidence method - Shewhart, W. (1939) “Statistical Method from the Viewpoint of Quality Control” - Jeffreys, H. (1939) “Theory of Probability” - Shannon, C.E. (1948) “A Mathematical Theory of Communication” - Kolmogorov, A.N. (n.d.) on probability axioms	- United States Department of Defense (2006) “Risk Management Guide for DoD Acquisition” - Haskins, C. ed. (2007) “INCOSE Systems Engineering Handbook v. 3.1” - RAND Corporation (2007) “Evaluating Uncertainty in Cost Estimates” - United States Air Force (2007) “Cost Risk and Uncertainty Analysis Handbook” - International Society of Parametric Analysis (2008) “Parametric Estimating Handbook” - United States National Aeronautics and Space Administration (2008 & 2015) “Cost Estimating Handbook”	- United Kingdom Ministry of Defence (2009) “The Forecasting Guidebook Version 4” - United Kingdom HM Treasury (2011) “The Green Book: Appraisal and evaluation in central government.” - RAND Corporation (2013) “Making Good Decisions Without Predictions. Robust Decision Making for Planning Under Deep Uncertainty” - United States Naval Center for Cost Analysis (2014) “Joint Agency Cost Schedule Risk and Uncertainty Hand Book” - United Kingdom Ministry of Defence (2014) “JSP 507 Investment Appraisal and Evaluation Part 2: Guidance”

2.5.5 Uncertainty Quantification Metrics Identified

In respect to uncertainty quantification metrics identified in industry guides (i.e. reports, standards or technical guidelines), journal papers, conference contributions, and PhD theses, methods for identifying or quantifying uncertainty or variables influencing the magnitude or behaviour of uncertainty metrics were not considered. Particular care was taken to focus on the metrics describing data patterns and not their interpretation, i.e. a (strange) attractors or thresholds in a dataset or the concept of randomness are considered as behaviour of data versus being an objective metric. This focus led not only to the identification of metrics (as defined by having a specific unit of measure), but also to the identification of metric "families" to which these metrics can be sorted.

2.5.5.1 Uncertainty Metrics Commonly Applied

The concept of manufacturing innovation covers a very wide field of systems whereby a clear separation needs to be made between incremental advancements of established technologies and the leaps of innovation as explored by Allen (2003). While the case studies examined in this paper focus primarily on more significant incremental advancement the review did include some where fundamental research in the sciences is / was still in early stages (i.e. public case studies and in particular those based on novel physics developed by U.S. NASA for space travel propulsion).

A review of commonly used uncertainty quantification metrics begins by visiting the cost estimator of today who is faced by the challenge of determining the uncertainty of a manufacturing innovation related cost estimate. The estimator will face the common situation that the innovation context to be estimated might be summarised as "… harsh and non-forgiving. New programs often uncover the unknown unknowns. Early flights of a new system have often revealed problems of which the designers were unaware." (Bertin & Cummings, 2003). The technical baseline estimate has already been created based on a work breakdown structure where each task has been assigned to the relevant supply chain units with the request for commitment to a single point estimate they are to provide. These single point estimates are then aggregated and a contingency added on top. This result becomes the estimated total cost for planning and forecasting purposes from a business perspective. Various stage gates in the relevant whole product life cycle management process are then progressed through as the innovation rises in technology readiness level and the cost estimate may be revisited regularly. The cost estimate will change over time and this change may well be significant enough to challenge the overall initial commercial proposition. The more robust the prediction of this change, i.e. the description of the change dynamics over time, the more effectively cost and commercial control mechanisms can be put in place. Important to note is that while currently available techniques (i.e. use of probability density functions) may be used for elements of the work breakdown structure, and while these may be revisited at regular points in the whole product life cycle, the cost estimation process typically ends at this point. While this perspective might appear to do injustice to many efforts made by cost estimators, it appears to be daily reality for most considering the time and resource constraints in place and, perhaps most importantly, the expectations of business decision makers, i.e. “give me a number to work with” as quickly as possible.

2.5.5.2 Comparing Applied to Available Metrics

An industry survey by Black (2008) which was completed 10 years previously as well, succinctly summarises that “Aerospace program cost overruns and schedule slides have created considerable angst, funding issues, and negative headlines. Accordingly, DoD and NASA increasingly emphasise the importance of cost risk management and “cost realism” i.e. “data-driven” estimates”. Although uncertainty quantification is becoming more and more objective, the survey respondents do note that subjective methods still dominate 60% of the time with all the issues related to expert judgment of uncertainty (Goldenson & Stoddard, 2013) or differing stakeholder risk perspectives (Hall et al., 2013). In the industry survey by Black (2008), it is further notable that from a metric perspective only standard single data centre driven statistics are mentioned as being used by respondents, while the scarcity of historical data was raised by 75% of respondents as the most significant hurdle to uncertainty quantification. It is unclear whether this scarcity refers to data as a whole, or data which follows only single data centre characteristics. Almost 2 / 3 of all cost estimations are hereby conducted in Microsoft ® Excel versus in professional cost estimation tools such as COCOMO (Boehm, B., 1981) or PRICE H (see also https://www.pricesystems.com/).

Applied metrics are also increasingly influenced by the most representative possible metric contributions which cluster in the period of 2005-2009 and are predominantly published by U.S. governmental space and defence organisations. U.K. publications hereby typically refer to U.S. resources regarding estimation details while embedding such in the local context of government regulation and terminology. The metric focus is based on those associated with single modal probability density functions and the methodologies involved make a clear separation between the generation of technical baseline estimates and the ensuing cost risk process. Of particular note, perhaps, is that default Central Limit Theorem based probability density functions are still typically recommended as starting points and parametric techniques commonly applied.

In general this current industry practice can be considered as a response to the U.S. General Accounting Office's report to the Subcommittee on Space and Aeronautics, Committee on Science, House of Representatives on "Lack of Disciplined Cost-Estimating Process Undermines NASA's Ability to Effectively Manage Its Programs" (2004) which identified major causes of cost growth including incomplete cost risk assessment, acquisition workforce problems, corporate-directed actions, competitive environment, and flawed initial program planning. The ensuing RAND report "Improving the Cost Estimation of Space Systems Past Lessons and Future Recommendations" (Younossi et al., 2008) then consolidated this into a set of recommendations that triggered first the U.S. Air Force "Cost Risk and Uncertainty Analysis Handbook" (2007) and then the U.S. NASA "Cost Estimating Handbook" (2008 and revised 2015) including relevant efforts by the U.S. Space Systems Cost Analysis Group (2005). A key recommendation of the following U.S. Government Accountability Office report (2009) hereby was to focus on the "inherent" uncertainty in an estimate. The U.S. Air Force "Cost Risk and Uncertainty Analysis Handbook" (2007) presents cost uncertainty analysis as that step in the cost estimation method which applies the "Formal Risk Assessment of System Cost Estimates" (FRISK) method (Young, 1992) to identify the impact and probability of various variables on the technical baseline estimate. The technical baseline estimate is determined in advance and should not include uncertainties, but focus on determining most likely single point estimates (often using default distributions for orientation). The FRISK method then determines the uncertainty of the technical baseline estimate in order to recommend financial provisioning for such in budgeting processes. Based on the default shape of the probability density function most fitting to the overall risk profile the metrics suggested for uncertainty quantification are the interquartile range, probability density function bounds, the co-efficient of dispersion, standard deviation and skew.

Similar to the U.S. Air Force “Cost Risk and Uncertainty Analysis Handbook” (2007), the U.S. NASA “Cost Estimating Handbook” (2008) proposes a methodology which clearly separates between the cost estimate, called “life cycle cost” point estimate, and the cost estimate uncertainty which is determined through a cost risk determination process. In comparison to the U.S. Air Force, the method is then extended to the six U.S. NASA phases of the project life cycle and the concept of cost readiness levels applied. While no specific cost risk policy is put forward, guidance is recommended through the relevant U.S. NASA Policy Directives, U.S. NASA Procedural Requirements and Cost Risk Volume 2 in the U.S. NASA “Cost Estimating Handbook” (2008). It is “NPR 8000.4 Risk Management Procedural Requirements” which outlines the relevant risk management process including the calculation of risks and uncertainties. Important to remember is that in contrast to the small series focus of the U.S. Air Force “Cost Risk and Uncertainty Analysis Handbook“ (2007), the approach of the NASA is designed for application to major space flight projects where the unit of one dominates. Other factors discussed by the U.S. Air Force “Cost Risk and Uncertainty Analysis Handbook” (2007) are also of relevance, although an extension is made in respect to emphasising the need for deriving the cumulative density function itself. FRISK (Young, 1992) is again put forward as the relevant risk assessment methodology. In addition several commercially available cost modelling tools are recommended including the U.S. NASA “Air Force Cost Model NAFCOM” (2002), PRICE H by Price Systems, SEER H by Galorath and COCOMO. In respect to estimation software it is also important to note that due to methodological and mathematical calculation differences results for similar calculations may differ widely or be prone to generic user errors (Smith & Shu-Ping, 2005). Further notable contributions in this timeframe were by Fox et al. (2008) and Arena et al., (2006).

The perspectives taken by U.K. based organisations are grounded on "The Orange Book Management of Risk – Principles and Concepts" as published by HM Treasury in 2004. This lays out the high level fundamental perspectives of risk and uncertainty to be considered. In 2009 this was followed by the fourth version of the U.K. MoD "The Forecasting Guidebook Version 4" (2009) which significantly increased in rigor in comparison to the previous versions by separating carefully the estimation and forecasting processes, emphasising the fundamental importance of Bayesian and parametric methods of predicting cost and schedule whereby the concept of uncertainty is clearly linked to the outcomes of a three point estimating technique.

In 2011 the "Green Book" (U.K. HM Treasury, 2011) re-emphasised the importance of the "base case" (which can be considered to equal the baseline technical estimate in U.S. based publications), clearly assigns forecasting inaccuracy to the influence of optimism bias and recommends reductions in innovation in order to increase cost estimate accuracy as indicated by the suggestion that for large or complex projects simpler alternatives should be developed wherever possible and consideration should be given to breaking down large, ambitious projects into smaller ones with more easily defined and achievable goals (U.K. HM Treasury, 2011).

The publication "JSP 507 Investment Appraisal and Evaluation Part 2: Guidance" (U.K. MoD, 2014) then marks the most intensive attempt at differentiation from U.S. based publications by clearly linking the concept of uncertainty to the influence of optimism bias, beginning to introduce specific manners to visualise uncertainty (i.e. boundary visualisations), recommending confidence levels, three point range generation techniques and referring to U.K. based sources for further details versus such published in the U.S. context.

The U.S. NASA approach is the most stringently codified method available and is designed for the cost estimation of typically single units for a single mission or very small series (i.e. reusable launch vehicles). In respect to small series (i.e. production units of several hundred) the U.S. Air Force cost estimation handbook provides solid orientation. In respect to innovative high value manufacturing products in particular a gap emerges however. Commonly accepted cost estimation methodologies for pure research and development projects also do not exist. The U.S. NASA "Cost Estimating Handbook" (2008) section "1-7. The Cost Estimating and Budgeting Connection" illustrates how single mode probability density functions are used to estimate cost ranges, whereby skew increases over the estimation process with kurtosis decreasing. Volume 2 is then specifically focused on cost risk. In section 2.2.2 the activity "Quantify Cost Estimating Uncertainty" is specifically mentioned. In this volume the U.S. NASA explicitly emphasises the importance of "…distinguishing between uncertainty (lack of knowledge or decisions regarding program definition or content) and risk (the probability of a predicted event occurring and its likely effect or impact on the program)" (U.S. NASA Cost Estimating Handbook, 2008, Volume 2, Page 2-2). From a general project perspective efforts do remain relevant in respect to estimation "short-cuts" (Chapman & Ward, 2000).

Based on the approach of NASA (2008) the starting point for the determination of cost estimate uncertainty is a single point estimate for the technical baseline cost. The next steps are determining the co-efficient of dispersion, deriving the cumulative density function and determining confidence levels. The probability density function of the program's total cost is hence derived from the single point estimate, the single point estimate probability, and the co-efficient of dispersion. Combining this function with the single point estimate and the confidence level then determines the "risk dollars" to be allocated as a measurement of cost estimate uncertainty. This is followed by a sensitivity analysis which enhances the determined uncertainty with factors such as the uncertainty of all cost estimating relationships and economic factors. Due to the low technical readiness level of most products in U.S. NASA efforts standard probability density functions are recommended (although without theoretical grounding for the recommendation) and thoroughly described including guidance under which conditions they should be used and benchmarks of relevance. Similar can be found in the U.S. Space Systems Cost Analysis Group publication "Space Systems Cost Risk Handbook: Applying the Best Practices in Cost Risk Analysis to Space System Cost Estimates" (2005) and U.S. Air Force approach (2007). In the practice of estimators, this available spectrum of approaches however typically reduces to the triangular distribution since it is fairly simple to characterise; the estimator only needs to produce three points: a reference point (sometimes called the “most likely”), a pessimistic point (upper boundary) and an optimistic point (lower boundary). Determination of the boundaries is then most often the result of an expert opinion elicitation process (U.S. Air Force, 2007). The U.S. Space Systems Cost Analysis Group report (2005) provides similar examples and guidance on technical risk distributions while the U.S. Air Force approach (2007) provides guidance and examples of selecting single modal uncertainty distribution shapes and bounds for the subjective assessment of technical input risk. All sources attempt to provide benchmark data from various programs for orientation purposes as well.

Key future concepts related to possible uncertainty quantification metrics tentatively pointed to in the literature are:

- In the conceptual area of entropy, the efforts in general build on the work of Shannon (1948) in information theory with a special focus on information transmission, whereby Zurek (1989) expands this solidly into reflections on algorithmic randomness, while Uffink (1990) hardens the mathematical underpinnings and linkages to physics, and Grenn et al. (2014) make first attempts to transfer the entropy principles into the systems engineering space.

- In the conceptual area of complex adaptive systems, the most notable efforts appear to be around the concepts of complex adaptive systems engineering (White, 2009) where especially human factors and collaboration influences gain prominence in seeking to understand overall complex engineering efforts. This then maps closely with reflections concerning the manner in which engineering environments develop from chaotic, through complex and complicated to the simpler structures found in industrial series manufacturing (Snowden & Boone, 2007).

- In the conceptual area of uncertain threshold response the emphasis remains similar to adaptive robust design approaches where the basic perceptions of risk levels in scenarios and robust versus optimal approaches are discussed (Morgan & Henrion, 1990; Lempert & Collins, 2007; Lempert et al., 2013). There are at the same time links here to the questions of scenario management and system dynamics especially as related to deep uncertainty. At the same time various related concepts can be included here such as uncertainty propagation methods (Lee & Chen, 2009) and the Bayesian calibration of computer models (Kennedy & O´Hagan, 2001; Hamdan et al., 2009; Minunno et al., 2013).

- In the conceptual area of deep uncertainty the fields of general policy analysis from the perspective of adaptive robust design (Hamarat et al., 2013), and dynamic scenario discovery (Kwakkel et al., 2013) form current areas of especially relevant research in addition to the further developments from the perspective of exploratory modelling and analysis (Lempert et al., 2003; Popper et al., 2005; Groves & Lempert, 2007; Von Krauss et al., 2008; Hall et al., 2012; Kwakkel et al., 2013; Stockdale, 2013; Wasim et al., 2013).

While certain conceptual areas can be identified, it must also be differentiated between the metric of choice and the method chosen for its presentation.

2.5.5.3 Static versus Dynamic Uncertainty Quantification Metrics

Revisiting earlier questions considering the static and dynamic nature of uncertainty quantification metrics, the question also arises which metrics may be more suitable than others for representing dynamic changes in uncertainty. Unfortunately no specific literature resource could be identified, in this respect, resulting in a focus on drawing upon potentially suitable analogies such as the pictures of state and dynamic spaces. It is important to note, therefore, that uncertainty quantification metrics in practice focus on values at specific points in time versus on how these values change between points in time.

2.6 Towards a Typology of Uncertainty Quantification Metrics

In the specific context of uncertainty quantification in cost estimation for innovative high value manufacturing products, the applied and available metrics provide a first typology for reflection on cost estimation paradigms. Available metrics indicate that the primary orientation given to the estimators stems from industry guides, company guidelines or from the techniques embedded in cost estimation software being used. Available metrics serve as a framework for guiding the work of the estimator. In this respect, as mentioned previously, it is the generation of a single point estimate with a high level of confidence which is the goal. Per se a deterministic paradigm is seen in practice which, in highly industrialised contexts, serves the organisation well since Central Limit Theorem applicability can be accepted. The less industrial the context however, the less the deterministic paradigm can confidently be accepted as being sufficient. These confidence influencers have several characteristics related to computational constraints, normalising to Central Limit Theorem based probability density functions, multiple plausible futures, set based typology and metric taxonomy:

- Computational restraints: Significant efforts are made to increase the reliability of the single point estimate through more and more rigorous engineering break-down cost estimation approaches, the assumption being that the more robustly the estimator can describe what is being built and how, the more robustly they can estimate the cost, or at least identify the key cost estimating relationships to open the path to probabilistic parametric approaches. The development and deployment of such efforts into operational contexts is, however, significantly constrained by generally available computational resources and the inherent complexity of designing cost simulation models that not only cover individual components, but the iterative aggregations of these into (sub-) assemblies, propulsion systems, airframes, mission paths, etc., as a whole. Indeed, it might be also be argued that the more information is available, the lower the ability to recognise patterns due to computational restraints (Kostko, 1993).

- Normalising to the Central Limit Theorem: A second characteristic is the increasing acceptance of basic probabilistic approaches in the use of probability density functions as discussed by the selection of best fitting probability density function where a decision tree centred on the continuity of the data being can be used so that the estimator, in the end, choses from a range of pre-selected probability density functions. The focus lies on finding the best fitting default probability density function to which the data can be normalised to. This approach can help the estimator make the relevant choice of probability density function to normalise to, however the branching criteria are not given objective thresholds. The same applies to questions concerning confidence level or skew. Wheeler (2012) builds on Shewart (1931 & 1939) in that the starting point for selecting the most suitable probability density function is the question of data homogeneity. This is used as a starting point for exploring the suitability of diverse metrics to point to relevant default probability density functions. The role of the Central Limit Theorem as put forward by Laplace in 1810 is also critically examined. Kurtosis and skew squared then become guiding criteria for separating between mound-, U- and J-shaped distributions. A threshold for the applicability of default probability density functions is suggested through definition of an "impossible region" where high skew squared values meet low kurtosis values. Important to note as well is that Wheeler (2012) emphasises the value of analysis approaches being the identification of changes, i.e. from a dynamic perspective, versus the more static "snapshot" of uncertainty statistics typically encountered. Almost 100 years apart, Wheeler (2012) and Shewart (1931) can both be considered as modern thinkers. The goal however remains the development of a single point estimate using probability density functions.

- Multiple plausible futures: The third, emerging, characteristic sees the estimation method less as an alternative to the previously raised characteristics, but extends these to encompass multiple plausible future scenarios both from an engineering perspective in the sense of trade-off analyses, and also in respect to varying contextual conditions such as developments in the market, the economy or legislature. Underlying this characteristic are developments in computational capability that allow for pattern recognition approaches in big data situations while at the same time making newer techniques, such as fuzzy thinking available in order to make sense of that data (Zadeh, 1965; Zadeh, 1978; Klir & Folger, 1988; Zadeh & Kacprzyk, 1992; Kostko, 1993; Klir & Yuan, 1995; Abebe et al., 2000; Baguley, 2004). While this perspective has matured to state of practice in general policy analysis (Lempert et al., 2003; Popper et al., 2005; Groves & Lempert, 2007; Von

Krauss et al., 2008; Hall et al., 2012; Hamarat et al., 2013; Kwakkel & Pruyt, 2013) and does find its place in systems engineering contexts in the form of trade-off analyses, the challenges of linking this trade-off analysis with relevant cost simulation from an engineering break-down perspective remain formidable. While parametric analysis promises “good enough” techniques, resistance to such generalisations in the engineering communities that are focused on high level of detail and exactness are often significant.

- Set based typology: A fourth characteristic is related to the typology of uncertainty concepts in their own right, i.e. how to categorise these different types of uncertainty and their interrelationships. While various typologies for interpreting uncertainty quantification have been proposed (Bedford & Cooke, 2001; Wierman, 2010) the context and literature review suggest to this researcher that the interlocking dimensions of hindsight, insight and foresight are well suited for dynamic long-term contexts such as those represented by innovative high value manufacturing products.

    - Hindsight captures the perspective of forward uncertainty quantification where patterns of historical parametric volatility are propagated into the future in order to explore how uncertainty will manifest itself.

    - Foresight is based on the concept of inverse uncertainty quantification where the performance of a mathematical model of future behaviour is compared to actual performance whereby the difference is understood as uncertainty.

    - Insight is then based on observing the degree to which forecasts based on hindsight and / or insight are able to predict actual values and the corrections made to relevant forward or inverse uncertainty quantification approaches used.

There thus appears to be less of a discourse regarding the “best” approach to uncertainty quantification in cost estimation for innovative high value manufacturing products, and more the slow emergence of a process for inferring a coherent set of measures starting with basic data understanding, through pattern recognition and various different metrics as the relevant information becomes more and more visible and understood. This view might then be generalised towards an uncertainty quantification typology as illustrated by the Venn diagram in **Error! Reference source not found.**. It is these sets (and sub-sets) which can then be considered as dimensions relevant for uncertainty quantification.

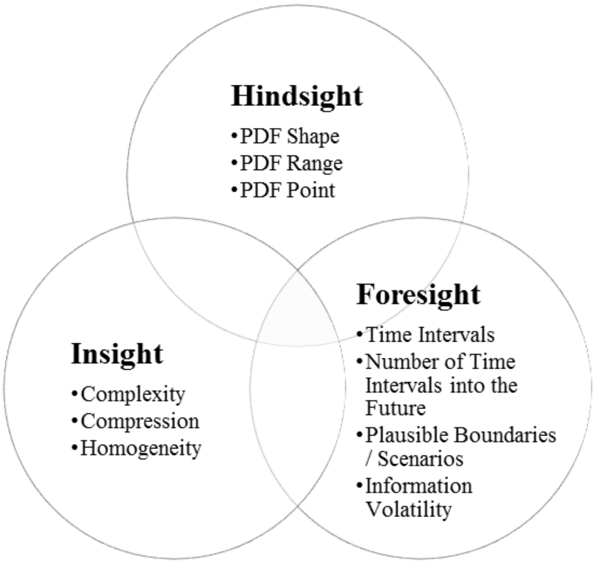

Figure 2-1: Uncertainty quantification typology

- The set “hindsight” contains uncertainty quantification metrics which admit the Central Limit Theorem. Examples of metric families belonging to this set are point, range, and shape.

- The set “insight” contains uncertainty quantification metrics describing the state of estimation parameters at the time of estimate and which are expected to change before the estimate can be verified. Examples of metric families belonging to this set are complexity, compression and homogeneity.
- The set “foresight” contains uncertainty quantification metrics defining the time-window of the estimate and the plausible future scenarios which is of particular

importance since it contains the boundary definitions for the propagation of uncertainty in the estimate. Examples of metric families belonging to this set are the chosen time intervals, the number of time intervals the estimate looks into the future, plausible boundaries and information volatility based on technical and cost readiness.

- Metric taxonomy: The fifth characteristic refers to the metrics identified in the literature review as aggregated into the taxonomy described in Table 2-3:

Table 2-3: Uncertainty quantification metric taxonomy

	Metric Family						
	Point	Range	Shape	Homogeneity	Compression	Complexity	Other
Specific Uncertainty Quantification Metrics	Single Point Estimate	Actuarial Central Estimate	Anderson Darling	Auto-Correlation	Entropy	Augmented Data Patterns	Colors
		Bayes Risk	Beta Coefficient	Cellular Automaton Rules		Degrees of Freedom	Data Harmonics
		Cumulative Distrib. Function	Cond. Tail Expectation	Correlation Co-Efficient		Neural Networks	Smell
		Confidence Interval	Kurtosis	Fuzzy Sets		Sensitivity	Taste
		Interquartile Range	Minimum Unbiased % Error	Rank Correlation			Time Criticality
		Mean / Median / Mode	Sample Size	RV Co-Efficient			Tactile Quality
		Minimax	Probability Density Function				
		Mean Square Error	P-Value				
		Probability	Co-Efficient of Dispersion				
		Three Point Estimate	Root Mean Square Deviation				
			Standard Deviation				
			Skew				
Generic Uncertainty Quantification Metrics	Business Value / Statistics / Thresholds / Volatility						

For this purpose, the concept of metric families is used in respect to general areas of metrics which exhibit conceptual closeness to clusters of principles. The basic clusters of principles which are deemed relevant relate to point and range estimates, shape, and information homogeneity, compression and complexity. Several metrics could not be specifically associated with these clusters however (metric family "Other"), while certain metrics were also identified as being generically relevant across a number of principle clusters.

2.7 Uncertainty Quantification Probability Field Framework

The literature review suggests that multiple uncertainty quantification metrics are available from various theoretical backgrounds and that their suitability is based on the degree that these are able to recognise a pattern in the available information which can then be propagated defensibly over the required time-frame. Foresight determines the most relevant uncertainty quantification metric, therefore the time-frame for the estimate (i.e. the number of whole product life cycle phases covered before validation occurs) and the volatility of the information available for pattern recognition (i.e. the technical and cost readiness levels at the time of estimate). Since both factors change over time, the uncertainty quantification metrics available for choice should also be mathematically coherent and offer clear thresholds for attraction to admit iterative maturation of the uncertainty quantification estimate. The estimator may also be able to use such a framework for understanding the requirements for the next most exact uncertainty quantification metric and working to meet those as the uncertainty quantification estimate matures towards the point where it can be validated.

Figure 2-2 illustrates these probability fields from a framework perspective. The confidence in the uncertainty quantification is highest at the bottom left where it is measured by a single point estimate and lowest at the top right where complexity metrics find application. The estimator should typically start at the top right and work to progress their estimate down to the bottom left using the uncertainty quantification metrics shown in Table 2-3 in order to continuously improve cost readiness levels.

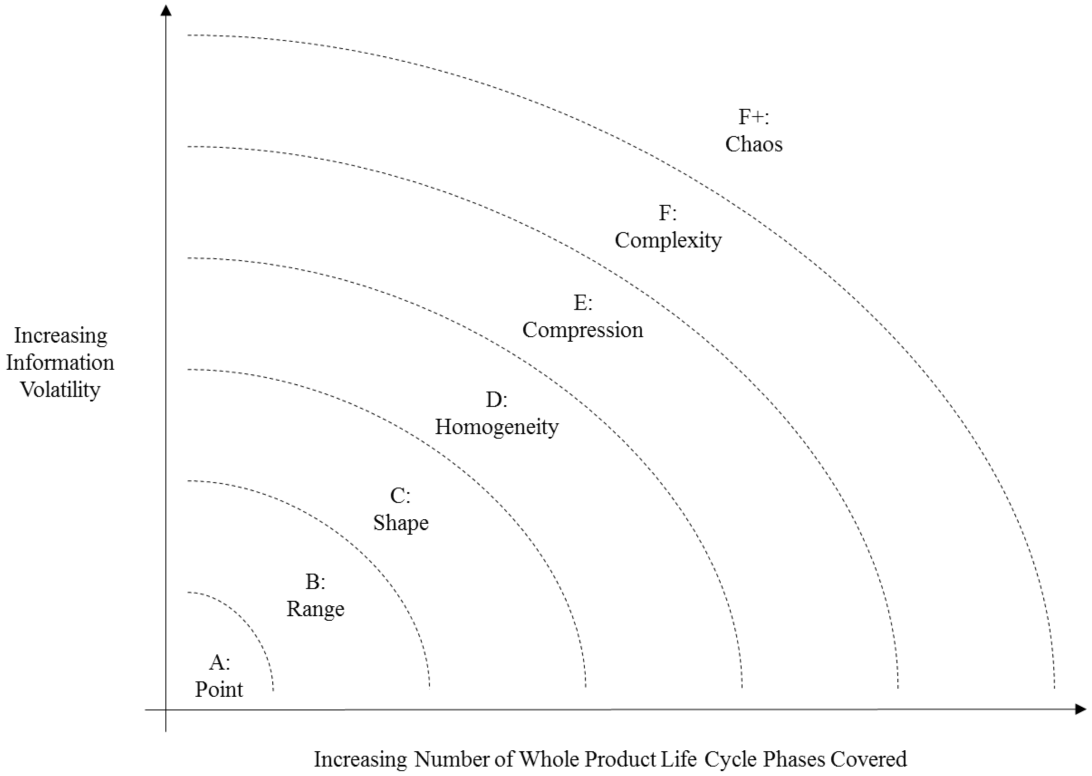

Figure 2-2: Uncertainty quantification probability field framework

Figure 2-2 highlights two fundamental dimensions of uncertainty quantification which frequently are raised as important influencers of confidence in cost estimates. For one, the further into the future an estimate is intended to be valid for, the more it must be assumed that the data being used to propagate will be subject to volatility in quality, content and density. Hence we can safely assume that the original data quality will decay in relevant density over time. The time intervals in the whole product life cycle of innovative high value manufacturing products are defined by models such as put forward by the International Standards Organisation (2015), the U.K. MoD (2009 & 2014) or the U.S. NASA (2015). Especially the phase changes are hereby of interest since that is where a significant amount of uncertainty is injected due to changes in methods, tools, techniques and reference data. The timeline of Figure 2-2 focuses on the number of whole product life cycle phases the estimate is intended to cover whereby the “number” is intended to describe the number of phase changes of relevance. In general, the probability field clusters might best be described from the perspectives of deterministic and bivalent (A), probabilistic and bivalent (B), probabilistic and multivalent (C), fuzzy and multivalent (D), complex and multivalent (E), chaotic and multivalent (F), or chaotic (F+) whereby the specific attributes for the boundaries between these remain unclear to a degree.

The degree of expected change (volatility) is generally determined from expert opinion and the quality of the assessment depends to a great degree on how well detailed the relevant attributes are. Assuming that the required data gathered in is a risk or stage gate register, the individual line items can be assessed or aggregated profiles from a higher perspective utilized. Metric families of relevance for similar clusters can therefore be redefined as follows:

- Complexity (CM): At the point of highest volatility and longest predication time-frame, the uncertainty quantification metric family of complexity appears most relevant. Within this metric family the metric degrees of freedom appears most suitable for uncertainty quantification. In this situation, the number of relevant variables affecting the uncertainty is determined, including their range of potential values. Then the maximum number of potential combinations is calculated and this factor applied to the technical baseline estimate to determine the probability field. The maximum number of combinations may be reduced through the development of more exact variable relationships based on analogy. While large ranges emerge it must be remembered that these cover the estimate across (almost) all whole product life cycle phases and often also the most plausible future scenarios. For example according to Price et al. (2006) "…typical airframe load models have approximately 200,000 degrees of freedom…" from a technical baseline estimate perspective whereby these are reduced primarily by deciding which degrees of freedom are "locked" and subject to formal change management, which degrees of freedom are linked to plausible future scenarios (and subjected to formal change management) and which are purposefully considered out-of-scope. The previous metric family hereby defines the boundaries of relevant information evaluated.

- Compression (CR): As more information is gathered about plausible future scenarios, variables affecting the uncertainty of the technical baseline estimate and the relevant project along the life cycle, a point is reached where the compression family of metrics becomes usable to generate more robust uncertainty quantification than the complexity approach. The most suitable metric in this family appears to be information entropy. The previous metric family hereby defines the boundaries of relevant information evaluated.

- Homogeneity (HG): The next level of the volatility / time-frame probability fields marks a transition to the homogeneity family of uncertainty quantification metrics, whereby the quantification approach shifts to fuzzy sets (Zadeh, 1978; Klir & Folger, 1988; Zadeh & Kacprzyk, 1992; Klir, & Yuan, 1995; Klir & Wierman, 1998). In essence the fuzzy set method of clustering the degree to which a data point belongs to a cluster is determined whereby the output is the number of clusters (single figure) and average degree of membership for data to each cluster (single figure per cluster). It is particularly at this level that the first (classical) probability density function patterns emerge although they are typically multi-model / cluster relationships that are not normalised to achieve state of practice single modal or linear relationships. The previous metric family hereby defines the boundaries of relevant information evaluated.

- Shape (SH): Shape is based on a custom probability density function generated from the available information and returns the uncertainty as “shape” and “scale” deviation from a separately chosen default probability density function. The deviation of the custom probability density function from the “normal” distribution values in % is then transferred to the three point estimate. The primary challenge encountered is limitations of standard statistical software packages which quickly reach performance limits due to complexity challenges of the computations. The previous metric family hereby defines the boundaries of relevant information evaluated.

- Range (R): The range uses the same approach as the single point estimate, but returns the complete range of uncertainty calculated by a Monte Carlo simulation. The difference to the single point estimate is that here a cumulative density function is used to indicate the uncertainty at various confidence levels and the confidence level chosen subjectively determines the single point estimate plus a certain % in order to raise the confidence level to 100%. The previous metric family hereby defines the boundaries of relevant information evaluated.

- Point (SPE): The single point estimate assigns a single uncertainty value to the technical baseline estimate, i.e. 5% and is based on the use of a Monte Carlo simulation using the technical baseline estimate as the best case and expert opinion for determining the most likely and worst values along with a default probability density function chosen such as a normal or triangular distribution. This is suitable in areas of low information volatility and when estimating within a single life-cycle phase. The most likely result of the Monte Carlo simulation output is used as the single point uncertainty estimate. The single point estimate may have a small default contingency added by decision makers or industry practice. The previous metric family hereby defines the boundaries of relevant information evaluated.

2.8 Data Analysis with (Polar) Force Fields

This investigation presents the findings of applying polar force fields to small cost data in order to forecast the uncertainty of cost estimates for high value manufacturing products. The attribute of "polar" hereby signifies that invariants are applied to the force field in order to create a specific type of shape.

The application of the polar force field method for visualising and quantifying cost variance, with the ensuing use of vector algebra to arrive at forecasting algorithms is based upon applying principles from physics to the field of cost estimation. Specifically the geometric space created by joining the vertices of cost variance dimensions, when represented as a polar force field, is considered to represent a probability space, the attributes of which, such as symmetry, provide indications as to the future shape of that space. The literature review (Schwabe et al., 2015b) failed to identify uncertainty quantification metrics suited to visualising and quantifying cost estimate uncertainty if cost variance data from only a single time period was available. The exploration of possible visualisation approaches then led to the examination of spider charts and an investigation of whether the shape of cost variance, when visualised as a spider chart, changed in a predictable manner of time, which was affirmed in respect to their symmetry (Schwabe et al., 2016a). Symmetry was hereby defined as the relationship between the actual and maximum area of the shape. Efforts to determine why changes in symmetry occurred in a predictable manner occurring in parallel then led to an investigation of dependencies between cost variance dimensions which are traditionally visualised as cost estimating relationships, or cost dependency models (Schwabe et al., 2015a). This researcher then examined the degree to which the spider chart visualisation could be considered as a specific layout form of a cost estimating relationship. The approach of converting the spider chart axes into vectors then emerged as an experimental path leading to a much simpler forecasting approach compared to the previously chosen approach based on symmetry. The consideration of the polar force field as a specific layout of a cost estimating relationship furthermore significantly eased discussion of the use of geometry / shape for exploring small cost data with stakeholders in research and industry, since the concept of dependency models is well known and widely used in the field of cost estimation. The degree to which the underlying principles of force fields, as used in physics, are applicable to the cost estimation context are the primary subject of future work recommendations, and based upon the possible consideration that cost is one attribute of the whole product life cycle which could be considered as a living system in its entirety (Settani et al., 2014; White, 2009).

Shapes are objects which can be described through their topology. Following Carlsson (2009), the study of topologies can be understood (from the perspective of the investigation) as the interpretation of the geometry / shape of a vector space which is declared to represent a probability field (Uffink, 1990). This literature review initially focused on identifying contributions dealing with the use of force fields in cost uncertainty quantification. A search on keywords related to force fields in conjunction with cost uncertainty or cost variance demonstrated that there is a lack of research work in this area. The primary link between the two concepts appears to be engineering geometry changes for cost optimisation or the geometrical evaluation of cost variance data when this is represented using default probability density functions such as Normal, logarithmic or Weibull distributions. When explored from the perspective of geometry and uncertainty analysis, the emphasis discovered appears to lie in the exploration of scientific measurement uncertainty. While geometrical data analysis is commonly used in the engineering, mathematics, natural sciences, big data and meteorology domains, its application to cost engineering requires more research efforts.

The lack of research is surmised to be due especially due to the insufficient and imprecise information related to whole product life cycle cost estimation for innovative high value manufacturing products. Further reasons may also be the biased nature of available information due to the heavy reliance on qualitative input, the inconsistency or unknown consistency of the data due to its fragmentation as related to sources, techniques, methods and responsibilities across the whole product life cycle. Finally, an important reason may be the general consensus that the "right", as in most relevant, information and its interdependencies is not known.

In relation to the quantification of uncertainty a review of industry guides, standards and reports in the field of cost estimation also found that emphasis is typically placed on the use of probability density functions, closely followed by related statistics based on the Central Limit Theorem such as the inter-quartile range, the co-efficient of dispersion or standard deviation. These insights are also supported by investigations conducted by Ghanmi et al. (2000) and the U.S. Naval Center for Cost Analysis (2014).

A further area of research explored related to data novelty detection and pointed to the need for defining these patterns a priori (Ghanmi et al., 2000). Here, geometrical symmetry represents the regularity of relevance and deviations represent novelties. Data points which reduce the symmetry of a geometry might be considered as deviations worthy of investigation. The analysis of the data set consisted of activities related to raw data representation, boundary definition, geometric data representation, data quantification, data visualisation, data decoupling, and data analysis.

2.9 Overall Research Gaps and their Significance

While the literature review suggested that the variously indicated metrics are most suitable for varying levels (or rather cluster ranges) of information density, these metrics also appear to have varying suitability for the description and containment of multiple plausible future scenarios, i.e. the deep uncertainty paradigm, whereby this again may help the estimator argue against progressing to a next threshold as long as the number of such scenarios are not reduced in and of themselves. It could be argued that the further the estimation context moves to the bottom left the more plausible future scenarios are guarded against.

Current approaches to whole product life cycle uncertainty calculation / estimation are struggling to produce robust and objective results because, disregarding the multi-modal context of the object of analysis being estimated, they:

- … focus on metrics of central tendency and measures of dispersion which find their origins in traditional utility analyses that emphasises the value of optimal versus sub-optimal solutions based on the Law of Large Numbers,

- … assume a static single versus dynamic multiple plausible future scenarios, and

- … assume predictable versus emergent contexts.

It is especially the mental models associated with traditional utility analysis that assume the validity of historical propagation for future projection which obscure the influence of changing context for innovative high value manufacturing products – whole product life cycles however happen neither within simple nor complicated contexts, but in complex if not on the threshold to chaotic ones (Snowden & Boone, 2007).

The dynamic emergent nature of the future is nothing unknown to past thinkers and authors. The much quoted economist Frank Knight (Knight, 1921) wrote about the concept of uncertainty:

"It is a world of change in which we live, and a world of uncertainty. We live only by knowing something about the future; while the problems of life, or of conduct at least, arise from the fact that we know so little. This is true of business as of other spheres of activity. The essence of the situation is action according to opinion, of greater or less foundation and value, neither entire ignorance nor complete and perfect information, but partial knowledge."

Lempert et al. (2003) phrase this as "Deep uncertainty exists when analysts do not know, or the parties to a decision cannot agree on, (1) the appropriate models to describe the interactions (2) the probability distributions to represent uncertainty about key variables and parameters in the models, and / or (3) how to value the desirability of alternative outcomes."

Karl Pearson, a founding father of modern statistics, was a strong advocate of the use of visual representations for data analysis and deeply believed in the use of geometry for arriving at statistical conclusions, indeed mentioning "Most statistical conclusions which can be obtained by arithmetic, can also be achieved by geometry, and many conclusions can be formed which it would be difficult to reach except by geometry." (Karl Pearson quoted in Ziliak, 2012) We perhaps also need to remind ourselves that "Statistics was graphical at its formal inception…" Ziliak (2012)

The states of uncertainty quantification thus demonstrate that a gap exists when estimating under conditions of small cost data where regression based techniques are not applicable. In light of this situation the opportunities of spatial geometry with an emphasis on the role of force fields to address the small cost data challenge are investigated.

The research gap identified in the investigation can hence be summarised as the lack of cost estimation techniques and relevant uncertainty quantification metrics for small cost data conditions.

The estimator of today has little guidance grounded in theory when it comes to the choice of the most suitable metric to quantify cost estimate uncertainty. This leads to the assumption that general statistical techniques (which build on the Central Limit Theorem) are applicable and default probability density functions which are commonly used in the peer community are chosen. Software based cost estimation tools also put these state of practice choices in the forefront.

From the perspective of the research study where small cost data is defined as the data from a single time period, a statistical analysis cannot confirm its non-random nature. From the perspective of the research problem this emphasises the need for expanding the range of available analysis techniques when examining discrete data of small cost data nature.

2.10 Summary

Chapter 2 presented the results of the literature review with an emphasis on the metrics for uncertainty quantification identified, an introduction to force field analysis, and a discussion of the research gaps identified and their significance.

Chapter 3 discusses the research method applied based on the research context, presents the elements of the adopted methodology and the reasons for their adoption, and explores the application of the methodology in practice with a focus on the four primary cycles of knowledge conversion.

CHAPTER 3: RESEARCH PRINCIPLES AND METHODOLOGY

3.1 Introduction

This chapter discusses the research principles and method, applied based on the research context, presents the elements of the adopted methodology and the reasons for their adoption, and explores the application of the methodology in practice with a focus on the four primary cycles of knowledge conversion.

3.2 Research Context

The research context is cost uncertainty forecasting for innovative high value manufacturing products under conditions of small cost data. The investigation exemplifies this context based on the U.S. DoD “Selected Acquisition Reports Summary Tables” (SAR) between 1970 and 2013 (U.S. DoD SAR). This publicly available data reports on manufacturing products which are (medium) high technology as defined by the OECD (SIC codes 20, 21, 25.4, 26, 27, 28, 29, 30.2, 30.3, 30.4, 30.6). They are also the result of significant research and development investments as described by the U.K. Blue Book (2011), the U.K. Blue Book 2011 Dataset (2011) and summarised by the industry landscape research report by the U.K. Cambridge Institute for Manufacturing (2012). From a cost uncertainty perspective this study considers innovativeness as a condition of products or services where no (repeatable), robust verified cost model exist. This may (re-) occur at multiple times during the whole product life cycle. The lack of such a model is indicated primarily by unplanned future cost variance with an unknown quantity. This is measured as changes in the compounded cost variance over time. All products in the US DoD SAR show evidence of such cost changes over time and can thus hence be considered as innovative to a varying degree. The higher the cost variance over time the more innovative a product is considered to be. An additional advantage is that this data has been subject to extensive third party analysis using parametric and regression based estimation techniques which lead to results widely used for estimating in practice. These reports summarise the latest estimates of cost and schedule on major defence acquisition program cost, schedule, and performance changes for calendar year reporting periods submitted to the U.S. Congress. Furthermore, the total program cost estimates provided in the SAR include research and development, procurement, military construction, and acquisition-related operations and maintenance. Case study data represents an amalgamation of data across various phases of the whole product life cycle for many differing products with aero (space), land and sea mission paths which share the attributes of innovativeness. Source data at the aggregated level of the U.S. DoD is provided in Appendix D for exemplary purposes.

In the SAR Summary Tables, the focus was placed on the tables representing base year cost variance and "to date" change figures from the base year were used. Decimals were rounded to full numbers, and absolute figures were used (therefore disregarding whether the variance was positive or negative). In this time period the cost variance factors reported on in the U.S. DoD SAR varied to a degree as highlighted by Table 3-1:

Table 3-1: Cost variance factor periods.

Period	Reported Cost Variance Factors (Dimensions)
1970	Economic, Schedule, Engineering, Estimating, Other, Support, Unpredictable
1971-1974	Economic, Quantity, Schedule, Engineering, Estimating, Other, Support, Unpredictable, Contractor cost overrun, Contract performance incentive
1975-1978	Economic, Quantity, Schedule, Engineering, Estimating, Other, Support, Program change related escalation, Contractor cost overrun
1979-1985	Economic, Quantity, Schedule, Engineering, Estimating, Other, Support, Program change related escalation
1986-2013	Quantity, Schedule, Engineering, Estimating, Other, Support

Important to note is that due to the differing number of variance categories assessed, each period is assumed to represent fundamentally different topologies from the perspective of polar force fields. Breaks in their continuity are assumed by the investigation to prevent coherent analysis across them.

The initial definition of data boundaries was thus performed in order to create a continuous set of data with the same financial baseline. This consisted of over 2000 forecastable events in the time period 1986-2013. Initial investigation of sample size requirements determined that since the data set being examined could not be verified to follow the Law of Large Numbers on any attribute, a corresponding determination of a minimum sample size attribute was not hence admissible. The sample represented the complete population. The cost variance factors used by the SAR Summary Tables in the period 1986-2013 can be interpreted as follows as discussed by Breaux et al. (2012):

- Quantity (QU): A cost variance that is due to a change in the number of units of an end item of equipment.

- Schedule (SC): Costs resulting from change in procurement or delivery schedule, completion date, or intermediate milestone for development or production.

- Engineering (EN): Cost increases or decreases that are due to an alteration in the physical or functional characteristics of a system or item delivered.

- Estimating (ES): Changes due solely to the correction of previous estimating errors or to refinements of a current estimate.

- Other (OT): Cost variances that are due to unforeseeable events not covered in any other category (e.g. natural disaster or strike).

- Support (SU): Any change in cost, regardless of reason, associated with support equipment for the major hardware item (defined as any work breakdown structure element not included in flyaway, rollaway, or sail-away costs).

3.3 Research Principles, Methodology and Reasons for Adoption

The research study progressed through four stages: Discovery, Prototyping, Validation and Integration and Application, whereby the application of the research findings marks a return to the Discovery phase.

Each stage can be understood as an evolving process of sense-making stages which can each be described through the knowledge conversion processes of Socialisation, Externalisation, Combination and Prototyping (SECI) and the knowledge spiral as put forward by Nonaka and Takeuchi (1995) and visualised as research principles in Figure 3-1.

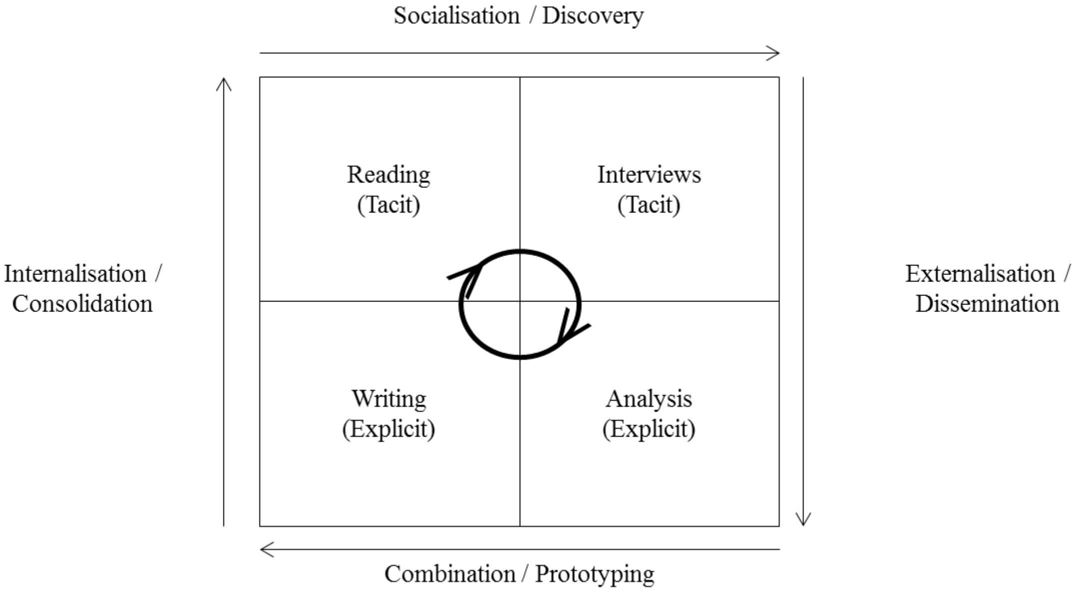

Figure 3-1: Research principles adopted (based on Nonaka and Takeuchi (1995)

The central knowledge spiral describes this researcher´s activities as a dynamic process which integrates specific activities such as reading, interviews, analysis, and writing in order to create tacit and explicit knowledge, which is then internalised, socialised, externalised and combined in order to evolve a coherent understanding of the research aim and objectives. In this respect, achievement of the objectives is an iterative and dynamic process where all elements are deemed interdependent.

Fundamental to understanding the research principles chosen, is to understand that the emphasis was placed on the generation of the tacit knowledge in order to enable this researcher to discover patterns of data behaviour with its ensuing externalisation in order to support verification and validation and the generation of contributions to knowledge in the form of outputs and findings.

The starting point for the design, development and implementation of the research principles was the understanding that the research process represents one of knowledge creation whereby knowledge can be considered as consisting of a tacit capacity to act and an explicit externalisation of that capacity through action. Following Nonaka and Takeuchi (1995), knowledge creation is considered the result of a spiral process of interaction between tacit and explicit knowledge.

The framework put forward by Nonaka and Takeuchi (1995) consists of four processes describing the interaction between tacit and explicit knowledge. These processes are called: socialisation, externalisation, combination and internalisation. These processes are completed sequentially in a repetitive manner whereby each repetition builds on the knowledge created in the previous cycle. This "spiralling" can then be considered to be occurring in a context shared by participants called "ba".

In the first process of socialisation tacit knowledge is shared with others through activities such as mentoring, observation and practice. The aim is to recreate the tacit knowledge held by one individual in another individual as closely as possible respecting that to a degree the knowledge will always be unique. An example of such an activity is the observation of a cost estimator forecasting cost estimate uncertainty for an innovative high value manufacturing product such as an aerospace engine.

In the second process of externalisation tacit knowledge is converted into explicit conceptual knowledge through images or words for example. Externalisation occurs through a dialogue between individuals. An example of such an activity is a review of the observed forecasting process with the cost estimator in order to validate observations made.

In the third process combination occurs which involves the connection of different types of explicit knowledge through an exchange process. Combination occurs in dialog which may be oral or written. An example of such a research activity would be to compare and contrast the observed forecasting process with state of art practices in order to assess the maturity of the process and identify interventions to improve this.

In the fourth process internalisation involves converting explicit knowledge into tacit knowledge. This process essentially concerns “learning by doing” which is the application of knowledge. Previous knowledge conversion processes are internalised in order to create new tacit knowledge. An example of such a research activity would be the implementation of identified improvement interventions and to observe the impact these have on the forecasting process and its outcomes.

The completion of one cycle of the knowledge conversion process can be considered as the starting point for the next whereby this builds on new knowledge gained during the preceding cycle. The concept of “ba” then refers to the shared context that participants in the knowledge conversion process have developed in their interactions and is fundamental for ensuring the efficiency of the next conversion cycle. An example of this would be that the observer and the cost estimator develop a shared appreciation for the potential value of the improvement activities and the benefits of increasing process maturity in light of industry standards. Table 3-2 provides and overview of the knowledge creation processes, related techniques and relevant research activities.

Table 3-2: Overview of knowledge creation processes, techniques and research activities

Process	Technique	Research Activity
Socialisation	Mentoring	Literature reviews, conversations, (semi-) structured interviews, and surveys, game playing and thought experiments.
	Observation	Analysis of case study data, review of inputs gained through (semi-) structured interviews, game playing and thought experiments.
	Practice	Development and application of mathematical models designed to emulate data behaviour.
Externalisation	Dialog	Conversations, (semi-) structured interviews, game playing and thought experiments.
Combination	Dialog	Conversations, documents.
Internalisation	Learning by doing	Game playing, application of mathematical models.

3.4 Primary Cycles of Knowledge Creation and their Transitions

The research methodology for investigating the research problem and hypothesis testing was operationalised across four phases which represented cycles in the knowledge conversion process, as illustrated in Figure 3-1. Each cycle built on evolving knowledge and focused on Discovery (understanding the context), Prototyping (iterative development of the framework), Validation (academic and industrial), and Integration and Application of all research findings. These four elements provided a guideline for achieving the research aim, study objectives, and intended deliverables. The central element of prototyping was chosen in order to emphasise the integrating role of learning by doing.

The first cycle of the research study (Discovery) was marked through completion of three (semi-) structured interview series, completion of three exploratory case studies (focused on an aerospace manufacturer, publicly available case studies and the U.S. DoD SAR Summary Tables between 1970 and 2013). Furthermore, a group was created on the social networking platform LinkedIn in order to create a community of practice supporting research efforts during the course of the study. Results from the aerospace manufacturing case study were presented in a conference (Schwabe et al., 2014a) as was research progress at the conference for Calculating and Communicating Uncertainty, on Wednesday 28 January, 2015, dstl with University of Southampton and Public Health England, London, U.K. (unpublished). The primary knowledge gained was related to the relevance and nature of the concept of probability spaces in exploring cost estimate uncertainty.

The second cycle of the research study (Prototyping) represented a focused effort to build on the first cycle and the concept of probability spaces. From the perspective of the objectives, the first dependency model and propagation models based on case study data were created. These insights then served as the foundation for creation of a serious game and the first of several iteratively developing Microsoft ® Excel desktop demonstrators. The concepts of cost estimate uncertainty as a probability space matured along with the initial examination of geometrical attributes such as symmetry in order to describe its propagation. The picture of cloud uncertainty emerged and was explored in particular through its translation to immersive visualisation spaces. Towards the end of this phase the concepts of small cost data and short spatial string analysis from the perspective of computational complexity as put forward by Kolmogorov emerged (Soler-Toscano et al., 2014). From a validation perspective the serious game was deployed towards the middle of the phase and repeated at regular intervals up to the middle of the following phase. In respect to publication and dissemination two journal articles were published (Schwabe et al., 2015b & 2016a) and one conference presentation (2015a) held. Two (semi-) structured interview series with an emphasis on exploring applied principles of the probability space were conducted.

The third cycle (Validation) was marked by completion of the serious game series including embedded (semi-) structured interviews, the completion of a web-based tool for creating immersive visualisations based on the previously developed Microsoft ® Excel based demonstrator and the development of a symmetrisation framework for forecasting cost estimate uncertainty. The emphasis was thus on developing a relevant mathematical model. Two conference presentations were held (Schwabe et al., 2016b & 2016c) and the research progress presented to the Special Interest Group Uncertainty Quantification and Management in High Value Manufacturing during a workshop. An initial version of the thesis was compiled and integrated based on the concept of symmetry propagation in cost variance when visualised as simplex geometries in the form of spider charts. Two further (semi-) structured interview series were conducted with an emphasis on general understanding of the use of geometrical principles in estimating and forecasting.

A significant evolution in the research dynamics in that the previous focus on the symmetries of the simplex geometry represented by the perimeter of the spider-chart representation was refined in the following phase to a view that this in fact represented a polar force field with unique topological attributes (i.e. invariant spatial centre, dimensional sequence and radial degree). This polar force field was of a state space nature so that any forecast was in essence a state space to state space forecast leaving open the question of the dynamic space perspective with its unifying / translation function. This evolution led to a significantly different view of the research results to date including a complete re-alignment of findings and their interpretation embedded in a final Microsoft ® Excel demonstrator. Furthermore several planned journal contributions were withdrawn in order to focus further dissemination efforts on the revised research perspective.

The fourth and final cycle of Integration and Application ended by consolidating all research findings in order to create an integrated framework with underlying mathematical model. Besides representing the knowledge conversion processes in their own rights it can also be considered as an integration of the three previous cycles. Furthermore, relevant was a final series of (semi-) structured interviews presenting the research findings to key stakeholders of the investigation as a whole. Finally, a case study using the U.K. MoD A400M was conducted in order to investigate how the research findings could be applied to a different context than the case study data used to develop such. This resulted in a more prominent inclusion of U.K. based reference sources within the study as a whole. The implementation of the research findings at a major aerospace manufacturer then represented a step from verification and validation to actual application in practice. Conceptual development continued regarding the dynamic space view of small cost data certainty in conjunction with the corresponding state space translation space.

3.5 Research Method

The overall application of the research methodology is illustrated in Figure 3-2. Each stage of Discovery, Prototyping, Validation and Integration and Application represents a knowledge cycle in its own right and as the stages evolved the knowledge cycle were re-applied under consideration of the previous.

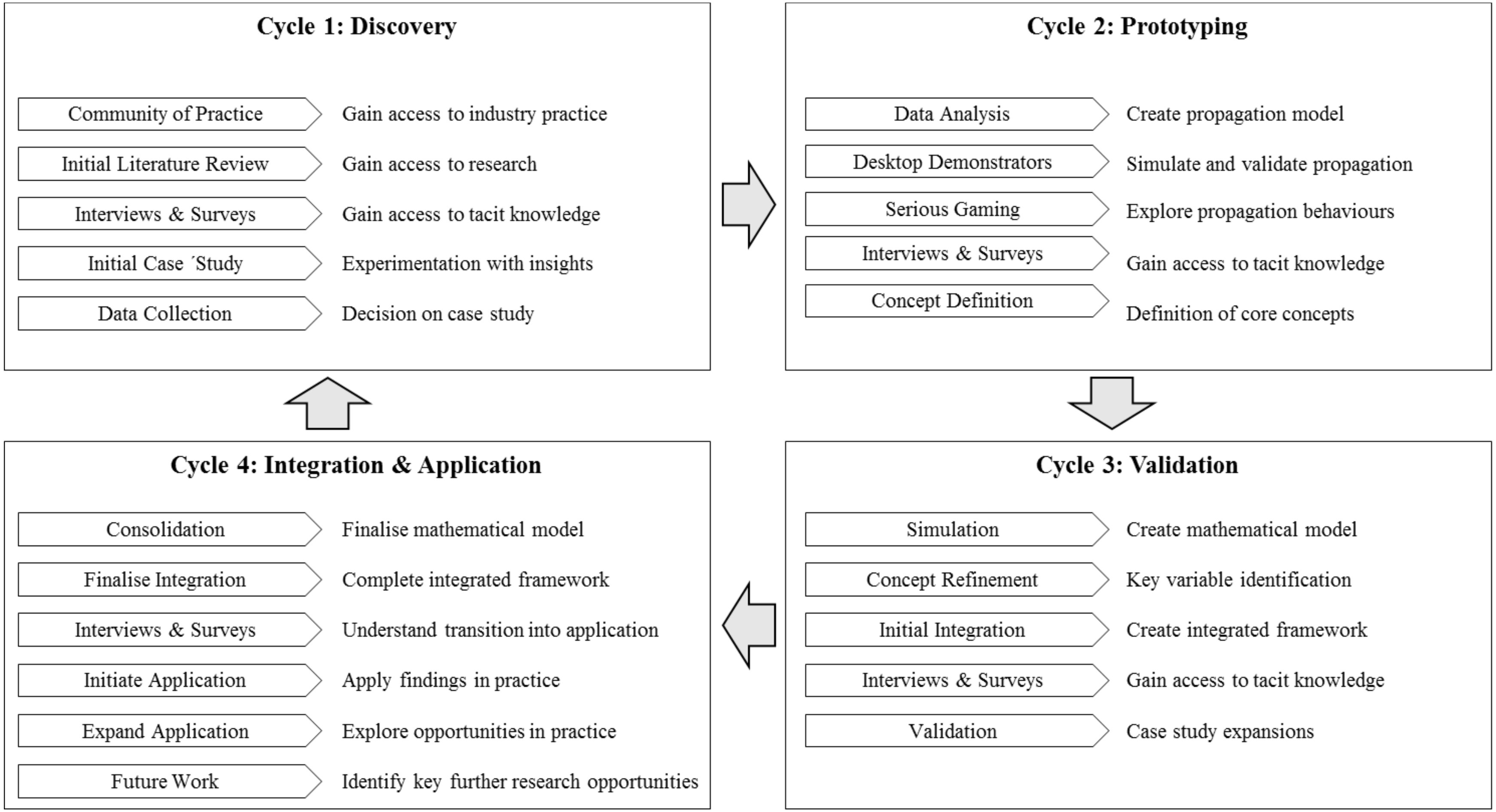

Figure 3-2: Research method

3.6 Summary

Chapter 3 discussed the research method applied based on the research context, presented the elements of the adopted methodology and the reasons for their adoption, and explored the application of the methodology in practice with a focus on the four primary cycles of knowledge conversion.

Chapter 4 is focused on discussing current practice and challenges in respect to forecasting the propagation of cost estimate uncertainty under conditions of small cost data. Currently, relevant guidelines and standards are introduced followed by a closer examination of their application in practice with a special view on their relationship to the contracting lifecycle. Gaps and challenges in practice are identified and correlated with the research gaps identified in the literature review. Finally, recommendations are made in respect to aligning theory and practice.

CHAPTER 4: CURRENT PRACTICE AND CHALLENGES

4.1 Introduction

This chapter focuses on discussing current practice and challenges in respect to forecasting the propagation of cost estimate uncertainty under conditions of small cost data. Currently, relevant guidelines and standards are introduced followed by a closer examination of their application in practice. Gaps and challenges in practice are identified and correlated with the research gaps identified in Chapter 2. Finally, recommendations are made in respect to aligning theory and practice.

The review of current practice and the challenges it faces was based on the literature reviews and enhanced through semi-structured interviews and surveys with interviewees and participants active in different industries from aerospace through power generation to defence. Industrial practice was segmented based on the nature and units of products manufactured therefore prototypes, units of one and single / multiple series.

4.2 Industrial Practice: Cost of Single and Multiple Series Manufacturing

Quantifying cost estimate uncertainty in industrial practice while influenced by (non-) governmental guidelines and standards demonstrates unique attributes primarily due to the commercial nature of the organisations involved. This is coupled with influence of a wide spectrum of available contracting mechanisms although the application of such is heavily limited in practice. These attributes can be examined from the perspective of the units of manufacturing as defined by prototypes, space systems and single and multiple series manufacturing. Single units are “one off” manufacturing efforts for units or single series therefore where no further units or series are expected to be produced. Multiple series manufacturing is about using the insights generated by a single series in order to enhance the product(s) and manufacture new series. The significant difference from a cost perspective is that estimation errors for single units or single series cannot be “evened out” through pricing or cost adjustments in further series. The estimate for single units and single series is hence of much greater commercial significance than for multiple series.

Prototypes are manufactured during early life cycle phases of a product and typically represent the first example of an operating product. The manufacture of prototypes occurs outside normal series manufacturing. The costs for product prototypes are typically contained within the research and development budgets of an organisation. Since the prototype is not sold (commercialised), it represents a capital expenditure which is not directly offset by revenue (internal or external). The prototype is hence more an integral part of the cost estimation process than an input to it. The research efforts presented in this paper do not focus on supporting cost estimation in prototypes.

Units of one in the context of space systems, while sharing the innovative nature of prototypes differ from these in that the unit, is intended for actual mission deployment however further units are generally not planned for manufacture. Similar to the prototype the complete unit is not intended for “sale”; there are hence no future opportunities for producing such units at lower cost. Cost hereby describes a budget initially assigned for the manufacture of the unit of one (U.S. NASA, 2015). While multiple techniques for cost discovery and containment exist, the fundamental challenge of not being able to leverage learning with manufacturing ramp-up remains. The research efforts and findings do not focus on supporting cost estimation in space systems although the mentioned guidelines are extended in practice to single and multiple series manufacturing contexts.

The manufacture of series of units is common in defence and industrial contexts whereby first series are often considered to be commercial loss leaders due to their novel nature and restrictions of contracting mechanisms typically used (i.e. fixed price with economic adjustment as commonly mentioned in personal interviews). The first series is hence fraught by significant cost estimate uncertainty while the commercial terms have been fixed, often leading to cost overruns which are irrecoverable within that first series. The manufacturing organisations are hence challenged to commercialise in a manner that allows for the first series to provide detailed cost information, not exceed overruns which might typically be carried by a research and technology budget, and set a solid foundation for the negotiation of price changes in future series where the relevant investments need to be recovered, whereby such does not strain customer relationships through significant price increases. The research efforts and findings primarily support cost estimation in single and multiple series manufacturing.

4.3 Aligning Practice and Theory

The results of the literature review clearly indicated that in practice, manufacturing organisations primarily rely on techniques that have been applied for over 150 years (state of practice) whereby the adoption of best practice (state of art) as demonstrated by U.S. publications from 2007 onward is only beginning. At the same time many (especially discrete) techniques have evolved since the advent of industrialisation, and while representing the state of the possible, are at early stages of the adoption process especially in manufacturing environments that are still working towards industrial maturity.

The alignment of practice and theory therefore suggests that the primary focus needs to be on accelerating the adoption of insights generated through theory into practice. For the context of the research study, this is then tightly coupled with the need for an industrial maturity which permits a faster validation of new approaches. Specifically, this researcher considers the speed of adoption to be tightly linked to the time window within which the cost estimate (uncertainty) forecasts can be objectively validated. The smaller this time window the more interest might be given to it by organisations. This then also points to some fundamental questions concerning organisational design in that, for example, cost accountability across a whole product life cycle will be regularly transferred between functions and individuals who themselves cycle in and out of accountabilities frequently. In addition, when taking a whole product life cycle view and considering that most products examined in the research study will have whole product life cycle lengths of several decades, there will by default be very few individuals in organisations who have experienced such cycles in their entirety and can hence provide seasoned judgement on their behaviour.

This researcher suggests that in order to support and accelerate the alignment of theory and practice, the research findings are considered to support knowledge elicitation and sense making in the formulation of analogies and / or expert opinions – which has been emphasised through the creation of the serious game.

The key challenges can hence be summarised as:

- No formally agreed and enforced minimum standards for cost estimation under conditions of small data which limits the assessment of comparative estimation accuracies and embedded uncertainties.

- Central Limit Theorem based techniques dominate in the analysis of data which pre-supposes propagation of patterns from past to present without relevant evidence of this being the case.

- Cost data is often highly confidential / sensitive company internal information not available to the public which prevents independent third party verification.

- Cost estimation approaches are applied across the whole product life cycle in inconsistent manners which challenges the continuity of data required for regression based estimation techniques.

4.4 The Condition of Small Cost Data

This section describes the attributes of small cost data upon which the investigation is focused, presents a technique for identifying when the condition of small cost data exists based on the principles of Kolmogorov complexity for short strings (Soler-Toscano et al., 2014), then applies the technique to case study data in order to illustrate the prevalence and relevance of this condition in the forecasting of cost estimate uncertainty and closes with an overview of the process for determining the presence of small cost data conditions.

4.4.1 Attributes of Small Cost Data and Computational Complexity

The investigation focuses on the condition of having cost data from only one time period to base estimation and forecasting upon. This condition is what is referred to as "small cost data." Figure 4-1 summarises this view followed by an explanation of its elements:

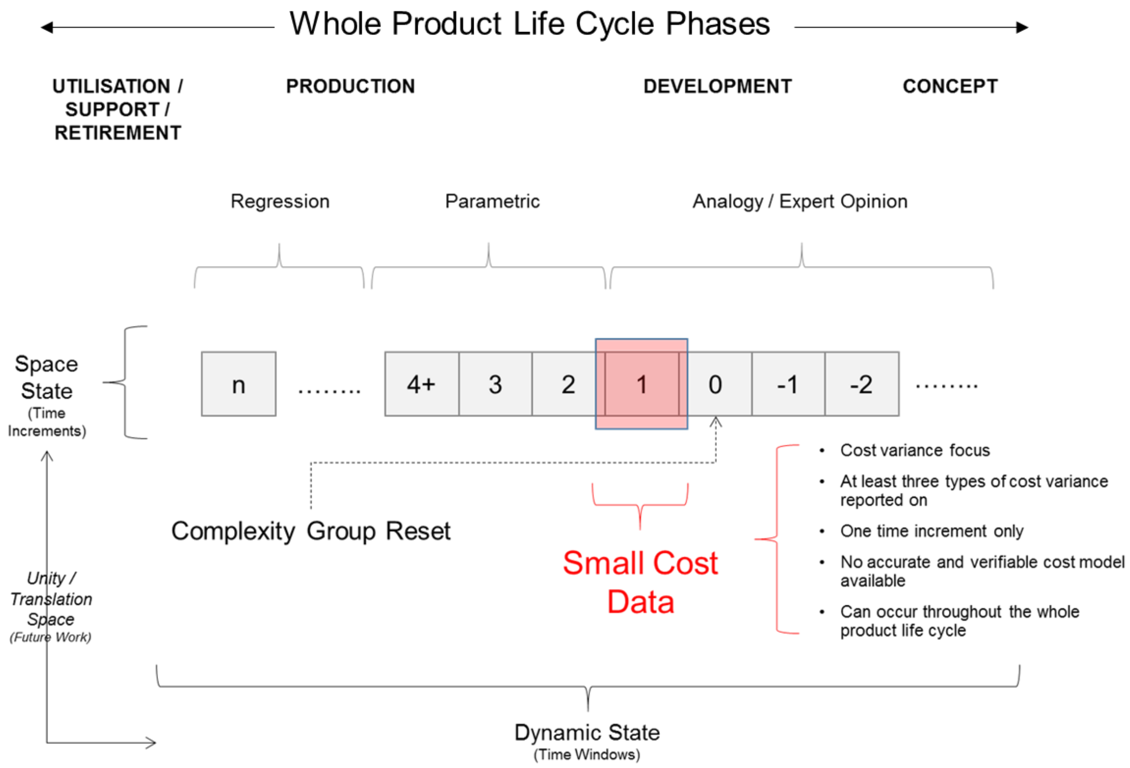

Figure 4-1: Attributes of small cost data

In the production / utilisation / support / retirement phases of the whole product life cycle, sufficient data is often available to apply regression based techniques. This means that there are enough comparable data sets available for unit or support costs so that the minimum prior information requirements of the Central Limit Theorem, and thus the Law of Large Numbers, can be applied. The exact minimum number of available data sets required is subject to debate, however, for purposes of the research study it should be assumed that enough for generating a robust and repeatable cost model is present, which can be considered as starting with around 15 data sets. During the development / concept phase such a cost model is typically not yet present and analogy / expert opinion in combination with parametric techniques are usually applied. In order to apply a parametric technique, a "change" in data is needed and that is given at a minimum if two comparable data sets (with common computational complexity class) are available. Under optimal conditions four comparable data sets are sufficient to move from parametric to regression based techniques. The concept of “optimal” is hereby drawn from principles of computational complexity for short strings as put forward by Kolmogorov and enhanced by Soler-Toscano et al. (2014). If one or less comparable data sets are available then current practice only provides analogy and expert opinion approaches for estimation.

The time periods -1 and -2 or less represent data sets before baselining of an estimate and under certain conditions these appear usable for forecasting as well although this is not explicitly explored in this investigation. Zero comparable data sets represent a specific condition when a computational complexity group is reset whereby this may happen at any point in the whole product life cycle. This is important since it points to when cost variance patterns start and stop during the whole product life cycle. Each time increment represents a state space view of cost variance / estimate uncertainty while a series of time increments begins pointing to a dynamic view and a relevant unity / translation space is then needed to ensure alignment with the state space view (which is recommended as future work by the investigation).

The metric of Kolmogorov complexity signifies the degree of compression a binary string can be subject to whereby compression is understood as the process of converting a sequence of bits into the description of the pattern represented by that bit sequence. The bit sequence is hence transformed into a program that can generate exactly that bit sequence. The program can be considered to consist of a descriptor language which explains how a sequence of instructions is applied by a Turing Machine in order to generate the bit string. The data of interest is the arithmetic cost variance, specifically across at least three dimensions of cost variance. This data needs to cover iterative and topologically discrete time intervals prior to the point in time where the cost estimate is being performed.

The investigation uses principles of Kolmogorov complexity as applied to short strings by Soler-Toscano et al. (2014) as an indicator for determining when conditions of small cost data exist based on changes in total cost (variance) between time periods, i.e. if the total cost for time period one is “100” and the total cost for time period two is “200” so that the change in cost from time period one to time period two is declared to have a bit value of “1” due to the increase in total cost while a decrease (or lack of change) would be declared to have a bit value of “0”. For two time periods, the string of relevance is hence “10”. If the total cost rises in the third time period then that time period would receive a bit strong value of “1” and the total string of relevance would have a value of “101”. The specific assignment of bit values to data value changes is for exemplary purposes only.

The first boundary suggested by Kolmogorov complexity Soler-Toscano et al. (2014) is that the data from at least 42 discrete time intervals is required before pattern recognition approaches can be applied for forecasting purposes. This means that a 42 character bit string is required before non-random patterns can be confirmed with an acceptable degree of statistical certainty. This includes the application of standard regression techniques. The second boundary suggested by short string Kolmogorov complexity is that depending on the length of the bit string the actual complexity score of individual bit strings can be grouped into groups of identical complexity. Single and double bit sequences each share the same Kolmogorov complexity. Three bit sequences are the first bit strings can be structured into different groups of identical complexity as described in Table 4-1. All permutations of a three bit string with the values “1” and “0” are presented, their complexity score given and identical complexity scores assigned to groups.

Table 4-1: Kolmogorov complexity groups for short strings based on Soler-Toscano et al. (2014)

Sequence	Complexity	Group
111	5.40	1
000	5.40	1
110	5.45	2
100	5.45	2
011	5.45	2
001	5.45	2
101	5.51	3
010	5.51	3

Soler-Toscano et al. (2014) calculate the the complexity score based on the frequency with which the bit sequence appears in an experiment running a Turing machine with random sampling over a predetermined period of time and number of samples. Different sequences hence appear with different frequency in this experiment, and this is the basis for the ranking. The less frequent the pattern appears the more complex the sequence is assumed to be. It is then with the fourth element of the bit string that a first determination of stability can be made. This researcher, therefore, suggests that while at time interval zero, no techniques other than analogy or expert opinion are currently feasible, starting with the second time intervals parametric models become applicable and with the fourth time interval the consistence of the underlying complexity groups can be verified (Schwabe et al., 2016b).

4.4.2 Changing Complexity Groups Create Conditions of Small Cost Data

The string for any cost variance factor can now be examined for relevant changes in complexity groups. The complexity group is based on an auto-correlation approach using a three bit sliding window which is the minimum size needed to confirm stability of a complexity group across the minimum of two time windows. Based on the assumption that the cost estimator begins the estimation process with the cost variance data of a single time period (therefore the change from the baseline at t=1 to t=2 is known) a “sliding window” approach can be applied in order to identify when a relevant complexity group (pattern) starts and ends. The complexity is measured by Kolmogorov complexity (*Km*) for the binary string in brackets (i.e. “011) and can be calculated as discussed by Soler-Toscano et al. (2014) as shown in Figure 4-2 where the string “0110100” is decomposed into overlapping three bit strings and the Kolmogorov complexity calculated for each.

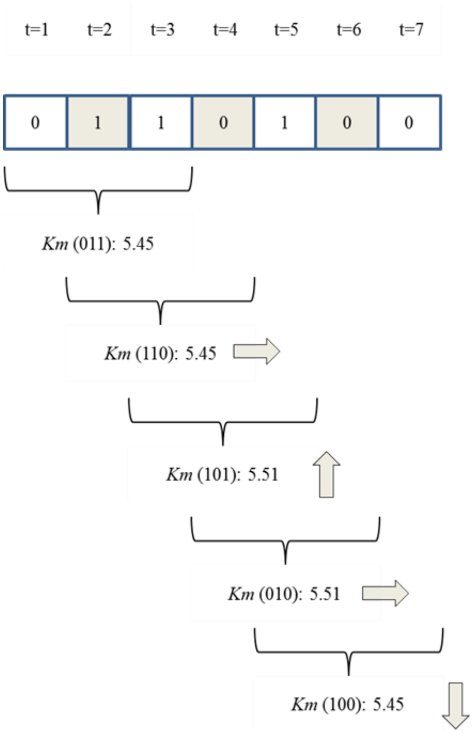

Figure 4-2: Exemplary pattern separation using short string complexity

The change in complexity group can thus be interpreted as a “pulse” as visualised in Figure 4-3:

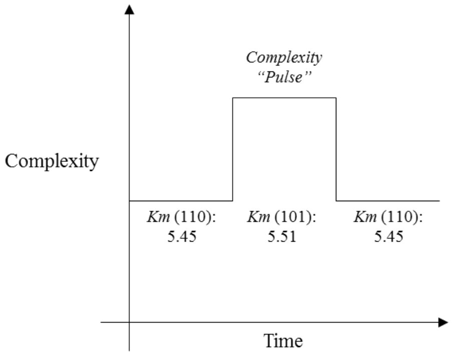

Figure 4-3: Exemplary complexity pulse

The initial complexity is 5.45 (three bit complexity group II) and rises to complexity 5.51 (three bit complexity group III), to then return to a value of 5.45 (three bit complexity group II). This change can be visualised as a "pulse" indicating where the complexity change reached a threshold value to the next higher state, remained in the higher state for two sliding window periods and then again reach a threshold state where the next lower stage was passed to. An increase in complexity group suggests that the pattern appears less frequently than the previous one in the experiments of Soler-Toscano et al. (2014) so that it may take greater computational power to identify such in polynomial time.

Important to recognise as well is that besides differences in computational power required to determine different three bit strings, as the total string grows the computational power required to determine whether the string as a whole exhibits a non-random pattern grows logarithmically as shown in Figure 4-4 for bit strong lengths of two to 10 in an exemplary manner. Generally speaking, a string with one bit is by default random since each bit value ("1" or "0") has a 50% probability, a two bit string has 2^2 possible permuations, a three bit string has 2^3 possible permutations and so on.

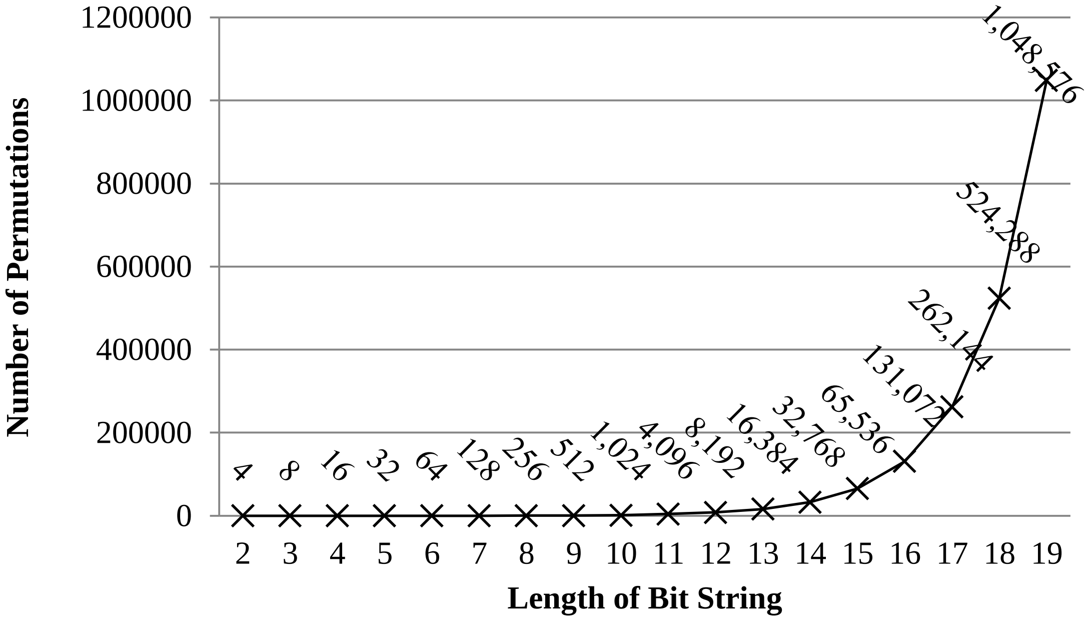

Figure 4-4: Exemplary growth of permutations with bit string length

For exemplary purposes, a longer string of “1001111111011” as illustrated in Table 4-2 is used to exemplify how the three bit sliding window approach leads to changes in complexity groups. This strong has 13 bits, 16,384 possible permutations and 10 three bit sliding windows. Larger sliding windows could be used however a window length of three represents the smallest possible group that will indicate changes in complexity.

Table 4-2: Exemplary bit string analysis “1001111111011”

Three Bit String	Complexity	Complexity Group
100	5.45	2
001	5.45	2
011	5.45	2
111	5.40	1
111	5.40	1
111	5.40	1
111	5.40	1
110	5.45	2
101	5.51	3
011	5.45	2

The emphasis is hence moved from identifying the pattern itself to identifying that point in time where the pattern changes. The limitation is that at least three time periods of information need to be available before the method can be applied. The sliding window consists of overlapping three bit strings for which the Kolmogorov complexity is calculated and assigned to a complexity group. The condition of small cost data exists until that time increment of cost variance data where two three bit sliding windows have the same complexity group.

4.4.3 Assessing the Prevalence and Relevance of Conditions of Small Cost Data

Table 4-3 applies the method to case study data from the U.S. DoD SAR for the time period 1986-2013 (U.S. DoD, 2015). The case study data is analysed to determine whether the cost variance for an accounting time period is higher (“1”), lower (“0”) or equal (“0”) to the previous time period. If the year of the baseline estimate changes then a “1” is also assigned. These classifications are for exemplary purposes only.

Table 4-3: Exemplary complexity strings and groups from case study data

Time Increment	Absolute Cost Variance (USD$M)	Cost Variance Trend (“1” = increase; “0”=decrease or unchanged)	Complexity String (sliding window size 3)	Complexity Group
1986	112,733	N/A	N/A	N/A
1987	85,882	0	0	N/A
1988	115,081	1	01	N/A
1989	92,968	0	010	3
1990	84,783	0	100	2
1991	90,068	1	001	2
1992	55,148	0	010	3
1993	64,580	1	101	3
1994	45,418	0	010	3
1995	52,484	1	101	3
1996	63,285	1	011	2
1997	85,939	1	111	1
1998	101,016	1	111	1
1999	117,376	1	111	1
2000	127,229	1	111	1
2001	162,505	1	111	1
2002	177,869	1	111	1
2003	201,927	1	111	1
2004	245,456	1	111	1
2005	159,672	0	110	2
2006	263,012	1	101	3
2007	257,726	0	010	3
2008	264,185	1	101	3
2009	290,521	1	011	2
2010	289,536	0	110	2
2011	242,056	0	100	2
2012	142,301	0	000	1
2013	81,752	0	000	1

Figure 4-5 illustrates the results of evaluating the data from the case study data shown in Table 4-3 as complexity group of the sliding window for cost variance data from 1989 to 2013:

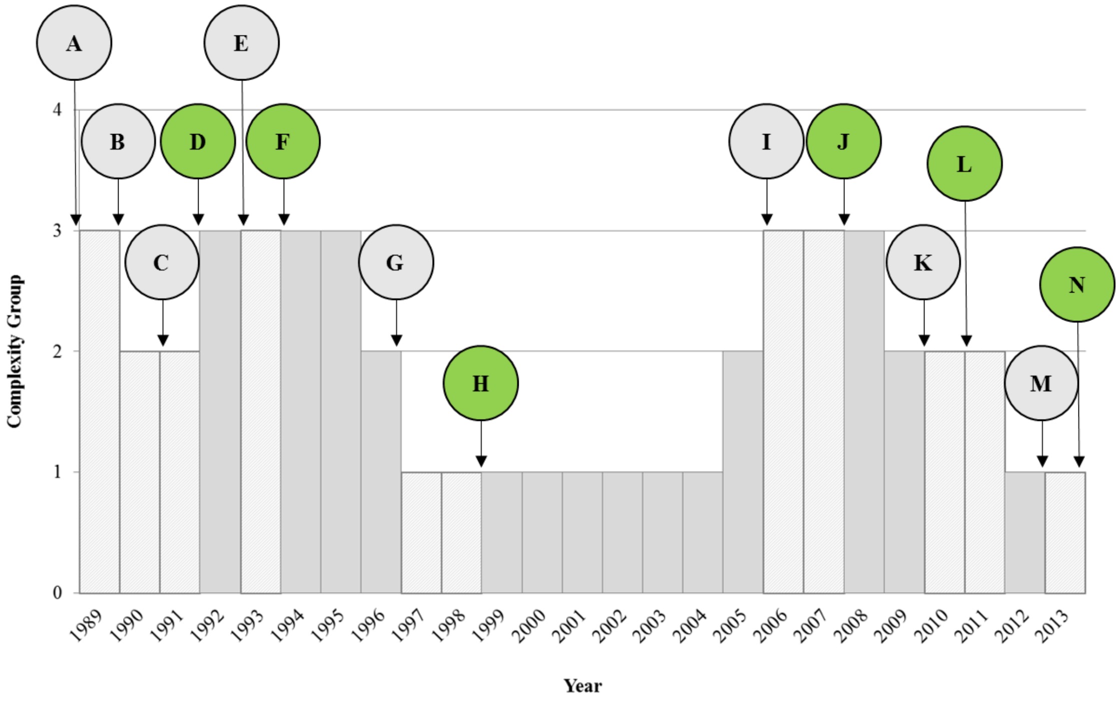

Figure 4-5: Exemplary complexity group propagation

The figure is interpreted as follows:

- Point A: At the beginning of 1989 the first two time increments of data are available (not shown on the graph) and the sliding window approach applied. Small cost data conditions exist by default since no patterns over at least two discrete time intervals are known. These conditions also exist for the preceding two time periods.

- Point B: At the end of 1989 the first complexity group is known. Small cost data conditions remain.

- Point C: At the end of 1990 the second complexity group is known and has changed from the previous year. Small cost data conditions remain.

- Point D: At the end of 1991 the complexity group has remained unchanged for two years. Small cost data conditions no longer exist, and previous insights regarding cost estimating relationships can be input into a parametric estimation model.

- Point E: At the end of 1992 it is determined that the complexity group has changed. Small cost data conditions return.

- Point F: At the end of 1993 the complexity group is unchanged from the previous year. Small cost data conditions no longer exist, and previous insights regarding cost estimating relationships can be input into a parametric estimation model.

- Point G: At the end of 1996 the complexity group changes again. Small cost data conditions return.

- Point H: At the end of 1998 two periods of stable complexity are again identified. Small cost data conditions no longer exist, and previous insights regarding cost estimating relationships can be input into a parametric estimation model.

- Point I: At the end of 2005 the complexity group again changes. Small cost data conditions return.

- Point J: At the end of 2007 the complexity group has remained stable for two periods. Small cost data conditions no longer exist, and previous insights regarding cost estimating relationships can be input into a parametric estimation model.

- Point K: At the end of 2009 the complexity group has again changed. Small cost data conditions return.

- Point L: At the end of 2010 two periods of stable complexity group are again determined. Small cost data conditions no longer exist, and previous insights regarding cost estimating relationships can be input into a parametric estimation model.

- Point M: At the end of 2012 a change in complexity group is again identified. Small cost data conditions return.

- Point N: At the end of 2013 two periods of stable complexity group are again determined. Small cost data conditions no longer exist and previous insights regarding cost estimating relationships can be input into a parametric estimation model.

Green (dark) shaded circles therefore when small cost data conditions begin and grey shaded columns indicate when small cost data conditions end. In 11 of 25 time periods (44%), conditions of small cost data were therefore present.

4.4.4 Process for Determining the Presence of Small Cost Data Conditions

The process applied for determining whether small cost data conditions exist can thus be described as follows:

1. Determine cost variance captured

2. Update last figure of binary string depending on change in cost variance

3. Update and evaluate the three interval binary string for its complexity group

4. Compare the complexity group to the complexity groups of the two previous time increments

    a. If the complexity group has changed then small cost data conditions exist

    b. If the complexity group has not changed then small cost data conditions do not exist.

In summary, therefore the moment the complexity group changes, the minimum a priori data counter must be reset to “0” and the recommended estimation technique must be re-evaluated. The moment a complexity group repeats itself two times the estimator can move to parametric estimation techniques. Generic times when a complexity group changes can be considered to include events such as a change in whole product life cycle phase, a major milestones applied during a project, after re-baselining of an estimate, significant changes in schedule or requirements, a change in responsible cost estimator or a change in key assumptions.

4.5 Industry Consideration of Small Cost Data Conditions

This section has demonstrated an approach to identifying when small cost data conditions exist and suggests the selection of a cost estimating method based the number of time periods for which a three bit sliding window approach for calculating complexity maintains its complexity group. Regression approaches are suggested to require at least 41 time periods of cost data where the three bit sliding window calculates an unchanging complexity group. Parametric approaches are suggested to require at least four time periods of cost data where the three bit sliding window calculates an unchanging complexity group for. For less than four time periods, the study suggests that the use of analogy and expert opinion is suitable, although the research findings offer a specific quantitative technique as a preferred alternative.

4.6 Summary

Chapter 4 focused on discussing current practice and challenges in respect to forecasting the propagation of cost estimate uncertainty under conditions of small cost data. Currently relevant guidelines and standards were introduced followed by a closer examination of their application in practice with a special view on their relationship to the contracting lifecycle. Gaps and challenges in practice were identified and correlated with the research gaps identified in the literature review. Finally, recommendations were made in respect to aligning theory and practice.

Chapter 5 presents the developed integrated forecasting framework. The framework is presented from an infographic, process and mathematical perspective with growing detail level. The forecasting algorithms are described and the process model for creating the dependency model for propagation of uncertainty over time introduced.

CHAPTER 5: INTEGRATED FRAMEWORK FOR ESTIMATING COST UNCERTAINTY

5.1 Introduction

This chapter presents the integrated framework developed in the research study including a dependency model derived from the forecasting process. The dependency model is used to provide an explanation of why the forecasting process is able to generate robust estimates.

5.2 Framework Description

The research hypothesis was investigated by using an integrated framework for forecasting cost uncertainty using geometrical principles. The framework consists of steps for “Calculation” which prepares data for forecasting by calculating it’s absolute and relative values, “Composition” which converts that data to the form required by the method, “Forecasting” to predict cost estimate uncertainty and “Explanation” for understanding the forecasts behaviour using a system dynamics model.

5.2.1 Forecasting Framework

Figure 5-1 provides a high level overview of the framework. The steps describe the path from the input of the number of cost variance reasons and their magnitude to the calculation of a most likely forecast and its explanation through a dependency model.

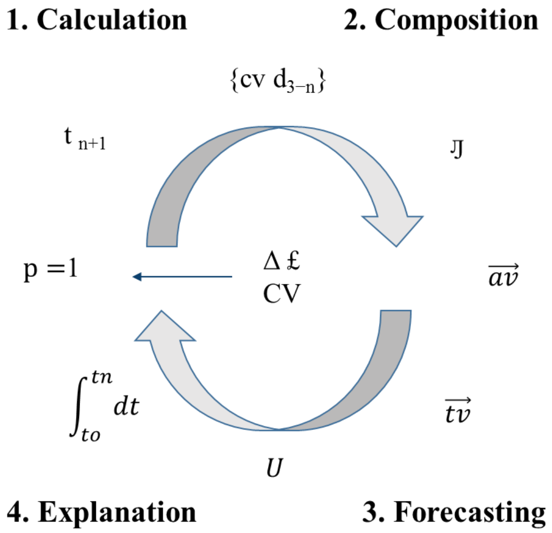

Figure 5-1: Forecasting framework

The left hand side of Figure 5-1 is of “arithmetical” (therefore consisting of numbers) and of a “state space” nature (therefore focused on a static view of a single time period) as shown in Table 5-1.

The example is drawn from the U.S. DoD Air Force C-17A Globemaster III cost variance data reported on in the SAR for 2002 and 2003. Figures in Table 5-1 in normal font indicate increases in costs for the cost variance reason mentioned, and figures in cursive font indicate decreases in cost for the cost variance reason mentioned.

Table 5-1: Exemplary arithmetical cost variance data C-17A Globemaster III in 2002

Attribute	Value #1 Absolute (USD$M)	Value #1 Relative	Value #2 Absolute (USD$M)	Value #2 Relative
Year of Baseline	1996	N/A	1996	N/A
Year of Reported Cost Variance	2002	N/A	2003	N/A
Cost Δ due to Δ in Quantity	*2,512*	*11.6%*	*2,512*	*11.8%*
Cost Δ due to Δ in Schedule	983	4.5%	983	4.6%
Cost Δ due to Δ in Engineering	315	1.4%	372	1.7%
Cost Δ due to Δ in Estimating	13,177	60.6%	13,074	61.2%
Cost Δ due to Δ in Other	411	1.9%	411	1.9%
Cost Δ due to Δ in Support	4,354	20%	4,007	18.8%

From a geometric / vector perspective, the initial challenge encountered is how to treat different types of cost variance in that some increase total costs (normal font), and some decrease total cost (cursive font). The presented framework converts cost variance into vectors that share starting coordinates which are declared to be topologically invariant. The starting coordinates are a single point which represents the centre of the vector space. For these reasons, each vector, by default, must share a common prefix (therefore be positive or negative) which is considered to be an attribute of the common invariant vector scale. For purposes of this investigation, the presented method thus considers only the absolute value of cost variance. The framework furthermore converts these absolute values into relative values (the individual cost variance values are thus transformed into percentages of total) in order to ease the comparison of geometrical shapes generated whereby this does not affect the value of the actual geometrical attributes evaluated.

The various reasons for cost variance (symbolised by "$\{cv\ d_{3\text{-}n}\}$") are considered as "dimensions" of cost variance and explored through geometrical visualisation. The individual dimensions are visualised as vectors and a method for composing these into a topologically coherent polar force field applied (as indicated by the symbol for a musical note "♫"). The right hand side of Figure 5-1 is thus of geometrical nature as shown in Figure 5-2. The state space nature remains. Each vector with a solid line represents the cost variance value of a specific cost variance dimension and the dashed line represents the aggregation of the cost variance vectors.

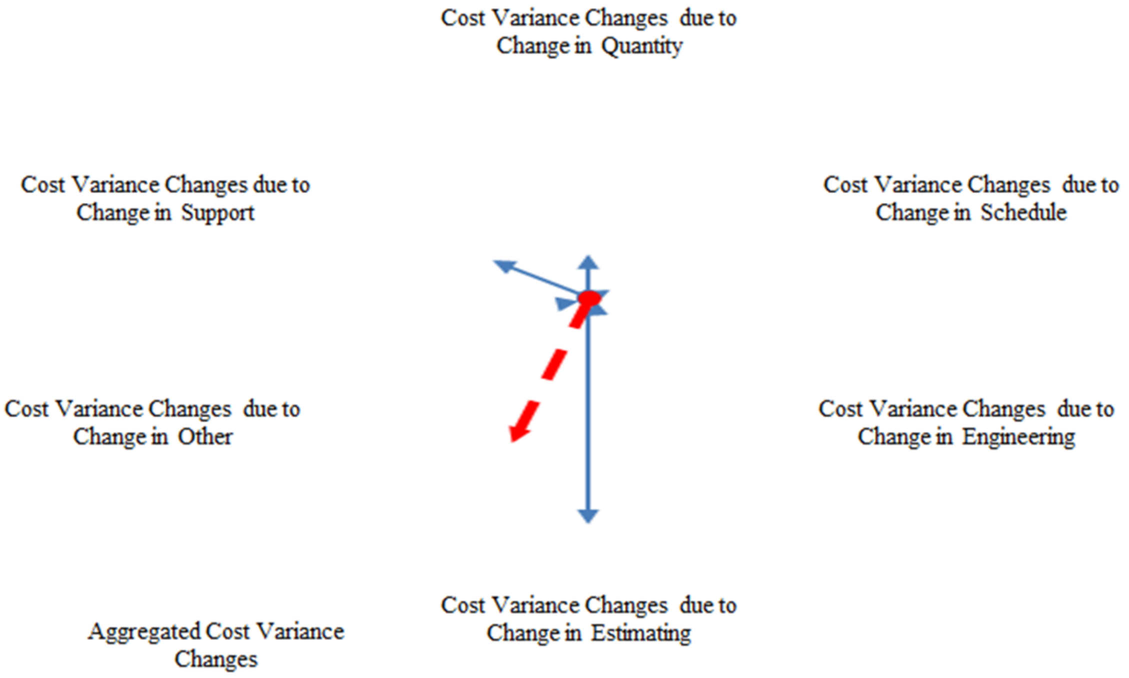

Figure 5-2: Exemplary geometrical cost variance data C-17A Globemaster III in 2002

Using only the basic operator of addition the vectors are consolidated in order to create an aggregated vector (symbolised by the vector "$\overrightarrow{av}$" in Figure 5.1) which is shown by the dashed arrow in the vector graph. Three basic transformation vectors (symbolised by the vector "$\overrightarrow{tv}$" in Figure 5.1) or algorithms, are used to forecast the attributes of the future individual / aggregated vectors for upper limit, a lower limit and a mode.

The results of the three transformation methods or algorithms are then used as inputs for worst case, best case and most likely values to a Monte Carlo simulation in conjunction with the selection of a triangular distribution (see also Section 2.5.5.2) in order to generate a three point estimate as an expression of uncertainty (symbolised by the character "U" in Figure 5.1) depending on the desired statistical confidence level. To then explain the dynamics of the generated forecasts, the arithmetical data is converted into a system dynamics model (symbolised by the integral "$\int_{to}^{tn} dt$" in Figure 5.1). This final step enables the simulation of different cost estimate uncertainty propagation scenarios and experimentation on how total cost uncertainty is influenced if management interventions are made to change the cost uncertainty of individual cost variance elements.

5.2.2 Forecasting Method

Figure 5-3 refines the framework to a process model. The reason for the sequence is given by the underlying mathematical model which generates a most likely estimate and requires a specific sequence of calculation starting from the input values (cost variance dimensions and their values for a single time interval).

Each step of the method is detailed through calculation steps which are then used as the basis for the mathematical model. The calculation steps derive from converting arithmetic inputs of cost variance reasons and their values (n) to a shape and then to an analysis of the geometrical shape of that data for the calculation of an arithmetic most likely value.

The forecasting method is thus of purely iterative nature and does not contain feedback loops although the dependency model presented as an explanation for the forecast is based upon such in order to support defining the relationships of the cost variance reasons over time.

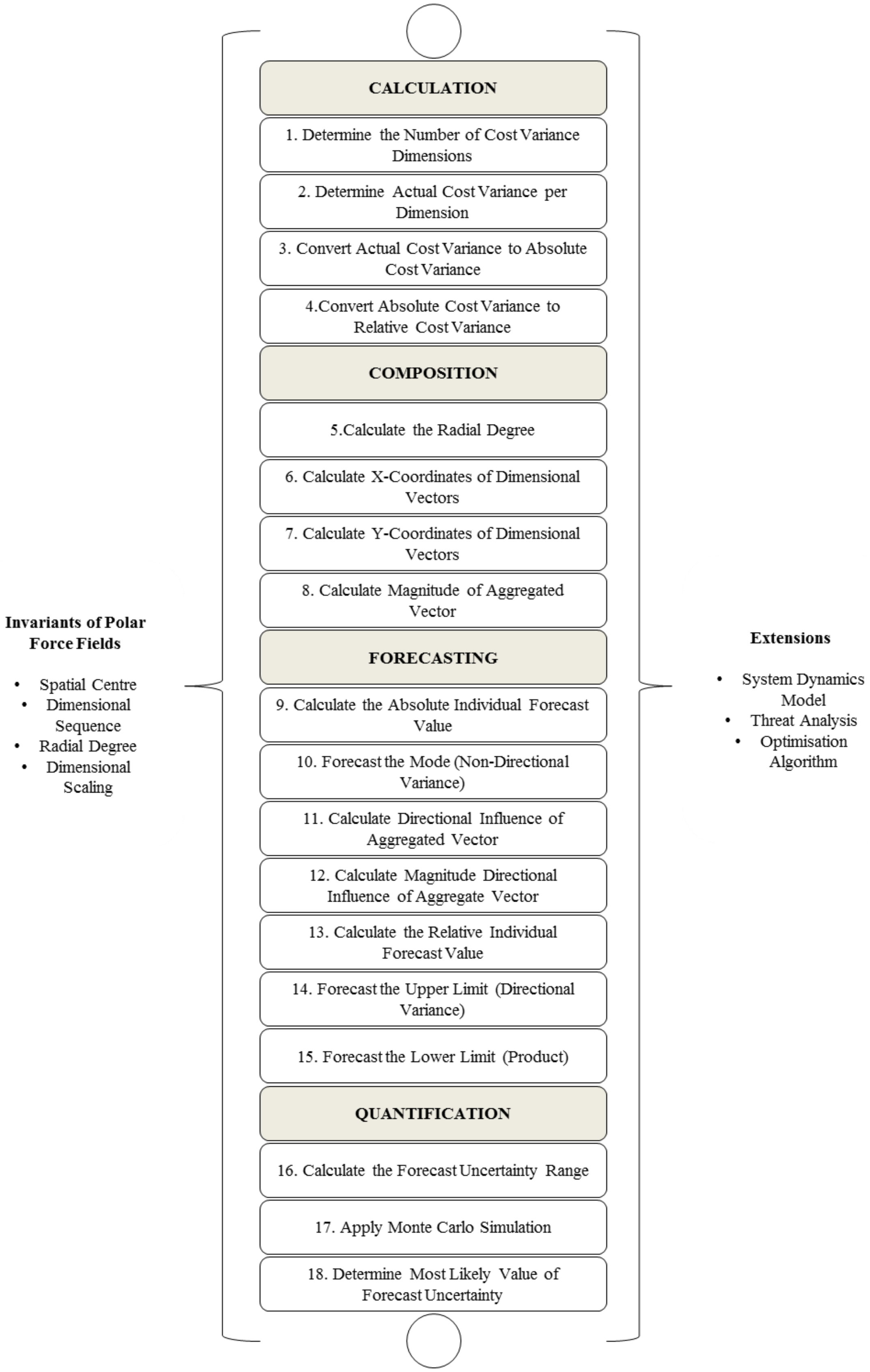

Figure 5-3: Forecasting method

The specific elements of the forecasting method are sequentially defined as follows:

1. Determine the number of cost variance dimensions (n) from the source data. These correspond to the different types of cost variance reported on as demonstrated in the case studies. This is the first of two inputs to the mathematical model and is initially needed for determining the actual cost variance (cva) and the radial degree (rd). The variable propagates through the complete mathematical model.

2. Determine the actual cost variance (cva) for each dimension from the source data. This is the second of two inputs to the mathematical model and is needed for determining the absolute cost variance (cvab).

3. Convert actual cost variance to absolute cost variance (cvab) for each dimension (n) since polar vector graphs will only accept absolute values. Note that this means that increases and decreases in cost variance are treated equally. This is needed for calculating relative cost variance (cvr). The Microsoft Excel ® function "ABS" is used to convert any negative "-" values to positive "+" values:

    - Equation: $\text{cvab}_{(\text{n actual})} = ABS(\text{cva}_{(\text{n actual})})$

4. Convert the actual absolute cost variance (cvab) to relative cost variance (cvr) for each dimension (n). Note that this step is relevant because it supports the visual comparison of different vector spaces by scaling the different spaces in an identical manner. The operation does not affect the value of geometrical attributes used for forecasting purposes. This is needed for the calculation of the forecast relative cost variance required for calculating the forecast mode (m) value of total cost and for calculating the upper limit (b) value of total cost. This is furthermore needed for calculating the x- and y- end-coordinates of the actual absolute cost variance vector (cvab) for each dimension (n):

    - Equation: $\text{cvr}_{(\text{1-n actual})} = \text{cvab}_{(\text{1-n actual})} \,/\, \sum_{1}^{n} cvab_{(\text{actual})}$

5. Calculate the radial degree (rd). This is needed in order to calculate the x- and y- end-coordinates of the actual absolute cost variance (cvab) vectors for each dimension (n):

   - Equation: $\mathrm{rd} = 360/\mathrm{n}$

6. Calculate the x- end-coordinates of the individual actual absolute cost variance (cvab) vectors and the x- end-coordinate of the actual aggregate vector (av) end. These are needed for calculating relative cost variance (cvr) and the x- end-coordinate of the dimensional vector (x_{di}).

   - (6a) Equation: $\mathrm{x}_{\mathrm{cvab\ (1\text{-}n\ actual)}} = \mathrm{COS(RADIANS(rd))^{*}cvr_{(1\text{-}n\ actual)})}$

   - (6b) Equation: $\mathrm{x}_{\mathrm{av\ (actual)}} = \sum_{1}^{n} x_{\mathrm{cvab(actual)}}$

7. Calculate the y- end-coordinates of the individual actual absolute cost variance (cvab) vectors and the y- end-coordinates of the actual aggregate vector (av) end. These are needed for calculating relative cost variance (cvr) and the y- end-coordinate of the dimensional vector (y_{di}).

   - (7a) Equation: $\mathrm{y}_{\mathrm{cvab\ (1\text{-}n\ actual)}} = \mathrm{SIN(RADIANS(rd))^{*}cvr_{(1\text{-}n\ actual)})}$

   - (7b) Equation: $\mathrm{y}_{\mathrm{av\ (actual)}} = \sum_{1}^{n} y_{\mathrm{cvab\ (actual)}}$

8. Calculate the magnitude of the aggregated vector (AV_{m}). The aggregate vector is the sum of the actual cost variance vectors and its magnitude is needed to calculate the forecast value of relative cost variance (cvr) for each dimension (n):

   - Equation: $\mathrm{AV}_{\mathrm{m\ (actual)}} = \sqrt{}(\Delta \mathrm{x}_{\mathrm{av\ (actual)}}{}^{2} + \Delta \mathrm{y}_{\mathrm{av\ (actual)}}{}^{2})$

9. Calculate the absolute cost variance forecast value for each dimension (n). This is needed to calculate the mode (m) value of forecast total cost variance:

   - Equation: $cvr_{(1\text{-}n\ \text{forecast})} = cvr_{(1\text{-}n\ \text{actual})} + (cvr_{1\text{-}(n\ \text{actual})} * ((AV_{m\ (\text{actual})}/100))$

10. Calculate the forecast mode (c) value of the aggregated vector. This is needed as an input into the Monte Carlo simulation used to determine the most likely value (ML):

    - Equation: $a = \sum_{1}^{n} cvr_{(\text{forecast})} / n$

11. Calculate the x- and y- end-coordinates of the directional influence of the aggregated vector (x_{di}). This is needed to calculate the length of the directional influence of the aggregated vector (di_{av}) on each individual dimension (n):

    - (11a) Equation: $x_{di\ (1\text{-}n\ \text{actual})} = x_{av\ (1\text{-}n\ \text{actual})} - x_{cvab\ (1\text{-}n\ \text{actual})}$

    - (11b) Equation: $y_{di\ (1\text{-}n\ \text{actual})} = y_{av\ (1\text{-}n\ \text{actual})} - y_{cvab\ (1\text{-}n\ \text{actual})}$

12. Calculate the magnitude of the directional influence of the aggregate vector (di_{av}). This is needed to calculate the relative value of forecast cost variance (cvrd) for each dimension (n) as the basis for calculating the upper limit (b):

    - Equation: $di_{av\ (\text{actual})} = \sqrt{}(\Delta x_{di\ (\text{actual})}{}^2 + \Delta y_{di\ (\text{actual})}{}^2)$

13. Calculate the relative value of forecast cost variance (cvrd) for each dimension (n) as the basis for calculating the upper limit (b):

    - Equation: $cvrd_{(1\text{-}n\ \text{forecast})} = cvr_{(1\text{-}n\ \text{actual})} + (cvr_{(1\text{-}n\ \text{actual})} + di_{av\ (\text{actual})})$

14. Calculate the forecast upper limit value (b). This is needed for calculating the forecast uncertainty range (ur), as an input into the Monte Carlo simulation (P(x)) for forecasting the most likely value (ML) and for calculating the lower limit multiplier ($iv_{m\ forecast}$):

    - Equation: $b = \sum_{1}^{n} cvrd_{(forecast)} / n$

15. Calculate the lower limit (a). This is needed to calculate the uncertainty range (ur), and as an input into the Monte Carlo simulation (P(x)) for forecasting the most likely value (ML). This includes calculating the forecast lower limit multiplier (iv_m) for the individual vector forecasts:

    - (15a) Equation: $iv_{m\ forecast} = \sum_{1}^{n}(b * c)$
    - (15b) Equation: $a = iv_{m\ forecast} / n$

16. Calculate the forecast uncertainty range (ur). The uncertainty range is the difference between the upper and lower limit.

    - Equation: $ur_{(forecast)} = b - a$

17. Apply a Monte Carlo simulation with the upper limit (b) as the worst case, the lower limit (a) as the best case and the mode (c) as the most likely with a triangular distribution. This probability density function (P(x)) is a continuous one where the lower limit (a) is smaller than the upper limit (b) and the mode (c) is >a and <b. The limits are connected to the mode with straight lines. This is required as an input into deciding on the most likely value (ML):

- Equation: $$P(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} \text{ for } a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(b-c)} \text{ for } c < x \leq b \end{cases}$$

18. Determine the most likely value (ML) of the forecast uncertainty for desired levels of statistical confidence. The most likely value (ML) is a function of the probability density function input into the Monte Carlo simulation using the upper limit (b), the lower limit (a) and the mode (c):

    - Equation: $ML_{(\text{forecast})} = \{a, b, c, PDF\}$

5.2.3 Forecasting Process (Mathematical Model)

The steps of the forecasting method where used to create a mathematical model suitable for programming a software demonstrator. Figure 5-4 illustrates a detailed sequenced mathematical process model and Appendix E presents this model in the form of a programmed Microsoft ® Excel template which was used for performing data analysis. The U.S. DoD case study was analysed with a six dimensional template. The U.K. MoD case study was analysed with a 13 dimensional template. The eight dimensional template was used for the implementation case study. The mathematical process model describes the input and transaction sequence required to arrive at the intended output of a most likely cost estimate uncertainty value in step 18:

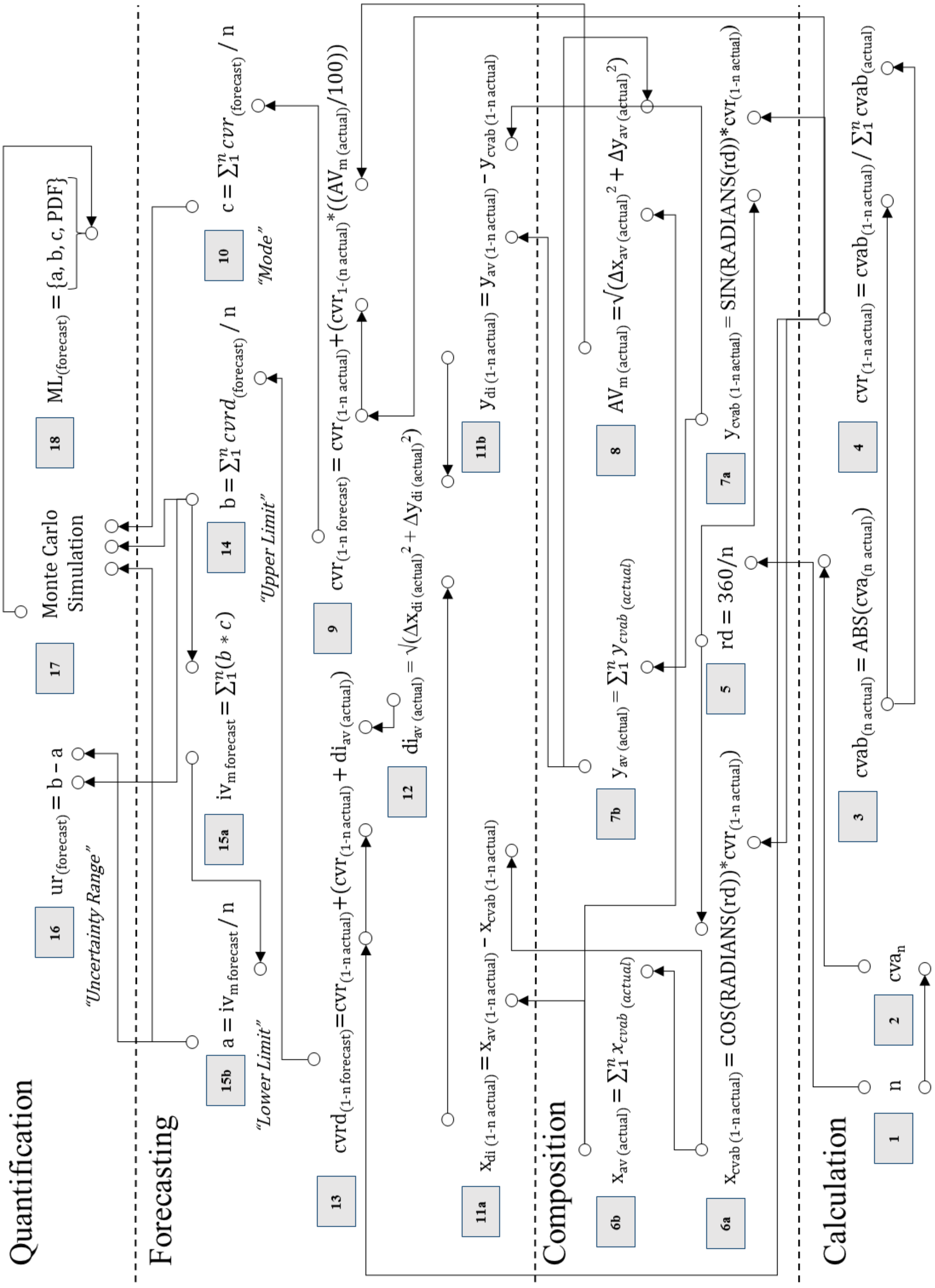

Figure 5-4: Forecasting process (mathematical model)

5.2.4 Forecasting Algorithms for Upper Limit, Lower Limit and Mode

Three different methods are used to determine the upper limit, lower limit and mode of the triangular distribution input into a Monte Carlo simulation in order to determine the most likely cost estimate uncertainty value for total cost variance at the desired confidence level as shown in Figure 5-5:

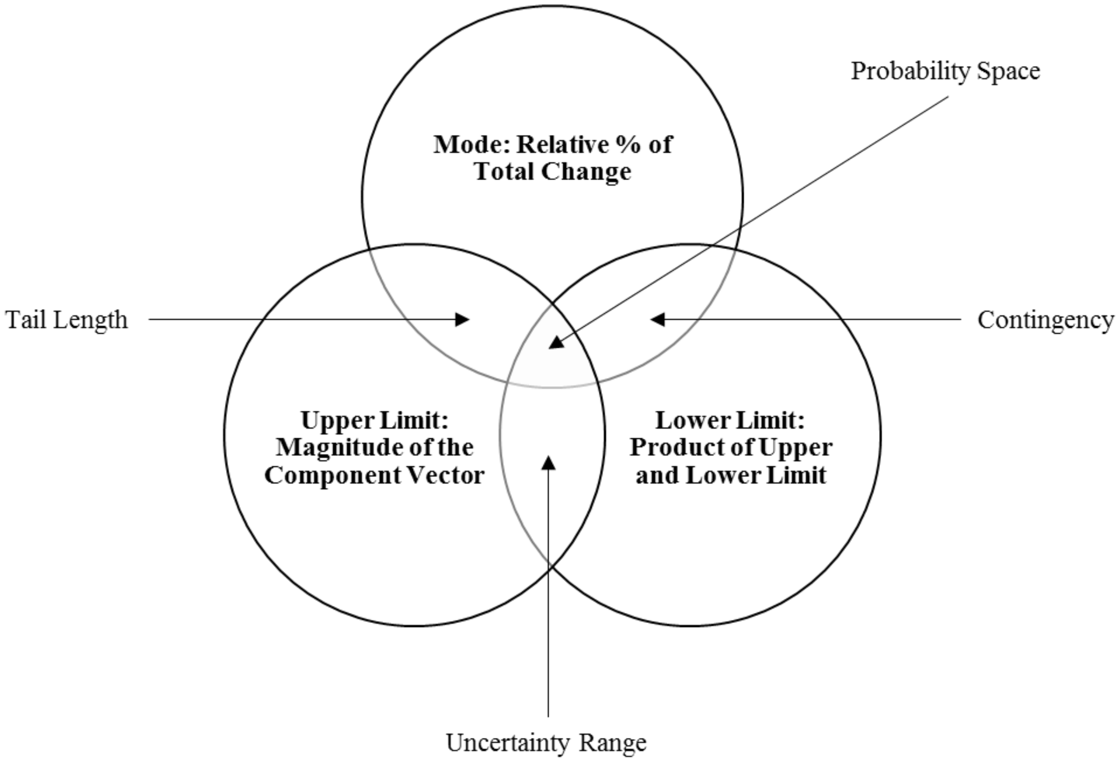

Figure 5-5: Forecasting methods and their inter-relationship

The choice of the forecasting algorithms derives from the intent to use basic mathematical operators of addition and multiplication as starting points for understanding the behaviour of geometric attributes of small cost data vector spaces. The operator of subtraction is not applied since this would inject the possibility of vectors with negative values into the force field approach and such cannot be supported by the polar approach chosen. These algorithms are intended as an initial experimental configuration subject to ongoing optimisation in respect to forecasting robustness as the amount of available patterns for examination increases. The classification as algorithms for the upper limit, mode and lower limit are based on analysis results regarding the forecast of total value which consists of the sum of all individual vector forecasts. The use of the sum of all individual vector forecasts represents only a starting point for the use of vector spaces in cost uncertainty forecasting, and is subject to further investigation from the perspective of an optimisation problem related to the invariants of the framework. The algorithms can be summarised as follows:

- Mode Algorithm (MA): The method for forecasting the mode calculates the relative % of total change represented by each cost variance vector, uses that to determine the relative proportion of the aggregated vector, adds the magnitude of that to the individual cost (variance) vector and adds the forecast value of all individual vectors in order to forecast the mode value of total future cost variance.

- Upper Limit Algorithm (ULA): The method for forecasting the upper limit calculates the magnitude of the component vector of the aggregated vector which has the same radial degree as the current state individual cost variance vector and adds it to the magnitude of each individual vector. For this the x- and y-coordinate differentials of the end points of the two vectors are calculated and added to the end point of the relevant individual current cost (variance) vector. The sum of the individual vector forecast is added in order to forecast upper limit of total future cost variance.

- Lower Limit Algorithm (LLA): The method for forecasting the lower limit multiplies the results of the ULA and the MA. This represents the lower limit of the total future cost variance.

The intersection of the mode and upper limit perspectives can be considered to provide an indication of the tail length of the probability distribution which is relevant to understanding the potential role of outliers. The intersection of the mode and lower limit perspectives are an indication of the amount of contingency that might be applicable from a business perspective. The intersection of the upper and lower limit perspectives gives the uncertainty range of relevance. In a Monte Carlo simulation, the lower limit would equate to the “best case” value, the upper limit would equate to the “worst case” value, and the mode would equate to the “most likely” value.

The classification of the algorithms as pertaining to mode, upper limit and lower limit was drawn from comparing forecast results to actual cost variance values in the U.S. DoD case study.

5.3 Dependency Model Development

Based on the forecasts generated by the integrated framework, a generic method for deriving a dependency model using cost variance data from the case study research to explain the results of the polar force field forecasting and uncertainty quantification was developed. Input-output model definitions are provided, the process of the input-output model described and an exemplary view of correlation, impact evaluation, determination of propagation sequence, integrated components and the results of an overall exemplary simulation provided.

5.3.1 Input-Output Model

The input-output model illustrated in Figure 5-6 explains why the integrated polar force field forecasting framework is able to generate robust estimates. As such the input output model creates a dependency model based on the actual and forecast values of the individual cost variance values. In this respect, the model is not an integral part of the framework and tried to address the question raised during verification and validation concerning why the presented framework is able to forecast with robustness.

Based on the data available for absolute cost variance, a covariate analysis is performed in order to identify the correlation function between all cost variance variables. The slope of the linear correlation function was used to determine the value of future impact, while the co-efficient of correlation is used to determine the sequence of impacts between the variables and their relative speed. Impact, sequence and speed were then used to quantify cost variance propagation.

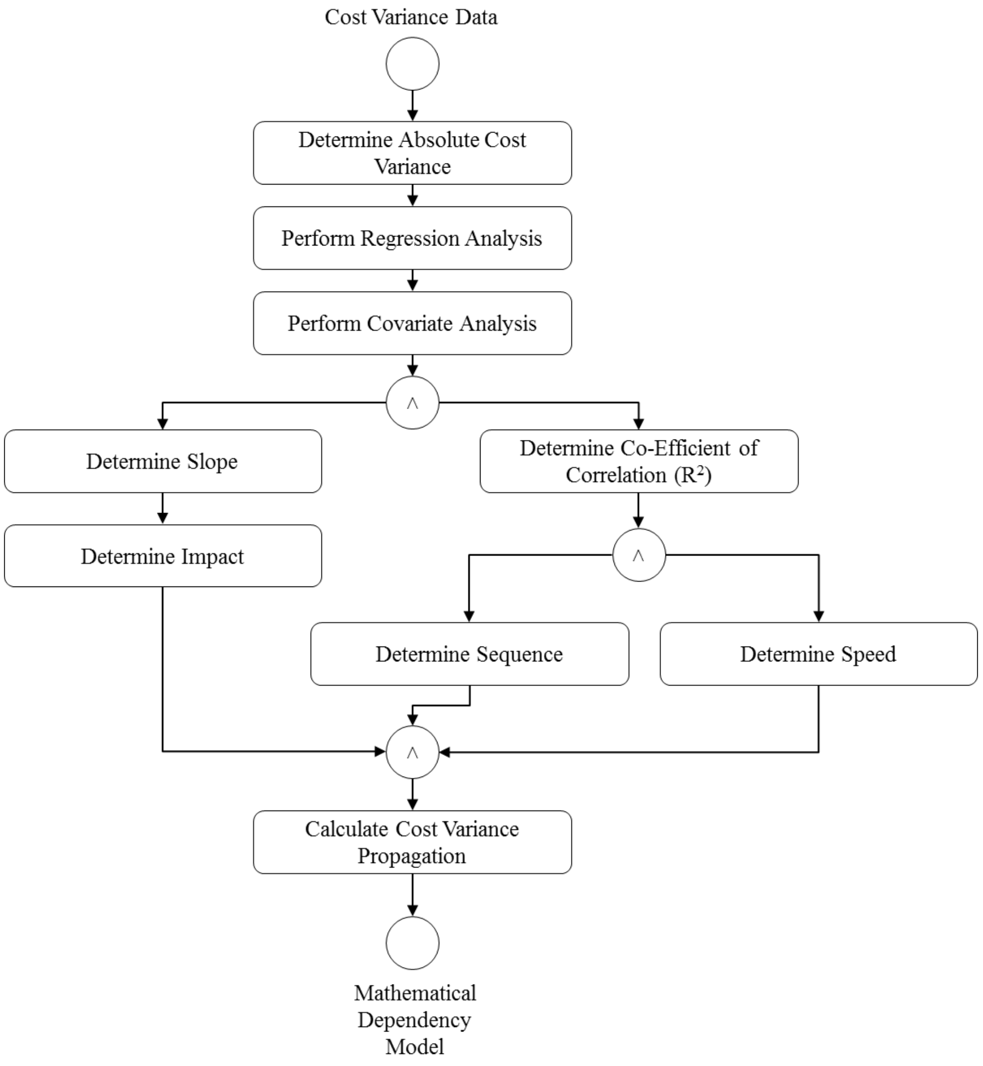

Figure 5-6: Dependency model – input-output model

Slope describes the direction and steepness of the linear trend line used to describe data. Impact states that the greater the slope of a correlation trend line between two cost variance factors the greater the impact of the cost variance factor on the y-axis is on the cost variance factor on the x-axis. For sequence, the higher the co-efficient of correlation between a pair of cost variance factors the earlier in the overall simulation sequence the cost variance propagation is scheduled. Speed refers to the higher the impact of one cost variance factor on another cost variance factor the faster the impact is declared to occur.

The input-output model uses simplified regression and co-variate analysis on case study data to identify a dependency model of the cost variance factors used for developing the integrated vector space forecasting framework. As in the Composition element of the integrated polar force field forecasting framework, cost variance is considered as absolute (therefore disregarding whether the cost variance increases or decreases total cost variance). For each cost variance factor, the case study data is analysed to determine its distribution over time in respect to current values (t=1) and the future value (t=2). The linear trend line for the distribution is chosen to describe the relevant propagation behaviour. For each possible pair of cost variance factors, the relevant case study data is correlated for its distribution over time in respect to values for t=1 and the value at t=2. The linear trend line for the correlation is chosen to describe the relevant correlation. The formula for the linear trend line of each possible cost variance pair, is evaluated to determine its slope. The higher the slope of each possible cost variance pair the greater the impact of one cost variance factor on the other is considered. The formula for the linear trend line of each possible cost variance pair is evaluated to determine the co-efficient of correlation in order to determine whether or not the cost variance pair is to be included in the dependency model.

The sequence of cost variance factors impacting each other in the dependency model is determined based on the co-efficient of correlation between the cost variance pairs.The speed of cost variance factors impacting each other in the dependency model is determined in a relative manner based on the ranking of the impact. The cost variance propagation in the dependency model is given by a sequential calculation of all correlated cost variance pairs in the sequence determined and based on the following equation in generic dependency model notation:

Cost Variance $_{(t=n)} = \int_{t_o}^{t_n} [Inflow(s) - Outflow(s)]ds.$

This approach in and of itself points to the previously mentioned focus of the research study on change over time (flows) versus change at different points in time (stocks).

5.3.2 Exemplary Overview of Cost Variance Correlation

The available historical cost variance data for multiple cost variance dimensions was correlated using the default linear trend-line function in Microsoft ® Excel. Based upon the correlation results a dependency model was created. Correlations were performed for the value of one variable at t=0 and the value of the second variable at t=1. Sample input data is shown in Table 5-2 for the U.S.DoD Navy CV Helo (SH-60F) with a financial base year of 1988. The source of the data was the relevant U.S.DoD SAR. Analysis boundaries for the example are indicated by grey shaded cells.

Table 5-2: Sample cost variance data boundaries for U.S. DoD Navy CV Helo (SH-60F)

Year	Quantity (USD$M)	Schedule (USD$M)	Engineering (USD$M)	Estimating (USD$M)	Other (USD$M)	Support (USD$M)
1988	0	2	34	11	0	*67*
1989	0	*9*	72	13	0	22
1990	0	22	22	326	0	149
1991	0	0	41	*185*	0	12
1992	*315*	*4*	22	43	0	*138*
1993	*616*	*7*	*69*	*214*	0	*241*

An example of a simple linear regression analysis is provided in Figure 5-7:

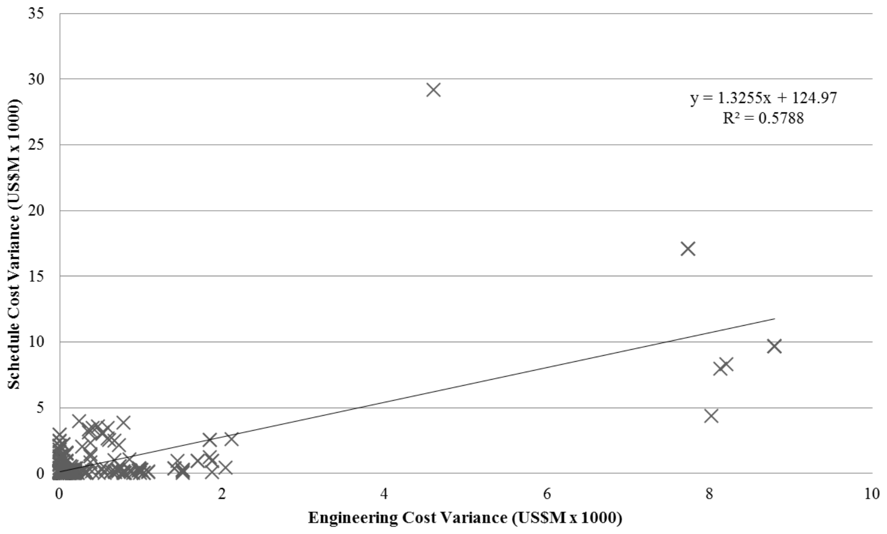

Figure 5-7: Exemplary engineering t=0 to schedule t=1 relationship

The calculated correlations of all variable pairs to each other are listed in Table 5-3:

Table 5-3: Variable correlation

Variable 1	Variable 2	Equation linear trend line	R^2
Schedule	Engineering	y = 1.2578x + 214.99	0.4368
Support	Engineering	y = 0.2178x + 316.13	0.1824
Estimating	Engineering	y = 0.8416x + 1275.9	0.1726
Quantity	Engineering	y = 0.0892x + 432.4	0.0579
Other	Engineering	y = -0.001x + 20.819	0.0008
Schedule	Estimating	y = 3.4186x + 857.18	0.4926
Support	Estimating	y = 0.0961x + 261.56	0.2324
Engineering	Estimating	y = 1.2218x + 1336.2	0.1959
Quantity	Estimating	y = 0.3338x + 1219.7	0.1239
Other	Estimating	y = 0.0048x + 10.245	0.1116
Estimating	Other	y = 0.0077x + 6.7607	0.1724
Support	Other	y = 2.3829x + 409.37	0.0307
Schedule	Other	y = 0.0057x + 18.759	0.0064
Engineering	Other	y = -0.0009x + 21.31	0.0005
Quantity	Other	y = 0.0001x + 20.499	6.00E-05
Estimating	Quantity	y = 0.2477x + 1168.3	0.1182
Support	Quantity	y = 0.0525x + 318.53	0.0838
Schedule	Quantity	y = 0.0494x + 217.87	0.07
Engineering	Quantity	y = 0.0791x + 375.8	0.0576
Other	Quantity	y = -0.0001x + 20.516	0.0001
Engineering	Schedule	y = 1.2221x + 98.315	0.572
Estimating	Schedule	y = 2.317x + 901.99	0.431
Support	Schedule	y = 0.5539x + 237.26	0.3886
Quantity	Schedule	y = 0.0611x + 247.5	0.0824
Other	Schedule	y = 0.0053x + 18.014	0.0068
Schedule	Support	y = 0.7329x + 249.66	0.4441
Estimating	Support	y = 0.1644x + 201.46	0.3338
Engineering	Support	y = 0.3277x + 313.76	0.2764
Quantity	Support	y = 0.0605x + 355.16	0.0797
Other	Support	y = 2.5022x + 455.82	0.0255

5.3.3 Exemplary Overview of Cost Variance Impact

The correlation results then rank the future impact based on the strength of the variable relationships in relation to the linear correlation line slope as illustrated in Figure 5-8.

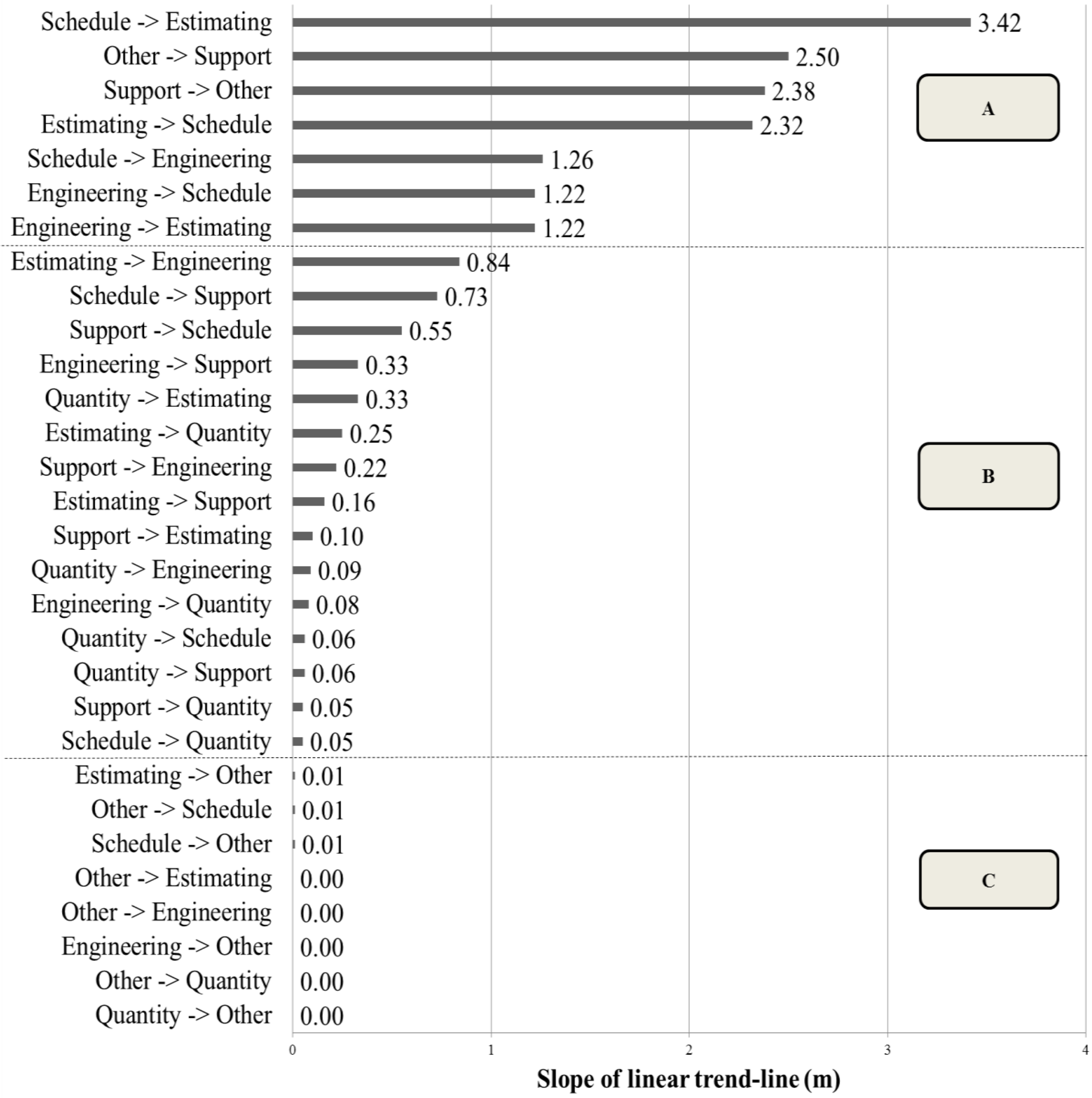

Figure 5-8: Correlation ranking - degree of future impact

For variable pairs in area A the first variable grows faster than the second. For variable pairs in area B the second grows faster than the first. Variable pairs in area C are disregarded for purposes of simplification since their value at one decimal point accuracy is zero.

5.3.4 Exemplary Overview of Cost Variance Propagation Sequence

The results of the correlation are used to rank the the relationships between the variables based on the value of their correlation co-efficient, as shown in Figure 5-9. It is assumed that the greater the correlation, the stronger / more dominant the correlation and that the correlation can therefore be used for determining sequence of impacts.

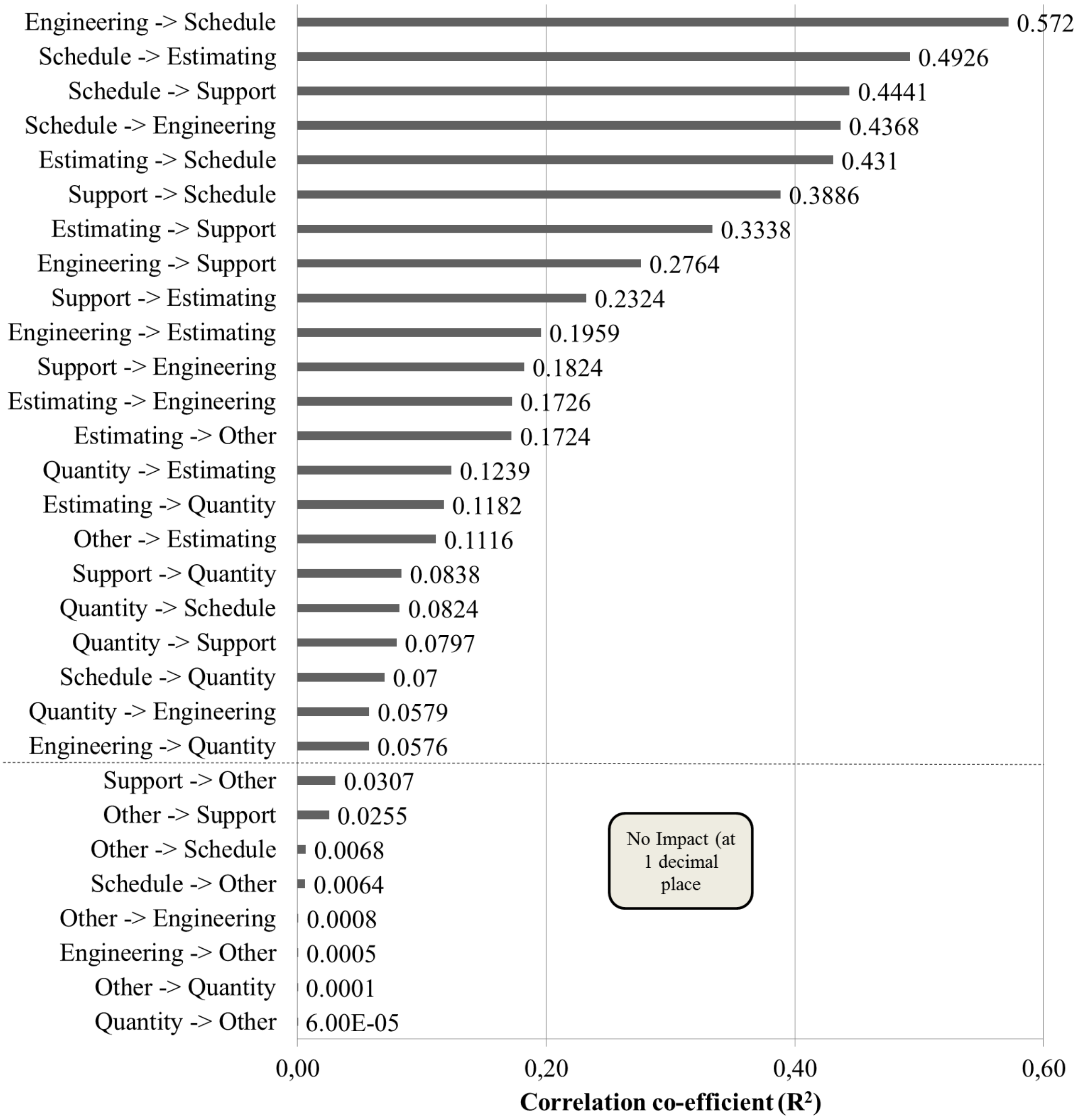

Figure 5-9: Correlation ranking - sequence of future impact

5.3.5 Exemplary Overview of Cost Variance Dependency Model

Based upon the previously identified correlation rankings for the degrees of future impact, speed and sequence a dependency model can be created as shown in Figure 5-10:

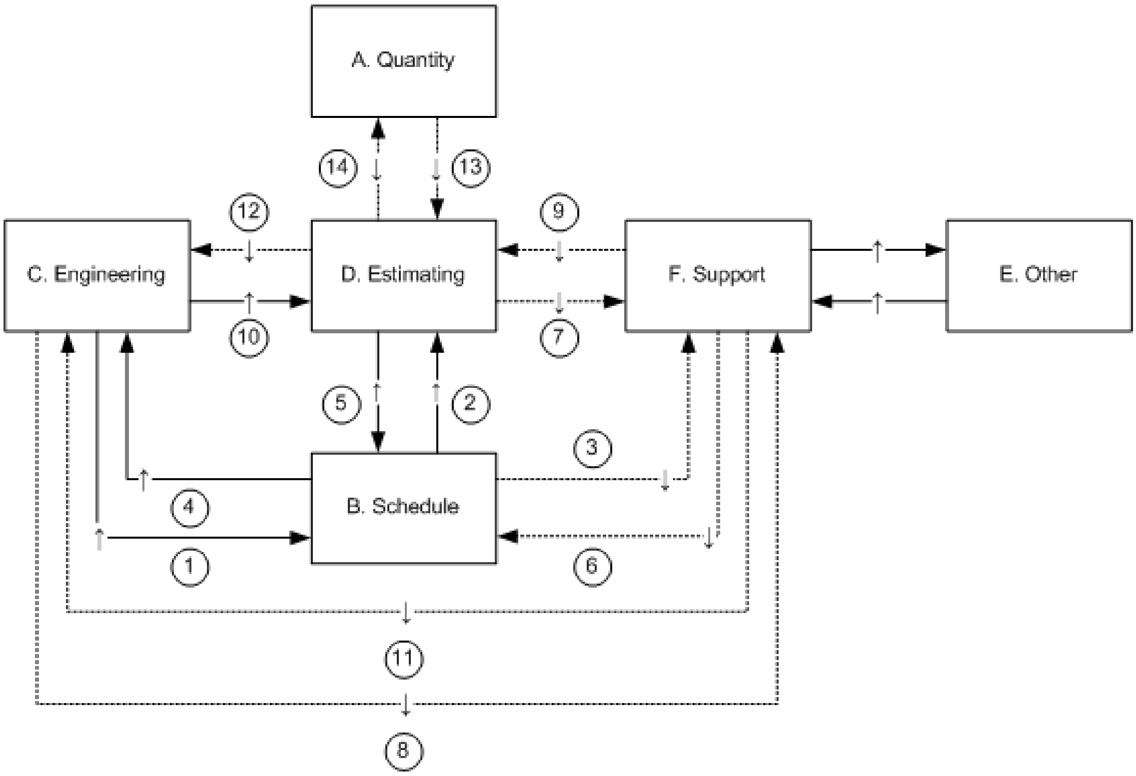

Figure 5-10: Dependency model based on case study data

The arrows connecting cost variance dimensions indicate from which factor an input arises / to which factor an output is delivered, and the numbers indicate the overall sequence of these inputs / outputs within the simulation. A dotted connecting arrow / "↓" symbol indicates a decreasing influence (therefore the impacted variable grows slower than the triggering variable – decelerated variance) and a solid arrow / "↑" indicates an increasing influence (therefore the triggering variable grows faster than the impacted variable – accelerated variance). In this respect, each cost variance variable can now be described based upon inputs and outputs including the sequence of these being generated or received.

A summary of inputs and outputs for the variables is shown in Table 5-4:

Table 5-4: Summary of inputs and outputs

	Number of Inputs Acc-elerating	Number of Outputs Acc-elerating	Number of Inputs De-celerating	Number of Outputs De-celerating	Number of Inputs Total	Number of Outputs Total	Number of Con-nections Total
Cost variance due to Δ in Quantity	0	0	1	1	1	1	2
Cost variance due to Δ in Schedule	2	2	1	1	3	3	6
Cost variance due to Δ in Engineering	1	2	2	1	3	3	6
Cost variance due to Δ in Estimating	2	1	3	3	4	4	8
Cost variance due to Δ in Other	1	1	0	0	1	1	2
Cost variance due to Δ in Support	0	0	3	3	3	3	6
SUM	6	6	10	9	15	15	28
AVERAGE	1	1	1.67	1.5	2.5	2.5	4.67

5.4 Summary

This chapter presented the developed integrated forecasting framework. The framework was presented from a high level view, process and mathematical perspective with growing detail level. The forecasting algorithms were described and the process model for creating the dependency model for propagation of uncertainty over time introduced.

Chapter 6 provides an in-depth view of verification and validation efforts of the research results. Quantitative verification and validation efforts based on data analysis of three exemplary case studies are presented. This is followed by qualitative insights gathered through (semi-) structured interviews and surveys assessed and a reflection of the contribution of inputs gathered during serious game plays. Finally, the role of the thought experiment is discussed.

CHAPTER 6: VERIFICATION AND VALIDATION

6.1 Introduction

The four outputs of the investigation were subject to verification and validation exercises through the data analysis, (semi-) structured interviews and surveys, game playing and a thought experiment. Due to their integrated nature all components are integrated into a single chapter.

6.2 Verification and Validation through Data Analysis

Based on data from the U.S. DoD SAR three case studies were chosen in order to exemplify the performance of the framework. This was on the level of an individual product (the U.S. DoD Air Force C-17A Globemaster III due to this being the longest running project reported on thus allowing for the greatest degree of time-series examination / comparison) through aggregated products in a complete domain the product belonged to (the U.S. Armed Forces Air Force) and through aggregated domains in the U.S. DoD as a whole. The investigation presents all forecasts for all products reported on in the time period 1986 to 2013, with these three case studies serving only as exemplary ones and chosen to represent different levels of aggregation. Data for all other available projects is included for comparative purposes. The source data was chosen due its public availability to allow independent verification of study results and the availability of independent third party reference tables based on such. For each case study the technique is applied to each forecastable event using the three individual forecasting methods integrated in them and the forecast accuracy assessed and evaluated. Finally, a critical comparison of the results against best practice third party reference tables as represented by the Joint Agency Cost Schedule Risk and Uncertainty Hand Book, specifically on p.58 Table 3-3 NCCA SAR Growth Factors: Since 1969/ Since 1980/ Since 1990 (Effective December 2011) whereby the “Since 1990” reference figures for “Mean Cost Growth Factor, Procurement Estimates at MS C” are used (U.S. Naval Center for Cost Analysis, 2014) is performed.

6.2.1 Data Source: U.S. DoD Selected Acquisition Reports (SAR)

The first case study examines the U.S. DoD Air Force C-17A Globemaster III from 2002 to 2003. The second case study examines all reported projects for the U.S. Air Force from 1986 to 2013. The third case study examines all reported projects for the U.S. DoD from 1990 to 2013. Each case study is presented with an exemplary forecast for a single time period, exemplary results of forecasting for each available time period with a forecasting method (specifically the mode calculation algorithm), the results of forecasting for each available time period with all forecasting methods and a comparison of the forecast results to best practice third party reference tables. Important to note is that for each case study, the verification and validation of forecasts is made using series of single data sets and without any reference to data or forecasts from a previous time period.

6.2.2 Case Study 1: U.S. Air Force C-17A Globemaster III

The first case study uses cost variance data for the U.S. DoD Air Force C-17A Globemaster III military transport aircraft. The first validation exercise uses the cost variance data from 2002 in order to forecast the cost variance data in 2003. The second validation exercise repeats the forecast for each individual forecastable event available using an exemplary forecasting method (specifically the mode calculation algorithm) to illustrate the detailed results achieved. A forecastable event for purpose of the research study occurs when cost variance data is available for two following time periods so that forecasts made based on the first can be compared to the actual values reported on in the second. The third validation exercise shares exemplary results of using all three forecast algorithms to all forecastable events and the fourth validation exercise compares the overall forecasting results with the results of best practice third party reference tables. Table 6-1 lists the source data used from the U.S. DoD SAR for the C-17A Globemaster III:

Table 6-1: Source data U.S. DoD Air Force C-17A Globemaster III

Reporting Period	Baseline Year	Quantity (USD$M)	Schedule (USD$M)	Engineering (USD$M)	Estimating (USD$M)	Other (USD$M)	Support (USD$M)	Net Change Sum (USD$M)
1986	1981	0	172	124	*356*	0	1,053	1,705
1987	1981	0	187	138	*195*	0	960	1,480
1988	1981	0	187	222	1,102	0	900	2,411
1989	1981	0	187	222	1,792	0	850	3,051
1990	1981	*4,778*	0	11	2,047	0	*913*	7,749
1991	1981	*4,778*	0	11	2,380	0	*745*	7,914
1992	1981	*4,778*	0	56	4,248	0	*507*	9,589
1993	1981	*10,355*	169	32	2,091	191	*2007*	14,845
1994	1981	*10,355*	169	32	1,854	191	*1,769*	14,370
1995	1996	*8,928*	641	100	7,610	342	*623*	18,244
1996	1996	*8,928*	641	130	7,613	342	*926*	18,580
1997	1996	*8,928*	641	130	7,774	342	*728*	18,543
1998	1996	*7,360*	725	163	9,505	342	*511*	18,606
1999	1996	*7,360*	725	250	9,605	342	*552*	18,834
2000	1996	*7,360*	725	250	9,605	342	*552*	18,834
2001	1996	*2,512*	1,016	267	11,601	411	4,118	19,925
2002	1996	*2,512*	983	315	13,177	411	4,354	21,752
2003	1996	*2,512*	983	372	13,074	411	4,007	21,359
2004	1996	*2,512*	983	372	14,028	411	1,980	20,286
2005	1996	*2,512*	983	372	14,081	411	2,024	20,383
2006	1996	*825*	1,418	372	13,678	411	2,483	19,187
2007	1996	*825*	1,418	372	13,640	411	2,397	19,063
2008	1996	*825*	1,418	372	13,640	411	2,397	19,063
2009	1996	2,550	2,047	402	14,948	446	2,726	23,119

6.2.2.1 Exemplary Forecast 2002 / 2003

Using the integrated polar force field forecasting method presented in Table 6-2 shows the results of using cost variance data from 2002 for forecasting the cost variance in 2003.

Table 6-2: Source and forecast data U.S. DoD Air Force C-17A Globemaster III 2002 / 2003

Year	Results of Mode Algorithm (USD$M)	Results of Upper Limit Algorithm (USD$M)	Results of of Lower Limit Algorithm (USD$M)	Actual Cost Variance (USD$M)	Forecast Error (Actual Cost Variance to Results of Mode Algorithm (USD$M)
2002	20,042	34,869	19,981	21,752	1,710
2003	21,869	38,535	21,810	21,359	-510

Table 6-3 then shows the details of the forecast and accuracy achieved. Cells for the forecast results are shaded green if they are greater than 75% and less than 125%. They are shaded yellow if they are between 50% and 75% or between 125% and 150%. They are shaded red if they are below 50% or above 150%. These ranges were set for exemplary purposes and guided by relevant input related to thresholds common in practice received in a series of semi-structured interviews. Green shading suggests that an estimate could be considered as “good enough” in respect to the actual future cost. Yellow shading suggests that an estimate is not accurate enough for decision making and red shading suggests that the actual cost represents a totally inacceptable “cost blowout” in respect to the estimate that would threaten the future of the project as a whole. Note that if the actual value at t=2 is "0" then the value is set to "1" for calculation purposes in order to avoid division by "0" error. This will however result in a significant forecast deviation indicated by red shading. Numbers in cursive font are figures that decrease cost variance. Numbers in normal font are figures that increase cost variance.

Table 6-3: Cost estimate uncertainty forecast results – U.S. DoD Air Force C-17A Globemaster III 2002 / 2003

Col./ Line	A	B	C	D	E	F	G	H	I	J
		Δ CV due to Δ in Quantity (USD $M)	Δ CV due to Δ in Schedule (USD $M)	Δ CV due to Δ in Engineering (USD $M)	Δ CV due to Δ in Estimating (USD $M)	Δ CV due to Δ in Other (USD $M)	Δ CV due to Δ in Support (USD $M)	Avg. Δ CV due to Δ in all CV Dimensions (USD $M)	Std. Deviation (STDEV.P / USD$M)	Total Δ CV (USD $M)
1	Actual Value at t=1	*2,512*	983	315	13,177	411	4,354	4,118	4,496	21,752
2	Actual Value at t=2	*2,512*	983	372	13,074	411	*4,007*	3,560	4,445	21,359
3	Mode Forecast	2,525	988	317	13,248	413	4,377	3,645	4,520	21,869
4	Upper Limit Forecast	4,105	1,464	447	24,598	581	7,340	6,422	8,475	38,535
5	Lower Limit Forecast	2,523	991	322	13,191	418	4,365	3,635	4,498	21,810
6	Average Forecast Value	3,051	1,148	362	17,012	471	5,361	4,567	5,831	27,405
7	Mode Forecast Accuracy	101%	101%	85%	101%	101%	109%	100%	7%	102%
8	Upper Limit Forecast Accuracy	163%	149%	120%	188%	141%	183%	158%	24%	55%
9	Lower Limit Forecast Accuracy	100%	101%	87%	101%	102%	109%	100%	7%	98%

The data for lines 1 and 2 is drawn directly from the relevant U.S. DoD SAR for 2002 and 2003 whereby the averages and sums in columns H, I and J are calculated independently. Lines 3, 4 and 5 represent the values of the three different forecasting methods and line 6 presents the average forecast value across all three forecasting methods. Lines 7, 8 and 9 represent the accuracy of the forecasts made in lines 3, 4 and

5. Line 9 presents the average accuracy across all three forecast accuracies. The forecast results can be described as follows in an exemplary manner:

- Based on the average forecast accuracy (line 10) the cost variance factor that demonstrated the greatest standard deviation (uncertainty) were “Support” (134%) followed by “Estimating” (130%), “Quantity” (121%), “Schedule” (117%), “Other” (115%) and “Engineering” (97%).

- Important to note is that the STDEV.P function in Microsoft ® Excel assumes that the analysed data represents the complete sample population.

6.2.2.2 Full Case Study Data

The full case study data for the U.S. DoD Air Force C-17A Globemaster III is shown in Table 6-4:

Table 6-4: Case study data – U.S. DoD Air Force C-17A Globemaster III - all forecastable events – absolute cost variance values

Year of Baseline	Year of Reported Cost Variance	Δ CV due to Δ in Quantity (USD$M)	Δ CV due to Δ in Schedule (USD$M)	Δ CV due to Δ in Engin-eering (USD$M)	Δ CV due to Δ in Estima-ting (USD$M)	Δ CV due to Δ in Other (USD$M)	Δ CV due to Δ in Support (USD$M)	Actual Total CV (USD$M)
1981	1986	0	172	124	*356*	0	1,053	1,705
1981	1987	0	187	138	*195*	0	960	1,480
1981	1988	0	187	222	1,102	0	900	2,411
1981	1989	0	187	222	1,792	0	850	3,051
1981	1990	*4,778*	0	11	2,047	0	*913*	7,749
1981	1991	*4,778*	0	11	2,380	0	*745*	7,914
1981	1992	*4,778*	0	56	4,248	0	*507*	9,589
1981	1993	*10,355*	169	32	2,091	191	*2,007*	14,845
1981	1994	*10,355*	169	32	1,854	191	*1,769*	14,370
1996	1995	*8,928*	641	100	7,610	342	*623*	18,244
1996	1996	*8,928*	641	130	7,613	342	*926*	18,580
1996	1997	*8,928*	641	130	7,774	342	*728*	18,543
1996	1998	*7,360*	725	163	9,505	342	*511*	18,606
1996	1999	*7,360*	725	250	9,605	342	*552*	18,834
1996	2000	*7,360*	725	250	9,605	342	*552*	18,834
1996	2001	*2,512*	1,016	267	11,601	411	4,118	19,925
1996	2002	*2,512*	983	315	13,177	411	4,354	21,752
1996	2003	*2,512*	983	372	13,074	411	4,007	21,359
1996	2004	*2,512*	983	372	14,028	411	1,980	20,286
1996	2005	*2,512*	983	372	14,081	411	2,024	20,383
1996	2006	*825*	1,418	372	13,678	411	2,483	19,187
1996	2007	*825*	1,418	372	13,640	411	2,397	19,063
1996	2008	*825*	1,418	372	13,640	411	2,397	19,063
1996	2009	2,550	2,047	402	14,948	446	2,726	23,119

6.2.2.3 Exemplary Mode Forecast All Forecastable Events

The presented method was then applied to each available data set for the U.S. DoD Air Force C-17A Globemaster III using each forecasting method. Table 6-5 summarises the forecasting results using the mode algorithm:

Table 6-5: Mode forecasting accuracy U.S. DoD Air Force C-17A Globemaster III

Year	Δ CV due to Δ in Quantity	Δ CV due to Δ in Schedule	Δ CV due to Δ in Engineering	Δ CV due to Δ in Estimating	Δ CV due to Δ in Other	Δ CV due to Δ in Support	Avg. Δ CV due to Δ in All Dim.	Standard Deviation (STDEV.P)	Total CV
1986/1987	0%	98%	96%	195%	0%	117%	84%	68%	123%
1987/1988	0%	108%	67%	19%	0%	115%	52%	48%	66%
1988/1989	0%	105%	105%	64%	0%	111%	64%	48%	83%
1989/1990	0%	19,415%	2,095%	91%	0%	97%	3,616%	7,105%	41%
1990/1991	102%	0%	102%	87%	0%	124%	69%	50%	99%
1991/1992	101%	0%	20%	57%	0%	149%	55%	55%	84%
1992/1993	47%	0%	177%	206%	0%	26%	76%	84%	65%
1993/1994	101%	101%	101%	114%	101%	114%	105%	6%	104%
1994/1995	117%	27%	32%	25%	56%	286%	90%	93%	79%
1995/1996	101%	101%	77%	101%	101%	68%	91%	14%	99%
1996/1997	101%	101%	101%	99%	101%	128%	105%	10%	101%
1997/1998	122%	89%	80%	82%	101%	143%	103%	23%	100%
1998/1999	101%	101%	66%	100%	101%	93%	93%	13%	99%
1999/2000	101%	101%	101%	101%	101%	101%	101%	0%	101%
2000/2001	295%	72%	94%	83%	84%	13%	107%	88%	95%
2001/2002	101%	104%	85%	89%	101%	95%	96%	7%	92%
2002/2003	101%	101%	85%	101%	101%	109%	100%	7%	102%
2003/2004	101%	101%	101%	94%	101%	203%	117%	39%	106%
2004/2005	101%	101%	101%	100%	101%	98%	100%	1%	100%
2005/2006	306%	70%	101%	104%	101%	82%	127%	81%	107%
2006/2007	101%	101%	101%	101%	101%	104%	101%	1%	101%
2007/2008	101%	101%	101%	101%	101%	101%	101%	0%	101%
2008/2009	33%	70%	93%	92%	93%	88%	78%	22%	83%
	🟩	🟥	🟥	🟩	🟨	🟩	🟥		🟩
STDEV.P	75%	3,943%	410%	40%	45%	53%	719%	1,442%	17%
Ranking STDEV.P	5	1	2	6	3	4			

In respect to forecasting total cost variance the most inaccurate forecast (outlier) was made for 1990 using the cost variance data from 1989 (3,616%). In that forecast significant inaccuracies are found in the forecasts of four of the six cost variance factors. Investigation of forecasts with significant inaccuracy (i.e. red shaded cells) identified improperly forecast reductions of cost to be based on cost variance values of zero US$M as the most evident cause for inaccuracy.

6.2.2.4 Exemplary Mode, Upper and Lower Limit Forecast All Forecastable Events

The results of applying all forecasting methods to the U.S. DoD Air Force C-17A Globemaster III data with an emphasis on total cost variance and its growth rate for actual source and generated forecast data is shown in Table 6-6. The growth rate factor calculated is then the basis of the later comparison to independent third party reference tables.

Table 6-6: Forecasting accuracy and comparison for U.S. DoD Air Force C-17A Globemaster III

Y	Mode Forecast	Upper Limit Forecast	Lower Limit Forecast	Act.	FE#1	AGR-A	AGR-R	FGR#1 -A	FGR#1-R	CGF-AFA-BY$P-MSC	CGF-AA-BY$P-MSC
1987	1,822	3,034	1,763	1,480	-342	N/A	N/A	N/A	N/A	N/A	N/A
1988	1,597	2,642	1,540	2,411	814	931	63%	-225	-72%	2,604	2,700
1989	2,528	4,152	2,463	3,051	523	640	27%	931	35%	3,295	3,417
1990	3,168	5,408	3,113	7,749	4,581	4,698	154%	640	15%	8,369	8,679
1991	7,866	14,161	7,808	7,914	106	165	2%	4,640	86%	8,547	8,864
1992	8,031	14,647	7,969	9,589	1,558	1,675	21%	223	3%	10,356	10,740
1993	9,706	18,148	9,627	14,845	5,139	5,256	55%	1,675	11%	16,033	16,626
1994	14,962	27,050	14,918	14,370	-592	-475	-3%	5,256	29%	15,520	16,094
1995	14,487	26,367	14,445	18,244	3,757	3,874	27%	-475	-2%	19,704	20,433
1996	18,361	33,410	18,282	18,580	219	336	2%	3,874	15%	20,066	20,810
1997	18,697	33,540	18,618	18,543	-154	-37	0%	336	1%	20,026	20,768
1998	18,660	33,768	18,580	18,606	-54	63	0%	-37	0%	20,094	20,839
1999	18,723	34,174	18,645	18,834	111	228	1%	63	0%	20,341	21,094
2000	18,951	34,422	18,873	18,834	-117	0	0%	228	1%	20,341	21,094
2001	18,951	34,422	18,873	19,925	974	1,091	6%	0	0%	21,519	22,316
2002	20,042	34,869	19,981	21,752	1,710	1,827	9%	1,091	3%	23,492	24,362
2003	21,869	38,535	21,810	21,359	-510	-393	-2%	1,827	5%	23,068	23,922
2004	21,476	37,783	21,418	20,286	-1,190	-1,073	-5%	-393	-1%	21,909	22,720
2005	20,403	36,776	20,352	20,383	-20	97	0%	-1,073	-3%	22,014	22,829
2006	20,500	36,944	20,449	19,187	-1,313	-1,196	-6%	97	0%	20,722	21,489
2007	19,304	34,755	19,257	19,063	-241	-124	-1%	-1,196	-3%	20,588	21,351
2008	19,180	34,540	19,134	19,063	-117	0	0%	-124	0%	20,588	21,351
2009	19,180	34,540	19,134	23,119	3,939	4,056	21%	0	0%	24,969	25,893
STDEV.P	6,870	12,364	6,870	6,501	1,781	1,805	35%	1,683	20%	637,13	660,73
Rel. STDEV.P	47%	47%	47%	42%	218%	183%	207%	213%	230%	36%	36%

Abbreviations used are: Y:Year/ /Act.:Actual/FE#1:Forecast Error (Actual to Forecast Method #1)/AGR-A:Actual Growth Rate Absolute/AGR-R:Actual Growth Rate Relative/FGR#1-A: Forecast Growth Rate Method #1 Absolute/FGR#1-R:Forecast Growth Rate Method #1 Relative/CGF-AFA-BY$P-MSC:Cost Growth Factor Air Force Aircraft BY$ Procurement MS C (1.08)/CGF-AA-BY$P-MSC:Cost Growth Factor All Aircraft BY$ Procurement MS C (1.12)

The accuracy of the forecasting results is visualised in Figure 6-1. Starting in 1992 the upper limit algorithm consistently generates the highest forecast value, the lower limit algorithm generates the lowest forecast value and mode algorithm generates a value between the upper limit and the lower limit being closest to the value of the results generated by the lower limit algorithm and the actual value. Thus of 22 forecasts the forecasting algorithms were correct 18 times (82%).

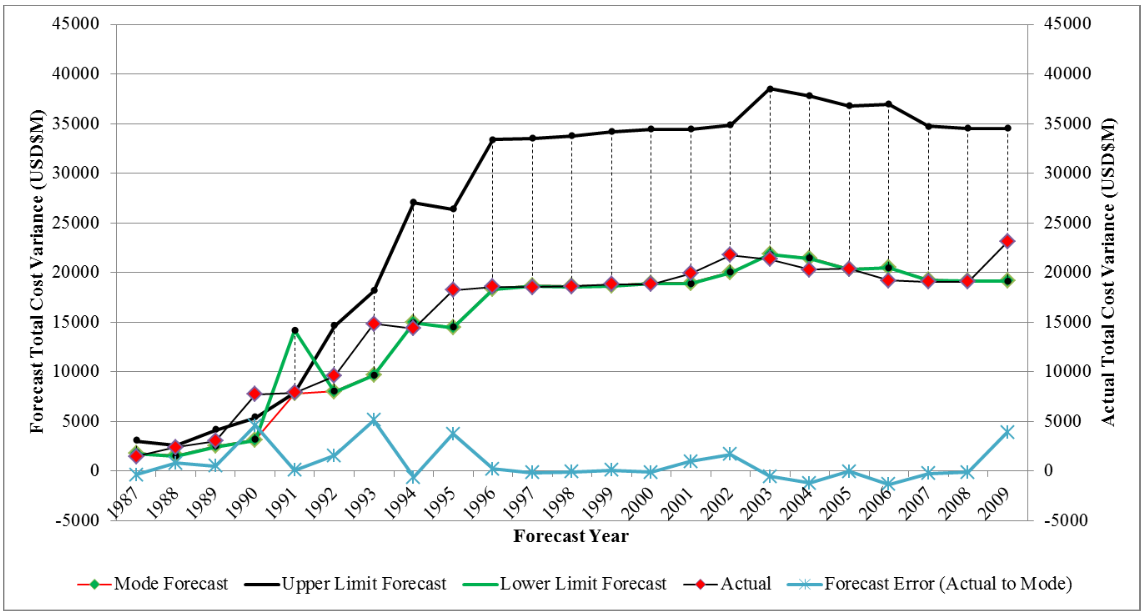

Figure 6-1: Accuracy of forecasting for U.S. DoD Air Force C-17A Globemaster III using polar force field forecasting

6.2.2.5 Comparison of Forecasting Results with Independent Reference Tables

The independent third party reference tables used are contained in the Joint Agency Cost Schedule Risk and Uncertainty Hand Book (U.S. Naval Center for Cost Analysis, 2014). The average actual total cost variance growth rate identified by this investigation and indicated in Table 6-6 is 17% (1.17) and the average forecast total cost variance growth rate is 9% (1.09). Based on the independent third party reference tables, the mean cost growth factor for procurement estimates at milestone C for Air Force Aircraft is a factor of 1.08 and for all Forces a factor of 1.12.

Figure 6-2 visualises the comparative growth rates. The first case study demonstrates the forecasting accuracy achieved by the presented technique using the U.S. DoD Air Force C-17A Globemaster III. Forecast results and their accuracies are presented for each individual forecastable event and the results of the three forecasting methods contrasted. The results of the most accurate forecast method (mode algorithm) for all forecastable events are then compared to results based on applying independent third party reference tables and their suggested cost growth rates to the same data. The results of the case study verification and validation effort indicate that the presented forecasting technique is, without reliance on historical data, able to forecast cost variance at t=2 using only cost variance data from t=1 with a greater degree of accuracy than the independent third party reference tables.

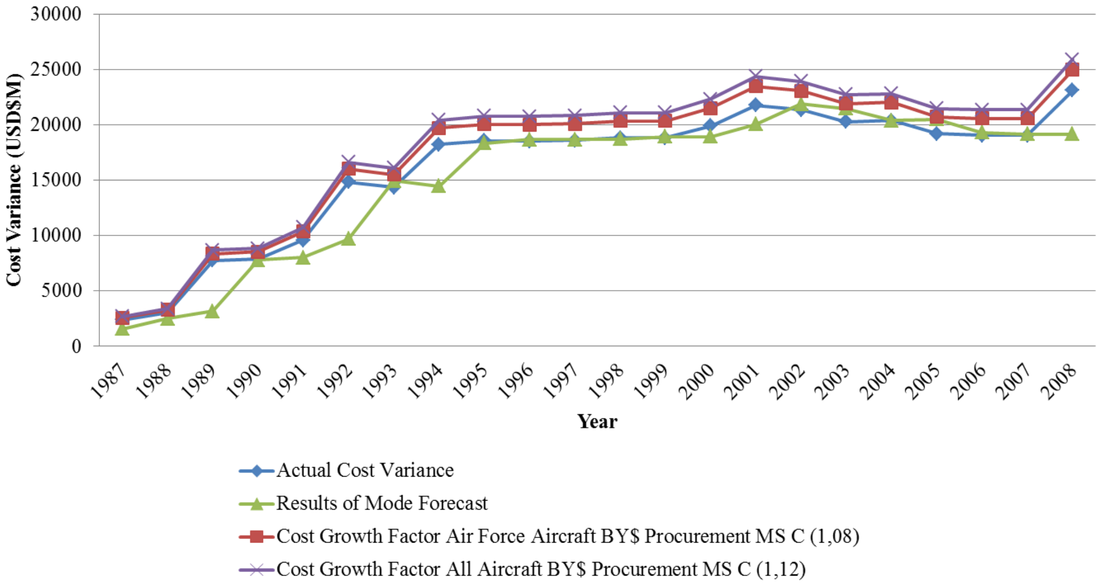

Figure 6-2: Comparative cost growth overview for U.S. DoD Air Force C-17A Globemaster III

In 11/22 (50%) of the forecasts, the mode forecast is lower than the actual cost while in 9/22 (41%) the mode forecast is equal to the actual cost whereby the difference to the reference values remains relatively constant in 18/22 (82%) of the forecasts.

6.2.3 Case Study 2: U.S. Air Force - All Projects

The second case study applies the presented method individually to all reported projects for the U.S. DoD Air Force from 1986 to 2013 and then aggregates the results achieved.

6.2.3.1 Full Case Study Data

The full case study data is shown in Table 6-7:

Table 6-7: Case study data – U.S. DoD Air Force - all forecastable events – absolute cost variance values

Year of Reported Cost Variance	Δ CV due to Δ in Quantity (USD$M)	Δ CV due to Δ in Schedule (USD$M)	Δ CV due to Δ in Engineering (USD$M)	Δ CV due to Δ in Estimating (USD$M)	Δ CV due to Δ in Other (USD$M)	Δ CV due to Δ in Support (USD$M)	Actual Total CV (USD$M)
1986	17,128	1,974	6,735	3,094	772	5,231	34,934
1987	9,607	1,596	7,423	2,821	8,197	7,921	37,565
1988	14,007	1,199	9,881	964	7,920	7,581	41,552
1989	11,356	1,042	6,683	3,351	5,991	8,970	37,393
1990	5,683	1,265	4,283	8,046	5,865	10,654	35,796
1991	8,300	1,420	3,249	9,658	7,574	8,851	39,052
1992	3,837	1,698	3,819	14,971	28	5,321	29,674
1993	14,241	5,134	4,289	9,699	219	3,719	37,301
1994	19,367	3,694	2,885	8,980	231	2,832	37,989
1995	14,548	5,096	534	18,480	342	1,860	40,860
1996	17,028	5,532	998	19,885	342	841	44,626
1997	23,010	5,040	360	17,748	342	442	46,942
1998	20,600	5,933	180	20,324	342	721	48,100
1999	20,555	6,801	1031	21,261	342	988	50,978
2000	20,555	6,813	1031	21,267	349	986	51,001
2001	5,447	6,149	3,425	31,067	430	6005	52,523
2002	11,712	3,519	7,930	32,693	442	5,674	61,970
2003	11,703	3,700	8,358	41,454	423	4,636	70,274
2004	28,425	5,409	7,561	45,814	423	1,095	88,727
2005	3,432	3,083	5,205	40,276	423	5,583	58,002
2006	2,668	3,395	5,038	45,893	423	7,669	65,086
2007	4,807	3,865	5,158	30,428	411	7,451	52,120
2008	2,058	4,650	5,227	31,909	411	7,199	51,454
2009	15,899	4,813	6,706	37,576	487	10,116	75,597
2010	20,092	3,201	6,310	21,335	41	7,857	58,836
2011	11,642	3,294	4,511	14,397	41	6,267	40,152
2012	20,031	1,208	6,056	50,095	57	5,908	83,355
2013	18,845	2,233	4,897	448	0	6,293	32,716

6.2.3.2 Exemplary Mode Forecast All Forecastable Events

The presented method was then applied to each available data set for the U.S. DoD Air Force using each forecasting method in order to determine the influencers of forecasting inaccuracy. Table 6-8 summarises the forecasting results for the U.S. DoD Air Force using the mode algorithm to illustrate the approach taken.

Table 6-8: Mode forecasting accuracy U.S. DoD Air Force - all projects (1986-2013)

Year	Δ CV due to Δ in Quantity	Δ CV due to Δ in Schedule	Δ CV due to Δ in Engineering	Δ CV due to Δ in Estimating	Δ CV due to Δ in Other	Δ CV due to Δ in Support	Avg. Δ CV due to Δ in All Dim.	Standard Deviation (STDEV.P)	Total CV
1986/1987	179%	124%	91%	110%	9%	66%	97%	52%	93%
1987/1988	69%	134%	75%	294%	104%	105%	130%	76%	91%
1988/1989	124%	115%	148%	29%	133%	85%	106%	39%	111%
1989/1990	200%	83%	157%	42%	102%	84%	111%	52%	105%
1990/1991	69%	89%	132%	84%	78%	121%	95%	23%	92%
1991/1992	217%	84%	85%	65%	27,131%	167%	4,625%	10065%	132%
1992/1993	27%	33%	89%	155%	13%	144%	77%	57%	80%
1993/1994	74%	139%	149%	108%	95%	132%	116%	26%	98%
1994/1995	134%	73%	542%	49%	68%	153%	170%	171%	93%
1995/1996	86%	92%	54%	93%	100%	222%	108%	53%	92%
1996/1997	74%	110%	278%	112%	100%	191%	144%	70%	95%
1997/1998	112%	85%	200%	88%	100%	61%	108%	44%	98%
1998/1999	100%	87%	18%	96%	100%	73%	79%	29%	95%
1999/2000	100%	100%	100%	100%	98%	100%	100%	1%	100%
2000/2001	378%	111%	30%	69%	81%	16%	114%	122%	97%
2001/2002	47%	175%	43%	95%	98%	106%	94%	44%	85%
2002/2003	100%	95%	95%	79%	105%	123%	99%	13%	88%
2003/2004	41%	69%	111%	91%	100%	424%	139%	129%	79%
2004/2005	829%	176%	145%	114%	100%	20%	231%	272%	153%
2005/2006	129%	91%	104%	88%	100%	73%	97%	17%	89%
2006/2007	56%	88%	98%	151%	103%	103%	100%	28%	125%
2007/2008	234%	83%	99%	96%	100%	104%	119%	52%	102%
2008/2009	13%	97%	78%	85%	85%	71%	71%	27%	68%
2009/2010	79%	151%	106%	176%	1,190%	129%	305%	397%	129%
2010/2011	173%	97%	140%	148%	100%	126%	131%	27%	147%
2011/2012	58%	273%	75%	29%	72%	106%	102%	80%	48%
2012/2013	106%	54%	124%	11,198%	5,708%	94%	2,881%	4247%	255%
STDEV.P	155%	46%	97%	2,096%	5,171%	75%	981%	1891%	37%
Ranking STDEV.P	3	6	4	2	1	5			

In respect to forecasting total cost variance the most inaccurate forecast (outlier) was made for 1992 using the cost variance data from 1991 (4,625%). In that forecast, significant inaccuracies are found in the forecasts of three of the six cost variance factors. Investigation of the remaining forecasts with significant inaccuracy (i.e. red shaded cells) identified improperly forecast reductions of cost to be the most evident cause for inaccuracy or forecasts based on cost variance values of zero US$M.

6.2.3.3 Exemplary Mode, Upper and Lower Limit Forecast All Forecastable Events

The results of applying all forecasting methods to the U.S. DoD Air Force data with an emphasis on total cost variance and its growth rate for actual source and generated forecast data is shown in Table 6-9. The growth rate factor calculated is then the basis of the latter comparison to independent third party reference tables.

Table 6-9: Forecasting accuracy and comparison for U.S. DoD Air Force – all projects (1986 to 2013)

Y	Mode Forecast Algorithm	Upper Limit Forecast Algorithm	Lower Limit Forecast Algorithm	Act.	FE#1	AGR-A	AGR-R	FGR#1-A	FGR #1-R	CGF-AFA-BY$P-MSC	CGF-AD-BY$P-MSC
1987	35,051	55,718	34,989	37,565	2,514	N/A	N/A	N/A	N/A	N/A	N/A
1988	37,682	53,187	37,603	41,552	3,870	1,356	3%	2,631	8%	44,876	46,53
1989	41,669	62,132	41,594	37,393	-4,276	-8,146	-22%	8,945	17%	40,384	41,88
1990	37,510	54,172	37,435	35,796	-1,714	2,562	7%	-7,960	-13%	38,660	40,09
1991	35,913	52,435	35,840	39,052	3,139	4,853	12%	-1,737	-3%	42,176	43,73
1992	39,169	57,040	39,095	29,674	-9,495	-12,634	-43%	4,605	9%	32,048	33,23
1993	29,791	47,756	29,724	37,301	7,510	17,005	46%	-9,284	-16%	40,285	41,77
1994	37,418	57,251	37,343	37,989	571	-6,939	-18%	9,495	20%	41,028	42,54
1995	38,106	62,826	38,040	40,860	2,754	2,183	5%	5,576	10%	44,129	45,76
1996	40,977	69,544	40,896	44,626	3,649	895	2%	6,718	11%	48,196	49,98
1997	44,743	77,129	44,663	46,942	2,199	-1,450	-3%	7,585	11%	50,697	52,57
1998	47,059	83,747	46,985	48,100	1,041	-1,158	-2%	6,618	9%	51,948	53,87
1999	48,217	84,304	48,137	50,978	2,761	1,720	3%	556	1%	55,056	57,09
2000	51,095	87,307	51,015	51,001	-94	-2,855	-6%	3,003	4%	55,081	57,12
2001	51,118	87,330	51,038	52,523	1,405	1,499	3%	24	0%	56,725	58,82
2002	52,640	89,097	52,580	61,970	9,330	7,925	13%	1,767	2%	66,928	69,40
2003	62,087	102,055	62,022	70,274	8,187	-1,143	-2%	12,957	15%	75,896	78,70
2004	70,391	120,013	70,333	88,727	18,336	10,149	11%	17,959	18%	95,825	99,37
2005	88,844	155,843	88,773	58,002	-	-49,178	-85%	35,830	30%	62,642	64,96
2006	58,119	102,231	58,073	65,086	6,967	37,809	58%	-53,613	-34%	70,293	72,89
2007	65,203	115,349	65,157	52,120	-	-20,050	-38%	13,119	13%	56,290	58,37
2008	52,237	87,232	52,178	51,454	-783	12,300	24%	-28,118	-24%	55,570	57,62
2009	51,571	87,576	51,518	75,597	24,026	24,809	33%	345	0%	81,645	84,66
2010	75,714	122,893	75,643	58,836	-	-40,904	-70%	35,316	40%	63,543	65,89
2011	58,953	91,859	58,869	40,152	-	-1,923	-5%	-31,033	-25%	43,364	44,97
2012	40,269	60,755	40,186	83,355	43,086	6,1887	74%	-31,104	-34%	90,023	93,35
2013	83,472	148,185	83,410	32,716	-	-9,3842	-	87,431	144%	35,333	36,64
STD	28,530	14,989	14,986	15,078	16,716	27,739	65%	25,274	33%	16,350	16,95
Rel. ST-DEV	34%	29%	29%	30%	-8,408 %	-1,354%	-591 %	673%	405%	30%	30%

The accuracy of the forecasting results is visualised in Figure 6-3. In all 26 forecasts the upper limit algorithm generates the highest forecast and the lower limit algorithm generates the lowest forecast while 19/22 mode algorithm forecasts generate a value between the upper and lower forecast values.

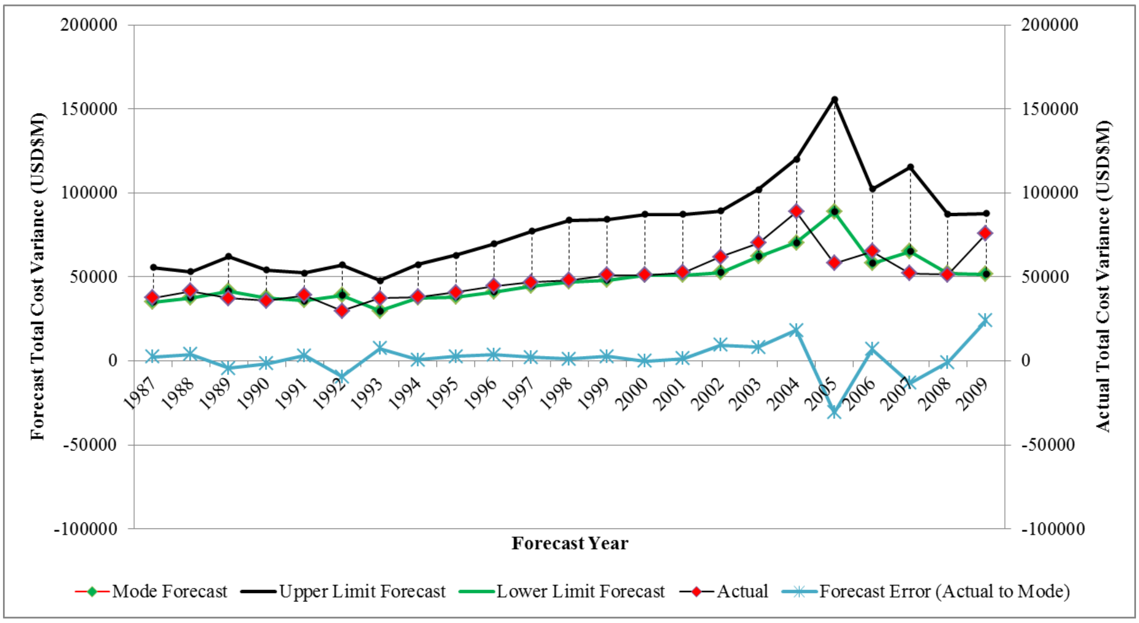

Figure 6-3: Accuracy of forecasting for U.S. DoD Air Force using polar force field forecasting

6.2.3.4 Comparison of Forecasting Results with Independent Reference Tables

The independent third party reference tables used are contained in the Joint Agency Cost Schedule Risk and Uncertainty Hand Book, specifically on p.58 Table 3-3 NCCA SAR Growth Factors: Since 1969 / Since 1980 / Since 1990 (Effective December 2011) whereby the “Since 1990” reference figures for “Mean Cost Growth Factor, Procurement Estimates at MS C” are used (U.S. Naval Center for Cost Analysis, 2014). The average actual total cost variance growth rate identified by this investigation and indicated in Table 6-9 is -11% (0.89) and the average forecast total cost variance growth rate is 8% (1.08). Based on the independent third party reference tables the mean cost growth factor for procurement estimates at milestone C for the United States Department of Defense Air Force overall is a factor of 1.29 and for all Forces a factor of 1.28. Both actual and forecast cost variance growth rates hence fall within a bandwidth given by third party independent research based on regression methods. Figure 6-4 visualises the comparative growth rates:

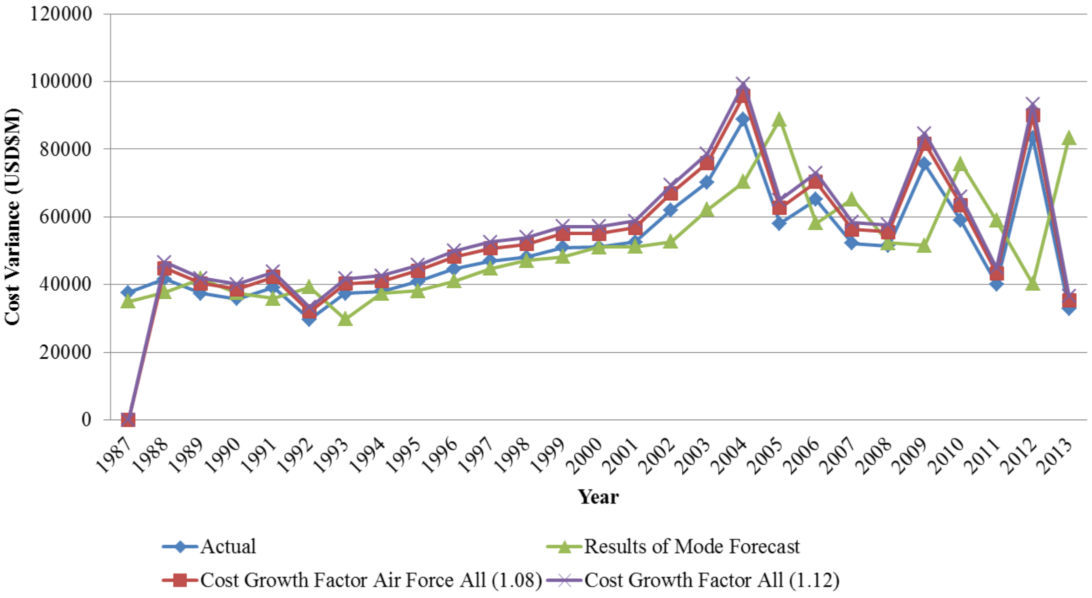

Figure 6-4: Comparative cost growth overview for U.S. DoD Air Force

The second case study demonstrates the forecasting accuracy achieved by the presented technique using the U.S. DoD Air Force. Forecast results and their accuracies are presented for each individual forecastable event within the chosen data boundaries, and the results of the three forecasting methods contrasted. The results of the most accurate mode forecast method for all forecastable events are then compared to results based on applying independent third party reference tables and their suggested cost growth rates to the same data.

The results of the case study verification and validation effort suggest that the presented forecasting technique is, without reliance on historical data, able to forecast cost variance at t=2 using only cost variance data from t=1 with a greater degree of accuracy than the independent third party reference tables.

6.2.4 Case Study 3: U.S. DoD – All Projects

The third case study applies the presented method individually to all reported projects for the U.S. DoD from 1990 to 2013 and then aggregates the results achieved.

6.2.4.1 Full Case Study Data

The full case study data is shown in Table 6-10:

Table 6-10: Case study data – U.S. DoD - all forecastable events – absolute cost variance values

Year of Reported Cost Variance	Δ CV due to Δ in Quantity (US$M)	Δ CV due to Δ in Schedule (US$M)	Δ CV due to Δ in Engin-eering (US$M)	Δ CV due to Δ in Estima-ting (US$M)	Δ CV due to Δ in Other (US$M)	Δ CV due to Δ in Support (US$M)	Actual Total CV (US$M)
1990	0	20	26	1,171	823	2	2,042
1991	3,432	8,797	12,564	22,750	9,554	11,277	68,374
1992	0	0	53	7,020	0	204	7,277
1993	283	40	256	15,991	0	97	16,667
1994	133	82	317	14,384	0	202	15,118
1995	1,436	425	528	1,179	581	256	4,405
1996	0	805	639	1,349	0	118	2,911
1997	0	550	982	1,100	0	148	2,780
1998	888	1,237	2,152	4,911	0	142	9,330
1999	1,533	1,505	2,229	10,461	0	620	16,348
2000	1,479	2,229	2,542	11,408	0	620	18,278
2001	0	3,096	5,452	11,118	8	0	19,674
2002	16,249	3,115	17,513	5,389	8	2,595	44,869
2003	16,249	11,216	22,867	20,652	8	2,735	73,727
2004	16,249	15,969	43,927	16,860	8	5,092	98,105
2005	16,646	16,211	42,545	21,083	8	7,894	104,387
2006	16,534	1,558	58,034	24,819	8	11,045	111,998
2007	21,793	15,581	58,083	36,134	8	746	132,345
2008	21,793	15,581	58,083	36,118	8	633	132,216
2009	20,109	15,806	52,230	52,768	8	6,288	147,209
2010	18,658	14,881	53,682	76,302	8	13,922	177,453
2011	31,741	15,652	54,105	100,307	8	10,148	211,961
2012	0	1,509	39,929	16,806	0	2,073	60,317
2013	0	1,328	41,025	13,191	0	2,701	58,245

6.2.4.2 Exemplary Mode Forecast All Forecastable Events

The presented method was then applied to each available data set for the U.S. DoD using each forecasting method in order to determine the influencers of forecasting inaccuracy. Table 6-11 summarises the forecasting results for the U.S. DOD using the mode algorithm in order to illustrate the approach taken.

Table 6-11: Mode forecasting accuracy U.S. DoD all projects (1990-2013)

Year	Δ CV due to Δ in Quantity	Δ CV Changes due to Schedule Δ	Δ CV Changes due to Eng. Δ	Δ Changes due to Δ in Estimating	Δ CV Changes due to Δ in Other	Δ CV Changes due to Support Δ	Avg. Δ CV due to All Dim. Δ	STDEV.P	Total CV
1990/1991	0%	0%	0%	5%	9%	0%	3%	4%	3%
1991/1992	343,786%	881,201%	23,746%	325%	957,030%	5,537%	368,604%	407,429%	941%
1992/1993	0%	0%	21%	45%	0%	214%	47%	76%	44%
1993/1994	214%	49%	81%	112%	0%	48%	84%	67%	111%
1994/1995	9%	19%	61%	1,229%	0%	80%	233%	446%	346%
1995/1996	147,403%	54%	85%	90%	596,39%	223%	34,582%	54,938%	155%
1996/1997	0%	152%	68%	128%	0%	83%	72%	58%	109%
1997/1998	0%	46%	48%	23%	0%	109%	38%	37%	31%
1998/1999	59%	83%	98%	48%	0%	23%	52%	33%	58%
1999/2000	104%	68%	88%	92%	0%	101%	76%	36%	90%
2000/2001	148,844%	72%	47%	103%	0%	62,396%	35,244%	55,670%	93%
2001/2002	0%	100%	31%	208%	101%	0%	73%	73%	44%
2002/2003	100%	28%	77%	26%	100%	95%	71%	32%	61%
2003/2004	100%	70%	52%	123%	100%	54%	83%	26%	75%
2004/2005	98%	99%	103%	80%	100%	65%	91%	14%	94%
2005/2006	101%	1,042%	73%	85%	100%	72%	245%	356%	93%
2006/2007	76%	10%	100%	69%	100%	1,482%	306%	527%	85%
2007/2008	100%	100%	100%	100%	100%	118%	103%	7%	100%
2008/2009	108%	99%	111%	69%	100%	10%	83%	35%	90%
2009/2010	108%	106%	97%	69%	100%	45%	88%	23%	83%
2010/2011	59%	95%	99%	76%	100%	137%	94%	24%	84%
2011/2012	3,175,847%	1,038%	136%	5,97%	800%	490%	529,818%	1,183,340%	352%
2012/2013	0%	114%	98%	1,28%	0%	77%	69%	51%	104%
STDEV.P	646,549%	179,673%	4,827%	257%	194,977%	12,691%	128,214%	226,788%	189%
Ranking STDEV.P	1	3	5	6	2	4			

In respect to forecasting total cost variance the most inaccurate forecast (outlier) was made for 1992 using the cost variance data from 1991 (368,604%). In that forecast, significant inaccuracies are found in the forecasts of all six cost variance factors. Investigation of the remaining forecasts with significant inaccuracy (i.e. red shaded cells) identified improperly forecast reductions of cost to be the most evident cause for inaccuracy or forecasts based on cost variance values of zero US$M.

6.2.4.3 Exemplary Mode, Upper and Lower Limit Forecast All Forecastable Events

The results of applying all forecasting methods to the U.S. DoD data with an emphasis on total cost variance and its growth rate for actual source and generated forecast data is shown in Table 6-12. The growth rate factor calculated is then the basis of the later comparison to independent third party reference tables.

Table 6-12: Forecasting accuracy and comparison U.S. DoD all projects (1990 to 2013)

Y	Mode Forecast Algorithm	Upper Limit Forecast Algorithm	Lower Limit Forecase Algorithm	Act.	FE#1	AGR-A	AGR-R	FGR#1-A	FGR#1-R	CGF-DA-BY$P-MSC	CGF-DA-BY$P-MSC
1991	2,159	4,013	2,131	6,8374	66,215	N/A	N/A	N/A	N/A	N/A	N/A
1992	68,491	98,388	68,419	7,277	-61,214	-127,429	-1,751%	94,375	2,352%	8,150	8,150
1993	7,394	14,449	7,374	16,667	9,273	70,487	422%	-83,940	-85%	18,667	18,667
1994	16,784	32,932	16,763	15,118	-1,666	-10,939	-72%	18,484	128%	16,932	16,932
1995	15,235	29,703	15,214	4,405	-10,830	-9,164	-208%	-3,229	-10%	4,933	4,933
1996	4,522	6,365	4,438	2,911	-1,611	9,219	317%	-23,338	-79%	3,260	3,260
1997	3,028	5,209	2,979	2,780	-248	1,363	49%	-1,156	-18%	3,113	3,113
1998	2,897	4,881	2,851	9,330	6,433	6,681	72%	-328	-6%	10,449	10,449
1999	9,447	16,323	9,397	16,348	6,901	468	3%	11,442	234%	18,309	18,309
2000	16,465	28,994	16,417	18,278	1,813	-5,088	-28%	12,671	78%	20,471	20,471
2001	18,395	32,496	18,346	19,674	1,279	-534	-3%	3,502	12%	22,034	22,034
2002	19,791	37,250	19,754	44,869	25,078	23,799	53%	4,754	15%	50,253	50,253
2003	44,986	75,168	44,923	73,727	28,741	3,663	5%	37,918	102%	82,574	82,574
2004	73,844	115,451	73,779	98,105	24,261	-4,480	-5%	40,283	54%	109,877	109,877
2005	98,222	158,867	98,166	104,387	6,165	-18,096	-17%	43,417	38%	116,913	116,913
2006	104,504	163,400	104,443	111,998	7,494	1,329	1%	4532	3%	125,437	125,437
2007	112,115	181,245	112,057	132,345	20,230	12,736	10%	17,845	11%	148,226	148,226
2008	132,462	218,784	132,409	132,216	-246	-20,476	-15%	37,538	21%	148,081	148,081
2009	132,333	218,760	132,280	147,209	14,876	15,122	10%	-23	0%	164,874	164,874
2010	147,326	239,298	147,269	177,453	30,127	15,251	9%	20,537	9%	198,747	198,747
2011	177,570	289,730	177,513	211,961	34,391	4,264	2%	50,432	21%	237,396	237,396
2012	212,078	350,864	212,020	60,317	-151,761	-186,152	-309%	61,134	21%	67,555	67,555
2013	60,434	115,211	60,403	58,245	-2,189	149,572	257%	-235,654	-67%	65,234	65,234
STDEV.P	62,078	100,911	62,072	60,671	39,675	60,441	4	62,357	5	69,478	69,478
Rel. ST-DEV.P	96%	95%	97%	91%	1,705%	-1,944%	-732%	1,234%	380%	93%	93%

The accuracy of the forecasting results is visualised in Figure 6-5. In 17/22 (70%) cases, the upper limit algorithm generates the highest forecast value, the lower limit algorithm generates the lowest forecast value 15/22 (68%) of the time and the mode algorithm generates a value between upper and lower limit 16/22 (73%) of the time while typically being closest to the value of the lower limit.

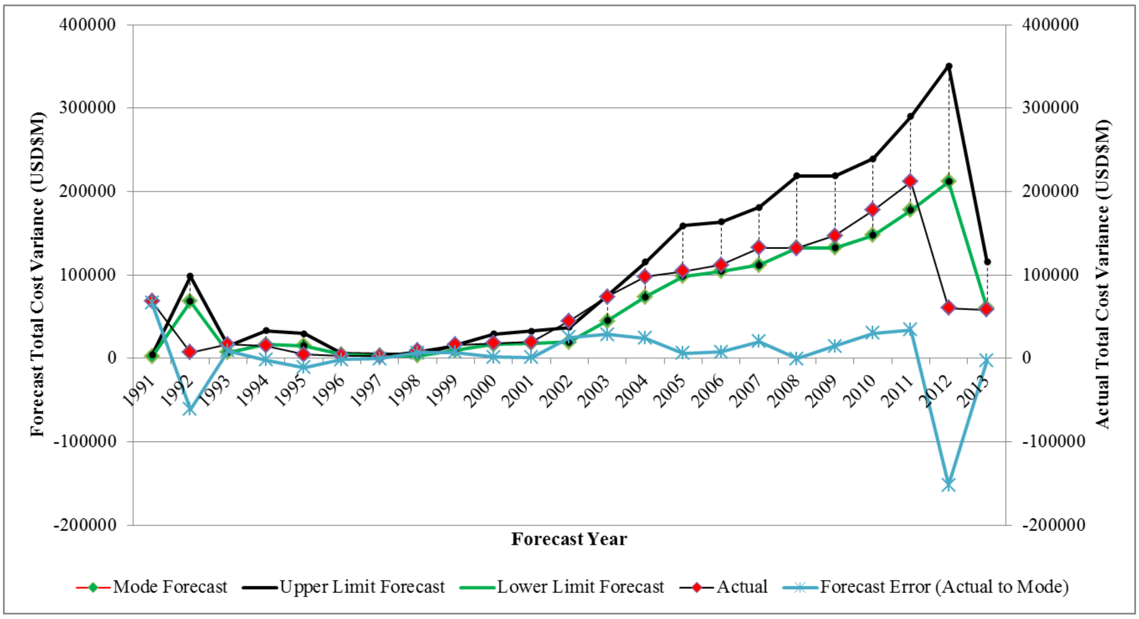

Figure 6-5: Accuracy of forecasting for U.S. DoD using polar force field forecasting

6.2.4.4 Comparison of Forecasting Results with Independent Reference Tables

The independent third party reference tables used are contained in the Joint Agency Cost Schedule Risk and Uncertainty Hand Book, specifically on p.58 Table 3-3 NCCA SAR Growth Factors: Since 1969 / Since 1980 / Since 1990 (Effective December 2011) whereby the “Since 1990” reference figures for “Mean Cost Growth Factor, Procurement Estimates at MS C” are used (U.S. Naval Center for Cost Analysis, 2014).

The average actual total cost variance growth rate identified by this investigation and indicated in Table 6-12 is 1% (1.01) and the average forecast total cost variance growth rate is 1% (1.01). Based on the independent third party reference tables the mean cost growth factor for procurement estimates at milestone C for the U.S. DoD is a factor of

1.28 and for all Forces a factor of 1.28. Both actual and forecast cost variance growth rates hence fall within a bandwidth given by third party independent research based on regression methods. Figure 6-6 visualises the comparative growth rates:

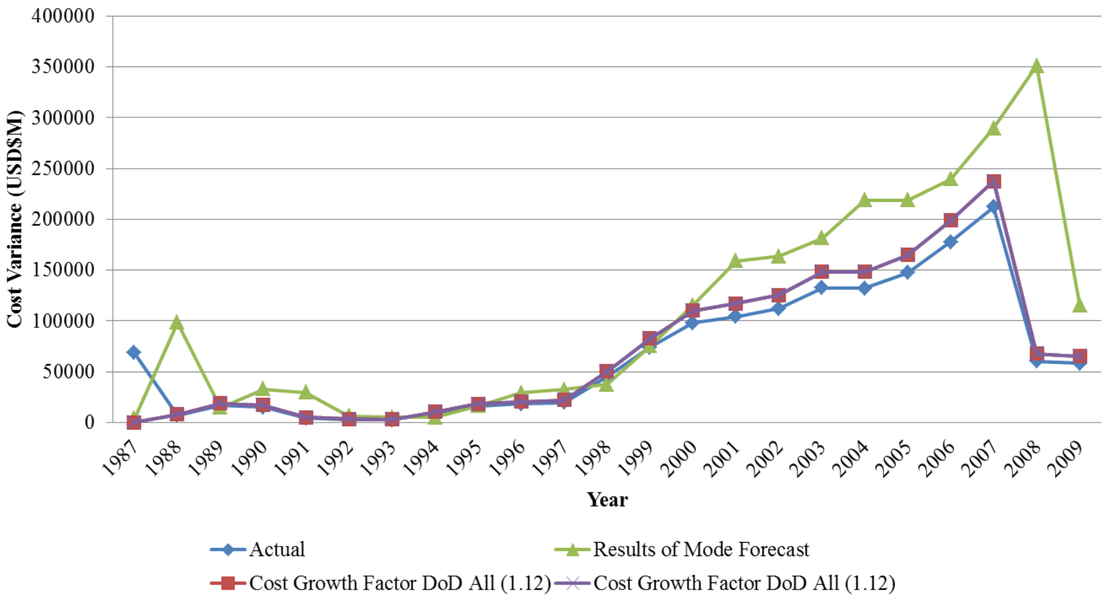

Figure 6-6: Comparative cost growth overview for U.S. DoD

The third case study demonstrates the forecasting accuracy achieved by the presented technique using the U.S. DoD. Forecast results and their accuracies are presented for each individual forecastable event within the chosen data boundaries and the results of the three forecasting methods contrasted. The results of the most accurate forecast method (mode algorithm) for all forecastable events are then compared to results based on applying independent third party reference tables and their suggested cost growth rates to the same data.

The results of the case study verification and validation effort suggest that the presented forecasting technique is, without reliance on historical data, able to forecast cost variance at t=2 using only cost variance data from t=1 with a greater degree of accuracy than the independent third party reference tables.

6.2.5 Explaining Uncertainty with a Dependency Model

In line with the reflections offered by Rothwell (2004), the metric of standard deviation was examined as a primary indicator of forecasting uncertainty on the level of individual cost variance dimensions whereby this, by default, sets the foundation for improved contingency setting by business decision makers. The cost variance factors were ranked from highest to lowest based on their standard deviation. A standard deviation of zero percent was given the highest ranking since this corresponded to a divide by zero error on cost variance. The lowest possible ranking would be given by the lowest standard deviation for a non-zero cost variance value. The "uncertainty" rankings based on standard deviation of the historical data sets for the case studies are consolidated in Table 6-13:

Table 6-13: Case study uncertainty rankings based on standard deviation of historical data

Case Study	Quantity	Schedule	Engin-eering	Estimating	Other	Support
C-17A Globemaster III	5	1	2	6	3	4
Air Force	3	6	4	2	1	5
Department of Defense	1	3	5	6	2	4
Average	3	3.33	3.66	5	2	4.33

The potential reasons for the ranking was examined using the dependency model created by Schwabe et al. (2016a) based upon an analysis of the same data sets. Figure 6-7 represents an integration of that dependency model with the technique presented in this investigation:

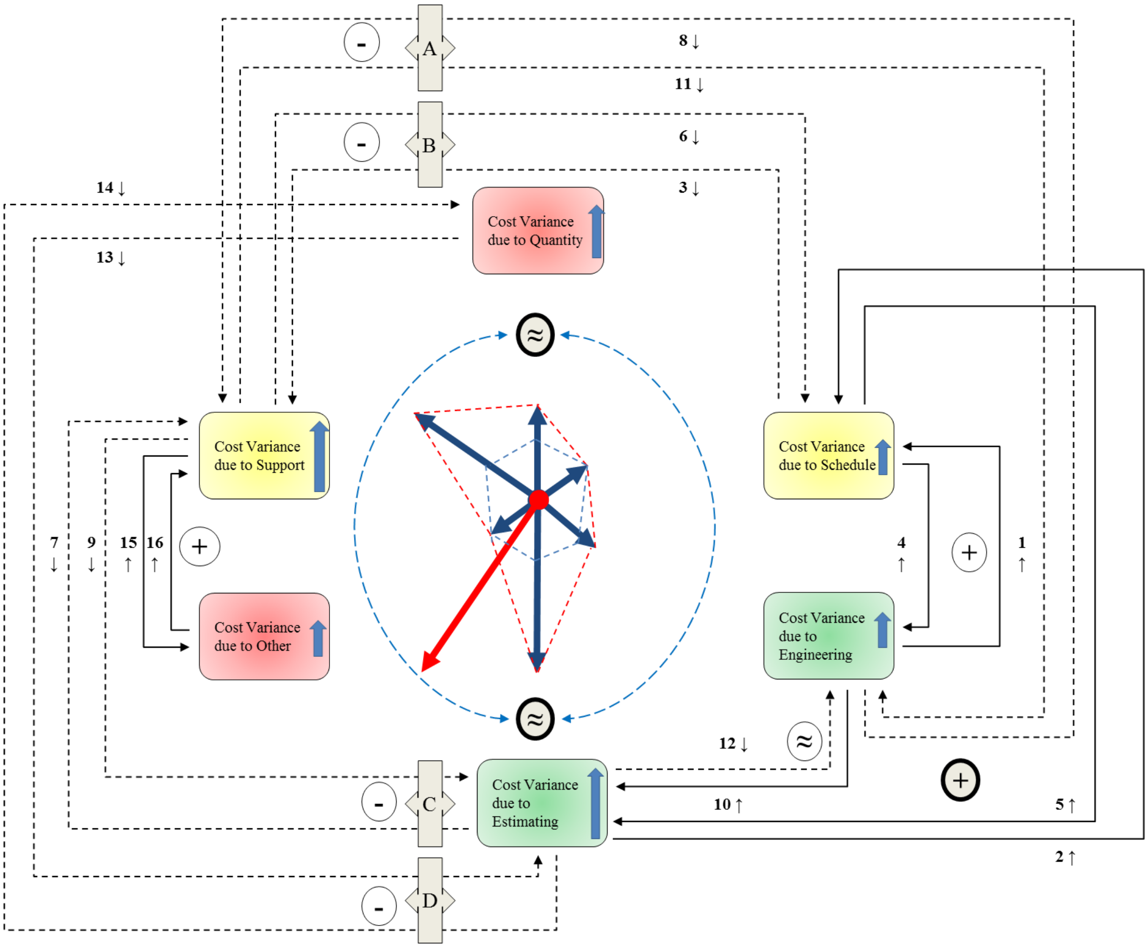

Figure 6-7: Adapted dependency model

The first step involved re-arranging the layout in order to mirror the manner in which the presented technique presents vector spaces. This improves the visual recognition of how the internal polar force field model is linked to the dependency model and to assist in visualising how it geometrically affects its dynamics. The linkages between cost variance factors might be considered as “springs” which reach as a whole to the internal “pulling and pushing” of the vector space. Table 6-14 summarises the input and outputs of the model.

Table 6-14: Dependency model input output overview

	Number of Inputs Acc-elerating	Number of Outputs Acc-elerating	Number of Inputs De-celerating	Number of Outputs De-celerating	Number of Inputs Total	Number of Outputs Total	Number of Con-nections Total
Quantity	0	0	1	1	1	1	2
Schedule	1	1	1	1	2	2	4
Engineering	1	2	2	1	3	3	6
Estimating	2	1	2	3	4	4	8
Other	1	1	0	0	1	1	2
Support	1	1	3	3	4	4	8
SUM	6	6	9	9	15	15	30
AVERAGE	1	1	1.5	1.5	2.5	2.5	5

Each vector could hence be considered to be exerting "pressure" on the networked dependency model leading to the (time delayed) diffusion of impact (Eigenvector) between the factors (dependency). The increase in cost variance, although of different magnitudes will then lead to a self-enforcing increase in cost variance in the positive feedback loop between "Estimating", "Engineering" and "Schedule". At the same time the increase in magnitude will be dampened by the interactions with changes due to "Quantity" and "Support" (interfaces A, B, C and D).

The use of the dependency model to explore the "uncertainty" rankings (based on statistical historical data analysis) derived from the vector based forecasting approach suggests the following potential reasons for the ranking being as identified:

- The two cost variance factors with the greatest forecasting uncertainty ("Quantity" and "Other") can be considered as peripheral to the dependency model in that they connect to only one other cost variance factor. This means that they are not subject to any (visible) dampening behaviour within the dependency model. Additionally, both factors are essentially independent from the whole product life cycle in that they are determined primarily by external factors.

- The two cost variance factors with the lowest forecasting uncertainty ("Estimating" and "Engineering") are integral components of a (self-enforcing) feedback loop which is dampened by interfaces due to "Support" and "Quantity".

- The overall feedback cycle (blue dashed line with the ≈ symbol at top and bottom expressing that the opposing dependencies are of opposite nature) excludes the factors with the highest forecasting uncertainty and evidences no significant weighting as enforcing or dampening so that a certain stable behaviour could be expected.

Important to note, however, is that stable behaviour describes the propagation pattern driven by the dependency model and not the direction of this variance propagation.

6.2.6 Comparison of Case Study Results

The results of the data analysis for the three case studies are summarised in Table 6-15 using a small sub-set of key statistics:

Table 6-15: Comparison of data analysis results for case studies

	Case Study #1: C-17A Globemaster III (1986-2009)	Case Study #2: U.S. Air Force (1986-2013)	Case Study #3: U.S. DoD (1990-2013)
Number of Forecasts	23	27	23
Number of Projects in Case Study	1	724	2050
Average Forecast Accuracy: Total Cost Variance	97%	105%	141%
Average Forecast Accuracy: Total Cost Variance STDEV.P	17%	37%	189%

Case study #1 reflected a single product (U.S. DoD Air Force C-17A Globemaster III) for which cost variance data was available for 23 individual sequential time periods from 1986 to 2009 resulting in a total of 23 sets of unique cost variance data available for 22 forecasts.

Case study #2 reflected all products reported on for a branch of the U.S. DoD Air Force for which cost variance data was available for 27 individual sequential time periods from 1986 to 2013 whereby an average of 27 unique products was reported on for each time period resulting in a total of 724 sets of unique cost variance data available for forecasting enabling 26 aggregated forecasts to be evaluated.

Case study #3 reflected all products reported on for the U.S. DoD for which cost variance data was available for 23 individual sequential time periods from 1991 to 2013 whereby an average of 89 unique products was reported on for each time period resulting in a total of 2050 sets of unique cost variance data available for forecasting.

Across the case studies, the forecasting accuracy drops significantly as the amount of forecasts increases and this is assumed to be due to case studies #2 and number #3 being based on aggregated cost variance sums (i.e. all cost variance for all products for a given unique time period) and hence compounding the inaccuracy of a forecast for a single product at a single point in time. The data for case study #1 represents a sub-set of the data for case study #2 which represents a sub-set of the data contained in case study #3.

The technique was also applied to 17 further projects randomly selected from the U.S. DoD data set. Table 6-16 aggregates the results of the experiments with consideration of the individual cost variance dimensions and Table 6-17 provides an aggregated view of forecasting results for all methods for all case studies and projects.

Table 6-16: Mode forecasting accuracy all projects

Project	Cost Variance Changes due to Change in Quantity	Cost Variance Changes due to Change in Schedule	Cost Variance Changes due to Change in Engineering	Cost Variance Changes due to Change in Estimating	Cost Variance Changes due to Change in Other	Cost Variance Changes due to Change in Support	Average Cost Variance Changes due to Change in All Cost Variance Dimensions	STD. for Average Cost Variance Changes due to Change in All Cost Variance Dimensions	Total Cost Variance
F-35 - Average All (2001-2010)	89%	290%	102%	103%	0%	2,201%	464%	906%	80%
V22 - Average All (1987-2006)	3,900%	365%	246%	528%	0%	707%	958%	1,750%	209%
Bradley - Average All (1994-2000)	35%	70%	116%	81%	0%	698%	167%	276%	83%
MSE - Average All (1986-1992)	174%	0%	0%	116%	0%	110%	67%	103%	118%
F22-A - Average All (1994-2006)	98%	77%	118%	74%	0%	95%	77%	61%	85%
F/A-18E/F - Average All (1992-1999)	1,804%	96%	160%	959%	0%	366%	564%	1,119%	158%
AEHF - Average All (2001-2011)	214%	120%	43%	174%	0%	911%	244%	437%	97%
E-2D AHE - Average All (2003-2011)	0%	57%	74%	185%	0%	67%	64%	78%	76%
Minuteman III - Average All (1994-2007)	79%	786%	193%	157%	0%	465%	280%	400%	154%
Longbow Apache - Average All (1995-2010)	134%	94%	77%	837%	0%	99%	207%	338%	156%
ARH - Average All (2005-2008)	49%	72%	35%	154%	0%	53%	60%	65%	49%
CVN 74/75 - Average All (1987-1997)	0%	127%	2,085%	90%	0%	1,670%	662%	1,428%	105%
Global Hawk - Average All (2001-2013)	117%	151%	106%	109%	0%	85%	95%	58%	105%
FCS - Average All (2003-2008)	0%	52%	76%	73%	0%	60%	43%	36%	68%
LCS - Average All (2004-2013)	2%	52%	37%	95%	0%	0%	31%	41%	62%
SSN 774 - Average All (1997-2013)	0%	10,823%	6,098%	163%	1,422%	203%	3,118%	4,085%	177%
DDG-51 - Average All (1987-2013)	91%	64%	84%	510%	0%	3,085%	639%	1,338%	91%
C-17A - Average All (1986-2009)	96%	961%	184%	99%	67%	104%	252%	353%	93%
STDEV.P	942%	2,446%	1,422%	264%	325%	841%	698%	976%	43%
Ranking STDEV.P	3	1	2	6	5	4			

Of note in all forecasts is that the accuracy of individual dimensional mode forecasts is generally significantly lower than the accuracy of total forecasts. Pending further analysis the reason for this is expected to lie in the directional component of the forecasting algorithm which leads to over-proportional growth of individual vectors within radial degrees of 180° of the aggregated vector and under-proportional growth of such that do not.

The total forecasting results for all forecasting experiments conducted using all three methods are shown in Table 6-17, Table 6-18 and Table 6-19. These forecasting experiments include the U.S. DoD (case study #3), all three branches of the U.S. DoD Armed Forces (the Air Force being case study #2) and 18 individual product level experiments (the U.S. DoD Air Force C-17A Globemaster III being case study #3).

Table 6-17: Summary of overall mode forecasting results

Project	Number of Forecasts made	Average Cost Variance Changes due to Change in All Cost Variance Dimensions	Standard Deviation for Average Cost Variance Changes due to Change in All Cost Variance Dimensions	Total Cost Variance	Standard Deviation for Total Cost Variance
US DoD (1990-2013)	23	4,2181%	128,214%	141%	189%
US Army (1986-2013)	27	2,510%	8,294%	105%	38%
US Navy (1986-2013)	27	471%	1,148%	108%	37%
US Air Force (1986-2013)	27	391%	981%	105%	37%
E-2D AHE (2003-2011)	7	64%	33%	76%	26%
AEHF (2001-2011)	10	244%	345%	97%	32%
ARH (2005-2008)	3	60%	52%	49%	27%
Bradley (1994-2000)	6	167%	230%	83%	28%
C17-A (1986-2009)	22	252%	734%	93%	17%
CVN 74/75 (1987-1997)	10	662%	1,252%	105%	23%
DDG-51 (1987-2013)	25	639%	2,481%	91%	21%
F/A-18E/F (1992-2012)	19	564%	1,320%	158%	283%
F22-A (1994-2006)	11	77%	30%	85%	18%
F35 (2001-2010)	9	464%	1,005%	80%	18%
FCS (2003-2008)	5	43%	29%	68%	49%
Global Hawk (2001-2013)	12	95%	36%	105%	25%
LCS (2004-2013)	8	31%	17%	62%	24%
Longbow Apache (1995-2010)	15	207%	508%	156%	251%
Minuteman III (1994-2007)	13	280%	540%	154%	68%
MSE (1986-1992)	6	67%	42%	118%	15%
SSN 774 (1997-2013)	16	3,118%	11,789%	177%	351%
V22 (1987-2006)	19	958%	2,910%	209%	416%

Table 6-18: Summary of overall upper limit forecasting results

Project	Number of Forecasts made	Average Cost Variance Changes due to Change in All Cost Variance Dimensions	Standard Deviation for Average Cost Variance Changes due to Change in All Cost Variance Dimensions	Total Cost Variance	Standard Deviation for Total Cost Variance
US DoD (1990-2013)	23	68,172%	211,831%	142%	335%
US Army (1986-2013)	27	3,714%	11,883%	65%	27%
US Navy (1986-2013)	27	738%	1,855%	64%	21%
US Air Force (1986-2013)	27	568%	1,385%	64%	20%
E-2D AHE (2003-2011)	7	96%	59%	181%	221%
AEHF (2001-2011)	10	405%	583%	67%	24%
ARH (2005-2008)	3	88%	78%	718%	882%
Bradley (1994-2000)	6	208%	262%	106%	62%
C17-A (1986-2009)	22	386%	1,113%	64%	20%
CVN 74/75 (1987-1997)	10	838%	1,552%	70%	28%
DDG-51 (1987-2013)	25	1,225%	4,788%	120%	319%
F/A-18E/F (1992-2012)	19	985%	2,474%	87%	68%
F22-A (1994-2006)	11	131%	58%	74%	31%
F35 (2001-2010)	9	686%	1,425%	80%	16%
FCS (2003-2008)	5	73%	50%	584%	873%
Global Hawk (2001-2013)	12	155%	55%	60%	14%
LCS (2004-2013)	8	53%	31%	121%	61%
Longbow Apache (1995-2010)	15	360%	928%	93%	104%
Minuteman III (1994-2007)	13	374%	823%	63%	28%
MSE (1986-1992)	6	115%	69%	49%	6%
SSN 774 (1997-2013)	16	5,791%	21,890%	64%	31%
V22 (1987-2006)	19	1,706%	5,289%	61%	33%

Table 6-19: Summary of lower limit overall forecasting results

Project	Number of Forecasts made	Average Cost Variance Changes due to Change in All Cost Variance Dimensions	Standard Deviation for Average Cost Variance Changes due to Change in All Cost Variance Dimensions	Total Cost Variance	Standard Deviation for Total Cost Variance
US DoD (1990-2013)	23	42,237%	128,165%	255%	633%
US Army (1986-2013)	27	2,568%	8293%	109%	46%
US Navy (1986-2013)	27	484%	1147%	103%	31%
US Air Force (1986-2013)	27	394%	989%	104%	29%
E-2D AHE (2003-2011)	7	489%	80%	198%	156%
AEHF (2001-2011)	10	524%	378%	118%	45%
ARH (2005-2008)	3	279%	132%	544%	556%
Bradley (1994-2000)	6	341%	184%	160%	84%
C17-A (1986-2009)	22	309%	763%	114%	34%
CVN 74/75 (1987-1997)	10	1,152%	947%	109%	26%
DDG-51 (1987-2013)	25	1,084%	2,535%	199%	451%
F/A-18E/F (1992-2012)	19	739%	1,317%	148%	121%
F22-A (1994-2006)	11	256%	108%	128%	48%
F35 (2001-2010)	9	588%	991%	133%	36%
FCS (2003-2008)	5	440%	90%	749%	983%
Global Hawk (2001-2013)	12	259%	38%	101%	22%
LCS (2004-2013)	8	774%	314%	193%	74%
Longbow Apache (1995-2010)	15	472%	693%	137%	118%
Minuteman III (1994-2007)	13	668%	553%	89%	34%
MSE (1986-1992)	6	639%	157%	90%	11%
SSN 774 (1997-2013)	16	3,440%	11,859%	120%	57%
V22 (1987-2006)	19	1,296%	2,929%	111%	56%

6.3 Verification and Validation through (Semi-Structured) Interviews

During the complete period of the research study 145 individual semi-structured interviews were conducted and 85 responses to online and email surveys were gathered, as shown in Table 6-20.

Table 6-20: Overview of semi-structured interviews and surveys

Context	Number of Unique Participants	Comments
Cycle 1: Discovery Interview Series #1	40 semi-structured interviews and 12 survey responses	Interviews and email survey questions initiating the research study.
Cycle 1: Discovery Interview Series #2	19 semi-structured interviews	Interviews to explore emerging research hypothese and research gap identification.
Cycle 1: Discovery Interview Series #3	19 survey responses	Interviewees were members of a company risk management community. The aim of the interviews was to aid in interpretation of analysis results of the enterprise risk database.
Cycle 2: Prototyping Interview Series #4	17 semi-structured interviews	Interviews with cost estimation professionals representing 4 aerospace manufacturing companies, 1 solution provider for parametric cost estimation tools, 1 automobile manufacturer and 2 cost estimation associations.
Cycle 2: Prototyping Interview Series #5	54 survey responses	Survey was provided online and responses gathered were anonymous. Due to anonymity particular information about survey respondents could not be gathered.
Cycle 2: Prototyping Interview Series #6	47 semi-structured interviews	16 serious games delivered with 48 participants whereby multiple round and final debriefs with each participant and their groups were performed in each game.

Context	Number of Unique Participants	Comments
Cycle 2: Prototyping Interview Series #7	7 semi-structured interviews	In-depth interviews outside of the serious game plays to explore forward / back uncertainty propagation behaviour.
Cycle 3: Validation Interview Series #8	11 semi-structured interviews	The intent of the interviews was to identify guidelines for pragmatically deploying the presented technique in practice in order to support of cost forecasting efforts.
Cycle 4: Integration and Application Interview Series #9	10 semi-structured interviews	In-depth interviews presenting and discussing research findings with key stakeholders

For purposes of verification and validation of the research findings, the focus is placed upon insights gained during the 10 semi-structured interviews performed in Cycle 4 where the forecasting framework as developed in the investigation and applied in practice were discussed in depth. Table 6-21 describes the key attributes of the interviewees:

Table 6-21: Key attributes of interviewees

Role	Experience (Years)	Domain
Cost Estimating Expert	6	Defence Strategy
Chief of Project Estimation	18	Aerospace Manufacturing
Chief Project Engineer	6	Aerospace Manufacturing
Design Methods Specialist	14	Aerospace Manufacturing
Principle Reliability and Modelling Specialist	12	Defence Manufacturing

Role	Experience (Years)	Domain
Product Development and Cost Engineering Tools & Methods Manager	20	Automobile Manufacturing
Professor of Operations Management	9	University / Research
Research Fellow Advance Cost Modelling Methods	8	University / Research
Senior Lecturer Lecturer in Manufacturing Engineering	10	University / Research
System and Lifecycle Cost Engineer	12	Aerospace Manufacturing

The semi-structured interviews were performed by presenting the final research presentation containing summaries of all case studies and then discussing a series of prepared questions. In parallel to these, an opportunity was given to focus on specific questions and interests of the interviewee. The prepared questions were:

1. What interest do you have in a forecast of cost uncertainty considering your role in the organisation? This question was asked in order to understand how relevant the interview subject was to the interviewee. Only if the subject was highly relevant could verification and validation feedback be considered as suitable for review and reflection.

2. What ranges does your organisation use to signify when an estimate is within tolerance, out of tolerance but acceptable and out of tolerance but not acceptable? This question was asked in order to verify the general suitability of green, amber and red scoring classification applied to the case study data analyse including the concept of estimate robustness in respect to uncertainty ranges identified.

3. Is your organisation required to estimate cost uncertainty under conditions of small cost data? This question was asked to verify that forecasting cost uncertainty under

small cost data conditions was relevant to the organisation the interviewee worked for.

4. What estimation techniques does your organisation use when needing to estimate with only a single data set? This question was asked in particular to verify the fundamental insight of the investigation that no specific cost uncertainty estimation techniques for the small cost data context were available in practice.

5. How reliable do you consider common techniques for estimation such as analogy / expert opinion, parametrics and regression to be when only a single data set is available? This question was asked to validate the importance of having a specific cost uncertainty estimation technique for addressing conditions of small cost data.

6. What techniques do you use to simulate plausible future scenarios? The purpose of this question was to verify the degree that the interviewee´s organisation used single point technical baseline estimates as the basis for forecasting cost uncertainty.

7. For how many time (accounting) periods do you typically make forecasts for? The intent of this question was to verify that the iterative approach across time periods proposed by the presented framework for estimating cost uncertainty mirrored the approaches used in practice.

8. To what degree do you differentiate between positive and negative cost variance? The purpose of this question was to verify the assumption of the investigation that any deviance from budget, whether positive or negative, was relevant for estimating cost uncertainty (regardless of whether these even out during the whole product life cycle).

All respondents voiced a keen interest in being able to forecast the uncertainty of cost estimates (especially in relation to innovative high value manufacturing products) in a robust manner over varying specific and groups of whole product life cycle phases. Their primary interest was in understanding improved approaches to contingency setting and being able to identify cost propagation behaviour, which had a high potential of leading to significant cost overruns. In particular, one respondent shared that "…turning the sometimes nebulous concept of unbounded unknown-unknown into a more manageable and bounded Known-Unknown…" was a highly valuable question to explore in order "…to forecast future cost and schedule overruns with a reasonably accuracy…" (Chief of Project Estimation, aerospace manufacturing company, personal communication, 2018).

While the concept of accuracy can be understood from a variety of perspectives and requires a verification against actual cost, the underlying question of relevance is in fact whether the estimate is within tolerance, out of tolerance but acceptable or out of tolerance but not acceptable. In practice, tolerance levels and thresholds are often indicated through traffic light systems (i.e. the green, amber and red scoring classification applied to the case study data analysis). The traffic light approach was confirmed as relevant and commonly used although the specific thresholds varied to some degree between organisations the respondents were active in. Multiple benchmarks for comparing cost estimates against were used in the sense that some compared them to planned budgets, some monitored them against actual cost figures and others used the traffic light system to indicate non-financial measures such as qualitative assessments of the cost estimation team competency as an indicator of cost estimate robustness.

In respect to the extent that the organisations of the respondents need to estimate cost uncertainty under conditions of small cost data, the most significant amount of discussion emerged due to the concept of small cost data conditions being largely unknown to the respondent in practice. While the concept was understood the respondents found it difficult to acknowledge its existence in practice due to the relevant attributes not being actively monitored (i.e. the dependence of regression statistics on a certain amount of homogeneous prior information or the changing complexity of prior information). While the discussions did lead to an acknowledgement that forecasting cost uncertainty under small cost data conditions was relevant to the organisation, the respondents worked for and the significance of the challenge to established cost estimation approaches understood, the ramifications of shifting the estimation paradigms applied in practice were considered significant although one respondent suggested that "… adding the presented estimation approach to the existing portfolio of techniques may serve best to introduce the way of thinking in the form of a socialisation process of the principles." (System and Lifecycle Cost Engineer, aerospace manufacturing company, personal communication, 2018)

The novelty of the small cost data condition for respondents transferred itself to challenges in answering the question concerning which estimation techniques their organisations used when needing to estimate with only a single data set. Respondents affirmed that if only a single data set was available (in the form of small cost data) analogy and / or expert opinion estimating techniques were used to expand the data set in order to either remain at the level of an analogy or expert opinion estimate or in order to have sufficient data for applying parametric and / or (preferably) regression based techniques. Especially these comments served as qualitative validation of the research gaps identified and the view that in practice "…numerous work-arounds are used to make data regressible" (Chief Project Engineer, aerospace manufacturing company, personal communication, 2018).

Based on the insights gained in responses to the question concerning which estimation techniques their organisations used when needing to estimate with only a single data set the question of how reliable, such approaches were considered to be followed naturally. While respondents acknowledged that the techniques used in practice were not fully suitable, they did consider their results to be within the bounds of the inherent accuracies of the estimates in any case and sufficient for decision making purposes. This view was mirrored by all respondents in the sense that estimates and their uncertainty generated in this manner were "good enough" (Principle Reliability and Modelling Specialist, defence manufacturing, personal communication, 2018) for decision making. This perspective does, however, lie in contrast to estimating experience regarding especially innovative high value manufacturing products where excessive cost overruns are known to occur with regularity. Interestingly, respondents also agreed that the cost estimates of such products are often not met although it was suggested that since in practice senior management is aware of these risks they still "accept" estimates in the sense of allowing them to flow into decision making. From the perspective of the research, especially this question validates the underlying assumption of the research study that while estimators acknowledge the existence of small cost data tacitly, they will continue to estimate with contemporary techniques as long as the accuracy of the forecasts is not made a key quality criteria required for decision making. Indeed, it may be surmised that in many cases estimates for such products are produced more for learning and orientation purposes (especially in first series manufacturing) rather than to help make informed business decisions.

The somewhat critical insights gained from the previous interview question might, however, be seen as alleviated by common management practice to examine multiple plausible manufacturing scenarios in the early stages of the whole product life cycle for innovative high value manufacturing products. In this respect, the respondents often considered the simulation of such plausible multiple scenarios to be an important part of contextualising the accuracy of forecasts of cost and its uncertainty. While the final result of an estimation activity was typically described as being a single point estimate, and the assignment of relevant contingency to be the responsibility of business decision makers, respondents did agree that in practice it was normal to provide a most likely single point estimate at 60% confidence level based on the application of Monte Carlo simulation with T-shaped distributions. Of interest at this point is that the estimate was at all times focused on unit or support costs and did not examine the development of the cost estimate (and its uncertainty) over the course of the whole product life cycle up to that point in time where the estimate could be verified.

The respondents focus on estimating unit and support costs then led to the insight that estimates were not made for specific accounting periods. Nonetheless, changes in the estimate were generally considered to be monitored and flow into annual budgeting decisions whereby the authorisation of such budgets was then seldom tied to the overall uncertainty and preferably linked to business imperatives. In many cases concerning products with major cost overruns, the respondents mentioned optimism bias of being able to meet final estimates as being a dominant factor in continued progress approvals during the whole product life cycle stage gates typically encountered.

The final prepared interview question concerned the degree that respondents differentiate between positive and negative cost variance, and all respondents confirmed that any deviance from budget, whether positive or negative, was relevant for estimating cost and its uncertainty. Based upon this, the respondents agreed that it was acceptable to use a polar vector space model which considered all input values as absolute figures for estimation. In this respect, respondents shared the opinion that actual cost typically exceeds estimated cost and indeed exceeds uncertainty boundaries set by estimators so that the scenario of overall cost being less than estimated was a low probability scenario that the presented framework did not necessarily need to support.

In summary, the responses received during the semi-structured interview supported the assumptions and findings of the investigation. No respondents challenged the logic of the presented framework, while the reasons for its robustness on a total cost (uncertainty) prediction level were the subject of significant interest. Of additional interest was the question why the forecast of individual cost variance (uncertainty) reasons proved significantly less robust than the forecast of the total cost variance (uncertainty) although the latter was the sum of the former. In response to the question regarding why the framework proved as robust as it was the dependency model explanation was considered acceptable but worthy of further exploration. In response to the question regarding why forecasts of individual cost variance (uncertainty) were significantly less robust than those for total cost variance (uncertainty), the respondents suggested that this may be due to the state space nature of the forecasts generated and that a dynamic space view might support in resolving this difference. This researcher supports this view and has focused recommendations for future research on this question. As mentioned by one respondent, "…this approach may well provide some useful insights on the influence of financial constraints and soft systems on technical outcomes (and either support decision making and or, in some cases, help set the foundation for project and programme success or failure)." (Cost Estimator, defence strategy, personal communication, 2018).

Of final note is that respondents from universities / research (see Table 6-21) consistently struggled to accept that the identified application of cost estimation approaches in practice were applied as they were while respondents from industry (see Table 6-21) consistently struggled to accept that there was value in adopting best practice methods in a rigorous manner. In this respect, this researcher draws the conclusion that forecasting of cost estimates and their uncertainty to date in practice primarily serves in supporting a complex decision making process that has emergent characteristics while research attempts to introduce scientific rigor to what is essentially a complex (if not chaotic) living systems concept. In the words of one respondent it may well be argued that cost estimation is "…more of an art than a science." (Product Development and Cost Engineering Tools & Methods Manager, automobile manufacturing, personal communication, 2018)

6.4 Verification and Validation of Framework Results through Game Playing

6.4.1 Overview

In order to support the research investigation a serious game in the form of a case-study based board game simulation of cost uncertainty propagation was developed and applied during Cycle 2 of the research methodology. It is highlighted in this section because of its fundamental influence on the research study as a whole. The aim of the game from a participant perspective was to learn more about "beating the dynamics" of cost propagation as identified in the case study data analyses presented in this investigation. The objectives of the game from a participant perspective were to observe (un-) managed cost uncertainty propagation, identify events that typically occur, define actions that are usually taken, maintain cost variance within agreed bands and share experiences about what works and what does not work in practice. From a research perspective the game served to verify and validate many of the research findings with a special emphasis on the elicitation of relevant tacit knowledge in a collaborative setting.

6.4.2 Description of the Serious Game

A serious game is a simulation of a real world problem for educational purposes where two or more actors are required to collaborate in order to resolve a problem (Susi et al., 2007) and heuristics for learning evolve (Mousavi & Gigerenzer, 2014). Serious games represent a versatile and effective method for developing relevant new knowledge and skills through education and training among participants which is particularly helpful when addressing management and leadership challenges (Allal-Chérif & Makhlouf, 2016). To a degree, serious games can also be considered as thought experiments enabling debate about current understanding and simulating the consequences of differing assumptions regarding future scenarios (Squire & Jenkins, 2003) within not only the game micro-world but the larger industrial context as well. Such games also "...encourage collaboration among players and thus provide a context for peer-to-peer teaching and for the emergence of learning communities" and "…are about choices and consequences, and good educational games force players to form theories and test their thinking against simulated outcomes" (Squire & Jenkins, 2003). Serious games also appear helpful in respect to creating a shared reality regarding the problem addressed among the stakeholders, independent of the degree that they are facilitated (Pando-Garcia et al., 2016). A serious game can thus be classified as a problem analysis approach and in this case is of sequential nature since a time axis is followed and the game follows principles of perfect, yet incomplete information. The game is furthermore finite and discrete. When representing a complex problem, the game helps explore the conundrum evolved from an ill-defined to a well-defined problem by combining multiple problem solving strategies into a collaborative learning experience. This then also helps address common barriers, such as confirmation or optimism bias to problem solving. In relation to cost estimation and scenario planning, similarities can be seen with the concepts of robust decision making under conditions of deep uncertainty as discussed by Lempert et al. (2006), Mahnovski (2007), Augusdinata (2008), Hamarat et al. (2013) and Lempert et al. (2013).

6.4.3 Game Delivery

The game was delivered 16 times to groups of programme and project managers with budget accountability, cost estimators, financial forecasters, executive decision makers and others seeking to understand how cost will propagate over time. A total of 48 individuals participated. The fundamental assumptions were that everyone wanted to forecast project budgets robustly; robust budgets being budgets that meet the needs of a project and do not need to be renegotiated.

The game posed the following challenge for one or more teams of about three to five participants each:

- A company is preparing to bid for a novel large engineering project.

- The technical baseline cost estimate has been completed.

- The risk assessment is being finalised.

- Past experience suggests that “the cost will only go upward”.

- Business executives are worried that an inaccurate bid for unit and support costs will break the company.

The task of the participating teams was to determine how the cost estimate will definitely develop into the future and to recommend a suitable contingency in % of the baseline estimate. The game was over when the assigned budget was exhausted. At the beginning of each round the participants ensured total cost variance was below contingency / break-even. If cost variance exceeded contingency / break-even the budget could be used to reduce cost variance below that contingency. The team which lasted the longest “won” the game. If more than one team completed all rounds, the team with the lowest total cost variance in the final round won.

The game development proceeded along a set of activities focused on:

- Determine the dependency model and propagation over time for cost variance based upon case study research.

- Visualise the propagation of cost variance over time based on the data behaviour.

- Identify typical risk threats to cost variance and their impact.

- Identify typical risk opportunities to cost variance and their impact.

- Identify typical actions related to cost variance and their impact.

- Determine potential questions of relevance when exploring the dependency model and cost variance propagation.

- Perform simulations to ensure the developed logic and visualisations are coherent.

- Validate simulation through field trials and refine accordingly.

Gaming experiences with less than three players demonstrated a significant lack in the variety of perspectives needed to explore a suitable spectrum of game playing approaches. The gaming experiences with more than five participants demonstrated challenges in keeping all parties involved and engaged. The technical baseline cost estimate and risk assessment were assumed to have been completed and yielding a single point estimate. Independent of the estimate the participant (team) was provided with a contingency of 100 fictional financial units and tasked to take actions to maintain the cost variance below an agreed threshold over a period of five rounds, representing calendar years, without fully consuming the contingency assigned. The participant (team) hereby played against an agent making decisions based on (a) the propagation pattern of cost variance over time as determined by case study research (b) the impact of typical events during the whole product life cycle as calculated by a dependency model determined by case study research and (c) the impact of actions taken by the participant (team) as calculated by the same dependency model. The agent was simulated by the facilitator of the game using simplified patterns of observed data behaviour in the case study data. This was also the agent operating the thought experiment. The actions of the participant (team) could consist of using contingency directly to reduce cost variance experienced or by using contingency to conduct actions which reduced such. In order to achieve the primary aim the participant (team) engaged in semi-structured discussions before, during and after each round in order to agree on observed behaviour of the simulation, negotiate decision making strategies and decided upon specific actions. The pay-off matrix and corresponding strategies were hereby revisited at the beginning and end of each round.

The participant (team) was then challenged to discover the dependency model of the cost variance factors through gameplay over the course of several rounds, to determine possible strategies for optimum management of cost variance propagation and to test the effectiveness of these strategies in future rounds of gameplay or future games as a whole. As the knowledge of the rules governing the actions of the simulated agent evolved, the suitability of this knowledge for decision support purposes matured.

Game results were continuously recorded on a game board as illustrated in Figure 6-8:

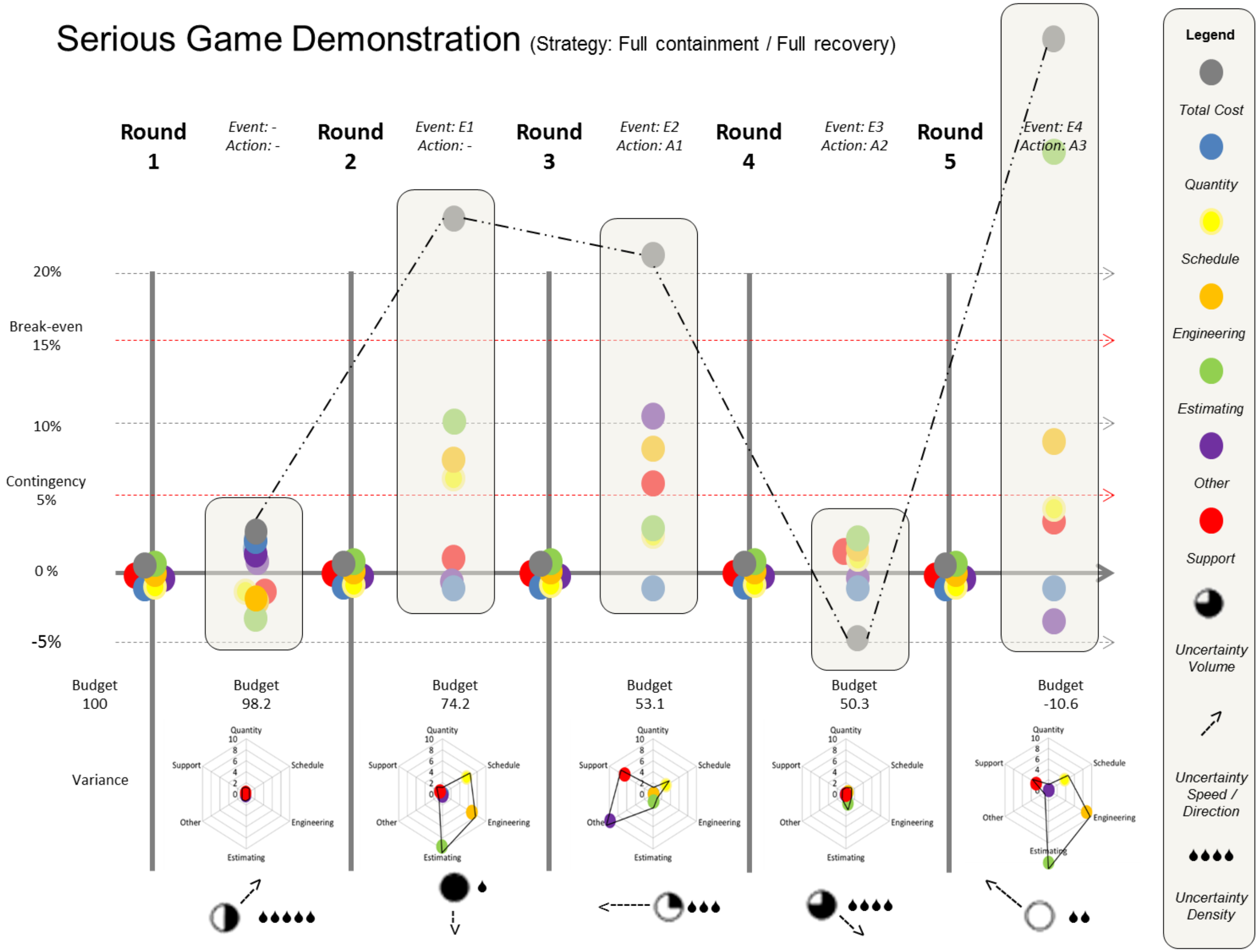

Figure 6-8: Exemplary completed game board

The game board can be understood as follows:

- Individual and total cost variance dimensions are indicated by coloured spheres as shown in the legend on the right of the figure. Their value is given by their y-axis coordinate.

- The y-axis indicates the degree of cost variance manifested at the end of a round. The cost variance begins with a value of zero for all cost variance dimensions.

- The x-axis represents the progression of the game with the round one starting conditions at the far left and then proceeding through five game rounds plus the condition after round five events and actions. Cost variance is shown for the times after events and actions in-between the rounds and then the final cost variance at the end of the round after (potentially) applying any financial contingency to adjust the manifested cost variance.

- Events and actions are recorded at the top of the timeline in accordance to when they occur.

- The starting value of the contigency is shown on the left side of the timeline ("100") and its change recorded on the game board in the round that is played to the right of that.

- A spider chart representation of cost variance is generated for each round whereby filled circles are used as symbols for the relative amount of total cost variance.

- The relative density of the spider chart representation was symbolised as a series of teardrops whereby these were generated based upon the ratio of actual to maximum area of the shape within the perimeters of the spider chart values. This represented the principle of symmetry progress and the correlation of symmetry to cost variance uncertainty.

- A directional arrow is used to suggest the speed and direction of total cost variance change from a spider chart perspective. This arrow can be considered a pre-cursor to the aggregated vector of the polar force field.

The dynamics of the exemplary game can be explained as follows:

- Cost variance in round one starts at “0” for all cost variance factors.

- During each round cost variance for each factor changes due to events and due to propagation behaviour determined by the dependency model.

- At the end of each round the participant (team) uses contigency to reduce the total cost variance to “0”.

- Cost variance in round five increases to a value which cannot be compensated by the contingency remaining at that time.

6.4.4 Forecasting and Uncertainty Quantification

The serious game included the visualisation of cost variance through the use of spider charts. The use of spider charts evolved into the use of vector spaces and polar force fields at a later stage in the research study when game plays were no longer being conducted. The discussion of the changing spider chart geometries, in particular when supported by the relevant Microsoft ® Excel based software demonstrator, were fundamental to preparing a shift to vector spaces and then polar force fields and while novel to the participants serves as effective anchors to discussions around cost estimate uncertainty propagation. While the central theme of geometrical symmetry that was used to discuss the spider chart shape proved a significant comprehension challenge to the participants many discussions did then lead to questions around why cost estimate uncertainty propagation should (not) demonstrate patterns.

Key concepts related to the spider chart representation explored in this activity were:

- Volume: Area within the perimeter of the spider chart. For gaming purposes this was scored as large, medium or low.

- Speed: The number of time periods required for the complete cost variance from a single time period to propagate into the future. For gaming purposes this was scored as high, medium or low.

- Direction: Which quadrant of the spider chart will show the greatest growth of cost variance area. For gaming purposes this was scored as top left, top right, bottom left and bottom right.

- Density (symmetry): The ratio between the actual and maximum area of the spider chart whereby the maximum area is given when the perimeter of the shape has equal face lengths. For gaming purposes this was scored as high, medium or low.

The aggregated results for completed spider charts are shown in Table 6-22:

Table 6-22: Average aggregated spider chart results from game plays

	Number of Completed Rounds a Forecast was Based On			
Indicators	2	3	4	5
Volume	Low	Medium	Medium	Large
Speed	Low	Medium	Medium	High
Direction	Top right	Bottom left	Bottom left	Bottom right
Density	Low	Medium	Medium	Medium

In summary, this researcher considers the verification and validation efforts through the serious game to have been highly affirmative of the uncertainty quantification approach developed although this was at a stage preceding the polar force field visualisation and quantification approach. In this respect, game plays directly addressed potential solutions to resolving the research gap and exploring techniques for exploring the hypothesis.

6.4.5 Explaining Uncertainty with a Dependency Model

The dependency model was fundamental to the behaviour of the serious game and the supporting Microsoft ® Excel based software demonstrator. Each round debrief included a specific discussion of how the dependency model could be used to explain the data behaviour and tacit knowledge was continuously elicited in order to explore the relevance of the model. In this respect the qualitative debrief discussions of the serious game continuously verified and validated the research gaps, the hypothesis and the evolving research findings.

Of particular note were activities completed where participants were asked to "Update dependency model template – which dependencies do you expect and which did you notice?" Since the actual case study based dependency model was only shown at the end of the game a number of insights developed that supported the quantitative nature of the research based models. Attributes considered were:

- "Uncertainty" Rankings: The higher the standard deviation of the data analysis for the cost variance factor the higher the ranking.

- Positive Feedback Loops: A positive feedback loop consists of at least two cost variance factors where the influence on the other cost variance factors is overall of increasing nature.

- Negative Feedback Loops: A negative feedback loop consists of at least two cost variance factors where the influence on the other cost variance factors is overall of decreasing nature.

- Balancing Loops: A balancing feedback loop consists of at least two cost variance factors where the influence on the other cost variance factors of increasing and decreasing nature negates themselves whereby no weighting of the impacts was considered.

- Propagation Sequence (Rank): The propagation sequence ranks the impact of cost variance factors on each other by the degree of this influence. The higher the influence the higher the rank.

- Propagation Speed (Rank): The propagation speed ranks the speed with which the impact of cost variance factors acts on each other. The greater the speed of the impact the higher the rank.

- Accelerating Impact: Accelerating impact increases the cost variance per time period for the impacted cost variance factor.

- Decelerating Impact: Decelerating impact decreases the cost variance per time period for the impacted cost variance factor.

- Most Central: The centrality of cost variance factors is given by the number of cost variance factors impacted by a cost variance factors. The more cost variance factors are impacted the more central the cost variance factor.

- Least Central: The centrality of cost variance factors is given by the number of cost variance factors impacted by a cost variance factors. The less cost variance factors are impacted the less central the cost variance factor.

6.5 Verification and Validation of Framework Results through a Thought Experiment

A thought experiment for supporting the verification and validation of the hypothesis and corresponding contributions to knowledge was developed and applied in seven semi-structured interviews in the Cycle 2 prototyping interview series of the research method. It is highlighted in this section because of its fundamental influence on the research study as a whole.

6.5.1 Description of the Thought Experiment

The thought experiment considered the whole product life cycle as a closed system with an intelligent agent that passed uncertainty between time periods based on eigenvectors and dependencies introduced as illustrated in an exemplary manner in Figure 6-9. The initial time period is indicated by “t=1” and the next time period is indicated by “t=2”. The agent nominally automated the activity of the facilitator in the serious game since it was based on specific behaviours of data observed in the case studies.

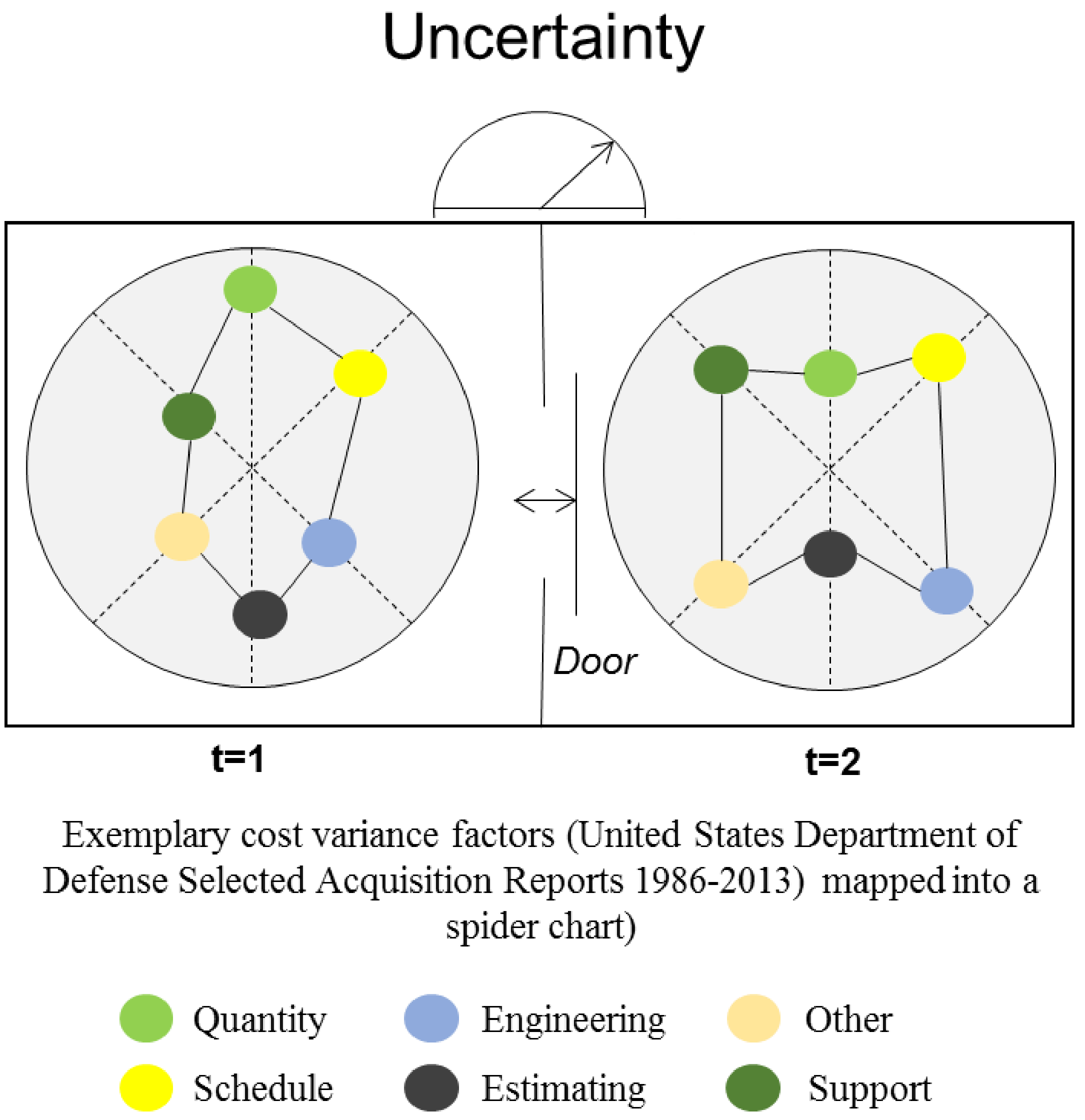

Figure 6-9: Thought experiment

The thought experiment was presented in the form of a “machine” where cost variance is visualised as a spider chart geometry at two different points in time whereby this can be considered as an abstracted version of a polar force field (although that view had not yet evolved when the experiment was designed). The transitions between the different points in time were controlled by an intelligent agent that opened / closed a door between the time periods and permits cost estimate uncertainty to propagate in both directions between two individual time periods. The thought experiment worked as follows:

- At time period 1 a project has an uncertainty related to the future cost variance for a series of dimensions represented as coloured spheres.

- As time period 2 approaches a virtual “door” opens between time period 1 and time period =2 so that cost variance can pass from time period 1 to time period 2 while at the same time cost variance can return from time period 2 to time period 1.

- Whether cost variance passes the door and what position it will have in the time it is travelling to is determined by a series of rules derived from the case study research.

- The transformation is governed by the eigenvector attributes of the individual cost variance dimension and the dependency between it and the other cost variance dimensions.

Generic rules governing data behaviour identified in the data analysis can be summarised as follows:

- (Forward) Eigenvector attributes: The cost variance variable is not completely accounted for in the time period it originates in. An example might be a series of technical factors the cost variance impact of which cascades from product system through assembly and sub-assembly level to component level which is a time consuming process.

- (Forward) Dependency attributes: A cost variance variable changes and due to another cost variable being dependent upon it that cost variable changes after a time delay. An example might be changes in quantity in time period 1 which result in changes in technical factors (at a cost) in time period 2. The delay being due to the time required to determine the engineering changes required in order to maintain affordable cost and the investments required for the relevant technical changes.

- (Backward) Eigenvector attributes: The cost variance variable value determined requires re-estimation of its value in a previous time period. An example might a technology that was originally estimated at cost x in time period 1, in time period 2 the original estimate is revised and requires revision of the previously estimated cost. Another example might be in-period revision of underlying cost indices.

- (Backward) Dependency attribute: A cost variance variable changes and due to another cost variable being dependent upon it, that cost variable changes after a time delay whereby this change requires re-estimation of the triggering values in the previous time period. Another example might be in-period revision of underlying cost indices.

The agent opened the door between time periods whenever a project aged sufficiently to move to the next time period (i.e. at the end of its accounting period or whole product life cycle phase), or when an in-period cost variance value change triggered re-estimation of baselines.

The thought experiment was used to support serious game playing and multiple semi-structured interviews where the maturity of the discussion warranted relevant in-depth explorations of data behaviour. In this respect, it represented a deeper investigation than was typically conducted during the serious games.

While the thought experiment was designed to explore the dependencies of cost variance factors between two discrete time intervals, the visualisation form was based on the geometrical / topological research perspective taken. Although not explicitly discussing polar force fields, the visualisation represented an abstract form of spider charts with all information except of the relative vertex values removed in order to help the interviewee focus on the patterns of relative positions versus related to a coordinate system. At this stage of the research the polar force field visualisation had also not yet evolved. All interviewee questions focused on exploring the potential “influence” of cost variance factors on each other upon the effect of the “door” being opened and the events which might trigger this occurring.

In relation to the polar force fields visualisation approach formulated at a later stage of the research study a major difference was that no specific techniques for correlation of cost variance changes over time was made. Insights generated were then used to continue development of the model as a whole.

6.5.2 Forecasting and Uncertainty Quantification

The concept of uncertainty quantification was presented to interviewees only as a basis for the visualisation of cost variance whereby its propagation dynamics were to be examined through the experiment. The visualisation of cost estimate uncertainty was based on a spider chart representation cost variance in a similar way to the way this was done in the serious game.

6.5.3 Explaining Uncertainty with a Dependency Model

Key questions discussed as part of the thought experiment and with an emphasis on exploring cost estimate uncertainty with a dependency model were:

- How does the value of a cost variable at time period 1 influence its own value at time period 2? This concept was termed “Forward Eigenvector.”

- How does the value of a cost variable at time period 1 influence the value of another cost variable at time period 2? This concept was termed “Forward Dependency.”

- How does the value of a cost variable at time period 2 influenced its own value at time period 1? This concept was termed “Reverse Eigenvector.”

- How does the value of a cost variable at time period 2 influenced the value of another cost variable at time period 1? This concept was termed “Reverse Dependency.”

This behaviour can thus be categorised in two directions; the first being forward in time from t=1 to t=2 and the second being backward in time from t=2 to t=1.

The forward direction was fully affirmed in respect to eigenvector and dependency. The backward direction was initially viewed in a reserved manner although specific examples from the data analysis then made this data behaviour more transparent. Relevant examples are shown in Table 6-23:

Table 6-23: Examples of forward and backward cost estimate uncertainty propagation

Source / Type	Forward		Backward	
	Eigenvector	Dependency	Eigenvector	Dependency
Data Analysis	Technical changes	Quantity -> Engineering	Revision of baseline estimate	Cost redistribution -> Estimating
Practice	Billing cycle delays	Engineering -> Estimating	Cost recognition cycle delays	Support -> Estimating

In respect to forward propagation, the most common data behaviour observed was technical changes triggering (eigenvector) cost changes over multiple accounting periods and quantity changes triggering engineering changes over multiple time periods (dependency) with corresponding cost variance propagation. Additional examples often raised during interviews were deviations between planned and forecast billing cycles so that cost variance caused in one accounting period was in fact not accounted for in that period, but in a later or multiple later ones. This cost variance was of forward Eigenvector nature. The frequency of engineering changes leading to changes in estimating with corresponding changes in cost estimate uncertainty was highlighted in relation to forward dependency changes.

In respect to backward propagation, the most common data behaviour observed was the revision of baseline estimates as an example of Eigenvector behaviour and the retroactive redistribution of planned or incurred costs across different cost variance factors and accounting periods (both being examples of dependency based cost estimate uncertainty propagation.) These causes for cost estimate uncertainty factors related to the practical use of financial processes which is not explicitly found as a cost variance factor tracked by the case study data. Examples from practice for backward cost estimate uncertainty propagation proved difficult to identify during the interviews. Indicators suggested for backward Eigenvector behaviour included an affirmation that billing / cost recognition events were rarely forecast correctly and hence generally the forecasts and actuals were not synchronised. In respect to backward dependency the closest example found was that incurred support costs can lead to retroactive changes in estimating which is again coupled to a relevant instability of financial processes. The insights related to the backward propagation of cost estimate uncertainty led to further investigation into the perceived reasons for the instability of relevant financial processes which identified the cost estimation conundrum as a descriptor for the manner in which cost estimate uncertainty propagates across the whole product life cycle.

6.6 Implementation at an Aerospace Manufacturing Company

6.6.1 Overview

The presented framework was also applied to estimates for the whole project cost of several dozen platform similar innovative high value manufacturing products at a major aerospace manufacturer. The intent of this case study was not only to examine whether the presented framework was applicable to a different context with eight dimensions, but also to explore the applicability to a specific industrial cost estimating activity within a unique aerospace manufacturing company scenario. As a result of the effort, the framework is in the process of being integrated into the company processes for cost estimation.

6.6.2 Total Cost Growth Curves in Research and Development

The challenge addressed was identifying early whole product life cycle cost estimates for research and development of innovative high value manufacturing products which resulted in significant cost overruns. In order to illustrate the context of the effort

Figure 6-10 illustrates anonymised total unit cost growth curves of six exemplary innovative high value manufacturing products over time based on examples from the organisations. Each line represents the total research and development cost of a specific product from the start to the end where a handover is made to production.

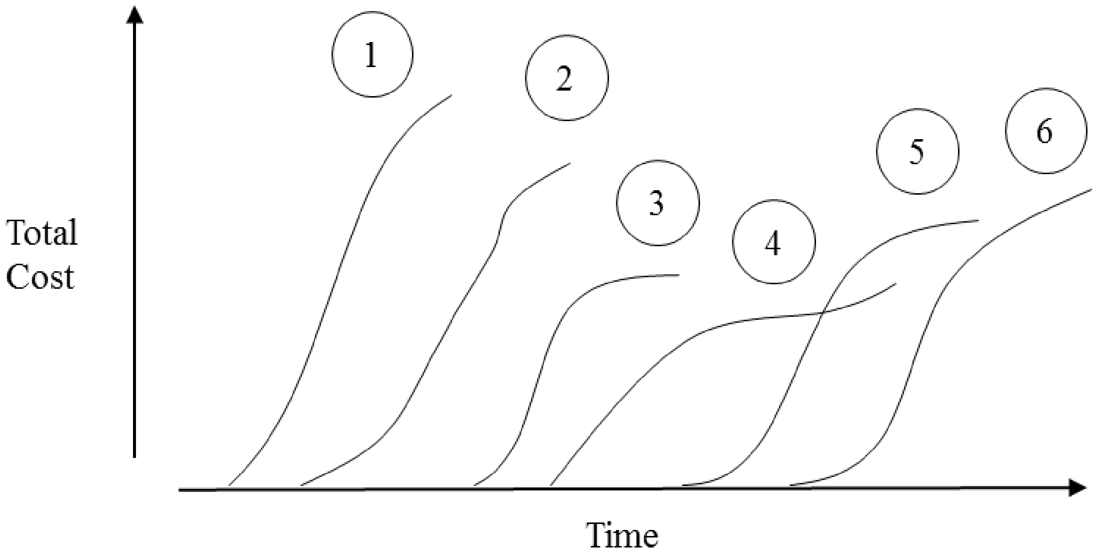

Figure 6-10: Exemplary total cost growth curves during research and development from industry

Managing these cost growth curves is about maintaining total cost growth within budget limits, ensuring a hand-over into an operational phase within expected time-lines and ensuring that the burn-rate of financial and human resources meets the relevant planning of the relevant organisation(s). The effectiveness of cost growth curve management then depends on understanding the interdependency of cost variance factors whereby the more innovative the product the less information for these factors is available by default (small cost data conditions thus exist). By representing data in a geometrical manner (specifically as polar force fields), the framework presented in this paper significantly increases the amount of information that can be drawn from minimum data (therefore data from a single time period) and can thus contribute to increasing the appetite for innovation by reducing the uncertainty of cost estimates and increasing visibility of relevant actionable management levers.

6.6.3 Geometrical Analysis

Cost estimates were available for each of the eight sub-assemblies of 61 products while the actual cost for the whole product was available for assessing the accuracy of the estimate. For the purpose of this case study the sub-assemblies are termed A-H. The presented method was applied to each estimate in order to determine whether any specific (set of) geometrical attributes of the vector space correlated to differences between actual and estimated cost, and in order to create a whole product cost dependency model without reliance on prior information. Initially for each estimate the current state vector graph was generated using the relative cost estimate for each sub-assembly as shown in an exemplary manner for one project in Figure 6-11:

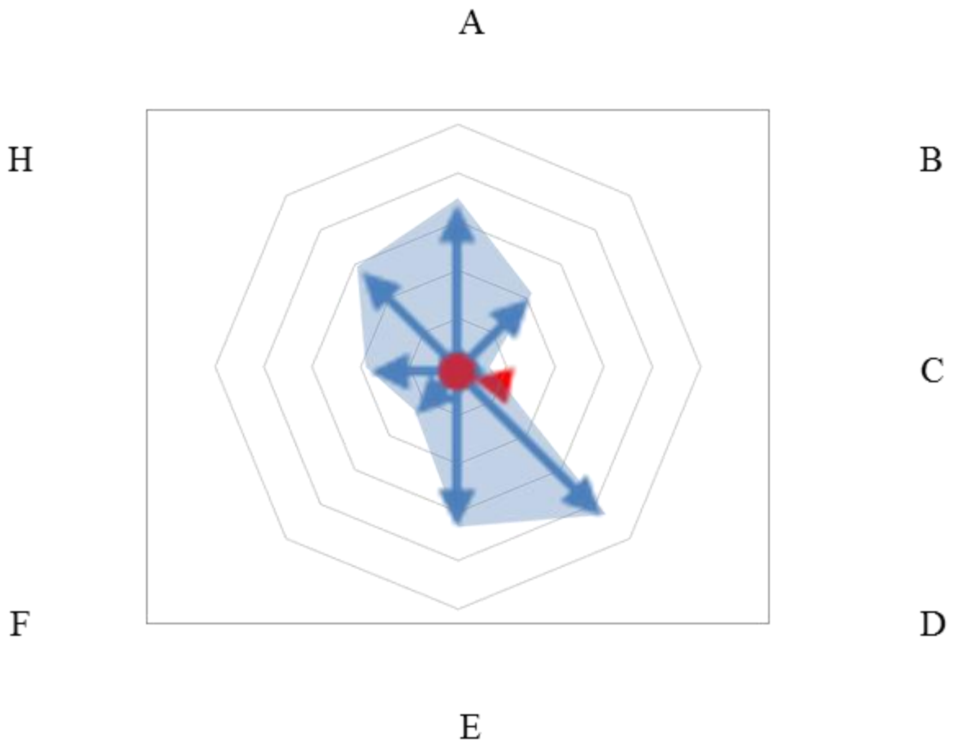

Figure 6-11: Exemplary current state vector graph

The geometrical attributes of the current state vector graph were then determined for each estimate. For the actual current state vector graph in Figure 6-11 this was:

- Direction of the aggregated vector: 114°

- Magnitude of the aggregated vector: 2% of Actual

- Geometrical Symmetry: 74%

The forecasting technique was then applied to each estimate resulting in forecast attributes for each estimate. For the actual current state vector graph in Figure 6-11 this was a geometrical forecast accuracy for the mode of 1% and a geometric uncertainty range of +187%.

The actual arithmetic forecasting accuracy for the example shown in Figure 6-11 was 2%. Figure 6-12 to Figure 6-15 illustrate the relationships of key geometrical attributes to the actual arithmetic forecasting accuracy for all estimates.

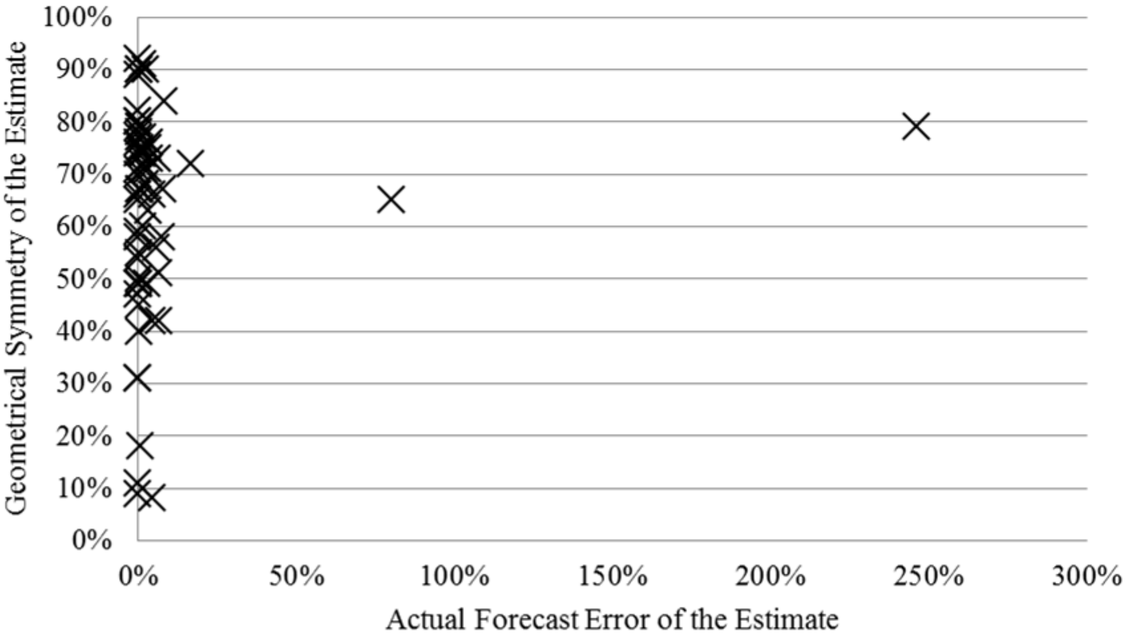

Figure 6-12: Estimate error v. vector space symmetry

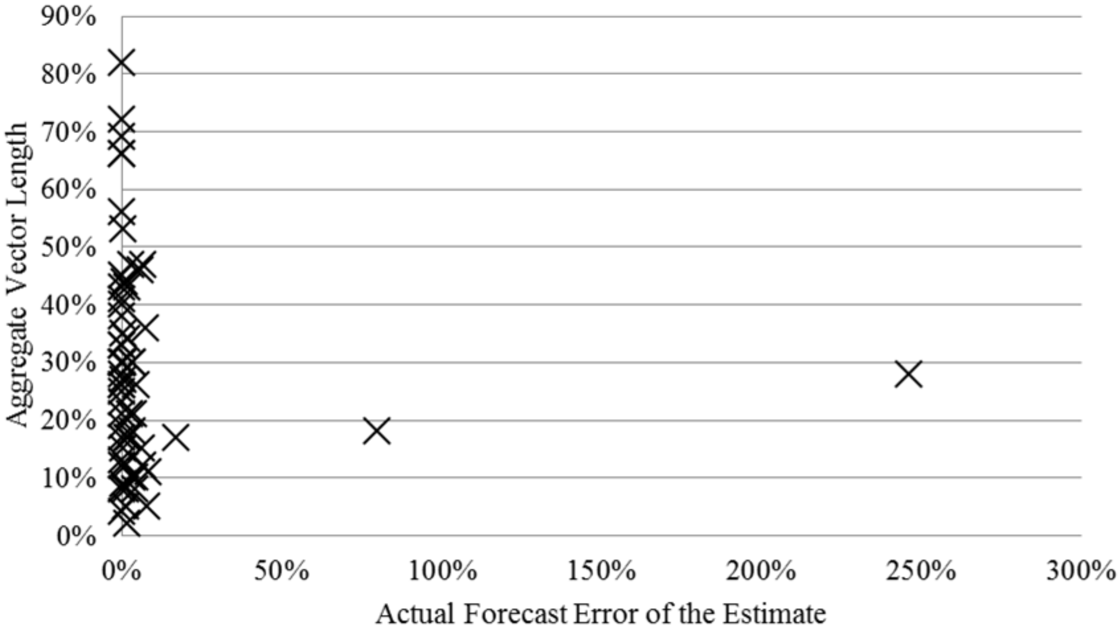

Figure 6-13: Estimate error v. aggregated vector length

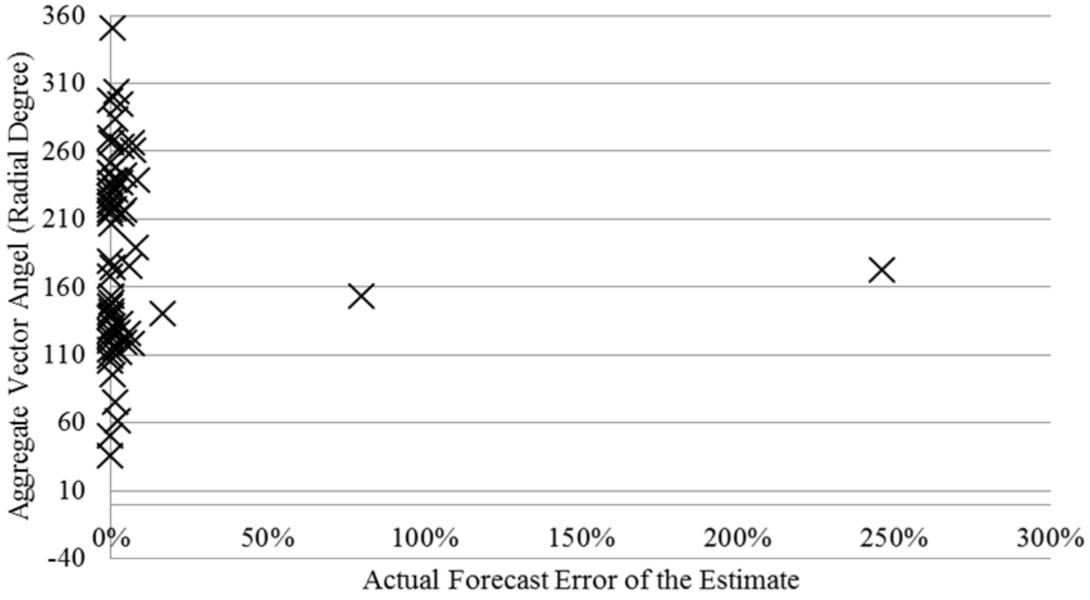

Figure 6-14: Estimate error v. aggregate vector angle

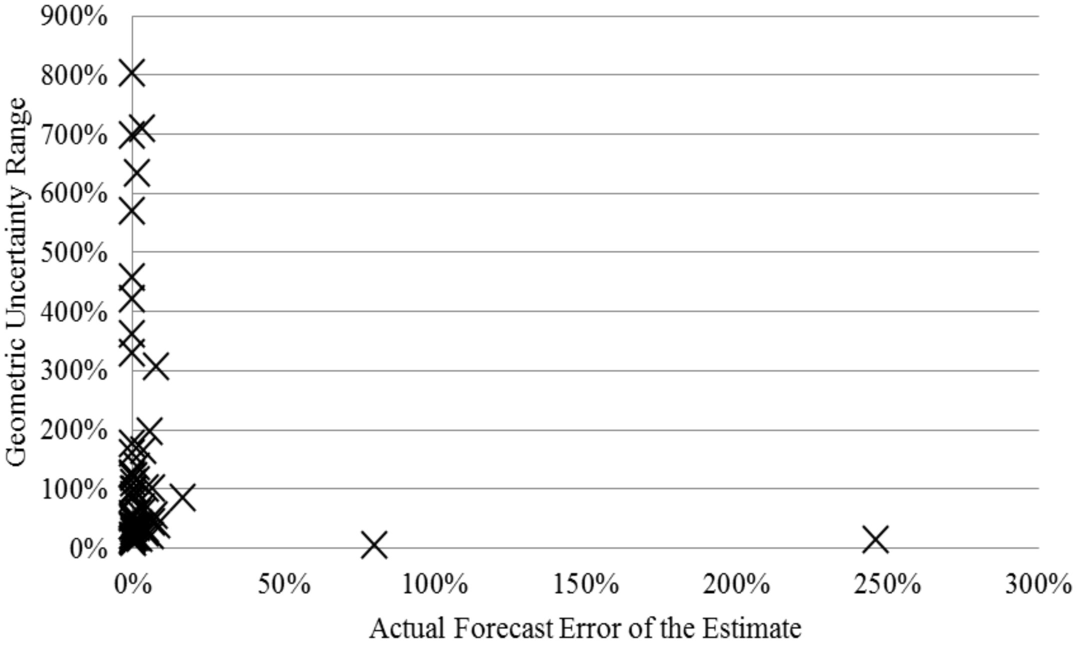

Figure 6-15: Estimate error v. geometric uncertainty range

6.6.4 Geometrical Attributes Suggesting Deviations

Based on assessing the 61 available estimates those likely to exceed a 10% cost estimate accuracy could be identified with 75% (3 of 4) accuracy using the following “AND” criteria. The 10% limit was agreed as an experimental threshold based upon guidance of the accountable cost estimation expert (Chief of Project Estimation, aerospace manufacturing company, personal communication, 2017).

- The direction of the aggregated vector is between 140° and 172° as shown by the amber segment of the compass visualisation in Figure 25:

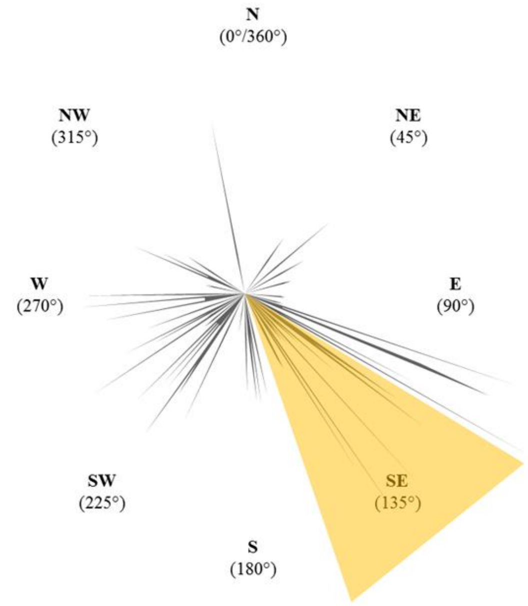

Figure 6-16: Radial distribution of actual vector space aggregated vector angle

Every line in the compass visualisation used in Figure 6-16 represents an aggregated vector describing the direction and relative magnitude of a cost estimate forecast using the polar force field model.

The radial degree of aggregated vector (rd_{av}) is calculated as follows:

    - $rd_{av\ actual}$ = =IF(90-DEGREES(ATAN2(($x_{av\ (actual)}$ -100);(100- $y_{av\ actual}$)))<0;(270+(180-DEGREES(ATAN2(($x_{av\ (actual)}$ -100);(100- $y_{av\ actual}$)))));90-DEGREES(ATAN2(($x_{av\ (actual)}$ -100);(100- $y_{av\ actual}$))))

- The aggregated vector length is between 17% and 28%
- The geometric uncertainty range is between 5% and 85%.
- The vector space symmetry is between 65% and 80%. While the direction and magnitude of the aggregated vector and the uncertainty range are calculated by the presented research method the attribute of symmetry is drawn from relevant work by Schwabe et al. (2016b) and calculated as follows:

1. Calculate actual area of polar force field segment ($AS_{n\ actual}$)
   - $AS_{n\ actual} = 0.5*(cvr_{n\ forecast})*(cvr_{n+1\ forecast})*SIN(rd)$
2. Calculate actual area of polar force field (A_{actual})
   - $A_{actual} = \sum_1^n AS_{actual}$
3. Calculate perimeter of polar force field (P_{actual})
   - $P_{actual} = \sum_1^n SQRT\ ((cvrn_{forecast})+(cvrn+1_{forecast})-2*((cvrn_{forecast})*(cvrn+1_{forecast})*COS(45)))$
4. Calculate reference area of polar force field ($A_{reference}$)
   - $A_{reference} = (0.5*(P_{actual} / n)^2*SIN(rd))$
5. Calculate symmetry (s)
   - $s_{forecast} = A_{actual} / A_{reference}$

6.6.5 Benefits and Way Forward

The technique was able to identify the three of four estimates where actual cost eventually exceeded the estimate by more than 10% based solely on the geometrical attributes of the current state vector space. The whole product cost dependency model was generated without previous information in a form suitable for detailed system dynamics simulations.

6.7 Summary

Chapter 6 provided an in-depth view of verification and validation efforts performed when applying the outputs of the investigation to its findings. The four research findings were subject to verification and validation exercises through the sources data analysis, (semi-) structured interviews and surveys, game playing and a thought experiment.

Chapter 7 concludes the investigation by initially discusses the research findings by revisiting the research context, fulfilment of the research objectives, and examining the findings of the research study and the nature of these as contributions to knowledge. Quality, generalisability, and implications of the findings are discussed followed by a review of the benefits for research and industry and boundaries of the study as a whole.

CHAPTER 7: DISCUSSION

7.1 Introduction

This chapter discusses the research findings by revisiting the research context and examining the findings of the research study and the nature of these as contributions to knowledge. Quality, generalisability, and implications of the findings are discussed followed by a review of the benefits for research and industry and boundaries of the study as a whole.

7.2 Research Context

The research context is defined as the forecasting of cost estimate uncertainty under conditions of small cost data for innovative high value manufacturing products. Specifically in the context of this investigation this means that cost variance data for at least three causes of variance for a single period of time is available. Uncertainty is defined as the two point range between a best case and a worst case as calculated by the highest and lowest forecast values generated by the vector based forecast methods. This section revisits the research problem, the research question, the research hypothesis and the research gaps as discussed in Chapter 1.

The research problem addressed is that in current practice and in particular for innovative high value manufacturing products, the forecasting of cost estimate uncertainty occurs under conditions of small cost data where parametric and / or regression based forecasting techniques are not reasonably applicable since often only data for a single time period is available for forecasting and forecasting accuracy is considered fundamental for reducing innovation hesitance and achieving competitive advantage.

The research problem gives rise to the research question. The research question is whether the geometry (shape) of small cost data is a viable technique for forecasting the propagation of cost estimate uncertainty over time. The concept of shape refers to the geometry exhibited when small cost data is visualised as a polar force field. “Viable” means that the technique is at least as repeatable, robust and fast as current practice.

The research question is investigated through a research hypothesis. The research hypothesis is that if the arithmetic state space of actual cost variance is represented as a polar force field then the state and (simplified) dynamic space of future cost variance can be derived through principles of vector algebra.

The research hypothesis is initially investigated through a series of literature reviews. The literature reviews focused on identifying metrics for visualising, quantifying and forecasting cost estimate uncertainty which had been used historically, were currently recommended in practice, were actually applied in industry and such that were maturing for possible future application in industry.

The research gap identified was primarily that all uncertainty quantification metrics identified in the literature review depended on the use of more than one historical data set. Further literature review determined that the default alternative to identified metrics is geometrical metrics and that these do not find consideration in the field of cost estimation although such are widely established in other sciences. Polar force fields hereby represent a distinct sub-set of geometries. In summary:

- Neither parametric or regression based cost estimation techniques are reasonably applicable for forecasting cost uncertainty under the condition of small cost data which does not provide the minimum information required for these approaches.

- Qualitative approaches such as analogy or expert opinion do not provide sufficiently robust results for forecasting cost uncertainty under conditions of small cost data as evidenced by the frequency of significant overruns of budgeted costs.

7.3 Research Findings

The findings of the research study are focused on the visualisation, quantification and forecasting of cost estimate uncertainty for small cost data through a framework consisting of steps for calculating, composing, forecasting and quantifying a most likely uncertainty range. Based on the research gaps identified the key research findings are:

- A method for preparing arithmetic small cost data for geometricisation (steps 1,2,3 and 4 of the forecasting method presented in section 5.1).

- A method for composing a polar force field based on small cost data (steps 5,6,7 and 8 of the forecasting method presented in section 5.1).

- A method for forecasting the propagation of cost variance based on the shape of the polar force field (steps 9,10,11,12,13,14 and 15 of the forecasting method presented in section 5.1).

- A method for quantifying the uncertainty of the forecast based on the forecast propagation of the polar force field (steps 16, 17 and 18 of the forecasting method presented in section 5.1).

- A method for creating a dependency model helping to explain the results generated by the integrated polar force field forecasting framework (section 5.2).

The first four contributions are presented as an integrated framework and the fifth contribution offered to explain the forecasting behaviour of such.

A method and its process model for preparing arithmetic small cost data for geometricisation (steps 1,2,3 and 4 of the forecasting method presented in section 5.1) is presented. The primary steps involved are performing any necessary conversions to absolute numbers since polar vector graphs only accept positive numbers and the conversion of these absolute numbers into relative figures in order to ease geometric comparability of forecasts for different time periods and products.

A method and its process model are presented for composing a polar force field based on small cost data prepared for geometricisation (steps 5,6,7 and 8 of the forecasting method presented in section 5.1). The process model consists of a series of activities to transform cost variance from a technical baseline estimate due to three or more causes into a polar force field. The sum of all cost variance vectors is represented by an aggregated vector which is declared to describe the force acting on each individual cost variance vector and determining the future value of these. The actual polar force field is input to the next element of the integrated framework. The polar force field is defined by a series of invariants and independent variables.

A method and its process model for forecasting of small cost data using polar force fields is presented (steps 9,10,11,12,13,14 and 15 of the forecasting method presented in section 5.1). The process model consists of a series of activities to forecast cost variance as defined in the Composition phase. Forecasting algorithms for the upper limit, lower limit and mode of a T-shaped distribution suitable for input into a Monte Carlo simulation are presented.

A method and its process model for quantifying the uncertainty of the forecast is presented (steps 16, 17 and 18 of the forecasting method presented in section 5.1). The process model consists of a series of activities to quantify the uncertainty of the forecast made in the Forecast phase. The use of a traditional Monte Carlo simulation is suggested to enhance the uncertainty indications offered by the differing results of the three forecast methods as discussed in the verification and validation of the research effort. In order to support this the shape of a custom triangular probability distribution is defined through dependent variables. This element of the integrated framework is designed to support diffusion of the research findings by providing a novel technique for seeding the information needed by a Monte Carlo simulation based on relevant evidence and respecting the small cost data nature being examined.

Finally, a dependency model and its process model helping to explain the forecasting behaviour of the integrated polar force field forecasting framework is presented (section 5.2). Based on the data available for absolute cost variance a regression analysis was performed in order to identify the correlation function between all cost variance variables. The slope of the linear correlation function was used to determine the value of future impact, while the co-efficient of correlation was used to determine the sequence of impacts between the variables and their relative speed. Impact, sequence and speed were then used to quantify cost variance propagation.

7.4 Contributions to Knowledge

Reflections on the degree to which the research findings can be considered to represent original, relevant and significant contributions to knowledge need to start with an examination of the findings in light of the research problem defined. The research problem essentially poses the challenge of synchronising forward and inverse uncertainty quantification approaches, whereby the starting conditions of small cost data challenge the reasonable use of any approaches relying on the Central Limit Theorem. This limit then challenges the appropriate use of statistical approaches and only leaves geometric approaches for investigation.

While the effort involved in verification is relatively straight-forward (assuming the same techniques are applied as in the estimation), the effort involved in the creation of an estimate depends on a multiplicity of factors that often strain computational resources if needing to be estimated in polynomial time. This constraint appears lifted by the research findings in that the focus on simple geometric form (polar force field) progression reduces the relevant factors to a minimum. In this respect, the hypothesis drives the research in a unique direction by focusing on starting conditions that are independent of the use of statistical forecasting approaches and emphasises polar force fields as a simplification perspective and dimensional reduction method.

The research study determined that small cost data can be classified as a short string from the perspective of computational complexity and as such does not contain sufficient information for identifying patterns that can be used for forecasting purposes. The ability to identify patterns is fundamental to the applicability of parametric and regression based forecasting techniques, so that an objective measure (Kolmogorov complexity as applied to short strings) was identified for discounting their use under conditions of small cost data. This insight then determined the need for an alternative approach whereby geometrical perspectives were chosen for investigation since these preceded the development of statistics and regression historically.

The insight regarding the nature of small cost data and hence the choice of geometrical perspectives raised the challenge of how to represent such in a geometrical manner and how an estimation technique might be developed from that. An investigation into techniques used for the visualisation of uncertainty initially identified the spider chart as a technique for converting small cost data into a simplex geometry, and set the foundation for the discovery of symmetry for describing cost estimate uncertainty propagation. Further investigation into the nature of spider charts then determined the vector space nature of such so that an evolution in geometric representation could be achieved with ensuing correlation to a potentially generic dependency model. The use of polar force fields then created the pre-requisites for simple vector aggregations / compositions and based on that the development of simple forecasting methods. This then evolved into an understanding that vector spaces are force fields and that the invariants defined classify the chosen approach as a polar force field in particular.

The discovery of patterns in small cost data is by default not feasible using arithmetic approaches due to its short string nature. Polar force field based forecasting methods presented in this investigation were able to achieve a reasonably robust estimate in respect to future total cost variance, but struggled to achieve such on the individual level of cost variance dimensions. The forecasting accuracy for total cost variance hence suggests that a pattern is emulated by the forecasting methods whereby these were in essence overall continuous growth of total cost variance for all dimensions, and relative directional growth for each individual cost variance.

Cost estimate uncertainty is typically the result of the addition of cost probability density functions at a component level and may be enhanced by risk considerations. The uncertainty is hereby set at the time of baselining for the technical estimate, and then usually addressed by a fixed contingency. This approach, however, does not do justice to the fluctuating nature of cost estimate uncertainty as it progresses through the whole product life cycle. This dynamic behaviour of propagation is highlighted to sensitise for the limitations of existing cost uncertainty management and containment approaches, which can be reduced through deeper understanding of the dependency models which are approximated by the polar force field model. Important to note is that the concept of “dynamic” here refers to a change in cost variance or uncertainty over time, and not the change in a dynamic state space as defined for purposes of future work.

The use of the dependency model proved central for translating the behaviour of the polar force field model to the heterogeneous stakeholder audience that was interacted with. Due to it having been developed directly through data analysis, it was challenged less frequently than concepts and insights generated through qualitative methods. Additionally, it provided stakeholders with an opportunity to identify actions / interventions to not only improve forecasting accuracy and contingency calculation, but also to make the relevant whole product life cycle cost more forecastable in the first place.

The (in-)variant metrics of the polar force field model (composition and forecasting) are drawn from the geometrical approach taken by the framework, and are not represented in the future metrics identified in the literature review.

The uncertainty quantification approach which translates the results of the polar force field forecasting approach to a three point range estimate using a Monte Carlo simulation then creates the opportunity for applying those identified state of future metrics although the concerns regarding applicability of Central Limit Theorem based techniques remain.

7.5 Quality, Generalisability and Implications of Findings

In this section the researcher critically examines the quality, generalisability and implications of the findings for theory and practice including a discourse into the potential business impact of wide-spread adoption.

At the outset of the investigation, this researcher designed a research methodology based upon review of relevant literature including university guidance and exemplary theses. Surveys, (semi-structured) interviews, workshops, case study data of public and confidential nature, serious game plays and general discussions were used to collect and evaluate data. Collected data was aggregated and analysed using techniques emerging from multiple research efforts. Due to the emergent nature of the investigation, an initially planned normalised database across multiple data sources could not be realised with the available resources however the consolidation of insights and data in a series of published articles did support the creation of a data set that was suitable for analysis. For each of the research findings, efforts were made to ensure that the data utilised was publically available and could be reviewed in a manner which allowed for replication of insights and results.

Principles applied to ensure quality of research are:

- The extent to which the research addresses the aim and objectives set out to achieve.
- The degree to which the research effort included the participation of relevant stakeholders from theory and practice.
- Evidence that research findings were reviewed and supported (at least in principle) by a relevant peer audience.
- The extent to which the research methods were clearly articulated and adherence to these critically monitored.
- The ability to demonstrate an objective view of the research progress.
- The ability to acknowledge the qualitative nature of insights drawn.

The research aim and objectives set at the beginning of the research study experienced a degree of change during the time-frame of the investigation. The change was primarily due to the growing insights and competence of this researcher in the field of investigation and is considered to be a natural result of an extended research effort. In this respect, the valid concern arises that data gathered at various stages may lose coherence as the research perspectives shift over time. This researcher has critically examined this concern and concludes that the changes to the research aim and objectives were primarily related to increased focus (hence remaining within the original intent of the investigation) and that whenever data from earlier research phases was drawn upon its applicability was carefully reviewed and considered. In this respect the investigation fully addresses the objectives the research study set out to achieve.

The participation of relevant stakeholders from theory and practice proved a continuous challenge during the investigation which this researcher does not believe to have been resolved satisfactorily. In general, the greatest degree of participation can be seen when stakeholders share a common purpose and in the context of research, this is generally evident in traditional project settings. Since the investigation occurred outside of a traditional project setting, it consistently proved difficult to engage constructively with stakeholders. The impact of this lack of satisfactory participation is also not considered to be alleviated through peer reviewed publication since the general quality of such is a separate field of critical reflection.

While multiple research findings were published in peer reviewed journals and conferences, this researcher questions the quality of these reviews in respect to the content of the findings. While it is appreciated that the peer reviews were primarily related to formal presentation and discussion of research results, this researcher remains concerned as to the quality of content related feedback received through this process. In contrast, the experiences of experimentation with stakeholders were deemed highly beneficial although these related primarily to operational perspectives versus critical review of underlying research. The creation of a LinkedIn group (see https://www.linkedin.com/groups/6939117) focused on cost risk and uncertainty to alleviate this lack of quality interaction did not reduce these concerns although over 70 professional experts in the field joined and followed regular updates made by this researcher.

This researcher continuously attempted to maintain an objective view of the research progress and avoid falling prey to cognitive biases or filters that might unduly influence the intent to produce objective research. Continuous discussion of research progress with multiple stakeholders served as an effective way to ensure objectivity and correct behaviours that might result in skewing of research results. During the complete course of the investigation this researcher believes he was able to acknowledge the degree to which qualitative insights were drawn was understood and subject to explicit reflection.

The research findings evolved out of an investigation primarily focused on products manufactured for the U.S. DoD. The data used was publically available. While validation and verification of insights through commercial non-military data was also performed this researcher considers this to have been of limited value due to the unique context giving rise to the data examined and the generally confidential nature of that data. This researcher believes that the principles of the research findings are applicable across the complete range of high value manufacturing products.

While this researcher´s understanding of the potential implications of the research findings for theory and practice continues to evolve the verification and validation efforts have suggested two questions that may in particular be important to explore:

- In respect to theory the investigation suggests that conditions of small cost data dominate when estimating for products especially when these exhibit a high degree of innovativeness. This state has not been intensively explored to date and its attributes may impact our understanding of uncertainty quantification as a whole. This researcher suggests that the findings of the investigation offer the opportunity of an innovative view of cost variance which makes explicit the tacit interpretations often applied by experts in the field. It is the opinion of this researcher that the primary novelty of findings is in fact related to giving expert opinion and analogy a more quantitative foundation.

- In respect to practice the investigation considers the greatest part of practice where estimation techniques relying on the Central Limit Theorem are applied to data sets not meeting minimum criteria for such as not reasonably robust. The impact on practice is expected to be a greater acknowledgement that the results of existing techniques need to be carefully questioned, and that permission is given to doubt these in a pragmatic manner. The concept of the shape of data as an alternate view of the estimating process is provided which bears the potential of leading to a more effective decision support approach by business decision makers.

7.6 Benefits For Research and Practice

The benefits of the findings of the investigation are determined by the viability of the solution offered for the problem presented at the outset. The fundamental problem addressed is innovation hesitance in respect to high value manufacturing products which leads to a loss of competitive advantage and risks the future of organisations. The primary cause for this is put forward as the inaccuracy of relevant cost estimate uncertainty estimates. During the course of the investigation, this researcher determined that the estimation inaccuracy was primarily due to conditions of small cost data. In order to address this condition, an integrated framework was developed and validated. The potential benefits of this framework need to be seen from two perspectives, therefore in respect to research and industry.

The potential benefits of the investigation for research can be understood as contributions to closing the research gaps identified. In particular, the introduction of polar force field forecasting techniques is considered to not only be novel, but also open up a wealth of alternate research directions in the field.

By identifying the estimating condition of small cost data, the investigation has uncovered a unique state which is commonly encountered, but the significance of which has not been recognised to date. The potential benefit for research is that this state has now been clearly identified and described and its significance for practice highlighted.

Based on the lack of alternatives to the Central Limit Theorem under conditions of small cost data, the investigation has suggested and investigated perspectives offered by polar force fields. By setting the foundation for (re-) introducing spatial geometry as an alternative to arithmetic techniques, research is encouraged to critically reflect on the dependency which has developed over time in the field in respect to having “enough” data to work with approaches based on the Central Limit Theorem (and ignoring its lack even if apparent). The investigation suggests that polar force fields, in fact, present a more reasonable perspective on cost estimate uncertainty under conditions of small cost data. As data increases and matures, the natural progression is to parametrics and then to regression based forecasting techniques. In summary, the investigation provides a potential benefit to research by introducing a new research context, providing new tools for investigating this context, and extending the range of perspectives for interpreting the context.

The primary potential benefits of the research findings and contributions to knowledge for industry can be understood as the availability of a method for quantifying, visualising and forecasting cost estimate uncertainty for innovative high value manufacturing products, which is independent of the Central Limit Theorem and is at least as fast and accurate as established alternative technique such as parametrics or regressions. In addition, it requires significantly less data and may reduce the cost of preparing robust and viable estimates in the first place. The use of the Monte Carlo simulation then allows for an eased transfer of the framework into practice in order to permit decisions at required confidence levels.

7.7 Research Boundaries

This section identifies the boundaries of the research in respect to the hypothesis, the research method applied and the findings of the study.

The primary boundary of the research is given by the hypothesis itself. As represented by the whole product life cycle, the boundaries drawn by a hypothesis will, by default, de-emphasise the influence of external factors and conversely highlight the influence of internal dynamics. This can also be seen in the fact that the greatest determinants of cost estimate uncertainty propagation identified by the dependency model, are primarily influenced by factors outside of the system of relevance.

The boundaries of the research method also need to be considered from the perspective of the activities involved and the manner in which these activities were interlinked. The activities of the research method involved conducting literature reviews, data selection and analysis, performing (semi-structured) interviews, game playing, a thought experiment, holding presentations to a variety of audiences and conducting surveys (through interviews and (online) surveys). The specific perspectives are:

- Literature reviews: The identification of literature relevant to the various phases of the research study was based on the use of key terms to support searches and the review of references in literature identified whereby this was initially based on titles only. Literature identified for potentially more thorough review based on key words or titles was then examined through review of the relevant abstracts. Only when the abstracts suggested relevance was the text of the paper reviewed. Three key limitations were seen in that: (a) the volume of potentially relevant literature was typically too high to review fully within time and resource constraints; (b) key word taxonomies may have been incomplete and evolved over the course of a research study, and (c) titles and abstracts may not have been sufficient to reflect relevant knowledge contained in the literature itself. The research study attempted to address this through an evolving taxonomy of key words which were embedded in automated search queries that provided daily updates of any relevant literature available on the Internet. Search queries provided up-to-date information regarding titles and sources of literature published which were both helpful in increasing review efficiency.

- Data selection: The selection of data suited for analysis is fundamental to the quality of a research study. This researcher attempted to reduce concerns by identifying data which is publically available and also subject to previous investigations so that insights from those studies could be considered and independent verification of data analysis was enabled.

- Data boundaries: A sub-set of available case study data was drawn based on creating a data set for analysis which was topologically coherent in respect to the cost variance dimensions reported on. The presented research findings are therefore valid for this sub-set of data only and need to be validated in a wider scope within available case study data.

- Data analysis: Since a very large spectrum of data analysis techniques exist, any choice of such, by default, leads to boundaries. While the presented techniques have sought to emphasise basic mathematical operators for the sake of simplicity and consciously minimised the amount of statistical analyses present in the deliverables a large variety of such tools was applied during data discovery in order to gain a deeper understanding of data structures and behaviours. The boundary is hence set by default and can only be mitigated to the degree that validations across multiple perspectives are performed especially in respect to subjective validation of insights generated through interaction with experts. In this respect, the serious game developed played a pivotal role in enabling the elicitation of such expert opinion.

- (Semi-structured) interviews: The boundaries related to interviews are significant and primarily related to the degree that a representative sample of interviewees is found, the extent that the questions are sufficiently focused and the manner in which the interviewer is able to gather a relevant and appropriate input which then allows for correlation with other inputs received via interviews. Qualitative in nature and highly sensitive to bias this researcher considers the interviews to be primarily helpful in validating the problem of relevance whereby the more the questions focused on research details (therefore requiring substantial expert knowledge for interpretation), the less useful the input that was solicited. The primary value of input gained through (semi-) structured interviews was hence guidance in what the relevant applied challenges were and through that a deeper appreciation of the cost estimation maturity of the relevant organisation. These insights then helped develop deliverables designed for easier adoption and diffusion.

- Game playing: A serious game was developed and used to support elicitation of expert opinion regarding the findings of the research study. As with any experimental technique, the boundaries primarily lie in the design and application including the degree to which the results influence the results of the research study overall. In the context of the research study, the serious game provided an alternate interview context and method that allowed this researcher to initially educate the participants in the behaviour of the data examined and the opportunity to examine their own perceptions and experiences through a more objective lens. The ensuing conversations among participants, especially when following the scripted debriefing paths after each round, helped focus conversations and elicit the opinions required for validating the research results and designing the deliverables for more effective dissemination and diffusion in environments of potential application.

- Thought experiment: In order to improve knowledge elicitation in interviews and game playing, a thought experiment was developed to visualise the key dynamics being investigated. The boundaries of the thought experiment lie primarily in their effect in focusing the attention of participants on a specific question of relevance and hence by default leading to the blending out of other perspectives which might be relevant and of importance. This boundary was addressed by at times embedding the thought experiment in the serious game which had sensitised the participants to the broader context. Additionally, the insights gained were challenged by perspectives similar to those relevant for interviews and managed in a similar manner.

- Surveys (embedded in interviews and (online) surveys): Multiple surveys were conducted during the course of the research effort. Surveys were included in (semi-structured) interviews, distributed as documents via email or provided online. While surveys conducted in interviews or via email exchange allowed a qualitative assessment of the respondent belonging to the target group of interest (i.e. based on their role in their organisation), this was not the case in online surveys where anonymity of the responses needed to be maintained (i.e. demographic details were voluntary and their visibility subject to explicit consent). The boundary raised by the anonymity of responses is a serious one in respect to qualifying the results received. The boundary was addressed by using survey feedback only to validate findings from more quantitative research activities (i.e. data analysis).

The manner in which research activities are interlinked can also lead to boundaries since the research findings can be considered to have an emergent nature so that results obtained at early stages of the research may not align with results obtained at later stages. The sequence of activities over time is thus relevant and also the manners in which the results evolve / mature in one activity influence the next. These boundaries were addressed through careful assessment of how information gathered was integrated into the research findings and also through a consistent focus on the analysis of the data sets through techniques developed in order to maintain an objective as possible view.

The boundaries related to the findings of the study also need to be considered in light of the research activities related to them. In respect to the nature of small cost data, the primary challenge needs to be seen in the fact that little research has been conducted into forecasting based on short strings and especially in respect to data sets with a unit of one. While the number of data sets required for meeting the requirements of the Central Limit Theorem can be considered to lie between four and 41, and that between two and four data sets some degree of parametric estimation is enabled, the existence of only one data set in cost uncertainty estimation is not researched and typically relegated to the realm of expert opinion or analogies. In respect to quantifying, visualising and forecasting cost estimate uncertainty the primary boundary is considered to be in the simplified data analysis techniques chosen. The polar force field forecast models are limited to basic mathematical operators while the dependency model is derived from linear trend applications. In both cases, the boundaries become evident in the accuracy of forecasts although this researcher considers especially these limitations outweighed by the advantage of making the novel principles more accessible to a wider audience.

For the propagation tendency of cost estimate uncertainty the boundaries are linked to the quality of the data being used and especially to the restricted amount of context information that is available. While the mathematical techniques are transparent and generate reasonable results repeatedly, the calibration of the results suffers from not being aligned thoroughly to the underlying data sets caused primarily by research constraints during the investigation.

The dependency model approach suffered primarily from the cases studies representing dimensions of cost variance which were not monitored by the stakeholders in their organisations. To the greatest degree no specific cost variance factors of this type were monitored and if so, exceptionally, then these were of a different nature.

7.8 Summary

This chapter discussed the research findings by revisiting the research context, and examining the findings of the research study and the nature of these as contributions to knowledge. Quality, generalisability, and implications of the findings are discussed followed by a review of the benefits for research and industry and boundaries of the study as a whole.

Chapter 8 concludes the investigation and provides recommendations for future research. Emphasis is placed on how the set objectives were achieved.

CHAPTER 8: CONCLUSIONS AND FUTURE WORK

8.1 Introduction

This chapter concludes the research study and provides recommendations for future research. Emphasis is placed on how the set objectives were achieved.

8.2 Fulfilment of Research Objectives

The research has achieved the aims and objectives outlined in Chapter 1. The hypothesis developed to shape the investigation has been explored and confirmed within the boundaries of the limitations identified.

8.2.1 Objective #1: Capture and Understand Current Methods and Metrics

The initial objective was focused on capturing and understanding current methods and metrics for estimating cost uncertainty in the high value manufacturing industry through literature review and industrial interaction. The purpose of this activity was to gain a deeper understanding of how the research problem is addressed in practice. In order to achieve this objective and fulfil the purpose of it the following activities were completed:

- Performance of an initial exploration of the dynamic nature of risk and uncertainty. “Dynamic” was hereby considered to describe the change of the value of uncertainty over time from a state space perspective. (Schwabe et al., 2014a)

- Evaluation of reports on 44 publically available innovative high value aerospace manufacturing projects (Schwabe et al., 2014b).

- Creation of initial overview of research perspectives (Schwabe et al., 2015c).

- Completion of an in-depth literature research (Schwabe et al., 2015b).

- Validation of the findings of the literature research and case study review by conducting a series of (semi-structured) interviews.

- Aggregated, investigated and prepared a set of reference data for the investigation.
- Created and facilitated a community of practice on the LinkedIn platform ("Cost Risk and Uncertainty") for continuous dissemination of knowledge into a relevant community of practitioners. As of March 2018 over 70 individuals from a variety of organisations and with an active interest in the research questions are members.

The objective is considered to have been fulfilled.

8.2.2 Objective #2: Key Metrics of Cost Uncertainty

The second objective was focused on classifying the key metrics for visualising, quantifying and forecasting cost estimate uncertainty and its propagation. The purpose of this activity was to determine whether attributes suitable for investigation from the perspective of the research hypothesis were available or such were in need of redefinition for properly examining the research questions. In order to achieve this objective and fulfil the purpose the following activities were completed:

- Completion of an exploration of the dynamic nature of uncertainty from a state space perspective (Schwabe et al., 2015a).

- Developed a polar force field based method for the visualisation, quantification and forecasting of cost estimate uncertainty for innovative high value manufacturing products.

- Developed a dependency model for forecasting cost estimate uncertainty propagation (Schwabe et al., 2016a)

The objective is considered to have been fulfilled.

8.2.3 Objective #3: Visualising, Quantifying and Forecasting Cost Uncertainty

The third objective was to develop a framework for visualising, quantifying and forecasting cost uncertainty and its propagation in the form of a mathematical model. In order to achieve this objective and fulfil the purpose the following activities were completed:

- Developed an approach for selecting cost estimation techniques for innovative high value manufacturing products (Schwabe et al., 2016b)

- Developed an approach for short interval control for the cost estimate uncertainty baseline of innovative high value manufacturing products using a complexity based approach (Schwabe et al., 2016c)

- Developed a mathematical model based on case study data.

- Operationalised the mathematical model with a Microsoft ® Excel based simulation tool.

- Created an integrated framework assembling research findings into a coherent process model.

- Operationalised the integrated framework in Microsoft ® Excel as a step-by-step process model

The objective is considered to have been fulfilled.

8.2.4 Objective #4: Validate and Verify the Framework

The last objective was focused on validating and verifying the framework with a real life case study by using a desktop demonstrator. Two desktop simulators were developed in support of the contribution for visualising, quantifying and forecasting of cost estimate uncertainty and also for quantifying the propagation tendency of cost estimate uncertainty. The desktop demonstrators were created using Microsoft ® Excel.

The first demonstrator uses a vector based input output model in order to convert actual cost variance into future cost variance. Visual Basic is used to generate the relevant vector visualisations. The second demonstrator uses actual cost variance and trends drawn from regression analysis of case study data in order to forecast the most likely cost uncertainty propagation, the symmetry propagation and the uncertainty range. Both demonstrators were part of field-trials of the integrated framework at an aerospace manufacturing company.

Through validation and verification efforts associated with the last objective the developed uncertainty quantification framework was confirmed as a viable enhancement to established cost estimation methods for innovative high value manufacturing products within the limitations discussed.

The objective is considered to have been fulfilled.

8.3 Conclusions

This researcher considers the hypothesis as acceptably investigated, validated and verified. While the context and its limitations present a very specific situation this researcher considers the principles developed as suitable for generalisation across many different scenarios and providing a valuable enhancement to both research and practice.

The presented framework represents an example of a paradigm shift in the visualisation, quantification and forecasting of cost estimate uncertainty leading to more robust estimates of cost uncertainty for innovative high value manufacturing products under conditions of small cost data. Instead of continuing to try and adapt parametric and / or regression techniques to amounts of data which are too small to meet the minimum data requirements of the Central Limit Theorem, a shift to geometric approaches in the form of polar force fields which do not depend on these pre-conditions is presented.

The key contributions to knowledge can be summarised as:

- Definition of the forecasting condition of “small cost data” which represents a previously unexamined object of analysis in cost uncertainty estimation.

- Determination that geometry provides an alternative view of small cost data that is not dependent on the principles of the Law of Large Numbers. This represents a novel technique for working with scarce data that has previously been subject to estimation techniques more suited to large amounts of data.

- Evidence that the geometrical view of small cost data in the form of a polar force field can be used to robustly forecast cost uncertainty. This can be considered as a successful initial effort to transfer basic principles of physics to the whole product life cycle.

- Translation of forecasting results into contemporary forecasting techniques, which represents an important prerequisite for further dissemination of research findings into the practice of cost uncertainty estimation.

The integrated polar force field framework presented in this investigation can be compared and contrasted to the uncertainty quantification categories and metrics identified in the relevant taxonomy and framework developed during the literature review. The latter are primarily related to statistical techniques applied to arithmetic state spaces. The metrics resulting from use of the integrated polar force field framework are however primarily related to vector algebra applied to geometric state spaces which represents a differing perspective.

The uncertainty quantification metrics related to the integrated polar force field framework can be categorised into two application areas the polar force field and uncertainty quantification using the Monte Carlo simulation. While the uncertainty quantification metrics related to the Monte Carlo simulation are statistical in nature and are applied to the arithmetic state space given by the three forecast results provided by the integrated polar force field framework, it is the group of geometric metrics related to the framework which cannot be clearly aligned and indeed draw upon elements of all metric families. The metrics relevant for examination in the framework can be understood as follows across the findings:

- Composition: The key attributes of the polar force field (the coordinate system and its scaling) are held invariant through fixation of the spatial centre, the dimensional sequence, the radial degree and dimensional scaling, the number of cost variance dimensions and the manner in which the cost variance calculation is performed. Since the actual arithmetic state space values are used for the composition of the polar force field, no uncertainty metrics are related to this element of the integrated framework. It should be noted, however, that if these metrics were to be considered as dynamic (versus invariant) they could represent metrics of uncertainty if these changed between the time periods being forecast for. A caveat of relevance, however, is related to the cost variance calculation since it could be argued that if the actual arithmetic value of a cost variance dimension is zero, then, based on case study forecasting results, the relevant forecast for the individual cost variance dimension becomes significantly more uncertain due to the divide by zero error encountered by the mathematical model. Furthermore, it could be argued that due to the innate growth behaviour of the mathematical model and the cost variance calculation approach of considering only absolute cost variance values, an additional degree of uncertainty is injected into the forecasting results as a whole. From the Composition perspective (in relation to the uncertainty quantification metric taxonomy), the relevant metrics could be considered to be related to the specific uncertainty quantification metrics of sample size (“Shape” family), degrees of freedom (“Complexity” family) and to the generic uncertainty quantification metrics of business value and thresholds.

- Forecasting: This element of the integrated polar force field framework, since based on principles of vector algebra, proves unique in comparison to the uncertainty quantification metrics identified in the literature reviews. The primary metrics related to uncertainty are the dependent variables of Eigenvector, torque and symmetry which can only be assigned as new entries to the specific uncertainty quantification metric family “Other”. Significant uncertainty can also be considered to derive from the focus of the vector transformation methods on basic operators of vector algebra (therefore addition and multiplication) whereby these would also be assigned to the uncertainty quantification metric family “Other”.

- Uncertainty Quantification: The uncertainty quantification element of the integrated polar force field framework marks a translation of polar force field paradigms to traditional statistical analysis of arithmetic state spaces. The arithmetic results of the three forecasting methods (therefore the absolute magnitude of the forecast individual cost variance dimension vectors forecast) are input as best case, most likely and worst case into a Monte Carlo simulation hence representing a three point estimate ("Range" family) followed by the selection of a triangular probability density function ("Shape" family). Running a relevant Monte Carlo simulation then achieves a single point estimate ("Point" family) at the desired confidence level using a cumulative distribution function ("Range" family).

- Dependency Model: Finally, the dependency model introduced as a potential explanation for the forecasting behaviour of the integrated polar force field forecasting framework fully includes all common generic uncertainty quantification metrics such as business value, statistics, thresholds and volatility.

The presented integrated polar force field forecasting framework can be seen to represent an innovative approach to uncertainty quantification since it builds on uncertainty quantification metrics related to vector algebra which were not identified in the literature review thus leading to the assignment to the "Other" family in the relevant taxonomy. The forecasting activities of the framework begin to translate these paradigms into a traditional view based on the quantitative seeding of a Monte Carlo Simulation while the explanatory dependency model is firmly anchored in contemporary practice.

The nature of the presented technique in relation to established cost estimation approaches such as analogy / expert opinion, parametrics and regression can be understood based on a variety of attributes. The attributes chosen for their comparative description are based on the insights gained during the investigation and may serve as orientation for the cost estimator / forecaster when deciding on which estimation method to focus on primarily and which to apply in a supporting / validating manner. A tabular summary of the estimation methods and their comparative attributes can be found in Appendix F.

The polar force field method presented in this investigation is considered to reside between analogy / expert opinion and parametrics based on the minimum amount of data needed for its robust application. The alignment of the other methods against these attributes was summarised in Chapter 2. The method is reasonably used when only one data set of cost variance for basing an estimate upon is available. The method can furthermore be used primarily up to that point where enough data is available to apply parametric approaches (therefore four or more data sets with complementary short string complexity groups). In relation to the probability field best suited for the presented framework is suited for forecasts across a large number of whole product life cycle phases especially where data volatility is projected to be highest (therefore chaotic multivalent). As the forecast ranges and projected information volatility drop (driven by increased data availability), the framework remains relevant although established approaches to compression (pattern recognition), homogeneity, shape, range and point lead to it moving from being the preferred to a supporting forecasting approach. Important to note, however, is that the amount of relevant data does not grow by default across the whole product life cycle and indeed a "reset" based on changes in computational complexity may recur regularly leading to a renewed preference for the presented framework. This is where chaos and multivalence predominate although overlaps with complex and multivalent conditions can be expected. While the dependency model appears to provide a reasonable quantitative explanation of forecasting behaviour with polar force fields, it must be remembered that this dependency model is based upon a high level explanation of that social system that is the whole product life cycle. Social systems consist of individual participants hence classifying it as a sub-set of living systems and subject to its characteristics such as emergence. In respect to organisation, the polar force field method is based on the principle that the current or actual geometric shape of the vector space is used to organise the data for forecasting. Regarding review altitude, the polar force field method quantifies cost estimate uncertainty based on the cost variance dimensions reported on in the arithmetic source data. The examined case study demonstrates six such dimensions. The granularity is thus relatively higher than analogy / expert opinion but lower than such given by a dependency model as used by the parametric method. Due to being based on the reported arithmetic cost variance dimensions the polar force field method can primarily be considered to provide a first quantitative definition of possible scenarios within the boundaries provided by analogy / expert opinion. The polar field method is based on the definition of a probability space defined by the outer limits of the relative vector space. While the aggregated vector suggests a change in centre of probability over time, this change needs to be seen both from a total and an individual cost variance vector perspective so that multiple centres of probability can be considered to exist. The polar force field is primarily focused on revealing changes in future cost variance and the uncertainties associated with it. Due to the focus on small cost data, the dependence on historical similarity is minimised while the lack of sufficient data for parametric approaches considered in that high level approximations from the vector space dependency model are drawn via the dependency model creation technique. The primary strength of the polar force field method is that the estimating history for only one time period is required although this strength is highly mitigated in practice through the unavailability of relevant structured cost variance data and the unfamiliarity of the method to the estimation community. The accuracy of the polar force field method is considered to be greater than 60% based on case study data analysis and relevant for the single time interval forecast of total cost variance. The use of the method for single time interval forecasts until the minimum conditions for parametric methods are met (therefore four time intervals of consequent historical data with a common computational complexity class) avoids the compounding of forecast errors by forecasting based on forecasts while limiting the time horizon of forecasts significantly. The reason for this limitation is that the forecasts are based on the change of state spaces while recommended future work emphasises the need to move to dynamic forecasts based on the relevant translation functions from the state to state forecast principle.

8.4 Recommendations for Future Research

This researcher believes that while each research finding is worthy of future research in respect to reduction of limitations, refinement of accuracy, improvement of quality and increase of generalisability, the primary recommendation is to investigate the nature of the space state from the perspective of its dynamic and unity / translation states. In simple terms this refers to investigating the effect of changing constants used in the framework (i.e. radial degree) to variables and vice-versa through the use of translation techniques (i.e. layout algorithms). A tabular summary of the relevant constants and variables is shown in Appendix G. This researcher believes that this work will lead to improvements in the accuracy of forecasting individual cost variance dimensions.

The polar force field attributes are separated into invariants and variables which can be viewed from the perspective of state and dynamic space pictures. The forecasting methods introduced in this investigation are hereby based on the state space and their behaviour will need to be examined from the other pictures in order to create a unified view of their interdependencies.

The recommended future direction is best understood when considering the unity / translation space in relation to invariants and variables of the polar force field attributes.

The unity / translation space as defined by its invariants is applied to the constants of the state space picture (therefore those attributes which are independent of time) in order to convert these to time dependent variables as the dynamic space emerges. The time dependent variables are thus converted to constants independent of time. The conversion principles of the unity / translation space for invariants and variables are intended to allow for the coherent and lossless conversion of the state space picture of polar force fields to a dynamic space picture of polar force fields in both directions.

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Appendix A: Terms and Definitions

Term	Definition
Actual Cost Variance	The change in cost for each time period per cost variance dimension.
Actual Cost Variance Vector	Describes the magnitude and direction of the vector for cost variance for a cost variance dimension.
Actual Prediction Error	The difference between a forecast and actual value.
Actuarial Central Estimate	The financial estimate of an actuary for a series of plausible future scenarios based on a dynamic risk management approach which is used for determining insurance and capital reserve requirements. This metric belongs to the metric family “Range”.
Advanced Aerospace Propulsion System	Any airframe propulsion system not yet in current series production and which represents a step-change improvement on existing airframe propulsion system.
Agent	A set of decision and transformation rules triggered at pre-defined points in time and applied to cost variance at two iterative time periods.
Aggregate Cost Variance Vector	Created by adding all cost variance vectors.
Analogy	A comparison used to compensate for the lack of information.
Anderson Darling (AD)	A statistical test used to assess the degree to which a sample data set follows a specific probability density function. It is commonly used to determine which type of probability density function most closely matches the distribution of the sample data set, whereby the test is deemed most appropriate for small numbers of sample data points. This metric belongs to the metric family “Shape”.
Archetype	Also termed a “canonical” example this concept refers to the fundamental recurring patters or conditions a context can be explained by.
Area	That surface within the perimeter of a polytopal reference geometry created by visualising the cost variance as vertex values at a specific point in time.
Arithmetic State Space	That pre-determined set of numerical values a process may arrive at.
Assumptions	The agreed state of the context the cost estimate is being performed in and for.
Augmented Data Patterns (ADP)	Metrics related to patterns of data presented in augmented / immersive reality spaces. Due to the (dynamic) presentation of data in 3-dimensional and / or immersive spaces new opportunities are presented for pattern matching and recognition. This metric belongs to the metric family “Complexity”.
Auto-Correlation (AC)	The cross-correlation of a data distribution with itself over sliding time-windows as a tool for finding repeating patterns. This metric belongs to the metric family “Homogeneity”.
Baseline Estimate	The agreed cost of producing a unit or delivering agreed support services. This cost consists of costed technical line items (often called the technical baseline estimate) and a risk contingency.

Term	Definition
Bayes Risk (BR)	The minimum area of error due to overlapping decision boundaries of multiple probability density functions. This metric belongs to the metric family “Range”.
Beta Co-Efficient (BC)	Describes the number of standard deviations a dependent variable may change as the predictor variables change. Often also called standardised co-efficient. This metric belongs to the metric family “Shape”.
Bivalent	A condition where only two alternatives exist.
Business Value (BV)	An umbrella term describing all perspectives related to financial performance, i.e. earned value management, break-even, or value for money. This metric belongs to all metric families.
Cellular Automaton Rules (AR)	Dynamic data arrays whose iteration patterns depend on specific rules governing the propagation behaviour of data points based on data array attributes (especially the behaviour / values of data point neighbours). This metric belongs to the metric family “Homogeneity”.
Central Limit Theorem (CLT)	The scientific principle based on the Law of Large Numbers which states that under certain conditions the arithmetic mean of a sufficiently large population will exhibit a normal distribution.
Chaotic	The apparently random state of a system where patterns cannot be identified but are surmised to exist.
Closed system	A physical system with no exchanges across its boundaries.
Cloud Symmetry	See planar symmetry.
Co-Efficient of Dispersion (R^2)	Represents the proportion of variation in the response data which accepts regression analysis techniques. This metric is often also called the co-efficient of variation, co-efficient of determination or index of dispersion and is closely related to the concept of entropy. This metric belongs to the metric family “Shape”.
Cognitive Bias	A pattern of human behaviour whereby rationality is replaced by inductive inference based on opinion. The most prevalent form of cognitive bias in the context of the study is optimism bias.
Colours (CO)	The use of colours to indicate data values, i.e. traffic lights (red, amber, green) to communicate the status of a system. This metric belongs to the metric family “Other”.
Competitive Advantage	Attributes of an organisation which enable it to perform better than its competitors. In the context of the investigation this is understood to be primarily due to the ability to invest more successfully in innovation.
Complex	A system with a high level of computational complexity which may obscure the identification of (especially emergent) behavioural patterns.
Complex Adaptive System	A system which cannot be understood through understanding of its parts alone.
Complexity (Kolmogorov)	As defined by Kolmogorov this metric quantifies the length of the shortest computer program that reproduces a specific binary string.
Complexity (CM)	Describes the extent that a system is liable to exhibit emergent behaviour

Term	Definition
	which is not predictable based on the understanding of its components. This metric family set contains the uncertainty quantification metrics augmented data patterns, degrees of freedom, neural networks and sensitivity.
Complexity Group	The Kolmogorov complexity shared by different binary strings of equal length.
Composite Symmetries	An aggregation of fundamental symmetry types.
Composition	That phase of the integrated polar force field framework which concerts arithmetic state space cost variance into geometric state space cost variance.
Compression (CR)	Describes the extent that information can be encoded using less data than the source message. One metric example related to compression is that of statistical redundancy. This metric family set contains the uncertainty quantification metric of entropy.
Computational Complexity	A factor relating to the effort needed to identify the pattern in a set of values. In this investigation the principles put forward by Kolmogorov are used in that the complexity is determined based on the length of the string being analysed and the length of the program needed to operate the string from a Turing perspective.
Conditional Tail Expectation (CTE)	A risk measure associated with the value at risk. Also known as tail value at risk. This metric belongs to the metric family “Shape”.
Confidence Level (CL)	Describes the reliability with which a certain value can be found within a data set. This metric belongs to the metric family “Range”.
Connectedness	Whether the data sets represent an unbroken set of time slices.
Contingency	The financial figure assigned to an estimate in order to compensate for cost estimate uncertainty.
Continuity	Whether the number of cost variance / uncertainty dimensions and their financial baseline is the same for all time slices evaluated.
Conundrum	A decision situation where the pay-off matrix of alternatives does not reward any participant in the decision.
Correlation	A description of the interdependency between variables.
Correlation Co-Efficient (CC)	A metric describing the strength and direction of the vector relationship between two variables. Common measures are the Pearson product-moment, the Spearman or Kendall tau rank correlations, and the Goodman and Kruskal gamma values. This metric belongs to the metric family “Homogeneity”.
Cost Containment Strategy	Actions taken to maintain cost variance within desired thresholds.
Cost Dependency	The correlation between cost variance factors.
Cost Diffusion	The propagation of cost variance over time.
Cost Dimension	The cost variance type reported on, i.e. quantity, schedule, engineering, estimating, other, and support.
Cost Estimate	The forecast of future cost (propagation).
Cost Estimate Uncertainty	Manifested and unintended future cost variance with an unknown quantity.
Cost Estimating Relationship	Describes the parametric interdependencies of variables affecting a cost

Term	Definition
(CER)	estimate.
Cost Estimation Technique	A method used to predict future cost.
Cost Readiness Level (CRL)	A measure of the usability and quality of a cost estimate.
Cost Risk	Potential unintended future cost variance with a probability of <100% and an estimated value.
Cost Uncertainty	Unplanned future cost variance of an unknown quantity.
Cost Variance	The absolute difference between the financial baseline and reported cost at any point in time.
Cost Variance Calculation	How cost variance data is prepared for vectorisation.
Cost Variance Dimension	The type of different cost variance factors to be considered by the forecasting model. The total number of these is used for calculating the even radial distribution and the number of cost variance labels and values is considered.
Cost Variance Propagation	The pattern describing the change in cost variance over time.
Cost Variance Vector	The magnitude and direction of cost variance for a cost variance dimension as related to the spatial centre.
Cubature	The numerical computation of multiple integrals, i.e. the aggregation of integrals describing multiple discrete time intervals.
Cumulative Distribution Function (CDF)	Refers to the use of cumulated s-curves. This metric belongs to the metric family “Range”.
Data Harmonics (DH)	Refers to the harmonics of data which has been sonified. This metric belongs to the metric family “Other”.
Deep Uncertainty (DU)	A decision-making situation where Knightian uncertainty, conflicting divergent paradigms and emergent decision making are relevant, i.e. “The presence of one or more of the following three elements: (1) Knightian uncertainty: multiple possible future worlds without known relative probabilities; (2) Multiple divergent but equally valid world-views, including values used to define criteria of success; and (3) Decisions which adapt over time and cannot be considered independently.” (Hallegatte & Shah et al., 2012)
Defensible	The condition when an uncertainty estimate can be decomposed into a set of coherent elements which are realistic and understandable for experienced business decision makers.
Degrees of Freedom (DF)	The minimum number of values which need to be specified to determine all the data points in a distribution. This metric belongs to the metric family “Complexity”.
Demon	A mediating agent enacting a thought experiment.
Density	The ratio between the actual area and the reference area represented by a most symmetric (regular cyclical) reference polygon at each time slice.
Dependency Model	A description of the interdependencies between multiple variables often presented in the notation of system dynamics or cost estimating relationships.
Deterministic	A paradigm based on the belief of cause and effect so that every cause will have known number and type of effects.

Term	Definition
Dilemma	A decision situation offering alternatives which are all unacceptable.
Dimensional Scaling	The relative degree that the scales of the individual cost variance dimensions match. This scaling remains constant in both the current and the future vector space.
Dimensional Sequence	The radial clockwise sequence of cost variance vectors starting from 0°. This sequence remains constant in both the current and future vector space. The number of the dimension is given by the sequence these numbers are reported on in source data.
Dimensions	The types of cost variance measured.
Dynamic Space	The paradigm that the invariants of a probability space will change over time.
Eigenvector	The direction and attributes of transformation a cost variance dimension will progress through without specific external intervention.
Entropy (EP)	The dispersion of information across a probability field. This metric belongs to the metric family “Compression”.
Estimated Prediction Error (EPE)	The three point uncertainty range associated with an unverified actual prediction error.
Financial Baseline	The financial value of the initial cost estimate (dimensions) used for planning purposes.
Forecast	Predictions of the future development of the baseline estimate.
Forecast Cost Variance	The change in cost predicted at a future time for a cost variance dimension.
Forecast Window	The time period between the time of estimation and the time estimated for.
Fuzzy Sets (FS)	Describes the relationships between data sets based on their degree of membership. This metric belongs to the metric family “Homogeneity”.
Game Theory	The study of mathematical models describing the behavioural relationship between decision makers typically assumed to be intelligent and rational.
Geometric State Space	That pre-determined set of spatial values a process may arrive at.
Geometricise	The process of converting arithmetic to geometrical information.
Half-Life (HL)	Describes the time required for the accuracy of a metric to drop by 50%.
High Value Manufacturing Products	Products which are the result of “…the application of leading edge technical knowledge and expertise…” and result in “…the creation of products, production processes, and associated services which have strong potential to bring sustainable growth and high economic value…” (United Kingdom Technology Strategy Board, 2012).
Homogeneity (HG)	Describes the degree to which assumptions regarding statistical properties can be applied across the probability field. This metric family set contains the uncertainty quantification metrics auto-correlation, cellular automaton rules, correlation co-efficient, fuzzy sets, rank correlation and RV co-efficient.
Human Dynamics	The behaviour of complex human systems over time.
Independent Variables	Used to prepare data for processing through the input output model. Specifically these variables relate to the cost variance calculation applied to the input data, the specific cost variance dimensions of relevance and the scaling of the cost variance vectors:

Term	Definition
Information Density	The degree to which data clusters within a co-ordinate system.
Information Entropy	The diffusion of information through any type of system. In the context of the investigation the emphasis is placed on the increasing symmetry of the polar force field over time.
Innovation Hesitance	The unwillingness to invest in products without a verified and accurate cost model.
Innovative	A condition of products or services where no (repeatable), robust verified cost model exist. This may (re-) occur at multiple times during the whole product life cycle.
Interquartile Range (IQR)	The range of values in a percentile, i.e. quartile. This metric belongs to the metric family “Range”.
Invariant	An attribute that does not change through transformation.
Kurtosis (K)	A measure of the peakedness of a distribution. This metric belongs to the metric family “Shape”.
Law of Large Numbers	A principle which proposes that if an experiment is conducted a sufficient number of times the average result of the experiment will normalise to a single value.
Layout Algorithm	A process that determines the position of the vertices and edges of geometrical shapes.
Length	The number of past / future whole product life cycle phases for which an estimate is completed. Alternatively the number of historical time-windows can be used.
Leptokurtic	A measure for the length of the tail of a distribution.
Living System	A social system that exhibits self-organising behaviour.
Machine	A system transforming an input into an output in an iterative series of pre-defined manipulations with repeatable outcomes.
Mass	The product of volume and density.
Mean / Median / Mode (MEM)	The average and the middle values in a set of data. This metric belongs to the metric family “Range”.
Mean Square Error (MSE)	Describes the variance in a set of data after normalisation based on differences in the means. This metric belongs to the metric family “Range”.
Minimax (MM)	The minimum and the maximum values / boundaries of a data range, whereby the “most likely” value is often included as a third reference point. This metric belongs to the metric family “Range”.
Minimum a Priori Data	The historical cost variance known in advance of estimation which suffices for the application of standard regression techniques.
Minimum Unbiased Percentage Error (MUPE)	An error regression metric helping to understand the relationship between individual observation error and magnitude of the observation. This metric belongs to the metric family “Shape”.
Monte Carlo Simulation	A mathematical process applied to probabilistic problems based on repeated random sampling.
Most Likely Value	The middle value input to a Monte Carlo simulation which is considered to

Term	Definition
	be the most probable outcome of a scenario.
Multivalent	A condition where more than two alternatives exist.
Neural Networks (NN)	A network structure of interdependent variables and commonly described by the composite metric of nonlinear weighted sum. This metric belongs to the metric family “Complexity”.
Normalisation	The adjustment of data towards a predefined set of attributes.
Novel	See innovative.
Open Complex System	A group of dependent variables that form a purposeful whole interacting with its environment and exhibits unpredictable behaviour.
Parametricise	The creation of a dependency model based on multiple correlations.
Pattern	Any series of repeating data sequences that allow for the compression of the information to a smaller size and its ensuing lossless decompression. The opposite of a pattern is randomness.
Pattern Recognition	The ability to calculate the computational complexity of a string of relevant information and in relation to small cost data the computational complexity of a short string.
Pay-Off Matrix	A table describing the returns associated with all possible actions for the participants in a decision.
Perimeter	The absolute length of the edges of the polytopal reference geometry created by visualising the cost variance dimensions at a specific point in time. This represents the boundary of the point cloud created by cost variance data.
Plausible Future Scenario	One of multiple product conditions which stakeholders consider to have a high probability of being achieved.
Point	An estimate with zero uncertainty, i.e. at 100% confidence. This metric family set contains the uncertainty quantification metric single point estimate.
Point Cloud	An n-dimensional probability space boundaried by a response surface.
Polar Force Field	A vector space with topological invariants related to the spatial centre, the dimensional sequence, the radial degree and dimensional scaling.
Polynomial Time	That time within which the operator of a process requires an output of such.
Polytope	A geometric object with flat sides.
Prior Information	The probability distribution function applied to a data set before the identification of relevant evidence.
Probability	Probability and the related concept of likelihood describe the degree to which an event can be expected to take place. This metric belongs to the metric family “Range”.
Probability Density Function (PDF)	A function describing the distribution of continuous data in a probability field. This metric belongs to the metric family “Shape”.
Probability Field (PF)	The range of values under consideration of deep uncertainty principles. The range can be described by a variety of metrics. Also referred to as uncertainty spaces, Hilbert spaces or hyper-spheres.
Probability Space	That probability field within which cost variance data exists as a point cloud.

Term	Definition
Probabilistic	The determination of the likelihood with which an event will occur. This is the opposite of deterministic.
P(robability)-Value (PV)	The degree of statistical significance for an observed relationship. This metric belongs to the metric family “Shape”.
Quadrature	The process used to determine the area of a shape.
Quantification	The use of a numerical or visual metric to communicate the relative amount and pattern of data in a data set.
Radial Degree	The central degree between adjacent cost variance vectors.
Range	The (dynamic) difference between an upper and a lower bound. This metric family set contains the uncertainty quantification metrics: actuarial central estimate, Bayes risk, cumulative density function, confidence level, inter-quartile range, mean / median / mode, minimax, mean square error, probability and three point estimate.
Rank Correlation (RC)	A measurement describing the degree of similarity between different rankings. This metric belongs to the metric family “Homogeneity”.
Reference Cost Variance	The forecast value suggested by relevant third party reference tables that are used to assess vector space forecast accuracy.
Reference Shape	The polytopal geometry used for the evaluation of symmetry.
Response Surface	The surface of a wrapper applied to a point cloud in order to convert it into a geometric shape.
Review Altitude	The granularity with which an analysis is performed. The greater the granularity the lower the review altitude.
Risk	The probability of a predicted threat or opportunity occurring.
Root Mean Square Deviation (RMSD)	Also referred to as the standard error of the mean, root mean square deviation describes the relationship between the sample and population mean as the basis for creating confidence intervals. This metric belongs to the metric family “Shape”.
R(andom) V(ariable) Co-Efficient (RVC)	Describes the closeness of two sets of points represented in matrix form. This metric belongs to the metric family “Homogeneity”.
Robust forecasts	Forecasts that meet the needs of a cost estimating activity.
Sample Size (N)	The number of data points being analysed. This metric belongs to the metric family “Shape”.
Scenario	A future use case for a product or service for which a business model has been created.
Sensitivity (S)	The degree of influence between inter-dependent factors. This metric belongs to the metric family “Complexity”.
Serious Game	The simulation of a real world problem for educational purposes where two or more actors are required to collaborate in order to resolve a problem.
Shape (SH)	Variables characterising the form of a function. This metric family set contains the uncertainty quantification metrics: Anderson Darling, beta-co-efficient, conditional tail expectation, kurtosis, minimum unbiased percentage error, sample size, probability density function, p-value, co-

Term	Definition
	efficient of dispersion, root mean square deviation, standard deviation and skew.
Silhouette	The outline of a probability space as given by connecting the outer vertices in a sequential and circular manner.
Single Point Estimate (SPE)	A calculation with an uncertainty of "0". This metric belongs to the metric family "Point".
Size	Based on the approximation of a cloud uncertainty time slice as a polygon derived from a spider chart, the actual area of this shape.
Skew (SK)	Describes the difference between the left and right hand tails of a single modal distribution. This metric belongs to the metric family "Shape".
Small Cost Data	Exists if the estimation occurs with a data set from a single time period.
Smell (SM)	The use of olfactory approaches to indicate data values. While this human sense plays a fundamental role in navigating and sense-making its transfer into purposeful communication and alert systems for data pattern remains hesitant. This metric belongs to the metric family "Other".
Social System	That network of relationship between individuals on a spatial and temporal scale.
Spatial Centre	The fixed centre of the vector coordinate system and topologically invariant in the presented technique. The spatial centre is the origin of all vectors and shown in the notation [x_{start} y_{start}]. Due to the topological invariance declared in the research study it is fixed at [0 0].
Spatial Geometry	The description of data populations using polytopes.
Spatial Scale	The size of organisation for which cost estimation and forecasting efforts are performed for.
Spatial String	A series of values describing an attribute of a topological space.
Stability	The consistency of the complexity group over time.
Standard Deviation (SD)	Describes the variance of a response based on statistical noise and is also called the standard error. This metric belongs to the metric family "Shape".
State of Art	Capabilities available for use in industrial practice.
State of Future	Capabilities that are maturing towards use in industrial practice.
State of Past	Capabilities historically used in industrial practice.
State of Present	Capabilities currently used in practice.
State Space	The paradigm that the invariants of a probability space will not change over time.
Statistical Forecasting Technique	Methods applied to arithmetic data in order to estimate and forecast its propagation / behaviour.
Statistics	General statistical descriptions of data such as t-stat, f-stat, z-stat, or chi square. This metric belongs to all metric families.
Symmetrisation	The use of the symmetry of an incomplete shape to forecast the missing parts of that geometric form.
Symmetry	The degree to which a shape is invariant to being transformed across a reference point. The research study partially considers this as the ratio

Term	Definition
	between the actual area and the maximum area.
System	A group of dependent variables that form a purposeful whole. An open system interacts with the environment while a closed system does not interact with the environment.
System of Systems (SoS)	A collection of interdependent (sub-) components which enables results no sub-part of the system can achieve on its own.
Tactile Quality (TQ)	The use of haptic approaches to indicate data values. While this human sense plays a fundamental role in navigating and sense-making its transfer into purposeful communication and alert systems for data patterns is only progressing slowly outside of steering systems such as in aircraft. This metric belongs to the metric family “Other”.
Taste (T)	The use of gustatory senses to indicate data values. While this human sense plays a fundamental role in navigating and sense-making its transfer into purposeful communication and alert systems for data patterns remains hesitant. This metric belongs to the metric family “Other”.
Technical Baseline (Cost) Estimate (TBE)	The single point engineering cost estimate that is input into the cost risk assessment process.
Technology Readiness Level (TRL)	A measure used to assess the maturity of a technology and scaled from basic technology research through to in-service operations.
Temporal Scale	The time-window for which an estimate and / or forecast is made.
Thought Experiment	A technique for investigating concepts which is based on a structured process of examination through deduction and inference in order to gain deeper knowledge of the dynamics of the context.
Three Point Estimate (TPE)	An estimate which contains a worst, best and most likely value or boundaries. This metric belongs to the metric family “Range”.
Thresholds (TR)	Defines a step-change of a metric usually based on the switch of attractors. This metric belongs to all metric families.
Time Criticality (TC)	The time for which an estimate is expected to maintain a certain accuracy or confidence. This metric belongs to the metric family “Other”.
Topology	The polytopal geometry created by the n-dimensional surface of a point cloud.
Topological Invariants	Define the attributes of the force field which are held constant between time intervals.
Torque	The angle of the aggregated vector in relation to each cost variance vector.
Total Cost Variance	The magnitude of the aggregated vector.
Transformation Method	These are the techniques used to apply the transformation vector to each individual cost variance vector.
Transformation Vector	Based on the transformation method chosen the vector applied to each individual actual cost variance vector in order to forecast its future value.
Translation Space	That function which reversibly and losslessly converts a state space into a dynamic space.
Triangular Probability Density	A continuous probability distribution with a lower limit, a mode and an upper

Term	Definition
Function	limit whereby the three values have a linear relationship.
Uncertainty	Unintended cost variance with an unknown impact at a future point in time.
Uncertainty Propagation (UP)	The actual iterative change in uncertainty of the technical baseline estimate from the time of estimation to the time of verification.
Uncertainty Quantification (UQ)	The process of determining the single point actual prediction error of a technical baseline estimate.
Uncertainty Quantification Metric	An attribute used to describe uncertainty in a quantitative manner.
Uncertainty Range	The difference between the highest and the lowest forecast values generated by the three forecast methods presented in the research study.
Uniform Density (UD)	The maximum entropy probability distribution in a normal distribution.
Unity Space	See translation space.
Utility Analysis	The assessment of decisions based on economic principles.
Vector (Euclidean)	A geometric object described by magnitude and direction.
Vector Algebra	Algebraic operations on Euclidean vectors.
Vector Space	The probability space created by joining the end points of all cost variance vectors in a radial manner.
Volatility (V)	A measure used to describe the extent that data is expected to change over time intervals. This metric belongs to all metric families.
Volume	The aggregated actual size of the time slices.
Whole Product Life Cycle	The phases of a product from concept, through development, production, utilisation and support to retirement.

Appendix B: Case Study U.K. MoD Royal Air Force A400M Transport Aircraft

B.1 Overview

This addendum applies the integrated polar force field framework to the U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase. The addendum is intended to provide a comparative example to the main case studies in respect to applying the polar force field framework.

The case study is based on cost variance data reported on in the U.K. NAO Major Projects Reports (https://www.nao.org.uk/search/type/report/sector/defence). The case study approach is the same as the quantitative one used in the main investigation. The first validation exercise uses the cost variance data from 2002 in order to forecast the cost variance data in 2003. The second validation exercise repeats the forecast for each individual forecastable event available using the mode forecasting method to illustrate the detailed results achieved. The third validation exercise shares exemplary results of using all three forecasting methods to all forecastable events and the fourth validation exercise (in light of lacking best practice third party reference tables as in the main case study) compares the overall forecasting results against a default defence inflation rate of 3.8% as suggested by the index numbers for main categories of MoD expenditure for 2005 / 2006 in the U.K. Defence Statistics Bulletin No. 10 (Jones & Woodhill, 2010, p. 29). The uncertainty of the results generated by the forecast methods is then quantified using a Monte Carlo simulation in the same manner as in the main investigation. The full case study data for the U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase is shown in Table B-1:

Table B-1: Case study data – U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase - all forecastable events – absolute cost variance values

	Δ CV due to Δ in Corporate decisions		Δ CV due to Δ in Project or Technical Issues				Δ CV due to Δ in Macro-Economic or Accounting Adjustments			Δ CV due to Δ in Other				
Year of Re-ported Cost Var-iance (UK£M)	Δ CV due to Δ in Changed Capa-bility Require-ment (UK£M)	Δ CV due to Δ in Budgetary Factors (UK£M)	Δ CV due to Δ in Technical Factors (UK£M)	Δ CV due to Δ in Procure-ment Processes (UK£M)	Δ CV due to Δ in Procure-ment Processes – International Collaboration (UK£M)	Δ CV due to Δ in Contracting Process (UK£M)	Δ CV due to Δ in Inflation (UK£M)	Δ CV due to Δ in Exchange Rate (UK£M)	Δ CV due to Δ in Accounting Adjustments and Redefinitions (UK£M)	Δ CV due to Δ in Receipts (UK£M)	Δ CV due to Δ in Change in Associated Project (UK£M)	Δ CV due to Δ in HM Treasury Reserve (UK£M)	Δ CV due to Δ in Risk Differential (UK£M)	Total Δ CV (UK£M)
2002	*319*	*54*	0	*65*	0	227	0	*142*	0	0	0	0	*119*	926
2003	*310*	*74*	*46*	*65*	0	384	*10*	*232*	*1*	0	0	0	*119*	1241
2004	*310*	*67*	13	*65*	0	353	*10*	*10*	*43*	0	0	0	*116*	987
2005	*313*	*67*	*7*	65	0	353	*2*	*49*	*42*	0	0	0	*116*	1014
2006	*312*	*90*	*2*	65	0	353	12	5	*43*	0	0	0	*116*	998
2007	*320*	*90*	27	65	0	353	12	5	*51*	0	0	0	*116*	1039
2008	*333*	*93*	88	65	0	353	12	*11*	*77*	0	0	0	*116*	1148
2009	*333*	*93*	88	65	0	353	12	*11*	*77*	0	0	0	*116*	1148
2010	*333*	*93*	88	65	0	353	12	*11*	*77*	0	0	0	*116*	1148
2011	*355*	*94*	84	65	0	353	12	*10*	0	0	0	0	0	973
2012	*355*	*94*	80	57	175	353	12	*10*	0	0	0	0	0	1136
2013	*329*	*77*	*10*	55	175	345	10	*8*	0	0	0	0	0	1009
2014	*329*	*77*	*10*	55	175	345	*24*	*51*	0	0	0	0	0	1066

Table B-2 quotes the definitions for the cost variance factors used by the U.K. NAO Major Projects Report 2013 Appendices and project summary sheets (2013).

Table B-2: Definition of cost variance factors U.K. NAO

Cost Variance Factor	Definition
Corporate Decisions	"Corporate decisions, that is decisions that are taken at the top of the Department by senior management or ministers."
Corporate Decisions / Changed Capability Requirement	"Variations due to changes in the customer's requirement for the equipment, flowing from operational reassessment rather than budgetary factors or because of support to current operations."
Corporate Decisions / Budgetary Factors	"Variations due to changes in the customer's requirement for equipment, flowing from changed budgetary priorities."
Project or Technical Issues	"Project/technical issues reflect variations at a lower project level."
Project or Technical Issues / Technical Factors	"Variations which are due to changes in technical ability to deliver the project."
Project or Technical Issues / Procurement Processes	"Variations due to changes associated with the contractual process including time taken in contract negotiations and placing contracts, effect of comparing contractor bids to estimates and variations due to changes in overall procurement strategy, e.g. change to collaborative options, or from competitive to single source."
Project or Technical Issues / Procurement Processes – International Collaboration	"As above, but relating to international contract negotiations."
Project or Technical Issues / Contracting process - not included from 2009 onwards	"Variations due to changes associated with the contractual process, including time taken in contract negotiations and placing contracts, international contract negotiations and effect of comparing contractor bids with estimates."
Macro-Economic or Accounting Adjustments	"Macro-economic or accounting adjustments, mainly resulting from changes the Department makes in assumptions regarding exchange rates and inflation."
Macro-Economic or Accounting Adjustments / Inflation	"Variations due to changes in inflation assumptions."
Macro-Economic or Accounting Adjustments / Exchange Rate	"Variations due to changes in exchange rate assumptions."
Macro-Economic or Accounting Adjustments / Accounting Adjustments and Redefinitions	"Variations that do not reflect any substantive change, and result from changes to accounting rules, or adjustments to reflect changes in defining terms."
Other / Receipts	"Variations due to changes in expectation of receipts, e.g. liquidated damages, commercial exploitation levy."
Other / Change in Associated Project	"Variations due to changes in an associated project, e.g. availability of equipment from another project for trials."
Other / HM Treasury Reserve	"Recovery of additional costs incurred in support of current operations."
Risk Differential	The contingency added to an estimate

B.2 Exemplary Forecast 2002 / 2003

Table B-3 shows the results of using arithmetic cost variance data from 2002 for forecasting the arithmetic cost variance in 2003 based on the presented vector algebra technique.

Table B-3: Cost estimate uncertainty forecast results – U.K. MoD Royal Air Force A400M transport aircraft post-main-gate phase

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
		CV Factor #1	CV Factor #2	CV Factor #3	CV Factor #4	CV Factor #5	CV Factor #6	CV Factor #7	CV Factor #8	CV Factor #9	CV Factor #10	CV Factor #11	CV Factor #12	CV Factor #13	Avg. Variance	STDEV. P	Total CV
0	Level 0 CV Taxonomy	Corporate Decisions		Project or Technical Issues		Macro-Economic or Accounting Adjustments		Other									
	Level 1 CV Taxonomy	Δ CV due to Δ in Changed Capability Requirement (UK£M)	Δ CV due to Δ in Budgetary Factors (UK£M)	Δ CV due to Δ in Technical Factors (UK£M)	Δ CV due to Δ in Procurement Processes (UK£M)	Δ CV due to Δ in Procurement Processes – International Collaboration (UK£M)	Δ CV due to Δ in Contracting Process (UK£M)	Δ CV due to Δ in Inflation (UK£M)	Δ CV due to Δ in Exchange Rate (UK£M)	Δ CV due to Δ in Accounting Adjustments and Redefinitions (UK£M)	Δ CV due to Δ in Receipts (UK£M)	Δ CV due to Δ in Change in Associated Project (UK£M)	Δ CV due to Δ in HM Treasury Re-serve (UK£M)	Δ CV due to Δ in Risk Differential (UK£M)	Avg. Δ CV due to Δ in all CV Dimensions (UK£M)	STDEV. P (UK£M)	Total CV due to Δ in all CV Dimensions (UK£M)
1	Actual Value at t=1	319	54	0	65	0	227	0	142	0	0	0	0	119	71	100	926
2	Actual Value at t=2	310	74	46	65	0	384	10	232	1	0	0	0	119	95	126	1241
3	Mode Forecast	356	60	0	72	0	253	0	158	0	0	0	0	0	69	112	900
4	Upper Limit Forecast	455	75	0	95	0	394	0	215	0	0	0	0	0	95	153	1233
5	Lower Limit Forecast	322	56	2	68	2	232	0	0	0	0	0	0	0	53	100	683
7	Mode Forecast Accuracy	115%	81%	0%	111%	0%	66%	0%	68%	0%	0%	0%	0%	0%	34%	45%	442%
8	Upper Limit Forecast	147%	101%	0%	146%	0%	103%	0%	93%	0%	0%	0%	0%	0%	45%	59%	589%

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
		CV Factor #1	CV Factor #2	CV Factor #3	CV Factor #4	CV Factor #5	CV Factor #6	CV Factor #7	CV Factor #8	CV Factor #9	CV Factor #10	CV Factor #11	CV Factor #12	CV Factor #13	Avg. Variance	STDEV. P	Total CV
0	Level 0 CV Taxonomy	Corporate Decisions		Project or Technical Issues		Macro-Economic or Accounting Adjustments		Other									
	Level 1 CV Taxonomy	Δ CV due to Δ in Changed Capability Requirement (UK£M)	Δ CV due to Δ in Budgetary Factors (UK£M)	Δ CV due to Δ in Technical Factors (UK£M)	Δ CV due to Δ in Procurement Processes (UK£M)	Δ CV due to Δ in Procurement Processes – International Collaboration (UK£M)	Δ CV due to Δ in Contracting Process (UK£M)	Δ CV due to Δ in Inflation (UK£M)	Δ CV due to Δ in Exchange Rate (UK£M)	Δ CV due to Δ in Accounting Adjustments and Redefinitions (UK£M)	Δ CV due to Δ in Receipts (UK£M)	Δ CV due to Δ in Change in Associated Project (UK£M)	Δ CV due to Δ in HM Treasury Re-serve (UK£M)	Δ CV due to Δ in Risk Differential (UK£M)	Avg. Δ CV due to Δ in all CV Dimensions (UK£M)	STDEV. P (UK£M)	Total CV due to Δ in all CV Dimensions (UK£M)
	Accuracy	🟨	🟩	🟥	🟨	🟥	🟩	🟥	🟩	🟥	🟥	🟥	🟥	🟥			
9	Lower Limit Forecast Accuracy	104%	76%	5%	105%	0%	60%	0%	0%	0%	0%	0%	0%	0%	27%	41%	350%
		🟩	🟩	🟥	🟩	🟥	🟩	🟥	🟨	🟥	🟥	🟥	🟥	🟥			

Based on the average forecast accuracy (line 10) the cost variance factor that demonstrated the greatest standard deviation (uncertainty) were “Technical Factors” (2%) followed by “Procurement Processes – International Collaboration”, “Inflation”, “Receipts”, “Associated Project”, “HM Treasury Reserve” and “Risk Differential” each with 0% due to zero cost variance values in 2002. Important to note is that the STDEV.P function in Microsoft ® Excel assumes that the analysed data represents the complete sample population.

B.3 Exemplary Visualisation 2002 / 2003

Figure B-1 shows an exemplary visualisation of the polar force field vector space for the arithmetic cost variance data from 2002.

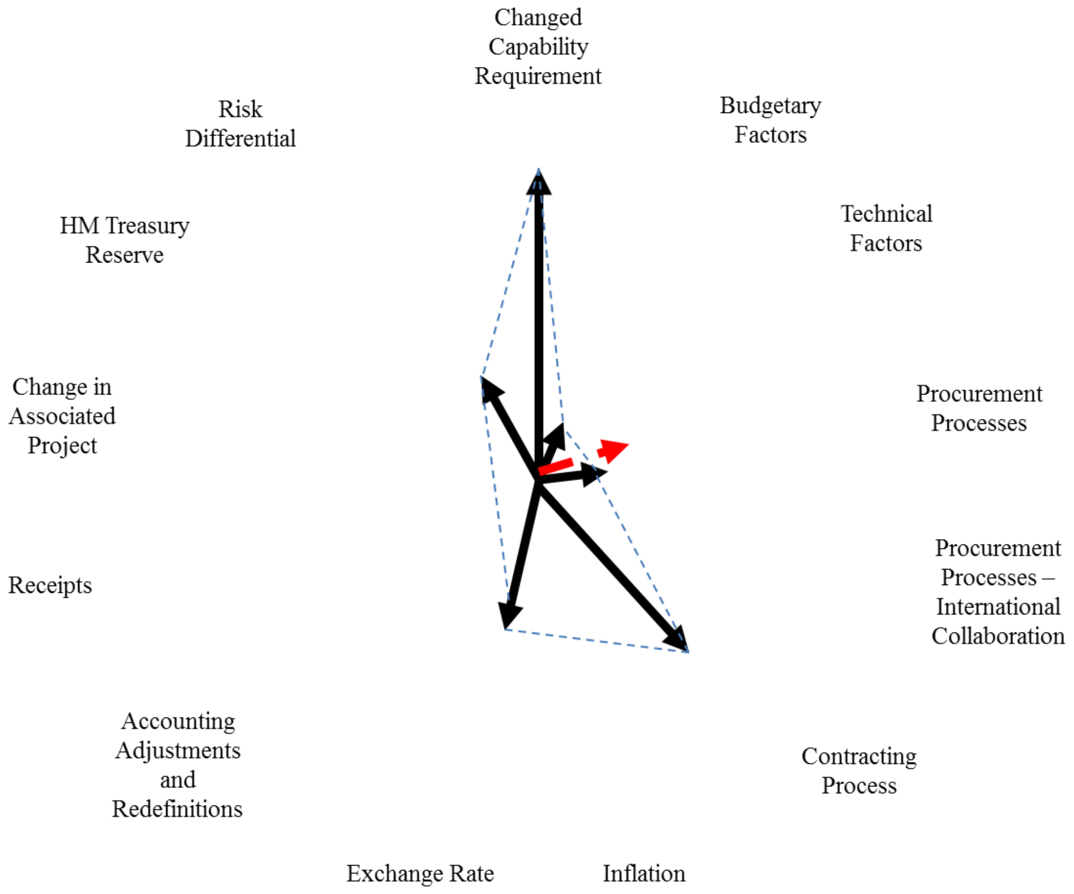


Figure B-1: Polar force field visualisation of cost variance in 2002 for the U.K. MoD A400M post-main-gate phase

B.4 Exemplary Forecast All Forecastable Events using Method #1

The presented method was then applied to each available data set for the U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase using each forecasting method in order to determine the influencers of forecasting inaccuracy. Table B-4 summarises the forecasting results using method #1 in order to illustrate the approach taken.

Table B-4: Mode forecasting accuracy U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase

	Δ CV due to Δ in CR	Δ CV due to Δ in BF	Δ CV due to Δ in TF	Δ CV due to Δ in PP	Δ CV due to Δ in PPI	Δ CV due to Δ in CP	Δ CV due to Δ in I	Δ CV due to Δ in ER	Δ CV due to Δ in AAR	Δ CV due to Δ in R	Δ CV due to Δ in Change in AP	Δ CV due to Δ in HTR	Δ CV due to Δ in RD	Avg. Δ CV due to Δ in All Dim.	Std. Devia-tion (STDEV. P)	Total CV
2002/2003	115%	81%	0%	111%	0%	66%	0%	68%	0%	0%	0%	0%	0%	34%	45%	442%
2003/2004	109%	120%	384%	109%	0%	118%	109%	2520%	3%	0%	0%	0%	0%	267%	658%	3470%
2004/2005	110%	111%	206%	111%	0%	111%	554%	23%	113%	0%	0%	0%	0%	103%	145%	1338%
2005/2006	111%	82%	387%	111%	0%	111%	18%	1083%	108%	0%	0%	0%	0%	155%	286%	2010%
2006/2007	108%	111%	8%	111%	0%	111%	111%	111%	93%	0%	0%	0%	0%	59%	53%	763%
2007/2008	106%	107%	34%	110%	0%	110%	110%	50%	73%	0%	0%	0%	0%	54%	48%	700%
2008/2009	109%	109%	109%	109%	0%	109%	109%	109%	109%	0%	0%	0%	0%	67%	53%	874%
2009/2010	109%	109%	109%	109%	0%	109%	109%	109%	109%	0%	0%	0%	0%	67%	53%	874%
2010/2011	103%	108%	114%	109%	0%	109%	109%	120%	0%	0%	0%	0%	0%	59%	55%	773%
2011/2012	111%	111%	117%	127%	0%	111%	111%	111%	0%	0%	0%	0%	0%	61%	57%	798%
2012/2013	118%	134%	876%	113%	109%	112%	131%	137%	0%	0%	0%	0%	0%	123%	222%	1731%
2013/2014	111%	111%	111%	111%	111%	111%	46%	17%	0%	0%	0%	0%	0%	105%	52%	727%
STDEV.P	4%	14%	236%	5%	41%	13%	135%	705%	51%	0%	0%	0%	0%	61%	173%	812%
Ranking STDEV.P	8	9	3	7	4	10	6	2	5	1	1	1	1			

In respect to forecasting total cost variance the most inaccurate forecast (outlier) was made for 2004 using the cost variance data from 2003 (267%). In that forecast significant inaccuracies are found in the forecasts of eight of the thirteen cost variance factors. For all datasets the total cost variance forecast has an average accuracy of 1,208% and STDEV.P of 812%. The forecast of the individual cost variance dimensions has an average accuracy of 96% with a STDEV.P of 61%. Investigation of remaining forecasts with significant inaccuracy (i.e. red shaded cells) identified improperly forecast reductions of cost to be based on cost variance values of zero UK£M as the most evident cause for inaccuracy.

B.5 Exemplary Forecast All Forecastable Events using Methods #1, #2 and #3

The results of applying all forecasting methods to the U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase data with an emphasis on total cost variance and its growth rate for actual source and generated forecast data are shown in Table B-5:

Table B-5: Forecasting accuracy and comparison for U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase

Y	Mode Forecast Algorithm	Upper Limit Forecast Algorithm	Lower Limit Forecast Algorithm	Act.	FE#1	AGR-A	AGR-R	FGR#1-A	FGR#1-R	CGF-(3.8%)
2002/2003	900	1233	683	1241	341	N/A	N/A	N/A	N/A	N/A
2003/2004	1219	1720	895	987	-232	-254	-20,47%	319	25,85%	1288
2004/2005	965	1417	827	1014	49	27	2,74%	-254	-14,74%	1025
2005/2006	992	1434	823	998	6	-16	-1,58%	27	1,93%	1053
2006/2007	976	1437	842	1039	63	41	4,11%	-16	-1,13%	1036
2007/2008	1018	1495	875	1148	130	109	10,49%	42	2,89%	1078
2008/2009	1128	1643	952	1148	20	0	0,00%	110	7,37%	1192
2009/2010	1128	1643	952	1148	20	0	0,00%	0	0,00%	1192
2010/2011	1128	1643	952	973	-155	-175	-15,24%	0	0,00%	1192
2011/2012	1080	1756	982	1136	56	163	16,75%	-48	-2,89%	1010
2012/2013	1243	1998	1145	1009	-234	-127	-11,18%	163	9,29%	1179
2013/2014	1116	1779	1022	1066	-50	57	0,0565	-127	-0,0636	1047
STDEV.P	101	197	112	82	152	118	0	142	0	89
Rel. STDEV.P	9%	12%	12%	8%	13061%	-741%	-1326%	723%	482%	8%

The accuracy of the forecasting results is visualised in Figure B-2:

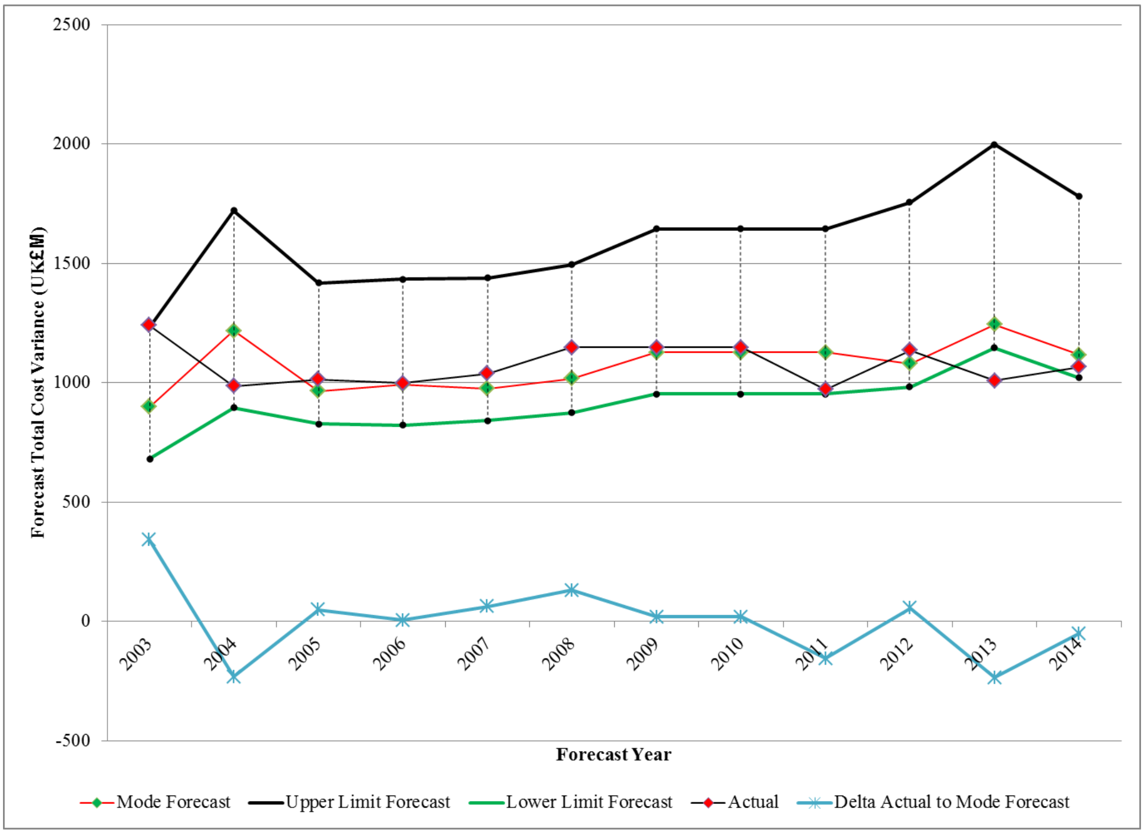


Figure B-2: Accuracy of forecasting for U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase - using polar force field forecasting

In all cases the upper limit forecast algorithm generates the highest forecast value, the lower limit forecast algorithm generates a forecast value between actual and mode and the mode forecast algorithm generates a value between the upper and lower limit.

B.6 Comparison of Forecasting Results with Default Defence Inflation Rate

A default defence inflation rate of 3.8% as suggested by the index numbers for main categories of MoD expenditure for 2005/2006 in the U.K. Defence Statistics Bulletin No. 10 (Jones & Woodhill, 2010, p. 29) was used to review the results of the forecasting methods. Figure B-3 visualises the comparative growth rates:

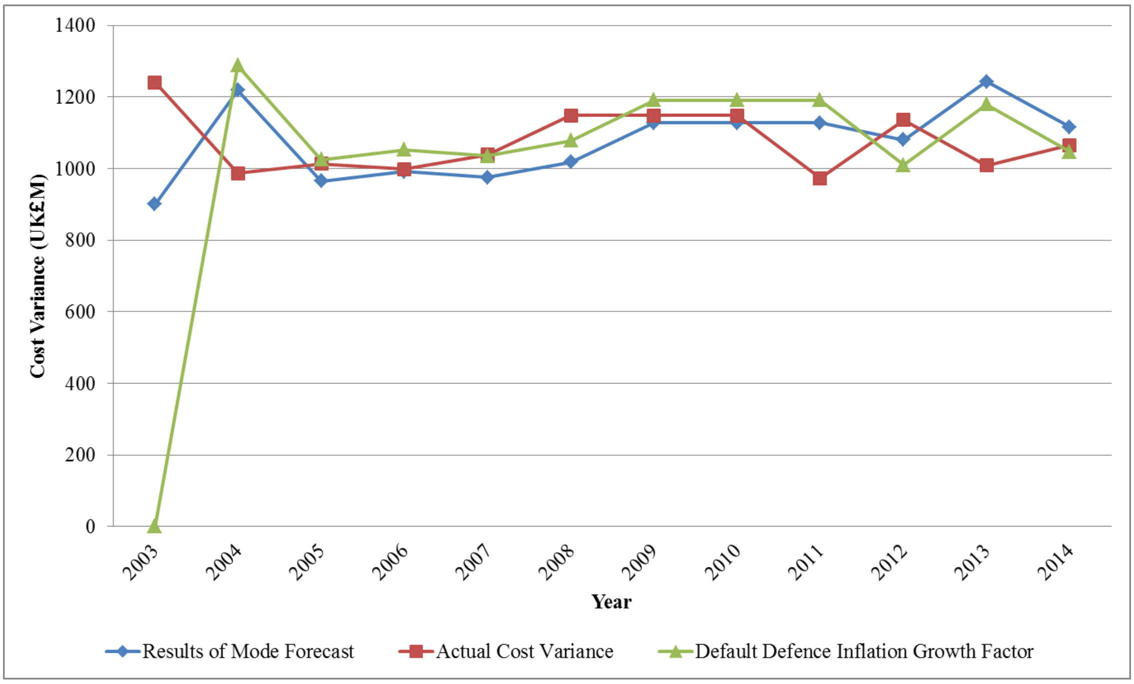


Figure B-3: Comparative cost growth overview for U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase

B.7 Dependency Model

The method was applied to the case study data and the calculated correlations of all variable pairs to each other with a minimum R^2 of 0.1 as shown in Table B-6:

Table B-6: Variable correlation (ranked from highest R^2 value downward)

Variable 1	Variable 2	Equation Linear Trend Line	R^2
Procurement Processes	Procurement Processes – International Collaboration	y = -18.505x + 1203.3	0.9869
Procurement Processes – International Collaboration	Risk Differential	y = -0.6x + 105	0.6747
Procurement Processes	Risk Differential	y = 11.103x - 616.99	0.6659
Budgetary Factors	Technical Factors	y = 2.0981x - 129.79	0.54
Changed Capability Requirement	Technical Factors	y = 1.7511x - 530.83	0.50
Changed Capability Requirement	Risk Differential	y = -2.5161x + 903.53	0.47
Accounting Adjustments and Redefinitions	Risk Differential	y = 1.1264x + 45.156	0.4327
Changed Capability Requirement	Budgetary Factors	y = 0.5361x - 93.543	0.38
Budgetary Factors	Contracting Process	y = 1.5682x + 216.23	0.3265
Budgetary Factors	Exchange Rate	y = -2.9163x + 281.15	0.3225
Procurement Processes – International Collaboration	Accounting Adjustments and Redefinitions	y = -0.2349x + 41.1	0.3032
Procurement Processes	Accounting Adjustments and Redefinitions	y = 4.3459x - 241.51	0.2992
Budgetary Factors	Inflation	y = 0.2249x - 7.6246	0.2829
Procurement Processes	Inflation	y = -0.6601x + 52.255	0.2334
Inflation	Risk Differential	y = -4.6428x + 130.77	0.2174
Exchange Rate	Accounting Adjustments and Redefinitions	y = -0.2107x + 40.611	0.1936
Changed Capability Requirement	Exchange Rate	y = -1.8491x + 647.34	0.17
Budgetary Factors	Accounting Adjustments and Redefinitions	y = 0.9899x - 49.329	0.1621
Changed Capability Requirement	Procurement Processes – International Collaboration	y = 2.0029x - 614.55	0.16
Technical Factors	Accounting Adjustments and Redefinitions	y = 0.3197x + 18.261	0.1373
Technical Factors	Contracting Process	y = 0.3417x + 330.19	0.1259
Changed Capability Requirement	Procurement Processes	y = -0.0944x + 93.722	0.12
Changed Capability Requirement	Inflation	y = 0.1159x - 27.123	0.10
Inflation	Exchange Rate	y = -3.6784x + 82.306	0.0918
Exchange Rate	Risk Differential	y = 0.2053x + 72.005	0.0627
Contracting Process	Accounting Adjustments and Redefinitions	y = 0.221x - 44.521	0.0609

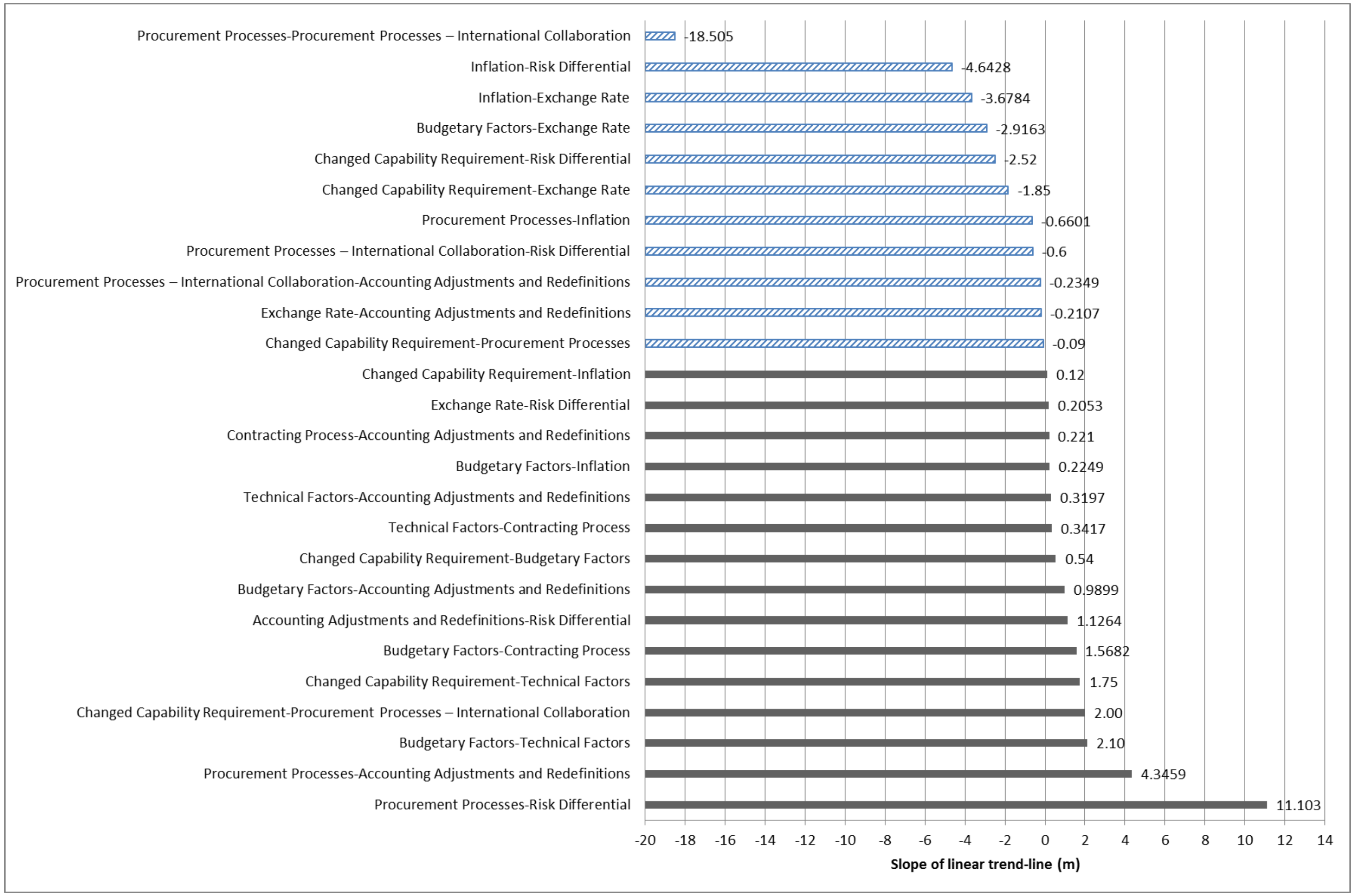


Figure B-4: Correlation ranking – degree of future impact

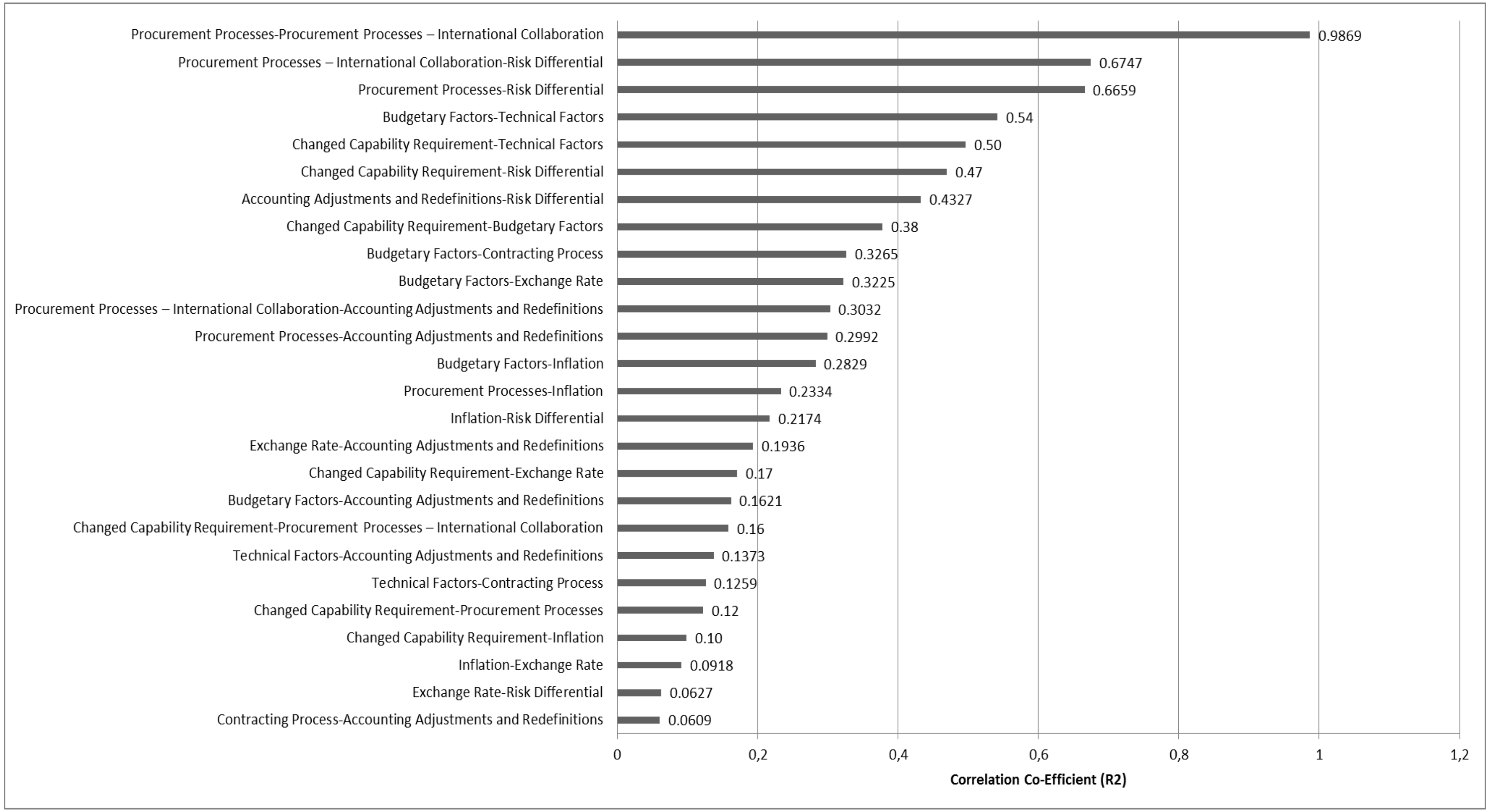


Figure B-5: Correlation ranking – sequence of future impact

Based upon the correlation rankings identified a dependency model can be created as shown in Figure B-6 to illustrate the manner in which the variables interact including their sequence and impact on each other. An exemplary polar force field as shown in Figure B-1 is included to highlight the way in which such may be seen to influence the layout of such.

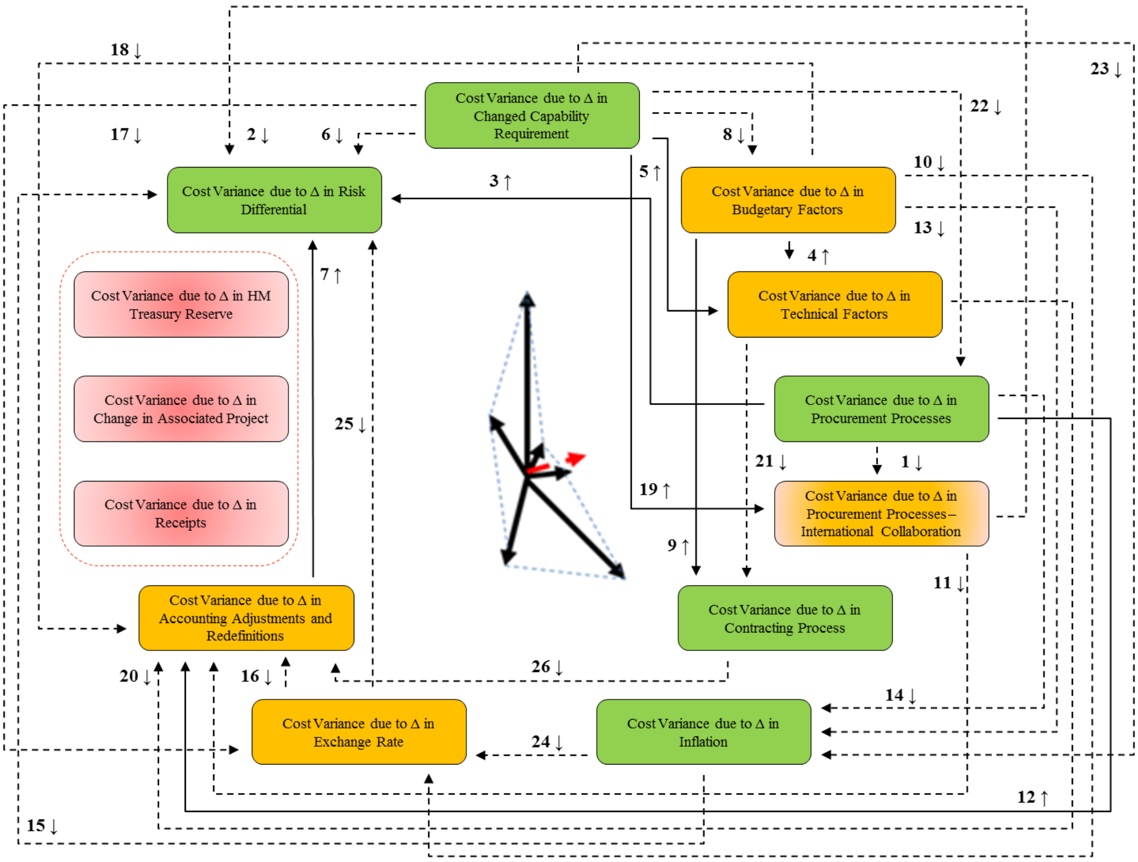


Figure B-6: Dependency model

Of note in the dependency model is that no feedback loops can be identified. A summary of inputs and outputs for the variables in shown in Table B-7:

Table B-7: Summary of inputs and outputs

	Number of Inputs Acc-elerating	Number of Outputs Acc-elerating	Number of Inputs De-celerating	Number of Outputs De-celerating	Number of Inputs Total	Number of Outputs Total	Number of Con-nections Total
Cost variance due to Δ in Changed Capability Requirement	0	2	0	5	0	7	7
Cost variance due to Δ in Budgetary Factors	0	2	1	3	1	5	6
Cost variance due to Δ in Technical Factors	2	0	0	2	2	2	4
Cost variance due to Δ in Procurement Processes	0	2	1	2	1	4	5
Cost variance due to Δ in Procurement Processes – International Collaboration	1	0	1	2	2	2	4
Cost variance due to Δ in Contracting Process	1	0	1	1	2	1	3
Cost variance due to Δ in Inflation	0	0	3	2	3	2	5
Cost variance due to Δ in Exchange Rate	0	0	3	2	3	2	5
Cost variance due to Δ in Accounting Adjustments and Redefinitions	1	1	5	0	6	1	7
Cost variance due to Δ in Receipts	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Cost variance due to Δ in Change in Associated Project	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Cost variance due to Δ in HM Treasury Reserve	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Risk Differential	2	0	4	0	6	0	6
SUM	7	7	19	19	26	26	52
AVERAGE	0,7	0,7	1,9	1,9	2,6	2,6	5,2

B.8 Case Study Comparison

The results case study of the U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase can now be compared to the results of the main investigation.

B.9 Conclusion

The case study demonstrates the forecasting accuracy achieved by the presented technique using the U.K. MoD Royal Air Force A400M transport aircraft for the time period from 2002 to 2014 in the post-main-gate phase. Forecast results and their accuracies are presented for each individual forecastable event within the chosen data boundaries and the results of the three forecasting methods contrasted. The results of the most robust forecast method (mode algorithm) for all forecastable events are then compared to results based on applying independent third party reference tables and their suggested cost growth rates to the same data.

The results of the case study verification and validation effort suggest that the presented forecasting technique is, without reliance on historical data, able to forecast cost variance at time period 2 using only cost variance data from time period 1 with a reasonable degree of accuracy in relation to the independent third party reference tables although the need for an application of standard correction factors appears more relevant than in the main cases studies of the investigation.

Appendix C: Exemplary Uncertainty Quantification Metrics in Literature Review

Source	Date	Type	Discussed Metrics
Abebe, A.J. et al.	2000	Conference	CDF, FS, IQR, PDF, MEM, TPE
Alexander et al.	2004	Report	CDF, CI, MM, PR, SD
Andersson, B.A. et al.	2013	Report	CI, PDF, RVC, TPE
Ansari, S. et al.	2006	Journal	PR
Arena, M.V. et al.	2006	Guide	CC, CDF, IQR, MEM, N, PDF, PR, RC, SD, TPE
Asiedu, Y., Gu, P.	1998	Journal	CDF, CI, MM, PDF, TPE
Augusdinata, B.	2008	Thesis	BR, BV, CDF, CI, DF, IQR, MEM, MM, PDF, PR, PV, R^2, S, SD, SK, STAT, V
Aven, T.	2013	Journal	BR, BV, CI, MEM, N, PDF, SPE
Baguley, P.	2004	Thesis	FS
Banazadeh, A. & Jafari, M.H.	2013	Journal	N, PDF, R^2, S
Bankole, O. et al.	2012	Journal	CI, IQR, MEM, MM, PDF, SPE, V
Bearman, N.E.	2013	Thesis	ADP, CO, DH, SB, SM, T, TQ
Black, H.M.	2008	Survey	CDF, MEM, PDF, SD, TPE
Celaya et al.	2012	Conference	BR, FS, N, PDF, R^2, S
Chalupnik, M.J. et al.	2013	Journal	BV, DF, PDF, S
Curran, R. & Raghunathan, S.	2004	Journal	CDF, FS, IQR, MM, PDF, PR, R^2, TPE
DeCarlo, L.T.	1997	Journal	BV, DF, IQR, K, MEM, N, PDF, PV, SD, SH, SK, STAT
Dieckmann et al.	2010	Journal	CI, IQR, MEM, PDF, PR, R^2, RVC, S, SK, SPE, TPE
Durugbo, C. et al.	2010	Journal	IQR, N, PR
Dysert, L.R.	2008	Conference	R^2, STAT
Erkoyuncu, J.A.	2011	Thesis	CDF, CI, MM, PDF, R^2, SD, SPE, TPE
Erkoyuncu, J.A. et al.	2011	Journal	CDF, DF, FS, MM, PR, PV, S, SD
Erkoyuncu, J.A. et al.	2013	Journal	BV, CDF, CI, DF, FS, MM, PDF, R^2, S, SK, TPE
Faller, W. & Schreck, S.J.	1996	Journal	NN
Ferguson, R. et al.	2011	Guide	BR, CDF, MEM, MM, MSE, PDF, PR, R^2, S, SD, SK
Fiori, A.M.	2008	Journal	CDF, DF, IQR, K, MEM, N, PDF, SD, SK
Galvao, A.F. et al.	2013	Journal	BV, IQR, K, N, PDF, PV, R^2, S, SD, SK
Galway, L.A.	2007	Report	BR, CI, IQR, MM, PDF, PR
Goddard GSFC-STD-0002	2009	Guide	PR
Goh Y.M. et al.	2010	Journal	CDF, FS, IQR, PDF, PR, R^2
Golkarl, A. & Crawley, E.F.	2014	Journal	BV, DF, IQR, MEM, MM, MSE, N, PDF, PR, R^2, RC, S, SD

Source	Date	Type	Discussed Metrics
Grenn, M.W. et al.	2014	Journal	DF, EP, PR, V
Haase, N. et al.	2013	Journal	CI, IQR, MEM, PR, PV, RMSD, S, SD
Hallegatte, S. et al.	2012	Report	BR, BV, DF, PDF, PR, SD
Hamarat, C. et al.	2013	Journal	BV, CC, DF, IQR, MEM, PDF, TR
Haskins, C., ed.	2007	Guide	BV, CI, PDF, PR, R^2, S
Hillson, D.A.	2005	Conference	BV, IQR, MEM, MM, PR, SPE, TPE
Hofmann, M.	2005	Journal	CC, DF, SD
International Society of Parametric Analysis	2008	Guide	DF, IQR, K, R^2, SD, STAT
ISO/IEC 15288	2008/2015	Standard	PDF, TR
Kennedy, M.C. & O´Hagan, A.	2001	Journal	BR, CI, MEM, PDF, R^2, RVC, S, TPE
Khodakarami, V. & Abdi, A.	2014	Journal	BR, BV, CDF, IQR, MEM, MM, PDF, R^2, RVC, SD
Kreye, M.E. et al.	2012	Journal	CI, DF, FS, IQR, MEM, MM, PDF, PR, PV, SPE, STAT
Kwakkel, J.H. et al.	2013	Journal	BV, CC, CDF, DF, IQR, MEM
Kwakkel, J.H. & Pruyt, E.	2013	Journal	DF, IQR, PDF, PV, R^2, TPE, V
Lee, S.H. & Chen, W.	2009	Journal	CI, DF, K, MEM, N, PDF, R^2, RVC, SD, SK
Lempert, R.J. et al.	2006	Report	BR, BV, CC, DF, PDF, TR
Lempert, R.J. & Collins, M.T.	2007	Journal	BR, BV, CI, DF, FS, IQR, MM, PDF, PR, S, SD, TC, TR
Mahnovski, S.	2007	Thesis	BV, PDF, PR
Marion, T.J. & Meyer, M.H.	2011	Journal	BC, CDF, MEM, PV, R^2, SD, STAT
NATO RTO-TR-SAS-069	2009	Guide	CDF, DF, IQR, MEM, PDF, PR, S, SPE, TPE
Niazi, A. et al.	2006	Journal	FS
Nilchiani, R. & Rifkin, S.	2013	Report	BR, BV, CDF, PDF, PR, R^2
Patt, A.G. & Schrag, D.P.	2003	Journal	CI, IQR, PR, PV, S, SPE
Price, M. et al.	2006	Journal	CDF, DF, IQR, PDF, PR, R^2
Rakow, T.	2010	Journal	PR, RB
RAND Project Air Force	2007	Guide	R^2, RMSD , STAT
Rech, J.E. & Yan, R.	n.d.	Guide	ACE, BV, CDF, CI, DF, IQR, MEM, MM, PDF, PR, R^2,RC, S, SD, STAT, TC, TPE
Rittel, H.W. & Webber, M.	1973	Journal	CC, DF, N
Rostami, J. et al.	2013	Journal	BV, CDF, IQR, MM, PV, R^2
Roy, R. & Sackett, P.	2003	Report	BV, DF, PDF, PR, R^2, SPE
Scales, J.A. & Tenorio, L.	2001	Journal	BR, IQR, MEM, MM, MSE, N, PDF, PV, R^2, SD
Smart, C.B.	2014	Journal	BV, CDF, CI, CTE, K, MEM, PDF, R^2, RVC, SD, SK

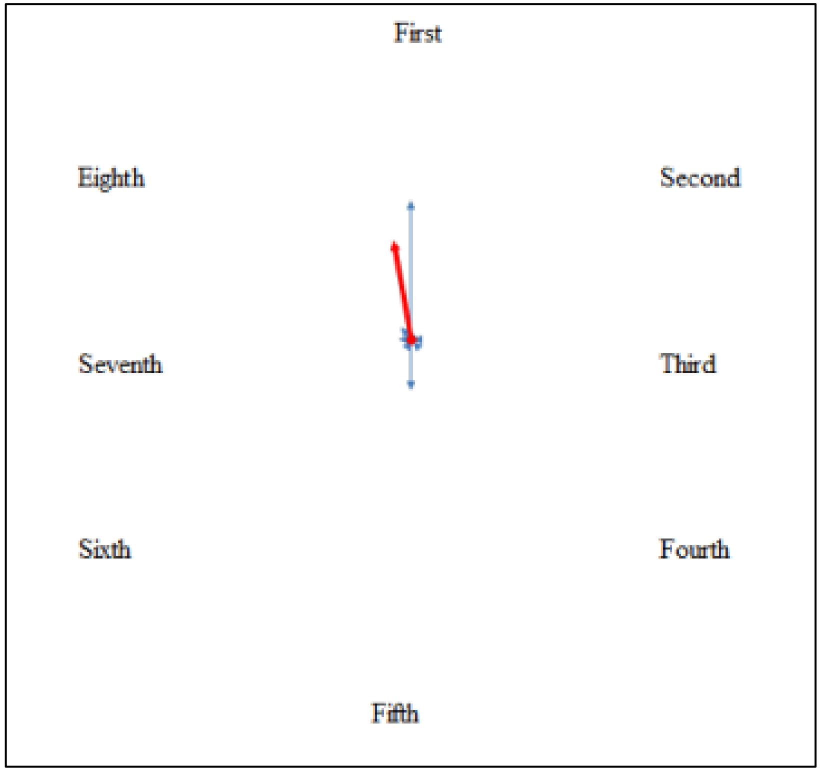
Source	Date	Type	Discussed Metrics
Smit, M.C.	2012	Journal	CI, MEM, PDF, R^2, S, TPE
Spackova, O. et al.	2013	Conference	BR, CI, IQR, MEM, PDF, R^2, SD, SK
Tammineni, S.V. et al.	2009	Journal	CDF, CI, PDF, R^2, S, SD, TPE
Trivailo, O. et al.	2012	Journal	CI, IQR, MUPE, PDF, PR, PV, R^2
Uffink, J.B.M.	1990	Thesis	CDF, DF, EP, IQR, K, MEM, PDF, PR, SD, SH, SK
United States Air Force Cost Risk and Uncertainty Handbook	2007	Guide	IQR, PDF, R^2, SD, SK
United States Government Accountability Office	2009	Report	BV, CI, IQR, PR, SPE
United States National Aeronautics and Space Administration Cost Estimating Handbook	2008	Guide	CDF, CI, IQR, PDF, R^2, SD, SK
United States Space Systems Cost Analysis Group Space Systems Cost Risk Handbook	2005	Guide	BV, CI, IQR, K, MEM, MM, PDF, R^2, SK, SPE, TPE
Wheeler, D.J.	2012	Conference	AC, IQR, K, MEM, PDF, PR, PV, R^2, SD, SK
Xu, Y. et al.	2012	Journal	BV, FS, IQR, PDF, R^2, S, SD, STAT
Yao, W. & Chen, X.	2011	Journal	AD, BR, CDF, FS, IQR, K, MEM, MM, MSE, PDF, PR, S, SD, SH, SK
Yoe, C.	2000	Report	BV, CDF, CI, DF, IQR, MEM, MM, N, PDF, PR, S, SK, SPE, STAT
Younossi, O. et al.	2008	Report	PDF, PR
Zadeh, L.A.	1965	Journal	AR, FS, PDF, PR

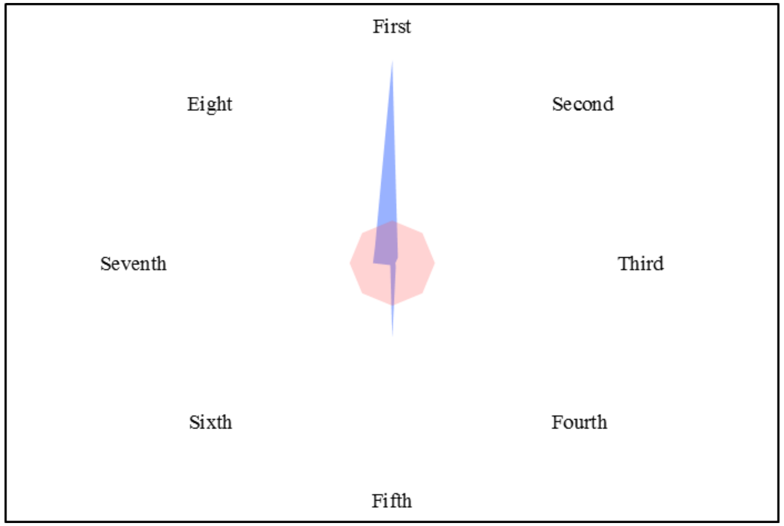
Appendix D: U.S. DoD Aggregated Exemplary Source Data Overview

Reporting Period (ending December)	Quantity (US$M)	Schedule (US$M)	Engineering (US$M)	Estimating (US$M)	Other (US$M)	Support (US$M)
1986	72,810	6,460	15,729	*2,493*	*696*	14,545
1987	46,441	5,525	14,241	*7,473*	*9,810*	*2,392*
1988	78,291	6,436	17,792	*1,903*	*9,463*	*1,196*
1989	48,649	5,009	15,839	4,921	*8,253*	*10,297*
1990	17,036	13,212	16,428	16,122	*8,126*	*13,859*
1991	*25,126*	8,797	12,564	22,750	*9,554*	*11,277*
1992	*12,110*	11,256	12,516	17,158	*797*	*1,311*
1993	*30,532*	14,017	11,930	3,375	250	*4,476*
1994	*16,973*	11,927	9,156	3,942	176	3,244
1995	*3,090*	11,065	4,873	32,348	*229*	879
1996	*9,969*	12,613	4,861	33,229	342	*2,271*
1997	*30,805*	11,931	2,955	33,402	342	*6,504*
1998	*28,964*	13,072	8,608	43,191	313	*6,868*
1999	*28,043*	14,499	12,464	54,642	777	*6,951*
2000	*31,837*	15,381	12,867	58,614	784	*7,746*
2001	22,364	10,965	25,677	94,897	894	7,708
2002	*12,960*	8,117	47,290	102,207	906	6,389
2003	*9,043*	16,928	47,860	116,758	762	10,576
2004	*23,442*	25,146	73,302	111,189	77	12,300
2005	840	27,913	94,310	11,890	778	23,941
2006	432	31,994	91,098	105,687	*937*	32,864
2007	*2,651*	32,800	91,989	109,095	2,200	18,991
2008	*9,090*	33,545	92,007	109,472	2,200	17,871
2009	13,256	26,907	73,342	138,170	2,613	36,233
2010	27,714	21,833	74,409	125,852	1,836	37,892
2011	*2,902*	24,248	67,531	124,486	1,830	21,059
2012	33,221	4,743	54,354	40,907	1,839	7,237
2013	15,647	6,915	53,882	731	1,782	2,795

Appendix E: Eight Dimensional Estimate Template

1/A	B	C	D	E	F	G	H	I	J	K
2	Geometrical Forecasting of Cost Estimate Uncertainty (Eight Dimensions - Formula Template)									
3	Note: This template contains the formulas for a Microsoft Excel (® which forecasts cost estimate uncertainty using the geometrical attributes of cost variance from eight dimensions. Data entry is required ONLY for the names of these dimensions (cells B3-I3) and the value of the cost variance for those dimensions (cells B4-I4) - these cells for manual data entry have a thick black border for orientation purposes. The current (supported) version of the spreadsheet can be obtained from the authors of the article this template is contained in. The Microsoft Excel ® version of this template contains controls allowing for the creation / deletion of a vector graph within this template and the completion of a Monte Carlo simulation for calculating the most likely value(s). Note that some (empty) lines have been removed to improve legibility.									
4	Name of Cost Dimensions	First	Second	Third	Fourth	Fifth	Sixth	Seventh	Eighth	Aggregated Vector
5	Absolute Value of Cost Dimensions	<Value First>	<Value Second>	<Value Third>	<Value Fourth>	<Value Fifth>	<Value Sixth>	<Value Seventh>	<Value Eighth>	=SUM(B4:I4)
6	Angle	=0*(360/8)	=1*(360/8)	=2*(360/8)	=3*(360/8)	=4*(360/8)	=5*(360/8)	=6*(360/8)	=7*(360/8)	=IF(90-DEGREES (ATAN2((J9-100);(100-J10)))<0;(270+(180-DEGREES (ATAN2((J9-100);(100-J10)))));90-DEGREES (ATAN2((J9-100);(100-J10))))
7	Relative Absolute Value of Cost Dimensions	=(B4/SUM($B$4:$I$4))*100	=(C4/SUM($B$4:$I$4))*100	=(D4/SUM($B$4:$I$4))*100	=(E4/SUM($B$4:$I$4))*100	=(F4/SUM($B$4:$I$4))*100	=(G4/SUM($B$4:$I$4))*100	=(H4/SUM($B$4:$I$4))*100	=(I4/SUM($B$4:$I$4))*100	=J11/SUM(B6:I6)
8	Coordinate x_s (Centre)	100	100	100	100	100	100	100	100	100
9	Coordinate y_s (Centre)	100	100	100	100	100	100	100	100	100
10	Coordinate x_e (End)	=100	=(100+COS(RADIANS(45))*C6)	=100+D6	=100+COS(RADIANS(45))*E6	100,00	=100-COS(RADIANS(45))*G6	=100-H6	=100-COS(RADIANS(45))*I6	=100+SUM(B9:I9)-800
11	Coordinate y_e (End)	=100-B6	=100-(SIN(RADIANS(45))*C6)	100	=100+SIN(RADIANS(45))*E6	=100+F6	=100+SIN(RADIANS(45))*G6	100	=100-SIN(RADIANS(45))*I6	=100+SUM(B10:I10)-800

1/A	B	C	D	E	F	G	H	I	J	K
12	Vector Length from Centre (Relative %)	=SQRT((B9-100)*(B9-100)+((100-B10)*(100-B10)))/100	=SQRT((C9-100)*(C9-100)+((100-C10)*(100-C10)))/100	=(D9-100)/100	=SQRT((E9-100)*(E9-100)+((100-E10)*(100-E10)))/100	=(F10-100)/100	=SQRT((100-G9)*(100-G9)+((100-G10)*(100-G10)))/100	=SQRT((100-H9)*(100-H9)+((100-H10)*(100-H10)))/100	=SQRT((100-I9)*(100-I9)+((100-I10)*(100-I10)))/100	=SQRT(ABS((100-J9)*(100-J9)+((100-J10)*(100-J10))))/100
13	(Inner) Angle Difference to Aggregated Vector	=IF(B5-J5>180;(-1)*(360-(B5-J5));B5-J5)	=IF(C5-J5>180;(-1)*(360-(C5-J5));C5-J5)	=IF(D5-J5>180;(-1)*(360-(D5-J5));D5-J5)	=IF(E5-J5>180;(-1)*(360-(E5-J5));E5-J5)	=IF(F5-J5>180;(-1)*(360-(F5-J5));F5-J5)	=IF(G5-J5>180;(-1)*(360-(G5-J5));G5-J5)	=IF(H5-J5>180;(-1)*(360-(H5-J5));H5-J5)	=IF(I5-J5>180;(-1)*(360-(I5-J5));I5-J5)	=SUM(B12:I12)/8
14	**Vector Space**									
15										
16										
17	**Symmetry**									

1/A	B	C	D	E	F	G	H	I	J	K
18	**Name of Cost Dimensions**	**First**	**Second**	**Third**	**Fourth**	**Fifth**	**Sixth**	**Seventh**	**Eighth**	**Aggregated Vector**
19	Actual Area	= 0.5*(B4)*(C4)*SIN(45) *10000	= 0.5*(C4)*(D4) *SIN(45) *10000	= 0.5*(D4)*(E4)*SIN(45) *10000	= 0.5*(E4)*(F4)*SIN(45) *10000	= 0.5*(F4)*(G4) *SIN(45) *10000	= 0.5*(G4)*(H4) *SIN(45) *10000	= 0.5*(H4)*(I4)*SIN(45) *10000	= 0.5*(I4)*(B4)*SIN(45) *10000	=SUM(B18:I18)
20	Actual Edge Length	= SQRT((B4*B4)+(C4*C4)-2*(B4*C4*COS(45)))	= SQRT((C4*C4)+(D4*D4)-2*(C4*D4*COS(45)))	= SQRT((D4*D4)+(E4*E4)-2*(D4*E4*COS(45)))	= SQRT((E4*E4)+(F4*F4)-2*(E4*F4*COS(45)))	= SQRT((F4*F4)+(G4*G4)-2*(F4*G4*COS(45)))	= SQRT((G4*G4)+(H4*H4)-2*(G4*H4*COS(45)))	= SQRT((H4*H4)+(I4*I4)-2*(H4*I4*COS(45)))	= SQRT((I4*I4)+(B4*B4)-2*(I4*B4*COS(45)))	=SUM(B19:I19)
21	Reference Edge Length	=$J$19/8	=$J$19/8	=$J$19/8	=$J$19/8	=$J$19/8	=$J$19/8	=$J$19/8	=$J$19/8	=SUM(B20:I20)
22	Reference Area	= (0.5*($B$20)*($B$20)*SIN(45))	= (0.5*($B$20)*($B$20)*SIN(45))	= (0.5*($B$20)*($B$20)*SIN(45))	= (0.5*($B$20)*($B$20)*SIN(45))	= (0.5*($B$20)*($B$20)*SIN(45))	= (0.5*($B$20)*($B$20)*SIN(45))	= (0.5*($B$20)*($B$20)*SIN(45))	= (0.5*($B$20)*($B$20)*SIN(45))	=SUM(B21:I21)*10000
23	Symmetry	=ABS(J18/J21)								
24										
25										
26										
27										
28										
29										
30										
31										
32										
33										
40	**Name of Cost Dimensions**	**First**	**Second**	**Third**	**Fourth**	**Fifth**	**Sixth**	**Seventh**	**Eighth**	**Average**

1/A	B	C	D	E	F	G	H	I	J	K
41	**Forecast Method #1 (Mode (c) - Non-Directional)**									
42	Actual Relative Vector Length (%)	=B11	=C11	=D11	=E11	=F11	=G11	=H11	=I11	=AVERAGE(B41:I41)
43	Forecast Method #1 - Forecast Relative Vector Length (%)	=B41+(B41*($J$41/100))	=C41+(C41*($J$41/100))	=D41+(D41*($J$41/100))	=E41+(E41*($J$41/100))	=F41+(F41*($J$41/100))	=G41+(G41*($J$41/100))	=H41+(H41*($J$41/100))	=I41+(I41*($J$41/100))	=AVERAGE(B42:I42)
44	Forecast Method #1 - Change in Relative Vector Length (%)	=B42-B41	=C42-C41	=D42-D41	=E42-E41	=F42-F41	=G42-G41	=H42-H41	=I42-I41	=AVERAGE(B43:I43)
46	**Forecast Method #2 (Upper Limit (b) - Directional)**									
47	Delta X From Aggregated Vector	=J9-B9	=J9-C9	=J9-D9	=J9-E9	=J9-F9	=J9-G9	=J9-H9	=J9-I9	=AVERAGE(B46:I46)
48	Delta Y From Aggregated Vector	=J10-B10	=J10-C10	=J10-D10	=J10-E10	=J10-F10	=J10-G10	=J10-H10	=J10-I10	=AVERAGE(B47:I47)
49	Length of Directional Influencing Vector	=SQRT((B46*B46)+(B47*B47))/1000	=SQRT((C46*C46)+(C47*C47))/1000	=SQRT((D46*D46)+(D47*D47))/1000	=SQRT((E46*E46)+(E47*E47))/1000	=SQRT((F46*F46)+(F47*F47))/1000	=SQRT((G46*G46)+(G47*G47))/1000	=SQRT((H46*H46)+(H47*H47))/1000	=SQRT((I46*I46)+(I47*I47))/1000	=AVERAGE(B48:I48)
50	Forecast Method #2 - Forecast Relative Vector Length (%)	=(B11+B48)	=(C11+C48)	=(D11+D48)	=(E11+E48)	=(F11+F48)	=(G11+G48)	=(H11+H48)	=(I11+I48)	=AVERAGE(B49:I49)
51	Forecast Method #2 - Change in Relative Vector Length (%)	=B49-B11	=C49-C11	=D49-D11	=E49-E11	=F49-F11	=G49-G11	=H49-H11	=I49-I11	=AVERAGE(B50:I50)
53	**Forecast Method #3 (Lower Limit (a) - Product)**									
54	Absolute % Change	=B43	=C43	=D43	=E43	=F43	=G43	=H43	=I43	=AVERAGE(B53:I53)

1/A	B	C	D	E	F	G	H	I	J	K
	Forecast in Method 1									
55	Absolute % Change Forecast in Method 2	=B50	=C50	=D50	=E50	=F50	=G50	=H50	=I50	=AVERAGE(B54:I54)
56	Forecast Method #2 - Forecast Relative Vector Length (%)	=B41*B56	=C41*C56	=D41*D56	=E41*E56	=F41*F56	=G41*G56	=H41*H56	=I41*I56	=AVERAGE(B55:I55)
57	Forecast Method #3 - Change in Relative Vector Length (%)	=B54*B53	=C54*C53	=D54*D53	=E54*E53	=F54*F53	=G54*G53	=H54*H53	=I54*I53	=AVERAGE(B56:I56)
59	**Threat Assessment Based on Geometrical Attributes of Forecasted Aggregate Vector (Calibrated based on Proprietary Industry Dataset of 63 Estimates)**									
60	**Attribute**	**Value / Threat**	Centre RED Range	Range Width	RED Range (Absolute)	GREEN Low %	AMBER Low %	RED Range	AMBER High %	GREEN High %
61	Direction of the aggregated vector (°)	=J5	156	10%	17	<124	123-139	140-172	171-189	>188
62	Magnitude of the aggregated vector	=J11*100	23	20%	5	<12	11-18	17-28	27-34	>33
63	Geometrical Symmetry (%)	=B22	73	10%	7	<58	57-66	65-80	79-88	>87
64	Uncertainty Range (%)	=J50-J56	45%	40%	40%	N/A	<6%	5%-85%	>84%	N/A
65	Most Likely (80%) Confidence at Next Time Interval	='Monte Carlo'!AB48								

Appendix F: Comparative Method Review

Method Type / Attribute	Analogy / Expert Opinion	Polar Force Field	Parametrics	Regression
Required periods of estimating history	0	1-3	4-41	42+
Probability field best suited for	F+	E & F	B, C & D	A
Based on	Human dynamics	Living systems principles	System dynamics principles	Law of large numbers
Organises with	Cognitive bias	Shape	Relationships of variables	Probability density functions
Review altitude	Very high	High	Low	Very low
Scenarios supported	Unknown	Possible	Plausible	Most likely
References	Stories	Multiple centres of probability spaces	Co-variate correlations with high statistical significance	Single centre of two-dimensional data distribution
Reveals	Lessons learned	Uncertainty	Dependencies	Statistical confidence
Primary strength	Speed	Minimum data points required	Readily understood	Based on significant and relevant experience
Primary weakness	Suitability of analogy	Unavailability of information and principles unfamiliar to estimators	Degree of abstraction and normalisation requirements	Reliance on the Central Limit Theorem paradigm
Typical accuracy	<50%	>60%	>80%	>95%

Appendix G: Future work – State and Dynamic Space Pictures

State Space Picture	Unity / Translation Space Picture	Dynamic Space Picture
Invariant Polar Force Field Attributes		
Constant (Independent of Time)		Variable (Dependent of Time)
• Computational Complexity • Cost Variance Calculation • Cost Variance Dimensions • Dimensional Scaling • Dimensional Sequence • Monte Carlo Probability Density Function • Radial Degree • Reference Forecast Cost Variance • Spatial Centre	• Dependency Model • Information Entropy Principles • Layout Algorithm (Force Directed) • Response Surface Modelling Methodology	• (Cloud) Symmetry • Actual Cost Variance Vector • Aggregate Cost Variance Vector • Eigenvector • Forecast Cost Variance • Forecast Cost Variance Vector • Monte Carlo Probability Density Function • Most Likely Cost Variance • Radial Degree • Spatial Centre • Torque • Uncertainty Range
Variable Polar Force Field Attributes		
Variable (Dependent of Time)		Constant (Independent of Time)
• (Planar) Symmetry • Actual Cost Variance • Actual Cost Variance Vector • Aggregate Cost Variance Vector • Eigenvector • Forecast Cost Variance • Forecast Cost Variance • Most Likely Cost Variance • Torque • Transformation Vectors • Uncertainty Range • Vector	• Computational Complexity • Symmetry (Planar to Cloud) • Torque	• Actual Cost Variance • Computational Complexity • Cost Variance Calculation • Cost Variance Dimensions • Dimensional Scaling • Dimensional Sequence • Reference Forecast Cost Variance • Transformation Vectors