

Investigating the Relationship between High-Yield Bonds and Equities and its Implications for Strategic Asset Allocation during the Great Recession

Abstract

In this paper, we focus on investing in US high-yield bonds during the period 2007-13, a period that covers the Great Recession in the aftermath of the Global Financial Crisis of 2007-08. Firstly, we use the Fama and French three-factor model to delve into the relationship between the risk-adjusted returns of high-yield bonds and equity market risk factors. Secondly, we gauge the extent to which the risk-adjusted returns of high-yield bonds are significantly higher than equity and investment-grade bonds' risk-adjusted returns. Thirdly, by using a modified version of the Black-Litterman model, we explore the asset allocation to high-yield bonds, accounting for investors' risk tolerance. Our findings suggest that equity market risk factors have significant explanatory power for high-yield bonds' risk-adjusted returns whilst the hypothesis of superior returns on high-yield bonds over investment-grade corporate bonds and equities cannot be supported. Our key contribution relates to the strategic asset allocation to high-yield bonds. Our results suggest that the share of high-yield bonds does not exceed 4.1% of total assets in a global market portfolio over the period 2007-13. Notably, the share of high-yield bonds in a simulated portfolio remains relatively small and stable on a risk-adjusted basis, irrespective of an investor's risk profile or the phase of the business cycle.

JEL Classification codes: C10; C61; G11

Keywords: *Fama-French three-factor model, Black-Litterman model, High-yield bonds, Asset allocation, Global Financial Crisis.*

1. Introduction

A major way that portfolios can effectively reduce risk is by combining investments whose returns do not move in tandem. Sometimes, a subset of assets will go up in value while another will go down. The fact that these may offset each other creates the diversification benefit that is attributed to portfolios. However, an important issue is the sensitivity to correlation risk, whereby the securities' returns in the portfolio can change in an unfavorable manner, especially during times of market stress.

During the GFC, the diversification benefits were relatively small. Evidently, all major global equity indices declined in unison. The lesson is that, although portfolio diversification generally does reduce risk, it does not necessarily provide the same level of risk reduction during times of severe market turmoil as it does when the economy and markets are operating smoothly. In fact, if either the economy or markets collapse, then any diversification benefits could simply be illusory.

Herein, data on past returns from indices tracking the US equity and bond markets are used. We explore the risk-adjusted returns of high-yield bonds in relation to equities, as well as the risk-adjusted returns of high-yield bonds relative to investment-grade bonds. The contribution of the paper is twofold: First, to the best of our knowledge, this is the first study to explore the relationship between high-yield-bond risk-adjusted returns over the period of the Global Financial Crisis (December 2007- June 2009) using the Fama and French three-factor model (Fama and French, 1993). In this context, we investigate whether the risk-adjusted returns of high-yield bonds can be explained effectively using equity market risk factors. Secondly, we use Black and Litterman's (1992) model for all major asset classes represented by twenty-five benchmark indices that represent an equal number of sub-asset classes, simulating the global market portfolio over 2007-13, a period that includes both phases of the business cycle. Thereby, we analyze what proportion of their portfolios investors of different risk tolerances should dedicate to high-yield bonds.

The rest of the paper is organized as follows: Section 2 provides an overview of related literature, whilst section 3 elaborates on the data and methodological framework adopted in the empirical investigation. Section 4 discusses the results of our analysis, whilst section 5 provides some concluding remarks.

2. Literature Review

High-yield bonds are deemed to lie somewhere between investment-grade corporate debt and equity securities. High-yield bond prices are volatile and, compared to those of investment-grade bonds, tend to be less affected by interest rates, instead exhibiting a relationship with equity market changes. The stock-to-bond return relationship has certainly received much research attention. Shiller and Beltratti (1992) find changes in stock prices to be highly correlated with long-term bond yields, whilst Campbell and Ammer (1993) demonstrate a weak correlation. More recently, a strong time dependency in the stock-bond correlation has been claimed (Gulko, 2002; Jones and Wilson, 2004; Cappiello, Engle and Sheppard, 2006). IMF (2015) states that, in 2008-09, following the recent crisis, global asset market correlation jumped to around 80%. Before this (1997-2007), such correlation lay at around 45%, close to what it had been historically. A crash has in the past always been accompanied by high correlations, due to the markets being most strongly influenced by panic. However, following this recent crisis, despite heavily increasing asset prices, correlations stayed at a level well above what had been seen prior to the crisis, at about 70%, which suggests a high degree of

inter-connection within the global asset management industry. This left investors few options for portfolio diversification.

2.1 The presence of an equity element in high-yield bonds and the business cycle

Contemporary work on corporate debt is heavily based on Black and Scholes (1973) and Merton (1974). According to Merton (1974), one can think of risky corporate bond holders as riskless bond holders who have issued put options on the equity of the firm in question. With an increase in volatility, we see an increase in the value of the put options, such that equity holders benefit at the expense of bond holders. As Bookstaber and Jacob (1986) find, when long-term corporate bonds decline in quality, there is an increase in their returns' correlation with those on common stock. Blume and Keim (1987) make a similar observation on the risk-return characteristics of junk bonds. Ramaswami (1991) shows that non-investment-grade bonds' return variance is more heavily affected than that of investment-grade bonds by sector-industry and firm-specific factors. Also, Regan (1990), who includes phases of the business cycle in his study, suggests that junk bond and low-quality stock performance tends to depend on the riskiness of the individual companies in question rather than swings in the capital markets.

Cornell and Green (1991) find that junk bond returns are less sensitive to movements in long-term interest rates and become more sensitive to equity risk than investment-grade bond returns are, whilst they also suggest that high-yield bond returns are more influenced by fluctuations in interest rates, and less by changes in stock prices, in contraction periods as compared to expansion periods. Similar results are reported by Kihn (1994), while Zivney, Bertin and Torabzadeh (1993) hint that Cornell and Green (1991) overstated the actual sensitivity of investment-grade bond returns to equity market risk factors. The results of Shane (1994) and Reilly and Wright (2002) suggest that junk bond returns are more sensitive to equity risk and less sensitive to movements in interest rates during phases of weak economic activity, while Patel, Evans and Burnett (1998) find that high-yield bond returns are significantly influenced by changes in stock prices over such phases. Along the same lines, Domian and Reichenstein (2008) find that high-yield bonds embed investment-grade bond, stock, and even cash characteristics, while the equity constituent of their returns exhibits a small-cap equity tilt. Using panel data for the late 1990s, Campbell and Taksler (2003) find that equity risk is an important factor that explains corporate bond yield spreads. Both the studies of Blume, Keim, and Patel (1991) and Blume and Keim (1991) suggest that

non-investment-grade bonds exhibit a similar pattern to a value-weighted portfolio of common stocks of the NYSE.

In one of the most influential studies in finance, Fama and French's (1993) regressions of bond returns on both equity market and debt market risk factors indicate that only junk bond returns produce significant coefficients on the equity market risk factors. Fridson (1994) extends the literature on whether there is an equity component in non-investment-grade bonds, while Fjelstad *et al.* (2005) investigate the role of junk bonds and emerging-market debt in a well-diversified portfolio of a US institutional investor. Coaker II (2007) examines the volatility of the correlation among eighteen different sub-asset classes, emphasizing weakly correlated assets, and Jirasakuldech, Emekter and Lee (2008) study the non-random-walk behavior of the US stock and bond returns. In a study on the historic changes in the junk bond market, Reilly, Wright and Gentry (2009) find a significant correlation between high-yield bond and small-cap stock returns. More recently, Zhang and Wu (2014) find a positive correlation but a weak relationship between high-yield bond and the corresponding common stock returns over long periods of time¹.

2.2 Performance of high-yield bonds compared to equities and investment-grade bonds

It is argued by Blume and Keim (1987) that the returns on non-investment-grade bonds are greater than those on investment-grade bonds but lower than stock returns. At the same time, the volatility of equity returns is higher than that of high-yield and investment-grade bond returns, possibly because the prices of non-investment-grade bonds do not adjust rapidly to new information as is the case with stock and investment-grade bonds (Blume and Keim, 1987). In Altman and Heine (1990), the returns on junk bonds are found to be greater than those on investment-grade bonds, while the standard deviation of returns is approximately equal for both. Altman and Heine (1990), meanwhile, claim that the performance of non-investment-grade bonds is higher than that of equities, although the volatility of returns is higher for stocks than for junk bonds. According to Regan (1990), when the economy is expanding the performance of high-yield bonds is superior to that of investment-grade bonds, but when it is contracting that of investment-grade bond returns is superior to that of non-investment-grade bonds.

¹ Zhang and Wu (2014) ascribe this to the asymmetric response of non-investment-grade bond returns to the information revealed by the equity market, depending on the market conditions.

Among the benchmark studies in the related literature, Cornell and Green (1991) and Blume, Keim and Patel (1991) suggest that junk bond returns outstrip investment-grade bond returns and lag behind common stock returns, exhibiting the lowest standard deviation of returns among these particular asset classes². In a later study, Kihn (1994) argues that junk bonds do not demonstrate significantly higher returns than investment-grade bonds in a downturn. Later, Patel, Evans, and Burnett (1998) argue that junk bonds present higher risk and poorer performance than investment-grade bonds during periods of weak economic activity³.

Briere and Szafarz (2008) determine that a crisis-robust portfolio is one that exhibits the lowest volatility in both expansion and contraction periods. The same authors find that junk bond returns are inferior to investment-grade bond returns in the long run, while the standard deviation of high-yield bond returns is lower than that of investment-grade bonds in the long run⁴. According to the argument of Zivney, Bertin and Torabzadeh (1993), studies prior to 1990 omitted to take account of the two factors that contribute the most to fixed-income securities' return volatilities: capital gains or losses due to interest rate movements and the rate of coupon reinvestment. In line with other authors, Zivney, Bertin, and Torabzadeh (1993) find that junk bond returns are higher than investment-grade bond returns⁵. Fjelstad *et al.* (2005) find that junk bond returns trail equity returns, while their standard deviation is nearly half the standard deviation of stock returns. The results of Fjelstad *et al.* (2005) are supported by the studies of Trainor Jr. and Wolfe (2006) and Bekkers, Doeswijk and Lam (2009)⁶. Contrary to previous studies, Reilly, Wright and Gentry (2009) suggest that the standard deviation of non-investment-grade bond returns is much higher than that of investment-grade bonds, while both asset classes present roughly similar returns⁷.

² The study by Cornell and Green (1991) covered the period 1960 -1976 whereas the one by Blume, Keim and Patel (1991) examined the period 1977-1989.

³ The results of Patel, Evans and Burnett (1998) which comprises daily returns over the eight years from January 1987 to December 1994, provide support to the findings of Regan (1990) that high-yield bonds present greater returns than investment-grade bonds.

⁴ Interestingly, Briere and Szafarz (2008), using three categories of bonds (sovereign, investment grade corporate, and high yield corporate) in the U.S. and Eurozone for the period 1998-2007, find that high-yield bonds outperform investment-grade bonds throughout periods of dilated economic activity, while they exhibit lower standard deviation in their returns. On the other hand, the authors show that this is true even though investment-grade bond returns are less volatile than non-investment-grade bond returns during periods of contraction.

⁵ However, Zivney, Bertin and Torabzadeh (1993) support the view that, on a risk-adjusted basis, the returns of high-yield bonds are not statistically significantly higher than investment-grade bond returns. Also, the authors argue that the standard deviation of non-investment-grade bond returns is almost the same as that of investment-grade bonds throughout uptrend periods, while it becomes three times higher during downtrend periods, reaching the standard deviation of stock returns.

⁶ In Bekkers, Doeswijk and Lam's study the sample covers the period from 1984 to 2008.

⁷ The dataset used in this study consists of monthly data covering the period 1985 through August 2009.

By assuming that returns are normally distributed, Li, McCarthy and Pantalone (2014) find that high-yield bonds exhibit superior returns to investment-grade bonds, while Manzi and Rayome (2016) provide evidence that non-investment-grade bond returns are superior to equity returns, but exhibit lower standard deviation than stock returns⁸.

2.3 High-yield bonds and their effect on portfolio allocation

Markowitz (1952) provided the foundations of modern portfolio theory. Although mean-variance optimization (MVO) represented a theoretical breakthrough at the time, applying it can be tricky. Black and Litterman (1992) combined MVO with the Capital Asset Pricing Model (CAPM) to resolve this issue. Among other favorable aspects, the Black-Litterman model (BL) eliminates the problem of portfolios that are highly concentrated among a handful of assets. Relatedly, Brinson, Hood and Beebower (1986), Brinson, Singer and Beebower (1991) and Ibbotson and Kaplan (2000) emphasize strategic asset allocation's criticality to investment performance.

In general, researchers have not dealt extensively with portfolio allocation in respect to non-investment-grade bonds. Blume and Keim (1987), Kihn (1994), Reilly, Wright and Gentry (2009), Manzi and Rayome (2016) and others have used the promise of enhanced returns and diversification benefits to argue that investment-grade bond and equity investors should also see junk bonds as reliable investments. Kihn (1994) adds that periods of economic contraction contribute to reducing the volatility of non-investment-grade bond returns in the long run. Hence, investors should consider adding non-investment-grade bonds to their portfolios. Fjelstad *et al.* (2005) make the argument that high-yield bonds can be a means for low-equity-exposure investors to obtain significant returns on their portfolios. They demonstrate that high-yield bonds ought not to be classified separately when allocating funds to different assets. Similarly, Trainor Jr. and Wolfe (2006) emphasize using exact percentages rather than approximations to determine the proportion of investors' portfolios that should be made up of non-investment-grade bonds⁹. From BL model results, Trainor Jr. and Wolfe (2006) demonstrate that these percentages fall markedly, depending on the high-yield-to-

⁸ In their study, Li, McCarthy and Pantalone compare the returns on investment-grade bonds to the returns on high-yield bonds over the period from January 1997 through mid-August 2011 whilst the study by Manzi and Rayome (2016) uses data spanning September 1, 2004 through August 31, 2014.

⁹ Sampling monthly returns for six different asset and sub-asset classes over the entire period of 1986-2004 and employing MVO, Trainor Jr. and Wolfe (2006) conclude that the percentage in question ranges strikingly, from 2% in portfolios with expected returns of 5%, to 42% in portfolios with expected returns of 11%, while it falls significantly to just 12% in portfolios with expected returns of 12%, and thereafter becomes negligible. This decline can be explained by the fact that junk bond expected returns do not exceed 11% and, therefore, the role of high-yield bonds in portfolios with expected returns of 12% or higher is restricted.

investment-grade bond return spread¹⁰. Briere and Szafarz (2008) highlight the flight-to-quality effect, in which all securities' return volatilities increase through the economic cycle. Thus, a bond portfolio including around 4.22% high-yield bonds as a share of total assets will typically survive downturns more effectively than a 'safe' portfolio. Bekkers, Doeswijk and Lam (2009), based on the application of three different methodologies, and a portfolio comprising ten different asset and sub-asset classes, come to the conclusion that including high-yield bonds increases portfolio value.

Thus far, the literature implies that high-yield bonds contain an equity-like component, and play a key role in portfolio construction and investment decisions. However, it is not unambiguous regarding the link between the risk tolerance of the investor and the consequent proportion of their portfolio that should be made up of high-yield bonds.

3. Data and Methodology

3.1 Data

Debt instruments lie at the core of the research interest of this study. The fact that the global debt market is largely dominated by US fixed-income securities explains why the US debt market provides the platform upon which our empirical investigation is built.¹¹

To define the phases of the business cycle, we rely on National Bureau of Economic Research (NBER, 2010) data. In particular, the most recent economic downturn is taken to be December 2007 to June 2009, whilst the latest expansion period began in July 2009, i.e. immediately following the Great Recession. Data frequency is key to separating critical information from noise. We have chosen monthly, seeking to remove volatile fluctuations. In addition, we use benchmark indices, i.e. portfolios of securities, representing particular markets, which means overall performance is tracked and aggregate changes measured relatively accurately. Appendix A presents the indices and sources used, while in Appendix B the correlation matrix of the indices is presented.

Besides the excess rate of return, we have computed the nominal risk-free rate of return, using that part of the returns associated with the risk embedded in the sub-asset classes studied. The nominal risk-free rate of return benchmark that is most commonly quoted is the

¹⁰ The authors argue that high-yield bonds should be included in a portfolio only if their return spread over investment-grade bond returns equals or exceeds 150 basis points.

¹¹ According to the Bank for International Settlements (2016), the market capitalization of the global debt market was roughly \$97.73 trillion at the end of the second quarter of 2016. At the same time, the value of the US fixed-income securities was approximately \$37.90 trillion, accounting for about 39% of the total value of outstanding debt universally.

three-month US Dollar (USD) London Interbank Offered Rate (LIBOR)¹². After collecting the annual three-month USD LIBOR at the end of each month from Bloomberg Professional, we then proceed to compute the three-month USD LIBOR at the end of each month¹³.

Realistically, private investors cannot include every single global investment in their portfolios. However, they can invest in a broad range of sub-asset classes in order to maintain a portfolio that closely replicates the global market portfolio. According to Maginn *et al.* (2007), the selection criteria for sub-asset classes for this purpose are that they should be homogeneous, diversifying and non-overlapping. Furthermore, the authors highlight that the total market capitalization of the particular sub-asset classes should make up the biggest possible fraction of the overall wealth of the investors, while each sub-asset class should carry the capacity to absorb a considerable portion of a potential investor's capital. The 25 indices that meet the selection criteria and mirror the global market portfolio are shown in Appendix A. The constructed portfolio comprises of all major asset classes, namely equities, bonds, commodities, real estate and private equity investments, and cash, weighted on the basis of their respective market capitalization. As such, equities account for about 27.1% of the portfolio and are represented by the first 9 benchmark indices, while bonds – both investment-grade and high-yield ones – account for around 66.4% of the portfolio and are represented by the following 9 indices. The remaining 6.5% of the constructed portfolio represents cash equivalents, commodities, real estate and private equity investments which are reflected in the last 7 indices shown in Appendix A.

3.2 Research methodology

In determining the extent to which equity market risk factors explain high-yield-bond risk-adjusted returns, we utilise the Fama and French three-factor model (FF) as follows:

$$R_{i,t} - RFR_t = \alpha_i + \beta_{i, RM - RFR} (RM_t - RFR_t) + \beta_{i, SMB} SMB_t + \beta_{i, HML} HML_t + u_{i,t}$$

where $R_{i,t}$ is the rate of return of asset i , RFR_t is the risk-free rate of return, $R_{i,t} - RFR_t$ is the rate of excess return of asset i , α_i is the rate of active return of asset i or the actual return of asset i minus the risk-free rate of return, $\beta_{i, RM - RFR}$ is the level of exposure of asset i to

¹² LIBOR is the average interbank interest rate that leading banks charge each other for short-term loans in the London money market. Unlike the three-month US Treasury Bill (T-bill) interest rate, the three-month USD LIBOR is not theoretically a risk-free rate. However, the three-month USD LIBOR is regarded as a better indicator of the nominal risk-free rate of return than the three-month US T-bill interest rate, mainly because it is an ideal hedging vehicle. Another drawback of the three-month US T-bill interest rate is that it is artificially kept at low levels, for tax and regulatory reasons.

¹³ Monthly three-month USD LIBOR = $(1 + \text{Annual three-month USD LIBOR})^{1/12} - 1$.

overall market risk, RM_t is the rate of return of the overall market, $RM_t - RFR_t$ is the rate of excess return of the overall market, $\beta_{i, SMB}$, is the level of exposure of asset i to company size risk or the company size risk factor coefficient of asset i , SMB_t is the company size risk factor during period t , which is computed as the average rate of return of the 30% of stocks with the smallest market capitalization during period t minus the average rate of return of the 30% of stocks with the largest market capitalization during period t , $\beta_{i, HML}$ is the book-to-market equity risk factor during period t , which is computed as the average rate of return of the 50% of stocks with the highest book-to-market equity ratio during period t minus the average rate of return of the 50% of stocks with the lowest book-to-market equity ratio during period t , and finally, $u_{i,t}$ is the nonsystematic risk of asset i during period t or the part of the rate of excess return of asset i that cannot be explained by the overall market, company size and book-to-market equity risk factors during period t or the random error term of asset i during period t . Data for the factors that feed into the FF model are sourced from Kenneth R. French's Data Library¹⁴.

Even though the academic literature in this area clearly supports the presence of an equity component in non-investment-grade bonds, it does not clarify the strength of this relationship. To test the explanatory power of the equity market risk factors on high-yield-bonds risk-adjusted returns, we apply the FF model over the two distinct phases of the business cycle.

The required rate of return on an investment is the minimum acceptable rate of return that compensates investors for the period over which they commit their capital, for the expected rate of inflation, for the changes in the conditions of the capital markets, and for the level of risk embedded in the specific investment. The three factors other than risk affect all investments equally, and constitute the underlying determinants of the nominal risk-free rate of return, which is common for all investments. In contrast, risk is a unique composite of the uncertainty around each particular investment, and its measure is referred to as the risk premium. The required rate of return of an investment is, therefore, equal to the sum of the nominal risk-free rate of return and the risk premium of the specific investment, typically calculated by the CAPM. Hence, if the investor-estimated price or rate of return of an asset is higher than the market price, the asset is considered undervalued and vice versa. The risk-return trade-off constitutes a primary criterion of the quality of a prospective investment, and an essential component in the investment decision, suggesting the purchase of undervalued securities and the sale of overvalued financial instruments.

¹⁴ Available at: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Currently, the majority of the literature focuses on the reliable risk-adjusted performance of high-yield bonds as compared to stocks and investment-grade bonds when risk is measured by the volatility of their returns, thus hinting that high-yield bonds are systematically underpriced securities. Therefore, it is worth exploring the underlying relationships by determining whether high-yield bonds are systematically undervalued during the contraction and expansion periods of 2007-13.

To this end, a two-sample t-test of means is used. Initially, the normality assumptions of the two-sample t-tests of means are evaluated by applying the Shapiro-Wilk test to each of the selected samples of high-yield bond, stock, and investment-grade bond risk-adjusted returns. The statistical significance of the superior performance of high-yield bonds compared to equities or investment-grade bonds is evaluated using two-sample t-tests of means on each of the corresponding pairs of the selected samples. The entire process is conducted for the economic expansion (2000-07) and contraction (2008-13) period.

Portfolio optimization is one of the main elements of investment and portfolio management. This involves selecting the weights of the various sub-asset classes in investors' portfolios so as to provide investors with their highest possible expected rate of return, for their given risk tolerance. One of the most important aspects of successful portfolio optimization is asset allocation¹⁵.

Risk tolerance is unique to each investor and is a key parameter in investment and portfolio management. Precisely, it refers to the investment return volatility an investor is willing to accept, which is likely based on individual aspects such as family responsibilities, age, net worth, cash reserves, expected income and insurance coverage.

Since its publication in 1992, the BL model has gained momentum as a popular portfolio optimization tool, thanks to its efficiency and ability to construct optimal portfolios for different investors depending on their risk profile. The bulk of studies in this area suggest that high-yield bonds play a critical role in the investment and portfolio management process. However, the related literature does not provide clear evidence on the extent to which high-yield bonds should be included in investors' portfolios depending on their risk tolerance. We

¹⁵ This refers to the traditional long-term asset allocation strategy that seeks to exploit full benefit from diversification across the different sub-asset classes based on their historical risk-return features, but without any adjustment of the sub-asset-class mix aimed at taking advantage of temporary changes in the capital market, as is seen in tactical asset allocation.

deal with this ambiguity by examining the potential allocation of an investor's portfolio to high-yield bonds depending on their risk tolerance. For this reason, we use a modified version of the BL model to construct a series of optimal portfolios that cover the entire range of risk profiles of investors. The modified version of the BL model is run for twenty-five different sub-asset classes and the entire process is conducted twice, once for each period, namely the economic expansion and economic contraction periods. Then, we determine the share of investors' portfolios that can be allocated to high-yield bonds, according to the investors' risk tolerance, for the two phases of the business cycle¹⁶.

The adjusted closing prices of the sampled indices are collected from Bloomberg Professional, the Barclays Guides & Factsheets, and Cambridge Associates Private Investment Benchmarks. Subsequently, the monthly total returns of the particular indices at the end of each month are computed as follows:

$$MTR_{j,m} = (ACP_{j,m} - ACP_{j,m-1}) / ACP_{j,m-1}$$

where $MTR_{j,m}$ is the monthly total return of index j at the end of month m , $ACP_{j,m}$ is the adjusted close price of index j at the end of month m , $ACP_{j,m-1}$ is the adjusted close price of index j at the end of month $m-1$, and j is the index subscript.

We compute the monthly total returns at the end of each month in each quarter using the following formula:

$$MTR_{q1,q2,q3} = (1 + QTR_q)^{1/3} - 1$$

where $MTR_{q1,q2,q3}$ is the monthly total return of Cambridge Associates Global ex-US Private Equity & Venture Capital Index at the end of months q1, q2 and q3 of quarter q and QTR_q is the quarterly total return of Cambridge Associates Global ex-US Private Equity & Venture Capital Index at the end of quarter q. For each sampled index, the monthly excess return over the nominal risk-free rate of return, at the end of each month, is given by the monthly total return of that index minus the monthly three-month USD LIBOR at the end of that month.

Assuming, for each sample index, aggregate market capitalization equals overall market capitalization of the global market portfolio, we compute the market weights of all the sampled indices in the global market portfolio, at the end of each month. Due to data availability issues, we assume all these market weights to remain constant, that is, we assume the market capitalizations of all the sampled indices to have been altered in proportion to

¹⁶ The detailed process and computations are described in the Technical Appendix. The programming code can be provided upon request for replication purposes.

each other over our study’s sample period. The market capitalization and corresponding weights are presented in Appendix A.

The coefficients of the FF model are estimated by regressing the Barclays US Corporate High-Yield Bond Index excess returns on the Fama and French three factors, using the ordinary least squares (OLS) method in STATA.

The reliability of each of the two-sample t-tests of means and the validity of the corresponding conclusions about the statistical significance of the superior performance of high-yield bonds over equities or investment-grade bonds relies on a set of assumptions. These assumptions are evaluated by performing the Shapiro-Wilk test individually on each of the samples of the Barclays US Corporate High-Yield Bond Index, MSCI USA Index and Barclays US Corporate Bond Index excess returns.

The statistical significance of the superior performance of the Barclays US Corporate High-Yield Bond Index over the MSCI USA or Barclays US Corporate Bond Indices is evaluated by performing two-sample t-tests of means on each of the corresponding pairs of the specific samples. For each of the two-sample t-tests of means, the corresponding selected samples are considered independent of each other, and the standard deviations of the corresponding population excess returns are considered unknown and unequal to each other.

Last, a modified version of the BL model for twenty-five different sub-asset classes as reflected in the benchmark indices shown in Appendix A is estimated in MATLAB¹⁷.

4. Estimation Results and Discussion

The OLS estimators of the coefficients of the FF model for both phases of the business cycle are presented in Table 1.

Table 1. OLS estimates from Fama and French three-factor model: Dependent variable: $R_t - RFR_t$

	Expansion		Contraction	
	Coefficients	(p-values)	Coefficients	(p-values)
$RM_t - RFR_t$	0.367	(0.000)***	0.784	(0.000)***
SMB_t	- 0.024	(0.801)	0.648	(0.029)**

¹⁷ This process is described in detail in the technical appendix. The programming code can be provided upon request for replication purposes.

HML _t	0.015	(0.841)	- 0.474	(0.032)**
c	0.525	(0.002)***	0.783	(0.045)**
Adjusted R ²	0.57		0.72	
F-test	2.354	(0.021)	3.782	(0.000)
B-G (p-value)	0.862		0.897	
B-P (p-value)	0.998		0.973	
VIF	1.13		0.98	

Notes: (1) The Breusch-Godfrey (B-G) and the Breusch-Pagan (B-P) tests for serial correlation and heteroscedasticity respectively were employed; (2) ***, ** denote significance at the 1% and 5% levels of significance, respectively.

On the basis of the results obtained for the upturn of the business cycle, the overall market risk is found to be highly significant and of the right sign. In other words, the Barclays US Corporate High-Yield Bond Index excess return is associated with the overall market return. The two remaining variables are found to be insignificant, implying that the company size risk and book-to-market equity risk factors do not have any significant impact on the overall market return.

As far as the downturn in the cycle is concerned, all explanatory variables are found to be significant. More specifically, the overall market and company size risk factor coefficients bear positive signs, whilst the book-to-market equity risk factor coefficient is negative. In other words, the Barclays US Corporate High-Yield Bond Index excess returns are positively associated with the overall market and company size risk factors, while there is an inverse relationship between that index's excess returns and the book-to-market equity risk factor.

In view of the above, it can be argued that the presence of an equity component in high-yield bonds is so strong that researchers, financial institutions, and investors can rely on and take advantage of it, employing equity market risk factors in order to estimate non-investment-grade-bond risk-adjusted returns both during periods of economic expansion and of economic contraction. It can also be sustained that, during economic contraction, the explanatory power of all factors suggested by Fama and French for the Barclays US Corporate High-Yield Bond Index excess returns is statistically significant, whilst only the overall market risk factor explains that index's excess returns when the economy is in the boom cycle.

Moreover, the generated evidence is in line with the research findings of Shane (1994), Patel, Evans and Burnett (1998) and Reilly and Wright (2002), according to which non-investment-grade bond returns are more sensitive to equity risk over periods when economic activity is shrunk, compared to periods when economic activity is galvanized. The evidence established

by Cornell and Green (1991) and Kihn (1994), however, stands in stark contrast to our findings, in that junk bond returns are found to be less sensitive to equity risk in periods when the level of economic activity is on the downturn, than in periods when it is on the upturn.

Table 2 presents the p-values of the Shapiro-Wilk statistics during both phases of the cycle, for the excess returns calculated from the samples of the Barclays US Corporate High-Yield Bond Index, MSCI USA Index, and Barclays US Corporate Bond Index. On the basis of the results generated, all indices originate from normally distributed populations, rendering the assumptions of the two-sample t-tests of means valid.

Table 2. Normality test on the excess returns of the three indices

<i>Excess returns</i>	<i>Expansion</i>	<i>Contraction</i>
	<i>Shapiro-Wilk statistic (p-values)</i>	
Barclays US Corporate High-Yield Bond Index	0.302	0.599
MSCI USA Index	0.492	0.815
Barclays US Corporate Bond Index	0.512	0.160

Table 3. Two-sample t-test of means

<i>Samples</i>	<i>Expansion</i>	<i>Contraction</i>
	<i>Two-sample t-test of means (p-values)</i>	
Barclays US Corporate High-Yield Bond Index & MSCI USA Index	0.342	0.157
Barclays US Corporate High-Yield Bond Index & Barclays US Corporate Bond Index	0.065	0.454

Moreover, the results presented in Table 3 suggest that, during both the expansion and contraction periods of the cycle, the means of the Barclays US Corporate High-Yield Bond Index excess returns are not significantly higher than those of either the MSCI USA Index or the Barclays US Corporate Bond Index.

On the basis of the reported results, it can be argued that non-investment-grade bonds are not statistically significantly undervalued compared to equities and investment-grade bonds in either the economic expansion or contraction period. Therefore, there is no inefficiency in the market that investors could exploit by adopting an appropriate investment strategy over both phases of the business cycle.

As far as the performance of high-yield bonds is concerned vis-à-vis equities or investment-grade bonds on a risk-adjusted basis, we find that high-yield bonds are fairly priced relative to equities and investment-grade bonds, during both phases of the cycle (for more on this, see Blume, Keim and Patel (1991) and Fjelstad *et al.* (2005)). In the same spirit, Cornell and Green (1991), Zivney, Bertin and Torabzadeh (1993) and Li, McCarthy and Pantalone (2014) find that the debt market is efficiently priced, as junk bonds do not perform significantly better than investment-grade bonds on a risk-adjusted basis, whilst Kihn (1994) reaches the same conclusion when economic activity shrinks.

Table 4 shows a summary of the model results regarding optimal allocation of a portfolio to high-yield bonds based on investor’s risk profile.

Table 4. Composition of high-yield bonds in an investor’s optimal portfolio

Risk profile:	<i>Risk-averse</i>	<i>Conservative</i>	<i>Moderate</i>	<i>Moderately aggressive</i>	<i>Aggressive</i>
<i>Expansion</i>	3.81%	3.72%	4.15%	4.04%	4.02%
<i>Contraction</i>	4.09%	3.94%	3.78%	4.03%	4.07%

The results presented in Table 4 suggest that high-yield bonds should constitute 3.81%, 3.72%, 4.15%, 4.04% and 4.02% of the total composition of the optimal portfolios of investors with the different risk tolerance profiles, moving from the most risk-averse to the most risk-taking. In the period of economic expansion, the risk compensation in the different investors’ optimal portfolios are 0.97%, 1.33%, 1.61%, 1.87% and 2.08%, moving from the “risk-averse” to the “aggressive” risk profile. Evidently, as we move from risk-averse to aggressive, the additional risk taken is compensated to a greater and greater degree. In the period of contraction, the risk compensation is 0.45%, 0.96%, 1.35%, 1.62% and 1.86%, moving from risk-averse to aggressive.

These results show that the modified BL model is validated. In particular, the allocations to high-yield bonds, by risk tolerance level, are fairly similar in the expansion and contraction phases. Our results also imply that the high-yield bonds’ allocation does not differ greatly between the risk-averse and aggressive cases. Rather, they may indicate a random walk process for the allocation. This suggested pattern may reflect that, in a well-diversified portfolio, each investor’s portfolio risk depends, not on each individual asset’s risk, but on the risk of the entire asset mix. The latter may be seen in that the proportion of high-yield

bonds never exceeds 4.15% of total assets, irrespective of the risk appetite of the global market portfolio, and regardless of the business cycle.

There are several related studies to which our results do not align. Our results do, however, coincide considerably with those of Bekkers, Doeswijk and Lam (2009). In the latter, 3.2% to 6.6% of investors' portfolios are suggested to be allocated to junk bonds, based on MVO combined with an optimal portfolio built so as to replicate the global market portfolio. When Bekkers, Doeswijk and Lam (2009) apply those two methodologies individually, though, they suggest that non-investment-grade bonds should constitute 0%-14% and 1.1% of investors' total portfolios, respectively. Likewise, Trainor Jr. and Wolfe (2006) recommend allocating 0% to 42% and 0% to 26% of investors' portfolios to non-investment-grade bonds, based on MVO and the BL model respectively. It is reasonable to suggest that our results differ from those of previous studies because of the timeframes used, leading to changing correlations between the asset classes. Fundamental macroeconomic and financial market changes will affect investors' risk assessments and the relationship between stocks and bonds (Andersson, Krylova and Vähämaa 2008).

5. Conclusions

In this paper we explore the nature of investment decisions in relation to high-yield bonds during periods of economic expansion and contraction. Even though the existing academic literature leaves no room for disputing the presence of an equity component in high-yield bonds, its strength over the period 2007-13 has not been evaluated effectively. In this context, we provide evidence by regressing high-yield-bond risk-adjusted returns on the Fama and French three factors. We find that the risk-adjusted returns of the high-yield bonds can be explained by equity market risk factors in both the expansion and contraction phases of the cycle. Furthermore, it is deduced that equity market risk factors have greater explanatory power for non-investment-grade-bond risk-adjusted returns during periods of galvanized economic activity, as compared to periods when economic activity is stifled. We further provide evidence that non-investment-grade-bond risk-adjusted returns are not significantly higher than equity and investment-grade-bond risk-adjusted returns during either phase of the cycle.

By modifying the BL model, we devise a novel risk measure that is effective when used in the portfolio optimization process. We find that between 3.72% and 4.15% of investment portfolios should be allocated to high-yield bonds in the expansion period, and between

3.78% and 4.07% in the downturn period, depending on the investor's risk tolerance profile. In simple terms, our results downplay the perception that high-yield bonds provide statistically significantly higher returns than equities and investment-grade bonds, on a risk-adjusted basis. Given this, high-yield bonds do not seem to merit favorable treatment, relative to other financial instruments, in the asset allocation process for a global market portfolio.

It is worth emphasizing that, in carrying out this study, we have assumed consistent investor utility across all wealth levels, and constant utility throughout the entire investment horizon, albeit such an assumption does not hold up in reality. Moreover, friction costs were not included in our empirical study, and these would certainly influence investment strategy and portfolio allocation in practice. While adding in such costs could make the analysis much more complex, doing so would not necessarily improve the validity or reliability of the results.

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Appendix A: Indices used, market capitalizations and market weights of the sampled indices.

Index	Market capitalization (USD trn)	Market weight (%)
MSCI USA Index	13.24	8.99
MSCI USA Small Cap Index	2.86	1.94
MSCI EAFE Index	9.74	6.61
MSCI EAFE Small Cap Index	1.88	1.28
MSCI Emerging Markets Index	3.05	2.07
MSCI Emerging Markets Small Cap Index	0.61	0.41
MSCI USA High Dividend Yield Index	5.89	4.00
MSCI EAFE High Dividend Yield Index	1.97	1.34
MSCI Emerging Markets High Dividend Yield Index	0.70	0.48
Barclays US Treasury Bond Index	12.46	8.46
Barclays US Agency Debenture Index	1.97	1.34
Barclays US Municipal Bond Index	3.78	2.57
Barclays US Treasury Inflation Protected Securities (TIPS) Index	1.08	0.73
Barclays US Mortgage Backed Securities (MBS) Index	10.18	6.91
Barclays US Corporate Bond Index	7.09	4.81
Barclays US Corporate High-Yield Bond Index	1.34	0.91
Barclays Developed Markets ex-US Hard Currency Aggregate Bond Index	41.23	28.00
Barclays Emerging Markets Hard Currency Aggregate Bond Index	18.60	12.63
Bloomberg Precious Metals Index	0.54	0.37
Bloomberg Commodity ex-Precious Metals Index	2.13	1.45
S&P US REIT Index	0.84	0.57
S&P Global ex-US REIT Index	0.72	0.49
S&P Listed Private Equity Index	1.31	0.89
Cambridge Associates Global ex-US Private Equity & Venture Capital Index	1.11	0.75
Barclays US Treasury Bill 1-3 Month Term Index	2.93	1.99
Global Market Portfolio	147.25	100.00

Data sources: Bloomberg Professional Services; Barclays Guides and Factsheets; Cambridge Associates Private Investment Benchmark.

Appendix B: Correlation matrix of excess returns of indices used.

	MSCI USA Index	MSCI USA Small Cap Index	MSCIEAFE Index	MSCIEAFE Small Cap Index	MSCI Emerging Markets Index	MSCI Emerging Markets Small Cap Index	MSCI USA High Dividend Yield Index	MSCIEAFE High Dividend Yield Index	MSCI Emerging Markets High Dividend Yield Index	Barclays US Treasury Bond Index	Barclays US Agency Debenture Index	Barclays US Municipal Bond Index	Barclays US Treasury Inflation Protected Securities (TIPS) Index	Barclays US Mortgage Backed Securities (MBS) Index	Barclays US Corporate Bond Index	Barclays US Corporate High-Yield Bond Index	Barclays Developed Markets ex-US Hard Currency Aggregate Bond Index	Barclays Emerging Markets Hard Currency Aggregate Bond Index	Bloomberg Precious Metals Index	Bloomberg Commodity ex-Precious Metals Index	S&P US REIT Index	S&P Global ex-US REIT Index	S&P Listed Private Equity Index	Cambridge Associates Global ex-US Private Equity & Venture Capital Index	Barclays US Treasury Bill 13 Month Term Index (Cash Equivalent)
MSCI USA Index	1.000																								
MSCI USA Small Cap Index	0.949	1.000																							
MSCIEAFE Index	0.886	0.817	1.000																						
MSCIEAFE Small Cap Index	0.852	0.814	0.961	1.000																					
MSCI Emerging Markets Index	0.813	0.804	0.866	0.865	1.000																				
MSCI Emerging Markets Small Cap Index	0.779	0.775	0.842	0.857	0.965	1.000																			
MSCI USA High Dividend Yield Index	0.903	0.849	0.791	0.748	0.730	0.686	1.000																		
MSCIEAFE High Dividend Yield Index	0.847	0.746	0.967	0.904	0.799	0.773	0.814	1.000																	
MSCI Emerging Markets High Dividend Yield Index	0.795	0.778	0.869	0.874	0.988	0.946	0.724	0.806	1.000																
Barclays US Treasury Bond Index	-0.505	-0.573	-0.419	-0.408	-0.357	-0.345	-0.327	-0.335	-0.309	1.000															
Barclays US Agency Debenture Index	-0.140	-0.140	-0.095	-0.183	0.038	0.038	-0.062	-0.030	0.034	0.140	1.000														
Barclays US Municipal Bond Index	-0.209	-0.189	-0.168	-0.153	0.022	0.057	-0.230	-0.153	0.011	0.104	0.673	1.000													
Barclays US Treasury Inflation Protected	-0.028	-0.097	0.072	0.089	0.187	0.190	0.052	0.110	0.212	0.652	-0.020	0.041	1.000												
Barclays US Mortgage Backed Securities	-0.195	-0.287	-0.086	-0.071	-0.002	-0.011	-0.047	-0.033	0.046	0.790	0.051	0.035	0.750	1.000											
Barclays US Corporate Bond Index	0.159	0.075	0.307	0.292	0.376	0.398	0.274	0.331	0.397	0.542	0.109	0.082	0.680	0.633	1.000										
Barclays US Corporate High-Yield Bond Index	0.744	0.717	0.804	0.773	0.839	0.848	0.711	0.779	0.821	-0.248	0.002	0.027	0.278	0.069	0.583	1.000									
Barclays Developed Markets ex-US Hard	-0.166	-0.252	-0.025	-0.008	0.015	0.053	-0.027	0.025	0.059	0.704	0.129	0.150	0.571	0.639	0.700	0.113	1.000								
Barclays Emerging Markets Hard	0.501	0.433	0.578	0.547	0.694	0.677	0.434	0.575	0.683	-0.040	0.260	0.299	0.310	0.170	0.495	0.701	0.152	1.000							
Bloomberg Precious Metals Index	0.257	0.257	0.249	0.297	0.412	0.382	0.161	0.204	0.417	0.134	-0.078	-0.110	0.475	0.282	0.260	0.227	0.095	0.400	1.000						
Bloomberg Commodity ex-	0.693	0.627	0.705	0.678	0.702	0.657	0.621	0.701	0.713	-0.381	-0.074	-0.217	0.028	-0.131	0.153	0.588	-0.222	0.563	0.455	1.000					
S&P US REIT Index	0.764	0.753	0.724	0.688	0.690	0.666	0.782	0.728	0.687	-0.180	-0.067	-0.163	0.229	0.112	0.430	0.748	0.128	0.529	0.233	0.516	1.000				
S&P Global ex-US REIT Index	0.641	0.637	0.586	0.548	0.556	0.534	0.694	0.601	0.552	-0.051	-0.060	-0.156	0.276	0.184	0.444	0.652	0.200	0.459	0.202	0.408	0.948	1.000			
S&P Listed Private Equity Index	0.908	0.906	0.912	0.911	0.828	0.801	0.813	0.847	0.818	-0.434	-0.197	-0.213	0.060	-0.110	0.258	0.792	-0.081	0.523	0.263	0.619	0.756	0.636	1.000		
Cambridge Associates Global ex-US Private	0.470	0.450	0.511	0.512	0.494	0.485	0.365	0.478	0.498	-0.414	-0.030	-0.042	-0.112	-0.202	0.010	0.387	-0.201	0.278	0.125	0.393	0.285	-0.034	0.460	1.000	
Barclays US Treasury Bill 13 Month Term	-0.040	-0.051	-0.018	-0.033	-0.086	-0.030	-0.049	0.036	-0.127	-0.087	-0.042	-0.018	-0.065	-0.141	-0.161	-0.043	-0.206	-0.261	-0.131	-0.090	-0.102	-0.163	-0.093	0.171	1.000

TECHNICAL APPENDIX

Computation of the risk aversion coefficient

Given the data limitations, we assume that the equilibrium risk premiums of all sampled indices remain constant throughout the time period of our study. Consequently, the risk aversion coefficient λ that remains constant over the time span in question is computed as follows:

$$\lambda = \text{ERP}_j / \sigma_j^2 w_j$$

where ERP_j is the equilibrium risk premium of index j ,

σ_j^2 is the variance of index j and

w_j is the market weight of index j .

The sampled index that is used for the computation of the risk aversion coefficient λ is the MSCI EAFE Index.

Computation of the implied equilibrium excess returns of the sampled indices

Assuming that the returns of all sub-asset classes are normally distributed and the market is in equilibrium or, in other words, the movements of the prices of all sub-asset classes reflect the homogeneous expectations for the performance of those respective sub-asset classes, the global market portfolio is the optimal mean-variance portfolio of all risky sub-asset classes. Consequently, investors should hold the optimal mean-variance portfolio that replicates the global market portfolio plus an amount of cash or leverage, the level of which varies and enables investors to match the volatility of their portfolios to their risk tolerance. In this setting, the aggregate market capitalization of the sub-asset classes that are held by all investors equals the market capitalization of the global market portfolio.

The aforementioned assumptions enable a reverse optimization process, carried out by computing the implied equilibrium excess returns of the sampled indices that make the global market portfolio optimal. According to the CAPM, the specific implied equilibrium excess returns are considered fair values of the expected returns of the sampled indices. The implied equilibrium excess returns of the sampled indices are computed from the CAPM as follows:

$$\Pi = \lambda \Omega w$$

where Π is the vector of implied equilibrium excess returns of the sampled indices, w is the vector of market weights of the sampled indices and Ω is the covariance matrix of the sampled indices.

Setting the forward-looking view of the expected excess returns for the sampled indices

The forward-looking views of the expected excess returns of the sampled indices are set in accordance with the macroeconomic dynamics, and with the market results and fundamentals that are associated with the performance of the sampled indices. If the forward-looking views deviate from their corresponding implied equilibrium excess returns, investors should hold portfolios that are different from the optimal mean-variance portfolio that replicates the global market portfolio. Putting this differently, in the strategic asset allocation process, the forward-looking views of the expected excess returns of the sampled indices tilt investors' portfolios away from the optimal mean-variance portfolio that replicates the global market portfolio, so as to enhance their performance. Meanwhile, the magnitude of that divergence depends on the degree of confidence in those forward-looking views. In the authors' experience, it is not feasible to set up the forward-looking views of the expected excess returns of the sampled indices in a reliable

manner. In this case, the Black-Litterman model suggests using the neutral values of the implied equilibrium excess returns of the sampled indices, under the strong assumption that the market is in equilibrium.

Computation of the adjusted monthly excess returns of the sampled indices

The adjusted monthly excess returns of the sampled indices are the expected returns of the sampled indices as given by the modified version of the Black-Litterman model, reflecting the historical performance of the indices. They are computed by shifting the monthly excess returns of the sampled indices in order that their mean values coincide with the implied equilibrium excess returns of the specific indices, as follows:

$$AMER = MER - \mu + \Pi$$

where AMER is the vector of adjusted monthly excess returns of the sampled indices, MER is the vector of monthly excess returns of the sampled indices and μ is the vector of means of the monthly excess returns of the sampled indices.

Computation of the adjusted annual excess returns of the sampled indices

The adjusted annual excess returns of the sampled indices are computed using the following formula:

$$AAER = (1 + AMER)^{12} - 1$$

where AAER is the vector of adjusted annual excess returns of the sampled indices.

Setting the time length N of the collected sub-sample of adjusted annual excess returns of the sampled indices

The time length N of the sub-sample of adjusted annual excess returns of the sampled indices should be selected in accordance with the degree of desired regularization of the financial outcome of the resampling technique that is applied in part of the portfolio optimization process. More specifically, if a very small value of N is chosen, the adjusted annual logarithmic excess returns of the sub-sample of adjusted annual excess returns of the sampled indices will not be accurate estimates of the expected returns of the sampled indices. In this case, the weights of each sub-asset class, across all optimal portfolios representing each level of investor risk tolerance, will present a large amount of variance, and the final optimal portfolio will be ‘excessively’ diversified. On the other hand, if a large value of N is selected, those weights will be almost identical to each other. As such, the effect of averaging them in order to compute the weight of the corresponding sub-asset class in the final optimal portfolio for each investor risk tolerance profile will be diminished. For this reason, we opted to assign moderate values for N . In particular, for the economic expansion period, N is set at 3 years, while for the economic contraction period it is set at 1 year.

Setting the risk tolerance coefficient of each risk tolerance profile

The risk tolerance coefficient T of each risk tolerance profile of an investor should be set in such a way as to enable the behavioral finance utility functions U_1 of all such profiles to cover the entire spectrum of risk profiles. In addition, the gap between any two consecutive risk tolerance profiles should not be so narrow that the differences between them are not clear, while not being so wide that an investor’s attitude to risk is not adequately captured by either of the two closest profiles. According to Barclays Bank PLC, the spectrum should, ideally, be divided into five different risk tolerance profiles of investor, setting $T = (0.2, 0.4, 0.6, 0.8, 1)$, with $T = 0.2$ and $T = 1$ assigned to the most risk-averse and most risk-taking investors, respectively.

Setting the number of resampling simulations

According to the resampling technique that is applied in part of the portfolio optimization process, the number of resampling simulations M of the portfolio optimization process should be so large that the process reaches convergence. Therefore, an adequately high number of resampling simulations was used.

A sub-sample of adjusted annual excess returns of the sampled indices, with time length N , is collected from the vector of adjusted annual excess returns of the sampled indices AAER, using a simple random sampling method.

Computation of the adjusted annual logarithmic excess returns of the collected sub-sample of adjusted annual excess returns of the sampled indices

The adjusted annual logarithmic excess returns of the sub-sample of adjusted annual excess returns of the sampled indices are computed as follows:

$$AALER = \log (1 + \Sigma w \text{ AAER})$$

where $AALER$ is the vector of adjusted annual logarithmic excess returns of the collected sub-sample of adjusted annual excess returns of the sampled indices.

Introduction of the behavioral finance utility function for each risk profile

The use of a behavioral finance utility function for each risk tolerance profile constitutes our modification of the Black-Litterman model. Taking into account all the factors that influence investors' financial decisions, it is assumed that investors hold stable and rational preferences regarding the outcome of their investment and portfolio management process. The expression of these preferences, in the form of a concave risk-averse behavioral finance utility function, captures investors' attitude to risk in a more refined way than does the mean-variance analysis. More specifically, a behavioral finance utility function with this form rewards potentially positive deviations away from the expected financial outcome of the investment and portfolio management process, penalizing only potentially negative deviations. In contrast, the mean-variance analysis penalizes positive and negative deviations equally. Therefore, the use of a behavioral finance utility function provides a more comprehensive measure of risk than variance, which is defined as behavioral variance σ_B^2 . Moreover, the use of a behavioral finance utility function relaxes the normality assumption regarding the returns of sub-asset classes, enabling the consideration of additional sub-asset classes with non-normal return distributions.

A behavioral finance utility function describes how investors' utility changes across the range of potential financial outcomes of the investment and portfolio management process. Certainly, this relationship is influenced by investors' wealth level, as well as potential changes to it over the investment horizon. We opt to use an exponential constant relative risk aversion utility function of logarithmic returns, which is in line with the assumptions that investors' utility is consistent across all investor wealth levels and remains constant throughout the investment horizon.

The behavioral finance utility function U_1 of each investor risk tolerance profile that fits the specified requirements reasonably well is formulated as follows:

$$U_1 = 1 - \frac{1}{N} \Sigma e^{-\frac{2 AALER}{T}}$$

Programming reasoning imposes the attribution of this utility function in MATLAB as $U_2 = -U_1$.

Generation of the optimal portfolio for each risk tolerance profile

Assuming that investors hold stable and rational preferences regarding the financial outcome of the investment and portfolio management process, they invest their capital in the portfolio that maximizes their expected utility, in line with their personal risk tolerance. This is why the optimal portfolio w_{opt} of each investor risk tolerance profile is the one that maximizes the corresponding behavioral finance utility function U_1 . Since the utility function is denoted in MATLAB by $U_2 = -U_1$, the optimal portfolio w_{opt} of each investor risk tolerance profile is equivalently provided by minimizing the corresponding behavioral finance utility function U_2 . The modified version of the Black-Litterman model that we use is subject to two constraints: The sum of the weights of all sub-asset classes in the optimal portfolio w_{opt} of each investor risk tolerance profile should equal one and no short sales can take place.

Application of a resampling technique to part of the portfolio optimization process

The application of a resampling technique to part of the portfolio optimization process constitutes our second modification of the Black-Litterman model. An inherent issue with that model is that the optimal portfolios w_{opt} of the different investor risk tolerance profiles are likely to be highly sensitive to the financial data, parameters and assumptions employed, since they will seek to exploit any inconsistencies or arbitrage opportunities as much as possible. Furthermore, the allocation of the optimal portfolios w_{opt} for the different investor risk tolerance profiles may not evolve smoothly across the various sub-asset classes, due to distortions arising from specific past events. These potential issues are addressed to some extent by applying a resampling technique to part of the portfolio optimization process. This method enhances diversification by generating a final optimal portfolio $w_{final\ opt}$ for each risk tolerance profile in accordance with a number of equally likely scenarios that jointly reflect all the available information about the past, instead of relying on a single past case that may not be repeated identically in the future.

Generation of the final optimal portfolio for each risk tolerance profile

The final optimal portfolio $w_{final\ opt}$ for each investor risk tolerance profile is generated as follows:

$$w_{final\ opt} = \frac{1}{M} \sum_{k=1}^M w_{opt}$$

Computation of the behavioral variance of the final optimal portfolio of each risk tolerance profile

The behavioral variance σ_B^2 of the final optimal portfolio $w_{final\ opt}$ of each investor risk tolerance profile is computed as follows:

$$\sigma_B^2 = \frac{T^2}{2} \ln E \left(e^{\frac{2E(AALER)ALAAER}{T}} \right)$$

Computation of the risk compensation of the final optimal portfolio for each risk tolerance profile

The risk compensation c of the final optimal portfolio $w_{final\ opt}$ for each investor risk tolerance profile is computed as follows:

$$c = \sigma_B^2 / T$$

Evaluation of the modified version of the Black-Litterman model

If the risk compensations c in the final optimal portfolios $w_{final\ opt}$ for the different investor risk tolerance profiles rise as the risk tolerance coefficient T increases, then the modified version of the Black-Litterman model must be reliable, as too must be our inferences on the extent to which different investors' portfolios should be allocated to high-yield bonds depending on their risk tolerance.