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Unified Multi-Objective Optimization Scheme for Aeroassisted Vehicle Trajectory Planning

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In this work, a multi-objective aeroassisted trajectory optimization problem with mission priority constraints is constructed and studied. To effectively embed the priority requirements into the optimization model, a specific transformation technique is applied and the original problem is then transcribed to a single-objective formulation. The resulting single objective programming model is solved via an evolutionary optimization algorithm. Such a design is unlike most traditional approaches, where the nondominated sorting procedure is required to be performed to rank all the objectives. Moreover, in order to enhance the local search ability of the optimization process, a hybrid gradient-based operator is introduced. Simulation results indicate that the proposed design can produce feasible and high-quality flight trajectories. Comparative simulations with other typical methods were also performed and the results show that the proposed approach can achieve a better performance in terms of satisfying the pre-specified priority requirements.

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I. Introduction

The design of flight vehicle optimal trajectory is among the most important and difficult components of modern guidance and control systems [1–3]. Due to the uncertainties in the flight conditions and multiple constraints, it is still difficult to design a robust and efficient algorithm such that the vehicle can fly along an optimum path and fulfill different mission requirements [4, 5]. To effectively solve the problem, techniques based on optimal control theory are commonly used and various mission scenarios have been studied during the past decades [4]. For example, Fahroo and Ross [6] developed a Chebyshev pseudospectral method for solving general trajectory optimization problems with control and state constraints. In their follow-up work [7], a pseudospectral knotting technique was constructed in order to solve nonsmooth optimal control problems. Liu et al. [8] applied convex programming methods to solve trajectory optimization problems in flight entry phase. However, all reported investigations target a single objective. In many practical spacecraft guidance systems, multiple mission performance indices and different priority requirements must frequently be considered during the trajectory planning phase. This brings the development of multi-objective trajectory optimization techniques.

The problem addressed in this research is an optimal flight path design for a constrained multi-objective aeroassisted vehicle trajectory planning problem, where the objective functions are specified with different priority requirements. These type of problems are becoming popular since multiple practical requirements can be taken into account during the design phase. For instance, in [9] the authors applied a Multi-Objective Evolutionary Algorithm (MOEA) to solve a two-objective reentry problem. Although the objective could be optimized based on the definition of pareto-optimal, the computational burden caused by the optimization process was high. Gao et al. [10] calculated the optimal control for a multi-objective spacecraft rendezvous problem. In their work, the multi-objective optimal control problem was transcribed into a convex optimization problem subject to linear matrix inequality constraints. However, the formulation can hardly be extended to solve the multi-objective optimal control problems with simultaneous consideration of priority requirements.

Due to these issues, an extended optimization approach, named Fuzzy Goal Programming-based Gradient Hybrid Genetic Algorithm (FGP-GHGA), is introduced and applied in this paper. In order to construct the priority constraints explicitly, an FGP technique [11, 12] is firstly adopt to fuzzify the objective functions and reformulate the problem. Following that, an enhanced evolutionary optimization algorithm is used to calculate the optimal control sequences. The FGP-GHGA approach designed in this paper does not relay on the designer's physical understanding of the problem. Another important feature of the proposed method is that it unifies the mission objectives, constraints and preemptive priorities in one optimization formulation such that the optimization process can then be simplified. Compared with traditional MOEAs, the proposed approach will not apply the nondominant principle, which implies that the computational complexity can be reduced significantly. Furthermore, it can optimize different objectives as well as meeting the designer's preference.

The motivation for the use of evolutionary optimization algorithms relies on their ability in dealing with local optimal solution and control constraints, that naturally arise in nonlinear optimal control problems [1, 4, 5]. Contributions made to apply evolutionary optimization techniques can be found in literatures. For instance, a constrained space plane reentry problem was solved in [13], wherein a Genetic Algorithm (GA) was applied to generate the optimal reentry trajectories. Similarly, in [14] a low-thrust interplanetary trajectory problem was formulated and solved via a modified GA. Pontani and Conway [15] investigated an optimal finite-thrust rendezvous trajectory problem. In their work, a Particle Swarm Optimization (PSO) algorithm was applied to solve the rendezvous optimal control problem. The main advantage with evolutionary optimization methods is that it is simple to understand and easy to apply. Besides, it is more likely than traditional gradient-based methods to locate the global optimum solution. Therefore, in this study, an enhanced GA is introduced to optimize the transcribed optimization model. Compared with traditional GA, it uses a hybrid evolutionary strategy and tends to have better local searching ability.

It is worth noting that in [16], the authors designed a multi-objective algorithm, namely Interactive Fuzzy Physical Programming (IFPP) method, to solve the multi-objective trajectory optimization problem. This method was analyzed as an effective tool to drive different objectives into the preference regions. However, its optimization model is largely depended on the designer's knowledge of the problem, and it tends to be sensitive with respect to the aspiration levels and the preference regions. When priority constraints are taken into account, the IFPP approach might not be as effective as the one developed in this study. This will be further discussed in the simulation section of this paper.

II. Basic Formulation of MOPs

Some mathematical preliminaries are necessary to facilitate the presentation of the main results. A typical Multi-objective Optimization Problem (MOP) can be expressed as follows [17]:

$$\begin{array}{lll} \text{Find design variables} & x = [x_1, x_2, ..., x_n] & \\ \text{To minimize objective functions} & f(x) = [f_1, f_2, ..., f_m] & \\ \text{subject to} & x_{min} \leq x \leq x_{max} & \\ & h_i(x) = 0 & i = 1, 2, ..., E \\ & g_j(x) \leq 0 & j = 1, 2, ..., I \end{array} \tag{1}$$

where n and m are the number of decision variables and objectives (e.g. $x \in \Re^n$, $f \in \Re^m$). E and I are the number of equality constraints $h(x) = [h_1(x), h_2(x), ..., h_E(x)]^T$ and inequality constraints $g(x) = [g_1(x), g_2(x), ..., g_I(x)]^T$, respectively. x_{min} and x_{max} stand for the lower and upper bounds of the decision variables.

In most practical MOPs, it is hard for the designer to find a solution that can optimize all the objective functions, since some of the objectives are usually contradicting. Therefore, the goal of MOPs is to find a good compromise between different objectives (performance indices). This leads to the definition of pareto-optimal solution. A solution that is pareto-optimal means no other solution can be found in the current search space that can improve all the performance indices. Currently, most of the existing studies are focusing on the development or implementation of MOEA for general MOPs [18–22]. This type of technique is effective for analyzing the relations between objectives and generate the pareto front. However, since all the objectives are involved in the optimization iteration and rank sorting process, the computational complexity can be high. Moreover, if the priority factors are required to be taken into account, the MOEA-based approach might need to relay on the interactive process, which is still a challenging problem for the decision makers.

Due to these drawbacks and challenges, in this paper a transcription strategy is proposed and applied to handle the mission-dependent priority constraints and reduce the computational complexity. This strategy is based on the FGP theory and the original multi-objective formulation is reformulated to a Single-Objective Problem (SOP). Compared with MOEA strategies studied in [18, 19], the present method has the capability to handle the priority requirement and does not rely on the time-consuming rank sorting process. It should be noted that another typical transcription technique that has been widely used is the weighted sum method. However, as analyzed in [16], weighted sum algorithm might not reflect the true compromise between different objectives (e.g. a higher weight value may not produce a higher satisfaction degree). Compared with the weighted-sum approach, the FGP optimization model has the capability to directly reflect the magnitude of goal attainment with respect to different objectives. The transcribed programming model is then solved via an evolutionary optimization technique, which will be detailed in the next section.

III. FGP-based Gradient Hybrid Genetic Algorithm

Considering the preemptive priorities associated with each objective function, the MOP model to be analyzed is given by:

$$\begin{aligned}
&\text{Find design variables} && x = [x_1, x_2, ..., x_n] \\
&\text{To minimize objective functions} && f(x) = [f_1, f_2, ..., f_m] \\
&\text{subject to} && x_{min} \leq x \leq x_{max} \\
& && x \in \mathscr{F}, \quad f(x) \in \mathscr{P}
\end{aligned} \tag{2}$$

where $\mathscr{F}$ is defined as $\mathscr{F} = \{x | h(x) = 0, g(x) \leq 0\}$. $\mathscr{P} = \{f(x) | P(f_i(x)) \geq P(f_j(x))\}$, in which $P(\cdot)$ represents priority factors of the different objectives. The inequality $P(f_i(x)) \geq P(f_j(x))$ means the priority of the ith objective is higher than the jth objective.

In order to deal with the pre-specified priority constraints, the fuzzy relations are firstly introduced. Generally, there are three typical fuzzy relations (e.g. "$\preceq$", "$\succeq$" and "$\simeq$") between the objectives f_i and their goal values f_i^* [11, 12]. The fuzzy relation "$\preceq$" denotes the requirements of fuzzy objective should be less or equal to the goal value (expected value), and the membership function associated with it can be given by:

$$\mu_{f_i}(x) = \begin{cases} 0, & f_i(x) \geq f_i^{max}; \\ 1 - \frac{f_i(x) - f_i^*}{f_i^{max} - f_i^*}, & f_i^* \leq f_i(x) \leq f_i^{max}; \\ 1, & f_i(x) \leq f_i^*. \end{cases} \tag{3}$$

where (f_i^*, f_i^{max}) is the tolerant region of the objective f_i. Similarly, for "$\succeq$" and "$\simeq$", the membership functions are given by Eq.(4) and Eq.(5), respectively.

$$\mu_{f_i}(x) = \begin{cases} 0, & f_i(x) \leq f_i^{min}; \\ 1 - \frac{f_i^* - f_i(x)}{f_i^* - f_i^{min}}, & f_i^{min} \leq f_i(x) \leq f_i^*; \\ 1, & f_i(x) \geq f_i^*. \end{cases} \tag{4}$$

$$\mu_{f_i}(x) = \begin{cases} 0, & f_i(x) \leq f_i^{min}; \\ 1 - \frac{f_i^* - f_i(x)}{f_i^* - f_i^{min}}, & f_i^{min} \leq f_i(x) \leq f_i^*; \\ 1, & f_i(x) = f_i^*; \\ 1 - \frac{f_i(x) - f_i^*}{f_i^{max} - f_i^*}, & f_i^* \leq f_i(x) \leq f_i^{max}; \\ 0, & f_i(x) \geq f_i^{max}. \end{cases} \tag{5}$$

In Eqs.(3)-(5), the value of f_i^* (goal value) can be calculated by solving the corresponding single objective optimization problem. For example,

$$f_i^* = \arg\min_{x \in \mathscr{F}} f_i, \quad \text{subject to} \quad x_{min} \leq x \leq x_{max} \tag{6}$$

The SOP formulation shown in Eq.(6) is solved for $i = 1, 2, ..., m$. Assuming that x_i^* is the optimal solution for the ith SOP, the lower and upper limits of the objective f_i (e.g. f_i^{min} and f_i^{max}) can be obtained by performing $f_i^{min} = \min(f_1(x_i^*), f_2(x_i^*), ..., f_m(x_i^*))$ and $f_i^{max} = \max(f_1(x_i^*), f_2(x_i^*), ..., f_m(x_i^*))$, respectively. These values are obtained using the same approach stated in [23]. Hence μ can be used as the satisfactory degree of the objectives and its value can directly reflect the magnitude of achieving the goal value.

Following the introduction for the definition of satisfactory degree, by constructing several inequalities the priority constraints arising from the MOPs can be obtained. Since it can be expected that an objective with a high priority has larger μ value, the original priority constraint (e.g.

$P(f_i(x)) \geq P(f_j(x)))$ can then be transcribed to:

$$\mu_{f_j}(x) - \mu_{f_i}(x) \leq 0 \tag{7}$$

where $i, j = 1, 2, ..., m$, $i \neq j$.

A. FGP Optimization Formulation

The FGP model can be constructed based on the fuzzy relationships, satisfactory degree and priority constraints. Defining the deviation parameters $p_i = f_i(x) - f_i^* \geq 0$ and $q_i = f_i^* - f_i(x) \geq 0$, the membership functions of fuzzy relations $\preceq$ and $\succeq$ become $\mu_{f_i}(x) = 1 - \frac{p_i}{f_i^{max} - f_i^*}$ and $\mu_{f_i}(x) = 1 - \frac{q_i}{f_i^* - f_i^{min}}$, respectively. Suppose the objectives have the following fuzzy relationships:

$$\begin{aligned}
& f_i(x) \preceq f_i^*, \quad i = 1, 2, ..., k_1 \\
& f_j(x) \succeq f_j^*, \quad j = k_1 + 1, k_1 + 2, ..., k_2 \\
& f_k(x) \simeq f_k^*, \quad k = k_2 + 1, k_2 + 2, ..., m
\end{aligned} \tag{8}$$

A general Goal Programming (GP) optimization formulation can firstly be constructed as follows [11, 24]:

$$\begin{aligned}
\text{Minimize} \quad & J = \textstyle\sum_{i=1}^{k_1}(p_i + q_i) + \sum_{j=k_1+1}^{k_2}(p_j + q_j) + \sum_{k=k_2+1}^{m}(p_k + q_k) \\
\text{subject to} \quad & f_i(x) + p_i - q_i = f_i^*, \quad i = 1, 2, ..., k_1 \\
& f_j(x) + p_j - q_j = f_j^*, \quad j = k_1, k_1 + 1, ..., k_2 \\
& f_k(x) + p_k - q_k = f_k^*, \quad k = k_2, k_2 + 1, ..., m \\
& p_i, p_j, p_k, q_i, q_j, q_k \geq 0, \quad p_i \cdot q_i = 0, p_j \cdot q_j = 0, p_k \cdot q_k = 0 \\
& x_{min} \leq x \leq x_{max}, \quad x \in \mathscr{F}
\end{aligned} \tag{9}$$

Without loss of generality, let us assume that the priority of the objective $f_i(x)$ is higher than $f_j(x)$ and is lower than f_k. By applying Eqs.(3)-(7) and the general GP model (9), the original MOP shown in Eq.(2) is then reformulated to an FGP formulation given by Eq.(10).

$$\begin{cases} \text{Minimize} & J = \frac{1}{m}[\sum_{i=1}^{k_1} \frac{p_i}{f_i^{max}-f_i^*} + \sum_{j=k_1+1}^{k_2} \frac{q_j}{f_i^*-f_i^{min}} \\ & \quad + \sum_{k=k_2+1}^{m} (\frac{q_k}{f_k^*-f_k^{min}} + \frac{p_k}{f_k^{max}-f_k^*})] + \beta \\ \text{subject to} & f_i(x) + p_i - q_i = f_i^*, \quad i = 1, 2, ..., k_1 \\ & f_j(x) + p_j - q_j = f_j^*, j = k_1, k_1+1, ..., k_2 \\ & f_k(x) + p_k - q_k = f_k^*, k = k_2, k_2+1, ..., m \\ & x_{min} \le x \le x_{max}, \quad x \in \mathscr{F} \\ & p_i \le f_i^* - f_i^{min}, p_j \le f_j^* - f_j^{min}, p_k \le f_k^* - f_k^{min} \\ & q_i \le f_i^{max} - f_i^*, q_j \le f_j^{max} - f_j^*, q_k \le f_k^{max} - f_k^* \\ & p_i, p_j, p_k, q_i, q_j, q_k \ge 0 \\ & p_i \cdot q_i = 0, p_j \cdot q_j = 0, p_k \cdot q_k = 0 \\ & \frac{p_i}{f_i^{max}-f_i^*} - \frac{q_j}{f_i^*-f_i^{min}} \le \beta \\ & [\frac{q_k}{f_k^*-f_k^{min}} + \frac{p_k}{f_k^{max}-f_k^*}] - \frac{p_i}{f_i^{max}-f_i^*} \le \beta \end{cases} \tag{10}$$

where $\beta \in [-1, 0]$. After introducing the deviation parameters, the dimension of the optimization problem has increased to include the p and q. The last two inequalities in Eq.(10) are the explicit expressions of Eq.(7) (priority constraints). The first term in the modified objective function J can be treated as the deviations of different objectives to their desired values. Minimizing this term is equivalent to maximizing the satisfactory degree for each objective. It is important to remark that the term β entailing in the objective and the last two priority constraints is designed for the case when it is desired to have a "much higher" relationship. For example, the priority of the objective $f_i(x)$ is much higher than $f_j(x)$, which can be expressed as $P(f_i(x)) \gg P(f_j(x))$. Therefore, minimizing β can result in a larger deviation regarding the satisfactory degree between f_i and f_j. If there is no such specific requirement, this parameter can be removed from the programming model or set to zero.

One main advantage of using the transformed model given by Eq.(10) is that the pre-specified priority requirements can be involved in the optimization process explicitly. Furthermore, if Evolutionary Algorithms (EA) are applied to solve the optimization model, the time-consuming non-dominant sorting procedure [19] is no longer necessary since the original MOP is transformed to an extended SOP formulation. This can reduce the worst-case computational complexity of the algorithm significantly (this will be further analyzed in the simulation section of this paper).

B. Gradient-based Hybrid Genetic Algorithm

Following the construction of the optimization model, the next step is to find an effective optimization algorithm. In recent years, numerous algorithms have been proposed for solving the general nonlinear optimization problems. There are two major classes of optimization algorithms: the gradient-based techniques and heuristic methods. A detailed introduction including the advantages and disadvantages of these algorithms can be found in [4, 25]. This paper applies an augmented genetic algorithm to solve the FGP model given by Eq.(10). In order to enhance the searching ability of the GA, a gradient-based local search strategy is proposed and embedded in the algorithm framework, hence the name Gradient-based Hybrid Genetic Algorithm (GHGA).

Prior to introducing in detail the local search operation, a brief description of the constraint-handling procedure is elaborated. It is well known that for heuristic approaches, a major challenge is to implement a constraint handling strategy that can directly reflect the magnitude of the solution infeasibility. The constraint handling procedure used for the GHGA is based on the constraint violation degree V (similar with the satisfactory degree). For instance, the violation degree for inequality constraints "$\leq$" ($g_j \leq g_j^*$, $j = 1, ..., I$) and equality constraints ($h_k = h_k^*$, $k = 1, ..., E$) can be defined as follows [25]:

$$\mu_{g_j} = \begin{cases} 0, & g_j \leq g_j^*; \\ \frac{g_j - g_i^*}{g_j^{max} - g_j^*}, & g_j^* \leq g_j \leq g_j^{max}; \\ 1, & g_j \geq g_j^{max}. \end{cases} \quad \mu_{h_k} = \begin{cases} 1, & h_k \geq h_k^{max}; \\ \frac{h_k - h_k^*}{h_k^{max} - h_k^*}, & h_k^* \leq h_k \leq h_k^{max}; \\ 0, & h_k = h_k^*; \\ \frac{h_k^* - h_k}{h_k^* - h_k^{min}}, & h_k^{min} \leq h_k \leq h_k^*; \\ 1, & h_k \leq h_k^{min}. \end{cases} \tag{11}$$

where g_j is the value of jth constraint for each individual, whereas (g_j^*, g_j^{max}) and (h_k^{min}, h_k^{max}) stand for the tolerance regions. These tolerance regions can be assigned by the users. For example, in terms of the equality constraint $h_k = h_k^*, h_k^* > 0$, h_k^{min} and h_k^{max} can be set as $0.5h_k^*$ and $2h_k^*$, respectively. Similarly, for the inequality constraint $g_j \leq g_j^*, g_j^* > 0$, g_j^{max} can be assigned as $2g_j^*$. Based on Eq.(11), the total violation degree for each individual among the population V can be obtained via $V = \sum_{j=1}^{I} \mu_{g_j} + \sum_{k=1}^{E} \mu_{h_k}$. In this way, priorities can be given to feasible individuals and individuals with a small value of V in the selection process. On the basis of this, the augmented objective function (fitness function) becomes:

$$J_{aug} = \begin{cases} J, & \text{if } V = 0; \\ J + J_{max}V, & \text{if } V > 0. \end{cases} \tag{12}$$

where J_{max} is the worst objective value among the current generation.

The gradient-based operator is then introduced. If J and V are first-order continuous partial differential in the feasible region, the gradient vectors of J and V, known as Jacobian vectors, have the form: $\nabla J(x) = [\frac{\partial J(x)}{\partial x_1}, \frac{\partial J(x)}{\partial x_2}, ..., \frac{\partial J(x)}{\partial x_n}]^T$ and $\nabla V(x) = [\frac{\partial V(x)}{\partial x_1}, \frac{\partial V(x)}{\partial x_2}, ..., \frac{\partial V(x)}{\partial x_n}]^T$. To find a direction for minimizing the objective and constraint violation, the following equation is applied to calculate the search direction e:

$$e = -\left(\frac{\nabla J(x)}{\|\nabla J(x)\|} + \frac{\nabla V(x)}{\|\nabla V(x)\|} \right) \tag{13}$$

It follows from Eq.(13) that if e is chosen as the search direction, a decrease in the augmented objective function J_{aug} is expected. This conclusion can be easily proven by performing the inner product $\langle e, -(\nabla J(x)/\|\nabla J(x)\|) \rangle$ or $\langle e, -(\nabla V(x)/\|\nabla V(x)\|) \rangle$. After the local search direction is determined, a new candidate solution x_{G+1} of the previous generation x_G is obtained by the gradient operator:

$$x_{G+1} = x_G + s_G e \tag{14}$$

where s_G can be treated as the step length along the direction e. This expression is equivalent to the line search process that commonly used in gradient optimization algorithms. The determination of the step size parameter s_G is based on the Goldstein condition [25]. That is,

$$J_{aug}(x_G) + c_1 s_G \nabla J_{aug}(x)e \leq J_{aug}(x_{G+1}) \leq J_{aug}(x_G) + c_2 s_G \nabla J_{aug}(x)e \tag{15}$$

with $0 < c_1 < c_2 < 1$. The first inequality term is applied to control the step length, whereas the second term is the general sufficient decrease condition.

The proposed GHGA method uses a gradient-based hybrid operator that combines the local gradient operator with the crossover and mutation operators. A new population is then created.

For simplicity reasons, this hybrid operator is abbreviated in the following equation:

$$w_1 Oper_{Cro} \oplus w_2 Oper_{Mut} \oplus w_3 Oper_{Grad} \tag{16}$$

where $w_1, w_2, w_3 \geq 0$ are the probabilities. Eq.(16) indicates that the next group of candidates are generated by the crossover $Oper_{Cro}$, mutation $Oper_{Mut}$ and local gradient operators $Oper_{Grad}$ with the probability w_1, w_2 and w_3, respectively. The algorithm will firstly initialize a random number $rand$, if this number is less or equal than $w_i, i = 1, 2, 3$, then the algorithm will perform the corresponding operation. The crossover and mutation operations are widely used in the traditional GA [13, 14]. In this study, a local gradient operation is embedded in the algorithm framework. Hence, the local searching ability of the algorithm can be improved. The combination of the above three operations is expected to facilitate global expansion of the search space without sacrificing good quality local solutions.

Remark 1 *It should be noted that according to the definition of satisfactory and violation degrees, J_{aug} given by Eq.(15) might not be differentiable at some x. This is because μ_f and μ_g defined in Eq.(3) and Eq.(11) are only piecewise continuous but not smooth functions. This indicates that the gradient information for J and V may not be applicable directly to Eq.(13) [5]. To deal with this issue, a smooth function is considered to replace μ_f and μ_g in practical implementations. Take the violation degree μ_g as an example, the smooth function can be described as:*

$$\delta(\mu_g, m, n) = \begin{cases} 0, & \mu_g < -m; \\ (\mu_g + m)^2/4m, & -m \leq \mu_g \leq m; \\ \mu_g, & m < \mu_g < 1 - n; \\ (-\mu_g^2 + 2(1+n)\mu_g - (n-1)^2)/4n, & 1 - n \leq \mu_g \leq 1 + n; \\ 1, & \mu_g > 1 + n. \end{cases} \tag{17}$$

where m and n are two small positive parameters. Note that $\delta(\mu_g, m, n)$ function is continuously first-order differentiable for any μ_g. Eq.(17) also implies that

$$\lim_{m \to 0^+, n \to 0^-} \delta(\mu_g, m, n) = \mu_g \tag{18}$$

C. General Framework of the Proposed Algorithm

In order to better show the structure of the proposed FGP-based GHGA method scheme, the overall procedure is illustrated in the Pseudocode (see Algorithm 1).

Algorithm 1 Part 1

1: **procedure** (FGP transformation)

2: Perform a fuzzification for different design objectives based on Eq.(3)-(5).

3: Build the membership function for each objective function.

4: Formulate the priority constraint based on line 1-2 and Eq.(7).

5: Construct the fuzzy goal programming model according to Eq.(10).

6: Output the transformed SOP model.

7: **end procedure**

Algorithm 1 Part 2

1: **procedure** (GHGA method)

2: Set the control parameters for GHGA and initialize the first population P with population size N_p

3: Calculate the value of the augmented fitness function for each individual among P based on Eq.(10)-(12)

4: **for** $G := 1, 2, ..., G_{max}$ **do**

5: (a). Choose the best number of the current population as P_{father}

6: (b). Generate the offspring generation $P_{offspring}$ by applying gradient-based hybrid operator

7: (c). Set $P = P_{father} \bigcup P_{offspring}$

8: (d). Perform elite selection

9: **end for**

10: Evaluate all $x_i \in P$

11: **return** x_{best}

12: **end procedure**

Algorithm 1 Part 3

procedure (Gradient-based hybrid operator)

  for $i := 1, 2, ..., N_p$ **do**

    if $rand(1) \leq w_1$ **then**

      Perform crossover operation operator $Oper_{Cro}$ to obtain x_i^{G+1}

    end if

    if $rand(1) \leq w_2$ **then**

      Perform mutation operation operator $Oper_{Mut}$ to obtain x_i^{G+1}

    end if

    if $rand(1) \leq w_3$ **then**

      /*Gradient operation $Oper_{Grad}$*/

      (a). Compute the local search direction e by using Eq.(13)

      (b). Perform the local search according to Eq.(14) and Eq.(15) to obtain x_i^{G+1}

    end if

  end for

end procedure

IV. Multi-Objective Aeroassisted Vehicle Trajectory Optimization

A. Dynamics

This section presents the mission scenario simulated in this investigation. It should be noted that the mission scenario is similar with the one proposed in [16]. For completeness, a brief description is recalled. Taking the rotation of the Earth into account, the following three degree-of-freedom equations of motion represent the flight dynamics of the vehicle [16, 26]:

$$
\begin{aligned}
&\dot{r} = V \sin\gamma \\
&\dot{\theta} = \frac{V \cos\gamma \sin\psi}{r \cos\phi} \\
&\dot{\phi} = \frac{V \cos\gamma \cos\psi}{r} \\
&\dot{V} = \frac{2T\cos\alpha - \rho V^2 S C_D}{2m} - g\sin\gamma + \Omega^2 r \cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\psi\cos\psi) \\
&\dot{\gamma} = \frac{2T\sin\alpha + \rho V^2 S C_L \cos\sigma}{2mV} + (\frac{V^2 - gr}{rV})\cos\gamma + 2\Omega\cos\phi\sin\psi + \Omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\cos\psi\sin\phi) \\
&\dot{\psi} = \frac{\rho V^2 S C_L \sin\sigma}{2mV\cos\gamma} + \frac{V}{r}\cos\gamma\sin\psi\tan\phi + \frac{\Omega^2 r\cos\phi\sin\phi}{\cos\gamma} - 2\Omega(\tan\gamma\cos\psi\cos\phi - \sin\phi) \\
&\dot{m} = -\frac{T}{I_{sp} g}, \quad \dot{\alpha} = K_\alpha(\alpha_c - \alpha), \quad \dot{\sigma} = K_\sigma(\sigma_c - \sigma), \quad \dot{T} = K_T(T_c - T)
\end{aligned}
\tag{19}
$$

in which r is the radial position; θ, ϕ, γ and ψ are the longitude, latitude, flight path angle (FPA) and azimuth angle, respectively. C_L and C_D are the lift and drag coefficients, whereas m and ρ stand for the mass of the vehicle and the density of the atmosphere. $\Omega = 7.2921151e^{-5}$rad/s is the Earth's rotation rate. It can be seen from Eq.(19) that the state equations have been augmented by adding the angle of attack α, bank angle σ and thrust T. α_c, σ_c and T_c can be treated as the demanded angle of attack, bank angle and thrust variables, respectively. These three equations permit to limit the control rate and will have positive influences in the optimization process when evolutionary algorithms are chosen as the optimizer [13].

B. Constraints and Objectives

The path constraints entailing in the optimization model are the aerodynamic heating $\dot{Q}$, dynamic pressure q and normal acceleration n_z. These three path constraints can be calculated according to [26]:

$$\dot{Q} = K_Q \rho^{0.5} V^3, \quad q = \frac{1}{2}\rho V^2, \quad n_z = \frac{\sqrt{L^2 + D^2}}{m g_0} \tag{20}$$

where L and D are the lift and drag accelerations, respectively. g_0 represents the gravitational acceleration at sea level, while k_q is a constant depending on the geometry of the thermal protection system.

In the past, early studies on spacecraft trajectory optimization problems usually focussed on single objective. However, in order to achieve more practical requirements, this type of problem should be constructed to contain multiple objectives and this is where nowadays the majority of research is focusing on. Therefore, to take more of the mission requirements into account, four objectives are considered in this investigation. That is,

$$\begin{cases} \min f_1 = t_f \\ \min f_2 = \int_{t_0}^{t_f} \dot{Q}(t) dt \\ \max f_3 = m(t_f) \\ \max f_4 = V(t_f) \end{cases} \tag{21}$$

In Eq.(21), the first objective f_1 is designed to minimize the time duration so as to complete the mission in the shortest possible time interval. In addition, as indicated in [16], minimizing the total amount of aerodynamic heating is also considered as one of the mission objectives since the vehicle structure integrity is largely affected by this performance index. The third mission objective f_3 is set to maximize the final mass value, which is equivalent to minimizing the fuel consumption. Moreover, for this mission, to ensure the aeroassisted vehicle has a greater terminal velocity (higher kinetic energy) to perform several continuous missions, the final objective f_4 is chosen as maximizing the final velocity value.

C. Priority Requirements

As stated in Section.III of this paper, in practice, for a MOP problem, it is a challenge to optimize all the objective at the same time. Therefore, priority factors should be assigned to different objectives. Specifically, for the mission scenario considered in this research, if the primary task for the vehicle is to maximize final mass such that it can perform further tasks, then the priority factor with respect to f_3 (maximizing the terminal mass value) should be higher than the others. On the other hand, if it is desirable for the flight vehicle to complete a reconnaissance mission in the shortest time possible, then reducing the flight time duration might have the highest priority factor. In order to provide a good illustration of the proposed FGP-GHGA algorithm capability in handling the multi-objective trajectory optimization problem with priority constraints, the following six cases that highlight different aspects of the mission are considered:

Case 1: The priority factor should satisfy: $P(f_1), P(f_2) > P(f_3) > P(f_4)$.

Case 2: The priority factor should satisfy: $P(f_1), P(f_2) > P(f_4) > P(f_3)$.

Case 3: The priority factor should satisfy: $P(f_1), P(f_2) > P(f_3) \gg P(f_4)$.

Case 4: The priority factor should satisfy: $P(f_1), P(f_2) > P(f_4) \gg P(f_3)$.

Case 5: The priority factor should satisfy: $P(f_3) > P(f_1), P(f_2) > P(f_4)$.

Case 6: The priority factor should satisfy: $P(f_3) > P(f_1), P(f_2) \gg P(f_4)$.

Combining the above definitions of vehicle dynamics, path constraints, objective functions and priority requirements; the multi-objective entry optimal control problem to be solved is complete.

Then the proposed FGP-GHGA algorithm is applied to solve this problem in order to obtain the optimized trajectories.

V. Simulation Results

This section presents the numerical simulation results obtained using the FGP-GHGA algorithm developed in the previous sections and applied to the multi-objective aeroassisted vehicle trajectory planning problem. The main aim of the simulations is to illustrate the effectiveness of the proposed strategy in satisfying the pre-assigned priority constraints as well as achieving safe and stable flight.

A. Parameters Specification

The vehicle-dependent parameters, reference values of the states and controls, and control parameters of the GHGA algorithm are tabulated in Table.1. The initial conditions for the vehicle are assigned as: $h_0 = 260000$ft, $\theta_0 = 0°$, $\phi_0 = 0°$, $V_0 = 25600$ft/s, $\gamma_0 = -1.064°$, $\psi_0 = 90°$, $m_0 = 6109.43$slug, $\alpha_0 = 17°$, $\sigma_0 = -75°$ and $T_0 = 0$N, whereas the boundary conditions at the minimum altitude point (time instant t_1) and final boundary point (time instant t_f) are set to: $h_{t_1} = 164000$ft, $\gamma_{t_1} = 0°$ and $h_{t_f} = 260000$ft, respectively [25]. Besides, the state variable at time instants t_1 and t_f should be less than a certain limit (accuracy level). These constraints are set as:

$$
\begin{aligned}
e_{h_1} &= |h_{t_1} - h(t_1)| \leq 500ft \\
e_{h_f} &= |h_{t_f} - h(t_f)| \leq 500ft \\
e_{\gamma_1} &= |\gamma_{t_1} - \gamma(t_1)| \leq 0.1deg
\end{aligned}
\tag{22}
$$

The upper limits associated with the path constraints are set to $\dot{Q} \leq 200$BTU, $q \leq 280$lb and $n_z \leq 2.5$. All the numerical simulations carried out in this investigation are experimented using Matlab 2016a under Windows 7 and Intel(R) Core(TM) i7-4790 CPU, 3.60GHZ, with 12.00 GB RAM.

Table 1 Parameters used in the simulation

States	Values/ranges	Controls	Values/ranges	GHGA parameters	Values
Altitude, h	$[164000, 260000ft]$	Angle of attack, α	$[0, 40°]$	Population size, N_P	300
Longitude, θ	$[-180, 180°]$	Bank angle, σ	$[-90, 1°]$	Iteration number, $Iter$	10000
Latitude, ϕ	$[-180, 180°]$	Thrust, T	$[0, 2 \times 10^6 N]$	w_1	0.8
Velocity, V	$[10, 35000ft/s]$	α_c	$[0, 40°]$	w_2	0.2
FPA, γ	$[-90, 90°]$	σ_c	$[-90, 1°]$	w_3	0.6
Azimuth, ψ	$[-90, 90°]$	T_c	$[0, 2 \times 10^6 N]$	c_1	0.2
Mass, m	$[1527.3, 6109slug]$	Terminal time, t_f	$[500, 2500s]$	c_2	0.8

B. Discretization of the problem

To solve the optimal control problem, an important procedure is to discretize/parametrize the continuous-time system. Currently, there are two types of discretization methods: collocation techniques [6, 27] and shooting techniques [1, 28]. This paper applies the shooting-based technique to parameterize the continuous-time dynamics. That is, only the control variable is discretized at temporal nodes $[t_0, t_1, ..., t_f]$. Then, the state variable is obtained by performing the numerical integration (e.g. Runge-Kutta methods).

Let us assume the number of temporal nodes is N_k, for the problem considered in this research, the problem decision variable can be expressed as $x = [\alpha_{c,1}, ..., \alpha_{c,N_k}, \sigma_{c,1}, ..., \sigma_{c,N_k}, T_{c,1}, ..., T_{c,N_k}]^T$. An attempt is also made to combine other discretization techniques such as the direct collocation or pseudospectral methods with the GHGA optimization method. However, this attempt failed since for direct methods using polynomials, both the control and state variables will be discretized. Subsequently, the equations of motion will be transcribed to a series of equality constraints (algebraic equations). If an optimization problem contains a large number of equality constraints, the evolutionary solver might use a large amount of iterations to capture the true behaviour or even fail to satisfy all the constraints. Therefore, when evolutionary algorithm is chosen to optimize the trajectory, it is suggested to use collocation methods with a relatively small temporal set or apply shooting-based discretization schemes to transcribe the continuous-time problem.

C. Overall Analysis of Relationships Between Different Objectives

According to the mission objectives formulated in Section.IV of this paper (see Eq.(21)), it can be observed that f_3 and f_4 are two contradicting objectives. Specifically, maximizing the terminal velocity can only be achieved at the expense of fuel consumption. This conclusion can also be verified by the dynamic equations of the vehicle velocity and mass. Moreover, it is worth mentioning that the main parameter responsible for the increase in the total amount of aerodynamic heating (f_2) is the dynamic pressure. From Eq.(20), dynamic pressure is a function of air density and velocity. Since the air density in the entry phase is relatively small compared to the velocity, it can be concluded that the f_2 and f_4 are also contradicting objectives. This implies that increasing the satisfactory degree of f_2 will result in a decrease in the satisfactory degree of f_4. On the other hand, according to the definition of f_2 in Eq.(21), the total amount of aerodynamic heating is largely affected by the flight time duration. For example, longer mission duration may result in a larger value of total amount of aerodynamic heating. Therefore, f_1 and f_2 are highly correlated objectives.

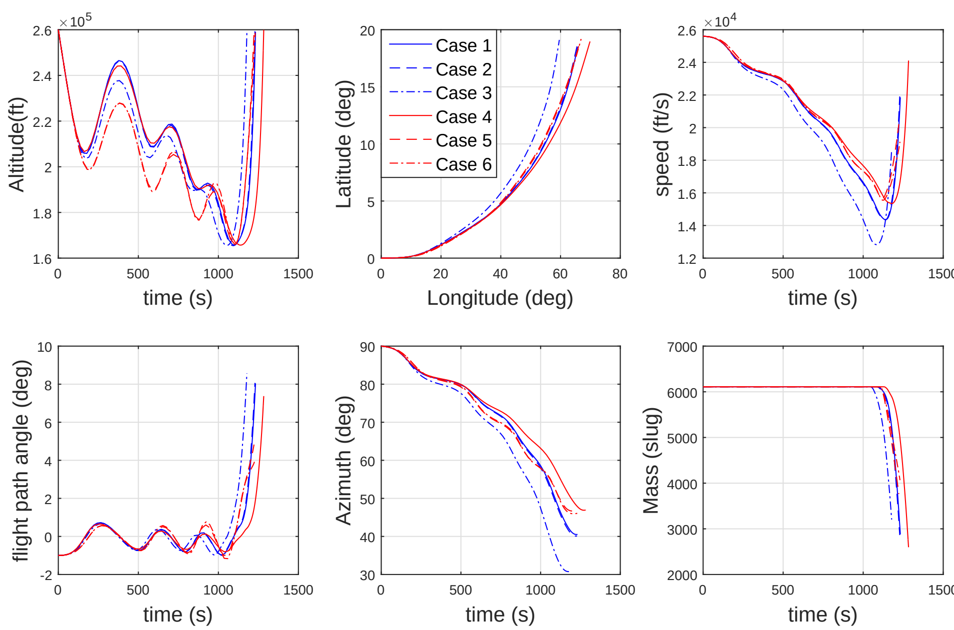

Fig. 1 State profiles obtained for different cases

D. Case Study

To construct the FGP model, the optimal results for each single objective programming problem are firstly generated using the GHGA algorithm. This step is used to determine the numerical values of f_i^*, f_i^{min} and f_i^{max}. The general strategy are stated in Section.III. In order to better describe this strategy, implementation steps are summarised as follows:

Step 1: For $i = 1, 2, 3, 4$, construct the SOP formulation given by Eq.(6).

Step 2: Solve the SOP model via GHGA to obtain x_i^* and f_i^*.

Step 3: For $j = 1, 2, 3, 4$, $j \neq i$, calculate $f_j(x_i^*)$.

Step 4: Set $i = i + 1$ and go back to Step 2.

Step 5: Output $f_i^{min} = \min(f_1(x_i^*), f_2(x_i^*), ..., f_m(x_i^*))$; $f_i^{max} = \max(f_1(x_i^*), f_2(x_i^*), ..., f_m(x_i^*))$.

By formulating the single objective programming problem based on Eq.(21), it is calculated that the optimum solution values associated with each objective are: $f_1^* = 850.31$, $f_2^* = 72.83$, $f_3^* = 4296.7$ and $f_4^* = 29297.01$, while the corresponding worst-case values are: $f_1^{max} = 2086.2$, $f_2^{max} = 219.35$, $f_3^{min} = 1527.3$ and $f_4^{min} = 15011.9$, respectively.

Based on the optimal and worst-case solutions, the FGP model can then be constructed (see Eq.(10)). The improved genetic algorithm is then applied to solve the FGP model. It should be noted that since stochastic algorithm is chosen to optimize the results, it is not enough to analyze the simulation results in only one trial. Therefore, ten trials were conducted independently and the best solution is presented. Fig.1 shows the optimal time history with respect to the state variables. The optimal control trajectories obtained using the proposed FGP-GHGA algorithm are plotted in Fig.2(a)-Fig.2(c), whereas the three path constraint profiles are given in Fig.2(d)-Fig.2(f). The average running time for the optimization algorithm is around 6h 19m (22740.42s). From the path constraint profiles, it can be concluded that the structural and thermal safety of the aeroassisted vehicle is guaranteed, which is the prerequisite for the validity of an approach to trajectory optimization.

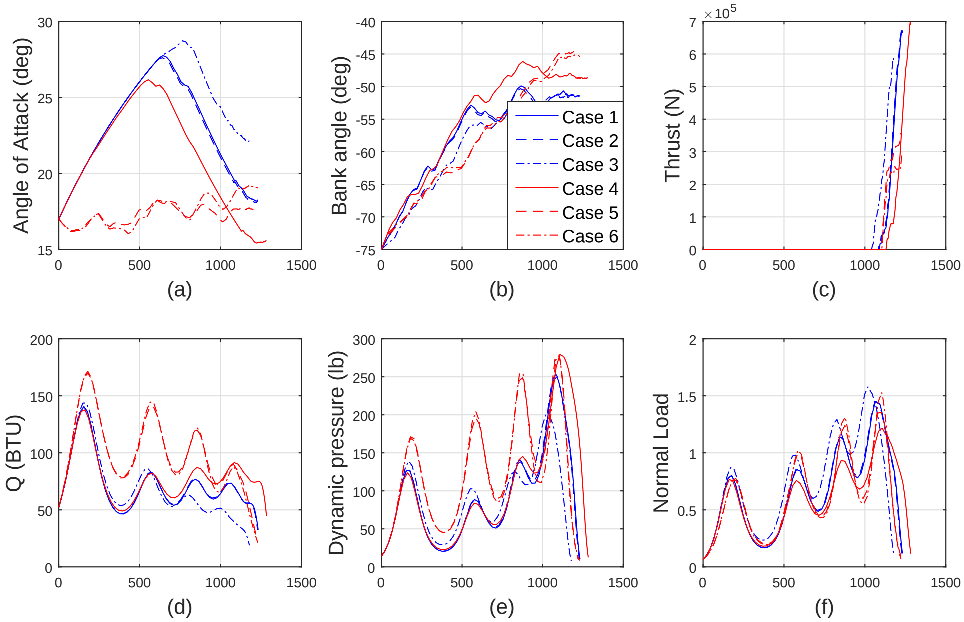

Fig. 2 Control and constraint profiles obtained for different cases

Table 2 Satisfactory degree values for each case

	μ_{f_1}	μ_{f_2}	μ_{f_3}	μ_{f_4}
Case 1	0.6931	0.9453	0.4803	0.4788
Case 2	0.6927	0.9450	0.4737	0.4829
Case 3	0.7356	0.9757	0.5982	0.2552
Case 4	0.6501	0.8861	0.3760	0.6369
Case 5	0.6994	0.7066	0.8037	0.3642
Case 6	0.6935	0.7095	0.9147	0.2884

It can be observed that in the results presented in Fig.1 and Fig2, a difference in the optimal trajectories between Cases.1 to 4 and Cases.5 to 6 can be found. This can be explained by the fact that since the primary task for Case.5 and Case.6 is to achieve a higher satisfactory degree value for minimizing the fuel consumption, the vehicle tends to maneuver relying more on the aerodynamic forces rather than the engine thrust. Therefore, for Case.5 and Case.6, the controls (especially the angle of attack and the thrust) will not experience a significant increase. The corresponding satisfactory degree values of each mission case are tabulated in Table.2, from where it can be seen that the proposed FGP-GHGA approach can offer satisfactory performance for all the cases in the absence of priority requirements. Based on all the figures and tables, it can be concluded that the proposed technique can be effective to generate credible solutions for the multi-objective trajectory optimization problem. Specifically, the state and control trajectories can be smooth and are all in their tolerant regions. Besides, the path constraints can be guaranteed and the pre-specified priority requirements can also be achieved.

VI. Comparison with Existing Designs

A. Comparison with Multi-objective Results of [16]

In [16], the authors proposed an IFPP formulation for solving the multi-objective trajectory planning problem. This approach was analyzed as an efficient and effective tool to handle a specific preference requirement. Different with the problem considered in this paper, the mission scenario investigated in [16] required that the objective should be moved to a pre-specified tolerable/desirable region. Based on the decision maker's physical knowledge of the problem, the IFPP method can drive different mission objectives into their pre-specified tolerant regions successfully (mainly by adjusting the aspiration level and preference functions through its interactive process). However, for the preference requirements considered in this study, the IFPP method might not be as effective as the FPG-GHGA algorithm. This is because the current IFPP design does not have the capability to deal with the priority constraints directly. Moreover, all the definitions of the preference regions are largely depended on the designer's experience. If the tolerable/desirable regions specified by the designers are not accurate, then the results cannot be credible. Although the IFPP method can use its interactive process to adjust the aspiration level and preference functions, it may need several tentative trials and this will result in large computational demand.

Therefore, it is proposed that if the primary task of the mission is to drive all the mission objectives into their pre-designed tolerable, desirable, or highly desirable regions, then it is advantageous to use the IFPP method developed in [16] for solving the problem. On the other hand, if the priority constraints are required to be considered in the mission optimization model, the FGP-GHGA method proposed in this paper might be more effective.

B. Comparison with MOEAs

For general MOPs, current studies are mainly focused on the application of MOEAs for solving this type of problem. Numerous updates and modifications have been made for this type of approach over the past decade [18, 19]. For the purposes of comparison, an Improved Nondominated Sorting Genetic Algorithm II (I-NSGA-II) developed in [19], coupled with the gradient local search operation process is used in this work. The approximated pareto front results are plotted in Fig.3, where the pareto front results are projected onto three planes: minimizing the total amount of aerodynamic heating versus minimizing the time duration, maximizing the terminal velocity versus minimizing the fuel consumption, and finally minimizing the total amount of aerodynamic heating versus maximizing terminal velocity.

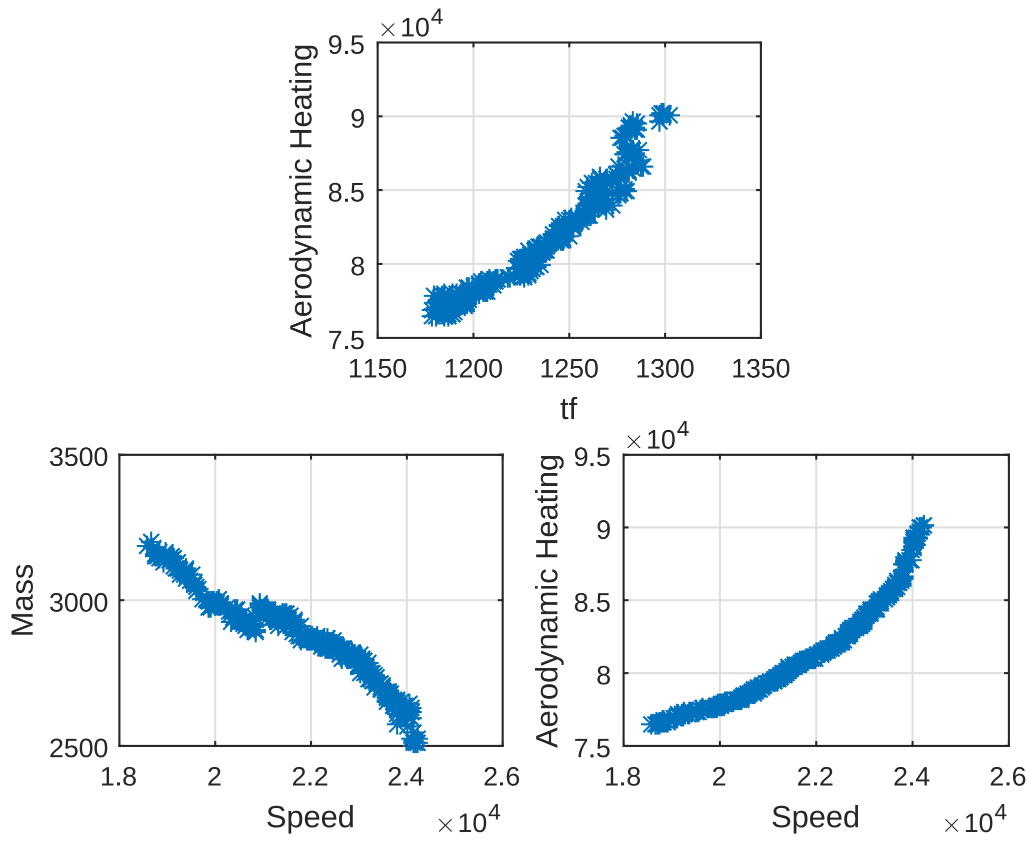

Fig. 3 Pareto front solutions obtained using I-NSGA-II [19]

As can be seen from Fig.3, the relationship between different mission objectives is presented and the results follow the analysis stated in Section.V of this paper. Therefore, the results confirm that the MOEA-based approach can be used to reflect the contradicting or correlated relationships of the multi-objective trajectory planning problem. Once the pareto front is generated, the obtained solution is then presented to the decision-maker such that the designer can select one candidate solution that can meet the pre-specified priority requirements (e.g. Cases.1 to 6). Based on the results shown in Fig.3, the first front set (rank 1) obtained by the I-NSGA-II can be extracted and used to calculate satisfactory degrees with respect to different objectives. Applying Eq.(3) and Eq.(4), the calculated satisfactory degrees with respect to different objectives are $\mu_{f_1} \in [0.6348, 0.7356]$, $\mu_{f_2} \in [0.8821, 0.9757]$, $\mu_{f_3} \in [0.3433, 0.5982]$, and $\mu_{f_4} \in [0.2499, 0.6459]$, respectively. These values are only used as an indicator to assess the solution distribution and the result indicates that not all the mission cases can be achieved by selecting candidates from the obtained I-NSGA-II results. For example, from the obtained pareto set, we cannot find a candidate solution that can satisfy the priority requirement for Case.5 ($P(f_3) > P(f_1), P(f_2) > P(f_4)$) or Case.6 ($P(f_3) > P(f_1), P(f_2) \gg P(f_4)$).

Typically, a main challenge faced by MOEAs is that it has the restriction of dimensionality in solving problems containing more than three objectives. This is because the current domination principle which is usually used and embedded in the MOEA framework lacks the ability to provide an adequate selection pressure and emphasize feasible solutions [21, 28]. In other words, the selection pressure can hardly be allocated to each objective uniformly, thereby resulting in poor diverse representation of the pareto front. Consequently, based on the obtained pareto results shown in Fig.3, it can be concluded that the MOEA-based methodology may fail to generate a well-distributed pareto front for the trajectory planning problem investigated in this paper.

C. Computational complexity of different multi-objective algorithms

In terms of the computational complexity, as indicated in [19], the worst-case computational complexity of NSGA-II algorithm is $\mathcal{O}(MN_p^2)$, where M represents the number of mission objectives. For the designed approach, the calculation and fuzzification of objectives require $\mathcal{O}(M^2N_p)$ computations. $\mathcal{O}(N_pE)$ and $\mathcal{O}(N_pI)$ computations are required for the constraint handling process. The computational complexity of the gradient-based hybrid operator is largely depended on the gradient operator. This procedure can be divided into two parts: local search and gradient estimation.

Calculating the directional vector e is rather straightforward and it requires $\mathcal{O}(n)$ computations. To calculate the gradient itself, the computational complexity should be $\mathcal{O}(nF)$, where F is the complexity for one single gradient evaluation. Therefore, taking into account all the above computations, the overall worst-case computational complexity of the proposed FGP-GHGA method is $\mathcal{O}(N_p^2 + nF)$, which is generally lower than the I-NSGA-II approach (e.g. $\mathcal{O}(MN_p^2 + MnF)$).

D. Potential applications of different multi-objective algorithms

From an application point of view, the solution obtained using the proposed FGP-GHGA method and MOEA-based algorithms can be used in different ways. It is well known that an important practical implementation of the pre-designed trajectory is the design of online guidance law. One main class of guidance methods is the reference-tracking guidance [29]. That is, the guidance law is achieved by tracking a reference trajectory. Therefore, the FGP-GHGA solution can be used to provide a high quality reference trajectory for the online tracking algorithms. Recently, a database-based online guidance scheme is designed for the entry vehicles [30]. In this guidance scheme, a large database of optimal trajectories is firstly generated, and a subset of trajectories is then selected by the onboard algorithm. For this case, the set of pareto-optimal solutions calculated using the MOEA-based approach can be used to provide an alternative to constructing the trajectory database.

VII. Conclusion

In this paper, a multi-objective aeroassisted trajectory optimization problem with mission priority constraints has been solved via an FGP-GHGA algorithm. This approach unifies the mission objectives, constraints and preemptive priorities in one optimization formulation. The new formulation aims to minimize the deviation between the objective and its goal value as well as satisfying the pre-specified priority requirements. Simulation results were carried out to demonstrate the effectiveness of the proposed design.

An important advantage coming from the implementation of this approach is the nondominant sorting procedure which is usually required in most traditional MOEAs is no longer necessary. As a result, the computational complexity of the proposed design can be decreased significantly. This aspect, together with the good performance achieved, suggest that it is advantageous to apply the proposed FGP-GHGA technique for solving the multi-objective aeroassisted vehicle trajectory optimization problem with priority constraints.

References

[1] Lu, P., Brunner, C. W., Stachowiak, S. J., Mendeck, G. F., Tigges, M. A., and Cerimele, C. J., "Verification of a Fully Numerical Entry Guidance Algorithm," *Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 2, 2017, pp. 230–247, doi:10.2514/1.G000327.

[2] Padhi, R., Chawla, C., and Das, P. G., "Partial Integrated Guidance and Control of Interceptors for High-Speed Ballistic Targets," *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 1, 2013, pp. 149–163, doi:10.2514/1.61416.

[3] Harl, N. and Balakrishnan, S. N., "Reentry Terminal Guidance Through Sliding Mode Control," *Journal of Guidance, Control, and Dynamics*, Vol. 33, No. 1, 2010, pp. 186–199, doi:10.2514/1.42654.

[4] Conway, B. A., "A Survey of Methods Available for the Numerical Optimization of Continuous Dynamic Systems," *Journal of Optimization Theory and Applications*, Vol. 152, No. 2, 2012, pp. 271–306, doi:10.1007/s10957-011-9918-z.

[5] Loxton, R. C., Teo, K. L., Rehbock, V., and Yiu, K. F. C., "Optimal control problems with a continuous inequality constraint on the state and the control," *Automatica*, Vol. 45, No. 10, 2009, pp. 2250–2257, doi:http://dx.doi.org/10.1016/j.automatica.2009.05.029.

[6] Fahroo, F. and Ross, I. M., "Direct Trajectory Optimization by a Chebyshev Pseudospectral Method," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 1, 2002, pp. 160–166, doi:10.2514/2.4862.

[7] Ross, I. M. and Fahroo, F., "Pseudospectral Knotting Methods for Solving Nonsmooth Optimal Control Problems," *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 3, 2004, pp. 397–405, doi:10.2514/1.3426.

[8] Liu, X., Shen, Z., and Lu, P., "Entry Trajectory Optimization by Second-Order Cone Programming," *Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 2, 2015, pp. 227–241, doi:10.2514/1.G001210.

[9] Gang, C., Min, X., Zi-ming, W., and Si-lu, C., *RLV Reentry Trajectory Multi-Objective Optimization Design Based on NSGA2 Algorithm*, American Institute of Aeronautics and Astronautics, pp. 1–6, Guidance, Navigation, and Control and Co-located Conferences, 2005, doi:10.2514/6.2005-6131.

[10] Gao, H., Yang, X., and Shi, P., "Multi-Objective Robust H-infinity Control of Spacecraft Rendezvous," *IEEE Transactions on Control Systems Technology*, Vol. 17, No. 4, 2009, pp. 794–802, doi:10.1109/TCST.2008.2012166.

[11] Hu, C.-F., Teng, C.-J., and Li, S.-Y., "A fuzzy goal programming approach to multi-objective optimization problem with priorities," *European Journal of Operational Research*, Vol. 176, No. 3, 2007, pp. 1319–1333, doi:http://dx.doi.org/10.1016/j.ejor.2005.10.049.

[12] Li, S. and Hu, C., "Two-Step Interactive Satisfactory Method for Fuzzy Multiple Objective Optimization With Preemptive Priorities," *IEEE Transactions on Fuzzy Systems*, Vol. 15, No. 3, 2007, pp. 417–425, doi:10.1109/TFUZZ.2006.887463.

[13] Yokoyama, N. and Suzuki, S., "Modified Genetic Algorithm for Constrained Trajectory Optimization," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 1, 2005, pp. 139–144, doi:10.2514/1.3042.

[14] Englander, J. A. and Conway, B. A., "Automated Solution of the Low-Thrust Interplanetary Trajectory Problem," *Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 1, 2017, pp. 15–27, doi:10.2514/1.G002124.

[15] Pontani, M. and Conway, B. A., "Optimal Finite-Thrust Rendezvous Trajectories Found via Particle Swarm Algorithm," *Journal of Spacecraft and Rockets*, Vol. 50, No. 6, 2013, pp. 1222–1234, doi:10.2514/1.A32402.

[16] Chai, R., Savvaris, A., Tsourdos, A., and Xia, Y., "An interactive fuzzy physical programming for skip entry problem," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. PP, No. 99, 2017, pp. 1–13, doi:10.1109/TAES.2017.2696281.

[17] Liu, G.-P., Yang, J.-B., and Whidborne, J.-F., *Multiobjective Optimisation and Control*, Engineering Systems Modeling and Control Series, 2003.

[18] Bosman, P. A. N., "On Gradients and Hybrid Evolutionary Algorithms for Real-Valued Multiobjective Optimization," *IEEE Transactions on Evolutionary Computation*, Vol. 16, No. 1, 2012, pp. 51–69, doi:10.1109/TEVC.2010.2051445.

[19] Chen, B., Zeng, W., Lin, Y., and Zhang, D., "A New Local Search-Based Multiobjective Optimization Algorithm," *IEEE Transactions on Evolutionary Computation*, Vol. 19, No. 1, 2015, pp. 50–73, doi:10.1109/TEVC.2014.2301794.

[20] Hu, W., Yen, G. G., and Luo, G., "Many-Objective Particle Swarm Optimization Using Two-Stage Strategy and Parallel Cell Coordinate System," *IEEE Transactions on Cybernetics*, Vol. 47, No. 6, 2017, pp. 1446–1459,
doi:10.1109/TCYB.2016.2548239.

[21] Jiang, S. and Yang, S., "Evolutionary Dynamic Multiobjective Optimization: Benchmarks and Algorithm Comparisons," *IEEE Transactions on Cybernetics*, Vol. 47, No. 1, 2017, pp. 198–211, doi:10.1109/TCYB.2015.2510698.

[22] Deb, K. and Jain, H., "An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point-Based Nondominated Sorting Approach, Part I: Solving Problems With Box Constraints," *IEEE Transactions on Evolutionary Computation*, Vol. 18, No. 4, 2014, pp. 577–601,
doi:10.1109/TEVC.2013.2281535.

[23] Chai, R., Savvaris, A., and Tsourdos, A., "Fuzzy physical programming for Space Manoeuvre Vehicles trajectory optimization based on hp-adaptive pseudospectral method," *Acta Astronautica*, Vol. 123, 2016, pp. 62–70,
doi:http://dx.doi.org/10.1016/j.actaastro.2016.02.020.

[24] Zhang, H., "A goal programming model of obtaining the priority weights from an interval preference relation," *Information Sciences*, Vol. 354, No. Supplement C, 2016, pp. 197–210,
doi:https://doi.org/10.1016/j.ins.2016.03.015.

[25] Chai, R., Savvaris, A., Tsourdos, A., Chai, S., and Xia, Y., "Improved Gradient-Based Algorithm for Solving Aeroassisted Vehicle Trajectory Optimization Problems," *Journal of Guidance, Control, and Dynamics*, pp. 1–9,
doi:10.2514/1.G002183.

[26] Senses, B. and Rao, A. V., "Optimal Finite-Thrust Small Spacecraft Aeroassisted Orbital Transfer," *Journal of Guidance, Control, and Dynamics*, Vol. 36, No. 6, 2013, pp. 1802–1810,
doi:10.2514/1.58977.

[27] Garg, D., Patterson, M., Hager, W. W., Rao, A. V., Benson, D. A., and Huntington, G. T., "A unified framework for the numerical solution of optimal control problems using pseudospectral methods," *Automatica*, Vol. 46, No. 11, 2010, pp. 1843–1851,
doi:http://dx.doi.org/10.1016/j.automatica.2010.06.048.

[28] Chai, R., Savvaris, A., Tsourdos, A., and Chai, S., *Solving Multi-objective Aeroassisted Spacecraft Trajectory Optimization Problems Using Extended NSGA-II*, American Institute of Aeronautics and Astronautics, AIAA SPACE Forum, 2017, doi:10.2514/6.2017-5193.

[29] Dai, J. and Xia, Y., "Mars atmospheric entry guidance for reference trajectory tracking," *Aerospace Science and Technology*, Vol. 45, 2015, pp. 335–345, doi:http://dx.doi.org/10.1016/j.ast.2015.06.006.

[30] Sagliano, M., Mooij, E., and Theil, S., "Onboard Trajectory Generation for Entry Vehicles via Adaptive Multivariate Pseudospectral Interpolation," *Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 2, 2017, pp. 466–476, doi:10.2514/1.G001817.