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A New Command Shaping Guidance Law using Lagrange Multiplier

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Abstract: This article presents a new command shaping guidance law by change of Lagrange multiplier (LM), called CSGL-LM. The Schwarz inequality approach is used to solve the optimal guidance problems considering both terminal constraints on interception and impact angle control. LM is introduced to combine two terminal constraints into a single equation. The main idea of this paper is to use LM as a design parameter for shaping the guidance command as well as controlling the terminal constraints. The guidance command of CSGL-LM is given a unified functional form of the time-to-go, the state variables, and LM. Therefore, through an appropriate choice of LM, we can achieve various shapes of the guidance commands for the interception case, as well as the impact angle control case. As illustrative examples, this paper also shows that a class of previous guidance laws is just one of particular solutions of CSGL-LM. Numerical simulations are performed to validate the properties of CSGL-LM, compared with the conventional guidance law.

Keywords: Lagrange Multiplier, Guidance Law, Command Shaping, Schwarz inequality

1. INTRODUCTION

The main goal of the guidance systems is to deliver the vehicles to a target position. In the application of the missile systems, the interception of a target is the most important requirement for the guidance systems. The proportional navigation guidance (PNG) in Ben-Asher et. al. (1998) and Zarchan (2007) has been widely accepted in many missile systems due to its effectiveness and simplicity in practice. Considering only target interception, PNG has been regarded as an attractive solution.

Recently, as battlefields and targets have been continuously diversified and modernised, the missile guidance system is demanded to take such changes into account. Consequently, the requirements of the guidance systems have gradually adapted to these changes. For the anti-tank or anti-ship missile systems, the terminal impact angle constraint is widely requested to maximize the lethality of the warhead. For the surface-to-air missile systems, the flight path angle constraint at the beginning of homing phase has been widely considered to achieve an advantageous initial homing position against a highly manoeuvring target.

The guidance law intended to achieve this additionally constraint is called the impact angle control guidance (IACG) law. Over the past several years, there have been extensive research activities on IACG in Idan et. al. (1995), Ryoo et. al. (2005) and Lee et. al. (2013a, 2013b). These works have been performed in the optimal control framework because it enables to handle the terminal constraints easily and to provide a state-feedback form solution.

In the field of the guidance technology, command shaping capability while keeping the terminal constraints has been widely demanded to meet various operational goals of guidance. However, in most of previous guidance laws such as PNG or IACG, the guidance command is decided by a selection of guidance gain. This leads to no degree of freedom remained for shaping the guidance command. In order to solve this issue, numerous command shaping guidance laws have been devised. Most of these works have been also performed under the optimal control framework. The underlining idea of these works was to introduce a weighting function to the control energy so as to provide an additional degree of freedom for shaping the guidance command. Various weighting functions have been successfully applied: for example time-to-go functions in Ryoo et. al. (2006), Ohlmeyer et. al. (2006) exponential function in Rusnak et. al. (1996), Ryu et. al. (2015) and sinusoidal function in Lee et. al. (2015). Also, in the reference Lee et. al. (2013a), the authors have proven that any positive functions can be used to shape the guidance command in this framework.

These approaches were to successfully provide the command variability, but there was still a point to be improved. That is, in this framework, guidance laws for the interception and for the impact angle control are not given by a unified form. Thus, guidance laws for the interception and the impact angle control should be separately designed. This implies that command shaping across the terminal constraints is not allowed in this approach unless the guidance configuration changes. It would be desirable to obtain a guidance law for the interception and the impact angle control in a unified form such that command shaping across the terminal constraints is possible. In general, the guidance phases consist of several sub-phases such as the initial phase, the mid-course phase, and the terminal homing phase. Each guidance phase requires different terminal constraints. Therefore, if a guidance command is given by a unified form, such different guidance phases and their constraints can be handled with a single guidance command without changing guidance configuration and a complicated logic.

To this end, this paper proposes a new command shaping guidance law across the terminal constraints, named as CSGL-LM. In this study, Schwarz inequality approach is used to solve the optimal guidance problem. In this approach, LM is used to combine two terminal constraints. The main idea of the proposed approach is to use LM as a design parameter to shape the guidance command. Since the role of LM is also to combine two terminal constraints, we can satisfy the terminal constraints and at the same time shape the guidance command through an appropriate choice of the LM value. As illustrative examples, this paper shows that CSGL-LM can be converted to other guidance laws under a specific LM setting.

The rest of this paper is organised as. In section 2, the engagement kinematics is explained. CSGL-LM is derived in section 3. Numerical simulations are performed in section 4. Finally, section 5 concludes this study.

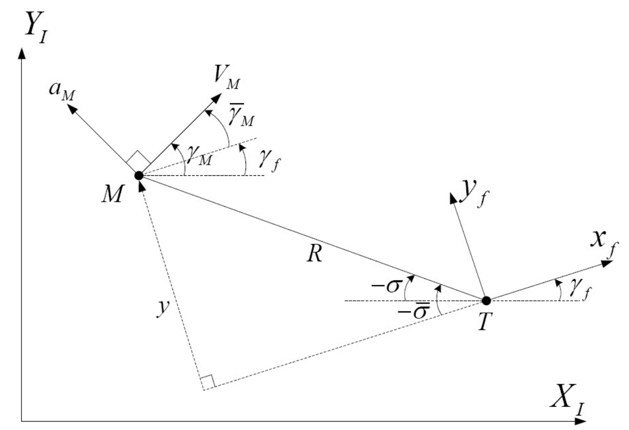


Fig. 1. Planar engagement kinematics.

2. ENGAGEMENT KINEMATICS

This section devotes to describe the planar engagement kinematics for deriving CSGL-LM. Fig. 1 shows the engagement kinematics for a stationary target. In this engagement kinematics, $\left(X_I, Y_I\right)$ represents the inertial frame and $\left(X_f, Y_f\right)$ represents the reference frame. The reference frame is rotated from the inertial reference frame by the desired impact angle (i.e., γ_f) and introduced for linearization purpose. The missile and target are denoted by M and T, respectively. The line-of-sight (LOS) angle and the relative distance between M and T are denoted by σ and R. The flight path angle, the velocity, and the normal acceleration of missile are given by γ_M , V_M , and a_M , respectively. In the missile systems, the normal acceleration is considered as the input variable and leads to change of the flight direction. Additionally, the variables with bar such as $\bar{\sigma}$ and $\bar{\gamma}_M$ represent the angles measured from the reference frame and the physical meanings are the same. The variable y denotes the lateral position perpendicular to x_f -axis. In Fig.1, the nonlinear engagement kinematics with respect to y is given by

$$\begin{aligned} \dot{y} &= V_M \sin \bar{\gamma}_M = v \\ \dot{\bar{\gamma}}_M &= a_M / V_M \end{aligned} \tag{1}$$

where the variable v is defined to be the lateral velocity perpendicular to x_f -axis. In order to achieve the interception as well as the satisfaction of the desired impact angle, the constraints at the final time are given by

$$y\left(t_f\right) = v\left(t_f\right) = 0 \tag{2}$$

where t_f represents the final time. Under the constant velocity assumption and small angle approximation of $\bar{\gamma}_M$, the above equation can be linearized as follows:

$$\begin{aligned} \dot{y} &= v \\ \dot{v} &= a_M \end{aligned} \tag{3}$$

This equation can be rewritten in the state-space form as

$$\dot{x} = Ax + Bu \tag{4}$$

where

$$x = \left[x_1, x_2\right]^T = \left[y, v\right]^T, u = a_M, A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{5}$$

Note that this linearized kinematics has been widely used to devise optimal guidance laws from many researchers in Ryoo et. al. (2005, 2006) and Lee et. al. (2013a, 2013b, 2014, 2015).

3. GUIDANCE LAW DESIGN

2.1 Derivation of CSGL-LM

This section provides the derivation of CSGL-LM. During the course of years, there have been a lot of approaches to devise new guidance laws. Among them, the optimal control approach has a great attention because it is systematically well-posed. Just solving optimal guidance problem can provide a practical and a capturability-proven guidance law. In addition, a guidance law obtained is given by a state-feedback form. This property can be a merit for implementing it.

In the optimal control approach, Schwarz inequality is one of ways to obtain the optimal solution. In the process of Schwarz inequality method, LM is introduced to merge two constraints into one equation. In this approach, there is a degree of freedoms in choosing LM. Accordingly, the main idea of this paper is to use LM as a design parameter that enables various shapes of the guidance command as well as control the terminal constraints.

Based on the linear control theory, the general solution of Eq. (4) is determined as

$$x(t_f) = \Phi(t_{go})x(t) + \int_t^{t_f} \Phi(t_f - \tau)B(\tau)u(\tau)d\tau \tag{6}$$

where $\Phi(t_{go})$ is the state transition matrix, which is obtained as

$$\Phi(t_{go}) = e^{A(t_{go})} = \begin{bmatrix} 1 & t_{go} \\ 0 & 1 \end{bmatrix} \tag{7}$$

where $t_{go} = t_f - t$ represents the time-to-go that is the remaining time of the interception. For a stationary target, t_{go} can be simply determined as

$$t_{go} = \frac{R}{V_M} \tag{8}$$

Substituting Eqs. (5) and (7) into Eq. (6) yields:

$$\begin{aligned} x_1(t_f) &= x_1(t) + x_2(t)t_{go} + \int_t^{t_f} (t_f - \tau)u(\tau)d\tau \\ x_2(t_f) &= x_2(t) + \int_t^{t_f} u(\tau)d\tau \end{aligned} \tag{9}$$

Then, imposing the terminal constraints to Eq. (9) gives

$$\begin{aligned} x_1(t) + x_2(t)t_{go} &= -\int_t^{t_f} (t_f - \tau)u(\tau)d\tau \\ x_2(t) &= -\int_t^{t_f} u(\tau)d\tau \end{aligned} \tag{10}$$

As shown in Eq. (10), we have two equations. Here, we introduce LM λ in order to combine above two equations as follows:

$$x_1(t) + x_2(t)(t_{go} - \lambda) = \int_t^{t_f} \left[\lambda - (t_f - \tau)\right]u(\tau)d\tau \tag{11}$$

Then, applying Schwarz inequality to Eq. (11) provides:

$$\left[x_1(t) + x_2(t)(t_{go} - \lambda)\right]^2 \le \int_t^{t_f} \left[\lambda - (t_f - \tau)\right]^2 d\tau \int_t^{t_f} u^2(\tau)d\tau \tag{12}$$

Rearranging above equation gives:

$$\frac{\left[x_1(t) + x_2(t)(t_{go} - \lambda)\right]^2}{\int_t^{t_f} \left[\lambda - (t_f - \tau)\right]^2 d\tau} \le \int_t^{t_f} u^2(\tau)d\tau \tag{13}$$

For convenience's sake, let the left-hand side of Eq. (13) be defined as

$$J = \frac{\left[x_1(t) + x_2(t)(t_{go} - \lambda)\right]^2}{\int_t^{t_f} \left[\lambda - (t_f - \tau)\right]^2 d\tau} \tag{14}$$

By using this definition, we can rewrite Eq. (13) as follows:

$$J \le \int_t^{t_f} u^2(\tau)d\tau \tag{15}$$

Then, we can readily observe that the physical quantity of the right-hand side of Eq. (15) is the control energy and the left-hand side of Eq. (15) represents the minimum value of the control energy. From Eq. (15), when the equality holds, the control energy can achieve its minimum value. According to Schwarz inequality, if there is a linear relationship between $\lambda - (t_f - \tau)$ and $u(\tau)$, then the equality holds as:

$$u(\tau) = K\left[\lambda - (t_f - \tau)\right] \tag{16}$$

where K is a constant value. In order to determine K, we substitute Eq. (16) into Eq. (10).

$$K = \frac{x_1(t) + x_2(t)t_{go}}{\int_t^{t_f} \left[(t_f - \tau)^2 - \lambda(t_f - \tau)\right]d\tau} = \frac{x_1(t) + x_2(t)t_{go}}{(1/3)t_{go}^3 - (\lambda/2)t_{go}^2} \tag{17}$$

Substituting Eq. (17) into Eq. (16) gives:

$$u(t) = \frac{\left(x_1(t) + x_2(t)t_{go}\right)\left[\lambda - t_{go}\right]}{(1/3)t_{go}^3 - (\lambda/2)t_{go}^2} \tag{18}$$

By using Eq. (5), the guidance command can be rewritten with the original variables as

$$a_M = \frac{\left(y + vt_{go}\right)\left[\lambda - t_{go}\right]}{(1/3)t_{go}^3 - (\lambda/2)t_{go}^2} \tag{19}$$

As shown in Eq. (19), CSGL-LM is given by the function of the lateral position and velocity, the time-to-go, and LM. As mentioned in before, in the proposed approach, LM can be used to vary the guidance command as well as control the terminal constraints.

2.2 Discussion of CSGL-LM

In the previous, we observe that CSGL-LM is given by the function of LM. In this section, we will show that various shapes of the guidance commands across the terminal constraints can be achieved using the proposed solution according to selections of LM.

We suppose that LM is given by a linear function of the time-to-go in order to match the physical unit from Eq. (11).

$$\lambda = S(y, v, t_{go})t_{go} \tag{20}$$

Wher $S(y, v, t_{go})$ represents the shaping function of LM, which is possibly given by a function of state variables and the time-to-go.

First, if we choose $S(y, v, t_{go})$ as a function of time-to-go (not dependent on the state variables), then we have

$$S\left(y,v,t_{go}\right)=f\left(t_{go}\right),\quad \text{where } f\left(t_{go}\right)\geq 0 \tag{21}$$

By substituting Eq. (21) with Eq. (20) into Eq. (19), the guidance command can be expressed as follows:

$$a_M=-N'\left(f\left(t_{go}\right)\right)\frac{\left(y+vt_{go}\right)}{t_{go}^2} \tag{22}$$

where $N'\left(f\left(t_{go}\right)\right)$ is the time-varying gain which is given by

$$N'\left(f\left(t_{go}\right)\right)=\frac{6\left(1-f\left(t_{go}\right)\right)}{2-3f\left(t_{go}\right)} \tag{23}$$

Note that in that case CSGL-LM can become the well-known proportional navigation guidance (PNG) with the time-varying gain. Therefore, according to selection of any positive functions $f\left(t_{go}\right)$, we can generate various guidance commands while keep satisfying the interception condition.

Here, there are some interesting properties for the specific choice of $f\left(t_{go}\right)$. Especially, $f\left(t_{go}\right)$ is chosen as a constant value $f\left(t_{go}\right)=\alpha$, where $\alpha\geq 0$, then we have

$$N'=\frac{6\left(1-\alpha\right)}{2-3\alpha} \tag{24}$$

In that case, the time-varying gain becomes a constant value which exactly corresponds to the navigation constant of the conventional PNG. The above equation can be rewritten with respect to α as follows:

$$\alpha=\frac{2N'-6}{3N'-6} \tag{25}$$

Therefore, through appropriate choices in α, we can mimic any values of the navigation constant. From Eq. (24), when the value of α approaches zero, then we have

$$\lim_{\alpha\to 0}N'=3 \tag{26}$$

This result is natural because the condition of $\alpha=0$ (that is $\lambda=0$) in Eq. (11) means that it does not take the constraint regarding the impact angle into account in the derivation. Accordingly, the guidance problem becomes the optimal interception problem and it is well-known that the PNG with N = 3 is the optimal solution for the interception case. Additionally, as $\alpha\to\infty$, we have

$$\lim_{\alpha\to\infty}N'=2 \tag{27}$$

Next, let us assume that the shaping function of LM is given by the trajectory dependent function as

$$S\left(y,v,t_{go}\right)=\frac{p_1y+p_2vt_{go}}{p_3y+p_4vt_{go}} \tag{28}$$

where p_1, p_2, p_3, and p_4 are constant values. First, we choose these parameters as follows

$$p_1=3,\ p_2=1,\ p_3=6,\ p_4=3 \tag{29}$$

Then, CSGL-LM becomes the following form

$$a_M=-6\frac{y}{t_{go}^2}-4\frac{v}{t_{go}} \tag{30}$$

This result is identical to the optimal impact angle guidance law (OGL) as studied in Ryoo et. al. (2005). Also, LM with Eqs. (20), (28), and (29) can satisfy the condition of $\partial J/\partial\lambda=0$. It means that this shaping function of LM gives the optimal solution from the control energy minimization standpoint.

Additionally, we choose p_1, p_2, p_3, and p_4 as follows:

$$\begin{aligned}
p_1&=N^2+5N+3\\
p_2&=2N+1\\
p_3&=\frac{3\left(N+4\right)\left(N+1\right)}{2}\\
p_4&=3\left(N+1\right)
\end{aligned} \tag{31}$$

where $N\geq 0$. In that case, substituting Eq. (28) with Eq. (31) into Eq. (19) provides

$$a_M=-\left(N+2\right)\left(N+3\right)\frac{y}{t_{gp}^2}-2\left(N+2\right)\frac{v}{t_{go}} \tag{32}$$

CSGL-LM is the same as the time-to-go weighted optimal guidance law (TWOGL) in Ryoo et. al. (2006) and Ohlmeyer et. al. (2006).

Finally, we choose p_1, p_2, p_3, and p_4 as follows:

$$\begin{aligned}
p_1&=\left(n+1\right)\left(m+1\right)+n+m\\
p_2&=n+m\\
p_3&=\frac{3}{2}\left[\left(m+1\right)\left(n+1\right)+m+n+1\right]\\
p_4&=\frac{3}{2}\left(m+n+1\right)
\end{aligned} \tag{33}$$

where $n>m\geq 0$. In that case, substituting Eq. (28) with Eq. (33) into Eq. (19) gives

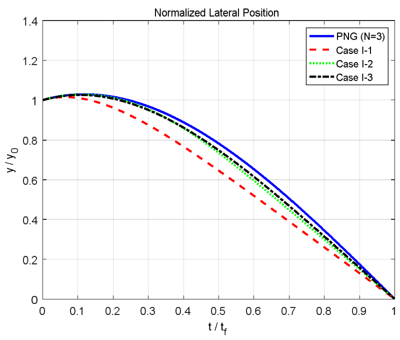
$$a_M=-\left(m+2\right)\left(n+2\right)\frac{y}{t_{gp}^2}-\left(n+m+3\right)\frac{v}{t_{go}} \tag{34}$$

CSGL-LM in this setting is equivalent to the time-to-go polynomial guidance (TPG) as studied in Lee et. al. (2013b).

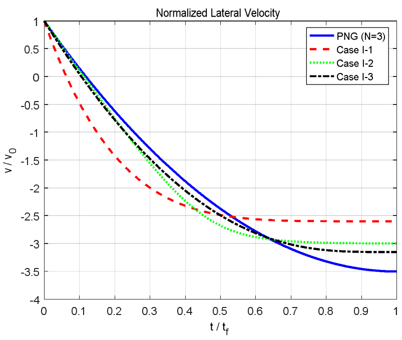
Accordingly, through specific choices in LM, CSGL-LM can contain a class of previous guidance laws. Therefore, CSGL-LM can be regarded as a more general guidance law compared with other guidance laws. Also, we can observe that CSGL-LM can be converted to both the interception guidance law such as PNG and IACG laws such as OGL, TWOGL, and TPG. Therefore, CSGL-LM can provide various guidance commands across the terminal constraints.

4. NUMERICAL SIMULATIONS

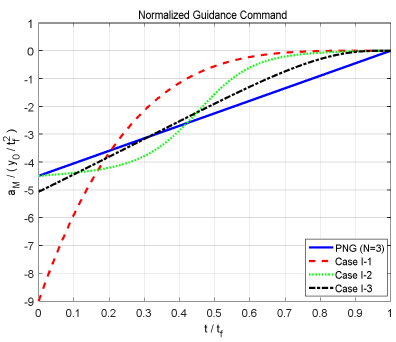
In this section, we perform two linear simulations in order to demonstrate the properties of CSGL-LM from the command shaping standpoint. In the first simulation, LM is chosen by a function which is independent of the state variables as shown in Eqs. (20) and (21). The second simulation is conducted to determine the characteristics of CSGL-LM when LM is designed by a function which is dependent on the state variables as shown in Eqs. (20) and (28). In these simulations, CSGL-LM is compared with PNG with N = 3 in order to show the superiority of CSGL-LM in the term of the command variability.



(a) Normalized Lateral Position



(b) Normalized Lateral Velocity



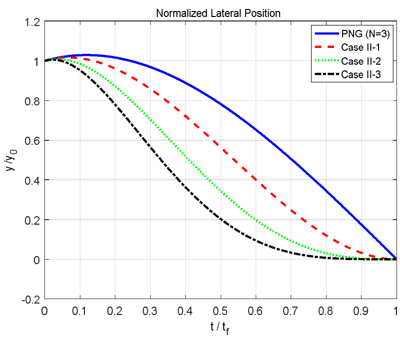
(c) Normalized Guidance Command

Fig. 2. The simulation results when λ is independent of the state variables

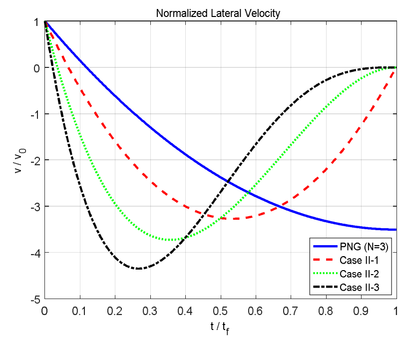
Table 1. Design parameters for simulation I

Nomenclature	Design of $f(t_{go})$
Case I-1	0.5
Case I-2	$0.5\sin((\pi / t_f) t_{go})$
Case I-3	$0.5\exp(-t_{go} / t_f)$

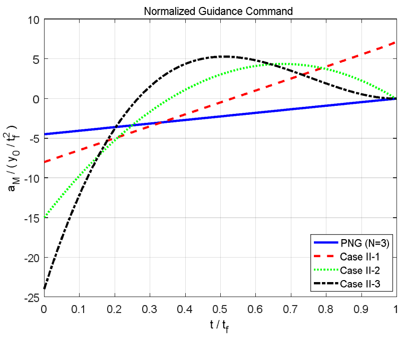
Fig. 2 (a), (b), and (c) show the normalized position, velocity, and guidance command in the first simulation. In that case, the shaping functions of LM are designed as provided in Table 1. As shown in Fig. 2(a), the lateral position converges to zero for all cases. It means that CSGL-LM can successfully intercept a target. In these cases, we can readily observe that the lateral velocity does not approach zero at the final time as shown in Fig. 2 (b). Therefore, in this setting of LM (e.g., independent of the state variables), the terminal constraint on the interception is only possible. These results match with the analytical results as provided in above section. Fig. 2 (c) shows that compared with the conventional PNG, CSGL-LM can produce various patterns of the guidance command while satisfying the interception condition. Especially, in Case I-2, CSGL-LM demands a small guidance command at the initial phase and the guidance command quickly approaches zero in the terminal phase. This command shape is desirable for reducing the sensitivity against to the initial heading error as well as ensuring the operational margin to cope with unexpected situation in the vicinity of a target.



(a) Normalized Lateral Position



(b) Normalized Lateral Velocity



(c) Normalized Guidance Command

Fig. 3. The simulation results when λ is dependent on the state variables.

Table 2. Design parameters for simulation II

Nomenclature	Parameters
Case II-1	$P_1 = 3, P_2 = 1, P_3 = 6, P_4 = 3$
Case II-2	$P_1 = 9, P_2 = 3, P_3 = 15, P_4 = 6$
Case II-3	$P_1 = 17, P_2 = 5, P_3 = 27, P_4 = 9$

Fig. 3 (a), (b), and (c) describe the normalized position, velocity and guidance command in the second simulation. The shaping function is designed as shown in Eq. (28) and their parameters are chosen as provided in Table 2. As shown in Fig. 3 (a) and (b), both the lateral position and velocity converge to zero as the time-to-go goes to zero. Therefore, through specific choices in the shaping function of LM (e.g., dependent on the state variables), CSGL-LM can be converted to IACG laws. As shown in Fig. 3 (c), the simulation results show that CSGL-LM can produce various shapes of the guidance command according to the combinations of the shaping function's parameters while keeping the impact angle control capability.

In this section, we reveal that CSGL-LM can become interception guidance law as well as impact angle control guidance law according to selection of shaping function of LM. Additionally, by changes of parameters of shaping function, CSGL-LM enables to produce various shapes of the guidance command. These properties of CSGL-LM are desirable for achieving various and extensive operational goals of guidance.

5. CONCLUSIONS

This paper suggests a new command shaping guidance law using LM. In this study, the optimal guidance problems with the terminal constraints of interception and impact angle control are considered. Schwarz inequality approach is used to obtain solution of given optimal guidance problem. In the proposed method, LM is used to combine two constraints into one equation as well as provide additional degree of freedom in shaping the guidance command. The proposed guidance law called CSGL-LM is given by the function of the time-to-go, the state variables and LM. We show that for a specific selection of LM, CSGL-LM can become PNG law with a time-varying gain as well as a class of impact angle control guidance laws. Therefore, this observation provides that CSGL-LM is a unified solution across the terminal constraints. Also, CSGL-LM is a more generalized guidance law compared with other guidance laws.

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