# Applying a Modified Smith Predictor-Bilinear Proportional Plus Integral Control for Directional Drilling\*

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**Abstract:** Recently, a Bilinear Proportional plus Integral (BPI) controller was proposed for the control of directional drilling tools commonly used in the oil industry. However, there are delays in the measurement signals which reduces the system performance. Here, the BPI controller is extended by addition of a modified Smith predictor. The effectiveness, robustness and stability of the proposed modified Smith Predictor (SP)-BPI controller are analysed. Transient simulations are presented and compared with that of the earlier BPI controller. From the results, it can be surmised that the proposed modified SP-BPI controller significantly reduces the adverse effects of disturbances and time delay on the feedback measurements with respect to stability and performance of the directional drilling tool.

Keywords: Directional drilling, Attitude control, Bilinear, Feedback delay, Disturbances

## 1. INTRODUCTION

Rotary steerable drilling and directional drilling tools are now commonly used for exploiting smaller, difficultto-commercialize reservoirs and for extending the life of existing oilwells (Pedersen et al., 2009). Steerable tools facilitate the orientation of the wellbore propagation to be directed as desired either by steering the borehole downhole via the application of a Rotary Steerable System (RSS) drilling tool or by passive steering control from the surface via the application of fixed-bend positive displacement motors (Yonezawa et al., 2002; Kuwana et al., 1994). By either perspective, directional drilling is primarily attitude control, that is, the control of the inclination and azimuth (Genevois et al., 2003).

A typical RSS directional drilling system with its primary elements are shown in Fig. 1. The bottomhole assembly (BHA) in combination with the drilling string can be regarded as a long flexible prop-shaft that conveys torque to the bit. During drilling operations, mud which acts as a means to transport cuttings and for lubrication (amongst other functions), is pumped from the earth's surface to the borehole through the center of the BHA and drillstring, and then returns to the earth's surface through the annular space between the wellbore and the BHA and drillstring. The drillstring and BHA are suspended from the block at the surface. These place weight on the bit actuator of the drill rig. The top drive rotational actuator is also located at the surface.

The development of a generic tool-independent attitude control algorithm for application to directional drilling tools is described in this paper. The significance of attitude control of directional drilling tools is highlighted in

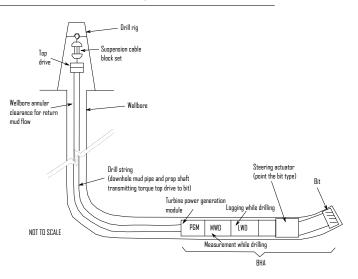


Fig. 1. Schematic of the RSS directional drilling system components

Genevois et al. (2003) and Yonezawa et al. (2002), where control techniques are proposed for the orientation of the borehole propagation based on holding the toolface angle. Genevois et al. (2003) highlighted the desire for closed-loop "shoot and forget systems" and illustrated the azimuth control to be the major challenge. Most of the attitude control techniques described in literature are elucidated based on specific tool structures. Typical examples are by Yinghui and Yinao (2000), Genevois et al. (2003) and Yonezawa et al. (2002). Another interesting example presented by Kuwana et al. (1994) is an attitude controlling system which uses two-way telemetry communication links with the surface. The steering correction, calculated from the telemetry, is determined and then manually downlinked to the tool. Other control strategies recently de-

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veloped include a hybrid approach consisting of two levels of automation for trajectory control of the tool (Matheus et al., 2014), a dynamic state-feedback controller design for 3D directional drilling systems (van de Wouw et al., 2016), a robust Proportional plus Integral (PI) feedback controller (Panchal et al., 2010), an optimal  $H_{\infty}$  controller (Bayliss and Whidborne, 2015) and a linear quadratic Gaussian controller (Bayliss et al., 2015).

When modelling physical systems, the dynamics are often approximated as being linear models obtained by a firstorder Taylor series approximation of the nonlinear model at a particular point of operation. It is clear that such linear models might be inaccurate over a wider range of operation; hence, bilinear models have been proposed to more accurately describe the nonlinear systems (see Bruni et al. (1974) and Schwarz and Dorissen (1989)). Bilinear models can characterize nonlinear properties more correctly than linear models; hence, broaden the range of adequate performance. This paper presents the development of a bilinear model of the directional drilling tool through the application of Carleman bilinearization technique, described in Ghasemi et al. (2014). The generalized state space representation of a Multiple-Input Multiple-Output (MIMO) bilinear system is expressed as (Kim and Lim, 2003):

$$\dot{x} = Ax + \left(B + \sum_{i=1}^{N} x_i M_i\right) u \tag{1}$$

where N is the number of expansion terms and augmented states,  $u \in \mathbb{R}^{m \times 1}$  denotes the control vector, A, B, C and  $M_i$  are constant matrices of suitable dimensions and  $x \in \mathbb{R}^{n \times 1}$  represents the vector of state variables.

The PI and BPI controllers proposed by Panchal et al. (2010) and Inyang et al. (2016), respectively, show good performances for the attitude control of the directional drilling tool but are not sufficiently robust to cope with the disturbances and long time delay on the feedback measurements. These disturbances are as a result of varying rock formations, a proclivity for the tool to drift horizontally, and to drop towards a vertical orientation due to gravity. While the long time delay arises because the sensor that measures the attitude change is, by necessity, located some distance (occasionally, several tens of feet) behind the bit. To handle these disturbances and lengthy time delay on the feedback measurements with respect to stability and performance, this paper proposes a more robust controller for the attitude control of directional drilling tool by applying the modified Smith Predictor (SP)-Bilinear Proportional plus Integral (BPI) controller, which is a combination of the modified SP, proposed by Normey-Rico et al. (1997), and the BPI controller.

Extending the previous work of Inyang et al. (2016), this paper presents a strategy for the design of a modified SP-BPI controller which has the control objective, once employed, of automatically holding the attitude of the directional drilling tool at nominally constant values during oilwell propagation. Here, the modified SP-BPI controller is considered to provide improved performance over a broader range of directional drilling tool operations. The remaining part of this paper begins with highlighting the system model, succeeded by system bilinearization, control design and then simulation results. The design and analysis of the modified SP-BPI controller are performed using MATLAB and its associated Control System Toolbox commands, while Simulink is subsequently used to perform the transient simulations.

## 2. SUMMARY OF EARLIER WORK

## 2.1 Tool Kinematics

The system model is derived from kinematic considerations as illustrated in Fig. 2 and detailed in Panchal et al. (2010). The resulting governing equations can be stated as follows (Panchal et al., 2010):

$$\dot{\theta}_{\rm inc} = V_{\rm rop} \left( U_{\rm dls} \cos U_{\rm tf} - V_{\rm dr} \right) \tag{2}$$

$$\dot{\theta}_{\rm azi} = \frac{V_{\rm rop}}{\sin \theta_{\rm inc}} \left( U_{\rm dls} \sin U_{\rm tf} - V_{\rm tr} \right) \tag{3}$$

where  $V_{\rm rop}$  denotes the rate of penetration, a time-varying parameter,  $V_{\rm tr}$  denotes the turn rate bias disturbance,  $V_{\rm dr}$ denotes the drop rate disturbance ( $V_{\rm dr} = \alpha \sin \theta_{\rm inc}$ ),  $\alpha$  is a constant,  $U_{\rm dls}$  denotes the curvature ( $K_{\rm dls} \times duty$  cycle), also known as "dogleg severity",  $K_{\rm dls}$  denotes the openloop curvature capability of the tool,  $U_{\rm tf}$  denotes the toolface angle control input,  $\theta_{\rm azi}$  denotes the azimuth angle and  $\theta_{\rm inc}$  denotes the inclination angle.

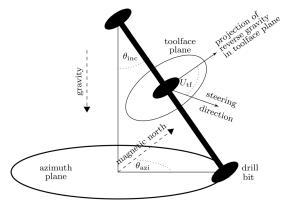


Fig. 2. Typical steering and attitude parameters of directional drilling tool

As detailed in Panchal et al. (2010), engineering constraints include the control inputs  $U_{\rm dls}$  and  $U_{\rm tf}$  being discretized into duty cycles known as "drilling cycles" and that the tool-face input  $U_{\rm tf}$  is subject to first order lag dynamics. Additionally, the on-tool feedback measurements of  $\theta_{\rm inc}$  and  $\theta_{\rm azi}$  are subject to pure delays dependent on  $V_{\rm rop}$  as a consequence of the relevant sensors being spatially offset from the drill bit (the inertial datum). These controller and sensor dynamics are ignored for the controller design.

#### 2.2 Partially Linearized and Decoupled System

The MIMO open-loop system can be partially linearized and decoupled using the following transformation:

$$U_{\rm tf} = {\rm ATAN2}\left(U_{\rm azi}, U_{\rm inc}\right) \tag{4}$$

$$U_{\rm dls} = K_{\rm dls} \sqrt{\left(U_{\rm azi}\right)^2 + \left(U_{\rm inc}\right)^2} \tag{5}$$

where  $U_{azi}$  and  $U_{inc}$  are virtual control of azimuth and inclination, respectively. Substituting (4) and (5) into (2) and (3) with the turn rate bias and drop rate disturbances removed, yields the open-loop dynamics:

$$\dot{\theta}_{\rm inc} = V_{\rm rop} K_{\rm dls} U_{\rm inc} \tag{6}$$

$$\dot{\theta}_{\rm azi} = \frac{V_{\rm rop}}{\sin \theta_{\rm inc}} K_{\rm dls} U_{\rm azi} \tag{7}$$

Therefore, the control transformations, (4) and (5), partially linearize and decouple the governing equations.

#### 3. SYSTEM BILINEARIZATION

In this section, the Carleman bilinearization technique (Ghasemi et al., 2014) is applied to the partially linearized and decoupled system, (6) and (7), to obtain a bilinear model of the directional drilling tool. Equations (6) and (7) are re-written as follows:

$$\dot{\theta}_{\rm inc} = a U_{\rm inc} \tag{8}$$
$$\dot{\theta}_{\rm azi} = \frac{a}{\sin \theta_{\rm inc}} U_{\rm azi}$$

where  $a = V_{rop} K_{dls}$ . Defining an augmented state vector for the Carleman bilinearization as:

$$x^{\otimes} = [x_1, x_1^{(2)}, x_1^{(3)}, x_1^{(4)}, \dots, x_1^{(N)}, x_2]^T$$
  
where  $x_1 = \theta_{\text{inc}}, x_2 = \theta_{\text{azi}}$  and  $x_1^{(i)} = \frac{d}{dt} \left[ (x_1)^i \right]$ 

 $i\dot{x}_1 x_1^{(a-1)}$  leads to an extended bilinear state space system:  $\dot{x}_1 = a U_{\rm inc}$  (9)

$$\dot{x}_2 = aU_{\text{azi}} \csc(x_1) = aU_{\text{azi}} \sec(\pi/2 - x_1) = aU_{\text{azi}} \sum_{i=1}^{\infty} b_i x_1^{(i)}$$

where  $b_i$  are the coefficients of the Taylor series expansion of sec  $(\pi/2 - x_1)$ . The Taylor series expansion of sec  $(\pi/2 - x)$  is given as:

$$\sec(\pi/2 - x) = 1 + \frac{1}{2}(\pi/2 - x)^2 + \frac{5}{24}(\pi/2 - x)^4 + \frac{61}{720}(\pi/2 - x)^6 + \dots + \frac{(-1)^n E_{2n}(\pi/2 - x)^{2n}}{(2n)!} \quad (10)$$

where  $E_{2n}$  is the Euler number (Rade and Westergren, 1999, pp 81-99).

Expanding (9), yields the following bilinear system:

$$\dot{x}_{1} = aU_{\text{inc}}$$
(11)  
$$\dot{x}_{1}^{(2)} = 2x_{1}\dot{x}_{1} = 2ax_{1}U_{\text{inc}}$$
  
$$\dot{x}_{1}^{(3)} = 3x_{1}^{(2)}\dot{x}_{1} = 3ax_{1}^{(2)}U_{\text{inc}}$$
  
$$\vdots \qquad \vdots$$
  
$$\dot{x}_{1}^{(N)} = Nx_{1}^{(N-1)}\dot{x}_{1} = Nax_{1}^{(N-1)}U_{\text{inc}}$$
  
$$\dot{x}_{2} = aU_{\text{azi}}\sum_{i=1}^{N}b_{i}x_{1}^{(i)}$$

which is in the form of (1), where  $A = [], u = [U_{\text{inc}}, U_{\text{azi}}]^T$ and  $x = x^{\otimes}$ .

## 4. CONTROL DESIGN

With the extension of the work of Inyang et al. (2016), the proposed modified SP-BPI controller design, shown in Fig. 3, is a combination of a BPI controller and a modified SP presented in Normey-Rico et al. (1997). The BPI controller is a combination of a bilinear compensator and a standard linear PI controller. The modified SP is incorporated to account for disturbances and long time delay on the feedback measurements.

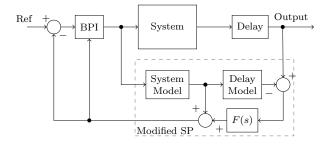


Fig. 3. Modified SP-BPI control scheme

#### 4.1 PI Controller

The PI control for the inclination and azimuth control channels are as follows, respectively:

$$U_{\rm inc} = k_{\rm pi}e_{\rm inc} + k_{\rm ii}\int_0^t e_{\rm inc}dt \tag{12}$$

$$\widetilde{U}_{azi} = k_{pa}e_{azi} + k_{ia}\int_0^t e_{azi}dt$$
(13)

where  $U_{azi}$  is the control input to the bilinear compensator;  $e_{inc} = r_{inc} - \theta_{inc}$  and  $e_{azi} = r_{azi} - \theta_{azi}$ ;  $r_{azi}$  and  $r_{inc}$  are the nominal operating points for azimuth and inclination, respectively.

The gains for the PI controllers in the azimuth and inclination feedback loops can be expressed as follows (Panchal et al., 2010):

$$k_{\rm ii} = \frac{\omega_{\rm i}^2}{a}, \qquad \qquad k_{\rm pi} = \frac{\sqrt{2\omega_{\rm i}}}{a} \qquad (14)$$

$$k_{\rm ia} = \frac{\omega_{\rm a}^2}{a \csc \theta_{\rm inc}}, \qquad \qquad k_{\rm pa} = \frac{\sqrt{2}\omega_{\rm a}}{a \csc \theta_{\rm inc}} \qquad (15)$$

which are dependent on  $a = V_{\rm rop} K_{\rm dls}$  and the chosen natural frequencies of the azimuth and inclination feedback loops dynamics,  $\omega_{\rm a}$  and  $\omega_{\rm i}$ , respectively.

#### 4.2 Bilinear Compensator

The azimuth feedback loop can be expressed based on (9) as follows:

$$\dot{\theta}_{\mathrm{azi}} = a \sum_{i=1}^{\infty} b_i x_1^{(i)} (k_{\mathrm{pa}} e_{\mathrm{azi}} + k_{\mathrm{ia}} \int_0^t e_{\mathrm{azi}} dt) \qquad (16)$$

Based on (10) and (11), (16) can further be expressed as:

$$\dot{\theta}_{azi} = a(1 + \frac{1}{2}\beta^2 + \frac{5}{24}\beta^4 + \dots)(k_{pa}e_{azi} + k_{ia}\int_0^t e_{azi}dt) \quad (17)$$

where  $\beta = \pi/2 - \theta_{inc}$ . To account for the nonlinearity in (17), a bilinear compensator is proposed for the azimuth feedback loop and is given as:

$$\frac{U_{\rm azi}}{\tilde{U}_{\rm azi}} = \frac{1}{1 + \frac{1}{2}\beta^2 + \frac{5}{24}\beta^4}$$
(18)

The bilinear compensator, in combination with PI, facilitates the ensuing controller (BPI) to sustain a required degree of control throughout a broader scope of operation about the tuning point compared with that obtained by the PI controller.

#### 4.3 Modified Smith Predictor

The modified SP is designed based on the work of Normey-Rico et al. (1997) as shown in Fig. 3, where the system model, delay model and F(s) are implemented. F(s)denotes a stable low-pass filter with unitary static gain (F(0) = 1). In the work of Normey-Rico et al. (1997), the major drawback of SP proposed by Smith (1959) is the poor performance as a result of dead-time uncertainties. These dead-time uncertainties are often prevalent in the process industry (including oil and gas industry), hence, the improvement of the robustness of the SP scheme is carried out by incorporating F(s), such that it acts on the difference between the output of the tool and its prediction, and

$$F(s) = \frac{1}{T_f s + 1} \tag{19}$$

where  $T_f$  is a tuning parameter of F(s).  $T_f$  is tuned with the consideration of the trade-off between disturbance rejection and robustness. As the value of  $T_f$  increases, good robustness characteristics are obtained. Conversely, a poorer disturbance rejection characteristics is obtained as the value of  $T_f$  increases (Normey-Rico et al., 1997; Albertos et al., 2015). However, in the absence of disturbances, the closed-loop system nominal performance remains unmodified by the incorporation of F(s). Also, F(s) has no effect on the closed-loop when the system and the system model are equal (Normey-Rico et al., 1997).

The robustness and stability of the modified SP with a linear controller (PI controller) is presented in Normey-Rico et al. (1997). Interestingly, the modified SP works effectively with a bilinear controller (BPI controller), and its effectiveness, robustness, and stability are shown in the simulation results in the subsequent section.

## 5. SIMULATION RESULTS

To demonstrate the effectiveness, robustness and stability of the proposed controller, simulations of the proposed modified SP-BPI controller with the dynamics of (2) and (3) and feedback delay are performed based on the modified SP-BPI control scheme shown in Fig. 3. For comparison purposes, the simulation responses of the BPI controller are also provided based on the work of Inyang et al. (2016) with delay implemented. The design parameters and operating point values used for the simulations are summarized in Table 1.

Referring to Fig. 3, the system and system model are implemented based on (2) and (3), while delay and delay model are implemented as  $e^{-\tau_d s}$  and  $e^{-\tau_m s}$ , respectively; where  $\tau_d$  and  $\tau_m$  are denoted as time delay and modeled time delay, respectively.

The  $\tau_d$  is dependent on  $V_{\rm rop}$  and the distance of the on tool attitude sensing unit from the tool,  $d_t$  and it is given by

$$\tau_d = \frac{d_t}{V_{\rm rop}} \tag{20}$$

Furthermore, to show the robustness and disturbance rejection of the proposed modified SP-BPI controller,  $\tau_m$  is chosen such that it is not equal to  $\tau_d$  (see Table 1); while  $V_{\rm dr}$  and  $V_{\rm tr}$  are ignored for the system model.

Table 1. Design Parameters and Operating Point Values

Parameter	Value
$\theta_{\rm inc}, \theta_{\rm azi}$	$\pi/6 \text{ rad } (30^\circ), \pi/2 \text{ rad } (90^\circ)$
$K_{\rm dls}$	$8^{\circ}/100$ ft $(4.5809 \times 10^{-3} \text{ rad/m})$
$V_{\rm rop}$	200 ft/hr (1.0158 m/min)
$r_{\rm inc}, r_{\rm azi}$	$\pi/6 + 0.015$ rad, $\pi/2 + 0.015$ rad
$\omega_{\mathrm{i}}$	0.0121  rad/min
$\omega_{\mathrm{a}}$	0.0151  rad/min
$T_{f}$	7.5 min
$d_t$	9.998 ft (3.0474 m)
$ au_d$	3 min
$ au_m$	0.1 min
$V_{ m dr}$	$0.0154^{\circ}/100 \text{ ft} (8.8094 \times 10^{-6} \text{ rad/m})$
$V_{\rm tr}$	$0.0077^{\circ}/100$ ft $(4.4048 \times 10^{-4} \text{ rad/m})$

The inclination and azimuth responses to step changes, from  $\pi/6$  rad to  $\pi/6+0.015$  rad and  $\pi/2$  rad to  $\pi/2+0.015$ rad, respectively, of the BPI controller are shown in Fig. 4. The attitude response of the BPI exhibits oscillatory characteristics. Hence, the inclination and azimuth responses of the directional drilling tool does not converge to the desired angles of  $\pi/6 + 0.015$  rad and  $\pi/2 + 0.015$  rad, respectively, because the BPI controller is unable to handle the adverse effects of disturbances and time delay on the feedback measurements.

The inclination and azimuth responses to step changes, from  $\pi/6$  rad to  $\pi/6+0.015$  rad and  $\pi/2$  rad to  $\pi/2+0.015$ rad, respectively, of the modified SP-BPI controller are shown in Fig. 5. The azimuth and inclination responses of the directional drilling tool converges to the desired angles of  $\pi/2+0.015$  rad and  $\pi/6+0.015$  rad, respectively. Hence, the proposed modified SP-BPI controller reduces the adverse effects of disturbances and time delay on the feedback measurements with respect to stability and performance, as compared with the BPI controller.

The inclination and azimuth errors for the modified SP-BPI controller are shown in Fig. 6. The modified SP-BPI controller is able to converge the azimuth and inclination errors directly to zero within 1 min.

#### 5.1 High-Fidelity Model Simulation

The high-fidelity model simulation of the drilling cycle, shown in Fig. 7 was implemented by Inyang et al. (2016). The magnetometers and accelerometers used to derive the inclination and azimuth measurements are explicitly included along with all the system delays and lags previously discussed and detailed in Panchal et al. (2010). Based on these results, a Simulink transient simulation was created to test the directional drilling tool using the proposed attitude control algorithm to hold azimuth and inclination at the desired angles without interaction between the inclination and azimuth. The high-fidelity model architecture is shown in Fig. 8.

To engineer a variable curvature,  $U_{\rm dls}$ , the toolface actuation,  $U_{\rm tf}$  (control input) is discretized into duty cycles known as "drilling cycles" of period,  $t_{\rm cycle}$ . To approximate

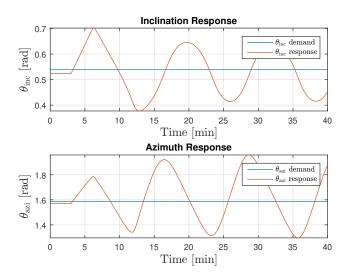


Fig. 4. BPI attitude response

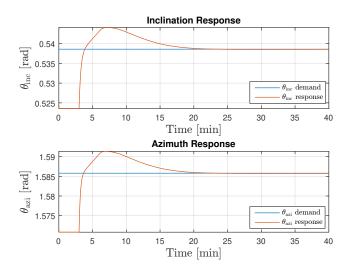


Fig. 5. Modified SP-BPI attitude response

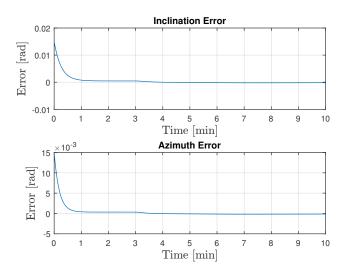


Fig. 6. Attitude error for modified SP-BPI controller

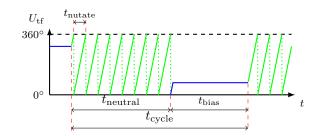


Fig. 7. Drilling cycle definition

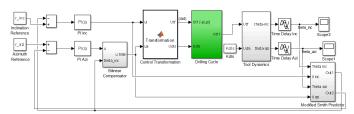


Fig. 8. Simulink diagram of high-fidelity model simulation scheme

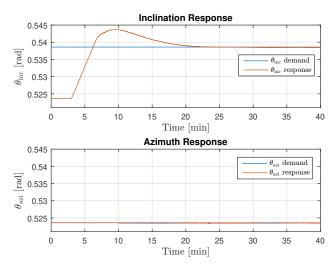


Fig. 9. Modified SP-BPI attitude response with azimuth response set at its initial angle of  $\pi/6$  rad

the  $U_{\rm dls}$  control input,  $t_{\rm cycle}$  is proportioned into the bias,  $t_{\rm bias}$  and neutral,  $t_{\rm neutral}$  phases as shown in Fig. 7. In the  $t_{\rm neutral}$  phase, the  $U_{\rm tf}$  is cycled at a constant rate of period,  $t_{\rm nutate}$ ; while in the  $t_{\rm bias}$  phase, the  $U_{\rm tf}$  is a servocontrolled constant which approximates the  $U_{\rm dls}$  control input as  $U_{\rm dls} = (t_{\rm bias}/t_{\rm cycle})K_{\rm dls}$ .

Based on the simulation of the high-fidelity model, the inclination response to step change from  $\pi/6$  rad to  $\pi/6 + 0.015$  rad and the azimuth response set at its initial angle of  $\pi/6$  rad are shown in Fig. 9; while the azimuth response to step change from  $\pi/6$  rad to  $\pi/6 + 0.015$  rad and the inclination response set at its initial angle of  $\pi/6$  rad are shown in Fig. 10. The proposed attitude control algorithm still holds the inclination and azimuth of the directional drilling tool at the desired angles of  $\pi/6 + 0.015$  rad and  $\pi/6 + 0.015$  rad and  $\pi/6 + 0.015$  rad, respectively, without interaction between the inclination and azimuth.

The inclination and azimuth errors based on the simulation of the high-fidelity model are shown in Fig. 11.

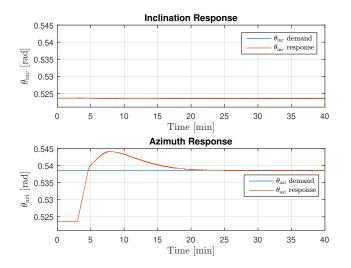


Fig. 10. Modified SP-BPI attitude response with inclination response set at its initial angle of  $\pi/6$  rad

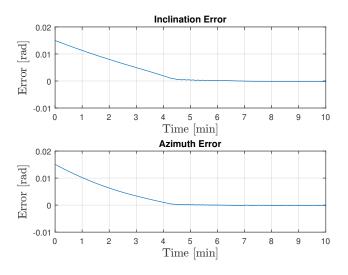


Fig. 11. Attitude error for modified SP-BPI controller with  $\rm HFM$ 

The proposed attitude control algorithm still converges the inclination and azimuth errors directly to zero within 4.5 min.

## 6. CONCLUSIONS

This paper presents a bilinear model of the directional drilling tool. It proposes a modified SP-BPI controller for directional drilling attitude control. The potential beneficial aspects gained by implementing the proposed modified SP-BPI controller include the significant reduction of the adverse effects of disturbances and time delay on the feedback measurements with respect to stability and performance of the directional drilling tool. In terms of robustness and disturbance rejection, the proposed modified SP-BPI controller is able to handle time delay of 3 min, with up to 96.67% uncertainty of the predicted time delay, and with drop-rate disturbance and turn-rate bias disturbance, in the attitude control of the directional drilling tool. Theoretical stability proof of the proposed modified SP-BPI controller remains an open problem.

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