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Optimal evacuation solutions for large-scale scenarios*

Daniel Dressler, Gunnar Flötteröd, Gregor Lämmel, Kai Nagel, Martin Skutella

1 Introduction

Evacuation, the process of moving people out of potentially dangerous areas, is a key response to many threats. *Planning* such an evacuation is therefore important, especially in large-scale emergencies, where routing becomes non-trivial. This paper deals with the optimization and simulation of the evacuation process. We draw our data from the study of the city of Padang in Indonesia, with its high threat of tsunami waves.

Ford and Fulkerson (1962) introduced *flows over time* (dynamic flows), which have become integral to evacuation planning. In the standard flow over time model, each arc a has a constant transit time $\tau(a)$. Flow entering the tail of the arc at time θ leaves the head of the arc at time $\theta + \tau(a)$. Capacity constraints limit the flow rate on each arc. Hamacher and Tjandra (2002) survey objectives in evacuation planning and discuss flow over time models in detail. Note that this model can be solved both deterministically (with every flow unit being deterministically allocated to a unique path with unique characteristics) and stochastically (with every flow unit selecting a path from a choice set with a certain probability, and the path characteristics being stochastic as well). While the former approach is more convenient from a mathe-

Daniel Dressler, Martin Skutella
Institute for Mathematics, TU Berlin, Str. des 17. Juni 136, 10623 Berlin, Germany, e-mail: {dressler, skutella}@math.tu-berlin.de

Gunnar Flötteröd
Transport and Mobility Laboratory, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland, e-mail: gunnar.floetteroed@epfl.ch

Gregor Lämmel, Kai Nagel
Institute for Land and Sea Transport Systems, TU Berlin, Salzufer 17-19, 10587 Berlin, Germany, e-mail: {laemmel, nagel}@vsp.tu-berlin.de

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matical programming perspective, the latter is of greater realism in that it explicitly accounts for uncertainty in the modeling. We apply both models in that we compute deterministic flows which we then evaluate in a stochastic simulation environment.

Problem Definition. We consider the EVACUATION PROBLEM (EP), both in a deterministic and a stochastic setting. An instance consists of a directed graph $G = (V,A)$, flow rate capacities $u : A \to \mathbb{R}_{\geq 0}$, travel times $\tau : A \to \mathbb{R}_{\geq 0}$, and optionally time windows $W : A \to \{[i, j) : i < j\}$, restricting when an arc is available. (Outside this time window it has zero capacity.) Demands $d : V \to \mathbb{R}$ determine which vertices are sources and which are sinks. We assume that each sink is essentially uncapacitated. The desired output is a feasible flow over time satisfying the demands of the sources with *minimum total travel time*. Note that we only consider flows over time without storage of flow at vertices. The stochastic model differs from its deterministic counterpart in that flow units are assigned probabilistically to routes, but no randomness in the network parameters (free flow travel times, capacities) is accounted for.

Literature Overview. The EP can be solved either based on a deterministic or a stochastic model. We first consider rigorous mathematical programming approaches that assume a deterministic model and then consider simulation based approaches that also cope with stochasticity, although merely in a heuristic manner.

Evacuation Planning with Deterministic Models. A justification for the objective of the EP is given by Jarvis and Ratliff (1982), because minimizing the total travel time also maximizes the amount of flow that has already reached the sinks *at each time step* (if we ignore time windows). This property also defines an Earliest Arrival Flow (EAF). Most practical approaches to flow over time problems rely on time-expanded networks, with the notion of time built explicitly into the graph, at the price of a pseudo-polynomial size. A time-expanded network uses one copy of the original network per time step, with a copy of an arc a of length $\tau(a)$ pointing from time layer $\theta \in \mathbb{Z}$ to time layer $\theta + \tau(a)$. Thus, static flows on the time-expanded network correspond to flows over time and vice versa. Then, a standard MINIMUM COST FLOW computation on the time-expanded network, with costs equal to the transit times, yields an answer to the EP.

Tjandra (2001, 2003) considers specialized algorithms for the EP and related problems, in particular when the network parameters may change over time. Our combinatorial algorithm uses similar ideas to exploit the repeating structure of the time-expanded network, but works on different assumptions. Furthermore, our large-scale instances require additional techniques to achieve acceptable run times. The QUICKEST TRANSSHIPMENT PROBLEM is a relaxation of the EP, which just asks for the minimum egress time. A highly non-trivial but polynomial time algorithm for this problem is given by Hoppe and Tardos (2000). Fleischer and Skutella (2007) present a polynomial time approximation scheme using a coarser time-expanded network. Their approach also yields approximate EAFs. These results are certainly useful, but the required precision is higher than we usually require.

Evacuation Planning with Stochastic Models. The EP can also be considered from the perspective of dynamic traffic assignment (DTA), see, e. g., Peeta and Ziliaskopoulos (2001). In order to account for stochasticity, we concentrate on simulation-based approaches that generate realizations of possible network states. We consider two possible directions to look into: The simulation of a Nash equilibrium (NE) and an approximate simulation-based system optimal (SO) assignment.

An approximate NE can be simulated by alternately evaluating a route choice model and a network loading model until mutual consistency is attained. This procedure has a long history in simulation-based DTA, see Nagel and Flötteröd (2009). Essentially, each simulated evacuee iteratively optimizes its personal evacuation plan. After each iteration, every evacuee calculates the cost of the most recently executed plan. Based on this cost, the evacuee revises the most recently executed plan. Some evacuees generate new, "best-response" plans using a time-dependent router. If the cost function uses the travel time of the last iteration, the system converges towards an approximate NE, where the main source of imprecision is due to the stochastic environment in which the best responses are computed.

Peeta and Mahmassani (1995) show that an identical simulation logic can be applied to the simulation of a SO assignment if the link travel times are replaced by the marginal link travel times, which essentially represent the increase in total travel time generated by a single additional agent entering a link. Lämmel and Flötteröd (2009) present a both computationally and conceptually very simple approximation of these marginal travel times.

Our contribution. We investigate evacuation plans with the following objectives: a Nash Equilibrium (NE), in which no evacuee benefits from choosing a different but the assigned route and a System Optimum (SO), in which the total travel time is minimized. The approach taken here is to solve the EP combinatorially using a customized MINIMUM COST FLOW algorithm. We then proceed to refine the solution either towards an NE or an SO in the terms of the simulation. This tests and accounts for the robustness of the deterministically computed plans in a stochastic environment.

2 The Instance and its Solution

We demonstrate our results on an instance that models a tsunami threatening the city of Padang in Indonesia. In case of a tsunami, the population should evacuate the shore line and flee to higher areas. We have a detailed network describing downtown Padang and trim this to the area below the height threshold. The resulting network has 4,444 nodes and 12,436 arcs, covering roughly 32 square kilometers.

It is assumed that all inhabitants leave on foot, with an average walking speed of 1.66 m/s. Derived from this, each arc has a flow rate capacity proportional to the smallest width of the corresponding street. The simulation uses a storage capacity that is computed from the area of the street. The scenario assumes a tsunami warning at 3:00 am, so that the 254,970 inhabitants all start at their respective homes. The first flooding starts at 3:20 am and water covers almost a third of the modelled area by 3:40 am. The evacuation plan completes around 3:50 am. The time windows remove arcs from the network when they are first covered by water.

Optimal Combinatorial Solution. Our algorithm for the EP has the main benefits that it can handle fine discretizations and almost arbitrarily large time horizons, and does not require an estimate of the egress time. The memory consumption is modest compared to standard algorithms. We need to store the underlying instance for the time-expanded network only once. We can also store the flow over time, i.e., the function for each arc, in intervals with constant values. Since by definition the evacuated areas of the network are never used again, there will at least be long intervals of zeros. Similarly, the flow on bottleneck arcs will usually be at full capacity. But this not only reduces memory consumption, the intervals also enable a different algorithmic approach.

The associated MINIMUM COST FLOW PROBLEM in the time-expanded network can be solved by the SUCCESSIVE SHORTEST PATH algorithm, which starts with a zero flow and iteratively adds flow on shortest paths to it. In our case, the shortest path computation only needs to determine what the earliest reachable copy of a sink is. For this, we need to propagate reachability in the network, which we also store in intervals: If a vertex v is reachable during some interval $[\theta_1, \theta_2)$, and there is some arc $a = (v,w)$, then w will be reachable at times $[\theta_1 + \tau(a), \theta_2 + \tau(a))$, assuming that the copies of arc a are available for the entire interval. If not, only certain subintervals can be propagated, which can be computed efficiently from the flow intervals.

We use a breadth-first search starting with reachable intervals $[0,\infty)$ from all (not yet empty) sources. If there are multiple sinks, we can also find multiple shortest paths, and we can hope to add flow on all of them. We also tried alternative search styles (e.g., starting at the sinks, or a simultaneous search from the sources and the sinks), but we found the forward search from the sources to deliver reliably good results. Dynamically joining consecutive intervals in the search also helps. In addition, we may guess shortest paths by repeating successful paths that arrived one time step earlier. Indeed, in the case of a single source, an optimum solution can be found by repeating a certain set of paths as long as possible. The addition of repeated paths to our algorithm closely mirrors this behavior. Further, we use heuristics for choosing which paths to add if they are not all compatible.

Simulation-based Approach. The simulation framework is based on the MATSim DTA microsimulation (see, e. g., Lämmel et al (2010) for the evacuation version). It essentially implements the same network model as assumed in the mathematical programming approach, only that the integer flow units are now replaced by microscopically simulated agents with potentially complex internal behavioral rules.

When feeding the solution obtained with the combinatorial approach into MATSim, the solution quality deteriorates because of the stochastic system environment. We then refine these plans within the simulation-based environment. In particular, the EAF solution dictates an exit plan for each agent and an order in which they should leave the sources. These plans become the starting solution for MATSim, but the order is deliberately removed. We believe this to be more realistic. In effect, agents following their plans might suddenly meet in a bottleneck, which would not have occurred with the original departure times.

As mentioned before, we have two possible directions to look into: The simulation of a Nash equilibrium (NE) and an approximate simulation-based system optimal (SO) assignment. For the NE, we deploy the iterative technique described

Instance				CPLEX 12.1		our algorithm	
	step Δ	T^*	T	Time	Mem	Time	Mem
Padang	10s	313	350	0:08:30	0.8 GB	0:19:05	1.1 GB
Padang	3s	1051	1100	0:47:56	2.2 GB	1:03:13	1.1 GB
Padang	1s	3221	3225	4:32:48*	6.8 GB	2:22:09	1.2 GB
Building	1s	809	850	0:23:36	3.2 GB	0:00:20	0.4 GB

Table 1 Reported times are user times in h:mm:ss on a PC with a 2x2.13 GHz Core2Duo CPU, running openSUSE 11.1 (x86_64). (*Estimated from 3:29:51 on a 2x.3.0 GHz Core2Duo CPU.)

by Nagel and Flötteröd (2009), whereas for the SO, we resort to the approximate solution procedure of Lämmel and Flötteröd (2009).

3 Results and Conclusion

We implemented our combinatorial algorithm for the EP in Sun Java 1.6 and use the network simplex implementation of ILOG CPLEX 12.1 for comparison.

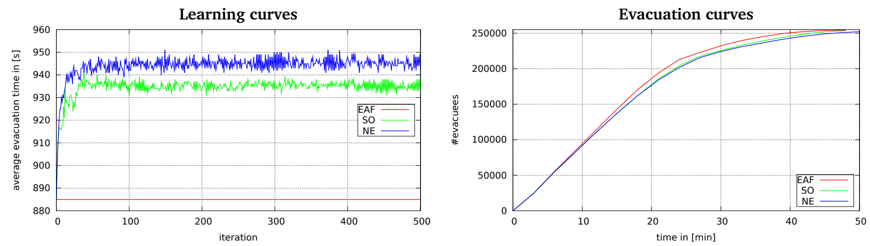
We present results for the instance of Padang, as well as another instance modeling a 20-story building in high detail. It consists of 5,966 nodes and 21,937 edges, but only 369 people are inside. Our findings are summarized in Table 1. Note that in all cases we had to provide CPLEX with a time-expanded network for a specific time horizon T, which we chose near the optimum T^*.

The general conclusion is that less agents and a greater time horizon favor our algorithm. The former effect is seen very clearly in the building instance, the latter in the Padang series. The following figures for the Padang instance with 1 second time steps might explain the good results: Despite the time horizon of 3221, in the worst case the reachability is stored in $85|V|$ intervals, the flow in $73|A|$ intervals. Finally, the many possible sinks (exits to higher ground) let the algorithm find 100 or more suitable paths at once, greatly decreasing the number of iterations required.

We then used the routes computed combinatorially for Padang ($\Delta = 1s$) as initial routes in MATSim. The computed average travel time was $844s$, while the same routes in MATSim result in $884s$. This difference of 5% seems quite acceptable, and may stem from the rounded capacities and discarded starting times, leading to additional disturbances.

The iterative process of MATSim then refines this solution, reaching an almost stable state with avg. travel times $945s$ (NE) and $935s$ (SO), respectively. The decrease in the solution quality and the fluctuation is due to the continuous stochasticity in the simulation environment that occurs as soon as route-replanning is enabled (hence it does not take effect in a plain network loading of the optimal plans). The learning curves for NE and SO are shown in Fig. 1(a). For comparison, the results for the start solution in MATsim are also plotted over the 500 learning iterations. Nonetheless, this procedure yields a NE solution that is relatively close to the optimal solution as illustrated in Fig. 1(b).

Concluding, we compute solutions that are optimal in the analytical model, which differs from the model of the learning framework in some details. We observe that the EP solution changes only slightly when replayed without the departure times



(a) Avg. evacuation time over iteration number (b) Evacuation curves

Fig. 1 Results of the MATSim simulation for EAF, NE and SO approach.

of the individual flow units in the stochastic simulation environment. This leads us to believe that it is indeed a good solution. The NE and SO solutions of the simulation framework are also of good quality, which is evident from the optimum combinatorial solution. Let us remark that NE and SO solutions of similar quality can also be obtained by the simulation-framework without optimized initial routes. This simpler methodology is also robust with respect to almost arbitrary sources of randomness.

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