

# Continuous–Discrete Time-Observer Design for State and Disturbance Estimation of Electro-Hydraulic Actuator Systems

Sofiane Ahmed Ali, Arnaud Christen, Steven Begg, and Nicolas Langlois

**Abstract**—In this paper, a continuous–discrete time observer which simultaneously estimates the unmeasurable states and the uncertainties for the electro-hydraulic actuator (EHA) system is presented. The main feature of the proposed observer is the use of an intersample output predictor which allows the users to increase the frequency acquisition of the piston position sensor without affecting the convergence performance. The stability analysis of the proposed observer is proved using Lyapunov function adapted to hybrid systems. To show the efficiency of our proposed observer, numerical simulations and experimental validation involving a control application, which combines the designed observer and a PI controller for the purpose of piston position tracking problem, are presented.

**Index Terms**—Continuous–discrete time observers, disturbance observer (DOB), electro-hydraulic actuator (EHA), intersample output predictor, sampled data measurements.

## I. INTRODUCTION

DU TO a high power to weight ratio and their ability to generate high torques/forces outputs, electro-hydraulic actuator (EHA) systems are widely used in several industrial applications [1]–[5]. Despite this advantage, the EHA systems suffer from some drawbacks due principally to their structure. Indeed, the EHA systems are subject to various uncertainties such as model parametric variations [6], [7], highly nonlinear dynamic behavior [8], potential faults such as internal leakage [9], and hard damage affecting their functioning. In the last years, the increasing demand of high precision control for EHA systems renders the development of advance controls' methods necessary to meet the actual requirements in terms of tracking performance.

Despite their actual dominance, the traditional proportional integral derivative (PID) controllers are not robust enough to

Manuscript received March 31, 2015; revised August 5, 2015; accepted January 24, 2016. This work was supported by the Combustion Engine for Range-Extended Electric Vehicle (CEREVE) Project which is funded by the European Union's INTERREG IVA France-Manche-England Programme.

S. Ahmed Ali, A. Christen, and N. Langlois are with the Department of Electrical Engineering, IRSEEM/École Supérieure d'Ingénieurs en Génie Électrique (ESIGELEC), 76801 Rouen, France (e-mail: sofiane.ahmedali@esigelec.fr; Arnaud.Christen@esigelec.fr; nicolas.langlois@esigelec.fr).

S. Begg is with the Department of Electrical Engineering, University of Brighton, Brighton, BN2 4GJ, U.K. (e-mail: S.M.Begg@brighton.ac.uk).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TIE.2016.2531022

counteract the effect of the uncertainties affecting the EHA systems. Therefore, the focus of the researchers has been shifted toward developing nonlinear closed-loop control methods in order to improve the tracking performance for the EHA systems. In the past decades, several nonlinear control techniques have been developed in the literature such as feedback linearization [7], [10] and sliding mode control [11]–[14]. In [6], a novel integration of adaptive control and integral robust feedback was proposed for hydraulic systems with considering all possible modeling uncertainties, and an excellent tracking performance was achieved, which is the first solution for theoretically asymptotic stability with unmatched disturbances for hydraulic systems; others nonlinear controllers such as robust/adaptive robust controllers [15]–[20], [37], [38] and backstepping control [21]–[24] were also proposed. These methods have already proved their efficiency to improve the tracking performance of the EHA systems facing modeling uncertainties, parametric variations, and external disturbances.

However, all aforementioned techniques are full-state feedback ones, i.e., the designed controllers assume that all states of the EHA systems are available for measurements. From practical of point of view, this assumption may not be realistic for some hydraulic systems. Indeed, for many hydraulics applications, only the position signal of the actuator is measured via sensor. The other states like velocity and hydraulic pressure are not measured because of the cost-reduction and the space limitation; therefore, states and disturbances observers have recently received in the literature more and more attention.

Several states and disturbances observers were developed by some researchers in the past decade. The idea behind developing these observers is to use the states and the disturbances estimation provided by these observers in order to synthesized an output-feedback controllers which compensate the internal and the external disturbances affecting the EHA systems. At this stage, we can distinguish between two main approaches in the literature. The first approach consists in developing only a state estimator (i.e., an observer) which estimates the unmeasurable state of the EHA systems. These observers ignore both the internal disturbances like parametric variations, modeling uncertainties, and the external disturbances such as the load and the friction torque affecting the hydraulic application. Those types of observers can be found in the work developed by the authors in [25]–[28]. The second approach developed by the authors in [29]–[31] assumes that the states of the EHA systems are measurable and synthesize a disturbance observer (DOB)

83 which estimates the mechanical and the hydraulic disturbances  
 84 affecting the system. These estimations are incorporated then in  
 85 a nonlinear closed-loop controller which compensates the effect  
 86 of the disturbance and improves the tracking performance of the  
 87 desired position for the EHA systems.

88 Recently, the authors in [32] proposed a novel framework  
 89 for the purpose of simultaneous estimation of the unmeasurable  
 90 states and the unmodeled disturbances, and then resulting in an  
 91 excellent output feedback nonlinear robust backstepping con-  
 92 troller for hydraulic systems, by developing an extended state  
 93 observer (ESO) [33] and robust backstepping design. In this  
 94 work, the authors consider that the main uncertainties affecting  
 95 the EHA systems come from the hydraulic part. Therefore, they  
 96 synthesized an observer based on the well-known techniques  
 97 of ESOs [33] which estimates the unmeasurable state and  
 98 the hydraulic disturbances of the EHA systems. The proposed  
 99 observer is also robust facing the mechanical disturbances gen-  
 100 erated by the load driven by the considered EHA system in this  
 101 paper.

102 In the case of hydraulic applications, the main drawback of  
 103 the designed observers [25]–[32] is that they assume that the  
 104 measured variable is continuous. In practical situations, this  
 105 measured variable which is given by the position sensor is sam-  
 106 pled. In other words, the piston positions are available for the  
 107 observer at only sampling times  $t_k$  fixed by the sampling rate  
 108 (i.e., the frequency acquisition) of the sensor. This frequency  
 109 can affect the convergence of the proposed when it comes to the  
 110 matter of implementation of the proposed observer on digital  
 111 signal processors (DSPs).

112 Following the design in [32], the authors in [34] designed  
 113 a sampled data observer which deals with the problem of  
 114 discrete time-measurements for the EHA system. The pro-  
 115 posed observer retains the same benefits which characterize the  
 116 observer proposed in [32] in terms of simultaneous estimation  
 117 of the unmeasurable states and the internal disturbances affect-  
 118 ing the EHA system. The proposed observer involves in its  
 119 structure an intersampled output predictor [35] which ensures  
 120 continuous time estimation of the states and the exponential  
 121 convergence of the observation errors. Moreover, the sampling  
 122 period of the data acquisition of the observer can be augmented  
 123 independently from the frequency acquisition of the sensor.  
 124 However, the designed observer in [34] suffers from two major  
 125 drawback. The first one concerns the Lyapunov function pro-  
 126 vided to prove the exponential convergence of the proposed  
 127 observer. Indeed, the authors in [34] demonstrated the expo-  
 128 nential convergence of the observer only locally between two  
 129 sampling periods. In addition, the performance of the proposed  
 130 observer were validated only in simulations and no experi-  
 131 mental validation of the observer is provided. Comparing to  
 132 the work of the author in [34], two main contributions were  
 133 provided. The first contribution consists in designing a novel  
 134 Lyapunov function based on small gain arguments which guar-  
 135 anty a global exponential convergence of the proposed observer.  
 136 In addition, the maximum sampling period  $T_{\max}$  derived from  
 137 this function is less restrictive comparing to the one derived in  
 138 [34]. The second one is that experimental results performed on  
 139 the experimental test rig of the Brighton University is provided  
 140

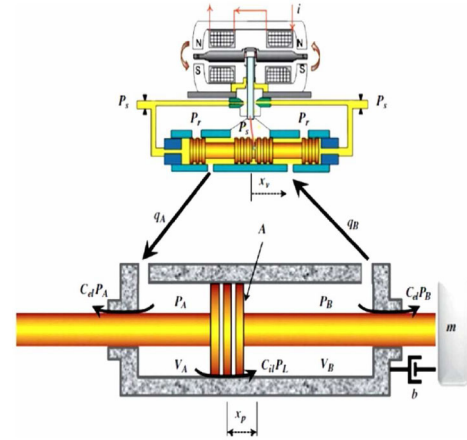


Fig. 1. Schematic of the EHA.

F1:1

for this observer. This is in our acknowledged the first time that  
 such observers were designed and tested experimentally for the  
 EHA systems.

This paper is organized as follows. The EHA modeling  
 issues and the problem formulation are presented in Section II.  
 Section III presents the continuous–discrete time observer for  
 the EHA system. Numerical simulations and experimental val-  
 idation showing the effectiveness of our proposed observer are  
 presented in Section IV. Section V contains the conclusion and  
 the future works.

## II. EHA MODELING

151

The schematic of the EHA studied in this paper is depicted in  
 Fig. 1 [26], [29]. The EHA system contains usually three parts,  
 namely the electrical, the mechanical, and the hydraulic part.  
 These parts represent an interconnected subsystem in such a  
 way that the dynamic of each subsystem influences the dynam-  
 ics of the others. The electrical part of the EHA system is a  
 servo-valve (top of Fig. 1) which controls the fluid dynamics  
 inside the chambers. The spool valve is driven by the electri-  
 cal input current  $u$  of a torque motor. The displacement of the  
 spool valve  $x_v$  together with the load pressure  $P_L$  controls the  
 fluid dynamic inside two chambers A and B which constitute  
 the hydraulic part of the EHA system. The mechanical part of  
 the EHA system is a cylindrical piston which is modeled as  
 a classical mass-spring system. The position of the cylindrical  
 piston  $x_p$  obeys to the fundamental principle of dynamics.

### A. State-Space Representation of the EHA

167

Considering the following states variable:  $x =$   
 $[x_1, x_2, x_3]^T = [x_p, \dot{x}_p, P_L]^T$ , the state-space representa-  
 tion of the EHA system can be written under the following  
 form [26], [29], [31]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{A_p}{m}x_3 \\ \dot{x}_3 = -\alpha x_2 - \beta x_3 + \gamma \sqrt{P_s} - \text{sign}(u)x_3 u \end{cases} \quad (1)$$

171

where  $x_p$  is the piston position (m).  $\dot{x}_p$  (m/s) is the piston velocity and  $P_L$  (Pa) is the pressure load inside the chambers of the hydraulic part.  $k$  is the load spring constant (N/m),  $b$  is the viscous damping coefficient [N/(m/s)], and  $A_p$  is the cylinder bore (m<sup>2</sup>).  $P_s$  is the supply pressure (Pa).  $\alpha, \beta, \gamma$  are the hydraulic coefficients of the EHA model. These coefficients depend on the flow characteristics of the EHA system. For more details about the expression of the hydraulic coefficients  $\alpha, \beta, \gamma$  and the modeling issues of the EHA system, the reader is referred to the work of the authors [26], [29] and their corresponding literature.

### B. Modeling Uncertainties and Time-Varying Disturbances Affecting the EHA System

In [29] and [31], the authors distinguished between two types of disturbances  $d_1$  and  $d_2$  which can affect the EHA system. The first one  $d_1$  is the mechanical disturbance which is the result of lumping together the modeling parametric uncertainties, the load charge  $F_{\text{Load}}$ , and the friction force  $F_{\text{friction}}$  acting on the mechanical part of the EHA system. As reported by the authors in [32], the second term  $d_2$  does not hold the same significance as  $d_1$ . Indeed,  $d_2$  represents the parametric deviation over the hydraulic coefficients  $\alpha, \beta, \gamma$  and potential leakage affecting the hydraulic device of the EHA system. These parameters are also sensitive to temperature inside the EHA system. Taking into account these issues, the disturbed EHA model can be written as follows [29]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{A_p}{m}x_3 - \frac{d_1}{m} \\ \dot{x}_3 = -\alpha x_2 - \beta x_3 + \gamma \sqrt{P_s - \text{sign}(u)x_3 u} + d_2 \end{cases} \quad (2)$$

where  $d_1(t)$  and  $d_2(t)$  are expressed as follows [31]:

$$\begin{aligned} d_1(t) &= -\Delta \frac{k}{m}x_1 - \Delta \frac{b}{m}x_2 - \Delta \frac{A_p}{m}x_3 + F_{\text{Load}} + F_{\text{friction}} \\ d_2(t) &= -\Delta \alpha x_2 - \Delta \beta x_3 + \Delta \gamma \sqrt{P_s - \text{sign}(u)x_3 u}. \end{aligned} \quad (3)$$

The  $\Delta$  symbolizes the considered parametric uncertainties affecting the mechanical and the hydraulic part of the EHA system. System (2) can be expressed under the following compact form:

$$\begin{cases} \dot{x} = Ax + \varphi(x, u) + B_d d \\ y = Cx = x_1 \end{cases} \quad (4)$$

where  $x \in \mathbf{R}^3$  and  $y \in \mathbf{R}$  represent, respectively, the state vector and the measured piston position  $x_1 = x_p$ . The vector  $u \in \mathbf{R}$  describes the set of admissible inputs.  $d(t) \in \mathbf{R}^2$  denotes the vector of the disturbances which affect the EHA.  $B_d$  with dimensions  $3 \times 2$ . The matrices  $A, B_d, C$ , and vector  $\varphi(x, u)$  have the following structure:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{A_p}{m} \\ 0 & 0 & 0 \end{pmatrix}$$

$$B_d = \begin{pmatrix} 0 & 0 \\ \frac{-1}{m} & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = (1 \ 0 \ 0)$$

$$\varphi(x, u) = \begin{pmatrix} 0 \\ -\frac{k}{m}x_1 - \frac{b}{m}x_2 \\ -\alpha x_2 - \beta x_3 + \gamma \sqrt{P_s - \text{sign}(u)x_3 u} \end{pmatrix}.$$

### C. Problem Formulation

For system (4), the piston position is available for measurement only at each sampling times  $t_k$  imposed by the frequency acquisition (the sampling period) of the sensor manufacturer. In this paper, we have to design a robust sampled data observer which simultaneously estimates the unmeasurable states  $x_2, x_3$ , and the hydraulic disturbance term  $d_2$  of system (4). The designed observer must deal with the sampling phenomenon of the measured piston position  $x_p$  and must be robust facing the mechanical disturbance term  $d_1(t)$ . Under these considerations, system (4) is rewritten as follows:

$$\begin{cases} \dot{x} = Ax + \varphi(x, u) + B_d d \\ y(t_k) = Cx(t_k) = x_1(t_k). \end{cases} \quad (5)$$

System (5) combines a continuous dynamic behavior for the states  $x_1, x_2, x_3$  between two sampling times  $[t_k, t_{k+1}]$  and an updated step for the state  $x_1$  which occurs at the sampling times  $t = t_k$ .

## III. CONTINUOUS–DISCRETE TIME-OBSERVER DESIGN FOR THE EHA SYSTEM

In this section, we design a continuous–discrete time observer for the EHA system. Since  $d_2$  is the main disturbance term, we use the well-known technique of the augmented state system in order to estimate it. Following this, we add an extended variable  $x_4 = d_2$  such as  $\dot{x}_4 = h(t)$  to system (5) so that the augmented state system can be written as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \overline{\varphi(\bar{x}, u)} + \delta(t) \\ y = \bar{C}\bar{x} = x_1 \end{cases} \quad (6)$$

where  $\bar{x} = [x_1, x_2, x_3, x_4]$  and

$$\bar{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{A_p}{m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overline{\varphi(\bar{x}, u)} = \begin{pmatrix} 0 \\ -\frac{k}{m}x_1 - \frac{b}{m}x_2 \\ -\alpha x_2 - \beta x_3 + \gamma \sqrt{P_s - \text{sign}(u)x_3 u} \\ 0 \end{pmatrix}$$

$$\delta(t) = \begin{pmatrix} 0 \\ \frac{-d_1}{m} \\ 0 \\ h \end{pmatrix}$$

$$\bar{C} = (1 \ 0 \ 0 \ 0).$$

## 233 A. Observer Design

234 In this paper, our proposed observer will be designed under  
235 the same assumptions taken in [32].

236 *Assumption 1:* The disturbance term  $d_1(t)$  is bounded by  
237 a real unknown constant  $\mu_1$  such that  $(|d_1(t)| < \mu_1)$  and the  
238 function  $h(t)$  is bounded by a real unknown constant  $\mu_2$  such  
239 that  $(|h(t)| < \mu_2)$ .

240 *Remark 1:* This assumption means that the mechanical dis-  
241 turbance and the derivative of the hydraulic disturbances affect-  
242 ing the EHA system are bounded by some unknown constants.  
243 From a practical point of view, the EHA system is a physical  
244 system which is BIBS (bounded input bounded state). So, it is  
245 quite reasonable to consider such assumption.

246 *Assumption 2:* In their practical range of parametric varia-  
247 tions, the functions  $\overline{\varphi_2(\bar{x}, u)} = -\frac{k}{m}x_1 - \frac{b}{m}x_2$  and  $\overline{\varphi_3(\bar{x}, u)} =$   
248  $-\alpha x_2 - \beta x_3 + \gamma\sqrt{P_s - \text{sign}(u)}x_3u$  are locally (inside com-  
249 pact set) Lipschitz with respect to  $(x_1, x_2, x_3)$ , i.e.,  $\exists\beta_0 > 0$ ,  
250 such that

$$|\overline{\varphi(X, u)} - \overline{\varphi(Y, u)}| \leq \beta_0 \|X - Y\|, \quad i = 2, 3. \quad (7)$$

251 *Remark 2:* At this point, we mention that the function  
252  $\overline{\varphi_2(\bar{x}, u)}$  is globally Lipschitz with respect to  $x_2, x_3$ . The function  
253  $\overline{\varphi_3(\bar{x}, u)}$  is differentiable everywhere except at  $u = 0$ ,  
254 however, and as stated by the authors in [32], this function is  
255 continuous and its derivative exists in the left and the right side  
256 of  $u = 0$  and it is finite. Hence, we can find a compact set so  
257 that  $\overline{\varphi_3(\bar{x}, u)}$  is locally Lipschitz.

258 Based on [35], let us consider the following continuous-  
259 discrete time observer:

$$\begin{cases} \dot{\hat{x}} &= \bar{A}\hat{x} + \overline{\varphi(f(\hat{x}), u)} - \theta\Delta_\theta^{-1}K(\bar{C}\hat{x} - w(t)) \\ \dot{w}(t) &= \bar{C}\left(\bar{A}\hat{x} + \overline{\varphi(f(\hat{x}), u)}\right) \quad t \in [t_k, t_{k+1}) \quad k \in \mathbb{N} \\ w(t_k) &= y(t_k) = x_1(t_k). \end{cases} \quad (8)$$

The function  $f$  is a saturation function which is introduced to  
guaranty that the estimated states  $\hat{x}$  remains inside the compact  
set so that the Lipschitz constant  $\beta_0$  always exists. The  $\Delta_\theta$  is a  
diagonal matrix  $4 \times 4$  defined by

$$\Delta_\theta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\theta} & 0 & 0 \\ 0 & 0 & \frac{1}{\theta^2} & 0 \\ 0 & 0 & 0 & \frac{1}{\theta^3} \end{pmatrix}$$

260 and the vector gains  $K \in \mathbb{R}^{4 \times 1}$  are chosen so that the matrix  
261  $(\bar{A} - K\bar{C})$  is Hurwitz. The vector  $\hat{x}$  is the continuous-time  
262 estimate of the system state  $\bar{x}$ . The vector  $w(t)$  represents the  
263 prediction of the output between two sampling times. The pre-  
264 diction  $w(t)$  is updated (reinitialized) at each sampling instant  
265  $t = t_k$ .

## 266 B. Observability Analysis

From the structure of matrices  $\bar{A}, \bar{C}$  in system (6), it can  
be easily checked that the pair  $(\bar{A}, \bar{C})$  is observable. Hence,  
their exists two matrices  $P, Q$  such that the following Lyapunov  
function is satisfied:

$$P(\bar{A} - K\bar{C}) + (\bar{A} - K\bar{C})^T P \leq -\mu \mathbb{I}_n$$

where  $\mu > 0$  is a free-positive constant and  $P$  is a symmetric  
positive definite matrix.

*Remark 3:* Comparing to the work of the authors in [26],  
[32], the novelty in the designed observer (8) is the introduc-  
tion of the intersample output predictor term  $w(t)$  [35] in the  
correction term. The dynamic of this predictor is simply a copy  
of the dynamics of system states equations. The role of the out-  
put predictor term is to provide a continuous time prediction of  
the output measured variable  $y(t)$ . Indeed, since the measured  
output variable  $y(t)$  is sampled, its values  $y(t_k)$  are available  
for the observer only at sampling times  $t = t_k$ . Comparing to  
constant-gain zero-order-hold (ZOH) approaches which main-  
tain  $y(t_k)$  constant between the sampling times, the output  
predictor term  $w(t)$  will provide a continuous time estimation  
of  $y(t)$  as it is the case in continuous time-observer design  
framework.

Now, we are able to state the main results of this paper.

*Theorem 1:* Consider the EHA system (6), and suppose that  
assumptions (1–2) holds, given a sampling period  $T$ , choose  
 $\sigma_0, \sigma_1, \sigma_2$  as in (17), define  $\sigma_3 = Te^{\sigma T} \frac{2\sigma_1(\theta + \beta_0)}{\sigma_0 \sqrt{\lambda_{\min}(P)}}$  then sys-  
tem (8) is an exponential sampled data observer for system  
(6) with the following properties: the vector of the observa-  
tion error  $\|\bar{e}_x\|$  converges exponentially toward a ball whose  
radius  $R = \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)(1 - \sigma_3)}}$ . Moreover, there exists a real  
positive bounded  $T_{\max}$  satisfying inequality (34), so that for all  
 $T \in (0, T_{\max})$ , the radius of the ball can be made as small as  
desired by choosing large values of  $\theta$  and  $k_{i=1, \dots, 4}$ .

*Proof 1:* The proof of this theorem 1 is inspired from the  
work of the authors in [35]. Let us now define the following  
observer  $e_{\bar{x}}$  and the output  $e_w(t)$  errors as follows:

$$\begin{cases} e_{\bar{x}}(t) = \hat{x} - \bar{x} \\ e_w(t) = w(t) - y(t) = w(t) - \bar{C}\bar{x}. \end{cases} \quad (9)$$

Combining (6) and (8), we can easily check that for the EHA  
system (6), the following properties are satisfied:  $\theta\Delta_\theta^{-1}\bar{A}\Delta_\theta =$   
 $\theta\bar{A}$  and  $\Delta_\theta^{-1}K\bar{C} = \Delta_\theta^{-1}K\bar{C}\Delta_\theta$ . Introducing the well-known  
change in coordinate in the high gain literature  $\bar{e}_x = \Delta_\theta e_x$   
yields the following dynamics of the state and the output errors:

$$\begin{cases} \dot{\bar{e}}_x = \theta(\bar{A} - K\bar{C})\bar{e}_x + \Delta_\theta(\overline{\varphi(f(\hat{x}), u)} - \overline{\varphi(\bar{x}, u)}) \\ \quad + \theta K e_w - \Delta_\theta \delta(t) \\ \dot{e}_w = \theta \bar{e}_{x2} + (\overline{\varphi_1(f(\hat{x}), u)} - \overline{\varphi_1(\bar{x}, u)}). \end{cases} \quad (10)$$

Let us now consider the following candidate Lyapunov  
quadratic function  $V = \bar{e}_x^T P \bar{e}_x$ :

$$\begin{aligned} \dot{V} &\leq -\mu\theta\|\bar{e}_x\|^2 + 2\bar{e}_x^T P \Delta_\theta (\overline{\varphi(f(\hat{x}), u)} - \overline{\varphi(\bar{x}, u)}) \\ &\quad + 2\theta\bar{e}_x^T P K e_w(t) - 2\bar{e}_x^T P \Delta_\theta \delta. \end{aligned} \quad (11)$$

Taking into account Assumptions (1–2) we have

$$\begin{aligned} \dot{V} &\leq -\mu\theta\|\bar{e}_x\|^2 + 4\beta_0\lambda_{\max}(P)\|\bar{e}_x\|^2 + 2\theta\|PK\|\|\bar{e}_x\|\|e_w(t)\| \\ &\quad + 4\lambda_{\max}(P)\|\bar{e}_x\|\|\xi \end{aligned} \quad (12)$$

where  $\xi = \sqrt{\mu_1^2 + \mu_2^2}$ .

306 Using the well-known property

$$\lambda_{\min}(P) \|\bar{e}_{\bar{x}}\|^2 \leq V \leq \lambda_{\max}(P) \|\bar{e}_{\bar{x}}\|^2 \quad (13)$$

307 we derive

$$\begin{aligned} \dot{V} \leq & -\mu\theta \frac{V}{\lambda_{\max}(P)} + \frac{4\beta_0\lambda_{\max}(P)V}{\lambda_{\min}(P)} \\ & + 2\theta\|PK\| \sqrt{\frac{V}{\lambda_{\min}(P)}} |e_w(t)| + 4\lambda_{\max}(P) \sqrt{\frac{V}{\lambda_{\min}(P)}} \xi. \end{aligned} \quad (14)$$

308 Now choosing the parameter  $\theta$  such that  $\theta > \theta_0$  with  $\theta_0 =$   
309  $\sup \left\{ 1, \frac{8\beta_0\lambda_{\max}^2(P)}{\mu\lambda_{\min}(P)} \right\}$ , we have

$$\begin{aligned} \dot{V} \leq & -\mu\theta \frac{V}{2\lambda_{\max}(P)} + 2\theta\|PK\| \sqrt{\frac{V}{\lambda_{\min}(P)}} |e_w(t)| \\ & + 4\lambda_{\max}(P) \sqrt{\frac{V}{\lambda_{\min}(P)}} \xi. \end{aligned} \quad (15)$$

310 Considering now the function  $W = \sqrt{V}$ , then we obtain

$$\begin{aligned} \dot{W} \leq & -\mu\theta \frac{W}{4\lambda_{\max}(P)} + \theta\|PK\| \frac{|e_w(t)|}{\sqrt{\lambda_{\min}(P)}} \\ & + 2 \frac{\lambda_{\max}(P)}{\sqrt{\lambda_{\min}(P)}} \xi. \end{aligned} \quad (16)$$

311 Let us set

$$\begin{cases} \sigma_0 = \frac{\mu\theta}{4\lambda_{\max}(P)} \\ \sigma_1 = \frac{\theta\|PK\|}{\sqrt{\lambda_{\min}(P)}} \\ \sigma_2 = 2 \frac{\lambda_{\max}(P)}{\sqrt{\lambda_{\min}(P)}}. \end{cases} \quad (17)$$

312 Integrating (16), then

$$\begin{aligned} W(t) \leq & e^{-\sigma_0(t-t_0)} W(t_0) + \sigma_1 e^{-\sigma_0 t} \int_{t_0}^t e^{\sigma_0 s} |e_w(s)| ds \\ & + \sigma_2 e^{-\sigma_0 t} \int_{t_0}^t e^{\sigma_0 s} \|\xi(s)\| ds. \end{aligned} \quad (18)$$

313 Multiplying both sides of (18) by  $e^{\sigma t}$  and using the fact that  
314  $e^{-(\sigma_0-\sigma)t} < 1$  we derive

$$\begin{aligned} e^{\sigma t} W(t) \leq & M(t_0) + \sigma_1 e^{-(\sigma_0-\sigma)t} \int_{t_0}^t e^{\sigma_0 s} |e_w(s)| ds \\ & + \sigma_2 e^{-(\sigma_0-\sigma)t} \int_{t_0}^t e^{\sigma_0 s} \|\xi(s)\| ds \end{aligned} \quad (19)$$

315 where  $M(t_0) = e^{\sigma_0 t_0} W(t_0)$ .

316 On the other hand, we have

$$\begin{aligned} e^{\sigma t} W(t) \leq & M(t_0) + \sigma_1 e^{-(\sigma_0-\sigma)t} \int_{t_0}^t e^{(\sigma_0-\sigma)s} e^{\sigma s} |e_w(s)| ds \\ & + \sigma_2 e^{-(\sigma_0-\sigma)t} \int_{t_0}^t e^{(\sigma_0-\sigma)s} e^{\sigma s} \|\xi(s)\| ds \end{aligned} \quad (20)$$

317 or

$$e^{\sigma t} W(t) \leq M(t_0) \quad (21)$$

$$\begin{aligned} & + \sigma_1 e^{-(\sigma_0-\sigma)t} \left( \int_{t_0}^t e^{(\sigma_0-\sigma)s} ds \right) \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|e_w(s)\|) \\ & + \sigma_2 e^{-(\sigma_0-\sigma)t} \left( \int_{t_0}^t e^{(\sigma_0-\sigma)s} ds \right) \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\xi(s)\|) \end{aligned} \quad (22)$$

which leads to

$$\begin{aligned} e^{\sigma t} W(t) \leq & M(t_0) \\ & + \frac{\sigma_1}{\sigma_0 - \sigma} \sup_{t_0 \leq s \leq t} (e^{\sigma s} |e_w(s)|) \\ & + \frac{\sigma_2}{\sigma_0 - \sigma} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\xi(s)\|). \end{aligned} \quad (23)$$

Now taking  $0 < \sigma < \sigma_0/2$ , we derive

$$\begin{aligned} \sup_{t_0 \leq s \leq t} (e^{\sigma s} W(s)) \leq & M(t_0) \\ & + 2 \frac{\sigma_1}{\sigma_0} \sup_{t_0 \leq s \leq t} (e^{\sigma s} |e_w(s)|) \\ & + 2 \frac{\sigma_2}{\sigma_0} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\xi(s)\|) \end{aligned} \quad (24)$$

and

$$\begin{aligned} W(t) \leq & e^{-\sigma t} M(t_0) + 2 \frac{\sigma_1}{\sigma_0} \sup_{t_0 \leq s \leq t} (e^{-\sigma(t-s)} |e_w(s)|) \\ & + 2 \frac{\sigma_2}{\sigma_0} \sup_{t_0 \leq s \leq t} (e^{-\sigma(t-s)} \|\xi(s)\|) \end{aligned} \quad (25)$$

which leads to

$$\begin{aligned} \|\bar{e}_{\bar{x}}\| \leq & e^{-\sigma t} \frac{M(t_0)}{\sqrt{\lambda_{\min}(P)}} \\ & + \frac{2\sigma_1}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{-\sigma(t-s)} |e_w(s)|) \\ & + \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{-\sigma(t-s)} \|\xi(s)\|) \end{aligned} \quad (26)$$

and

$$\begin{aligned} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_{\bar{x}}\|) \leq & \frac{M(t_0)}{\sqrt{\lambda_{\min}(P)}} \\ & + \frac{2\sigma_1}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{\sigma s} |e_w(s)|) \\ & + \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\xi(s)\|). \end{aligned} \quad (27)$$

On the other hand, we have from (10) the following expression  
of  $|e_w(t)|$ :

$$|e_w(t)| = \int_{t_k}^t |\theta \bar{e}_{\bar{x}2} + (\overline{\varphi_1(f(\hat{x}), u)} - \overline{\varphi_1(\bar{x}, u)})| ds. \quad (28)$$

Multiplying again both sides of (28) by  $e^{\sigma t}$  and taking into  
account assumptions 1–2, we have

$$e^{\sigma t} |e_w(t)| \leq e^{\sigma t} (\theta + \beta_0) \int_{t_k}^t e^{-\sigma s} e^{\sigma s} \|\bar{e}_{\bar{x}}(s)\| ds \quad (29)$$

which leads to

$$\begin{aligned} e^{\sigma t} |e_w(t)| \leq & e^{\sigma t} (\theta + \beta_0) \left( \int_{t_k}^t e^{-\sigma s} ds \right) \\ & \sup_{t_k \leq s \leq t} (e^{\sigma s} \|\bar{e}_{\bar{x}}(s)\|) ds \end{aligned} \quad (30)$$

328 taking into account that  $e^{-\sigma s} < 1$ , we derive that

$$\begin{aligned} \sup_{t_k \leq s \leq t} e^{\sigma s} |e_w(s)| &\leq T e^{\sigma T} (\theta + \beta_0) \\ \sup_{t_k \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|) ds & \end{aligned} \quad (31)$$

329 since  $\sup_{t_k \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|) \leq \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|)$  and  
330 taking into account that  $t > t_0, t_1, \dots, t_k$  we derive that

$$\begin{aligned} \sup_{t_0 \leq s \leq t} e^{\sigma s} |e_w(s)| &\leq T e^{\sigma T} (\theta + \beta_0) \\ \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|) ds & \end{aligned} \quad (32)$$

331 Combining (32) with (27) we have

$$\begin{aligned} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_x\|) &\leq \frac{M(t_0)}{\sqrt{\lambda_{\min}(P)}} \\ &+ T e^{\sigma T} \frac{2\sigma_1}{\sigma_0 \sqrt{\lambda_{\min}(P)}} (\theta + \beta_0) \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|) ds \\ &+ \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{\sigma(s)} \|\xi(s)\|) \end{aligned} \quad (33)$$

332 setting  $\sigma_3 = T e^{\sigma T} \frac{2\sigma_1(\theta + \beta_0)}{\sigma_0 \sqrt{\lambda_{\min}(P)}}$  then selecting  $T_{\max}$  satisfying  
333 the following the small gain condition:

$$T_{\max} e^{\sigma T_{\max}} \frac{2\sigma_1(\theta + \beta_0)}{\sigma_0 \sqrt{\lambda_{\min}(P)}} < 1 \quad (34)$$

334 we have

$$\begin{aligned} \|\bar{e}_x\| &\leq e^{-\sigma t} \frac{M(t_0)}{\sqrt{\lambda_{\min}(P)(1 - \sigma_3)}} \\ &+ \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)(1 - \sigma_3)}} \sup_{t_0 \leq s \leq t} \|\xi(s)\|. \end{aligned} \quad (35)$$

335 This complete the proof of Theorem 1.

336 **Remark 4:** Contrary to ([34], (35) demonstrates the global  
337 exponential convergence of the vector of the observation error  
338  $\|\bar{e}_x\|$  toward a ball whose radius depends on the magnitude of  
339 the disturbance vector  $\xi$ . In addition, the maximum sampling  
340 period  $T_{\max}$  derived in (34) is less restrictive comparing to the  
341 one derived in [34] which depends on the computation of a  
342 bounded positive function  $\psi(t)$  (see (13) in [34]).

343 **Remark 5:** The radius of the ball  $R$  is defined such that  $R =$   
344  $\frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)(1 - \sigma_3)}}$ . We also notice that in the case where there  
345 is no mechanical disturbances (i.e.,  $d_1 = 0$ ) and the hydraulic  
346 disturbances are constant or equal to 0, we have an exponential  
347 convergence of the observation error  $\|\bar{e}_x\|$  toward 0. Looking at  
348 the expression of the maximum sampling period  $T_{\max}$  in (34),  
349 we can easily see that when  $\sigma$  tends to zero,  $T_{\max} \simeq \frac{1}{\theta}$ . Hence,  
350 augmenting  $\theta$  will diminish the value of  $T_{\max}$ . On the other  
351 hand, large values of parameter  $\theta$  will contribute to reduce the  
352 radius  $R$  and hence to improve the performance of our observer.  
353 However, it is well known that the high gain observers litera-  
354 ture, augmenting the values of  $\theta$  will lead to the undesirable  
355 peaking phenomenon which consists in an impulsive behavior  
356 of the states estimation trajectory around initial conditions.

TABLE I  
NUMERICAL PARAMETER VALUES FOR THE EHA SYSTEM

Parameters	Value
$m$	0.5
$b$	0
$k$	$5.651110 \times 10^5$
$A_p$	$5.058 \times 10^{-4}$
$k_v$	$1.333 \times 10^{-5}$
$\alpha$	$3.257 \times 10^{10}$
$\beta$	2.146
$\gamma$	$7.169 \times 10^9$
$P_s$	$2.1 \times 10^7$

TABLE II  
PARAMETERS OF THE HYBRID OBSERVER

Parameter	$\theta$	$K = \begin{pmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{pmatrix}$	$T_s$

#### IV. SIMULATIONS AND EXPERIMENTAL RESULTS

##### A. Numerical Simulation of the Hybrid Observer Coupled With PI Controller for the EHA System Subject to Mechanical and Hydraulic Disturbances

The performance of the proposed observer will be evaluated first under MATLAB/Simulink Software. For the purpose of comparison, the numerical simulations were performed on the EHA system validated experimentally by the authors in [26] and [29]. The model parameters' values are shown in Table I. In this numerical simulations, we will demonstrate the effectiveness of our proposed observer in terms of states/disturbances estimation and positioning control. In [29], the authors considered a sinusoidal reference position signal  $x_{1d} = 0.008 \sin(2\pi t)$ . For the purpose of tracking  $x_{1d}$ , a PI controller was employed and combined with the proposed observer (8) so that the novel PI control law  $u$  is expressed as follows:

$$u = K_p(w(t) - x_{1d}) + K_i \int (w(t) - x_{1d}) \quad (36)$$

where  $x_1 = x_p$  is the piston position and  $K_p = 3.18 \times 10^{-2}$ ,  $K_i = 100$  are the PI gains. The PI controller gains were tuned in order to track. The numerical simulations were performed using the Runge–Kutta solver with a fixed step size  $T_{\text{sim}} = 10^{-4}$  s. The parameters of the hybrid observer are summarized in Table II where  $T_s$  is the sampling period of our proposed hybrid (continuous–discrete time) observer.

The values of the observer parameters used in this simulation are  $\theta = 1000$ ,  $K = (10, 35, 49\ 426, 23\ 724)$  and  $T_s = 1$  ms.

The evaluation of our observer is performed under the consideration that both mechanical and hydraulic disturbances affect the considered EHA system in this paper. For the mechanical disturbance term  $d_1$ , we have taken the same one considered by the authors [29]. To show the robustness of our observer facing the mechanical disturbances, we considered it in the simulation not from the beginning but at  $t = 10$  s. Hence, the term  $d_1$  in the disturbed model of the EHA in (2)

T1:1  
T1:2

T2:1  
T2:2

357

358

359

360

361

362

363

364

365

366

367

368

369

370

371

372

373

374

375

376

377

378

379

380

381

382

383

384

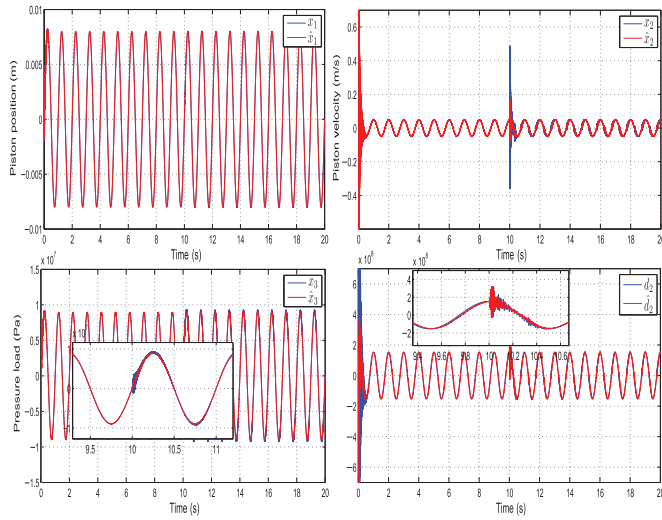
385

386

387

388

389



F2:1 **Fig. 2.** Estimation of  $x_1$ ,  $x_2$ ,  $x_3$ ,  $d_2$  for  $\theta = 1000$  and  $T_s = 1$  ms with  
 F2:2 mechanical and hydraulic disturbances.

390 is expressed as follows:

$$d_1(t) = \begin{cases} 0, & \text{if } t < 10 \text{ s} \\ 294 \sin(62.83x_1) + 20 \operatorname{sign}(x_2), & \text{if } t \geq 10 \text{ s}. \end{cases}$$

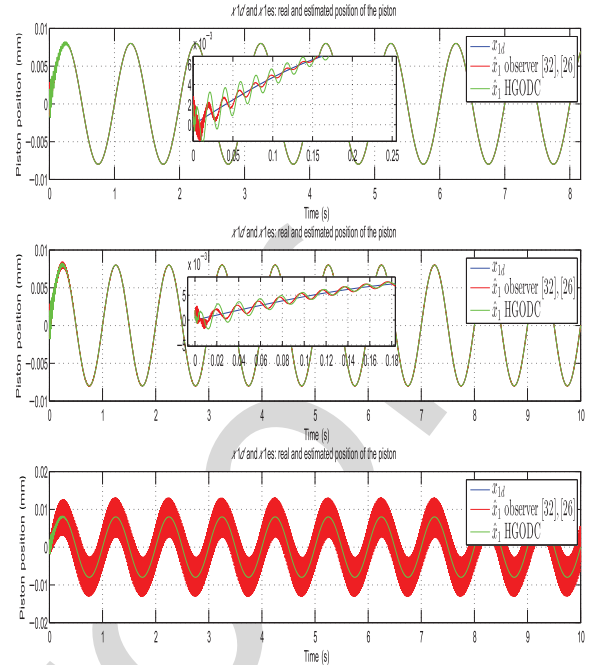
391 We also assume in this simulation that 10% additive para-  
 392 metric variation affects the hydraulic coefficients  $\gamma$ ; hence  
 393 (see Section II), the hydraulic disturbance term  $d_2$  takes the  
 394 following form:

$$d_2(t) = 10\% \sqrt{P_s - \operatorname{sign}(u)x_3u}.$$

395 From **Fig. 2**, we can see that the tracking performance of  
 396 the reference  $x_{1d}$  even in the presence of the mechanical dis-  
 397 turbance at  $t = 10$  s is achieved correctly by the PI controller  
 398 (36). The robustness of the PI controller facing the mechan-  
 399 ical disturbance can be also seen in **Fig. 2** where we can see  
 400 that this disturbance has no effect on the tracking performance  
 401 of the motion reference trajectory  $x_{1d}$ . For the estimation of  
 402 the piston velocity  $x_2$ , the pressure load  $x_3$ , and the hydraulic  
 403 disturbance term  $d_2$ , we can see the effect of the mechanical  
 404 disturbance (see **Fig. 2** top right, bottom left, and right) which  
 405 consists in a deviation of the states estimation trajectory occur-  
 406 ring at  $t = 10$  s. Meanwhile, this deviation is quickly rejected  
 407 by the observer, thanks to the large value of parameter  $\theta$  taken  
 408 in this simulation. As mentioned in Remark 5, large values of  
 409 parameter  $\theta$  will lead to a better rejection of the mechanical and  
 410 the hydraulic disturbance term, however, this will amplify the  
 411 peaking phenomenon which consists in an impulsive behavior  
 412 of the trajectory of the states estimation at the beginning of the  
 413 simulation (see **Fig. 2**).

### 414 **B. Performance Comparison With the Observer** 415 **Designed in [26] and [32]**

416 To show the performance of our proposed observer, we have  
 417 performed a comparison with the observers designed in [26]  
 418 and [32]. Indeed, the observers [26], [32] have the same high  
 419 gain like observer structure as the one considered in the design



F3:1 **Fig. 3.** Comparison of position tracking performance between our  
 F3:2 observer [high gain observer discrete-continuous (HGODC)] ( $T_s =$   
 F3:3 1 ms) and observers [26], [32] (top:  $T_s = 0.1$  ms; middle:  $T_s = 0.5$  ms;  
 F3:4 bottom:  $T_s = 1$  ms).

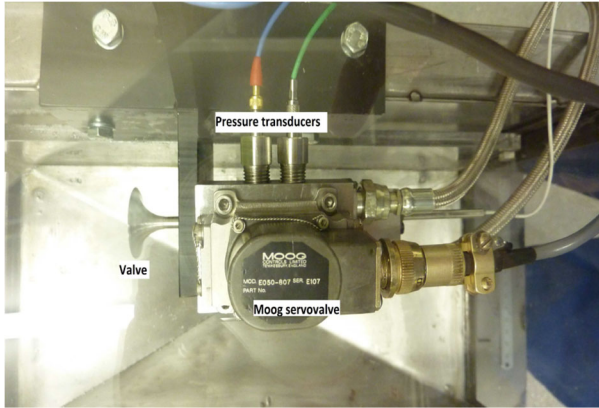
of our observer. By taking into account the sampling effect in  
 the structure of these two observers, a continuous–discrete time  
 version of the observers designed in [26] and [32] can be written  
 as follows:

$$\dot{\hat{x}} = \bar{A}\hat{x} + \overline{\varphi(f(\hat{x}), u)} - H(\bar{C}\hat{x}(t) - y(t_k)). \quad (37)$$

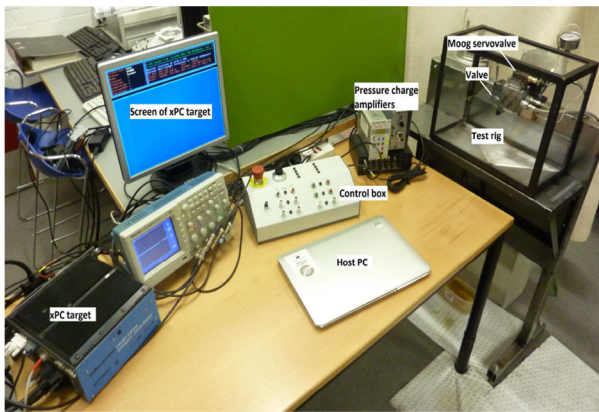
We notice that in the case of our observer  $H = \theta \Delta_\theta^{-1} K$ . The  
 structure of (37) uses the sampled data  $y(t_k)$  in the correction  
 term since that continuous measured variable  $y(t)$  is available  
 only at sampled instants  $t = t_k$ . The simulations presented in  
**Fig. 3** show the performance of observer (8) and observer (37)  
 in terms of position tracking performances. For our proposed  
 observer (named HGODC), we have fixed the value of  $T_s$  to  
 1 ms. For observer (37), three values were taken ( $T_s = 0.1$ ,  
 0.5, and 1 ms). Looking at **Fig. 3** (top), we can see that even  
 if observer (37) performs better in the transitory regime, our  
 observer has quite the same performance. Recalling that in this  
 case,  $T_s = 0.1$  ms for observer (37) which is the same sampling  
 period as the one of the solver, we can say that our observer  
 recovers the performances of continuous time observers. When  
 augmenting the sampling period of observer (37) to 0.5 ms,  
 we can see that for observer (37), the performance degrades.  
 Finally, when the two observers have the same sampling peri-  
 ods ( $T_s = 1$  ms), observer (37) diverges and the PID controller,  
 which is based on the estimation provided by observer (37),  
 fails to track the desired trajectory  $x_{1d}$ .

### 444 **C. Experimental Validation**

To illustrate the performance of our proposed observer, an  
 experimental test rig platform has been set up and photographed



F4:1 Fig. 4. Moog servo-valve and the EHA actuator assembly.



F5:1 Fig. 5. Control system of the experimental test rig of the EHA system.

447 in Figs. 4 and 5. The test rig was constructed in the Brighton  
448 University to investigate the performance of the EHA assem-  
449 bly and the control parameters influencing the motion of the  
450 poppet valve. The test rig comprised of three main subsystems:  
451 a hydraulic oil pressure supply; a hydraulic valve actuation  
452 assembly; and the servo-valve control signal and valve position  
453 interface.

454 Hydraulic oil from a large tank was supplied to a smaller  
455 reservoir coupled to a high-pressure pump and accumulator.  
456 An electromagnetic pressure-limit switch was used to regulate  
457 the supply of high-pressure oil to the hydraulic valve actuation  
458 assembly via an oil filter. The supply pressure was regulated to  
459 70 bar  $\pm$  2 bar by a pressure-limit switch.

460 The actuator body housed a double-acting hydraulic piston,  
461 oil-sealing end plates, and the high-pressure oil supply  
462 and return feed lines. A continuous-proportional (four-way)  
463 directional servo-valve (Moog series 31) was used to control  
464 the flow rate of hydraulic oil to the hydraulic piston by  
465 means of a proportional electromagnetic servo control signal.  
466 The interchangeable poppet valve head was attached to one  
467 end of the hydraulic piston and a linear variable differential  
468 transducer (LVDT) was mounted to the opposite end to record  
469 the change in valve position. The calibration factor for the  
470 amplified output of the LVDT sensor (Lord MicroStrain) was  
471 2.97 mm/V  $\pm$  0.005 mm/V. Two piezoelectric gauge pressure

TABLE III  
EHA PARAMETER VALUES FOR THE EXPERIMENTAL TEST RIG

Parameters	Value
$m$	0.05
$k$	2000
$b$	0.1398
$A_p$	0.0614
$k_v$	0.02
$\alpha$	28.2226
$\beta$	0.0063
$\gamma$	0.0029
$P_s$	$7 \times 10^6$

472 transducers (Kistler type 6125 transducer and type 5011 ampli-  
473 fier) were used to measure the instantaneous and difference in  
474 oil pressures in the supply and return chambers either sides of  
475 the hydraulic piston. The pressure transducer was calibrated to  
476 20 bar/V. The full-scale error in the transducer was  $\pm 3$  bar. The  
477 value of the oil pressure at the instant of initial piston motion  
478 was used as the gauge reference pressure.

479 The control system for the electro-hydraulic valve system  
480 was based on a real-time simulation and testing platform  
481 (hardware in the loop, HIL); MathWorks MATLAB Simulink  
482 and xPC Target application and a real-time target machine  
483 (Speedgoat GmbH). Positional feedback of the valve was deter-  
484 mined from the LVDT sensor output. The actuation of the  
485 directional servo-valve was achieved using a current driver sig-  
486 nal rated to  $\pm 50$  mA. The displacement of the poppet valve is  
487 comprised between [20–32] mm. Based on the physical param-  
488 eters of the experimental test rig [36], the nominal values of the  
489 EHA model parameters were identified and listed in Table III.

490 In the following experiments, the parameters' values of  
491 the hybrid observer for this experiment are  $\theta = 500$ ,  $K =$   
492 (2.8, 2.87, 1.0423, 0.1710), and  $T_s = 1$  ms.

#### D. PID Control Design for the Experimental Test Rig

493 In order to track the motion reference  $x_{1d}$ , the following  
494 PID control law  $u$  with a velocity feedforward action was  
495 implemented  
496

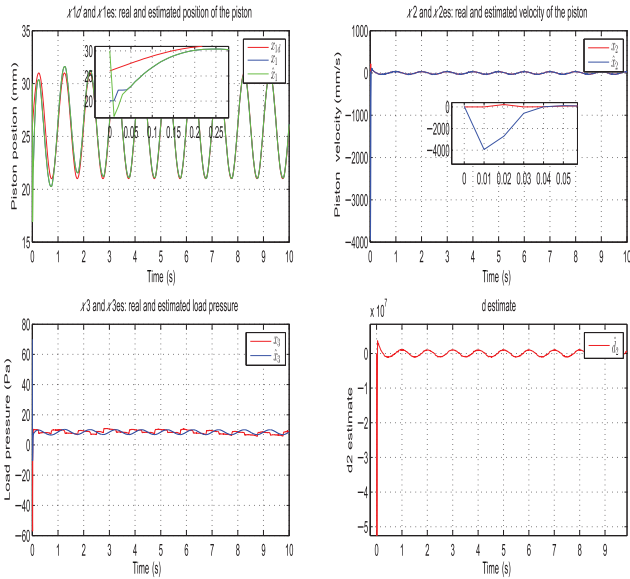
$$u = K_p(x_{1d} - w(t)) + K_i \int (x_{1d} - w(t)) + K_d \frac{d}{dt}(x_{1d} - w(t)) + K_f \dot{x}_{1d} \quad (38)$$

497 where  $K_p = 0.54$ ,  $K_i = 1.93$ ,  $K_d = 0.04$ ,  $K_f = 1$ . As it was  
498 the case in the simulation section, the implemented control law  
499  $u$  contains the output prediction term  $w(t)$ . We mention that for  
500 this experimental validation, we used the same Runge–Kutta  
501 solver with the same fixed step size  $T_{\text{sim}} = 10^{-4}$  as in the  
502 numerical simulations section. The experimental validation was  
503 conducted with a sampling period  $T_s = 1$  ms which is 10 times  
504 bigger than the fixed step size of the solver.

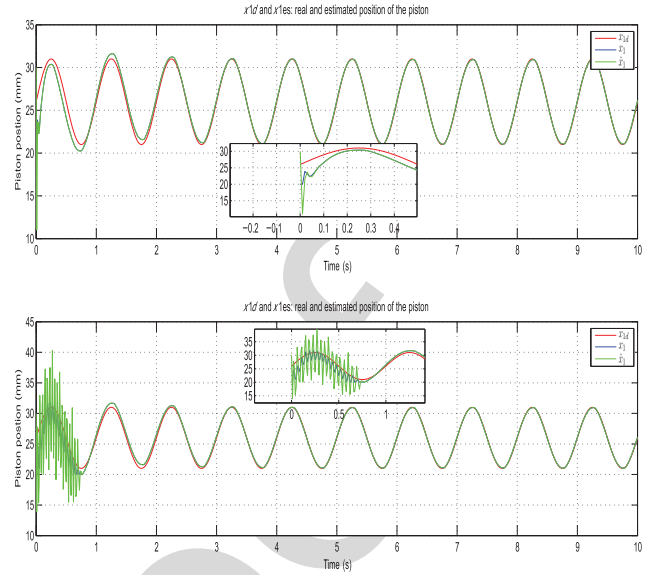
#### E. Experimental Performances of the Hybrid Observer Without Disturbance

505 In this section, we investigate the performance of the hybrid  
506 observer for state estimation and piston position tracking  
507  
508





F6:1 **Fig. 6.** Estimation and tracking performance of  $x_1, x_2, x_3, d_2$  for  
 F6:2  $\theta = 500, T_s = 1$  ms for EHA system without disturbances.



**Fig. 7.** Estimation and tracking performance of  $x_1$  for  $\theta = 500$ . (Top) F7:1  
 $T_s = 2$  ms. (Bottom)  $T_s = 3$  ms. F7:2

509 motion trajectory  $x_{1d} = 26 + 5 \sin(2\pi t)$ . Since the considered  
 510 EHA system does not drive any mechanical load, we have theoret-  
 511 ically  $d_1 \simeq 0$ . We also mention that we have used the  
 512 same nominal values of the EHA system when implementing the  
 513 hybrid observer.

514 In Fig. 6 (top left), we show the performance of the hybrid  
 515 observer in terms of tracking performances and state estimation  
 516 of the piston position  $x_1$ . We can see in Fig. 6 (top left) that both  
 517 the tracking performance and the state estimation are achieved  
 518 correctly by the hybrid observer. For the state estimation of the  
 519 piston position  $x_1$ , the convergence of the hybrid observer is  
 520 achieved with small convergence rate [less than 0.05 s when  
 521 looking to the zoom of Fig. 6 (top left)]. We can see also that the  
 522 tracking performance of the motion reference  $x_{1d}$  by the PI con-  
 523 troller, which uses the output predictor  $w(t)$ , is also achieved  
 524 correctly.

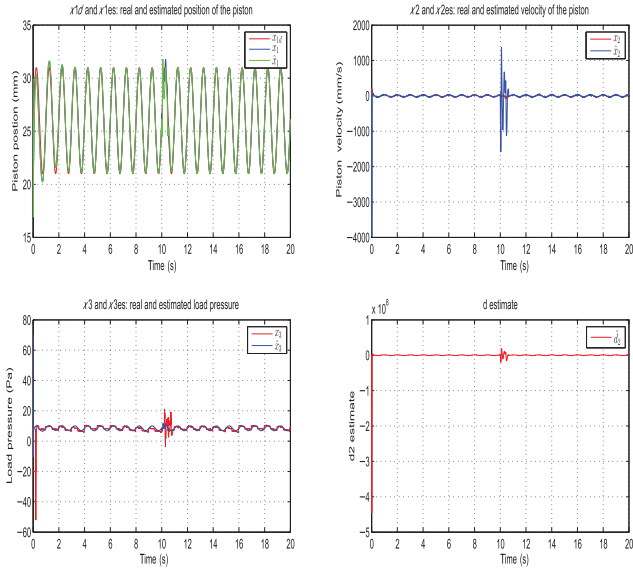
525 Fig. 6 (top right) shows the state estimation of the piston  
 526 velocity  $x_2$ . We can see in Fig. 6 (top right) that our hybrid  
 527 observer provides a very good estimation of the real piston  
 528 velocity  $x_2$ . A quick look to Fig. 6 (top right) shows that the  
 529 effect noise, which comes from the numerical differentiation  
 530 used to obtain the real piston, has been attenuated by our hybrid  
 531 observer.

532 In Fig. 6 (bottom left), we present the estimation results of  
 533 the hydraulic pressure state  $x_3$  by our proposed observer. First,  
 534 we can observe from Fig. 6 (bottom left) that our observer  
 535 provides a good estimation of the hydraulic pressure state  
 536  $x_3$  despite the variations in the hydraulic parameters and the  
 537 hydraulic disturbance which affects the functioning of the EHA  
 538 system. The effects of these disturbances can be viewed. In  
 539 Fig. 6 (bottom right) where we can see that even if there is  
 540 no mechanical load driven by the EHA system, the estimated  
 541 disturbance term  $\hat{d}_2$  is not equal to 0. Indeed, the difficulty of  
 542 capturing the hydraulic parameters ( $\alpha, \beta, \gamma$ ) and the internal  
 543 leakage occurring on the EHA system generates automatically  
 544 the disturbance term  $d_2$ . For the reader, we mention that it was

very difficult for us to plot in Fig. 6 (bottom right) the real  
 545 hydraulic disturbance term  $d_2$  for the reasons explained above.  
 546 Finally, we can observe in Fig. 6 (bottom left) that there is  
 547 small phase lag between the real and the estimated hydraulic  
 548 pressure  $x_3$ . This observation is quite interesting because of the  
 549 discrepancies between the numerical simulations and the exper-  
 550 imental validation of our observer. This discrepancies come  
 551 from the difficulty of capturing exactly the hydraulic param-  
 552 eters of the EHA system and the fact that the dynamic of the  
 553 electrical part of the EHA system has been neglected in the  
 554 EHA model. In addition, it appears that the PID control is not  
 555 able to compensate it. Taking into account that the kistler pres-  
 556 sure transducers give a relative and not an absolute pressures  
 557 values in each chamber of the hydraulic actuator, we can say  
 558 that the estimated hydraulic pressures provided by our observer  
 559 are good.  
 560

#### F. Effect of the Sampling Period on the Performance of the Hybrid Observer

561 To compute the maximum allowable sampling period  $T_{\max}$   
 562 of the hybrid observer, we can proceed following two poss-  
 563 ible manners. The first one is to compute  $T_{\max}$  analytically  
 564 using the expression in (34); however, this will necessitate to  
 565 know the constant  $\beta_0$  which is practically very difficult to deter-  
 566 mine. The second one is to start with a sampling period  $T_s$  and  
 567 increasing it until the observer diverges. We proceed follow-  
 568 ing the second manner. In Fig. 7, we present the experimental  
 569 results of the estimated piston position  $x_1$  and the tracking per-  
 570 formance of the piston position reference  $x_{1d}$ . We mention that  
 571 we did not report the experimental results concerning the esti-  
 572 mations of the piston velocity  $x_2$ , the hydraulic pressure  $x_3$ ,  
 573 and the hydraulic disturbances  $d_2$ . The reason is that they are  
 574 characterized by the same dynamic behavior as the results pre-  
 575 sented in Fig. 7. When increasing  $T_s$  to 2 ms, we can observe  
 576 from the top of Fig. 7 that the estimated piston position and  
 577  
 578



F8:1 **Fig. 8.** Estimation and tracking performance of  $x_1, x_2, x_3, d_2$  for  
 F8:2  $\theta = 500, T_s = 1$  ms for EHA system with disturbances.

579 the tracking performance are quite the same as it is the case of  
 580 of  $T_s = 1$  ms. The difference concerns the convergence speed  
 581 which is slower in the case of  $T_s = 1$  ms. When increasing  $T_s$   
 582 to 3 ms, we can observe that the performances of the hybrid  
 583 observer are affected only in the transitory regime (see bottom  
 584 of Fig. 7). Indeed, the oscillations observed in the bottom of  
 585 Fig. 7 are due to the increase in the sampling period  $T_{max}$  to  
 586 3 ms which clearly affects the transitory regime for our hybrid  
 587 observer. In the permanent regime, the hybrid observer which  
 588 provides the output predictor term  $w(t)$  for the PID controller  
 589 performs well in the case of estimation and the tracking per-  
 590 formance. From this, we can deduce that in the case of this  
 591 experimental results,  $T_{max} \simeq 2$  ms.

### 592 G. Experimental Performances of the Hybrid Observer 593 With Disturbance

594 To investigate the performance of our observer in the pres-  
 595 ence of disturbance, an additional disturbance term  $d_3 = 2x_{1d}$   
 596 is inserted in the control input at  $t = 10$  s; meanwhile, the  
 597 new control input sent to the control board is  $u_1 = u + 2x_{1d}$ ,  
 598 where  $u$  is the previous control calculated by the PID controller.  
 599 According to the structure of the model of the EHA system,  
 600 this disturbance will be added to the previously hydraulic dis-  
 601 turbance term  $d_2$  and will change the dynamic of the states  
 602 ( $x_1, x_2, x_3, x_4$ ) of the EHA system. We can see from Fig. 8 that  
 603 both tracking performances and states estimation are achieved  
 604 correctly by our observer. At  $t = 10$  s, we can see the influ-  
 605 ence of the disturbances on the performances of our observer.  
 606 Despite its occurrence, we can clearly say that: first, the PID  
 607 controller is robust facing this disturbance; since that the PID  
 608 control law  $u$  uses the predictor term  $w(t)$  provided by our  
 609 observer, this will demonstrate the easiness of the incorpora-  
 610 tion of our observer in a control scheme; second, our observer  
 611 succeeds to estimate the states and the disturbances affecting  
 612 the EHA system after ( $t = 10$  s).

## V. CONCLUSION AND FUTURE WORK

In this paper, a continuous–discrete time observer is designed  
 for the EHAs system subject to discrete time measurement and  
 mechanical and hydraulic disturbances. The exponential con-  
 vergence of the proposed observer is proven using a classical  
 quadratic Lyapunov function based on small gain arguments.  
 The proposed observer is combined with PID controller for the  
 purpose of tracking motion reference trajectory of the piston  
 position for the EHA system. The simulation results and the  
 experimental validation of our proposed observer demonstrate  
 its efficiency in terms of tracking performance and distur-  
 bance estimation. In our future works, we plan to synthesize an  
 output feedback controllers based on the designed continuous–  
 discrete time observer in this paper. The resulting controllers  
 will improve the positioning control for the EHAs system.

## REFERENCES

- [1] J. Yao, Z. Jiao, and S. Han, “Friction compensation for low velocity control of hydraulic flight motion simulator: A simple adaptive robust approach,” *Chin. J. Aeronaut.*, vol. 26, no. 3, pp. 841–822, Jun. 2013.
- [2] J. Yao, Z. Jiao, B. Yao, Y. Shang, and W. Dong, “Nonlinear adaptive robust control of electrohydraulic load simulator,” *Chin. J. Aeronaut.*, vol. 25, no. 5, pp. 766–775, Oct. 2012.
- [3] W. Sun, H. Gao, and O. Kaynak, “Adaptive backstepping control for active suspension systems with hard constraints,” *IEEE/ASME Trans. Mechatronics*, vol. 18, no. 3, pp. 1072–1079, Jun. 2013.
- [4] Y. Pi and X. Wang, “Observer-based cascade control of a 6-DOF parallel hydraulic manipulator in joint space coordinate,” *Mechatronics*, vol. 20, no. 6, pp. 648–655, Sep. 2010.
- [5] W. Sun, Y. Zhao, J. Li, L. Zhang, and H. Gao, “Active suspension control with frequency band constraints and actuator input delay,” *IEEE Trans. Ind. Electron.*, vol. 59, no. 1, pp. 530–537, Jan. 2012.
- [6] J. Yao, Z. Jiao, D. Ma, and L. Yan, “High-accuracy tracking control of hydraulic rotary actuators with modeling uncertainties,” *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 2, pp. 633–641, Apr. 2014.
- [7] H. A. Mintsas, R. Venugopal, J.-P. Kenne, and C. Belleau, “Feedback linearization-based position control of an electrohydraulic servo system with supply pressure uncertainty,” *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 4, pp. 1092–1099, Jul. 2012.
- [8] H. E. Merritt, *Hydraulic Control Systems*. Hoboken, NJ, USA: Wiley, 1967.
- [9] J. Yao, G. Yang, and D. Ma, “Internal leakage fault detection and tolerant control of single-rod hydraulic actuators,” *Math. Prob. Eng.*, vol. 2014.
- [10] J.-H. Kwon, T.-H. Kim, J.-S. Jang, and I.-S. Lee, “Feedback linearization control of a hydraulic servo system,” in *Proc. SICE-ICASE Int. Joint Conf.*, 2006, pp. 455–460.
- [11] H.-M. Chen, J.-C. Renn, and J.-P. Su, “Sliding mode control with varying boundary layers for an electro-hydraulic position servo system,” *Int. J. Adv. Manuf. Technol.*, vol. 26, no. 1, pp. 117–123, 2005.
- [12] M. A. Ghazy, “Variable structure control for electrohydraulic position servo system,” in *Proc. IEEE 27th Annu. Conf. Ind. Electron. Soc.*, 2001, pp. 2194–2198.
- [13] C. Guan and S. Pan, “Adaptive sliding mode control of electro-hydraulic system with nonlinear unknown parameters,” *Control Eng. Pract.*, vol. 16, no. 11, pp. 1275–1284, Nov. 2008.
- [14] Y. Lin, Y. Shi, and R. Burton, “Modeling and robust discrete-time sliding mode control design for a fluid power electro-hydraulic actuator (EHA) system,” *IEEE/ASME Trans. Mechatronics*, vol. 18, no. 1, pp. 1–10, Feb. 2013.
- [15] A. G. Loukianov, J. Rivera, Y. Orlov, and E. Teraoka, “Robust trajectory tracking for an electrohydraulic actuator,” *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3523–3531, Sep. 2009.
- [16] B. Yao, F. Bu, J. Reedy, and G. T. C. Chiu, “Adaptive robust motion control of single-rod hydraulic actuators: Theory and experiments,” *IEEE/ASME Trans. Mechatronics*, vol. 5, no. 1, pp. 79–91, Mar. 2000.
- [17] B. Yao, F. Bu, and G. T. C. Chiu, “Nonlinear adaptive robust control of electro-hydraulic systems driven by double-rod actuators,” *Int. J. Control.*, vol. 74, no. 8, pp. 761–775, Aug. 2001.

- 680 [18] G. Cheng and P. Shuangxia, "Nonlinear adaptive robust control of  
681 single-rod electro-hydraulic actuator with unknown nonlinear param-  
682 eters," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 3, pp. 434–445,  
683 May 2008.
- 684 [19] C. Wang, Z. Jiao, S. Wu, and Y. Shang, "Nonlinear adaptive torque control  
685 of electro-hydraulic load system with external active motion disturbance,"  
686 *Mechatronics*, vol. 24, no. 1, pp. 32–40, Feb. 2014.
- 687 [20] Y. Cungui and Q. Xianwei, "Simplified adaptive robust motion control  
688 with varying boundary discontinuous projection of hydraulic actuator,"  
689 *Math. Prob. Eng.*, vol. 2014.
- 690 [21] A. Alleyne and R. Liu, "A simplified approach to force control for electro-  
691 hydraulic systems," *Control Eng. Pract.*, vol. 8, no. 12, pp. 1347–1356,  
692 Dec. 2000.
- 693 [22] C. Kaddissi, J.-P. Kenne, and M. Saad, "Identification and real-time  
694 control of an electrohydraulic servo system based on nonlinear backstep-  
695 ping," *IEEE/ASME Trans. Mechatronics*, vol. 12, no. 1, pp. 12–22, Feb.  
696 2007.
- 697 [23] C. Kaddissi, J.-P. Kenne, and M. Saad, "Indirect adaptive control of  
698 an electrohydraulic servo system based on nonlinear backstepping,"  
699 *IEEE/ASME Trans. Mechatronics*, vol. 16, no. 6, pp. 1171–1177, Dec.  
700 2011.
- 701 [24] K. K. Ahn, D. N. C. Nam, and M. Jin, "Adaptive backstepping control of  
702 an electrohydraulic actuator," *IEEE/ASME Trans. Mechatronics*, vol. 19,  
703 no. 3, pp. 987–995, Jun. 2014.
- 704 [25] P. Nakkarat and S. Kuntanapreeda, "Observer-based backstepping force  
705 control of an electrohydraulic actuator," *Control Eng. Pract.*, vol. 17,  
706 no. 8, pp. 895–902, 2009.
- 707 [26] K. Wonhee, W. Daehee, S. Donghoon, and C. Chung, "Output feedback  
708 nonlinear control for electro-hydraulic systems," *Mechatronics*, vol. 22,  
709 no. 6, pp. 766–777, Sep. 2012.
- 710 [27] X. Wang, X. Sun, S. Li, and H. Ye, "Output feedback domination  
711 approach for finite-time force control of an electrohydraulic," *IET Control  
712 Theory*, vol. 6, no. 7, pp. 921–934, 2011.
- 713 [28] H. Khan, S. C. Abou, and N. Sepehri, "Nonlinear observer-based fault  
714 detection technique for electro-hydraulic servo-positioning systems,"  
715 *Mechatronics*, vol. 15, no. 9, pp. 1037–1059, 2005.
- 716 [29] K. Wonhee, W. Daehee, S. Donghoon, and C. Chung, "Disturbance-  
717 observer-based position tracking controller in the presence of biased sinu-  
718 soidal disturbance for electrohydraulic actuators," *IEEE Trans. Control  
719 Syst. Technol.*, vol. 21, no. 6, pp. 2290–2298, Nov. 2013.
- 720 [30] C. S. Kim and C. O. Lee, "Speed control of an overcentered variable  
721 displacement hydraulic motor with a load torque observer," *Control Eng.  
722 Pract.*, vol. 4, no. 11, pp. 1563–1570, 1996.
- 723 [31] W. Daehee, K. Wonhee, S. Donghoon, and C. Chung, "High-gain distur-  
724 bance observer-based backstepping control with output tracking error  
725 constraint for electro-hydraulic systems," *IEEE Trans. Control Syst.  
726 Technol.*, vol. 23, no. 6, pp. 758–795, Mar. 2014.
- 727 [32] J. Yao, Z. Jiao, and D. Ma, "Extended-state-observer-based output feed-  
728 back nonlinear robust control of hydraulic systems with backstepping,"  
729 *IEEE Trans. Ind. Electron.*, vol. 61, no. 11, pp. 6285–6293, Nov. 2014.
- 730 [33] J. Yao, Z. Jiao, and D. Ma, "Adaptive robust control of dc motors with  
731 extended state observer," *IEEE Trans. Ind. Electron.*, vol. 61, no. 7,  
732 pp. 3630–3637, Jul. 2014.
- 733 [34] S. Ahmed Ali, "Sampled data observer based inter-sample output pre-  
734 dicator for electro-hydraulic actuators," *ISA Trans.*, vol. 58, pp. 421–433,  
735 2015.
- 736 [35] I. Karafyllis and C. Kravaris, "From continuous-time design to sampled-  
737 data design of observers," *IEEE Trans. Autom. Control*, vol. 54, no. 9,  
738 pp. 2169–2174, Sep. 2009.
- 739 [36] A. Karakayis and S. Begg, "Investigation of control system strategies for  
740 hydraulic valve actuation in an IC engine Adil Karakayis," M.S. thesis,  
741 Dept. Autom. Eng., Univ. Brighton School Comput., Eng. Math., Div.  
742 Eng. Product Des., Brighton, U.K., 2014.
- 743 [37] W. Sun, Z. Zhao, and H. Gao, "Saturated adaptive robust control for active  
744 suspension systems," *IEEE Trans. Ind. Electron.*, vol. 60, no. 9, pp. 3889–  
745 3896, Sep. 2013.
- 746 [38] W. Sun, H. Gao, and B. Yao, "Adaptive robust vibration control of full-car  
747 active suspensions with electrohydraulic actuators," *IEEE Trans. Control  
748 Syst. Technol.*, vol. 21, no. 6, pp. 2417–2422, Nov. 2013.



**Sofiane Ahmed Ali** was born in Algiers, 749  
Algeria, in 1977. He received the B.Sc. degree 750  
in electrical engineering from the University 751  
of Technology Houari Boumediene, Algiers, 752  
Algeria, in 2001, and the M.Sc. and Ph.D. 753  
degrees in electrical and computer engineer- 754  
ing from the University of Le Havre, Le Havre, 755  
France, in 2004 and 2008, respectively. 756

In 2008, he was appointed as a Research 757  
and Development Engineer with Renault. Since 758  
2010, he has been a Teaching and Research 759

Assistant Professor with the École Supérieure d'Ingénieurs en Génie 760  
Électrique (ESIGELEC), Rouen, France. His research interests include 761  
sliding mode control, nonlinear observers and fault-tolerant control, and 762  
diagnosis in the field of mechatronics devices. 763



**Arnaud Christen** was born in France, in 764  
1991. He received the Baccalaureate degree 765  
in science (with honors) in 2009, before fol- 766  
lowing a two-year preparation in mathemat- 767  
ics and physics for entrance to the French 768  
Engineering Schools. He received the dual M.S. 769  
degree in control theory (electrical engineer- 770  
ing) and mechatronics from École Supérieure 771  
d'Ingénieurs en Génie Électrique (ESIGELEC), 772  
Rouen, France, and the University of Rouen, 773  
Rouen, France, in 2015. 774



**Steven Begg** received the B.Eng. degree (with 775  
honors) in mechanical engineering and the Ph.D. 776  
degree from the University of Brighton, Brighton, 777  
U.K., in 2003. 778

He is a Reader, Fellow of the Higher 779  
Education Academy, and the Course Leader 780  
for the Automotive Engineering undergraduate 781  
degree pathways in the School of Computing, 782  
Engineering and Mathematics, University of 783  
Brighton. He is the Leader of the Experimental 784  
Fluid Mechanics Research Group and a Member 785

of the Advanced Engineering Centre at Brighton, U.K. He has led 786  
applied research programmes (EPSRC, DfT, DTI, TSB, and EU) as 787  
well as industrial consultancy projects, in the fields of automotive engi- 788  
neering, fluid mechanics, and optical diagnostic techniques, for over 21 789  
years. 790



**Nicolas Langlois** received the Ph.D. and HDR 791  
(habilitation to supervise research) degrees in 792  
automatic control and signal processing from the 793  
University of Rouen, Rouen, France, in 2001 and 794  
2008, respectively. 795

In 2000, he joined the Graduate School 796  
of Electrical Engineering, ESIGELEC, Rouen, 797  
France. He is currently the Head-in-Charge 798  
of skills acquisition through research of 799  
ESIGELEC, where he teaches courses on 800  
control systems and digital signal processing. 801

He has also the Head of the "Automatic Control and Systems" research 802  
team at the research institute IRSEEM since 2008. His research 803  
interests include fault-tolerant control. 804

## QUERIES

- Q1: Please provide expansion for “PI.”
- Q2: Please provide expansion for “IRSEEM.”
- Q3: As per IEEE style, vectors have been changed to boldface italic. Please check whether all the occurrences are identified correctly and specify the missed out occurrences.
- Q4: Please provide complete details of Refs. [9] and [20].
- Q5: Please provide the location for Renault in the biography section of the author Sofiane Ahmed Ali.
- Q6: Please provide the institution name and location for the Baccalaureate degree of the author Arnaud Christen.
- Q7: Please spell out the term “ESIGELEC.”
- Q8: Please provide field of study for the Ph.D. degree of author “Steven Begg.”
- Q9: Please expand “IRSEEM” in the biography section.

IEEE PROOF

# Continuous–Discrete Time-Observer Design for State and Disturbance Estimation of Electro-Hydraulic Actuator Systems

Sofiane Ahmed Ali, Arnaud Christen, Steven Begg, and Nicolas Langlois

**Abstract**—In this paper, a continuous–discrete time observer which simultaneously estimates the unmeasurable states and the uncertainties for the electro-hydraulic actuator (EHA) system is presented. The main feature of the proposed observer is the use of an intersample output predictor which allows the users to increase the frequency acquisition of the piston position sensor without affecting the convergence performance. The stability analysis of the proposed observer is proved using Lyapunov function adapted to hybrid systems. To show the efficiency of our proposed observer, numerical simulations and experimental validation involving a control application, which combines the designed observer and a PI controller for the purpose of piston position tracking problem, are presented.

**Index Terms**—Continuous–discrete time observers, disturbance observer (DOB), electro-hydraulic actuator (EHA), intersample output predictor, sampled data measurements.

## I. INTRODUCTION

DU TO a high power to weight ratio and their ability to generate high torques/forces outputs, electro-hydraulic actuator (EHA) systems are widely used in several industrial applications [1]–[5]. Despite this advantage, the EHA systems suffer from some drawbacks due principally to their structure. Indeed, the EHA systems are subject to various uncertainties such as model parametric variations [6], [7], highly nonlinear dynamic behavior [8], potential faults such as internal leakage [9], and hard damage affecting their functioning. In the last years, the increasing demand of high precision control for EHA systems renders the development of advance controls' methods necessary to meet the actual requirements in terms of tracking performance.

Despite their actual dominance, the traditional proportional integral derivative (PID) controllers are not robust enough to

Manuscript received March 31, 2015; revised August 5, 2015; accepted January 24, 2016. This work was supported by the Combustion Engine for Range-Extended Electric Vehicle (CEREEV) Project which is funded by the European Union's INTERREG IVA France-Manche-England Programme.

S. Ahmed Ali, A. Christen, and N. Langlois are with the Department of Electrical Engineering, IRSEEM/École Supérieure d'Ingénieurs en Génie Électrique (ESIGELEC), 76801 Rouen, France (e-mail: sofiane.ahmedali@esigelec.fr; Arnaud.Christen@esigelec.fr; nicolas.langlois@esigelec.fr).

S. Begg is with the Department of Electrical Engineering, University of Brighton, Brighton, BN2 4GJ, U.K. (e-mail: S.M.Begg@brighton.ac.uk).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TIE.2016.2531022

counteract the effect of the uncertainties affecting the EHA systems. Therefore, the focus of the researchers has been shifted toward developing nonlinear closed-loop control methods in order to improve the tracking performance for the EHA systems. In the past decades, several nonlinear control techniques have been developed in the literature such as feedback linearization [7], [10] and sliding mode control [11]–[14]. In [6], a novel integration of adaptive control and integral robust feedback was proposed for hydraulic systems with considering all possible modeling uncertainties, and an excellent tracking performance was achieved, which is the first solution for theoretically asymptotic stability with unmatched disturbances for hydraulic systems; others nonlinear controllers such as robust/adaptive robust controllers [15]–[20], [37], [38] and backstepping control [21]–[24] were also proposed. These methods have already proved their efficiency to improve the tracking performance of the EHA systems facing modeling uncertainties, parametric variations, and external disturbances.

However, all aforementioned techniques are full-state feedback ones, i.e., the designed controllers assume that all states of the EHA systems are available for measurements. From practical point of view, this assumption may not be realistic for some hydraulic systems. Indeed, for many hydraulics applications, only the position signal of the actuator is measured via sensor. The other states like velocity and hydraulic pressure are not measured because of the cost-reduction and the space limitation; therefore, states and disturbances observers have recently received in the literature more and more attention.

Several states and disturbances observers were developed by some researchers in the past decade. The idea behind developing these observers is to use the states and the disturbances estimation provided by these observers in order to synthesized an output-feedback controllers which compensate the internal and the external disturbances affecting the EHA systems. At this stage, we can distinguish between two main approaches in the literature. The first approach consists in developing only a state estimator (i.e., an observer) which estimates the unmeasurable state of the EHA systems. These observers ignore both the internal disturbances like parametric variations, modeling uncertainties, and the external disturbances such as the load and the friction torque affecting the hydraulic application. Those types of observers can be found in the work developed by the authors in [25]–[28]. The second approach developed by the authors in [29]–[31] assumes that the states of the EHA systems are measurable and synthesize a disturbance observer (DOB)

83 which estimates the mechanical and the hydraulic disturbances  
 84 affecting the system. These estimations are incorporated then in  
 85 a nonlinear closed-loop controller which compensates the effect  
 86 of the disturbance and improves the tracking performance of the  
 87 desired position for the EHA systems.

88 Recently, the authors in [32] proposed a novel framework  
 89 for the purpose of simultaneous estimation of the unmeasurable  
 90 states and the unmodeled disturbances, and then resulting in an  
 91 excellent output feedback nonlinear robust backstepping con-  
 92 troller for hydraulic systems, by developing an extended state  
 93 observer (ESO) [33] and robust backstepping design. In this  
 94 work, the authors consider that the main uncertainties affecting  
 95 the EHA systems come from the hydraulic part. Therefore, they  
 96 synthesized an observer based on the well-known techniques  
 97 of ESOs [33] which estimates the unmeasurable state and  
 98 the hydraulic disturbances of the EHA systems. The proposed  
 99 observer is also robust facing the mechanical disturbances gen-  
 100 erated by the load driven by the considered EHA system in this  
 101 paper.

102 In the case of hydraulic applications, the main drawback of  
 103 the designed observers [25]–[32] is that they assume that the  
 104 measured variable is continuous. In practical situations, this  
 105 measured variable which is given by the position sensor is sam-  
 106 pled. In other words, the piston positions are available for the  
 107 observer at only sampling times  $t_k$  fixed by the sampling rate  
 108 (i.e., the frequency acquisition) of the sensor. This frequency  
 109 can affect the convergence of the proposed when it comes to the  
 110 matter of implementation of the proposed observer on digital  
 111 signal processors (DSPs).

112 Following the design in [32], the authors in [34] designed  
 113 a sampled data observer which deals with the problem of  
 114 discrete time-measurements for the EHA system. The pro-  
 115 posed observer retains the same benefits which characterize the  
 116 observer proposed in [32] in terms of simultaneous estimation  
 117 of the unmeasurable states and the internal disturbances affect-  
 118 ing the EHA system. The proposed observer involves in its  
 119 structure an intersampled output predictor [35] which ensures  
 120 continuous time estimation of the states and the exponential  
 121 convergence of the observation errors. Moreover, the sampling  
 122 period of the data acquisition of the observer can be augmented  
 123 independently from the frequency acquisition of the sensor  
 124 position without affecting the convergence of the observer.  
 125 However, the designed observer in [34] suffers from two major  
 126 drawback. The first one concerns the Lyapunov function pro-  
 127 vided to prove the exponential convergence of the proposed  
 128 observer. Indeed, the authors in [34] demonstrated the expo-  
 129 nential convergence of the observer only locally between two  
 130 sampling periods. In addition, the performance of the proposed  
 131 observer were validated only in simulations and no experi-  
 132 mental validation of the observer is provided. Comparing to  
 133 the work of the author in [34], two main contributions were  
 134 provided. The first contribution consists in designing a novel  
 135 Lyapunov function based on small gain arguments which guar-  
 136 anty a global exponential convergence of the proposed observer.  
 137 In addition, the maximum sampling period  $T_{\max}$  derived from  
 138 this function is less restrictive comparing to the one derived in  
 139 [34]. The second one is that experimental results performed on  
 140 the experimental test rig of the Brighton University is provided

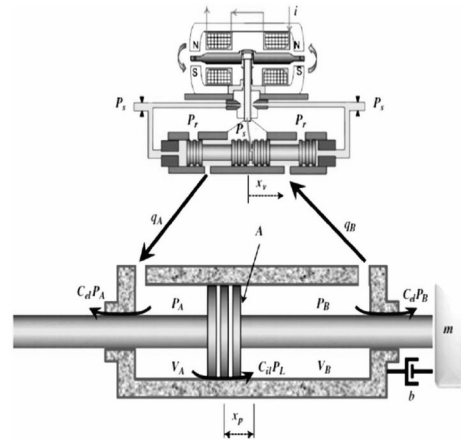


Fig. 1. Schematic of the EHA.

F1:1

for this observer. This is in our acknowledged the first time that  
 such observers were designed and tested experimentally for the  
 EHA systems.

This paper is organized as follows. The EHA modeling  
 issues and the problem formulation are presented in Section II.  
 Section III presents the continuous–discrete time observer for  
 the EHA system. Numerical simulations and experimental val-  
 idation showing the effectiveness of our proposed observer are  
 presented in Section IV. Section V contains the conclusion and  
 the future works.

## II. EHA MODELING

151

The schematic of the EHA studied in this paper is depicted in  
 Fig. 1 [26], [29]. The EHA system contains usually three parts,  
 namely the electrical, the mechanical, and the hydraulic part.  
 These parts represent an interconnected subsystem in such a  
 way that the dynamic of each subsystem influences the dynam-  
 ics of the others. The electrical part of the EHA system is a  
 servo-valve (top of Fig. 1) which controls the fluid dynamics  
 inside the chambers. The spool valve is driven by the electri-  
 cal input current  $u$  of a torque motor. The displacement of the  
 spool valve  $x_v$  together with the load pressure  $P_L$  controls the  
 fluid dynamic inside two chambers A and B which constitute  
 the hydraulic part of the EHA system. The mechanical part of  
 the EHA system is a cylindrical piston which is modeled as  
 a classical mass-spring system. The position of the cylindrical  
 piston  $x_p$  obeys to the fundamental principle of dynamics.

### A. State-Space Representation of the EHA

167

Considering the following states variable:  $x =$   
 $[x_1, x_2, x_3]^T = [x_p, \dot{x}_p, P_L]^T$ , the state-space representa-  
 tion of the EHA system can be written under the following  
 form [26], [29], [31]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{A_p}{m}x_3 \\ \dot{x}_3 = -\alpha x_2 - \beta x_3 + \gamma \sqrt{P_s} - \text{sign}(u)x_3 u \end{cases} \quad (1)$$

171

172 where  $x_p$  is the piston position (m).  $\dot{x}_p$  (m/s) is the piston veloc-  
 173 ity and  $P_L$  (Pa) is the pressure load inside the chambers of the  
 174 hydraulic part.  $k$  is the load spring constant (N/m),  $b$  is the vis-  
 175 cious damping coefficient [N/(m/s)], and  $A_p$  is the cylinder bore  
 176 ( $\text{m}^2$ ).  $P_s$  is the supply pressure (Pa).  $\alpha, \beta, \gamma$  are the hydraulic  
 177 coefficients of the EHA model. These coefficients depend on  
 178 the flow characteristics of the EHA system. For more details  
 179 about the expression of the hydraulic coefficients  $\alpha, \beta, \gamma$  and  
 180 the modeling issues of the EHA system, the reader is referred  
 181 to the work of the authors [26], [29] and their corresponding  
 182 literature.

### 183 B. Modeling Uncertainties and Time-Varying 184 Disturbances Affecting the EHA System

185 In [29] and [31], the authors distinguished between two types  
 186 of disturbances  $d_1$  and  $d_2$  which can affect the EHA system.  
 187 The first one  $d_1$  is the mechanical disturbance which is the  
 188 result of lumping together the modeling parametric uncertain-  
 189 ties, the load charge  $F_{\text{Load}}$ , and the friction force  $F_{\text{friction}}$  acting  
 190 on the mechanical part of the EHA system. As reported by the  
 191 authors in [32], the second term  $d_2$  does not hold the same  
 192 significance as  $d_1$ . Indeed,  $d_2$  represents the parametric deviation  
 193 over the hydraulic coefficients  $\alpha, \beta, \gamma$  and potential leakage  
 194 affecting the hydraulic device of the EHA system. These param-  
 195 eters are also sensitive to temperature inside the EHA system.  
 196 Taking into account these issues, the disturbed EHA model can  
 197 be written as follows [29]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{A_p}{m}x_3 - \frac{d_1}{m} \\ \dot{x}_3 = -\alpha x_2 - \beta x_3 + \gamma \sqrt{P_s - \text{sign}(u)x_3 u} + d_2 \end{cases} \quad (2)$$

198 where  $d_1(t)$  and  $d_2(t)$  are expressed as follows [31]:

$$\begin{aligned} d_1(t) &= -\Delta \frac{k}{m}x_1 - \Delta \frac{b}{m}x_2 - \Delta \frac{A_p}{m}x_3 + F_{\text{Load}} + F_{\text{friction}} \\ d_2(t) &= -\Delta \alpha x_2 - \Delta \beta x_3 + \Delta \gamma \sqrt{P_s - \text{sign}(u)x_3 u}. \end{aligned} \quad (3)$$

199 The  $\Delta$  symbolizes the considered parametric uncertainties  
 200 affecting the mechanical and the hydraulic part of the EHA sys-  
 201 tem. System (2) can be expressed under the following compact  
 202 form:

$$\begin{cases} \dot{x} = Ax + \varphi(x, u) + B_d d \\ y = Cx = x_1 \end{cases} \quad (4)$$

203 where  $x \in \mathbf{R}^3$  and  $y \in \mathbf{R}$  represent, respectively, the state vec-  
 204 tor and the measured piston position  $x_1 = x_p$ . The vector  $u \in$   
 205  $\mathbf{R}$  describes the set of admissible inputs.  $d(t) \in \mathbf{R}^2$  denotes  
 206 the vector of the disturbances which affect the EHA.  $B_d$  with  
 207 dimensions  $3 \times 2$ . The matrices  $A, B_d, C$ , and vector  $\varphi(x, u)$   
 208 have the following structure:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{A_p}{m} \\ 0 & 0 & 0 \end{pmatrix}$$

$$B_d = \begin{pmatrix} 0 & 0 \\ \frac{-1}{m} & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = (1 \ 0 \ 0)$$

$$\varphi(x, u) = \begin{pmatrix} 0 \\ -\frac{k}{m}x_1 - \frac{b}{m}x_2 \\ -\alpha x_2 - \beta x_3 + \gamma \sqrt{P_s - \text{sign}(u)x_3 u} \end{pmatrix}.$$

### C. Problem Formulation

209 For system (4), the piston position is available for measure-  
 210 ment only at each sampling times  $t_k$  imposed by the frequency  
 211 acquisition (the sampling period) of the sensor manufacturer. In  
 212 this paper, we have to design a robust sampled data observer  
 213 which simultaneously estimates the unmeasurable states  $x_2,$   
 214  $x_3$ , and the hydraulic disturbance term  $d_2$  of system (4). The  
 215 designed observer must deal with the sampling phenomenon of  
 216 the measured piston position  $x_p$  and must be robust facing the  
 217 mechanical disturbance term  $d_1(t)$ . Under these considerations,  
 218 system (4) is rewritten as follows:  
 219

$$\begin{cases} \dot{x} = Ax + \varphi(x, u) + B_d d \\ y(t_k) = Cx(t_k) = x_1(t_k). \end{cases} \quad (5)$$

220 System (5) combines a continuous dynamic behavior for the  
 221 states  $x_1, x_2, x_3$  between two sampling times  $[t_k, t_{k+1}]$  and an  
 222 updated step for the state  $x_1$  which occurs at the sampling times  
 223  $t = t_k$ .

## 224 III. CONTINUOUS–DISCRETE TIME-OBSERVER DESIGN 225 FOR THE EHA SYSTEM

226 In this section, we design a continuous–discrete time  
 227 observer for the EHA system. Since  $d_2$  is the main distur-  
 228 bance term, we use the well-known technique of the augmented  
 229 state system in order to estimate it. Following this, we add an  
 230 extended variable  $x_4 = d_2$  such as  $\dot{x}_4 = h(t)$  to system (5) so  
 231 that the augmented state system can be written as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \overline{\varphi(\bar{x}, u)} + \delta(t) \\ y = \bar{C}\bar{x} = x_1 \end{cases} \quad (6)$$

232 where  $\bar{x} = [x_1, x_2, x_3, x_4]$  and

$$\bar{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{A_p}{m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overline{\varphi(\bar{x}, u)} = \begin{pmatrix} 0 \\ -\frac{k}{m}x_1 - \frac{b}{m}x_2 \\ -\alpha x_2 - \beta x_3 + \gamma \sqrt{P_s - \text{sign}(u)x_3 u} \\ 0 \end{pmatrix}$$

$$\delta(t) = \begin{pmatrix} 0 \\ \frac{-d_1}{m} \\ 0 \\ h \end{pmatrix}$$

$$\bar{C} = (1 \ 0 \ 0 \ 0).$$

### 233 A. Observer Design

234 In this paper, our proposed observer will be designed under  
235 the same assumptions taken in [32].

236 *Assumption 1:* The disturbance term  $d_1(t)$  is bounded by  
237 a real unknown constant  $\mu_1$  such that  $(|d_1(t)| < \mu_1)$  and the  
238 function  $h(t)$  is bounded by a real unknown constant  $\mu_2$  such  
239 that  $(|h(t)| < \mu_2)$ .

240 *Remark 1:* This assumption means that the mechanical dis-  
241 turbance and the derivative of the hydraulic disturbances affect-  
242 ing the EHA system are bounded by some unknown constants.  
243 From a practical point of view, the EHA system is a physical  
244 system which is BIBS (bounded input bounded state). So, it is  
245 quite reasonable to consider such assumption.

246 *Assumption 2:* In their practical range of parametric varia-  
247 tions, the functions  $\overline{\varphi_2(\bar{x}, u)} = -\frac{k}{m}x_1 - \frac{b}{m}x_2$  and  $\overline{\varphi_3(\bar{x}, u)} =$   
248  $-\alpha x_2 - \beta x_3 + \gamma\sqrt{P_s - \text{sign}(u)}x_3u$  are locally (inside com-  
249 pact set) Lipschitz with respect to  $(x_1, x_2, x_3)$ , i.e.,  $\exists\beta_0 > 0$ ,  
250 such that

$$\overline{\varphi(X, u)} - \overline{\varphi(Y, u)} \leq \beta_0 \|X - Y\|, \quad i = 2, 3. \quad (7)$$

251 *Remark 2:* At this point, we mention that the function  
252  $\overline{\varphi_2(\bar{x}, u)}$  is globally Lipschitz with respect to  $x_2, x_3$ . The func-  
253 tion  $\overline{\varphi_3(\bar{x}, u)}$  is differentiable everywhere except at  $u = 0$ ,  
254 however, and as stated by the authors in [32], this function is  
255 continuous and its derivative exists in the left and the right side  
256 of  $u = 0$  and it is finite. Hence, we can find a compact set so  
257 that  $\overline{\varphi_3(\bar{x}, u)}$  is locally Lipschitz.

258 Based on [35], let us consider the following continuous-  
259 discrete time observer:

$$\begin{cases} \dot{\hat{x}} &= \bar{A}\hat{x} + \overline{\varphi(f(\hat{x}), u)} - \theta\Delta_\theta^{-1}K(\bar{C}\hat{x} - w(t)) \\ \dot{w}(t) &= \bar{C}\left(\bar{A}\hat{x} + \overline{\varphi(f(\hat{x}), u)}\right) \quad t \in [t_k, t_{k+1}) \quad k \in \mathbb{N} \\ w(t_k) &= y(t_k) = x_1(t_k). \end{cases} \quad (8)$$

The function  $f$  is a saturation function which is introduced to  
guaranty that the estimated states  $\hat{x}$  remains inside the compact  
set so that the Lipschitz constant  $\beta_0$  always exists. The  $\Delta_\theta$  is a  
diagonal matrix  $4 \times 4$  defined by

$$\Delta_\theta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\theta} & 0 & 0 \\ 0 & 0 & \frac{1}{\theta^2} & 0 \\ 0 & 0 & 0 & \frac{1}{\theta^3} \end{pmatrix}$$

260 and the vector gains  $K \in \mathbb{R}^{4 \times 1}$  are chosen so that the matrix  
261  $(\bar{A} - K\bar{C})$  is Hurwitz. The vector  $\hat{x}$  is the continuous-time  
262 estimate of the system state  $\bar{x}$ . The vector  $w(t)$  represents the  
263 prediction of the output between two sampling times. The pre-  
264 diction  $w(t)$  is updated (reinitialized) at each sampling instant  
265  $t = t_k$ .

### 266 B. Observability Analysis

From the structure of matrices  $\bar{A}, \bar{C}$  in system (6), it can  
be easily checked that the pair  $(\bar{A}, \bar{C})$  is observable. Hence,  
their exists two matrices  $P, Q$  such that the following Lyapunov  
function is satisfied:

$$P(\bar{A} - K\bar{C}) + (\bar{A} - K\bar{C})^T P \leq -\mu \mathbb{I}_n$$

where  $\mu > 0$  is a free-positive constant and  $P$  is a symmetric  
positive definite matrix.

*Remark 3:* Comparing to the work of the authors in [26],  
[32], the novelty in the designed observer (8) is the introduc-  
tion of the intersample output predictor term  $w(t)$  [35] in the  
correction term. The dynamic of this predictor is simply a copy  
of the dynamics of system states equations. The role of the out-  
put predictor term is to provide a continuous time prediction of  
the output measured variable  $y(t)$ . Indeed, since the measured  
output variable  $y(t)$  is sampled, its values  $y(t_k)$  are available  
for the observer only at sampling times  $t = t_k$ . Comparing to  
constant-gain zero-order-hold (ZOH) approaches which main-  
tain  $y(t_k)$  constant between the sampling times, the output  
predictor term  $w(t)$  will provide a continuous time estimation  
of  $y(t)$  as it is the case in continuous time-observer design  
framework.

Now, we are able to state the main results of this paper.

*Theorem 1:* Consider the EHA system (6), and suppose that  
assumptions (1–2) holds, given a sampling period  $T$ , choose  
 $\sigma_0, \sigma_1, \sigma_2$  as in (17), define  $\sigma_3 = Te^{\sigma T} \frac{2\sigma_1(\theta + \beta_0)}{\sigma_0\sqrt{\lambda_{\min}(P)}}$  then sys-  
tem (8) is an exponential sampled data observer for system  
(6) with the following properties: the vector of the observa-  
tion error  $\|\bar{e}_x\|$  converges exponentially toward a ball whose  
radius  $R = \frac{2\sigma_2}{\sigma_0\sqrt{\lambda_{\min}(P)(1-\sigma_3)}}$ . Moreover, there exists a real  
positive bounded  $T_{\max}$  satisfying inequality (34), so that for all  
 $T \in (0, T_{\max})$ , the radius of the ball can be made as small as  
desired by choosing large values of  $\theta$  and  $k_{i=1, \dots, 4}$ .

*Proof 1:* The proof of this theorem 1 is inspired from the  
work of the authors in [35]. Let us now define the following  
observer  $e_{\bar{x}}$  and the output  $e_w(t)$  errors as follows:

$$\begin{cases} e_{\bar{x}}(t) = \hat{x} - \bar{x} \\ e_w(t) = w(t) - y(t) = w(t) - \bar{C}\bar{x}. \end{cases} \quad (9)$$

Combining (6) and (8), we can easily check that for the EHA  
system (6), the following properties are satisfied:  $\theta\Delta_\theta^{-1}\bar{A}\Delta_\theta =$   
 $\theta\bar{A}$  and  $\Delta_\theta^{-1}K\bar{C} = \Delta_\theta^{-1}K\bar{C}\Delta_\theta$ . Introducing the well-known  
change in coordinate in the high gain literature  $\bar{e}_x = \Delta_\theta e_x$   
yields the following dynamics of the state and the output errors:

$$\begin{cases} \dot{\bar{e}}_x = \theta(\bar{A} - K\bar{C})\bar{e}_x + \Delta_\theta \left( \overline{\varphi(f(\hat{x}), u)} - \overline{\varphi(\bar{x}, u)} \right) \\ \quad + \theta K e_w - \Delta_\theta \delta(t) \\ \dot{e}_w = \theta \bar{e}_{x2} + \left( \overline{\varphi_1(f(\hat{x}), u)} - \overline{\varphi_1(\bar{x}, u)} \right). \end{cases} \quad (10)$$

Let us now consider the following candidate Lyapunov  
quadratic function  $V = \bar{e}_x^T P \bar{e}_x$ :

$$\begin{aligned} \dot{V} &\leq -\mu\theta\|\bar{e}_x\|^2 + 2\bar{e}_x^T P \Delta_\theta \left( \overline{\varphi(f(\hat{x}), u)} - \overline{\varphi(\bar{x}, u)} \right) \\ &\quad + 2\theta\bar{e}_x^T P K e_w(t) - 2\bar{e}_x^T P \Delta_\theta \delta. \end{aligned} \quad (11)$$

Taking into account Assumptions (1–2) we have

$$\begin{aligned} \dot{V} &\leq -\mu\theta\|\bar{e}_x\|^2 + 4\beta_0\lambda_{\max}(P)\|\bar{e}_x\|^2 + 2\theta\|PK\|\|\bar{e}_x\|\|e_w(t)\| \\ &\quad + 4\lambda_{\max}(P)\|\bar{e}_x\|\xi \end{aligned} \quad (12)$$

where  $\xi = \sqrt{\mu_1^2 + \mu_2^2}$ .



306 Using the well-known property

$$\lambda_{\min}(P) \|\bar{e}_{\bar{x}}\|^2 \leq V \leq \lambda_{\max}(P) \|\bar{e}_{\bar{x}}\|^2 \quad (13)$$

307 we derive

$$\begin{aligned} \dot{V} \leq & -\mu\theta \frac{V}{\lambda_{\max}(P)} + \frac{4\beta_0\lambda_{\max}(P)V}{\lambda_{\min}(P)} \\ & + 2\theta\|PK\| \sqrt{\frac{V}{\lambda_{\min}(P)}} |e_w(t)| + 4\lambda_{\max}(P) \sqrt{\frac{V}{\lambda_{\min}(P)}} \xi. \end{aligned} \quad (14)$$

308 Now choosing the parameter  $\theta$  such that  $\theta > \theta_0$  with  $\theta_0 =$   
309  $\sup \left\{ 1, \frac{8\beta_0\lambda_{\max}^2(P)}{\mu\lambda_{\min}(P)} \right\}$ , we have

$$\begin{aligned} \dot{V} \leq & -\mu\theta \frac{V}{2\lambda_{\max}(P)} + 2\theta\|PK\| \sqrt{\frac{V}{\lambda_{\min}(P)}} |e_w(t)| \\ & + 4\lambda_{\max}(P) \sqrt{\frac{V}{\lambda_{\min}(P)}} \xi. \end{aligned} \quad (15)$$

310 Considering now the function  $W = \sqrt{V}$ , then we obtain

$$\begin{aligned} \dot{W} \leq & -\mu\theta \frac{W}{4\lambda_{\max}(P)} + \theta\|PK\| \frac{|e_w(t)|}{\sqrt{\lambda_{\min}(P)}} \\ & + 2 \frac{\lambda_{\max}(P)}{\sqrt{\lambda_{\min}(P)}} \xi. \end{aligned} \quad (16)$$

311 Let us set

$$\begin{cases} \sigma_0 = \frac{\mu\theta}{4\lambda_{\max}(P)} \\ \sigma_1 = \frac{\theta\|PK\|}{\sqrt{\lambda_{\min}(P)}} \\ \sigma_2 = 2 \frac{\lambda_{\max}(P)}{\sqrt{\lambda_{\min}(P)}}. \end{cases} \quad (17)$$

312 Integrating (16), then

$$\begin{aligned} W(t) \leq & e^{-\sigma_0(t-t_0)}W(t_0) + \sigma_1 e^{-\sigma_0 t} \int_{t_0}^t e^{\sigma_0 s} |e_w(s)| ds \\ & + \sigma_2 e^{-\sigma_0 t} \int_{t_0}^t e^{\sigma_0 s} \|\xi(s)\| ds. \end{aligned} \quad (18)$$

313 Multiplying both sides of (18) by  $e^{\sigma t}$  and using the fact that  
314  $e^{-(\sigma_0-\sigma)t} < 1$  we derive

$$\begin{aligned} e^{\sigma t}W(t) \leq & M(t_0) + \sigma_1 e^{-(\sigma_0-\sigma)t} \int_{t_0}^t e^{\sigma_0 s} |e_w(s)| ds \\ & + \sigma_2 e^{-(\sigma_0-\sigma)t} \int_{t_0}^t e^{\sigma_0 s} \|\xi(s)\| ds \end{aligned} \quad (19)$$

315 where  $M(t_0) = e^{\sigma_0 t_0}W(t_0)$ .

316 On the other hand, we have

$$\begin{aligned} e^{\sigma t}W(t) \leq & M(t_0) + \sigma_1 e^{-(\sigma_0-\sigma)t} \int_{t_0}^t e^{(\sigma_0-\sigma)s} e^{\sigma s} |e_w(s)| ds \\ & + \sigma_2 e^{-(\sigma_0-\sigma)t} \int_{t_0}^t e^{(\sigma_0-\sigma)s} e^{\sigma s} \|\xi(s)\| ds \end{aligned} \quad (20)$$

317 or

$$e^{\sigma t}W(t) \leq M(t_0) \quad (21)$$

$$\begin{aligned} & + \sigma_1 e^{-(\sigma_0-\sigma)t} \left( \int_{t_0}^t e^{(\sigma_0-\sigma)s} ds \right) \sup_{t_0 \leq s \leq t} (e^{\sigma s} |e_w(s)|) \\ & + \sigma_2 e^{-(\sigma_0-\sigma)t} \left( \int_{t_0}^t e^{(\sigma_0-\sigma)s} ds \right) \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\xi(s)\|) \end{aligned} \quad (22)$$

which leads to

$$\begin{aligned} e^{\sigma t}W(t) \leq & M(t_0) \\ & + \frac{\sigma_1}{\sigma_0 - \sigma} \sup_{t_0 \leq s \leq t} (e^{\sigma s} |e_w(s)|) \\ & + \frac{\sigma_2}{\sigma_0 - \sigma} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\xi(s)\|). \end{aligned} \quad (23)$$

Now taking  $0 < \sigma < \sigma_0/2$ , we derive

$$\begin{aligned} \sup_{t_0 \leq s \leq t} (e^{\sigma s}W(s)) \leq & M(t_0) \\ & + 2 \frac{\sigma_1}{\sigma_0} \sup_{t_0 \leq s \leq t} (e^{\sigma s} |e_w(s)|) \\ & + 2 \frac{\sigma_2}{\sigma_0} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\xi(s)\|) \end{aligned} \quad (24)$$

and

$$\begin{aligned} W(t) \leq & e^{-\sigma t}M(t_0) + 2 \frac{\sigma_1}{\sigma_0} \sup_{t_0 \leq s \leq t} (e^{-\sigma(t-s)} |e_w(s)|) \\ & + 2 \frac{\sigma_2}{\sigma_0} \sup_{t_0 \leq s \leq t} (e^{-\sigma(t-s)} \|\xi(s)\|) \end{aligned} \quad (25)$$

which leads to

$$\begin{aligned} \|\bar{e}_{\bar{x}}\| \leq & e^{-\sigma t} \frac{M(t_0)}{\sqrt{\lambda_{\min}(P)}} \\ & + \frac{2\sigma_1}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{-\sigma(t-s)} |e_w(s)|) \\ & + \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{-\sigma(t-s)} \|\xi(s)\|) \end{aligned} \quad (26)$$

and

$$\begin{aligned} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_{\bar{x}}\|) \leq & \frac{M(t_0)}{\sqrt{\lambda_{\min}(P)}} \\ & + \frac{2\sigma_1}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{\sigma s} |e_w(s)|) \\ & + \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\xi(s)\|). \end{aligned} \quad (27)$$

On the other hand, we have from (10) the following expression  
of  $|e_w(t)|$ :

$$|e_w(t)| = \int_{t_k}^t |\theta \bar{e}_{\bar{x}2} + (\overline{\varphi_1(f(\hat{x}), u)} - \overline{\varphi_1(\bar{x}, u)})| ds. \quad (28)$$

Multiplying again both sides of (28) by  $e^{\sigma t}$  and taking into  
account assumptions 1–2, we have

$$e^{\sigma t} |e_w(t)| \leq e^{\sigma t} (\theta + \beta_0) \int_{t_k}^t e^{-\sigma s} e^{\sigma s} \|\bar{e}_{\bar{x}}(s)\| ds \quad (29)$$

which leads to

$$\begin{aligned} e^{\sigma t} |e_w(t)| \leq & e^{\sigma t} (\theta + \beta_0) \left( \int_{t_k}^t e^{-\sigma s} ds \right) \\ & \sup_{t_k \leq s \leq t} (e^{\sigma s} \|\bar{e}_{\bar{x}}(s)\|) ds \end{aligned} \quad (30)$$

328 taking into account that  $e^{-\sigma s} < 1$ , we derive that

$$\begin{aligned} \sup_{t_k \leq s \leq t} e^{\sigma s} |e_w(s)| &\leq T e^{\sigma T} (\theta + \beta_0) \\ \sup_{t_k \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|) ds & \end{aligned} \quad (31)$$

329 since  $\sup_{t_k \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|) \leq \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|)$  and  
330 taking into account that  $t > t_0, t_1, \dots, t_k$  we derive that

$$\begin{aligned} \sup_{t_0 \leq s \leq t} e^{\sigma s} |e_w(s)| &\leq T e^{\sigma T} (\theta + \beta_0) \\ \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|) ds & \end{aligned} \quad (32)$$

331 Combining (32) with (27) we have

$$\begin{aligned} \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_x\|) &\leq \frac{M(t_0)}{\sqrt{\lambda_{\min}(P)}} \\ &+ T e^{\sigma T} \frac{2\sigma_1}{\sigma_0 \sqrt{\lambda_{\min}(P)}} (\theta + \beta_0) \sup_{t_0 \leq s \leq t} (e^{\sigma s} \|\bar{e}_x(s)\|) ds \\ &+ \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)}} \sup_{t_0 \leq s \leq t} (e^{\sigma(s)} \|\xi(s)\|) \end{aligned} \quad (33)$$

332 setting  $\sigma_3 = T e^{\sigma T} \frac{2\sigma_1(\theta + \beta_0)}{\sigma_0 \sqrt{\lambda_{\min}(P)}}$  then selecting  $T_{\max}$  satisfying  
333 the following the small gain condition:

$$T_{\max} e^{\sigma T_{\max}} \frac{2\sigma_1(\theta + \beta_0)}{\sigma_0 \sqrt{\lambda_{\min}(P)}} < 1 \quad (34)$$

334 we have

$$\begin{aligned} \|\bar{e}_x\| &\leq e^{-\sigma t} \frac{M(t_0)}{\sqrt{\lambda_{\min}(P)(1 - \sigma_3)}} \\ &+ \frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)(1 - \sigma_3)}} \sup_{t_0 \leq s \leq t} \|\xi(s)\|. \end{aligned} \quad (35)$$

335 This complete the proof of Theorem 1.

336 *Remark 4:* Contrary to ([34], (35) demonstrates the global  
337 exponential convergence of the vector of the observation error  
338  $\|\bar{e}_x\|$  toward a ball whose radius depends on the magnitude of  
339 the disturbance vector  $\xi$ . In addition, the maximum sampling  
340 period  $T_{\max}$  derived in (34) is less restrictive comparing to the  
341 one derived in [34] which depends on the computation of a  
342 bounded positive function  $\psi(t)$  (see (13) in [34]).

343 *Remark 5:* The radius of the ball  $R$  is defined such that  $R =$   
344  $\frac{2\sigma_2}{\sigma_0 \sqrt{\lambda_{\min}(P)(1 - \sigma_3)}}$ . We also notice that in the case where there  
345 is no mechanical disturbances (i.e.,  $d_1 = 0$ ) and the hydraulic  
346 disturbances are constant or equal to 0, we have an exponential  
347 convergence of the observation error  $\|\bar{e}_x\|$  toward 0. Looking at  
348 the expression of the maximum sampling period  $T_{\max}$  in (34),  
349 we can easily see that when  $\sigma$  tends to zero,  $T_{\max} \simeq \frac{1}{\theta}$ . Hence,  
350 augmenting  $\theta$  will diminish the value of  $T_{\max}$ . On the other  
351 hand, large values of parameter  $\theta$  will contribute to reduce the  
352 radius  $R$  and hence to improve the performance of our observer.  
353 However, it is well known that the high gain observers litera-  
354 ture, augmenting the values of  $\theta$  will lead to the undesirable  
355 peaking phenomenon which consists in an impulsive behavior  
356 of the states estimation trajectory around initial conditions.

TABLE I  
NUMERICAL PARAMETER VALUES FOR THE EHA SYSTEM

Parameters	Value
$m$	0.5
$b$	0
$k$	$5.651110 \times 10^5$
$A_p$	$5.058 \times 10^{-4}$
$k_v$	$1.333 \times 10^{-5}$
$\alpha$	$3.257 \times 10^{10}$
$\beta$	2.146
$\gamma$	$7.169 \times 10^9$
$P_s$	$2.1 \times 10^7$

TABLE II  
PARAMETERS OF THE HYBRID OBSERVER

Parameter	$\theta$	$K = \begin{pmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{pmatrix}$	$T_s$

#### IV. SIMULATIONS AND EXPERIMENTAL RESULTS

##### A. Numerical Simulation of the Hybrid Observer Coupled With PI Controller for the EHA System Subject to Mechanical and Hydraulic Disturbances

The performance of the proposed observer will be evaluated  
first under MATLAB/Simulink Software. For the purpose of  
comparison, the numerical simulations were performed on the  
EHA system validated experimentally by the authors in [26]  
and [29]. The model parameters' values are shown in Table I.

In this numerical simulations, we will demonstrate the  
effectiveness of our proposed observer in terms of states/  
disturbances estimation and positioning control. In [29], the  
authors considered a sinusoidal reference position signal  $x_{1d} =$   
 $0.008 \sin(2\pi t)$ . For the purpose of tracking  $x_{1d}$ , a PI controller  
was employed and combined with the proposed observer (8) so  
that the novel PI control law  $u$  is expressed as follows:

$$u = K_p(w(t) - x_{1d}) + K_i \int (w(t) - x_{1d}) \quad (36)$$

where  $x_1 = x_p$  is the piston position and  $K_p = 3.18 \times$   
 $10^{-2}$ ,  $K_i = 100$  are the PI gains. The PI controller gains were  
tuned in order to track. The numerical simulations were per-  
formed using the Runge–Kutta solver with a fixed step size  
 $T_{\text{sim}} = 10^{-4}$  s. The parameters of the hybrid observer are sum-  
marized in Table II where  $T_s$  is the sampling period of our  
proposed hybrid (continuous–discrete time) observer.

The values of the observer parameters used in this simulation  
are  $\theta = 1000$ ,  $K = (10, 35, 49\ 426, 23\ 724)$  and  $T_s = 1$  ms.

The evaluation of our observer is performed under the con-  
sideration that both mechanical and hydraulic disturbances  
affect the considered EHA system in this paper. For the  
mechanical disturbance term  $d_1$ , we have taken the same one  
considered by the authors [29]. To show the robustness of our  
observer facing the mechanical disturbances, we considered  
it in the simulation not from the beginning but at  $t = 10$  s.  
Hence, the term  $d_1$  in the disturbed model of the EHA in (2)

T1:1  
T1:2

T2:1  
T2:2

357

358

359

360

361

362

363

364

365

366

367

368

369

370

371

372

373

374

375

376

377

378

379

380

381

382

383

384

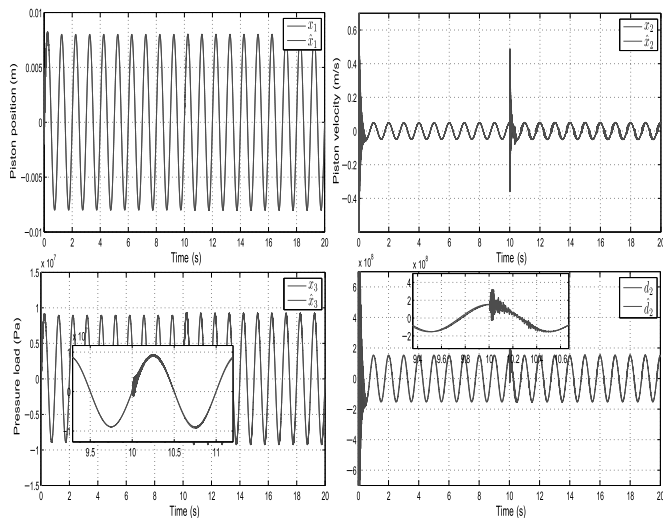
385

386

387

388

389



F2:1 Fig. 2. Estimation of  $x_1$ ,  $x_2$ ,  $x_3$ ,  $d_2$  for  $\theta = 1000$  and  $T_s = 1$  ms with  
 F2:2 mechanical and hydraulic disturbances.

390 is expressed as follows:

$$d_1(t) = \begin{cases} 0, & \text{if } t < 10 \text{ s} \\ 294 \sin(62.83x_1) + 20 \operatorname{sign}(x_2), & \text{if } t \geq 10 \text{ s}. \end{cases}$$

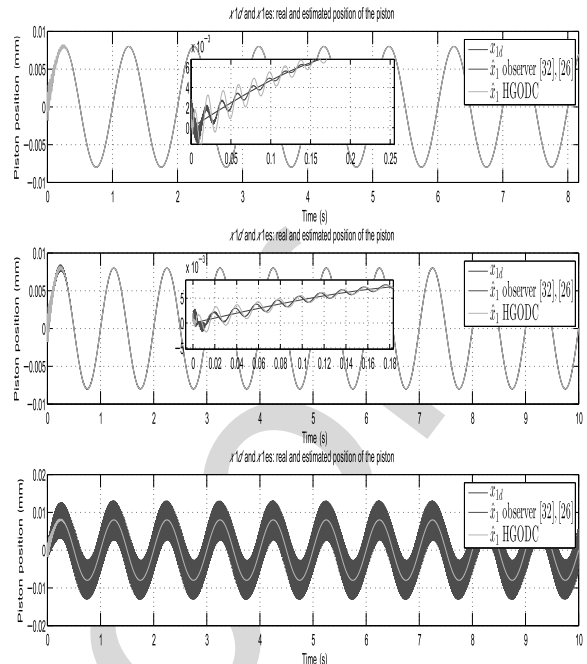
391 We also assume in this simulation that 10% additive para-  
 392 metric variation affects the hydraulic coefficients  $\gamma$ ; hence  
 393 (see Section II), the hydraulic disturbance term  $d_2$  takes the  
 394 following form:

$$d_2(t) = 10\% \sqrt{P_s - \operatorname{sign}(u)x_3u}.$$

395 From Fig. 2, we can see that the tracking performance of  
 396 the reference  $x_{1d}$  even in the presence of the mechanical dis-  
 397 turbance at  $t = 10$  s is achieved correctly by the PI controller  
 398 (36). The robustness of the PI controller facing the mechan-  
 399 ical disturbance can be also seen in Fig. 2 where we can see  
 400 that this disturbance has no effect on the tracking performance  
 401 of the motion reference trajectory  $x_{1d}$ . For the estimation of  
 402 the piston velocity  $x_2$ , the pressure load  $x_3$ , and the hydraulic  
 403 disturbance term  $d_2$ , we can see the effect of the mechanical  
 404 disturbance (see Fig. 2 top right, bottom left, and right) which  
 405 consists in a deviation of the states estimation trajectory occur-  
 406 ring at  $t = 10$  s. Meanwhile, this deviation is quickly rejected  
 407 by the observer, thanks to the large value of parameter  $\theta$  taken  
 408 in this simulation. As mentioned in Remark 5, large values of  
 409 parameter  $\theta$  will lead to a better rejection of the mechanical and  
 410 the hydraulic disturbance term, however, this will amplify the  
 411 peaking phenomenon which consists in an impulsive behavior  
 412 of the trajectory of the states estimation at the beginning of the  
 413 simulation (see Fig. 2).

#### 414 B. Performance Comparison With the Observer 415 Designed in [26] and [32]

416 To show the performance of our proposed observer, we have  
 417 performed a comparison with the observers designed in [26]  
 418 and [32]. Indeed, the observers [26], [32] have the same high  
 419 gain like observer structure as the one considered in the design



F3:1 Fig. 3. Comparison of position tracking performance between our  
 F3:2 observer [high gain observer discrete-continuous (HGODC)] ( $T_s =$   
 F3:3 1 ms) and observers [26], [32] (top:  $T_s = 0.1$  ms; middle:  $T_s = 0.5$  ms;  
 F3:4 bottom:  $T_s = 1$  ms).

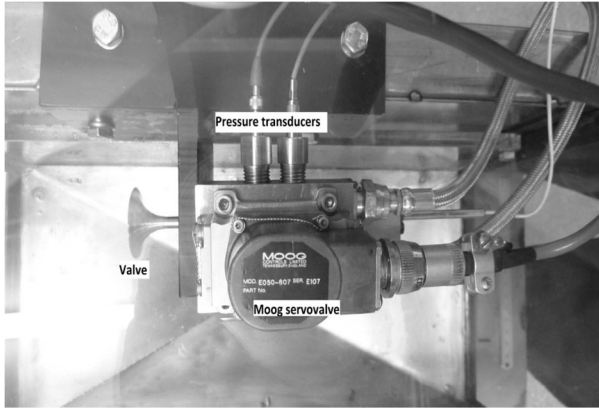
of our observer. By taking into account the sampling effect in  
 the structure of these two observers, a continuous–discrete time  
 version of the observers designed in [26] and [32] can be written  
 as follows:

$$\dot{\hat{x}} = \bar{A}\hat{x} + \overline{\varphi(f(\hat{x}), u)} - H(\bar{C}\hat{x}(t) - y(t_k)). \quad (37)$$

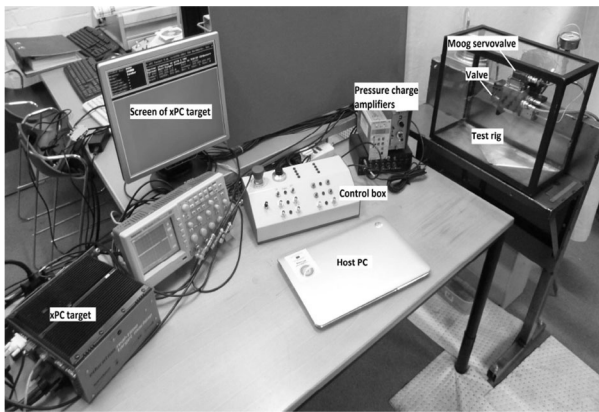
We notice that in the case of our observer  $H = \theta \Delta_\theta^{-1} K$ . The  
 structure of (37) uses the sampled data  $y(t_k)$  in the correction  
 term since that continuous measured variable  $y(t)$  is available  
 only at sampled instants  $t = t_k$ . The simulations presented in  
 Fig. 3 show the performance of observer (8) and observer (37)  
 in terms of position tracking performances. For our proposed  
 observer (named HGODC), we have fixed the value of  $T_s$  to  
 1 ms. For observer (37), three values were taken ( $T_s = 0.1$ ,  
 0.5, and 1 ms). Looking at Fig. 3 (top), we can see that even  
 if observer (37) performs better in the transitory regime, our  
 observer has quite the same performance. Recalling that in this  
 case,  $T_s = 0.1$  ms for observer (37) which is the same sampling  
 period as the one of the solver, we can say that our observer  
 recovers the performances of continuous time observers. When  
 augmenting the sampling period of observer (37) to 0.5 ms,  
 we can see that for observer (37), the performance degrades.  
 Finally, when the two observers have the same sampling peri-  
 ods ( $T_s = 1$  ms), observer (37) diverges and the PID controller,  
 which is based on the estimation provided by observer (37),  
 fails to track the desired trajectory  $x_{1d}$ .

#### 444 C. Experimental Validation

To illustrate the performance of our proposed observer, an  
 experimental test rig platform has been set up and photographed



F4:1 Fig. 4. Moog servo-valve and the EHA actuator assembly.



F5:1 Fig. 5. Control system of the experimental test rig of the EHA system.

447 in Figs. 4 and 5. The test rig was constructed in the Brighton  
 448 University to investigate the performance of the EHA assem-  
 449 bly and the control parameters influencing the motion of the  
 450 poppet valve. The test rig comprised of three main subsystems:  
 451 a hydraulic oil pressure supply; a hydraulic valve actuation  
 452 assembly; and the servo-valve control signal and valve position  
 453 interface.

454 Hydraulic oil from a large tank was supplied to a smaller  
 455 reservoir coupled to a high-pressure pump and accumulator.  
 456 An electromagnetic pressure-limit switch was used to regulate  
 457 the supply of high-pressure oil to the hydraulic valve actuation  
 458 assembly via an oil filter. The supply pressure was regulated to  
 459 70 bar  $\pm$  2 bar by a pressure-limit switch.

460 The actuator body housed a double-acting hydraulic piston,  
 461 oil-sealing end plates, and the high-pressure oil supply  
 462 and return feed lines. A continuous-proportional (four-way)  
 463 directional servo-valve (Moog series 31) was used to control  
 464 the flow rate of hydraulic oil to the hydraulic piston by  
 465 means of a proportional electromagnetic servo control signal.  
 466 The interchangeable poppet valve head was attached to one  
 467 end of the hydraulic piston and a linear variable differential  
 468 transducer (LVDT) was mounted to the opposite end to record  
 469 the change in valve position. The calibration factor for the  
 470 amplified output of the LVDT sensor (Lord MicroStrain) was  
 471 2.97 mm/V  $\pm$  0.005 mm/V. Two piezoelectric gauge pressure

TABLE III  
 EHA PARAMETER VALUES FOR THE EXPERIMENTAL TEST RIG

T3:1  
 T3:2

Parameters	Value
$m$	0.05
$k$	2000
$b$	0.1398
$A_p$	0.0614
$k_v$	0.02
$\alpha$	28.2226
$\beta$	0.0063
$\gamma$	0.0029
$P_s$	$7 \times 10^6$

472 transducers (Kistler type 6125 transducer and type 5011 ampli-  
 473 fier) were used to measure the instantaneous and difference in  
 474 oil pressures in the supply and return chambers either sides of  
 475 the hydraulic piston. The pressure transducer was calibrated to  
 476 20 bar/V. The full-scale error in the transducer was  $\pm 3$  bar. The  
 477 value of the oil pressure at the instant of initial piston motion  
 478 was used as the gauge reference pressure.

479 The control system for the electro-hydraulic valve system  
 480 was based on a real-time simulation and testing platform  
 481 (hardware in the loop, HIL); MathWorks MATLAB Simulink  
 482 and xPC Target application and a real-time target machine  
 483 (Speedgoat GmbH). Positional feedback of the valve was deter-  
 484 mined from the LVDT sensor output. The actuation of the  
 485 directional servo-valve was achieved using a current driver sig-  
 486 nal rated to  $\pm 50$  mA. The displacement of the poppet valve is  
 487 comprised between [20–32] mm. Based on the physical param-  
 488 eters of the experimental test rig [36], the nominal values of the  
 489 EHA model parameters were identified and listed in Table III.

490 In the following experiments, the parameters' values of  
 491 the hybrid observer for this experiment are  $\theta = 500$ ,  $K =$   
 492 (2.8, 2.87, 1.0423, 0.1710), and  $T_s = 1$  ms.

#### D. PID Control Design for the Experimental Test Rig

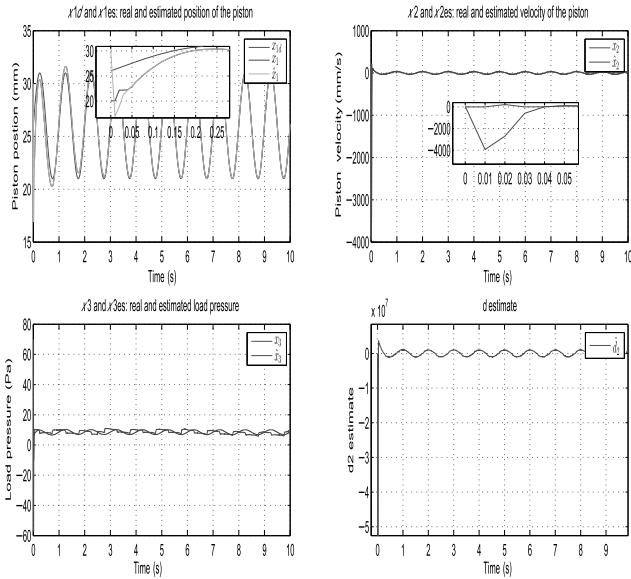
493 In order to track the motion reference  $x_{1d}$ , the following  
 494 PID control law  $u$  with a velocity feedforward action was  
 495 implemented  
 496

$$u = K_p(x_{1d} - w(t)) + K_i \int (x_{1d} - w(t)) + K_d \frac{d}{dt}(x_{1d} - w(t)) + K_f \dot{x}_{1d} \quad (38)$$

497 where  $K_p = 0.54$ ,  $K_i = 1.93$ ,  $K_d = 0.04$ ,  $K_f = 1$ . As it was  
 498 the case in the simulation section, the implemented control law  
 499  $u$  contains the output prediction term  $w(t)$ . We mention that for  
 500 this experimental validation, we used the same Runge–Kutta  
 501 solver with the same fixed step size  $T_{sim} = 10^{-4}$  as in the  
 502 numerical simulations section. The experimental validation was  
 503 conducted with a sampling period  $T_s = 1$  ms which is 10 times  
 504 bigger than the fixed step size of the solver.

#### E. Experimental Performances of the Hybrid Observer Without Disturbance

505 In this section, we investigate the performance of the hybrid  
 506 observer for state estimation and piston position tracking  
 507  
 508



F6:1 Fig. 6. Estimation and tracking performance of  $x_1, x_2, x_3, d_2$  for  
 F6:2  $\theta = 500, T_s = 1$  ms for EHA system without disturbances.

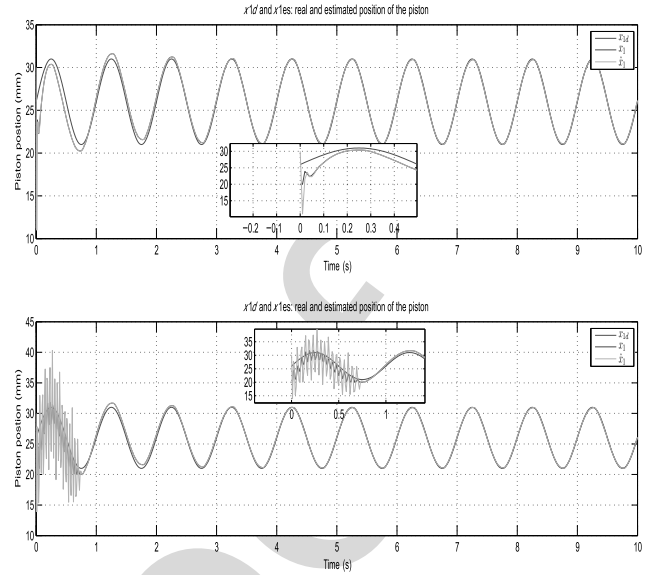


Fig. 7. Estimation and tracking performance of  $x_1$  for  $\theta = 500$ . (Top)  $T_s = 2$  ms. (Bottom)  $T_s = 3$  ms.

509 motion trajectory  $x_{1d} = 26 + 5 \sin(2\pi t)$ . Since the considered  
 510 EHA system does not drive any mechanical load, we have theoret-  
 511 ically  $d_1 \simeq 0$ . We also mention that we have used the  
 512 same nominal values of the EHA system when implementing the  
 513 hybrid observer.

514 In Fig. 6 (top left), we show the performance of the hybrid  
 515 observer in terms of tracking performances and state estimation  
 516 of the piston position  $x_1$ . We can see in Fig. 6 (top left) that both  
 517 the tracking performance and the state estimation are achieved  
 518 correctly by the hybrid observer. For the state estimation of the  
 519 piston position  $x_1$ , the convergence of the hybrid observer is  
 520 achieved with small convergence rate [less than 0.05 s when  
 521 looking to the zoom of Fig. 6 (top left)]. We can see also that the  
 522 tracking performance of the motion reference  $x_{1d}$  by the PI con-  
 523 troller, which uses the output predictor  $w(t)$ , is also achieved  
 524 correctly.

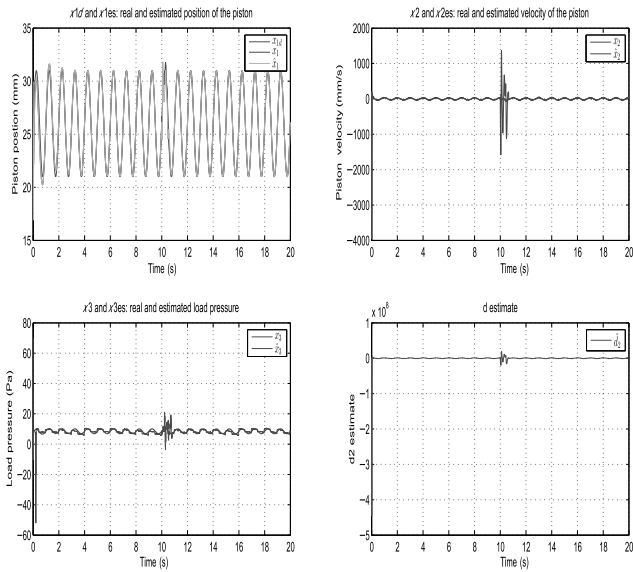
525 Fig. 6 (top right) shows the state estimation of the piston  
 526 velocity  $x_2$ . We can see in Fig. 6 (top right) that our hybrid  
 527 observer provides a very good estimation of the real piston  
 528 velocity  $x_2$ . A quick look to Fig. 6 (top right) shows that the  
 529 effect noise, which comes from the numerical differentiation  
 530 used to obtain the real piston, has been attenuated by our hybrid  
 531 observer.

532 In Fig. 6 (bottom left), we present the estimation results of  
 533 the hydraulic pressure state  $x_3$  by our proposed observer. First,  
 534 we can observe from Fig. 6 (bottom left) that our observer  
 535 provides a good estimation of the hydraulic pressure state  
 536  $x_3$  despite the variations in the hydraulic parameters and the  
 537 hydraulic disturbance which affects the functioning of the EHA  
 538 system. The effects of these disturbances can be viewed. In  
 539 Fig. 6 (bottom right) where we can see that even if there is  
 540 no mechanical load driven by the EHA system, the estimated  
 541 disturbance term  $\hat{d}_2$  is not equal to 0. Indeed, the difficulty of  
 542 capturing the hydraulic parameters ( $\alpha, \beta, \gamma$ ) and the internal  
 543 leakage occurring on the EHA system generates automatically  
 544 the disturbance term  $d_2$ . For the reader, we mention that it was

545 very difficult for us to plot in Fig. 6 (bottom right) the real  
 546 hydraulic disturbance term  $d_2$  for the reasons explained above.  
 547 Finally, we can observe in Fig. 6 (bottom left) that there is  
 548 small phase lag between the real and the estimated hydraulic  
 549 pressure  $x_3$ . This observation is quite interesting because of the  
 550 discrepancies between the numerical simulations and the exper-  
 551 imental validation of our observer. This discrepancies come  
 552 from the difficulty of capturing exactly the hydraulic param-  
 553 eters of the EHA system and the fact that the dynamic of the  
 554 electrical part of the EHA system has been neglected in the  
 555 EHA model. In addition, it appears that the PID control is not  
 556 able to compensate it. Taking into account that the kistler pres-  
 557 sure transducers give a relative and not an absolute pressures  
 558 values in each chamber of the hydraulic actuator, we can say  
 559 that the estimated hydraulic pressures provided by our observer  
 560 are good.

#### F. Effect of the Sampling Period on the Performance of the Hybrid Observer

561 To compute the maximum allowable sampling period  $T_{\max}$   
 562 of the hybrid observer, we can proceed following two poss-  
 563 ible manners. The first one is to compute  $T_{\max}$  analytically  
 564 using the expression in (34); however, this will necessitate to  
 565 know the constant  $\beta_0$  which is practically very difficult to deter-  
 566 mine. The second one is to start with a sampling period  $T_s$  and  
 567 increasing it until the observer diverges. We proceed follow-  
 568 ing the second manner. In Fig. 7, we present the experimental  
 569 results of the estimated piston position  $x_1$  and the tracking per-  
 570 formance of the piston position reference  $x_{1d}$ . We mention that  
 571 we did not report the experimental results concerning the esti-  
 572 mations of the piston velocity  $x_2$ , the hydraulic pressure  $x_3$ ,  
 573 and the hydraulic disturbances  $d_2$ . The reason is that they are  
 574 characterized by the same dynamic behavior as the results pre-  
 575 sented in Fig. 7. When increasing  $T_s$  to 2 ms, we can observe  
 576 from the top of Fig. 7 that the estimated piston position and  
 577 tracking performance are still good. When increasing  $T_s$  to 3 ms,  
 578 we can observe from the bottom of Fig. 7 that the estimated piston position and  
 579 tracking performance are still good.



F8:1 Fig. 8. Estimation and tracking performance of  $x_1, x_2, x_3, d_2$  for  
 F8:2  $\theta = 500, T_s = 1$  ms for EHA system with disturbances.

579 the tracking performance are quite the same as it is the case of  
 580 of  $T_s = 1$  ms. The difference concerns the convergence speed  
 581 which is slower in the case of  $T_s = 1$  ms. When increasing  $T_s$   
 582 to 3 ms, we can observe that the performances of the hybrid  
 583 observer are affected only in the transitory regime (see bottom  
 584 of Fig. 7). Indeed, the oscillations observed in the bottom of  
 585 Fig. 7 are due to the increase in the sampling period  $T_{\max}$  to  
 586 3 ms which clearly affects the transitory regime for our hybrid  
 587 observer. In the permanent regime, the hybrid observer which  
 588 provides the output predictor term  $w(t)$  for the PID controller  
 589 performs well in the case of estimation and the tracking per-  
 590 formance. From this, we can deduce that in the case of this  
 591 experimental results,  $T_{\max} \simeq 2$  ms.

### 592 G. Experimental Performances of the Hybrid Observer 593 With Disturbance

594 To investigate the performance of our observer in the pres-  
 595 ence of disturbance, an additional disturbance term  $d_3 = 2x_{1d}$   
 596 is inserted in the control input at  $t = 10$  s; meanwhile, the  
 597 new control input sent to the control board is  $u_1 = u + 2x_{1d}$ ,  
 598 where  $u$  is the previous control calculated by the PID controller.  
 599 According to the structure of the model of the EHA system,  
 600 this disturbance will be added to the previously hydraulic dis-  
 601 turbance term  $d_2$  and will change the dynamic of the states  
 602 ( $x_1, x_2, x_3, x_4$ ) of the EHA system. We can see from Fig. 8 that  
 603 both tracking performances and states estimation are achieved  
 604 correctly by our observer. At  $t = 10$  s, we can see the influ-  
 605 ence of the disturbances on the performances of our observer.  
 606 Despite its occurrence, we can clearly say that: first, the PID  
 607 controller is robust facing this disturbance; since that the PID  
 608 control law  $u$  uses the predictor term  $w(t)$  provided by our  
 609 observer, this will demonstrate the easiness of the incorpora-  
 610 tion of our observer in a control scheme; second, our observer  
 611 succeeds to estimate the states and the disturbances affecting  
 612 the EHA system after ( $t = 10$  s).

## V. CONCLUSION AND FUTURE WORK

In this paper, a continuous–discrete time observer is designed  
 for the EHAs system subject to discrete time measurement and  
 mechanical and hydraulic disturbances. The exponential con-  
 vergence of the proposed observer is proven using a classical  
 quadratic Lyapunov function based on small gain arguments.  
 The proposed observer is combined with PID controller for the  
 purpose of tracking motion reference trajectory of the piston  
 position for the EHA system. The simulation results and the  
 experimental validation of our proposed observer demonstrate  
 its efficiency in terms of tracking performance and distur-  
 bance estimation. In our future works, we plan to synthesize an  
 output feedback controllers based on the designed continuous–  
 discrete time observer in this paper. The resulting controllers  
 will improve the positioning control for the EHAs system.

## REFERENCES

- [1] J. Yao, Z. Jiao, and S. Han, “Friction compensation for low velocity control of hydraulic flight motion simulator: A simple adaptive robust approach,” *Chin. J. Aeronaut.*, vol. 26, no. 3, pp. 841–822, Jun. 2013.
- [2] J. Yao, Z. Jiao, B. Yao, Y. Shang, and W. Dong, “Nonlinear adaptive robust control of electrohydraulic load simulator,” *Chin. J. Aeronaut.*, vol. 25, no. 5, pp. 766–775, Oct. 2012.
- [3] W. Sun, H. Gao, and O. Kaynak, “Adaptive backstepping control for active suspension systems with hard constraints,” *IEEE/ASME Trans. Mechatronics*, vol. 18, no. 3, pp. 1072–1079, Jun. 2013.
- [4] Y. Pi and X. Wang, “Observer-based cascade control of a 6-DOF parallel hydraulic manipulator in joint space coordinate,” *Mechatronics*, vol. 20, no. 6, pp. 648–655, Sep. 2010.
- [5] W. Sun, Y. Zhao, J. Li, L. Zhang, and H. Gao, “Active suspension control with frequency band constraints and actuator input delay,” *IEEE Trans. Ind. Electron.*, vol. 59, no. 1, pp. 530–537, Jan. 2012.
- [6] J. Yao, Z. Jiao, D. Ma, and L. Yan, “High-accuracy tracking control of hydraulic rotary actuators with modeling uncertainties,” *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 2, pp. 633–641, Apr. 2014.
- [7] H. A. Mintsas, R. Venugopal, J.-P. Kenne, and C. Belleau, “Feedback linearization-based position control of an electrohydraulic servo system with supply pressure uncertainty,” *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 4, pp. 1092–1099, Jul. 2012.
- [8] H. E. Merritt, *Hydraulic Control Systems*. Hoboken, NJ, USA: Wiley, 1967.
- [9] J. Yao, G. Yang, and D. Ma, “Internal leakage fault detection and tolerant control of single-rod hydraulic actuators,” *Math. Prob. Eng.*, vol. 2014.
- [10] J.-H. Kwon, T.-H. Kim, J.-S. Jang, and I.-S. Lee, “Feedback linearization control of a hydraulic servo system,” in *Proc. SICE-ICASE Int. Joint Conf.*, 2006, pp. 455–460.
- [11] H.-M. Chen, J.-C. Renn, and J.-P. Su, “Sliding mode control with varying boundary layers for an electro-hydraulic position servo system,” *Int. J. Adv. Manuf. Technol.*, vol. 26, no. 1, pp. 117–123, 2005.
- [12] M. A. Ghazy, “Variable structure control for electrohydraulic position servo system,” in *Proc. IEEE 27th Annu. Conf. Ind. Electron. Soc.*, 2001, pp. 2194–2198.
- [13] C. Guan and S. Pan, “Adaptive sliding mode control of electro-hydraulic system with nonlinear unknown parameters,” *Control Eng. Pract.*, vol. 16, no. 11, pp. 1275–1284, Nov. 2008.
- [14] Y. Lin, Y. Shi, and R. Burton, “Modeling and robust discrete-time sliding mode control design for a fluid power electro-hydraulic actuator (EHA) system,” *IEEE/ASME Trans. Mechatronics*, vol. 18, no. 1, pp. 1–10, Feb. 2013.
- [15] A. G. Loukianov, J. Rivera, Y. Orlov, and E. Teraoka, “Robust trajectory tracking for an electrohydraulic actuator,” *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3523–3531, Sep. 2009.
- [16] B. Yao, F. Bu, J. Reedy, and G. T. C. Chiu, “Adaptive robust motion control of single-rod hydraulic actuators: Theory and experiments,” *IEEE/ASME Trans. Mechatronics*, vol. 5, no. 1, pp. 79–91, Mar. 2000.
- [17] B. Yao, F. Bu, and G. T. C. Chiu, “Nonlinear adaptive robust control of electro-hydraulic systems driven by double-rod actuators,” *Int. J. Control.*, vol. 74, no. 8, pp. 761–775, Aug. 2001.

- 680 [18] G. Cheng and P. Shuangxia, "Nonlinear adaptive robust control of  
681 single-rod electro-hydraulic actuator with unknown nonlinear param-  
682 eters," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 3, pp. 434–445,  
683 May 2008.
- 684 [19] C. Wang, Z. Jiao, S. Wu, and Y. Shang, "Nonlinear adaptive torque control  
685 of electro-hydraulic load system with external active motion disturbance,"  
686 *Mechatronics*, vol. 24, no. 1, pp. 32–40, Feb. 2014.
- 687 [20] Y. Cungui and Q. Xianwei, "Simplified adaptive robust motion control  
688 with varying boundary discontinuous projection of hydraulic actuator,"  
689 *Math. Prob. Eng.*, vol. 2014.
- 690 [21] A. Alleyne and R. Liu, "A simplified approach to force control for electro-  
691 hydraulic systems," *Control Eng. Pract.*, vol. 8, no. 12, pp. 1347–1356,  
692 Dec. 2000.
- 693 [22] C. Kaddissi, J.-P. Kenne, and M. Saad, "Identification and real-time  
694 control of an electrohydraulic servo system based on nonlinear backstep-  
695 ping," *IEEE/ASME Trans. Mechatronics*, vol. 12, no. 1, pp. 12–22, Feb.  
696 2007.
- 697 [23] C. Kaddissi, J.-P. Kenne, and M. Saad, "Indirect adaptive control of  
698 an electrohydraulic servo system based on nonlinear backstepping,"  
699 *IEEE/ASME Trans. Mechatronics*, vol. 16, no. 6, pp. 1171–1177, Dec.  
700 2011.
- 701 [24] K. K. Ahn, D. N. C. Nam, and M. Jin, "Adaptive backstepping control of  
702 an electrohydraulic actuator," *IEEE/ASME Trans. Mechatronics*, vol. 19,  
703 no. 3, pp. 987–995, Jun. 2014.
- 704 [25] P. Nakkarat and S. Kuntanapreeda, "Observer-based backstepping force  
705 control of an electrohydraulic actuator," *Control Eng. Pract.*, vol. 17,  
706 no. 8, pp. 895–902, 2009.
- 707 [26] K. Wonhee, W. Daehee, S. Donghoon, and C. Chung, "Output feedback  
708 nonlinear control for electro-hydraulic systems," *Mechatronics*, vol. 22,  
709 no. 6, pp. 766–777, Sep. 2012.
- 710 [27] X. Wang, X. Sun, S. Li, and H. Ye, "Output feedback domination  
711 approach for finite-time force control of an electrohydraulic," *IET Control  
712 Theory*, vol. 6, no. 7, pp. 921–934, 2011.
- 713 [28] H. Khan, S. C. Abou, and N. Sepehri, "Nonlinear observer-based fault  
714 detection technique for electro-hydraulic servo-positioning systems,"  
715 *Mechatronics*, vol. 15, no. 9, pp. 1037–1059, 2005.
- 716 [29] K. Wonhee, W. Daehee, S. Donghoon, and C. Chung, "Disturbance-  
717 observer-based position tracking controller in the presence of biased sinu-  
718 soidal disturbance for electrohydraulic actuators," *IEEE Trans. Control  
719 Syst. Technol.*, vol. 21, no. 6, pp. 2290–2298, Nov. 2013.
- 720 [30] C. S. Kim and C. O. Lee, "Speed control of an overcentered variable  
721 displacement hydraulic motor with a load torque observer," *Control Eng.  
722 Pract.*, vol. 4, no. 11, pp. 1563–1570, 1996.
- 723 [31] W. Daehee, K. Wonhee, S. Donghoon, and C. Chung, "High-gain distur-  
724 bance observer-based backstepping control with output tracking error  
725 constraint for electro-hydraulic systems," *IEEE Trans. Control Syst.  
726 Technol.*, vol. 23, no. 6, pp. 758–795, Mar. 2014.
- 727 [32] J. Yao, Z. Jiao, and D. Ma, "Extended-state-observer-based output feed-  
728 back nonlinear robust control of hydraulic systems with backstepping,"  
729 *IEEE Trans. Ind. Electron.*, vol. 61, no. 11, pp. 6285–6293, Nov. 2014.
- 730 [33] J. Yao, Z. Jiao, and D. Ma, "Adaptive robust control of dc motors with  
731 extended state observer," *IEEE Trans. Ind. Electron.*, vol. 61, no. 7,  
732 pp. 3630–3637, Jul. 2014.
- 733 [34] S. Ahmed Ali, "Sampled data observer based inter-sample output pre-  
734 dicator for electro-hydraulic actuators," *ISA Trans.*, vol. 58, pp. 421–433,  
735 2015.
- 736 [35] I. Karafyllis and C. Kravaris, "From continuous-time design to sampled-  
737 data design of observers," *IEEE Trans. Autom. Control*, vol. 54, no. 9,  
738 pp. 2169–2174, Sep. 2009.
- 739 [36] A. Karakayis and S. Begg, "Investigation of control system strategies for  
740 hydraulic valve actuation in an IC engine Adil Karakayis," M.S. thesis,  
741 Dept. Autom. Eng., Univ. Brighton School Comput., Eng. Math., Div.  
742 Eng. Product Des., Brighton, U.K., 2014.
- 743 [37] W. Sun, Z. Zhao, and H. Gao, "Saturated adaptive robust control for active  
744 suspension systems," *IEEE Trans. Ind. Electron.*, vol. 60, no. 9, pp. 3889–  
745 3896, Sep. 2013.
- 746 [38] W. Sun, H. Gao, and B. Yao, "Adaptive robust vibration control of full-car  
747 active suspensions with electrohydraulic actuators," *IEEE Trans. Control  
748 Syst. Technol.*, vol. 21, no. 6, pp. 2417–2422, Nov. 2013.



**Sofiane Ahmed Ali** was born in Algiers, 749  
Algeria, in 1977. He received the B.Sc. degree 750  
in electrical engineering from the University 751  
of Technology Houari Boumediene, Algiers, 752  
Algeria, in 2001, and the M.Sc. and Ph.D. 753  
degrees in electrical and computer engineer- 754  
ing from the University of Le Havre, Le Havre, 755  
France, in 2004 and 2008, respectively. 756

In 2008, he was appointed as a Research 757  
and Development Engineer with Renault. Since 758  
2010, he has been a Teaching and Research 759

Assistant Professor with the École Supérieure d'Ingénieurs en Génie 760  
Électrique (ESIGELEC), Rouen, France. His research interests include 761  
sliding mode control, nonlinear observers and fault-tolerant control, and 762  
diagnosis in the field of mechatronics devices. 763Q5



**Arnaud Christen** was born in France, in 764  
1991. He received the Baccalaureate degree 765  
in science (with honors) in 2009, before fol- 766  
lowing a two-year preparation in mathemat- 767  
ics and physics for entrance to the French 768  
Engineering Schools. He received the dual M.S. 769  
degree in control theory (electrical engineer- 770  
ing) and mechatronics from École Supérieure 771  
d'Ingénieurs en Génie Électrique (ESIGELEC), 772  
Rouen, France, and the University of Rouen, 773  
Rouen, France, in 2015. 774Q6



**Steven Begg** received the B.Eng. degree (with 775  
honors) in mechanical engineering and the Ph.D. 776Q7  
degree from the University of Brighton, Brighton, 777  
U.K., in 2003. 778

He is a Reader, Fellow of the Higher 779  
Education Academy, and the Course Leader 780  
for the Automotive Engineering undergraduate 781  
degree pathways in the School of Computing, 782  
Engineering and Mathematics, University of 783  
Brighton. He is the Leader of the Experimental 784  
Fluid Mechanics Research Group and a Member 785

of the Advanced Engineering Centre at Brighton, U.K. He has led 786  
applied research programmes (EPSRC, DfT, DTI, TSB, and EU) as 787  
well as industrial consultancy projects, in the fields of automotive engi- 788  
neering, fluid mechanics, and optical diagnostic techniques, for over 21 789  
years. 790



**Nicolas Langlois** received the Ph.D. and HDR 791  
(habilitation to supervise research) degrees in 792  
automatic control and signal processing from the 793  
University of Rouen, Rouen, France, in 2001 and 794  
2008, respectively. 795

In 2000, he joined the Graduate School 796  
of Electrical Engineering, ESIGELEC, Rouen, 797  
France. He is currently the Head-in-Charge 798  
of skills acquisition through research of 799  
ESIGELEC, where he teaches courses on 800  
control systems and digital signal processing. 801

He has also the Head of the "Automatic Control and Systems" research 802  
team at the research institute IRSEEM since 2008. His research 803  
interests include fault-tolerant control. 804Q8

## QUERIES

- Q1: Please provide expansion for “PI.”
- Q2: Please provide expansion for “IRSEEM.”
- Q3: As per IEEE style, vectors have been changed to boldface italic. Please check whether all the occurrences are identified correctly and specify the missed out occurrences.
- Q4: Please provide complete details of Refs. [9] and [20].
- Q5: Please provide the location for Renault in the biography section of the author Sofiane Ahmed Ali.
- Q6: Please provide the institution name and location for the Baccalaureate degree of the author Arnaud Christen.
- Q7: Please spell out the term “ESIGELEC.”
- Q8: Please provide field of study for the Ph.D. degree of author “Steven Begg.”
- Q9: Please expand “IRSEEM” in the biography section.

IEEE PROOF