Modelling of a Two-Phase Vortex Ring Flow Based on the Fully Lagrangian Approach

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Abstract
The injection of a jet is studied within the framework of a 2D transient 'gas-particle' model. The modelling is based on a Eulerian-Lagrangian Approach: carrier phase parameters are calculated using the Direct Numerical Simulation (DNS), discrete phase parameters are calculated using the Fully Lagrangian Approach. Two applications are considered: the injection of liquid into a two-phase cloud, and two-phase jet injection into a quiescent gas. In the vortex flow, the dispersed medium forms folds, and the concentration field becomes multivalued. In order to assess concentration fields, it is suggested that the Lagrangian number density fields are re-mapped to Eulerian.

Introduction
Two-phase vortex-ring flows are widely observed in engineering and environmental conditions (e.g. [1, 2]). Also, two-phase vortex-ring-like structures have been identified in direct injection internal combustion engines [3]. In such flows, the admixture forms high concentration regions with folds (local zones of crossing particle/droplet trajectories) and caustics.
The dynamics of vortex rings have been of a great interest to many researchers [1, 4]. In recent studies (e.g. [5]), a mathematical model of a viscous vortex ring was developed and validated against DNS simulations [6]. The investigation of the properties of a two-phase flow using the analytical model is presented in [7]. The aim of the current study is to perform simulations of a two-phase vortex ring flow based on a coupled Fully Lagrangian Approach [8] and DNS modelling. According to [9], the only method capable of calculating the particle concentration field, without using excessive computer power, is the one suggested by Osiptsov, here referred to as the Fully Lagrangian Approach (FLA). Our study is focussed on further investigation of these types of flows based on the FLA. Particular attention is paid to the details of the mixing process within regions of high particle concentration, which can potentially lead to the formation of unfavourable zones of high fuel vapour concentration in internal combustion engines, when particles are identified with fuel droplets.

Two-phase vortex ring flow
We consider an axially symmetric transient flow of a two-phase gas-particle mixture, interacting with a vortex ring. The vortex ring is formed by the injection of a column of liquid into a quiescent medium. A cylindrical coordinate system is introduced: the directions of injection and vortex ring propagation are along the z-axis, the axis of symmetry, the origin of the reference frame is in the plane of the inlet (see Figure 1). The problem of dispersed phase dynamics in a vortex ring field is studied in the framework of the one-way coupled, two-fluid approach [10].

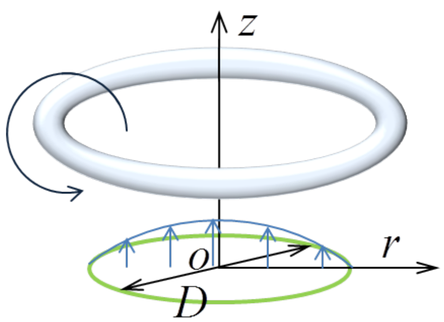

Figure 1. Flow diagram and the coordinate system.

Modelling of the carrier phase dynamics

The carrier phase is assumed to be an incompressible viscous gas. The inlet diameter D and the maximum velocity at the inlet U_{max} are used as the length and velocity scales for the problem. The non-dimensional time is introduced as $t = t^* U_{max}/D$, and the Reynolds number is defined as $\mathrm{Re} = U_{max} D/\nu$. The Specified Discharge Velocity (SDV) model [11] is used to specify the inlet velocity. According to this model, the velocity at the inlet is approximated by the following expression:

$$
\begin{aligned}
&U_{sdv}(r,t) = U_{max} U_p(t) U_{CL}(t) U_b(r,t),\\
&U_p = \begin{cases} \frac{1}{2}\left[1-\cos\left(\pi \frac{t}{t_1}\right)\right], & t \le t_1 \\ \frac{1}{2}\left[1+\cos\left(\pi \frac{t-t_1}{t_2-t_1}\right)\right], & t_1 \le t \le t_2 \end{cases}\\
&U_{CL}(t) = \frac{1}{1 - \dfrac{8}{\sqrt{\pi \mathrm{Re}}}\sqrt{t} + \dfrac{8}{\mathrm{Re}} t},\\
&U_b = \frac{1}{2}\left\{1 + \tanh\left[\frac{1}{4\Theta(t)}\left(1 - \frac{r}{R_{jet}(t)}\right)\right]\right\},\\
&\Theta(t) = \frac{\sqrt{2}-1}{\sqrt{\pi}} B(t),\ R_{jet}(t) = \frac{1}{2} - 0.477 B(t),\ B(t) = \frac{2}{\sqrt{\mathrm{Re}}}\sqrt{t},
\end{aligned}
\tag{1}
$$

where $U_p(t)$ is the piston velocity programme as suggested in [11, 12], normalized by the maximum velocity U_{max}, $U_{CL}(t)$ describes the evolution of the centreline velocity, $U_b(r,t)$ describes radial distribution of velocity; $t_1 = 1.57$, $t_2 = 2.26$; $\Theta(t)$ is the momentum thickness and $R_{jet}(t)$ is the discharge jet radius. The stroke ratio (ratio of the length of the column of fluid injected over the diameter of the column) is $\int_0^{t_2} U_p(t)dt = 1.13$, which is less than the formation number of optimum vortex ring [13]. This corresponds to the formation of a vortex ring without a trailing jet.

The incompressible Navier-Stokes equations in cylindrical coordinates are solved numerically using a second-order method. See [6, 14, 15] for more details.

Fully Lagrangian Approach for the dispersed phase

Particles are approximated by a cloud of identical spheres, treated as a pressureless continuum; the effects of particles on the carrier phase are ignored. For the modelling of the interphase momentum exchange, the aerodynamic drag is approximated by the corrected Stokes drag force [16]. The correction takes into account the effect of the finite Reynolds number for the flow around each particle (Re_d). Thus, the expression for the force acting on a particle is presented as:

$$
\mathbf{f}^*_{ad} = 6\pi\sigma\mu\left(\mathbf{v}^* - \mathbf{v}^*_d\right)\psi_{ad}, \tag{2}
$$

where $\psi_{ad} = 1 + \mathrm{Re}_d^{2/3}/6$; $\mathrm{Re}_d = 2\rho\sigma\left|\mathbf{v}^* - \mathbf{v}^*_d\right|/\mu$; the asterisk indicates the dimensional parameters; subscript 'd' refers to the dispersed phase parameters; ρ and μ are the gas density and viscosity, respectively; σ is the particle radius.

The non-dimensional dispersed phase parameters are obtained using the same length, velocity and time scales as for the carrier phase. In addition to those, non-dimensional droplet number density is introduced as $n_d = n^*_d/n_{d0}$, where n_{d0} is characteristic (initial) number density of admixture.

The dispersed phase is modelled using the Fully Lagrangian Approach [8], which makes it possible to calculate all of the dispersed phase parameters, including number density, from the solutions to the systems of ordinary differential equations along chosen particle trajectories. According to the FLA, the Lagrangian variables are the coordinates of initial particle positions r_{d0}, z_{d0}. The equations for the dispersed phase in cylindrical coordinates take the form:

$$
n_d r_d |J| = n_{d0} r_{d0} \tag{3a}
$$

$$
\frac{\partial \mathbf{r}_d}{\partial t} = \mathbf{v}_d, \quad \frac{\partial \mathbf{v}_d}{\partial t} = \beta(\mathbf{v} - \mathbf{v}_d)\psi_{ad}, \tag{3b}
$$

$$
\frac{\partial J_{ij}}{\partial t} = q_{ij}, \tag{3c}
$$

$$
\frac{\partial q_{ij}}{\partial t} = \beta\left(\frac{\partial v_i}{\partial r} J_{rj} + \frac{\partial v_i}{\partial z} J_{zj} - q_{ij}\right)\psi_{ad} + \beta\left(v_i - v_{di}\right)\frac{\partial \psi_{ad}}{\partial z_{j0}}, \tag{3d}
$$

$$
\frac{\partial \psi_{ad}}{\partial z_{j0}} = \frac{1}{9}\frac{\mathrm{Re}_{d0}}{\mathrm{Re}_d^{1/3}}\frac{1}{|\mathbf{v} - \mathbf{v}_d|}\left((u - u_d)\left(\frac{\partial u}{\partial r} J_{rj} + \frac{\partial u}{\partial z} J_{zj} - q_{rj}\right) + (v - v_d)\left(\frac{\partial v}{\partial r} J_{rj} + \frac{\partial v}{\partial z} J_{zj} - q_{zj}\right)\right),
$$

where

$$J_{ij} = \frac{\partial z_i}{\partial z_{j0}}, \quad q_{ij} = \frac{\partial v_{di}}{\partial z_{j0}},$$

$$\beta = \frac{6\pi\sigma\mu D}{mU_{max}}, \quad \mathrm{Re}_d = \mathrm{Re}_{d0} \left|\mathbf{v} - \mathbf{v}_d\right|, \quad \mathrm{Re}_{d0} = \frac{2\sigma U_{max}}{\nu},$$

indices i and j take values of r or z. In Equaions (3), particle parameters: number density n_d, radius-vector $\mathbf{r}_d$, and velocity $\mathbf{v}_d$, are the functions of the Lagrangian variables r_{d0}, z_{d0} and time t. J is the Jacobian of the transform from Eulerian to Lagrangian coordinates. Equation (3a) is the continuity equation in Lagrangian variables; (3b) are momentum balance equations along chosen particle trajectories; (3c) and (3d) are additional equations to calculate the Jacobian components. The equations for the components are derived from (3b) by differentiation with respect to r_{d0} and z_{d0}. In Equaions (3), the derivatives in the left-hand side of the equations are partial since the corresponding parameters are functions of three variables r_{d0}, z_{d0} and t. For a chosen particle trajectory, i.e. for constant r_{d0}, z_{d0}, Equations (3) can be treated and solved as ODE. The initial conditions are presented as:

$$\begin{aligned} & r_d = r_{d0}, \quad z_d = z_{d0}, \\ & u_d = u_{d0}, \quad v_d = v_{d0}, \quad n_d = 1, \\ & J_{ij} = \delta_{ij}, \quad q_{ij} = \frac{\partial v_{di0}}{\partial z_j}. \end{aligned} \tag{4}$$

System (3) of ordinary differential equations with initial conditions (4) was solved using the 4-th order Runge-Kutta method. In (3), the values of the carrier phase velocity components and their derivatives were calculated numerically using interpolation of the tabular data by the second order polynomials.

Results and discussion

Interaction of a vortex ring with a cloud of droplets

We consider vortex ring propagation through a cloud of droplets. In Table 1, the values for non-dimensional β and Re_{d0} are presented for a range of sizes of water droplets in an air mixture, D = 0.1 m, $U_{max} = 3$ m/s ($\mathrm{Re} = 20\,000$). The values of β equal to 0.1, 1, and 10, and $\mathrm{Re} = 20\,000$ were selected in order to investigate the properties of the flow. The clouds of droplets are subjected to significant deformations, leading to the formation of regions with high values of number density. The regions of singularity of the particle number density are formed at the edges of the folds of the dispersed phase concentration field.

Table 1. Dimensionless values of parameters β and Re_{d0} for water droplets in air; $D = 0.1$m, $U_{max} = 3$ m/s, $\mathrm{Re} = 20\,000$.

σ^*, μm	β	Re_{d0}
10	27	4
50	1.1	20
100	0.27	40

The problem formulation corresponds to the following initial conditions for the dispersed phase:

$$\begin{aligned} & r_d = r_{d0}, \quad z_d = z_{d0}, \\ & u_d = 0, \quad v_d = 0, \quad n_d = 1, \\ & q_{ij} = 0, \quad J_{ij} = \delta_{ij}, \end{aligned} \tag{5}$$

where δ_{ij} is the Kronecker delta.

Some results are presented in Figures 2 – 4. The number density of the dispersed phase is capped at 4 to improve visualisation of the results. As air is injected, droplets interact with the jet; the dynamics of the droplets depends on their mass and size. Smaller droplets ($\beta = 10$) are pushed away by the jet and the vortex, forming a core without droplets (see Figure 2, right). At the edge of the two-phase region a thin layer of high number density is formed. In the case of larger droplets ($\beta = 0.1$), these are also entrained into the central region of the flow, with high number density near the axis. In the case of $\beta = 1$, the dispersed phase performs a complex pattern forming rolls, which originate from the central part of the domain.

We consider the case of $\beta = 1$ in more detail. As the dispersed media 'rolls', the cloud of droplets folds and the number of folds increases with time. The evolution of the cloud of droplets and its folds is presented in Figure 3. Folds in the dispersed media correspond to multivalued dispersed parameter fields. The particle number density has a singularity at the edge of the fold (see Figure 2). The FLA allows us to simulate two-phase flows in the case of multivalued particle parameter fields (folds) and to correctly calculate particle concentration at the edges of the fold. Note that this singularity is integrable and the collisionless model of particles remains valid when the initial particle

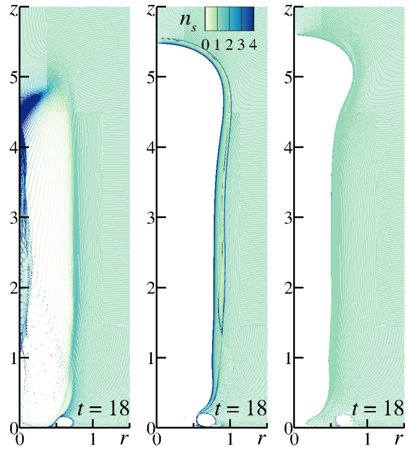

Figure 2. Particle number density at $t = 18$, left to right: $\beta = 0.1,\ 1,\ 10,\ \mathrm{Re} = 20\,000$.

volume fraction is sufficiently small (see [17] for the details). In order to calculate the number density at a Eulerian point, one needs to calculate the sum:

$$n_d(\mathbf{r}, t) = \sum_{i=0}^{N} n_{di}(\mathbf{r}, t), \tag{6}$$

where N is equal to the number of folds at a Eulerian point $\mathbf{r}$ at time instant t. Each change in the sign of the Jacobian corresponds to the intersection between particle trajectories leading to the formation of a fold. In Figure 3, the evolution of the cloud of droplets is presented. The number of folds is shown by different colours. In order to show a clearer picture of the number density field, this field was recalculated to Eulerian coordinates using Equation 6 (see Figure 4). As mentioned above, the particle number density rapidly increases on the caustics (edges of the folds of the particle concentration field).

Two-phase jet injection

In this case, droplets were injected together with the column of liquid forming a two-phase jet. Since the injection of liquid is simulated using the SDV model (1), the droplets were initialised at the inlet with the following initial conditions:

$$\begin{aligned} & t = t_0: \quad r_d = r_{d0}, \quad z_d = 0, \\ & u_d = 0, \quad v_d = 1, \quad n_d = 1, \\ & q_{ij} = 0, \quad J_{ij} = \delta_{ij}. \end{aligned} \tag{7}$$

In Figure 5, the evolutions of the jet and particle number density are presented. A mushroom-like structure is formed with higher values of concentration of droplets on the mushroom cap. As in the previous case, there is a singularity in the number density field at the edge of the fold.

Conclusions

Using the Fully Lagrangian Approach (FLA) for the dispersed phase coupled with the DNS solver for the carrier phase, an axially symmetric transient particle-laden flow in a vortex ring has been investigated.
Two problem formulations have been considered: injection of a two-phase jet into a vortex ring field and interaction of a vortex ring with a cloud of droplets. In the case when droplets are identified with fuel droplets, these zones of particle accumulation are expected to lead to the formation of zones of high fuel vapour concentration. It has been observed that in cases when the dispersed medium forms folds, caustics with singularities and local zones of particle accumulation in the concentration fields appear. Accurate calculations of the number density in these zones could not be performed if the analysis was based on the conventional rather than the Fully Lagrangian Approach.

Acknowledgements

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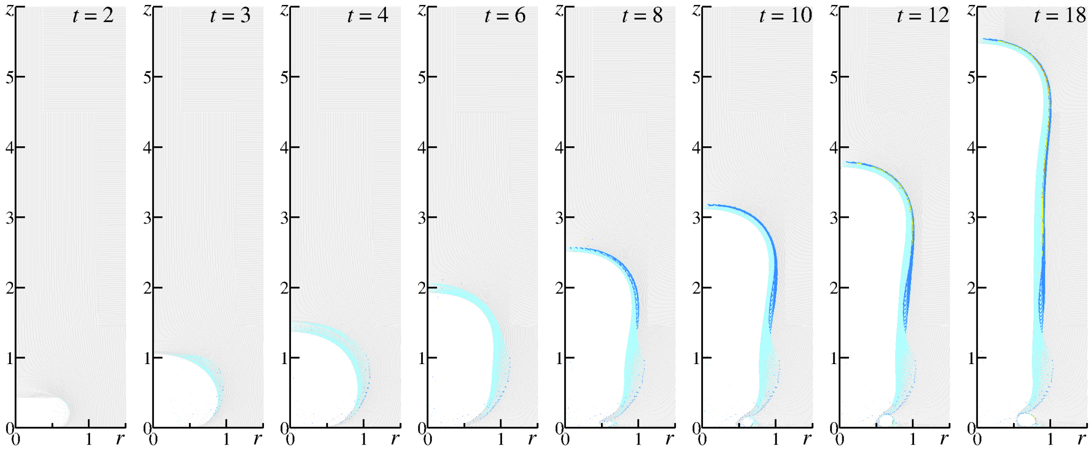

Figure 3. Evolution of the cloud of droplets; number of folds is indicated by colours: grey – 0, light blue – 1, dark blue – 2, yellow – 3, orange – 4, red – 5; $\beta = 1,\ \mathrm{Re} = 20\,000$.

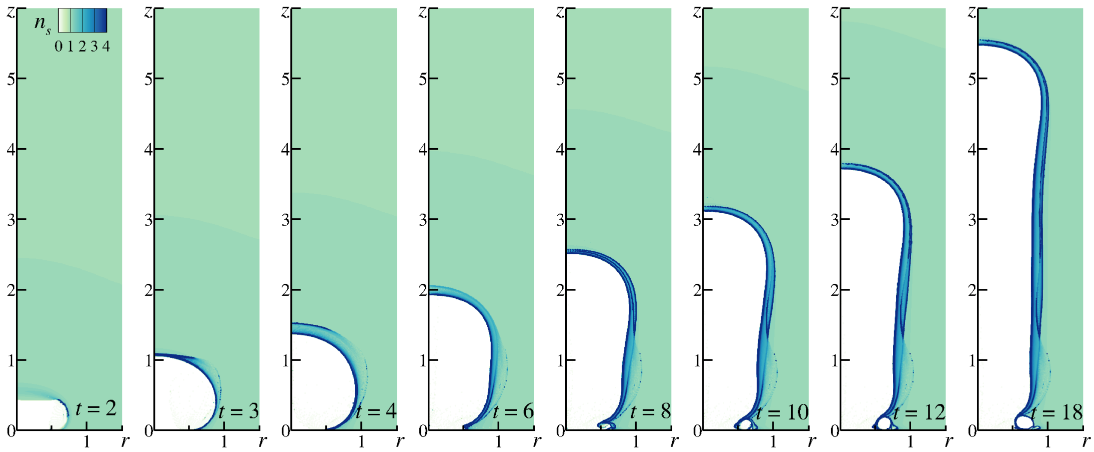

Figure 4. Evolution of the number density field; $\beta = 1,\ \mathrm{Re} = 20\,000$.

Nomenclature

$B(t)$	used in the SDV model 1
D	diameter of the inlet $[\mathrm{m}]$
DNS	Direct Numerical Simulations
$\mathbf{f}_{ad}$	aerodynamic drag force
FLA	Fully Lagrangian Approach also known as Osiptsov's method [8]
J	Jacobian and Jacobian components
m	mass of a particle/droplet of radius σ
N	number of folds in dispersed media
n_d	particle/droplet number density
q	denote partial derivatives of Jacobian components with respect to Lagrangian coordinates 3c
R	radius of the vortex ring $[\mathrm{m}]$
R_{jet}	discharge jet radius used in the SDV model 1
Re	Reynolds number
(r, z)	coordinates of the cylindrical coordinate system
SDV	Specified Discharge Velocity model
t	time $[\mathrm{s}]$
$\mathbf{v} = (u, v)$	velocity
U	axial velocity used in defining the SDV model
β	droplet inertia parameter
δ	the Kronecker delta
$\Theta(t)$	momentum thickness used in the SDV model 1
μ	dynamic viscosity $[\mathrm{Pa\ s}]$
ν	kinematic viscosity $[\mathrm{m^2\ s^{-1}}]$

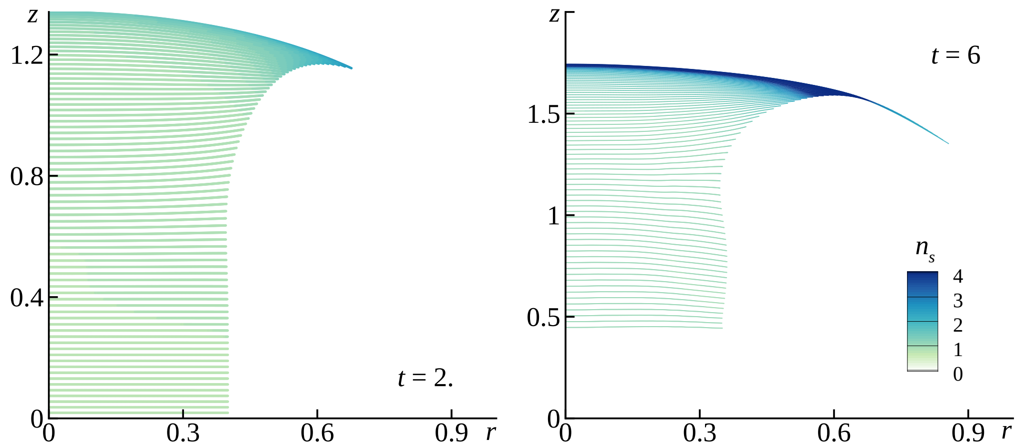

Figure 5. Two-phase jet: evolution of the number density field; $\beta = 1,\ \mathrm{Re} = 20\,000$.

ρ	density $[\mathrm{kg\ m^{-3}}]$
σ	droplet radius
ψ	correction function
Subscripts	
ad	aerodynamic drag
b	refers to the radial distribution of velocity at the inlet U_b 1
CL	centreline
d	dispersed phase parameter
i, j	indices
max	maximum value
p	piston parameter
0	initial value
Superscripts	
$*$	dimensional parameter

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