Deriving Displacement from a 3 axis Accelerometer

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Abstract

This paper describes a formal approach to derive displacement from a 3 axis accelerometer. It forms part of a larger research aspiration striving to afford computer users, including gamers, full physical immersion into 3D environments utilising affordable and practical motion sensor technologies. To derive displacement from the accelerometer a motion sensing device was built utilising a standard Windows PC compatible gamepad and software device driver. The accelerometer replaced the gamepad's analogue joysticks and was mounted in a black box small enough to be held in the hand. A software application was written that collected data, in real time, from the motion sensing device. A series of experiments were performed, the data captured and analysed. It was observed that specific motion characteristics correlated with oscillations in the data. A hypothesis was formed and the focus of this paper is on data analysis with a view to test the hypothesis.

Keywords

Accelerometer, Position Measurement, Multiplicative, Oscillation, Additive

1. Introduction

The Nintendo Wii™, Sony's Playstation 3™ and Microsoft's Xbox 360™ all feature accelerometers. Adam Champy, from Analog Devices Inc, defines three categories of motion gaming applications; Tilt Threshold, Gesture Recognition and Position Measurement [2]. Arcade racing games often use an accelerometer to aid steering. The gamer can use the gamepad as if it were a steering wheel. These types of games can be categorised as Tilt Threshold [2] [6]. Nintendo's Wii remote employs an accelerometer [4]. To play Wii Sports Tennis the gamer swings the remote as if it were a tennis racket. These types of games can be categorised as Gesture Recognition [2]. The goal of this paper was to derive displacement from 3 axis accelerometers; displacement pertaining to positional vectors changing over time. This is categorised as Position Measurement [2]. Adam Champy states "A challenging question facing designers is whether accelerometers can be successfully employed to measure position changes" [2].

There are a number of problems employing accelerometers to yield position measurement or displacement. The most notable with respect to this paper is their use to determine displacement over time. Adam Champy explains that by double integrating constant acceleration "Errors increase with the square of time; the error after a 1000 seconds is 1,000,000 times greater than that at 1 second. Any small offset errors in the acceleration measurement, especially with consumer-grade devices, will soon produce an intolerable error level.”[2]. He illustrates this with the following:

$$x = x_0 + \frac{dX}{dt}\bigg|_0 t + \frac{1}{2}\,at^2$$

Figure 1. Formula for constant acceleration

Bertis Rasco acknowledged problems with accuracy of accelerometer data and proposed Kalman Filters as a solution [5]. This solution might be regarded as “over kill”. Those familiar with Kalman Filters are likely to have come across them in association with safety-critical systems [3]; where system failure may result in death or serious injury etc. To debate whether Nintendo’s Wii be regarded as a safety-critical system is beyond the scope of this paper, but perhaps a necessary one, but since Bertis Rasco published his proposal Nintendo addressed this problem with the introduction of the Wii Motion Plus. This is an addition to the existing Wii remote that contains a multi-axis gyroscopic sensor [1]. Not only do Nintendo claim it will “more accurately and quickly reflect motions in 3D space” but afford “true 1:1 response in their game play” [4].

2. Data Analysis

To yield useful data for analysis a number of experiments were performed with an initial focus on how the accelerometer was moved during a session and the frame rate at which data was captured. It was ascertained single discrete movements generated optimum data for analysis at a frame rate of 60fps. Figure 3 represents typical data obtained from experiments. It reflects movement of the accelerometer, right, along the x axis. The oscillations would vary depending on which direction the accelerometer was moved. If the accelerometer was moved left, along the x axis, then the oscillation in Figure 3 would have been inverted; starting with a trough and ending with a peak. If the accelerometer was moved with greater force the same oscillation was plotted. While the accelerometer was not moving, at rest, the plot was flat. However, the oscillation was always plotted irrespective of the direction of the discrete movement or the force applied.

2.2 Oscillation

In section 1.1 Adam Champs expressed concern double integrating constant acceleration illustrating any small offset errors in the acceleration measurement is multiplicative i.e. “the error after a 1000 seconds is 1,000,000 times greater than that at 1 second” [2]. The data yielded during experimentation oscillated and therefore was not constant; it did not produce a continuous flat line. Acceleration increased and decreased over time for each discrete movement. Consequently, acceleration data had to be double integrated at each and every interval accumulating displacement over time. It was thus hypothesised offset errors would be additive and not multiplicative.

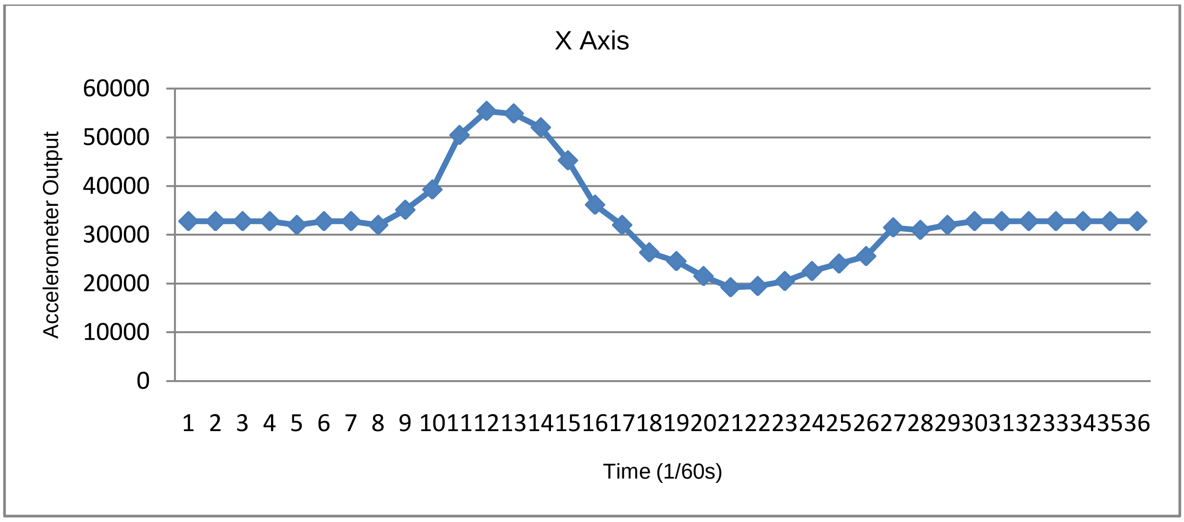

Figure 3. Data captured from session – accelerometer moving right

2.2 Analysis

Double integrating the data was undertaken in two steps. The first step was to integrate acceleration. Acceleration or the derivative of velocity was integrated with respect to time to yield the velocity. The second step was to integrate velocity. Velocity or the derivative of displacement was integrated with respect to time to yield displacement. Figure 4 represents an abstraction of data yielded from the accelerometer as illustrated in Figure 3. It focuses on three coordinates (t_A, a_A), (t_B, a_B) and (t_C, a_C) indicative of the oscillation. For the purposes of clarity the coordinates were plotted from point (0, 0) where the accelerometer was regarded at rest. The following integration refers to Figure 4.

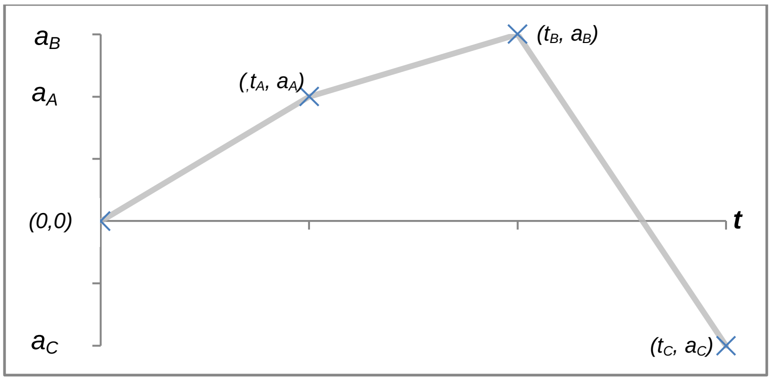

Figure 4. Characteristic Data from the Accelerometer

2.2.1 Integrating Acceleration and Velocity from Rest

Velocity v_A at interval t_A is the integral acceleration, expressed as the equation of the line, with respect to time:

$$v_A - v_0 = \int_0^{t_A} \frac{a_A}{t_A}\, t \ dt = \frac{a_A}{t_A}\,\frac{t^2}{2} \ \Bigg]_0^{t_A} equates\ to\ v_A = \frac{a_A\, t_A}{2}$$

Displacement x_A at interval t_A is the integral of velocity, expressed as the equation of the line, with respect to time:

$$x_A - 0 = \int_0^{t_A} \frac{a_A}{2}\, t \ dt = \frac{a_A}{2}\,\frac{t^2}{2} \ \Bigg]_0^{t_A} equates\ to\ x_A = \frac{a_A\, t_A^2}{4}$$

2.2.2 Integrating Acceleration and Velocity

Velocity v_B at interval t_B is the integral acceleration, expressed as the equation of the line, with respect to time:

$$v_B - v_A = \int_{t_A}^{t_B} \left[\ \frac{a_B - a_A}{t_A}\, t + (2a_A - a_B) \ \right] dt, equates\ to, v_B - v_A = \frac{1}{2} t_A\, (a_B + a_A)$$

Displacement x_B at interval t_B is the integral velocity, expressed as the equation of the line, with respect to time:

$$x_B - x_A = \int_{t_A}^{t_B} \left[\ \frac{v_B - v_A}{t_B - t_A}\, t + (2v_A - v_B) \ \right] dt, equates\ to, x_B - x_A = \frac{1}{2} t_A\, (v_B + v_A)$$

2.2.3 Integrating deceleration and velocity

Velocity v_C at interval t_C is the integral acceleration, expressed as the equation of the line, with respect to time:

$$v_C - v_B = \int_{t_B}^{t_C} \left[\ \frac{a_C - a_B}{t_C - t_B}\, t + (3a_B - 2a_C) \ \right] dt, equates\ to, v_C - v_B = \frac{1}{2} t_A\, (a_C + a_B)$$

Displacement x_C at interval t_C is the integral acceleration, expressed as the equation of the line, with respect to time:

$$x_C - x_B = \int_{t_B}^{t_C} \left[\ \frac{v_C - v_B}{t_C - t_B}\, t + (3v_B - 2v_C) \ \right] dt, equates\ to, x_C - x_B = \frac{1}{2} t_A\, (v_C + v_B)$$

3. Results

The results of analysis are summerised in table 1. The results illustrate velocity and displacement is accumulated over time and time is a constant. Figure 5 presents a general formula for displacement. Displacement or position measurement can be yielded directly from accelerometer output, at each interval, over time. Thus displacement is accumulative and any offset error is additive.

Table 1. Velocity / Displacement

Acc.	Velocity $v_{n+1} - v_n = \frac{1}{2}\, t\, (a_{n+1} + a_n)$	Displacement $x_{n+1} - x_n = \frac{1}{2}\, t\, (v_{n+1} + v_n)$
a_A	$\frac{a_A\, t_A}{2}$	$\frac{a_A\, t_A^2}{4}$
a_B	$\frac{a_A\, t_A}{2} + \frac{1}{2} t_A \left[\; a_B + a_A \;\right]$	$\frac{a_A\, t_A^2}{4} + \frac{1}{2} t_A \left[\; v_B + v_A \;\right]$
a_C	$\frac{a_A\, t_A}{2} + \frac{1}{2} t_A \left[\; a_B + a_A \;\right]$ $+ \frac{1}{2} t_A \left[\; a_C + a_B \;\right]$	$\frac{a_A\, t_A^2}{4} + \frac{1}{2} t_A \left[\; v_B + v_A \;\right]$ $+ \frac{1}{2} t_A \left[\; v_C + v_B \;\right]$

$$\boldsymbol{x_{n+1} - x_n = \left[\frac{1}{2} a_{n+1} + \frac{3}{2}\, a_n + 2 \sum_{j=1}^{n-1} a_j \right] \frac{t^2}{2}}$$

Figure 5. General formula for displacement where acceleration changes for each time interval

7. Further Work

During experimentation it was observed the data was reasonably consistent for prescribed movement. However, measurement offset needs to be calibrated. Accelerometers have optimum operating conditions [6]. These conditions affect measurement offsets. Temperature, for example, can affect the consistency of their measurement over time. It is a requirement of this research that the accelerometer is held in the hand. It is likely the longer it is held in the hand the greater its operating temperature will become. Therefore, how long it is held in the hand and the affect this has on measurement needs to be established, among other influences, to calibrate measurement offset.

Analysis of complex movement is also regarded as essential for the wider aspirations of this research. Experimentation thus far has focused on discrete movement; the accelerometer moving in a single direction. The broader context of this research requires the accelerometer be moved in multiple directions for a given movement. The oscillation in Figure 3 not only plots a discrete movement, right, along the X axis but the accelerometer resetting itself after movement; the trough. It is anticipated the accelerometer resetting itself will have to be considered to derive reliable displacement at each time interval for a given movement in multiple directions.

Conclusion

As long as acceleration measurements change for each time interval the detriment of offset, over time, can be radically reduced as any offset error is additive. Offset error can be further reduced by increasing the time between intervals at which the accelerometer is measured. During experimentation measurements were taken every 1/60th of a second. It is anticipated if measurements are taken every 1/30th of a second then offset errors can be further reduced by 50%; this is also true for constant acceleration [2].

The context within which an accelerometer is employed will determine if acceleration is to be regarded as constant or not. Accelerometers typically measure acceleration as units of g-force [6]. This is a relatively small force and it is sometimes easy to generate forces exceeding an accelerometer's measurement parameters. During experimentation it was observed that the impact force generated dropping the accelerometer was beyond its optimum measurement capability. If the context generates a force equal to or beyond an accelerometer's optimum measurement capability then it is reasonable to presume constant acceleration.

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