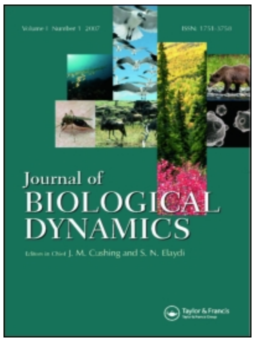

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Optimal control and cost effectiveness analysis for Newcastle disease eco-epidemiological model in Tanzania

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ABSTRACT

In this paper, a deterministic compartmental eco- epidemiological model with optimal control of Newcastle disease (ND) in Tanzania is proposed and analysed. Necessary conditions of optimal control problem were rigorously analysed using Pontryagin's maximum principle and the numerical values of model parameters were estimated using maximum likelihood estimator. Three control strategies were incorporated such as chicken vaccination (preventive), human education campaign and treatment of infected human (curative) and its' impact were graphically observed. The incremental cost effectiveness analysis technique used to determine the most cost effectiveness strategy and we observe that combination of chicken vaccination and human education campaign strategy is the best strategy to implement in limited resources. Therefore, ND can be controlled if the farmers will apply chicken vaccination properly and well in time.

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1. Introduction

Mathematical control theory is a basic principle that underlies the analysis and design of control systems. This theory is used to influence the objects behaviour so as to achieve a desired goal [24] and determine whether the species persist or extinct in natural system. It is also important for decision making regarding intervention programmes [14]. Modelling infectious diseases in species provides an important insight into disease behaviour and control measures while the epidemiological data and economic cost of controlling infectious diseases provides essential elements in evaluating the relevance of the intervention programmes. Currently, mathematical techniques are well linked with biological process of disease transmission and the epidemics of infectious diseases among humans and other animals resulting from the transmission of a pathogen either through hosts or environment [7]. The interaction between human and animals or among animals themselves may results into disease transmission which destabilize the ecosystem. The studies of [3,6,9,17,26] employed modelling techniques to analyse ecological aspect of interacting species of various animals.

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Tanzania poultry population is estimated to be 69 millions which comprises of traditional or family poultry and commercial poultry systems, about 90% of the poultry in Tanzania are chickens. The agricultural sector contributes 30% to the national gross domestic product (GDP) of which livestock sector contributes 18% of the agricultural GDP. Chickens only contribute 16% of livestock GDP, 3% of agricultural sector GDP and 1% of national GDP which is a significant contribution to the national economy [11]. Newcastle disease (ND) is one of the animal diseases which affect mostly domestic animals and chickens are most susceptible. It can cause a mortality rate of about 90–100% in chicken population. Incessantly loosing chickens due to ND may affect the quantity and quality of food for people on marginal diets while chickens play a vital role by providing an important source of high-quality nutrition and income at very little cost [10]. It is extremely difficult to assess the prevalence of ND at given time because in some areas the outbreaks are not reported. Moreover it extremely occur especially in rural and remote areas [2]. Vaccination provides a dormant pathogen in susceptible population which allows the vaccinated animal to produce strong antibodies against the weaker pathogen [3]. Currently, the phenomenon of prey predator ecosystem is well-studied with and without disease infection and these species in ecosystem do not exist alone [13] and the mechanisms of saving the population from extinction is biologically controlled in ecosystem [6]. The existence of ND in chickens is a big loss to human (farmers). Controlling ND threats require early preparation before the outbreak becomes overwhelming. Modelling tool plays a big role in epidemiology by providing a concrete mechanism for understanding spreads of the disease and suggesting effective control measures [8]. The intervention programmes are used in planning, implementing, evaluating, prevention, therapy and control measures [23]. The eradication of disease in the environment does not only depend on medical issues, but also on the ability of understanding the transmission dynamics of a particular disease and the application of the optimal control strategies and the implementation of logistic policies [8].

This particular study is motivated by a significant contribution of chicken 1% of GDP in national level in Tanzania [11], regardless of many obstacles such as poultry disease, poor quality feeds and inadequate technical support services. The national sample census of agriculture 2012 [5] indicates that the most dangerous poultry disease is ND which causes a very big drainage loss in many families, industries, organization and or individuals that really rely on poultry. It is against this background that this study is therefore undertaken as an attempt to apply the optimal control theory in minimizing the spread of ND and the cost of implementing control strategies. In order to achieve this goal, we use the following control parameters: chicken vaccination (u_1), education campaign (u_2) and treatment rate of infected human (u_3) as time dependent variables. In the next section, we derive the model that describes the dynamics of ND.

2. Model formulation

In this section, we formulate and analyse a mathematical model of ND in Tanzania. The modelled populations include chickens and human being. The epidemiological model comprises of five subclasses namely susceptible chicken $S_1(t)$, infected chicken $I_1(t)$, susceptible human $S_2(t)$, infected human $I_2(t)$ and human recovery class $R_2(t)$. The model presented under the following assumptions: The growth rate of chicken population follows a logistic function with intrinsic growth rate r and carrying capacity k. The chicken population gets infection when it comes into contact with other infected chicken and this contact process is assumed to follow the simple mass action kinetics with β_1 as the force of infection while human get infection with the force of infection β_2. Natural death rate of chicken μ_1 and induced death rate due to disease m reduces the chicken population. The human population suffers loss due to the natural death rate μ_2 and increases due to recovery rate θ through treatment rate γ. The predation functional response of the human towards susceptible as well as infected chicken is assumed to follow Michaelis–Menten kinetics and is modelled using a Holling type-II functional response with predation coefficients b_1, c_1, b_2, c_2 and half saturation constant a_1, a_2, n_1 and n_2. Consumed susceptible and infected chicken are converted into human with efficiency $\alpha_1, \alpha_2, \alpha_3$ and α_4. Basing on these assumptions, we formulate the model as

$$\begin{aligned}
\frac{dS_1}{dt} &= r\left(1-\frac{S_1}{k}\right)S_1 - \beta_1 S_1 I_1 - \frac{b_1 S_1 S_2}{a_1+S_1} - \frac{b_2 S_1 I_2}{a_2+S_1},\\
\frac{dI_1}{dt} &= \beta_1 S_1 I_1 - mI_1 - \mu_1 I_1 - \frac{c_1 I_1 S_2}{n_1+I_1} - \frac{c_2 I_1 I_2}{n_2+I_1},\\
\frac{dS_2}{dt} &= \frac{\alpha_1 b_1 S_1 S_2}{a_1+S_1} + \frac{\alpha_2 c_1 I_1 S_2}{n_1+I_1} - \beta_2 S_2 I_1 - \mu_2 S_2 + \theta R_2,\\
\frac{dI_2}{dt} &= \beta_2 S_2 I_1 - \gamma I_2 - \mu_2 I_2 + \frac{\alpha_3 b_2 S_1 I_2}{a_2+S_1} + \frac{\alpha_4 c_2 I_1 I_2}{n_2+I_1},\\
\frac{dR_2}{dt} &= \gamma I_2 - \theta R_2 - \mu_2 R_2.
\end{aligned} \tag{1}$$

We introduce the time dependent controls in the model (1) for the aim of controlling ND and study the strategies that curtail ND epidemic in poultry. For the optimal control problem, we consider the following model equations

$$\begin{aligned}
\frac{dS_1}{dt} &= r\left(1-\frac{S_1}{k}\right)S_1 - (1-u_1)\beta_1 S_1 I_1 - \frac{b_1 S_1 S_2}{a_1+S_1} - \frac{b_2 S_1 I_2}{a_2+S_1},\\
\frac{dI_1}{dt} &= (1-u_1)\beta_1 S_1 I_1 - (m+\mu_1)I_1 - \frac{c_1 I_1 S_2}{n_1+I_1} - \frac{c_2 I_1 I_2}{n_2+I_1},\\
\frac{dS_2}{dt} &= \frac{\alpha_1 b_1 S_1 S_2}{a_1+S_1} + \frac{\alpha_2 c_1 I_1 S_2}{n_1+I_1} - (1-u_2)\beta_2 S_2 I_1 - \mu_2 S_2 + \theta R_2,\\
\frac{dI_2}{dt} &= (1-u_2)\beta_2 S_2 I_1 - (u_3+\gamma)I_2 - \mu_2 I_2 + \frac{\alpha_3 b_2 S_1 I_2}{a_2+S_1} + \frac{\alpha_4 c_2 I_1 I_2}{n_2+I_1},\\
\frac{dR_2}{dt} &= (u_3+\gamma)I_2 - \theta R_2 - \mu_2 R_2,
\end{aligned} \tag{2}$$

where

(i) $u_1(t)$ the control variable based on chicken vaccination
(ii) $u_2(t)$ the control variable based on human education campaign
(iii) $u_3(t)$ the control variable to measure the effectiveness of treatment of infected human.

We apply control theory as a mathematical tool that is used to make decision involving complex biological situations [12]. The purpose of introducing controls in the model is to find the optimal level of the intervention strategy preferred to reduce the spreads and cost of implementation of the control. The control variables $u_1(t)$, $u_2(t)$ and $u_3(t)$ are minimized subject to the differential equations (2) and formulate the objective functional as

$$J = \min_{u_1,u_2,u_3} \int_0^{t_f} \left(B_1 I_1 + B_2 I_2 + \frac{1}{2} A_1 {u_1}^2 + \frac{1}{2} A_2 {u_2}^2 + \frac{1}{2} A_3 {u_3}^2 \right) \mathrm{d}t, \tag{3}$$

where t_f is the final time, $B_1 I_1$, $B_2 I_2$ are the cost associated with chicken vaccination and treatment of infected human respectively while A_1, A_2 and A_3 are relative cost weight for each individual control measure. The objective function (3) involved in minimizing of the number of infected chickens as well as the cost for applying control strategies. In this paper, a quadratic function which satisfies the optimality conditions is considered for measuring the control cost as applied by [14–16,18–20,21,25]. Then the optimal controls $u_1^*(t)$, $u_2^*(t)$ and $u_3^*(t)$ exists such that

$$J(u_1^*(t), u_2^*(t), u_3^*(t)) = \min\{J(u_1(t), u_2(t), u_3(t)) | u_1(t), u_2(t), u_3(t) \in \cup\}, \quad \text{where}$$
$$\cup = \{(u_1(t), u_2(t), u_3(t))\} \text{ are measurable,}$$
$$a_i \leq (u_1(t), u_2(t), u_3(t)) \leq b_i, \quad i = 1, \ldots 5\, a_i = 0,\ b_i = 1,\ t \in [0, t_f] \text{ is the closed set.}$$

The optimal control must satisfy the necessary conditions that are formulated by Pontryagin's Maximum Principle [11]. This principle converts the system of Equations (2) and (3) into a problem of minimizing point-wise a Hamiltonian (H), with respect to $u_1(t)$, $u_2(t)$, $u_3(t)$ as

$$\begin{aligned} H &= B_1 I_1 + B_2 I_2 + \frac{1}{2} A_1 {u_1}^2 + \frac{1}{2} A_2 {u_2}^2 + \frac{1}{2} A_3 {u_3}^2 \\ &+ \lambda_1 \left\{ r\left(1 - \frac{S_1}{k}\right) S_1 - (1-u_1)\beta_1 S_1 I_1 - \frac{b_1 S_1 S_2}{a_1 + S_1} - \frac{b_2 S_1 I_2}{a_2 + S_1} \right\} \\ &+ \lambda_2 \left\{ (1-u_1)\beta_1 S_1 I_1 - (m + \mu_1) I_1 - \frac{c_1 I_1 S_2}{n_1 + I_1} - \frac{c_2 I_1 I_2}{n_2 + I_1} \right\} \\ &+ \lambda_3 \left\{ \frac{\alpha_1 b_1 S_1 S_2}{a_1 + S_1} + \frac{\alpha_2 c_1 I_1 S_2}{n_1 + I_1} - (1-u_2)\beta_2 S_2 I_1 - \mu_2 S_2 + \theta R_2 \right\} \\ &+ \lambda_4 \left\{ (1-u_2)\beta_2 S_2 I_1 - (u_3 + \gamma) I_2 - \mu_2 I_2 + \frac{\alpha_3 b_2 S_1 I_2}{a_2 + S_1} + \frac{\alpha_4 c_2 I_1 I_2}{n_2 + I_1} \right\} \\ &+ \lambda_5 \{ (u_3 + \gamma) I_2 - (\theta + \mu_2) R_2 \}, \end{aligned} \tag{4}$$

where λ_i, $i = 1, 2, 3, 4, 5$ are the co-state variables associated by S_1, I_1, S_2, I_2, R_2. The adjoint equations are obtained by

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}t} = -\frac{\partial H}{\partial i}, \tag{5}$$

with transversality condition

$$\lambda_i(t_f) = 0. \tag{6}$$

From Equation (4) we obtain the following adjoint equations

$$\begin{aligned}
\frac{\partial H}{\partial S_1} &= -\lambda_1 \left(\frac{r}{k}(k - 2S_1) - (1 - u_1)\beta_1 I_1 - \frac{b_1 S_2 a_1}{(a_1 + S_1)^2} - \frac{b_2 I_2 a_2}{(a_2 + S_1)^2} \right) - \lambda_2 (1 - u_1)\beta_1 I_1 \\
&\quad - \lambda_3 \frac{\alpha_1 b_1 S_2 a_1}{(a_1 + S_1)^2} - \lambda_4 \frac{\alpha_3 b_2 I_2 a_2}{(a_2 + S_1)^2}, \qquad (7)\\
\frac{\partial H}{\partial I_1} &= -B_1 + \lambda_1 (1 - u_1)\beta_1 S_1 \\
&\quad - \lambda_2 \left((1 - u_1)\beta_1 S_1 - m - \mu_1 - \frac{c_1 S_2 n_1}{(n_1 + l_1)^2} - \frac{c_2 l_2 n_2}{(n_2 + l_1)^2} \right) \\
&\quad - \lambda_3 \left(\frac{\alpha_2 c_1 S_2 n_1}{(n_1 + l_1)^2} - (1 - u_2)\beta_2 S_2 \right) - \lambda_4 \left(\frac{\alpha_4 c_2 I_2 n_2}{(n_2 + I_1)^2} + (1 - u_2)\beta_2 S_2 \right), \qquad (8)\\
\frac{\partial H}{\partial S_2} &= \frac{\lambda_1 b_1 S_1}{a_1 + S_1} + \frac{\lambda_2 c_1 I_1}{n_1 + I_1} - \lambda_3 \left(\frac{\alpha_1 b_1 S_1}{a_1 + S_1} + \frac{\alpha_2 c_1 I_1}{n_1 + I_1} - (1 - u_2)\beta_2 I_1 - \mu_2 \right) \\
&\quad - \lambda_4 (1 - u_2)\beta_2 I_1, \qquad (9)\\
\frac{\partial H}{\partial I_2} &= -B_2 + \lambda_1 \frac{b_2 S_1}{a_2 + S_1} + \lambda_2 \frac{c_2 I_1}{n_2 + I_1} - \lambda_4 \left(\frac{\alpha_3 b_2 S_1}{a_2 + S_1} + \frac{\alpha_4 c_2 I_1}{n_2 + I_1} - (u_3 + \gamma) - \mu_2 \right) \\
&\quad - \lambda_5 (u_3 + \gamma), \qquad (10)\\
\frac{\partial H}{\partial R_2} &= \lambda_5 (\theta + \mu_2) - \lambda_3 \theta. \qquad (11)
\end{aligned}$$

The optimality of the control problem is obtained by

$$u_i^*(t) = \frac{\partial H}{\partial u_i}, \qquad (12)$$

where $i = 1, 2, 3$. The solution of $u_1^*(t)$, $u_2^*(t)$ and $u_3^*(t)$ are presented in compact form as

$$\begin{aligned}
u_1^*(t) &= \max \left\{ 0, \min \left\{ 1, \frac{\beta_1 S_1 I_1 (\lambda_2 - \lambda_1)}{A_1} \right\} \right\}, \\
u_2^* &= \max \left\{ 0, \min \left\{ 1, \frac{\beta_2 S_2 I_1 (\lambda_4 - \lambda_3)}{A_2} \right\} \right\}
\end{aligned}$$

and

$$u_3^* = \max \left\{ 0, \min \left\{ 1, \frac{I_2 (\lambda_4 - \lambda_5)}{A_3} \right\} \right\}.$$

3. Parameter estimation of ND model

Ordinary differential equations (ODEs) are widely used in ecology to describe the dynamical behaviour of systems of interacting populations. However, systems of ODEs rarely provide quantitative solutions that are close to real field observations or experimental data because natural systems are often subject to environmental noise and ecologists are

Table 1. Distribution of Chicken death cases data due to ND per district for 2014.

Month	Kongwa	Chamwino	Mkalama	Singida	Ikungi	Total
Jan	214	195	189	233	129	960
Feb	123	157	158	237	124	799
March	164	178	168	141	136	787
April	176	219	218	147	195	955
May	138	136	180	139	182	775
June	248	186	299	162	245	1140
July	204	125	86	97	197	709
Aug	234	145	186	102	182	849
Sept	308	271	278	257	243	1357
Oct	354	201	345	762	362	2024
Nov	362	234	456	750	395	2197
Dec	398	365	481	654	308	2206

often uncertain about the correct parameterization [4]. Therefore, it is important to estimate model parameters for numerical simulations. In this section, we present the data of chickens death cases collected from five districts in two regions (Singida and Dodoma) in Tanzania for 2014 as summarized in Table 1. The method used to estimate parameters in this section is maximum likelihood (ML) where real data of chicken death cases from Kongwa, Chamwino, Ikungi, Singida and Mkalama districts were used.

3.1. The maximum likelihood estimator (MLE)

The idea of ML method is to maximize the likelihood function. In this paper, the likelihood function is the sum of squares of residual (SSR) defined as

$$L(\theta) = \sum_{i=1}^{N} (y_i - {y_i}^{\mathrm{est}})^2, \tag{13}$$

where $\{y_i\}_{i=1}^{N}$ is the real data and $\{y_i^{\mathrm{est}}\}_{i=1}^{N}$ is the solution of model equations (1) at a given parameter value. The numerical results for MLE for the ND model parameters are summarized in Table 2.

Table 2. Estimated parameter values for ND model.

Symbol	Literature value	Source	Estimate value (per month)
α_i, $i = 1, \ldots, 4$	0.25, 0.6, 0.8, 0.6	[9], Estimated	0.3093, 0.6078, 0.8939, 0.6008
β_1	0.1	[22]	0.1495
k	500	[17]	500
μ_2	0.01	[3]	0.0244
c_1	0.02	Estimated	0.0202
r	10	[3]	10
$a_i, i = 1, 2$	0.25, 0.8	Estimated	0.2481, 0.6306
$n_i, i = 1, 2$	0.03, 0.5	Estimated, [17]	0.0304, 0.503
b_1	0.4	[17]	0.4085
b_2	0.6	[9]	0.6019
m	0.6	Estimated	0.5968
μ_1	0.02	[3]	0.02488
θ	0.4	Estimated	0.4048
γ	0.6	Estimated	0.612
β_2	0.012	Estimated	0.0119
c_2	0.05	Estimated	0.4974
B_1, B_2	10, 10	Estimated	–
A_1, A_2, A_3	30, 20, 10	[3]	–

4. Numerical results

4.1. Optimal control

In this section we study numerically the effects of optimal control strategies such as chicken vaccination, education campaign and treatment of infected human in the spread of ND. The solution of the optimal control problem was obtained by solving the optimality system of state and adjoint systems through forward–backward sweep method. The adjoint systems (7–11) were solved by fourth order Runge–Kutta scheme using the forward solution of the state equations. The optimality condition is satisfied through the convex update of the previous control values. We describe the controls in the following strategies using the parameter values in Table 2.

4.1.1. Strategy A: control with chicken vaccination (u_1)

With strategy A, only chicken vaccination u_1 is applied to control the system while other controls are set to zero. In Figures 1–3, the effect of chickens vaccination and its' positive impact is revealed. Figure 1(a) and 1(b) shows a significant difference in susceptible chicken population and stabilizes around the carrying capacity $k = 500$ while infected chicken are gradually decreasing to zero. A significant difference is also observed in human population as shown in Figure 2(a) and 2(b) and the control profile suggests that the control u_1 to be at the highest level for about seven months in a year before dropping to lower bound (see Figure 3). This result shows that the optimal control measure is effective in chicken and human populations and hence the community is disease free.

4.1.2. Strategy B: control with education campaign in human population (u_2)

The purpose of education campaign strategy is to explore the awareness of the disease, mode of transmission, prevention and control measures in community. Figures 4–6

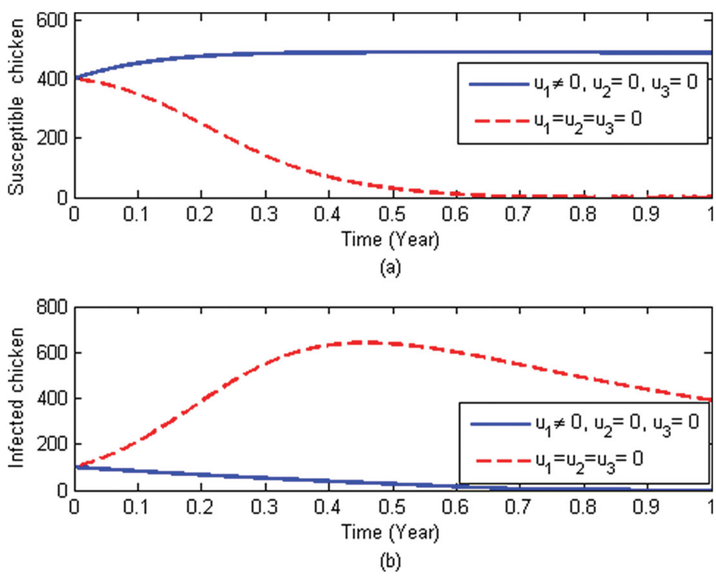

Figure 1. Simulations of the model showing the effect of chicken vaccination in chicken population.

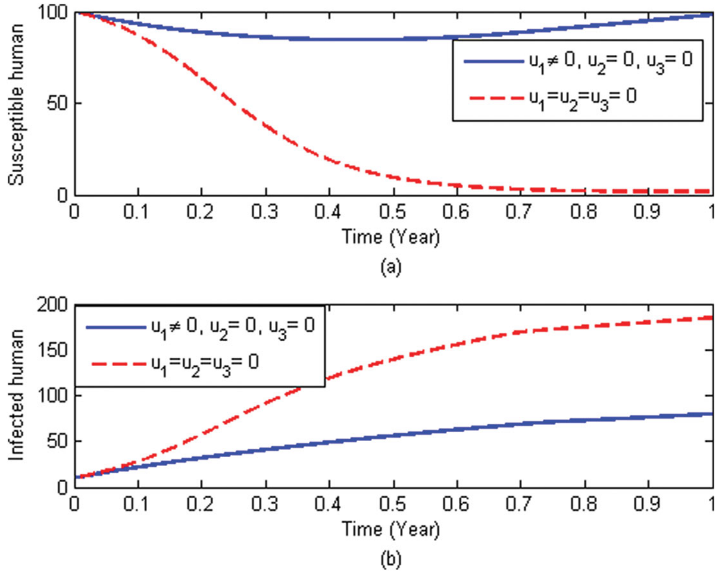

Figure 2. Effect of chicken vaccination in human population.

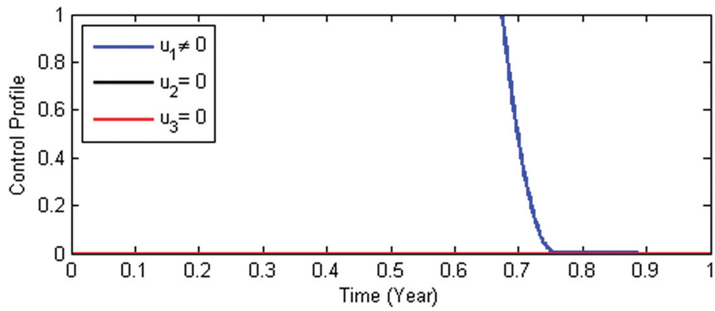

Figure 3. Control profile for the effect of chicken vaccination.

describes the effect of implementing education campaign in human and the impact is visible in human population (see Figure 5(a) and 5(b)) while the control profile maintained at its upper bound for the interval of almost one year (see Figures 6 and 7).

4.1.3. *Strategy C: control with treatment of infected human* (u_3)

When only control u_3 is applied while others are set to zero, the significant effect occurs on the class of human populations (see Figures 8 and 9). The control profile shows high increase to the upper bound and remains effective for long before gradually decreasing to lower bound (see Figure 9(b)). This result shows that the chicken population is not free from the disease. The treatment control strategy is not effective without vaccination of susceptible chicken and hence it is not preferable to the community as the control measure for ND.

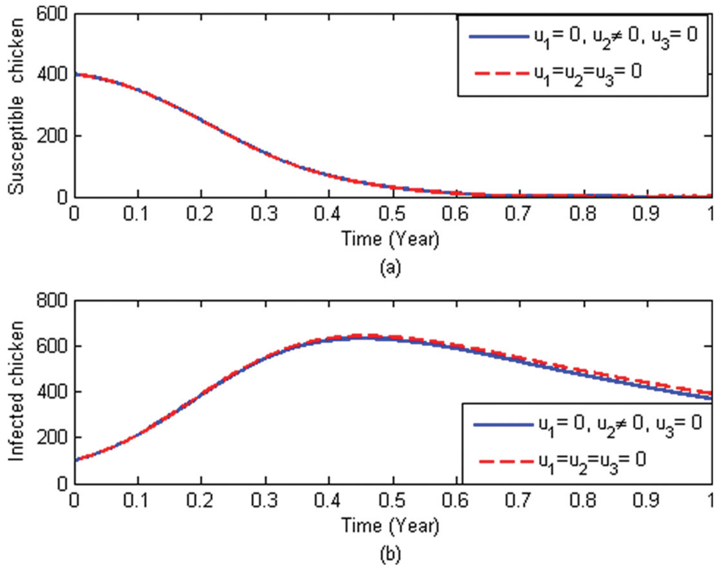

Figure 4. Simulations of the model showing the effect of control strategy B.

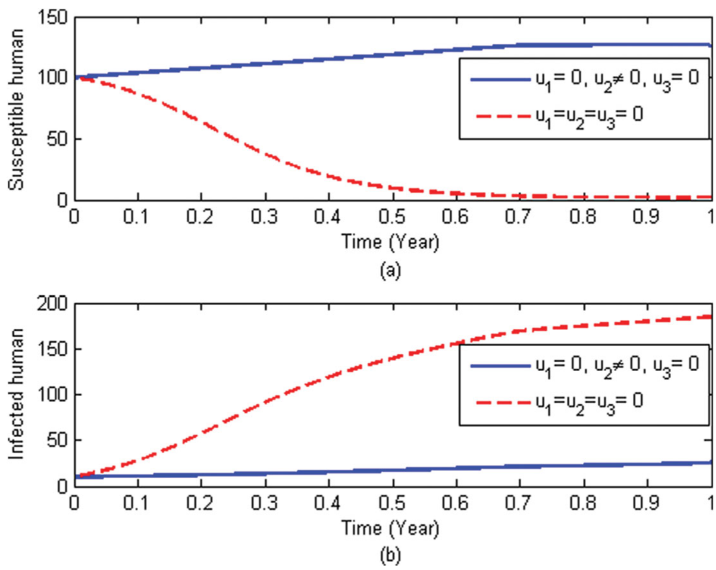

Figure 5. Simulations of the model showing the effect of control strategy B.

4.1.4. Strategy D: combination of education campaign in human (u_2) and treatment of infected human (u_3)

The numerical results shows that the susceptible human population increases while infected human population gradually decreases as illustrated in Figure 10. The presence of treatment and education in the community will somehow reduce the spread of disease moreover the strategy seems less effective. From Figure 11, we observed the control profile with different upper bounds and at the end both gradually decreases to lower bound.

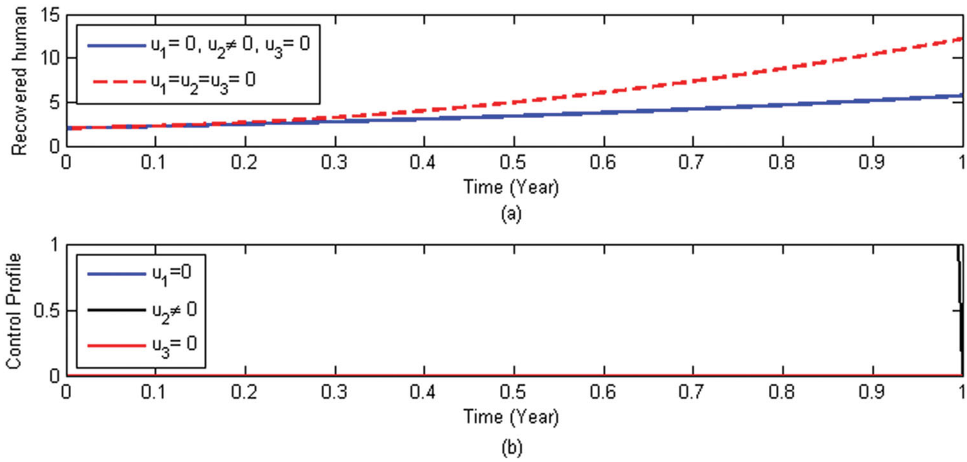

Figure 6. Control profile for strategy B.

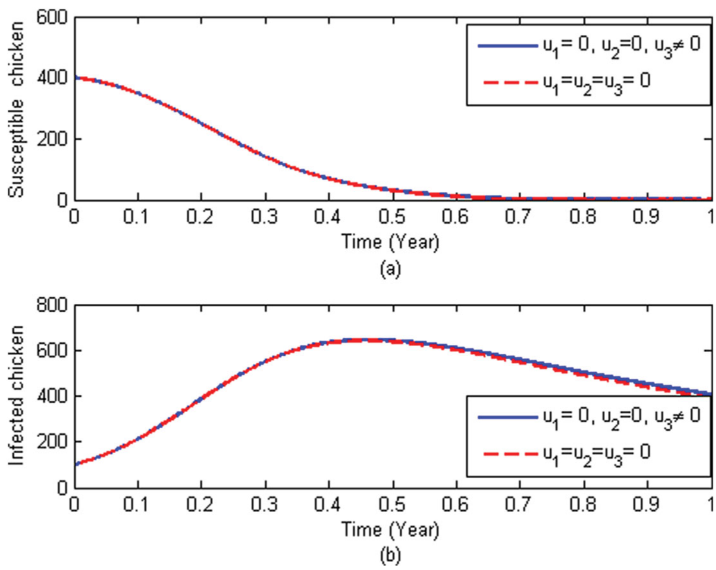

Figure 7. Simulations of the model showing the effect of treatment on infected human toward chicken populations.

4.1.5. Strategy E: control with combination of chicken vaccination (u_1) and treatment of infected human (u_3)

With this strategy, a positive impact is observed in both chicken and human populations. In Figures 12 and 13 we observe the control strategies results in decreasing the number of infected chicken and infected human while increasing the susceptible chicken and human respectively. The control profile for chicken vaccination (u_1) is at it's optimal level for about seven months to ensure that the community is disease free (see Figure 14) while (u_3) maintained at upper bound for about eleven months and finally both dropped to zero.

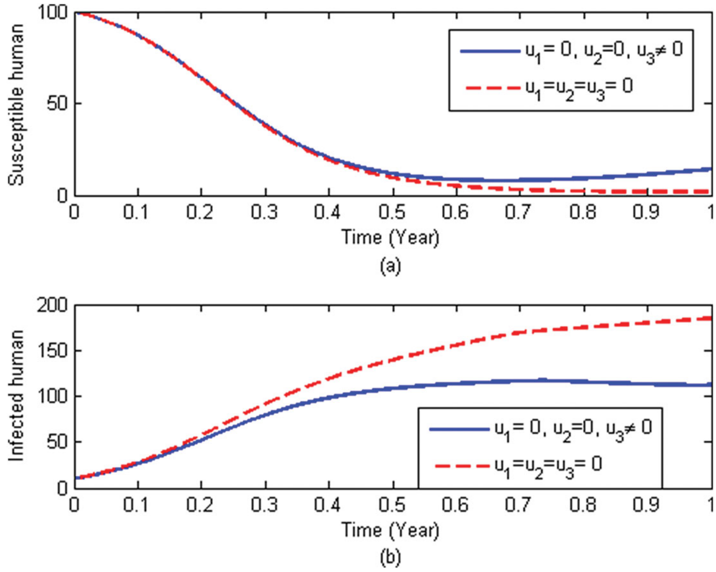

Figure 8. Simulations of the model showing the effect of treatment on the infected human.

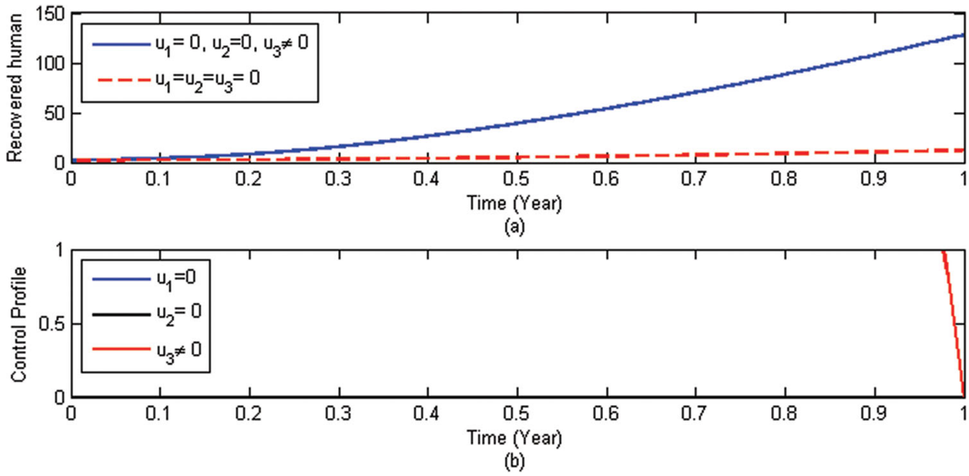

Figure 9. Simulations showing recovery and control profile for treatment strategy.

4.1.6. *Strategy F: combination of chicken vaccination ($\mathbf{u_1}$) and education campaign in susceptible human ($\mathbf{u_2}$)*

The controls (u_1) and (u_2) are used to optimize the objective function (J) while (u_3) is set to zero. Figures 15 and 16 show a significant difference when the controls (u_1) and (u_2) are applied. This result shows that the presence of controls saves the population and enables community to benefit from chicken. The control profile suggests that the control (u_1) to be at the upper bound for about seven months in a year before dropping to lower bound while control (u_2) to be at the upper bound for about seven months and two weeks before tends to zero (see Figure 17).

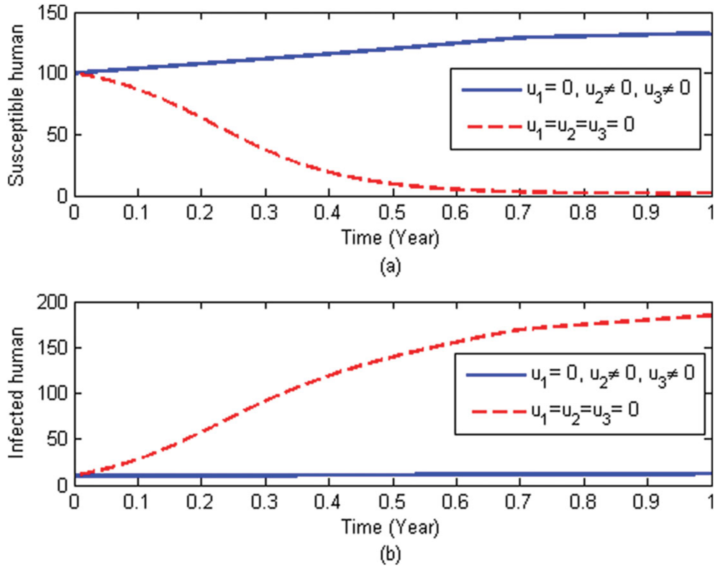

Figure 10. Simulations of the model showing the effect of education campaign and treatment on the infected human (strategy D) on the dynamics of ND in human population.

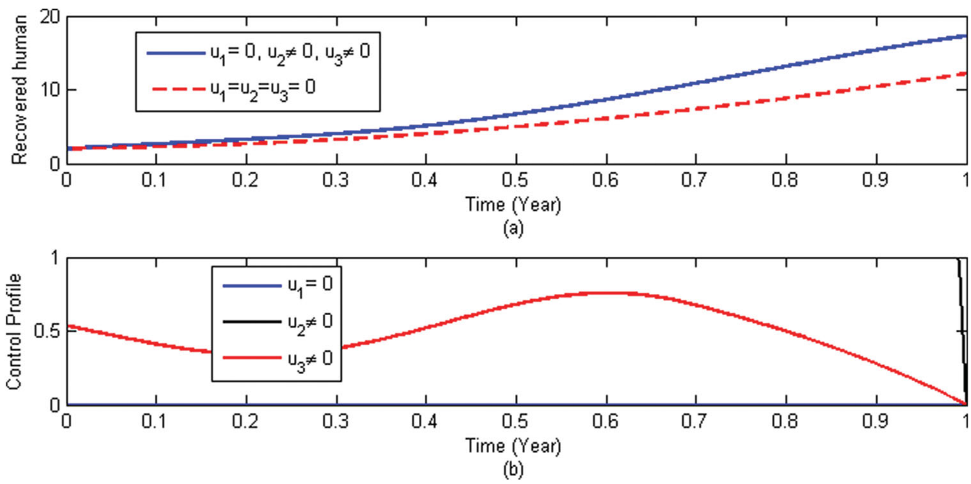

Figure 11. Simulations of the model showing the effect of education campaign and treatment on the infected human (strategy D) on the dynamics of ND.

4.1.7. Strategy G: control with combination of chicken vaccination (u_1) and education campaign in susceptible human (u_2) and treatment of infected human (u_3)

In this strategy, the combination of three strategies (u_1), (u_2) and (u_3) used to optimize the objective function (J) and then analysed its impact in chicken and human populations. Figures 18 and 19 shows the impact of with and without control application in the model. The significant difference is observed in both chicken and human population. In Figure 20,

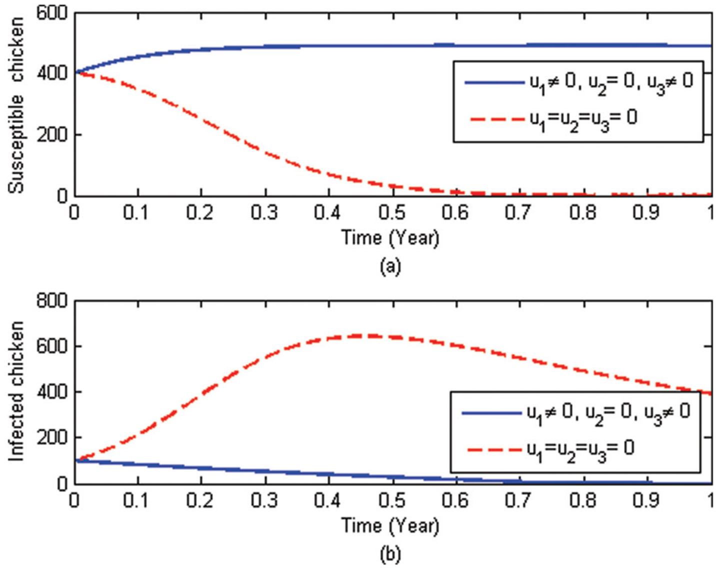

Figure 12. Simulations of the model showing the effect of vaccination and treatment of infected human (strategy E) in chicken population.

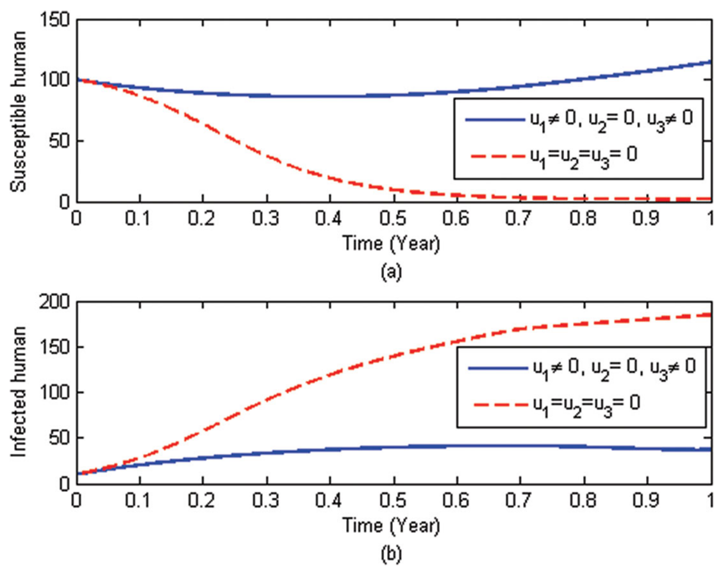

Figure 13. Simulations of the model showing the effect of vaccination and treatment of infected human (strategy E) in human population.

the control (u_1) is maintained at the upper bound until about the end of intervention, (u_2) to be at the upper bound for about four month before dropping to zero while (u_3) oscillates in between lower and upper bounds for the entire period of intervention. From the numerical simulations it is not easily to conclude the best strategy for implementation

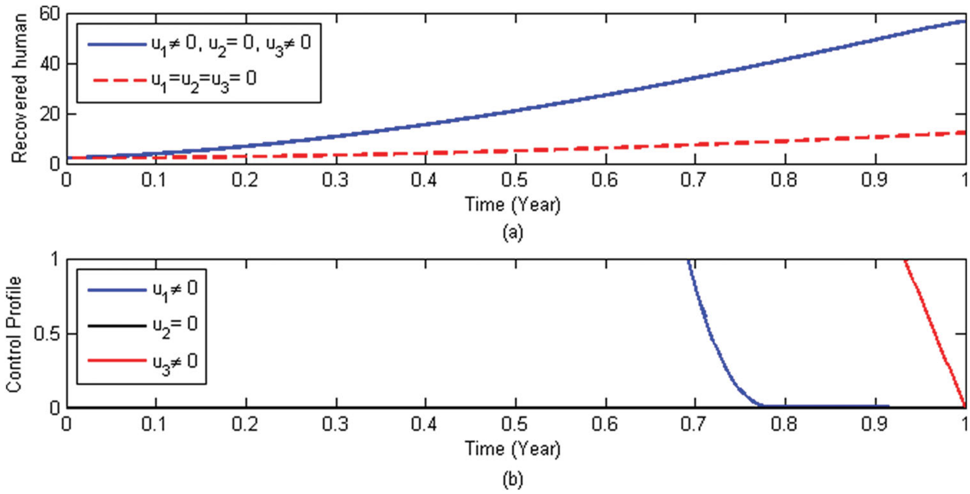

Figure 14. Simulations of the model showing the effect of vaccination and treatment of infected human (strategy E) in control profile.

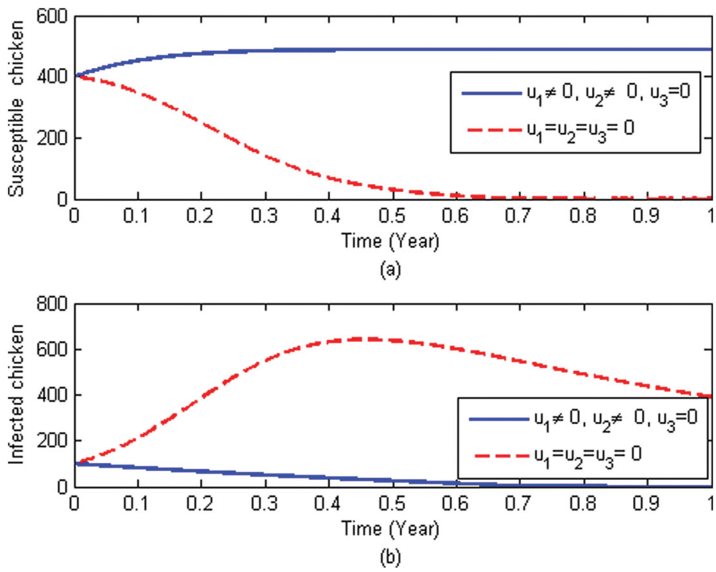

Figure 15. Simulations of the model showing the effect of chicken vaccination and education campaign in human strategy (F) in chicken population.

with limited resources. In the next section evaluates the cost effectiveness analysis (CEA) for each strategy.

5. Cost-effective analysis

In order to make decision on which intervention to choose, we evaluate the economic implications of ND control strategies using CEA technique. The CEA helps us to determine

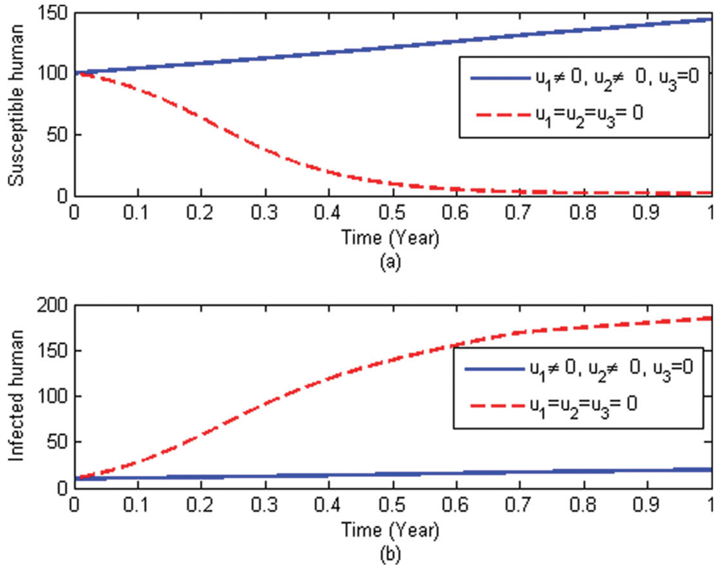

Figure 16. Simulations of the model showing the effect of vaccination and education campaign (strategy F) in human population.

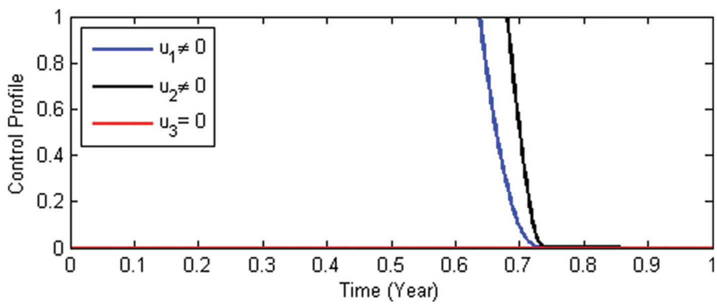

Figure 17. Simulations of the model showing the effect of vaccination and treatment of infected human (strategy F) in control profile.

and propose the most cost effective strategy to implement in limited resources. We evaluate the cost using incremental cost effectiveness ratio (ICER) which used to compare the differences between the costs and health outcomes of the two competing intervention strategies. Each intervention is compared with the next less effective alternative [14]. The infectious averted is computed by taking the difference between the total number of species individuals without control and the total number of species individuals with control. The total control costs $A_1u_1^2$, $A_2u_2^2$ and $A_3u_3^2$ (where A_i for $i = 1, \ 2, \ 3$ are relative cost weight for each individual control measure, while $u_1, \ u_2, \ u_3$ are the chicken vaccination costs ($), costs for human education campaign ($) and costs for treatment of infected human ($) respectively) are calculated and estimated in ($) USD over the period of one year respectively. The control strategies are ranked in order of increasing infection averted as presented in Table 3.

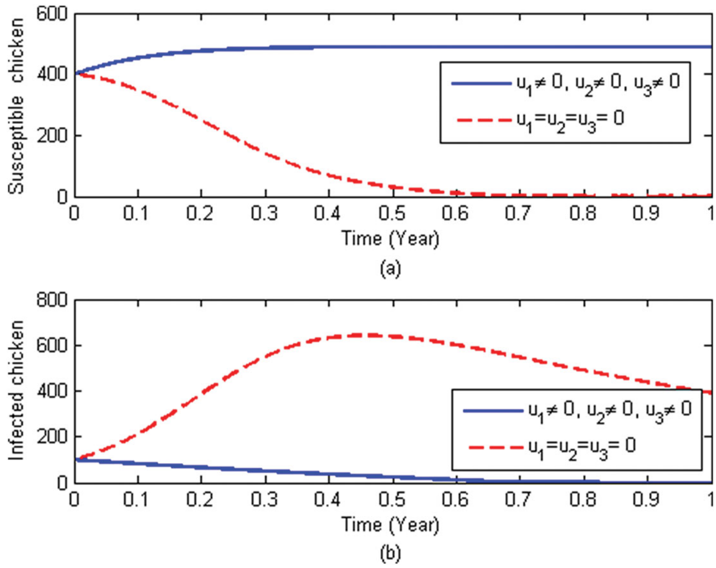

Figure 18. Simulations of the model showing the effect of vaccination, education and education campaign (strategy G) in chicken population.

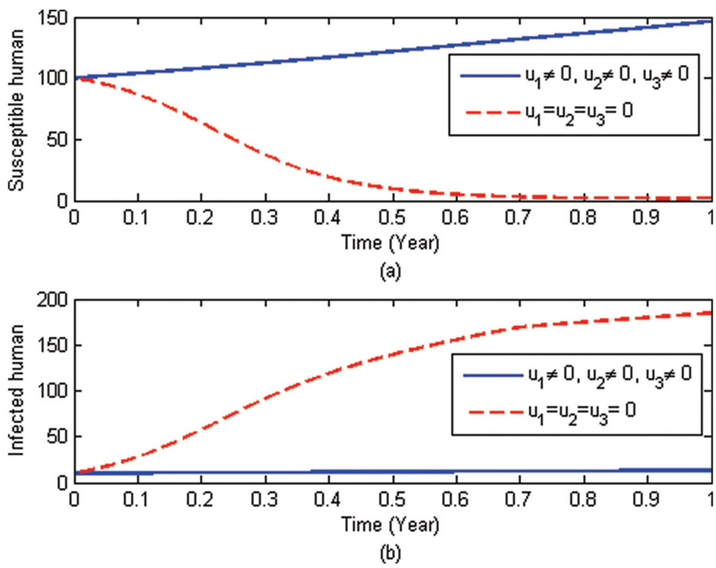

Figure 19. Simulations of the model showing the effect of vaccination, education and education campaign (strategy G) in human population.

We calculate and compare the cost effectiveness ratio (ICER) for strategy B and strategy C as follows:

Strategies	Total infections averted	Total costs ($)	ICER
Strategy B	0.3021	87.1431	288.4578
Strategy C	0.6625	47.2157	−110.786

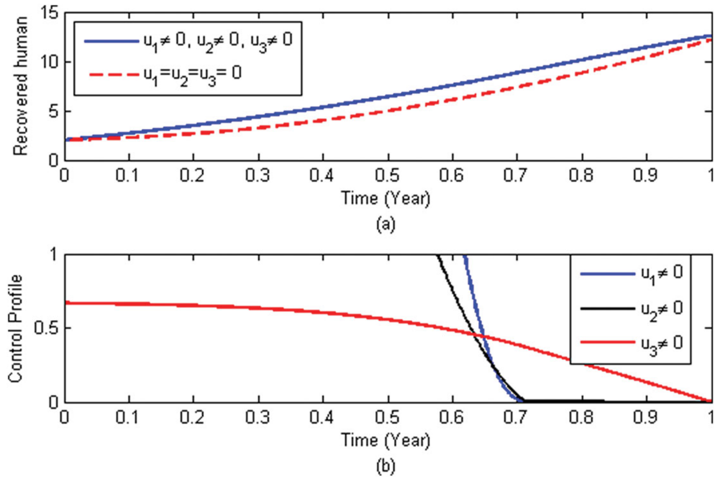

Figure 20. Simulations of the model showing the effect of vaccination education and treatment of infected human (strategy G) in control profile.

Table 3. Control strategies in order of increasing averted.

Strategies	Total infections averted	Total costs ($)	*J* ($)
Strategy B	0.3021	87.1431	4.9826×10^4
Strategy C	0.6625	47.2157	5.6626×10^4
Strategy D	0.9553	131.0979	5.6353×10^4
Strategy G	456.5505	255.9165	4.6908×10^3
Strategy F	456.5515	147.8035	4.6803×10^3
Strategy A	456.8543	148.6328	5.1308×10^3
Strategy E	456.8662	191.8063	5.1757×10^3

The ICER is calculated as follows

$$\text{ICER(B)} = \frac{87.1431}{0.3021} = 288.4578, \quad \text{ICER(C)} = \frac{47.2157 - 87.1431}{0.6625 - 0.3021} = -110.786.$$

The comparison between strategies C and B shows a cost saving of $110.786 for strategy C over strategy B. The negative ICER for strategy C indicates that strategy B is strongly dominated and less effective than strategy C. Therefore, strategy B is excluded from the set of alternatives. We exclude B and compare strategy C and D, and ICER recalculated as follows

Strategies	Total infections averted	Total costs ($)	ICER
Strategy C	0.6625	47.2157	71.2689
Strategy D	0.9553	131.0979	286.4829

The comparison between strategies C and D indicate that strategy D is strongly dominated and is more costly than strategy C as ICER(C) $<$ ICER(D) then strategy D is excluded in set of alternative hence C and G are compared.

Strategies	Total infections averted	Total costs ($)	ICER
Strategy C	0.6625	47.2157	71.2689
Strategy G	456.5505	255.9165	0.45779

The comparison shows that ICER(G) $<$ ICER(C), hence strategy C is more costly and excluded in the set of alternatives. We compare strategies G and F.

Strategies	Total infections averted	Total costs ($)	ICER
Strategy G	456.5505	255.9165	0.5605
Strategy F	456.5515	147.8035	−108113

The negative ICER for strategy F shows that the strategy G is more costly and less effective than strategy F. Therefore, the strategy G is excluded from the set of alternatives and we compare strategies F and A as follows

Strategies	Total infections averted	Total costs ($)	ICER
Strategy F	456.5515	147.8035	0.3237
Strategy A	456.8543	148.6328	2.7388

The strategy A is strongly dominated and is more costly than strategy F as then strategy A is excluded in set of alternative. Strategies F and E are compared.

Strategies	Total infections averted	Total costs ($)	ICER
Strategy F	456.5515	147.8035	0.3237
Strategy E	456.8662	191.8063	139.8246

Comparison between strategies E and F shows that strategy E is more costly and less effective than strategy F as ICER(F) $<$ ICER(E). Therefore strategy E is excluded from the set of alternatives and strategy F is cost effectiveness. Now, basing on these results we therefore conclude that strategy F (chicken vaccination and human education) is most cost effective of all strategies for ND.

6. Discussion and conclusion

In this paper, a deterministic model with optimal control for ND in Tanzania was derived and analysed to examine the best strategy for controlling ND in susceptible chicken and human in Tanzania poultry activities. The Pontryagin's maximum principle used to derive and analyse the necessary conditions for optimal control strategies such as chicken vaccination (u_1), human education campaign (u_2) and treatment on infected human(u_3) for minimizing the spread of ND. Numerically, the model was rigorously analysed. Graphically, strategies A, E, F, G shows a significant different in chicken populations while B, C, D, F and G its' positive impact observed in human populations. The CEA results suggest the combination of chicken vaccination and human education campaign as the most cost-effective strategy in case of limited resources. In 2012 the Office International des Epizooties (OIE) reported that ND can be controlled through chicken vaccination and bio security measures [1], this results concurred with our findings. However this conclusion should be taken with high precautions because of the uncertainties around the geographical location especially in rural and remote areas. Basing on strategy F which is sufficient to combat the ND epidemic we advise the Ministry of Livestock and Fisheries in Tanzania through regional, district, division, ward and village veterinary officers to take extra incentive in ensuring that all chickens are vaccinated properly and timely.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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