

# DEVELOPMENT OF A DYNAMIC NETWORK DEA MODEL TO MEASURE PRODUCTION LINE'S PERFORMANCE: A CONCEPTUAL PAPER

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## ABSTRACT

Production line in manufacturing industry usually is made up of several processes and must go through performance measurement to determine whether they are efficient or inefficient. The extended Data Envelopment Analysis (DEA) which is the Network Data Envelopment Analysis (NDEA) is developed to look inside the production line and find the source of inefficiency of each sub process. However, the model can only measure the efficiency of the production line for current time periods only without considering the past time periods and detect any changes of performances that might occur during the time periods. In this paper, we proposed a Dynamic Network DEA model that can be used to measure the performance of the same production line by taking into account any changes according to time. We treat these changes as a Decision Making Units (DMUs) which is the entity that are going to be measured. This dynamic network model only considered the data inputs from different time periods. Our goals of developing the DNDEA model on the production line are to identify the inputs and outputs required and to consider the relationship and connection between each of the processes in the production line and thus measure the performance of the entire production line. The expected outcome of this paper is to propose a conceptual model that can be used for performance measurement in manufacturing production line dynamically.

*Keywords: Dynamic Network DEA, Performance Measurement, Manufacturing System*

## INTRODUCTION

In the manufacturing industry, a production line is quite an efficient setup for manufacturing and assembling large quantities of products. For example, in the automotive industry, a car is composed of thousands of components and still hundreds of cars are being produced on the production line every single day. If each car was to be assembled individually rather than on a production line, it could take months to produce just one car. Other than quantity, the quality of the product as well as the performance of production line must also be taken into consideration during the products manufacturing. There are many tools that appear great for performance measurement, which one of them is called Data Envelopment Analysis (DEA).

DEA deal with measurements of decision making units (DMU) to find the relative efficiency based on multiple inputs and outputs. The traditional DEA treats the DMU mainly as a black box, within which inputs are supplied to produce outputs and measuring the efficiency of a DMU as a whole unit without taking into account the sub DMUs inside the structure. It also usually neglects carry-over activities between two consecutive terms and only

focus on the separate time period independently aiming local optimization in a single period, even if these models can consider the time change effect [1].

Our approach, however, used the combination of dynamic and network DEA model that can overcome the limitation above by measuring the whole DMU, considering the inside of the DMU as well. Thus, in this paper, we proposed a conceptual dynamic network DEA (DNDEA) model that can be implemented for performance measurement of the network structured production line for different time scales in details. By using this model, each of the processes in the production will be measured to find the efficiencies and thus measuring the whole performance of the production line from one period to one another.

## LITERATURE REVIEW

### Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA), first introduced by Charnes et al. (1978), has been proved to be a useful tool in evaluating relative performance of homogeneous decision-making units (DMU) in a multiple-input multiple-output setting [2]. A few of the characteristics that make it powerful is that it can

handle multiple input and multiple output models and doesn't require an assumption of a functional form relating inputs to outputs. It also estimates the efficiency index by calculating the ratio of weighted outputs to weighted inputs, and the input and output weights are decided according to the best interests of the DMU being evaluated.

- DEA can handle multiple input and multiple output models.
- It doesn't require an assumption of a functional form relating inputs to outputs.
- DMUs are directly compared against a peer or combination of peers.
- Inputs and outputs can have very different units. For example, X1 could be in units of lives saved and X2 could be in units of dollars without requiring an a priori tradeoff between the two.

**Previous Models of Dynamic Network Data Envelopment Analysis (DNDEA)**

Färe and Grosskopf (1996) introduced the dynamic network DEA model which used the formulation of storable inputs to allow a synchronism between the appearance of inputs and the use of inputs in the dynamic production model [2]. These network DEA models allow the researcher to study the “inside” of the usual black box technology both in static and dynamic ways.. They achieve this by providing a very general framework for specifying the inner workings of the black box. The basic idea of the network model is thus to “connect” processes, providing a single model framework for multi-stage production (with intermediate products, for example) or multi-period production. This situation has typically been handled in the DEA literature as a rather ad hoc series of DEA problems, or through the use of multiple stages [3].

Apart from that, Tone and Tsutsui (2014) has developed a dynamic model with network structures within the slack-based measures. Divisions are connected by links and consecutive periods are connected by carry-overs. In the dynamic situation, carry-overs appear in many enterprises. For example, in financial institutions, non-performing loans and profit earned forward are respectively undesirable and desirable carry-overs [3]. Also in medical institutions, numbers of bed and hospital bond are non-discretionary and undesirable ones. Meanwhile, in the electric power industry, the representative carry-overs are generation capacity, transmission line length and distribution transformer [2]. They apply the DNSBM model to a data set consists of 21 U.S. electric utilities over five years and compare the results with those given by the dynamic SBM (DSBM) model. This example deals with the

production side of relevant enterprises but not financial side, as shown in Fig.1.

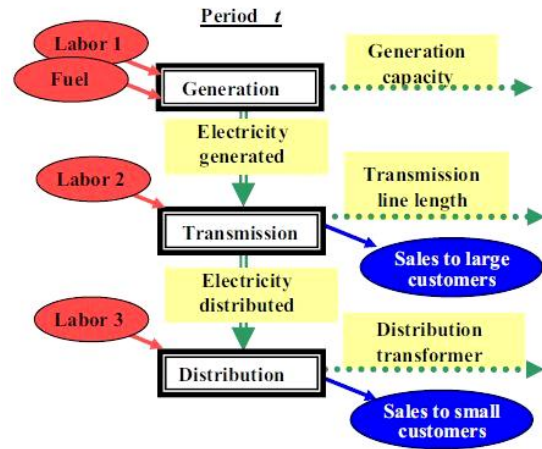
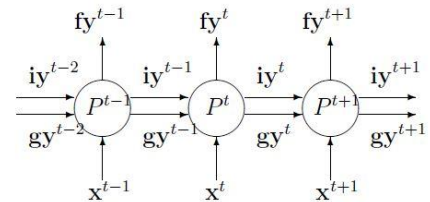


Fig.1 Vertically integrated electric power company

Bogetoft, Färe et al. (2009) introduced the dynamic technology which is their basic model for estimation of optimal private and public investment. The dynamics of our technology are modeled as the choice of consuming total output in the period of production or instead diverting some current production toward adding to the next period capital stock [4]. To provide some intuition for their model, assume that we have three time periods  $t-1$ ;  $t$ ;  $t+1$  and that there is a technology  $P^\tau$ ,  $\tau = t-1; t; t+1$ . In addition at each  $\tau$  there are some exogenous inputs  $x^\tau$  and final outputs  $fy^\tau$ . The final output is that part of total production  $y^\tau$  that is not allocated to private  $iy^\tau$  or public  $gy^\tau$  investments, i.e.,

$$y^\tau = fy^\tau + iy^\tau + gy^\tau \tag{1}$$

Final outputs



Exogenous inputs

Fig.2 The dynamic technology.

The bottom vertical arrows indicate the exogenous inputs into the respective technologies  $P^\tau$ . The top vertical arrows indicate the final output from each technology. They define the objective function over these final outputs, just as in the Ramsey model. The two horizontal arrows entering each technology represent the private and public

investment from the previous period. Thus, decisions concerning consumption versus investment in one period have consequences in the ensuing periods. One attractive feature of the dynamic model illustrated in Fig 2 is that it can be implemented as a dynamic activity analysis or DEA model. Given that we have  $k = 1, \dots, K$  observations of  $m = 1, \dots, M$  outputs ( $y_1, \dots, y_M$ ) and  $n = 1, \dots, N$  inputs ( $x_1, \dots, x_N$ ) in each period  $t$ , the model may be written for the three period case as;

$$\begin{aligned} & P(x^{t-1}, x^t, x^{t+1}, iy^{t-2}, gy^{t-2} \\ & = \{(fy^{t-1}, fy^t, (fy^{t+1} + iy^{t+1} \\ & + gy^{t+1})) \end{aligned} \quad (2)$$

$$\begin{aligned} & fy_m^{t-1} + iy_m^{t-1} + gy_m^{t-1} \\ & \leq \sum_{k=1}^K z_k^{t-1} (fy_{km}^{t-1} \\ & + iy_{km}^{t-1} + gy_{km}^{t-1}), \forall m, \end{aligned} \quad (3)$$

$$\sum_{k=1}^K z_k^{t-1} x_{kn}^{t-1} \leq x_n^{t-1}, n = 1, \dots, N, \quad (4)$$

$$\sum_{k=1}^K z_k^{t-1} iy_{km}^{t-2} \leq iy_m^{t-2}, m = 1, \dots, M \quad (5)$$

$$\sum_{k=1}^K z_k^{t-1} gy_{km}^{t-2} \leq gy_m^{t-2}, m = 1, \dots, M \quad (6)$$

$$z_k^{t-1} \geq 0, k = 1, \dots, K, \quad (7)$$

$$\begin{aligned} & fy_m^t + iy_m^t + gy_m^t \\ & \leq \sum_{k=1}^K z_k^t (fy_{km}^t + iy_{km}^t \\ & + gy_{km}^t), \forall m, \end{aligned} \quad (8)$$

$$\sum_{k=1}^K z_k^t x_{kn}^t \leq x_n^t, n = 1, \dots, N, \quad (9)$$

$$\sum_{k=1}^K z_k^t iy_{km}^{t-1} \leq iy_m^{t-1}, m = 1, \dots, M, \quad (10)$$

$$\sum_{k=1}^K z_k^t gy_{km}^{t-1} \leq gy_m^{t-1}, m = 1, \dots, M, \quad (11)$$

$$z_k^t \geq 0, k = 1, \dots, K, \quad (12)$$

$$\begin{aligned} & fy_m^{t+1} + iy_m^{t+1} + gy_m^{t+1} \\ & \leq \sum_{k=1}^K z_k^{t+1} (fy_{km}^{t+1} \\ & + iy_{km}^{t+1} + gy_{km}^{t+1}), \forall m, \end{aligned} \quad (13)$$

$$\sum_{k=1}^K z_k^{t+1} x_{kn}^{t+1} \leq x_n^{t+1}, n = 1, \dots, N, \quad (14)$$

$$\sum_{k=1}^K z_k^{t+1} iy_{km}^t \leq iy_m^t, m = 1, \dots, M \quad (15)$$

$$\sum_{k=1}^K z_k^{t+1} gy_{km}^t \leq gy_m^t, m = 1, \dots, M \quad (16)$$

$$z_k^{t+1} \geq 0, k = 1, \dots, K, \quad (17)$$

where the  $z_k^{t+1}$  are intensity variables for  $k = 1, \dots, K$ ,  $\tau = t-1, \dots, t+1$  in our example. The  $P$  technology for period  $t-1$  is modeled by the (3)-(7), where (3) is the output constraint, (4) is the input constraint, (5)-(6) is the intertemporal investment constraints and (7) is the constraint on the intensity variables. Similarly, the period  $t$  technology is modeled by (8)-(12) and the  $t+1$  technology by (13)-(17). Note that each period's technology has its own intensity variables  $z_k^{t+1}$ ;  $\tau = t-1; t; t+1; k = 1, \dots, K$ . Fare and Grosskopf (1996) have shown that the dynamic technology inherits its properties from the single period sub technologies, so in this case the dynamic technology exhibits constant returns to scale and strong disposability of inputs and outputs [4].

Hashimoto et al. (2013) then developed a weighted dynamic network model (WDM) by adopting a slacks based similar to Tone and Tsutsui (2010, 2012) that focuses on the following matters.

- WDM does not include slacks of divisions or sub processes in the objective function of the optimization problem.
- WDM allows for joint outputs produced by more than one division.
- WDM incorporates aged variables in each period.

As an illustration in Fig 3, they assess the dynamic efficiency or productivity performance of Japanese prefectures. Their framework specifies that a prefecture's production process is expressed as a two parallel network system that allows resources to be reallocated between periods so that larger final outputs can be achieved through intertemporal optimization [5].

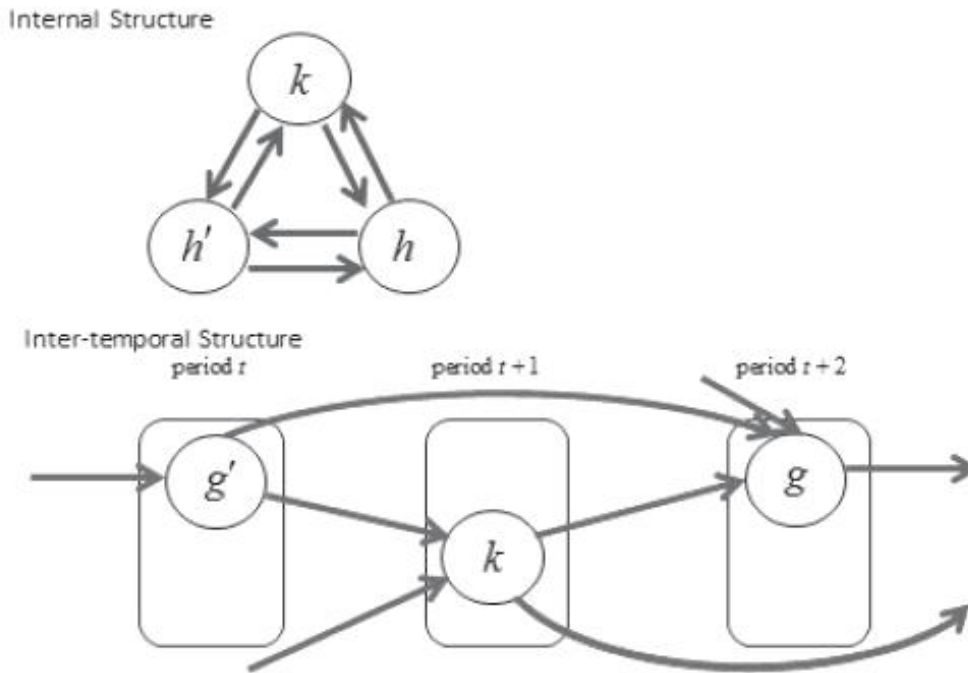


Fig.3 Dynamic Network Structure

Another research done by You and Jie (2016) applied the dynamic network slacks-based measurement model with free links and fixed links to evaluate the operational performance of 31 electric power supply companies in China from 2010 to 2012. Their approach allows for the consideration of the group heterogeneity of electric power transmission and also considers the new structural reform in the power grid of China in the near future, i.e., the separation of the transmission division and the distribution division [6].

## RESEARCH APPROACH

In this paper, we implemented the DNDEA model to a manufacturing production line in one of a company in Malaysia. In this model, we treated the production line as the DMU, which is the entity that will be measured to find the efficiency, while the processes in the production line is treated as the sub DMUs. This model enables us to study the relationship between each of the processes in the production line, thus measuring the whole production line in a different time scale.

This research begins by gathering the data set required considering the past and current time periods, which consist of the inputs consumed and the outputs produced by the DMU and sub DMUs. Unlike the previous dynamic models developed by other researchers, our proposed model does not consider any carry-overs because there are no stocks

or products kept at the end of the process. The production line should not have stocks or products, causing the production line to be inefficient that will put the company in big losses. All of the stocks and products are produced based on customer's demand and delivered straight away to the customers. Therefore, there will be no links between the production line in a certain period. Theoretically, there are four inputs that literally used in the manufacturing production line, which are the 4M; Manpower, Money, Machine and Method [7]. However, we are going to neglect the Method because it is not a numeric value and thus, it cannot be measured.

There are also other possible inputs that need to be considered in the production line which is the 2T; Time and Training [8]. Time is an index to show how long it takes to complete a certain task where the shorter time consumed is preferred than longer time while training is an activity that can improve the workers' skill that lead to better performance of productivity compared to the workers with less skills [7]. During this research, we might consider other inputs and outputs consumed in the manufacturing industry other than the 4M2T and account the relationship between each of the production line's processes.

Prior to conceptualizing the model, the processes in the production line were studied for better understanding in presenting the structures of the model. At the end of this research, we will compare

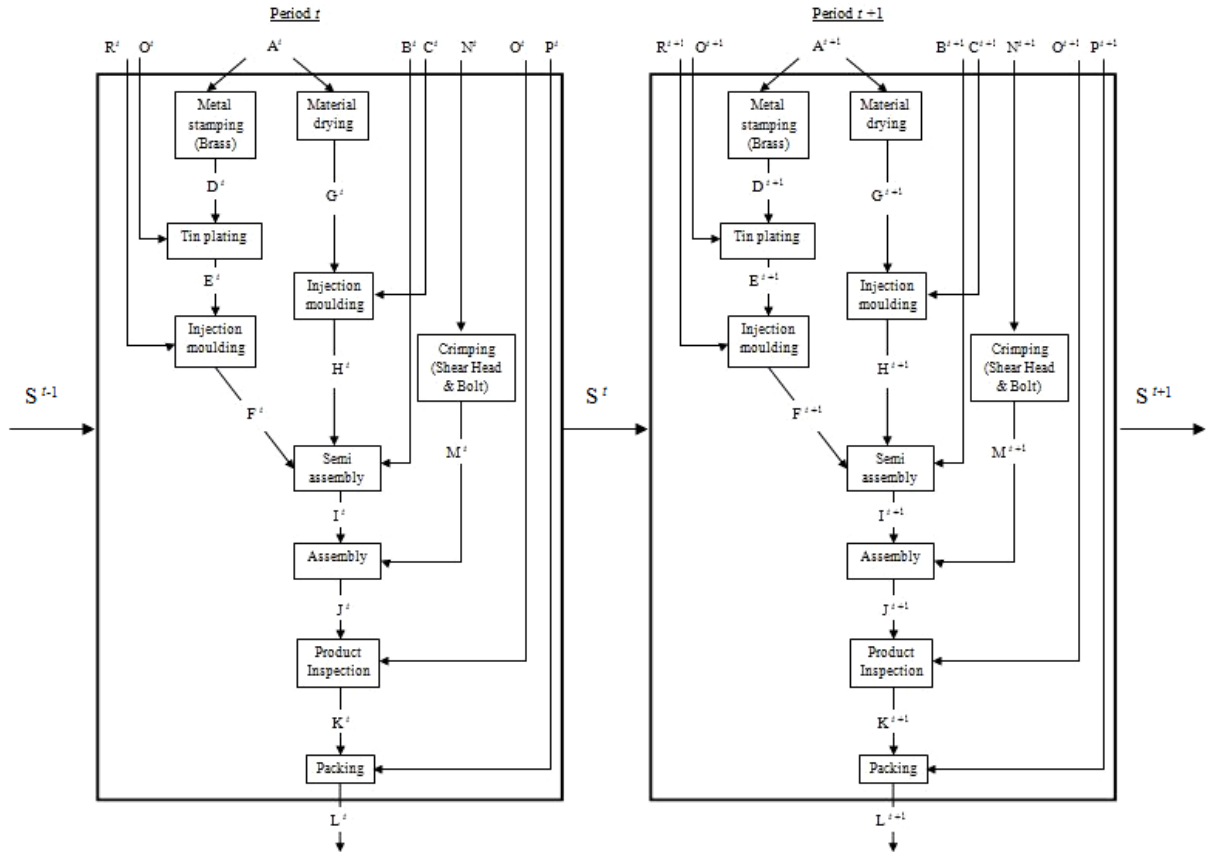


Fig.4 The DMU model for manufacturing production line.

between the DMUs from different periods to observe the efficiencies measured.

**CONCEPTUAL MODEL FOR POWER DISTRIBUTION PRODUCTION LINE**

As a case study, the DNDEA model was implemented on one of the manufacturing production line that produced electrical parts for power distribution. This production line does describe a network structured model with a combination of a series and parallel structure to form a complex structured model, which enable us to find the source of efficiencies in details. As illustrated in Fig 4, there are 10 processes consist of direct and indirect processes that are denoted as the sub DMUs inside the two production lines, which denoted as the DMUs from two different time scales. Since this model does not consider any carry overs, we illustrated this model to measure the efficiency of the production line, which denoted as S, from period to period and obtain the results, whether they affect each other in terms of the performance and compare which of the production line are efficient or inefficient.

In this model, some of the sub DMUs are linked to the other sub DMUs based on the inputs they consumed and the outputs they produced.

Table 1 Inputs and outputs determined.

Items	Inputs/Outputs
A	Material, Money, Machine, Manpower, Time
B	Machine, Manpower, Time
C	Machine, Time
D	Time, Number of Parts Produced
E	Time, Number of Parts Produced
F	Time, Number of Parts V
G	Time, Number Of First Piece Sample
H	Time, Number of Parts W
I	Time, Number of Semi Finished Product X
J	Time, Number of Semi Finished Product X
K	Time, Number of Finished Product Z, Number of Rejected Parts
L	Time, Number of Finished Product Z
M	Time, Number of Parts Y
N	Material, Money, Manpower, Time
O	Machine, Manpower
P	Machine, Manpower
Q	Machine, Manpower
R	Machine, Manpower

The numbers and types of inputs and outputs

used may vary according to the sub DMUs determined, refer to Table 1.

## CONCLUSION

We proposed the Dynamic Network DEA model to look inside the DMU, allowing greater insight as for measuring the efficiency of the production line. In this research, the model proposed is a combination of both series and parallel structures to form a complex structure. This model applies to the DMUs that consist of several sub DMUs, some of which consume the outputs produced by other sub DMUs and some of which produce the inputs consumed by another sub DMUs. The relationship between the sub DMUs will also be considered during the performance measurement of the production line. Our Dynamic Network DEA model allows for either an input orientation or an output orientation, and it's the only also can used to measure the network structures of the DMU for a different time scale. The model proposed is based on a case study of manufacturing production line that produced electrical parts power distribution and might differ from other production line, especially the relationship between the processes.

To complete the research, this model will be applied as a benchmark for different time scales. Then the results will be computed as to acknowledge which time scale of the production is more efficient and assist the company to appraise the production line's performance.

## ACKNOWLEDGEMENTS

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