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Improving Short-Length LDPC Codes with a CRC and Iterative Ordered Statistic Decoding

(Invited Paper)

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***Abstract*—We present a CRC-aided LDPC coding scheme that can outperform the underlying LDPC code under ordered statistic decoding (OSD). In this scheme, the CRC is used jointly with the LDPC code to construct a candidate list, instead of conventionally being regarded as a detection code to prune the list generated by the LDPC code alone. As an example we consider a $(128, 64)$ 5G LDPC code with BP decoding, which we can outperform by 2 dB using a $(128, 72)$ LDPC code in combination with a 8-bit CRC under OSD of order $t = 3$. The proposed decoding scheme for CRC-aided LDPC codes also achieves a better performance than the conventional scheme where the CRC is used to prune the list. A manageable complexity can be achieved with iterative reliability based OSD, which is demonstrated to perform well with a small OSD order.**

I. Introduction

In this era, we have witnessed the unprecedented emergences of new services and applications that require exponentially growing data and billions of devices in the network. Among different research and standardization efforts, the design of fifth-generation (5G) new radio (NR) wireless systems stands out to address these challenges. Two service categories in 5G, namely machine-to-machine communications (M2M) and ultra reliable communication (URC) call for traffic types with short packet transmission [1]. Hence, the attention to channel codes at short-to-moderate block lengths with good performance has been rising again.

Low-density parity-check (LDPC) codes [2] have been selected for data channels in 5G. In designing the LDPC codes of short lengths, well-established techniques such as density evolution, which models the asymptotic behavior of the codes, become less accurate. A detailed bibliography of improved constructions in designing LDPC codes of short-to-moderate length can be found in [3], but these constructions are heuristic.

While the sum-product (SP) algorithm, a sub-optimal iterative algorithm for decoding LDPC codes, remains appealing due to its low complexity, maximum likelihood (ML) optimal decoding may prove itself a feasible solution for short block length. Ordered statistic decoding (OSD) [4] is capable of achieving near ML performance for any linear code with a given generator matrix. The idea of OSD is to build a list of codeword candidates based on the most reliable independent positions (MRIPs), also referred to as most reliable basis (MRB), and apply an ML search within the list only. The use of the MRB achieves the smallest list error probability among all bases [5]. Several improvements [6] – [9] have been proposed to provide more flexible complexity and performance trade-offs during the past decade.

Aiming at reducing the complexity, in [10] an iterative reliability based-(IRB-) OSD algorithm was proposed. Compared to OSD, the list order t in IRB-OSD can be greatly reduced without compromising the performance. Promising results are shown for codes of medium and high length.

A cyclic redundant check (CRC) is an error-detecting code to verify the integrity of raw data. It is applied, for example, in the data channel of 5G NR [11] [12] to enhance the overall undetected block error rate (UBLER) of the coding system. In [13] [14], a remarkable performance is observed for CRC-aided polar codes under list decoding, where the CRC hereby is not used for error detection, but contributes directly to a stronger overall code. This phenomenon is a key element that helps polar codes to outperform LDPC codes at practical code lengths. In [15], in addition to polar codes, Gallager's regular LDPC codes are found to achieve near optimal performance for short blocks when combined with a CRC.

Inspired by this observation, in this work we investigate CRC-aided LDPC coding schemes based on two exemplary list decoders, namely the OSD as well as the IRB-OSD. Specifically, in Section II, a coding scheme is introduced where the CRC is used jointly with the LDPC code to construct the candidate list in the OSD. A potential coding gain compared to the conventional scheme is demonstrated. In Section III the scheme is decoded by IRB-OSD to reduce the complexity while maintaining a good performance. It turns out that a CRC-aided 5G LDPC code can achieve a performance comparable with a CRC-aided polar code with list decoding at the cost of manageable complexity. With an increased list order t, IRB-OSD has a competitive performance with a BCH code, which is quoted in the literature for having the best performance known at the given length and rate [1].

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II. Ordered Statistic Decoding in CRC-aided Coding Schemes

In this section an overview of OSD is firstly given for BPSK modulated signaling over the AWGN channel. Then two CRC-aided LDPC coding schemes based on OSD are presented and compared in terms of the number of candidates in the list that are valid CRC codewords. It is demonstrated that the scheme in which the CRC is used jointly with the LDPC code to construct a candidate list has a better performance as well as a lower complexity than the other.

A. Overview of OSD

Consider a binary information message sequence $\mathbf{u} = [u_1, u_2, \ldots, u_k]$. It is encoded into $\mathbf{v} = [v_1, v_2, \ldots, v_n]$ of length $n > k$ via a $k \times n$ generator matrix $\mathbf{G}$. In the BPSK case, the coded sequence $\mathbf{v}$ is mapped to a symbol sequence $\mathbf{x} \in \{-1, +1\}^n$ by $\mathbf{x} = 1 - 2\mathbf{v}$ and transmitted over the AWGN channel. The receiver gets the output sequence $\mathbf{y} = [y_1, y_2, \ldots, y_n]$ where $y_i = x_i + n_i$, and n_i denotes the real Gaussian noise with zero mean and variance $N_0/2$. Define the log-likelihood ratio (LLR) $r_i = \log \frac{P(v_i=0)|y_i)}{P(v_i=1|y_i)}$ for $i = 1, 2, \ldots n$. It is a reliability measure of how likely the transmitted bit v_i is equal to 0 or 1.

In OSD, the LLR vector $\mathbf{r} = [r_1, r_2, \ldots, r_n]$ is sorted in decreasing order of reliability and the resulting vector is $\mathbf{r}'$, i.e., $\mathbf{r}' = \pi_1(\mathbf{r})$ with $|r'_1| > |r'_2| > \ldots > |r'_n|$, where π_1 is a permutation of the set $\{1, 2, \ldots, n\}$ accounting for the sorting of LLR values. Let us permute the columns of $\mathbf{G}$ based on π_1 into $\mathbf{G}' = \pi_1(\mathbf{G})$. Now form the matrix $\mathbf{G}''$ as follows: select the k linearly independent columns of $\mathbf{G}'$ with largest LLR values and put them in decreasing order to form the first k columns of $\mathbf{G}''$. The remaining $n-k$ columns of $\mathbf{G}'$, ordered by decreasing reliability, are used to form the next $n-k$ columns of $\mathbf{G}''$. Denote by π_2 a permutation of the set $\{1, 2, \ldots, n\}$ corresponding to this column swapping process, i.e., $\mathbf{G}'' = \pi_2(\mathbf{G}') = \pi_2(\pi_1(\mathbf{G}))$. Lastly, perform Gaussian elimination on $\mathbf{G}''$ to obtain the systematic form $\mathbf{G}_{sys}$, where the first $k \times k$ submatrix is an identity matrix. Let $\tilde{\mathbf{r}} = \pi_2(\pi_1(\mathbf{r}))$, and $\tilde{\mathbf{u}}$ be the bitwise hard decisions on the first k elements of $\tilde{\mathbf{r}}$. Then the vector $\tilde{\mathbf{u}}$ corresponds to the MRIP of $\tilde{\mathbf{r}}$.

During the OSD algorithm, k-bit test error patterns are added to $\tilde{\mathbf{u}}$ with increasing Hamming weight $\leq t$, where t is also called *order* of the OSD. For example, $[0\ 0\ \ldots\ 0]$ is the only weight-0 test error pattern, and there are k weight-1 test error patterns which are $[1\ 0\ \ldots\ 0]$, $[1\ 0\ \ldots\ 0]$, $\ldots$,$[0\ 0\ 0 \ldots 1]$ where every single bit in the information position is inverted at a time. Build a list $\mathcal{L}$ of size $|\mathcal{L}| = \sum_{i=0}^{t} \binom{k}{i}$ consisting of candidate codewords $\tilde{\mathbf{v}}_i$ by re-encoding $\tilde{\mathbf{u}}$ with test pattern $\mathbf{e}_i$, for $i = 1, 2, \ldots, |\mathcal{L}|$, i.e., $\tilde{\mathbf{v}}_i = (\tilde{\mathbf{u}} + \mathbf{e}_i)\mathbf{G}_{sys}$. Perform an ML search within the list by choosing $\tilde{\mathbf{v}}^*$ such that $1 - 2\tilde{\mathbf{v}}^*$ has the closest Euclidean distance with $\tilde{\mathbf{r}}$. Finally obtain $\hat{\mathbf{v}}$, the OSD estimate, from $\tilde{\mathbf{v}}^*$ by the inverse permutation $\pi_1^{-1}\pi_2^{-1}$, i.e., $\hat{\mathbf{v}} = \pi_1^{-1}(\pi_2^{-1}(\tilde{\mathbf{v}}^*))$.

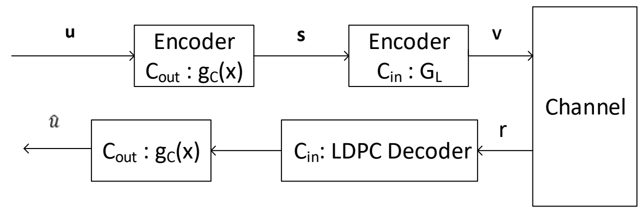

Fig. 1. CRC-aided LDPC codes: Scheme 1

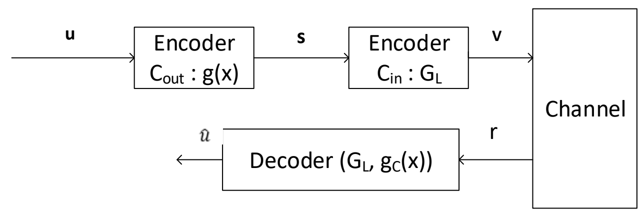

Fig. 2. CRC-aided LDPC codes: Scheme 2

B. CRC-aided LDPC codes: two coding schemes

A conventional CRC-aided LDPC coding scheme is depicted in Fig. 1, which we refer to as Scheme 1 in the following. As Fig. 1 shows, the information sequence $\mathbf{u}$ of length k is first encoded by a CRC of length m as outer code, resulting in the code sequence $\mathbf{s}$. Then $\mathbf{s}$ is encoded systematically by an LDPC code as the inner code and the resulting code sequence $\mathbf{v}$ of length n is modulated and transmitted over the channel. Let the vector $\mathbf{v} = [\mathbf{s}\ \mathbf{p}]$ be composed of the CRC code sequence $\mathbf{s}$ and the LDPC parity sequence $\mathbf{p}$. Also let $\hat{\mathbf{v}} = [\hat{\mathbf{s}}\ \hat{\mathbf{p}}]$ be the output delivered by the LDPC decoder, and $\hat{s}(x)$ be the polynomial representation of $\hat{\mathbf{s}}$. If $g_C(x)$, the CRC generator polynomial, divides $\hat{s}(x)$, then it implies that the errors in $\hat{s}(x)$ remain undetected, and we say this candidate passes the CRC test. Otherwise the candidate fails the test and an error is detected. Given $P_{u,L}$, the UBLER of the LDPC decoder, the UBLER of Scheme 1, P_u, is

$$P_u = 2^{-m} P_{u,L}. \tag{1}$$

Equation (1) assumes that each LDPC codeword appears at the output of decoder with the equal probability. A more detailed analysis on UBLER taking into account of weight distribution of the LDPC code is given in [16].

An alternative coding scheme is shown in Fig. 2, which we refer to as Scheme 2. The encoding process is identical to that in Scheme 1. In the decoding process, the decoder at the receiver takes in both $\mathbf{G}_L$ and $g_C(x)$ and delivers $\hat{\mathbf{u}}$ as the output. Denote by $\mathbf{G}_C$, a k by $k+m$ matrix which is the generator matrix of the CRC. Then define the overall code $\mathcal{C}$, with the generator matrix $\mathbf{G} = \mathbf{G}_C\mathbf{G}_L$. The joint decoding of CRC code and LDPC code can be equivalently seen as decoding on $\mathcal{C}$, the overall code and we refer to this system as Scheme 2.

C. Comparing the number of candidates under OSD

The block error rate (BLER) P_B in a list decoding algorithm can be formulated as

$$P_B = P_m + (1 - P_m)P_e \ , \tag{2}$$

where P_m is the probability that the candidate list misses the correct codeword, and P_e denotes the conditional probability that given the correct codeword is in the list, it is not delivered as the final estimate $\hat{\mathbf{u}}$ by the list decoder, i.e., an error occurs during the ML search of the list. A list with more candidates lowers the missing rate P_m. In the following we enumerate the number of candidates in the list under OSD of order t for the two CRC-aided decoding schemes.

In Scheme 1, a hard-decision information vector $\tilde{\mathbf{u}}$ is obtained through MRIPs based on $\mathbf{G}_L$, which takes $k+m$ bits as input to re-encode, thus the number of codeword candidates is $\sum_{i=0}^{t}\binom{k+m}{i}$. Assume that all LDPC codewords can be re-encoded in the list with equal probability, then after the CRC checking there are $N_1(t) = 2^{-m}\sum_{i=0}^{t}\binom{k+m}{i}$ candidates in the list. While for Scheme 2, the vector $\tilde{\mathbf{u}}$ is obtained through MRIPs based on $\mathbf{G}$, which takes k bits as input, it follows that the number of candidates is $N_2(t) = \sum_{i=0}^{t}\binom{k}{i}$.

Example 1. If an overall code has information bit length $k = 64$ and is based on an $m = 8$ bit CRC, Table I shows the number of candidates in the OSD list for the two CRC-aided schemes according to the enumeration $N_i(t)$ for $i = 1, 2$. For order $t = 2$, there are 10.3 candidates in Scheme 1 versus 2080 in Scheme 2, and for order $t = 3$, the number is 243.2 in Scheme 1 versus 43744 in Scheme 2. Simulations are also conducted to find the number of candidates passing the CRC test averaged over all SNRs of interest in Scheme 1, and the results are very close to the $N_1(t)$ for $t = 2$ and 3. It confirms the assumption that all LDPC codewords have nearly equal probability in the OSD list built by the re-encoding process.

TABLE I
Number of candidates in the OSD list for the two schemes

Scheme i	$N_i(2)$	simulated $t=2$	$N_i(3)$	simulated $t=3$
$i=1$	10.3	10.9	243.2	243.6
$i=2$	2081	2081	43745	43745

■

Note that having less candidates in the list does not translate to a lower complexity. In fact, the number of re-encodings to generate an OSD list is $\sum_{i=0}^{t}\binom{k+m}{i}$ in Scheme 1, which is larger than $\sum_{i=0}^{t}\binom{k}{i}$ in Scheme 2. Also to identify the MRIPs during the OSD, it consumes more operations on a generator matrix of larger dimension, which is the case for $\mathbf{G}_L$ in Scheme 1 versus $\mathbf{G}$ in Scheme 2. In addition, Scheme 2 saves the process of CRC checking, which is needed in Scheme 1. So we conclude that given the same OSD order, the complexity of Scheme 1 is larger than that of Scheme 2.

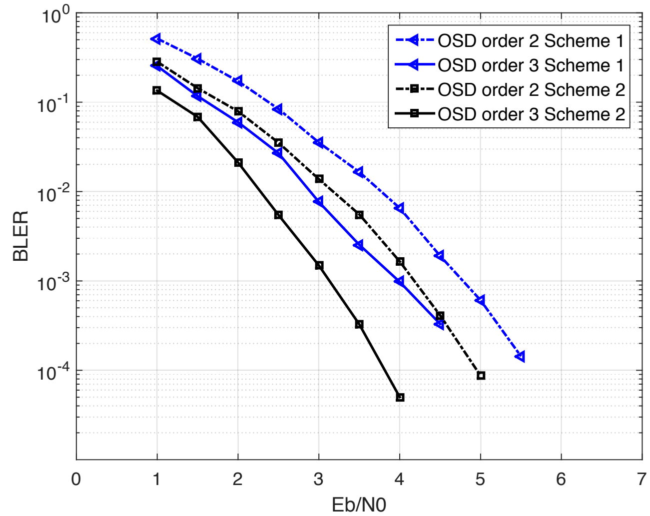

Fig. 3. Performance comparison between Scheme 1 and 2

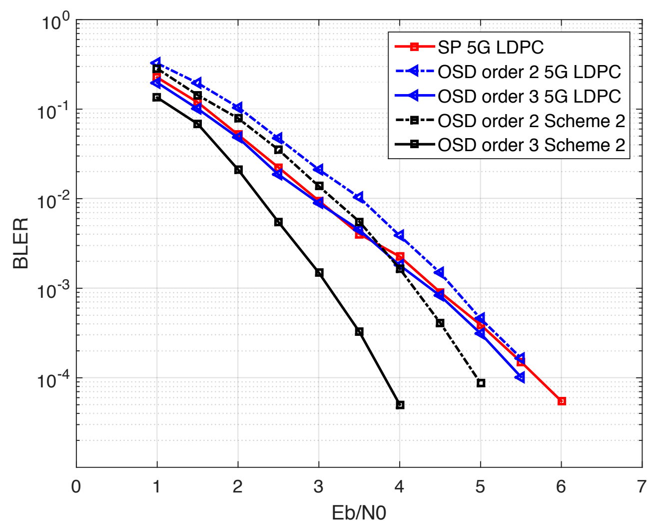

Fig. 4. Performance of CRC-aided Scheme 2 and $(128, 64)$ 5G LDPC

D. Performance of OSD in CRC-aided coding schemes

Fig. 3 shows a BLER performance comparison between the two coding schemes. For an order of $t = 2$, Scheme 2 achieves a 0.5 dB gain over Scheme 1 at a BLER of 10^{-4}. While for an order of $t = 3$, a coding gain of 1 dB is achieved with Scheme 2 compared to Scheme 1.

In the following, we show the performance of a single 5G LDPC code versus its CRC-aided counterpart (Scheme 2) to demonstrate the better performance of the latter one. Specifically, a single $(128, 64)$ 5G LDPC code [17] is compared with Scheme 2, where a $(128, 72)$ LDPC code is used together with a 8-bit CRC. The LDPC code is constructed by taking the first 9 information nodes and 9 parity nodes in the 5G base graph 2 (the first 2 information nodes are punctured and not transmitted over the channel), then lifting the base matrix with size 8.

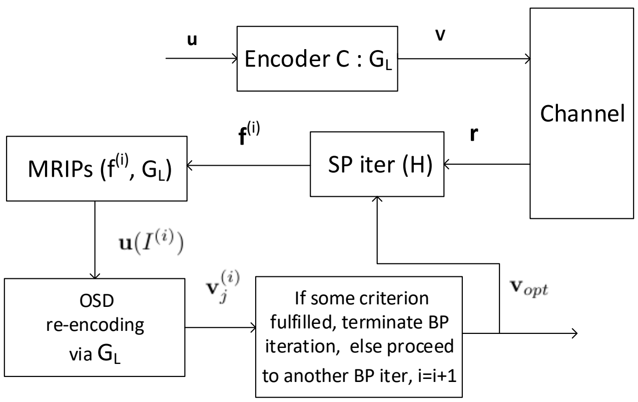

Fig. 5. Flow chart of the IRB-OSD

Fig. 4 shows the BLER performance. For reference, the SP decoding performance of the single $(128, 64)$ 5G LDPC code is also simulated. In the single LDPC coding scheme, an improvement of 0.5 dB can be obtained when the OSD order is increased from 2 to 3 at a BLER of 10^{-2}. At a BLER of 10^{-4}, both get similar performance to that of SP. For Scheme 2, more than 1dB improvement is obtained by increasing the OSD order, and it outperforms the 5G LDPC code by 0.5 dB and 1.5 dB, respectively, for OSD order $t = 2$ and 3, at a BLER of 10^{-4}. The superior performance observed for Scheme 2 implies that the weight spectrum of the overall code $\mathcal{C}$ is improved by the CRC.

III. Iterative Reliability Based OSD in CRC-aided Coding Schemes

In Section II, a remarkable performance is achieved with CRC-aided Scheme 2 under OSD. However, it is at the cost of re-encoding a vast number of candidates in the list. It is shown in [4] that the list size is exponentially increasing with the OSD order t, and the complexity of the algorithm is $O(k^t)$. In fact, OSD is a universal decoder and can be used to decode any linear code. To reduce the complexity of the CRC-aided LDPC scheme, IRB-OSD, a list decoding algorithm that exploits the structure of LDPC codes, is considered in this section. An overview of IBR-OSD is presented, followed by a performance and complexity comparison.

A. Overview of IRB-OSD

A flow chart of IRB-OSD is illustrated in Fig. 5. Given the channel LLR vector $\mathbf{r}$ at the receiver, an SP iteration is performed and at the i-th iteration the decoder delivers $\mathbf{f}^{(i)}$, the *a posteriori* probability LLRs for each bit. Then reliable information is sorted based on $\mathbf{f}^{(i)}$ and $\mathbf{G}_L$, and the k MRIPs are identified. Denote by $I^{(i)}$ the set of indices corresponding to the MRIPs and $\mathbf{u}(I^{(i)})$ the hard decision bits from $\mathbf{f}^{(i)}$ with the index set $I^{(i)}$. Then OSD is realized by re-encoding test patterns and the information set $\mathbf{u}(I^{(i)})$. For each candidate codeword in the list, find the corresponding Euclidean distance with respect to the received vector $\mathbf{r}$. If $\mathbf{v}_j^{(i)}$, the j-th codeword in the list at the i-th SP iteration has the distance $d_j^{(i)}$ such that $d_j^{(i)} < d_{opt}$, where d_{opt} is the minimal Euclidean distance found so far, set $\mathbf{v}_{opt} = \mathbf{v}_j^{(i)}$ and $d_{opt} = d_j^{(i)}$. If a maximum number of SP iterations is reached, then halt the program, otherwise perform another SP iteration and set $i = i + 1$.

A few stopping criteria are also presented in [10] to reduce the complexity. Denote by $\mathbf{v}_f^{(i)}$ the hard decision estimate from $\mathbf{f}^{(i)}$ and $\mathbf{H}$ the parity check matrix of the LDPC code, then if $\mathbf{v}_f^{(i)}\mathbf{H}^T = 0$, where $\mathbf{H}^T$ denotes the transpose of $\mathbf{H}$, simply terminate the program without going through the OSD step. In addition, if d_{opt} keeps unchanged for a consecutive number of iterations, then $\mathbf{v}_{opt}$ will be delivered as the final estimate. The rationale is that the SP decoder may have reached its convergence.

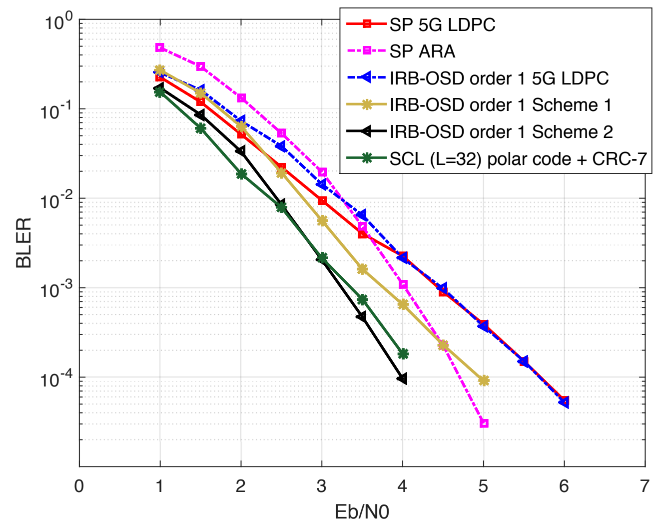

Fig. 6. Performance of $(128, 64)$ 5G LDPC, ARA, and CRC-aided coding schemes under SP and IRB-OSD

B. Performance of IRB-OSD in CRC-aided schemes

Fig. 6 shows the BLER performance of different $(128, 64)$ coding schemes under SP and IRB-OSD. At this length, we also compare the SP performance between the 5G LDPC code and an accumulate-repeat-accumulate (ARA) LDPC code [18], which has a low decoding threshold as the block length goes to infinity. At a BLER above 10^{-2}, the 5G LDPC code outperforms the ARA code, but at a BLER of 10^{-4}, the ARA code achieves about 1 dB coding gain over the 5G LDPC counterpart. Note that the 5G LDPC codes are designed primarily for the enhanced mobile broadband (eMBB) scenario in 5G for which the target error rate is 10^{-2}. Under the IRB-OSD of order 1, the performance of the 5G LDPC code without CRC is not improved upon SP.

The performance of Scheme 1 under IRB-OSD becomes better than that of SP decoding of the 5G LDPC code below a BLER of 10^{-2}, but is still worse than the ARA code at a BLER of 10^{-4}. On the other hand, the performance of Scheme 2 under IRB-OSD outperforms the ARA code and 5G LDPC code by around 0.6 dB and 1.5 dB at a BLER of 10^{-4}. For

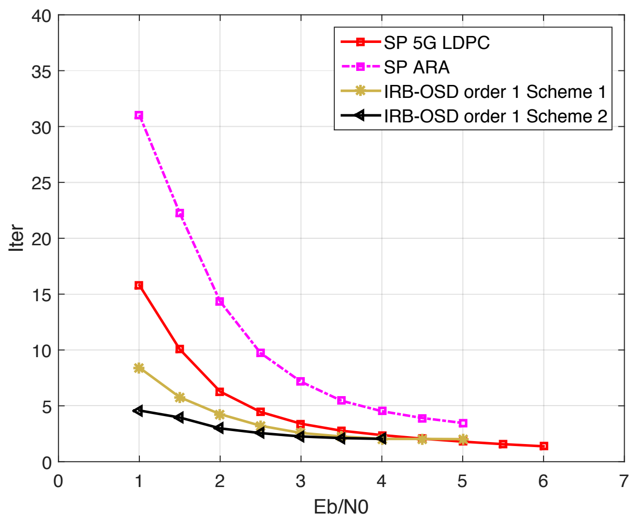

Fig. 7. Number of iterations for $(128, 64)$ 5G LDPC code, ARA code, and CRC-aided coding schemes under SP and IRB-OSD

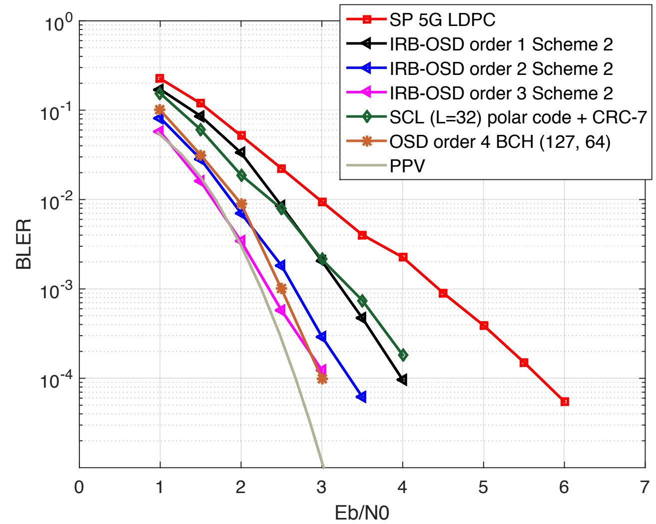

Fig. 8. Performance of Scheme 2 under IRB-OSD in different orders

comparison, the performance of a polar code with 7-bit CRC under successive cancellation list (SCL) decoding [19] is also included in this figure, and it has a performance comparable with Scheme 2 under IRB-OSD.

Fig. 7 shows the average number of iterations in decoding 5G LDPC codes, ARA codes, and CRC aided schemes under SP and IRB-OSD. The SP decoder terminates as soon as the hard-decision estimate satisfies the syndrome condition, while for IRB-OSD, if the optimal list candidates coincide with each other from two consecutive iterations, then IRB-OSD terminates. So the minimum number of iterations for IRB-OSD is 2. For the SP decoder, the ARA code has a better performance than the 5G code at the cost of a higher iteration number. The IRB-OSD requires a much smaller iteration number than SP, namely 8.4 and 4.6 at an SNR of 1 dB for Scheme 1 and 2, respectively, compared to 15.8 for SP. The iteration number for SP decreases to 2.4 around 4 dB, which is very close to that in Scheme 1 and 2. Given the order $t = 1$ of IRB-OSD, namely 64 candidates in the list, the overall complexity of IRB-OSD is manageable, and the complexity gap to SP is even smaller at low SNR.

The computational cost of IRB-OSD results mainly from the iterations of SP and the re-encodings in OSD of order t. Numbers of operations are presented in [10] for a rate 1/2 LDPC code of length n with column weight J decoded by IRB-OSD. For OSD of order $t = 1$, it requires nk^2 binary additions and k^2 real additions, while $11nJ - 9n$ real multiplications, $n(J + 1)$ real divisions and $n(3J + 1)$ real additions are required for one iteration of SP.

Tables II and III list the number of operations for Scheme 2 under IRB-OSD and the 5G LDPC code under SP at an SNR of 1 dB. The iteration numbers for IRB-OSD, 4.6, and 15.8 for SP are extracted from Fig. 7 and used in Table II and III, respectively. We also assume an average column weight of $J = 3.22$ in Table II and $J = 3.27$ in Table III.

As Tables II and III show, the number of binary additions is 2.4×10^6 for IRB-OSD. The number of real additions in IRB-OSD, 2.5×10^4, is about the same as 2.2×10^4 in SP. The SP requires 8.6×10^3 and 5.5×10^4 real multiplications and divisions, respectively, which are three times as many as for IRB-OSD. This is because SP iterates three times as much as IRB-OSD at 1dB.

We also compare the two algorithms at 4 dB, where both the IRB-OSD and SP take 2 iterations. Then the IRB-OSD has an extra computational cost compared to that of SP, which is two times of OSD re-encodings. As Table IV shows, it takes 1.1×10^6 binary additions and 8.2×10^3 real additions. From Fig. 6 the extra complexity in IRB-OSD achieves a BLER of 10^{-4}, which is about 20 times smaller than that of SP at 4dB.

As the last example, Fig. 8 shows the performance of Scheme 2 under IRB-OSD for different orders t. Since we mainly focus on the ML performance of the overall code, complexity is not the concern in this case. At order $t = 2$, it outperforms the order $t = 1$ counterpart by more than 0.5 dB at a BLER of 10^{-4}. At order $t = 3$, a competitive performance is shown

TABLE II
Number of operations under IRB-OSD for Scheme 2 at SNR=1dB

	OSD	SP	IRB-OSD
Binary addition	2.4×10^6	0	2.4×10^6
Real addition	1.9×10^4	6.3×10^3	2.5×10^4
Real multiplication	0	2.5×10^3	2.5×10^3
Real division	0	1.6×10^4	1.6×10^4

TABLE III
Number of operations under SP for $(128, 64)$ LDPC code at SNR=1dB

	SP
Real addition	2.2×10^4
Real multiplication	8.6×10^3
Real division	5.5×10^4

TABLE IV
Number of OSD operations under the IRB-OSD for Scheme 2 at SNR=4dB

	OSD
Binary addition	1.1×10^6
Real addition	8.2×10^3

between Scheme 2 and a $(127, 64)$ BCH code under OSD of order $t = 4$. The BCH code is one of the codes known to have the best performance at this length [1]. Fig. 8 also includes a Polyanskiy-Poor-Verdú (PPV) bound [20] which characterizes the maximal achievable channel coding rate at a given block length and error probability. At an SNR of $1, 1.5$, and 2 dB, the corresponding simulated BLERs are on top of the PPV bound.

IV. Conclusion

In this work we improve short 5G LDPC codes by introducing CRC-aided LDPC coding schemes of equal overall length and rate. The resulting overall codes show an excellent performance under near-ML OSD decoding, which demonstrates that the improvements that can be observed for CRC-aided coding schemes are not unique to polar codes with list decoding. A competitive performance is observed if the CRC is used jointly with the 5G LDPC code to construct candidate the list in the OSD. With the IRB-OSD, the performance can be achieved at manageable complexity.

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