

Macroeconomic Risk and Debt Overhang*

Hui Chen

Gustavo Manso

MIT Sloan School of Management

University of California at Berkeley

November 30, 2016

Abstract

Since corporate debt tends to be riskier in recessions, transfers from equity holders to debt holders that accompany corporate decisions also tend to concentrate in recessions. Such systematic risk exposures of debt overhang have important implications for corporate investment and financing decisions, and for the ex ante costs of debt overhang. Using a calibrated dynamic capital structure model, we show that the costs of debt overhang become higher in the presence of macroeconomic risk. We also provide several new predictions on how the cyclicalities of a firm's assets in place and growth options affect its investment and capital structure decisions.

JEL Classification: G31, G33

*We are grateful to Santiago Bazzdrusch, Ian Dew-Becker, David Mauer, Erwan Morellec, Stew Myers, Chris Parsons, Michael Roberts, Antoinette Schoar, Neng Wang, Ivo Welch, and seminar participants at MIT, Federal Reserve Bank of Boston, Boston University, Dartmouth, University of Lausanne, University of Minnesota, the Risk Management Conference at Mont Tremblant, the Minnesota Corporate Finance Conference, the Western Finance Association Meeting at Victoria, and the American Economic Association Meeting at San Diego for their comments. Send correspondence to Gustavo Manso, University of California at Berkeley, 545 Student Services Building # 1900, Berkeley, CA 94705, United States; telephone: 510-643-6623. E-mail: manso@berkeley.edu.

Introduction

How do firms make investment decisions? The classic net present value (NPV) rule prescribes that we value an investment opportunity by forecasting its future cash flows and discounting future cash flows at rates that appropriately reflect the embedded risks. However, deviations from the first-best can arise due to market frictions, such as agency problems.¹ Most of the existing studies of agency problems primarily focus on the cash-flow effects of agency conflicts, while treating the discount rates as exogenous (often by adopting risk-neutral settings). In this paper, we demonstrate the important interactions between the cash-flow channels and discount rate channels. In particular, the presence of macroeconomic risk and time-varying risk premiums affects the timing and size of investment distortions, which endogenously determines the discount rate that should be used to evaluate such distortions. The ex ante magnitude of agency costs can become significantly higher as a result.

We focus on a classic type of agency problem, *debt overhang*. [Myers \(1977\)](#) argued that, in the presence of risky debt, equity holders of a levered firm underinvest, because a fraction of the value generated by their new investment will accrue to the existing debt holders. Thus, from equity holders' point of view, investment decisions not only depend on the cash flows from investment, but also the transfers between different stake holders. We connect the investment distortions to the cyclicity of assets-in-place and growth options. Moreover, we quantify the impact of macroeconomic risk on the ex ante costs of debt overhang in a dynamic model.

In the context of debt overhang, the intuition for how macroeconomic risks and agency problems interact is as follows. First, recessions are times of high marginal utilities, and this means that the distortions caused by agency problems during such times will affect investors more than in booms. Second, the size of agency conflict due to debt overhang (as measured by the potential transfer from equity holders to debt holders) depends on the riskiness of debt. It is well documented that corporate credit spreads are strongly countercyclical. Specifically, credit spreads tends to rise significantly in aggregate bad times. Thus, for a given investment opportunity, the transfers from equity holders to debt holders in a typical procyclical firm

¹See [Stein \(2003\)](#) for a recent survey on this massive body of research.

will tend to concentrate in bad times. Taken together, these two effects both raise the ex ante costs of debt overhang and cause larger distortions to investment.²

To demonstrate these effects, we combine a calibrated asset pricing model that generates realistic implications for asset prices, and a simple capital structure/investment model that captures the interactions between agency conflicts and macroeconomic conditions. These interactions are endogenous due to agents' ability to respond to changing macroeconomic conditions through their investment and financing decisions (e.g., delaying rather than deserting an investment; choosing a lower leverage). We adopt a stochastic discount factor that generates time-varying risk prices as macroeconomic conditions change. For the firm, the cash flows from assets-in-place and growth options have time-varying expected growth rates, conditional volatility, and jumps that coincide with changes in macroeconomic conditions. We then examine the agency costs of debt for firms with different leverage, different present value of growth option (PVGO), and different systematic risk exposure for their assets-in-place and growth options.

Our model shows that debt overhang costs are substantially higher when macroeconomic risk is taken into account. In our benchmark case, the debt overhang costs for a low leverage firm peak at less than 0.5% of the total firm value without macroeconomic risk, while these costs peak at 2.7% or 3.6% in booms and recessions, respectively, in the presence of macroeconomic risk. For a high leverage firm, the debt overhang costs peak at 5.1% without macroeconomic risk, while these costs peak at 8.5% or 10.7% in boom and recessions, respectively, with macroeconomic risk.

The impact of macroeconomic risk on debt overhang depends on the cyclicity of cash flows from assets-in-place and growth opportunities. More cyclical cash flows from the assets-in-place increase the probability that the firm will underinvest during recessions, when marginal utilities are higher, thus amplifying the impact of macroeconomic risk on the agency cost of debt. The effect of more cyclical cash flows from growth opportunities is ambiguous. On the one hand, more cyclical cash flows from growth opportunities increase the probability

²To the extent that the concentration of debt overhang in bad times affects aggregate investment and output, it can amplify the macroeconomic shocks and the fluctuations in risk premiums, which will further strengthen the two channels above. We do not examine this general equilibrium effect in this paper.

that firms will underinvest during recessions. On the other hand, the cost from delaying investment in recessions is lower. In our calibrated model, either of the two effects may dominate.

Another implication from the dynamic model is that debt overhang in bad times can also significantly distort investment decisions in good times, which we refer to as the *dynamic overhang effect*. In anticipation of poor economic conditions in the future, equity holders can become reluctant to invest, even though debt is currently relatively safe. Thus, when we make the firm more cyclical (for example, by making its growth rate higher in the good state and lower in the bad state), the conditional agency cost in the good state can rise rather than fall, which is in sharp contrast with the prediction of a static model. The more persistent the states are, the less the debt overhang problem in the bad states will propagate to the good states, and hence the bigger the differences in the conditional agency costs between good and bad states.

The macroeconomic risk in debt overhang will also affect firms' financing decisions. We compute the optimal leverage using the trade-off between tax benefits and costs of debt overhang.³ Based on our calibration, the optimal interest coverage for a firm with a relatively valuable growth option is 1.25 in the case without macroeconomic risks. After taking macroeconomic risks into account, the interest coverage rises to 2.47, while the market leverage drops from 54% to 37%. Furthermore, even with the firm's endogenous response in choosing a moderate leverage ratio, the ex ante agency costs are still quite sizable in the presence of macroeconomic risks.

Besides raising the costs of debt overhang and causing more delay in investment, macroeconomic risk can also lead to a new type of risk-shifting incentives for equity holders. Specifically, equity holders will want to reduce the transfer to debt holders by synchronizing the cash flows from investment with those from the assets-in-place. For example, the equity holders of a procyclical firm might prefer to invest in procyclical projects, even if these projects have

³We take the type of debt contract (consol bond) as given in this paper. [Stulz and Johnson \(1985\)](#), [Berkovitch and Kim \(1990\)](#), [Hackbarth and Mauer \(2012\)](#), and [Diamond and He \(2014\)](#) are among the papers that examine how the agency conflict can be (partially) resolved through contracting and financing adjustments.

lower NPVs. This result can be viewed as a general form of asset substitution, whereby in the presence of risky debt equity holders not only want to make risky investments in general, but especially in investments that minimizes the transfer to debt holders in bad times. This result can explain why a highly levered firm (e.g., a large bank) might not want to diversify its investments or hedge its market risk exposure, but instead load on assets with high systematic risks.⁴

In summary, our model produces the following testable predictions. First, the model predicts that underinvestment is more severe in recessions than in booms for firms with more cyclical assets-in-place or more cyclical growth options. Second, firms with more cyclical assets-in-place have higher agency costs of debt, and therefore should take on less debt. Third, firms with procyclical (countercyclical) assets-in-place have a bias to invest in procyclical (countercyclical) projects.

Our paper builds on a growing literature bringing macroeconomic risk into corporate finance. [Almeida and Philippon \(2007\)](#) used a reduced-form approach to measure the ex ante costs of financial distress. They show that the NPV of distress costs rises significantly after adjusting for the credit risk premium embedded in the losses. [Hackbarth, Miao, and Morellec \(2006\)](#), [Bhamra, Kuehn, and Strebulaev \(2010\)](#), and [Chen \(2010\)](#) used structural models to link capital structure decisions to macroeconomic conditions. A contemporaneous and independent paper by [Arnold, Wagner, and Westermann \(2013\)](#) extended the model of [Hackbarth, Miao, and Morellec \(2006\)](#) with real options to show that firms with growth options are more likely to default in recessions than those without growth options and thus should have higher credit spreads. They assumed agents are risk neutral (no risk premium), and they did not measure the costs of debt overhang. [Lamont \(1995\)](#) studied a static reduced-form model of debt overhang with macroeconomic conditions. He focused on the multiplicity of equilibria that arises in a general equilibrium model in which firms make financing and investment decisions.

Our paper contributes to the literature on dynamic investment and financing decisions of

⁴The result also can be applied to asset sales. [Diamond and Rajan \(2011\)](#) argue that debt overhang might make impaired banks reluctant to sell those bad assets with high systematic risk.

the firm. [Mello and Parsons \(1992\)](#), [Mauer and Triantis \(1994\)](#), [Leland \(1998\)](#), [Mauer and Ott \(2000\)](#), [Décamps and Faure-Grimaud \(2002\)](#), [Hennessy \(2004\)](#), [Titman, Tompaidis, and Tsyplakov \(2004\)](#), [Childs, Mauer, and Ott \(2005\)](#), [Ju and Ou-Yang \(2006\)](#), [Moyen \(2007\)](#), [Manso \(2008\)](#), [Morellec and Schuerhoff \(2010\)](#), [Hackbarth and Mauer \(2012\)](#), and [Sundaresan and Wang \(2015\)](#) are among those that developed dynamic models of investment to study distortions produced by debt financing. The bulk of these papers found that agency costs are typically below 1%. They do not consider, however, macroeconomic risk and its impact on the agency cost of debt.

Our paper also is related to the real options literature that studies dynamic investment decisions of the firm. [McDonald and Siegel \(1986\)](#), for example, studied the timing of an irreversible investment decision. [Dixit \(1989\)](#) analyzed entry and exit decisions of a firm whose output price follows a geometric Brownian motion. [Dixit and Pindyck \(1994\)](#) provided a survey of this literature. [Guo, Miao, and Morellec \(2005\)](#) studied a real options problem with regime shifts, but did not consider debt financing.

Recent studies have introduced defaultable debt into real business cycle models with investment (e.g. [Gomes and Schmid, 2010](#); [Miao and Wang, 2010](#); [Gourio, 2013](#)). They highlighted the role of credit risk in amplifying aggregate technology shocks and helped explain the predictive power that corporate bond spreads have for future investment and other economic activities documented in [Philippon \(2009\)](#) and [Gilchrist, Yankov, and Zakrajsek \(2009\)](#). Our paper differs from these studies by focusing on the debt overhang problem with long-term debt and lumpy investment. The partial equilibrium setting allows us to analytically characterize the impact of macroeconomic risk on investment.

Our analysis focuses on the debt overhang problem in a firm, but the insight on the interactions of macroeconomic risks and debt overhang has wide applications. As highlighted by the recent financial crisis in the U.S. and the European sovereign debt crisis, the important effects of debt overhang on the real economy go through multiple channels, including households, firms, and governments (see, e.g., [Philippon, 2010](#); [Reinhart, Reinhart, and Rogoff, 2012](#); [Dyran, 2012](#); [Mian, Rao, and Sufi, 2013](#); [Chen, Michaux, and Roussanov, 2013](#)).

While earlier studies have separately examined the impact of macroeconomic risk on

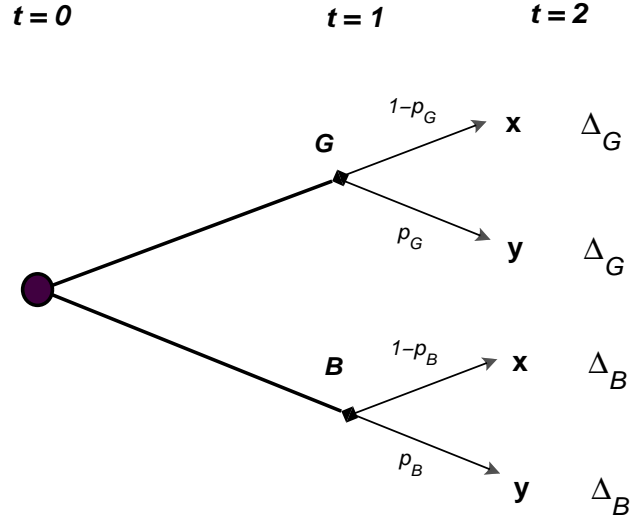


Figure 1: A Two-period Example.

investment (e.g., [Guo, Miao, and Morellec, 2005](#)) and financing (e.g., [Hackbarth, Miao, and Morellec, 2006](#); [Chen, 2010](#)), we emphasize the interactions between investment and financing in the presence of business-cycle fluctuations in cash flows and risk prices.

1 Two-Period Example

We first study a simple two-period model that illustrates the interplay between macroeconomic conditions and debt overhang. This simple model will help with the intuition behind the results obtained in the dynamic model, which we develop in the next section.

The economy can be in one of two aggregate states $s \in \{G, B\}$ at $t = 1$. The time-0 price of a one-period Arrow-Debreu security that pays \$1 at $t = 1$ in state s is given by Q_s . Since the marginal utility in the bad state is higher than the marginal utility in the good state, agents will pay more for the Arrow-Debreu security that pays off in the bad state than in the good state: $Q_B > Q_G$. For simplicity, we assume that the risk-free interest rate is 0, so that $Q_G + Q_B = 1$.

At $t = 2$, the firm's assets-in-place produce cash flow x with probability $1 - p_s$ and y

with probability p_s , where $x > y$. The different realizations of cash flow in a given aggregate state reflect firm-specific shocks, and the dependence of probability p_s on the aggregate state captures the impact of aggregate shocks on assets-in-place.

The firm has zero-coupon debt with face value F , $y < F \leq x$, which matures at time $t = 2$. Absolute priority is satisfied. Thus, if the firm does not produce enough cash flow to pay back debt holders, then debt holders seize the realized cash flow of the firm (no bankruptcy costs). $y < F$ makes debt risky and without which there will be no debt overhang.

Let's first assume that the equity holders of the firm can choose whether or not to undertake an investment I *after* learning the state s of the economy at $t = 1$. The investment produces an additional cash flow of $I + \Delta_s$, realized at the same time as the cash flows from assets-in-place. We assume that $\Delta_s > 0$ so that the investment opportunity has a positive NPV regardless of Q_s .

We now derive conditions under which equity holders will undertake the investment. The equity value of the firm when the manager makes the investment is

$$-I + (1 - p_s)(x + I + \Delta_s - F) + p_s(y + I + \Delta_s - F) \quad (1)$$

if $y + I + \Delta_s \geq F$, and

$$-I + (1 - p_s)(x + I + \Delta_s - F) \quad (2)$$

if $y + I + \Delta_s < F$. The equity value of the firm when equity holders choose not to make the investment is

$$(1 - p_s)(x - F). \quad (3)$$

It follows that equity holders will make the investment if

$$p_s \times \min(F - y, I + \Delta_s) < \Delta_s. \quad (4)$$

The left-hand side of the inequality gives the expected value of the transfer from equity holders to existing debt holders after the investment is made. Thus, equity holders will only

make the investment if the expected transfer is less than the NPV of the investment, so that the “overhang-adjusted NPV” is positive. It is easy to see that a higher leverage (larger F relative to y) will tend to increase the transfer, making the above condition harder to satisfy.

We define the indicator function Ω_s as

$$\Omega_s \equiv \begin{cases} 0 & \text{if } p_s \times \min(F - y, I + \Delta_s) < \Delta_s, \\ 1 & \text{otherwise.} \end{cases} \quad (5)$$

The function is equal to one if the equity holders do not undertake the investment opportunity, and zero otherwise.

Since the only source of agency cost in this example is the (present value of) foregone investment opportunities with positive NPV, the ex ante agency cost will be

$$A = Q_G \Omega_G \Delta_G + Q_B \Omega_B \Delta_B, \quad (6)$$

which is the sum over the two states of the product of the Arrow-Debreu prices Q_s ; the indicator function Ω_s which is equal to one when underinvestment occurs; and the losses Δ_s from underinvestment.

To assess the impact of variations in state prices on the agency cost of debt, we subtract the agency cost of debt when $Q_G = Q_B$ from (6) to obtain:

$$\left(\frac{1}{2} - Q_G\right) (\Omega_B \Delta_B - \Omega_G \Delta_G). \quad (7)$$

In the following discussions, we say that the assets-in-place are *procyclical* if $p_G < p_B$. We say that the growth option is *procyclical* if $\Delta_G > \Delta_B$.

Since $Q_G < \frac{1}{2}$, stronger cyclicity in the state prices will exacerbate the agency cost of debt if $\Omega_B \Delta_B > \Omega_G \Delta_G$.

Keeping all else constant, more cyclical cash flows from assets-in-place, i.e., lower p_G and higher p_B , makes the condition for investment (4) easier to satisfy in state G but harder in state B . As a result, underinvestment becomes more concentrated in the bad state,

exacerbating the costs of debt overhang when macroeconomic risk is taken into account.

Next, keeping all else constant, more cyclical cash flows $I + \Delta_s$ from the investment also make the condition for investment (4) easier to satisfy in state G , but harder in state B . However, it has the additional effect of reducing the potential loss if the investment is not made in state B . Therefore, the effect of stronger cyclicality of the growth option on the costs of debt overhang is ambiguous.⁵

So far the investment we consider is riskless: its cash flow is constant after investment is made. We now consider a risky investment opportunity that is only exposed to aggregate shocks. This is accomplished by assuming that the investment I is made at $t = 0$ as opposed to $t = 1$, while the cash flows from investment at $t = 2$ remain the same. When would equity holders make the investment? The condition is

$$Q_G p_G \min(F - y, I + \Delta_G) + Q_B p_B \min(F - y, I + \Delta_B) < Q_G \Delta_G + Q_B \Delta_B. \quad (8)$$

The right-hand side of the inequality gives the NPV of the investment, and the left-hand side again gives the expected transfer from equity holders to debt holders. In the case in which the cash flow from new investment is sufficiently high to make the existing debt risk-free in both states, the inequality (8) simplifies to

$$Q_G p_G (F - y) + Q_B p_B (F - y) < Q_G \Delta_G + Q_B \Delta_B.$$

In this case, the cyclicality of the growth option does not matter for the investment decision (only the NPV matters). The cyclicality of assets-in-place does matter for investment, as higher p_B and lower p_G will raise the total value of transfer.

However, if the cash flow from new investment is not enough to pay off the debt holders

⁵Growth opportunities can be either procyclical or countercyclical in practice. On the one hand, there may be less investment opportunities during recessions due to slower growth of the overall economy. On the other hand, financial distress and fire sales may provide profitable investment opportunities for firms.

in the states with low cash flows from assets-in-place, then the condition becomes

$$Q_{GP}G(I + \Delta_G) + Q_{BP}B(I + \Delta_B) < Q_G\Delta_G + Q_B\Delta_B.$$

Holding the NPV constant, making the investment opportunity more procyclical means raising Δ_G while lowering Δ_B so that $Q_G\Delta_G + Q_B\Delta_B$ is unchanged. If $Q_{GP}G < Q_{BP}B$ (e.g., when the assets-in-place are procyclical), then a more procyclical investment can lower the expected transfer from equity holders to debt holders, making equity holders more willing to make such an investment. In fact, the stronger the cyclicity of the investment, the better off the equity holders. Finally, it is also easy to check that when the assets-in-place are countercyclical, equity holders would prefer to invest in countercyclical growth options.

To summarize, our two-period model provides the following predictions:

- More cyclical assets-in-place make underinvestment more likely in bad times, exacerbating the costs of debt overhang when macroeconomic risk is taken into account.
- More cyclical investment opportunities also make underinvestment more likely in bad times. The overall effect on the costs of debt overhang when macroeconomic risk is taken into account is ambiguous.
- Among the growth options that are not too profitable (so that debt is still risky), equity holders would prefer to invest in ones that have the same cyclicity as their assets-in-place.

2 A Dynamic Model of Debt Overhang

In this section, we set up a dynamic capital structure model with investment to assess the quantitative impact of macroeconomic risk on investments and the costs of debt overhang.

2.1 Model Setup

2.1.1 The economy

We consider a simple economic environment that features business-cycle fluctuations in the level, the expected growth rate, and the volatility of firm cash flows. In addition, risk prices also vary over the business cycle, reflecting investors' different attitudes towards risks in good and bad times.

The economy has two aggregate states, $s_t = \{G, B\}$, which represent booms and recessions, respectively. The state s_t follows a continuous-time Markov chain, where within a small period Δ the probability of the economy switching from state G (boom) to state B (recession) is approximately equal to $\lambda_G \Delta$, while the probability of switching from state B to G is approximately $\lambda_B \Delta$. The long-run probability of the economy being in state G is $\lambda_B / (\lambda_G + \lambda_B)$.

We specify an exogenous stochastic discount factor (SDF), which captures business-cycle fluctuations in the risk-free rate and the risk prices:

$$\frac{dm_t}{m_t} = -r(s_t) dt - \eta(s_t) dW_t^m + \delta_G(s_t) (e^\kappa - 1) dM_t^G + \delta_B(s_t) (e^{-\kappa} - 1) dM_t^B, \quad (9)$$

with

$$\delta_G(G) = \delta_B(B) = 1, \quad \delta_G(B) = \delta_B(G) = 0,$$

where W^m is a standard Brownian motion that generates small systematic shocks; M^G and M^B are compensated Poisson processes with intensities λ_G and λ_B , respectively, which generate large shocks in the economy.⁶

The first two terms in the SDF process are standard. The instantaneous risk-free rate is $r(s_t)$, and the risk price for Brownian shocks is $\eta(s_t)$, both of which could change value when the state of the economy changes. The last two terms in (9) introduce jumps in the SDF

⁶Chen (2010) (Proposition 1) shows that such a stochastic discount factor can be generated in a consumption-based model when the expected growth rate and volatility of aggregate consumption follow a discrete-state Markov chain, and the representative agent has recursive preferences. His calibration is based on the long-run risk model of Bansal and Yaron (2004).

that coincide with a change of state in the Markov chain specified earlier. For example, if the current state is G , a positive relative jump size ($\kappa > 0$) will imply that the SDF jumps up when the economy moves from a boom into a recession. The value κ determines the risk price for the large shocks in the economy.

While it would be interesting to endogenize the stochastic discount factor in a general equilibrium model, we focus on the partial equilibrium setting in this model because it allows us to analytically characterize the impact of business-cycle risks on debt overhang problem.

2.1.2 Firms

A firm has assets-in-place that generate cash flow y_t^a , which we assume to be conditionally affine in an underlying state variable x_t ,

$$y_t^a = a_0(s_t) + a_1(s_t)x_t, \quad (10)$$

where x follows a Markov-modulated diffusion process:

$$\frac{dx_t}{x_t} = \mu(s_t)dt + \sigma_m(s_t)dW_t^m + \sigma_f dW_t^f. \quad (11)$$

Here, W^f is a standard Brownian motion that is independent of W^m ; $\mu(s_t)$ and $\sigma_m(s_t)$ determine the expected growth rate and systematic volatility of cash flow; and σ_f determines the idiosyncratic volatility, which is assumed to be constant over time for simplicity.

The affine functional form for cash flow in (10) provides the flexibility to capture the impact of business cycles on cash flows in several dimensions. First, consider the case with $a_0(s_t) = 0$ and $a_1(s_t)$ being a constant (normalized to 1). Then $y_t^a = x_t$ is the cash flow of the firm, with the expected growth rate $\mu(s_t)$ and systematic volatility $\sigma_m(s_t)$. In this case, one can characterize the cyclical nature of assets-in-place through the conditional moments of growth rates. For example, assets-in-place can have procyclical growth rate ($\mu(G) > \mu(B)$) and countercyclical systematic volatility ($\sigma_m(G) < \sigma_m(B)$). These shocks on the conditional moments have permanent effects on cash flow.

Second, when $a_1(s_t)$ is allowed to change value, the level of cash flow can jump by a factor of $a_1(B)/a_1(G)$ when the economy enters into a recession, reflecting a significant change in asset productivity. We can thus set $a_1(G) > a_1(B) > 0$ to capture the procyclicality of assets-in-place. The effects of these shocks on cash flow are transitory, as they are reversed when the aggregate state changes.

Third, the term $a_0(s_t)$ allows cash flow to move independently of x_t . In the special case in which $a_1(s_t) = 0$ and $a_0(s_t)$ is constant, the cash flow from assets-in-place becomes riskless.

Besides assets-in-place, the firm has a growth option. Exercising the growth option requires a one-time lump-sum cost ϕ and generates cash flow y_t^g ,

$$y_t^g = g_0(s_t) + g_1(s_t)x_t. \quad (12)$$

Equation (12) captures the cyclicity of growth option in similar ways as Equation (10) does for assets-in-place. We assume that investment is irreversible.⁷

The firm has debt in the form of a consol with coupon c . We first take the firm's debt level as given and focus on the effects of existing debt on investments. We do not restrict our analysis exclusively to the case of optimal leverage because it is well documented that leverage ratios often drift far away from optimal levels, which can be due to adjustment costs (see, e.g., [Leary and Roberts, 2005](#)) or debt overhang (see, e.g., [Admati, DeMarzo, Hellwig, and Pfleiderer, 2015](#)). Thus, as long as the arrival of growth options is not strongly dependent on financial leverage, it makes sense to examine the impact of debt overhang on investment for a wide range of leverage ratios. Later in [Section 4](#), we compute the optimal capital structure, which demonstrates the impact of debt overhang on capital structure.

At each point in time, the firm makes coupon payment, pays taxes at rate τ , and then distributes the remaining cash flow to equity holders (no internal cash holdings).⁸ We assume that the absolute priority rule applies at the time of default. Equity value will be zero. Debt holders take over the firm, including the growth option (if not exercised yet), and implement

⁷[Manso \(2008\)](#) shows that agency cost of debt depends on the degree of investment reversibility. The bulk of the previous literature that study debt overhang assumes irreversible investment.

⁸See e.g, [Bolton, Chen, and Wang \(2011, 2015\)](#) for dynamic models of endogenous cash holdings.

the first-best policies for the all-equity firm, but lose a fraction $1 - \rho(s_t)$ of the value due to financial distress.⁹ Evidence on bond recovery rates and asset fire sales suggest that the firm recovery rate is procyclical, i.e., $\rho(G) > \rho(B)$.

The agency problem stems from the assumption that the firm acts in the interest of its equity holders. It chooses the optimal timing of default and investment to maximize the value of equity. For simplicity, we assume that investment is entirely financed by equity holders, and there are no ex post renegotiations between debt holders and equity holders. In particular, we rule out the possibility of financing the investment with new senior debt. (likely restricted by covenants in practice). [Hackbarth and Mauer \(2012\)](#) argue that it could be in the interest of existing debt holders to allow for new senior debt to finance investment. However, such priority structures become difficult to implement when there is uncertainty about the quality of investment. Ex post renegotiations can be quite costly due to the free-rider problem among debt holders and the lack of commitment by equity holders.

2.2 Model solution

Before presenting the solution, we first introduce some notations. The value of equity before investment in state s is denoted by $e_s(x)$. The value of equity after investment is $E_s(x)$. Similarly, the value of debt before and after investment is $d_s(x)$ and $D_s(x)$, respectively.

As shown in earlier models of real options and dynamic capital structure, the optimal investment policy is summarized by a pair of investment boundaries $\{x_G^u, x_B^u\}$. The firm invests when x_t is above x_s^u , while the economy is in state s . The default policy is summarized by two pairs of default boundaries: $\{x_G^d, x_B^d\}$ are the thresholds of default before investment is made, while $\{x_G^D, x_B^D\}$ apply after investment.

Taking the set of default and investment boundaries as given, the value of equity and debt can be solved analytically. The following proposition summarizes the results for equity valuation. The solution for defaultable debt is in a similar form (see [Appendix A](#) for more

⁹Alternatively, one can assume that debt holders lose the growth option in bankruptcy, and only recover a fraction of the value from assets-in-place. This assumption does not affect the investment policy equity holders choose, but does change the costs of bankruptcy.

details). While the ordering of the default and investment boundaries is endogenous, we assume the following ordering to simplify the presentation of the solution:

$$x_G^D < x_B^D,$$

and

$$x_G^d < x_B^d < x_G^u < x_B^u.$$

This ordering holds when leverage is not too high, and the cash flows from the firm's assets-in-place and growth option are sufficiently procyclical. It has the intuitive implication that firms default earlier and invest later in bad times. The solution is easily modified for different orderings.

Proposition 1. The value of equity after investment is given by:

$$E_G(x) = \begin{cases} 0 & x \in (0, x_G^D] \\ \sum_{j=1}^2 w_{1,j}^E x^{\alpha_j} + h_1^E(G)x + k_1^E(G) & x \in [x_G^D, x_B^D) \\ \sum_{j=1}^4 w_{2,j}^E \theta_j(G) x^{\beta_j} + h_2^E(G)x + k_2^E(G) & x \in [x_B^D, \infty), \end{cases} \quad (13)$$

$$E_B(x) = \begin{cases} 0 & x \in (0, x_B^D] \\ \sum_{j=1}^4 w_{2,j}^E \theta_j(B) x^{\beta_j} + h_2^E(B)x + k_2^E(B) & x \in [x_B^D, \infty). \end{cases} \quad (14)$$

The value of equity before investment is given by:

$$e_G(x) = \begin{cases} 0 & x \in (0, x_G^d] \\ \sum_{j=1}^2 w_{1,j}^e x^{\alpha_j} + h_1^e(G)x + k_1^e(G) & x \in [x_G^d, x_B^d) \\ \sum_{j=1}^4 w_{2,j}^e \theta_j(G) x^{\beta_j} + h_2^e(G)x + k_2^e(G) & x \in [x_B^d, x_G^u) \\ E_G(x) - \phi & x \in [x_G^u, \infty), \end{cases} \quad (15)$$

$$e_B(x) = \begin{cases} 0 & x \in (0, x_B^d] \\ \sum_{j=1}^4 w_{2,j}^e \theta_j(B) x^{\beta_j} + h_2^e(B)x + k_2^e(B) & x \in [x_B^d, x_G^u) \\ \sum_{j=1}^2 w_{3,j}^e x^{\gamma_j} + h_3^e(B)x + k_3^e(B) + \sum_{j=1}^4 \omega_{3,j}^e x^{\beta_j} & x \in [x_G^u, x_B^u) \\ E_B(x) - \phi & x \in [x_B^u, \infty). \end{cases} \quad (16)$$

The coefficients $\alpha, \beta, \gamma, \theta, h^E, k^E, h^e, k^e, w^E, w^e, \omega^e$ are given in [Appendix A](#).

Next, we discuss the conditions that determine the optimal default and investment policies. Whenever the optimal default boundaries after exercising the growth option $\{x_G^D, x_B^D\}$ are in the interior region (above 0), they satisfy the smooth-pasting conditions (see [Krylov, 1980](#); [Dumas, 1991](#), for details):

$$\lim_{x \downarrow x_G^D} E'_G(x) = 0, \quad (17)$$

$$\lim_{x \downarrow x_B^D} E'_B(x) = 0. \quad (18)$$

Intuitively, these conditions equate the marginal benefit and cost of immediate default at the optimal threshold conditional on the aggregate state. Since E_G and E_B are given in closed form in (13) and (14), these smooth-pasting conditions render two nonlinear equations for x_G^D and x_B^D that can be solved numerically.

Similarly, the optimal default and investment boundaries $\{x_G^d, x_B^d, x_G^u, x_B^u\}$ satisfy four smooth-pasting conditions:

$$\lim_{x \downarrow x_G^d} e'_G(x) = 0, \quad (19)$$

$$\lim_{x \downarrow x_B^d} e'_B(x) = 0, \quad (20)$$

$$\lim_{x \uparrow x_G^u} e'_G(x) = \lim_{x \downarrow x_G^u} E'_G(x), \quad (21)$$

$$\lim_{x \uparrow x_B^u} e'_B(x) = \lim_{x \downarrow x_B^u} E'_B(x), \quad (22)$$

which again translate into a system of nonlinear equations in $\{x_G^d, x_B^d, x_G^u, x_B^u\}$.

2.2.1 Agency cost measure

Before defining our measure of agency cost, we introduce some additional notation. Let $v_s(x; x_G^u(c), x_B^u(c), c)$ denote the total firm value (equity plus debt) before investment, where the investment thresholds $\{x_G^u(c), x_B^u(c)\}$ and the default thresholds are all optimally chosen

from the perspectives of the equity holders (and determined by (17)-(22)). Next, suppose the firm maintains the same default boundaries but commit to a different investment policy as characterized by investment thresholds $\{u_G, u_B\}$. Its firm value becomes $v_s(x; u_G, u_B, c)$.

The first-best investment policy is achieved by maximizing firm value instead of equity value. We denote the corresponding optimal investment thresholds as $\{\bar{x}_G^u, \bar{x}_B^u\}$, which will be independent of the firm's debt level (coupon).

One way to define the costs of underinvestment is to measure how much the value of the growth option to the firm differs under the first- and second-best investment policy (see e.g., [Hackbarth and Mauer, 2012](#)). It can be expressed as

$$ac_s(x_0; c) = \frac{v_s(x_0; \bar{x}_G^u, \bar{x}_B^u, c) - v_s(x_0; x_G^u(c), x_B^u(c), c)}{v_s(x_0; x_G^u(c), x_B^u(c), c)}, \quad (23)$$

which measures the value lost due to adopting a risky debt-induced suboptimal investment policy (as fraction of the second-best first value). This measure not only takes into account the direct effect of delayed investment, but also the feedbacks of investment distortions on the firm's default policy. The costs of bankruptcy and the ex-ante tax benefits of debt are a result of this.

Next, consider an all-equity firm ($c = 0$). By comparing the firm value under the first-best investment policy and the value when it commits to never exercise the growth option (i.e., by setting the investment thresholds u_G and u_B at $+\infty$), we get a measure of the value of the growth option that is independent of a firm's capital structure, which we refer to as PVGO,

$$PVGO_s(x_0) = v_s(x_0; \bar{x}_G^u, \bar{x}_B^u, c = 0) - v_s(x_0; +\infty, +\infty, c = 0). \quad (24)$$

Later, PVGO will be an important consideration when we measure the agency costs for the cross section of firms.

2.2.2 Static investment option

So far, we have modeled the growth options as American options. The firm (equity holders) decides when to make the investment, and the costs of debt overhang on investment are caused by delays in investments. Alternatively, we can model the growth option as a static, take-it-or-leave-it project. In this case, the firm decides whether to invest in the given project immediately. The costs of debt overhang show up as the deviation of the investment policy from the first best, where the investment decision is made to maximize the total firm value.

The intuition for how macroeconomic risks amplify the costs of debt overhang in this case resembles that in the two-period example in [Section 1](#). When the firm has risky debt in place, the value of investment for equity holders would be equal to the NPV of investment minus the transfer to debt holders, which leads equity holders to value the investment with a discount. Naturally, this discount is likely to be larger when debt is more risky. Furthermore, the size of the discount varies with the cyclicalities of assets-in-place and growth option, which generates predictions on what types of projects equity holders would prefer to invest in.

We measure the agency cost in this case as follows. Let $e_s^n(x_0; c)$ denote the equity value for a firm with coupon c , assuming that the firm commits to never exercise the investment option. Let $e_s(x_0; c)$ be the equity value of the firm with the investment option and coupon c , immediately before the investment is made. Then, the difference between the two, $e_s(x_0; c) - e_s^n(x_0; c)$, is the value of the investment option to the equity holders, which is similar to the PVGO measure in the case of dynamic investment options. It also is the cutoff lump-sum cost that equity holders will be willing to pay to make the investment.

Under the first best, the cutoff investment cost will be equal to the difference between the total firm value for the firm immediately before investment, $v_s(x_0; c)$, and the total firm value when the firm commits to not making the investment, $v_s^n(x_0; c)$. In the presence of risky debt, the investment project not only brings additional cash flows, but also helps reduce the default risk and thus reduces the bankruptcy costs and raises the tax benefits. Thus, its value under the first best will be higher than its NPV.

Then, we express the agency cost as the gap between the two two cutoff investment costs,

$$ac_s^{static}(x_0; c) = 1 - \frac{e_s(x_0; c) - e_s^n(x_0; c)}{v_s(x_0; c) - v_s^n(x_0; c)}. \quad (25)$$

Having described the model and its solution, next we examine its quantitative implications.

3 Debt Overhang and Investment

In this section, we first discuss the calibration strategy, and then analyze the quantitative effects of macroeconomic risk on the costs of debt overhang.

3.1 Model calibration

Our calibration strategy follows [Chen \(2010\)](#), who used a nine-state Markov chain to model the dynamics of aggregate consumption in the long-run risk model of [Bansal and Yaron \(2004\)](#) and then derived the stochastic discount factor using recursive preferences. There are two main differences in our model. First, we use two aggregate states instead of nine. Second, we assume a constant annual inflation rate of $\pi = 3\%$ instead of modeling a stochastic price index.

We calibrate the transition intensities of the two states by matching the average duration of NBER expansions and recessions. During the period of 1854 to 2009, the average length of an expansion is 38 months, while the average length of a recession is 17 months, which yield $\lambda_G = 0.32$ and $\lambda_B = 0.71$. As a result, the unconditional probability of being in an expansion and a recession state are 0.69 and 0.31, respectively.

Given λ_G and λ_B , we then calibrate the expected growth rate of firm cash flows $(\mu(G), \mu(B))$ to match the first two moments of the unconditional distribution of conditional expected growth rates of corporate dividend. Specifically, the calibration of [Bansal and Yaron \(2004\)](#) implies that the mean of the conditional expected growth rate of real aggregate dividend is 1.8% per year, and the standard deviation is 1.75%. Assuming $\mu(G) > \mu(B)$, we

obtain $\mu(G) = 5.97\%$ and $\mu(B) = 2.18\%$ by matching these two moments and adjusting for the 3% annual inflation rate.

Similarly, the systematic volatility of cash flows are calibrated to match the first two moments of the unconditional distribution of conditional volatility of dividend growth, which gives $\sigma_m(G) = 9.82\%$ and $\sigma_m(B) = 17.39\%$ (assuming $\sigma_m(G) < \sigma_m(B)$). To gauge whether these parameter values are reasonable, we can compare them with the moments of the growth rates for aggregate corporate profits before taxes (nominal, seasonally adjusted). Based on the data from the Bureau of Economic Analysis, the annualized standard deviation of the growth rates for aggregate corporate profits is 10.8% in expansions and 21.7% in recessions, reasonably close to our calibration.

The risk-free rate ($r(G), r(B)$) is calibrated the same way. The mean and standard deviation of the real risk-free rate in the data are 0.86% and 0.97% based on [Chen \(2010\)](#). Matching these two moments and then adjusting for the constant inflation rate gives $r(G) = 4.51\%$ and $r(B) = 2.41\%$ (assuming $r(G) > r(B)$).

The remaining parameters for the SDF include the prices of the Brownian shocks ($\eta(G), \eta(B)$) and the relative jump size of the SDF (κ), which do not have easily-measurable counterparts in the data. We set $\kappa = \ln(2.5)$, which is consistent with the average relative jump size across states implied by the calibration in [Chen \(2010\)](#). We then calibrate $\eta(G)$ and $\eta(B)$ by targeting the following asset pricing moments: the unconditional equity premium (6.3%), the average volatility of market portfolio return (19.4%), the average Sharpe ratio of the market portfolio (0.33). For the market portfolio, we assume the dividend process is the same as x_t in Equation (11), with the idiosyncratic volatility σ_f calibrated to give an average correlation of 0.71 between the Brownian shocks for the dividend of the market portfolio and the SDF.

[Chen, Collin-Dufresne, and Goldstein \(2009\)](#) and [Chen \(2010\)](#) showed that both the prices of systematic risks and the amount of systematic risk exposures in a firm can significantly affect the pricing of corporate claims. They use the market Sharpe ratio and the equity Sharpe ratio for individual firms as key statistics to gauge whether these two quantities are reasonable. For this reason, unless stated otherwise, we always recalibrate the idiosyncratic

Table 1: Model calibration

The table reports the calibrated parameters and the model-generated moments of the equity market. $E(r_m - r_f)$ denotes the annualized equity premium. $\sigma(r_m - r_f)$ denotes the annualized volatility of the market excess return. The effective tax rate is $\tau = 25\%$. We use the notation $E[\chi_s]$ and $\sigma(\chi_s)$ to denote the unconditional mean and standard deviation of a random variable χ_s whose value only depends on the state s_t .

Variable	State G	State B	Mean	SD
	χ_G	χ_B	$E[\chi_s]$	$\sigma(\chi_s)$
<i>A. Calibrated parameters</i>				
λ_s	0.32	0.71	-	-
$r(s_t)$	4.51	2.41	3.86	0.97
$\eta(s_t)$	0.17	0.43	0.25	0.12
$\mu(s_t)$	5.97	2.18	4.80	1.75
$\sigma_m(s_t)$	9.82	17.39	12.16	3.50
$\rho(s_t)$	0.83	0.57	0.75	0.12
<i>B. Asset pricing implications</i>				
$E(r_m - r_f)$	4.75	10.42	6.50	2.62
$\sigma(r_m - r_f)$	16.05	22.02	17.82	2.76
$E(r_m - r_f)/\sigma(r_m - r_f)$	0.30	0.47	0.35	0.08

volatility of cashflow σ_f for a levered firms so that the initial Sharpe ratio of equity is 0.25, roughly the median firm-level Sharpe ratio in the data.

We set the effective tax rate $\tau = 25\%$, which is lower than the typical corporate tax rate (of 35%) to reflect that the tax benefit of corporate debt at the firm level is partially offset by the tax disadvantage of debt at investor level (where the tax rate on interest income is higher than that on equity income). The recovery rates $\rho(G)$ and $\rho(B)$ are chosen to match the unconditional mean firm recovery rate of 75% and standard deviation of 12%. All the resulting parameter values are summarized in panel A of [Table 1](#), where the means and standard deviations are computed using the stationary distribution of the Markov chain. The asset pricing implications of the stochastic discount factor are in panel B.

3.1.1 Calibration for the case without macroeconomic risks

A key objective of our paper is to compare the agency costs between the cases with and without macroeconomic risks, which refer to the business-cycle fluctuations in cash-flow dynamics $(\mu(s_t), \sigma_m(s_t))$, recovery rates in bankruptcy $(\rho(s_t))$, interest rates $(r(s_t))$, and risk prices $(\eta(s_t), \kappa)$. For $\mu(s_t), \sigma_m(s_t), \rho(s_t)$, and $r(s_t)$, we simply set their values in the case without macroeconomic risks to their unconditional means. We also set κ to 0. Finally, we set η to 0.4, which is higher than the average Sharpe ratio of the market portfolio of 0.35.¹⁰

Next, we examine the quantitative implications of our model. We first analyze the case of static (take-it-or-leave-it) investment options in Section 3.2, and then study the case of dynamic investment options in Sections 3.3 and 3.4.

3.2 Static Investment Model

As a benchmark, we assume that $a_1(G) = a_1(B) = 1$, $a_0(G) = a_0(B) = 0$, $g_1(G) = g_1(B) = 0.4$, and $g_0(G) = g_0(B) = 0$, and for normalization, we set the fixed cost of investment ϕ to 0. Thus, the take-it-or-leave-it investment opportunity will increase the firm's cash flows by 40%. Suppose the firm has coupon $c = 0.4$, which implies an initial interest coverage of 2.5. The agency cost in state G is 13%, meaning that this investment option is valued at a 13% discount by the equity holders due to agency conflicts. In state B , the discount for the same investment option is 14%. If we raise the coupon to $c = 1.0$ (an interest coverage of 1), the agency cost rises to 40% in state G and 45% in state B .

Figure 2 reports the investment discount for the firm as we vary the cyclicity of assets-in-place and growth option. We present the results for the case in which the initial state is the good state. The agency costs tend to be higher in the bad state, but the results will be qualitatively similar. The left panels are for the case of relatively low leverage, with the coupon of the consol fixed at $c = 0.4$. In the right panels, the coupon is fixed at $c = 1.0$. In comparison, the firm's initial cash flow x_0 is normalized to 1.

¹⁰A lower value for η , such as $\eta = 0.35$, would further reduce the agency costs in the case without macroeconomic risks.

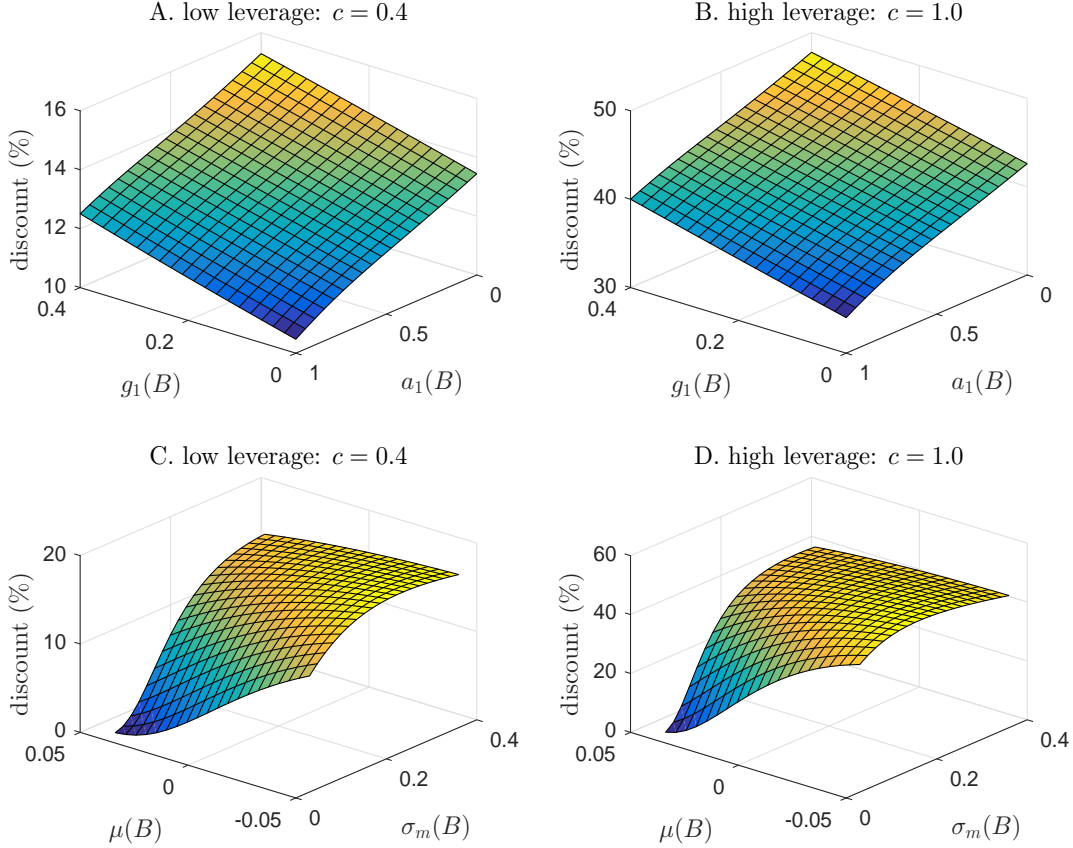


Figure 2: **Agency costs with static investment option.** The top panels show how the agency cost changes with the cyclicalities of the assets-in-place and growth option (through $a_1(s)$ and $g_1(s)$). The bottom panels show how the agency cost changes with the business-cycle variations in the conditional moments of cash flows ($\mu(s)$ and $\sigma_m(s)$).

We first examine how the investment discount changes with the cyclicalities of assets-in-place and growth option via the transitory business-cycle shocks $a_1(s)$ and $g_1(s)$. We perform a “mean-preserving spread” for the cash flows of the assets-in-place and the investment option. Specifically, we vary the value of $a_1(B)$ between 0 and 1, and solve for the corresponding value for $a_1(G)$ such that the expected NPV of the cashflows from assets-in-place is unchanged. Similarly, we vary the value of $g_1(B)$ between 0 and 0.4, and solve for $g_1(G)$ such that the NPV of the cashflows from the growth option is unchanged. Recall that for the benchmark firm $a_1(G) = a_1(B) = 1$ and $g_1(G) = g_1(B) = 0.4$. Lowering $a_1(B)$ ($g_1(B)$) and raising $a_1(G)$ ($g_1(G)$) will make the assets-in-place (growth option) more procyclical.

In panels A and B of Figure 2, we see that the agency cost rises as the firm’s assets-

in-place become more procyclical (smaller $a_1(B)$), but the opposite happens as the growth option becomes more procyclical (smaller $g_1(B)$). With moderate leverage, the agency cost is relatively small, ranging from 10 to 16%. With high leverage, not only is the average level of agency cost significantly higher, but it also becomes more sensitive to changes in the cyclical of assets-in-place and growth option (it ranges from 35% to 48%).

Intuitively, when cash flow from assets-in-place are low compared to the coupon payment, debt becomes relatively more risky, which means a bigger part of the value generated by the investment option will be transferred to debt holders. Holding the growth option fixed, making assets-in-place more cyclical increases the probability of such transfers in the bad state, while lowering their probability in the good state. The net effect is higher expected total transfer because of the higher risk prices associated with the bad state. Put differently, due to debt overhang, stronger cyclical of assets-in-place makes the part of cash flows equity holders receive from the investment project more risky and hence lowers their valuation of the project, even though the total cash flow from the project remains unchanged.

The effects of changing cyclical for the growth option depend on the cyclical of assets-in-place. When a firm's assets-in-place are procyclical, debt is more risky in the state B . In this case, having a more procyclical growth option reduces the transfer to debt in the bad state, hence lowering the agency cost. However, if the firm's assets-in-place are countercyclical instead, then debt could be more risky in the good state. In that case, having a more procyclical growth option will raise the agency cost.

Such interactions between the cyclical of assets-in-place and growth option are a form of asset substitution in the presence of macroeconomic risk. The standard asset substitution argument ([Jensen and Meckling, 1976](#)) is that equity holders of a levered firm prefers to invest in projects with cash flows that are more correlated with assets-in-place. Higher correlation raises the volatility of the firm overall, and reduces the amount of transfer to debt holders. With macroeconomic risk, equity holders not only care about the average correlation, but also want to line up the cyclical of the investment with that of assets-in-place. For example, a highly levered procyclical firm, such as a large financial institution, will have strong incentive to invest in assets with high systematic risk exposures, even if these assets have lower NPV.

Such incentives can have severe consequences for the aggregate economy, as highlighted by the 2008-09 financial crisis.¹¹

Next, we examine the effects of firm cyclicalities on agency costs by changing the amount of business-cycle variations in the conditional moments of cash flow growth rates. As reported in [Table 1](#), the conditional expected growth rates for the benchmark firm are $\mu(G) = 5.97\%$ and $\mu(B) = 2.18\%$, while the conditional systematic volatilities of cash flows are $\sigma_m(G) = 9.82\%$ and $\sigma_m(B) = 17.39\%$. We perform mean-preserving spreads for μ and σ_m , varying $\mu(B)$ and $\sigma_m(B)$ while keeping their unconditional means unchanged. In our model, such changes affect the cyclicalities of assets-in-place and growth option simultaneously. As explained above, the cyclicalities of assets-in-place affects debt riskiness in the absence of the growth option, which is the source of agency conflict, while the effects of the cyclicalities of growth option depends on how it aligns with that of assets-in-place.

The results are shown in panels C and D of [Figure 2](#). The lowest agency cost occurs when $\mu(B)$ is high and $\sigma_m(B)$ is low. In these cases, cash flows from assets-in-place are relatively safe in the bad state, which drives default risk to very low levels and thus largely removes the agency conflicts. When we reduce $\mu(B)$ and increase $\sigma_m(B)$, both of which make the cash flows more procyclical, the agency cost rises significantly, especially in the case with high leverage. Not surprisingly, the marginal effect of changing $\mu(B)$ ($\sigma_m(B)$) is smaller when $\sigma_m(B)$ is high ($\mu(B)$ is low), which already makes cash flows risky in the bad state.

In summary, the above results are quite informative about the effects of macroeconomic risks on the agency costs of debt. Even with the same debt level, firms with different cyclicalities in their assets-in-place or growth options will have significantly different levels of agency costs, resulting in significantly different amount of investment distortions.

¹¹As we show in [Section 4](#), allowing for endogenous leverage may mitigate this problem to some extent, but it is still subject to the problem of a lack of incentive to reduce leverage after the leverage has risen (see e.g., [Admati, DeMarzo, Hellwig, and Pfleiderer \(2015\)](#)).

3.3 Dynamic debt overhang

While the case of take-it-or-leave-it investment opportunity allows for easy comparison with the NPV rule, in practice firms usually have the ability to choose when to invest. An investment opportunity that is rejected by equity holders under current market conditions could become attractive again in the future, for example, when debt becomes less risky. Thus, it is important to take into account the option to wait when measuring the costs of debt overhang.

As mentioned when we define the measure of agency cost in Section 2.2, the costs of debt overhang depend on the value of the growth option itself. If the growth option is too far out of the money, the firm is unlikely to invest soon regardless of whether it has debt in place or not. In this case, the agency costs as defined in Equation (23) will be (essentially) zero. As the value of the growth option increases, the investment thresholds are likely to drop. If the growth option is sufficiently in the money, the optimal investment thresholds can be below x_0 both under the first best (no debt) and the second best ($c > 0$). In this case, the firm invests immediately, and there will be no difference in the actual investment thresholds under the first and second best. Then, the agency costs will again be zero.

For the benchmark firm, we assume $a_0(G) = a_0(B) = 0$, and set $a_1(G) = 1.54$, $a_1(B) = 0.77$ for assets-in-place, so that the unconditional average value of assets-in-place is the same as when $a_1(G) = a_1(B) = 1$, but cash flow falls by 50% when the economy transitions from state G to state B .

For the growth option, we assume $g_0(G) = g_0(B) = 0.14$ and $g_1(G) = g_1(B) = 1$. By adding a component $g_0(s) > 0$ which is independent of x_t , and by making g_1 the same across the two states, we are making the cash flows from the growth option relatively safe. As the results from the case of static investment options show, safer growth options will tend to be beneficial to risky debt and thus lead to larger agency costs. The NPV of this riskless component will be on average 20% of the total growth option. We then vary PVGO (as defined in (24)) by changing the fixed cost of investment ϕ , but recalibrating σ_f each time to fix the Sharpe ratio of equity at 0.25.

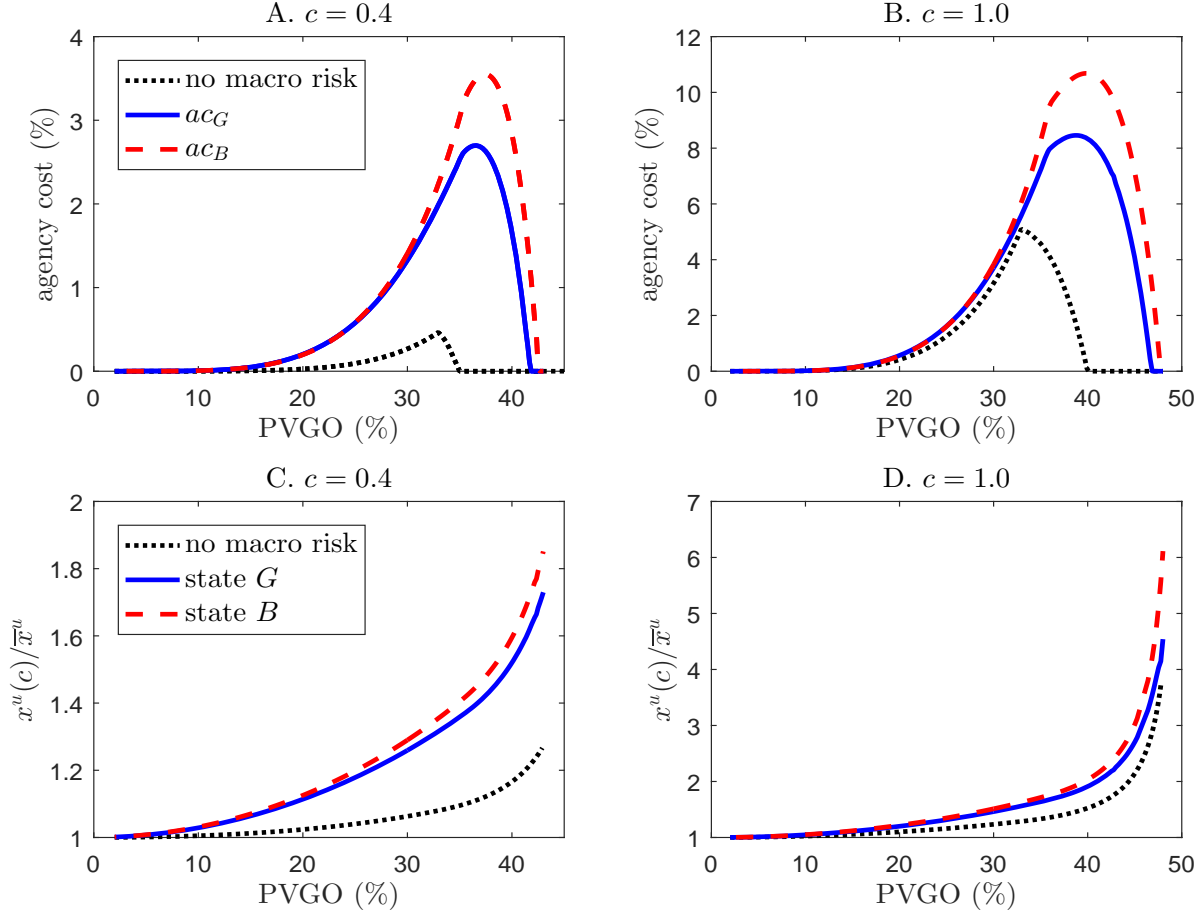


Figure 3: **Costs of debt overhang.** This figure plots the costs of debt overhang (in percentage of first-best firm value) for investments with different PVGO. ac_G and ac_B are the conditional costs of debt overhang in good and bad state. The market Sharpe ratio in the “no macro” case is matched to the average market Sharpe ratio in the case with macro risk. The equity Sharpe ratio is always fixed at 0.25 through recalibration of σ_f .

Next, to turn off macroeconomic risk (for comparison), we fix μ , σ_m , a_0 , a_1 , g_0 , g_1 at their respective unconditional means for the benchmark firm. The calibration for the stochastic discount factor is discussed in Section 3.1. The fixed cost ϕ and idiosyncratic volatility σ_f are calibrated to generate different levels of PVGO while keeping the equity Sharpe ratio at 0.25.

In Figure 3, we plot the agency costs for growth options with PVGO ranging from 0 to 50% of the first best firm value. Panels A and B show the agency costs for a low leverage firm ($c = 0.4$) and a high leverage firm ($c = 1.0$). The firm’s initial cash flow x_0 is again normalized to 1. Panels C and D reveal the sources of the agency costs by showing the gaps

between the first-best and second-best investment thresholds. As expected, the costs of debt overhang are close to zero when the value of the growth option is either very low or high, but rise up for intermediate values. In that region, the delay in investment relative to first best becomes sizable, which results in significant losses in firm value.

In the absence of macroeconomic risks, the agency costs are low. As panel A shows, for a firm with low leverage, the agency cost is close to 0 in most of the cases, and peaks at less than 0.5% of the total firm value. Consistently, panel C shows that the delay in investment relative to the first best is also quite limited in the absence of macroeconomic risks, with the second-best investment threshold only about 8% higher than the first best.

Once we take macroeconomic risk into account, the agency cost can become substantially higher for the low-leverage firm. It peaks when PVGO is at about 37%, where the agency cost rises to 2.7% in state G and 3.6% in state B . The investment boundaries with risky debt in states G and B become 40% and 48% higher than under the first best, respectively. When the leverage of the firm is higher, as we see in panel B of Figure 3, the impact of macroeconomic risks on the agency cost becomes somewhat less pronounced. The agency cost peaks at 5.1% of the firm value without macroeconomic risk, at which point the second-best investment threshold is about 29% higher than the first-best; it peaks at 8.5% and 10.7%, respectively, in states G and B with macroeconomic risk, where the second-best investment thresholds are 81% and 101% higher than their first-best counterparts.

These results show that macroeconomic risks indeed have important effects on debt overhang, especially for firms with relatively low leverage. It is reminiscent (and indeed a mirror image) of the “credit spread puzzle” (see [Huang and Huang, 2012](#)): traditional Merton-style structural models tend to overvalue investment-grade bonds (after matching the historical average default rates and recovery rates), but can do a better job on speculative-grade bonds (bonds issued by firms with high default risks). [Chen \(2010\)](#) argues that such failures are largely due to these models missing the comovements in the conditional default probability, losses given default, and the prices of risks over the business cycle, which generate a sizable credit risk premium for investment-grade bonds. When a model is overvaluing the defaultable debt, it will also underestimate the transfer from equity holders to debt holders

Table 2: Agency costs: the SDF

The table reports the comparative statics of conditional credit spreads and conditional agency costs with respect to various parameters for the stochastic discount factor. In each case, $c = 0.4$, while ϕ and σ_f are calibrated to fix the PVGO at 40% of the first-best firm value and the average equity Sharpe ratio at 0.25. The remaining parameters are reported in Table 1. For comparison, in the benchmark case, $\kappa = \ln(2.5)$, $E[\eta(s_t)] = 0.25$, $\sigma(\eta(s_t)) = 0.12$, $\pi = 3\%$, $\lambda_G = 0.32$, and $\lambda_B = 0.71$.

	Credit spread (bps)		Agency costs (%)		Avg. ac (%)
	$sprd_G(x_0)$	$sprd_B(x_0)$	$ac_G(x_0)$	$ac_B(x_0)$	
Benchmark	155	175	1.7	2.8	2.0
$\kappa = \ln(3.0)$	223	243	3.0	4.1	3.4
$E[\eta(s_t)] = 0.3$	219	243	2.9	4.1	3.3
$\sigma(\eta(s_t)) = 0.16$	208	234	2.7	4.0	3.1
$\pi = 2\%$	233	255	5.3	6.8	5.7
$\lambda_G = 0.11, \lambda_B = 0.24$	115	164	0.1	3.2	1.1

resulting from the investment, thus underestimating the cost of debt overhang ex ante.

The results in Figure 3 also highlight the dynamic debt overhang effects, which are absent from the static model in Section 1. The conditional agency costs in the good and bad state, $ac_G(x_0)$ and $ac_B(x_0)$, are not that far apart, despite the fact that business-cycle fluctuations in the level and conditional moments of cash flows imply that the benchmark firm is in a better than average condition in state G . When in state G , even though the cash flows are currently higher and are expected to grow faster, equity holders are still reluctant to invest because they are concerned that the state of the economy might change, which can make debt substantially more risky and raise the amount of wealth transfer from equity holders to debt holders through investment. Thus, debt overhang in this state comes mainly from concern of future wealth transfer in a state with worse conditions, which is different from the concern of immediate wealth transfer when debt is already under water. This dynamic overhang effect will become weaker when we make the two states more persistent.

We also examine the effects of systematic risk on the costs of debt overhang by raising

the price of jump risks κ , the average price of Brownian risk $\eta(s_t)$ and its variation across the two states, and the persistence of the two aggregate states. The results are reported in Table 2. The benchmark firm has the same calibration discussed above. In the benchmark case and in each of the comparative statics, the fixed cost of investment ϕ and idiosyncratic volatility of cash flows σ_f are re-calibrated to match a 40% PVGO and equity Sharpe ratio of 0.25.

In the benchmark case, the average agency cost across the two states is 2%. The conditional credit spreads for the consol bond is 155 bps in state G and 175 bps in state B . If we increase the price of jump risk κ from $\ln(2.5)$ to $\ln(3)$, the average agency cost rises to 3.4%, while the credit spreads in the two states rise to 223 and 243 bps, respectively. Similarly, when we increase either the mean or volatility of the risk price for Brownian shocks, $E[\eta(s_t)]$ and $\sigma(\eta(s_t))$, the agency costs in the two states will rise, as do the credit spreads. Next, lowering the rate of inflation from 3% to 2% has the effect of lowering the nominal growth rate of cash flows, which increases the firm's default risk and credit spreads, and in turn the agency costs. In addition, the lower nominal risk-free rate further increases the ex ante agency costs. These results consistently demonstrate the strong linkage between debt pricing and the agency cost.

Finally, if we increase the persistence of the two states by lowering λ_G to 0.11 and λ_B to 0.23 (making both states three times more persistent than before), the dynamic debt overhang effects start to diminish. The agency costs in the two states become further apart. The costs rise to 3.2% in state B , but fall to 0.1% in state G , making the average agency costs lower as well.

Having demonstrated the overall effect of business cycle risks on the costs of debt overhang, we next decompose the effects into two parts, one through assets-in-place, the other through growth option.

3.4 Assets-in-place and growth option

As discussed in the static model in Section 1, the cyclicity of assets-in-place and growth option have different effects on the agency costs of debt. To examine these effects in the dynamic model, we consider the following comparative statics in Table 3.

Table 3: AGENCY COSTS: CYCLICALITY OF ASSETS-IN-PLACE AND GROWTH OPTION

The table reports the comparative statics of conditional credit spreads and conditional agency costs with respect to the cyclicality of assets-in-place and growth option. In each case, $c = 0.4$, while ϕ and σ_f are calibrated to fix the PVGO at 40% of the first-best firm value and the average equity Sharpe ratio at 0.25. The remaining parameters are reported in Table 1. For comparison, the benchmark firm has $a_1(B)/a_1(G) = 0.5$, $g_1(B)/g_1(G) = 1.0$, $\sigma(\mu_t) = 1.75\%$, $\sigma(\sigma_{m,t}) = 3.5\%$, $g_0(s_t) = 0.14$, and $g_1(s_t) = 1$.

	credit spread (bps)		agency costs (%)		avg. ac (%)
	$sprd_G(x_0)$	$sprd_B(x_0)$	$ac_G(x_0)$	$ac_B(x_0)$	
benchmark	155	175	1.7	2.8	2.0
$a_1(B)/a_1(G) = 1$	112	128	0.0	1.2	0.4
$g_1(B)/g_1(G) = 0.25$	163	223	1.1	2.5	1.5
$\sigma(\mu_t) = 2.5\%$	208	235	2.6	4.0	3.1
$\sigma(\sigma_{m,t}) = 7.0\%$	279	319	3.6	4.9	4.0
$g_0 = 0.21, g_1 = 0.87$	130	149	2.0	3.6	2.5

Recall that for the benchmark firm, $a_1(B)/a_1(G) = 0.5$ and $g_1(B)/g_1(G) = 1$. If we make the assets-in-place less cyclical (see the case in which $a_1(B)/a_1(G) = 1$), cash flows become safer, which helps lower the default risk. As a result, the credit spreads fall in both states, as do the agency costs. In fact, the second-best investment boundary in state G drops below $x_0 = 1$, the assumed initial cash flow level, implying that the firm will invest immediately in state G . As a result, the conditional agency cost in state G drops to zero in this case.

Next, we examine the relation between the cyclicality of growth option and the costs of debt overhang. Strong pro-cyclicality raises the value of the growth option in the good state, but lowers it in the bad state, which has the effect of making default less likely in the good state but more likely in the bad state. Indeed, as we see in the case with $g_1(B)/g_1(G) = 0.25$, the gap between the credit spreads in states G and B widens, with the spreads rising by 8 bps in state G from the benchmark case, but by 48 bps in state B . However, the higher credit spreads do not necessarily imply higher agency costs. This is because a more cyclical growth option also lowers the potential for wealth transfer to debt holders in the bad state.

This second effect tends to lower the agency costs. As we see in [Table 3](#), the average agency cost indeed drops in this case.

When we increase the variation of the conditional moments of cash flows, both the assets-in-place and growth option become more cyclical. Thus, while debt becomes more risky in state B , the conditional value of the growth option also becomes lower in state B . The agency cost can potentially go up or down depending on which of the two competing effects dominates. As [Table 3](#) shows, when we increase the volatility of μ_t (the expected growth rate of cash flow) from 1.75% to 2.5% (without changing the average expected growth rate), both the credit spreads and the conditional agency costs are higher in both states. The results are similar when we raise the volatility of $\sigma_{m,t}$ from 3.5% to 7.0%.

Finally, we examine what happens to the costs of debt overhang when the cash flow from growth option becomes safer relative to assets-in-place. To do so, we increase the value of the risk-free component of the growth option to $g_0(s_t) = 0.21$, while adjusting $g_1(s_t)$ downward to 0.87 to keep the average NPV of the growth option constant. Such a change raises the agency costs in both states, to 2% and 3.6%.

This result is quite intuitive. Part of the cash flow from the growth option loads on the same shock as assets-in-place (from x_t). Thus, when debt becomes risky (x_t is low), so will be the cash flow from the growth option, which reduces the wealth transfer to debt holders, hence limiting the costs of debt overhang. If the cash flow from growth option is uncorrelated with that from assets-in-place, in particular if the growth option is riskless, then the debt overhang problem will become more severe. Macroeconomic risk further strengthens this effect by (1) making debt more risky in state B and (2) the wealth transfer in state B more costly for equity holders ex ante.

4 Debt Overhang and Capital Structure

In this section, we investigate the endogenous choice of capital structure based on the trade-off between tax benefits and agency costs. The capital structure that maximizes the market value received by the initial owners for sale of equity and debt in state s with initial cash

Table 4: Optimal leverage and debt overhang

The table reports the optimal coupon, initial interest coverage and market leverage, agency cost, and the 5-year conditional investment and default probabilities in the two states ($p_G^5(x_0), p_B^5(x_0)$). In the case “no macro”, there are no business-cycle variations in the cash flows, and the market Sharpe ratio is 0.35.

	Coupon c	Coverage ratio $\frac{X_0}{c}$	Leverage $\frac{d_0}{d_0+e_0}$	Credit spread (bps)		Agency costs (%)	
				$sprd_G(x_0)$	$sprd_B(x_0)$	$ac_G(x_0)$	$ac_B(x_0)$
<i>A1. High PVGO</i>							
No macro	0.80	1.25	0.54	44		0.0	
With macro	0.40	2.47	0.37	168	189	2.6	3.6
<i>A2. Low PVGO</i>							
No macro	0.80	1.25	0.60	113		0.3	
With macro	0.60	1.65	0.50	353	380	0.6	0.6
<i>B. Cyclicalilty of assets-in-place and growth option</i>							
Benchmark	0.40	2.47	0.37	168	189	2.6	3.6
$\frac{a_1(B)}{a_1(G)} = 1$	0.43	2.31	0.39	177	198	2.8	3.8
$\frac{g_1(B)}{g_1(G)} = .25$	0.29	3.44	0.31	93	151	0.0	1.6

flow x_0 can be determined from the coupon rate c^* solving

$$\sup_c \{e_s(x_0, c) + d_s(x_0, c)\}. \quad (26)$$

In what follows, we will focus on the optimal capital structure decision made in state G , and we normalize $x_0 = 1$.

We consider two firms with different levels of PVGO. Specifically, we assume one firm has a lumpsum investment cost of $\phi = 9$, and the cost doubles for the other, $\phi = 18$. All the other parameters are the same as the benchmark firm, and the idiosyncratic volatility of cash flows is fixed at 25%. As a result, the first firm has a growth option with PVGO of about 38% of the first-best firm value, while the second has PVGO of about 20%.

For the firm with relatively high PVGO (see panel A1), the optimal coupon is 0.8 when

there are no macroeconomics risks, which corresponds to an initial interest coverage of 1.25, and initial market leverage of 54%. Despite the high leverage, the firm's credit spread is only 44 bps, and the agency cost is essentially 0. In the presence of macroeconomic risks, the optimal coupon for a firm choosing its capital structure in state G decreases to 0.4. This translates into a rise in the interest coverage to 2.47, and a drop in the initial market leverage to 37%. This significant drop in optimal leverage is due to both higher ex ante agency costs (from 0% in the absence of macroeconomic risks to 2.6% in state G and 3.6% in state B), and higher ex ante costs of bankruptcy (see the discussion in [Chen, 2010](#)). Notice that the agency costs for this firm are sizable. This result shows that while the ability to choose optimal (lower) leverage does help reduce the agency costs to some extent, the resulting agency costs can still be quite meaningful, especially for firms with valuable growth options.

Panel A2 of [Table 4](#) shows the optimal leverage choice when the value of the growth option is only around 20% of the first-best firm value. Interestingly, the optimal coupon is almost unchanged from the high PVGO case when there are no macroeconomic risks. This is because the agency costs are negligible and thus not a main factor in the trade-offs for optimal leverage. The lower PVGO makes the firm's debt more risky, but its equity value drops even more, resulting in a higher market leverage ratio.

In the presence of macroeconomic risks, the optimal coupon chosen in state G rises to 0.6 for the firm with low PVGO, substantially higher than that for the firm with high PVGO. This high sensitivity of optimal coupon to changes in PVGO results from the fact that the agency costs of debt are also sensitive to the changes in PVGO, which we have seen in [Figure 3](#). Also notice that, even with a higher leverage, the agency cost for the second firm is lower due to the lower PVGO. This result highlights the fact that firm leverage could be a misleading indicator of the magnitude of agency cost due to its endogeneity.

Next, in panel B of [Table 4](#), we examine how changes in the cyclicity of assets-in-place and growth option affects the firm's leverage choice, which extends a part of the comparative statics exercises in [Table 3](#) to the setting of optimal leverage. The benchmark for this exercise is the firm with high PVGO as discussed in panel A1.

Compared to the benchmark firm, which has $a_1(B)/a_1(G) = 0.5$, a firm that has less

cyclical assets-in-place ($a_1(B)/a_1(G) = 1.0$) chooses a higher coupon (0.43), which implies a higher market leverage (39%). Consistent with our earlier findings in Section 3.4, all else equal, less cyclical assets-in-place imply lower default risks and lower costs of debt overhang, which lead the firm to take on more leverage. It is possible that the higher leverage then results in higher credit spreads and higher agency costs, as we see in panel B.

Next, a firm that has a more cyclical growth option ($g_1(B)/g_1(G) = 0.25$) than the benchmark firm (which has $g_1(B)/g_1(G) = 1$) chooses a lower coupon of 0.29, which results in lower leverage (31%), lower credit spreads, and lower agency costs. Again recall our analysis in Section 3.4 that, all else equal, stronger cyclicity of the growth option can result in lower agency cost, but it will also raise the firm's default risk. In our model, this effect is stronger and it leads the firm to choose a lower leverage, which further reduces the agency costs.

5 Concluding Remarks

Using a dynamic model of capital structure with investment decisions and macroeconomic risk, we showed that the agency cost of debt due to debt overhang increases substantially when macroeconomic risk is taken into account. For example, in our benchmark case, the debt overhang costs for a low leverage firm peak at 0.7% when macroeconomic risk is not taken into account, while these costs peak at 2.7% or 3.5% in booms and recessions, respectively, when macroeconomic risk is taken into account.

We also showed that investment and capital structure decisions, as well as debt overhang costs depend on the cyclicity of cash flows from assets-in-place and growth opportunities. More cyclical cash flows from assets-in-place make underinvestment more likely in bad times, exacerbating the costs of debt overhang when macroeconomic risk is taken into account. More cyclical cash flows from growth opportunities also make underinvestment more likely in bad times, but the overall effect on the costs of debt overhang when macroeconomic risk is taken into account is ambiguous. Moreover, among the growth options that are not too profitable (so that debt is still risky), equity holders would prefer to invest in ones that have the same cyclicity as their assets-in-place. Finally, we showed that macroeconomic risk

significantly impacts the optimal capital structure of the firm.

Several questions remain unanswered. For example, what is the effect of macroeconomic risk on different agency conflicts, such as asset substitution ([Jensen and Meckling, 1976](#)) or free cash-flow ([Jensen, 1986](#))? Because in bad times firms are usually closer to default, the asset substitution problem may be more prevalent in bad times. If this is indeed the case, asset substitution costs will be amplified by macroeconomic risk as well. On the other hand, the free cash flow problem may be more prevalent in good times, when there is more cash available to be diverted. If this is the case, free cash flow costs are reduced if macroeconomic risk is taken into account. We leave these questions to future research.

Appendix

A Model Solution

We value debt and equity under the risk-neutral probability measure \mathbb{Q} . The process for x_t under \mathbb{Q} becomes

$$\frac{dx_t}{x_t} = \tilde{\mu}(s_t)dt + \sigma(s_t)d\tilde{W}_t, \quad (\text{A1})$$

where

$$\tilde{\mu}(s_t) = \mu(s_t) - \eta(s_t)\sigma_m(s_t), \quad (\text{A2})$$

$$\sigma(s_t) = \sqrt{\sigma_m^2(s_t) + \sigma_f^2}, \quad (\text{A3})$$

and \tilde{W}_t is a standard Brownian motion under \mathbb{Q} . In addition, the transition intensities of the Markov chain under \mathbb{Q} become

$$\tilde{\lambda}(G) = \lambda_G e^\kappa, \quad \tilde{\lambda}(B) = \lambda_B e^{-\kappa}. \quad (\text{A4})$$

Thus, if the stochastic discount factor m_t jumps up when the economy changes from state G to B ($\kappa > 0$), then $\tilde{\lambda}(G) > \lambda_G$, while $\tilde{\lambda}(B) < \lambda_B$. Intuitively, the jump risk premium in the model makes the duration of the good state shorter and bad state longer under the risk-neutral measure.

Next, we derive the solution for equity value in [Proposition 1](#).

A.1 Value of Equity

A.1.1 After Investment

After the firm exercises the investment option, the problem becomes the same as the static capital structure model with two aggregate states. Here we only derive the solution for the case in which the default boundaries satisfy $x_G^D < x_B^D$, and it is straightforward to extend the solution for the case in which $x_G^D \geq x_B^D$.

Taking x_G^D and x_B^D as given, the value of equity can be solved in two regions: $J_1 = [x_G^D, x_B^D)$ and

$J_2 = [x_B^D, \infty)$. For $x \in J_1$, the firm has not defaulted yet in state G , but has already defaulted in state B . Thus, $E_B(x) = 0$ in this region, while $E_G(x)$ satisfies the following ODE,

$$(r(G) + \tilde{\lambda}(G))E_G = (1 - \tau)(\ell_G(x) - c) + \tilde{\mu}(G)x E_G' + \frac{1}{2}\sigma^2(G)x^2 E_G'', \quad (\text{A5})$$

where for simplicity of notation we denote the cash-flow after investment as

$$\ell_s(x) = (a_0(s) + g_0(s)) + (a_1(s) + g_1(s))x. \quad (\text{A6})$$

The solution to the homogeneous equation in the ODE (A5) is

$$E_G(x) = w_{1,1}^E x^{\alpha_1} + w_{1,2}^E x^{\alpha_2}, \quad (\text{A7})$$

where

$$\alpha_1, \alpha_2 = -\sigma^{-2}(G) \left[\left(\tilde{\mu}(G) - \frac{\sigma^2(G)}{2} \right) \pm \sqrt{\left(\tilde{\mu}(G) - \frac{\sigma^2(G)}{2} \right)^2 + 2r(G)\sigma^2(G)} \right], \quad (\text{A8})$$

and the particular solution is $h_1^E(G)x + k_1^E(G)$, where

$$h_1^E(G) = \frac{(1 - \tau)(a_1(G) + g_1(G))}{r(G) + \tilde{\lambda}(G) - \tilde{\mu}(G)}, \quad (\text{A9a})$$

$$k_1^E(G) = \frac{(1 - \tau)(a_0(G) + g_0(G) - c)}{r(G) + \tilde{\lambda}(G)}. \quad (\text{A9b})$$

Next, for $x \in J_2$, the firm is not in default yet in either state, and $E_G(x)$ and $E_B(x)$ satisfy a system of ODEs:

$$(r(G) + \tilde{\lambda}(G))E_G = (1 - \tau)(\ell_G(x) - c) + \tilde{\mu}(G)x E_G' + \frac{1}{2}\sigma^2(G)x^2 E_G'' + \tilde{\lambda}(G)E_B, \quad (\text{A10a})$$

$$(r(B) + \tilde{\lambda}(B))E_B = (1 - \tau)(\ell_B(x) - c) + \tilde{\mu}(B)x E_B' + \frac{1}{2}\sigma^2(B)x^2 E_B'' + \tilde{\lambda}(B)E_G. \quad (\text{A10b})$$

The homogeneous equations from the ODE system (A10a-A10b) can be formulated as a quadratic

eigenvalue problem (see [Chen \(2010\)](#) for details), and the solution is given by

$$E_s(x) = \sum_{j=1}^4 w_{2,j}^E \theta_j(s) x^{\beta_j}, \quad (\text{A11})$$

where β_j and θ_j are the j -th eigenvalue and (part of the) eigenvector for the following standard eigenvalue problem:

$$\begin{bmatrix} 0 & \mathbf{I} \\ -2\Sigma^{-1}(\tilde{\Lambda} - \mathbf{r}) & -(2\Sigma^{-1}\tilde{\mu} - \mathbf{I}) \end{bmatrix} \begin{bmatrix} \theta_j \\ \varphi_j \end{bmatrix} = \beta_j \begin{bmatrix} \theta_j \\ \varphi_j \end{bmatrix}, \quad (\text{A12})$$

where \mathbf{I} is a 2×2 identity matrix, $\mathbf{r} = \mathbf{diag}([r(G), r(B)]')$, $\tilde{\mu} = \mathbf{diag}([\tilde{\mu}(G), \tilde{\mu}(B)]')$, and $\Sigma = \mathbf{diag}([\sigma^2(G), \sigma^2(B)]')$. From [Barlow, Rogers, and Williams \(1980\)](#), we know that there are exactly 2 eigenvalues with negative real parts, and 2 with positive real parts.

Next, the particular solutions will be in the form $\mathbf{h}_2^E x + \mathbf{k}_2^E$, where

$$\mathbf{h}_2^E = (1 - \tau) (\mathbf{r} - \tilde{\mu} - \tilde{\Lambda})^{-1} (\mathbf{a}_1 + \mathbf{g}_1), \quad (\text{A13})$$

$$\mathbf{k}_2^E = (1 - \tau) (\mathbf{r} - \tilde{\Lambda})^{-1} (\mathbf{a}_0 + \mathbf{g}_0 - c\mathbf{1}). \quad (\text{A14})$$

The coefficients $\{\mathbf{w}_1^E, \mathbf{w}_2^E\}$ are determined by the following boundary conditions for given default boundaries x_G^D, x_B^D . First, the absolute priority rule implies that the value of equity at default should be equal to zero,

$$\lim_{x \downarrow x_G^D} E_G(x) = 0, \quad (\text{A15})$$

$$\lim_{x \downarrow x_B^D} E_B(x) = 0. \quad (\text{A16})$$

Next, the value of $E_G(x)$ must be continuous and smooth at the boundary of regions J_1 and J_2 (see [Karatzas and Shreve, 1998](#)), which implies

$$\lim_{x \uparrow x_B^D} E_G(x) = \lim_{x \downarrow x_B^D} E_G(x), \quad (\text{A17})$$

$$\lim_{x \uparrow x_B^D} E_G'(x) = \lim_{x \downarrow x_B^D} E_G'(x). \quad (\text{A18})$$

Finally, to rule out bubbles, we also impose the following conditions:

$$\lim_{x \uparrow +\infty} \frac{E_G(x)}{x} < \infty, \quad (\text{A19})$$

$$\lim_{x \uparrow +\infty} \frac{E_B(x)}{x} < \infty. \quad (\text{A20})$$

The boundary conditions (A15-A20) lead to a system of linear equations for $\{\mathbf{w}_1^E, \mathbf{w}_2^E\}$, which can be solved in closed form.

A.1.2 Before Investment

Before the investment is made, we have conjectured that $x_G^d < x_B^d < x_G^u < x_B^u$, which gives 3 relevant regions for cash flow x_t : $I_1 = [x_G^d, x_B^d)$, $I_2 = [x_B^d, x_G^u)$, and $I_3 = [x_G^u, x_B^u)$. Again, we can solve for $e_G(x)$ and $e_B(x)$ analytically when taking $x_G^d, x_B^d, x_G^u, x_B^u$ as given.

In region I_1 , the firm has already defaulted in state B . Thus, $e_B(x) = 0$ in this region. In state G , $e_G(x)$ satisfies the same ODE as (A5), except that before investment, the firm's cash flow at time t becomes $a_0(G) + a_1(G)x_t$ instead of $(a_0(G) + g_0(G)) + (a_1(G) + g_1(G))x_t$. The solution is

$$e_G(x) = w_{1,1}^e x^{\alpha_1} + w_{1,2}^e x^{\alpha_2} + h_1^e(G)x + k_1^e(G), \quad (\text{A21})$$

where α is the same as in the post-investment case, and

$$h_1^e(G) = \frac{(1 - \tau) a_1(G)}{r(G) + \tilde{\lambda}(G) - \tilde{\mu}(G)}, \quad (\text{A22})$$

$$k_1^e(G) = \frac{(1 - \tau) (a_0(G) - c)}{r(G) + \tilde{\lambda}(G)}. \quad (\text{A23})$$

In region I_2 , the firm has not defaulted or made investment in either state, and $e_G(x)$ and $e_B(x)$ satisfy the same ODE system as (A10a-A10b), again with instantaneous profit $\ell_s(x)$ replaced by $a_0(s) + a_1(s)x$. The solution is

$$e_s(x) = \sum_{j=1}^4 w_{2,j}^e \theta_j(s) x^{\beta_j} + h_2^e(s)x + k_2^e(s), \quad (\text{A24})$$

where the values of β and θ are the same as in the post-investment case, and

$$\mathbf{h}_2^e = (1 - \tau) \left(\mathbf{r} - \tilde{\mu} - \tilde{\Lambda} \right)^{-1} \mathbf{a}_1, \quad (\text{A25})$$

$$\mathbf{k}_2^e = (1 - \tau) \left(\mathbf{r} - \tilde{\Lambda} \right)^{-1} (\mathbf{a}_0 - c\mathbf{1}). \quad (\text{A26})$$

In region I_3 , the firm will have already made the investment in state G . In state B , $e_B(x)$ satisfies:

$$(r(B) + \tilde{\lambda}(B))e_B = (1 - \tau) (a_1(B)x + a_0(B) - c) + \tilde{\mu}(B)xe'_B + \frac{1}{2}\sigma^2(B)x^2e''_B + \tilde{\lambda}(B)(E_G - \phi). \quad (\text{A27})$$

The last term captures the fact that the firm will invest immediately if the state changes from B to G . the solution to the homogeneous equation in ODE (A27) is

$$e_B(x) = w_{3,1}^e x^{\gamma_1} + w_{3,2}^e x^{\gamma_2}, \quad (\text{A28})$$

where

$$\gamma_1, \gamma_2 = -\sigma^{-2}(L) \left[\left(\tilde{\mu}(B) - \frac{\sigma^2(B)}{2} \right) \pm \sqrt{\left(\tilde{\mu}(B) - \frac{\sigma^2(B)}{2} \right)^2 + 2r(B)\sigma^2(B)} \right], \quad (\text{A29})$$

and we can verify that the particular solution is $h_3^e(B)x + k_3^e(B) + \sum_{j=1}^4 \omega_{3,j}^e x^{\beta_j}$, where

$$h_3^e(B) = \frac{(1 - \tau) a_1(B) + \tilde{\lambda}(B) h_2^E(G)}{r(B) + \tilde{\lambda}(B) - \tilde{\mu}_B}, \quad (\text{A30a})$$

$$k_3^e(B) = \frac{(1 - \tau) (a_0(B) - c) + \tilde{\lambda}(B) (k_2^E(G) - \phi)}{r(B) + \tilde{\lambda}(B)}, \quad (\text{A30b})$$

$$\omega_j = \frac{\tilde{\lambda}(B) w_{2,j}^E \theta_j(G)}{r(B) + \tilde{\lambda}(B) - \tilde{\mu}(B)\beta_k - \frac{1}{2}\sigma_B^2 \beta_k (\beta_k - 1)}. \quad (\text{A30c})$$

The values of the coefficients \mathbf{w}^e are determined by the following boundary conditions. First,

the value of equity is 0 at default:

$$\lim_{x \downarrow x_G^d} e_G(x) = 0, \quad (\text{A31})$$

$$\lim_{x \downarrow x_B^d} e_B(x) = 0. \quad (\text{A32})$$

Next, $e_G(x)$ and $e_B(x)$ must be piecewise \mathcal{C}^2 ,

$$\lim_{x \uparrow x_B^d} e_G(x) = \lim_{x \downarrow x_B^d} e_G(x), \quad (\text{A33})$$

$$\lim_{x \uparrow x_B^d} e'_G(x) = \lim_{x \downarrow x_B^d} e'_G(x), \quad (\text{A34})$$

$$\lim_{x \uparrow x_G^u} e_B(x) = \lim_{x \downarrow x_G^u} e_B(x), \quad (\text{A35})$$

$$\lim_{x \uparrow x_G^u} e'_B(x) = \lim_{x \downarrow x_G^u} e'_B(x). \quad (\text{A36})$$

Finally, at the two investment boundaries x_G^u and x_B^u , the value-matching conditions imply

$$\lim_{x \uparrow x_G^u} e_G(x) = \lim_{x \downarrow x_G^u} E_G(x) - \phi, \quad (\text{A37})$$

$$\lim_{x \uparrow x_B^u} e_B(x) = \lim_{x \downarrow x_B^u} E_B(x) - \phi. \quad (\text{A38})$$

The coefficients $\{\mathbf{w}_1^e, \mathbf{w}_2^e\}$ are determined by the boundary conditions (A31-A38) for given default and investment boundaries $\{x_G^d, x_B^d, x_G^u, x_B^u\}$, which leads to a system of linear equations that can be solved in closed form.

For a given coupon and the default and investment boundaries, we can also price the defaultable debt ($d_s(x)$ and $D_s(x)$) in closed form the same way as we do equity. The values of the coefficients \mathbf{w}^D are determined by the following boundary conditions. First, there are the value-matching at default:

$$\lim_{x \downarrow x_D(G)} D_G(x) = \alpha(G) V_G^{AE}(x_D(G)), \quad (\text{A39a})$$

$$\lim_{x \downarrow x_D(B)} D_B(x) = \alpha(B) V_B^{AE}(x_D(B)). \quad (\text{A39b})$$

Next, $D_G(x)$ needs to be piecewise \mathcal{C}^2 , which implies

$$\lim_{x \uparrow x_D(B)} D_G(x) = \lim_{x \downarrow x_D(B)} D_G(x) \quad (\text{A40})$$

$$\lim_{x \uparrow x_D(B)} D'_G(x) = \lim_{x \downarrow x_D(B)} D'_G(x) \quad (\text{A41})$$

Finally, to rule out bubbles, we have

$$\lim_{x \uparrow +\infty} \frac{D_G(x)}{x} < \infty, \quad (\text{A42})$$

$$\lim_{x \uparrow +\infty} \frac{D_B(x)}{x} < \infty. \quad (\text{A43})$$

The remaining unknowns are $\{w_{1,1}^D, w_{1,2}^D, w_{2,1}^D, w_{2,2}^D\}$, which can be solved via a system of linear equations implied by the boundary conditions above. Further details are available upon request.

References

- Admati, A., P. DeMarzo, M. Hellwig, and P. Pfleiderer, 2015, “The Leverage Ratchet Effect,” Research papers, Stanford University.
- Almeida, H., and T. Philippon, 2007, “The Risk-Adjusted Cost of Financial Distress,” *Journal of Finance*, 62, 2557–2586.
- Arnold, M., A. Wagner, and R. Westermann, 2013, “Growth options, macroeconomic conditions, and the cross-section of credit risk,” *Journal of Financial Economics*, 107, 350–85.
- Bansal, R., and A. Yaron, 2004, “Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles,” *Journal of Finance*, 59, 1481–1509.
- Barlow, M., L. Rogers, and D. Williams, 1980, *Wiener-Hopf Factorization for Matrices, in Séminaire de Probabilités XIV, Lecture Notes in Math. 784* . pp. 324–331, Springer-Verlag, Berlin.
- Berkovitch, E., and E. Kim, 1990, “Financial Contracting and Leverage Induced Over- and Under-Investment Incentives,” *Journal of Finance*, 45, 765–794.
- Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev, 2010, “The Levered Equity Risk Premium and Credit Spreads: A Unified Framework,” *Review of Financial Studies*, 23, 645–703.
- Bolton, P., H. Chen, and N. Wang, 2011, “A Unified Theory of Tobin’s q , Corporate Investment, Financing, and Risk Management,” *Journal of Finance*, 66, 1545–1578.
- Bolton, P., H. Chen, and N. Wang, 2015, “Debt, Taxes, and Liquidity,” Working paper, Columbia University and Massachusetts Institute of Technology.
- Chen, H., 2010, “Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure,” *Journal of Finance*, 65, 2171–2212.
- Chen, H., M. Michaux, and N. Roussanov, 2013, “Houses as ATMs? Mortgage Refinancing and Macroeconomic Uncertainty,” NBER Working Papers 19421, National Bureau of Economic Research, Inc.

- Chen, L., P. Collin-Dufresne, and R. Goldstein, 2009, “On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle,” *Review of Financial Studies*, 22, 3367–3409.
- Childs, P., D. Mauer, and S. Ott, 2005, “Interactions of Corporate Financing and Investment Decisions: The Effects of Agency Conflicts,” *Journal of Financial Economics*, 76, 667–690.
- Décamps, J., and A. Faure-Grimaud, 2002, “Excessive Continuation and Dynamic Agency Cost of Debt,” *European Economic Review*, 46, 1623–1644.
- Diamond, D., and Z. He, 2014, “A Theory of Debt Maturity: The Long and Short of Debt Overhang,” *Journal of Finance*, 69, 719–762.
- Diamond, D. W., and R. G. Rajan, 2011, “Fear of Fire Sales, Illiquidity Seeking, and Credit Freezes,” *Quarterly Journal of Economics*, 126, 557–591.
- Dixit, A., 1989, “Entry and Exit Decisions Under Uncertainty,” *Journal of Political Economy*, 97, 620–638.
- Dixit, A., and R. Pindyck, 1994, *Investment under uncertainty*, Princeton University Press, Princeton.
- Dumas, B., 1991, “Super contact and related optimality conditions,” *Journal of Economic Dynamics and Control*, 4, 675–685.
- Dynan, K., 2012, “Is a Household Debt Overhang Holding Back Consumption,” *Brookings Papers on Economic Activity*, 44, 299–362.
- Gilchrist, S., V. Yankov, and E. Zakrajsek, 2009, “Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets,” *Journal of Monetary Economics*, 56, 471–493.
- Gomes, J. F., and L. Schmid, 2010, “Equilibrium Credit Spreads and the Macroeconomy,” Working Paper, University of Pennsylvania and Duke University.

- Gourio, F., 2013, “Credit Risk and Disaster Risk,” *American Economic Journal: Macroeconomics*, 5, 1–34.
- Guo, X., J. Miao, and E. Morellec, 2005, “Irreversible Investment with Regime Shifts,” *Journal of Economic Theory*, 122, 37–59.
- Hackbarth, D., and D. Mauer, 2012, “Optimal Priority Structure, Capital Structure, and Investment,” *Review of Financial Studies*, 25, 747–796.
- Hackbarth, D., J. Miao, and E. Morellec, 2006, “Capital Structure, Credit Risk, and Macroeconomic Conditions,” *Journal of Financial Economics*, 82, 519–550.
- Hennessy, C., 2004, “Tobin’s Q, Debt Overhang, and Investment,” *Journal of Finance*, 59, 1717–1742.
- Huang, J.-Z., and M. Huang, 2012, “How much of the Corporate-Treasury Yield Spread is Due to Credit Risk,” *Review of Asset Pricing Studies*, 2, 153–202.
- Jensen, M., 1986, “Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers,” *American Economic Review*, 76, 323–329.
- Jensen, M., and W. Meckling, 1976, “Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure,” *Journal of Financial Economics*, 3, 305–360.
- Ju, N., and H. Ou-Yang, 2006, “Asset Substitution and Underinvestment: A Dynamic View,” Discussion paper, Duke University.
- Karatzas, I., and S. Shreve, 1998, *Brownian motion and stochastic calculus*, Springer, New York, second edn.
- Krylov, N., 1980, *Controlled Diffusion Processes*, Springer, Berlin.
- Lamont, O., 1995, “Corporate-Debt Overhang and Macroeconomic Expectations,” *American Economic Review*, 85, 1106–1117.

- Leary, M. T., and M. R. Roberts, 2005, “Do Firms Rebalance Their Capital Structures?,” *Journal of Finance*, 60, 2575–2619.
- Leland, H., 1998, “Agency Costs, Risk Management, and Capital Structure,” *Journal of Finance*, 53, 1213–1243.
- Manso, G., 2008, “Investment Reversibility and Agency Cost of Debt,” *Econometrica*, 76, 437–442.
- Mauer, D., and S. Ott, 2000, “Agency Costs, Underinvestment, and Optimal Capital Structure: The Effect of Growth Options to Expand,” in M. Brennan, and L. Trigeorgis (ed.), *Project Flexibility, Agency, and Competition* . pp. 151–179, Oxford University Press, Oxford.
- Mauer, D., and A. Triantis, 1994, “Interactions of Corporate Financing and Investment Decisions: A Dynamic Framework,” *Journal of Finance*, 49, 1253–1277.
- McDonald, R., and D. Siegel, 1986, “The value of waiting to invest,” *Quarterly Journal of Economics*, 101, 707–728.
- Mello, A., and J. Parsons, 1992, “Measuring the Agency Cost of Debt,” *Journal of Finance*, 47, 1887–1904.
- Mian, A., K. Rao, and A. Sufi, 2013, “Household Balance Sheets, Consumption, and the Economic Slump,” *The Quarterly Journal of Economics*, 128, 1687–1726.
- Miao, J., and P. Wang, 2010, “Credit Risk and Business Cycles,” Working Paper, Boston University and Hong Kong University of Science and Technology.
- Morellec, E., and N. Schuerhoff, 2010, “Dynamic Investment and Financing Under Personal Taxation,” *Review of Financial Studies*, 23, 101–146.
- Moyen, N., 2007, “How Big is the Debt Overhang Problem?,” *Journal of Economic Dynamics and Control*, 31, 433–472.
- Myers, S., 1977, “Determinants of Corporate Borrowings,” *Journal of Financial Economics*, 5, 147–175.

- Philippon, T., 2009, “The Value of Waiting to Invest,” *Quarterly Journal of Economics*, 124, 1011–1056.
- Philippon, T., 2010, “Debt Overhang and Recapitalization in Closed and Open Economies,” *IMF Economic Review*, 58, 157–178.
- Reinhart, C. M., V. R. Reinhart, and K. S. Rogoff, 2012, “Public Debt Overhangs: Advanced-Economy Episodes since 1800,” *Journal of Economic Perspectives*, 26, 69–86.
- Stein, J. C., 2003, “Agency, information and corporate investment,” in G.M. Constantinides, M. Harris, and R. M. Stulz (ed.), *Handbook of the Economics of Finance*, vol. 1 of *Handbook of the Economics of Finance*, . chap. 2, pp. 111–165, Elsevier.
- Stulz, R., and H. Johnson, 1985, “An Analysis of Secured Debt,” *Journal of Financial Economics*, 14, 501–521.
- Sundaresan, S., and N. Wang, 2015, “Dynamic Investment, Capital Structure, and Debt Overhang,” *Review of Corporate Finance Studies*, 4, 1–42.
- Titman, S., S. Tompaidis, and S. Tsyplakov, 2004, “Market Imperfections, Investment Flexibility, and Default Spreads,” *Journal of Finance*, 59, 165–205.